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THE TRANSLATION PLANES OF ORDER 49 AND THEIR AUTOMORPHISM GROUPS

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Abstract. Using isomorphism invariants, we enumerate the translation planes of order 49 and determine their automorphism groups.

1. Introduction

We describe the enumeration of the translation planes of order 49 and the computation of their automorphism groups. We follow the same pattern of classification used by the second author in [10] to handle the translation planes of order 27, but we use slightly more refined methods.

The classification of the translation planes of order 49 was also obtained previously by R. Mathon and G. Royle [15]. However, these authors use quite different methods. In particular our solution to the isomorphism problem, a significant component of any enumeration of projective planes of a fixed order, is entirely different. We systematically use isomorphism invariants to solve this problem. This permits a different search strategy than that used in [15], resulting in a significant reduction in the computational effort. As a consequence, the complete enumeration of isomorphism classes of the spread sets corresponding to the translation planes of order 49, can be repeated by anyone with standard computing resources.

The isomorphism invariants we use originate from an invariant of general (finite) projective planes which was proposed by J. H. Conway and investigated by C. Charnes in [3]; see [4]. To distinguish the isomorphism classes of translation planes, we use the *fingerprint*, the *Leitzahl* and *Kennzahl*, which are defined in [4] and [10], respectively. These invariants can be computed for the translation planes of order 49 without much overhead. Furthermore, our invariants can be used to determine the automorphism groups of the planes. Therefore, they are of independent interest, and we give in §3 a self-contained description of the invariants and of our algorithm for generating the spread sets.

We provide in §7 a detailed description of the automorphism groups of the translation planes of order 49. For each plane we give the following information: the order of the automorphism group; the order of the center; the order of the Fitting factor group; orders of the factors of the derived series; orders of the factors of the composition series; orders of the factors of the lower central series. Structural information of this kind is useful in the study of the geometrical properties of the planes and should suffice to identify each group. The orders of the groups were computed in two independent ways. First by U. Dempwolff, who determined the

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elements of $GL_4(7)$ which leave invariant each spread set. The group orders were computed from these generators with a program written by U. Dempwolff. Independently, GAP [16] routines were used to obtain the aforementioned data from the generating matrices.

In [6] we announced the classification of the translation planes of order 49 whose automorphism groups contain involutory homologies. These planes were used as a check on the correctness of the complete enumeration – each involutory homology plane had to occur in the final list of planes. Finally, the number of isomorphism classes of translation planes of order 49 we obtain, 973 up to polarity, agrees with the enumeration in [15].

In §5 we identify the involutory homology planes and some other planes which have appeared in the literature.

2. Definitions and notation

We recall briefly the results and notation used in the enumeration of translation planes; details can be found in [10] and [13].

Let $W = V \oplus V$, $V = GF(p^n)$, be a $2n$-dimensional vector space over $GF(p)$. A collection $\mathcal{S} = \{V_\infty, V_0, \ldots, V_m\}$ of mutually disjoint n-dimensional subspaces of W is called a *partial spread*. If $m = p^n - 1$, then $\mathcal{S}$ is a *spread* and describes a translation plane of order p^n. By choosing a basis in W we can write $V_\infty = \{(0, v) | v \in V\}$, $V_0 = \{(v, 0) | v \in V\}$ and $V_i = \{(v, vt_i) | v \in V\}$, where $t_i \in G = GL_n(p)$ and $t_1 = 1$. We call $S = \{t_0, t_1, \ldots, t_m\}$ a *spread set* with respect to the coordinate triple $(\infty, 0, 1)$.

A basic property of spread sets is: $(*)$ $\det(t_i - t_j) \neq 0$ for $0 \leq i < j \leq m$. Conversely, each set $S \subseteq G \cup \{0\}$ satisfying $(*)$ defines a spread set. This description determines a spread set only up to conjugacy; see [10]. Replacing (V_∞, V_0, V_1) by some other triple (V_i, V_j, V_k) defines a spread set S'; this is a *coordinatization* of S with respect to (i, j, k).

The possible coordinatizations are obtained from each other by the successive application of the following operations:

$O_1(x)$**:** $S \to xSx^{-1}$ $(x \in G)$,
$O_2(i)$**:** $S \to t_i^{-1}S$ $(1 \leq i \leq m)$,
O_3**:** $S \to S^{-1}$,
O_4**:** $S \to 1 - S$.

Definition 2.1. Two (partial) spread sets S_1, S_2 are said to be *equivalent* if and only if S_1 can be obtained from S_2 by a successive application of operations $O_1(x), \ldots, O_4$.

The problem of enumerating the isomorphism classes of translation planes of order p^n reduces to the problem of determining a set of representatives of the equivalence classes induced by the above relation.

These equivalence operations can also be used to determine the automorphisms of the translation planes. For suppose that $t \in GL_{2n}(p)$ is an automorphism of $\mathcal{S}$ and that t maps (V_∞, V_0, V_1) to (V_i, V_j, V_k). To determine t amounts to finding a sequence of equivalence operations which take a spread set S (with respect to $(\infty, 0, 1)$) onto a spread set S' (with respect to (i, j, k)). To ease the computations, we also use the more crude *weak equivalence*, which is defined as:

Definition 2.2. S is *weakly equivalent* to S' if and only if S or S^T is equivalent to S'.

3. Invariants

We describe some invariants of (weak) equivalence. They are a decisive tool for the practical computation of the equivalence classes of spread sets. Let $\{x_0 = 0, x_1, \ldots, x_m\}$ be a spread set.

Fingerprint [4], [10]. For $x \in GF(p)^{n\times n}$ set $[x] = (\frac{\det x}{p}$, the Legendre symbol, if $\det x \neq 0$ and $[x] = 0$ otherwise. Define an $(m+2)\times(m+2)$-matrix $Q = (q_{ij})$ by

$$q_{ij} = \Big|\sum_{k=0}^{m}[x_i - x_k][x_j - x_k] + 1\Big|, \qquad 0 \le i, j \le m;$$

$$q_{i\infty} = q_{\infty i} = \Big|\sum_{k=0}^{m}[x_i - x_k]\Big|, \qquad 0 \le i \le m,\ q_{\infty\infty} = m + 1.$$

Then the multiset of entries of $Q = Q(S)$ is an invariant of equivalence.

Fingerprints were used in [4] to determine the *canonical* forms of the ovoids which correspond (by the Klein correspondence) to translation planes of order p^2; see also [7]. The automorphism groups of translation planes can be determined in this way; see [4] and [5].

Leitzahl [6].For $x \in GF(p)^{n\times n}$ define $\ll x \gg = 1$ if $\det x = 1$ and $\ll x \gg = 0$ otherwise. Set

$$\ell(S) = \sum_{0\le i<j\le m}\ \sum_{\substack{k\neq i,j\\ k=0}}^{m} \ll (x_i - x_k)(x_k - x_j)^{-1} \gg .$$

Then $\ell(S)$ is invariant with respect to all equivalence operations except possibly O_3. Thus, if S_1 and S_2 are two coordinatizations of a spread set with respect to (a, b_1, c_1) and (a, b_2, c_2), respectively, then $\ell(S_1) = \ell(S_2)$.

Kennzahl [10].For $x \in GF(p)^{n\times n}$ define $((x)) = 1$ if $\det\ x = 0$ and $((x)) = 0$ otherwise. Set

$$k(S) = \sum_{1\le i<j\le m} ((x_i + x_j)).$$

Then $k(S)$ is invariant with respect to all equivalence operations except possibly O_4. Thus, if S_1 and S_2 are two coordinatizations of a spread set with respect to (a_1, b_1, c_1) and (a_2, b_2, c_2), respectively, then $k(S_1) = k(S_2)$ if $\{a_1, b_1\} = \{a_2, b_2\}$.

Finally, we denote by $c(S)$ the multiset of numbers $|S \cap C|$, where C ranges over the conjugacy classes of G. Clearly, $c(S)$ is a conjugacy invariant.

Next, we give an algorithm which uses the above mentioned invariants to test the equivalence of two spread sets S, S'.

Algorithm

Step 1: If $Q(S) \neq Q(S')$, then stop. Otherwise:
Step 2: For $a \in \{\infty, 0, \ldots, m\}$ compute one coordinatization S_a with respect to $(a, *, *)$ (for an arbitrary admissible $*$). If always $\ell(S_a) \neq \ell(S')$, then stop. Otherwise:
Step 3: For $b \in \{\infty, 0, \ldots, m\} \setminus \{a\}$ compute one coordinatization S_{ab} with respect to $(a, b, *)$. If always $k(S_{ab}) \neq k(S')$, then repeat Step 2 with $a := a + 1$ if possible. If not, then stop. Otherwise:
Step 4: For $c \in \{\infty, 0, \ldots, m\} \setminus \{a, b\}$ compute a coordinatization S_{abc} of S with respect to (a, b, c). If always $c(S_{abc}) \neq c(S')$, then repeat Step 3 with $b := b + 1$ (or $b := b + 2$ if $a := b + 1$) if possible. If not, then stop. Otherwise:
Step 5: Attempt to find a $x \in G$ such that $xS_{abc}x^{-1} = S'$.
Stop.

For more details regarding this algorithm see [10]. Precisely the same procedure (excluding Step 1), is used to determine the generators for the automorphism group of $\mathcal{S}$ (belonging to S). Again the details can be found in [10].

4. The search and results

The search for the representatives of the equivalence classes of spread sets of translation planes of order 49 routinely follows the search for the spread sets of the translation planes of order 27, described in [10]. In the order 49 case, we have used a similar classification of starter sets $S = \{t_0, t_1 = 1, \ldots, t_m\}$, where $6 \leq m \leq 12$. We choose $t_2, \ldots, t_6$ which fixes a particular subspace of V, say $t_i = \binom{*\,*}{0\,*}$.

If k is the maximum number of scalar matrices in a coordinatization of S, we have to distinguish the six cases: $k = 1, \ldots, 6$. In contrast to the search used by R. Mathon and G. Royle [15], we did not require a result like their Lemma 3.1. Instead we determined all completions in every case to gain extra control. (Distinguishing the isomorphism classes of completions is inexpensive with our methods.) Our final enumeration matches completely the enumeration described in [15].

There are precisely 973 representatives of spread sets up to weak equivalence. There are 374 spread sets which have the property that S is inequivalent to S^T (i.e., represent pairs of mutually polar planes). Thus, there are 1347 representatives of spread sets up to equivalence. For each representative spread set we have computed a set of matrix generators – elements of $GL_4(7)$, which leave the spread set invariant. From these generators we computed the data contained in §7.

4.1. **Control.** In such a large enumeration problem, it is important to first establish a control set. Therefore, we have independently determined the spread sets of the translation planes of order 49 which admit involutory homologies. There are precisely 154 such spread sets up to weak equivalence; see [6]. Heuristic reasons explained in [10] and [13] show that the identification of the 154 spread sets among the complete list of spread sets will provide a reasonable test of the correctness of the enumeration.

5. Identification of some known planes

In this section we identify some planes of order 49 which have previously appeared in the literature with those occurring in our list. (For the notation see §7.)

As in [10], we have checked our list of spread sets for possible *symplectic spreads*. It turns out that only the Desarguesian spread $9cu$ has this property.

Planes with involutory homologies [6]:

$0aa$	$0ab$	$0ac$	$0ad$	$0ae$	$0af$	$0ah$	$0ai$	$0ak$	$0am$
$0ao$	$0aq$	$0as$	$0av$	$0aw$	$0ax$	$0ay$	$0bb$	$0be$	$0bk$
$0bl$	$0br$	$0bs*$	$0bx$	$0cc$	$0cg$	$0ch$	$0ck$	$0cl$	$0cm$
$0cs$	$0cz$	$0df$	$0dk$	$1ab$	$1ac$	$1af$	$1am*$	$1an$	$1aq$
$1bd$	$1be$	$1bq$	$1br$	$1ck*$	$1cm$	$1cs*$	$1ct$	$1dl$	$1dm$
$2bt$	$2bu$	$2cv$	$2dc$	$2dd*$	$3af*$	$3an$	$3ao$	$3ap$	$3aq$
$3as$	$3db$	$3dp$	$4ad$	$4an$	$4bi$	$4bm$	$4cj*$	$4db$	$4ds$
$5af$	$5an$	$5aw$	$5bq$	$5br$	$5bs$	$5bt$	$5cs*$	$5cu$	$6am$
$6au$	$6cb$	$6ce*$	$6cs$	$6dj*$	$6dl$	$6dm$	$7ab$	$7am*$	$7ao$
$7be$	$7bi$	$7bs$	$7bz$	$7cs$	$7cy$	$7dd$	$7df$	$7dp$	$7dq*$
$7ds*$	$8ag*$	$8al$	$8as$	$8at$	$8bl$	$8ce$	$8ch$	$8co$	$8cw$
$8cx$	$8de$	$8dh$	$8dl$	$8dq$	$9aa$	$9aj$	$9ak$	$9at$	$9aw$
$9az$	$9bg$	$9bh$	$9bi$	$9bj$	$9bk$	$9bl$	$9bm$	$9bp$	$9bq*$
$9br$	$9bt$	$9bv$	$9bw$	$9by$	$9bz$	$9ca$	$9cc$	$9cd$	$9ce$
$9cg$	$9ch$	$9cj$	$9ck$	$9cl$	$9cm$	$9cn$	$9co$	$9cp$	$9cq$
$9cr$	$9cs$	$9ct$	$9cu$						

Heimbeck planes with quaternion group of homologies [11]:

Name	Name in [11]	Order	Orbits on l_∞
$9cq$	I	13824	2 48
$0df$	II	6912	2 48
$9ct$	III	11520	10 40
$3an$	IV	9216	2 48
$0af$	V	2304	2 16 32
$0ac$	VI	3072	2 16 32
$4ad$	VII	768	2 16 16 16
$0ad$	VIII	2304	2 16 32
$1be$	IX	1536	2 16 32
$9cp$	X	576	1 1 48

Heimbeck plane with a shear [12]: $8dl$, 2016, 1 7 42.

Non-Desarguesian planes with nonsolvable group:

Name	Name in the literature	Order	Orbits on l_∞
$0ab$	Hall	32256	8 42
$0au$	Mason	4320	20 30
$0ba$	S_5-Type [7]	1440	10 40
$9bn$	Mason	4320	20 30
$9cs$	S_5-Type [7] (also Korchmaros)	2880	20 30
$9ct$	Mason-Ostrom	11520	10 40

Non-Desarguesian flag transitive planes [2]:

Name	Name in [2]	Order	Orbits on l_∞
$0an*$	$[L_1] \cup [L_1'], \ [L_1] \cup [L_1'']$	600	50
$1dr$	$[L_3] \cup [L_3']$	600	50

Planes as projections of 8-dimensional ovoids: Certain translation planes of order 49 arise (by the Klein correspondence) from 6-dimensional ovoids which are 'sections' of ovoids in $\Omega^+(8,7)$. There are two ovoids in $\Omega^+(8,7)$ which are invariant under the Weyl groups $W(D_7)$ [17], and $W(E_7)$ [8]. These give 16 isomorphism classes of translation planes which are indexed by the orbits of $W(D_7)$ and $W(E_7)$ acting on the isotropic vectors of $\Omega^+(8,7)$; see [8] and [3]. We identify these planes below. An entry in the 'ovoid' column indicates that the plane corresponds to a section of the listed 8-dimensional ovoid.

Name	Ovoid	Orbits on l_∞	Name	Ovoid	Orbits on l_∞
$0ad$	D_7 E_7	2 16 32	$0ca$	D_7	6 8 12 24
$0ae$	D_7 E_7	6 8 36	$1aq$	D_7 D_7 D_7	4 6 16 24
$0ag$	D_7 D_7 E_7	2 8 12 12 16	$6cs$	D_7 D_7	2 8 12 12 16
$0ak$	D_7	2 12 36	$9bn$	D_7 D_7 E_7	20 30
$0au$	E_7 E_7	20 30	$9cn$	D_7 E_7	8 18 24
$0ba$	D_7 D_7 E_7	10 40	$9cq$	D_7 E_7	2 48
$0be$	D_7 D_7 E_7	3 3 8 36	$9cs$	D_7 E_7	20 30
$0bl$	D_7	6 12 32	$9ct$	D_7 E_7	10 40

6. The structure of the automorphism groups

To describe the structure of the automorphism groups of the 1347 translation planes of order 49, it suffices to consider only the 973 representatives up to polarity. Since a dual spread has the same abstract group in its contragradient representation, the two groups have the same orders.

For each representative spread set we calculate the following: the order of the automorphism group; the order of the center; the order of the Fitting factor group; orders of the factors of the derived series; orders of the factors of a composition series; orders of the factors of the lower central series. We also indicate which spreads are self-polar (S isomorphic to S^T). This data was obtained with GAP [16] with the help of the generators obtained by U. Dempwolff. The GAP routines were prepared mechanically using AWK [1] from the generators to prevent errors from creeping in.

6.1. **Non-Abelian composition factors.** Tables I – X, given in §7, show that the automorphism groups of the non-Desarguesian planes of order 49 have only the following non-Abelian composition factors: $PSL_2(5)$, $PSL_2(7)$ and $PSL_2(9)$. Here, $PSL_2(7)$ gives the Hall plane, see [9], while $PSL_2(9)$ is of a Mason type, see [7]. Planes with $PSL_2(5)$ relate to the ones we considered in [7]. By the automorphism group, we mean the subgroup of $GL_4(7)$ generated by the spread stabilizer and the kernel homologies.

7. The tables

We present the automorphism groups of the translation planes of order 49 as follows. The 973 groups are split into 10 tables: Table I – X. The first 9 tables account for 900 groups, split into blocks of 100, the last table contains 73 groups. Each table contains the following information. The first five columns contain respectively: the name of the group – a name with an asterisk indicates a polar pair; the order of group; the order of the center; the order of the derived subgroup; the order of the Fitting factor group. The last three columns contain respectively: orders of the factor groups of the composition series; orders of the factors of the derived subgroup series; orders of the factors of the lower central series.

We omit a group from the tables if it is the group of central homologies (of order 6) and the corresponding plane is self-polar. For example, in Table I, $1at$ is omitted. (There are 246 such planes up to polarity.) In case that $G = Z(G)$ has order 6 (example $1df$), we give no composition series and no derived series. If $G = Z(G)$ has order greater than 6 (example $0as$), we omit the derived series.

7.1. **The spread sets.**

The complete list of spread sets of the translation planes of order 49 is available at the following ftp site:

ftp://www.mathematik.uni-kl.de/pub/Math/Algebra/49_planes

Other information regarding these planes can also be found at this site.

Table I. 0aa–0dv.

$0aa$	960	6	160	10	[3, 2, 5, 2, 2, 2, 2, 2]	[6, 5, 16, 2]	[6]
$0ab$	32256	6	1344	336	[2, 3, 2, 168, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0ac$	3072	6	128	1	[2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2]	[24, 8, 16]	[24, 4, 2, 2, 2, 2, 2]
$0ad$	2304	6	192	6	[2, 3, 2, 3, 2, 2, 2, 2, 2, 2]	[12, 3, 16, 4]	[12]
$0ae$	1728	6	72	12	[2, 3, 2, 3, 2, 2, 2, 3, 2]	[24, 9, 4, 2]	[24]
$0af$	2304	6	192	6	[2, 3, 2, 3, 2, 2, 2, 2, 2, 2]	[12, 3, 16, 4]	[12]
$0ag$	288	6	24	6	[3, 2, 2, 3, 2, 2, 2]	[12, 3, 4, 2]	[12]
$0ah$	288	12	24	6	[2, 3, 2, 3, 2, 2, 2]	[12, 3, 4, 2]	[12]
$0ai$	192	6	8	1	[2, 2, 2, 2, 3, 2, 2]	[24, 8]	[24, 4, 2]
$0aj$	432	6	36	8	[2, 3, 2, 2, 2, 3, 3]	[12, 4, 9]	[12, 2, 2]
$0ak$	864	6	36	8	[2, 2, 3, 2, 2, 3, 3, 2]	[24, 4, 9]	[24, 2, 2]
$0al$	96	6	8	1	[2, 2, 2, 3, 2, 2]	[12, 8]	[12, 2, 2, 2]
$0am$	1536	6	32	1	[2, 2, 2, 2, 2, 3, 2, 2, 2, 2]	[48, 32]	[48, 4, 4, 2]
$0an*$	600	6	25	4	[2, 3, 2, 2, 5, 5]	[24, 25]	[24]
$0ao$	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
$0ap$	288	6	24	2	[2, 3, 3, 2, 2, 2, 2]	[12, 24]	[12, 2, 2, 2]
$0aq$	192	6	4	1	[2, 2, 2, 3, 2, 2, 2]	[48, 4]	[48, 2, 2]
$0ar$	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
$0as$	24	24	1	1	[2, 2, 3, 2]	[]	[24]
$0at$	288	6	24	2	[2, 3, 3, 2, 2, 2, 2]	[12, 24]	[12, 2, 2, 2]
$0au$	4320	6	720	720	[2, 360, 3, 2]	[6]	[6]
$0av$	768	6	32	1	[2, 2, 2, 3, 2, 2, 2, 2, 2]	[24, 32]	[24, 4, 4, 2]
$0aw$	288	12	24	6	[2, 3, 2, 3, 2, 2, 2]	[12, 3, 4, 2]	[12]
$0ax$	1536	6	64	1	[2, 2, 3, 2, 2, 2, 2, 2, 2, 2]	[24, 64]	[24, 2, 2, 2, 2, 2, 2]
$0ay$	384	6	16	1	[2, 2, 2, 3, 2, 2, 2, 2]	[24, 16]	[24, 2, 2, 2, 2]
$0az$	72	6	6	2	[2, 3, 2, 2, 3]	[12, 6]	[12, 2]
$0ba$	1440	6	120	120	[2, 60, 2, 2, 3]	[12]	[12]
$0bb$	864	6	36	8	[2, 2, 3, 2, 2, 2, 3, 3]	[24, 4, 9]	[24, 2, 2]
$0bc$	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
$0bd$	72	6	6	2	[2, 3, 2, 3, 2]	[12, 6]	[12, 2]
$0be$	864	6	72	12	[3, 2, 3, 2, 3, 2, 2, 2]	[12, 9, 4, 2]	[12]
$0bf$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$0bg$	576	6	24	2	[2, 2, 3, 2, 3, 2, 2, 2]	[24, 24]	[24, 2, 2, 2]
$0bh$	36	6	3	2	[2, 3, 3, 2]	[12, 3]	[12]
$0bi$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$0bj$	48	12	2	1	[2, 3, 2, 2, 2]	[24, 2]	[24, 2]
$0bk$	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
$0bl$	1152	6	96	6	[2, 3, 2, 2, 2, 3, 2, 2, 2]	[12, 12, 4, 2]	[12, 2, 2]
$0bm$	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bn$	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
$0bo$	12	12	1	1	[3, 2, 2]	[]	[12]
$0bp$	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bq$	2016	6	168	12	[3, 2, 2, 7, 3, 2, 2, 2]	[12, 21, 4, 2]	[12]
$0br$	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bs*$	96	6	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bt$	96	6	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bu$	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$0bv$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$0bw$	72	12	3	2	[2, 3, 2, 2, 3]	[24, 3]	[24]
$0bx$	576	6	48	6	[2, 3, 2, 2, 3, 2, 2, 2]	[12, 6, 4, 2]	[12, 2]
$0by$	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
$0bz*$	288	6	12	2	[2, 2, 3, 3, 2, 2, 2]	[24, 12]	[24, 2, 2]

TABLE I. (continued).

0*ca*	576	6	24	2	[2, 2, 3, 2, 2, 2, 2, 3]	[24, 24]	[24, 2, 2, 2]
0*cb*	12	12	1	1	[2, 3, 2]	[]	[12]
0*cc*	72	12	3	2	[2, 3, 2, 2, 3]	[24, 3]	[24]
0*cd*	48	6	4	1	[2, 2, 2, 3, 2]	[12, 4]	[12, 2, 2]
0*ce*∗	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
0*cf*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*cg*	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
0*ch*	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
0*ci*	48	6	4	1	[2, 2, 2, 3, 2]	[12, 4]	[12, 2, 2]
0*cj*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*ck*	576	6	24	6	[2, 2, 3, 2, 3, 2, 2, 2]	[24, 3, 4, 2]	[24]
0*cl*	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
0*cm*	192	12	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
0*cn*	72	6	6	2	[3, 2, 2, 2, 3]	[12, 6]	[12, 2]
0*co*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
0*cp*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*cq*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
0*cr*∗	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
0*cs*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*ct*	216	6	18	4	[3, 2, 3, 2, 2, 3]	[12, 18]	[12, 2]
0*cu*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*cv*	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
0*cw*	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
0*cx*∗	12	12	1	1	[2, 3, 2]	[]	[12]
0*cy*	336	6	28	2	[2, 3, 2, 2, 2, 7]	[12, 28]	[12, 2, 2]
0*cz*	384	6	8	1	[2, 2, 2, 3, 2, 2, 2, 2]	[48, 8]	[48, 2, 2, 2]
0*da*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
0*db*	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
0*dc*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
0*dd*	24	24	1	1	[3, 2, 2, 2]	[]	[24]
0*de*	12	12	1	1	[2, 3, 2]	[]	[12]
0*df*	6912	6	576	36	[2, 3, 2, 3, 3, 2, 2, 2, 2, 2, 2]	[12, 9, 16, 4]	[12]
0*dg*	144	6	12	2	[3, 2, 3, 2, 2, 2]	[12, 12]	[12, 2, 2]
0*dh*	12	12	1	1	[3, 2, 2]	[]	[12]
0*di*	72	6	6	2	[2, 3, 2, 2, 3]	[12, 6]	[12, 2]
0*dj*	24	24	1	1	[2, 3, 2, 2]	[]	[24]
0*dk*	192	12	8	1	[2, 2, 2, 2, 3, 2, 2]	[24, 8]	[24, 4, 2]
0*dl*	144	6	24	6	[2, 3, 2, 2, 3, 2]	[6, 3, 4, 2]	[6]
0*dm*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
0*dn*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
0*do*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
0*dp*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
0*dq*	12	12	1	1	[3, 2, 2]	[]	[12]
0*dr*	12	12	1	1	[2, 3, 2]	[]	[12]
0*dt*	576	6	24	2	[2, 2, 2, 2, 3, 3, 2, 2]	[24, 24]	[24, 2, 2, 2]
0*du*∗	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
0*dv*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]

Table II. 1aa–1dv.

1aa	432	6	36	8	[2, 2, 3, 2, 2, 3, 3]	[12, 4, 9]	[12, 2, 2]
1ab	24	24	1	1	[2, 3, 2, 2]	[]	[24]
1ac	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1ad	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
1ae	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1af	768	6	32	1	[2, 2, 2, 3, 2, 2, 2, 2, 2]	[24, 32]	[24, 4, 4, 2]
1ag	12	12	1	1	[2, 3, 2]	[]	[12]
1ah	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
1ai	12	12	1	1	[2, 3, 2]	[]	[12]
1aj*	12	12	1	1	[2, 3, 2]	[]	[12]
1ak	12	12	1	1	[2, 3, 2]	[]	[12]
1al	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1am*	72	12	3	2	[2, 3, 2, 2, 3]	[24, 3]	[24]
1an	96	6	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
1ao	12	12	1	1	[2, 3, 2]	[]	[12]
1ap	12	12	1	1	[3, 2, 2]	[]	[12]
1aq	1152	6	48	6	[2, 2, 3, 2, 2, 3, 2, 2, 2]	[24, 6, 4, 2]	[24, 2]
1ar	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1as*	12	12	1	1	[2, 3, 2]	[]	[12]
1av	12	12	1	1	[3, 2, 2]	[]	[12]
1aw*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1ax	108	6	9	2	[3, 2, 2, 3, 3]	[12, 9]	[12]
1ay	72	6	6	2	[2, 3, 3, 2, 2]	[12, 6]	[12, 2]
1az	12	12	1	1	[3, 2, 2]	[]	[12]
1ba	12	12	1	1	[2, 3, 2]	[]	[12]
1bb	24	24	1	1	[2, 3, 2, 2]	[]	[24]
1bc	12	12	1	1	[2, 3, 2]	[]	[12]
1bd	24	24	1	1	[2, 2, 3, 2]	[]	[24]
1be	1536	6	64	1	[2, 2, 2, 3, 2, 2, 2, 2, 2, 2]	[24, 8, 8]	[24, 4, 2, 2, 2, 2]
1bf	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1bg	144	6	24	6	[3, 2, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
1bi	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1bj	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1bk	24	24	1	1	[2, 2, 3, 2]	[]	[24]
1bl	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1bn	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
1bo	12	12	1	1	[2, 3, 2]	[]	[12]
1bp*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1bq	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
1br	192	6	4	1	[2, 2, 3, 2, 2, 2, 2]	[48, 4]	[48, 2, 2]
1bs	12	12	1	1	[3, 2, 2]	[]	[12]
1bt	12	12	1	1	[2, 3, 2]	[]	[12]
1bu	12	12	1	1	[2, 3, 2]	[]	[12]
1bv	24	24	1	1	[2, 2, 3, 2]	[]	[24]
1bw	96	6	8	1	[2, 3, 2, 2, 2, 2]	[12, 8]	[12, 2, 2, 2]
1bx	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1bz*	12	12	1	1	[2, 3, 2]	[]	[12]
1ca	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1cb	12	12	1	1	[2, 3, 2]	[]	[12]
1cc	24	24	1	1	[3, 2, 2, 2]	[]	[24]
1cd	96	6	8	1	[2, 2, 2, 3, 2, 2]	[12, 8]	[12, 2, 2, 2]
1ce	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
1cf	12	12	1	1	[2, 3, 2]	[]	[12]
1cg	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
1ch*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1ci*	12	12	1	1	[3, 2, 2]	[]	[12]
1cj*	12	12	1	1	[3, 2, 2]	[]	[12]
1ck*	24	24	1	1	[3, 2, 2, 2]	[]	[24]
1cl	12	12	1	1	[2, 3, 2]	[]	[12]
1cm	192	6	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
1cn	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
1co*	12	12	1	1	[3, 2, 2]	[]	[12]
1cp	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
1cq*	12	12	1	1	[3, 2, 2]	[]	[12]
1cr*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1cs*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1ct	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1cu	12	12	1	1	[2, 3, 2]	[]	[12]
1cv*	12	12	1	1	[3, 2, 2]	[]	[12]
1cw*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
1cx	12	12	1	1	[3, 2, 2]	[]	[12]
1cy	216	6	9	4	[2, 3, 2, 2, 3, 3]	[24, 9]	[24]
1cz*	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
1da	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
1db*	12	12	1	1	[2, 3, 2]	[]	[12]
1dc	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
1dd*	12	12	1	1	[2, 3, 2]	[]	[12]
1de*	12	12	1	1	[3, 2, 2]	[]	[12]
1df*	6	6	1	1	[]	[]	[6]
1dg*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
1dh*	12	12	1	1	[2, 3, 2]	[]	[12]
1di	12	12	1	1	[2, 3, 2]	[]	[12]
1dj*	12	12	1	1	[2, 3, 2]	[]	[12]
1dk*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1dl	96	6	4	1	[3, 2, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
1dm	3072	6	128	1	[2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2]	[24, 128]	[24, 4, 4, 4, 2]
1dn	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
1dp	108	6	9	2	[3, 2, 3, 3, 2]	[12, 9]	[12]
1dq*	12	12	1	1	[3, 2, 2]	[]	[12]
1dr	600	6	25	4	[3, 2, 2, 2, 5, 5]	[24, 25]	[24]
1ds*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
1dt	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
1du	12	12	1	1	[2, 3, 2]	[]	[12]
1dv*	12	12	1	1	[2, 3, 2]	[]	[12]

Table III. 2aa–2dv.

2aa*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2ab	12	12	1	1	[2, 3, 2]	[]	[12]
2ad*	6	6	1	1	[]	[]	[6]
2ae*	48	6	4	1	[3, 2, 2, 2, 2]	[12, 4]	[12, 2, 2]
2af*	12	12	1	1	[2, 3, 2]	[]	[12]
2ag	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
2ah	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
2ai	12	12	1	1	[2, 3, 2]	[]	[12]
2ak	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
2al*	6	6	1	1	[]	[]	[6]
2an	12	12	1	1	[2, 3, 2]	[]	[12]
2ao*	6	6	1	1	[]	[]	[6]
2ap	192	12	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
2aq	12	12	1	1	[2, 3, 2]	[]	[12]
2ar*	12	12	1	1	[2, 3, 2]	[]	[12]
2as*	12	12	1	1	[3, 2, 2]	[]	[12]
2at*	6	6	1	1	[]	[]	[6]
2au*	12	12	1	1	[2, 3, 2]	[]	[12]
2av	12	12	1	1	[2, 3, 2]	[]	[12]
2aw	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
2ax*	6	6	1	1	[]	[]	[6]
2ay*	12	12	1	1	[2, 3, 2]	[]	[12]
2az	12	12	1	1	[2, 3, 2]	[]	[12]
2ba*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
2bb*	6	6	1	1	[]	[]	[6]
2bc	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2bd	12	12	1	1	[3, 2, 2]	[]	[12]
2be*	12	12	1	1	[2, 3, 2]	[]	[12]
2bg	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
2bh*	6	6	1	1	[]	[]	[6]
2bi	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
2bj*	6	6	1	1	[]	[]	[6]
2bk	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2bl	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
2bm	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
2bn	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
2bq*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
2br*	12	12	1	1	[3, 2, 2]	[]	[12]
2bs	12	12	1	1	[3, 2, 2]	[]	[12]
2bt	144	6	6	2	[2, 2, 3, 2, 2, 3]	[24, 6]	[24, 2]
2bu	144	6	6	2	[2, 3, 3, 2, 2, 2]	[24, 6]	[24, 2]
2bv	24	24	1	1	[2, 3, 2, 2]	[]	[24]
2bw	12	12	1	1	[3, 2, 2]	[]	[12]
2bx	24	24	1	1	[3, 2, 2, 2]	[]	[24]
2by	12	12	1	1	[2, 3, 2]	[]	[12]
2bz*	6	6	1	1	[]	[]	[6]
2ca	12	12	1	1	[3, 2, 2]	[]	[12]
2cb	12	12	1	1	[2, 3, 2]	[]	[12]
2cd	12	12	1	1	[3, 2, 2]	[]	[12]
2ce	24	24	1	1	[2, 3, 2, 2]	[]	[24]
2cf*	6	6	1	1	[]	[]	[6]
2cg*	12	12	1	1	[3, 2, 2]	[]	[12]
2ch	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2ci	24	24	1	1	[3, 2, 2, 2]	[]	[24]
2cj	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
2ck	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
2cl	12	12	1	1	[2, 3, 2]	[]	[12]
2cm*	6	6	1	1	[]	[]	[6]
2cn	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2co	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
2cp	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2cq	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
2cr*	12	12	1	1	[3, 2, 2]	[]	[12]
2cs*	6	6	1	1	[]	[]	[6]
2ct*	6	6	1	1	[]	[]	[6]
2cu	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
2cv	24	24	1	1	[2, 2, 3, 2]	[]	[24]
2cy	96	6	8	1	[2, 3, 2, 2, 2, 2]	[12, 8]	[12, 2, 2, 2]
2cz*	12	12	1	1	[2, 3, 2]	[]	[12]
2da	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
2db*	6	6	1	1	[]	[]	[6]
2dc	144	6	6	2	[2, 3, 2, 3, 2, 2]	[24, 6]	[24, 2]
2dd*	24	24	1	1	[2, 3, 2, 2]	[]	[24]
2de	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2df*	6	6	1	1	[]	[]	[6]
2dg*	12	12	1	1	[3, 2, 2]	[]	[12]
2dh*	12	12	1	1	[3, 2, 2]	[]	[12]
2di*	12	12	1	1	[2, 3, 2]	[]	[12]
2dj	12	12	1	1	[3, 2, 2]	[]	[12]
2dk	24	24	1	1	[3, 2, 2, 2]	[]	[24]
2dl*	12	12	1	1	[2, 3, 2]	[]	[12]
2dm*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
2dn	48	6	4	1	[3, 2, 2, 2, 2]	[12, 4]	[12, 2, 2]
2do	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
2dq*	6	6	1	1	[]	[]	[6]
2dr*	12	12	1	1	[2, 3, 2]	[]	[12]
2ds*	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
2dt*	6	6	1	1	[]	[]	[6]
2du	12	12	1	1	[3, 2, 2]	[]	[12]
2dv*	12	12	1	1	[2, 3, 2]	[]	[12]

Table IV. 3aa–3dv.

$3aa*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3ab*$	288	6	24	6	[3, 2, 2, 3, 2, 2, 2]	[12, 3, 4, 2]	[12]
$3ac$	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
$3ad*$	144	6	24	6	[3, 2, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
$3ae*$	6	6	1	1	[]	[]	[6]
$3af*$	24	24	1	1	[2, 2, 3, 2]	[]	[24]
$3ag$	12	12	1	1	[3, 2, 2]	[]	[12]
$3ai*$	24	24	1	1	[2, 2, 3, 2]	[]	[24]
$3aj*$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$3ak*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3al*$	6	6	1	1	[]	[]	[6]
$3an$	9216	6	384	2	[2, 2, 2, 3, 2, 2, 2, 2, 3, 2, 2, 2]	[24, 24, 16]	[24, 4, 2, 2, 2, 2, 2]
$3ao$	288	12	12	2	[2, 3, 2, 2, 2, 2, 3]	[24, 12]	[24, 2, 2]
$3ap$	24	24	1	1	[2, 3, 2, 2]	[]	[24]
$3aq$	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
$3ar$	12	12	1	1	[3, 2, 2]	[]	[12]
$3as$	192	12	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
$3at*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3av*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3aw$	12	12	1	1	[2, 3, 2]	[]	[12]
$3ax*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3ay$	12	12	1	1	[2, 3, 2]	[]	[12]
$3az*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3ba*$	6	6	1	1	[]	[]	[6]
$3bb*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3bc*$	6	6	1	1	[]	[]	[6]
$3bd*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3be$	12	12	1	1	[3, 2, 2]	[]	[12]
$3bg*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3bh$	12	12	1	1	[2, 3, 2]	[]	[12]
$3bi*$	6	6	1	1	[]	[]	[6]
$3bk*$	6	6	1	1	[]	[]	[6]
$3bn$	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
$3bo$	12	12	1	1	[3, 2, 2]	[]	[12]
$3bq$	12	12	1	1	[2, 3, 2]	[]	[12]
$3br*$	6	6	1	1	[]	[]	[6]
$3bt*$	6	6	1	1	[]	[]	[6]
$3bu*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3bv$	12	12	1	1	[2, 3, 2]	[]	[12]
$3bw$	12	12	1	1	[2, 3, 2]	[]	[12]
$3bx$	12	12	1	1	[2, 3, 2]	[]	[12]
$3by*$	6	6	1	1	[]	[]	[6]
$3cb*$	6	6	1	1	[]	[]	[6]
$3cc*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3cd*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3cf$	12	12	1	1	[2, 3, 2]	[]	[12]
$3cg*$	6	6	1	1	[]	[]	[6]
$3ch$	12	12	1	1	[2, 3, 2]	[]	[12]
$3ci*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3cj$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$3ck$	12	12	1	1	[2, 3, 2]	[]	[12]
$3cm*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3cn*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3co*$	24	24	1	1	[2, 3, 2, 2]	[]	[24]
$3cr*$	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
$3cs*$	6	6	1	1	[]	[]	[6]
$3ct*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3cu*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3cv*$	6	6	1	1	[]	[]	[6]
$3cw*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3cx*$	96	6	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
$3cy*$	6	6	1	1	[]	[]	[6]
$3cz*$	6	6	1	1	[]	[]	[6]
$3da*$	6	6	1	1	[]	[]	[6]
$3db$	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
$3dc*$	6	6	1	1	[]	[]	[6]
$3dd*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3de$	12	12	1	1	[2, 3, 2]	[]	[12]
$3df$	12	12	1	1	[2, 3, 2]	[]	[12]
$3dh$	12	12	1	1	[2, 3, 2]	[]	[12]
$3di*$	6	6	1	1	[]	[]	[6]
$3dk*$	6	6	1	1	[]	[]	[6]
$3dl*$	12	12	1	1	[2, 3, 2]	[]	[12]
$3dm$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$3dn*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3do*$	12	12	1	1	[3, 2, 2]	[]	[12]
$3dp$	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
$3dq$	12	12	1	1	[3, 2, 2]	[]	[12]
$3dr$	24	24	1	1	[3, 2, 2, 2]	[]	[24]
$3ds$	12	12	1	1	[2, 3, 2]	[]	[12]
$3dt$	12	12	1	1	[2, 3, 2]	[]	[12]
$3du$	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
$3dv$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]

Table V. 4aa–4dv.

4ab	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
4ac*	24	24	1	1	[3, 2, 2, 2]	[]	[24]
4ad	768	6	32	1	[2, 2, 2, 3, 2, 2, 2, 2, 2]	[24, 8, 4]	[24, 4, 2, 2, 2]
4af*	6	6	1	1	[]	[]	[6]
4ag*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4ah	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
4ai*	6	6	1	1	[]	[]	[6]
4ak*	6	6	1	1	[]	[]	[6]
4al	12	12	1	1	[3, 2, 2]	[]	[12]
4am*	12	12	1	1	[3, 2, 2]	[]	[12]
4an	48	12	2	1	[3, 2, 2, 2, 2]	[24, 2]	[24, 2]
4ap*	6	6	1	1	[]	[]	[6]
4aq*	6	6	1	1	[]	[]	[6]
4ar	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4as*	6	6	1	1	[]	[]	[6]
4at	12	12	1	1	[3, 2, 2]	[]	[12]
4au*	12	12	1	1	[3, 2, 2]	[]	[12]
4av*	12	12	1	1	[3, 2, 2]	[]	[12]
4aw*	6	6	1	1	[]	[]	[6]
4ax*	6	6	1	1	[]	[]	[6]
4ay	12	12	1	1	[3, 2, 2]	[]	[12]
4az	12	12	1	1	[3, 2, 2]	[]	[12]
4ba	12	12	1	1	[3, 2, 2]	[]	[12]
4bb	12	12	1	1	[3, 2, 2]	[]	[12]
4bc	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4bd*	6	6	1	1	[]	[]	[6]
4be*	6	6	1	1	[]	[]	[6]
4bf*	12	12	1	1	[3, 2, 2]	[]	[12]
4bg*	6	6	1	1	[]	[]	[6]
4bh*	12	12	1	1	[3, 2, 2]	[]	[12]
4bi	432	6	18	4	[2, 2, 3, 2, 2, 3, 3]	[24, 18]	[24, 2]
4bj	192	6	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
4bk	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
4bl	12	12	1	1	[3, 2, 2]	[]	[12]
4bm	96	6	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
4bn	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4bo	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4bp*	12	12	1	1	[3, 2, 2]	[]	[12]
4bq	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4br	72	6	6	2	[2, 3, 2, 2, 3]	[12, 6]	[12, 2]
4bs*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
4bt*	6	6	1	1	[]	[]	[6]
4bu	144	6	24	6	[2, 3, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
4bv	12	12	1	1	[3, 2, 2]	[]	[12]
4bw*	12	12	1	1	[3, 2, 2]	[]	[12]
4bx*	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
4by*	6	6	1	1	[]	[]	[6]
4bz*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4ca*	6	6	1	1	[]	[]	[6]
4cb	12	12	1	1	[3, 2, 2]	[]	[12]
4cc*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4ce*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
4cf*	6	6	1	1	[]	[]	[6]
4cg*	6	6	1	1	[]	[]	[6]
4ch*	6	6	1	1	[]	[]	[6]
4ci	144	6	24	6	[3, 2, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
4cj*	288	6	12	2	[3, 2, 2, 2, 2, 2, 3]	[24, 12]	[24, 2, 2]
4ck*	12	12	1	1	[3, 2, 2]	[]	[12]
4cl	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
4cm*	12	12	1	1	[3, 2, 2]	[]	[12]
4cn	12	12	1	1	[3, 2, 2]	[]	[12]
4co*	12	12	1	1	[3, 2, 2]	[]	[12]
4cp	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
4cq*	6	6	1	1	[]	[]	[6]
4cr*	6	6	1	1	[]	[]	[6]
4cs*	12	12	1	1	[3, 2, 2]	[]	[12]
4ct*	6	6	1	1	[]	[]	[6]
4cu*	6	6	1	1	[]	[]	[6]
4cw*	6	6	1	1	[]	[]	[6]
4cy*	12	12	1	1	[3, 2, 2]	[]	[12]
4cz	24	24	1	1	[2, 3, 2, 2]	[]	[24]
4da	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
4db	192	6	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
4dd*	12	12	1	1	[3, 2, 2]	[]	[12]
4de*	6	6	1	1	[]	[]	[6]
4df*	6	6	1	1	[]	[]	[6]
4dg*	6	6	1	1	[]	[]	[6]
4dh	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4di	96	6	8	1	[2, 2, 3, 2, 2, 2]	[12, 8]	[12, 2, 2, 2]
4dk*	6	6	1	1	[]	[]	[6]
4dl*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
4dm*	12	12	1	1	[3, 2, 2]	[]	[12]
4do*	12	12	1	1	[3, 2, 2]	[]	[12]
4dp*	6	6	1	1	[]	[]	[6]
4dq	24	24	1	1	[3, 2, 2, 2]	[]	[24]
4dr	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
4ds	24	24	1	1	[3, 2, 2, 2]	[]	[24]
4dt	12	12	1	1	[2, 3, 2]	[]	[12]
4dv	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]

TABLE VI. 5aa–5dv.

5ab*	6	6	1	1	[]	[]	[6]
5ac*	12	12	1	1	[3, 2, 2]	[]	[12]
5ad*	12	12	1	1	[2, 3, 2]	[]	[12]
5ae*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
5af	48	12	2	1	[2, 3, 2, 2, 2]	[24, 2]	[24, 2]
5ag*	12	12	1	1	[2, 3, 2]	[]	[12]
5ah	12	12	1	1	[3, 2, 2]	[]	[12]
5ak*	6	6	1	1	[]	[]	[6]
5am*	12	12	1	1	[2, 3, 2]	[]	[12]
5an	24	24	1	1	[2, 3, 2, 2]	[]	[24]
5ao	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
5ap*	6	6	1	1	[]	[]	[6]
5aq	12	12	1	1	[3, 2, 2]	[]	[12]
5ar*	6	6	1	1	[]	[]	[6]
5as*	6	6	1	1	[]	[]	[6]
5at*	12	12	1	1	[2, 3, 2]	[]	[12]
5au*	12	12	1	1	[3, 2, 2]	[]	[12]
5av	36	6	3	2	[2, 3, 2, 3]	[12, 3]	[12]
5aw	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
5ax*	6	6	1	1	[]	[]	[6]
5ay	12	12	1	1	[3, 2, 2]	[]	[12]
5az	144	6	6	2	[2, 2, 3, 2, 2, 3]	[24, 6]	[24, 2]
5ba*	6	6	1	1	[]	[]	[6]
5bb*	144	6	24	6	[3, 2, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
5bc	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
5bd	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
5be	12	12	1	1	[3, 2, 2]	[]	[12]
5bf	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
5bi*	6	6	1	1	[]	[]	[6]
5bj	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
5bl*	12	12	1	1	[2, 3, 2]	[]	[12]
5bm	12	12	1	1	[2, 3, 2]	[]	[12]
5bn*	12	12	1	1	[2, 3, 2]	[]	[12]
5bo*	6	6	1	1	[]	[]	[6]
5bp*	12	12	1	1	[3, 2, 2]	[]	[12]
5bq	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
5br	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
5bs	192	6	8	1	[2, 2, 2, 3, 2, 2, 2]	[24, 8]	[24, 4, 2]
5bt	96	12	4	1	[2, 2, 2, 2, 3, 2]	[24, 4]	[24, 4]
5bu*	12	12	1	1	[3, 2, 2]	[]	[12]
5bv	60	6	5	2	[3, 2, 2, 5]	[12, 5]	[12]
5bx*	6	6	1	1	[]	[]	[6]
5by*	6	6	1	1	[]	[]	[6]
5bz*	6	6	1	1	[]	[]	[6]
5ca*	12	12	1	1	[2, 3, 2]	[]	[12]
5cb*	6	6	1	1	[]	[]	[6]
5cc	12	12	1	1	[2, 3, 2]	[]	[12]
5cd*	12	12	1	1	[2, 3, 2]	[]	[12]
5ce*	12	12	1	1	[3, 2, 2]	[]	[12]
5cf*	12	12	1	1	[2, 3, 2]	[]	[12]
5cg*	12	12	1	1	[2, 3, 2]	[]	[12]
5ch	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
5ci*	12	12	1	1	[2, 3, 2]	[]	[12]
5cj*	6	6	1	1	[]	[]	[6]
5cm*	6	6	1	1	[]	[]	[6]
5cn*	6	6	1	1	[]	[]	[6]
5co*	12	12	1	1	[2, 3, 2]	[]	[12]
5cp*	36	6	3	2	[2, 3, 3, 2]	[12, 3]	[12]
5cq*	12	12	1	1	[3, 2, 2]	[]	[12]
5cs*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
5cu	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
5cv	12	12	1	1	[3, 2, 2]	[]	[12]
5cw	12	12	1	1	[3, 2, 2]	[]	[12]
5cx*	6	6	1	1	[]	[]	[6]
5cy*	6	6	1	1	[]	[]	[6]
5cz*	6	6	1	1	[]	[]	[6]
5da*	6	6	1	1	[]	[]	[6]
5dc*	12	12	1	1	[2, 3, 2]	[]	[12]
5dd*	12	12	1	1	[2, 3, 2]	[]	[12]
5de*	6	6	1	1	[]	[]	[6]
5df	12	12	1	1	[3, 2, 2]	[]	[12]
5dg*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
5dh*	6	6	1	1	[]	[]	[6]
5di*	6	6	1	1	[]	[]	[6]
5dj*	12	12	1	1	[2, 3, 2]	[]	[12]
5dk*	12	12	1	1	[2, 3, 2]	[]	[12]
5dl*	12	12	1	1	[3, 2, 2]	[]	[12]
5dn	12	12	1	1	[2, 3, 2]	[]	[12]
5do*	6	6	1	1	[]	[]	[6]
5dp*	12	12	1	1	[2, 3, 2]	[]	[12]
5dq*	12	12	1	1	[3, 2, 2]	[]	[12]
5dr*	6	6	1	1	[]	[]	[6]
5ds	12	12	1	1	[2, 3, 2]	[]	[12]
5dt	144	6	12	2	[2, 3, 2, 2, 3, 2]	[12, 12]	[12, 2, 2]
5du*	12	12	1	1	[2, 3, 2]	[]	[12]
5dv*	12	12	1	1	[3, 2, 2]	[]	[12]

Table VII. 6aa–6dv.

$6aa*$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6ab*$	6	6	1	1	[]	[]	[6]
$6ac$	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
$6ad*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6af$	12	12	1	1	[2, 3, 2]	[]	[12]
$6ag*$	6	6	1	1	[]	[]	[6]
$6ah*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6ai*$	6	6	1	1	[]	[]	[6]
$6aj*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6ak*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6al*$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6am$	96	12	4	1	[2, 2, 2, 3, 2, 2]	[24, 4]	[24, 2, 2]
$6an*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6ap*$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6aq$	12	12	1	1	[2, 3, 2]	[]	[12]
$6ar$	12	12	1	1	[2, 3, 2]	[]	[12]
$6as*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6at*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6au$	144	12	6	2	[3, 2, 2, 2, 2, 3]	[24, 6]	[24, 2]
$6av*$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$6aw$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6ax$	12	12	1	1	[3, 2, 2]	[]	[12]
$6ay$	12	12	1	1	[3, 2, 2]	[]	[12]
$6az$	12	12	1	1	[2, 3, 2]	[]	[12]
$6bb*$	6	6	1	1	[]	[]	[6]
$6bc*$	6	6	1	1	[]	[]	[6]
$6bd*$	6	6	1	1	[]	[]	[6]
$6bf*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6bg*$	6	6	1	1	[]	[]	[6]
$6bh$	12	12	1	1	[2, 3, 2]	[]	[12]
$6bi*$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6bj*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6bk$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$6bl$	288	6	24	2	[2, 3, 2, 2, 3, 2, 2]	[12, 24]	[12, 2, 2, 2]
$6bm$	12	12	1	1	[3, 2, 2]	[]	[12]
$6bo*$	6	6	1	1	[]	[]	[6]
$6bp*$	6	6	1	1	[]	[]	[6]
$6bq*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6br*$	6	6	1	1	[]	[]	[6]
$6bs*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6bt$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6bu$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6bv*$	6	6	1	1	[]	[]	[6]
$6bw*$	6	6	1	1	[]	[]	[6]
$6bx$	12	12	1	1	[2, 3, 2]	[]	[12]
$6by*$	6	6	1	1	[]	[]	[6]
$6bz$	12	12	1	1	[3, 2, 2]	[]	[12]
$6ca*$	6	6	1	1	[]	[]	[6]
$6cb$	192	6	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
$6cd$	72	6	6	2	[2, 3, 2, 2, 3]	[12, 6]	[12, 2]
$6ce*$	24	24	1	1	[3, 2, 2, 2]	[]	[24]
$6cf$	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
$6ch*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6ci*$	6	6	1	1	[]	[]	[6]
$6cj*$	6	6	1	1	[]	[]	[6]
$6ck*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6cl*$	6	6	1	1	[]	[]	[6]
$6cm*$	6	6	1	1	[]	[]	[6]
$6cq*$	12	12	1	1	[3, 2, 2]	[]	[12]
$6cr$	12	12	1	1	[2, 3, 2]	[]	[12]
$6cs$	576	6	24	6	[2, 2, 3, 3, 2, 2, 2, 2]	[24, 3, 4, 2]	[24]
$6ct$	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
$6cu$	12	12	1	1	[2, 3, 2]	[]	[12]
$6cv$	24	24	1	1	[3, 2, 2, 2]	[]	[24]
$6cw$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6cx*$	6	6	1	1	[]	[]	[6]
$6cz$	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
$6da*$	6	6	1	1	[]	[]	[6]
$6db*$	6	6	1	1	[]	[]	[6]
$6dd$	12	12	1	1	[3, 2, 2]	[]	[12]
$6de*$	6	6	1	1	[]	[]	[6]
$6df*$	6	6	1	1	[]	[]	[6]
$6dg*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6dh*$	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
$6dj*$	144	12	6	2	[2, 3, 2, 2, 2, 3]	[24, 6]	[24, 2]
$6dk$	18	18	1	1	[3, 3, 2]	[]	[18]
$6dl$	12	12	1	1	[2, 3, 2]	[]	[12]
$6dm$	192	6	4	1	[2, 2, 3, 2, 2, 2, 2]	[48, 4]	[48, 2, 2]
$6do$	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
$6dp$	12	12	1	1	[2, 3, 2]	[]	[12]
$6dq$	12	12	1	1	[2, 3, 2]	[]	[12]
$6dr$	12	12	1	1	[3, 2, 2]	[]	[12]
$6ds*$	12	12	1	1	[2, 3, 2]	[]	[12]
$6dt$	24	24	1	1	[3, 2, 2, 2]	[]	[24]
$6du$	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
$6dv*$	6	6	1	1	[]	[]	[6]

TABLE VIII. 7aa–7dv.

7ab	48	12	2	1	[2, 2, 2, 3, 2]	[24, 2]	[24, 2]
7ac	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
7ad	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
7ae	144	6	12	2	[2, 3, 2, 2, 3, 2]	[12, 12]	[12, 2, 2]
7ag*	12	12	1	1	[2, 3, 2]	[]	[12]
7ah*	6	6	1	1	[]	[]	[6]
7ai*	12	12	1	1	[2, 3, 2]	[]	[12]
7aj*	12	12	1	1	[3, 2, 2]	[]	[12]
7ak*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
7am*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
7an	12	12	1	1	[2, 3, 2]	[]	[12]
7ao	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
7ap*	12	12	1	1	[3, 2, 2]	[]	[12]
7aq*	6	6	1	1	[]	[]	[6]
7ar*	6	6	1	1	[]	[]	[6]
7as*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
7at*	6	6	1	1	[]	[]	[6]
7au*	6	6	1	1	[]	[]	[6]
7av*	6	6	1	1	[]	[]	[6]
7ay	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
7ba*	12	12	1	1	[2, 3, 2]	[]	[12]
7bc	72	6	6	2	[3, 2, 2, 3, 2]	[12, 6]	[12, 2]
7bd	72	6	6	2	[3, 2, 2, 2, 3]	[12, 6]	[12, 2]
7be	576	6	24	2	[2, 2, 3, 2, 2, 2, 2, 3]	[24, 24]	[24, 2, 2, 2]
7bf	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
7bg	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
7bi	12	12	1	1	[2, 3, 2]	[]	[12]
7bj*	24	24	1	1	[3, 2, 2, 2]	[]	[24]
7bk*	6	6	1	1	[]	[]	[6]
7bl*	12	12	1	1	[2, 3, 2]	[]	[12]
7bm*	6	6	1	1	[]	[]	[6]
7bn*	12	12	1	1	[3, 2, 2]	[]	[12]
7bo	12	12	1	1	[2, 3, 2]	[]	[12]
7bp*	6	6	1	1	[]	[]	[6]
7bq*	24	24	1	1	[2, 3, 2, 2]	[]	[24]
7br*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
7bs	96	6	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
7bt*	12	12	1	1	[2, 3, 2]	[]	[12]
7bu*	6	6	1	1	[]	[]	[6]
7bv	12	12	1	1	[2, 3, 2]	[]	[12]
7by*	576	6	24	2	[2, 2, 2, 2, 3, 2, 2, 3]	[24, 24]	[24, 2, 2, 2]
7bz	192	6	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
7ca	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
7cf*	12	12	1	1	[2, 3, 2]	[]	[12]
7cg	12	12	1	1	[2, 3, 2]	[]	[12]
7ci*	12	12	1	1	[2, 3, 2]	[]	[12]
7cj*	12	12	1	1	[2, 3, 2]	[]	[12]
7cl	12	12	1	1	[2, 3, 2]	[]	[12]
7cm*	12	12	1	1	[2, 3, 2]	[]	[12]
7cn*	12	12	1	1	[2, 3, 2]	[]	[12]
7cp*	12	12	1	1	[2, 3, 2]	[]	[12]
7cq	12	12	1	1	[2, 3, 2]	[]	[12]
7cr	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
7cs	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
7ct*	6	6	1	1	[]	[]	[6]
7cu*	6	6	1	1	[]	[]	[6]
7cw*	12	12	1	1	[2, 3, 2]	[]	[12]
7cx*	6	6	1	1	[]	[]	[6]
7cy	96	6	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
7cz	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
7da	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
7dc*	12	12	1	1	[3, 2, 2]	[]	[12]
7dd	576	6	24	2	[2, 2, 3, 2, 2, 2, 2, 3]	[24, 24]	[24, 2, 2, 2]
7de	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
7df	12	12	1	1	[2, 3, 2]	[]	[12]
7dh*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
7dj	12	12	1	1	[2, 3, 2]	[]	[12]
7dk*	6	6	1	1	[]	[]	[6]
7dm	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
7dn*	12	12	1	1	[2, 3, 2]	[]	[12]
7do*	6	6	1	1	[]	[]	[6]
7dp	576	6	24	6	[2, 2, 3, 3, 2, 2, 2, 2]	[24, 3, 4, 2]	[24]
7dq*	96	6	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
7dr	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
7ds*	24	24	1	1	[2, 3, 2, 2]	[]	[24]
7dt*	12	12	1	1	[3, 2, 2]	[]	[12]
7du*	12	12	1	1	[3, 2, 2]	[]	[12]
7dv	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]

Table IX. 8aa–8dv.

8aa	12	12	1	1	[3, 2, 2]	[]	[12]
8ab	12	12	1	1	[3, 2, 2]	[]	[12]
8ac*	12	12	1	1	[3, 2, 2]	[]	[12]
8ae	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8af	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
8ag*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8ah	12	12	1	1	[2, 3, 2]	[]	[12]
8ai*	6	6	1	1	[]	[]	[6]
8aj*	24	24	1	1	[2, 3, 2, 2]	[]	[24]
8ak*	12	12	1	1	[3, 2, 2]	[]	[12]
8al	96	6	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
8am*	12	12	1	1	[3, 2, 2]	[]	[12]
8an*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8ap*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8aq*	12	12	1	1	[2, 3, 2]	[]	[12]
8ar*	6	6	1	1	[]	[]	[6]
8as	1152	6	48	6	[2, 2, 2, 3, 2, 3, 2, 2, 2]	[24, 6, 4, 2]	[24, 2]
8at	384	6	16	1	[2, 2, 3, 2, 2, 2, 2, 2]	[24, 16]	[24, 4, 2, 2]
8au	72	6	8	3	[3, 3, 2, 2, 2]	[9, 4, 2]	[9]
8aw*	6	6	1	1	[]	[]	[6]
8ax	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
8ay*	12	12	1	1	[3, 2, 2]	[]	[12]
8az	12	12	1	1	[3, 2, 2]	[]	[12]
8ba	12	12	1	1	[3, 2, 2]	[]	[12]
8bf	24	24	1	1	[2, 2, 3, 2]	[]	[24]
8bg*	12	12	1	1	[2, 3, 2]	[]	[12]
8bh*	12	12	1	1	[2, 3, 2]	[]	[12]
8bi	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8bj*	6	6	1	1	[]	[]	[6]
8bk	12	12	1	1	[2, 3, 2]	[]	[12]
8bl	12	12	1	1	[2, 3, 2]	[]	[12]
8bm*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
8bn*	6	6	1	1	[]	[]	[6]
8bo	24	24	1	1	[2, 3, 2, 2]	[]	[24]
8bp	24	24	1	1	[2, 2, 3, 2]	[]	[24]
8bt*	12	12	1	1	[2, 3, 2]	[]	[12]
8bu	18	18	1	1	[3, 3, 2]	[]	[18]
8bv*	18	18	1	1	[3, 2, 3]	[]	[18]
8bw*	12	12	1	1	[3, 2, 2]	[]	[12]
8bx	12	12	1	1	[3, 2, 2]	[]	[12]
8by*	12	12	1	1	[3, 2, 2]	[]	[12]
8bz	72	6	6	2	[2, 3, 2, 3, 2]	[12, 6]	[12, 2]
8ca	12	12	1	1	[2, 3, 2]	[]	[12]
8cc*	6	6	1	1	[]	[]	[6]
8cd	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8ce	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 4]
8cf*	12	12	1	1	[2, 3, 2]	[]	[12]
8cg*	12	12	1	1	[3, 2, 2]	[]	[12]
8ch	384	12	16	1	[3, 2, 2, 2, 2, 2, 2, 2]	[24, 16]	[24, 4, 4]
8ci*	12	12	1	1	[2, 3, 2]	[]	[12]
8cj*	12	12	1	1	[3, 2, 2]	[]	[12]
8ck*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8cl	12	12	1	1	[3, 2, 2]	[]	[12]
8cm*	6	6	1	1	[]	[]	[6]
8cn	12	12	1	1	[3, 2, 2]	[]	[12]
8co	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
8cp*	6	6	1	1	[]	[]	[6]
8cq	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
8cr*	12	12	1	1	[2, 3, 2]	[]	[12]
8cs	12	12	1	1	[2, 3, 2]	[]	[12]
8ct	432	6	72	6	[3, 2, 3, 3, 2, 2, 2]	[6, 9, 4, 2]	[6]
8cu*	12	12	1	1	[2, 3, 2]	[]	[12]
8cv*	12	12	1	1	[2, 3, 2]	[]	[12]
8cw	48	12	2	1	[3, 2, 2, 2, 2]	[24, 2]	[24, 2]
8cx	24	24	1	1	[2, 3, 2, 2]	[]	[24]
8cy	24	6	2	1	[3, 2, 2, 2]	[12, 2]	[12, 2]
8cz	12	12	1	1	[3, 2, 2]	[]	[12]
8da*	12	12	1	1	[2, 3, 2]	[]	[12]
8db*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8dc	12	12	1	1	[2, 3, 2]	[]	[12]
8dd	12	12	1	1	[2, 3, 2]	[]	[12]
8de	192	12	8	1	[2, 3, 2, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
8df*	12	12	1	1	[2, 3, 2]	[]	[12]
8dh	576	6	24	2	[2, 2, 3, 3, 2, 2, 2, 2]	[24, 24]	[24, 2, 2, 2]
8di*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
8dj*	12	12	1	1	[2, 3, 2]	[]	[12]
8dk	48	12	2	1	[2, 2, 3, 2, 2]	[24, 2]	[24, 2]
8dl	2016	6	168	12	[2, 7, 3, 2, 3, 2, 2, 2]	[12, 21, 4, 2]	[12]
8dm*	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
8dn	18	18	1	1	[3, 3, 2]	[]	[18]
8do	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8dp*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8dq	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8dr	24	24	1	1	[2, 2, 3, 2]	[]	[24]
8ds	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8dt	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
8du	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
8dv*	12	12	1	1	[3, 2, 2]	[]	[12]

Table X. 9aa–9cu.

9*aa*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
9*ab*	72	6	8	3	[3, 3, 2, 2, 2]	[9, 4, 2]	[9]
9*ac*∗	12	12	1	1	[3, 2, 2]	[]	[12]
9*ad*∗	12	12	1	1	[2, 3, 2]	[]	[12]
9*af*	12	12	1	1	[2, 3, 2]	[]	[12]
9*ag*∗	12	12	1	1	[2, 3, 2]	[]	[12]
9*ah*	96	6	8	1	[2, 3, 2, 2, 2, 2]	[12, 8]	[12, 2, 2, 2]
9*ai*∗	48	6	4	1	[2, 2, 3, 2, 2]	[12, 4]	[12, 2, 2]
9*aj*	240	6	10	4	[2, 3, 2, 2, 2, 5]	[24, 10]	[24, 2]
9*ak*	48	12	2	1	[3, 2, 2, 2, 2]	[24, 2]	[24, 2]
9*al*	12	12	1	1	[3, 2, 2]	[]	[12]
9*am*	12	12	1	1	[2, 3, 2]	[]	[12]
9*an*∗	12	12	1	1	[2, 3, 2]	[]	[12]
9*ao*	48	6	4	1	[3, 2, 2, 2, 2]	[12, 4]	[12, 2, 2]
9*ap*	144	6	24	6	[3, 2, 3, 2, 2, 2]	[6, 3, 4, 2]	[6]
9*aq*∗	24	6	2	1	[2, 2, 3, 2]	[12, 2]	[12, 2]
9*ar*∗	36	6	3	2	[3, 2, 3, 2]	[12, 3]	[12]
9*as*∗	6	6	1	1	[]	[]	[6]
9*at*	192	6	8	1	[3, 2, 2, 2, 2, 2, 2]	[24, 8]	[24, 4, 2]
9*av*	72	12	3	2	[2, 2, 3, 3, 2]	[24, 3]	[24]
9*aw*	216	12	9	2	[2, 3, 2, 2, 3, 3]	[24, 9]	[24]
9*ax*∗	36	6	3	2	[2, 3, 3, 2]	[12, 3]	[12]
9*ay*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
9*az*	96	12	4	1	[2, 2, 3, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*ba*	24	6	2	1	[2, 3, 2, 2]	[12, 2]	[12, 2]
9*bb*	144	6	12	2	[2, 2, 3, 3, 2, 2]	[12, 12]	[12, 2, 2]
9*bc*	12	12	1	1	[2, 3, 2]	[]	[12]
9*bd*	18	18	1	1	[3, 3, 2]	[]	[18]
9*be*	18	18	1	1	[3, 3, 2]	[]	[18]
9*bf*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
9*bg*	48	12	2	1	[2, 3, 2, 2, 2]	[24, 2]	[24, 2]
9*bh*	192	6	8	1	[2, 2, 3, 2, 2, 2, 2]	[24, 8]	[24, 2, 2, 2]
9*bi*	384	6	8	1	[2, 2, 2, 3, 2, 2, 2, 2]	[48, 8]	[48, 4, 2]
9*bj*	192	6	4	1	[2, 2, 2, 3, 2, 2, 2]	[48, 4]	[48, 2, 2]
9*bk*	144	12	6	2	[2, 3, 2, 2, 2, 3]	[24, 6]	[24, 2]
9*bl*	96	12	4	1	[3, 2, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*bm*	96	12	4	1	[3, 2, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*bn*	4320	6	720	720	[2, 360, 2, 3]	[6]	[6]
9*bo*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
9*bp*	864	6	72	12	[2, 3, 2, 3, 3, 2, 2, 2]	[12, 9, 4, 2]	[12]
9*bq*∗	96	12	4	1	[3, 2, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*br*	1536	6	64	1	[2, 2, 3, 2, 2, 2, 2, 2, 2, 2]	[24, 64]	[24, 2, 4, 4, 2]
9*bs*	12	12	1	1	[3, 2, 2]	[]	[12]
9*bt*	24	24	1	1	[2, 2, 3, 2]	[]	[24]
9*bu*	60	6	5	2	[2, 3, 5, 2]	[12, 5]	[12]
9*bv*	1152	6	48	6	[2, 2, 2, 3, 2, 3, 2, 2, 2]	[24, 6, 4, 2]	[24, 2]
9*bw*	48	6	4	1	[2, 3, 2, 2, 2]	[12, 4]	[12, 2, 2]
9*bx*	96	6	8	1	[2, 3, 2, 2, 2, 2]	[12, 8]	[12, 2, 2, 2]
9*by*	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*bz*	864	6	36	8	[2, 2, 2, 3, 2, 2, 3, 3]	[24, 4, 9]	[24, 2, 2]
9*ca*	192	12	8	1	[2, 2, 2, 2, 2, 3, 2]	[24, 8]	[24, 4, 2]
9*cb*	288	6	24	2	[2, 2, 3, 3, 2, 2, 2]	[12, 24]	[12, 2, 2, 2]
9*cc*	576	6	24	6	[2, 3, 2, 2, 3, 2, 2, 2]	[24, 3, 4, 2]	[24]
9*cd*	96	12	4	1	[2, 3, 2, 2, 2, 2]	[24, 4]	[24, 2, 2]
9*ce*	480	6	20	4	[2, 2, 5, 3, 2, 2, 2]	[24, 20]	[24, 2, 2]
9*cf*	36	6	3	2	[3, 2, 2, 3]	[12, 3]	[12]
9*cg*	768	6	32	1	[2, 3, 2, 2, 2, 2, 2, 2, 2]	[24, 32]	[24, 4, 4, 2]
9*ch*	1152	6	96	6	[2, 3, 2, 2, 2, 3, 2, 2, 2]	[12, 12, 4, 2]	[12, 2, 2]
9*ci*	288	6	12	2	[2, 2, 2, 3, 2, 3, 2]	[24, 12]	[24, 2, 2]
9*cj*	384	6	16	1	[2, 3, 2, 2, 2, 2, 2, 2]	[24, 16]	[24, 2, 2, 2, 2]
9*ck*	1152	6	24	2	[2, 2, 2, 3, 2, 2, 3, 2, 2]	[48, 24]	[48, 2, 2, 2]
9*cl*	120	6	5	4	[2, 3, 2, 2, 5]	[24, 5]	[24]
9*cm*	384	12	16	1	[2, 2, 3, 2, 2, 2, 2, 2]	[24, 16]	[24, 4, 2, 2]
9*cn*	1728	6	72	12	[2, 3, 2, 2, 3, 3, 2, 2, 2]	[24, 9, 4, 2]	[24]
9*co*	1152	6	48	6	[2, 2, 3, 2, 2, 3, 2, 2, 2]	[24, 6, 4, 2]	[24, 2]
9*cp*	576	12	24	2	[2, 2, 3, 2, 2, 2, 2, 3]	[24, 24]	[24, 4, 2]
9*cq*	13824	6	1152	72	[2, 3, 2, 2, 3, 3, 2, 2, 2, 2, 2, 2]	[12, 2, 9, 16, 4]	[12, 2]
9*cr*	576	12	24	2	[2, 2, 3, 3, 2, 2, 2, 2]	[24, 24]	[24, 2, 2, 2]
9*cs*	2880	6	240	120	[2, 3, 2, 60, 2, 2]	[12, 2]	[12, 2]
9*ct*	11520	6	1920	120	[2, 60, 2, 2, 2, 2, 2, 3]	[6]	[6]
9*cu*	11289600	48	940800	235200	[2, 2, 3, 2, 2, 2, 58800, 2]	[12, 8]	[12, 2, 2, 2]

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