MATHEMATICS OF COMPUTATION
Volume 67, Number 222, April 1998, Pages 867–875
S 0025-5718(98)00967-3

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

8[65Kxx, 90Cxx]—*Primal-dual interior-point methods*, by Stephen J. Wright, SIAM, Philadelphia, PA, 1997, xx+289 pp., 25 cm, softcover, $37.00

During the last twelve years, a minor explosion has occurred in the literature on optimization, caused by the impact of Karmarkar's projective algorithm for linear programming. Over a thousand papers have been published on the resulting methods, called interior-point algorithms. Early papers provided extensions and clarification of Karmarkar's original method, and some preliminary (and mixed) computational experiments. The connection to classical barrier methods in nonlinear (and linear programming) was established. Two key early papers were Megiddo [2] and Renegar [4]; Megiddo proposed a symmetric primal-dual path-following approach, while Renegar analyzed a simple primal-only path-following method that required only $O(\sqrt{n}\ln(1/\epsilon))$ iterations to attain precision ϵ in the worst case, compared to Karmarkar's $O(n\ln(1/\epsilon))$; here n is the number of inequality constraints.

Monteiro and Adler and independently Kojima, Mizuno, and Yoshise realized Megiddo's program by developing $O(\sqrt{n}\ln(1/\epsilon))$ symmetric primal-dual algorithms. Significantly, both showed how these methods could be extended beyond linear programming: Monteiro and Adler treated convex quadratic programming, while Kojima et al. dealt with the monotone linear complementarity problem, which subsumes linear and convex quadratic programming. Implementations of these primal-dual methods, differing from the theoretical versions by taking much longer steps, began to be developed. The work of Lustig, Marsten, and Shanno was particularly significant here; they showed a practical way to deal with the difficulty of not having an initial strictly feasible iterate from which to start, and exhibited very favorable computational results. Mehrotra introduced a predictor-corrector variant whose ideas have been adopted by all successful primal-dual codes. For a recent review of the state of the art in computational experiments, see the survey article by Lustig, Marsten, and Shanno [1]. At the same time as these developments, Nesterov and Nemirovskii were investigating a very general theoretical approach to certain convex programming problems using primal algorithms based on so-called self-concordant barrier functions; their work culminated in the brilliant but technically demanding monograph [3].

The field of interior-point methods has now reached a fairly mature state, and it is clearly time that some generally accessible books dealt with the subject. A few texts in operations research or mathematical programming include some material on Karmarkar's method or the (primal or dual) affine-scaling method, but it now seems clear that the methods of choice for linear programming and the extensions mentioned above are primal-dual methods. Several books have appeared dealing with interior-point methods in the last year or so, e.g., those of Roos, Terlaky, and

Vial [5], Saigal [6], and Vanderbei [7]. The current book of Stephen Wright, concentrating on primal-dual methods, provides a highly readable and up-to-date account of the field. It is accessible to numerical analysts and applied mathematicians with some background in optimization, and is also suitable for an advanced graduate class. Before discussing it in depth, let me make some general remarks about the two main classes of methods for linear programming: simplex and interior-point algorithms.

The feasible region of a linear programming problem is a convex polyhedron. Dantzig's well-known simplex method exploits the fact that the extremum of a linear function over such a set must be attained at an extreme point; it iterates among these extreme points, moving along the edges of the feasible region. It has proven remarkably efficient over the last fifty years, as problem sizes have risen from a few tens to those involving tens of thousands of (equality) constraints and hundreds of thousands of (nonnegative) variables. It is a totally astounding fact that the number of iterations required typically remains a small multiple of the number of constraints, possibly multiplied by a term that is logarithmic in the number of variables, although the performance degrades as the problems reach the largest scale indicated above. Theoretical studies on the diameter of polytopes and on probabilistic analyses of variants of the simplex method give a partial, but far from convincing, justification for its efficiency. Consideration of small examples suggests that it might be more efficient to travel through the interior (or relative interior) of the feasible region to reach an optimal solution. While several such methods were tried (and found wanting) in the fifties, this view is the basis of the new interior-point methods. These also exhibit outstanding (and partially inexplicable) behavior by using sophisticated techniques to generate efficient search directions, and seem to require a number of iterations rising extremely slowly, from ten or so for the smallest problems to a hundred or so for the largest. Each iteration is much more expensive than a simplex iteration (depending very much on efficient sparse linear algebra techniques and the structure of the problem), paving the way for serious competition between the two classes of methods. At present, it appears that interior-point methods are slightly slower for small and medium-sized problems, comparable on large problems, and superior (depending on the structure of the problem) for very large scale problems.

Wright's book starts with an excellent introduction, describing the main classes of primal-dual methods and the issues that will be dealt with in future chapters. Chapter 2 gives some background on linear programming and interior-point methods. It is generally clear, but not necessarily to one with no prior linear programming exposure. The development relies on the Karush-Kuhn-Tucker optimality conditions, which are discussed further in an appendix. In particular, the central path, which plays a fundamental role in interior-point methods, is defined and its existence established. It is somewhat surprising that the main theorem here does not prove that there is a unique solution to the central path equations for each positive τ, nor is it shown (a simple consequence of the implicit function theorem) that the set of solutions forms a path. An excellent historical overview of the development of interior-point methods is given. Chapter 3 then provides a good informal treatment of complexity theory. There is some confusion of rational and real-number complexity; the author suggests ensuring that the coefficient matrix A has full row rank by performing a QR factorization, but this will destroy rationality of the entries.

Chapters 4, 5 and 6 describe the main methods of the book; potential-reduction, path-following, and infeasible-interior-point algorithms. Each of these chapters provides a beautifully written and concise development of the basic motivation, properties, and convergence behavior of these methods, with technical lemmas well planned and motivated so that the reader is led to a clear understanding of the subject. The only change I would like is for the equations defining the search directions to be repeated, rather than continually referring back to Chapter 1. Also, Theorem 6.1 on the last class of methods really needs an explicit assumption that the primal and dual linear programming problems do have feasible solutions.

The next three chapters give important further refinements and extensions of the basic methods. Chapter 7 deals with superlinear convergence, a subject to which the author has made significant contributions. This rather technical subject is very cleanly presented, including treatment of both theoretical and practical algorithms with fast asymptotic convergence. Then Chapter 8 shows how various extensions of linear programming, in particular linear complementarity and quadratic programming problems, can be dealt with using the same basic tools and algorithms. There is a short discussion of the very active area of semidefinite programming also. Chapter 9 discusses refinements of the methods that permit the detection of infeasibility. Unfortunately, the proof of Theorem 9.1 in the appendix is not quite complete, and the proof of Theorem 9.3 refers to Theorem 2.2, which should be Corollary 2.2 and does not quite apply. Nevertheless, the topic is well treated.

Finally, the author covers practical aspects, in particular Mehrotra's predictor-corrector algorithm, and details of implementations, in the final two chapters. These give excellent background for the issues that really determine the effectiveness of primal-dual interior-point methods for solving large-scale problems in practice.

Two appendices give some background results and proofs that are postponed from the main development for clarity and an excellent listing of software available. This reviewer has to compliment the author particularly on his efforts to make the book as up-to-date as possible (there are several references to papers that appeared in 1996) and his intention to maintain a web site keeping the list of available software current. (The author has already performed a singular service to the interior-point methods community by running a web site for dissemination of announcements of new papers and meetings and maintaining an archive of preprints.)

While discussing only a subset of interior-point methods (although, as indicated above, primal-dual methods are by far the most important in practice), and confining himself in the main to linear programming for simplicity, the author provides an exceptionally broad and clear treatment of this very significant area of optimization. Many authors concentrate on polynomial complexity issues, theoretical discussion of asymptotic convergence, or practical aspects of implementations. Steve Wright gives a very balanced and excellently written treatment of all these topics. I highly recommend this book to both specialists in these methods and general readers interested in this fascinating area of computational mathematics.

References

[1] I. J. Lustig, R. E. Marsten, and D. F. Shanno, *Interior point methods for linear programming: Computational state of the art*, ORSA J. Comput. **6** (1994), 1–14. CMP 94:08

[2] N. Megiddo, *Pathways to the optimal set in linear programming.* In N. Megiddo, editor, Progress in Mathematical Programming: Interior Point and Related Methods, pages 131–158, Springer-Verlag, New York, 1989. MR **90c:**90147

[3] Yu. E. Nesterov and A. S. Nemirovskii, *Interior-point polynomial algorithms in convex programming: Theory and algorithms*, SIAM Publications, SIAM, Philadelphia, PA, 1994. MR **94m:**90005

[4] J. Renegar, *A polynomial-time algorithm, based on Newton's method for linear programming*, Math. Programming **40** (1988), 59–93. MR **89b:**90130
[5] C. Roos, T. Terlaky, and J. P. Vial, *Theory and algorithms for linear optimization*: *An interior point approach*, John Wiley, Chichester, 1997. CMP 97:13
[6] R. Saigal, *Linear programming*: *A modern integrated analysis*, Kluwer Academic Publishers, Boston, 1995. MR **97h:**90003
[7] R. J. Vanderbei, *Linear programming*: *Foundations and extensions*, Kluwer Academic Publishers, Boston, 1996. MR **97i:**90002

Michael J. Todd
School of Operations Research
Cornell University
Ithaca, NY 14853

9[65-01, 65-04]—*Computational mathematics in engineering and applied science: ODEs, DAEs, and PDEs*, by William E. Schiesser, CRC Press, Boca Raton, FL, 1994, xii+587 pp., 26 cm, $74.95

This is an engineering book on scientific computation, a book on computing by Fortran programming. More than half of the contents are source codes (with many comment lines), data files, and output information. There are no convergent theories.

The main stream of the book is the method of lines. The basic idea in the method of lines for PDEs is to replace the spatial derivatives with algebraic approximations, thereby leaving only the time derivatives. The procedure produces an ODE at each spatial grid, and the resulting system of ODEs can be solved by a known ODE solver. Basically, the method of lines is a systematical way of using ODE solvers to integrate PDEs.

The methodology is illustrated through detailed examples in Fortran 77 coding. The underlying mathematics of the methods are not discussed in detail, and theoretical analyses are not provided. Effectiveness and validity of the methods are discussed through observation from the output of the computation, and the error analyses of computed solutions are presented numerically as part of the example applications.

All examples are coded in the same format and presented in the following structure: (1) start from an example which includes a differential equation and initial and/or boundary conditions; (2) explain the coding, list all subroutines and data files, and comment on their purposes; (3) list some output and draw some conclusions from computing experience. Almost every example calls for some library routines, usually, differential routines and integration routines. Instead of writing an entire code from the very beginning, the author proposes to use quality library routines which have achieved the status of international standards. However, the author does not recommend the uninformed use of library routines, since some knowledge of differential equation characteristics and numerical methods will invariably lead to more effective use of existing packages.

The intention of the book is not to provide the state-of-art methods, rather it is to introduce some practical methods proved to be effective in dealing with some typical problems. Therefore, the book is a source of some practical tools for numerical methods of ODEs and PDEs. Because of its elementary contents, the book is also suitable for beginners.

The reader may request Fortran source codes from the author for problems and library routines discussed in the book. Since the codes are for PCs, a Unix user may need some minor editing after downloading a source code, for example, to get rid of

^M

at the end of each line.

The organization of the book is according to equations' types, rather than methods. Chapter 1 is a brief summary (only 16 pages) of various types of ODEs, DAEs, and PDEs. Chapter 2 occupies almost one third of the contents of the book which covers the numerical solution of ODEs and DAEs. The Runge-Kutta methods are introduced and discussed in detail. The use of a nonlinear algebraic equations solver SNSQE and the use of a library routine RKF45 are illustrated by an orbit problem. The issue of stability is briefly discussed. Multiple-step methods of both explicit (for nonstiff equations) and implicit (for stiff equations) are discussed in some detail. This chapter also includes some examples of unconventional uses of ODE integrators.

The remaining three chapters are devoted to PDEs. Chapters 3 and 4 discuss the PDE with first order time derivative while Chapter 5 considers PDEs with zero or second order in time. Chapter 3 has only two sections with the first one for PDEs containing first order spatial derivative and the second one for PDEs having second order spatial derivatives.

Chapter 4 is the longest chapter of the book with almost 200 pages. Many examples are demonstrated including diffusion equations, convective diffusion equations, the one-dimensional Burgers' equation and its modification, a PDE with mixed partial derivatives (this is an example with two spatial variables), an example that has strong nonlinearity, and others. A variety of topics are discussed in this chapter, including nonlinear PDE solutions with DAE solvers, nonuniform spatial grids in one-dimension, diffusion in two regions, and band-width reduction in the method of lines. Other than the finite difference method, this chapter also discusses the finite volume method briefly and linear finite elements in detail for a model parabolic equation with one spatial variable.

Chapter 5 basically considers wave equations with one spatial variable and the two-dimensional Poisson equation. For the latter, a time derivative is added to the equation in order to apply the method of lines.

In addition, Appendix C lists some applications in practice using the methods discussed in the book.

The book contains many interesting examples, including some nonlinear problems from practical applications. However, most examples for PDEs have only one spatial variable. For those examples with two spatial derivatives, domains are all regular. Nonrectangular domains and general meshes are not addressed. There is no example for three spatial derivatives.

As a final remark, the book does not have enough subsections and the arrangement of the material makes it a little difficult to locate the needed information later.

Zhimin Zhang
Department of Mathematics
Texas Tech University
Lubbock, TX 79409

10[33-01]—*Special functions: An introduction to the classical functions of mathematical physics*, by Nico M. Temme, John Wiley & Sons, Inc., New York, NY, 1966, xii+374 pp., 24 cm, hardcover, $54.95

Anyone who wants to learn about special functions or wants to teach a course in the subject can choose from a variety of superb books, [and], [car], [hoc], [leb], [nik], [rai], [spa], [sri]. All of these books display a personal slant on the subject; in their design and choice of material they are far more unresemblant than books on most mathematical topics. Perhaps the subject appeals especially to the iconoclast. Rainville's book stresses formal identities satisfied by the special functions, and the main tool used in obtaining these identities is what is called eponymously Sister Celine's technique (for an interesting biography of Sister Mary Celine Fasenmyer, a student of Earl Rainville, the reader should consult the whimsical and brilliant oddity [pet]). Carlson's book stresses hypergeometric functions, and its primary tool is Dirichlet averaging. Nikiforov and Uvarov's book makes much of finite difference techniques, so is able to offer a unique and highly unified theory of orthogonal polynomials of a discrete variable (though, I think, few readers will have the patience to indulge the authors's finicky derivations). Lebedev's book, a beautiful little opus from which I have taught from time to time, makes frequent and shamelessly ingenious use of multiple integration techniques to obtain special function identities, and furthermore has, for each special function discussed, a section on the applications of that function. And so on and on. In addition, there are many books treating specialized topics: orthogonal polynomials, group theoretic methods in special functions, integral transforms, asymptotics of special functions, elliptic functions, etc. And there are massive enchiridions, too: [erd], [luk].

The question is: do we need yet another book on special functions? Yes, we always need more good books. Temme's book is a very good one, indeed. Though it contains material that is pretty standard, I found intriguing items I have never seen before. The plan of the book, too, is standard. Chapter 1, on Bernoulli, Euler and Stirling numbers, begins with a fascinating puzzle that has been elucidated in a recent and celebrated paper [bor]:

We all know that

$$\frac{\pi}{2} = 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}.$$

Taking 50 000 terms of the sum gives the astonishing result

$$2\sum_{n=1}^{50\ 000} \frac{(-1)^{n-1}}{2n-1}$$
$$= 1.5707\ \overline{8}6326\ 79489\ \overline{7}6192\ 31321\ \overline{1}9163\ 975\overline{20\ 5}2098\ 58\overline{331\ 46876}$$

where I have placed a bar over the figures that are in error. Why astonishing? Experience tells us that if one digit of an answer is in error, all the following will be, too. What can account for the strange distribution of erroneous digits in the above figure? A sly idea: to begin the book with a problem so intriguing and involving it would vivify the most enervated reader. In this chapter Temme points out that the same phenomenon occurs with Euler's series for $\ln 2$ and reveals that the mystery is explicable in terms of the properties of Euler numbers and of a little-known procedure called Boole's summation method, an analog of Euler's summation formula which uses Euler, rather than Bernoulli, polynomials:

Let f have $k \geq 1$ continuous derivatives. Then

$$f(1) = \frac{1}{2}\sum_{i=0}^{k-1}\frac{E_i(1)}{i!}[f^{(i)}(1) + f^{(i)}(0)] + R_k,$$

$$R_k = \frac{1}{2(k-1)!}\int_0^1 f^{(k)}(x)E_{k-1}(x)dx,$$

where $E_k(x)$ is the Euler polynomial.

The chapter concludes with an abundance of challenging and interesting exercises, a feature characteristic of the book.

Rather than devote Chapter 2 to a class of special functions, the author declares time out to explain an assortment of techniques that are useful in obtaining results in special functions, for instance, integrating series termwise, interchanging the order of integration in double integrals, expanding integrals asymptotically, Watson's lemma, the saddle point method.

Chapter 3 introduces the Gamma function. In an unusual move, the author states, but does not prove, the Bohr-Mollerup theorem: *the gamma function is the only positive logarithmically convex function with $f(1) = 1$ that satisfies the recursion formula $f(x+1) = xf(x)$.* As far as I know, this theorem is not stated nor proved in other books on special functions. However, the proof of the theorem is neither lengthy nor demanding [con], and I would like to have seen it here. The author lists the usual properties of $\Gamma(z)$, but sometimes includes unusual proofs, for instance, Legendre's duplication formula is proved from the integral representation of the Beta function. Having developed the theory of the Euler-Maclaurin summation procedure, the author has established the foundation for an elaborate and satisfying explanation of Stirling's formula. This chapter contains the most appealing treatment of the Gamma function available in an introductory text.

In Chapter 4, in an obviously eclectic mood, the author strikes out into the territory of differential equations, but it makes a great deal of sense to do so. Almost all the special functions are solutions of differential equations. He talks about the wave equation—separating variables—then about differential equations in the complex plane, namely, second order differential equations with three singular points, including the hypergeometric equation. He explains how to develop solutions in the neighborhood of singular points, e.g., the method of Frobenius. An understanding of the hypergeometric function, the subject of Chapter 5, depends on this sedulously prepared infrastructure.

Succeeding chapters treat, in a fairly traditional manner, orthogonal polynomials, the confluent hypergeometric function, Legendre functions, Bessel functions. In Chapter 10 the author discusses separating the wave equation in various coordinate systems; this, too, is an unusual touch, and he shows how the various special functions previously discussed arise out of this process.

The author's special area of expertise is the confluent hypergeometric function, and, in particular, the incomplete gamma functions, and, still more in particular, the asymptotic theory of these functions. This makes him eminently qualified to discuss, in Chapter 11, special statistical distribution functions, since most of these functions are special cases of the confluent hypergeometric functions. No other elementary book contains a description of the uniform asymptotic expansion of these functions, a very valuable feature.

Chapter 12 contains a brief treatment of elliptic functions, and the final chapter, Chapter 13, discusses the numerical computation of special functions, including the use of recurrence relations, Miller's algorithm, and techniques for computing with continued fractions. Most other books maintain an attitude of sublime indifference to the actual computation of special functions.

All in all, the book is a superb achievement. It is beautifully and clearly written, the material is cannily organized and each topic takes its carefully designated place in the overall scheme of things. The idea of first describing an assembly of techniques that will facilitate the analysis of the special functions to follow is a very fruitful one, and it makes the theory look much more coherent than in other books, which often have the ad hoc quality of a potpourri of special functions thrown together with no unifying pattern underlying the exposition.

I think the book would be ideal as a text in a one semester or even a two quarter course. The author has included applications, but not so many as to encumber the book, a fault I found in the otherwise admirable Lebedev.

References

[and] Larry C. Andrews, *Special functions for engineers and applied mathematicians*, Macmillan, New York, 1985. MR **86g:**33001

[bor] J. M. Borwein, P. B. Borwein, and K. Dilcher, *Pi, Euler numbers and asymptotic expansions*, Amer. Math. Monthly **96** (1989), 681–687. MR **91c:**40002

[car] B. C. Carlson, *Special functions of applied mathematics*, Academic Press, New York, 1977. MR **58:**28707

[con] John B. Conway, *Functions of one complex variable*, Springer-Verlag, New York, 1995. MR **96i:**30001

[erd] A. Erdélyi, et al., *Higher transcendental functions*, 3 v., McGraw-Hill, New York, 1953. MR **15:**419i; MR **16:**586c

[hoc] Harry Hochstadt, *The functions of mathematical physics*, Wiley-Interscience, New York, 1971. MR **58:**17241

[leb] N. N. Lebedev, *Special functions and their applications*, Prentice-Hall, Englewood Cliffs, NJ, 1965. MR **30:**4988

[luk] Yudell Luke, *The special functions and their approximations*, 2 v., Academic Press, New York, 1969. MR **39:**3039; MR **40:**2909

[nik] A. F. Nikiforov and V. B. Uvarov, *Special functions of mathematical physics*, Birkhäuser, Boston, MA, 1988. MR **89h:**33001

[pet] M. Petkovsek, H. M. Wilf, and D. Zeilberger, $A = B$, A. K. Peters, Wellesley, MA, 1966. MR **97j:**05001

[rai] Earl D. Rainville, *Special functions*, Macmillan, New York, 1960. MR **21:**6447

[spa] Barry Spain, *Functions of mathematical physics*, Van Nostrand Reinhold, New York, 1970.

[sri] H. M. Srivastava, *Special functions in queuing theory and related stochastic processes*, Academic Press, New York, 1982. MR **83m:**60129

Jet Wimp
Department of Mathematics and Computer Science
Drexel University
Philadelphia, PA 19104
E-mail address: jwimp@mcs.drexel.edu

11[44A10, 44A20, 44-00, 33-00]—*Tables addenda for Laplace transforms*, by R. G. Buschman, Buschman, Langlois, OR, 1996, vi+169 pp., 28 cm, vinyl cover, spiral bound, $17.50

This addenda includes three tables for direct transforms, inverse transforms and two-dimensional inverse transforms. The material provides an addenda to the tables of G. E. Roberts and H. Kaufman, *Table of Laplace Transforms* (1966) and D. Voelker and G. Doetsch, *Die zweidimensionale Laplace-Transformation* (1950). It is also useful in connection with F. Oberhettinger and L. Badii, *Tables of Laplace Transforms*. The format and notations follow Roberts and Kaufman.

The introduction gives a short overview of new contributions in the literature concerning boundary value problems, heat and diffusion problems, and so on, that can be solved by use of the Laplace transforms.

Nico M. Temme