

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of *Mathematical Reviews*.

**23[65-01, 65-04].**—DAVID KAHANER, CLEVE MOLER & STEPHEN NASH, *Numerical Methods and Software*, Prentice-Hall Series in Computational Mathematics, Prentice-Hall, Englewood Cliffs, N.J., 1989, xii + 495 pp., 24 cm. Price \$50.00.

This book is a sequel to the widely acclaimed *Computer Methods for Mathematical Computations* by G.E. Forsythe, M.A. Malcom, and C.B. Moler. Although only one of the original authors is represented, its format is retained: presenting numerical methods along with useable software that implements the methods. The major distinction is that the present volume provides software that is close to state-of-the art, even though this means that not all details of the implementation can be presented at the level of the text.

To quote from the Introduction, "This book is written for students of science and engineering. It is intermediate between a cookbook and a numerical analysis text. The reader is assumed to have completed two years of university mathematics including differential and integral calculus as well as a little matrix theory and differential equations." To indicate the scope of this book, a list of the chapter titles follows: 1. Introduction; 2. Computer Arithmetic and Computational Errors; 3. Linear Systems of Equations; 4. Interpolation; 5. Numerical Quadrature; 6. Linear Least-Squares Data Fitting; 7. Solution of Nonlinear Systems; 8. Ordinary Differential Equations; 9. Optimization and Nonlinear Least Squares; 10. Simulation and Random Numbers; 11. Trigonometric Approximation and the Fast Fourier Transform. The book can be viewed as either a user's guide to numerical software or as a textbook on numerical methods. In this reviewer's opinion, it is a success on both counts.

As a software guide, this book contains complete prologues for twenty-one user-callable Fortran subprograms implementing the numerical methods described in the text, and there are numerous references to sources of additional software. Although the prologues are somewhat uneven in style, presumably due to the many sources from which the software was obtained, all contain sufficient guidance for the intelligent use of the subprograms. There is a list of these subprograms by chapter in the front endpapers, with calling sequences,

brief descriptions, and references to the complete prologues in the text. The text is organized in such a way that only the details needed for understanding the prologues is provided prior to their appearance.

Machine-readable copies of the subprograms are provided with the book on a double-density DOS diskette for the IBM PC and compatible computers. The information provided with the diskette indicates that the software is also available in either 5.25 or 3.5 inch high-density form or on a Macintosh diskette. The supplied version is single-precision, with the exception of UNCMND, the double-precision equivalent of the unconstrained minimization routine UNCMIN. A double-precision version of the complete collection is available from the authors. The supplied diskette contains a total of 14 files. Excellent information on the contents of the remaining files is given in *readme.nms*. Twelve files contain source code and data files, which have been packed in order to fit on a single diskette, and the remaining file, *arce.com*, is a utility provided to do the unpacking. This worked fine in this reviewer's environment, and he was soon using the software to solve the sample problems. The software package itself (ten files which unpack to 26) consists of a total of 13,990 lines of text in 143 subprograms. The other two files contain the sample programs discussed in the text and several associated data sets.

As a textbook, the scope of coverage is appropriate for a two-semester numerical methods course, but the authors provide guidance as to how a one-semester course may be achieved by suitable selection from the material provided. Numerous problems are given at the end of each chapter. Most of these are computational in nature. Because of the quality of the software, it is possible to give quite realistic examples and problems, even though a high level of mathematical sophistication is not required. Very few proofs are given, but references are generally provided. In addition to the standard material found in most numerical mathematics texts, this book provides excellent advice on how to detect when a method has failed to provide an adequate solution to the problem posed. Most chapters are quite understandable, although this reviewer had trouble following Chapters 8 and 11 (possibly due to their remoteness from his areas of expertise). The text includes numerous references to current literature, if the reader would like more details on the methods presented. This reviewer especially appreciated the occasional "Historical Perspective" sections, which provide insights into the origin of some of the methods and the lives of their developers that are not generally provided in textbooks. These breathe life into the otherwise dry subject of numerical mathematics.

This book generally escapes the problems of uneven style that plague multiple-authored texts. There are some minor stylistic differences between chapters, but these are generally confined to the form of reference to the bibliography and other chapters of the book. While there are some typographical errors, they are small in number for a first edition, and most do not detract from the understanding of the material. A list of errata is available from the reviewer. The book includes an extensive index. This reviewer was disappointed to find that

the first topic he looked up (Interpolation, visually-pleasing) had two incorrect page references: there is no material on this topic on pages 106 or 116 (try 112 and 114). However, a random selection over a dozen other index entries revealed no additional errors.

All in all, this reviewer highly recommends *Numerical Methods and Software* to any scientist or engineer who would like insight into current mathematical software or to those who find themselves in the position of teaching a course on numerical mathematics. This reviewer hopes that *he* has the opportunity to teach from this text!

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**24[65-02, 65F05, 65F10].**—ALEKSANDR A. SAMARSKII & EVGENII S. NIKOLAEV, *Numerical Methods for Grid Equations*, Translated from the Russian by Stephen G. Nash, Vol. I: *Direct Methods*, Birkhäuser, Basel, 1989, xxxv + 242 pp., Vol. II: *Iterative Methods*, Birkhäuser, Basel, 1989, xv + 502 pp., 24 cm. Price \$260.00.

These volumes are devoted to the solution of systems of equations that arise in applying the finite difference method to problems of mathematical physics, mainly to boundary value problems for second-order elliptic equations. They are focused on iterative methods, although direct methods are also discussed. The aim is to gather in one place information on iterative methods for solving difference equations. The book has primarily been written for students of applied mathematics at the Moscow State University. The revised second edition was issued in the Soviet Union in 1987.

There are two volumes. The first volume (Chapters 1-4) deals with the application of direct methods to the solution of difference equations, the second volume (Chapters 5-15) considers iterative methods.

Chapter 1 provides the necessary foundations for solving linear difference equations. Chapter 2 describes some variants of the Gauss elimination method for solving one-dimensional 3- and 5-point difference equations. In Chapter 3 the cyclic reduction method is studied, and Chapter 4 deals with the separation of variables method (FFT method) for solving Poisson's difference equations in a rectangle.

The theory of iterative methods is introduced in Chapter 5. Iterative methods are considered as operator difference schemes. This approach has many advantages. The method does not depend on a choice of particular basis functions and on a representation of the operators in this basis. The authors introduce the

concept of a *canonical form* of an iterative method. It is possible to compare two different iterative methods by comparing their canonical forms. Studying the convergence rate of an iterative method, it is sufficient to know only some properties of the difference scheme operators such as symmetry, positive definiteness and lower and upper estimates of the spectrum of the operators. Necessary concepts from functional analysis are provided.

Chapters 6 and 7 discuss two- and three-level iterative methods (with Chebyshev parameters). Iterative methods of variational type (also the conjugate gradient method) and Gauss-Seidel and SOR methods are the subject of Chapters 8 and 9. Chapter 10 deals with the alternate triangular method. The authors consider it as a fundamental method in the book. The method was very effective for solving the Dirichlet problems in an arbitrary region at that time (up to 1980), but nowadays more effective methods are known (mainly the domain decomposition or multigrid methods). The next five chapters investigate the alternating direction (ADI) methods, iterative methods for non-self-adjoint equations with indefinite and singular operators, iterative methods for solving nonlinear difference equations and for solving elliptic equations in curvilinear coordinates. In Chapter 14 example solutions of elliptic grid equations are given (multi-dimensional problems, schemes for equations of elasticity theory, etc.).

The mathematical level of the text is very high. Apart from some introductory lemmas and theorems in Chapters 1 and 5, every theorem and lemma has a proof. The book is directed at advanced readers. However, it contains many remarks and examples illustrating the methods discussed. (Poisson's equation on a square region with the Dirichlet boundary conditions is a model problem which is intensively used in comparing various methods.) The methods considered are investigated from a mathematical point of view. For iterative methods the convergence rate and the choice of optimal parameters, for which the convergence rate is maximal, are studied. Estimates are given for the number of iterations required. The implementation aspects of the methods are briefly discussed. Some computational comparisons of the methods are given for the model problem.

The book deals only with difference scheme equations. The finite element method is not touched upon, although it is a very active area of research. Nevertheless, the book presents the broad picture of iterative methods and contains a large number of iterative methods with their detailed analyses. It is recommended for those acquainted with fundamentals of functional analysis and finite difference methods. The book is a classic, and should be a valuable addition for practitioners as well as students in the field.

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**25[65-01, 65M05, 65M10, 65M15, 65N05, 65N10, 65N15, 65N20].**—JOHN C. STRIKWERDA, *Finite Difference Schemes and Partial Differential Equations*, Cole Mathematics Series, Wadsworth & Brooks, Pacific Grove, California, 1989, xii + 386 pp., 24  $\frac{1}{2}$  cm. Price \$47.95.

According to the author's preface, this book is intended as an introductory graduate text on finite difference methods for students in applied mathematics, engineering and the sciences. The purpose is both to present basic material which is useful in scientific computing and to convey a theoretical understanding of the methods, assuming no mathematical background beyond advanced calculus. The contents are proposed to suffice for two one-semester courses, an introductory one, and a somewhat more advanced one.

The book starts out with five chapters essentially on finite difference methods for first-order scalar hyperbolic equations in one space dimension, with constant coefficients, where standard concepts such as convergence, consistency and stability as well as dissipation and dispersion are defined, exemplified and analyzed. The next three chapters, Chapters 6–8, deal with parabolic equations, systems of equations in higher dimensions, and second-order hyperbolic equations. The following three contain somewhat more advanced material: Chapter 9 discusses well-posedness and stability in  $L_2$ , including a nice proof of an extended version of the Kreiss stability theorem (which, incidentally, is then never applied!). Chapter 10 is on the Lax equivalence theorem and on convergence analysis and the dependence of convergence rates on the regularity of the solutions and data, and Chapter 11 reviews the Gustafsson-Kreiss-Sundström stability theory for discrete boundary conditions. Of the final three chapters, Chapter 12 contains a rudimentary treatment of finite difference methods for elliptic problems and Chapters 13 and 14 give elements of iterative methods for solving the resulting linear systems of equations.

The selection of the material naturally reflects the interest of the author, and this has given it an emphasis on methods for hyperbolic equations, which are studied in the style of the school of H. O. Kreiss. The treatment is almost entirely based on the use of Fourier transforms and assumes that the equations and difference schemes have constant coefficients. The main tool is Parseval's relation, and the results are expressed in terms of  $L_2$ -type estimates, so that, in particular, well-posedness and stability are always considered to be with respect to the  $L_2$  norm. Even the Lax-Richtmyer equivalence theorem is given in this framework and has thereby lost some of its elegance and generality. (The reviewer feels the author's extremely respectful comments on this result somewhat overstate its importance.) Since the author wants to keep the presentation accessible to students with limited mathematical knowledge, many interesting things have had to be omitted, and some are presented without proofs, such as most of the Gustafsson-Kreiss-Sundström theory.

The book is thus quite limited in its scope. No discussion of variable-coefficient problems is included beyond a few remarks to the effect that the

constant-coefficient results are of importance also for such cases, and nonlinear problems are not mentioned. Also, the section on elliptic problems is shorter and more sketchy than would be indicated by the importance of such problems. Here more attention is paid to the iterative solution of the resulting system of linear algebraic equations than the finite difference equations themselves, but even so, there is no mention of modern fast methods such as fast Fourier transform methods or multigrid methods.

One could question the usefulness, for the intended purpose, of a text with the composition of the present book at a time when a large fraction of the computations in engineering and science are done by finite elements, and when also those that are carried out by finite differences relate to problems with variable coefficients, nonlinearities, and complicated geometries. It appears to the reviewer that it is not natural to expect students of the type to which the book is directed to take two consecutive courses on this rather restricted part of the numerical PDE area.

Although the approach of the presentation is novel in some instances, most of the material dates back 15–20 years (more care could have been taken to give a correct historical account). The text contains a lot of nice mathematics, and for someone who was involved in finite differences at the time, it brings back nice memories. The presentation is clear and well organized, and although the reviewer feels that the selection of material is nonoptimal for the purposes stated, and not quite up to date, the book offers a good way of learning what it covers.

V. T.

**26[73–02, 65–02, 73K30, 73M05, 65N30, 45L10].**—T. A. CRUSE, *Boundary Element Analysis in Computational Fracture Mechanics*, Mechanics: Computational Mechanics, Vol. 1, Kluwer Academic Publishers, Dordrecht, 1988, xiii + 162 pp., 24  $\frac{1}{2}$  cm. Price \$67.00/Dfl.125.00.

This book presents a collection and review of the author's fundamental contributions to the boundary element method with special emphasis on fracture mechanics. This specific point of view is the fascination of this book as well as its limitation. It is well written and self-contained. From fracture mechanics as well as from boundary integral methods, only those aspects are selected which are of principal importance for computational boundary element methods and which stem from the original work of the author. Hence, the topics are restricted to static fracture mechanics and to classical stationary plasticity. Dynamical aspects and dynamic viscoplasticity are not dealt with. As the author emphasizes, this is not a reference book but an interesting monograph which can serve as an introduction to the field as well as a source of interesting details for the specialist. The mathematical analysis is classical; neither modern

variational techniques and results are treated, nor is asymptotic error analysis. The discretizations here are restricted to collocation methods. However, many numerical results illustrate the strength and also the limitation of these boundary element methods.

The first chapter is devoted to historical remarks on boundary integral methods and the boundary element method. This introduction is particularly set by the author's view. I am convinced that any mathematical existence proof which is constructive can always be converted into a computational algorithm—contrary to the author's belief. In the brief review on boundary integral equations, I could not find the enormous contribution by C. F. Gauss, who already in 1839 used the Fredholm integral equation of the first kind to prove existence of the solution of the Dirichlet problem for the Laplacian. (The proof was completed by Feller in the twenties.) Also I could not find references to the thorough analysis and solution algorithm for second-kind Fredholm equations and convex regions by C. Neumann (hundred years ago), to the relation of the boundary integral equation method with the method of reduction to the boundary, founded by Sobolev in the thirties, which started the modern theory of pseudodifferential operators on manifolds; I miss the contributions by F. Noether to the foundation of Cauchy singular boundary integral equations in two, and by Mikhlin, in three dimensions with their applications to elasticity problems. The numerical treatment of boundary integral equations has a longer history than mentioned in the historical chapter. An early version, the panel method used in flow problems, goes back to Lavrentyev in 1932 and was already used for large-scale computations in flow problems and acoustics by Hess and Smith in 1967 and in electromagnetic field computations by Poggio and Miller in 1973. On the other hand, I have learned a lot about the history of boundary integral equations in elasticity from this chapter, which I appreciated very much.

Chapter 2 gives a brief introduction to the boundary value problems for, and the local behavior of, ideal elastic fields near crack tips and crack edges for simplest crack geometries. This includes the Williams expansion and special stress intensity factors.

The third chapter introduces the construction of solutions via boundary potentials defined in terms of fundamental solutions for isotropic as well as anisotropic elastic materials. Somigliana's identity is carefully exemplified as well as the so-called direct and indirect formulations of boundary integral equations. A brief introduction to piecewise polynomial boundary elements and analytic and numerical integration completes this chapter. Mesh refinement at crack tips is heuristically introduced, but mesh grading related to the singular behavior of the stress field is not systematically analyzed.

Chapter 4 deals with the modelling of the crack within the boundary element analysis. Since the hypersingular formulation is here avoided, the direct formulation requires a domain decomposition which imbeds the crack into an interior boundary. The relation between the stress intensity factor and the energy

release rate provides a method whose boundary displacement approximations are not related to the specific crack tip behavior. For a better approximation at the crack tips, quarter-point elements and the augmentation method are used.

In Chapter 5, the fundamental solution is chosen to be the Green function for the whole exterior to a straight crack. Then the boundary integral equation on the crack degenerates to a representation formula and is to be solved only on the exterior boundary. Now, the singular behavior of stress and displacement as well as the stress intensity factors are completely inherent in the Betti representation formula. This method proved to be most accurate and efficient. Here it is also presented for anisotropic materials. For piecewise linear boundary elements, the integrations are performed analytically.

Chapter 6 contains the extension of the method to elastoplastic crack problems by introducing artificial forcing terms corresponding to the nonlinear stress contributions in the integral equations. As long as these terms are small—which corresponds to small regions of plasticification—they can be treated iteratively. Here, also an incremental approach is used. The additional volume integrals for piecewise linear displacements on triangulations of the plasticity domain are evaluated analytically. The resulting algorithm is extremely fast, and the numerical results for test problems are excellent; they are compared with finite element results obtained with ADINA.

In Chapter 7, the displacement discontinuity method leads to the hypersingular boundary integral equations on the crack. As is known from recent analysis, the hypersingular boundary integral operator defines a pseudodifferential operator of order  $+1$ , hence, the collocation boundary element method can only be conforming for continuously differentiable trial functions. For such elements, polar coordinates and integration by parts in the radial direction is used for the evaluation of the hypersingular integrals. At the crack tip in two, and crack edge in three dimensions, the special form of the singularities is incorporated into the trial spaces. The resulting numerical scheme provides very convincing results for elliptic cracks in three dimensions.

In Chapter 8, the boundary element method of Chapter 5 is modified for being utilized by the weight function method of Bueckner and Rice with the necessary weight functions belonging to the specific two-dimensional crack problem. The strength of the method is demonstrated for three reference problems.

This book will surely find many friends among advanced students, and also among researchers in mechanics and in numerical and applied mathematics.

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27[35-02, 73-02, 76-02, 22Exx].—C. ROGERS & W. F. AMES, *Nonlinear Boundary Value Problems in Science and Engineering*, Mathematics in Science and Engineering, vol. 183, Academic Press, Boston, 1989, xiii + 416 pp., 23  $\frac{1}{2}$  cm. Price \$69.96.

Has the ascendancy of the computer and numerical analysis in the late twentieth century signalled the end of the era of exact solutions of differential equations? On the contrary! Not only is the subject alive, but it is currently in the midst of an incredible renaissance, with many new methods, new solutions and new insights appearing in the last 10–20 years. Indeed, exact solutions still remain the touchstone of physical applications, and, even more to the point, are also crucial in the proper development of numerical methods. Explicit exact solutions (even nonphysical ones) are crucial tools for checking the accuracy of numerical integration schemes, and serve as key benchmarks for comparison and evaluation of competing numerical packages. New physical phenomena (e.g., shock waves, black holes, interacting solitons, cavitation of elastic materials, scattering phenomena, etc.) are often first detected or are epitomized in their simplest form by suitable exact solutions of particular model systems. (Nobel prizes have been awarded for exact solutions of the equations of physics!) Moreover, exact solutions often serve as asymptotics for more complicated solutions after the decay of irrelevant transients. The discovery of solitons, the application of Lie group methods, Bäcklund and reciprocal transformations, canonical form theory, Bergman series, etc., etc., have all vastly enlarged the researcher's arsenal of techniques. Moreover, the recent arrival of sophisticated and powerful computer algebra systems will provide yet another stimulus to the continued development and expansion of the methods to yet more complicated problems.

The book under review is a paean to the exact solution, supplied with a cornucopia of examples, methods, and physical applications. It contains a wealth of interesting and unusual examples of special solution techniques for a variety of boundary value problems arising in a wide range of physical systems, including fluid mechanics, elasticity, heat conduction, gas dynamics, meteorology and so on. It is divided into four chapters: Bäcklund and reciprocal transformations, Bergman expansion methods in moving boundary and Stefan problems, model constitutive laws of elasticity, and applications of symmetry group methods, although the different methods often interact in unexpected ways. Of particular interest to those interested in computations are sections on the use of group methods to compute eigenvalues of nonlinear boundary value problems by exact shooting, and to devising numerical finite difference schemes incorporating group invariance. There is a useful appendix listing the symmetry groups of a large number of physically interesting problems, and an excellent list of references to recent research papers.

In the broad spectrum of mathematical exposition, this book lies almost at the polar opposite to the Bourbaki approach: instead of empty theory and abstract

generalizations, the book develops its topics by relying almost exclusively on particular examples. Each chapter consists of a number of sections, each of which presents a particular physical problem, some special techniques to generate solutions, and, often, physical consequences of the results. My own pedagogical tastes certainly run towards the particular, well-chosen example, although I felt this was perhaps carried to an extreme here. What is often lacking is any kind of general framework for the particular methods introduced, or a discussion of how to ever decide which of the many techniques available to apply to a new problem. Of course, in many cases, this approach is necessitated by the nature of the subject; many of the methods only work in particular instances, and (as in much of applied mathematics) one learns primarily through example. Only in the final section on symmetry groups is there an attempt to develop a general theory which can be readily ported to other contexts. Students especially will profit from the wide repertoire of methods and applications, although I would find it hard to use this book in a course other than as a supplement to more systematic texts. Nevertheless, I can recommend the book to anyone seeking to enlarge their "bag of tricks" for tackling complicated nonlinear problems.

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**28[15-02, 65F50].**—I. S. DUFF, A. M. ERISMAN & J. K. REID, *Direct Methods for Sparse Matrices*, Monographs on Numerical Analysis, Clarendon Press, Oxford University Press, New York, 1989, xiv + 341 pp., 23  $\frac{1}{2}$  cm. Price \$22.50 paperback.

This is a paperback edition (with corrections) of the 1986 edition of the book. See [1] for a review of the original edition.

W. G.

I. K. Turner, Review 3, Math. Comp. 52 (1989), 250-252.

**29[53-01, 65D05, 65D07, 65D10, 68U05].**—GERALD FARIN, *Curves and Surfaces for Computer Aided Geometric Design—A Practical Guide*, Computer Science and Scientific Computing, Academic Press, Boston, 1988, xv + 334 pp., 23  $\frac{1}{2}$  cm. Price \$39.95.

This book consists of a collection of material on parametric curves and surfaces used in fields known variously as "Computer Aided Design" and "Computer Aided Geometric Design". The topics covered are, in order of the chapters: the de Casteljau algorithm, Bézier curves, polynomial interpolation, B-spline curves, geometric continuity for curves, conic sections, rational Bézier and B-spline curves, tensor product and composite surface patches. Included

also in the text are two guest chapters on differential geometry, written by W. Boehm.

I have mixed reactions to this book. It is difficult to assess in what capacity the book would be most valuable. It is neither an undergraduate text, teaching curve and surface algorithm design, nor comprehensive enough to be a graduate text. Perhaps, its best use is as a browser to quickly gather working knowledge of some of the approaches taken in this rapidly advancing field.

The exposition is quite clear and understandable. There are also numerous figures and color pictures to add to the clarity. However, I find the book severely lacking in its attempt to provide a unified treatment of the main topics of geometric design. Most of the book deals with parametric Bézier and  $B$ -spline curves, with parametric surfaces treated only at the very end. What is even more disappointing is the lack of precise statements, backed up with proofs. Rare mention is made of the more general implicit form of curves and surfaces. Only scant statements are made of the type "A surface may be given by an implicit form  $f(x, y, z) = 0$  or, more useful for CAGD, by its parametric form." This statement is never elaborated further in the text.

The introductory Chapters 11 and 21 on Differential Geometry, though well written, lack references where the interested reader may further his study. Theorems such as Meusnier's and Euler's are explained by examples, with no formal statement of the theorems or indication where the proofs may be found.

The book lacks a binding theme and comes across as a potpourri of facts, piled one upon the other. A more modular approach would be highly desirable. Introductory facts about general curves and surfaces, leading to appropriate representations and data structures for them, and followed by fundamental algorithmic paradigms for manipulating those representations, would be a better way to organize the material.

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**30[49-06, 65K05, 90Cxx].**—MASAO IRI & KUNIO TANABE (Editors), *Mathematical Programming: Recent Developments and Applications*, Mathematics and Its Applications (Japanese Series), Kluwer, Dordrecht, 1989, ix + 382 pp., 23  $\frac{1}{2}$  cm. Price \$139.00/Dfl.290.00.

This volume contains the texts of one Plenary Lecture, two Memorial Lectures (in memory of Martin Beale and L. V. Kantorovich, respectively) and 10 Tutorial-and-Survey Lectures highlighting the state of the art in Mathematical Programming and its applications as of 1988. The lectures were delivered at

the 13th International Symposium on Mathematical Programming held on the Kasuga campus of Chuo University, Tokyo, August 29–September 2, 1988.

W. G.

**31[68U30].**—ERICH KALTOFEN & STEPHEN M. WATT (Editors), *Computers and Mathematics*, Springer, 1989, xiii + 326 pp., 24 cm. Price \$39.00.

*Computers and Mathematics '89* is the third in a series of international conferences devoted to the use of computers in mathematics and the mathematical sciences. It was held from June 13–17, 1989, at the Massachusetts Institute of Technology. This volume contains 36 papers covering a wide range of topics on mathematical computing. The main subject areas covered include symbolic computation (symbolic integration, computer-enhanced analysis, expert systems for learning mathematics), numerical analysis (differential equations, fractals, hyperbolic manifolds, differential geometry), group algorithms (fast group membership testing, cohomology, group representation theory), and numerical algebra (Jordan forms, algebraic varieties, symmetric matrices, quadratic forms, symmetric polynomials). There are also papers on other areas of mathematical computing.

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**32[11Y40, 68Q40].**—MICHAEL POHST (Editor), *Algorithmic Methods in Algebra and Number Theory*, Academic Press, London, 1987, 135 pp., 24  $\frac{1}{2}$  cm. Price: Softcover \$12.50.

This volume is a special issue of the *Journal of Symbolic Computation* dedicated to H. Zassenhaus on the occasion of his 75th birthday. It includes 14 publications, introducing algorithms from the fields of computational algebra and number theory, such as a principal ideal test in algebraic number fields (J. Buchmann & H. C. Williams), an analytic method of computing the class number of an algebraic number field (C. Eckhardt), a method to resolve Thue inequalities (A. Pethö), a procedure for generating nonsymmetric modular binary forms over  $\mathbf{Q}(\sqrt{2})$  (H. Cohn & J. I. Deutsch), and an extension of the LLL-algorithm for integral lattices (M. Pohst). Also, L. Cerlienco et al. and E. Kaltofen address computational aspects of polynomials such as irreducibility testing and computation of their measure. Two special cases of the inverse problem of Galois theory are solved by G. Malle and by H. Matzat & A. Zeh-Marschke, respectively. Other algorithms include methods for determining integral bases of algebraic number fields (E. Maus), constructing maximal orders

over Dedekind domains (D. Ford), and computing  $p$ -adic values in semisimple algebras over  $\mathbf{Q}$  (R. Böffgen & M. A. Reichert). W. Plesken outlines an algorithm to find soluble quotients of finitely presented groups. Finally, H. G. Folz & H. G. Zimmer analyze the rank of a certain matrix which is of importance in the investigation of elliptic curves.

The volume also contains a short biography and a bibliography of Professor Zassenhaus.

H. C. W.