NEW CULLEN PRIMES

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ABSTRACT. Numbers of the forms $C_n = n \cdot 2^n + 1$ and $W_n = n \cdot 2^n - 1$ are both called Cullen numbers. New primes C_n are presented for $n = 4713$, 5795, 6611, 18496. For W_n, several new primes are listed, the largest one having $n = 18885$. Furthermore, all efforts made to factorize numbers C_n and W_n are described, and the result, the complete factorization for all $n \leq 300$, is given in a Supplement.

1. INTRODUCTION

In 1905, the Reverend J. Cullen [6] called attention to the numbers $C_n = n \cdot 2^n + 1$, the particular case of $k \cdot 2^n + 1$ where $k = n$. He observed that C_n was composite for all n in the interval $1 < n < 100$, with the only possible exception of $n = 53$. Shortly afterwards, Cunningham [7] gave the prime factor 5591 for C_{53} and restated Cullen's assertion for $1 < n \leq 200$, now leaving $n = 141$ as the only uncertain case in the considered range. Cunningham pointed out that primes of the form $C_n = n \cdot 2^n + 1$ seemed to be remarkably rare.

Half a century later, in 1957, Robinson [22] showed that C_{141} was in fact a prime. Moreover, he established that this was the only prime C_n with $1 < n \leq 1000$. In our numerical investigation we were able to determine four additional primes C_n. It can now be asserted that C_n is prime for $n = 1$, 141, 4713, 5795, 6611, 18496, and for no other $n \leq 30000$.

The analogous numbers $W_n = n \cdot 2^n - 1$ have also been investigated. As to their primality, Riesel [21] found in 1968 that W_n is prime for $n = 2, 3, 6$, $30, 75, 81$, but for no other $n \leq 110$. Considerably extending this search, we could add to Riesel's list the primes W_n with $n = 115, 123, 249, 362, 384$, $462, 512, 751, 822, 5312, 7755, 9531, 12379, 15822, 18885$. All other values of $n \leq 20000$ yielded composite numbers.

A comprehensive study of divisibility properties of numbers C_n and W_n was presented by Cunningham and Woodall (abbreviated C & W in what follows) in 1917. On account of their classical paper [8], the notations C_n (for "Cunningham numbers") and W_n (for "Woodall numbers") have been used, while the term Cullen numbers is extended to both these varieties (cf. [13, 24]). C & W also initiated the project of actually factoring the considered numbers. This work has been extended over the past decades, inasmuch as increasingly

powerful factorization methods were developed. In a Supplement to this paper the complete factorization of numbers C_n and W_n is given for all $n \le 300$.

2. FACTOR TABLES

C & W's report includes the factorization of C_n and W_n for all $n \le 32$, except for the single number W_{30}, whose primality was not recognized. In the interval $32 < n \le 66$, the character of W_n remained doubtful only for $n = 39$, 42, 51. The factor table for the original Cullen numbers C_n has subsequently been completed at least up to $n \le 43$ by Beeger [1], to $n \le 61$ by E. Karst, D. H. Lehmer and J. S. Madachy (as reported in [13]), and to $n \le 101$ by Steiner [24]. Here, three errata have to be observed: one digit is missing in each of the factorizations given for $n = 91$, 97, 101.

Also from [13] we learn that the factorizations of W_n were completed for all $n \le 49$ by Karst and Madachy, and that Madachy also factored W_n for $n = 52$, 55, 56, 57, 59, 60, and 76. The decomposition of W_{50} attributed to Lehmer gives the cofactor $1197765858343217 = 30503527 \cdot 39266471$ as a prime, and the numbers W_{123} and W_{249} are falsely reported as composites. The factors of W_{51} were later given in [24]. Finally, [13] lists small factors of numbers C_n and W_n up to $n = 300$.

In the Supplement, the complete factorization of both C_n and W_n is extended to just that limit. The tables were compiled and carefully checked by the author of this paper. Factors for $n \le 210$ considered difficult to obtain at the time were provided by G. Löh and W. Niebuhr. Thus, Löh completed C_n for $102 \le n \le 171$ and W_n to $n \le 122$ in 1984, using the methods of [2, 18, 20], and Niebuhr completed C_n and W_n for $n \le 210$ in 1986 with his implementation of the multiple polynomial quadratic sieve (MPQS) based on the description given in [23].

In 1988, H. Suyama supplied the smallest factor of W_{278} and the 16-digit prime factor of W_{211}, enabling us to prove primality for the 51-digit cofactor of this number. Suyama used his implementation of the elliptic curve method (ECM), cf. [17]. With only the exception of W_{211}, the complete range of $210 < n \le 260$ was done by the author in 1991, using the excellent factorization and prime-testing routines distributed with version 8.15 of Y. Kida's UBASIC, as released in December 1990. For a description of UBASIC, see [19]. The programs ECMX, MPQSX, and MPQSHD, written by Kida, were run on an IBM PS/2-70 386 PC equipped with a mathematical coprocessor. The most demanding application, the decomposition of the 78-digit number C_{251} into its two prime factors with MPQSHD, required 264 hours.

The completion of the final segment $260 < n \le 300$ seemed too hard for UBASIC and was therefore not attempted, leaving unfinished 16 numbers C_n and 12 numbers W_n. They were finally accomplished with the assistance of Niebuhr and R. P. Brent. Niebuhr factorized 21 of the remaining composites, using his own programs for ECM and for MPQS. The latter was run on an IBM ES/9021-440 mainframe. The largest numbers thus treated were the 77-digit cofactors of C_{261}, W_{264}, W_{267}, and W_{289}, which required about 64 hours of CPU time on the average, and the 79-digit cofactor of C_{297}, which took 81 hours.

The other seven composites had 80, 81, 83, or 86 digits. All these were

attacked and finished with Brent's program MVFAC [4], a vectorized implementation of ECM. For details, see [3] and [17]. The program was run by Brent himself on a Fujitsu VP 2200/10 vector processor at the Computer Sciences Laboratory of the Australian National University, and, more intensively, on an SNI S100/10 (similar to a VP 2100/10) recently installed at the Computation Center of the University of Hamburg. Brent obtained the factorization of W_{277} (83-digit cofactor), and the author finished the remaining six numbers. The most notable factor determined with MVFAC was a 39-digit prime factor of C_{296} found after trying only 750 curves. It is also the largest "penultimate" factor contained in Tables I or II of the Supplement. The factorizations for $n \leq 300$ were completed on August 16, 1992.

Table I of the Supplement includes the factorization of $C_{128} = 2^{135} + 1$ obtained by Le Lasseur and published by Lucas [16] in 1878 (one digit in the largest factor misprinted), as well as Robinson's prime C_{141}. Table II includes the factorization of $W_{64} = 2^{70} - 1$ already contained in [16], that of $W_{128} = 2^{135} - 1$ established by P. Poulet in 1946 (see [14]), and the previously known primes. It also includes the factorizations of W_{100} and W_{256}, which, according to [13], had been completed by Karst and K. R. Isemonger, respectively. It should be noted that for $n = 100, 144, 196, 256$ the factorization of W_n was facilitated by an algebraic decomposition, as was the case historically for C_{128} and W_{128}.

C & W listed all 44 values of $n \leq 1000$ for which no factor of C_n was known and they showed that in every case the smallest factor p had to be > 1000. Precisely these C_n were tested for primality by Robinson to discover that only C_{141} was prime in that range. Apart from this prime, six more of the C_n in question, corresponding to $n = 233, 245, 251, 252, 285, 293$, are in Table I and thus completely factored. In seven cases with $n > 300$ the factorization of C_n could also be established:

$$C_{318} = 10939 \cdot 100429275849522701 \cdot \text{p78},$$
$$C_{436} = 1081501579 \cdot 43008589651 \cdot 3717967975567 \cdot \text{p102},$$
$$C_{579} = 4988803100279081295749323 \cdot \text{p153},$$
$$C_{634} = 2459 \cdot 1085629591 \cdot 2756181749 \cdot 7148901709 \cdot \text{p162},$$
$$C_{753} = 164834525239 \cdot \text{p219},$$
$$C_{921} = 2428711 \cdot 2719027 \cdot 4410839 \cdot \text{p261},$$
$$C_{933} = 197724478669 \cdot \text{p273}.$$

Here pN denotes an N-digit prime. The seven cofactors, like others to be mentioned below, were proved prime by using the procedure APRT-CL (Cohen-Lenstra version of Adleman-Pomerance-Rumely Test), programmed by K. Akiyama, which is included in Kida's UBASIC. The factorization of C_{579} was obtained through MVFAC.

For 27 of the remaining 30 Cullen numbers C_n, we give in Table 1 a prime factor. If by trial division a factor p was found below 10^7, the smallest factor of C_n is tabulated. If, however, a factor $p > 10^7$ is given, it was found by one of Pollard's methods or (for $n = 581, 648, 941$) by Brent's MVFAC and might not be the least. Thus, only three Cullen numbers C_n with $n \leq 1000$ have no known factor, which are for $n = 435, 453, 915$.

TABLE 1. Prime factors p of Cullen numbers $C_n = n \cdot 2^n + 1$

n	p	n	p
333	2423	711	367957
402	1117	713	3079
412	1091	778	33667759
473	365969	816	6451
516	20641	849	7103923
532	18313	869	69508729
533	95257	870	39869
580	2843	899	1633716607
581	2660379251641	900	18176209
587	60744852593	916	247451
588	203793838081	941	771733518497258330461570 8011
609	70038149	942	2598767
648	240383530451966593	953	381287
693	134409623339		

A prime factor $p < 10^7$ does exist for all but 41 of the composite numbers W_n with $n \leq 1000$. Six of these are completely factored since they have $n \leq 300$. The following six with $n > 300$ were factored by finding one factor (in the case of W_{885}, MVFAC determined a composite factor):

$$W_{369} = 7728415141 \cdot p104,$$

$$W_{463} = 141959514756636037 \cdot p125,$$

$$W_{672} = 96279477675861747760 6791593 \cdot p179,$$

$$W_{789} = 73849762727000585999 9837 \cdot p217,$$

$$W_{885} = 8353578155864671 \cdot 84769280380290403 \cdot p237,$$

$$W_{908} = 3057961301 \cdot p267.$$

For 23 of the remaining 29, a factor $p > 10^7$ is presented in Table 2. The factor given for $n = 366$ and the smaller factor of W_{463} were kindly supplied

TABLE 2. Prime factors p of Cullen numbers $W_n = n \cdot 2^n - 1$

n	p	n	p
332	12165323	675	1608969227141
350	5169607633	722	64952161193
366	62497690394803	723	128915821
386	119570203	765	1302342234491 38177
423	3537292571	795	80958347
522	40818521	824	1757762099
541	3241623612714017	866	10885241
564	26768197513	906	16556069
570	48623921	931	15127751
621	16127089673	932	114082154860229
663	60166683064673819	943	405108238890652513
669	73399869877		

by H. Suyama, who found them in 1988. Those for $n = 541$, 663, 765, 932, 943 are due to MVFAC, like the above factors of W_{672} and W_{789}. The numbers W_n without a known factor now occur for $n = 349$, 375, 668, 715, 951, 963.

There is presently a project underway attempting complete factorizations for the whole range $300 < n \leq 1000$. As of the end of February 1994, 147 numbers C_n and 142 numbers W_n had virtually been finished. A machine readable list with all known factors is available from the author, who would welcome any new factors that readers may wish to contribute.

3. DIVISIBILITY PROPERTIES

In pursuance of the original observations made by Cullen and by Cunningham, a glance at the factor table for C_n suggests that a prime p divides both the numbers C_{p-1} and C_{p-2}. A more general statement is in fact true. For a given odd prime p let $n_k = (2^k - k)(p-1) - k$, $k \geq 0$. This includes $n_0 = p - 1$ and $n_1 = p - 2$. The statement is that p divides C_{n_k} for all $k \geq 0$, and it is verified easily. Since $n_k \equiv -2^k \pmod{p}$, we have $n_k \cdot 2^{n_k} \equiv -2^{k+n_k} \equiv -2^{(2^k - k)(p-1)}$ $\pmod{p}$, which by Fermat's theorem is congruent to -1 modulo p. Therefore $C_{n_k} \equiv 0 \pmod{p}$. Obviously, n_k cannot be a multiple of p.

To describe the totality of numbers C_n that are divisible by a given prime p, two remarks are in order. First, the numbers $C_n \bmod p$ are periodic with period ph_p, where $h_p = \exp_p(2)$ is the smallest positive integer h such that $2^h \equiv 1 \pmod{p}$. As a consequence, if p divides C_n for some n, then p also divides C_{n+ph_p}. Secondly, from the definition of n_k it may be deduced that there are exactly h_p numbers n_k which are incongruent modulo ph_p, and no other n with $0 < n < ph_p$ gives a C_n with a prime factor p (cf. [8]). So, the totality of all n such that C_n is divisible by p is obtained as follows. Determine $n'_k = n_k \bmod ph_p$ for $k = 0, 1, \ldots, h_p - 1$, $0 < n'_k < ph_p$. Then, for every k, include $n = n'_k + rph_p$ for all $r \geq 0$.

In particular, the prime $p = 3$ divides all C_n with $n \equiv 1, 2 \pmod{6}$. As a further example, for $p = 11$, $h_p = 10$, we get $n'_k = 10$, 9, 18, 47, 6, 45, 24, 103, 52, 71, and all integers n congruent modulo 110 to any one of these.

A similar description can be given for the set of subscripts n for which W_n is divisible by p. Let $n_k = -(2^k + k)(p-1) - k$ (these are negative integers) for $k \geq 0$, determine $n'_k = n_k \bmod ph_p$ for $k = 0, 1, \ldots, h_p - 1$, $0 < n'_k < ph_p$, and, for every k, include $n = n'_k + rph_p$ for all $r \geq 0$. In this case, $p = 3$ divides all W_n with $n \equiv 4, 5 \pmod{6}$, and $p = 11$, $h_p = 10$ give $n'_k = 100$, 79, 48, 107, 16, 65, 64, 73, 102, 61, and all n congruent modulo 110 to any one of these.

Note that for $n = 65$, 64 we have two consecutive numbers W_n, and actually an infinity of such pairs, which are both divisible by the same prime $p = 11$. For numbers C_n this situation occurs for every prime p, as we have seen. For numbers W_n, however, this is restricted to the case that h_p is an even number. The minimality of h_p then implies that $2^{h_p/2} \equiv -1 \pmod{p}$ and thus also $2^{ph_p/2} \equiv -1 \pmod{p}$. It is now easily seen that $W_n \equiv 0 \pmod{p}$ for $n = ph_p/2 + p - 1$ and for $n = ph_p/2 + p - 2$.

Other divisibility rules are also contained in the given general description.

For instance, p divides $C_{(p+1)/2}$ and $W_{(3p-1)/2}$, or it divides $C_{(3p-1)/2}$ and $W_{(p+1)/2}$, according as the Jacobi symbol $(2/p)$ is -1 or $+1$. Again, this can be verified immediately. For more details, see [7] and [8].

In spite of the apparent similarity in the description of small factors for both sequences $\{C_n\}$ and $\{W_n\}$, considerably more composites are obtained for the original Cullen numbers C_n. This is primarily due to the fact that every prime $p > 3$ immediately gives four consecutive composite numbers C_n. In addition to C_{p-1} and C_{p-2}, always two neighboring numbers of the sequence are divisible by 3. If p is of the form $p = 6k + 1$, then 3 divides C_p and C_{p+1}, and in the case of $p = 6k - 1$, the prime 3 divides C_{p-3} and C_{p-4}. In the special situation of a twin prime pair p, $q = 6k \pm 1$, together a string of eight consecutive composites C_n occurs.

To give an indication of how the notable difference in the number of primes of the forms C_n and W_n comes about, let us first consider all values of $n \leq 110$ such that C_n has no prime factor $p \leq n + 2$. Here we get $n = 33$, 53, 75, where $n = 75$ can be eliminated through the prime factor $p = 2n - 1 = 149$. In the case of W_n, however, 32 values of n remain in the first step, of which eight are eliminated by a factor $p = 2n - 1$. The remainder of 24 includes, of course, the six values of $n \leq 110$ giving rise to the primes W_n determined by Riesel.

4. CULLEN PRIMES

In our search for new primes, we tested C_n for $n \leq 20000$ in 1984 and continued (without success) to $n \leq 30000$ in 1987. Similarly, W_n was tested up to $n \leq 15000$ in 1984, and to $n \leq 20000$ in 1987. In this case two more primes were found for $n = 15822$ and $n = 18885$. All other new Cullen primes were thus discovered in 1984. The methods used for proving primality are found in [5]. For $1000 < n \leq 20000$, a total of 632 numbers C_n and 1203 numbers W_n had actually to be tested after the sieving procedure. In addition, 321 numbers C_n with $20000 < n \leq 30000$ remained without known factor. We also considered the possibility of $n \cdot 2^n \pm 1$ forming a pair of twin primes. Such "Cullen twins" may indeed exist for some large n. But we showed (by testing individual numbers C_n) that this n must exceed $n = 41528$.

The prime $C_{18496} = 17^2 \cdot 2^{18502} + 1 = (17 \cdot 2^{9251})^2 + 1$ might be of some interest in relation to Conjecture E of Hardy and Littlewood [10], which states that an infinity of primes $N^2 + 1$ should exist (actually, the conjecture was given in a quantitative asymptotic form). At the time of its discovery, our prime seemed to be the largest one known of that expression. But now we know five larger ones, discovered by H. Dubner (unpublished), which are of the form $b^{2^m} + 1$, with b even. The largest of these primes is $200944^{2^{11}} + 1$, which has 10861 digits and was found in September, 1992. Incidentally, Dubner [9] has also considered a generalization of Cullen numbers given by $n \cdot b^n + 1$ for $b > 2$.

Regarding the primes W_n listed for $n > 110$, it appeared that four of them had already been known for some time. As a matter of fact, Williams and Zarnke [25] had found W_{115} to be prime, Jönsson [12] had found $181 \cdot 2^{363} - 1 = W_{362}$, Riesel [21] had given $3 \cdot 2^{391} - 1 = W_{384}$, and W_{512} turned out to be the Mersenne prime $M_{521} = 2^{521} - 1$ discovered by Robinson (see [15]).

A first result concerning the distribution at large of Cullen primes C_n was obtained by Hooley [11]. By applying sieve methods, he showed that the natural

density of positive integers $n \leq x$ for which C_n is a prime is of the order $o(x)$ for $x \to \infty$. In that sense, almost all Cullen numbers are composite. Suyama (communicated personally) has reworked Hooleys proof to show that it is applicable to any sequence of numbers $n \cdot 2^{n+a} + b$, where a, b are arbitrarily fixed integers. In particular, Hooley's result also follows for numbers W_n.

In the context of a search for prime factors of Fermat numbers our particular interest in Cullen numbers C_n emanated from the remark of C & W, supported by an explicit argument, that primes of the form C_n seemed likely as possible divisors of Fermat numbers. However, the four new primes C_n proved not to divide any of these.

Instead, there is some evidence for the likelihood that a prime W_n divides a Mersenne number M_p. We noticed that $W_2 = M_3$ and $W_{512} = M_{521}$, and that W_3 divides M_{11}, W_6 divides M_{191}, and W_{123} divides M_p for $p = 123 \cdot 2^{122} - 1$, a 39-digit prime. In all three cases where W_n properly divides M_p, we have $p = (W_n-1)/2 = n \cdot 2^{n-1}-1$, that is, p and the divisor $W_n = 2p+1$ are both primes.

To establish in general whether W_n divides any M_p or not, the complete factorization of $(W_n - 1)/2$, if available, can be used. In fact, a divisor W_n of M_p must also be of the form $2kp + 1$. Equivalently, $(W_n - 1)/2 = kp$, hence the particular subscript p should divide $(W_n - 1)/2$. So, for all different prime factors p of this number, one would have to check if $M_p \bmod W_n = 0$. But in practice a much easier test suffices, as Suyama has indicated in a personal communication. To see whether a prime q divides some Mersenne number or not, only an "evenly factored portion" of $q - 1$ and its cofactor need to be used.

Let q be a prime and $q - 1 = 2^{\alpha_0} \cdot p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots \cdot p_r^{\alpha_r} \cdot C$, where p_j are odd primes and C is composite. If $2^{p_j} \not\equiv 1 \pmod q$ for $1 \leq j \leq r$ and $2^C \not\equiv 1 \pmod q$, then q cannot be a factor of any Mersenne number. In the alternative case that $2^C \equiv 1 \pmod q$, q may be a factor of M_p, where p is some prime factor of C. Indeed, if for an unknown factor p of C we had $q \mid M_p$, that is $2^p \equiv 1 \pmod q$, then necessarily $2^C \equiv 1 \pmod q$.

For all primes $q = W_n$ with $30 \leq n \leq 822$ (except for the otherwise settled $n = 123$) this test was conclusive by using only one odd factor p_1 of W_n-1. In most cases, a very small p_1 was readily found, while for $n = 249$ and $n = 384$ the factors $p_1 = 708211533214392631$ resp. $p_1 = 12694590781$ determined by Suyama had to be used. It has also been checked that the Cullen number W_{5312} does not divide any Mersenne number, by calculating $2^C \bmod W_{5312}$ for $C = (W_{5312}-1)/(2 \cdot 3 \cdot 5^2)$. But here we had the interesting case that, nevertheless, $2^{C'} \equiv 1 \pmod{W_{5312}}$ for $C' = (W_{5312} - 1)/(2 \cdot 3)$. For W_{7755}, the test could not be carried out, because no factor of the composite number $(W_{7755} - 1)/2$ was found. Recently, Dubner has kindly tested the numbers W_n for $n = 9531$, 12379, 15822, 18885, without detecting a divisibility. In the case of $n = 15822$ he obtained $2^C \equiv 1 \pmod{W_{15822}}$, where $C = (W_{15822} - 1)/(2 \cdot 7)$ and no further factor of this number was known.

Of course, W_n may also be identical to a Mersenne number without being a prime. Generally, W_n is a Mersenne number whenever $n = 2^m$ and $m + 2^m$ is prime. This is the case for $m = 1$: $W_2 = M_3 = 7$; $m = 3$: $W_8 = M_{11} = 23 \cdot 89$; $m = 5$: $W_{32} = M_{37} = 223 \cdot 616318177$; $m = 9$: $W_{512} = M_{521}$; and also for $m = 15, 39, 75, 81, 89, 317, 701, 735$ and no other $m \leq 1310$.

5. BIOGRAPHICAL NOTE

When we searched the literature for references to Cullen numbers, we were intrigued by the fact that almost nothing could be found about Cullen as a person. Even mathematical or encyclopaedic dictionaries usually giving such information failed to disclose Cullen's complete first name or to mention the year of his birth or his death. Only the abbreviation "S. J." placed after his name in a footnote to [8] suggested that he was a Jesuit priest. That hint finally enabled us to locate the most appropriate and authoritative source for this matter, namely, the Department of Historiography and Archives of the English Province of the Society of Jesus. The archivist of that institution, Father T. G. Holt, was kind enough to provide the following biographical data, which we are pleased to present to the reader in unabridged form.

Father James Cullen, S. J., was born on April 19th 1867 at Drogheda in Ireland. At first he was educated privately, then by the Christian Brothers. Next he went to Trinity College, Dublin, to study pure and applied Mathematics. Afterwards he was in business for a while, but after a short period at an apostolic school in Ireland to learn Latin he entered the noviceship of the English Jesuits at Manresa House, Roehampton, London, as he had decided to become a priest. From 1892 till 1895 he was at Manresa in the noviceship and then for a year's study. From 1895 till 1897 he studied philosophy at the Jesuit house for philosophical studies at St Mary's Hall, Stonyhurst in Lancashire. From 1897 till 1901 he studied theology at the theologate of the English Jesuits at St Beuno's College in North Wales. He was ordained priest at St Beuno's College on July 31st 1901. In 1902 and 1903 he taught Mathematics to young Jesuits who had finished their noviceship at Manresa House. In 1905 he taught Mathematics at the Jesuit boarding school Mount St Mary's College in Derbyshire. It is said that he found difficulty in teaching. In 1906 he was sent to Stonyhurst College, also a boarding school, as accountant and later manager of the College farm and estate. Meanwhile he kept in touch with leading mathematicians and contributed articles to *Nature*, the *Mathematical Gazette* and the *Messenger of Mathematics*. In 1921 he left Stonyhurst and was appointed to supervise accounts of other English Jesuit houses. He died on December 7th 1933.

Finally, the author is greatly indebted to Father Bruno Brinkman of Heythrop College, University of London, for helping to establish contact with Father Geoffrey Holt, who compiled and wrote the enlightening biographical note of §5.

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Supplement to

NEW CULLEN PRIMES

WILFRID KELLER AND WOLFGANG NIEBUHR

TABLE I

Factorization of Cullen numbers $C_n = n \cdot 2^n + 1$

n	$C_n = n \cdot 2^n + 1$
1	3
2	3^2
3	5^2
4	$5 \cdot 13$
5	$7 \cdot 23$
6	$5 \cdot 7 \cdot 11$
7	$3 \cdot 13 \cdot 23$
8	$3 \cdot 683$
9	$11 \cdot 419$
10	$7^2 \cdot 11 \cdot 19$
11	$13 \cdot 1733$
12	$13 \cdot 19 \cdot 199$
13	$3^2 \cdot 11833$
14	$3 \cdot 157 \cdot 487$
15	$17 \cdot 29 \cdot 997$
16	$17 \cdot 61681$
17	$5^2 \cdot 19 \cdot 4691$
18	$11 \cdot 19 \cdot 107 \cdot 211$
19	$3 \cdot 17 \cdot 37 \cdot 5279$
20	$3^3 \cdot 103 \cdot 7541$
21	$23 \cdot 1914791$
22	$13 \cdot 23 \cdot 43 \cdot 7177$
23	$5 \cdot 151 \cdot 421 \cdot 607$
24	$5 \cdot 11 \cdot 1399 \cdot 5233$
25	$3 \cdot 17 \cdot 2459 \cdot 6689$
26	$3 \cdot 5 \cdot 7^2 \cdot 2373919$
27	$7 \cdot 29 \cdot 53 \cdot 336823$
28	$29^2 \cdot 8937209$
29	$31 \cdot 83 \cdot 6051013$
30	$17^2 \cdot 31 \cdot 59 \cdot 149 \cdot 409$
31	$3^2 \cdot 7 \cdot 61 \cdot 1483 \cdot 11681$
32	$3 \cdot 1777 \cdot 25781083$
33	$47 \cdot 6031230671$
34	$19 \cdot 23 \cdot 67 \cdot 1061 \cdot 18803$
35	$37 \cdot 32502455213$
36	$37^2 \cdot 1933 \cdot 934861$
37	$3 \cdot 5 \cdot 339016085231$
38	$3^2 \cdot 20879 \cdot 55586743$
39	$41 \cdot 3433 \cdot 152326961$
40	$41 \cdot 131611 \cdot 8150491$

n	$C_n = n \cdot 2^n + 1$
41	$13 \cdot 43 \cdot 1291 \cdot 124932557$
42	$23 \cdot 43 \cdot 83 \cdot 2250270487$
43	$3 \cdot 5 \cdot 163 \cdot 2633 \cdot 58752797$
44	$3 \cdot 5 \cdot 28909 \cdot 1785040943$
45	$11 \cdot 47 \cdot 2437 \cdot 1256655529$
46	$5 \cdot 31 \cdot 47 \cdot 139297 \cdot 3189821$
47	$7 \cdot 11 \cdot 43 \cdot 3593 \cdot 556021079$
48	$7 \cdot 379 \cdot 997 \cdot 5107973329$
49	$3^2 \cdot 3405499771822769$
50	$3 \cdot 163 \cdot 211 \cdot 563 \cdot 857 \cdot 1130809$
51	$53 \cdot 101 \cdot 21453725107033$
52	$7 \cdot 11 \cdot 17 \cdot 53 \cdot 1433 \cdot 2355599273$
53	$5591 \cdot 16747 \cdot 5098461301$
54	$29 \cdot 107 \cdot 7187 \cdot 110503 \cdot 394739$
55	$3 \cdot 109 \cdot 2185637 \cdot 2772596459$
56	$3^2 \cdot 1021 \cdot 439136496476653$
57	$5 \cdot 13 \cdot 59 \cdot 10740337 \cdot 199434979$
58	$31 \cdot 59 \cdot 101 \cdot 7002713 \cdot 12923089$
59	$37 \cdot 43 \cdot 61 \cdot 350446511482643$
60	$61 \cdot 2267 \cdot 1529093 \cdot 327141571$
61	$3 \cdot 19 \cdot 37 \cdot 41 \cdot 21557 \cdot 110359 \cdot 683759$
62	$3 \cdot 5104570123 \cdot 18671146721$
63	$5 \cdot 313993 \cdot 37011808436117$
64	$5^2 \cdot 29 \cdot 41 \cdot 113 \cdot 7416361 \cdot 47392381$
65	$67 \cdot 149 \cdot 2357 \cdot 367019 \cdot 277685929$
66	$5^3 \cdot 13 \cdot 67 \cdot 107 \cdot 131 \cdot 8353 \cdot 382030403$
67	$3^2 \cdot 413009 \cdot 7451263 \cdot 356987159$
68	$3 \cdot 7 \cdot 4841894619 \cdot 19738492 0991$
69	$7 \cdot 71 \cdot 193 \cdot 135979 \cdot 3122721481331$
70	$37 \cdot 47 \cdot 71 \cdot 139 \cdot 4815318361753391$
71	$11^2 \cdot 73 \cdot 173 \cdot 5113 \cdot 2145645254477$
72	$41 \cdot 73 \cdot 6011 \cdot 40163 \cdot 470557385537$
73	$3 \cdot 7 \cdot 89 \cdot 277 \cdot 13317528541 01535409$
74	$3^3 \cdot 130860199 \cdot 395621657653429$
75	$149 \cdot 367 \cdot 298013 \cdot 663557 \cdot 262026467$
76	$47^2 \cdot 79939 \cdot 27729643 \cdot 1172720609$
77	$5 \cdot 17 \cdot 79 \cdot 17328236803 \cdot 858236649523$
78	$41 \cdot 79 \cdot 78094529 \cdot 93197166057943$
79	$3 \cdot 59 \cdot 157 \cdot 3533 \cdot 3^2 930529 \cdot 1317026956 1$
80	$3 \cdot 67 \cdot 1960787 \cdot 2453393615236 40963$
81	$83 \cdot 13633 \cdot 984666689 \cdot 1757745 09403$
82	$43 \cdot 83 \cdot 163 \cdot 6816153221823204 40507$
83	$5 \cdot 23 \cdot 15373 \cdot 2205616169 \cdot 205864436623$
84	$5 \cdot 2971 \cdot 22637 \cdot 4831787703823228467$
85	$3^2 \cdot 15607 \cdot 24763 \cdot 132157 \cdot 144163 \cdot 49620299$

n	$C_n = n \cdot 2^n + 1$
86	$3 \cdot 5 \cdot 3989 \cdot 6687383 \cdot 69261971 \cdot 24008711$
87	$19 \cdot 89 \cdot 173 \cdot 22853 \cdot 2013705447963047603$
88	$89 \cdot 2137 \cdot 13607459 \cdot 1052327033714 3867$
89	$7 \cdot 79867 \cdot 372067 \cdot 264833583219205703$
90	$7 \cdot 47 \cdot 179 \cdot 1891878275724375961367971$
91	$3 \cdot 181 \cdot 9157 \cdot 74609 \cdot 607332797678367491$
92	$3^2 \cdot 94957781 \cdot 5330578725027984 81877$
93	$71 \cdot 79 \cdot 887 \cdot 127703 \cdot 1449647415604834793$
94	$7 \cdot 62869 \cdot 4230706069276050048396059$
95	$29 \cdot 97 \cdot 769973 \cdot 1737512608551063059489$
96	$97 \cdot 57735121 \cdot 1358122681230804234961$
97	$3 \cdot 5 \cdot 13 \cdot 283583 \cdot 1806689 \cdot 15384490196056309$
98	$3 \cdot 929 \cdot 260171 \cdot 4283214214886904432769$
99	$13 \cdot 101 \cdot 197 \cdot 35461 \cdot 684105110146470338953$
100	$101 \cdot 12006189421 \cdot 10453771468831515648 1$
101	$103 \cdot 2486072050930324184692625 96951$
102	$53 \cdot 103 \cdot 22619 \cdot 4188641917664387362988929$
103	$3^3 \cdot 5^2 \cdot 11 \cdot 1951 \cdot 2333 \cdot 57413 \cdot 538329365932625711$
104	$3 \cdot 5 \cdot 140624706585318248272700942249711$
105	$107 \cdot 39806598287540661579709558598723$
106	$5 \cdot 17^2 \cdot 71^2 \cdot 107 \cdot 211 \cdot 4157 \cdot 12579245807344233937$
107	$23 \cdot 109 \cdot 9967 \cdot 440854639 \cdot 157608309709749967$
108	$109 \cdot 20481577 \cdot 6861664297 \cdot 2287936306776469$
109	$3^2 \cdot 73 \cdot 61428359 \cdot 3417532243 \cdot 1538758185211703$
110	$3^2 \cdot 7 \cdot 3137736253 \cdot 7223292860541388297302 19$
111	$7^3 \cdot 113 \cdot 647 \cdot 659 \cdot 303139 \cdot 19064027 \cdot 301406215899$
112	$53 \cdot 107 \cdot 113 \cdot 51861119 \cdot 174983686965081854 45369$
113	$241 \cdot 17409703 \cdot 27967880066027439386153 8439$
114	$59 \cdot 227 \cdot 5663103129009631 \cdot 31217061335588719$
115	$3 \cdot 7 \cdot 229 \cdot 1361 \cdot 10909 \cdot 6040974323677 \cdot 1107496870 9153$
116	$3 \cdot 11 \cdot 19 \cdot 191 \cdot 2377 \cdot 8448443 \cdot 160686473 \cdot 24937413111887$
117	$5^2 \cdot 881 \cdot 8669 \cdot 17382493 \cdot 5857312177073603180401$
118	$61 \cdot 79 \cdot 1314346888817839 \cdot 619091110207218773$
119	$11 \cdot 19 \cdot 3463 \cdot 11329 \cdot 96455298848471130904167751 1$
120	$11 \cdot 983 \cdot 11789 \cdot 6974348643991 \cdot 179412997157803583$
121	$3^2 \cdot 19763 \cdot 50971 \cdot 42224697299 \cdot 84029110559486051$
122	$3 \cdot 71918637905 9 \cdot 3006468053486426691767 42057$
123	$5 \cdot 229 \cdot 821 \cdot 1146542269 9 \cdot 1213544938157707554 59727$
124	$5 \cdot 29931493 \cdot 176214954839 5458314332353602249$
125	$23 \cdot 127 \cdot 3023 \cdot 60212930843445299962811 9625733847$
126	$5 \cdot 127 \cdot 251 \cdot 1627 \cdot 4133471990403788192971 9526509387$
127	$3 \cdot 433 \cdot 16634280446096683933736941933731 54843$
128	$3^4 \cdot 11 \cdot 19 \cdot 331 \cdot 811 \cdot 15121 \cdot 87211 \cdot 18837001 \cdot 385838642647891$
129	$131 \cdot 683636977 \cdot 980307465963544964235926264027$
130	$67 \cdot 131 \cdot 93056049341 \cdot 21664670065515035509110253$

n	$C_n = n \cdot 2^n + 1$
131	$7 \cdot 50945131504734787101373737983697847726515127$
132	$7 \cdot 80024014833 \cdot 5019929924009 \cdot 2556606837555877927687$
133	$3 \cdot 29 \cdot 89 \cdot 18703884420012287356967741610783442439359$
134	$3 \cdot 11 \cdot 5801 \cdot 27997 \cdot 5444975308596630004327093119223757$
135	$23 \cdot 137 \cdot 269 \cdot 1817394275513994941 \cdot 381709829921016209$
136	$7 \cdot 137 \cdot 642183982348149971 \cdot 192371283457400055007 73$
137	$5 \cdot 29 \cdot 139 \cdot 1662897 \cdot 7122526232262432009988578545236219$
138	$71 \cdot 139 \cdot 405497 \cdot 1201593840849533737095030928668766 1$
139	$3^2 \cdot 19 \cdot 277 \cdot 1979 \cdot 325583013 \cdot 5451651523 7 \cdot 581759112652171901$
140	$3 \cdot 13 \cdot 12251 \cdot 411469 \cdot 9925540382009565345892001102626 01$
141	$393050634124102232863956703455542737154290483 3$
142	$73 \cdot 283 \cdot 523 \cdot 2621 \cdot 9385153 \cdot 2978710979175931428632455909$
143	$5 \cdot 9623 \cdot 4143608069 \cdot 79977211370662846933537861828927$
144	$5 \cdot 53 \cdot 1373 \cdot 349801 \cdot 1368841 \cdot 5345933 \cdot 27481361353 \cdot 125467039561$
145	$3 \cdot 97 \cdot 277 \cdot 499 \cdot 2393 \cdot 150091 \cdot 1431929 \cdot 1335527932327 \cdot 2306728188553$
146	$3^2 \cdot 5 \cdot 13 \cdot 2600807606089 \cdot 2594714440129 1 \cdot 329989714783078582883$
147	$31 \cdot 149 \cdot 293 \cdot 19378090652036144875295910472815908292551$
148	$149 \cdot 37447769501363 \cdot 23985387089791 \cdot 39427133267640296 17$
149	$19 \cdot 151 \cdot 42805751 \cdot 15459214075 1 \cdot 1917354724069 \cdot 2921010204449$
150	$151 \cdot 16091 \cdot 199447 \cdot 1375019 \cdot 32128793631982551221065580 57577$
151	$3 \cdot 17^2 \cdot 47 \cdot 1259 \cdot 639827737 \cdot 1635229069 \cdot 5835105553 \cdot 1376116144571$
152	$3 \cdot 7 \cdot 17 \cdot 67 \cdot 109 \cdot 21818809 \cdot 152546543509087132564981806454918 7$
153	$7 \cdot 19 \cdot 1307 \cdot 132607 \cdot 730690897 \cdot 453636682637 \cdot 2286363224856032 29$
154	$47 \cdot 79 \cdot 103 \cdot 307 \cdot 701 \cdot 22483 \cdot 5171164022290 1 \cdot 36751825179839702143$
155	$11 \cdot 157 \cdot 20063 \cdot 36299 \cdot 1034291 \cdot 14914093 \cdot 12251789447 \cdot 1749989995597$
156	$47 \cdot 157 \cdot 93460637836 9 \cdot 2066225280962497247971207030159067$
157	$3^3 \cdot 5 \cdot 7^2 \cdot 11 \cdot 139242096273379 \cdot 2830843655976733930778079912663$
158	$3 \cdot 19243104891523555089668185029743105975880311181585 1$
159	$317 \cdot 3517 \cdot 416107949358495440739 \cdot 250152176243212999440083$
160	$13 \cdot 6397 \cdot 8430041 \cdot 199458852483019 \cdot 16723119620808052397209728 19$
161	$17 \cdot 163 \cdot 2907271 \cdot 261594464257 3 \cdot 373463966441 9 \cdot 597937623646541 9$
162	$11 \cdot 83 \cdot 113 \cdot 163 \cdot 78524336642930939 \cdot 71718832838350779056816983 53$
163	$3 \cdot 5 \cdot 13 \cdot 139 \cdot 211 \cdot 52392423893769317 \cdot 63602844580695545403763318 3$
164	$3^2 \cdot 5^2 \cdot 73 \cdot 5821 \cdot 1050326180531 \cdot 3818879619342221653856268049862 3$
165	$167 \cdot 46207955958725553342008718064323199663373196568343$
166	$5^2 \cdot 17 \cdot 167 \cdot 269 \cdot 331 \cdot 1229 \cdot 14703383 \cdot 7514212799 \cdot 1809460264127522417287 7$
167	$13 \cdot 59 \cdot 101 \cdot 227 \cdot 2741 \cdot 6623272651 \cdot 102105933399 \cdot 113553555055259 \cdot 4497801465241949 7$
168	$13 \cdot 41 \cdot 263 \cdot 1193 \cdot 5351046957296294967143 \cdot 7024030521036567472342 9$
169	$3 \cdot 113 \cdot 2371 \cdot 11771611 \cdot 41616310457 \cdot 1222206583179 3 \cdot 26280319520686669 31$
170	$3 \cdot 97 \cdot 163 \cdot 95289941 \cdot 110210593399 \cdot 11355255505259 \cdot 4497801465241949 7$
171	$173 \cdot 135623 \cdot 2359451 \cdot 92455693844406120912442583813464913589 21$
172	$173 \cdot 489670204593126715298666683 \cdot 12154522980200506074974940 7$
173	$7^3 \cdot 67 \cdot 113 \cdot 855316291913 9 \cdot 932526629073429149874437530546907 51$
174	$7 \cdot 43 \cdot 89 \cdot 347 \cdot 379 \cdot 1182614792051019187653215427710083905181807 381$
175	$3^2 \cdot 349 \cdot 6106365265705407 \cdot 436954860034446908167985766266588164523$

n	$C_n = n \cdot 2^n + 1$
176	$3 \cdot 453233666209 \cdot 1239791025125899607113653582980660065227395 31$
177	$5 \cdot 29 \cdot 37 \cdot 179 \cdot 229 \cdot 58601 \cdot 263099021046281902327762292292994426967314 03$
178	$7 \cdot 13 \cdot 23 \cdot 179 \cdot 10409 \cdot 71817702441006255786659 \cdot 243688224134989561205669$
179	$181 \cdot 70019 \cdot 3431312315027799523547 3 \cdot 31504423983662126966196119 9$
180	$181 \cdot 45307 \cdot 336337820086813508684423918112864104768776725071943$
181	$3 \cdot 11 \cdot 67 \cdot 4698961 \cdot 1452194066821 \cdot 3676990847936189144165174329002614 3$
182	$3^3 \cdot 179 \cdot 353 \cdot 16607 \cdot 3351244652513 \cdot 266236575405447547779061912028697 31$
183	$5 \cdot 1687103736767 53 \cdot 26596745925372916948913023303749601907846 1$
184	$5 \cdot 19 \cdot 31 \cdot 84633443 \cdot 18101297569505073941351624729091167875711436 27$
185	$29 \cdot 31^2 \cdot 3623 \cdot 1196519931107497 \cdot 81054482601806621 \cdot 926476123489354759$
186	$5 \cdot 3691 \cdot 11454196829 \cdot 582890357931799323431 \cdot 148056191977189331673161469$
187	$3 \cdot 373 \cdot 1601 \cdot 701791 \cdot 75585331 \cdot 19083591564681 73 \cdot 2022664958314506886434 1$
188	$3 \cdot 17 \cdot 53 \cdot 61 \cdot 79 \cdot 49547 \cdot 11428170164954600003932035575707651653421531955 1$
189	$191 \cdot 39937 \cdot 7318601 \cdot 850076641 \cdot 6393623158493 \cdot 488752498681295030083539$
190	$97 \cdot 127 \cdot 191 \cdot 379 \cdot 3343526307580267530521277808589038492471198552571 71$
191	$193 \cdot 3106027024504808357234807197737416722164504919498201864 33$
192	$193 \cdot 11883 \cdot 994666835144756887 \cdot 5515294579803480236108947490874696 1$
193	$3^2 \cdot 3578059 \cdot 95891935319 \cdot 78464711343839266226176034421433809841912 13$
194	$3 \cdot 7 \cdot 239 \cdot 1579 \cdot 3440291 \cdot 258428384479 \cdot p36$
195	$7 \cdot 197 \cdot 389 \cdot 144271 \cdot 577503957059 \cdot p39$
196	$197 \cdot 1306853 \cdot 764614018537780340667688440227486383694030434193607 77$
197	$5 \cdot 13 \cdot 41 \cdot 97 \cdot 109 \cdot 12377333 \cdot 22037693 \cdot p40$
198	$101 \cdot 199 \cdot 395758163047017352278108978411301800213189550705025024103 80587$
199	$3 \cdot 7 \cdot 19 \cdot 397 \cdot p58$
200	$3^2 \cdot 89 \cdot 39559890596688394063636801 \cdot 1014241859111806744495284478673760 1$
201	$31 \cdot 181648470879949 \cdot 15223488831747250 7 \cdot 75358595060514022505942702318 41$
202	$103 \cdot 3368839219 \cdot 374190695691480412126386996809351225556348453920263 7$
203	$5^3 \cdot 73 \cdot 89 \cdot 331 \cdot 9708100959919126819787692624900260511576321906856757 11$
204	$5 \cdot 2492495026574897626245441249 \cdot 42086710083131525791428591843413799 7$
205	$3 \cdot 97 \cdot 137 \cdot 114013 \cdot 102402353 \cdot p47$
206	$3 \cdot 5 \cdot 7583 \cdot 17987 \cdot 5579464314665490724941619 \cdot p30$
207	$43 \cdot 709 \cdot 51871 \cdot 6700779177 \cdot 2698140802631396 09 \cdot p30$
208	$67^2 \cdot 4660549 \cdot 4183447538554517 \cdot p40$
209	$211 \cdot 64109 \cdot 155558938512817 \cdot 177957745253673881040940 9 \cdot p24$
210	$107 \cdot 211 \cdot 419 \cdot 701 \cdot 14096035847 \cdot 365134180927300543727 \cdot p26$
211	$3^4 \cdot 421 \cdot 521 \cdot 86499277 \cdot 6204442993066576799 \cdot p31$
212	$3 \cdot 19 \cdot 26479201931116897 \cdot 6745433348420303130802 9 \cdot p26$
213	$11 \cdot 13 \cdot 17 \cdot 31 \cdot 688787 \cdot 7177381 \cdot 190198081 \cdot p41$
214	$109 \cdot 233 \cdot 695889188243972 9 \cdot 24352829561682522854453 \cdot p25$
215	$7 \cdot 589967429 \cdot 1163966779 \cdot 22544929787 \cdot p39$
216	$7 \cdot 937 \cdot 7873 \cdot 212209 \cdot 228443 \cdot 89111069 \cdot 436988286291 1 \cdot p40$
217	$3 \cdot 5^4 \cdot 1399 \cdot 146056139 \cdot p54$
218	$3^2 \cdot 277 \cdot 347 \cdot 733 \cdot 839 \cdot 11964318119 9 \cdot p46$
219	$587 \cdot 129517 \cdot 7811923380407761 \cdot p45$
220	$7 \cdot 199 \cdot 18103586337773 \cdot 5298515667 9853 \cdot p40$

$C_n = n \cdot 2^n + 1$

n	$C_n = n \cdot 2^n + 1$
266	$3 \cdot 5^2 \cdot 349 \cdot 461 \cdot 25981 \cdot 34429 \cdot 38736789578839475898958001 \cdot p39$
267	$11 \cdot 37 \cdot 269 \cdot 1780361920474953713 \cdot p60$
268	$157 \cdot 269 \cdot 1486932233537747 \cdot p64$
269	$101 \cdot 271 \cdot 733 \cdot 88328239 \cdot 264727681 \cdot 334476454696144844621 \cdot p41$
270	$137 \cdot 271 \cdot 70831729 \cdot 56793431292459 \cdot 512697534125707566113 \cdot p36$
271	$3 \cdot 29 \cdot 541 \cdot 1300967 \cdot 208725859 \cdot 63588827359 \cdot 322811751482293647881 \cdot p34$
272	$3^2 \cdot 11 \cdot 53 \cdot 9561577711705862764264581308221 \cdot p50$
273	$19 \cdot 37 \cdot 148139 \cdot 135718328880164683 \cdot 7168668256270117427 \cdot p41$
274	$23 \cdot 139 \cdot 547 \cdot 6569 \cdot 1897367 \cdot 19799729220030924786873035843 \cdot p41$
275	$23 \cdot 277 \cdot p82$
276	$277 \cdot 14625971419341908904640781 \cdot p58$
277	$3 \cdot 5 \cdot 83 \cdot 359 \cdot 461 \cdot 617 \cdot 31033 \cdot 48991 \cdot p66$
278	$3 \cdot 7 \cdot 89 \cdot 1471 \cdot 34253 \cdot 381859 \cdot 4352019290125240214222680479 \cdot p42$
279	$7 \cdot 281 \cdot 557 \cdot 25261 \cdot p76$
280	$281 \cdot 61543 \cdot p80$
281	$283 \cdot 3564719 \cdot 236025301 \cdot 72774196049 \cdot 2409022417178297019319073 \cdot p34$
282	$283 \cdot 563 \cdot 15937 \cdot 81457 \cdot 216607 \cdot 10552853 \cdot 47082195578299225313557321 \cdot p35$
283	$3^2 \cdot 5 \cdot 7 \cdot 4007741 \cdot 19922095489 \cdot 23012605391 \cdot p58$
284	$3 \cdot 5 \cdot 82074069805086682207 \cdot p68$
285	$30904835831 \cdot 10504283644431287 \cdot 3177283904952691222562463378883 \cdot p32$
286	$5 \cdot 191 \cdot 571 \cdot 1600177320221252531594715923 \cdot p56$
287	$17 \cdot 23 \cdot 59 \cdot 61 \cdot 67 \cdot 653 \cdot 8527 \cdot 337045967 \cdot 160842582255424486737453797 \cdot p38$
288	$17 \cdot 379 \cdot 28599041 \cdot 51956280102547 \cdot p65$
289	$3 \cdot 41 \cdot 193 \cdot 937 \cdot 1697 \cdot 128473 \cdot 134683499681615123465185200459 \cdot p44$
290	$3^4 \cdot 58687 \cdot 598703908892041857497 \cdot 40898509318467427355331985863 \cdot p34$
291	$11 \cdot 17 \cdot 293 \cdot 830873 \cdot 1348760417 \cdot 174147908726094291201341 \cdot p48$
292	$293 \cdot 7867 \cdot 24810223 \cdot 150104039 \cdot 1144094201891 \cdot 349919902076896943293 \cdot p36$
293	$12010369901 \cdot 68407661168143771308920443 \cdot p55$
294	$79 \cdot 149 \cdot 587 \cdot 1277 \cdot 772953991446500226601 \cdot p62$
295	$3 \cdot 23 \cdot 541 \cdot 122507681 \cdot 108214271237041 \cdot p65$
296	$3 \cdot 13 \cdot 6254661727 \cdot 203286749356260617944192803394680763543 \cdot p42$
297	$5 \cdot 17 \cdot 61 \cdot 254289897 \cdot 76488038596885516779524537964477899 \cdot p44$
298	$151 \cdot 199 \cdot 691 \cdot 101691391 \cdot 220187193731253482966627 \cdot p54$
299	$7 \cdot 519037 \cdot 3190932476819 \cdot 4464189318176161338907 \cdot p52$
300	$7 \cdot 157 \cdot 4220221272271 \cdot p78$

$C_n = n \cdot 2^n + 1$

n	$C_n = n \cdot 2^n + 1$
221	$223 \cdot 313553 \cdot 30828930277 \cdot p51$
222	$13 \cdot 113 \cdot 223 \cdot 443 \cdot 344171 \cdot p56$
223	$3 \cdot 5 \cdot 171428753575820291 \cdot p52$
224	$3 \cdot 5 \cdot 131 \cdot 5250967 \cdot 186338701 \cdot 1948401751 \cdot 36355543037 \cdot p31$
225	$227 \cdot 761 \cdot 4241 \cdot 1072961 \cdot 113814671468528690819 \cdot p36$
226	$5 \cdot 11 \cdot 151 \cdot 191 \cdot 227 \cdot 9917196447643013 \cdot 73284847859761399627 \cdot p26$
227	$47 \cdot 229 \cdot 6209419 \cdot 62128127 \cdot p53$
228	$73 \cdot 229 \cdot 4789 \cdot 4986353 \cdot p57$
229	$3^2 \cdot 11^2 \cdot 301622720557 \cdot p57$
230	$3 \cdot 11 \cdot p71$
231	$131 \cdot 233 \cdot 461 \cdot 534923 \cdot p60$
232	$61 \cdot 233 \cdot 1619 \cdot 108670440655993 \cdot p52$
233	$17389 \cdot 2928794234257417 \cdot 269609587388273533 \cdot p36$
234	$467 \cdot p71$
235	$3 \cdot 4099 \cdot 1556886580277184344040121 \cdot p45$
236	$3^3 \cdot 7 \cdot 29 \cdot 17299 \cdot 321599692831 \cdot 251697875255953361345641 \cdot p31$
237	$5 \cdot 7 \cdot 239 \cdot 604957 \cdot 1724555797 \cdot 21728147227 \cdot 89540042563 \cdot 526901242589 \cdot p22$
238	$11 \cdot 239 \cdot 367 \cdot 11146451 \cdot p61$
239	$149 \cdot 241 \cdot 431 \cdot 2783631563 \cdot 1699619317441448124371 \cdot p37$
240	$137 \cdot 241 \cdot 17380141 \cdot p63$
241	$3 \cdot 7 \cdot 109 \cdot 251 \cdot 677 \cdot p67$
242	$3 \cdot 17 \cdot 523 \cdot 4730766374357138123 \cdot p53$
243	$5 \cdot 59 \cdot 58907 \cdot 588173 \cdot 183779976254802979327009873 \cdot p36$
244	$5 \cdot 11 \cdot 4517 \cdot 14234771 \cdot 103522546497268413031966177 \cdot p38$
245	$25935383 \cdot 5201121395131517 \cdot p55$
246	$5 \cdot 491 \cdot 47353 \cdot 10030753 \cdot 310516454471 \cdot p50$
247	$3^2 \cdot 85601 \cdot 16906867 \cdot p64$
248	$3 \cdot 5851 \cdot 294793 \cdot 456037 \cdot p62$
249	$241 \cdot 251 \cdot 4013 \cdot 91081 \cdot 856411 \cdot 2465222216963224591 \cdot 7776427048612116457 \cdot p22$
250	$127 \cdot 167 \cdot 251 \cdot 499 \cdot 425237 \cdot p63$
251	$30765339546509967970123309 \cdot p52$
252	$18424659817 \cdot 144533935823133455939 \cdot p48$
253	$3 \cdot 13 \cdot 29453 \cdot 16247207 \cdot 129471682943 \cdot p55$
254	$3^2 \cdot 53 \cdot 113 \cdot 8680459887401 \cdot p62$
255	$13 \cdot 41 \cdot 61 \cdot 257 \cdot 509 \cdot 32911 \cdot 35393 \cdot 34053463247 \cdot p50$
256	$97 \cdot 257 \cdot 673 \cdot 229153 \cdot 119782433 \cdot 43872038849 \cdot 16875081675650881 \cdot p32$
257	$5 \cdot 7 \cdot 53 \cdot 10429 \cdot 668929 \cdot 2416591963 \cdot p58$
258	$7^2 \cdot 23 \cdot 97 \cdot 131 \cdot 167 \cdot 337 \cdot 2297 \cdot 559667 \cdot 230313647 \cdot 364364467 \cdot p43$
259	$3 \cdot 43 \cdot 71 \cdot 83 \cdot 85381 \cdot p70$
260	$3 \cdot 23 \cdot 1690253 \cdot 5083453 \cdot 29861808013 \cdot 13221664979960397979 \cdot p37$
261	$229 \cdot 263 \cdot 337666820937693285382073012867537183 \cdot p41$
262	$7 \cdot 263 \cdot 523 \cdot 1906481897 \cdot 9473984967157274101778182337917 \cdot p34$
263	$5 \cdot 22786936409553397246797207757 \cdot p54$
264	$5^3 \cdot 5021 \cdot 592119738209 \cdot p65$
265	$3^3 \cdot 11 \cdot 17367004185637 \cdot p67$

TABLE II

Factorization of Cullen numbers $W_n = n \cdot 2^n - 1$

n	$W_n = n \cdot 2^n - 1$
1	1
2	7
3	23
4	$3^2 \cdot 7$
5	$3 \cdot 53$
6	383
7	$5 \cdot 179$
8	$23 \cdot 89$
9	$17 \cdot 271$
10	$3 \cdot 3413$
11	$23 \cdot 2503$
12	$23 \cdot 2137$
13	$5 \cdot 19^2 \cdot 59$
14	$5^4 \cdot 367$
15	$7^3 \cdot 1433$
16	$3 \cdot 5^2 \cdot 11 \cdot 31 \cdot 41$
17	$3 \cdot 47 \cdot 15803$
18	$79 \cdot 59729$
19	$13 \cdot 29 \cdot 26423$
20	$1879 \cdot 11161$
21	$13 \cdot 41 \cdot 53 \cdot 1559$
22	$3^3 \cdot 271 \cdot 12611$
23	$3 \cdot 7 \cdot 9187523$
24	$47 \cdot 67 \cdot 127867$
25	$7 \cdot 119837257$
26	$229 \cdot 661 \cdot 11527$
27	$5 \cdot 307 \cdot 2360833$
28	$3 \cdot 19 \cdot 2999 \cdot 43969$
29	$3^2 \cdot 733 \cdot 1009 \cdot 2339$
30	32212254719
31	$263 \cdot 983 \cdot 257503$
32	$223 \cdot 616318177$
33	$5 \cdot 217561 \cdot 260587$
34	$3 \cdot 5 \cdot 29 \cdot 10501 \cdot 127873$
35	$3 \cdot 587 \cdot 682902239$
36	$5 \cdot 7 \cdot 71 \cdot 22153 \cdot 44939$
37	$73 \cdot 69660839431$
38	$17 \cdot 518239 \cdot 1185617$
39	$1217 \cdot 17617482943$
40	$3^2 \cdot 79 \cdot 1747 \cdot 35407667$

n	$W_n = n \cdot 2^n - 1$
41	$3 \cdot 19 \cdot 4517 \cdot 350177899$
42	$6709 \cdot 161123 \cdot 170881$
43	$29 \cdot 191 \cdot 773 \cdot 883 \cdot 100043$
44	$7 \cdot 53 \cdot 2086404813893$
45	$71 \cdot 89 \cdot 412397 \cdot 607573$
46	$3 \cdot 7^2 \cdot 18251 \cdot 1206517519$
47	$3^4 \cdot 5 \cdot 3555949 \cdot 4593007$
48	$11 \cdot 1439 \cdot 8963 \cdot 95230081$
49	$97 \cdot 271 \cdot 281 \cdot 3734380121$
50	$47 \cdot 30503527 \cdot 39266471$
51	$228713 \cdot 257489 \cdot 1950071$
52	$3 \cdot 103 \cdot 18954679 \cdot 39984181$
53	$3 \cdot 5^2 \cdot 40703633 \cdot 156376397$
54	$5 \cdot 39014093 \cdot 4986800639$
55	$37 \cdot 1609 \cdot 4983497 \cdot 6679139$
56	$5 \cdot 941 \cdot 857646177709663$
57	$7 \cdot 113 \cdot 197 \cdot 52715933184389$
58	$3^2 \cdot 1857484646311031239$
59	$3 \cdot 23 \cdot 1151 \cdot 589481 \cdot 726486469$
60	$9377 \cdot 22259 \cdot 331422069413$
61	$11 \cdot 5908739 \cdot 2164073856799$
62	$13 \cdot 23 \cdot 37 \cdot 25845117340910969$
63	$479 \cdot 71843 \cdot 16885359197099$
64	$3 \cdot 11 \cdot 31 \cdot 43 \cdot 71 \cdot 127 \cdot 281 \cdot 86171 \cdot 122921$
65	$3^2 \cdot 7 \cdot 11 \cdot 59 \cdot 58651325105345017$
66	$41 \cdot 1097 \cdot 2973611 \cdot 36412375309$
67	$5^2 \cdot 7 \cdot 564997418461869521$
68	$13 \cdot 101 \cdot 863 \cdot 151631 \cdot 1156548163$
69	$137 \cdot 1583 \cdot 41866553 \cdot 4485905329$
70	$3 \cdot 37045301 \cdot 743606802296993$
71	$3 \cdot 1332789373 \cdot 41928107956153$
72	$434445574513 \cdot 782630568047$
73	$5 \cdot 11^2 \cdot 773 \cdot 1474272195907258831$
74	$5 \cdot 370423 \cdot 4014713 \cdot 187987488397$
75	$2833419889721787128217599$
76	$3^3 \cdot 5 \cdot 59 \cdot 151 \cdot 9043 \cdot 1983649 \cdot 266166367$
77	$3 \cdot 457007371 \cdot 8487033800154911$
78	$7 \cdot 59 \cdot 137 \cdot 416642574052867020851$
79	$11 \cdot 53 \cdot 28319 \cdot 507919 \cdot 5694502939577$
80	$131 \cdot 7382753096688323160125909$
81	$195845982777569926302400511$
82	$3 \cdot 13 \cdot 571 \cdot 17806262914077792864683$
83	$3^2 \cdot 17 \cdot 421 \cdot 1246218533873735736651$
84	$17 \cdot 37 \cdot 167 \cdot 379 \cdot 2687 \cdot 15188851247211937$
85	$13 \cdot 25294447918090700727323138363$

n	$W_n = n \cdot 2^n - 1$
86	$7^2 \cdot 13579444308487589750168947$
87	$5 \cdot 17 \cdot 409481053971 \cdot 38679080140583$
88	$3 \cdot 7 \cdot 59 \cdot 318743 \cdot 689620762407770951$
89	$3 \cdot 13 \cdot 23003 \cdot 61405961260570719575051$
90	$13 \cdot 85703541181295557493010629243$
91	$61 \cdot 103 \cdot 1087 \cdot 1583629493 \cdot 2083151293163\ 9$
92	$41 \cdot 1682843867 \cdot 6602607236430060157\ 3$
93	$5 \cdot 17^2 \cdot 6373891966978013318511753\ 79$
94	$3^2 \cdot 5 \cdot 12445010777 \cdot 3324601949501589203$
95	$3 \cdot 853 \cdot 2833 \cdot 19841 \cdot 2616331534989737841\ 7$
96	$5 \cdot 191 \cdot 436610833 \cdot 1824118038954999393917$
97	$193 \cdot 7963867112832788338418407443\ 1$
98	$17 \cdot 3089 \cdot 1064491273 \cdot 254871049 \cdot 2179981889$
99	$7 \cdot 5453983 \cdot 16435879380339981512878\ 31$
100	$3 \cdot 11 \cdot 13 \cdot 67 \cdot 199 \cdot 769 \cdot 10243 \cdot 25373 \cdot 2113879 \cdot 52457663$
101	$3^3 \cdot 83 \cdot 15737 \cdot 162525039511 \cdot 4467523587907\ 3$
102	$11 \cdot 19 \cdot 247464806189557874692128745423$
103	$180347 \cdot 25737919 \cdot 1110312353 \cdot 2026745671\ 87$
104	$521 \cdot 3733 \cdot 10845688970519084455592422891$
105	$491 \cdot 870301 \cdot 9967537290687835623087649$
106	$3 \cdot 97 \cdot 300112427 \cdot 9847102026050874282671\ 9$
107	$3 \cdot 5 \cdot 7 \cdot 11 \cdot 137 \cdot 515357 \cdot 1247327 \cdot 3085307 \cdot 55322764669$
108	$157 \cdot 161411101 \cdot 13830256552501951115047\ 1$
109	$7 \cdot 26325617 \cdot 2174155521 \cdot 176574864682964793$
110	$130651 \cdot 166844751811 \cdot 6550386407519435599$
111	$1097 \cdot 2843 \cdot 69193 \cdot 107251 \cdot 2708417 \cdot 4597163694127$
112	$3^2 \cdot 223 \cdot 289754483386090000695274334266393$
113	$3 \cdot 5 \cdot 75880099 \cdot 5689708397 \cdot 181200251723491271$
114	$5^2 \cdot 4391 \cdot 2156854809830099590085915063761$
115	$47769131098520414182480566622882483319$
116	$5^2 \cdot 919 \cdot 7331 \cdot 213011270207009 \cdot 268606281442027$
117	$37 \cdot 233 \cdot 2254954416500091574541447375671036\ 3$
118	$3 \cdot 4759 \cdot 677213 \cdot 4055638617872072047137294991$
119	$3^2 \cdot 13 \cdot 23 \cdot 139 \cdot 4789 \cdot 118139849 \cdot 3737203087037846364139$
120	$7 \cdot 17 \cdot 239 \cdot 1453 \cdot 3858984841330380459073439670\ 7603$
121	$241 \cdot 3140681898569 \cdot 42498524452770840370188$
122	$101 \cdot 11621 \cdot 55265544124597663838089108164247$
123	$1307960347852357241219938471 \cdot 34840479030801$
124	$3 \cdot 83 \cdot 24203685429734375829580602640493699779$
125	$3 \cdot 31 \cdot 157 \cdot 197 \cdot 307 \cdot 30930012653 \cdot 19466620916811 6568357$
126	$11 \cdot 1151 \cdot 62491603 \cdot 13554747131147755549323268061$
127	$5 \cdot 1980508153 \cdot 2182059211771041228855562167727$
128	$7 \cdot 31 \cdot 73 \cdot 151 \cdot 271 \cdot 631 \cdot 23311 \cdot 262657 \cdot 348031 \cdot 4997161783080\ 1$
129	$257 \cdot 2341 \cdot 81761 \cdot 2117501 \cdot 24191938471 \cdot 348040479030801$
130	$3^6 \cdot 7 \cdot 67153 \cdot 1794539 \cdot 28773911268598462123306 9219$

n	$W_n = n \cdot 2^n - 1$
131	$3 \cdot 127 \cdot 261323 \cdot 16835422570103 \cdot 2127522173130823178\ 3$
132	$31 \cdot 263 \cdot 13043 \cdot 72551 \cdot 9315260821455598950 8528585899$
133	$5 \cdot 1811 \cdot 433681662659423 \cdot 368792049903504448788799$
134	$5 \cdot 193 \cdot 150869 \cdot 4391918900022160\ 7 \cdot 45639678905560044\ 49$
135	$13 \cdot 3967 \cdot 47536927 \cdot 2398537577041438200943105679387$
136	$3 \cdot 5 \cdot 271 \cdot 291445778271070791978864706822741501846 3$
137	$3^2 \cdot 43 \cdot 1867 \cdot 41189 \cdot 339707 \cdot 23609617220656803600297 9689$
138	$37 \cdot 20441 \cdot 406253 \cdot 4543139 \cdot 34447841970043617742753 89$
139	$163 \cdot 6287 \cdot 19423 \cdot 38698120924997167 \cdot 125761278921765611$
140	$6841 \cdot 4062822730517 \cdot 70206927086422099207880098 7$
141	$7 \cdot 19 \cdot 79 \cdot 281 \cdot 1069 \cdot 84411423 \cdot 14752792445889689239 8594039$
142	$3 \cdot 2638921515159457071748393328930056111541011589$
143	$3 \cdot 127 \cdot 4185047983451284920261952379250283064770603$
144	$13 \cdot 9907 \cdot 36529 \cdot 478069 \cdot 2489413 \cdot 17674961049 \cdot 3244984056851$
145	$17 \cdot 263 \cdot 2865701 \cdot 480498143 \cdot 99979526359 \cdot 1050700744 1217757$
146	$359 \cdot 73811696978582107 \cdot 491487621200028133744084811$
147	$5 \cdot 71 \cdot 73875144651766764173083381602432245952403 6533$
148	$3^2 \cdot 54979 \cdot 177564092711756503 \cdot 60104161182428160672 38939$
149	$3 \cdot 7 \cdot 83 \cdot 3767 \cdot 457837 \cdot 3537135751215776257754812972 0017971$
150	$9533 \cdot 224574796922158798026584386255558869671050 03$
151	$7 \cdot 101 \cdot 609658844691937601244133469269798488242700821$
152	$1369393 \cdot 52648121647545442613 \cdot 12036270629590109 5699$
153	$5^2 \cdot 37573 \cdot 18597941882437866493655997941140521618852907$
154	$3 \cdot 5 \cdot 188183740063357946597 \cdot 12458527017773392156423164 77$
155	$3^3 \cdot 41 \cdot 681406627705570211101 \cdot 93848436671239802600367 77$
156	$5 \cdot 89 \cdot 311 \cdot 1913 \cdot 20004997 \cdot 495190453 \cdot 543322634897241176752429$
157	$47 \cdot 313 \cdot 14529660369137 \cdot 134187268764109724982453531968329$
158	$11 \cdot 7237 \cdot 1187780086293052576 \cdot 61053294546895796814302584 49$
159	$37349542741 3 \cdot 311086486312797781992676071608151931907$
160	$3 \cdot 107 \cdot 40289 \cdot 168391 \cdot 2375797 \cdot 1243852 9 \cdot 816495564329 \cdot 4450171110173$
161	$3 \cdot 4831 \cdot 1073424463 \cdot 1342972013 \cdot 13531253041 \cdot 1664640357 13696793$
162	$7^2 \cdot 431 \cdot 4289 \cdot 2394662731759 \cdot 650845427612519 \cdot 6708457 955477977$
163	$109 \cdot 1748438656602361516858761668459018877351706752667$
164	$64811 \cdot 287251 \cdot 24965913652051835753 \cdot 8250977467792656790231$
165	$1789 \cdot 405821444674969 \cdot 5057467303275727 \cdot 2101622996244839597$
166	$3^2 \cdot 47 \cdot 8117 \cdot 37265662921300 9 \cdot 1213507887800600198635857310011 7$
167	$3 \cdot 5^2 \cdot 23 \cdot 4969 \cdot 24113 \cdot 20931123 \cdot 7221404209968629394777782812953744 1$
168	$79328683868329 \cdot 792370220009792723930065632315 1444583$
169	$337 \cdot 375254633998506934037486184333909358197266626432 8271$
170	$7 \cdot 61 \cdot 131 \cdot 419 \cdot 9194441 \cdot 342813949 \cdot 1382014891 \cdot 24919450006796321673 47$
171	$11 \cdot 31 \cdot 577 \cdot 450581 \cdot 180330641 \cdot 3416114580511 \cdot 9371742480893273383 4521$
172	$3 \cdot 7 \cdot 13877 \cdot 3533237393560667623060608180334936327762 0667427 35383$
173	$3^2 \cdot 5 \cdot 1063 \cdot 6089 \cdot 1093366174909 \cdot 210588224891407 \cdot 3088546855389 6867527$
174	$5 \cdot 11 \cdot 17 \cdot 197 \cdot 587849 \cdot 42185262151 \cdot 912145501761563889 8992728792461803$
175	$11^2 \cdot 13 \cdot 1523 \cdot 349831299982607423371295165367657910 2590530523681$

n	$W_n = n \cdot 2^n - 1$
176	$5 \cdot 1231 \cdot 7121437 \cdot 3845884264827278163439134784699764600502721$
177	$13 \cdot 353 \cdot 33530183 \cdot 1868156007229 \cdot 7758826253161 \cdot 15202651323230236708 1$
178	$3 \cdot 101 \cdot 22506947712386816566648999928015705184903033125974577$
179	$3 \cdot 76943 \cdot 5941989564027789091625058330190429795817559814210 19$
180	$359 \cdot 72959 \cdot 218279 \cdot 469429 \cdot 13068169 \cdot 7865055470999309673533513465221$
181	$19 \cdot 401 \cdot 4409371 \cdot 16513273541186422445453216046701082014116010639$
182	$55529 \cdot 8193259 \cdot 2452189696262232365307289000131288183663255557$
183	$7 \cdot 11 \cdot 19 \cdot 397 \cdot 5052057592355743181651 \cdot 764605064950640310745761387983$
184	$3^3 \cdot 109 \cdot 367 \cdot 557 \cdot 3594686439045638327383 \cdot 2086241068266386874791025613$
185	$3 \cdot 891329 \cdot 548315371 \cdot 17285113404167811777 \cdot 3579801789474969290678711$
186	$23 \cdot 79316638775945830303090331743998817618172813265423330447961$
187	$5 \cdot 338251 \cdot 2502217077000520769118431 \cdot 866795640583981662472447287$
188	$19 \cdot 1735847 \cdot 3811455751 \cdot 156290590237 \cdot 178631417978669 \cdot 2101601795183 8813$
189	$11 \cdot 19 \cdot 149 \cdot 2663 \cdot 232846303 \cdot 19850182013 \cdot 112105648753621 \cdot p19$
190	$3 \cdot 71 \cdot 79 \cdot 347933 \cdot 4544910959 \cdot 35482797866677223 \cdot 3157964358923899736 32457$
191	$3^2 \cdot 7 \cdot 2843 \cdot 7573 \cdot 44195410600799884369178839524487490398768451322 31$
192	$383 \cdot 3146745517478440487353711669907538386640118079724790147457$
193	$5 \cdot 7^2 \cdot 23 \cdot 50811968782739099 3 \cdot 84626320386474911987033669426773401861$
194	$5 \cdot 89 \cdot 1094613695878661287048477735750896941338040055964248948331$
195	$47 \cdot 3064337 \cdot 2235287369 3 \cdot 3041697485428346091381996229032738263754517$
196	$3 \cdot 5 \cdot 71 \cdot 131 \cdot 439 \cdot 1150229 \cdot 2152037 \cdot 44653789 \cdot 309580846003 19 \cdot p21$
197	$3 \cdot 6480725792111775132047942 93 \cdot 2035309552772743054282950284776937 7$
198	$113 \cdot 12577 \cdot 15126017 \cdot 370019185792805617506426785110424487264918718258 3$
199	$43 \cdot 1741 \cdot 213577248311942524900718950867512227603176926633904560497$
200	$79 \cdot 36767 \cdot 110648069743959536184217267163532104168951032921165089 43$
201	$401 \cdot 9825469 \cdot 17051501929214396600161 \cdot 9615345300804499900039069819 9$
202	$3^2 \cdot 6349086310647768409304026279268 9 \cdot 227225335396994344666414887492407$
203	$3 \cdot 8953710468965671 \cdot 42705497066400255173 \cdot 2274977339335820902908063127$
204	$7 \cdot 251 \cdot 29852281027099928434472944945589571625682814864581771452685 9$
205	$19 \cdot 409 \cdot 9412319702383331 \cdot 1441217055780820030404071424337399828197539759$
206	$199 \cdot 41963347 \cdot 2537015557852079000105512529952608161235434218137898211$
207	$5 \cdot 5851 \cdot 5858553100 7 \cdot 24842138276349001977616182152060636979835322614007$
208	$3 \cdot 139 \cdot 5827 \cdot 7901 \cdot 20149 \cdot 108967 \cdot 124799 \cdot 806363 \cdot p35$
209	$3^5 \cdot 79 \cdot 197 \cdot 1143091 \cdot 66700928583636 11 \cdot 20104161840576676 3 \cdot p21$
210	$11 \cdot 73 \cdot 313 \cdot 37489 \cdot 84972609269 \cdot 159184389262884 9413 3 \cdot p26$
211	$1523978951215517 \cdot 45565125596760841223532329870594788847777993777216331$
212	$7 \cdot 11 \cdot 8929 \cdot p61$
213	$5 \cdot 1149794687 \cdot 1087552691851 \cdot 37631253393046957452 1 \cdot p26$
214	$3 \cdot 5^2 \cdot 7 \cdot 3889 \cdot 8816053 \cdot 2382649541384883242891 \cdot p33$
215	$3 \cdot 877 \cdot 18169 \cdot 46441 \cdot 113131 \cdot 6740387 \cdot 593952659 \cdot 51461022281 \cdot p24$
216	$5^2 \cdot 47 \cdot 431 \cdot 5011 \cdot 184007 \cdot 30406151611 3 \cdot 13358088912111577689 1 \cdot p22$
217	$11 \cdot 433 \cdot 3557 \cdot 6089 \cdot 9156265010915501 \cdot p41$
218	$13 \cdot 59 \cdot 97 \cdot 4739483727308617451399 \cdot p42$
219	$17 \cdot 31 \cdot 213091 \cdot 394914094415751 \cdot 19721568061802707483 \cdot p26$
220	$3^2 \cdot 17^2 \cdot 439 \cdot 198694031 \cdot 745392199571 \cdot p43$

n	$W_n = n \cdot 2^n - 1$
221	$3 \cdot 111519329 \cdot 27040856681920901126718274 1 \cdot p34$
222	$49943 \cdot 50671 \cdot p60$
223	$17 \cdot 73 \cdot 149 \cdot 6911 \cdot 219091 \cdot 4360189 \cdot 1314454769 \cdot p40$
224	$13 \cdot 1021 \cdot 14891 \cdot 901687013 \cdot 2978604757 \cdot p43$
225	$7 \cdot 23^2 \cdot 449 \cdot 29642662471 \cdot 289237669241 \cdot 6255586503605152 9 \cdot p26$
226	$3 \cdot 43 \cdot 53569 \cdot 146701 \cdot 155284141848661507524792754 1 \cdot p32$
227	$3^2 \cdot 5 \cdot 12113 \cdot 603397729419667959524043 \cdot p44$
228	$6221 \cdot 7937 \cdot 207924974333 \cdot 100387175866864487 \cdot 164688011024948317 \cdot p18$
229	$17 \cdot 457 \cdot 523 \cdot 3510469 \cdot 2587119135808 7 \cdot 590732643920597501 \cdot p27$
230	$24841 \cdot 16618159 \cdot p60$
231	$23 \cdot p71$
232	$3 \cdot 19 \cdot 463 \cdot 9151413510915654722 53 \cdot p47$
233	$3 \cdot 5 \cdot 7^2 \cdot 271 \cdot 8708959226622450926508685529 \cdot p40$
234	$5 \cdot 17 \cdot 3907 \cdot 65729 \cdot 1016318451341 \cdot p51$
235	$7 \cdot 157 \cdot 33751 \cdot 1140911 \cdot 3906407 \cdot p53$
236	$5 \cdot 11 \cdot 2339 \cdot 1469389 \cdot 3076741 \cdot 493269761 \cdot p47$
237	$179 \cdot 13729 \cdot p68$
238	$3^3 \cdot 13 \cdot 1277 \cdot p69$
239	$3 \cdot 30497 \cdot 2308067663211073 \cdot p54$
240	$479 \cdot 6403687 \cdot p66$
241	$13 \cdot 57527 \cdot 38691599345 28167 \cdot 78479371056589809289 \cdot p33$
242	$199 \cdot 112559 \cdot 18810095809 \cdot p58$
243	$619 \cdot 111965779 \cdot 62903688 81 \cdot 1440435493266785377186751 \cdot p31$
244	$3 \cdot 163 \cdot 487 \cdot 3310614977 \cdot 43397802341 \cdot 20235366730734468 71 \cdot p32$
245	$3^2 \cdot 13^2 \cdot 8857361882781395220427 \cdot 1281566868334874509091 57 \cdot p28$
246	$7 \cdot 13 \cdot 3269913466931 \cdot p62$
247	$5 \cdot 43 \cdot 859 \cdot 2797 \cdot 3529 \cdot 7559 \cdot p61$
248	$29 \cdot 197 \cdot 82651 \cdot 3695123 \cdot p62$
249	$p78$
250	$3 \cdot 283 \cdot 21517 \cdot p71$
251	$3 \cdot 61 \cdot 6007 \cdot 8761 \cdot 22568841573044207488225886861949 \cdot p36$
252	$137 \cdot 503 \cdot 90917 \cdot 100039717 \cdot p61$
253	$5^2 \cdot 15431938240 7 \cdot 19997800061321 \cdot 1714984311441837 \cdot p39$
254	$5 \cdot 7 \cdot 29 \cdot 3673 \cdot 33355889 31 \cdot 1069324064627 3 \cdot 461770367783497 \cdot p37$
255	$83 \cdot 393809253484254516467081 \cdot p56$
256	$3^2 \cdot 5 \cdot 7 \cdot 13 \cdot 17 \cdot 23 \cdot 67 \cdot 89 \cdot 241 \cdot 353 \cdot 397 \cdot 683 \cdot 2113 \cdot 7393 \cdot 20857 \cdot 312709 \cdot$ $599479 \cdot 4327489 \cdot 1761345169 \cdot 2931542417 \cdot 9861827395 3$
257	$3 \cdot 3347 \cdot p76$
258	$19 \cdot 4289 \cdot 620111 \cdot 8431567 \cdot 885136236304628711 \cdot p44$
259	$97 \cdot 173 \cdot 1583 \cdot 9923 \cdot 1177626413711 9 \cdot 20677553294440 1 \cdot p42$
260	$137 \cdot 439 \cdot p76$
261	$23 \cdot 521 \cdot 739 \cdot 17623 \cdot 719846891 \cdot p61$
262	$3 \cdot 37 \cdot 1291 \cdot 11182 7 \cdot p72$
263	$3^3 \cdot 2026647209 \cdot 60861242119597 \cdot 2867608272043515 47 \cdot p40$
264	$41 \cdot 3001 \cdot 9750070234553729470131822491838505523 \cdot p40$

$$W_n = n \cdot 2^n - 1$$

n	
265	$23 \cdot 137 \cdot 19373 \cdot p75$
266	$103 \cdot 433 \cdot 316508477 \cdot p70$
267	$5^2 \cdot 7 \cdot 4561 \cdot 7445848658467078427793481278853 \cdot p48$
268	$3 \cdot 11 \cdot 29 \cdot 179 \cdot 14153 \cdot 43783 \cdot p70$
269	$3 \cdot 47 \cdot 52817 \cdot 4474837 \cdot 3230125302316958737 \cdot p52$
270	$29 \cdot 730084679 \cdot p74$
271	$181 \cdot 233 \cdot 307 \cdot 48523 \cdot 170239 \cdot 125776109 \cdot 888611338973 \cdot p47$
272	$2416149541696444378932580061 \cdot p57$
273	$5 \cdot 29 \cdot 97 \cdot 2895349 \cdot 15257323515151 \cdot 547689773467 \cdot 1875950427661 \cdot p39$
274	$3^2 \cdot 5 \cdot 83 \cdot 2056111 \cdot p75$
275	$3 \cdot 7 \cdot 13 \cdot 149 \cdot 461 \cdot 17713 \cdot p74$
276	$5 \cdot 83 \cdot 503 \cdot 8401303 \cdot 13158661 \cdot 613852069 \cdot 2795911099269763687 \cdot p39$
277	$7 \cdot 487 \cdot 581336756412301829561710076899 \cdot p54$
278	$9988463834053 \cdot 1337234569900330723 \cdot p57$
279	$10973 \cdot 11119 \cdot p79$
280	$3 \cdot 31 \cdot p85$
281	$3^2 \cdot 11 \cdot 17 \cdot p84$
282	$109 \cdot 16759289843149 \cdot 168072493867 \cdot 1108813257321964411610451 \cdot p38$
283	$31 \cdot 269463881 \cdot p78$
284	$11 \cdot 1319 \cdot 398059 \cdot p79$
285	$11 \cdot 569 \cdot 43095878927 \cdot 3887901324779895549 \cdot 2969953626364566699011 \cdot p36$
286	$3 \cdot 263 \cdot 82781 \cdot 2103139 \cdot 9290893 \cdot 3221074277 \cdot 32596012900854379111 \cdot p39$
287	$3 \cdot 5 \cdot 19 \cdot 31 \cdot p85$
288	$7 \cdot 215972447 \cdot 21832833263333501909837455531663 \cdot p48$
289	$577 \cdot 11555664079 \cdot 567726560001676032818717557254210823 \cdot p41$
290	$19 \cdot 373 \cdot 2039 \cdot 537316698379259 \cdot 3024361100545903751433064483 \cdot p41$
291	$13 \cdot 15737 \cdot 310819 \cdot 85781290805200929027277954460971 \cdot p49$
292	$3^4 \cdot 15823 \cdot 18517 \cdot p80$
293	$3 \cdot 5 \cdot 11 \cdot 1031 \cdot 7587852017686325286503002427 \cdot p58$
294	$5 \cdot 77213 \cdot 2173953007511 \cdot 2528561667091 \cdot 1624989556931822910531716 23537 \cdot p31$
295	$197 \cdot 421 \cdot 305047 \cdot 122787799 \cdot 54111564049743951996576535684253 \cdot p42$
296	$5 \cdot 7 \cdot 433 \cdot 32309 \cdot 13464233 \cdot 17015857 \cdot 40625893949060953497331008751 \cdot p40$
297	$593 \cdot 55196717 \cdot 186037286461271729864663 \cdot p59$
298	$3 \cdot 7 \cdot 59 \cdot 100291 \cdot 588303929303 \cdot 55154894793997 \cdot p59$
299	$3^2 \cdot 11 \cdot 19 \cdot 2583351020133193 \cdot 14093940903798044237375107 \cdot p48$
300	$13 \cdot 599 \cdot 769 \cdot 5722763 \cdot p80$