REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of Mathematical Reviews starting with the December 1990 issue.


This volume represents the fifth edition of a well-known work. The fourth edition was reviewed in [1], and an errata notice was published in [2]. Since then, further errata notices have been published in [3]–[6].

Although other comprehensive collections of formulas for integrals and series have been published in the meantime, the present volume contains such a wealth of information that a new edition was to be welcomed, provided it had been prepared with the care merited by its contents and by taking into account the remarks and criticisms made in the reviews of earlier editions. Unfortunately, this is often not the case.

As mentioned in the preface, the entire volume has been reset, presumably using a modern text-processing system on a computer, though nothing is said about this. Some formulas and a few short sections have been added, in particular for some special functions and in the tables for integral transforms, but not to the extent one would expect from reading the preface. Some references, in particular to tables of integral transforms, have been added. Nevertheless, a number of important references are missing.

The contents of this table is well known and has been described in earlier reviews. In particular, Chapters 10 to 17, which constitute somewhat foreign material in this volume, were criticized rather severely in [1]. Therefore, it seems more appropriate to concentrate on aspects of editing and presentation. (An errata notice will be given separately.)

There are quite a number of remarks and suggestions in previous reviews or errata notices which have been neglected or taken into account in a rather careless fashion in preparing this edition. To begin with, although the translation has been criticized as too literal, many verbose and clumsy sections remain unchanged. It is surprising to see that the term degenerate (qualified “ridiculous” in [1]) instead of confluent for this hypergeometric function has been changed only in §9.2, but not in §7.6 and other places. The Index of Special Functions (pp. xli–xliv) and the Notations (p. xlv) are presented with little care.

In the formula part of the table, errors given in the errata notices have been corrected, but in some cases new misprints have been introduced by doing this.
Suggested new expressions which were intended to simplify certain results have often not been considered at all, or only formally, disregarding the table environment. A minimum of care should have been shown in introducing new formulas, e.g., the letter $\theta$ instead of $\vartheta$ for denoting the theta functions in 8.199(1)--(3) is to be deplored. Having defined symbols like $[x]$, $(\alpha)_n$, etc., and the value of empty sums and products in the Notations (p. xlv) should have eliminated the need for footnotes or explanations later. The convention (p. 264) of not indicating principal values should have been applied throughout.

The short tables for the Laplace, Fourier, and Mellin transforms in Chapter 17 merit special criticism. Their heterogeneity in layout and notation would suggest careless and superficial editing. Although they are announced in the preface as having been revised, the number of misprints and errors in the tables for the Laplace and Fourier transforms is larger than would usually be considered acceptable in a mathematical reference work.

There are other points of principle. Since the appearance of the fourth edition, expressions for certain integrals, for example of rational functions and powers of logarithms, have been published. These formulas contain a number of parameters and unify simple special cases given in the volume, for instance in 4.23--27, 4.33--35. Other newly evaluated integrals are certainly scattered in the literature, and one would have hoped that some of these might have been incorporated.

Further, it is difficult to understand why curiosities like the Gudermannian $\text{gd}(x)$ or the Lobachevski $\Pi(x)$ are presented in detail, while the polylogarithm $\text{Li}_n(x)$ and its generalization $S_{n,p}(x)$ due to Nielsen, both defined by integrals, with many functional relations, related integrals, and of importance in modern physics, continue to be ignored.

More specifically, infinite series should be replaced by their values whenever this is possible. For example, several of the series in 3.411 and 4.261 can be expressed in terms of $\zeta(n)$, as it has been done in some cases. Obviously superfluous or divergent integrals (the latter with the possible exception of cases which are difficult to recognize as such) should be deleted, keeping an empty number if necessary. Useless cross-references, e.g., in 3.727 and 3.735, and trivial numerical values should be suppressed.

As a rule, definite integrals should be given only when the corresponding indefinite integral cannot be expressed in closed form. For example, all the integrals in 3.351 can be obtained from the indefinite integrals in 2.32. By adding in 2.3 the formula [7, equation 7.4.32] for $\int \exp[-(ax^2 + 2bx + c)]\,dx$, which is important in applications, and which is curiously missing, the presentation in 3.321--23 could be simplified.

The restrictions should be reconsidered in a number of sections. For uniformity, they should always be enclosed in square brackets, and symbols like $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{N}_+$, $\in \mathbb{G}$, etc. should be used, as has been done on rare occasions.

Some remarks about the notation used in this volume seem appropriate. Although it has survived several editions, the function symbol $\text{Ei}$ defined in 8.2113. is unnecessary. Its use creates confusion and leads to superfluous comments, in particular in 3.35. It should be replaced by $\text{Ei}$, on the understanding that principal values are not indicated. The letter $N$ for the Bessel function of the second kind or Neumann function is rarely used in the English literature;
Y should be used instead. The Gaussian hypergeometric function $2F_1$ should be written in this form or simply as $F$, but not in both ways even on the same page. For the probability integral $\Phi(x)$, the term “error function erf $x$” is more appropriate, especially as it already occurs in several places and the letter $\Phi$ is used for two other functions. In particular, by using the complementary error function $\text{erfc} x = 1 - \Phi(x)$, a number of formulas can be simplified. The inconsistent notation for the theta functions in 6.16 and 8.18–19 should not have survived five editions. At the very least, the notation for elementary functions should be used consistently.

The excessive and arbitrary use of the possessive in function names is annoying. It is also unusual to extend the term Riemann zeta function to the generalized zeta function $\zeta(z, q)$.

In general, the formulas are given in a readable form; although, considering the ease with which typesetting can now be handled, a more skillful layout seems often possible. For example, a number of exponentials have an exponent that reaches down to the main line, e.g., in 7.386. This is unusual and unprofessional. There are also a number of typographical inconsistencies. Last but not least, a more economical, legible, and visually satisfying presentation of many formulas, illustrated by the simple example $\Gamma\left(\frac{1}{2} + \nu\right)$ instead of $\Gamma\left(\frac{1}{2} + \nu\right)$, should be adopted in a next, hopefully more carefully prepared, edition.

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There exists a large family of problems, deterministic or probabilistic in nature, which require the evaluation of an integral, with respect to the law of a stochastic process, of a functional on the space of the trajectories of the process. The book under review provides a good sampling of examples coming from physics, like Feynman integrals; some other less classical examples are: in
Random Mechanics (problems of crack propagation, for example), the computation of the probability that a stochastic process crosses a threshold during a fixed time interval; in Finance, the pricing of exotic options which depend on the trajectory of the underlying asset; in Neutronics, the computation of fluxes.

For a probabilist, a natural way of approximating such integrals is to develop Monte-Carlo methods based upon the simulation of stochastic processes. The present book gives a large overview on a different methodology: integrals with respect to countably additive measures, in particular the measures generated by random processes, are approximated by mean of deterministic procedures, some of them generalizing to infinite dimension the well-known quadrature formulae used in finite-dimensional spaces.

Chapters 1–3 are devoted to abstract considerations on cylindric measures, quasi-measures and measures induced by stochastic processes on general linear topological spaces. Chapter 2 focuses on Gaussian measures, with some notions on Wick products, while Chapter 3 focuses on integrals in product spaces.

Chapters 4, 5, 9, and 12 give a generic presentation of the main topics of the book: the approximation problem of functional integrals; as in the finite-dimensional case, the guiding principle is to construct formulae which preserve some important statistics of the underlying measure; interpolation or quadrature formulae, which are exact for the integration of specific functionals (polynomial, trigonometric, or other functionals), approximation of the characteristics functional preserving a given number of moments, formulae which are exact for polynomials of Wick powers, etc.

In Chapters 6–11 and 14, typical applications are described: integrals with respect to Gaussian measures, Wiener measure, laws of stationary processes with independent increments, laws of solutions of stochastic differential equations on the whole space or on manifolds. The crucial questions of convergence and convergence rate are investigated.

Chapter 13 briefly presents a completely different approach, the Monte-Carlo method, and to conclude, the authors in Chapter 15 exhibit some problems coming from physics which have solutions expressible in terms of functional integrals.

The reader is assumed to be familiar with functional integration, cylindric measures, etc. For such a reader, the presentation is excellent, extremely clear and easy to read; according to his own level of interest in the field, he will find pleasing reviews as well as technical discussions. To my mind, the numerical aspects are treated too lightly: the authors give no indication as to the limitations of the procedures and their numerical behavior; they do not give bases of comparison between them; and the proportion of Eastern European authors in the bibliography is probably too large. These remarks, however, should not detract from the very positive qualities of the book, which indeed gives a pleasant and instructive overview on the state of the art of new mathematical problems.

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The purpose of this research monograph is to present a comprehensive theory of mappings of A-proper type and its application to differential equations in the context of constructive (abstract numerical) functional analysis. To put the book in perspective, it is useful to trace the roots of the subject, which evolved in response to the following question posed by the author 27 years ago (see, e.g., [5]): "For what type of linear and nonlinear mapping T is it possible to construct a solution to a given operator equation (1) \( Tx = f \) as a strong limit of solutions \( x_n \) of finite-dimensional approximating equations \( T_n(x_n) = f_n \)?" In a series of papers in the 1960s (see [7] for references and perspective summary) the author studied this problem, and the notion that evolved from these investigations in 1967 is that of an A-proper mapping [6].

Let \( X, Y \) be Banach spaces, \( D \) a subset of \( X \) and \( T: D \subseteq X \rightarrow Y \) a possibly nonlinear mapping. Let \( \{X_n\} \) and \( \{E_n\} \) be sequences of oriented finite-dimensional spaces with \( X_n \subseteq X \), \( V_n \) an injective map of \( X_n \) into \( X \), and \( W_n \) a continuous linear map of \( Y \) onto \( E_n \) for each positive integer \( n \). An approximation scheme \( \Gamma = \{X_n, V_n; E_n, W_n\} \) for the equation (1) is said to be admissible provided that \( \dim X_n = \dim E_n \) for all \( n \), the distance between \( x \) and \( X_n \) tends to zero as \( n \rightarrow \infty \) for each \( x \) in \( X \), and \( \{W_n\} \) is uniformly bounded. Note that the spaces \( E_n \) are not required to be subspaces of \( Y \) and the sequence \( \{X_n\} \) is not assumed to be nested. Hence, for example, the finite element method can be used in the construction of admissible schemes. The existence of an admissible scheme for \( (X, Y) \) implies that \( X \) is separable, \( \bigcup_n X_n \) is dense in \( X \), but \( Y \) need not be separable. Examples of simple admissible schemes include various projection and Galerkin-type methods. If \( X \) is separable with dual \( X^* \), then \( (X, X^*) \) always has an admissible scheme. Also there is an admissible scheme for \( (X, X) \) whenever \( X \) has a Schauder basis.

The concepts of approximation properness (A-properness) of \( T: D \subseteq X \rightarrow Y \) and of the approximation solvability of equation (1) are defined in terms of a given admissible scheme for \( (X, Y) \). Equation (1) is said to be strongly (respectively, feebly) approximation solvable with respect to the admissible scheme \( \Gamma = \{X_n, V_n; E_n, W_n\} \) if for all sufficiently large \( n \), the equation (2), where \( f_n := W_n f \), has a solution \( x_n \in D_n := D \cap X_n \) such that \( x_n \rightarrow x_0 \) in \( D \) (respectively some subsequence of \( \{x_n\} \) converges to \( x_0 \) and \( T(x_0) = f \)). The equation (1) is uniquely approximation solvable if the approximate solutions \( x_n \) and the limit solution \( x_0 \) are unique. Clearly, approximation solvability implies solvability, but the converse need not be true. A mapping \( T: D \subseteq X \rightarrow Y \) is said to be A-proper with respect to \( \Gamma \) if and only if the restriction \( T_n \) of \( W_n T \) to \( D_n \), \( T_n: D_n \subseteq X_n \rightarrow E_n \), is continuous for each \( n \) and the following condition holds: If \( \{x_{nj}\} | x_{nj} \in D_{nj} \) \) is any bounded sequence such that \( \|T_{nj}(x_{nj}) - W_n(g)\| \rightarrow 0 \) as \( j \rightarrow \infty \) for some \( g \) in \( Y \), there exists a subsequence \( \{x_{nj(k)}\} \) of \( \{x_{nj}\} \) and \( x \in D \) such that \( x_{nj(k)} \rightarrow x \) as \( k \rightarrow \infty \) and \( T(x) = g \).
The motivation for the terminology "A-proper" resides in the connection with the well-known notion of a *proper* map. We recall that a map $T$ is *proper* if the inverse image of each compact set in $Y$ is compact in $X$. One can show that if $D \subset X$ is open and $T: \overline{D} \to Y$ is continuous and A-proper, then the restriction of $T$ to every bounded closed subset of $\overline{D}$ is a proper map.

The book is organized into five chapters. Chapter I provides an introduction to the general theory of A-proper and pseudo-A-proper maps, including examples and applications to constructive solvability of some 2nd-order differential equations. Chapter II develops the linear theory of A-proper maps and its application to the variational solvability of linear elliptic PDEs of even order. The author shows that the abstract results in this chapter are best possible. Chapter III establishes fixed-point and surjectivity (solvability) theorems for various important classes of A-proper-type maps. These results unify and extend earlier results on monotone and accretive maps. The author also shows how the Friedrichs linear extension theory can be generalized to the extensions of densely defined nonlinear operators in a Hilbert space, and provides applications of the theory to ODEs and PDEs. Chapter IV presents the generalized topological degree theory of A-proper maps developed by Browder and Petryshyn. The degree theory is applied to local bifurcation and asymptotic bifurcation problems. Finally, in Chapter V the author applies the abstract results to the solvability of boundary value problems of ODEs and PDEs and to bifurcation problems. The bibliography consists of 360 references to books and research papers.

The fields of nonlinear functional analysis and numerical functional analysis (or numerical analysis in abstract spaces) have developed extensively in the last thirty years. Moreover, the two fields have benefited immensely from their interactions. Some historical perspectives on landmarks in these fields are given in the book reviews [2] and [3], and in the books [1] and [4]. As the author states, the main thrust of the development of nonlinear functional analysis and its applications during the last 25 years has been in the direction of breaking out of the classical framework into a much wider field of noncompact operators, such as operators of monotone and accretive type, operators of set and ball-condensing type, and operators of approximation-proper type. There exist extensive treatments of the theories and applications of the first two classes of operators. The present monograph complements the existing literature by providing the first comprehensive study of approximation-solvability of equations involving A-proper type operators. The author is the recipient of the M. Krylov Award (1992) of the Ukrainian Academy of Sciences for originating and developing the theory of A-proper mappings and its manifold applications, and was elected by the Ukrainian Academy in 1992 as a foreign full member. The contributions of Krylov to numerical mathematics in particular are well known to readers of *Mathematics of Computation*.

The book should be of interest to researchers in the theory and applications of nonlinear functional analysis and also to abstract numerical analysts. Among the references, the journals which have the most citations (in terms of the number of cited references published in these journals) include the *Journal of Mathematical Analysis and Applications*, the *Bulletin of the American Mathematical Society*, *Transactions of the AMS*, *Journal of Functional Analysis*, *Nonlinear Analysis: TMA*, and *Soviet Mathematics Doklady*. The perspective of an analyst dominates the book. There are no references in the book to papers in
Mathematics of Computation. But this does not diminish the relevance of this monograph to numerical functional analysis. One is reminded of a statement by the late A. S. Householder, in his review of the book by Collatz [1] in Mathematical Reviews (MR 29, #2931): “It seems strange that this book should be the first of its kind, since it hardly needs to be said that ‘numerical mathematics’ must draw heavily from functional analysis.” It has been thirty years since Householder’s review. Time has only reinforced the relation between these two fields, which must continue to enrich each other by drawing heavily from each other.

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This book consists of a series of twenty-seven papers based on lectures given at the NATO Advanced Research Workshop held in Grenoble, France, in June 1992. A typical conference proceedings volume, it provides an overview of the most current research in the study of vortex flows. The emphasis is on both up-to-date mathematical models as well as numerical methods used to study the particular features of these flows.

The articles are not meant to provide an introduction to the specific subjects, but are rather a summary of recent results in the area. Extensive references serve as a useful guide to related literature. Intended readers are researchers and graduate students with interests in computational fluid dynamics, numerical analysis or applied mathematics in general.

The papers in the first part of the volume cover most recent developments in mathematical and numerical modeling for incompressible vortex flows. The second part treats mainly vorticity generation problems, related boundary layer and wake models in two dimensions. The third part concentrates on vortex
methods, hybrid finite difference vortex methods, contour dynamics and numerical experiments performed using these methods. Topics in the fourth part revolve around three-dimensional computations for incompressible flows: vortex rings, vortex sheets, dynamics of vortex tubes in turbulence are some of the subjects addressed here. Finally, the fifth part consists of four articles which focus on the numerical simulation of reacting and compressible flows.


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Interior-point methods have proved to be powerful and elegant techniques for solving linear programming problems, and their study is now being incorporated into introductory courses on linear programming. It is now possible to present some of the fundamental ideas concisely and with simple mathematical tools. Nevertheless, students often develop the idea that codes implementing interior-point methods need to be highly sophisticated, or else they will fail owing to ill-conditioning, or may behave poorly. This book makes a splendid contribution
towards the goal of educating a wide audience in the computational aspects of interior-point methods. It will show a student that a simple implementation and a few numerical precautions suffice to produce software that works.

The book comes with a diskette containing a software package implementing three interior-point methods: a primal method, a dual method and a primal-dual method. The software is designed to be used on IBM compatibles under DOS, and its primary goal is to be educational. It is not sophisticated and, as the author points out from the outset, this software is neither as versatile nor as powerful as the leading codes currently being used to solve very large problems. The book is a self-contained and very readable manual for the software that provides sufficient background in linear programming and interior-point methods to be used for instruction or for self-education. The book can be understood by students and professionals with a minimum background in linear programming and mathematics.

The author succeeds in making the presentation clear and simple. However, the emphasis on clarity makes the presentation too repetitive at places. Also missing, in my view, is a clear explanation of the strengths of the primal-dual approach. But these relatively minor flaws do not prevent this book, and the accompanying software, from being a valuable educational tool in linear programming.

J. J. N.


This is a very valuable book for anyone interested in using optimization codes, and for all those wishing to learn about the state-of-the-art in this field. The book lists most of the currently available software for solving various classes of optimization problems, such as linear programming, integer programming, network optimization and nonlinear optimization, and describes how the software can be obtained. Many readers will be pleasantly surprised to find that much of it is distributed freely on the Internet.

The book begins by classifying optimization problems in various categories, and gives a lucid and concise description of each of these categories. Readers with limited knowledge of optimization and numerical analysis will have very little difficulty reading through this summary, which gives a nice overview of the field. The book then proceeds to give a short description of each of the software packages, and a list of the software in each category. As explained by the authors, some areas of optimization, such as stochastic programming (or programming under uncertainty) are not included because general-purpose software is not yet readily available. Nevertheless, the book covers a wide range of problems, and one would hope that it finds wide dissemination in industry, where it can be extremely valuable. The book can also be used as a teaching aid in optimization courses.

J. J. N.
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS


This is a useful collection of essays on parallel computing, providing broad surveys of very fundamental subjects such as parallel programming paradigms and parallel computational complexity, as well as more specialized topics. It includes provocative introductions to several topics such as parallel algorithms for dynamic programming, branch and bound, discrete event simulation, packet routing, genetic algorithms, neural network algorithms, matrix algorithms, and others.

The book begins with a reasonably comprehensive survey of parallel programming paradigms. It is generally well written, except that there is very little comparison of different paradigms. This reviewer prefers surveys which are more than a list, even at the expense of the biases of the author creeping in. The other chapter on programming paradigms discusses a language/system called Divacon, which purports to be based on the familiar divide and conquer paradigm of algorithm design. Divide and conquer is obviously a great programming strategy, but the chapter does not convince this reviewer that a restricted model of divide and conquer is likely to be better than standard languages with simple constructs. The chapter does discuss interesting ideas, except that they are attributed to Divacon, when they were clearly developed earlier.

The chapter on parallel complexity theory is somewhat disappointing. One would expect a chapter titled "computational complexity" to be based on reasonably rigorous material: description (or perhaps mention) of interesting fast algorithms for problems that do not appear to parallelize well, and (formal) obstacles in parallelization, e.g. P-completeness, Kung's result on recurrences, and a listing of problems which have eluded parallel algorithms. Instead, one finds definitions of scalability, speedup, long informal discussion of spatial and temporal locality, detailed descriptions of the Bulk Synchronous PRAM model, which while important, does not have much to do with fundamental issues in parallel algorithm complexity.

The chapters dealing with specialized topics in parallel computation are more interesting, for instance the chapter on packet routing on arrays. The chapters on parallel branch and bound and discrete event simulation were informative as well as provocative, though sometimes only providing a bland survey without much comparison among approaches.

The book also leaves much to be desired as far as editing is concerned. Often, chapters repeat material from other chapters, and preliminary material (e.g., the discussion of speedup, scalability) that should occur in initial chapters occurs towards the end. The chapter on parallel derandomization techniques certainly discusses important and exciting ideas, but is hard to read because of grammatical errors, as well as repetition.

In spite of the drawbacks, the collection is useful. There are very few books which even attempt to encompass such a broad field; and the book should be of use to the general reader.

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