REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[30B70, 40A15, 65D15].—Lisa Lorentzen & Haakon Waadeland, *Continued Fractions with Applications*, Studies in Computational Mathematics, Vol. 3, North-Holland, Amsterdam, 1992, xvi+606 pp., $24\frac{1}{2}$ cm. Price $157.00/ Dfl.275.00.

There are three classic monographs on continued fractions. They are by H. S. Wall [5], O. Perron, [3, 4], and W. B. Jones and W. J. Thron [2]. There is surprisingly little overlap amongst the three with their different styles and emphasis. We now have a fourth, which is again very different. It falls somewhere between textbook and research monograph.

As the authors explain in the Preface, this book is not intended as a substitute for, or an updating of, any of the other three but, rather, as a companion. It is aimed at those in or near mathematics on the one hand, and senior or graduate level students on the other, who are curious about continued fractions.

The style is, for the most part, casual with many motivating and illustrative examples, both theoretical and numerical. A minimum of background is required to appreciate the material, and each but one of the twelve chapters concludes with some Problems, Remarks, and References. The problems are not overly difficult and emphasize the principles of the text. All this makes the book particularly useful as an undergraduate and graduate introductory text. However, even the expert will derive enjoyment and obtain new information and insights from its reading. I certainly did.

The original title of the book "A Taste of Continued Fractions" was perhaps more appropriate since, although the material is indeed tastefully presented, there are instances where the reader is given only the flavor of the subject and is instructed to go elsewhere for a fuller, more satisfying understanding. This is particularly true of Chapters VII, VIII, and IX. A serious omission in connection with Chapter VII is the lack of any reference to the recent developments in hypergeometric orthogonal polynomials, inspired by the work of R. Askey and J. Wilson [1]. Indeed, the book has no hypergeometric examples beyond ${}_2F_1$ or ${}_2\phi_1$. Also missing is the connection with the spectral theory of Jacobi matrices.

Chapter I contains motivating examples and three classical convergence theorems (Sleszynski-Pringsheim, van Vleck, and Worpitzky). Chapter II deals with basic notation and transformations. Chapters III, IV, and V are the main entrée, dealing with Convergence (Stern-Stolz, value sets, parabola and oval theorems,

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limit periodic cases), Three-term recurrences (Pincherle's and Auric's theorems and generalizations), and Correspondence, respectively. Chapters VI, VII, and VIII give a taste of the connections with hypergeometric functions, moments and orthogonality, and Padé approximants, respectively. The final four chapters have applications to number theory, zero-free regions of polynomials, digital filter theory, and differential equations. The Appendix is an (admittedly incomplete) catalogue of known continued fraction expansions. Although there are some references at the end of each chapter, there is, unfortunately, no overall bibliography. A Subject Index completes the text.

The book is recommended as a tasteful introduction to continued fractions, which will stimulate an appetite for further reading and prepare one for digesting current research on the subject.

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1. R. Askey and J. Wilson, *Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials*, Mem. Amer. Math. Soc. **319** (1985).
2. W. B. Jones and W. J. Thron, *Continued fractions: Analytic theory and applications*, Addison-Wesley, Reading, Mass., 1980.
3. O. Perron, *Die Lehre von den Kettenbrüchen, Band* I: *Elementare Kettenbrüche*, Teubner, Stuttgart, 1954.
4. O. Perron, *Die Lehre von den Kettenbrüchen, Band* II: *Analytisch-funktionentheoretische Kettenbrüche*, Teubner, Stuttgart, 1957.
5. H. S. Wall, *Analytic theory of continued fractions*, Van Nostrand, New York, 1948.

2[01–00, 11A55, 30B70, 40A15, 41A21, 65B10].—CLAUDE BREZINSKI, *History of Continued Fractions and Padé Approximants*, Springer Series in Computational Mathematics, Vol. 12, Springer-Verlag, Berlin, 1991, 551 pp., 24 cm. Price $79.00.

As the author admits in the Introduction to this book, he realized soon after having embarked on the project of writing a history of continued fractions that he had neither the time nor the inclination to write a complete history of the subject. Thus he restricted himself to presenting "a collection of facts and references about continued fractions." Moreover, "this history ends with the first part of our century, that is, 1939."

The contents of the book are fairly well described by the titles of the sections of the book, which are as follows:

1. Euclid's algorithm, the square root, indeterminate equations, history of notations.

2. Ascending continued fractions, the birth of continued fractions, miscellaneous contributions, Pell's equation.

3. Brouncker and Wallis, Huygens, number theory.

4. Euler, Lambert, Lagrange, miscellaneous contributions, the birth of Padé approximants.

5. Arithmetical continued fractions, algebraic properties, arithmetic, applications, number theory, convergence, algebraic continued fractions, expansion

methods and properties, examples and applications, orthogonal polynomials, convergence and analytic theory, Padé approximants, varia.

6. Number theory, set and probability theories, convergence and analytic theory, Padé approximants, extensions and applications.

All important aspects and applications of the theory of continued fractions are thus at least touched upon. The area most thoroughly covered is that of continued fraction approximation to power series, in particular, Padé approximation.

Appended to the text are two impressive bibliographies. The first consists of 2302 items which contain "almost all" contributions to the theory of continued fractions up to 1939. There are at least two instances (von Koch, Śleszyński) where a certain article is listed twice. Also Worpitzky's article of 1862 is missing.

A compilation extending to 1988 was recently published by the same author [1].

The second bibliography has 478 entries and lists books and articles containing historical material. There are also indexes of collected works, names, and subjects.

The information gathered by Brezinski is impressive and very useful to people interested in the field. Unfortunately, the author, at least in the reviewer's opinion, misjudges the English-speaking readers' ability to read French. Thus the inclusion of a large number of quotations (some of them several pages long) in French appears to be of little use to the average reader.

There are the usual typographical errors and some factual errors. Thus, Śleszyński in 1888 proved the criterion

$$K(a_n/b_n) \quad \text{converges if for all } n \geq 1, \quad |b_n| \geq |a_n| + 1.$$

The author (p. 189) (as have many mathematicians before him) credits Pringsheim (1898) with the result.

In 1917 Schur initiated a study of functions bounded in the unit disk. One of his main tools was a "continued-fraction-like" algorithm. Brezinski (p. 306) states that he used a certain continued fraction (which essentially appears to be due to Hamel). The approach used by Schur is credited (p. 290) to R. Nevanlinna.

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1. C. Brezinski, *A bibliography on continued fractions, Padé approximation, sequence transformation and related subjects*, Prensas Universitarias, Zaragoza, 1991.

3a [65–00, 65–04].—WILLIAM H. PRESS, SAUL A. TEUKOLSKY, WILLIAM T. VETTERLING & BRIAN P. FLANNERY, *Numerical Recipes in Fortran: The Art of Scientific Computing*, 2nd ed., Cambridge Univ. Press, Cambridge, 1992, xxvi+963 pp., 25 cm. Price $49.95.

3b [65–00, 65–04].—WILLIAM H. PRESS, SAUL A. TEUKOLSKY, WILLIAM T. VETTERLING & BRIAN P. FLANNERY, *Numerical Recipes in C: The Art of Scientific*

Computing, 2nd ed., Cambridge Univ. Press, Cambridge, 1992, xxvi+994 pp., 25 cm. Price $49.95.

The first edition of these widely known volumes has been reviewed respectively in [2, 3]. Virtually every chapter in the present edition has undergone reorganization or expansion, in text as well as in computer routines, reflecting newer developments in methodology and omissions in the first edition. The major addition is a new chapter on integral equations, inverse problems, and regularization. (Surprisingly, there is no reference to [1].) All in all, more than 100 new routines have been added, almost all of the old ones still being there, though often with improved codes. To compensate for this substantial growth in material, many topics deemed "advanced" are now set in smaller type. Even so, the volumes have swelled to nearly 1000 pages, from the original 700–800 pages.

W. G.

1. C. T. H. Baker, *The numerical treatment of integral equations*, Clarendon Press, Oxford, 1977.
2. F. N. Fritsch, Review **3**, Math. Comp. **50** (1988), 346–348.
3. W. Gautschi, Review **6**, Math. Comp. **52** (1989), 253.

4[65–01].—Kendall Atkinson, *Elementary Numerical Analysis*, 2nd ed., Wiley, New York, 1993, xiv+425 pp., 24 cm. Price $61.95.

For a review of the first edition, see [1]. In the present edition, the outlay and character of the text have remained the same. Three paragraphs have been added, one on the general fixed-point method for a single equation, and one each on iterative methods for solving systems of linear, respectively nonlinear, equations. Some other parts of the text have been rewritten and supplied with new examples and problems.

W. G.

1. M. Minkoff, Review **36**, Math. Comp. **47** (1986), 749.

5[68–06, 68Q40, 68U99].—Andreas Griewank & George F. Corliss (Editors), *Automatic Differentiation of Algorithms: Theory, Implementation, and Application*, SIAM Proceedings Series, Society for Industrial and Applied Mathematics, Philadelphia, 1991, xiv+353 pp., 25 $\frac{1}{2}$ cm. Price: Softcover $48.50.

Since 1981, when L. Rall's Lecture Note Volume on Automatic Differentiation appeared, efforts directed toward the design and implementation of automatic differentiation software have multiplied. Nevertheless, the awareness among computational scientists of the availability and use of these tools is still rather restricted, even though their potential applicability is almost unlimited. This has been largely due to the fact that no comprehensive presentation of this subject has been available. The volume compiled and edited by A. Griewank and G. F. Corliss from papers presented at the SIAM Workshop on the Automatic Differentiation of Algorithms (held in January of 1991 in Breckenridge, Colorado) should finally fill this gap.

Naturally, a multiauthored volume cannot have the uniformity of a monograph. On the other hand, given the variety of viewpoints and orientations represented by the contributions, a more complete presentation of the state-of-the-art of the subject area has been achieved than could possibly have been realized by a single author. It is true that there are some duplications, and not all individual articles are equally to the point; but such a volume is not meant to be read from beginning to end. Altogether, the result of the editors' effort is remarkable and deserves high commendation.

Perhaps the two most valuable articles are the last two: a taxonomy of automatic differentiation tools by D. W. Juedes, and a bibliography of automatic differentiation compiled by G. F. Corliss. Altogether, 29 different products have been classified by Juedes and their similarities and distinctions described; information about their availability has also been included. The bibliography is the union of all quotations by authors in the volume and of previously compiled bibliographies by various people.

Before we enter into an assessment of other parts of the book, it may be advisable to clearly state the objectives of automatic differentiation: in today's understanding—and that of our volume—they consist in the computation of the *values* of derivatives (or of related quantities like a Jacobian times a vector) from an algorithmic description of the computation of the values of the underlying function. Thus, the mere transformation of the symbolic string for an arithmetic expression into that for its derivative with respect to some parameter (a capability present in virtually all computer algebra systems) is not automatic differentiation except if accompanied by the generation of efficient code for the derivative evaluation. The excellent introductory article by Masao Iri gives a concise description of the fundamentals of automatic differentiation, with a very transparent presentation of its two basic modes: forward or bottom-up, and backward or top-down. The juxtaposition and distinction of these two modes recurs in many of the following articles in ever varying forms, so that their relative advantages and disadvantages cannot escape the reader. Also, the mechanism of the (at first sight) not so intuitive reverse mode is illuminated.

It is clearly not possible to do justice to, and appraise, each one of the altogether 32 sections of the volume. Instead, the reviewer will only point to a few contributions which struck a particular chord in him; for the others, he can only summarize topics and directions.

The paper by Y. G. Evtushenko exposes the close analogy between the establishment of the data sensitivity of results in arithmetic expressions and in differential equations, which sheds yet another light on the reverse mode. Ch. Lawson draws attention to repeated automatic differentiation of the inverse function (for its Taylor expansion) and shows how this is feasible even in the multivariate case. The important problem of computing Newton corrections without an explicit computation of the elements of the Jacobian or Hessian is treated in several contributions. This is taken further by M. Berz who treats the evaluation of Lie derivatives in differential algebras. Two papers (Corliss, Layne) consider the combination of automatic differentiation with validation in the context of Taylor series expansions for solutions of systems of ordinary differential equations, which results in guaranteed inclusions, possibly under perturbations like in satellite computations.

A good number of papers deal with techniques which make implementations of automatic differentiation more efficient, special attention being given to concurrent architectures. Language aspects are also considered, and a group of papers describes particular aspects of particular software products. Another group of papers comments on the advantages which the authors have been able to derive from the use of automatic differentiation in their particular application areas, which range from weather prediction to distributed dynamical systems.

Altogether, this volume offers the opportunity to the computational scientist for a first, or in-depth, encounter with automatic differentiation, an experience which he is well advised to seek.

H. J. S.

6[41–06, 65Dxx].—E. W. CHENEY, C. K. CHUI & L. L. SCHUMAKER (Editors), *Approximation Theory* VII, Academic Press, Boston, xx+249 pp., $23\frac{1}{2}$ cm. Price $59.95.

During the last ten or fifteen years, *Approximation Theory* as a research subject has undergone remarkable changes and has turned up a variety of interesting new facets. Practical applications as well as theoretical questions, partly emerging from other mathematical subjects, have motivated new types of problems, which to a large extent have replaced classical issues like Jackson and Bernstein theorems. While this makes the field perhaps less coherent, it does enhance its pivotal position as a link between several different areas.

These trends are clearly reflected in this book. The Texas Symposium on Approximation Theory has long become a central and traditional event. This seventh volume of its proceedings has broken with tradition in that this time it only contains the contributions by the invited speakers. I am tempted to say, though, that the book is a good example of "less is more". In fact, all the articles are survey-type articles. They are all well, and some even exceptionally well, written. While they still address classical subjects like approximation by algebraic polynomials or rational functions, they emphasize important connections with potential theory, consider not only classical accuracy questions but also issues of shape preservation, and deal with algorithmic aspects such as recursive interpolation processes. The reader will learn of important principles for studying approximation orders and cardinal interpolation for shift-invariant spaces. This latter setting is also inherent in several contributions to the theory of wavelets and its applications, for instance, to signal processing. One important issue in this context is data compression, which is also the central theme of knot removal techniques for spline curves and surfaces motivated by applications in Computer Aided Geometric Design. Last but not least, the question of denseness of systems of ridge functions or sigmoidal functions is discussed and related to the theory of neural networks.

So the ideas and principles discussed in these articles, combined with the many references to the original papers, make this volume a valuable source of information on several recent interesting developments in approximation theory and beyond.

The authors and their titles are as follows:

Carl de Boor, Approximation order without quasi-interpolants
Charles K. Chui, Wavelets and signal analysis
Albert Cohen, Wavelet bases, approximation theory, and subdivision schemes
Bo Gao, Donald J. Newman, and Vasil Popov, Approximation with convex rational functions
Martin Gutknecht, Block structure and recursiveness in rational interpolation
Kurt Jetter, Multivariate approximation from the cardinal interpolation point of view
Will Light, Ridge functions, sigmoidal functions, and neural networks
Tom Lyche, Knot removal for spline curves and surfaces
Vilmos Totik, Approximation by algebraic polynomials

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7[65–01, 65Mxx, 65Nxx].—William F. Ames, *Numerical Methods for Partial Differential Equations*, 3rd ed., Computer Science and Scientific Computation, Academic Press, Boston, 1992, xvi+451 pp., $23\frac{1}{2}$ cm. Price $59.95.

This is a third and significantly updated edition of a well-known textbook which first appeared in 1969, with a second edition in 1977. Of the six chapters of the second edition, the last one, "Weighted residuals and finite elements", has been disassembled and its contents, to quote the author, "... merged with the material on finite differences", so that "... they now constitute equal partners". Further, "Additional material has been added in the areas of boundary elements, spectral methods, the method of lines, and invariant methods."

The book covers a large number of topics of interest for applications in science and engineering, and contains about 300 problems and 650 references. The presentation is more descriptive than analytic, and thus introduces and discusses concepts and gives many recipes for numerical approaches but provides little in terms of theory and proofs.

In spite of the author's claim for equal partnership between finite differences and finite elements, the text is still strongly founded in the finite difference ideology of the time of its first edition, and many important new points of view and developments are omitted such as, e.g., the variational approach to boundary value problems, and domain decomposition and multigrid methods. The first sentence of the introduction, "Numerical calculation is commonplace today in fields where it was virtually unknown before 1950" is taken over from an earlier time and gives an antiquated impression, and the author's claim in the preface that "the references have been brought up to date" is not quite justified: among the 134 references in the chapter on "Elliptic Equations", 95 are from before 1969 and only 19 from the 1970s and 20 from 1980 and later.

In summary, in the opinion of the reviewer, the updating of this classical book from almost 25 years ago has not resulted in a text that is representative of today's philosophy and technology for the numerical solution of partial differential equations. In view of the rapid and fundamental development of this field during the last decades it would, in fact, have been very difficult to achieve such a goal.

V. T.

8[35–06, 35J60, 65–06, 65K10, 65L05, 65N06].—RANDOLPH E. BANK (Editor), *Computational Aspects of VLSI Design with an Emphasis on Semiconductor Device Simulation*, Lectures in Applied Mathematics, vol. 25, Amer. Math. Soc., Providence, RI, 1990, xiii+190 pp., $23\frac{1}{2}$ cm. Price $56.00.

These are the proceedings of the eighteenth AMS-SIAM Summer Seminar on Applied Mathematics, held at the Institute for Mathematics and Its Applications from 30 April to 7 May 1987.

The primary focus of the book is on process or device simulation in the design of VLSI (Very Large Scale Integrated circuits, such as the computer chips that make up a personal computer, workstation, or even supercomputer). Topics related to circuit-level simulations are also presented.

The so-called drift-diffusion model used in device simulations is a system of nonlinear partial differential equations. Several papers are devoted to an asymptotic analysis of the singular limit of these systems as one of the physical parameters (which is usually small in typical applications) tends to zero. There is also some work on existence theory for such systems.

The numerical treatment of the drift-diffusion models is not extensive in the book, although substantial examples are presented in the papers dealing with the asymptotic behavior of the singular limit. These models exhibit internal layers (and possibly boundary layers), and the asymptotic analysis is intended to improve numerical methods for such problems as well as provide qualitative information of independent interest.

The drift-diffusion model is known to be insufficiently detailed for some VLSI designs of current interest. Two papers explore more detailed models. One is based on the Boltzmann equation and involves no less than seven independent variables: space, time, and wave numbers. The relationship between this model and the drift-diffusion model is reviewed and some numerical experiments on a simplified model problem are presented. Another paper explores the inclusion of quantum mechanical effects that are influential especially for VLSI chips made of gallium arsenide. A model consisting of a system of nonlinear partial differential equations, not unlike the drift-diffusion system, is derived and numerical experiments are described.

One paper considers block Gauss-Seidel iterative algorithms for solving the steady-state version of the drift-diffusion model. Another considers parameter-continuation methods (together with multigrid solution of the linearized equations) for solving them.

One paper is devoted to solving the basic algebraic-ODE equations that comprise the circuit-level model of VLSI. These models arise via spatial averaging of the device-level description of VLSI, reducing an entire device to discrete point values of current or voltage.

There are other significant computational problems of VLSI design not covered in the book. Among these are logic- and register-level models, which are discrete both in space and time. Logic-level models arise by quantizing the current or voltage levels in the circuit-level models into Boolean variables, as well as averaging over the time variable. Register-level models are a further abstraction in which logic elements are grouped into functional units operating on bytes or words of data. Other problems of computational VLSI design are optimization of chip layout and routing of wires, as well as testing of final designs. However, the title of the book indicates that these topics are beyond its scope.

The overall quality of the articles is high, both in terms of historical perspective and technical contribution. This monograph is essential reading for anyone interested in computational simulation of VLSI behavior.

L. R. S.

9[60–06, 65–06, 68–06, 70–06, 80–06, 82–06, 90–06, 92–06, 93–06, 94–06].— MATTI HEILIÖ (Editor), *Fifth European Conference on Mathematics in Industry*, European Consortium for Mathematics in Industry, Vol. 7, Kluwer, Dordrecht, 1991, x+400 pp., $24\frac{1}{2}$ cm. Price \$139.00/Dfl.260.00.

This carefully edited volume is the fifth in a series of proceedings of European conferences on industrial mathematics. In format and organization it resembles many SIAM conference proceedings and recent publications of the University of Minnesota's Institute of Mathematics and Its Applications. It contains seven invited presentations and sixty-four contributed presentations. The invited papers average about ten pages, and the contributed about five. In addition, there is a special section, or minisymposium, focusing on problems related to the distribution of electric power.

Such a volume of necessity sacrifices depth for diversity. The diversity of subject matter, both from technological and mathematical vantage points, is far beyond the knowledge base of any given individual. Areas of application include electromagnetic field theory, fluid mechanics, materials science, chemical engineering, design, phase and shape transitions, systems analysis, process simulation, control theory, image processing, robotics, nondestructive testing, signal processing, robotics, image processing, oceanography, and technical education.

The editor emphasizes that there is no fixed body of knowledge, theory, or techniques properly forming an area called "industrial mathematics". They refer to industrial mathematics as an orientation and a process by which the science of mathematics and computation meets the world of technological application. This conference and its proceedings reflect a growing awareness of the importance of this process to the development of sophisticated technology by the scientific community at large.

The present volume should prove to be a valuable resource to mathematically oriented individuals engaged in applications related to research, education, and creative research management.

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10[11D09].—RANDALL L. RATHBUN, *Table of Equal Area Pythagorean Triangles, from coprimitive sets of integer generator pairs*, iii+199 pp., deposited in the UMT file.

Diophantus of Alexandria asks for three equal area right triangles [6, Ch. XX, p. 500], showing very old interest in this question of triangles of equal area. Dickson studies the problem, mentioning that Pierre de Fermat showed how to obtain as many rational triangles as one desires from a given one, all with the same area [3, Ch. IV, pp. 172–174]. This question of Diophantus was raised by Lewis Carroll of Alice in Wonderland fame, and answered by both J. P. McCarthy [8] and D. L. MacKay [7], who gave a short list of n-triangles of equal area for $1 < n < 6$. Taking up the problem, W. P. Whitlock, Jr. discusses the question at some length and provides two parametric solutions [10]. He additionally repeats D. L. MacKay's tables. Martin Gardner asked for more sets of triples of primitive Pythagorean triangles other than the one found by Charles Shedd in 1945 [4]. Most recently, Andrew Bremner considers the problem as a set of linear automorphisms upon an elliptic surface and gives a list of parametric solutions [2].

There appear to be two extensive lists of primitive Pythagorean triangles deposited in the UMT file, the first by A. S. Anema [1] and the second by Francis L. Miksa [9], arranged according to increasing areas.

The present table is the result of an extensive computer search to find all sets of Pythagorean triangles (primitive or not) that have equal area and are created from integer generator pairs (m, n) in $(*)$ such that their areas are equal:

$$(*) \quad A = 2mn, \quad B = m^2 - n^2, \quad C = m^2 + n^2; \qquad \text{Area} = AB/2.$$

The table lists 9916 sets of coprimitive generator pairs on 199 pages. There are 50 sets per page, and the relative indexed entry range is given at the top along with the actual generator pairs covered. All coprime opposite parity generator pairs are in bold, because they generate primitive triangles. Also, any area not divisible by 10 is printed in bold.

All 49,995,000 (m, n)-pairs from $(2, 1)$ to $(10000, 9999)$ were searched. The third page of the introduction gives a short summary page of all primitive triplets, all quadruplets, and all quintuplets of equal-area triangles found. There appear to be only five primitive triplets found so far, the author's main purpose for creating the table.

All sets of pairs provide solutions for the face diagonal type of rational cuboid solution, as noted in Problem **D18** of [5, pp. 97–103]. The author is making available an ASCII MSDOS version of the table on a floppy disk for a nominal fee upon the asking.

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1. A. S. Anema, *Primitive Phythagorean triangles with their generators and with their perimeters up to* 119,992, manuscript in the UMT file, MTAC **5** (1951), UMT **111**, p. 28.

2. Andrew Bremner, *Pythagorean triangles and a quartic surface*, J. Reine Angew. Math. **318** (1980), 120–125.

3. Leonard E. Dickson, *History of the theory of numbers*, Vol. II, Chelsea, New York, 1956.

4. Martin Gardner, *Simple proofs of the Pythagorean theorem and sundry other matters*, Mathematical Games, Scientific American, October 1964, Volume 211, #4, pp. 118–126.
5. Richard K. Guy, *Unsolved problems in number theory*, vol. 1, Springer-Verlag, New York, 1981.
6. Thomas Heath, *A history of Greek mathematics*, Volume II, from Aristarchus to Diophantus, Dover, New York, 1981.
7. D. L. MacKay, *Solution to Problem* E327[1938, 248] *proposed by Philip Franklin*, Amer. Math. Monthly **46** (1939), 169–170.
8. J. P. McCarthy, *A Lewis Carroll problem*, Mathematical Notes 1196, Math. Gaz. **20** (1936), 152–153.
9. Francis L. Miksa, *Table of primitive Pythagorean triangles, arranged according to increasing areas*, 5 vol. ms. comprising a total of 27+980 pp. deposited in UMT file; see Math. Comp. **23** (1969), Review **69**, p. 888.
10. W. P. Whitlock, Jr., *Rational right triangles with equal area*, Scripta Math. **9** (1943), 155–161; ibid., pp. 265–268.

11[11D09].—RANDALL L. RATHBUN & TORBJÖRN GRANLUND, *The Integer Cuboid Table, with Body, Edge, and Face Type of Solutions*, vii+399 pp. (2 vols.) + *The Integer Cuboid Auxiliary Table*, 100 pp., deposited in the UMT file.

The Integer Cuboid Table and its Auxiliary Table is the collation of computer searches for all three types of integer cuboids as noted in Problem **D18** of [1, pp. 97–103]. The range of the smallest edge is from 2 to 333,750,000 exhaustively, and 19,929 primitive cuboids were found (6800 body, 6749 face, and 6380 edge).

There exist earlier lists of cuboids, or cuboid generators, but they cover only one type of cuboid at a time, or list only the generators [2–8, 11]. Furthermore, they are not exhaustive over the dimensions of the cuboid, and have corrections [9, 10, 12]; hence this present table, which attempts to be accurate and complete both over the dimensions and type of cuboid solutions.

The new table is presented as a two-volume set, owing to its extensive length. There are 50 cuboids listed per page. At the top of the page is the indexed range of cuboids covered, both by their smallest edge and number of actual occurrence. Upon each line is listed the type of cuboid, B, F, E standing for body, face, and edge cuboids, respectively. Next is given the three edges and body diagonal of the actual cuboid. The primitive Pythagorean generator pairs for each type of cuboid are the last set of entries per solution. At the bottom of each page is given the subtotal of the B, F, E types and then a running total of all types found. The Auxiliary Table is indexed as a supplement to match the cuboid table, giving the irrationality of either the edge or diagonal of a selected cuboid in terms of a square and small excess or deficit, whichever is closer.

The seven-page introduction provides an adequate instruction about the integer cuboid problem, and introduces some properties of the Pythagorean generators associated with each type of solution. Additionally, a simple summarization is provided of the cuboid table, including other tables resulting from its study, such as cuboid solutions with pairs of common values, extended B, F, E solutions not covered in the current table, but derivable from an entry, etc.

The first author is making available a catalog and/or copy of one of these tables upon request. Additionally, an extensive bibliography of over 60 references on the integer cuboid is also available.

It is hoped that the Integer Cuboid Table will be extended through at least the first 1,000,000,000 integers for the smallest side.

Finally, the perfect cuboid was not found. If it exists, the smallest edge must be greater than 333,750,000.

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1. Richard K. Guy, *Unsolved problems in number theory*, vol. 1, Springer-Verlag, New York, 1981.
2. Maurice Kraitchik, *On certain rational cuboids*, Scripta Math. **11** (1945), 317–326.
3. ——, *Théorie des nombres*, Tome 3, Analyse Diophantine et Applications aux Cuboides Rationnels, Gauthier-Villars, Paris, 1947, pp. 122–131.
4. ——, *Sur les cuboides rationnels*, Proc. Internat. Congr. Math. 1954, vol. 2, Amsterdam, 1954, pp. 33–34.
5. Jean LaGrange, *Sur le cuboide entier*, Seminaire DeLange-Pisot-Poitou (Groupe d'étude de Théorie des Nombres), 17ème année, 1975/76, #G1, 5 pages.
6. Mohan Lal and W. J. Blundon, *Solutions of the Diophantine equations* $x^2 + y^2 = 1^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$, Math. Comp. **20** (1966), 144–147.
7. John Leech, *The location of four squares in an arithmetic progression, with some applications*, Computers in Number Theory (A. Oliver, L. Aitkin, and Brian J. Birch, eds.), Academic Press, London and New York, 1971, pp. 83–98.
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9. ——, Table Errata **554**—M. Kraitchik, *Théorie des nombres*, Tome 3, Analyse Diophantine et Applications aux Cuboides Rationnels, Gauthier-Villars, Paris, 1947, pp. 122–131; **555**—M. Kraitchik, *Sur les cuboides rationnels*, Proc. Internat. Congr. Math., 1954, vol. 2, Amsterdam, 1954, pp. 33–34; see Math. Comp. **32** (1978), 661.
10. Daniel Shanks, Corrigenda, M. Lal and W. J. Blundon, *Solutions of the Diophantine Equations* $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$, Math. Comp. **20** (1966), 144–147; see Math. Comp. **23** (1969), 219.
11. William Gideon Spohn, Jr., *Table of integral cuboids and their generators*, 4pp+45pp reduced-size computer printout, UMT file, Applied Physics Laboratory, Johns Hopkins University, Laurel, Maryland, 1978; see Math. Comp. **33** (1979), Review **4**, 428–429.
12. ——, Table Errata **558**—John Leech, *Five tables related to rational cuboids*, UMT **12**, Math. Comp. **32** (1978), 657–659; see Math. Comp. **33** (1979), 430.

12[11D09].—RANDALL L. RATHBUN & TORBJÖRN GRANLUND, *The Classical Rational Cuboid Table of Maurice Kraitchik*, revised and enlarged, ii+(3-page errata)+135pp., deposited in the UMT file.

Maurice Kraitchik first discussed the problem of certain rational cuboids in 1945, giving a table of 50 cuboids at the end of his article [1]. He had published two years later, in the third volume [2, pp. 122–131] of his *Théorie des Nombres*, 241 rational cuboids of the body type for the odd side less than 1,000,000. He added 16 more new cuboids in 1954 in his addendum [3]. John Leech discusses the errata of Kraitchik's tables, giving a list of misprints and omissions [4].

The present table is a new revision of Maurice Kraitchik's originals, completely corrected for all errors and omissions. It is further expanded by extensive computer search to completely cover all odd sides $< 333{,}750{,}000$ of the body type of rational cuboids that do exist. The new revision has two introduction pages, three pages of corrections to the original tables, and 135 pages of the new, expanded table.

The table lists 36 cuboids per page for a total of 4839 cuboids. Each page (except the last) has two columns of 18 cuboids listed in ascending order of the given odd side. All three sides of the cuboid are given in factored format along with their Pythagorean generators. The introduction explains the relation of the generators with the cuboid sides.

The errata for Kraitchik's original tables agrees with that of John Leech [4], with one noted exception, Entry #34 in Corrections to Tome 3, but is more extensive and thought to be totally complete, giving all misprints, transpositions, errors, and omissions that exist in the originals.

It is hoped by the first author that this table will adequately serve as a suitable revision of Kraitchik's own tables and stimulate further progress upon the rational cuboid problem.

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1. Maurice Kraitchik, *On certain rational cuboids*, Scripta Math. **11** (1945), 317–326.

2. ——, *Théorie des nombres*, Tome 3, Analyse Diophantine et Applications aux Cuboides Rationnels, Gauthier-Villars, Paris, 1947.

3. ——, *Sur les cuboides rationnels*, Proc. Internat. Congr. Math., 1954, vol. 2, Amsterdam, 1954, pp. 33–34.

4. John Leech, Table Errata **554**—M. Kraitchik, *Théorie des nombres*, Tome 3, Analyse Diophantine et Applications aux Cuboides Rationnels, Gauthier-Villars, Paris, 1947, pp. 122–131; **555**—M. Kraitchik, *Sur les cuboides rationnels*, Proc. Internat. Congr. Math., 1954, vol. 2, Amsterdam, 1954, pp. 33–34; see Math. Comp. **32** (1978), 661.