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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

4[65F15]—*The matrix eigenvalue problem: GR and Krylov subspace methods*, by David S. Watkins, Society of Industrial and Applied Mathematics, Philadelphia, PA, 2007, x + 442 pp., softcover, US$99.00, ISBN 978-0-898716-41-2

This book deals with numerical methods for the solution of eigenvalue problems. Problems treated include the standard problem $Ax = \lambda x$, the generalized problem $Ax = \lambda Bx$, and more in general, the product problem $A_1 A_2 A_3 \ldots A_p x = \lambda x$ (with some of the A_i possibly written as inverses of some other matrix). Special attention is given to certain matrix structures, such as symplectic or Hamiltonian matrices.

Most of the book deals with moderate size problems, namely, those whose matrices can be easily stored and manipulated, say up to hundreds or a few thousand variables. For these problems, several versions of the GR method (including QR and QZ) are presented. For larger matrices, Krylov subspace methods are described. This book is a very welcome addition to the literature.

David Watkins is well known for his expertise in structured eigenvalue problems and for his engaging lectures. In this book, he has been able to communicate on the printed page the excitement of his oral presentations. In the book, Watkins talks to the reader as if he were lecturing, and this works very well.

The book, however, does not cover all modern methods for eigenproblems. One notable omission is the Jacobi-Davidson method [3], which is included for instance in the *Templates* [1], but not even mentioned by Watkins. This is actually the only drawback of this otherwise excellent text.

The book starts with two introductory chapters, one with general material on matrices, and one with basic theory of eigenvalue problems. In my view, these 115 pages can be considered a complete Linear Algebra course by themselves, and a very good one indeed.

Then, Watkins launches into the meat of the tome, namely the GR methods. These are iterative methods, of which the QR method is a particular case. A sequence of similarity transformations is performed, first to obtain an upper Hessenberg matrix, and then to decrease the value of the elements in the subdiagonal, until they are small enough to be considered zeros. Thus, a triangular matrix is obtained from which the eigenvalues can be directly read. Watkins does a superb job in explaining the intricacies of these methods, guiding the reader through the process. This is no easy feat. (Let me mention here the recent book by Kressner [2] which goes more deeply into this subject.)

Watkins touches upon the relatively recent developments of very accurate solutions of these problems, in which all the algorithms are designed in such a way that no two numbers of the same sign are ever subtracted. Thus, no serious loss of significant digits due to cancellation can occur.

Two other important topics well covered are the implicit restarted Arnoldi method of Sorensen and the Krylov-Schur method of Stewart. This is probably the best description of these methods in the literature, together with a proof of the mathematical equivalence of these two methods.

In addition to the clarity of the exposition, there are many things I like in this book. Throughout the book, many proofs of theorems are given, and many others are left to the reader as exercises. The exercises are well designed, with sketches of those proofs, so that the reader can follow them and really understand the results. There are several sections titled "Why the method works", which do exactly that. There is a web site with Matlab programs that the reader can access and use. The extensive bibliography includes page numbers where the paper or book is mentioned in the text.

In summary, this book is a pleasure to read, and I recommend it both to graduate students and to scientists who are interested in an introduction to the state-of-the art in several aspects of eigenvalue computations.

References

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[2] Daniel Kressner. *Numerical Methods for General and Structured Eigenvalue Problems*, Lecture Notes in Computational Science and Engineering, vol. 46, Springer-Verlag, Berlin, Heidelberg, New York, 2005. MR2164298 (2006d:65002)

[3] Gerard L. G. Sleijpen and Henk A. van der Vorst, A Jacobi-Davidson iteration method for linear eigenvalue problems. *SIAM J. Matrix Anal. Appl.*, 17:401–425, 1996. MR1384515 (96m:65042)

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