A TABLE OF PRIMITIVE BINARY POLYNOMIALS

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Abstract. For those \( n < 5000 \) for which the factorization of \( 2^n - 1 \) is known, the first primitive trinomial (if such exists) and a randomly generated primitive 5- and 7-nomial of degree \( n \) in \( \mathbb{GF}(2) \) are given.

A primitive polynomial of degree \( n \) over \( \mathbb{GF}(2) \) is useful for generating a pseudorandom sequence of \( n \)-tuples of zeros and ones, see [8]. If the polynomial has a small number \( k \) of terms, then the sequence is easily computed. But for cryptological applications (correlation attack, see [5]) it is often necessary to have the primitive polynomials with larger values of \( k \) than one can find in the existing tables. For example, Zierler and Brillhart [10, 11] have calculated all irreducible trinomials of degree \( n \leq 1000 \), with the period for some for which the factorization of \( 2^n - 1 \) is known; Stahnke [7] has listed one example of a trinomial or pentanomial of degree \( n \leq 168 \); Zierler [12] has listed all primitive trinomials whose degree is a Mersenne exponent \( \leq 11213 = M_{23} \) (there, \( M_j \) denotes the \( j \)th Mersenne exponent); Rodemich and Rumsey [6] have listed all primitive trinomials of degree \( M_j \), \( 12 \leq j \leq 17 \); Kurita and Matsumoto [2] have listed all primitive trinomials of degree \( M_j \), \( 24 \leq j \leq 28 \), and one example of primitive pentanomials of degree \( M_j \), \( 8 \leq j \leq 27 \).

Here we give (see Table 1 in the Supplement section) one primitive binary \( k \)-nomial (\( k \)-term polynomial) of degree \( n \) (if such exists and the factorization of \( 2^n - 1 \) is known) for \( 2 \leq n \leq 5000 \), \( k \in \{3, 5, 7\} \). For chosen \( n \) and \( k \), we have the polynomial \( 1 + x^n + \sum x^a \), where \( a \) takes the values from the entry at the intersection of the row \( n \) and the column \( k \).

The 5- and 7-nomials listed in Table 1 were obtained using a random number generator. Randomly chosen \( k \)-nomials of degree \( n \) are checked for primitivity (see [9], for example) and rejected until a primitive polynomial is found. The trinomials were tested in the natural order.

The primitivity check is carried out using the factorizations of \( 2^n - 1 \) from [1], and also from [3] \( 2^{512} + 1 \), [4] \( 2^{484} + 1 \). These factorizations are known for all \( n \leq 310 \), and for some \( n \leq 2460 \), where \( 2^n - 1 \) is not a Mersenne prime. An asterisk in front of \( n \) in Table 1 means that \( 2^n - 1 \) contains "probably a prime" factor [1], i.e., a factor without the complete primality proof.

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