

APPENDIX A. ADDITIONAL COMPUTATIONAL TABLES

TABLE 9. Decomposition of polynomials into powers of linear forms

$n$	Degree $M$							
	1	2	5	10	20	30	40	50
2	2	3	11	33	129	279	491	775
3	3	6	40	205	1381	4306	9880	18970
4	4	10	103	831	9373	41373	122349	286893
5	5	15	221	2681	49586	305836	$1.2 \cdot 10^6$	$3.3 \cdot 10^6$
8	8	36	1226	42271	$3.1 \cdot 10^6$	$4.8 \cdot 10^7$	$3.7 \cdot 10^8$	$1.9 \cdot 10^9$
10	10	55	2917	181413	$3.0 \cdot 10^7$	$8.4 \cdot 10^8$	$1.0 \cdot 10^{10}$	$7.5 \cdot 10^{10}$
15	15	120	15338	$3.3 \cdot 10^6$	$3.2 \cdot 10^9$	$3.4 \cdot 10^{11}$	$1.2 \cdot 10^{13}$	$2.1 \cdot 10^{14}$
20	20	210	52859	$3.0 \cdot 10^7$	$1.4 \cdot 10^{11}$	$4.7 \cdot 10^{13}$	$4.2 \cdot 10^{15}$	$1.6 \cdot 10^{17}$
30	30	465	324076	$8.5 \cdot 10^8$	$4.7 \cdot 10^{13}$	$1.2 \cdot 10^{17}$	$5.5 \cdot 10^{19}$	$8.9 \cdot 10^{21}$
40	40	820	$1.2 \cdot 10^6$	$1.0 \cdot 10^{10}$	$4.2 \cdot 10^{15}$	$5.5 \cdot 10^{19}$	$1.1 \cdot 10^{23}$	$6.0 \cdot 10^{25}$
50	50	1275	$3.5 \cdot 10^6$	$7.5 \cdot 10^{10}$	$1.6 \cdot 10^{17}$	$8.9 \cdot 10^{21}$	$6.0 \cdot 10^{25}$	$1.0 \cdot 10^{29}$









TABLE 14. Integration of a monomial of prescribed degree with 2 effective variables using an iterated Laurent expansion

$n$	Degree										
	1	2	5	10	20	30	40	50	100	200	300
3	0	0	0	0	0	0	0	0	0.2	0.3	0.9
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.3</b>	<b>1.2</b>	<b>16</b>
	0	0	0	0.1	0.1	0.3	5.6	2.1	3.6	20	62
4	0	0	0	0	0.1	0.1	0.1	0.1	0.2	0.4	0.9
	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.1</b>	<b>0.1</b>	<b>0.3</b>	<b>0.3</b>	<b>0.5</b>	<b>2.1</b>	<b>11</b>	<b>47</b>
	0.1	0.1	0.1	0.1	0.2	4.2	2.5	3.0	5.4	38	166
5	0.1	0	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.6	1.3
	<b>0.1</b>	<b>0.1</b>	<b>0.1</b>	<b>0.1</b>	<b>0.3</b>	<b>0.3</b>	<b>0.5</b>	<b>0.7</b>	<b>3.6</b>	<b>21</b>	<b>86</b>
	0.1	0.1	0.1	0.2	6.4	2.3	2.8	3.7	9.8	80	271
6	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.4	1.0	1.9
	<b>0.1</b>	<b>0.1</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.6</b>	<b>0.8</b>	<b>1.1</b>	<b>7.2</b>	<b>39</b>	<b>100</b>
	0.2	0.1	0.2	3.4	2.5	2.6	3.8	4.7	30	269	550
7	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.6	1.2	
	<b>0.1</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.6</b>	<b>0.8</b>	<b>1.2</b>	<b>2.0</b>	<b>6.8</b>	<b>94</b>	
	0.2	0.2	3.9	2.5	2.8	3.4	4.2	6.7	26	260	
8	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.7	2.0	
	<b>0.2</b>	<b>0.3</b>	<b>0.3</b>	<b>0.4</b>	<b>0.7</b>	<b>1.2</b>	<b>1.9</b>	<b>2.9</b>	<b>16</b>	<b>99</b>	
	0.3	4.5	2.3	2.6	3.1	3.6	5.8	12	42	322	
10	0.3	0.3	0.3	0.4	0.4	0.5	0.6	0.6	1.1		
	<b>0.5</b>	<b>0.4</b>	<b>0.5</b>	<b>0.7</b>	<b>1.2</b>	<b>2.4</b>	<b>3.8</b>	<b>5.9</b>	<b>23</b>		
	4.2	2.3	2.5	3.0	3.9	6.5	11	17	77		
15	1.3	1.3	1.4	1.4	1.6	1.7	1.9	2.4			
	<b>1.4</b>	<b>1.5</b>	<b>1.7</b>	<b>2.1</b>	<b>4.0</b>	<b>7.3</b>	<b>11</b>	<b>20</b>			
	1.6	1.9	2.0	3.4	9.1	21	31	52			
20	4.0	4.0	4.1	4.2							
	<b>4.2</b>	<b>4.4</b>	<b>4.8</b>	<b>6.1</b>							
	4.4	5.6	5.7	9.3							
30	22	24	24	25	26						
	<b>25</b>	<b>25</b>	<b>27</b>	<b>32</b>	<b>47</b>						
	26	28	31	45	96						
40	71	88	90								
	<b>85</b>	<b>92</b>	<b>95</b>								
	92	102	103								
50	196	228	212	217							
	<b>231</b>	<b>239</b>	<b>244</b>	<b>261</b>							
	252	249	277	347							

TABLE 15. Integration of a monomial of prescribed degree with 2 effective variables using Taylor expansion

$n$	Degree										
	1	2	5	10	20	30	40	50	100	200	300
3	0	0	0	0	0	0	0	0	0	0	1.9
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.4</b>	<b>0.8</b>	<b>5.8</b>	<b>62</b>	
	0	0	0.1	0.1	0.2	0.6	1.3	2.6	27	359	
4	0	0	0	0	0	0	0	0	0		
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.2</b>	<b>0.4</b>	<b>0.7</b>	<b>1.8</b>	<b>11</b>		
	0	0	0.1	0.1	0.4	1.2	3.0	8.2	58		
5	0	0	0	0	0	0	0	0	0		
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.6</b>	<b>2.0</b>	<b>2.9</b>	<b>19</b>		
	0	0	0.1	0.2	0.7	2.2	6.7	15	110		
6	0	0	0	0	0	0	0	0	0		
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.7</b>	<b>0.9</b>	<b>2.0</b>	<b>4.4</b>	<b>31</b>		
	0	0	0.1	0.3	16	8.4	8.8	72	124		
7	0	0	0	0	0	0	0.2	0	0		
	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.5</b>	<b>1.6</b>	<b>4.3</b>	<b>5.9</b>	<b>52</b>		
	0	0.1	0.1	0.3	1.8	8.4	21	32	246		
8	0	0	0	0	0	0	0	0.3	0.1		
	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.9</b>	<b>2.7</b>	<b>3.6</b>	<b>9.3</b>	<b>65</b>		
	0.1	0.1	0.2	0.4	2.8	36	19	42	547		
10	0	0	0	0	0	0	0	0			
	<b>0</b>	<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>1.1</b>	<b>3.5</b>	<b>8.4</b>	<b>20</b>			
	0.1	0.1	0.2	0.6	7.0	22	40	92			
20	0	0	0	0	0	0	0	2.0			
	<b>0</b>	<b>0.1</b>	<b>0.4</b>	<b>2.8</b>	<b>12</b>	<b>40</b>	<b>74</b>	<b>121</b>			
	0.1	0.2	1.1	9.3	43	156	309	462			
30	0	0	0	0	0	0.1					
	<b>0</b>	<b>0.2</b>	<b>1.4</b>	<b>6.9</b>	<b>62</b>	<b>90</b>					
	0.1	0.5	5.6	29	205	583					
40	0	0	0.1	0.1	0.1						
	<b>0.1</b>	<b>0.4</b>	<b>3.3</b>	<b>10</b>	<b>71</b>						
	0.1	0.9	32	40	329						
50	0.1	0.1	0.1	0.1	0.1						
	<b>0.1</b>	<b>0.4</b>	<b>5.5</b>	<b>19</b>	<b>126</b>						
	0.1	1.5	39	65	525						

