

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

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10[35Bxx, 35Jxx, 35Kxx, 65Dxx, 65Mxx, 65Nxx, 76Dxx, 76Mxx]—
Numerical analysis for fluids, Part 3. Finite element methods for incompressible viscous flow, by Roland Glowinski, Handbook of numerical analysis, Vol. IX, edited by P. G. Ciarlet and J. L. Lions, North-Holland, Amsterdam, 2003, x+1176 pp., hardcover, US\$109.00, ISBN 0-444-51224-1

This excellent volume gives a complete and up-to-date presentation of numerical methods for solving many problems of incompressible flow. The emphasis is on Navier–Stokes equations, but the volume also treats other important flows such as particulate flows and visco-plastic flows. Furthermore, several methods and algorithms studied have a much wider range of applications than flow problems. The number of numerical methods and algorithms proposed is impressive, and they are illustrated by a very large set of numerical experiments. Although the volume is mostly focused on numerical methods, it includes proofs or sketches the proofs of results (existence, convergence, error estimates, etc.) whenever these proofs exist. Finally, it includes a very complete set of references (20 pages).

The volume is divided into ten chapters. Each chapter starts with a comprehensive synopsis that gives a clear idea of the chapter's contents. Here is a brief description of the contents of each chapter.

Chapter I. Chapter I is devoted first to the mechanical derivation of the Navier–Stokes equations in primitive variables, including boundary conditions and the stream-function vorticity formulation, which is very useful in two dimensions. Then it turns to the variational formulations of the Navier–Stokes equations and to existence and uniqueness or nonuniqueness results.

Chapter II. This chapter is devoted to a family of operator splitting methods for an abstract time-dependent problem with a nonlinear operator:

$$(1) \quad \frac{\partial \varphi}{\partial t} + A(\varphi, t) = 0, \quad \varphi(0) = \varphi_0.$$

The operator A is split into a sum of two operators, and several schemes are proposed for splitting in time (1). These are inspired by the Peaceman–Rachford and Douglas–Rachford schemes and the θ -scheme introduced by Glowinski. The chapter ends with an application to the Navier–Stokes problem, where one of the operators expresses the nonlinear convection term, possibly linearized, and the other operator expresses the divergence zero constraint.

Chapter III. In practice, the above splitting involves the solution of a linear or nonlinear advection-diffusion system of equations. Chapter III is devoted to the solution of such a system. The first half of the chapter discusses the theoretical aspects of advection-diffusion problems, linear or nonlinear: existence and uniqueness of solutions. The second half proposes numerical algorithms combining least-squares (because the problem is not symmetric) and conjugate-gradient methods for solving advection-diffusion equations. Summing up, this chapter gives a very good introduction to both algorithms.

Chapter IV. Now we come to the solution of the Stokes part in the splitting. In its standard formulation, the Stokes problem differs from the Laplace equation by its divergence constraint and Lagrange multiplier: the pressure. The chapter is mostly devoted to a number of important algorithms for decoupling the computation of the velocity from that of the pressure, such as Uzawa's algorithm, augmented Lagrangian methods, and conjugate-gradient algorithms.

The Stokes problem has a less standard formulation, suited to boundary conditions that involve the Cauchy stress (instead of adherence to the wall), called the generalized Stokes problem by Glowinski. He adapts the above algorithms to this formulation.

The last part of the chapter is devoted to penalty methods, that Glowinski calls artificial compressibility conditions, for weakly enforcing the zero divergence.

Chapter V. This chapter presents a thorough description, ranging from mathematical analysis to computer implementation, of several major finite-element schemes for the space discretization of first the steady Stokes and Navier-Stokes equations and next the time-dependent problems. It focuses particularly on the Hood-Taylor method and the implementation of the θ scheme.

Chapter VI. This chapter contains highly original material; namely the interpretation of the transport equation

$$(2) \quad \frac{\partial \varphi}{\partial t} + \mathbf{v} \cdot \nabla \varphi = 0,$$

with $\operatorname{div} \mathbf{v} = 0$, \mathbf{v} independent of time, as a “wave equation”

$$(3) \quad \frac{\partial^2 \varphi}{\partial t^2} - \mathbf{v} \cdot \nabla (\mathbf{v} \cdot \nabla \varphi) = 0.$$

This idea is due to Glowinski. The chapter begins by developing other operator splitting methods, and it includes an interesting relation to domain decomposition as a side product. Then this is applied to the time-dependent Navier-Stokes equations; one step of the splitting being an advection equation of the form (2) transformed into (3). Several schemes are discussed for solving this equation. The chapter ends with another valuable technique for solving the transport part of the Navier-Stokes system, namely the method of characteristics.

Chapter VII. This chapter is dedicated to the famous L^2 -projection methods of Chorin and Temam for decoupling the treatment of the convection from that of the divergence constraint. It gives a comprehensive discussion of the original schemes and the variations brought by subsequent authors.

Chapter VIII. This chapter, one of the most original of this volume, is devoted to a family of fictitious domain methods with Lagrange multipliers. Fictitious domain methods are very useful when the domain has a complex boundary or when the domain moves in time. The idea is to embed the original domain, say Ω , into a larger, simply shaped, fixed domain, say \mathcal{O} , which can be meshed uniformly. Then the original problem is reformulated so that it can be solved in \mathcal{O} and the original boundary conditions on $\partial\Omega$ recovered either exactly or approximately. There are several fictitious domain methods; the family of methods considered in this chapter is characterized by the fact that the original boundary conditions on $\partial\Omega$ are imposed weakly (but exactly) by a constraint and a Lagrange multiplier. Again, there are several ways for imposing this constraint, and this chapter discusses essentially two methods: the boundary constraint with boundary multiplier, and the volume constraint with distributed multiplier. Both methods are thoroughly described, analyzed, and applied to a variety of complex flow problems, in particular to particulate flow and sedimentation. The chapter includes a selection of impressive numerical experiments.

Chapter IX. This chapter collects a large number of numerical experiments, which are successful applications of several powerful methods described in the previous chapters. For instance, the wave-like approach (see Chapter VI) for the transport equation is applied to delicate problems, such as the wall-driven cavity problem and the channel with backward facing step. This wave-like approach and operator splitting schemes (see Chapter II) are applied to solve the Boussinesq equations modeling thermal convection phenomena. Fictitious domain methods (see Chapter VIII) are applied to simulate blood flow around the heart and, again, to particulate flow.

Chapter X. In this chapter, Glowinski discusses important methods that would have justified chapters of their own, but could not be developed fully in this volume for obvious lack of space. These include the famous stream-function-vorticity method for the Navier–Stokes equations in two dimensions and problems of optimal control for the incompressible Navier–Stokes equations, with a very interesting application to drag reduction.

Last but not least, this volume is very clear and so well written that it is even accessible at the level of Ph.D. students. It is a unique reference text on incompressible flow for researchers and engineers.

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