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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

29[65-06, 65M60, 65N30]—*Finite element methods: Fifty years of the Courant element*, M. Křížek, P. Neittaanmäki, and R. Stenberg (Editors), Lecture Notes in Pure and Applied Mathematics, Vol. 164, Dekker, New York, 1994, xxii+504 pp., 25 cm, softcover, $165.00

In a 1943 paper ([1]), Richard Courant discussed the numerical solution of partial differential equations using piecewise linear functions on triangles. This paper is considered to be one of the earliest about the finite element method, and the triangular linear element is also known as the Courant element. To commemorate the fiftieth anniversary of the publication of this paper, a conference was held at the University of Jyväskylä, Finland in 1993. The book under review contains the proceedings of the conference.

Besides the reprinted aforementioned paper by Courant, there are 45 papers, which cover many different aspects of contemporary finite element research. Theoretical topics include domain decomposition methods, error estimators, locking, the maximum-angle condition, mesh orientation, mixed finite element methods (h-version, p-version, least-squares methods), parabolic variational inequalities, and superconvergence.

Various applications are also discussed in these papers, which include compressible flows, convection-diffusion problems, elasticity and viscoelasticity, the Helmholtz equation, Maxwell equations, Navier-Stokes equations, plates, porous media, shape optimization, shells, Stokes equations, thermal flows, and turbulence.

There are also three papers which are historical in nature. The first is a paper by I. Babuška, which surveys the history of the finite element method. It also contains the results of an opinion poll concerning the major achievements of the past and the important problems for the present and the future. The second is by L. A. Oganesjan and V. Rivkind on finite element research in St. Petersburg, and the third is by V. Thomée on the development of finite element methods for parabolic problems.

Anyone who is working in the field of finite elements would certainly find some of the papers in this book interesting. This book will be a valuable addition to any library.

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30[34E15, 35B25, 73U05, 78A30, 80A25, 92C20]—*The boundary function method for singular perturbation problems*, by Adelaida B. Vasil′eva, Valentin F. Butuzov, and Leonid V. Kalachev, SIAM Studies in Applied Mathematics, Vol. 14, SIAM, Philadelphia, PA, 1995, xiv+221 pp., 26 cm, $58.50

A. B. Vasil′eva and V. F. Butuzov wrote three books, Vasil′eva and Butuzov [3, 4, 5], on singularly perturbed equations. The book under review is an updated and revised translation of the 1990 book with a new set of exercises, more illustrative examples, and other changes. For example, §17 on the acoustic oscillations in a medium with small viscosity in the 1990 book was replaced by §4.4 on the relaxation waves in the FitzHugh-Nagumo system; §3.4.5 with an example of nonisothermal chemical reaction and §4.3.1 on a one-dimensional model of semiconductor devices were not available in the 1990 book. Also, the references have increased from 121 items to 167 items.

The term *boundary function method*, appearing in the title of the book, may be vague to some people working in the area of singular perturbations. As stated in the Preface, it is also known as the *method of boundary layer correction*, which is a basic technique in seeking an asymptotic expansion, uniformly valid on the given domain, for a singularly perturbed problem. The core of this book reflects a cumulative work of investigating singularly perturbed systems by using this asymptotic technique at the Department of Physics of the Moscow State University since the pioneering paper [1] of A. N. Tikhonov; see Vasil′eva [2]. The main attention of this book is devoted to algorithms for constructing the asymptotic approximations of the solutions of problems motivated by applications in physics, engineering, chemistry, and biology. Justification of the asymptotic correctness of asymptotic solutions is given only for problems whose proofs are short. Otherwise outlines of the proofs are presented or references are mentioned where proofs can be found. The book is intended to be used in courses on asymptotic methods and applied mathematics at the undergraduate or graduate level, as well as for self-study by applied scientists who use asymptotic methods in their work.

There are four chapters. Chapter 1 presents basic ideas of regular and singular perturbations, asymptotic approximations, and asymptotic series along with examples to illustrate the notion of initial and boundary layers. Chapter 2 treats singularly perturbed differential equations, most of which are excerpted from [3, 4]. It begins with the statement of the Tikhonov theorem on the convergence of the solution of the singularly perturbed system of first-order differential equations $\mu\mathbf{z}'(t) = F(\mathbf{z}, \mathbf{y}, t), \mathbf{y}'(t) = f(\mathbf{z}, \mathbf{y}, t)$ defined in $t \in (0, T)$, subject to initial conditions, to a reduced solution as $\mu \downarrow 0$. An asymptotic algorithm is then developed to construct a uniform approximation involving initial layer functions with a remainder of order μ^{n+1}, as proved by Vasil′eva. For a corresponding boundary value problem with two components in $\mathbf{z}$ and a scalar function y defined in $t \in (0, 1)$, subject to initial conditions of y, z_1 at $t = 0$ and a terminal condition of z_2 at $t = 1$, a uniform solution with boundary layer functions at $t = 0$, $t = 1$ is constructed with a remainder of order μ^{n+1}. The next problem is a system with a small nonlinearity, $\mu\mathbf{x}'(t) = A(t)\mathbf{x} + \mu f(\mathbf{x}, t, \mu)$, defined in $t \in (0, T)$, subject to an initial condition. A uniform solution containing initial layer functions is constructed with a remainder of order μ^{n+1}. Attention is also paid to the case of using μ^2 to replace μ as the coefficient of $\mathbf{x}'(t)$, together with an example from chemical kinetics. The equation $\mu^2 z''(t) = F(z, t)$ defined in $t \in (0, 1)$, subject to homogeneous Dirichlet boundary

conditions, is studied for shock layer behavior as $\mu \downarrow 0$. The chapter concludes with the construction, in three cases, of the asymptotic expansion for the spike-type solution of the system $\mu^2 u''(x) = f(u,v), v''(x) = g(u,v)$ defined in $x \in (0,l)$, subject to homogeneous Neumann boundary conditions, along with a survey of the stability.

Chapter 3 deals with singularly perturbed partial differential equations. The first problem is a selfadjoint elliptic differential equation $\varepsilon^2 \Delta u - k^2(x,y)u = f(x,y,\varepsilon)$ defined in a bounded planar domain, subject to the homogeneous Dirichlet boundary condition. For a domain with smooth boundary, a uniform approximation involving boundary layer functions along the entire boundary is constructed with the remainder of order ε^{n+1}. In the case of the rectangle $(0,a) \times (0,b)$ as domain, corner layer functions are constructed at each corner in order to obtain a uniform solution. An equation of the form $\varepsilon^2 \Delta u - \varepsilon^\alpha A(x,y)u_y - k^2(x,y)u = f(x,y,\varepsilon)$ defined in the rectangle is also studied. The second type of equation is a system of parabolic equations $\varepsilon^2\{\mathbf{u}_t - a(x,t)\mathbf{u}_{xx}\} = f(\mathbf{u},x,t,\varepsilon)$ defined in $x \in (0,1)$, $t \in (0,T)$, subject to an initial condition and homogeneous Neumann boundary conditions at $x = 0$, $x = 1$. An asymptotic solution containing initial layer functions, boundary layer functions at $x = 0$, $x = 1$, and corner layer functions at points $(0,0)$, $(1,0)$ is constructed with the remainder of order ε^{n+1}. The third type of equation is given by $\varepsilon u_t + b(x)u_x - \varepsilon^2 a(x)u_{xx} = f(u,x,t,\varepsilon)$ defined in $x \in (0,1)$, $t \in (0,T)$, subject to an initial condition and homogeneous Neumann boundary conditions at $x = 0$, $x = 1$. Owing to an incompatibility between initial and boundary data at the inflow corner $(0,0)$ angular layer functions are constructed to obtain a uniform solution with a remainder of order ε^2. This smoothing procedure is also applied to other parabolic problems. The next problem is a system of elliptic equations $\varepsilon^2 \Delta \mathbf{u} = A(x,y)\mathbf{u} + \varepsilon^2 f(\mathbf{u},x,y,\varepsilon)$ defined in a bounded planar domain, subject to Dirichlet boundary conditions. For a domain having smooth boundary, a uniform solution having boundary layer functions along the entire boundary is constructed with a remainder of order ε^{n+1}. The method is also applied to a system of parabolic equations and an example from nonisothermal chemical reaction. The fifth type of equation is of the form $u_t + s(x,t)u + \varepsilon F(u,x,t,\varepsilon) + f(x,t) = \varepsilon^2 u_{xx}$ defined in $x \in (0,l)$, $t \in (0,\infty)$, subject to homogeneous Dirichlet boundary conditions at $x = 0$, $x = l$ and a periodic condition in time with the period 2π. An approximation with boundary layer functions at $x = 0$, $x = l$ is derived with a remainder of order ε^{n+1}. Several variants of this problem are also studied. The last class of problems includes a first-order hyperbolic equation of the form $\varepsilon\{u_t + \Gamma(x,t)u_x\} = a(x,t)u + f(x,t)$ together with a hyperbolic system of two first-order differential equations and telegraphic equations.

Chapter 4 provides asymptotic solutions for four applied problems. The combustion process with a first-order autocatalytic reaction consists of a system of two time-dependent reaction diffusion equations in the dimensionless form $\varepsilon\theta_t - a\theta_{xx} = (v_0 + v)(1-v)\exp(\theta), \varepsilon v_t - b v_{xx} = \varepsilon(v_0+v)(1-v)\exp(\theta)$ for $x \in (0,1), t \in (0,T)$, subject to the homogeneous initial conditions, the homogeneous Dirichlet boundary conditions for θ at $x = 0$, $x = 1$, and the homogeneous Neumann boundary conditions for v at $x = 0$, $x = 1$. A uniform solution involving initial layer functions is given with a remainder of order ε^{n+1}. For a heat conduction process in a thin rod, one derives the parabolic reaction diffusion equation of the form $\varepsilon^2 u_t - a(x)\{\varepsilon^2 u_{xx} + u_{yy}\} = \varepsilon^2 f(u,x,t)$ defined in the domain $x \in (0,1)$,

$y \in (0,1)$, $t \in (0,T)$, subject to an initial condition, the Dirichlet boundary conditions at $x = 0$, $x = 1$, and the Robin boundary conditions at $y = 0$, $y = 1$. A uniform approximation containing an initial layer function, boundary layer functions at $y = 0$, $y = 1$, and corner layer functions is constructed with the remainder of order ε. A one-dimensional semiconductor device is described as a system $\mu^2 E'(x) = p - n + N(x)$, $n'(x) = -nE + I_n$, $p'(x) = pE - I_p$ defined in $x \in (0,1)$, subject to the boundary conditions $n(0) = p(0)$, $n(1) = 1$, $p(1) = 0$. A uniform approximation of the zeroth order having boundary layer functions at $x = 0$, $x = 1$ is given. Its extension to a two-dimension model is provided as well. The fourth problem is a model for the propagation of excitation in a nerve axon, which can be formulated as the FitzHugh-Nagumo system $\varepsilon u_t - \varepsilon^2 u_{xx} = u(a-u)(u-1) - v + I$, $v_t = bv - \gamma v$. A uniform approximation for the relaxation wave solution (or periodic traveling wave solution) of this system involving shock layer functions is constructed with a remainder of order ε. This chapter is furnished with references to problems arising from optimal control, rigid body dynamics under electro- and hydrodynamic influences, a variety of semiconductor structures, molecular aerodynamics, theory of alloys, theory of neutrons, theory of epidemics, heat and mass transfer in a two-component medium, and acoustic oscillations in a medium with small viscosity.

Typesetting of mathematical equations in the book is better than that of its 1990 Russian counterpart. Some obvious typographical errors remain, however. Compared to other books in the area of singular perturbations, the strength of this book is to blend nicely systems of semilinear differential equations, linear and semilinear partial differential equations, as well as some applied problems. Experts in the area of singular perturbations and related fields should find part, if not all, of the book extremely useful. It can also serve as a textbook for a two-semester graduate course in singular perturbations.

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31[65-06, 76-06, 78-06, 35L05, 65C20]—*Mathematical and numerical aspects of wave propagation*, Gary Cohen (Editor), SIAM, Philadelphia, PA, and INRIA, Rocquencourt, France, 1995, xiv+808 pp., $25\frac{1}{2}$ cm, softcover, $106.00

These are the proceedings of the third international conference on the topic of the book title. The conference was held in April 1995 in French Riviera and featured 8 invited lectures and 88 contributed papers. The book contains short articles or abstracts of 7 invited lectures and 86 contributed papers.

The invited lectures are mostly reviews on different aspects of wave propagation, including multiple scattering, numerical methods for electromagnetic equations, large-time asymptotics for certain wave equations, homogenization and Wigner transforms (only an abstract), inverse problems via layer stripping, approximation via distributed approximating functionals, and applications to marine science. These papers may serve as a good introduction and reference source for people interested in the recent development of wave propagation. Not all the major areas in wave propagation, though, are covered in the invited lectures. For example, wavelets, an important tool in approximations for wave phenomena, are missing.

The contributed papers consist of all oral presentations in Part II and poster presentations in Part III. Part II is also divided into 12 sections, covering water waves, boundary integral equations, numerical methods, electromagnetism, homogenization and asymptotic analysis, absorbing boundary conditions, scattering, guided waves, parallel processing, domain decomposition methods, optimal control and inverse problems, and nonlinear waves. The division of papers into these categories is not sharp though. Many papers may actually fit in different sections. The quality of these contributed papers varies; however, most are good papers containing relevant and recent research results in wave propagation. Some are excellent papers containing important results. About half of the contributed papers are from France.

This book is of interest to researchers in the broad area of wave propagation.

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32[34-01, 34-04]—*Differential equations with Mathematica*, by Kevin R. Coombes, Brian R. Hunt, Ronald L. Lipsman, John E. Osborn, and Garrett J. Stuck, Wiley, New York, 1995, vi+218 pp., $23\frac{1}{2}$ cm, softcover, $20.95

The authors' preface and introduction give clear and accurate statements of their goals and the content of the book, some of which are quoted here.

"We designed this supplement to accompany" [1] ".... It could, however, easily be used in conjunction with most other ODE texts." True, but it would be easy to get a different impression because the text is so closely connected to [1]: there is a syllabus for use with [1], many problems are taken directly from [1], and there are a good many references to [1] in the review material. Because the content of a first course in ODEs is so standard, I believe the supplement could be used with any of the popular texts, certainly with [2], the text I have been using.

"This supplement changes the emphasis in the traditional ODE course by using a mathematical software system to introduce numerical methods, geometric interpretation, symbolic computation, and qualitative analysis into the course in a basic way." This book is based on Mathematica; there is a forthcoming version based on Maple. These are both general-purpose computing environments. Other authors have relied upon specialized software for computer experiments in ODE courses. Reviews of some of the possibilities can be found in [3]. There are advantages to both approaches: General-purpose software is harder to learn and more trouble to

apply in a particular context like ODEs. On the other hand, it provides capabilities not available in most packages specialized to ODEs and is a valuable tool for general computation. Some schools amortize the effort required to learn general-purpose software by using the software in a variety of courses.

Having chosen to work with Mathematica, the authors explain in Chapter 2 the interface on four popular platforms. They introduce the package in Chapter 3 and amplify its use in Chapter 8. The philosophy is to develop the necessary understanding of the package gradually and by means of examples. There are a Glossary of Mathematica commands and a chapter of Sample Notebook Solutions. The authors exploit the "Notebooks" in Mathematica that allow a student to include computations and graphics when writing up an experiment. "Engineers and scientists have to develop not only skills in analyzing problems and interpreting solutions, but also the ability to present coherent conclusions in a logical and convincing style." As an absolute beginner working entirely on my own, I found the authors' development of Mathematica to be quite satisfactory.

The authors' approach is to review a mathematical topic in a chapter and follow it with a substantial set of problems on the topic. The review is brief, clear, and illustrated with examples. There are six sets of problems. Along with the general instruction on using Mathematica, the reader has the examples and a sample solution to assist in solving the problems. I found the problems to be interesting illustrations of topics taken up in a standard first course on ODEs.

One of the virtues of a general-purpose package like Mathematica is that it is possible to solve ODEs symbolically as well as numerically. Besides direct solution by symbolic means, it is also possible to use Laplace transforms and series expansions. The authors take up all these techniques, but they emphasize the more generally applicable numerical solution. They supplement the usual ODE course by providing a rather nice chapter on numerical methods.

I have read this book carefully, checked all the examples of the text and the sample notebooks (including, thanks to Wolfram Research, the results stated for several versions of Mathematica), and worked problems from each of the sets. I encountered very few errors, none of which was serious.

References

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33[39A05, 44A12, 65T10, 94A12]—*The DFT: An owner's manual for the discrete Fourier transform*, by William L. Briggs and Van Emden Henson, SIAM, Philadelphia, PA, 1995, xvi+434 pp., $25\frac{1}{2}$ cm, softcover, $37.50

This is an enormously useful handbook on the discrete Fourier transform (DFT) written for scientists, engineers, and applied mathematicians. It starts with a few pages of historical and practical introduction and moves on to cover the theory and the computational aspects of the DFT and a sample of DFT applications. The authors succeed admirably in their ambitious aim to present a comprehensive treatment of many facets of the DFT.

Chapter 2 on various paths to the definition of the DFT and Chapter 3 on the basic properties of the DFT form the core of the book. The remaining seven chapters can be read more or less independently. Chapters 4 and 5 are devoted to symmetric DFTs and multidimensional DFTs, respectively. Chapter 6 discusses in great detail the pointwise and mean-square errors in the DFT, with the Poisson summation formula playing a central role in the analysis. An interesting selection of applications is presented in Chapter 7: difference equations, digital signal processing, seismic exploration, and image reconstruction in tomography. Transforms related to the DFT, such as the Laplace transform, the z-transform, the Chebyshev transform, and the Hartley transform, are treated in Chapter 8. The fundamental connections between the DFT and quadrature rules are explored in Chapter 9. The final chapter presents an overview of the fast Fourier transform. There is also a table of DFTs in an appendix, a bibliography of 166 items, and a very detailed index.

The authors have taken great care to give a lively and well-motivated account of the subject. There is a lot of illustrative material in the form of case studies, examples from applications, and numerical data. Each chapter concludes with a list of problems that ask for detailed proofs of theoretical results or computational work and sometimes offer an outlook on new territory. A welcome feature are the historical and biographical notes and remarks interspersed throughout the text. There are only very few irritating misprints: "Noble Prize" instead of "Nobel Prize" in the footnote on p. 45, and a sign error in the definition of the DFT on the inside front and back covers. The book can be strongly recommended both as a reference work for practitioners and as an introductory textbook.

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34[65-00, 65-04]—*A numerical library in C for scientists and engineers*, by H. T. Lau, CRC Press, Boca Raton, FL, 1995, xviii+795 pp., 26 cm, $69.95

During 1974–78 Dutch numerical mathematicians cooperated in building the so-called NUMAL library (NUMerical procedures in ALgol 60), a systematically arranged collection of over 400 numerical modules. The project was coordinated by the Mathematical Centre, Amsterdam. The library contained older collections of numerical algebra procedures developed in the sixties and was completed with results of current research and implementations of algorithms that were publicly available. With the publication (by the Mathematical Centre) of the whole NUMAL manual in their book series in 1981, the project practically ended, as the world was turning its back on Algol 60, and the interest in maintaining this library was rapidly waning. What remains fifteen years later is this manual printed from one-alphabet lineprinter output, and it does not at all look palatable now.

The book under review is the manual of a straightforward conversion of the complete NUMAL library into C. The book contains a diskette with all source code of routines and example programs.

Nearly all NUMAL procedures and their example programs have been converted into ANSI C, and all routine descriptions (purpose of the routine, specification of the parameters, information about the solution method) are now meeting modern standards of typesetting and readability. Owing to the availability of typesetting for mathematical formulas, the descriptions of purpose and method of many routines could actually be supplemented with further explanations that were (for NUMAL) only available in the referenced literature. The typesetting and rewriting of formulas is not perfect and errors occur (which probably do not trouble the reader much), but is a big improvement compared with the NUMAL manual. There is no new extension, algorithms have not been replaced or modified reflecting recent achievements, and the choice of parameters is still the choice that was found useful when employing Algol 60. C utilities are used for imitating the Algol dynamic memory management.

It is difficult to assess the usefulness of this work. One could say that the work was hardly called for, and the programming style in use for Algol 60 procedures twenty years ago by some contributors of the NUMAL library is probably not adhered by them any more. Experienced C programmers might find that the Algol 60 style of making and using arrays is error prone when applied to C implementations. What (relative) merits does this collection have compared with the set of routines known as Numerical Recipes, or compared with well-known libraries for which efforts are continuing to improve methods, the performance on new architectures, the ease of use for users that do not have a PhD in numerical analysis, and the attractive presentation of the computed results?

As can be perceived from the Introduction, the author appreciates the NUMAL-like setup of a library with very technical routines and accessible auxiliaries because of its possibilities for developing new research software using the library modules as building blocks. Also, students could be entertained with assignments to modify example programs in order to provoke some listed error exit of a routine, or they could learn by studying the implementations. This justification for the book is not very pretentious. However, the book is also a tribute to a group of pioneers in numerical software.

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35[01A70, 33-XX, 33C45, 41-XX, 41A55]—*Approximation and Computation: A festschrift in honor of Walter Gautschi*, R. V. M. Zahar (Editor), International Series of Numerical Mathematics, Vol. 119, Birkhäuser, Boston, 1994, xlvi+591 pp., 24 cm, $98.00

Nearly twelve months after the actual event, an International Symposium was held at Purdue University in December 1993 to celebrate the 65th birthday of Walter Gautschi. About 80 delegates from many parts of the world attended this Symposium and over thirty papers were presented. This volume, the 119th in the

ISNM series, is a commemorative issue in Walter's honour and comprises nearly all the papers that were given at the Symposium, together with a few more which were subsequently submitted.

During his distinguished career, Walter has worked principally in four areas of mathematics. These are: (1) Approximation theory, (2) Orthogonal polynomials, (3) Quadrature and (4) Special functions. A list of the thirty-eight papers to be found in this volume, classified according to these areas, appears at the conclusion of this review. Each paper has been refereed and, although Walter has written papers in English, French, German and Italian, every paper in this volume is written in English. But there is more to this volume than just a collection of papers and a photograph of Walter. There is a "Foreword" from John Rice, Head of the Department of Computer Sciences at Purdue, where Walter has spent about half his life. This is followed by an "Introduction" written by R. V. M. Zahar who was the first of Walter's seven Ph.D. students and who has done such a superb job of editing this volume. In his "Introduction", Zahar has given a brief review of each paper and the classification of the papers into the four subject areas is due to him.

However, for this reviewer, the *pièce de résistance* of the volume is Walter's 20-page "Reflections and Recollections", followed by a list of his publications. The latter makes interesting reading; 150 articles are given ranging from books to chapters in books, papers in refereed journals, Conference Proceedings, translations and a miscellaneous collection labelled "other". It is a most impressive record of mathematical activity which started in 1951 and continues unabated to this day (there are four articles in the "to appear" category). Under one item of the "other" category, one reads that Walter has had 97 book reviews published in *Mathematics of Computation* and no less than 450 reviews of technical articles and books published in *Mathematical Reviews*! As further evidence of his mathematical activity, John Rice remarks in his "Foreword", that Walter has served a total of 76 years on the editorial boards of various mathematical journals with the last nine years as Editor-in-Chief of *Mathematics of Computation*. This journal alone receives more than one paper each working day of the year and each has to be considered in some way by Walter. But a list of publications gives only an outline of anyone's career; Walter has added substance to this outline in his twenty autobiographical pages.

He starts out by saying that he had "no intention to give a talk" at the Symposium but "only after persistent persuasion" did he agree to give "an informal personal talk reflecting on my career". Persistence has been well rewarded, for the result is a fascinating account of the first half of Walter's life, his family background, his undergraduate years and the postgraduate years which set him on the mathematical track he has followed ever since. As a student at the University of Basel he worked for two years as Professor Ostrowski's assistant. What a marvellous mathematical training that must have been for a young man! His own first excursion into research was on a variant of a *graphical* method for solving ordinary differential equations which was published in 1951. In 1954, Walter was given a two-year travelling fellowship. The first year was spent in Rome, where he worked at Professor Picone's Institute for Computational Mathematics. The second year was spent at Aiken's Computation Laboratory at Harvard, where he worked on the Mark III computer, "a massive electronic computer with magnetic drum storage and lots of vacuum tubes which had a tendency to blow out every so often". The programming was in basic machine language. What nostalgia this evokes, and one wonders how many people, now under the age of 60, have had the doubtful

pleasure of having to write programmes this way but, in the pre-Fortran, pre-Algol days, there was no alternative. After Harvard, Walter went to the National Bureau of Standards in Washington, D.C., and while there contributed two Chapters to Abramowitz & Stegun's, "Handbook of Mathematical Functions".

After Washington, there followed a few years at Oak Ridge, Tennessee, where Walter worked with Alston Householder's group. Here he became fascinated by J. C. P. Miller's backward recurrence algorithm for the computation of Bessel functions and this led to his work on the minimal solution of three-term recurrence relations. It was also at this time that his interest in orthogonal polynomials and Gaussian quadrature began. For this we owe a debt of gratitude to an unknown Chemist who asked Walter how to construct Gauss quadrature rules associated with the weight function $w(x) = 4/(\pi^2 + \ln^2((1+x)/(1-x)))$ on the interval $(-1, 1)$. One is reminded of that other Chemist, Mendelieff, who asked Markoff how large the derivative of a polynomial, of a given degree, could become on the interval $[-1, 1]$. Perhaps we should all pay more attention to questions posed by our colleagues in Chemistry. Walter's thirty years or so at Purdue are, however, covered in a mere $2\frac{1}{2}$ pages. There is a need for a sequel covering those years, although his contribution to de Branges' proof of the Bieberbach conjecture has been well documented.

Although the list of thirty-eight papers is appended, I will say no more of them here. This volume has been very well produced and its Editor is to be congratulated on a magnificent tome which should be in every Numerical Analyst's library. To assist the reader, there is both a comprehensive Subject Index and an Author Index listing every reference given in the papers. Not surprisingly, but very fittingly, the list of references under the name of "Gautschi W." has substantially more entries than that of any other author.

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36[11Txx, 94A60, 94B60]—*Introduction to finite fields and their applications*, 2nd ed., by Rudolf Lidl and Harald Niederreiter, Cambridge Univ. Press, Cambridge, 1994, xii+416 pp., $23\frac{1}{2}$ cm, $44.95

This is the second edition of the well-known 1986 text by Lidl and Niederreiter. The latter is, in turn, based on their 1983 monograph in volume 20 of the Encyclopedia of Mathematics and its Applications.

The second edition hardly differs from the first one: the authors have updated the bibliography and have extended their historical and bibliographical notes. The main text remains unaltered: a discussion of the theory of finite fields with some applications to coding theory and cryptography. The approach is easygoing, with a student reader in mind: the algebraic prerequisites are minimal and each chapter contains a lot of exercises.

The book contains ten chapters. The first one contains a rather general algebraic introduction. Chapters 2 and 3 contain the basic theory. Chapters 4, 5, 6 and 7 are devoted to factorization algorithms for polynomials, exponential sums, linear recurrences and designs, respectively. Chapters 8 and 9 contain the applications to coding theory and cryptography. Finally, Chapter 10 contains some tables of finite fields and irreducible polynomials over finite fields.

The authors introduce Gaussian sums, linearized polynomials etc., but they do not discuss the deeper properties of finite fields and polynomial equations over finite fields. They do not even state the main consequences of the work of P. Deligne or even A. Weil, on varieties over finite fields. This is a pity, but perhaps understandable, since a full discussion of their results in a book of this sort seems quite difficult.

On the other hand, many of the recent results on finite field theory obtained by researchers in computational number theory and computer algebra are quite accessible and can easily be explained to anyone aware of the contents of Chapters 2 and 3 of this book. The authors have not included these new results in their second edition, but have left the book as it was: an easygoing introduction to the theory of finite fields.

René Schoof

37[11-02, 11Y05, 11Y11, 11Y16, 11Y35, 11Y40]—*Prime numbers and computer methods for factorization*, 2nd ed., by Hans Riesel, Progress in Mathematics, Vol. 126, Birkhäuser, Boston, 1994, xvi+464 pp., 24 cm, $69.50

The first edition was revised earlier [1]. Although both editions have the same number of pages, much new material has been placed in the second edition. Hardly any text from the first edition has been omitted from the second edition. The size has been maintained by replacing some tables by shorter ones and by using a smaller point size.

The long first-edition Chapter 5 on factorization has been split into two chapters in the second edition: "Classical Methods of Factorization" and "Modern Factorization Methods". Most of the new material appears in the second of these new chapters. There are new sections on the multiple-polynomial quadratic sieve, the elliptic curve method and the number field sieve (special and general). A lot of the background material for the book has been relegated to the eleven appendices. The second edition has new appendices on elliptic curves and higher algebraic number fields. With this new material, the second edition is an excellent introduction to all of the best-known factoring algorithms. The only omission I noticed was the FFT versions of the second steps of Pollard's $p-1$ method and the elliptic curve method.

The chapter on recognition of primes in the second edition treats elliptic curve primality proving, namely the tests of Goldwasser-Kilian and Atkin-Morain. Other new items reported in the second edition include: four new Mersenne primes 2^p-1, with $p = 110503, 216091, 756839$ and 859433; a new largest known twin prime pair $1692923232 \cdot 10^{4020} \pm 1$ found in 1993 by H. Dubner; some new first occurrences of large prime gaps and Jaeschke's new primality test for small numbers using strong pseudoprimes. It is reported that the Fermat number F_{22} is now known to be composite by Pepin's test. There is a section on Chebyshev's function $\theta(x)$. The first edition had tables of factors of $a^n \pm b^n$ for $1 \le b < a \le 10, \gcd(a,b) = 1$. The factor tables for $2^n \pm 1$ and $10^n \pm 1$ are retained in the second edition, but the other factor tables are replaced by tables of known factors of the generalized Fermat numbers $a^{2^n} + b^{2^n}$ for $a = 3, 4, 5, 6$, and $1 \le b < a,\ \gcd(a,b) = 1$, and also of $10^{2^n} + 1$ and $12^{2^n} + 1$. The table of Lucas' formulas for cyclotomic polynomials has been lengthened from degree 120 to 180, with its new coefficients contributed by Richard Brent. Many new references have been included.

Like the first edition, this clearly written text on factoring and prime testing will be welcomed by both novices and experts in the field.

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