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NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>572</td>
<td>October 22, 1960</td>
<td>Worcester, Massachusetts</td>
<td>Sept. 8</td>
</tr>
<tr>
<td>573</td>
<td>November 18-19, 1960</td>
<td>Nashville, Tennessee</td>
<td>Oct. 5</td>
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<td>574</td>
<td>November 19, 1960</td>
<td>Pasadena, California</td>
<td>Oct. 5</td>
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<tr>
<td>576</td>
<td>January 24-27, 1961</td>
<td>Washington, D. C.</td>
<td>Dec. 9</td>
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<tr>
<td></td>
<td>(67th Annual Meeting)</td>
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<td></td>
<td>April, 22, 1961</td>
<td>Stanford, California</td>
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<td></td>
<td>August, 1961</td>
<td>Stillwater, Oklahoma</td>
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<td></td>
<td>(66th Summer Meeting)</td>
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<td></td>
<td>November 17-18, 1961</td>
<td>Milwaukee, Wisconsin</td>
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<td></td>
<td>January, 1962</td>
<td>Cincinnati, Ohio</td>
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<tr>
<td></td>
<td>(68th Annual Meeting)</td>
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<tr>
<td></td>
<td>August, 1962</td>
<td>Vancouver, British Columbia</td>
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<tr>
<td></td>
<td>(67th Summer Meeting)</td>
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<tr>
<td></td>
<td>January, 1963</td>
<td>Berkeley, California</td>
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<tr>
<td></td>
<td>(69th Annual Meeting)</td>
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<tr>
<td></td>
<td>August, 1963</td>
<td>Boulder, Colorado</td>
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<tr>
<td></td>
<td>(68th Summer Meeting)</td>
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</tr>
<tr>
<td></td>
<td>August, 1966</td>
<td>New Brunswick, New Jersey</td>
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</tbody>
</table>

* The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

The NOTICES of the American Mathematical Society is published by the Society seven times a year, in February, April, June, August, October, November, and December. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (non available before 1958), and inquiries should be addressed to the American Mathematical Society, Ann Arbor, Michigan, or 190 Hope Street, Providence 6, Rhode Island.

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SIXTY-FIFTH SUMMER MEETING
and
THIRTY-NINTH COLLOQUIUM

Michigan State University
East Lansing, Michigan
August 30-September 2, 1960

The sixty-fifth Summer Meeting and the thirty-ninth Colloquium of the American Mathematical Society will be held at Michigan State University, East Lansing, Michigan, from Tuesday, August 30 to Friday, September 2, 1960. During the same week there will be meetings of the Mathematical Association of America and the Society for Industrial and Applied Mathematics. An outline of the Programs of these organizations is found under "Activities of Other Associations" within this issue of the NOTICES.

Professor S. S. Chern of the University of California, Berkeley, will deliver the Colloquium Lectures: "Geometrical structures on manifolds." The first of these lectures will be held on Tuesday, August 30 at 2:00 P.M., and will be followed by three others on Wednesday, Thursday, and Friday at 9:00 A.M. All of the lectures will be delivered in the Anthony Hall Auditorium.

The Committee to Select Hour Speakers for Summer and Annual Meetings has invited Professor Paul Halmos of the University of Chicago and Professor P. E. Conner of the University of Virginia to address the Society. Professor Halmos will speak at 1:30 P.M. on Thursday, September 1. His title is "Recent progress in ergodic theory." Professor Conner's lecture, "Involutions and equivariant maps," will be delivered at 1:30 P.M. on Friday, September 2. Both addresses will be delivered in the Anthony Hall Auditorium.

Sessions for contributed papers will be held at 3:30 P.M. on Tuesday, August 30, and at 3:00 P.M. on the last three days of the meeting. In addition, there will be two morning sessions at 10:30 A.M. on Thursday and Friday, September 1 and 2. As stated in the Preliminary Announcement of this meeting, there will, however, be no special session for papers which failed to meet the deadline.

The Council of the Society will meet at 4:00 P.M. on Tuesday, August 30 in the Conference Room of the Physics-Mathematics Building. There will be a Business Meeting of the Society at 10:15 A.M. on Wednesday, August 31, in Anthony Hall Auditorium.

At the Business Meeting the Society will be asked to vote on certain changes in the By-Laws concerning the position of Executive Director. These suggested changes were prepared by a Committee appointed by the President and have been approved by the Council,
and have been proposed in order to bring practice in the Society into line with that which has proved successful in other professional and learned societies.

It is proposed that the first sentence of Article II, Section 3, read as follows:

The Board of Trustees shall have the power to appoint such assistants and agents as may be necessary or convenient to facilitate the conduct of the affairs of the Society, and to fix the terms and conditions of their employment.

and Article VI, Sections 2 and 3, read as follows:

Section 2: The Executive Director shall be appointed by the Board of Trustees with the consent of the Council. The terms and conditions of his employment shall be fixed by the Board of Trustees.

Section 3: The Executive Director shall work under the immediate direction of a committee consisting of the President, the Secretary, and the Treasurer, of which the President shall be chairman ex-officio. The Executive Director shall attend meetings of the Board of Trustees, of the Council, and of the Executive Committee, but he shall not be a member of any of these bodies.

The complete By-Laws are to be found in the November, 1958 issue of the BULLETIN of the Society.

REGISTRATION, ROOMS, MEALS

Registration headquarters will be in the lobby of Snyder Hall, situated near the northeast corner of the campus on Bogue Street, approximately 300 yards south of East Grand River Avenue (U. S. Route 16). All persons attending the meeting are requested to register immediately upon arrival, whether or not they are staying in the dormitories. The registration fee for those attending the meeting is $2.00 for each member of any of the participating organizations and fifty cents for each accompanying adult. There is no registration fee for children. A directory of all persons attending the meetings and an information desk will be maintained at registration headquarters in Snyder Hall.

Dormitory facilities will be available to all those attending the meeting, and to their families. New arrivals will be accommodated at all hours of the day and night after 2:00 P.M. on Sunday, August 28. The cost of housing is $3.00 a day per person in double
rooms with separate beds or $18.00 per week. Single rooms when available will cost $4.50 per day or $27.00 per week. A state use tax of 4% is added to these room rates. Bedding, towels, and soap will be furnished and daily maid service with clean towel and clean linens each day will be provided. Children under 8 years of age will be charged half price for rooms, and there will be no charge for infants. Automatic washing and drying equipment and electric irons are available in the dormitories. Rooms may be occupied from two o'clock Sunday afternoon, August 28, until Sunday noon, September 4.

Cafeteria service will be maintained in the dormitory dining rooms during the meeting, beginning Sunday, August 28, at 5:30 P.M. Meal hours will be: breakfast 7:45 to 8:45 A.M.; lunch 12:15 to 1:15 P.M.; dinner 5:45 to 6:45 P.M.

RESERVATIONS

A reservation form will be found on the last page of these NOTICES. Persons desiring dormitory accommodations are requested to complete the form and mail it to Professor L. M. Kelly, Department of Mathematics, Michigan State University, East Lansing, Michigan, at the earliest possible moment unless they have availed themselves of the prior form found in the NOTICES which carried the Preliminary Announcement of this meeting.

HOTELS AND MOTELS

Persons desiring hotel or motel accommodations should make their reservations directly with the respective managements. Among Lansing hotels (about 4 or 5 miles from the meeting) are Hotel Olds ($6.50 single, $11.00 twin), Hotel Porter ($4.75 single, $8.50 twin), and Hotel Roosevelt ($5.50 single, $9.00 twin), quoted rates being the minima for rooms with single beds and twin beds respectively. Among East Lansing motels (about 2 miles from the meeting) are Amity Hall ($7.00 single, $10.00 twin, $15.00 suite), Holiday Inn ($7.50 single, $11.00 twin), Albert Pick Motor Hotel ($7.50 single, $11.00 twin, extra rollaway beds $2.50), and Poplars Inn ($7.50 single, $11.50 twin). The state use tax of 4% will be added to all of these rates.

ENTERTAINMENT AND RECREATION

The lounges and recreation rooms of Snyder-Phillips, the Conference Room of the Physics-Mathematics Building (Room 221), and the adjoining Mathematics Library will be open to the members and guests of the participating organizations.

Athletic facilities available to members and guests include table tennis in the dormitory recreation rooms, tennis courts near
the stadium on campus, outdoor and indoor swimming pools (restricted to persons aged 14 years or over) near the stadium on campus, and an 18-hole golf course on the southwestern corner of the campus. Visitors should bring their own sports equipment.

On Monday evening there will be an informal coffee hour at 8:00 P.M. in the Phillips Hall, Lower Lounge.

An informal tea will be held Tuesday afternoon from 4:00 to 6:00 P.M. in the Phillips Hall, Lower Lounge.

A chicken barbecue for members and guests of the participating organizations will be held Wednesday evening at 5:00 P.M. on the campus. The price will be $2.75 (plus tax) per person.

TRAVEL INFORMATION

East Lansing is four miles east of Lansing and eighty miles west of Detroit on U.S. Highway 16. Persons driving to Lansing from the South or Southwest will wish to use Highways 27 or 78.

Those driving from Wisconsin may cross Lake Michigan from Milwaukee or Muskegon by using the Wisconsin and Michigan Steamship Company Lines.

The Chesapeake and Ohio streamliner trains leave Detroit (Fort Street) and Grand Rapids twice daily, arriving at the Michigan Avenue Station in Lansing which is about a half mile east of the Capitol. The Grand Trunk Railway Line from Chicago to Fort Huron has service through Lansing and its Lansing station is about one mile south of the Capitol on Washington Avenue. The Hotel Olds, facing the Capitol, may be reached from the Grand Trunk Station by taxicab or Washington Avenue bus.

Buses marked College-Fisher leave the southeast corner of Washington and Michigan Avenues in Lansing (one block east of the Hotel Olds) about every 25 minutes, and pass the Chesapeake and Ohio Depot on the way to East Lansing. (Snyder Hall is near the northeast corner of the campus on Bogue Street approximately 300 yards south of the Bogue Street stop.) Greyhound buses come to East Lansing from Detroit (80 miles), Ann Arbor (60 miles), Battle Creek (55 miles), Chicago (220 miles) and other points.

Capital Airlines has direct service to Lansing from New York, Cleveland, Willow Run (Detroit), Chicago, Milwaukee and Minneapolis, and connects with other airlines. The Lansing Airport is 5 miles west of Lansing and 9 miles from East Lansing on U.S. Highway 16.

Those driving should note that there are no camping facilities within forty miles of the Lansing area.
<table>
<thead>
<tr>
<th>Time</th>
<th>Activities</th>
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<tbody>
<tr>
<td>SUNDAY, August 28</td>
<td>American Mathematical Society</td>
</tr>
<tr>
<td>2:00 P.M. - 8:00 P.M.</td>
<td>REGISTRATION - LOBBY OF SNYDER HALL</td>
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<tr>
<td>MONDAY, August 29</td>
<td>Mathematical Association of America</td>
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<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION - LOBBY OF SNYDER HALL</td>
</tr>
<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>BOOK AND JOURNAL EXHIBIT - LOWER LOUNGE OF SNYDER HALL</td>
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<tr>
<td>9:00 A.M.</td>
<td>Hedrick Lecture I, Ivan Niven - Anthony Hall Auditorium</td>
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<tr>
<td>2:00 P.M.</td>
<td>Hedrick Lecture II, Ivan Niven - Anthony Hall Auditorium</td>
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<tr>
<td>3:15 P.M.</td>
<td>Session on the Role of Abstract and Concrete Approaches in the Teaching of Mathematics: A. E. Ross, M. Kac - Anthony Hall Auditorium</td>
</tr>
<tr>
<td>7:30 P.M.</td>
<td>Board of Governors, 221 Physics - Mathematics Building</td>
</tr>
<tr>
<td>8:00 P.M.</td>
<td>INFORMAL COFFEE HOUR - PHILLIPS HALL, LOWER LOUNGE</td>
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**TIME TABLE**
(Eastern Standard Time)

<table>
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<tr>
<td>9:00 A.M.</td>
<td></td>
<td>Hedrick Lecture III, Ivan Niven - Anthony Hall Auditorium</td>
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<tr>
<td>10:00 A.M.</td>
<td></td>
<td>Business Meeting - Anthony Hall Auditorium</td>
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<tr>
<td>10:30 A.M.</td>
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<td>Report of Committee on the Undergraduate Program in Mathematics: R. C. Buck - Anthony Hall Auditorium</td>
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<tr>
<td>11:00 A.M.</td>
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<td>Lecture, W. J. Thron - Anthony Hall Auditorium</td>
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<tr>
<td>2:00 P.M.</td>
<td>Colloquium Lecture I, S. S. Chern, Anthony Hall Auditorium</td>
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<tr>
<td>3:00 P.M.</td>
<td>Sessions for Contributed Papers, Applied Mathematics, 146 Giltner Hall</td>
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<td></td>
<td>Topology, 118 Physics-Mathematics Building</td>
<td></td>
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<tr>
<td>4:00 P.M.</td>
<td>Council Meeting, Conference Room of Physics-Mathematics Building</td>
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<tr>
<td>4:00 P.M. - 6:00 P.M.</td>
<td>INFORMAL TEA - PHILLIPS HALL, LOWER LOUNGE</td>
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<tr>
<td>7:30 P.M.</td>
<td></td>
<td>Meeting of officers of the Sections of the Association - 221 Physics-Mathematics Building</td>
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<tr>
<td>Time</td>
<td>American Mathematical Society</td>
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<tr>
<td>9:00 A.M.</td>
<td>Colloquium Lecture II, S. S. Chern</td>
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<td></td>
<td>Anthony Hall Auditorium</td>
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<tr>
<td>10:15 A.M.</td>
<td>Business Meeting, Anthony Hall</td>
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<td></td>
<td>Auditorium</td>
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<td>1:30 P.M.</td>
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<td>2:45 P.M.</td>
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<tr>
<td>3:00 P.M.</td>
<td>Sessions for Contributed Papers, Analysis, 146 Giltner Hall</td>
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<td></td>
<td>Algebra, 118 Physics-Mathematics Building</td>
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<tr>
<td></td>
<td>Statistics and Probability, 116 Natural Science Building</td>
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<tr>
<td>5:00 P.M.</td>
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<tr>
<td>Time</td>
<td>American Mathematical Society</td>
<td>Society for Industrial and Applied Mathematics</td>
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<td><strong>EMPLOYMENT REGISTER - LOWER LOUNGE OF SNYDER HALL</strong></td>
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<td>9:00 A.M. - 5:00 P.M.</td>
<td><strong>BOOK AND JOURNAL EXHIBIT - LOWER LOUNGE OF SNYDER HALL</strong></td>
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<tr>
<td>9:00 A.M.</td>
<td>Colloquium Lecture III, S. S. Chern - Anthony Hall Auditorium</td>
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<tr>
<td>10:30 A.M.</td>
<td>Sessions for Contributed Papers</td>
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<td></td>
<td>Analysis, 146 Giltner Hall</td>
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<td></td>
<td>Algebra, 118 Physics-Mathematics Building</td>
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<td></td>
<td>Topology, 116 Natural Science Building</td>
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<tr>
<td>1:30 P.M.</td>
<td>Invited Address, P. R. Halmos - Anthony Hall Auditorium</td>
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<tr>
<td>3:00 P.M.</td>
<td>Sessions for Contributed Papers</td>
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<td></td>
<td>Analysis, 146 Giltner Hall</td>
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<td></td>
<td>Algebra, 118 Physics-Mathematics Building</td>
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<td></td>
<td>Topology, 116 Natural Science Building</td>
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<tr>
<td>8:00 P.M.</td>
<td></td>
<td>Session: J. P. Roth, S. Gorn - Anthony Hall Auditorium</td>
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**TIME TABLE**

(Eastern Standard Time)
# TIME TABLE
(Eastern Standard Time)

<table>
<thead>
<tr>
<th>FRIDAY, September 2</th>
<th>American Mathematical Society</th>
<th>Society for Industrial and Applied Mathematics</th>
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</thead>
<tbody>
<tr>
<td>9:00 A.M. - 2:00 P.M.</td>
<td><strong>REGISTRATION - LOBBY OF SNYDER HALL</strong></td>
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<td>9:00 A.M. - 2:00 P.M.</td>
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<td></td>
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<tr>
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<td><strong>BOOK AND JOURNAL EXHIBIT - LOWER LOUNGE OF SNYDER HALL</strong></td>
<td></td>
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<tr>
<td>9:00 A.M.</td>
<td>Colloquium Lecture IV, S. S. Chern - Anthony Hall Auditorium</td>
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</tbody>
</table>
| 10:30 A.M. | Sessions for Contributed Papers  
Analysis, 146 Giltner Hall  
Logic and Algebra, 118 Physics-Mathematics Building | |
| 1:30 P.M. | Invited Address, P. E. Conner - Anthony Hall Auditorium | |
| 2:40 P.M. | | Session: Martin Kruskal  
Anthony Hall Auditorium |
| 3:00 P.M. | Sessions on Contributed Papers  
Analysis, 146 Giltner Hall  
Geometry, 118 Physics-Mathematics Building | |
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

TUESDAY, 2:00 P.M.

Colloquium Lecture I, Anthony Hall Auditorium

Geometrical structures on manifolds (One hour)
Professor S. S. Chern, University of California, Berkeley

TUESDAY, 3:30 P.M.

Session on Applied Mathematics, 146 Giltner Hall

3:30 - 3:40
(1) A variational approach to a class of differential games
Dr. L. D. Berkovitz, RAND Corporation, Santa Monica, California (571-54)

3:45 - 3:55
(2) An investigation of the flow of a viscous compressible fluid past a semi-infinite flat plate at zero angle of attack
Dr. Gerald L. Davey, Hughes Aircraft Company, Culver City, California (571-86)

4:00 - 4:10
(3) Bounds on the nonlinear diffusion controlled growth rate of spherical precipitates
Mr. J. A. Morrison, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (571-63)
(Introduced by Dr. D. Slepian)

4:15 - 4:25
(4) A well posed problem for the backward heat equation
Dr. W. L. Miranker, IBM Research Laboratory, Yorktown Heights, New York (571-62)

4:30 - 4:40
(5) A two-body problem of classical electrodynamics
Mr. Rodney D. Driver, University of Minnesota (571-109)
(Introduced by Lawrence Markus)

4:45 - 4:55
(6) Some extensions of Bairstow's method
Dr. Herbert E. Salzer, Convair-Astronautics, San Diego, California (571-11)
TUESDAY, 3:30 P.M.

Session on Topology, 118 Physics-Mathematics Building

3:30 - 3:40
(7) Solution of a problem of L. B. Treybig
    Professor J. H. Roberts, Duke University (571-51)

3:45 - 3:55
(8) A converse of a theorem of R. H. Bing and its generalization
    Professor R. L. Wilder, University of Michigan
    (571-124)

4:00 - 4:10
(9) The limit of a sequence of homeomorphisms of $S^n$ onto itself
    Professor M. K. Fort, Jr., University of Georgia
    (571-42)

4:15 - 4:25
(10) The width of a tree-like continuum
     Professor C. E. Burgess, University of Utah
     (571-105)

4:30 - 4:40
(11) A characterization of the $n$-sphere
     Professors P. H. Doyle and J. G. Hocking, Michigan State University,
     (571-60)

4:45 - 4:55
(12) A regular semi-metric space for which there is no semi-metric under which all spheres are open
    Professor Robert W. Heath, University of Georgia
    (571-113)

5:00 - 5:10
(13) $S^3$ does not contain uncountably many mutually disjoint tori, no two of which are concentric
    Mr. C. H. Edwards, Jr., University of Tennessee
    (571-88)

WEDNESDAY, 9:00 A.M.

Colloquium Lecture II, Anthony Hall Auditorium
Geometrical structures on manifolds (One hour)
Professor S. S. Chern, University of California, Berkeley

WEDNESDAY, 10:15 A.M.

Business Meeting, Anthony Hall Auditorium
WEDNESDAY, 3:00 P.M.

Session on Analysis, 146 Giltner Hall
3:00 - 3:10
(14) On the relation between elimination and iteration methods in linear programming
Professor Edwin W. Titt, University of Arizona (571-134)

3:15 - 3:25
(15) On sectionally linear functions over an indefinite range
Professor H. W. E. Schwerdtfeger, McGill University (571-98)

3:30 - 3:40
(16) Extension properties of continuous functions, and the problem of mice
Professor R. C. Buck, University of Wisconsin and Institute for Defense Analyses, Princeton, New Jersey (571-138)

3:45 - 3:55
(17) Some classical theorems by methods of topological analysis
Professor Edwin H. Connell, Institute for Defense Analyses, Princeton, New Jersey (571-85)

4:00 - 4:10
(18) An extension of the Farkas theorem
Professor C. C. Braunschweiger and Mr. H. E. Clark, University of Delaware (571-31)

4:15 - 4:25
(19) Energy distribution of band-limited functions whose samples on a half line vanish
Dr. H. O. Pollak, Bell Telephone Laboratories, Murray Hill, New Jersey (571-64)

WEDNESDAY, 3:00 P.M.

Session on Algebra, 118 Physics-Mathematics Building
3:00 - 3:10
(20) Nested matrix multiplication tables
Professor E. P. Miles, Jr., Florida State University (571-92)

3:15 - 3:25
(21) Malcev algebras
Mr. Arthur A. Sagle, University of Chicago (571-97)

3:30 - 3:40
(22) On torsion-free linear systems
Dr. Uri Fixman, Yale University (571-110)
(Introduced by Dr. D. W. Dean)
3:45 - 3:55
(23) A restriction on the primitive operations of primal algebras which implies strict independence
Dr. Edward S. O'Keefe, Boeing Airplane Company, Seattle, Washington, (571-128)

4:00 - 4:10
(24) Derivations and embeddings in power series rings
Professor Nickolas Heerema, Florida State University (571-142)

4:15 - 4:25
(25) Direct summands of direct products of Abelian groups
Professor Elbert A. Walker, New Mexico State University (571-148)

4:30 - 4:40
(26) Subfields of the ring of $n \times n$ matrices over a finite field
Professor C. B. Hanneken, Marquette University (571-112)

WEDNESDAY, 3:00 P.M.

Session on Statistics and Probability, 116 Natural Science Building

3:00 - 3:10
(27) Optimal policy when the number of servers for a queue can be changed. Preliminary report
Professors Martin Fox and Herman Rubin, Michigan State University (571-125)

3:15 - 3:25
(28) A problem of arrangements
Dr. Leonard Tornheim, California Research Corporation, Richmond, California (571-135)

3:30 - 3:40
(29) On the measurability of stochastic process
Professor Mark E. Mahowald, Syracuse University (571-116)

3:45 - 3:55
(30) Some properties of stochastic independence. Preliminary report
Dr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey (571-8)

4:00 - 4:10
(31) Minimum sampling rate for stationary random processes
Professor L. L. Campbell, Assumption University of Windsor (571-84)

4:15 - 4:25
(32) On conditional expectations of location statistics
Professor Robert V. Hogg, University of Iowa (571-91)
4:30 - 4:40  
(33) A nonparametric ordering  
Dr. John S. White, General Motors Technical Center, Warren, Michigan (571-68)

4:45 - 4:55  
(34) Iterates of conditional expectation operators  
Professor D. L. Burkholder, University of Illinois and Dr. Y. S. Chow, IBM Research Center, Yorktown Heights, New York (571-70)

5:00 - 5:10  
(35) Gaussian probability measure defined by a generalized density  
Dr. William Clare Taylor, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland (571-133)

THURSDAY, 9:00 A.M.

Colloquium Lecture III, Anthony Hall Auditorium  
Geometrical structures on manifolds (One hour)  
Professor S. S. Chern, University of California, Berkeley

THURSDAY, 10:30 A.M.

Session on Analysis, 146 Giltner Hall  
10:30 - 10:40  
(36) A generalization of the Euler-Maclaurin summation formula  
Dr. Israel Navot, Technion, Haifa and Polytechnic Institute of Brooklyn (571-94)  
(Introduced by Professor Ronald M. Foster)

10:45 - 10:55  
(37) Collections of summability methods applied to orthogonal series  
Dr. Dan J. Eustice, National Aeronautic and Space Administration, Cleveland, Ohio (571-140)

11:00 - 11:10  
(38) Best approximators within a linear family on an interval  
Professor T. S. Motzkin, University of California, Los Angeles and Professor J. L. Walsh, Harvard University (571-117)

11:15 - 11:25  
(39) On Fourier series and inductive proof  
Dr. Daniel Shanks, David Taylor Model Basin, Washington, D. C. (571-131)
11:30 - 11:40
(40) Cluster sets of pseudo-meromorphic functions
Professor D. A. Storvick, University of Minnesota
(571-146)

11:45 - 11:55
(41) Orthogonal polynomials and polynomial approximation
Dr. Burton Wendroff, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (571-67)

THURSDAY, 10:30 A.M.

Session on Algebra, 118 Physics-Mathematics Building
10:30 - 10:40
(42) On loops represented by triplets
Professor Volodymyr Bohun-Chudyniv, Morgan State College (571-137)

10:45 - 10:55
(43) An extension of a theorem of A. V. Prasad
Dr. L. C. Egan, University of Oregon (571-40)

11:00 - 11:10
(44) A coding problem arising in data transmission
Professor R. C. Bose, University of North Carolina Case Institute of Technology and Mr. I. M. Chakravarti, Case Institute of Technology (571-82)

11:15 - 11:25
(45) Notes on a paper by Sanov, II
Dr. Ruth Rebekka Struik, University of British Columbia (571-132)

11:30 - 11:40
(46) Difference sets of the Hadamard type
Professor Albert Leon Whiteman, University of Southern California and Institute for Advanced Study (571-79)

11:45 - 11:55
(47) Generalized Kummer congruences for products of sequences (applications)
Dr. H. R. Stevens, Duke University (571-100)

THURSDAY, 10:30 A.M.

Session on Topology, 116 Natural Science Building
10:30 - 10:40
(48) A combinatorial cubical homology theory
Dr. J. P. Roth, IBM Research Center, Yorktown Heights, New York (571-145)
10:45 - 10:55
(49) Free topological groups
Professor B. R. Gelbaum, University of Minnesota
(571-90)

11:00 - 11:10
(50) Commutators in a complex semi-simple Lie group
Mr. S. Pasiencier and Professor H. C. Wang, Northwestern University, (571-29)

11:15 - 11:25
(51) The identity of a compact connected semi-group which is not a group is not a weak cutpoint
Professor Robert P. Hunter, University of Georgia
(571-114)

11:30 - 11:40
(52) The structure of Frobenius algebraic groups
Professor David Hertzig, Cornell University (571-127)

11:45 - 11:55
(53) Stationary points for solvable transformation groups
Professors John Greever and M. F. Tinsley, Florida State University, (571-141)

THURSDAY, 1:30 P.M.

Invited Address, Anthony Hall Auditorium
Recent progress in ergodic theory (One hour)
Professor P. R. Halmos, University of Chicago

THURSDAY, 3:00 P.M.

Session on Analysis, 146 Giltner Hall

3:00 - 3:10
(54) Continued fraction solutions of a Riccati equation
Professor E. P. Merkes, Marquette University and
Professor W. T. Scott, Northwestern University
(571-144)

3:15 - 3:25
(55) Potential forces which yield periodic motions of a fixed period
Professor Minoru Urabe, Hiroshima University; RIAS, Baltimore, Maryland; and Mathematics Research Center, University of Wisconsin (571-147)

3:30 - 3:40
(56) A geometrical approach to the second order linear differential equations
Dr. C. M. Petty, Lockheed Aircraft Corporation, Sunnyvale, California (571-150)
3:45 - 3:55
(57) Singular perturbations of two point boundary problems
Professor W. A. Harris, Jr., Mathematics Research Center, University of Wisconsin (571-126)

4:00 - 4:10
(58) Bifurcation of an invariant manifold from a periodic solution of a differential system. Preliminary report
Mr. Fred S. Van Vleck, University of Minnesota (571-122)

4:15 - 4:25
(59) Series solutions for systems of linear differential equations
Professor Charles Buck, University of Alabama (571-104)

THURSDAY, 3:00 P.M.

Session on Algebra, 118 Physics-Mathematics Building
3:00 - 3:10
(60) On high subgroups of Abelian torsion groups
Dr. John M. Irwin, New Mexico State University (571-143)
(Introduced by Dr. Elbert A. Walker)

3:15 - 3:25
(61) The alpha-completion of a ring of continuous real-valued functions. Preliminary report
Mr. E. C. Weinberg, Purdue University (571-136)

3:30 - 3:40
(62) Compact rings
Professor Seth Warner, Duke University (571-123)

3:45 - 3:55
(63) Reflective N-prime rings with the ascending chain condition
Professor E. H. Feller, University of Wisconsin-Milwaukee and Professor E. W. Swokowski, Marquette University (571-89)

4:00 - 4:10
(64) Linear quasi ordered semigroups
Professor N. J. Rothman, University of Rochester (571-120)

4:15 - 4:25
(65) The Šilov boundary induced by a certain Banach algebra
Dr. W. Wistar Comfort, Harvard University (571-2)
THURSDAY, 3:00 P.M.

Session on Topology, 116 Natural Science Building

3:00 - 3:10

(66) Extension of continuous functions in f_N
Professor N. J. Fine, University of Pennsylvania and
Institute for Advanced Study and Professor L. Gillman,
University of Rochester and Institute for Advanced
Study (571-41)

3:15 - 3:25

(67) Simplicial interior maps on the three-sphere
Professors P. T. Church and Erik Hemmingsen,
Syracuse University (571-107)

3:30 - 3:40

(68) On the Alexander polynomials of 2-spheres in 4-sphere.
Preliminary report
Dr. S. Kinoshita, Institute for Advanced Study (571-72)

3:45 - 3:55

(69) Unique representation of 3-spheres with handles
Professor D. E. Sanderson, Iowa State University
(571-130)

4:00 - 4:10

(70) The maximum connectivity of a graph
Professor Frank Harary, University of Michigan
(571-45)

4:15 - 4:25

(71) Two-dimensional continuous curves, I. Preliminary
report
Professors R. D. Anderson and J. E. Keisler,
Louisiana State University (571-80)

4:30 - 4:40

(72) Two-dimensional continuous curves, II. Preliminary
report
Professors R. D. Anderson and J. E. Keisler,
Louisiana State University (571-81)

4:45 - 4:55

(73) Locally affine spaces with nilpotent fundamental groups
Mr. J. C. Sanwal, Indiana University (571-121)
(Introduced by Professor L. Auslander)

FRIDAY, 9:00 A.M.

Colloquium Lecture IV, Anthony Hall Auditorium
Geometrical structures on manifolds (One hour)
Professor S. S. Chern, University of California,
Berkeley
FRIDAY, 10:30 A.M.

Session on Analysis, 146 Giltner Hall

10:30 - 10:40

(74) Unified treatment of martingales and ergodic theorems. I. Mean convergence
   Professor Gian-Carlo Rota, Massachusetts Institute of Technology (571-95)

10:45 - 10:55

(75) Two theorems on normal cones
   Professor H. H. Schaefer, University of Michigan (571-96)

11:00 - 11:10

(76) Banach algebras of multipliers
   Dr. F. T. Birtel, University of Notre Dame (571-69)

11:15 - 11:25

(77) The Liouville theorem in space
   Professor F. W. Gehring, University of Michigan (571-111)

11:30 - 11:40

(78) Adjointness as a functor
   Dr. Grosman Jay Clark, Lockheed Aircraft Corporation, Sunnyvale, California (571-139)

11:45 - 11:55

(79) On the reducibility of some linear differential operators. II
   Mr. Murray S. Klamkin, AVCO Corporation, Wilmington, Massachusetts and Professor Donald J. Newman, Yeshiva University (571-115)

FRIDAY, 10:30 A.M.

Session on Logic and Algebra, 118 Physics-Mathematics Building

10:30 - 10:40

(80) Recursive unsolvability of Post's "Tag" problem
   Dr. Marvin L. Minsky, Massachusetts Institute of Technology (571-9)

10:45 - 10:55

(81) Imaginary quadratic fields of class number one
   Professor Emil Grosswald, University of Pennsylvania and Institute for Advanced Study (571-44)

11:00 - 11:10

(82) Some remarks on a relative anti-closure property
   Mr. Albert A. Mullin, University of Illinois (571-10)

11:15 - 11:25

(83) Differentiable intrinsic functions of complex matrices
   Professor R. F. Rinehart, Duke University and Case Institute of Technology (571-50)
11:30 - 11:40
(84) A quasi algebra of matrices
Professor Ali R. Amir-Moéz and Professor Arnold L. Fass, Queens College (571-1)

11:45 - 11:55
(85) A generalization of the ring of formal polynomials
Professor George W. Patterson, University of Pennsylvania (571-129)

FRIDAY, 1:30 P.M.

Invited Address, Anthony Hall Auditorium
Involutions and equivariant maps (One hour)
Professor P. E. Conner, University of Virginia

FRIDAY, 3:00 P.M.

Session on Analysis, 146 Giltner Hall

3:00 - 3:10
(86) Arc cluster sets of bounded analytic functions
Professor P. T. Church, Syracuse University (571-36)

3:15 - 3:25
(87) Angular and tangential limits of Blaschke products and their successive derivatives
Professor G. T. Cargo, Syracuse University (571-106)

3:30 - 3:40
(88) Asymmetric prime ends
Dr. E. F. Collingwood, Alnwick, Northumberland, England and Professor George Piranian, University of Michigan (571-119)

3:45 - 3:55
(89) On the maximum number of deficient values of certain classes of meromorphic functions
Professor Albert Edrei, Syracuse University and Professor Wolfgang H. J. Fuchs, Cornell University (571-87)

4:00 - 4:10
(90) Interpolation by bounded analytic functions
Professor H. S. Shapiro and Professor A. L. Shields, Institute of Mathematical Sciences, New York University (571-99)

4:15 - 4:25
(91) A characterizing property for Bers system
Professor William V. Caldwell, University of Delaware (571-83)
FRIDAY, 3:00 P.M.

Session on Geometry, 118 Physics-Mathematics Building

3:00 - 3:10
(92) Complementary subspaces
    Professor T. K. Pan, University of Oklahoma (571-118)

3:15 - 3:25
(93) On the maximum number of points no three on one line in
    a projective plane of order 10
    Professor Esther Seiden, University of Illinois
    (571-149)

3:30 - 3:40
(94) Silhouette mathematics
    Professor R. S. Underwood, Texas Technological
    College (571-101)

3:45 - 3:55
(95) Three branch points on a surface in a space of ten dimen-
    sions
    Professor W. R. Hutcherson and Mr. J. W. Kenelly,
    University of Florida (571-7)

4:00 - 4:10
(96) An involution of period seventeen contained on a rational
    surface in a space of 11 dimensions
    Professor W. R. Hutcherson and Mr. J. W. Kenelly,
    University of Florida (571-6)

SUPPLEMENTARY PROGRAM
(To be presented by title)

(97) On the cohomology of an Eilenberg-MacLane space
    Professor William D. Barcus, Jr., Brown University

(98) On Lie algebras of derivations. Preliminary report
    Mr. Richard T. Barnes, Ohio State University

(99) Some remarks on the completeness of infinite valued
    predicate logic. Preliminary report
    Mr. L. P. Belluce, University of California, Los
    Angeles
    (Introduced by C. C. Chang)

(100) A weak completeness theorem for infinite valued predi-
    cate logic. Preliminary report
    Mr. L. P. Belluce and Professor C. C. Chang, Uni-
    versity of California, Los Angeles

(101) On coefficients of schlicht pseudo-conformal mappings
    in the space of two complex variables. I
    Professor Stefan Bergman, Stanford University (571-32)
(102) On coefficients of schlicht pseudo-conformal mappings in the space of two complex variables. II
Professor Stefan Bergman, Stanford University (571-33)

(103) Smooth surface interpolation
Professor Garrett Birkhoff, Harvard University and Dr. Henry L. Garabedian, General Motors Corporation, Warren, Michigan

(104) Asymptotic behavior of solutions of a system of functional equations
Dr. J. R. Blum, Dr. M. J. Norris and Dr. G. M. Wing, Sandia Corporation, Albuquerque, New Mexico (571-34)

(105) On the continuation of the solution of the equations of elasticity by reflection
Professor J. H. Bramble, U. S. Naval Ordnance Laboratory, White Oak, Maryland and University of Maryland, and Professor L. E. Payne, University of Maryland

(106) Sphere theorems for the equation of elasticity
Professor J. H. Bramble, U. S. Naval Ordnance Laboratory, White Oak, Maryland and University of Maryland

(107) Satellite orbit theory
Dr. J. L. Brenner, Stanford Research Institute, Menlo Park, California and Professor G. E. Latta, Stanford University (571-35)

(108) The monotone union of open n-cells is an open n-cell
Professor Morton Brown, University of Michigan (571-4)

(109) Perturbation of systems of ordinary differential operators
Professor John B. Butler, Jr., University of Arizona (571-12)

(110) On Dirichlet's problem for the half-space and the behavior of its solution on the boundary
Professor P. L. Butzer, Technical University, Aachen, Germany (571-55)

(111) On some theorems of Hardy, Littlewood and Titchmarsh
Professor P. L. Butzer, Technical University, Aachen, Germany (571-56)

(112) Criteria for Kummer's congruences
Professor L. Carlitz, Duke University (571-57)

(113) Some arithmetic properties of the lemniscate coefficients
Professor L. Carlitz, Duke University (571-58)

(114) Some orthogonal polynomials related to elliptic functions. II. Arithmetic properties
Professor L. Carlitz, Duke University (571-59)
A lemma on ultraproducts and some applications.
Preliminary report
Professor C. C. Chang, University of California, Los Angeles

Almost even functions of finite abelian groups
Professor Eckford Cohen, University of Tennessee (571-71)

A note on certain equally distributed sets of integers
Professor Eckford Cohen, University of Tennessee

Unitary functions (mod r)
Professor Eckford Cohen, University of Tennessee (571-37)

On the polyhedral Schoenflies theorem
Professor Morton L. Curtis and Mr. E. C. Zeeman, Caius College, Cambridge, England (571-13)

Homotopically homogeneous polyhedra
Professor Morton L. Curtis, Caius College, Cambridge, England (571-14)

Corollary to a proof due to Bing
Professor Morton L. Curtis, Caius College, Cambridge, England (571-15)

Shrinking continua in 3-space
Professor Morton L. Curtis, Caius College, Cambridge, England (571-16)

0-valued representations of polyadic algebras of infinite degree
Dr. Aubert Daigneault, University of Ottawa (571-17)

Dilations of polyadic algebras and of other algebraic systems
Dr. Aubert Daigneault, University of Ottawa (571-18)

A new formula for the volume of a polytope
Dr. A. C. Downing, Oak Ridge National Laboratory, Oak Ridge, Tennessee

Delay-differential equations
Mr. Rodney D. Driver, University of Minnesota (571-108)
(Introduced by Lawrence Markus)

On the spectrum of a multidiagonal infinite matrix operator
Dr. Peter Duren, Massachusetts Institute of Technology (571-38)

Extension of a result of Beurling on invariant subspaces
Dr. Peter Duren, Massachusetts Institute of Technology (571-39)

Exceptional real Lucas sequences
Professor L. K. Durst, Rice Institute (571-20)
(130) Ring domains in space  
Professor F. W. Gehring, University of Michigan

(131) Quasiconformal mappings in space  
Professor F. W. Gehring, University of Michigan

(132) Type I C*-algebras  
Professor James Glimm, Institute for Advanced Study and Massachusetts Institute of Technology (571-21)

(133) A q-series transformation  
Mr. H. W. Gould, West Virginia University (571-43)

(134) The Euler composition, V. Lie derivatives  
Professor H. W. Guggenheimer, University of Minnesota (571-22)

(135) Relations between complementary sequence spaces  
Dr. Günther W. Goes, Northwestern University

(136) A problem on partitions with a prime modulus p greater or equal to 3  
Professor Peter Hagis, Jr., Temple University (571-61)  
(Introduced by Emil Grosswald)

(137) Probability one convergence for age-dependent branching processes  
Dr. T. E. Harris, The RAND Corporation, Santa Monica, California (571-23)

(138) Indecomposable representations  
Professors Alex Heller and Irving Reiner, University of Illinois

(139) On outer automorphisms of fine P-groups  
Professor O. J. Huval, Southwestern Louisiana Institute

(140) Random walks with absorbing barriers and Toeplitz forms  
Dr. Harry Kesten, Hebrew University, Jerusalem (571-46)

(141) The scope of implicit alternating-direction methods  
Dr. Milton Lees, Institute for Advanced Study (571-47)

(142) Note on the metric transitivity of Fuchsian groups  
Professor Joseph Lehner, Michigan State University (571-73)

(143) Rational Tchebycheff approximation  
Mr. H. L. Loeb and Dr. E. W. Cheney, System Development Corporation and Space Technology Laboratories, Incorporated, Los Angeles, California (571-17)

(144) Continuous images of ordered continua  
Dr. Sibe Mardeljić and Dr. Pavle Papić, University of Zagreb, Yugoslavia (571-48)

(145) A semantic proof of the eliminability of descriptions  
Dr. Elliott Mendelson, Columbia University (571-24)
(146) On starlike hypergeometric functions
Professor E. P. Merkes, Marquette University and
Professor W. T. Scott, Northwestern University

(147) Concerning "Separation of variables". Preliminary report
Miss Joanne Monger, University of Texas (571-25)
(Introduced by Professor H. S. Wall)

(148) Irrational power series
Professor L. J. Mordell, St. Johns College, Cambridge, England (571-93)

(149) Normal ideals in local domains
Professor H. T. Muhly, University of Iowa (571-78)

(150) Inherently solution-seeking processes irrespective of initial values
Dr. C. A. Muses, Barth Foundation, Switzerland

(151) The dimensional continuum in Hilbert space
Dr. C. A. Muses, Barth Foundation, Switzerland

(152) The topology of the zeroth dimension
Dr. C. A. Muses, Barth Foundation, Switzerland

(153) Hyperspheres and dimensionality. Preliminary report
Dr. C. A. Muses, Barth Foundation, Switzerland

(154) Uniform distribution of sequences of integers
Professor Ivan Niven, University of Oregon (571-26)

(155) A sublinear Sturm-Liouville problem
Dr. George H. Pimbley, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (571-49)

(156) On the number of matrices with given characteristic polynomial
Professor Irving Reiner, University of Illinois

(157) A unified approach to certain problems of approximation and minimization
Dr. T. J. Rivlin, IBM Research Center, Yorktown Heights, New York and Professor H. S. Shapiro, Institute for Mathematical Sciences, New York University

(158) Characterization of convex systems of axioms
Dr. M. O. Rabin, Hebrew University, Jerusalem (571-65)

(159) Linear interpolating functions and best $L_p$ approximations
Dr. John R. Rice, National Bureau of Standards, Washington, D. C., (571-74)

(160) Interpolating function and best Tchebycheff approximations
Dr. John R. Rice, National Bureau of Standards, Washington, D. C. (571-75)

(161) Interpolating functions and best approximations on $[0,1]$
Dr. John R. Rice, National Bureau of Standards, Washington, D. C. (571-76)

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Applications of the subordination principle to univalent functions
Professor Malcolm S. Robertson, Rutgers, The State University

On patterns of growth of figures in two dimensions
Mr. R. G. Schrandt and Dr. Stanislaw M. Ulam, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

A hydrodynamic formulation of the prime number theorem
Dr. Daniel Shanks, David Taylor Model Basin, Washington, D. C.

Differential forms of bounded p-variation
Professor Victor L. Shapiro, University of Oregon and Rutgers, The State University

Continuous images of Borel sets
Professor Maurice Sion, University of California, Berkeley (571-27)

Rectangles in a convex region
Professor Sherman K. Stein, University of California, Davis (571-28)

Automorphisms of classical Lie algebras
Professor Robert Steinberg, University of California, Los Angeles

Generalized Kummer congruences for products of sequences
Dr. H. R. Stevens, Duke University

On class-division of integers modulo r
Miss M. Sugunamma, S. R. I. Venkateswara University, India (571-30)

(Introduced by Professor Alex Rosenberg)

Characters on inverse semi-groups
Professor Ronson J. Warne, Seton Hall University and Oak Ridge National Laboratory, Oak Ridge, Tennessee and Professor L. K. Williams, Southern University

Stable processes and integral equations. I
Professor Harold Widom, Cornell University (571-102)

Stable processes and integral equations. II
Professor Harold Widom, Cornell University (571-103)

A note on Riemannian covering manifolds
Dr. Joseph A. Wolf, University of Chicago and L'Université de Paris (571-3)

Recurrent tensors on a differentiable manifold
Professor Yung-Chow Wong, University of Hong Kong (571-53)
(176) Projectively flat spaces with recurrent curvature
Professor Yung-Chow Wong, University of Hong Kong,
and Mr. Kentaro Yano, Tokyo Institute of Technology
(571-77)

(177) Integral currents mod 2
Mr. William P. Ziemer, Brown University

Bloomington, Indiana
July 21, 1960

J. W. T. Youngs
Associate Secretary
The five hundred seventy-second meeting of the American Mathematical Society will be held on Saturday, October 22, 1960, at the College of the Holy Cross in Worcester, Massachusetts. All sessions will be in Haberlin Hall, which is the newly dedicated Science Center.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Shreeram Abhyankar of the Johns Hopkins University will address the Society at 2:00 P. M. in Room 106. The title of his address is "Introduction to Analytic Geometry."

Sessions for contributed papers will be held on both Saturday morning and Saturday afternoon.

The registration desk in the main foyer on the first floor of Haberlin Hall will be open from 9:00 A.M. till 3:30 P.M. The Faculty Lounge, which is Room 324, and Room 319 will be available as conversation rooms. Rest rooms and coat rooms are near the main foyer on the first floor.

Lunch at moderate prices will be served in the Cafeteria of Kimball Dining Hall.

Mail and telegrams for those attending the meetings may be addressed c/o Department of Mathematics, College of the Holy Cross, Worcester, Massachusetts.

Hotels and motels which are available to those attending the meeting include the following: the Bancroft Hotel in downtown Worcester; the Standish Motel in Auburn, Massachusetts, near Massachusetts Turnpike Exit 10; the Publick House, Sturbridge, Massachusetts, near the junction of Route 15 with the Massachusetts Turnpike at Exit 9; and the Elm Motel on Route 9, the Boston Turnpike, in Shrewsbury, Massachusetts, three miles east of Worcester center.

The College of the Holy Cross is located at the southern end of Worcester, approximately two miles from the business district. It is easily accessible by automobile, bus or taxicab. By automobile, drivers from east or west may take the Massachusetts turnpike to Exit 10, from which one drives in an easterly direction four miles on Southbridge Street (Route 12). The college is located on a high hill off Southbridge Street, the entrance to the college grounds being on College Street, which runs up the hill from Southbridge Street. Haberlin Hall, where the sessions will be held, is adjacent to College
Street. There are ample free parking areas near Haberlin Hall and on College Street. Drivers from the south may use Route 146 from Providence. The college is visible just after one crosses the Worcester city line. Bus service from downtown Worcester is available every half hour on the College Hill Bus (No. 10), which may be boarded at Main and Franklin Street near the Hotel Bancroft. The bus fare is $.20. The taxi fare from downtown is about $1.25.

Limited train service from New York is available from Grand Central Station. Direct air service is available from New York via Northeast Airlines. From the west, visitors may use the Mohawk line from Buffalo or Albany unless they wish to fly to Boston. The cab fare from the Worcester Airport to Holy Cross is about $2.25.

Further details about the meeting will appear in the October issue of these NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, September 8.

Bethlehem, Pennsylvania
July 14, 1960
ACTIVITIES OF OTHER ASSOCIATIONS

THE MATHEMATICAL ASSOCIATION OF AMERICA. The forty-first summer meeting of the MAA will be held at Michigan State University, East Lansing, Michigan, from Monday, August 29 to Wednesday, August 31, 1960, in conjunction with summer meetings of the AMS, the Society for Industrial and Applied Mathematics, the Pi Mu Epsilon Fraternity, and Mu Alpha Theta.

Sessions of the MAA will be held on Monday at 9:00 A.M. and 2:00 P.M., on Tuesday at 9:00 A.M., and on Wednesday at 2:45 P.M. All sessions will be held in Anthony Hall Auditorium (C-109) at Michigan State University. At the first three sessions, the ninth series of Earle Raymond Hedrick Lectures will be delivered by Professor Ivan Niven of the University of Oregon with the title: "Some Aspects of Diophantine Approximation".

The Board of Governors of the Association will meet on Monday evening at 7:30 in Room 221, Physics-Mathematics Building.

A meeting of officers of the Sections of the Association will be held on Tuesday evening at 7:30 in Room 221, Physics-Mathematics Building.

The Pi Mu Epsilon Fraternity will hold a luncheon and business meeting Tuesday noon. A session for seven 20 minute papers by students will be held at 2:45 P.M. in Room 315, Physics-Mathematics Building.

Mu Alpha Theta, the National High School and Junior College Mathematics Club, will hold a luncheon meeting Wednesday noon. All faculty sponsors of chapters are invited to attend.

The Society for Industrial and Applied Mathematics will have three sessions. The first, at which the von Neumann Lecture will be presented by Professor L. V. Ahlfors, of Harvard University, will be held Wednesday afternoon at 1:30 in the Anthony Hall Auditorium (C-109). The second will be held Thursday evening at 8:00 in Anthony Hall Auditorium, and the third will be held Friday afternoon at 2:40 in Anthony Hall Auditorium.

The Mathematical Sciences Employment Register will be located in Snyder Hall Dormitory near the registration desk. Book exhibits will be maintained in the same area.

A Program Committee consisting of J. C. Oxtoby, Chairman, W. E. Briggs, J. G. Herriot, K. O. May, and R. J. Wisner, has prepared the following program for the MAA meeting:

MONDAY, 9:00 A.M.

First Session, Anthony Hall Auditorium
9:00 - 10:00

The Earle Raymond Hedrick Lectures: Some Aspects of Diophantine Approximation, Lecture I
Professor Ivan Niven, University of Oregon
SESSION ON SPECIAL PROGRAMS IN THE TEACHING OF UNDERGRADUATE MATHEMATICS, 10:15-11:55 A.M.

10:15 - 10:35
The Honors Program at Princeton
Professor Albert W. Tucker, Princeton University

10:35 - 10:55
The New Program at Wesleyan
Professor Robert A. Rosenbaum, Wesleyan University

10:55 - 11:15
The Program at Dartmouth
Professor John G. Kemeny, Dartmouth College

11:15 - 11:35
The Undergraduate Thesis Program at Reed
Professor Lloyd B. Williams, Reed College

11:35 - 11:55
The Philips Visitor Program at Haverford
Professor James O. Brooks, Haverford College

MONDAY, 2:00 P.M.

Second Session, Anthony Hall Auditorium
2:00 - 3:00
The Earle Raymond Hedrick Lectures: Lecture II
Professor Niven

SESSION ON THE ROLE OF ABSTRACT AND CONCRETE APPROACHES IN THE TEACHING OF MATHEMATICS, 3:15-5:00 P.M.

3:15 - 4:05
Toward the Abstract: The Problem of Communication of Ideas
Professor Arnold E. Ross, University of Notre Dame

4:10 - 5:00
What Price Abstraction?
Professor Mark Kac, Cornell University

TUESDAY, 9:00 A.M.

Third Session, Anthony Hall Auditorium
The Earle Raymond Hedrick Lectures: Lecture III
Professor Niven

10:10 - 10:30
Business Meeting of the Association

10:30 - 10:50
Report of the Committee on the Undergraduate Program in Mathematics
Professor R. Creighton Buck, University of Wisconsin
11:00 - 12:00
Lecture: "Convergence Regions for Continued Fractions and Certain Other Infinite Processes"
Professor Wolfgang J. Thron, University of Colorado

WEDNESDAY, 2:45 P.M.

Fourth Session, Anthony Hall Auditorium
2:45 - 3:45
Lecture: Fourier Transforms
Professor Jacob Korevaar, University of Wisconsin

THE 1960 EASTERN JOINT COMPUTER CONFERENCE will be held December 13-15 at the Hotel New Yorker and the Manhattan Center in New York City. This tenth Eastern Joint meeting will stress papers describing significant and interesting accomplishments in the field. Contributed papers may pertain to any aspect of computer application or development, but should report achievement.

Anyone wishing to submit a paper for consideration by the Program Committee is requested to forward four copies of an abstract as well as four copies of a summary of the proposed paper. The summary should consist of approximately 1000 words and will be used by the Program Committee to evaluate and select papers. All copies of abstracts and summaries must be received on or before August 13, 1960 by the Chairman of the Program Committee, Elmer C. Kubie, Computer Usage Company, Incorporated, 18 East 41st Street, New York 17, New York.

In an attempt to build a program of the highest possible quality, no parallel sessions are planned and a $300 prize will be awarded for the best presentation of a paper at the Conference.

THE ASSOCIATION FOR SYMBOLIC LOGIC will meet at the Hotel Willard, Washington, D. C., on Tuesday, January 24, 1961, in conjunction with a meeting of the American Mathematical Society. Members of ASL desiring to submit abstracts will please do so in duplicate by November 15, 1960, sending them to the chairman of the Program Committee, Professor David Nelson, Department of Mathematics, The George Washington University, Washington 6, D. C.
THE EDUCATIONAL EXCHANGE PROGRAM under the auspices of the Board of Foreign Scholarships and the Department of State are described in a booklet issued in June by the Conference Board of Associated Research Councils, Committee on International Exchange of Persons, 2101 Constitution Avenue, Washington 25, D. C. The program was authorized by Public Law 584, 79th Congress (the Fulbright Act) and extended by Public Law 402, 80th Congress (the Smith-Mundt Act).

Awards under the program are handled by various co-operating agencies: university lecturing and advanced research appointments are made by the Committee on International Exchange of Persons; graduate study awards by the Institute of International Education, 1 East 67th Street, New York 21, New York; and appointments to teach in elementary and secondary schools by the United States Office of Education, Division of International Education, Department of Health, Education, and Welfare, Washington 25, D. C. For university lecturing and advanced research the eligibility requirements include United States citizenship, and in some cases, a knowledge of the language of the host country. For lecturing, there is an additional requirement of at least one year of college or university teaching experience, and for research the additional requirement of a doctoral degree or recognized professional standing. These awards are tenable in one country, usually for an academic year, and payable in the currency of the host country. There are additional allowances for the round-trip travel of the grantee, (but not for members of the grantee's family), a maintenance allowance to cover ordinary living expenses of the grantee and family while in residence abroad, and a small incidental allowance. The closing date for applications is October 1, 1960.

Application forms and additional information may be obtained by writing to the Committee on International Exchange of Persons at the address given above.

NSF POSTDOCTORAL FELLOWSHIPS AND SENIOR POSTDOCTORAL FELLOWSHIPS to be awarded on October 17 and December 12, 1960, respectively, have been announced. There are to be approximately 35 postdoctoral fellowships and 75 senior postdoctoral fellowships. The closing dates for receipt of application are September 6 and October 10, respectively.

Information concerning the NSF fellowship programs is so well known and readily available that we need not describe these programs in detail. However, more detailed information may be obtained by writing to The Fellowships Section, Division of Scientific
NATO POSTDOCTORAL FELLOWSHIPS IN SCIENCE were awarded to 41 United States citizens in May by the Department of State and the National Science Foundation.

This fellowship program was instituted by the North Atlantic Treaty Organization (NATO) in the belief that full development of science and technology is essential to the culture, economy, strength, and welfare of the Atlantic community. Designed to encourage further study abroad, these awards will enable the United States Fellows to study in ten foreign nations, while outstanding foreign scientists from other NATO countries will come to the United States for further training.

NATO Fellows receive a basic (12 month) stipend of $4,500. In addition, limited round-trip travel and dependency allowances are provided.

There were five awards under this program in mathematics, as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>Present Institution</th>
<th>Fellowship Institution</th>
</tr>
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<tbody>
<tr>
<td>GREENBERG, Leon</td>
<td>Brown University</td>
<td>University of Copenhagen (Denmark)</td>
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<tr>
<td>KRABBE, Gregers L.</td>
<td>Purdue University</td>
<td>University of Rennes (France)</td>
</tr>
<tr>
<td>MALTESE, George J.</td>
<td>Yale University</td>
<td>University of Göttingen (Germany)</td>
</tr>
<tr>
<td>ROTH, Karl H.</td>
<td>University of Southern California</td>
<td>University of Freiburg (Germany)</td>
</tr>
<tr>
<td>SWANN, Dale W.</td>
<td>Stanford University</td>
<td>University of Cambridge (England)</td>
</tr>
</tbody>
</table>

OEEC SENIOR VISITING SCIENCE FELLOWSHIP AWARDS were announced by NSF in June. There were 27 fellowships awarded under this new program founded by the Organization for European Economic Cooperation and administered for United States citizens by the National Science Foundation. The awards are designed to improve scientific work at the Fellow's home institution by training the Fellows in specialties that the institution desires to strengthen.

OEEC Fellows will receive a subsistence allowance of $10 per day, with tenures extending from eight weeks to ten months. In addition, round-trip air travel from a Fellow's home station to his institution abroad is provided.

There was only one award in mathematics under this program.
to Forman S. Acton from Princeton University who has designated the University of Cambridge as his fellowship institution.

THE NSF VISITING FOREIGN STAFF PROGRAM in mathematics brings internationally known scholar-teachers to this country to lecture and observe the summer institutes sponsored by the National Science Foundation.

Participating mathematicians, pictured here with three American professors, are (bottom row, l. to r.) Samuel Park of Korea, now with Rutgers, The State University; A. L. Blakers, University of Western Australia; Fernando Bertolini, University of Rome; Sigekatu Kuroda, University of Nagoya, Japan; (middle, l. to r.) W. H. Cockcroft, The University, Southampton, England; R. P. Bambah, University of Punjab, India; Cesar Abuauad, University of Chile; Wade Ellis, Oberlin College, director of the Visitors program in mathematics; Ernst Snapper, University of Indiana, consultant; (back row, l. to r.) Francisco M. Quesada, San Marcos University, Lima, Peru; Dr. M. Rueff, Ecole Polytechnique Federale, Zurich, Switzerland, and Byron E. Cohn, University of Denver. Dr. Cohn is coordinator of the overall Visiting Foreign Staff program for NSF institutes in the biological and physical sciences and in mathematics.

For the second year, Oberlin College was host and provided a headquarters for the program in mathematics. Mathematicians from nine countries met at the College June 20-24 for an orientation con-
ference before beginning a series of individual visits to institutes across the country. Between June 27 and August 12 they will visit 43 mathematics institutes in 29 states and the District of Columbia.

UNIVERSITY OF MINNESOTA DEPARTMENT OF STATISTICS. During the academic year 1958-1959 the Department of Statistics was established at the University of Minnesota. The Department has supplemented and coordinated the statistical activities of the University including: graduate curriculum, research, and consulting. The Department's organization involves direct appointments as well as joint appointments in mathematics and the sciences. Following is the current staff of the Faculties of Statistics: STATISTICS: L. Hurwicz, I. Olkin, D. Richter, I. R. Savage, M. Sobel; MATHEMATICS: G. Baxter, M. Donsker, B. Lindgren, S. Orey, W. Pruitt, E. Reich, F. Spitzer; AGRICULTURE: R. Comstock, C. Gates; BIOSTATISTICS: J. Bearman, J. Berkson, B. Brown, E. Johnson, R. McHugh; BUSINESS ADMINISTRATION: D. Hastings, J. Neter; INDUSTRIAL ENGINEERING: G. McElrath.

A SYMPOSIUM ON ENGINEERING APPLICATIONS OF PROBABILITY AND RANDOM FUNCTION THEORY will be held at Purdue University on November 15 and 16, 1960. The symposium will be sponsored by AMS.

The speakers and their topics are as follows:

1. Dr. M. Kac, Cornell University, Professor of Mathematics; Probability
2. Dr. A. J. F. Siegert, Northwestern University, Professor of Physics;
   Averages of Functionals for Various Processes
3. Dr. R. B. Murphy, Bell Telephone Laboratories, Statistical Quality Control;
   Reliability
4. Dr. A. M. Freudenthal, Columbia University, Professor of Civil Engineering;
   Structural Factors of Safety
5. Dr. E. T. Jaynes, Stanford University, Associate Professor of Physics;
   Application of Information Theory in Engineering
6. Dr. I. Dyer; Bolt, Beranek and Newman, Acoustical Consultant;
   Noise Generation from Aerodynamic Sources
7. Dr. R. Kalman, R. I. A. S., Research Mathematician;
   New Methods in Wiener Filtering Theory
8. Dr. E. Montroll, University of Maryland, Professor of Fluid Dynamics and Applied Mathematics;
   Traffic Flow
9. Dr. D. Slepian, Bell Telephone Laboratories, Research Mathematician; Problems in Electrical Noise and Detection Theory

10. Dr. S. S. Shu, Professor of Engineering Sciences, Purdue University; Introduction

The purpose of the Symposium is to bring before the research engineer and scientist current techniques and thoughts concerning important practical applications of probability and random function theory. The unifying theme of the conference is the application of probabilistic models to engineering problems.

Requests for further information should be addressed to either Professor J. L. Bogdanoff or Professor F. Kozin, co-chairmen of the Symposium, Division of Engineering Sciences, Purdue University, Lafayette, Indiana.

CONTINENTAL CLASSROOM, the NBC-TV program for college credit will be devoted to Contemporary Mathematics during 1960-1961. The first semester course, devoted to Modern Algebra, will be broadcast beginning September 26. The national teacher will be Professor John L. Kelley, Chairman of the Mathematics Department at the University of California, Berkeley. Another course, Probability and Statistics, will be offered during the second semester. Present plans are that Dr. Frederick Mosteller of Harvard University will teach this course.

To date, the station line-up has not been determined. However, a total of 159 stations has carried CONTINENTAL CLASSROOM during each of the past two years. This total represents coverage of 90.4% of the United States. It is expected that the line-up for Contemporary Mathematics will be approximately the same.

Sponsors of the new CONTINENTAL CLASSROOM course include Learning Resources Institute and the National Broadcasting Company. Financial backing is being provided by The Ford Foundation and industry.
ACKNOWLEDGMENTS TO REFEREES. Publication in the NOTICES of combined lists of referees has become an annual ceremony. This form of acknowledgment provides the referees with a measure of recognition, but at the same time protects their anonymity.

The indebtedness of the mathematical community to the individuals serving in this capacity was well expressed by Dr. Franz L. Alt when he was Chairman of the Editorial Board of the Association for Computing Machinery: "The publication of scientific journals would be impossible without the dedicated and unselfish efforts of referees. The job of a referee is a thankless one. He must necessarily remain anonymous, while others participating in various stages of publication can at least receive credit for their contribution. To read manuscript from a referee's viewpoint is often unpleasant and always time consuming. The combined good judgment of the referees is essential to making a journal successful."

The following two lists are based on information available at the Headquarters Offices of the Society as of April 30, 1960.

Referees of the BULLETIN, the PROCEEDINGS, and the TRANSACTIONS of the American Mathematical Society:


AFOSR SPONSORS NINE YOUNG SCIENTISTS for a year of advanced training. "The purpose of AFOSR's annual award is to advance America's scientific strength in areas of special Air Force interest," Colonel A. L. Gagge, AFOSR commander, explained: "These research associates will work under the supervision of top scientists. The young scientists we have chosen all show very unusual ability. I am sure we will benefit greatly from their fresh, original thinking and their enthusiasm for research."

The nine associates have earned their doctorate degrees, or will have earned them by the end of the current semester. Two of the nine awards were in mathematics, to: Dr. Bruno Harris, an assistant professor at Northwestern University at Evanston, Illinois, to work under Professor N. E. Steenrod at Princeton University; and Dr. James P. Jans, an assistant professor at the University of Washington at Seattle, to work under Dr. O. T. O'Meara at Princeton University.

DIFFERENTIAL EQUATIONS AND NONLINEAR MECHANICS is the subject of a symposium sponsored by the Air Force Office of Scientific Research and RIAS. The symposium will be held at the Air Force Academy, Colorado Springs, Colorado from July 31 through August 4, 1961.

Topics to be discussed are control theory, stability of motion, nonlinear oscillations, qualitative theory, etc. For further information and submission of abstracts write: OSR-RIAS Symposium, Mathematics Department, 7212 Bellona Avenue, Baltimore 12, Maryland.

SYSTEMS RESEARCH INSTITUTE, a major new educational effort, has been established by IBM. The new Institute, located at United Nations Plaza in New York City, will be the first of its kind in the computer industry. The purpose of this graduate-level school will be to train IBM people to find computer system solutions
to complex business and scientific problems. The curriculum will include such subjects as case studies in systems-design, workshops in systems planning, advanced programming, and business simulation techniques. The faculty will include senior IBM systems people and visiting lecturers from industry and leading universities.

The director of the Institute will be IBM vice president John C. McPherson; annual budget for this school is expected to be about $2 million. The first group of 30 students will be enrolled this fall.

In announcing the plans for the Institute, IBM President Thomas J. Watson, Jr. pointed out the necessity of sharply increasing the number of computer professionals in the United States.

COMPUTER ABSTRACTS ON CARDS, a new technical abstracting service in the field of electronic computers, has been announced by Cambridge Communications Corporation, 238 Main Street, Cambridge 42, Massachusetts. Abstracts of papers on computer equipment, programs, and mathematics are printed on 3" × 5" index cards to provide a cumulative, multiple-entry index to the computer literature. Approximately 2000 cards, abstracting papers published during 1959 and 1960, will constitute volume I to be published during the first year of publication. The price of volume I will be $100.

A SYMPOSIUM ON CONTEMPORARY SCIENCE IN COMMUNIST CHINA is being arranged for the 1960 AAAS Meeting, which is to be held in New York from December 26 to 31. The committee under the chairmanship of Dr. S. H. Gould, Executive Editor of MATHEMATICAL REVIEWS, Editor of the SOVIET MATHEMATICS - DOK-LADY and other AMS translation projects, has been working on plans for the Symposium and the selection of speakers. The program committee consists of representatives of a number of the major scientific societies. AMS is represented on the organizing committee for the Symposium by Professor Leon Cohen.

With the help of the National Science Foundation and William Locke, librarian at MIT, photo-copies are being made of the Chinese scientific literature of the past ten years and are being distributed to the prospective speakers. A grant from NSF will take care of the expenses of the very substantial amount of photo-copying, digesting and reviewing of the literature that will be required to make the reviews authoritative and up-to-date. Professor Marshall Stone will review recent Communist Chinese developments in mathematics and will be one of the principal speakers at the Symposium.

This Symposium is similar to one held at the 1951 annual meeting of AAAS in Philadelphia. The 1951 Symposium was on Soviet Science and the Proceedings were subsequently published by AAAS.
Proceedings of the Symposium on Contemporary Sciences in Communist China will be published by AAAS in 1961.

WEST VIRGINIA UNIVERSITY has acquired an IBM-650 Computer. In announcing its acquisition, President Elvis J. Stahr, Jr., said the Computer will give West Virginia University the basic tool for a research center. The Claude Worthington Benedum Foundation is contributing $10,000 a year over the next three years toward the cost of the system.

CONVENTIONS AND TAXES. Snapping up one association that called convention attendance tax-deductible, the Chicago District Director of the Internal Revenue Service issued a warning to the group and asked to have it published in the Association's bulletin. He took exception to this wording: "By combining business with pleasure, you can take a tax credit. Be good to yourself and let the government pay for your participation."

The tax official declared, "These two sentences carry the erroneous implication that all expenses incurred by a person going to a convention are deductible. This may result in the collection of deficiencies of tax and interest and possibly penalties from your members who rely on this statement. You may save your membership much inconvenience and perhaps some embarrassment by informing them of the correct treatment of convention expense for income tax purposes." Then he quoted the following from the official 1960 "Tax Guide for Small Business":

"Incidental personal expenses, such as those incurred for entertaining, sight-seeing, social visiting, etc., while at a convention or other business meeting are not deductible....Nor are those of a post-convention trip, the purpose of which is primarily recreational although some incidental sessions are scheduled for lectures, discussions or exhibitions related to your business interests....Expenses of your wife when she accompanies you are not deductible unless it can be adequately shown that her presence has a bona fide business purpose."

Should we stipulate that AMS meetings are not a pleasure?
BACKLOGS OF MATHEMATICAL RESEARCH JOURNALS

Information on this important matter is being published twice a year, in the February and the August issues of the NOTICES, with the kind cooperation of the respective editorial boards. To broaden the coverage of applied mathematics, we have added two additional journals to the group previously included in the survey.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table on the following page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (It should be noted that pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society's journals), and based on these factors: Manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (Q₁) and the third quartile (Q₃) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the NOTICES. The waiting times were measured by counting the months from receipt of manuscript in final revised form, to month in which the issue was received at the Headquarters Offices (not counting month of receipt of manuscript but counting month when issue was received at Headquarters Offices). It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.
<table>
<thead>
<tr>
<th>Journal</th>
<th>No. issues per year</th>
<th>Approx. No. pages published currently per year</th>
<th>Backlog 5/31/60 11/30/59</th>
<th>Estimated Current Waiting Time Months</th>
<th>Observed Waiting Time in latest issue</th>
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</thead>
<tbody>
<tr>
<td>American Journal</td>
<td>4</td>
<td>NR(a)</td>
<td>NR(a) NR(a)</td>
<td>NR(a)</td>
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<td>Annals of Math.</td>
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<td>0</td>
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<td>Annals of Math. Statistics</td>
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<td>13 15 16</td>
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<tr>
<td>Illinois Journal</td>
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<td>95 250</td>
<td>10-12</td>
<td>14 17 17</td>
</tr>
<tr>
<td>Journal of Math. Analyses and Applications</td>
<td>4</td>
<td>500</td>
<td>150</td>
<td>(b)</td>
<td>6-9</td>
</tr>
<tr>
<td>Journal of Math. and Mechanics</td>
<td>6</td>
<td>1,000</td>
<td>250 200</td>
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<td>(c) (c) (c)</td>
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<td>Journal of Math. and Physics</td>
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<td>Proceedings</td>
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<td>11</td>
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<td>Quarterly of Applied Mathematics</td>
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<td>10-12</td>
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<td>SIAM Journal</td>
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<td>SIAM Review</td>
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<td>300</td>
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<td>9-12</td>
<td>11 13 14</td>
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<tr>
<td>Transactions</td>
<td>12</td>
<td>2,200</td>
<td>310 430</td>
<td>11</td>
<td>11 13 14</td>
</tr>
</tbody>
</table>

Footnotes:  
(a) NR means that no response was received to a request for information.  
(b) Not included in the November 30 request for information.  
(c) Dates of receipt of manuscripts not indicated in this journal.
The National Science Foundation is celebrating its tenth anniversary this year. During the brief period of its existence it has grown rapidly. A knowledge of the Foundation's policies and procedures of but a few years ago may well be seriously out of date. It is hoped that the following account of its administrative structure and activities, as they relate to the mathematical sciences, may serve to smooth your future dealings with the NSF.

NSF In Perspective

The National Science Foundation Act was signed May 10, 1950 by President Truman. Its philosophy was enunciated by Chester I. Barnard, second chairman of the National Science Board, who said,

An ever-present danger inherent in any governmental organization for promotion of basic science lies in its propensity to exercise the kind and degree of control which is appropriate to research and development more closely related to immediate practical ends. The chief safeguard against this danger, outside the integrity and understanding of the Director and members of the Board, is the extensive, active cooperation of scientists who are not part of the regular staff of the Foundation. For wise judgment of the merits of specific research proposals the Foundation depends upon those most competent and respected in their various fields. Such advice is a personal thing, relating not only to subject matter, but to character, scientific competence, and integrity of those to whom support is to be given.

This philosophy has been most recently affirmed by Dr. Alan Waterman, the Director of the Foundation, in *Science*, May 6, 1960,
"National Science Foundation: A Ten-Year Resume".

In carrying out its obligations regarding the development of national science policy, the Foundation started from the promise that, in its broadest sense, national policy for science is a matter primarily to be determined by the scientists themselves. The scientists of the country are unquestionably the ones most capable of deciding what is best for progress in science, in the true meaning of the word. Policy in this sense should not be "master-minded" by the Federal government or any single agency.

Operations began in fiscal 1951 with an appropriation of $3.5 million. This figure enabled the Foundation during that first year of operation to make 97 grants for basic research -- of which only one was in mathematics. Foundation appropriations increased slowly but steadily until 1956, when they reached $16 million. In the next three years they grew exponentially to $136 million in 1959. The appropriation for the current fiscal year, 1961, is $175.8 million.

By June 30, 1960, the Foundation had made more than 8,000 grants for basic research in science totalling more than $170 million; it had made grants and contracts to provide basic research facilities totalling more than $40 million; it had expended for education programs and training in the sciences more than $175 million.

A point not clearly understood is the fact that grants in these various categories are administered by different groups within the Foundation. There follows a description of those activities of the NSF which might be of interest to mathematicians, together with the relevant subdivision of the Foundation.

Grants for support of research are made by the Mathematical Sciences Program Office, of the Mathematical, Physical and Engineering Sciences Division. Questions relating to education and training come within the jurisdiction of the Division of Scientific Personnel and Education. The programs supported by various subsets of that division include fellowships of many kinds, teacher training institutes, and curriculum studies. Among the concerns of the Office of Science Information Service are scientific publications, translations from unfamiliar foreign languages, research on information storage and retrieval, and machine translation. A program for development of graduate research laboratories, under the recently created Office of Institutional Programs, makes provision for improved research facilities. Office space for mathematicians is considered such a facility.
Assistance in the acquisition, maintenance, and operation of computing machines, as well as the support of computer science, is provided in the Computer Program, managed by the Mathematical Sciences Program Office. Most subdivisions of the Foundation can make grants for foreign travel for purposes related to their functions.

**Basic Research**

The general objectives of this program are support of mathematical research and, in collaboration with the Division of Scientific Personnel and Education, preparation of the next generation of mathematicians for research. The Foundation does not suggest research programs. Mathematicians seeking assistance are expected to submit a proposal describing the research they wish to undertake. Grants are based upon an evaluation of the merit of the proposed research and the competence of the investigators. However, a mathematician need not feel obliged to adhere strictly to the problems originally proposed, if he comes to the conclusion that other avenues of research will be more fruitful or more interesting. Insofar as it represents a change in emphasis, it should be noted that while research support was previously restricted in most cases to individual mathematicians, in recent years an increasing number of grants have been made for research to be undertaken by groups of mathematicians with related interests.

**TABLE I**

<table>
<thead>
<tr>
<th>Fiscal Year</th>
<th>Number of Grants</th>
<th>Total Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>1</td>
<td>$19,300</td>
</tr>
<tr>
<td>1953</td>
<td>20</td>
<td>104,500</td>
</tr>
<tr>
<td>1954</td>
<td>21</td>
<td>173,950</td>
</tr>
<tr>
<td>1955</td>
<td>48</td>
<td>562,400</td>
</tr>
<tr>
<td>1956</td>
<td>39</td>
<td>712,350</td>
</tr>
<tr>
<td>1957</td>
<td>64</td>
<td>1,038,900</td>
</tr>
<tr>
<td>1958</td>
<td>72</td>
<td>1,242,100</td>
</tr>
<tr>
<td>1959</td>
<td>87</td>
<td>1,634,450</td>
</tr>
<tr>
<td>1960</td>
<td>114</td>
<td>2,800,100</td>
</tr>
</tbody>
</table>

Table I shows the growth of basic research grants for mathematical sciences in the Foundation from fiscal year 1952 through 1960. During this time the Foundation has become one of the leading supporters of basic mathematical investigations in the United States.

Grants for support of research may provide for faculty research time, support of graduate students, supplement of partial
salary during sabbatical leave, support of research associates and visiting mathematicians, and various subsidiary items such as secretarial assistance, travel, and publication costs. In administering the grants program, recognition is given to the current shortage of teachers. In accordance with the strong recommendations of our mathematical advisors, it is intended that a man shall not be taken from teaching for more than two years in seven (one year in addition to the sabbatical), and that support of visiting mathematicians shall not disrupt teaching schedules and the direction of graduate students at their home institutions. Likewise, released time from teaching is ordinarily provided only when the teaching load is so heavy as to interfere substantially with the ability to do research.

**Education**

The training of future mathematicians for basic research and higher education is a major concern of those interested in mathematical sciences. At the graduate level the program of fellowships within the Division of Scientific Personnel and Education is intended to alleviate this pressing need for mathematicians. Table II illustrates the growth in both number and types of fellowship awards offered for studies in mathematical sciences, throughout the history of the Foundation. (See page 446 for Table II).

NSF has tried to be alert to the possibility of new methods of encouraging young and prospective mathematicians. The table on fellowships, showing the growth in nine years from two programs to seven, is a graphic example. There are others to note. With respect to undergraduate mathematics, for example, Carleton College is completing a four year experimental program comprising accelerated instruction aimed at training research mathematicians, terminating in undergraduate participation in research. Exploratory and experimental projects in elementary school mathematics are being carried on with NSF support by mathematicians and teachers at Stanford University. In 1958 The School Mathematics Study Group was organized to carry out an extensive program to improve mathematics teaching in elementary and secondary schools. Many eminent mathematicians and accomplished teachers are engaged in this project, which is directed by Professor E. G. Begle at Yale University. Sample texts and teacher guides are being tried experimentally and special manuals are being written. A related project at the University of Minnesota is testing the use of new material in courses of study taken by talented students in small schools lacking advanced work in mathematics.

Particular note should be made of the number of cooperative fellowships as given in Table II. Prior to establishment of these fellowships the tendency of graduate students in mathematics receiving NSF fellowships was to congregate at a few leading institutions.
### TABLE II

**NSF Fellowship Awards Offered in the Mathematical Sciences**

<table>
<thead>
<tr>
<th>Fiscal Year</th>
<th>Regular Graduate</th>
<th>Cooperative Graduate</th>
<th>Graduate Teaching</th>
<th>Regular Postdoctoral</th>
<th>Senior Postdoctoral</th>
<th>Science Faculty</th>
<th>Secondary School Teachers</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>50</td>
<td>-</td>
<td>-</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>58</td>
</tr>
<tr>
<td>1953</td>
<td>49</td>
<td>-</td>
<td>-</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>56</td>
</tr>
<tr>
<td>1954</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>72</td>
</tr>
<tr>
<td>1955</td>
<td>51</td>
<td>-</td>
<td>-</td>
<td>11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>62</td>
</tr>
<tr>
<td>1956</td>
<td>67</td>
<td>-</td>
<td>-</td>
<td>16</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>87</td>
</tr>
<tr>
<td>1957</td>
<td>86</td>
<td>-</td>
<td>-</td>
<td>16</td>
<td>6</td>
<td>21</td>
<td>-</td>
<td>129</td>
</tr>
<tr>
<td>1958</td>
<td>140</td>
<td>-</td>
<td>-</td>
<td>19</td>
<td>11</td>
<td>36</td>
<td>-</td>
<td>206</td>
</tr>
<tr>
<td>1959</td>
<td>143</td>
<td>128</td>
<td>71</td>
<td>30</td>
<td>6</td>
<td>58</td>
<td>253</td>
<td>689</td>
</tr>
<tr>
<td>1960</td>
<td>168</td>
<td>171</td>
<td>99</td>
<td>25</td>
<td>11</td>
<td>63</td>
<td>226</td>
<td>763</td>
</tr>
</tbody>
</table>

-- Not offered
Under the cooperative graduate fellowships a way is being found to exploit the competence of the excellent scientists and teachers at other institutions.

Institutes for training teachers of mathematics have comprised another important program of NSF's Division of Scientific Personnel and Education. In fiscal 1959, for example, Academic Year Institutes for teachers of science and mathematics were supported at 26 institutions, and similar institutes for teachers of mathematics alone were supported at five institutions. Summer Institutes and In-Service Institutes for teachers in elementary schools, high schools, and colleges have also been supported on both a regular and experimental basis.

In summary, the NSF was created to foster the scientific growth of the country. It can serve the mathematical community in numerous ways. It is hoped that this article may make this clear, and will improve communications between mathematicians and the NSF. A mathematician having any doubts concerning what the Foundation can or cannot do for him should not hesitate to resolve his doubts through a direct inquiry.


Marston Morse, Chairman of the U. S. National Committee for Mathematics (USNC) recently released the annual report of the committee for 1959-1960. Professor Morse is Vice President of the International Mathematical Union (IMU) and attended the meeting of the Executive Committee of the IMU at Paris on April 26-27, 1960. As a result, the USNC report contains a good deal of information on the activities of the IMU.

ICMI and USCMI. In 1958, the General Assembly of the IMU requested the Executive Committee of ICMI to study certain proposed New Terms of Reference. The Executive Committee, consisting of Professors Y. Akizuku, Japan; P. Alexandroff, USSR; H. Behnke, Germany; O. Frostman, Sweden; G. Kurepa, Yugoslavia; M. H. Stone, USA (President); and G. Walusinski, France, made a Report to the Executive Committee of IMU. New Terms of Reference of ICMI were adopted by the IMU Executive Committee at a meeting in Paris on April 26-27, 1960. They were approved by the USNC on May 25, 1960, and will become effective when a majority of the National Commission of Mathematics of the members nations of IMU approves them.

The USNC elected Professor John Moore as Chairman of the U. S. Commission on Mathematical Instruction (USCMI) effective July 1, 1960. The USCMI is a sub-committee of the Division of Mathematics created under USNC.
WORLD DIRECTORY. The World Directory of Mathematicians, published under the auspices of the International Mathematical Union with the co-operation of the Tata Institute of Fundamental Research at Bombay, attempted in the first edition in 1958 a complete listing of mathematicians who had at least two papers reviewed in MATHEMATICAL REVIEWS. Preparations for the second edition of the Directory are now being made. In October 1960, notices will be sent to all individuals listed in the Directory and to the societies for distribution to their members. The IMU has allocated $1,000 for the 1961 edition of the DIRECTORY; the lists remain the property of IMU. Closing date for entries is April 1961, and August 1961 is set for publication date.

Proofreading and the application of the criterion for listing will be undertaken by the Bureau. For good reasons, exceptions for the criterion may be made, and outside advice may be invited.

Professor P. Alexandroff is working on the possibility of including the names of Russian mathematicians in the listing. Many suggestions have been made to list additional information in the Directory, such as the "field of research" (which can be found in MATHEMATICAL REVIEWS), or adding names of important journals and editors (which would be a difficult choice). Although these impractical suggestions have been discarded, some information on IMU and its commissions may be published.

Of the total edition of 1000 copies, only 555 were sold. To arrange for a more adequate distribution of the Directory in the United States, the Providence offices of the American Mathematical Society have arranged to assist in the distribution to American Mathematical Society members (see page 461 of these NOTICES). The price for the 1961 Directory will be $1.50.

INTERNATIONAL CONFERENCES. Three Conferences sponsored by ICMI were scheduled during 1960-1961. June 1960: at Aarhus, Denmark: Professor J. G. Kemeny of Dartmouth was on the Organizing Committee. September 1960: At a regional meeting in Belgrade, Yugoslavia, which Professor Marshall H. Stone will attend, the topic will be "On Coordination of Mathematics and Physics Instruction". A regional meeting is planned to be held in Europe in the spring of 1961.

IMU LECTURESHIPS. Professor Rickart of Yale University will lecture at a conference in London on "Functional Analysis and Some of its Applications." The IMU has granted $700 to the London Mathematical Society for the organization of the Conference, and $1,000 has been allocated for additional lectureships elsewhere. An amount was granted by the IMU to the Bolyai Janos Mathematical
Society for a travel grant to Professor G. Szegö, of Stanford University, to give lectures in Hungary in August, 1960.

**SCIENTIFIC PUBLICATIONS.** A Committee, headed by Professor Köksma has been considering two principal problems: First, to investigate various techniques of mathematical printing and the possibilities of standardization. Dr. S. H. Gould of *Mathematical Reviews* and editors of other review journals may be invited to present their views on standard, current, and new symbols. Second, a Committee of Professors de Rham (Chairman), Jean Leray, and Lipman Bers, will be asked to study the possibility of editing a series of monographs, e.g. "Progress in ...... concerning various fields of pure and applied mathematics."

**SYMPOSIA SPONSORED BY IMU.** 1960: IMU made a grant of $4,000 and the NSF aided with travel grants for a Symposium in Israel in July on "Linear Spaces - Geometrical Aspects and applications to Analysis." Professor A. Dvoretzky was Chairman of the Organizing Committee. IMU granted $2,000 and the NSF aided with travel grants for a Symposium in June at Zürich, on "Differential Geometry and Topology". Professor H. Hopf was Chairman of the Organizing Committee.

1961: The Czechoslovak National Committee of Mathematics with the assistance of Polish mathematicians will conduct a symposium on "Topology and its Methods in Other Mathematical Disciplines" in September in Prague. The Academy of Sciences will cover the costs of invited scientists for about seven days in Prague; IMU has granted $3,000 for the symposium, and Professor Kazimierz Kuratowski of the University of Warsaw is assisting the Conference.

IMU granted $2,000 for a symposium on " Fonctions de Variables Complexes et Analyse Fonctionelle" to be held in Portugal in the spring. Chairman of the Organizing Committee is Professor J. Vicente Gonvalves; Secretary is Professor Sebastiao e Silva. Professors H. A. L. Behnke and H. Cartan will attend the Conference.

Professor J. Barkley Rosser is Chairman of the Committee on Space Research, at IMU. The committee has one member from the Netherlands, A. van Wigngaarden, and one eventual member from Russia. This Committee has been asked to recommend a representative of IMU on COSPAR from a country which does not already have a national representative of a Union, and further to stimulate and organize research leading eventually to an international conference, to be sponsored by IMU (in 1963 or later).

1962: The International Congress of Mathematicians will be held in Stockholm, August 15-22. The Congress will be preceded by a meeting of the Assembly of IMU.
PERSONAL ITEMS
(This section is reserved for members of the Society)

Professor A. A. ALBERT of the University of Chicago has
been appointed to the Eliakim Hastings Moore Distinguished Service
Professorship.

Professor GARRETT BIRKHOFF of Harvard University was
made Doctor Honoris Causa at the 4th Centenary of the University of
Lille, France, this June.

Professor M. R. HESTENES of the University of California
was awarded an honorary Doctor of Science Degree during com­
 mencement exercises held at Wartburg College June 7, 1960.

Associate Professor G. W. PATTERSON of the University of
Pennsylvania has been appointed Vice Chairman of the Electronic
Computers Committee of the Institute of Radio Engineers.

Associate Professor EDGAR REICH of the University of Min­
nesota will spend the academic year 1960-1961 at Aarhus University,
Denmark, as a Guggenheim Fellow and Fulbright Research Scholar.

Professor R. F. RINEHART of Case Institute of Technology
was awarded the honorary degree of Doctor of Science by Wittenberg
University at its June 6th commencement.

The following have joined the staff of the Army Department at
the University of Wisconsin, Madison, Wisconsin, for the Summer
of 1960:

Dr. N. ARTEMIADES, University of Wisconsin; Dr. A.
ERDELYI, California Institute of Technology; Dr. J. GURLAND, Iowa
State University; Dr. H. B. MANN, Ohio State University; Dr. M.
MARDEN, University of Wisconsin; Dr. W. RUDIN, University of
Wisconsin; Dr. H. SCHNEIDER, University of Wisconsin; Dr. J. C.
WILSON, Southern Illinois University.

Assistant Professor W. L. ALLEN of Fresno State College has
accepted a position as mathematician with Librascope Incorporated,
Glendale, California.

Dr. A. R. AMIR-MOEZ of Queens College has been appointed
to a visiting assistant professorship at the University of California,
Los Angeles.

Dr. K. W. ANDERSON of the University of Illinois has been
appointed to an assistant professorship at Harpur College.

Associate Professor RAFAEL ARTZY of the Israel Institute
of Technology has been appointed to an Associate Professorship at
the University of North Carolina.

Associate Professor A. G. AZPEITIA, on leave from the Uni­
versity of Massachusetts, has been appointed to a visiting associate
professorship at Brown University.

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Professor IACOPO BARSOTTI of the University of Pittsburgh has been appointed to a professorship at Brown University.

Assistant Professor F. G. BRAUER of the University of British Columbia has been appointed to an assistant professorship at the University of Wisconsin.

Dr. J. D. BROOKS of the University of Southern California has been appointed to an assistant professorship at the University of Arizona.

Dr. R. L. BROUSSARD of Douglas Aircraft Company, Incorporated has been appointed to an associate professorship at Portland State College.

Associate Professor F. E. BROWDER of Yale University has been appointed to a visiting professorship at the Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil, for the period May-September of 1960.

Professor L. H. BUNYAN of Rutgers, The State University, has retired with the title Professor Emeritus.

Dr. MARY H. CABELL of the University of Virginia has been appointed to an assistant professorship at Georgia Institute of Technology.

Associate Professor J. M. CALLOWAY of Carleton College has been appointed to a professorship at Kalamazoo College.

Assistant Professor F. W. CARROLL, Jr. of Purdue University has been appointed to an assistant professorship at the University of Wisconsin, Milwaukee.

Dr. SRISHTI D. CHATTERJI of Michigan State University has been appointed a lecturer at the University of Toronto.

Dr. PHILIP COOPERMAN of the University of Pittsburgh has accepted a position as director of research with Research-Cottrell, Incorporated, Bound Brook, New Jersey.

Professor WILHELM L. DAMKÖHLER of the University Mayor de San Andres has been appointed to a professorship at the Universidad de Cuyo, San Luis, Argentina.

Assistant Professor R. L. DAVIS of the University of Virginia has been appointed to an assistant professorship at the University of North Carolina.

Assistant Professor KAREL deLEEUW, on leave from Stanford University, has received a fellowship and will be at the Institute for Advanced Study for the 1960-1961 academic year.

Mr. J. W. DRANE of Newport News Shipbuilding and Dry Dock Company has been appointed to an associate professorship at Randolph-Macon College.

Associate Professor JOHN DYER-BENNET of Purdue University has been appointed to an associate professorship at Carleton College.
Assistant Professor E. O. ELLIOTT of the University of Nevada has accepted a position as staff member at Bell Telephone Laboratories, Murray Hill, New Jersey.

Dr. J. A. ERNEST has received a National Science Foundation Post Graduate Fellowship and will be at the Institute for Advanced Study for the 1960-1961 academic year.

Dr. ARWEL EVANS of Yale University has been appointed to an associate professorship at McGill University, Montreal 2, Quebec, Canada.

Visiting Professor TOMLINSON FORT of Emory University has been appointed to a professorship at the University of Miami.

Professor D. GAIER, on leave from the University of Giessen, has been appointed senior research fellow at the California Institute of Technology for a period of six months beginning September, 1960.

Dr. J. B. GARNER of Auburn University has been appointed to an assistant professorship at Louisiana Polytechnic Institute.

Dr. R. P. GILBERT of the University of Pittsburgh has been appointed to an assistant professorship at Michigan State University.

Dr. R. P. GILBERT of the University of Pittsburgh has been appointed to an assistant professorship at Michigan State University.

Dr. R. P. GILBERT of the University of Pittsburgh has been appointed to an assistant professorship at Michigan State University.

Dr. SEYMOUR GINSBURG of Hughes Aircraft Company has accepted a position as senior mathematician with the System Development Corporation, Santa Monica, California.

Dr. S. C. H. GITLER of Princeton University has been appointed a research associate at Brandeis University.

Dr. MALCOLM GOLDMAN of the University of Michigan has been appointed to a visiting assistant professorship at Reed College.

Assistant Professor D. S. GREENSTEIN of the University of Michigan has been appointed to an assistant professorship at Northwestern University.

Dr. BRANKO GRÜNBAUM of the Institute for Advanced Study has been appointed to a visiting assistant professorship at the University of California, Los Angeles.

Associate Professor H. W. GUGGENHEIMER of Washington State University has been appointed to an associate professorship at the University of Minnesota.

Dr. ROBERT W. HEATH of the University of North Carolina has been appointed to an assistant professorship at the University of Georgia.

Mr. R. E. HILL of the University of Oregon has accepted a position as applied programmer at the International Business Machines Corporation, New York, New York.

Dr. MAURICE HOROWITZ of Goodyear Aircraft Corporation has accepted a position as senior staff engineer with Magnavox Company, Fort Wayne, Indiana.
Dr. R. R. HUNZIKER of Convair has accepted a position as senior scientist with Radio Corporation of America, Melbourne Beach, Florida and also a lectureship with Brevard Engineering College.

Dr. G. P. JOHNSON of the Standard Oil Company of California has been appointed to an associate professorship at Wesleyan University.

H. L. JOHNSON of the University of Minnesota has accepted a position as mathematician with Wright Patterson Air Force Base, Ohio.

Professor R. E. JOHNSON of Smith College has been appointed to a professorship at the University of Rochester.

Mr. D. E. JULIE of the University of Wisconsin has accepted a position as applied science representative with International Business Machines Corporation, Minneapolis, Minnesota.

Assistant Professor COSTAS KASSIMATIS of Cornell University has been appointed to an associate professorship at North Carolina State College.

Dr. R. B. KELMAN is on leave of absence from International Business Machines Corporation in order to serve as a consultant to the President's Science Advisory Committee.

Miss ELEANOR KILLAM of Yale University has been appointed to an assistant professorship at the University of Massachusetts.

Associate Professor J. S. KLEIN of Lafayette College has been appointed to an associate professorship at Wilson College, Chambersburg, Pennsylvania.

Dr. A. G. KONHEIM of General Electric Company has accepted a position as mathematician with International Business Machines Corporation, Yorktown Heights, New York.

Dr. WALTER KOPPELMAN of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Pennsylvania.

Dr. D. L. KREIDER of Massachusetts Institute of Technology has been appointed to an assistant professorship at Dartmouth College.

Dr. L. H. LANGE of Valparaiso University has been appointed to an assistant professorship at San Jose State College.

Dr. MILTON LEES of the Institute for Advanced Study has been appointed to an assistant professorship at New York University.

Assistant Professor K. T. LEUNG of the University of Cincinnati has been appointed senior lecturer in Pure Mathematics at the University of Hong Kong for the academic year 1960-1961.

Assistant Professor H. M. LIEBERSTEIN of the University of Wisconsin has been appointed to an associate professorship at the University of Arizona.

Mr. J. H. LINDSAY of the University of British Columbia has been appointed a lecturer at the University of Toronto.
R. G. McIntyre of Standard Oil Company of Ohio has accepted a position as member of the technical staff with Texas Instruments Incorporated, Dallas, Texas.

Professor W. R. Mann, on leave from the University of North Carolina, has received a National Science Foundation Faculty Fellowship and will be at the University of California, Berkeley, for the 1960-1961 academic year.

Assistant Professor J. C. Mathews of Iowa State University has been appointed to an assistant professorship at the University of Oklahoma.

Dr. H. Frazier Mattson of the Air Force Cambridge Research Center has accepted a position as engineering specialist with Sylvania Electric Products, Incorporated, Waltham, Massachusetts.

Dr. J. E. Maxfield of the Naval Ordnance Test Station has been appointed to a professorship at the University of Florida.

Dr. Pinchas Mendelson of Columbia University has been appointed to an assistant professorship at the Polytechnic Institute of Brooklyn.

Mr. R. B. Merkel of Foothill College has been appointed an associate mathematician at the University of California, Berkeley, California.

Professor M. Morris of Case Institute of Technology has retired with the title professor emeritus.

Assistant Professor Benjamin Muckenhoupt of De Paul University has been appointed to an assistant professorship at Rutgers, The State University.

Associate Professor D. E. Myers of Millikin University has been appointed to an assistant professorship at the University of Arizona.

Professor J. M. H. Olmsted of the University of Minnesota has been appointed to a professorship at Southern Illinois University.

Dr. Narasimhachari Padma of Annamalai University has been appointed a visiting lecturer at Connecticut College.

Dr. L. G. Peck has resigned as Mathematical Director of Arthur D. Little, Incorporated and has established offices as a research consultant in the fields of information processing, operations research, mathematical physics, statistical analysis, organization and utilization of electronic computer facilities. These offices are located in the Statler Office Building, Boston, Massachusetts.

Dr. R. R. Phelps of the Institute for Advanced Study has been appointed a lecturer and assistant research mathematician at the University of California, Berkeley.

Assistant Professor Barth Pollak, on leave from Syracuse University, will be at the Institute for Defense Analyses, Princeton, New Jersey for the 1960-1961 academic year.
Mr. D. W. POUNDER of DeHavilland Aircraft of Canada Limited has accepted a position as applied science representative with International Business Machines Company, Limited, Ottawa, Ontario, Canada.

Mr. A. G. RAWLING of Johns Hopkins University has accepted a position as engineer with General Electric Company, Radnor, Pennsylvania.

Research Associate R. RIMHAK of Columbia University has been appointed to an assistant professorship at the University of British Columbia.

Dr. S. L. SALAS of Yale University has been appointed to an assistant professorship at Wesleyan University.

Associate Professor CHARLES SALTZER of Case Institute of Technology has been appointed to a professorship at the University of Cincinnati.

Dr. JAMES SANDERS of the University of Glasgow has been appointed to an assistant professorship at the University of Western Ontario.

Associate Professor H. H. SCHAEPFER of Washington State University has been appointed to an associate professorship at the University of Michigan.

Mr. ERNST SCHWANDT of Michigan State University has been appointed an associate mathematician in the Applied Physics Laboratory of Johns Hopkins University, Silver Spring, Maryland.

Assistant Professor BERTHOLD SCHWEIZER of the University of California has been appointed to an associate professorship at the University of Arizona.

Dr. L. A. SEGEL of the National Physical Laboratory, Teddington, England, has been appointed to an assistant professorship at Rensselaer Polytechnic Institute.

Mr. G. S. SILBERMAN of the University of Nevada has been appointed to a visiting assistant professorship at Kenyon College.

Assistant Professor MAURICE SION of the University of California, Berkeley, has been appointed to a professorship at the University of British Columbia.

Dr. PAUL SLEPIAN of Hughes Aircraft Corporation has been appointed to an associate professorship at the University of Arizona.

Associate Professor T. H. SOUTHARD of the University of California, Los Angeles, has been appointed to a professorship and chairman of the department at Alameda County State College, Hayward, California.

Dr. C. J. STANDISH, on leave from International Business Machines Corporation, has been appointed to a visiting associate professorship at North Carolina State College.

Assistant Professor J. A. STEKETEE of the University of Toronto has been appointed to a professorship at the Technological University of Delft, Netherlands.
Dr. C. R. STOREY, Jr. of Princeton University has been appointed to an assistant professorship at Florida State University.

Professor D. J. STRUIK of Massachusetts Institute of Technology has retired with the title professor emeritus.

The Board of Trustees of the College of the Holy Cross has announced the appointment of Very Reverend RAYMOND J. Swords, S. J. to President of the College.

Professor GERHARD TINTNER of Iowa State University will spend the summer at the Indian Statistical Institute, Calcutta, India.

Mr. George Van ZWALENBERG of Bowling Green State University has been appointed a visiting lecturer at Calvin College, Grand Rapids, Michigan.

Assistant Professor D. W. WALL has been appointed to an associate professorship at the State University of Iowa.

Dr. J. A. WARD of the Air Force Missile Development Center has accepted a position as chief of missile computation with Remington Rand UNIVAC Division, Sperry Rand Corporation, Washington, D. C.

Mr. DONALD WEHN of Princeton University has been appointed a lecturer at the University of California, Berkeley.

Assistant Professor J. H. WELLS, on leave from the University of North Carolina, has been appointed to a visiting assistant professorship at the University of California, Berkeley.

Dr. ELIZABETH WETHERELL of New York University has accepted a position as mathematician with International Business Machines Corporation, New York, New York.

Professor NORBERT WIENER of Massachusetts Institute of Technology has retired with the title Institute Professor Emeritus.

Dr. J. S. WHITE of Minneapolis Honeywell Regulator Company has accepted a position as senior research mathematician in the research laboratories of General Motors Corporation, Warren, Michigan.

Associate Professor J. C. WILSON of Central College has been appointed to an assistant professorship at Florida Presbyterian College.

Associate Professor HIDEHIKO YAMABE has been appointed to a professorship at Northwestern University.

Dr. BOHYUN YIM of Brown University has accepted a position as research scientist with Hydronautics, Incorporated, Rockville, Maryland.

Associate Professor A. D. ZIEBUR of Ohio State University has been appointed to an associate professorship at Harpur College.

The following promotions are announced:

S. ARAKI, Kyusyu University, to an assistant professorship.

B. F. BRYANT, Vanderbilt University, to an associate professorship.
R. J. BUEHLER, Iowa State University, to an associate professorship.

H. L. de RIVERA, Northeastern University, to an associate professorship.

K. A. FOWLER, San Jose State College, to an associate professorship.

T. A. JEEVES, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania, to fellow mathematician in the mathematics department.

ANNE L. LEWIS, University of North Carolina, to a professorship.

MASAO NARITA, International Christian University, Tokyo, Japan, to an assistant professorship.

Dr. C. L. RIGGS, Texas Technological College, to a professorship.

L. F. TAKÁCS, Columbia University, to an associate professorship.

J. R. WESSON, Vanderbilt University, to an associate professorship.

The following appointments to Instructorships are announced:

Air Force Institute of Technology: Mr. RICHARD L. PRATT; University of British Columbia: Dr. ROBERT C. THOMPSON; University of North Carolina: Dr. C. W. PATTY; Oberlin College: Mr. DAVID H. STALEY; Princeton University: Mr. ALLAN H. CLARK; Queens College: Mr. R. G. KRUTCHKOFF; St. Peter's College: Mr. K. T. BURKE; University of Wisconsin, Kenosha Extension: Mr. W. C. LORDAN.

Deaths:

Mr. HERMAN BETZ of the University of Missouri died on March 3, 1960 at the age of 78 years. He had been a member of the Society for 54 years.

Mr. J. J. DODD of Whitewater State College died on April 3, 1960 at the age of 27 years.

Mr. M. B. PORTER of the University of Texas died on May 27, 1960 at the age of 91 years. He had been a member of the Society for sixty-three years.
LETTERS TO THE EDITOR

Canberra, 1960 VI 13

Editor, the NOTICES

I am in Canberra since 5 weeks, will pay short visits to Brisbane, Sydney, Melbourne, Adelaide, Perth and leave Australia after July 20. At the Australian National University there is no department of mathematics at present, therefore I am at the department of statistics. The University has ambitious plans of getting a big department of mathematics. The Australian National University has no undergraduates, they have a flourishing physics department, (Prof. Oliphant) and the John Curtin medical school (also only postgraduate studies) + the observatory at Mount Stromlo, less than 10 miles from Canberra. There are many other scientific activities going on here the Csiro* has many departments here + the building of the Australian Academy of Sciences is also here. Prof. Dobzhansky of Columbia, a geneticist, was just here + preached to the Academy on genetical effect of radiation, Canberra University College, an undergraduate school, is also in this city.

In Oberwolfach I hear that there will be a mathematical meeting September 5-10 on diophantine approximation and additive number theory, if my trip to Samland does not materialise I will attend + afterwards go to Innsbruck. Unfortunately I can not go to the meeting in Israel early in July since I have some lectures in Melbourne at that date.

Kindest regards to all, P. E.

(Paul Erdos)

*Commonwealth Science and Industrial Research Organization.

Editor, the NOTICES

In sympathy with the spirit of the text of the recent letter [Notices, June, 1960] submitted by Professor deLeeuw may I make the following few comments? I have no bones to pick in either direction, but contend myself with possibly shedding some light on an aspect of these political situations? Firstly, Article I (AI) of the Amendments was not conceived nor has it ever been interpreted by the judiciary in its weakest sense. In its weakest sense it unmistakably contradicts, among other things, The Preamble. Unless we want to change some of our most basic laws, clearly one cannot, at least in the eyes of the law, always say whatever one wants to say. A first step toward a weak interpretation of AI would be to throw out all of our libel laws - most risky. Of course, AV is in a similar boat with respect to interpretation by the judiciary. The interpretation comes
only in the context of a mosaic of enforced laws.

I have held, for a long time, that the fundamental theorem of politics is that an inconsistent set of axioms is necessarily a complete set of axioms, but, still, I wonder how many of us will be voting in November? This is the fear of God that politicians understand.

A. A. Mullin

Editor, the NOTICES

Professor Lorch's letter in the April 1960 issue of the NOTICES complains of the use of the term "Iron Curtain countries" in the NOTICES. Perhaps he has a point; perhaps it is better for scientific publications to avoid the unnecessary use of terms which might be painful to scientists from other countries. But then Professor Lorch proceeds to cite instances of United States restrictions concerning the movement of people and goods across frontiers, in such a way that a visitor from Mars reading his letter would be tempted to conclude that Hungary, for example, is a land of freedom, while despotism reigns in America.

Most of us will concede that American immigration laws are not ideal, and that they are sometimes administered in a manner which is inconsistent with our libertarian heritage and principles. But the enormous difference between our restrictions relating to the crossing of our frontiers and the corresponding restrictions of the Soviet government, is this: the United States government makes no effort to prevent citizens from leaving the country, but only to prevent aliens from entering illegally, while the Soviet government is, and has been for many years, deeply concerned to prevent its scientists from escaping. It has been widely noted that when Russian scientists travel abroad they are seldom, if ever, accompanied by their wives or children. This absence of dependents accompanying Russian mathematicians evoked frequent comment at the International Congress of Mathematicians held in Edinburgh in 1958. The published "List of Members" of that Congress included the names of a hundred or two wives and children of American mathematicians. It listed 35 mathematicians from the Soviet Union, but not a wife or a child of one of them.

To give brutal statement to the brutal fact, it is a regular practice of the Soviet government to hold hostages to guarantee the return of its scientists who go abroad. Protests may be voiced against the accusation that the Soviet government engages in such savage and inhuman conduct as the holding of hostages; but if such protests are voiced, I am confident that numerous American scientists will be ready to testify, on the basis of their personal contacts
with Russian scientists, that the charge is accurate. And included among the witnesses will be a few former Russians who, despite all the precautions of the Soviet government, succeeded in escaping.

Abram V. Martin
MEMORANDA TO MEMBERS

THE MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The following item is repeated from the June, 1960 issue of the NOTICES, and gives a more detailed time schedule.

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Summer Meeting in East Lansing, Michigan, on August 30 to September 2, 1960. The Register will be conducted from 12:00 noon to 5:00 P.M. on August 30, and from 9:00 A.M. to 5:00 P.M. on August 31, September 1, and 2. The morning of Tuesday, August 30 will be devoted to the registration of both employers and applicants and to distribution of the listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence, Rhode Island, for application forms and for position description forms, which must be completed and returned to Providence not later than August 5, 1960 in order to be included free of charge in the listings at the meeting in East Lansing. Forms which arrive after this closing date, but before August 22, will be included in the listings at the meeting for a late registration fee of $3.00, and will also be included in the printed listings, but not until ten days after the meeting. The printed listings will be available for distribution both during and after the meeting. The prices are as follows: Position descriptions, $2.00; listing of applicants, academic only, $5.00; comprehensive listing of applicants, academic, industrial, and government, $20.00.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

WORLD DIRECTORY OF MATHEMATICIANS

At the suggestion of Professor Marston Morse, the Chairman of the United States National Committee for Mathematics and Vice-President of the International Mathematical Union, the American Mathematical Society has made arrangements to distribute copies of the World Directory of Mathematicians. The Directory is published under the auspices of the International Mathematical Union and is intended to contain the names of mathematicians who have published at least two papers which have been reviewed in MATHEMATICAL REVIEWS. The price of the Directory is $1.25, and all orders sent
to the Providence Office will be transmitted to the distributor, James Thin and Son, in Edinburgh. Since we are arranging for the distribution of the Directory as a service, it would be appreciated if you would enclose the $1.25 with your order.

THE 1960-1961 COMBINED MEMBERSHIP LIST

Members are advised that the deadline for changes of listings in the forthcoming issue of the COMBINED MEMBERSHIP LIST is October 14. If you were listed incorrectly in the 1959-1960 COMBINED MEMBERSHIP LIST, or if you have changed any part of your listing since October 14, 1959, please send the information requested below to reach the Headquarters Offices by October 14, 1960. Please note the following explanations of listings: Students are listed at the college or university without a job title; two concurrent permanent positions are separated by a semicolon; a position held while on leave is shown as a separate second listing; in all cases, mathematics department is understood unless otherwise specified.

To insure accurate listing, please give us the following information: name in full and highest earned degree; place of employment and job title (please give us the complete business name and address); if you will be on leave, say what your job will be, include the name and address of the employer and the duration of the temporary job; if you will hold two concurrent positions indicate your secondary employment, including job title, business name and address; and finally, your mailing address.
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Southern Illinois University, Carbondale, Illinois
Southern Methodist University, Dallas 5, Texas
Stanford University, Stanford, California
Swarthmore College, Swarthmore, Pennsylvania
Sweet Briar College, Sweet Briar, Virginia
Syracuse University, Syracuse 10, New York
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Beatley, R. See Birkhoff, G. D.


Boron, L. F. See Gelfond, A. O.

Botts, T. A. See McShane, E. J.

Bruhat, F. Algèbres de Lie et groupes de Lie. Résumé des leçons. (Textos de Matematica, No. 3.) Instituto de Física e Matemática, Universidade do Recife, 1959. 5 + 71 pp. (polycopiées)


Čerkudinov, S. A. See Artobolevskiĭ, I. I.

Chern, S. S. Complex manifolds. Textos de Matemática, No. 5. Instituto de Física e Matemática, Universidade do Recife, 1959. 5 + 181 pp. (polycopiées).

Churchman, C. W. See Management technology.

Cramér, H. Aus der neueren mathematischen Wahrscheinlichkeitslehre. (Arbeitsgemeinschaft für Forschung des Landes Nordrhein-Westfalen, Heft 76a.) Cologne, Westdeutscher Verlag, 1959. 28 pp. 2.60 DM.

Crane, R. R. See Management technology.


Eaves, J. C. See Parker, W. V.


Godement, R. Variétés différentiables. Résumé des leçons. (Textos de Matemática, No. 2.) Institute de Fisica e Matematica, Universidade do Recife, 1959. 5 + 51 pp. (polycopiées)


Inhelder, B. See Piaget, J.


Kemmer, N. See Fock, V.


Levitskii, N. I. See Artobolevski'i, I. I.


Lifschitz, E. M. See Landau, L. D.


Lunzer, E. A. See Piaget, J.


Maple, C. G. See Holl, D. L.


Meltcер, L. V. See Šumilovskii, N. N.


Pollard, H. See Bochner, S.


Ralston, A. See Mathematical methods for digital computers.

Reid, C. Introduction to higher mathematics for the general reader. New York, Crowell, 1960. 7 + 184 pp. $3.50.

Reid, W. H. See Landau, L. D.


Sario, L. See Ahlfors, L. V.


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Sykes, J. B. See Landau, L. D.

Szász, G. Bevezetés a hálóelmezetebe. [Introduction to lattice theory.] Budapest, Akadémiai Kiadó, 1959. 225 pp. 60 Ft.

Szeminska, A. See Piaget, J.

Tenenbaum, M. See Bochner, S.


Vinograde, B. See Holl, D. L.

Wilf, H. S. See Mathematical methods for digital computers.

A NEW RUSSIAN TRANSLATION

American Mathematical Society Translations
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The first article (reviewed in MATHEMATICAL REVIEWS vol. 13, p. 811) is the fundamental "Theory of algorithms" by A. A. MARKOV (1951).

The second, "Algebras of complexes" (MR vol. 19, p. 246) by A. A. ZUKOV (1957), presents a generalized background, with ap-
plications, for the theory of symmetric functions.

The third, "On the theory of locally nilpotent groups" (MR vol. 14, p. 1059) by V. S. Carin (1951), deals e.g. with the properties of complete groups (for \( n = 1, 2, 3, \ldots \), the \( n \)th root exists for every element) of matrices over the rationals and with related abstract groups.

The fourth, "Structure of locally compact groups and Hilbert's fifth problem" (MR vol. 21, p. 698) by V. M. Gluskov (1957), is an expository paper, but with some interesting new material, on the results of Montgomery, Zippin, Gleason and Yamabe.

The fifth, "Geometry of infinite uniform complexes and 8-dimensionality of point sets" (MR vol. 19, p. 300) by Yu. M. Smirnov (1956), gives a proof of the author's theorem on the relation between a point-set in Euclidean \( n \)-space and the \( \delta \)-dimension of its proximity space.

The sixth, "Differentiable mappings and the order of connectivity" (MR vol. 17, p. 288) by A. M. Rodnyanskii (1955), gives four theorems about the connectivity of \( fA \) where \( f \) is a differentiable mapping (with essentially non-vanishing Jacobian) and \( A \) is one or another of various subsets (simply connected, \( F_\sigma \) etc.) of Euclidean \( n \)-space.

The seventh, "The multiplicative structure of J-contractive matrix functions" (MR vol. 17, p. 958) by V. P. Potapov (1955), gives among many other results an extension to matrix-valued functions of the result of R. Nevanlinna for ordinary complex-valued functions whereby they are decomposed into a Blaschke product over their zeros inside the unit-circle and an exponential integral.

The eighth, "Combinatorial topology of non-closed sets" (MR vol. 17, p. 1120) by K. Sitnikov (1954-1955), gives several characterizations of the dimension of a subset of Euclidean \( n \)-space \( E^n \) and proofs of theorems on obstruction, separation and deformation in \( E^n \).

If $B$ is any algebra over a field $\Phi$, and if $A$ is any algebra containing $B$, then $A/B$ is semialgebraic ($s,a,$) if to each $a \in A$ there is a nonconstant polynomial $f_a(x)$ in $\Phi[x]$, with zero constant term, such that $f_a(a) \in B$; $A/B$ is $s,a,b,d,$ if $\{\deg f_a(x) | a \in A\}$ is a bounded set. **Theorems.** (1) If $A$ is a noncommutative division algebra, and if $A/B$ is $s,a,$, then $A$ is algebraic; if $A/B$ is $s,a,b,d,$, then $A$ is locally finite ($l.f.$) (2) If $B$ is a transcendental division algebra over $\Phi$ (from now on $\Phi \not\cong GF(2)$ or $GF(3)$), then $A/B$ is $s,a,$ if and only if $A = Q \oplus K$, where $K$ is an algebraic algebra, and $Q$ is a directly irreducible $s,a,$ extension of $B$. $N,a,s,c,$ on $Q \not\cong B$ are obtained which are strong enough to imply (3) the Jacobson radical $J$ of $Q$ is nil, $Q - J$ and $B$ are fields, $Q$ has prime characteristic $p$, and for each $q \in Q$, $q^{p^t} \in B$, $t = t(q)$; (4) If $B$ is a division algebra over $\Phi$, and if $B$ is either noncommutative or has characteristic 0, then $A/B$ is $s,a,$ if and only if $A = B \oplus K$. (5) If $A$ is a simple algebra with a minimal left ideal, and $B$ a subalgebra $\not\cong A$, then (i) $A$ is $l.f.$ when $A/B$ is $s,a,b,d,$, and (ii) $A$ is algebraic if $A/B$ is $s,a,$ and $\Phi$ is not countably infinite. (Received May 9, 1960.)

570-41. Carl Faith: A disjointness property of transcendental fields.

The method of proof of Herstein's theorem (Canad. J. Math. vol. 7 (1955) pp. 202-203) is applied to obtain the following: **Theorem (1).** If $L \supset K \supset \Phi$, $L \not\cong K$, are three fields such that (i) $L/K$ is not purely inseparable, and (ii) $L/\Phi$ is transcendental, then there exists a transcendental (over $\Phi$) $u \in L$ such that $K \cap \Phi(u) = \Phi$ where $\Phi(u)$ is the field generated by $u$ and $\Phi$, (1) contains Herstein's theorem, and, with the help of (1) of the abstract on semialgebraic extensions of division algebras (directly above, or below), has the following extension: **Theorem (2).** If $A$ is a noncommutative and transcendental division algebra over the field $\Phi$, and if $B$ is any division subalgebra $\not\cong A$, then there exists a transcendental $u \in A$ such that $B \cap \Phi(u) = \Phi$. The original proof of Herstein's theorem required a lemma (Nagata, Nakayama, Tuzuku) on valuations of fields whose proof made use of arithmetic theory of "algebraic function fields." In the pre-
sent paper use of this lemma is avoided, and an elementary proof of Herstein's theorem is given which requires only Lüroth's theorem, and divisibility properties of polynomials. (2) leads to a new "commutativity" theorem, and a new theorem on the generation of division algebras. (Received May 9, 1960.)

570-42. Adriano Garsia: An imbedding of closed Riemann surfaces in Euclidean space.

For every closed Riemann surface a conformally equivalent surface of 3 dimensional Euclidean space is constructed. In fact, the following theorem is proved: Theorem. Given any closed \( c^\infty \) surface \( \Gamma \) of genus \( g \geq 2 \) of 3-dimensional Euclidean space, every Riemann surface of the same genus can be obtained by a \( c^\infty \) deformation of \( \Gamma \) which displaces each point of \( \Gamma \) by an arbitrarily small amount. (Received May 9, 1960.)

570-43. J. C. Kiefer: Large deviations of the empiric d.f. of vector chance variables.

Let \( F \) be a distribution function (d.f.) on \( m \)-dimensional Euclidean space \( \mathbb{R}^m \), and let \( X_1, X_2, \ldots, X_n \) be independent chance vectors with common d.f. \( F \). The empiric d.f. \( S_n \) is a chance d.f. on \( \mathbb{R}^m \) defined as follows: \( n S_n(x) \) is the number of \( X_i \)'s, \( 1 \leq i \leq n \), for which each coordinate of \( X_i \) is \( \leq \) the corresponding coordinate of \( x \). Let \( D_n = \sup_{x \in \mathbb{R}^m} |S_n(x) - F(x)| \). The limiting d.f. of \( n^{1/2} D_n \) when \( m = 1 \) and \( F \) is continuous was first computed by Kolmogorov. It was proved by Dvoretzky, Kiefer, and Wolfowitz that, when \( m = 1 \),
\[(A) \quad P\{n^{1/2} D_n \geq r\} \leq C e^{-2r^2} \quad \text{for all } n > 0, \ r \geq 0. \]
Chung proved that, when \( m = 1 \) and \( F \) is continuous, \( (B) \), \( P\{\lim \sup_{n \to \infty} n^{1/2} D_n / (2^{-1} \log \log n)^{1/2} = 1\} = 1 \), and also gave the finer result characterizing upper and lower classes.

When \( m > 1 \), Kiefer and Wolfowitz proved that there are \( F \) for which \( (A) \) does not hold, but that there are positive constants \( c_m \) and \( c'_m \) for which
\[ P \{ n^{1/2} D_n \geq r \} \leq c'_m e^{-cmr^2} \quad \text{for all } F, \ n > 0, \ \text{and } r \geq 0. \]
The present paper proves: Theorem 1: For each \( m \) and \( \varepsilon > 0 \) there is a constant \( c(\varepsilon, m) \) such that \( P \{ n^{1/2} D_n \geq r \} \leq c(\varepsilon, m) e^{-(2-\varepsilon) r^2} \quad \text{for all } F, \ n > 0, \ \text{and } r \geq 0. \) Theorem 2: For each \( m \) and continuous \( F, \ (B) \) holds. (A finer characterization depends on \( F \).) (Received May 6, 1960.)
570-44. E. H. Connell: The existence of the second derivative by topological analysis.

It is shown by methods of topological analysis that if $f$ is a function of a complex variable, the existence of $f'$ implies the existence of $f''$. (Received May 26, 1960.)

Let \( J_k \) be a symmetry with respect to the subspace spanned by \( \{ \alpha_{k+1}, \ldots, \alpha_n \} \) of a unitary space with the basis \( \{ \alpha_1, \ldots, \alpha_n \} \), \( 0 < k < n \). Then to linear transformations \( A \) and \( B \) corresponds a product \( AJ_kB \), called quasi product of order \( k \) of \( A \) and \( B \). This idea suggests the study of new kinds of linear transformations and matrices such as quasi-unitary and quasi-hermitian matrices which have interesting properties. It is possible to define a quasi inner product and thence quasi orthonormal sets of vectors. This allows diagonalization of some quasi-hermitian matrices by use of quasi-unitary transformations. There is also a way of estimating a combination of sums of squares of some elements of a matrix minus sums of squares of other elements by eigenvalues of \( A^*J_kA \) and \( AJ_kA^* \). (Received March 28, 1960.)


Let \( \hat{G} \) denote the space of semi-characters on the commutative semigroup \( G \), and let \( \mathcal{L}_1(G) \) be the Banach algebra defined and studied by Hewitt and Zuckerman (The \( \mathcal{L}_1 \)-algebra of a commutative semigroup, Trans. Amer. Math. Soc., vol. 83 (1956) pp. 70-97). We pursue our earlier studies (see these Abstracts 545-9 and 557-42) into the nature of the Šilov boundary \( \partial \) induced in \( \hat{G} \) by the transform algebra \( \mathcal{L}_1(G)^\ast \), where \( \hat{X}(\alpha) = \sum_{x \in G} \alpha(x) X(x) \) for each \( \alpha \in \mathcal{L}_1(G) \) and \( X \in \hat{G} \). \( \hat{G} \) being topologized by point-wise convergence on \( G \).

Notations: (1) for \( X \in \hat{G}, S(X) = \{ x \in G | X(x) \neq 0 \} \); (2) for finite \( F \subset G, G_F = \{ \chi \in G | y \in F \Rightarrow \text{either } \exists x_1 \text{ and } x_2 \text{ in } S(X) \text{ for which } x_1 y = x_2 y \text{ and } X(x_1) \neq X(x_2), \text{ or } \exists x_1 \text{ and } x_2 \text{ in } S(X) \text{ and } z \notin S(X) \text{ for which } x_1 y z = x_2 y \} \); (3) \( \Gamma = \{ X \in \hat{G} | |X| \equiv 1 \text{ on } S(X) \} \). Theorem 1. \( \partial = \bigcap_{F \subset G \text{ finite}} \hat{X} \bigcap_{\{ F \}} \Gamma \). Theorem 2. The following statements are equivalent: (a) \( G \) is a union of groups; (b) \( \partial = \hat{G} \); (c) \( \Gamma = \hat{G} \). Our methods depend heavily on the Hewitt-Zuckerman decomposition theorems for \( G \) (loc. cit., especially 4.10 and 5.6.1), and (c) \( \Rightarrow \) (a) of Theorem 2 is deduced from a theorem of Ross (A note on
extending semicharacters on semigroups, Proc. Amer. Math. Soc. vol. 10 (1959) pp. 579-583). (Received April 12, 1960.)


Let \( p: N \rightarrow M \) be a Riemannian covering (covering where \( M \) and \( N \) are Riemannian manifolds and \( p \) is a local isometry), \( G \) the group of isometries of \( N \), \( D \) the group of deck transformations of the covering, and \( Z \) the centralizer of \( D \) in \( G \). If \( M \) is Riemannian homogeneous, then \( Z \) is transitive on \( N \) and every element of \( D \) is a Clifford translation of \( N \) (the distance between \( x \) and \( d(x) \) is independent of \( x \in N \)). The connected Riemannian homogeneous manifolds of constant sectional curvature are the hyperbolic spaces \( H^n \), the connected abelian Lie groups in flat metric, and the spherical space forms \( S^n/P \) where \( P \) is a cyclic or binary polyhedral group of Clifford translations of \( S^n \). The cyclic and binary polyhedral groups of Clifford translations of \( S^n \) are classified for each \( n \). The connected Riemannian symmetric manifolds of constant sectional curvature are the hyperbolic spaces, the connected abelian Lie groups in flat metric, the spheres, and the real projective spaces. The connected isotropic Riemannian manifolds of constant sectional curvature are the hyperbolic spaces, the Euclidean spaces, the spheres, and the real projective spaces. (Received April 8, 1960.)

571-4. Morton Brown: The monotone union of open \( n \)-cells is an open \( n \)-cell.

In a research announcement (Embeddings of spheres, Bull. Amer. Math. Soc. vol. 65 (1959) p. 65) Barry Mazur indicated that modulo the Generalized Schoenflies Theorem, the following theorem could be proved: "If the open cone over a topological space \( X \) is locally Euclidean at the origin, then it is topologically equivalent with Euclidean space." In an abstract submitted to a recent meeting Ronald Rosen has described an ingenious proof of this theorem based on the now known Schoenflies theorem. (See M. Brown, A proof of the Generalized Schoenflies Theorem, Bull. Amer. Math. Soc. vol. 66 (1960) pp. 74-76).

In the present paper the author proves a somewhat stronger theorem without employing the Schoenflies Theorem: Let \( S \) be a topological space which is the union of a sequence of open sets \( U_i \), where each \( U_i \) is homeomorphic to \( E^m \), and \( U_i \subset U_{i+1} \). Then \( S \) is homeomorphic to \( E^m \). (Received May 18, 1960.)

The invariant plane curves of order 17 are mapped [W. R. Hutcherson et N. A. Childress, Etude d'une involution cyclique de periode cinq, Acad. Roy. Belg. Bull. Cl. Sci. (1954) pp. 103-108] on a surface \( \varphi \) of order 17 in \( S_{10} \). This is a 17 to 1 correspondence between the plane \( \pi \) and the surface \( \varphi \). Assume a twelfth coordinate to be proportional to \( g(x_1, x_2, x_3) \), where \( g = 0 \) represents one of the above curves invariant under the homography \( x'_1 : x'_2 : x'_3 = x_1 : E^5 x_2 : E^7 x_3 \) when \( E^{17} = 1 \). Now each point on the plane \( \pi \) orders a point in \( S_{11} \) on a surface \( F \), by use of a transformation \( T_1 \), whose inverse in laboriously exhibited. Hence each point of \( F \) orders a point on the plane \( \pi \), making \( F \) rational. On this surface \( F \), each point is associated with 16 others of the set belonging to the involution of period seventeen (I_{17}). And these 17 points are mapped by the inverse transformation onto the plane \( \pi \) into 17 distinct points, which are a set of I_{17}. (Received May 20, 1960.)


One of the seventeen families of curves invariant under the homography \( x'_1 : x'_2 : x'_3 = x_1 : E x_2 : E^{15} x_3 \), where \( E^{17} = 1 \), is used in mapping the surface \( \varphi \) in \( S_{10} \). [W. R. Hutcherson and Stanley Frank, A certain branch point in a space of seven dimensions, Notices, Amer. Math. Soc. vol.6 (1959) p. 405 (abstract)].

The image of point \( 0_2(0,1,0) \) is \( 0_2(0,1,0,0,0,0,0,0,0,0,0) \). By use of two quadratic transformations on the special curves passing through \( 0_2 \), a study of points in the second and fifth order neighborhoods of \( 0_2 \) is made. The projective equivalent of these two neighborhoods is found on the surface \( \varphi \) in the first order neighborhood of the image point \( 0_2 \). The equations of a quintic and a quadric cone, which are tangent to \( \varphi \) at this point, were obtained. At
0^4(1,0,0,0,0,0,0,0,0,0,0), a similar investigation revealed the tangent element
to be a degenerate cubic cone (quadric cone and a plane). However, at
0^3(0,0,1,0,0,0,0,0,0,0,0), a degenerate quartic cone (two planes and a quadric
cone) was found as the tangent element. (Received May 20, 1960.)

Preliminary report.

For an arbitrary class of sets \( C \subset \mathcal{J} \) in a probability space \((\Omega, \mathcal{J}, P)\),
let \( C^S \) denote the class of \( \mathcal{J} \) sets independent of \( C : C^S = \{ \mathcal{T} : P\{T \cap C\} \}
= P\{T\} P\{C\} \) for every \( C \in C \). It can be shown that if \((\mathcal{J}, P)\) is nonatomic,
either \( C^{SS} = (C^S)^S \cup C \) is nowhere dense or \( C^{SS} = \mathcal{J} \) (metric \( d(A, B) = P\{A \cup B\} - (A \cap B)\} \). A consequence is that a necessary condition for
\( \mathcal{B}(C \cup C^S) = \mathcal{J} (\mathcal{B}(C) \neq \mathcal{J}) \) is that \( C^{SS} \) be nowhere dense \( (\mathcal{B}( \ ) \}
= smallest Borel field containing). (Received May 25, 1960.)


It is shown that the problems raised by Post in [Amer. J. Math. vol. 65,
pp. 196-215] are unsolvable, by reduction to Turing machines. A Godel-number­
ing argument shows that there exist Universal Turing machines with 2 semi­
infinitie tapes which do not write on their tapes (but can sense the ends). It is
then shown that such machines can be represented by monogenic normal canoni­
cal systems of the "Tag" form: Let \( a_1, ..., a_n \) be an alphabet and let \( A_1, ..., A_n \) be
words in that alphabet. Let \( N \) be an integer and let \( S \) be an "initial" string in
the alphabet. If a string begins with \( a_i \), remove the first \( N \) letters of the string
and attach \( A_i \) to the other end. It is undecidable for a given set of \( A_i \)'s and an
arbitrary \( S \), whether this operation when repeated, will ever yield a string of
length less than \( N \), or, for given \( A_i \)'s and \( S \), whether an arbitrary string will be
generated. (Received April 22, 1960.)

571-10. A. A. Mullin: Some remarks on a relative anti-closure
property.

Let \( (A, \cdot) \) denote a nonempty set \( A \) together with a closed binary compo­
sition law defined on \( A \). Call \( (A, \cdot) \) an algebraic system. By a mutant of \((A, \cdot)\)
is meant a subset \( M \) of \( A \) that satisfies the condition \( M \cdot M \subseteq \overline{M} \), where \( M \cdot M
= \{a \cdot b : a \in M, b \in M\} \) and \( M \) is the set of all elements of \( A \) not in \( M \). It is
shown (among other things) that every algebraic system has a mutant and a
maximal mutant, a subset of a mutant is a mutant, under a homomorphism. The inverse image of a mutant is a mutant, etc. Some group-theoretic interpretations are made, e.g., that all of the maximal mutants of any infinite cyclic group have the same cardinality, $K_0$, and whereas, in the additive group of integers, the set of all even integers is a subgroup, the set of all odd integers is a maximal mutant, etc. (Received March 24, 1960.)


Bairstow's method for improving an approximate real quadratic factor $(x^2 - px - q)$ of a polynomial with real coefficients which leaves a remainder $r(x)$, is to determine $\delta p$ and $\delta q$ to satisfy $0 = r(x) + (\partial r(x)/\partial p)\delta p + (\partial r(x)/\partial q)\delta q$. One extension is to determine $\delta p$ and $\delta q$ when the three second-order terms $2^{-1}[(\partial^2 r(x)/\partial p^2)(\delta p)^2 + 2(\partial^2 r(x)/\partial p \partial q)(\delta p)(\delta q) + (\partial^2 r(x)/\partial q^2)(\delta q)^2]$ are added to the right member. By taking advantage of polynomial congruences and the linearity in $x$ of every $\partial^{j+k} r(x)/\partial p^j \partial q^k$, only one extra division is needed besides the two required divisions of Bairstow's method. Another extension improves an approximate real quartic factor $(x^4 - px^3 - qx^2 - rx - s)$, considering only terms of the first order in $\delta p, \delta q, \delta r$ and $\delta s$. This latter method may be immediately generalized for approximate real factors of any degree. By employing polynomial congruences, no more than two divisions are necessary in any case. (Received April 20, 1960.)


Let $L_0$ be an ordinary $(\eta, \eta)$ matrix differential operator of order $h$, \( h\eta = 2\lambda = n \), on the interval \( 0 \leq t < \infty \) and let $q(t)$ be a diagonal matrix whose elements are real valued bounded continuous functions. Define $L_\epsilon = L_0 + \epsilon q$ where $\epsilon$ is a real parameter. Suppose the eigenvalue problem (i) $L_0 u = \lambda u$, $[\partial_j u](0) = 0, j = 1, \ldots, \nu$ is self adjoint (cf. K. Kodaira, Amer. J. Math. vol. 72, (1950) p. 540). We impose the following conditions relative to a finite real interval $\Delta$: (ii) $\int_0^\infty \Phi^2(t)|q(t)|dt \leq \gamma$ where $\Phi^2 = \sup (s_j \cdot s_j), j = 1, \ldots, n$, $\eta \in \Delta, s_j(t, \ell)$ fundamental solutions of (i) and $|q| = \sum \epsilon|s_j|$. (iii) $|M^{jk}(\ell + i\delta)| \leq K, j, k = 1, \ldots, n, \ell \in \Delta, 0 < \delta = \delta_0$ where $M^{jk}$ is the characteristic matrix of $L_0$. (iv) The spectral density matrix $\rho^{jk}(\ell)$ of $L_0$ is diagonal. (v) $\int_0^\infty |s_j|^2 |s_j|^2 |q| |\int_{s_j}^t f d\xi|^2 dt d\ell \leq P^2 ||f||^2, \int_\Delta \int_0^\infty |M^{jk}|^2 |s_j|^2 |q| |\int_{s_j}^t f d\xi|^2 dt d\ell \leq P^2 ||f||^2$. Theorem. If $L_0, q$ satisfy (ii), (iii), (iv), (v) then
for $|e| < (n^2 \lambda \nu)^{-1}$, $L_e$ determines a self adjoint operator $H_e$ with analytic spectral measure $F_e(\Delta')$, $\Delta' \subseteq \Delta$. Conditions under which (iv), (v) can be removed are investigated. (Received May 16, 1960.)


A combinatorial $n$-sphere is a finite simplicial complex which is piecewise linearly homeomorphic with the boundary of the standard $(n + 1)$-simplex. An embedding of $S^{n-1}$ in $S^n$ is of type I if $S^n$ can be represented as a combinatorial $n$-sphere with $S^{n-1}$ a subcomplex. An embedding of $S^{n-1}$ in $S^n$ is of type II if there is a simplicial decomposition of $S^n$ such that $S^{n-1}$ is a subcomplex which is a combinatorial $(n-1)$-sphere. M. H. A. Newman (to appear in Proc. Roy. Soc.) has shown that if the embedding is simultaneously of types I and II, then $S^n - S^{n-1}$ consists of two disjoint $n$-cells; i.e. the Schoenflies theorem holds. Now let $M$ be the double suspension of a Poincaré manifold. If $M = S^5$, then the Schoenflies theorem fails for embeddings of type II with $n = 5$. If $M = S^5$, then the Schoenflies theorem fails for embeddings of type I with $n = 6$. (Received May 19, 1960.)


For a space $X$ let $\hat{X} = \{ (x_1, x_2) | x_1 \neq x_2 \}$, and define $p: \hat{X} \to X$ by $p(x_1, x_2) = x_1$. The space $X$ is said to be homotopically homogeneous if $(\hat{X}, X, p)$ is a Hurewicz fiber space. Manifolds are homotopically homogeneous (h.h.) and an h.h. polyhedron is a generalized manifold, both homologically and homotopically. For any simplicial decomposition of an h.h. polyhedron, the boundaries of star neighborhoods of points are homology generalized manifolds with the homotopy type of a sphere. If the Poincaré Conjecture is true, then 4-dimensional h.h. polyhedra are manifolds, and lower dimensional ones are in any event. It is also shown that h.h. polyhedra are Kosinski $r$-polyhedra (Fund. Math. (1955)). The double suspension of a Poincaré manifold is an $r$-polyhedron (Curtis, Fund. Math. (1958)), but it is not known if it is h.h. (Received May 19, 1960.)


R. H. Bing (Ann. of Math. (1959)) has proved that a certain nonmanifold $B$ has the property that $B \times E^1 = E^4$. This note shows that essentially the same proof can be applied to a different, and in some ways a simpler, situation.
Let $\alpha$ be the knotted arc of Example 1.1 of Fox-Artin (Ann. of Math. (1948)). Let $X$ be the space obtained from $E^3$ by collapsing $\alpha$ to a point, $\beta: E^3 \to X$ being the natural map. If $x = \beta(\alpha)$, then $X - x$ is not simply connected, although $X$ is simply connected. It follows that $X$ is not locally euclidean at $x$. However, $X \not\simeq E^1 = E^4$. The space $X$ is nicer than the Bing space $B$ in the sense that it has only one bad point instead of a Cantor set of them, but $B$ is nicer than $X$ in the sense that it is a homotopy manifold whereas $X$ is not. (Received May 19, 1960.)


Theorem: Let $C$ be a continuum in $E^3$, and let $X$ be the quotient space obtained by collapsing $C$ to a point. If $X$ is a homotopy manifold, then $X$ is homeomorphic with $E^3$. The homotopy manifold referred to here is that defined by H. B. Griffiths (Michigan Math. J. (1953)). It is an easy corollary to this theorem that if one shrinks any set of continua to points such that the image points form a discrete set in $X$, and $X$ is a homotopy manifold, then $X$ is $E^3$. It is interesting to contrast this with the Bing space $B$ (Ann. of Math. (1957)) which is obtained from $E^3$ by shrinking arcs such that the set of image points in $B$ is a zero-dimensional set. For $B$ is a homotopy manifold (Curtis-Wilder, Trans. Amer. Math. Soc. (1959)), but it is not locally euclidean. The theorem is proved by using a version of the sphere theorem due to J. H. C. Whitehead (Bull. Amer. Math. Soc. (1958)). (Received May 19, 1960.)

571-17. Aubert Daigneault: 0-valued representations of polyadic algebras of infinite degree.

We prove the existence of 0-valued functional representations of any I-polyadic algebra of infinite degree. The proof uses the existence of dilations of such an algebra and follows the steps of the Henkin-Halmos proof in the locally finite case. Let $A^+$ be an $I^+$-dilation of $A$ such that the cardinality of $I^+$ is sufficiently large. For each $p$ in $A$ let $t_p$ be a one-one mapping of $I$ into $I^+ - I = X$ such that $t_p(I) \cap t_q(I) = \emptyset$ if $p \neq q$. Let $A'$ be the $I$-algebra obtained from $A^+$ by fixing the variables of $X$ and let $M_1$ be the (polyadic) ideal of $A'$ generated by the elements of the form $(\exists I)p + S(t'_p)q$ where $t'_p$ is the canonical extension of $t_p$ to $I^+$, i.e., $t'_p(I) = t_p$ and $t'_p(i) = i$ for $i$ in $X$. The process is repeated a transfinite number $\beta$ of times exceeding the cardinality of $I$. $X$ is kept fixed throughout and for a limit ordinal $\nu$ one sets $M_\nu = \cup M_{\nu'}$, $\nu' < \nu$. 

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If $M$ is a maximal (polyadic) ideal of $A'$ containing all $M_\alpha$ for $\alpha \leq \beta$ it is easy to show that $A \subset A'/M$ and that an 0-valued representation $f$ of $A'$ with domain $X$ is obtained by setting $fp(x) = S(x')p$ modulo $M$ for all $p$ in $A'$ and $x$ in $X^I$.

(Received May 31, 1960.)

571-18. Aubert Daigneault: Dilations of polyadic algebras and of other algebraic systems.

Let $A$ be an algebraic system admitting, for some set $I$, the semigroup $T(I)$ of all transformations of $I$ (i.e., mappings of $I$ into itself) as a semigroup of endomorphisms. For $t$ in $T(I)$ we denote by $S(t)$ the corresponding endomorphism of $A$. For a set $I^+ \supset I$ an $I^+$-dilation of $A$ is a minimal extension $A^+$ of $A$ admitting $T(I^+)$ as a semigroup of endomorphisms in such a way that if $t$ in $T(I^+)$ has the property that $t(i) = i$ for all $i$ in $I^+$ - $I$ then $S(t)|A = S(t)|I$.

Theorem: For any $A$ and $I^+ \supset I$, $A^+$ exists and is unique to within equivalence provided $I$ is infinite. If $\Lambda$ is a ring of polynomials with set of variables $I$ then extending $A$ to $A^+$ amounts to enlarging the set of variables to $I^+$. The support of an element $p$ in $A$ is the set of elements $i$ in $I$ for which there is an element $j$ in $I$ such that $S(i/j)p \neq p$. $A^+$ consists of all couples (t,p), t in $S(I^+)$, p in $A$, under a suitable equivalence relation defined using the concept of support. $A$ is injected into $A^+$ by means of the mapping $p \rightarrow (\delta,p)$ where $\delta$ is the identity transformation of $I^+$. Operations on $A$ extend to $A^+$ in a natural manner. The idea amounts to shifting the scene of action into I. If $A$ is an $I$-polyadic algebra, $A^+$ can be made into an $I^+$ - polyadic algebra. (Received May 31, 1960.)


Let $A_\alpha$ and $B_\alpha$ denote real-valued functions on a set $X$ satisfying
\[ \inf_{x \in X} A_\alpha(x) > 0 \quad \text{and} \quad \sup_{x \in X} B_\alpha(x) < \infty. \]
The minimum of the function $F(x)$
\[ = \sup_{x \in X} A_\alpha(x)/B_\alpha(x) \]
on $X$ is the limit of the descending sequence $F(x_k)$, where $x_k$ is taken to minimize the function $f_k(x) = \sup_{x' \in X} [A_\alpha(x') - F(x_k^{-1})B_\alpha(x')] / B_\alpha(x_k^{k-1})$.

The approximation in the Tchebycheff sense of a continuous function by a rational function is a problem of this type with $X \subset \mathbb{R}^n$, $A_\alpha$ and $B_\alpha$ being linear functions plus constants. The successive $x$'s may in this case be obtained by the methods of convex or linear programming. (Received May 9, 1960.)
571-20. L. K. Durst: **Exceptional real Lucas sequences.**

If \( \ell \) and \( m \) are nonzero integers and \( U_0 = 0, U_1 = 1, U_n = \ell U_{n-1} - m U_{n-2}, \) for \( n > 1 \), \((U)\) is called a **Lucas sequence**. An index \( n \), greater than 2, of \((U)\) is **exceptional** if \( p | U_n \) implies \( p | U_1 U_2 \ldots U_{n-1} \). If \( \ell > 0, \ell^2 > 4m, (\ell, m) = 1 \), and \((U)\) is not the Fibonacci sequence (\( \ell = 1, m = -1 \)), then six is the only possible exceptional index in \((U)\). Moreover, under these conditions, six is exceptional if and only if either (i) \( \ell = 2^{t+1} - 3r, m = (2^t - r)(2^t - 3r) \) where \( t \geq 1, r \leq r < (1/3)2^{t+1} \), or (ii) \( \ell = 3^s \lambda, m = 3^{2s-1} \lambda^2 - 2^t \) where \( s \geq 1, t \geq 0, \lambda \equiv \pm 1 \pmod{6} \) and \( 3^{2s-1} \lambda^2 < 2^{t+2} \). However, there exist infinitely many \((U)\) with \( \ell^2 > 4m, (\ell, m) > 1 \) and any prescribed finite set of exceptional indices. (Received April 25, 1960.)

571-21. James Glimm: **Type I C*-algebras.**

Let \( A \) be a separable C*-algebra. Then the following statements are equivalent. (a) \( A \) is type I. (b) \( A \) is GCR (see I. Kaplansky, The structure of certain operator algebras, Trans. Amer. Math. Soc. vol. 70 (1951) pp. 219-255). (c) Every irreducible image of \( A \) contains the completely continuous operators. (d) If \( \phi_1 \) and \( \phi_2 \) are any two irreducible representations of \( A \) such that kernel \( \phi_1 = \text{kernel} \phi_2 \) then \( \phi_1 \) is unitarily equivalent to \( \phi_2 \). (e) \( A \) has a smooth dual (see G. Mackey, Borel structures in groups and their duals, Trans. Amer. Math. Soc. vol. 85 (1957) pp. 134-165). The proof of the equivalence of (a),..., (e) is based upon the fact that if \( B \) is a C*-algebra with no nonzero GCR ideals then \( B \) contains strong approximations to certain matrix units of arbitrarily high finite order in the weak closure of \( B \). (The matrix units \( e_{ij} \) do not satisfy \( \sum_{1}^{} e_{ii} = 1 \).) It follows from (a) \( \iff \) (e) that a separable locally compact group is type I if and only if it has a smooth dual (Mackey's conjecture). (Received May 9, 1960.)

571-22. H. W. Guggenheimer: **The Euler Composition. V. Lie derivatives.**

It is well known that the difference of two connections is a tensor, and that the Lie derivative of a connection is a tensor. These two statements may be inverted and extended: **If the Lie derivative of a geometric object is a tensor, then the object is either a tensor (including scalars) or it transforms by the Euler composition (extended to tensor products).** We consider geometric objects as functionals of the coordinate mappings defining the manifold. By the
process of dragging along we may evaluate at one point the functional on
different mappings. We then show that if the difference of the functional of two
different mappings has tensor character, and it vanishes over the identity map-
ping, it is a tensor or an Euler object. (Received May 31, 1960.)

571-23. T. E. Harris: Probability one convergence for age-dependent
branching processes.

Consider the age-dependent branching process, introduced by Bellman
further by Harris (Proceedings of the Second Berkeley Symposium, 1951). Let
the life-length distribution \( G(t) \) have a density \( g(t) \) such that either (a) \( g \) is of
bounded total variation on \( (0, \infty) \) or (b) \( \int_0^{\infty} (g(t))^p \, dt < \infty \) for some real \( p > 1 \).

Let the probabilities \( q_r \) that an object is transformed into \( r \) objects satisfy
\[ m = \sum r q_r > 1, \quad \sum r^2 q_r < \infty. \]
Define \( a \) by \( m \int_0^{\infty} e^{-at} g(t) \, dt = 1 \). Let \( Z(x,t) \) be
the number of objects of age \( \leq x \) at time \( t \). Define \( A(x) = c \int_0^{\infty} e^{-at} (1 - G(t)) \, dt \),
choosing \( c \) so that \( A(\infty) = 1 \). **Theorem.** There is a random variable \( W \), having
finite positive variance, such that with probability one we have
\[ \lim_{t \to \infty} Z(x,t) e^{-at} = A(x)W. \]
This extends previous results on mean square convergence.
(Received May 23, 1960.)

571-24. Elliott Mendelson: A semantic proof of the eliminability of
descriptions.

The proofs by Hilbert-Bernays, Rosser, and Kleene, of the eliminability
of descriptions are all effective and syntactical, but rather complicated. How-
ever, by using the completeness theorem, it is possible to give a simple,
semantical, but noneffective, proof. The demonstration irritates the obvious,
intuitive way through which we come to see the truth of the theorem. (Received
May 9, 1960.)


Preliminary report.

Suppose \( g(x,y) \) is a function of two variables whose domain of definition
is the rectangular interval \( [a,b; c,d] \) consisting of all points \( (x,y) \) such that
\( a \leq x \leq b \) and \( c \leq y \leq d \), and suppose that \( g(x,y) \) is of bounded variation in the
sense of Hardy (On double fourier series, Quarterly J. Math. vol. 37 (1906)
pp. 53-79) on the rectangular interval \( [a,b; c,d] \) with \( g(a,y) = g(x,c) = 0 \). Then
there exist an infinite sequence of functions \( \{ f_p(x) \}_{p=1}^{\infty} \) of uniform bounded variation on the interval \([a,b]\) and an infinite sequence of functions \( \{ h_p(y) \}_{p=1}^{\infty} \) of uniform bounded variation on the interval \([c,d]\) such that if \((x,y)\) is a point of the rectangular interval \([a,b; c,d]\), then \( g(x,y) = \sum_{p=1}^{\infty} f_p(x) h_p(y) \), uniformly convergent on \([a,b; c,d]\). In particular the above theorem holds for a function \( g(x,y) \) of two variables whose domain of definition is the rectangular interval \([a,b; c,d]\) such that \( \partial^2 g(x,y)/\partial x \partial y \) is bounded on \([a,b; c,d]\), since under the above condition \( g(x,y) \) is of bounded variation on \([a,b; c,d]\). (Received May 16, 1960.)


For any infinite sequence of integers, say \( A = \{a_i\} \), define \( A(n,j,m) \) as the number of terms among \( a_1, a_2, a_3, \ldots, a_n \) that satisfy the congruence \( a_i \equiv j \pmod{m} \). Say that the sequence \( A \) is uniformly distributed modulo \( m \) in case \( A(n,j,m) \to 1/m \) as \( n \to \infty \) for \( j = 1, 2, \ldots, m \). Say that \( A \) is uniformly distributed in case it is uniformly distributed modulo \( m \) for every integer \( m \geq 2 \).

Properties of uniformly distributed sequences are investigated, and two of the principal results are as follows. Given any irrational \( \theta \), the sequence \( \{[n\theta]; n = 1, 2, 3, \ldots\} \) is uniformly distributed, where as usual \([n\theta]\) denotes the unique integer satisfying \( n\theta \leq [n\theta] + 1 \). This result is equivalent to the theorem of H. Weyl that the fractional parts \( n\theta - [n\theta] \) are uniformly distributed in the unit interval. Next consider a nonconstant polynomial \( f \) with integer coefficients. The sequence \( f(1), f(2), f(3), \ldots \) is uniformly distributed if and only if \( f \) has the form \( \pm x + c \). (Received May 17, 1960.)

571-27. Maurice Sion: Continuous images of Borel sets.

For a topological space \( X \), let \( K(X) \) denote the family of compact sets in \( X \); Borel \( K(X) \) the smallest family containing \( K(X) \) and closed under countable unions and difference of two sets; Borelian \( K(X) \) the smallest family containing \( K(X) \) and closed under countable unions and intersections. Let \( X \) have property I iff \( X \) is Hausdorff and the difference of two compact sets in \( X \) is a \( K_\sigma \). We have the following results. \( X \) has property I iff \( X \) is Hausdorff and Borel \( K(X) = \text{Borelian } K(X) \). If \( X \) has property I, then \( A \) is Borel \( K(X) \) iff \( A \) is the continuous one-to-one image of a \( K_\sigma \) for some \( X' \) that has property I and \( A \) is contained in some \( K_\sigma \). If \( X \) has property I, \( A \) is Borel \( K(X) \) and \( f \) is continuous and countable-to-one on \( A \) to some compact Hausdorff space \( Y \), then \( f(A) \) is Borelian \( K(Y) \) and \( f(A) \) has property I. (Received May 16, 1960.)

Let K be a convex closed region in the plane. Theorem: There is a rectangle in K whose area is \( \frac{1}{2} \) that of K. The proof consists of first showing that for any direction there is a parallelogram in K with area \( \frac{1}{2} \) the area of K, with one side of the parallelogram in the given direction; a continuity argument then establishes the existence of the rectangle. (Received May 12, 1960.)


Shoda proved that, in the group of all unimodular matrices over an infinite and algebraically closed field, each element is a commutator. For a complex semi-simple Lie group G, Goto showed that the set C of commutators is everywhere dense and the dimension of G-C is, at most, dim. G-2. It is the aim of the present paper to sharpen Goto's result and prove that G and C actually coincide. (Received June 3, 1960.)

571-30. M. Sugunamma: On class-division of integers modulo \( r \).

In this paper it is shown that all the integers modulo \( r^g \) can be divided into classes \( C_{d_1}, C_{d_2}, \ldots, C_{d_K} \) containing all those integers whose sth power g.c.d., with \( r^g \) is \( d_K \); where the sth power g.c.d. of any 2 numbers is the greatest sth power factor common to both. It is proved here that if \( C_{d_1} + C_{d_j} \) stands for all the numbers got by adding any number of \( C_{d_1} \) to any number of \( C_{d_j} \) then these classes combine by addition i.e., \( C_{d_1} + C_{d_j} = \sum_{1 \leq i \leq K} \gamma_{i,j}^{K} C_{d_{K}} \) and the coefficients \( \gamma_{i,j}^{K} \) are evaluated. Again if we consider the ordered sets of \( K' \) integers \((x_1,x_2,\ldots,x_{K'}) \) ranging over a complete residue system modulo \( r \), they can be divided into classes \( C_{d_1}, C_{d_2}, \ldots, C_{d_K} \) containing all the sets \((x_1,x_2,\ldots,x_{K'}) \) for which the g.c.d. of \( x_1,x_2,\ldots,x_{K'} \) with \( r \) is \( d_K \). If \((x_1,x_2,\ldots,x_{K'}) + (y_1,y_2,\ldots,y_{K'}) = (x_1 + y_1, x_2 + y_2,\ldots,x_{K'} + y_{K'}) \) and the addition between the classes is defined as above then it is proved that those classes combine by addition i.e., \( C_{d_1} + C_{d_j} = \sum_{1 \leq i \leq K} \gamma_{i,j}^{K} C_{d_{K}} \) and the coefficients \( \gamma_{i,j}^{K} \) are evaluated. A further generalization of these two results is given, from which the first results if we put \( K' = 1 \) and the second comes if we replace \( r^g \) by \( r \) and sth power g.c.d. by g.c.d. (Received June 3, 1960.)

Because of the duality of euclidean $n$-space the original Farkas Theorem (see J. Farkas, J. Reine Angew. Math. vol. 124 (1902) pp. 1-27) can be stated in either of the following equivalent ways: Theorem 1. Let $A$ be a nonempty finite subset of euclidean $n$-space $E$, let $A^*$ denote the set of all continuous linear functionals $y$ on $E$ such that $y(x) \geq 0$ for all $x$ in $A$, and let $B$ denote the set of all $x$ in $E$ such that $y(x) \leq 0$ for all $y$ in $A^*$. Then $B = A^c$ (where $A^c$ denotes the conical extension of $A$). Theorem 2. Let $A$ be a nonempty finite set of continuous linear functionals on euclidean $n$-space $E$, let $A^*$ denote the set of all elements $x$ in $E$ such that $y(x) \geq 0$ for all $y$ in $A$, and let $B$ denote the set of all continuous linear functionals $y$ on $E$ such that $y(x) \geq 0$ for all $x$ in $A^*$. Then $B = A^c$. Using some techniques from elementary functional analysis we generalize Theorem 1 to: Theorem 3. Let $A$ be any nonvoid subset of a locally convex space $E$. Let $A^*$ and $B$ be defined as in Theorem 1. Then $B = A^c$. An example is given to show that an analogous extension of Theorem 2 cannot be made unless a reflexivity condition is imposed on $E$. Examples are also given to show that the closure requirement is needed in the conclusion of Theorem 3. (Received June 21, 1960.)

571-32. Stefan Bergman: On coefficients of schlicht pseudo-conformal mappings in the space of two complex variables, I.

The author considers a schlicht mapping of a Reinhardt circular domain $\mathbb{C}^4$ onto a domain $\mathcal{D}^4$. Let $a_k = a_{k0} z_1 + a_{k1} z_2 + a_{k2} z_3 + a_{k3} z_4 + \ldots$, $|a_{k0} / a_{k1} - a_{k1} / a_{k0}| = 1$, be the function elements of the pair mapping $\mathcal{D}^4$ onto $\mathbb{C}^4$. (Here and in the following $k = 1,2,3$.) $\mathcal{D}^4$ omits four segments of analytic surfaces $z_k = h_k(z_{3-k}) + 2k_\kappa$ and $z_\kappa = h(z_{3-k}) + 2k_\kappa + d_\kappa$, $k_\kappa > 0$, $d_\kappa > 0$. The functions $h_\kappa$, $h_\kappa(0) = 0$, are chosen in such a way that the mapping $m = [z_\kappa - h_\kappa(z_{3-k})]$ is schlicht in $\mathcal{D}^4$. Generalizing the considerations of (1) J. Reine Angew. Math. vol. 162 p. 256; (2) Math. Z. vol. 29 p. 641, the author investigates the minimum $\lambda_{\mathcal{D}^4}(X_{00}, \ldots, X_{m_\nu n_\nu})$ of $\int_{\mathcal{D}^4} f(z_1, z_2)^2 d\omega_1 d\omega_2$ under the condition $f(0,0) = X_{00}, \ldots, (\partial X_{m_\nu n_\nu} / \partial z_1^\nu \partial z_2^\nu) z_1 z_2 = 0 = X_{m_\nu n_\nu}$. The minimum $\lambda_{\mathcal{D}^4}(X_{00}, \ldots, X_{m_\nu n_\nu})$ is a Hermitian form in $X_{m_\nu n_\nu}$. It depends on the $X_{m_\nu n_\nu}$, the kernel function $K_{\mathcal{D}^4}$ and the derivatives of $K_{\mathcal{D}^4}$ at the origin. These quantities are polynomials in $P(mn)$.

$= \int_{\mathcal{D}^4} \left| \xi_1^m \xi_2^n \right|^2 d\xi_1 d\eta_1 d\xi_2 d\eta_2$, $a_{k0}^\nu$ and $a_{k1}^\nu$. In this way one obtains for
\[ \lambda_{\mathcal{B}^4}(X_{00}, \ldots, X_{m_p n_p}) \] certain rational functions of \( p^{(m_p n_p)}, X_{m_p n_p}, a^{(n)}_{(m_p n_p)} \) and their conjugates, \((m_p n_p) = (00), (10), (01), \ldots, (m_p n_p)\). (The upper indices indicate the dimension of the manifold.) (Received June 13, 1960.)

571-33. Stefan Bergman: On coefficients of schlicht pseudo-conformal mappings in the space of two complex variables, II.

\( m \) transforms the domain \( \mathcal{B}^4 \) into the domain \( \mathcal{U}^4 = m(\mathcal{B}^4) \) in the \( v_1, v_2 \)-space. \( \mathcal{U}^4 \) omits four analytic planes \( v_K = 2k_K \) and \( v_K = 2k_K + d_K \). Let \( \mathcal{H}_1^2 \) and \( \mathcal{H}_2^2 \) be the projections of \( \mathcal{U}^4 \) on the \( v_1 \) and \( v_2 \) planes, respectively, and let \( \mathcal{J}_K^2 = \left( |v_K| = \rho \right), \rho > 0, \) sufficiently small. It is assumed that there exist domains \( \mathcal{L}_K^2 \) of the \( v_K \)-plane such that \( \mathcal{L}_K^2 \times \mathcal{J}_K^2 \) lie inside \( \mathcal{U}^4 \), and that the intersections \( \mathcal{U}^4 \cap (v_K = \text{const}), v_2 \in \mathcal{H}_2^2 - \mathcal{L}_K^2 \) can be connected with the origin \( v_1 = v_2 = 0 \) by a curve whose projection on the \( v_K \)-plane omits the segment \( 2k_K \leq \text{Re} v_K \leq 2k_K + d_K \). Under these conditions it is shown that there exists a domain \( \mathcal{A}^4, \mathcal{T}^4 \subseteq \mathcal{U}^4 \) and possessing the kernel function: Let \( \mathcal{J}_K^2 \) and \( \mathcal{J}_K^2 \) be two sheets of the Riemann surface with the branch points of the second order at \( v_K = 2k_K \) and \( v_K = 2k_K + d_K \) and let \( \mathcal{H}_K^2 \) be the domain \( \left[ |v_K| \leq \rho \right] \) lying in the second sheet. Then \( \mathcal{A}^4 = \mathcal{H}_1^2 \times \mathcal{H}_2^2, \mathcal{A}_K^2 = \mathcal{J}_K^2 \times \mathcal{J}_K^2 - \mathcal{H}_K^2 \). Using the kernel function of \( \mathcal{A}^4 = m^{-1}(\mathcal{A}^4) \) one obtains for the minimum \( \lambda_{\mathcal{B}^4}(X_{00}, \ldots, X_{m_p n_p}) \) an expression which depends only on \( \rho, h_K(z_{3-K}), k_K \) and \( d_K \). The Hermitian form \( H(X_{00}, \ldots, X_{m_p n_p}) = \lambda_{\mathcal{B}^4} - \lambda_{\mathcal{B}^4} \) is positive definite. For every \((m_p n_p)\) the coefficients of \( \lambda_{\mathcal{B}^4}(X_{00}, \ldots, X_{m_p n_p}) \) are rational functions \( p_{(m_p n_p)} \) of finitely many \( p(mn), a^{(n)}_{(mn)}, a^{(n)}_{(mn)} \). Applying the usual criteria in order that the form \( H \) is positive definite, one obtains upper bounds for the rational functions \( p_{(mn)}, a^{(n)}_{(mn)}, a^{(n)}_{(mn)} \). (Received June 13, 1960.)


Let \( f \) be a real-valued function of a real variable, whose domain contains a right ray, and \( \lambda \) be a real number such that \( \lim_{t \to \infty} (f(t + s) - f(t)) = \lambda s \) for all real \( s \). It is shown that \( f(t)/t \) need not have a limit as \( t \to \infty \); but that the assumption, in addition, of either measurability of \( f \) or boundedness of \( f \) on an appropriate set of intervals is adequate to assure that \( \lim_{t \to \infty} (f(t)/t) = \lambda \). (Received June 6, 1960.)

In this paper, closed-form, approximate formulas are obtained which describe the motion of a satellite of an oblate primary. An independent numerical integration suggests that the formulas maintain their validity for hundreds of revolutions, for typical initial conditions. This paper will appear in Proc. Roy. Soc. A. (Received June 6, 1960.)

571-36. P. T. Church: **Arc cluster sets of bounded analytic functions.**

Let $f$ be any map of the open unit disk $D$ into the sphere $S$, and let $\Gamma$ be an arc contained in $D$ except for an endpoint $p$ on the circle $\partial D$. The arc cluster set of $f$ on $\Gamma$, denoted by $C(f, \Gamma)$, is the set of points $y \in S$ such that there exists $z_n \in \Gamma$, $z_n \to p$, with $f(z_n) \to y$. **Theorem 1.** Given a continuum $C$ in the plane, there is a function $f(z)$, bounded and analytic on $D$ and a local homeomorphism (into) on the closed disk $\overline{D}$ except at $z = 1$, such that for every arc in $D$ approaching $z = 1$, the arc cluster set is $C$. **Theorem 2.** Given any continuum $C$ in the plane, there is a function $f(z)$, bounded and analytic on $D$ and a local homeomorphism (into) on $\overline{D}$ except at the points of a Cantor set on the circle $\partial D$, such that each arc cluster set at a point of the Cantor set is $C$. **Theorem 3.** The map $f$ can be chosen to be a homeomorphism (into) if and only if $C$ is the boundary of a simply connected region of $S$. The first two results are extensions of theorems of G. S. Young, Jr. (Notices Amer. Math. Soc. vol. 6 (1959) p. 868). (Received June 15, 1960.)

571-37. Eckford Cohen: **Unitary functions (mod r).**

For integers $n, r$ ($r > 0$), let $(n, r)_*$ denote the largest divisor $d$ of $n$ such that $d \delta = r$, $(d, \delta) = 1$. A function $f(n, r)$ is defined to be a unitary function of $n$ (mod $r$) if $f(n, r) = f((n, r)_*, r)$ for all $n$. This paper is concerned with the arithmetical and trigonometric inversion theory of the unitary functions (mod $r$). This class of functions forms a subclass of the even functions (mod $r$), a fact which is used in the development of the paper. The general theory developed is applied to obtain formulas for the number of compositions of $n$ (mod $r$), $n \equiv x_1 + \ldots + x_s$ (mod $r$), subject to the condition, $(x_i, r)_* = 1$, $i = 1, \ldots, s$. Solvability criteria for this congruence are readily deduced from the formulas obtained. Some of the concepts and results of the paper are restated in the terminology of abstract algebra. (Received June 13, 1960.)

Consider the bounded operator \( T = T_p \) generated in the sequence space \( \ell_p \) (1 \( \leq p \leq \infty \)) by a "multidiagonal" infinite matrix \( \| a_{ij} \|, 1 \leq i, j \leq \infty \), of complex numbers in which \( a_{i,i+k} = \beta_k, 0 \leq k \leq m \); \( a_{j,i+j} = \gamma_k, 1 \leq k \leq m \); and \( a_{ij} = 0 \), \(|i - j| > m\). Assume \( \beta_0 = 0 \) and \( \gamma_m \neq 0 \). The point and residual spectra of \( T \) are characterized by the number of roots of the polynomial \( f_T(z; \lambda) = \sum_{k=0}^{m-1} \gamma_m^{-k} z^k - \lambda z^m + \sum_{k=1}^{m} \beta_k z^{m+k} \) which satisfy \(|z| < 1\). Using the well-known convexity theorem of M. Riesz, it is further shown that the essential spectrum of \( T \) (which is the set of all \( \lambda \) such that the range of \( T - \lambda I \) is not closed) is contained in the union of a closed analytic curve and a finite set of points, both independent of \( p \). (Received June 6, 1960.)


A. Beurling has proved (Acta. Math. vol. 81 (1949) pp. 239-255) that the lattice of invariant subspaces of the shift operator \( S: (x_0, x_1, \ldots) \rightarrow (x_1, x_2, \ldots) \) in \( \ell_2 \) is isomorphic to the lattice \( \mathcal{F} \) of inner functions. Consider now, for fixed complex numbers \( \beta \neq 0 \) and \( \gamma \), the more general operator \( T: (x_0, x_1, \ldots) \rightarrow (y_0, y_1, \ldots) \), where \( y_0 = \beta x_1 \) and \( y_n = \gamma x_{n-1} + \beta x_{n+1}, n = 1, 2, \ldots \). By means of a conformal mapping technique, it is shown that the lattice of invariant subspaces of \( T \) is again isomorphic to \( \mathcal{F} \). (Received June 6, 1960.)


Let \( n \) be a positive integer and let \( \theta_n = [0,n,n,\ldots] = ((n^2 + 4)^{1/2} - n)/2; \) the notation \([a,b,c,\ldots]\) denotes the continued fraction expansion of an irrational number. For any positive integer \( m \) let \( c_m = \theta_n + n + P_{2m-1}/Q_{2m-1} \) where \( P_j/Q_j \) is the \( j \)th convergent to \( \theta_n \). Then if \( \theta = [a_0,a_1,a_2,\ldots] \) is such that \( a_j \neq n \) for infinitely many values of \( j \), we prove that there are at least \( m \) solutions in relatively prime integers \( p,q,r,\ldots \) to the inequality \(|\theta - p/q| \leq 1/(c_m q^2)\). Moreover, if \( \theta = \theta_n \), there are exactly \( m \) solutions and the equality sign is necessary. The basic ideas in the proof are essentially the same as A. V. Prasad, J. London Math. Soc. vol. 23 (1948) pp. 169-171, used for proving the special case of \( n = 1 \). Two corollaries concerning infinitely many approximations are (1) Theorem 9 of M. Müller, Arch. Math. vol. 6 (1955) pp. 253-258, and (2) if \( \theta \) satisfies the condition stated above but \( \theta \) is not equivalent to \( \theta_n \), then there
are infinitely many solutions to the inequality for any \( m \). (Received June 9, 1960.)

571-41. N. J. Fine and L. Gillman: Extension of continuous functions in \( \mathbb{N} \).

Let \( p \in \beta \mathbb{N} - \mathbb{N} \); assuming the continuum hypothesis \([\text{CH}]\), we show that \( \beta (\mathbb{N} - \mathbb{N} \setminus \{p\}) \neq \beta \mathbb{N} - \mathbb{N} \). (Notation and terminology are as in Gillman-Jerison, Rings of continuous functions.) This is a special case of Theorem 2 below. Theorem 1 \([\text{CH}]\). All open subsets of \( \beta \mathbb{R} - \mathbb{R} \) resp. \( \beta \mathbb{N} - \mathbb{N} \) are F-spaces resp. zero-dimensional F-spaces. Lemma. If \( Y \) is locally compact and real-compact, then each zero-set in \( \beta Y - Y \) is the closure of its interior. Theorem 2 \([\text{CH}]\). Let \( K = \beta Y - Y \), where \( Y \) is locally compact and \( \sigma \)-compact, and assume that \( K \) has just exp \( \mathcal{K}_0 \) zero-sets. (E.g., \( K = \beta \mathbb{R} - \mathbb{R} \) or \( \beta \mathbb{N} - \mathbb{N} \).) Then:

(a) No proper dense subset is C*-embedded (i.e., \( \beta X = K \) implies \( X = K \)).

(b) The following are equivalent for any open set \( S \): (i) \( S \) is C*-embedded in \( K \); (ii) \( S \) is a cozero-set; (iii) for all \( p \in K - S \), there exist a neighborhood \( V \) of \( p \) and a cozero-set \( H \subset S \) such that \( \text{int}(S \cap V - H) = \emptyset \). (c) If \( S \) is open but is not a cozero-set, then \( |\beta S - S| > \exp \exp \mathcal{K}_1 \). (d) If \( K \) is totally disconnected (e.g., \( K = \beta \mathbb{N} - \mathbb{N} \)), and if \( S \) is open but is not a cozero-set, then some two-valued function in \( C^*(S) \) has no continuous extension to all of \( K \). (Received June 22, 1960.)

571-42. M. K. Fort, Jr.: The limit of a sequence of homeomorphisms of \( S \) onto itself.

Let \( C \) be an arbitrary subset of the \( n \)-sphere \( S^n \). It is shown that there exists a sequence \( h_1, h_2, h_3, \ldots \) of homeomorphisms of \( S^n \) onto itself which converge pointwise to a function \( g \) for which \( g[S^n] = C \). This result gives an affirmative answer to a question raised by Borsuk and Ulam (see p. 46 of: S. M. Ulam, A collection of mathematical problems, Interscience Tracts in Pure and Applied Mathematics Number 8, New York, 1960). (Received June 7, 1960.)


Let a q-series transformation be defined by \( F(n) = q^n(n - 1)/2 \)

\[
\sum_{k=0}^{n} \binom{n}{k} q^{k} f(k) = \sum_{k=0}^{n} \left[ \frac{g(k)}{k} \right] q^{k} f(k) \text{.}
\]

Then if \( f(0) = 1 \), \( f(k) \) is independent of \( n \), and \( g(k) \) is a non-negative integer-valued function, it is shown that \( \sum_{n=0}^{\infty} (-1)^{n} F(n) u^n = \sum_{k=0}^{\infty} \left[ \frac{g(k)}{k} \right] u^k q^{(k-1)/2} f(k) \prod_{j=0}^{k-1} (1 - u q^j) \), where \( \binom{n}{k} \) is a q-binomial.
coefficient, defined by \[ \binom{n}{k} = \prod_{j=1}^{k} \frac{q^{n-j+1} - 1}{q^j - 1} \]. Consequences of this transformation are discussed, in particular in the case when \( q \to 1 \) the transformation implies certain convolution identities for binomial coefficients.

(Received June 15, 1960.)

571-44. Emil Grosswald: Imaginary quadratic fields of class number one.

Let \( S = \{3,4,7,8,11,19,43,67,163\} \). It is well-known that the nine imaginary quadratic fields of discriminant \( d = - k \), \( k \in S \) have class number \( h = 1 \) and that there can exist at most one more such field. If it exists, then its discriminant \( d \) satisfies \( d \leq - p \), \( p \equiv 3 \pmod{4} \), where \( p \) is a prime larger than \( 5 \times 10^9 \). For a long time, the existence of this tenth field of class number one has been considered highly unlikely. Here the (not surprising) result is proven that no such quadratic field exists, if certain \( L \)-series do not vanish close to \( s(= \sigma + it) = 1 \). More precisely, set \( L(s) = L_k(s, \chi) = \sum_{n=1}^{\infty} \frac{(- k/n)^n}{n^s} \); then it is shown that if \( k \equiv p \equiv 3 \pmod{4} \), \( p > 5 \times 10^9 \) imply that \( L(s) \neq 0 \) for \( \sigma > 1 - (30 \log k)^{-1} \) and \(|t| < 3 + (3 \log k)^{-1} \), then the tenth imaginary quadratic field of class number \( h = 1 \) does not exist. The method is a modification of Littlewood's estimate of the lower bound for \( L(1) \) (Proc. London Math. Soc. (2) vol. 27 (1928) pp. 358-372). (Received June 16, 1960.)

571-45. Frank Harary: The maximum connectivity of a graph.

In the second book on graph theory ever written, C. Berge, Théorie des graphes et ses applications, lists 14 unsolved problems, one of which is the following. "11. Quelle est la connexité maximum d'un graphe de \( n \) sommets et de \( m \) arêtes? L'intérêt de ce problème est analogue à celui de trouver le diamètre minimum d'un graphe." The connectivity of a graph \( G \) is the minimum number of points whose deletion results in a disconnected graph. (The graph with one point and no lines is here regarded as disconnected.) A \( p,q \) graph is one with \( p \) points and \( q \) lines. The problem cited asks for the maximum connectivity among all \( p,q \) graphs. The answer is \([2q/p]\) or 0 when \( q \geq p - 1 \) or \( q < p - 1 \) respectively. When \( q < p - 1 \), \( G \) is disconnected; when \( q = p - 1 \), \( G \) is a tree. The proof of the result for \( q > p - 1 \) is in two parts. First, the simple but useful lemma that the connectivity of \( G \) does not exceed the minimum degree of its points, establishes that \([2q/p]\) is an upper bound to the connectivity. Second, a collection of graphs is constructed which shows that this number is a
lower bound. The minimum connectivity of all $p,q$ graphs is also found, and is 0 or $q - (p - 1)(p - 2)/2$, whichever is larger. The minimum diameter is infinite if $q < p - 1$, it is 1 if $q = p(p - 1)/2$, and is 2 for all intermediate values $p - 1 \leq q < p(p - 1)/2$. (Received June 8, 1960.)


Let $X_1, X_2, \ldots$ be a sequence of integral valued independent random variables with for each $i$, $\Pr(X_i = k) = \Pr(X_i = -k) = c_k$ ($k$ integer) and for some $0 < \alpha \leq 2$, $0 < \lim_{r \to 0} t^\alpha (1 - E \exp itX_1) = Q < \infty$. Denote by $\|T(N)\|_{i,j}$ the matrix $\|c_{j-i}\|_{i,j} = 0,1, \ldots, N$. It is well known that one can derive properties of $(I - T(N))^{-1}$ from the properties of the process $S_N = \sum_{i=1}^{N} X_i$ and vice versa, cf. (1) F. L. Spitzer and C. J. Stone, A class of Toeplitz forms and their applications to probability theory, Illinois J. Math. (1960) and (2) Harry Kesten, On a theorem of Spitzer and Stone and random walks with absorbing barriers, Abstract 569-9, Notices Amer. Math. Soc. vol. 7 (1960) p. 259. In the present note the limiting behavior of $N^1 - \alpha (I - T(N))^{-1}$ is determined as well as the limiting behavior of $N^1 - \alpha/2 p_{N,k}$ for $1 < \alpha < 2$. The $p_{N,k}$ are the coefficients of the orthogonal polynomials corresponding to $1 - E \exp itX_1$. They satisfy $(I - T(N))^{-1} = \sum_{r=1}^{N} \max(k,j)p_{r,k}P_{r,j}$ (cf. (1)). The constants $D(\alpha,c)$ of (2) are found explicitly.

The results were found by using a recent result of R. K. Getoor concerning the absorption probabilities of a symmetric stable process with independent increments of index $\alpha$, in the presence of absorbing barriers (to be published). Conversely some results of this note are applied to the infinitesimal generators of such processes. (Received June 16, 1960.)


We verify that the basic implicit alternating-direction procedures for the numerical solution of parabolic partial differential equations are unconditionally $L^2$-stable, even when the coefficients are nonconstant and the underlying domain is nonrectangular. Our proof is based on certain energy inequalities for the difference equations. We also establish the convergence of several of the alternating-direction iterative methods for elliptic difference equations over nonrectangular domains. Again, the equations considered need not have constant coefficients. In the elliptic case, explicit estimates for the rate of convergence are obtained. (Received June 17, 1960.)
It is well-known that a (Hausdorff) space $X$, which is the image of an ordered continuum under a continuous mapping, is necessarily compact connected and locally connected. In this paper it is shown that $X$ must satisfy a fourth condition (independent of the three preceding ones), which is given by Theorem 1. Let $X$ be the image of an ordered continuum under a continuous mapping. Whenever $p: X \to Y$ is a mapping onto a (Hausdorff) space $Y$ and, for each $y \in Y$, $p^{-1}(y)$ is nowhere dense in $X$, then the weight of $Y$ equals the weight of $X$ (the weight of a space is the minimal cardinal number of a basis for the topology of the space). Among the consequences of this theorem one obtains Theorem 2. A product space $\prod X_\alpha$ of at least two nondegenerate continua $X_\alpha$ is the image of an ordered continuum if and only if each $X_\alpha$ is a (metric) Peano continuum and there are at most countably many factors. Theorem 3. Let $X$ be the image of an ordered continuum. If $X$ is $\mathcal{K}_r$-separable (i.e. has a dense subset of cardinality $\mathcal{K}_r$), then its weight $w(X)$ is at most $\mathcal{K}_r$. (Received June 6, 1960.)


The problem $(p(x)u_x)_x + \lambda^{-1}f(x,u) = 0$, $p(x) > 0$, $u(0) - ap(0)u_x(0) = 0$, $u(1) + bp(1)u_x(1) = 0$, $a,b > 0$, is studied in real $L_2(0,1)$. $f(x,u)$ is assumed to be odd and monotone increasing in $u$ and to possess derivatives $f_u f_{uu}$ where $f_u$ is monotone increasing, $u < 0$, monotone decreasing, $u > 0$. Moreover, uniformly in $0 \leq x \leq 1$, $|f(x,u_1) - f(x,u_2)| \leq K|u_1 - u_2|$, $f(x,u) = 0(|u|^\alpha)$ as $|u| \to \infty$ with $\alpha < 1$, and $\lim |u| \to 0 u^{-1}f(x,u) = f_u(x,0)$. Let $\{\mu_n\}$ be the set of eigenvalues for the linearized version of the problem, where $\lim_{n \to \infty} \mu_n = 0$, $\mu_n > 0$. The resolvent set, defined in the usual way, comprises the segments $\lambda < 0$, $\lambda > \mu_1$. The spectrum consists of the closed interval $(0, \mu_1]$. There exist precisely $n$ nontrivial continuous solutions for $\mu_{n+1} \leq \lambda \leq \mu_n$, $n = 1,2,\ldots$. These solutions branch off from the trivial solution at the bifurcation points $\frac{\mu_n}{\mu_1}$. A sequence of continuous "characteristic value functions" $\{r_n(\lambda)\}$ is defined on the spectrum such that $r_n(\lambda) \equiv 0$, $\mu_n \leq \lambda \leq \mu_1$, $r_n(\lambda) > 0$, $0 < \lambda < \mu_n$. These are generally decreasing with $\lambda$, becoming large as $\lambda \to 0$. $r_n(\lambda)$ gives the norm of the eigenfunction, which bifurcates at $\lambda = \mu_n$, as a function of $\lambda$. The set $\{r_n(\lambda)\}$ serves to illustrate the solution multiplicity pattern of the problem. Proof is by a synthesis of methods first exploited independently by I. Kolodner and M. A. Krasnoselskii. (Received June 13, 1960.)
A function with domain and range in $M^n_C$, the algebra of complex matrices, which admits all those automorphisms and antiautomorphisms of $M^n_C$ which leave the complex field elementwise invariant, is called *intrinsic*. Continuous intrinsic functions $F(X)$ on $M^n_C$ have previously been essentially characterized as $n$-ary functions, i.e. functions arising from a scalar function $f(\lambda_1, \sigma_1, ..., \sigma_{n-1})$ of $n$-complex variables in the following way. Let $f_A(A)$ denote the function of $\lambda$ only, $f(\lambda, \sigma_1[A], ..., \sigma_{n-1}[A])$, where $\sigma_i[A]$ denotes the $i$th elementary symmetric function of the eigenvalues of $A$. Then $F(A)$ is defined to be $f_A(A)$ as given by any of the equivalent classical definitions of the extension of a function of a single complex variable to $M^n_C$. The present paper derives criteria for the Hausdorff differentiability [c.f. W. O. Portmann, Proc. Amer. Math. Soc. vol. 10 (1959) pp. 101-105] of intrinsic functions on $M^n_C$ in terms of differentiability of the stem function $f(\lambda, \sigma_1, ..., \sigma_{n-1})$. An explicit formula for the Hausdorff derivative of an $n$-ary function is developed. The Hausdorff derivative is again an $n$-ary function on $M^n_C$, and the existence of the Hausdorff derivative on an open set of $M^n_C$ guarantees the existence of iterated Hausdorff derivatives of all orders on that set. (Received June 16, 1960.)


In a paper to appear in the Duke Math. J. (September, 1960) L. B. Treybig proves the following theorem. Hypothesis: $X$ is a connected metric space such that (1) no point separates $X$ and (2) if $p \in X$, $q \in X$, then in every neighborhood of $p$ there exists a compact continuum $M$ which separates $p$ from $q$. Conclusion: The topology of $X$ has a countable base. He raises the question as to whether or not the words "compact continuum $M$" in Hypothesis (2) may be replaced by "separable closed connected set $M$". The present paper answers this question in the affirmative. (Received June 13, 1960.)

WITHDRAWN.
571-53. Y. -C. Wong: Recurrent tensors on a differentiable manifold.

Let $M$ be a connected $C^\infty$ manifold of dimension $n$, and $B$ the total space of the bundle of frames over $M$. Let a linear connection be given on $M$, and let $B_0$ be the submanifold of $B$ generated by the points which can be joined to a given point in $B$ by sectionally smooth horizontal curves. The following results are proved: Theorem 1. A correspondence can be set up in a natural way between a tensor $S$ of type $(r,s)$ on $M$ and a set of $nr+s$ functions of a particular type on $B$. A tensor $S$ on $M$ is recurrent (i.e., its covariant derivative is equal to the tensor product of a covariant vector and $S$ itself) iff the restrictions to $B_0$ of its corresponding functions on $B$ are proportional to a set of constants. Corollary. A recurrent tensor on $M$ is either a zero tensor or has no zero.

Theorem 2. A linear connection on $M$ is of recurrent curvature (i.e., its curvature tensor is recurrent) iff the Lie product of every two basic vector fields on $B$ is such that the restriction to $B_0$ of its vertical component is co-directional with a fundamental vector field. A corresponding property holds for a linear connection with recurrent torsion. A result on covariantly constant tensors on $M$ similar to Theorem 1 leads easily to a theorem of K. Nomizu (Lie groups and differential geometry, p. 73) similar to Theorem 2 above. (Received June 6, 1960.)


A rough description of what is meant by a differential game is given in the introduction of Contributions to the theory of games, vol. III, Princeton University Press, Princeton, 1957, p. 12. The author presents a variational approach to a class of differential games having pure strategy solutions by relating such games to two Bolza problems with differential inequalities as added side conditions. Necessary conditions that must hold along an optimal path are derived from the theory of the related Bolza problems. These con-
ditions are (i) a multiplier rule, together with transversality conditions and jump, or corner, conditions, (ii) a local min-max condition that is related to the Weierstrass condition, and (iii) an analogue of the Clebsch condition. The continuity and differentiability properties of the value of the game are derived, and it is shown that wherever the value is differentiable, it satisfies an analogue of the Hamilton-Jacobi equation. Sufficient conditions are given in terms of the notion of a field and a local min-max condition. A method that, in principle, permits the construction of optimal strategies is given. (Received June 27, 1960.)


Dirichlet's problem for the half-space calls for a solution $\psi(x,y,z)$ of the initial value problem $\Delta \psi = 0 \ (-\infty < x,y < +\infty, z > 0), \psi(x,y,0) = f(x,y)$, the initial condition being satisfied in the form $\lim_{z \to 0} ||\psi(x,y,z) - f(x,y)||_1 = 0$, where $f(x,y)$ is a given function of the real variables $x,y$ and belongs to $L_1$ over the whole 2-dimensional plane $E_2$. The solution is given by $\psi_S(x,y,z) = (z/2\pi) \int_{E_2} f(x + u,y + v)[z^2 + u^2 + v^2]^{-3/2} dudv$. Let $f(\xi,\eta)$ be the double Fourier transform of $f(x,y)$ and $\tilde{g}(\xi,\eta)$ the Fourier-Stieltjes transform of $g(x,y)$, where $g$ is of bounded variation over $E_2$ (in sense of Hildebrandt and Schoenberg, Ann. of Math. (2) vol. 34 pp. 317-328 (1933)). Theorem: Let $f \in L_1(E_2)$. (i) If $||\psi(x,y,z) - f(x,y)||_1 = o(z) (z \downarrow 0)$, then $f(x,y) = 0$ almost everywhere in the $E_2$-plane. (ii) If $||\psi(x,y,z) - f(x,y)||_1 = O(z) (z \downarrow 0)$, then there exists a function $g(x,y)$ of bounded variation over the $E_2$-plane such that $(\xi^2 + \eta^2)^{1/2}f(\xi,\eta) = \tilde{g}(\xi,\eta)$ throughout the $(\xi,\eta)$-plane. The method of proof is based upon a Fourier transform method developed by the author (Arch. Rational Mech. Analysis, vol. 5 in print). The method is also applicable to the solution of the three-dimensional diffusion equation for an infinite solid. (Received June 23, 1960.)


E. C. Titchmarsh, G. H. Hardy and J. E. Littlewood established several results on Lipschitz classes in the $L_p$-metric for periodic functions (see Zygmund, Trigonometrical series, vol. I, p. 180). In this paper, corresponding results are shown for (nonperiodic) functions $f$ defined over $(-\infty,\infty)$ with
$f \in L_p(-\infty, \infty), 1 \leq p \leq 2$. Let $\|f\|_p = \left[ \int_{-\infty}^{\infty} |f(x)|^p \, dx \right]^{1/p}, D_p(f) = \|f(x + 2h) + f(x - 2h) - 2f(x)\|_p$, let $\hat{f}(v), \hat{g}(v)$ be the Fourier and Fourier-Stieltjes transforms of $f$ and $g$, respectively. **Theorem 1:** If $f \in L_1(-\infty, \infty)$ and $D_1(f) = o(h^2)(h \to 0)$, then $f$ is equal to the null-function in $(-\infty, \infty)$. The following two statements are equivalent: (i) $D_1(f) = O(h^2)(h \to 0)$; (ii) there is a function $g$ of bounded variation on $(-\infty, \infty)$ such that $\hat{g}(v) = v^2 \hat{f}(v)$, all $v$.

**Theorem 2:** If $f \in L_p(-\infty, \infty), 1 < p \leq 2$ and $D_p(f) = o(h^2)(h \to 0)$, then $f$ is equal to the null-function. The following three statements are equivalent: (i) $D_p(f) = O(h^2)(h \to 0)$; (ii) $v^2 \hat{f}(v)$ is the Fourier transform of a function $g \in L_p(-\infty, \infty)$; (iii) $f$ and $f'$ are absolutely continuous and $f'' \in L_p(-\infty, \infty)$. The method of proof is based upon a Fourier transform method developed by the author (Arch. Rational Mech. Analysis, vol. 5 in print). These results may also be stated for higher central as well as forward differences. (Received June 23, 1960.)

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Let $f(x) = \sum_{n=1}^{\infty} a_n x^n / n! = \sum_{k=1}^{\infty} A_k (e^x - 1)^k / k!$, where the $a_n$ are rational integers. It is proved that the $a_n$ satisfy \*$(*)$ \*$\sum_{s=0}^{T}(-1)^{s}S(T)^{s}_{s}a_{n+s}(p-1) \equiv 0 \pmod{p^r}$ (n $\geq$ r) if and only if the $A_k$ satisfy $A_k \equiv 0 \pmod{p^{k/p}}$, where $p$ is an arbitrary prime. More generally if $g(x) = \sum_{n=1}^{\infty} c_n x^n / n! = \sum_{k=1}^{\infty} C_k f^k(x) / k!$, where the $c_n$ are integral and $f(x)$ is a fixed series whose coefficients satisfy \*$(*$), then the $c_n$ satisfy $\sum_{s=0}^{T}(-1)^{s}S(T)^{s}_{s}c_{n+s}(p-1) \equiv 0 \pmod{p^r}$ (n $\geq$ r) if and only if the $C_k$ satisfy $C_k \equiv 0 \pmod{p^{k/p}}$. (Received June 24, 1960.)

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571-58. L. Carlitz: Some arithmetic properties of the lemniscate coefficients.

In a previous paper (Math. Z., vol. 72 (1960) pp. 307-318) the writer obtained congruences \$(\pmod{2^r})$ for the coefficients of the Jacobi elliptic function. In the present paper it is shown that these results can be improved considerably for the special case of the lemniscate function. For example, if $\phi(x) = sn(x,-1)$ \*$= \sum_{n=0}^{\infty} a_{4n+1} x^{4n+1} / (4n+1)!$ \*$\text{then} a_{4n+1} = 2^{2n} c_n$, \*$\text{where the} c_n \text{are odd integers} \text{that satisfy} \sum_{s=0}^{T}(-1)^{s}S(T)^{s}_{s}c_{n+s+w} \equiv 0 \pmod{2^{e-1}}$, \*$\text{where} 2^{e-1}\mid w \text{and} e = \text{re for} e > 1 \text{, while for} e = 1, \text{e = r or r + 1 according as r is even or odd}$. Again if $x/\phi(x) = \sum_{n=0}^{\infty} F_n x^{4n} / (4n)! \text{then} \sum_{s=0}^{T}(-1)^{s}S(T)^{s}_{s}2^{2n+2s+w}F_{n+s+w} \equiv u \pmod{2^{re}}$ \*$\text{where} 2^{e-1}\mid w$. (Received June 2, 1960.)
571-59. L. Carlitz: *Some orthogonal polynomials related to elliptic functions II. Arithmetic properties.*

In a previous paper (*Some orthogonal polynomials related to elliptic functions*, to appear in Duke Math. J.) the writer defined four sets of orthogonal polynomials by making use of the continued fraction expansions (due to Stieltjes and Rogers) of the definite integrals
\[ \int_0^{\infty} \text{sn}(u, k^2) e^{-zu} du, \text{sn}^2(u, k^2) e^{-zu} du, \]
\[ \int_0^{\infty} \text{cn}(u, k^2) e^{-zu} du, \int_0^{\infty} \text{dn}(u, k^2) e^{-zu} du. \]
The polynomials in question all have integral coefficients so that it is of interest to look for arithmetic properties.

It was found previously that the polynomials satisfy congruences of the form
\[ \sum_{s=0}^{r} (-1)^{r-s} f_{n+m}(x)f_{m} \equiv 0 \pmod{m^r}, \]
where \( m \) is arbitrary and \( r_1 = \left\lceil \frac{r+1}{2} \right\rceil \). It is now shown first that (*) can be replaced by the simpler congruence
\[ \sum_{s=0}^{r} (-1)^{r-s} f_{n+m}(x)f_{m} \equiv 0 \pmod{m^r}. \]
Moreover it is found that
\[ f_{p}(x) = x(x^{m} - (-1)^{m} W_{m}(k^2))^{2} \pmod{p}, \]
where the prime \( p = 2m + 1 \) and
\[ W_{m}(k^2) = \sum_{r=0}^{m} \frac{m^{2r}}{r!}. \]
It follows that \( \sum_{s=0}^{r} (-1)^{r-s} f_{n+sM}(x) \cdot \left\{ x(x^{m} - (-1)^{m} W_{m}(k^2))^{2} \right\} \equiv 0 \pmod{p^rM}, \)
where \( p^r | M \). The case \( p = 2 \) requires separate treatment. (Received June 24, 1960.)

571-60. P. H. Doyle and J. G. Hocking: *A characterization of the n-sphere.*

**Theorem 1.** Let \( M \) be an n-manifold and suppose there is a point \( p \in M \) such that, for every neighborhood \( U \) of \( p \), there exists a homeomorphism \( h \) of \( M \) onto itself such \( h(M - U) \subseteq U \). Then \( M \) is an n-sphere. The proof involves the application of recent results of Morton Brown to show that the manifold is the union of two n-cells whose intersection is their common boundary. (The manifold need not be assumed to be compact.) Indeed, the proof permits us to replace the n-manifold by a space which is simply locally Euclidean at the point \( p \). As a simple application of this theorem, we have **Theorem 2.** Let \( M \) be a noncompact n-manifold and suppose there is a point \( p \in M \) such that, for every neighborhood \( U \) of \( p \), there is a homeomorphism \( h \) of \( M - p \) onto itself such that \( h(M - U) \subseteq U - p \). Then \( M \) is homeomorphic to Euclidean n-space. (Received June 24, 1960.)

571-61. Peter Hagis, Jr.: *A problem on partitions with a prime modulus p greater than or equal to 3.*

Let \( p \geq 3 \) be a prime, and let \( a = \{ a_1, a_2, \ldots, a_r \} \), where \( 1 \leq a_i \leq (p - 1)/2, \)

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be a set of $r$ distinct integers. This paper is concerned with the problem of determining $p_a(n)$, the number of partitions of a positive integer $n$ into summands congruent to elements of $a$ or their negatives modulo $p$. This is a generalization of the problem first considered by Lehner, who took $p = 5$ and $r = 1$, and later studied by Livingood, who took $p > 3$ and $r = 1$. More recently both Petersson and Grosswald have also worked in this problem area. In the present paper the circle dissection method of Hardy and Rademacher is utilized, and a convergent series for $p_a(n)$ is obtained. The circle dissection method is also employed to obtain a convergent series for the number of partitions of $n$ where summands congruent to $p$ as well as $\pm a_i$ are permissible. The terms in both series involve certain finite sums of roots of unity and Bessel functions. As these series are extremely complicated, asymptotic formulas are also derived. Finally, some special cases are considered and in particular Rademacher's formula for $p(n)$, the number of unrestricted partitions of $n$, is obtained. (Received June 29, 1960.)


To reverse a diffusion process requires solving a heat equation with a negative coefficient of diffusivity (i.e. the backward heat equation). This equation presents a problem of instability which is classical. Namely, arbitrarily small changes in initial values can lead to arbitrarily large changes in final values. This state of affairs is summarized by the statement that the backward heat equation is not a well posed problem in the sense of Hadamard. In this paper we show that if the values of a diffusion process have bounded spectra, then the backward heat equation is a well posed problem and a convergent and stable iteration scheme for solving it is given. If the values of a diffusion process do not have this property then our iterations will not converge. But the projections of the iterates into any subspace with a fixed bounded spectrum will converge. Thus the backward heat equation is decomposed into a well posed part and a complementary improperly posed part. The methods introduced are applicable to wide classes of improperly posed problems. (Received June 24, 1960.)

571-63. J. A. Morrison: Bounds on the nonlinear diffusion controlled growth rate of spherical precipitates.

A nonlinear diffusion problem is considered in which there is a change of
Phase, the nonlinear diffusion coefficient \( D = D_0 g(c/c_0) \) being a function of the local concentration of solute. A radially symmetric precipitate which grows at a rate proportional to the square root of the time is considered in n-dimensions and the radius of the precipitate is \( r = s(D_0 t)^{1/2} \). It is supposed that \( g(v) > 0, 0 \leq v \leq 1 \). The diffusion equation is transformed into the ordinary differential equation: 
\[
 s^2 F'' + 2ng(v)(F')^2(1-1/n) = 0, 0 < v < 1,
\]
with boundary conditions \( F(0) = -1/\lambda, F'(0) = 1, F(1) = 0 \). Here \( \lambda = k c_0 \) where \( k^{-1} \) is the density of the precipitate, and \( 0 < \lambda < 1 \). A comparison theorem is established, stating: \( \lambda_1 \leq \lambda_2 \) and \( g_1(v) \geq g_2(v) \), \( 0 \leq v \leq 1 \), \( \Rightarrow s_1 \leq s_2 \). Uniqueness follows and the comparison theorem also provides bounds on \( s \), since \( s \) may be calculated when \( g(v) = \text{const.} \ (1 - \alpha v)^{-1} \). Other bounds on \( s \), involving functionals of \( g(v) \), are obtained. For example, if \( n = 1, \int_0^1 (1 - v)(1 - \lambda v)^{-1} g(v) dv \leq s^2 (1 - \lambda)/(2 \lambda^2) \leq \min \left[ \int_0^1 (1 - v)(1 - \lambda v)^{-1} g(v) dv, \int_0^1 g(v) dv \right] \). (Received June 21, 1960.)

571-64. H. O. Pollak: Energy distribution of band-limited functions whose samples on a half line vanish.

Let \( S \) be the class of functions \( f(t) \) such that (i) \( f(t) \in L^2(-\infty, \infty) \); (ii) \( f(t) = \int_{-\infty}^{\infty} F(x)e^{i xt} dx \), (iii) \( f(n) = 0 \) for \( n = 1, 2, 3, \ldots \). Let \( \Phi(N) = \sup_{f \in S} \int_{-\infty}^{\infty} f^2(t) dt \). It is shown, among other results, that \( \Phi(N) = 1/2 \) if \( N \geq 3/4 \), that \( \Phi(N) > 1/2 \) if \( N \leq 1/4 \), and that \( \Phi(N) < 1 \) for all real \( N \). Further, let \( S_m \) be the subclass of \( S \) of functions \( f(t) \) for which \( f(n) = 0 \) for \( n = -M, -M - 1, \ldots \), as well, and let \( \Phi_M(N) \) be the corresponding supremum. Various bounds for \( \Phi_M(N) \) are given; it follows in particular that if \( M/N \) is large, then \( \Phi_M(N) \) is close to 1/2. (Received June 24, 1960.)


A set of sentences \( H \) of an applied first order functional calculus with equality is called convex if the intersection of every two submodels of a model (of \( H \)) is either a model (of \( H \)) or empty. Sets of axioms for algebraic systems are usually convex; we show that in a sense the converse is also true and models of convex sets of axioms are, essentially, algebraic systems with respect to certain definable operations. Let \( N(k,y_1,\ldots,y_n,P(x_1,\ldots,x_r,y_1,\ldots,y_n)) \) be the first order formula saying that there are exactly \( k \) different \( n \)-tupels \( y_1,\ldots,y_n \) satisfying the formula \( P(x_1,\ldots,x_r,y_1,\ldots,y_n) \). Theorem: A necessary and sufficient condition for \( H \) to be convex is that for every sentence of the form \( \text{AE} \)
(say $\bigwedge_x \bigvee_y A(x,y)$) which is a consequence of $H$, there exist two sequences $C_1(x,u), \ldots, C_m(x,u), R_1(x,y,z), \ldots, R_m(x,y,z)$ of formulas not containing quantifiers ($u$ and $z$ stand for sequences of individual variables) and there exists a sequence $k_1, \ldots, k_m$ of positive integers such that (a) $\vdash_H \bigwedge_x [\bigwedge_u C_1(x,u) \lor \ldots \lor \bigwedge_u C_m(x,u)];$
(b) for $1 \leq i \leq m$, $\vdash_H \bigwedge_x [\bigwedge_u C_i(x,u)] \rightarrow N(k_i, y, \bigvee_x \bigvee_z [A(x,y) \lor R_i(x,y,z)]).$ In virtue of (b) the Skolem function corresponding to $\bigvee_y$ in $\bigwedge_x \bigvee_y A(x,y)$ becomes, upon adding the condition $R_i(x,y,z)$, a $k_i$ valued function of $x$. From our theorem it follows that the axioms of a convex set can be transformed into statements ensuring the existence of certain finitely valued functions (generalized algebraic operations). (Received June 24, 1960.)

571-66. WITHDRAWN

571-67. Burton Wendroff: **Orthogonal polynomials and polynomial approximation.**

Let the polynomials $P_k(x)$ satisfy the following: $P_k(x) = (x - a_k)P_{k-1}(x)$
- $\lambda_k P_{k-2}(x)$, $k \geq 1$, where $P_0 = 1$, $P_{-1} = 0$, and $a_k$ is real, $\lambda_k > 0$. By a theorem of Favard the $P_k$ are orthogonal on some interval with respect to some distribution. If the zeroes of all the $P_k$ form a bounded set Favard's theorem can be strengthened as follows: let $(a,b)$ contain all the zeroes of all the $P_k$ in its interior. Then the $P_k$ are orthogonal on $(a,b)$ with respect to some distribution. By applying this result it is shown that if $a < x_1 < \ldots < x_n < b$ there exists a sequence of polynomials $Q_k$ (where the degree of $Q_k$ is $k$) orthogonal on $(a,b)$ such that $Q_n(x) = (x - x_1) \ldots (x - x_n)$. Finally, if $f(x)$ is continuous on $(a,b)$ and $P$ is a real polynomial of degree $n$ such that $f - P$ has at least $n + 1$ changes of sign in $(a,b)$ then $\int_a^b (f - P)^2 \, dw$ is a minimum over all real polynomials of degree $n$ for some distribution $w$. (Received June 24, 1960.)
Let $X_1, \ldots, X_N$ be independent and identically distributed normal variables with mean $\mu > 0$ and variance 1. Let $Z = (Z_1, \ldots, Z_N)$ be a random vector with $Z_i = 1(0)$ if the $i$th smallest in absolute value of $X_1, \ldots, X_N$ is positive (negative).

I. R. Savage (Ann. Math. Statist. vol. 30 (1959) pp. 1018 - 1023) obtains a simple ordering of the probabilities of the $2^N$ possible values of $Z$ for $N = 3$ and a partial ordering for $N = 4$. In this paper the pair $z = (1001)$ and $z' = (0110)$ is considered and it is shown that, for $\mu > 0$, $\operatorname{Prob}(Z = z) > \operatorname{Prob}(Z = z')$. The method of proof is similar to that of Savage's Theorem 6.1 (op. cit., p. 1022).

(Received June 27, 1960.)


Let $A^m$ be the set of all complex-valued functions $f$ on $\mathcal{M}(A)$, the maximal regular ideal space of a commutative semi-simple Banach algebra $A$, such that $\mathcal{M}^*(A) \subseteq \mathcal{M}$. Each $f \in A^m$ is bounded and continuous and induces a bounded operator $f$ on $A$. [See Helgason, Ann. of Math. vol. 64 (1956) pp. 240-254.] $\mathcal{M}^m = \{f : f \in A^m\}$ and $A_0^m = \{f : f \in C_0(\mathcal{M}(A)) \cap A^m\}$ are studied. Both are commutative semi-simple Banach algebras (called algebras of multipliers). $\mathcal{M}(A)$ is homeomorphically embedded in $\mathcal{M}(A^m)$ and so in $\mathcal{M}(A_0^m)$ as an open set and as an open-and-closed set, respectively, both in the weak$^*$ topologies. Various characterizations of the operators in $\mathcal{M}^m$ are given and used to determine possible new multiplications on $A$ leaving $\mathcal{M}(A)$ unchanged. If $A$ is sup-normed, then $A_0^m(A^m)$ is sup-normed. If $\mathcal{M}(A) \neq \mathcal{M}(A^m)$, then $A$ is not regular, whereas if $A_0^m$ is regular, so is $A$. $f \in A_0^m$ is shown to maximize on the Silov boundary $B$ of $A$ in $\mathcal{M}(A)$. Using a result of Arens and Singer [Proc. Amer. Math. Soc. vol. 5 (1954) p. 740], values of multipliers $(A_0^m)$ at points off $B$ are represented as boundary integrals. Assuming that $\mathcal{M}^m(A_0)$ is dense in $C_0^R(B)$, it follows that, if $\mathcal{M}(A) \neq \mathcal{M}(A_0^m)$, there exists $f \in A_0^m$ such that $f$ maximizes on $\mathcal{M}(A_0^m) \setminus \mathcal{M}(A)$. These results are used to compare $\mathcal{M}(A), \mathcal{M}(A_0^m)$ and $\mathcal{M}(A^m)$.

(Received July 5, 1960.)


Theorem 1. Suppose that $T$ is a linear self-adjoint operator with norm less than or equal to one in $L_2$ of a positive measure space. If $x$ is in $L_2$ and $\lim \sum T^{2k}x/2^n$ (the sum is from 1 to $2^n$) exists almost everywhere, then
\[ \lim T^{2n}x = Qx \text{ almost everywhere, } Q \text{ being the strong limit of } \{T^{2n}\}. \]

**Corollary.** If \( T \) is a linear operator satisfying the conditions of the Dunford-Schwartz ergodic theorem (J. Rational Mech. Anal. vol. 5 (1956) p. 143) and, in addition, is self-adjoint in \( L_2 \), then, for each \( x \) in \( L_2 \), \( \lim T^{2n}x = Qx \text{ almost everywhere.} \)

**Application.** Let \((W,F,P)\) be a probability space. Let \( F_n \) be a sub-\( \sigma \)-field of \( F \), \( \overline{F_n} \) the smallest \( \sigma \)-field containing \( F_n \) and the collection of all sets \( A \) in \( F \) satisfying \( P(A) = 0 \), and \( T_n = E(\cdot | F_n) \). Let \( S_n = T_n \cdots T_2 T_1 \).

**Theorem 2.** If \( T_{2n-1} = T_1 \) and \( T_{2n} = T_2 \) for all \( n \), then, for each \( x \) in \( L_2(W,F,P) \), \( \lim S_nx = E(x|\overline{F_1} \cap \overline{F_2}) \text{ almost everywhere.} \)

The main part of the proof consists of observing that \( T = T_1 T_2 T_1 \) satisfies the conditions, hence the conclusion, of the above corollary. Related results are obtained. (Received July 5, 1960.)

571-71. Eckford Cohen: Almost even functions of finite abelian groups.

In his Eratosthenian averages, Wintner developed a Fourier theory of almost periodic arithmetical functions based on the Besicovitch theory of real functions. Like the later development of Delsarte, Wintner's theory depended upon additive characters and periodicity ideas. It is the purpose of this paper to reformulate the Wintner-Delsarte theory in a strictly arithmetical form, independent of the additivity of the integers. The almost periodicity property is replaced by the notion of almost parity. A function \( f(n) \) is defined to be almost even (B) if there exists a sequence of even functions \( f_k(n) \) of order \( \leq k \) such that \( \lim_{k \to \infty} \lim_{x \to \infty} 1/x \sum_{n \leq x} |f(n) - f_k(n)| = 0 \). (A function is said to be even of order \( \leq k \) if it is a sum of even functions (mod \( r \), \( r \geq k \).) In order to emphasize the essentially multiplicative nature of the theory, the domain of the paper is taken to be the multiplicative semigroup of the finite abelian groups. The results proved include analogues of Wintner's main results (ibid., \( \S 33 \) and 35). The methods of the paper are elementary. (Received June 22, 1960.)


Let \( S^2 \) be a polyhedral 2-sphere in 4-sphere. If the 1st elementary ideal of the Alexander matrix of \( S^2 \) in \( S^4 \) is principal, a generator \( \Delta(t) \) of this ideal is called the Alexander polynomial of \( S^2 \) in \( S^4 \). It is easy to see that \( |\Delta(1)| = 1 \). The following is proved: If \( f(t) \) is a polynomial with \( |f(1)| = 1 \), then there exists a 2-sphere in 4-sphere whose Alexander polynomial is equal to \( f(t) \). This can be also generalized to the case of \( S^n \) in \( S^{n+2} \) (\( n \geq 2 \)). **Remark:** In the
case of a knot the 1st elementary ideal of the Alexander matrix is always prin-
cipal and the Alexander polynomial \( \Delta(t) \) has been characterized by (i) \( |\Delta(1)| = 1 \)
and (ii) the coefficients of \( \Delta(t) \) are symmetric. (Received July 5, 1960.)

571-73. Joseph Lehner: Note on the metric transitivity of Fuchsian
groups.

metric transitivity of all (finitely generated) Fuchsian groups of the first kind
acting on the unit disk whose fundamental regions do not touch the boundary of
the disk. In this note it is shown how Seidel's proof can be extended to the case
of Fuchsian groups whose fundamental regions do touch the unit circle. The
extension is made possible by a geometrical lemma essentially due to Hedlund
(Duke Math. J., vol. 2 (1936) p. 538). Lemma. If \( a \) is a point on the unit circle
which is not a parabolic vertex of \( \Gamma \) and \( \lambda_a \) is the radius terminating at \( a \), there
is a constant \( r(0 < r < 1) \), depending only on \( \Gamma \), with the following property:
on \( \lambda_a \) there is a sequence of points \( z_i \rightarrow a \) having \( \Gamma \)-images \( z_i' \) all lying in the
disk \( |z| \leq r \). (Received July 5, 1960.)

571-74. J. R. Rice: Linear interpolating functions and best \( L_p \)
approximations.

Let \( P(A,x) = \sum_{i=1}^{n} a_i \phi_i(x) \) where the \( \phi_i(x) \) form a Tchebycheff set. Let a
real finite point set \( X \) and a function \( f(x), x \in X \) be given. \( P(A,x) \) is a strongly
interpolating function of \( f(x) \) on \( X \) if \( P(A,x)-f(x) \) has \( n \) strong sign changes on \( X \).
P(\( A,x \)) is a weakly interpolating function if \( P(A,x)-f(x) \) has \( n \) weak sign changes
p. 1525 for definition of weak and strong sign changes. \( P(A,x) \) is an exactly
interpolating function if \( P(A,x)-f(x) \) has \( n \) zeros in \( X \). Theorem: Let \( P(A,x),X, 
f(x) \) and \( 0 < q < 1 < p < \infty \) be given. Then we have the following pairs of identi-
cal sets: (1) \( \{ A | P(A,x)-f(x) \text{ minimizes a weighted } L_p \text{ norm}\} \), \( \{ A | P(A,x) \text{ strongly }
interpolates } f(x) \} \) (2) \( \{ A | P(A,x)-f(x) \text{ minimizes a weighted } L_1 \text{ norm}\} \),
\( \{ A | P(A,x) \text{ weakly interpolates } f(x) \} \) (3) \( \{ A | P(A,x)-f(x) \text{ minimizes a weighted } 
L_q \text{ norm}\} \), \( \{ A | P(A,x) \text{ exactly interpolates } f(x) \} \). (Received June 20, 1960.)

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571-75. J. R. Rice: Interpolating function and best Tchebycheff approximations.

Let \( F(A,x) \) be a continuous function for \( x \in [0,1] \) depending on a variable number \( m(A) \) of parameter \( A \) from the domain \( P \). \( F(A,x) \) is said to be varisolvent if it is a function unisolvent of variable degree [Notices Amer. Math. Soc. Abstract 560-40 vol. 6 (1959) p. 557]. \( F(A,x) \) interpolates \( f(x) \) strongly on a set \( S \) if \( F(A,x) - f(x) \) has \( m(A) \) strong sign changes on \( S \). Theorem 1. Let \( X \) be finite subset of \([0,1]\) and let \( f(x) \) be defined on \( X \). Then the following subsets of \( P \) are identical: \{ \( A \mid F(A,x) \) strongly interpolates \( f(x) \) on \( X \) \}, \{ \( A \mid F(A,x) \) is the best approximation to \( f(x) \) on \( X \) in a weighted Tchebycheff norm \}. Theorem 2. Let \( f(x) \) be continuous on \([0,1]\). Then the following subsets of \( P \) are identical: \{ \( A \mid F(A,x) \) strongly interpolates \( f(x) \) on \([0,1]\) \}, \{ \( A \mid F(A,x) \) is the best approximation to \( f(x) \) on \([0,1]\) in a weighted Tchebycheff norm \}. (Received June 20, 1960.)

571-76. J. R. Rice: Interpolating functions and best approximations on \([0,1]\).

The norm \( \delta \) is monotonically, i.e., given \( g(x), A = \{ x \mid g(x) \leq k \max |g(x)|, k < 1 \} \) and \( |h(x)| = |g(x)| \) for \( x \notin A \) and \( |h(x)| < |g(x)| \) for \( x \in A \) then \( \delta(h(x)) < \delta(g(x)) \). Let \( g_n(x) \to g(x) \) with \( 0 < g_n(x)/g(x) \leq 1 \) for \( x \in R_n = \{ x \mid |g(x)| \leq r(n) \} \), \( r(n) \to 0 \). Let \( Y \) be a closed subinterval of \([0,1]\) with \( 0 < g_n(x)/g(x) < 1 \) and if \( g(x_0) = \max |g(x)| \) then \( x_0 \in Y \). Set \( s_n(x) = g_n(x), x \in Y; s_n(x) = g(x), x \notin Y; t_n(x) = g_n(x), x \in R_n; t_n(x) = g(x), x \notin R_n \). \( \delta \) is a CLASS 2 norm if (*) \( \delta(t_n(x) - g(x)) / [\delta(s_n(x)) - \delta(g(x))] \to 0 \) for all \( g(x), g_n(x), \) etc. \( \delta \) is a CLASS 1 norm if \( \text{meas}(R_n) \to 0 \) implies (*). Let \( f_n(x) \to f(x) \) and set \( e_n = \max |f(x) - f_n(x)| \). \( Z = \text{closure of } \{ x \mid f(x) = f_n(x) \text{ for some } n \} \). Let \( I \) be a closed interval with \( I \cap Z \) empty. \( \{ f_n(x) \} \) converges regularly to \( f(x) \) if \( \min_{x \in I} |f(x) - f_n(x)| \geq k e_n \). \( F \) is regular if \( F(A_n,x) \to F(A,x) \) implies the convergence is regular. Let \( F \) be a regular varisolvent function. Theorem 1. If \( \delta \) is Class 2 and \( F(A^*,x) \) is the best approximation to \( f(x) \) on \([0,1]\) then \( F(A^*,x) \) strongly interpolates \( f(x) \).

Theorem 2. If \( \delta \) is Class 1 and \( F(A^*,x) \) is a best approximation to \( f(x) \) on \([0,1]\) then either \( F(A^*,x) - f(x) \) vanishes identically on a set of positive measure or \( F(A^*,x) \) weakly interpolates \( f(x) \). (Received June 20, 1960.)
Y.-C. Wong and Kentaro Yano: Projectively flat spaces with recurrent curvature.

Let $A_N$ be an affinely connected $N$-dimensional space with a symmetric connection. $A_N$ is projectively flat if there exists a coordinate system in terms of which the finite equations of the paths are linear. $A_N$ is of recurrent curvature if the covariant derivative of its curvature tensor is the tensor product of a nonzero covariant vector and the curvature tensor itself. Denoting by $P^*_N$ a projectively flat space with recurrent curvature, we prove that $P^*_N$ depends on 2 arbitrary functions of one variable or on 3 arbitrary functions of one variable according as its Ricci tensor is symmetric or not. The actual construction of the connection of the $P^*_N$ depends on the solution of a differential equation of the Riccati type and on the solution of a completely integrable system of differential equations. Projectively flat space with covariantly constant curvature tensor is also considered. We prove that it is a well-known type of projectively flat space characterized by its Ricci tensor being symmetric and covariantly constant. (Received June 20, 1960.)

H. T. Muhly: Normal ideals in local domains.

Let $(\mathcal{O}, \mathfrak{m})$ be an equicharacteristic analytically irreducible local domain that is such that its completion $\mathcal{O}$ is separably generated over a coefficient field $k$. It is assumed that both $\mathcal{O}$ and $\mathfrak{m}$ are integrally closed in their respective quotient fields. An $\mathcal{O}$-ideal $\mathfrak{a}$ is called normal if $\mathfrak{a}^r$ is integrally closed for each positive integer $r$. It is shown that for each $\mathcal{O}$-ideal $\mathfrak{a}$ there is an integer $g$ such that the integral closure of $\mathfrak{a}^g$ is a normal ideal. If $\mathfrak{a}$ is $\mathfrak{m}$-primary, if $\mathfrak{a}$ is superficial of degree $t$ relative to $\mathfrak{m}$, and if $a_1, a_2, \ldots, a_n$ is a basis for $\mathfrak{m}$, let $\mathcal{O}[\mathfrak{m} ; a^{-1}]$ denote the ring $\mathcal{O}[a_1/a, a_2/a, \ldots, a_n/a]$. The set of quotient rings taken with respect to the prime ideals of all rings $\mathcal{O}[\mathfrak{m} ; a^{-1}]$ (as $a$ varies over the set of superficial elements of $\mathfrak{m}$) is defined to be the variety of $\mathfrak{a}$. It is normal if $\mathfrak{a}$ is normal. Two normal ideals are projectively equivalent in the asymptotic sense of Samuel if and only if they have the same variety. The relation of dominance among varieties defines an order on the set of normal $\mathfrak{m}$-primary ideals that is shown to be a directing relation. (Received July 6, 1960.)

Let \( e = (p - 1,q - 1) \), where \( p \) and \( q \) are distinct primes, and put \((p - 1)(q - 1) = ef\). Let \( g \) be a primitive root of both \( p \) and \( q \). This paper investigates difference sets having parameters \( v = pq, k = (v - 1)/e, \lambda = (v - 1 - e)/e^2 \). In the case \( e = 2 \) such difference sets are of the Hadamard type. A. Brauer (Math. Z. vol. 58 (1953) pp. 219-225) proved Theorem 1: If \( e = 2, q = p + 2 \), then the numbers \( 1, g, g^2, \ldots, g^{f-1}, 0, q, 2q, \ldots, (p - 1)q \) constitute a difference set of the prescribed type. The major result of the present paper is Theorem 2: If \( e = 4, q = 3p + 2, \) and \( k \) is an odd square, then for a suitable choice of \( g \), the numbers \( 1, g, g^2, \ldots, g^{f-1}, 0, q, 2q, \ldots, (p - 1)q \) constitute a difference set of the prescribed type. In the example \( v = 901, k = 225, \lambda = 56 \) which satisfies the conditions of Theorem 2, the number \( g \) may be selected equal to 5. (Received July 6, 1960.)


A collection of simple closed curves (scs's) is called a T-set if it satisfies certain conditions, the main force of which is that if each scc is spanned by a 2-cell, the resulting space is the complement of a 2-cell on a torus. Let \( \{ F_i, \tau_i \} \) be an inverse mapping sequence of finite collections of scc's, the principal conceptual restrictions being: (i) \( F_1 \) is a single scc, (ii) \( \tau_i^{-1} \) of an element of \( F_i \) is a T-set contained in \( F_{i+1} \), and (iii) certain incidence relations. Let \( X \) be the inverse limit of the \( F_i \) under \( \tau_i \), and let \( Y(F) \) be the decomposition space of \( X \) determined by identifying those points of \( X \) whose coordinate scc's intersect for each \( i \). Any other such system, \( \{ G_i, \tau_i \} \), defines a space \( Y(G) \) which is homeomorphic with \( Y(F) \). The methods of proof are constructive and may be used to produce various homeomorphisms. \( Y(F) \) is called a T-cell, an example being a closed disc with an infinite, dense set of "handles" attached, subject to some mild and necessary conditions. (Received July 11, 1960.)


With a (2-dimensional) T-cell defined as suggested in the preceding abstract, the authors define T-spheres and T-manifolds as connected finite unions of nonoverlapping T-cells with no boundary. Orientable and non-orientable T-manifolds can be distinguished. It is shown that each orientable
T-manifold is homeomorphic to a T-sphere. The methods of proof are like those for the characterization of the T-cell. These methods also lead to the further theorem that a T-sphere is homogeneous; i.e., for any two points A and B of it, there exists a homeomorphism of the T-sphere onto itself carrying A onto B. Stronger homogeneity theorems can also be established. Possible applications include the imbedding of a T-sphere in 3-space, bounding two complementary domains; and demonstrating the existence of locally finite ε-partitions of 3-space by connected sets whose boundaries are T-spheres. (Received July 11, 1960.)


Let I denote the set of \(2^n\) integers, 1, 2, ..., \(2^n\) and B denote the set of \(2^n\) n-place binary sequences. Consider a (1,1) encoding \(f\) of I into B. Let \(a(i)\) be the binary sequence corresponding to \(i\) and \(a_j(i)\) the sequence which differs from \(a(i)\) only in the \(j\)th position, \(j = 1, 2, ..., n\). Let \(a(a)\) be the integer corresponding to the sequence \(a\) and let \(d_i(f) = \sum_{j=1}^{n} |i - a_j(i)|\). \(s(f) = \sum_{i=1}^{2^n} d_i(f)\), \(u(f) = \min_{i} d_i(f)\). It is known that \(\min_{i} s(f) = 2^n(2^n - 1)\). The problem is to find an encoding \(f_0\) such that \((i)\) \(s(f_0) = 2^n(2^n - 1)\) and consistent with this condition \(u(f_0)\) is as small as possible. We give a constructive method for obtaining an encoding \(f_0\) satisfying (i), for which \(u(f_0) = (2^{n+1} + 3n - 8)/6\) where 8 = 1 or 2 according as \(n\) is odd or even. We conjecture that there does not exist an encoding \(f_0\) satisfying (i) for which \(u(f_0) < (2^{n+1} + 3n - 8)/6\). (Received July 11, 1960.)

571-83. W. V. Caldwell: A characterizing property for Bers system.

Let \(D\) and \(\tilde{D}\) be conformally equivalent domains in the complex plane and let \(z = x(\xi, n) + iy(\xi, n)\) be an analytic map of \(\tilde{D}\) onto \(D\). Let \(\mathcal{L}\) be the system of first-order partial differential equations \(U_x = a(x,y)V_x + b(x,y)V_y\), \(-U_y = c(x,y)V_x + d(x,y)V_y\) where \(a, b, c,\) and \(d\) are Hölder-continuous and \(4bc - (a + d)^2 > 0\) in \(\tilde{D}\). (We will assume that \(\mathcal{L}\) is normalized so that \(b > 0\).) Let \(\mathcal{L}'\) be the system of first-order partial differential equations defined on \(\tilde{D}\) with coefficients \(\tilde{a}(\xi, n) = a(x(\xi, n), y(\xi, n)), \tilde{b}(\xi, n) = b(x(\xi, n), y(\xi, n)),\) etc., and let \(\mathcal{W}\) be the set of all functions \(f \circ z\) where \(f\) is a solution of \(\mathcal{L}'\). A necessary and sufficient condition for \(\mathcal{W}\) to consist of solutions to \(\mathcal{L}'\) is that \(\mathcal{L}\) be a Bers system (i.e., \(a = -d\) and \(b = c\). (Received July 11, 1960.)
571-84. L. L. Campbell: **Minimum sampling rate for stationary random processes.**

Let \( x_1(t), x_2(t), \ldots, x_N(t) \) be \( N \) sample functions of a stationary random process with mean zero, variance one, and spectral density \( S(f) \). It is shown that, as \( N \to \infty \), it is possible to construct an approximation to these functions with arbitrarily small error by using an average of \( \exp \left[ - \int_{-\infty}^{\infty} S(f) \log S(f) \, df \right] \) numbers per function per unit time. If \( S(f) \) has a constant value in the band \((-W, W)\) and vanishes outside, this sampling rate reduces to the well-known rate of \( 2W \) samples per function per unit time. If \( S(f) \) vanishes outside \((-W, W)\) but is not constant inside the interval the rate is lower. If \( S(f) \) is positive on an infinite interval the sampling rate is still defined whenever the above integral converges absolutely. (Received July 7, 1960.)

571-85. E. H. Connell: **Some classical theorems by methods of topological analysis.**

Using only the fact that differentiable functions are open, the following classical theorems of complex variable are proved: (1) If \( f \) is differentiable in a region \( R \), then \( f' \) is differentiable in \( R \). (2) If \( f_n \) is a sequence of differentiable functions converging to \( f \) uniformly in \( R \), then \( f \) is differentiable in \( R \). (3) If \( f \) is bounded and differentiable in \( R - z_0 \), then \( f \) may be defined at \( z_0 \) so that it is differentiable there. (Received July 8, 1960.)

571-86. G. L. Davey: **An investigation of the flow of a viscous compressible fluid past a semi-infinite flat plate at zero angle of attack.**

An appropriate boundary value problem is set up to describe the flow of a viscous, compressible fluid past a semi-infinite flat plate at zero angle of attack. The general Navier-Stokes equations are linearized and fundamental solution tensors found for both subsonic and supersonic flows. The various components of these tensors are determined explicitly by means of integral transforms. The behavior of these fundamental solutions are investigated for \( r (r^2 = x^2 + y^2) \) close to zero and for large values of \( r \). Once the fundamental solution tensor has been determined for the subsonic flow, an equivalent radiation problem is set up. By this means, an integral equation is determined which can be treated by the Wiener-Hopf technique. Although an analytic solution could not be found for this integral equation, bounds for the solution are obtained. It is shown that the bound for the solution is \( f(x) \leq C/x^{1/2}, (x > 0) \) which is the exact
solution for the same flow problem, but for a viscous, incompressible fluid.

(Received July 7, 1960.)

571-87. Albert Edrei and W. H. J. Fuchs: On the maximum number of deficient values of certain classes of meromorphic functions.

Let \( f(z) \) be a meromorphic function of finite order and let \( A(r) \) denote the annulus \( r \leq |z| \leq \sigma r \) \((\sigma > 1, \text{fixed})\). Assume that there exists an unbounded sequence of values of \( r \) such that the corresponding annuli \( A(r) \) have the following property: the zeros and poles of \( f(z) \) which lie in \( A(r) \) may be enclosed in \( q \ (\geq 0) \) sectors of total opening \( O((\log r)^{-1-\epsilon}) \) \( (\epsilon > 0) \); the position of the sectors may vary arbitrarily with \( r \). Then the only possible deficient values of \( f(z) \) are 0, \( \infty \) and at most \( q \) other values. With suitable modifications the theorem may be extended to infinite orders. It is also possible to replace \( \sigma \) by \( \sigma(r) \ (\sigma > 1) \) provided the latter function does not tend too rapidly to one.

(Received July 12, 1960.)

571-88. C. H. Edwards, Jr.: \( S^3 \) does not contain uncountably many mutually disjoint tori, no two of which are concentric.

If \( B_1 \) and \( B_2 \) are two solid tori (of genus 1) with \( B_2 \) interior to \( B_1 \) then \( B_1 \) and \( B_2 \) are said to be concentric if and only if \( Cl(B_1 - B_2) \) is the topological product of a torus and an interval. Two tori \( T_1 \) and \( T_2 \) in the 3-sphere \( S^3 \) are said to be concentric if there exist two concentric solid tori \( B_1 \) and \( B_2 \) in \( S^3 \) such that \( T_i = Bd B_i \), \( i = 1,2 \). It is shown that, if \( G \) is an uncountable collection of mutually disjoint tori in \( S^3 \), then there exists a sequence \( \{B_n\}_0^\infty \) of tame solid tori such that (1) \( Bd B_n \subset G \) for each \( n \), (2) \( B_{n+1} \subset Int B_n \) for \( n \geq 1 \), and (3) \( \cap_{n=1}^\infty B_n = B_0 \). The existence of pairs of concentric tori in \( G \) is then demonstrated by showing that, if \( \{B_n\}_0^\infty \) is any sequence of tame solid tori in \( S^3 \) satisfying conditions (2) and (3) above, then there exists an integer \( N \) such that \( B_i \) and \( B_j \) are concentric whenever \( i > j \geq N \). (Received July 8, 1960.)


For a ring \( R \) let \( N(R) \) denote its nil radical. A ring \( R \) is N-prime if \( R/N(R) \) is a prime ring and is reflective if it has the property that an element \( a \) is regular in \( R \) if and only if \( a + N(R) \) is regular in \( R/M(R) \). Theorem: If \( R \) satisfies the ascending chain condition for right and left ideals then the following
Statements are equivalent. (1) R is reflective and N-prime. (2) R has a right and left quotient ring Q(R) of the form $K_n$, where K is a completely primary ring which satisfies the ascending chain condition for right and left ideals. In addition, $Q(R/N(R)) \cong K_n/N(K_n)$. (Received July 7, 1960.)


Let X be a Tychonov space, $Q^*(X)[R^+(X)]$ the set of bounded, nonvanishing, continuous, quaternion-valued functions on X. The natural embedding $T(X)$ of X in $H_{Q^*}[H_{R^+}]$, the set of all homomorphisms of $I_{Q^*}[I_{R^+}]$ (the set of invertible elements of $Q^*(X)[R^+(X)]$) into $Q^*[R^+]$ ($H_{Q^*}[H_{R^+}]$ endowed with the weak* topology) is a free basis for the subgroup it engenders. This turns out to be the free [abelian] topological group $F(X)[\Lambda(X)]$ of X. $F(X)[\Lambda(X)]$ are maximally almost-periodic and $T(X)$ is a closed subset of $F(X)[\Lambda(X)]$. Applications of the method to the question of the complete systems of irreducible unitary representations [the character group] of $G_F[G^A]$ the compact groups associated to $F(X)[\Lambda(X)]$. Some of the results are known (Graev, Kakutani, Markov, Nakayama). The derivation is new and leads to simple proofs of other results. Attention is called to the parallel: "$R^+$ is the abelianization of $Q^*$" and "$\Lambda(X)$ is the abelianization of $F(X)$." (Received July 11, 1960.)


Let $(X_1,X_2,\ldots,X_n)$ be a random vector whose distribution of probability is symmetric about a point $(\theta,\theta,\ldots,\theta)$ in n-dimensional space. Let T and S be an odd location statistic and an even location-free statistic, respectively. If the expectation of T exists, then the conditional expectation of T, given $S = s$, is $\theta$. This implies that if the random weights $W_1$ and $W_2$ are functions of even location statistics so that $W_1 + W_2 = 1$ and if $T_1$ and $T_2$ are odd location statistics such that their means exist, then $W_1T_1 + W_2T_2$ is an unbiased estimator of $\theta$. (Received July 7, 1960.)

571-92. E. P. Miles, Jr.: Nested matrix multiplication tables.

Consider a doubly infinite matrix C whose elements $c_{j,k}$, $-\infty < j, k < \infty$ are defined recursively by (a): $c_{j,k+n} = \sum_{m=0}^{n-1} a_mc_{j,k+m}$, $a_0 \neq 0$ and (b): $c_{j+n,k} = \sum_{m=0}^{n-1} b_mc_{j,k+m}$, $b_0 \neq 0$ from the elements $c_{j,k} = \delta_{jk}$, $0 \neq j, k \leq n - 1$. Denote by $C_{j,k}$ the n by n submatrix of adjacent elements of C having $c_{j,k}$ as the lead element of its principal diagonal. From the definition of C one sees
that \( C_{0,0} = I_n \), while \( C_{0,1} = A \) is a companion matrix for \((a)\): \( x^n = \sum_{m=1}^{n} a_m x^m \) and \( C_{1,0} = B \) is a companion matrix for \((b)\): \( x^n = \sum_{m=0}^{n-1} b_m x^m \). Since \( a_0 \) and \( b_0 \) are nonzero, \( A \) and \( B \) are nonsingular matrices. The general solutions for \((a)\) and \((b)\) are expressible in terms of the roots of \((a)'\) and \((b)'\) respectively, that is, in terms of the eigenvalues for \( A \) and \( B \). The author has shown in his forthcoming paper, \textit{On Matrix Slide Rules} to appear in the Amer. Math. Monthly, that \( c_{0,k} = (c_{0,1})^k = A^k \), \( -\infty < k < \infty \). A more general result that \( c_{j,k} = c_{j,0} c_{0,k} = B^j A^k \), \( -\infty < j, k < \infty \) is established which enables one to think of \( C \) as a nested multiplication table displaying all matrix products \( B^j A^k \) for \( j \) and \( k \) integers. (Received July 11, 1960.)

571-93. L. J. Mordell: \textit{Irrational power series.}

Let \( \alpha \) be a real number, and let \( f(x) \) be a polynomial of degree \( \geq 1 \). Then \( F(x) = \sum_{n=0}^{\infty} f([n\alpha]) x^n \), where \([n\alpha]\) denotes the integer part of \( n\alpha \), is a rational function of \( x \) if and only if \( \alpha \) is a rational number. This is proved by Dr. Morris Newman in a paper to appear in Proc. Amer. Math. Soc. The result is implicit in Hecke's work. A proof different from the others is now given. It uses the Fourier series for the Bernoulli polynomials to transform \( F(x) \) into a doubly infinite series. Reducing this to a singly infinite series makes evident that when \( \alpha \) is irrational, \( x = e^{2\pi i n/\alpha} \) is a singularity of \( F(x) \) for all integers \( n \) except a finite number. (Received July 8, 1960.)

571-94. Israel Navot: \textit{A generalization of the Euler-Maclaurin summation formula.}

The pertinent error in the ordinary Euler-Maclaurin summation formula when applied to the approximate evaluation of an integral by summation, or to the approximate summation of a series by integration, is generally expressed by an asymptotic series in the differences between the end-point derivatives of the function on which the integral or sum operates, the coefficients being expressed in terms of Bernoulli numbers. This obviously excludes the application of the formula to integrable functions with a branch singularity at one of the end points where the derivatives from a certain order and up become infinite. A generalization of the summation formula which eliminates this difficulty is now presented. It replaces at the singular end point the Euler-Maclaurin series of derivatives by a series of derivatives of the analytic part of the function obtained by dividing out the singularity. The coefficients of this
series, which depend of course on the type of branch singularity, are expressed in terms of Riemann zeta functions with decreasing arguments. (Received July 7, 1960.)


Theorem. Let $T_\lambda$ be a family of bounded everywhere defined commuting linear operators on a reflexive Banach space $X$, indexed by a positive parameter $\lambda$. Assume that (a) the $T_\lambda$ are uniformly bounded in norm, (b) for $\xi \neq \lambda$ the $T_\lambda$ satisfy the identity $(\xi T_\lambda - \lambda T_\xi) T_\xi = (\xi - \lambda) T_\lambda T_\xi^2$, (c) if for some $x$ in $X$, $T_\lambda x = 0$ for all $\lambda$, then $x = 0$, and if for some $x^*$ in $X^*$, $T_\lambda x^* = 0$ for all $\lambda$, then $x^* = 0$. Then $\lim_{\lambda \to 0} T_\lambda x$ exists for all $x$ in $X$, and the $T_\lambda$ converge to a projection operator in the strong operator topology. The convergence of martingales follows as a special case by taking $T_\lambda$ to be a family of commuting projections such that $T_\lambda T_\xi = T_\xi$ for $\xi \neq \lambda$. The mean ergodic theorem can be obtained as a special case by choosing the $T_\lambda$ as follows. Let $U^t$ ($t > 0$) be a one-parameter semigroup of operators with $A$ as its infinitesimal generator. Set $T_\lambda = \lambda (A I - A)^{-1} = \int_0^\infty e^{-\lambda t}U^t dt$. Applying the theorem, one obtains the existence of $\lim_{\lambda \to 0} \int_0^\infty e^{-\lambda t}U^t x dt$ (Abel convergence). Under known conditions Abel convergence implies Cesàro convergence $\lim_{\lambda \to 0} (1/T) \int_0^\lambda U^t x dt$, which is the ergodic theorem. Other applications of the theorem include inverse martingales, and a new type of "mixed" ergodic-martingale theorem. Various generalizations of the theorem can be proved. (Received July 11, 1960.)

571-96. H. H. Schaefer: Two theorems on normal cones.

In 1939, M. G. Krein gave this definition of a normal (convex) cone $K$ (of vertex 0) in a Banach space $(E, \| \cdot \|)$: $K$ is normal if there exists $a > 0$ such that $\|x + y\| \leq a \|y\|$ for all $x, y \in K$. Let $K$ be a convex cone in a real (or complex) vector space $E$; for $A \subseteq E$, set $[A] = (A + K) \cap (A - K)$. If $\mathcal{F}$ is a filter on $E$, denote by $[\mathcal{F}]$ the filter with basis $[F]$: $F \in \mathcal{F}$. Definition: A convex cone $K$ (of vertex 0) in a topological vector space is normal if $\lim \mathcal{F} = 0$ implies $\lim [\mathcal{F}] = 0$ for every filter on $E$. Denote by $E$ a (Hausdorff) locally convex space, partially ordered with positive cone $K$. Theorem 1: Let $K$ be normal, and let $H$ be a directed (nonempty) subset of $E$. If the filter $\mathcal{F}(H)$ of sections of $H$ converges weakly, then it converges for the given topology on $E$. Theorem 2: Let $f$ be an analytic function with values in $E$, holomorphic at 0.
and such that \( f(z) = \sum_{n=0}^{\infty} a_n z^{n} \) has radius of convergence 1. If \( \{a_n : n \in N\} \subset K \) and \( K \) is normal, then \( z = 1 \) is singular for \( f \); if this singularity is a pole of order \( \ell \), there is no pole of \( f \) on \(|z| = 1\), of an order \( > \ell \). (Received July 8, 1960.)

571-97. A. A. Sagle: Malcev algebras.

A Malcev algebra (briefly M-algebra) is a nonassociative algebra satisfying the identities \( x^2 = 0 \), \((xy)(xz) = ((xy)z)x + ((yz)x)x + ((zx)x)y\). These algebras arise when commutation is taken as the new multiplication in an alternative algebra. In this paper extensions of these identities are obtained and a Lie algebra of transformations is introduced which, to a large extent, characterizes semi-simple M-algebras. An important result is the Theorem: Let \( A \) be an M-algebra of characteristic not 2 or 3 and let \( J(x, y, z) = (xy)z + (yz)x + (zx)y \). Set \( N = \{n \in A : J(n, A, A) = 0\} \), \( J(A) = \{\sum J(x_i, y_i, z_i) : x_i, y_i, z_i \in A\} \). Then \( N \) and \( J(A) \) are ideals of \( A \); furthermore, if \( A \) is semisimple, then \( A = N \oplus J(A) \). The product of two ideals of an M-algebra \( A \) need not be an ideal: however, if \( B \) and \( C \) are ideals of \( A \), then \( J(B, C, A) \) is an ideal of \( A \). This leads to the following: An ideal \( B \) of \( A \) is \( J \)-potent if defining \( J'(B) = J(B, B, B) \) and \( J^{k+1}(B) = J(J^k(B), J^k(B), J^k(B)) \), there exists an integer \( n \) such that \( J^n(B) = 0 \). Results concerning the radical of an M-algebra \( A \) are obtained and it is shown that the maximal solvable ideal is contained in the maximal \( J \)-potent ideal which in turn is contained in the radical. Derivations and the M-algebra obtained from the split Cayley-Dickson algebra are discussed in detail. (Received July 11, 1960.)

571-98. H. W. E. Schwerdtfeger: On sectionally linear functions over an infinite range.

In an earlier paper (Abstract 564-41, Notices Amer. Math. Soc., vol. 6 (1959) p. 784; Canad. Math. Bulletin vol. 3 (1960) pp. 41-57) an analytic representation has been discussed for an \( s, L \) function over a finite partition of a finite interval. The corresponding theory is now developed for an infinite partition \( P: \ldots < x_{-1} < x_0 < x_1 < x_2 < \ldots \) of the whole x-axis, or a half-axis, or a finite interval. With the basis functions \( \phi_n(x) = \phi(x - x_{n-1}) \) where \( \phi(x) = (1/2) (|x| + x) \), every \( s, L \) function \( f(x) \) over \( P \) can be represented in the form \( \sum_{n=-\infty}^{\infty} a_n \phi_n(x) \) where the coefficients \( a_n \) are subjected to complicated limit conditions. These can be avoided by introducing the orthogonal basis functions
\[\psi_n(x) = \phi_n(x) - (x_{n+1} - x_{n-1})/(x_{n+1} - x_n) \phi_{n+1}(x) + (x_n - x_{n-1})/(x_{n+1} - x_n) \phi_{n+2}(x)\]
(n = \ldots, -1, 0, 1, 2, \ldots). Every \(s, f(x)\) can be represented in the form

\[\sum_{-\infty}^{\infty} f(x_n)/(x_n - x_{n-1}) \psi_n(x)\]
where for every \(x\) not more than two terms of the sum are different from zero. This representation appears as an easy tool for the solution of certain linear difference equations. In particular if \(x_n = n\) the equation \(\Delta y = f(x)\) has an \(s, f(x)\), solution \(y = Sf(x)\), unique up to an additive constant, if and only if \(\sum_{n=0}^{N} f(n)\) is convergent. (Received July 11, 1960.)


The problem we consider is to characterize those sequences \(z_n\) in the unit circle such that, given an arbitrary bounded sequence \(w_n\) there is a bounded analytic function \(f\) with \(f(z_n) = w_n\). This problem has been solved by L. Carleson and D. J. Newman. We give a new proof and we show that these sequences \(z_n\) also have the property that, if \(f\) is in \(H_p\), then the sequence \(f(z_n)(1 - |z_n|^p)^{1/p}\) is in \(L_p\), and everything in \(L_p\) is obtained in this manner. (Received July 11, 1960.)

571-100. H. R. Stevens: Generalized Kummer congruences for products of sequences (applications).

Let \(p\) be a fixed rational prime. A sequence \(\{a(n)\}\) of numbers that are integral (mod \(p\)) satisfies a generalized Kummer's congruence if

\[\sum_{s=0}^{r} (-1)^s \binom{r}{s} \lambda^{r-s} a(n + sw) \equiv 0 \pmod{p^r}\]
for all \(n \geq r \geq 1\) and where \(\lambda\) is integral (mod \(p\)) and \(w\) is a fixed integer. Applying the definitions and theorems of a previous paper of H. R. Stevens (Generalized Kummer congruences for products of sequences, Duke Math. J., to appear), various specific sequences are shown to satisfy Kummer's congruence. For example, it is proved that

\[\sum_{s=0}^{r} (-1)^s \binom{r}{s} h^{r-s} E(h)(n + 2s) \equiv 0 \pmod{2^r}\]
where \(E(h)(n)\) is the \(h\)th order Euler number. Some weaker forms of Kummer's congruence are also obtained for Bernoulli numbers of higher order. (Received July 11, 1960.)


When one of the many possible loci of a quadratic equation in three variables, as mapped on a plane by the rules of extended analytic geometry, leaves an uncovered area on the plane, it invariably reveals the type of quadric surface usually associated with the equation. It is then defined as a silhouette
and consists of a conic section or an area bounded by one. Such silhouettes,
including the vacuous case, still hold for quadratic equations in $n$ variables when
linear plotting rules are used; but quadratic rules in general yield areas bound-
ed by adjoined parts of conic sections. Many loci can be found "by inspection"
after very short calculations. One algebraic windfall from the method is that
inconsistent equations can often be shown as such via simple demonstrations
that their loci do not overlap. Again, common solutions, if present, are obtain-
able by a systematic procedure in all cases involving linear-quadratic pairs,
and frequently in other cases. (Received July 7, 1960.)


Let $x(t), t \geq 0$, be the symmetric stable process with exponent $\alpha$ and
starting at a point $x_0$ in the interior of the unit interval: $E\{e^{i\xi x(t)}\}$
$= \exp(i\xi x_0 - t|\xi|^\alpha)$. Then $d/dxPr\{\max_{t \leq t} |x(t)| < 1, x(t) \leq x\}$
$= \sum e^{-t/\lambda_i} f_i(x_0)f_i(x)$, where $\lambda_1 \leq \lambda_2 \leq \ldots$ and $f_1, f_2, \ldots$ are the eigenvalues
and normalized eigenfunctions respectively of the integral equation on $(-1,1)$ with
symmetric kernel $K(x,y)$ given for $x < y$ by $\pi^{-1}\Gamma(\alpha)^{-1}\sin^{-1}a\pi(1 - y^2)^{\alpha/2}$
$\cdot \int_x^y (1 - u^2)^{-\alpha/2}(x - u)^{-1}(y - u)^{-1}du$. For $\alpha = 2$ (when $K = (1 + x)(1 - y)/2$ for
$x < y$) the result is classical, and for $\alpha = 1$ was proved by Kac and Pollard
(On the distribution of the maximum of partial sums of independent random
variables, Canad. J. Math. vol. 2 (1950) pp. 375-384). If $T$ is the time of first
passage to the exterior of the unit interval, all the moments of the distribution
of $T$ can be expressed in terms of the iterates of $K$. The first moment was
obtained by Elliot (Absorbing barrier processes connected with the symmetric
stable densities, Illinois J. Math. vol. 3 (1959) pp. 200-216) and the second by
Getoor (First passage times for symmetric stable processes in space, Notices
Amer. Math. Soc. vol. 7 (1960) p. 374). Also expressible in terms of the $\lambda_i$ and
$f_i$ is the joint distribution of $T$ and the place of first passage $x(T)$. In particular
it is shown that for $\alpha < 2$, $(d/dx)Pr\{x(T) \leq x\} = \pi^{-1}\sin^{-1}a\pi(1 - x_0^2)^{\alpha/2}$
$\cdot (x^2 - 1)^{-\alpha/2}|x - x_0|^{-1}$. (Received July 8, 1960.)

571-103. Harold Widom: Stable processes and integral equations. II.

Let $k(x)$ be non-negative and even with $\int_{-\infty}^{\infty} k(x)dx = 1$. Setting $\hat{k}(\xi)$
$= \int_{-\infty}^{\infty} e^{i\xi x}k(x)dx$, assume that as $\xi \to \infty$, $1 - \hat{k}(\xi) \sim c|\xi|^{-\alpha} (0 < c < \infty)$ with
$0 < \alpha \leq 2$. Denote by $\mu_1, \mu_2, \ldots$ and $g_1, g_2, \ldots$ the positive eigenvalues
and normalized eigenfunctions respectively of the integral operator on $(-1,1)$
with kernel $Ak(A(x - y))$. It has been conjectured (Kac, Murdock and Szegö, *On the eigenvalues of certain Hermitian forms*, J. Rat. Mech. Analysis vol. 2 (1953) pp. 767-800; see §1.7) that for $\alpha = 1, 2$ we have, as $\Lambda \to \infty$ and for fixed $i$, $1 - \mu_{i,\Lambda} \sim C\lambda_i A^{-\alpha}$, where the $\lambda_i$ are as in the previous abstract. This was proved by us in the case $\alpha = 2$ under slightly different hypothesis (on the eigenvalues of certain Hermitian operators, Trans. Amer. Math. Soc. vol. 88 (1958) pp. 491-522). Here we prove the conjecture and its analogues for all $\alpha$ in $0 < \alpha \leq 2$, with the hypotheses as stated. Moreover if the $g_{i,\Lambda}$ are multiplied by suitable normalizing constants then $g_{i,\Lambda} \to f_i$ in $L^2(-1,1)$ for all $i$ if $\alpha = 2$ and, for $\alpha < 2$, under the assumption that $\lambda_i$ is of multiplicity one. In all cases the convergence is uniform if also $k \in L^p$ for some $p > 1$. (Received July 8, 1960.)


Any system of ordinary linear differential equations of any order can be reduced to one of the matrix differential equations $X' = AX$ or $X' = AX + B$ where $A$ is a square matrix, $B$ is a matrix of the same number of rows as $A$, and $X'$ is the matrix whose elements are the derivatives, with respect, say, to $t$, of the corresponding elements of $X$. If the coefficients of the equations, the elements of $A$ and $B$, are functions of $t$ with derivatives of all orders, uniformly bounded in a certain closed neighborhood of $t = c$, then the Taylor's expansion of the solution $X$ represents $X$ in this neighborhood. The coefficients of the Taylor's series solution of $X' = AX$ are defined recursively as $f_k(A_c)/k!$, where $A_c$ is the value of $A$ at $t = c$ and $f_k(A)$ is defined recursively as $f_0(A) = I$, $f_k(A) = f_{k-1}(A)A + f'_{k-1}(A)$. The coefficients of the Taylor's series for the solution of $X' = AX + B$ can also be defined recursively. (An important special case of a set of equations of the form $X' = AX$ is the Frenet formulas of differential geometry.) (Received July 13, 1960.)

571-105. C. E. Burgess: The width of a tree-like continuum.

A tree is a finite coherent collection $G$ of open sets such that no subcollection of $G$ consisting of more than two elements is a circular chain. Let $G$ denote a tree in a metric space. For each chain $C$ in $G$, there is a number which is the maximum distance from a link of $G$ to the sum of the elements of $C$. Let $W(G)$ denote the least such maximum number for all chains in $G$. A number
w is called the width of a tree-like continuum M if, for any cofinal sequence
$G_1, G_2, \ldots$ of trees defining M, the sequence $W(G_1), W(G_2), \ldots$ converges to w.
It follows readily that every tree-like continuum has a width, every linearly
chainable continuum has width zero, and every tree-like triad has a positive
width. The following theorems are proved. (1) If $M_1, M_2, \ldots$ is a convergent
sequence of mutually exclusive tree-like continua in $E^2$ and, for each i, $w_i$ is
the width of $M_i$, then the sequence $w_1, w_2, \ldots$ converges to zero. (2) If, in the
above theorem, the sequence $M_1, M_2, \ldots$ converges homeomorphically, then each
$w_i$ is zero. (3) If H is an uncountable collection of mutually exclusive tree-like
continua in $E^2$, then all except a countable number of continua of H have width
zero. (Received July 13, 1960.)

571-106. G. T. Cargo: Angular and tangential limits of Blaschke products
and their successive derivatives.

Let $B(z)$ be a Blaschke product with zeros at $a_n \ (m = 1, 2, \ldots)$, and let
$R(m, \theta, \nu) = \{z: 1 - |z| \leq m |\arg z - \theta|^{\nu}; \ |z| < 1\}$. We say that $B(z)$ has a
$T_\nu$-limit ($\nu \geq 1$) at $e^{i\theta}$ if there exists a number L such that $B(z) \to L$ as
$z \to e^{i\theta}$ on $R(m, \theta, \nu)$ for each $m > 0$. Theorem 1. Let $\sum (1 - |a_n|^{1/\nu} |e^{i\theta} - a_n|^{\nu})$
$k = 1, 2, 3, \ldots$ has a $T_\nu/2k$-limit at $e^{i\theta}$. Theorem 2. Let $\sum (-|a_n|)^{\alpha} < \infty$
$(0 < \alpha < 1)$. Then corresponding to each $\nu$, there is an exceptional set $E_\nu$
whose capacity of order $\alpha$ is zero such that (i) $B(z)$ (as well as any subproduct) has
a $T_\nu$-limit on $W_\nu = \{z: |z| = 1\} - E_\nu$; (ii) $B^{(k)}(z)$ has a $T_\nu/2k$-limit on $W_\nu$.
Theorem 3. Theorem 2 (i) is the "best possible." When $\nu = 1$, Theorems 1
(i), 2(i), and 3 yield classical results due to Frostman on angular limits. When
$\nu = 2$ and $k = 1$, Theorems 1(ii) and 2(ii) reduce to known results on Carathéodo-
dory angular derivatives. Theorem 2 is a sharpened and generalized version
of some results (to be published soon) of J. R. Kinney on the tangential limits
of $B(z)$ and $B^{(1)}(z)$. Certain techniques used in the proofs are the same as
those of Frostman and Kinney. Conjecture: Theorem 2 (i) can be extended to
include all the functions (not just the Blaschke products) studied in Carleson's
thesis. (Received July 13, 1960.)

571-107. P. T. Church and Erik Hemmingsen: Simplicial interior maps
on the three-sphere.

In the following $f:S^n \to S^n$ is simplicial interior of degree d; its branch
set $B_f$ is the set of points $p$ at which $f$ is not a local homeomorphism; and the
exceptionality $e(p)$ is one less than the local degree of $f$ at $p$. For $n = 2$, $B_f$
consists of $2(d - 1)$ points counting exceptionality. The $(n - 2)$-dimensional
set $B_f$ is realizable as an $(n - 2)$-chain by assigning to each $(n - 2)$-simplex
its exceptionality as coefficient. The authors prove: I. The chain $B_f$ need not
be a cycle except mod 2. II. While, for $n \geq 3$ and $d = 2$, $B_f$ is connected (an
$(n - 2)$-homology sphere mod 2), for $n = 3$ and for each $d > 2$ and natural num­
ber $N$, there exists $f$ whose $B_f$ has $N$ components. Moreover, there exists
$f:S^3 \to S^3$ with $B_f$ a single knotted circle. III. For each $n \geq 3$, there exists a
finite to one interior map $g:S^n \to S^n$ such that (1)$\dim B_g = \dim f(B_g) = n - 2$,
(2) there is an uncountable subset $A$ of $B_g$ such that $\dim B_g$ at points of $A$ is
$n - 3$, and (3) $B_g$ is not locally connected. Thus for $n = 3$, $B_g$ has an infinite
number of point components. It appears that the results of I, II and III can be
extended; in particular most of the 3-dimensional results extend to dimension $n$.
(Received July 13, 1960.)


Consider the system of delay-differential equations $y'_j(t) = f_j(t, y(t),
y(t - \tau'_2(t, y(t))), y(t - \tau'_3(t, y(t))), \ldots, y(t - \tau'_m(t, y(t)))) (i = 1, 2, \ldots, n)$ where $y(t)$ stands for $(y_1(t), y_2(t), \ldots, y_n(t))$ and the delays, $\tau'_j(t, y(t)) (j = 2, 3, \ldots, m)$ are
known non-negative functions of $t$ and $y(t)$. One specifies $y(t) = \phi(t)$, a given
function, for $t_0 - a \leq t \leq t_0$ (where $a \geq 0$) and then tries to determine $y(t)$ for $t_0 < t < t_0 + \beta$ (where $\beta > 0$) so as to satisfy the delay-differential equations
for $t_0 < t < t_0 + \beta$ and to be continuous at $t_0$. By imposing continuity and
Lipschitz continuity conditions on the functions $f_i$, $\tau'_j$ and $\phi_i (i = 1, 2, \ldots, n,$
$j = 2, 3, \ldots, m)$ and by requiring $0 \leq \tau'_j(t, y) \leq t - t_0 + a (j = 2, 3, \ldots, m)$ in appropri­
ate regions one obtains a well-set problem, i.e. a unique solution, $y(t)$, exists
and the solution depends continuously on the initial data, $\phi(t)$, and on the right
hand sides of the equations, represented by $f_i (i = 1, 2, \ldots, n)$ and $\tau'_j (j = 2, 3, \ldots, m)$.
(Received July 13, 1960.)

571-109, R. D. Driver: A two-body problem of classical electrodynamics.

Consider that two-body problem of classical (nonquantum) electrodynam­
ics in which the two point charges move along the x-axis. The equations of
motion representing this problem can be transformed into a system of delay­
differential equations (see previous abstract). One specifies the initial trajec-
tories, $x_1(t)$ and $x_2(t)$, of the two charges for a finite interval of time, $t_0 - a \leq t \leq t_0$, subject to the following requirements: 1. The implicit equation $r_{ji} = |x_j(t_0) - x_i(t_0 - r_{ji}/c)|$, where $c$ is the speed of light, is solvable for $r_{ji}$ when $(j,i) = (2,1), (1,2)$, 2. $x_1(t_0) \neq x_2(t_0)$, and 3. the velocities $x'_1(t)$ and $x'_2(t)$ are Lipschitz continuous and have magnitudes less than $c$ for $t_0 - a \leq t \leq t_0$. Then the trajectories are determined for all $t > t_0$ unless and until the charges collide. If two point charges have the same sign they will not collide. If they have opposite signs they may or may not collide, depending on the initial data. If two charges collide at $t_0 + \beta$ then as $t \to t_0 + \beta$ the velocities of the charges approach $c$ and $-c$. In case the charges do not collide then $|x_2(t) - x_1(t)| \to \infty$ as $t \to \infty$. (Received July 13, 1960.)

571-110. Uri Fixman: On torsion-free linear systems.

For the definition of a (linear) system $(V,W;A,B) = (V,W)$ and related concepts see these Notices vol. 6, p. 429, Abstract 559-79. Denote $B_\emptyset = B - \emptyset A$ ($B_\emptyset = A$). A system $(V,W)$ is torsion-free (t.f) if $B_\emptyset v = 0$ implies $v = 0$. A notion of (t.f) rank is derived from the following dependence relation. An element $w \in W$ depends on a subset $L \subseteq W$ if $w \in Y$, where $(X,Y)$ is the smallest subsystem of $(V,W)$ such that $L \subseteq Y$ and $(V,W)/(X,Y)$ is t.f. If $(V,W)$ is t.f., $w \in W$, the height $H(w,\emptyset)$ is the l.u.b. of $0$ and the integers $k$ such that the equations $B_\emptyset v_i = w, B_\emptyset v_1 = B_\emptyset v_{i-1}$, $i = 2, ..., k, \emptyset \neq \emptyset$, are solvable. Two such functions $H(\emptyset,\emptyset)$ are called equivalent if $\Lambda = \{ \emptyset | H(\emptyset) \neq K(\emptyset) \}$ is finite, $H(\emptyset),K(\emptyset) < \infty$ for $\emptyset \in \Lambda$ and in case $H(\emptyset) < \infty$ for all $\emptyset$, $\sum_{\emptyset \in \Lambda} (H(\emptyset) - K(\emptyset)) = 0$. If $(V,W)$ is t.f. of rank 1, it is q.-sp. irr. and the functions $H(w,\emptyset), \emptyset \neq w \in W$, form a complete equivalence class which characterizes the isomorphism type of $(V,W)$. The ring of endomorphisms (obvious definition) of $(V,W)$ is isomorphic to the ring of rational functions regular on $\{ \emptyset | H(w,\emptyset) < \infty \}$. (Received July 13, 1960.)

571-111. F. W. Gehring: The Liouville theorem in space.

Let $R$ be a ring in 3-space whose boundary components $B_0$ and $B_1$ are nondegenerate continua. There exists a unique function $u$ which is continuous in $\overline{R}$, which is 0 and 1 on $B_0$ and $B_1$, which is absolutely continuous on almost all segments in $R$ parallel to the coordinate axes, and for which $\int (R) = \int |\nabla u|^2 \, d\omega$, where the integral is over $R$. If, in addition, bounds $m$ and $M$ exist so that $0 < m \leq |\nabla u| \leq M < \infty$ a.e. in $R$, then $u$ is real analytic in $R$. From this fact we obtain the following extensions of the Liouville theorem con-
cerning the conformal mappings in space. If \( y(x) \) is a 1-quasiconformal mapping of \( D \) onto \( D' \), i.e., if \( \text{mod } R' = \text{mod } R \) for all rings \( R, R' \subset D \), then \( y(x) \) is a Moebius transformation, i.e., maps spheres onto spheres. If \( y(x) \) is a homeomorphism of \( D \) onto \( D' \) and \( \lim \sup (\max |y(x) - y(x')|/\min |y(x) - y(x'')|) = 1 \) for all \( x \) in \( D \), then \( y(x) \) is a Moebius transformation. Here the maximum and minimum are taken for \( |x - x'| = |x - x''| = r > 0 \) and the lim sup is taken as \( r \to 0 \). Hence a homeomorphism maps spheres onto spheres if it maps infinitesimal spheres onto infinitesimal spheres. These results hold in \( n \)-space with appropriate modifications in the definition for the modulus of a ring \( R \).

(Received July 13, 1960.)

571-112. C. B. Hanneken: Subfields of the ring of \( n \times n \) matrices over a finite field.

Let \( \mathcal{M}_n \) be the ring of \( n \times n \) matrices over \( GF(p) \), \( \mathcal{J}_n \) be the group of nonsingular matrices of \( \mathcal{M}_n \), and let \( \mathcal{H}_n \) be the scalar matrices of \( \mathcal{M}_n \). Let \( g(x) \) be an irreducible polynomial of degree \( n \) over \( GF(p) \), and let \( C_g \in \mathcal{J}_n \) be the companion matrix of \( g \). Then \( C_g \) defines a subfield \( F = \{ \sum_{i=0}^{n-1} A_i C_g^i A_i \in \mathcal{H}_n \} \) of \( \mathcal{M}_n \) of order \( p^n \). There exist \( A \in \mathcal{J}_n \) of order \( n \) \( \exists A^{-1} F A = F \).

If \( \Gamma = \{ A \} \) and if \( F \) denotes the multiplicative group of \( F \), then \( H = \{ F, \Gamma \} \) is a subgroup of \( \mathcal{J}_n \) of order \( n(p^n - 1) \). Since subfields of \( \mathcal{M}_n \) are conjugate, the number of subfields of \( \mathcal{M}_n \) of order \( p^n \) is \( [\mathcal{J}_n : H] = p^{(n-1)/2} (p^{n-1} - 1) \ldots (p - 1)/n \). If \( n = 2 \), the subfields of \( \mathcal{M}_2 \) are characterized by the \( p(p - 1)/2 \) distinct irreducible quadratics over \( GF(p) \). The group \( G = \mathcal{J}_2/C_2 \), \( C_2 \) the center of \( \mathcal{J}_2 \), is of order \( p(p^2 - 1) \) and is isomorphic to the linear fractional transformation group over \( GF(p) \). If \( T \in G \) is of prime power order \( q^s, q > 2 \), then \( q^s [p, p - 1, or p - 1 according as the discriminant, \( D(C) \), of the characteristic polynomial of the matrix \( C \) defining \( T \) is 0, a square, or a non-square modulo \( p \). If \( p = q^s K - 1, (q, K) = 1 \), there are \( p(p - 1)/2 \) distinct subgroups of \( G \) of order \( q^s \). These subgroups are all cyclic and form a complete conjugate set of subgroups.

(Received July 13, 1960.)

571-113. R. W. Heath: A regular semi-metric space for which there is no semi-metric under which all spheres are open.

In this paper the question, "Does every semi-metric space have a semi-metric under which all spheres are open" is answered in the negative. The counter example is a connected, locally connected, regular semi-metric space.
which is hereditarily separable, weakly complete, and paracompact. (Received July 13, 1960.)

571-114. R. P. Hunter: The identity of a compact connected semigroup which is not a group is not a weak cut-point.

R. J. Koch has conjectured that the identity of a compact connected semigroup which is not a group is not a weak cut point. Using the theory of Lie groups it is shown that there are arbitrarily small continua which contain the identity and meet a proper ideal. This and a result of Koch yield the theorem. Koch had previously answered this conjecture when the semigroup was homogeneous or one dimensional. (Received July 13, 1960.)


The linear polynomial operator $P(D,X)$ (of nth-order) is usually defined to be reducible if it can be expressed in the form $P\{G(D,X)\}$ where $G(D,X)$ is a linear polynomial operator of order less than n. In a previous note with the same title as this abstract (Amer. Math. Monthly, April, 1959, pp. 293-295) the authors showed that the operators $x^{rn}D^{(r+1)n}$, $x^{(r+1)n}D^{rn}$, $D^{(r+1)n}x^{rn}$, and $D^{rn}x^{(r+1)n}$ were reducible and utilized this to solve certain nth-order linear differential equations. By extending the notion of reducibility, a wider class of reducible operators is found and these are used to solve certain other nth-order linear differential equations. (Received July 13, 1960.)


The following theorem is proved: Suppose $x$ and $y$ are metric compact spaces with regular Borel measures. Suppose $f(x)$ is continuous as a function of $y$ for each $x$ and suppose there exists a function $f(x,y)$ measurable in both variables together and for which $f(x) = f(x,y)$ almost everywhere for each $x$. Then $f(x,y)$ is measurable in both variables together. (Received July 25, 1960.)
571-117. T. S. Motzkin and J. L. Walsh: Best approximators within a linear family on an interval.

The behavior of functions \( p(x) \) of best approximation to a given \( f(x) \) is investigated, where \( p(x) \) belongs to a given linear family \( P \) and deviation is measured by a generalized norm \( \int_0^1 \tau(|f - p|)w(x)dx \) involving a transformer \( \tau(t) \) and a positive continuous weight function \( w(x) \). Properties of existence, uniqueness, oscillation, coincidence with \( f(x) \) on a set of positive measure, and orthogonality are emphasized. In particular conditions, some necessary, some sufficient, some both, are developed that a given function \( p(x) \) should be a best approximator. (Received July 13, 1960.)


Let \( V_n \) be a subspace in a Euclidean space \( E_m, m \geq n + 1 \). Let \( \xi \) be a unit normal to \( V_n \) and \( v \) a vector field on \( V_n \). There exists on \( V_n \) a family of asymptotic lines of the field with respect to the normal. The straight lines on \( v \) along these asymptotic lines generate a family of developable subspaces in \( E_m \), whose edges of regression form a subspace \( \bar{V}_n \) complementary to \( V_n \) for \( v \) with respect to \( \xi \) in \( E_m \). Some properties of \( \bar{V}_n \) are obtained to generalize those for \( n = 2, m = 3 \), given in a former paper by the author [Proc. Amer. Math. Soc. vol. 6 (1955) pp. 151-158; Math. Reviews vol. 16 p. 746]. (Received July 13, 1960.)

571-119. E. F. Collingwood and George Piranian: Asymmetric prime ends.

Each simply connected domain in the plane has at most countably many prime ends whose right and left wings do not coincide. On the other hand, to each countable set \( E \) on the unit circle \( C \) there corresponds a function which is holomorphic and univalent in the unit disk \( D \) and which has the property that it carries each point of \( E \) and no point of \( C \setminus E \) onto a prime end with unequal wings. (Received July 13, 1960.)

571-120. N. J. Rothman: Linear quasi ordered semigroups.

Let \( S \) be a compact Hausdorff topological semigroup, where \( S^2 = S \), and for each \( x \) and \( y \) in \( S \) either \( x \cup Sx \subseteq y \cup Sy \) or \( y \cup Sy \subseteq x \cup Sx \). For \( a \) in \( S \), denote by \( J_a \) the set of generators of the ideal generated by \( a \), and \( L_a \) the set of
generators of the left ideal generated by \( a \). **Theorem:** Each closed left ideal in \( S \) is a principal left ideal; for all \( a \) in \( S \), \( L_a = J_a \); and \( L_a \) is a semigroup equal to its kernel and hence, its structure is completely determined. **Theorem:** Let \( S \) be connected and let the idempotent elements commute. If \( e \) and \( f \) are idempotent elements with \( Sf \subseteq Se \), and \( e \) is the only idempotent element in \( Se - Sf \), then the Rees Quotient \( Se/Sf \) is isomorphic to \( G \times I/G \times \{0\} \), where \( G \) is the compact topological group \( L_e \) and \( I \) is a semigroup on the unit interval.

(Received July 13, 1960.)

571-121. J. C. Sanwal: **Locally affine spaces with nilpotent fundamental groups.**

Let \( X \) be an \( n \)-dimensional, compact, complete locally affine space. Auslander and Markus [Ann. of Math. vol. 62 (1955) pp. 139-151] proved that \( \pi_1(X) = \Gamma \) can be considered to be a group of affinities and the \( n \)-dimensional Euclidean space is the universal covering space of \( X \). Hence \( \Gamma \) has a natural matrix representation in \( GL(n + 1,R) \). It is shown that \( \Gamma \) is indecomposable in \( GL(n + 1,C) \) and if \( \Gamma \) is a nilpotent group then \( X \) is homeomorphic to the compact nilmanifold with fundamental group \( \Gamma \). (Received July 13, 1960.)


Assume the transformation \((T_\mu)\): \( y = (1 + \mu)A(\mu)x - Q(x, \mu) \), where \( x, Q \) are \( n \)-vectors, is a homeomorphism in a neighborhood of the origin. \( A(\mu) \) is an orthogonal matrix. If \( Q = P(x, \mu)A(\mu)x \), where \( P \) is real valued, \( 0 < P \leq M|x| \), \( \partial P/\partial x^i \) is continuous in \((x, \mu)\), and \( (\text{grad } P, x) > 0 \) for \( x \neq 0 \), then for small positive \( \mu \) there exists a Lipschitz continuous manifold \( S_\mu \) which is homeomorphic to the \( n \)-sphere, is convex with respect to the origin, and \( T^*_\mu S_\mu = S_\mu \). For \( n = 2 \) a stronger result is obtained. These results are applied to the problem of bifurcation of an invariant manifold in the \( x \)-space from a periodic solution \( \phi(t) \) of a differential system \((1) \) \( dx/dt = f(x, \mu) \) where \( x, f \) are \((n + 1)\)-vectors. It is assumed the characteristic exponents of \((1)\) based on \( \phi \) have zero real parts for \( \mu = 0 \). The imaginary parts may be zero for any or all values of \( \mu \). In particular, for systems of the form \((2) \) \( x' = B(\mu)x - xh(|x|^2, \mu) \), and \( w' = 1 \) together with the identification \( w \mapsto w + 1 \), conditions are given on \( B \) and \( h \) for the existence of an invariant manifold in the \((x, w)\)-space. (Received July 13, 1960.)
571-123. Seth Warner: Compact rings.

If \( A \) is a compact ring with identity, then \( A \) is a Noetherian ring (i.e., satisfies the ascending chain condition on left ideals) if and only if every left ideal is closed. If \( A \) is a compact commutative ring with identity, then every nonzero ideal of \( A \) is open if and only if either \( A \) is finite or else \( A \) is a Noetherian ring with at most one nonzero proper prime ideal. If \( A \) is a compact integral domain (with identity), then every nonzero ideal of \( A \) is open if and only if \( A \) is an open subdomain of a locally compact field. If \( Y \) is a locally compact and totally disconnected space, \( K \) a finite field, \( A \) the ring of continuous functions with compact support from \( Y \) into \( K \), then \( Y \) is the Stone-Čech compactification of a discrete subspace \( X \) if and only if \( A \) admits a compact topology compatible with its ring structure. (Received July 13, 1960.)


A converse is obtained of a theorem announced by R. H. Bing in these Notices, vol. 6 (1959) p. 838, Abstract 564-165, regarding the transformations of a 2-sphere in \( \mathbb{E}^3 \) into its complement. In addition, analogous results are found for the 2-manifold in \( \mathbb{E}^3 \) and for the \((n - 1)\)-dimensional generalized closed manifold \( "(n - 1) - \text{gcm}" \) in \( \mathbb{E}^n \). For extension of the converse to the case of the 2-manifold in \( \mathbb{E}^3 \), the Cantor set \( C \) (of Bing's abstract) must be assumed not to locally separate, as is shown by the example of the torus with a meridian circle pinched to a point. The general theorem for \( \mathbb{E}^n \) is: Let the \( 1c^k \), compact set \( S \) be a common boundary of (at least) two domains in \( \mathbb{E}^n \) \((n > 1)\) and such that if \( U \) is a component of \( \mathbb{E}^n - S \) and \( e > 0 \), then \( S \) contains a closed set \( C \) which is of at most dimension \( k \) and is not a local \( r \)-separating set of \( S \) for \( r \leq k \) and such that there exists an \( e \)-transformation of \( S \) into \( U \cup C \). For \( n = 2m \) or \( 2m + 1 \), \( k = m - 1 \), and in case \( n = 2m + 1 \), then \( H_m(S) \) is finitely generated. Then \( S \) is a closed, orientable \((n - 1) - \text{gcm}\). (Received July 13, 1960.)

571-125. Martin Fox and Herman Rubin: Optimal policy when the number of servers for a queue can be changed. Preliminary report.

Consider a queue with Poisson arrivals and departures. Assume the cost components (i) salary rate per server, (ii) loss per unit time per customer waiting for service, (iii) interest rate. The present paper gives an iterative
method for finding the optimal policy (in the sense of minimizing present value of the risk) for selecting the number of servers when the number of servers may be increased while the queue is in operation. We assume the parameters are known. The optimal policy is approximated by the optimal policy under the restriction that the number of servers may be increased only at the times of the first \( j \) arrivals and departures. Extensions of the method are indicated for the cases: (i) the number of servers may be decreased as well as increased, (ii) the parameters are unknown but have known a priori distributions.

(Received July 14, 1960.)

571-126. W. A. Harris, Jr.: **Singular perturbations of two point boundary problems.**

This paper is concerned with showing the relationship of the solution of a "complete" boundary problem \( \Omega(\varepsilon)(d/dt)x(t,\varepsilon) = A(t,\varepsilon)x(t,\varepsilon), \ R(\varepsilon)x(a,\varepsilon) + S(\varepsilon) \cdot x(b,\varepsilon) = c(\varepsilon), \) as \( \varepsilon \to 0+ \) to the solution of a related "degenerate" problem obtained by formally setting \( \varepsilon = 0. \) \( \Omega, \ A, \ R, \) and \( \ S \) are square matrices of order \( n_1 + m, \) \( \Omega(\varepsilon) = \text{diag}(I_{h_1}, \cdots, I_{h_p}), \) \( I_j \) the unit matrix of order \( n_j, \) \( m = \sum_{j=2}^{p} h_j, \) \( h_j \) integers, \( 0 < h_2 < h_3 < \cdots < h_p; \) \( x(t,\varepsilon) \) is a vector of dimension \( n_1 + m \) with asymptotic expansion with respect to \( \varepsilon; \) and \( \varepsilon > 0. \) A "regular" problem is defined and it is shown that for a regular problem the solution of the complete boundary problem has a limit as \( \varepsilon \to 0^+ \) which satisfies the degenerate differential system and \( n_1 \) of the degenerate boundary conditions. (Received July 14, 1960.)

571-127. David Hertzig: **The structure of Frobenius algebraic groups.**

An algebraic group \( G \) is called a Frobenius algebraic group if it has a proper algebraic subgroup \( H \) which is its own normalizer in \( G \) and is disjoint from all its conjugates (i.e., the intersection of any two conjugates of \( H \) is the identity \( e \)). Let \( U \) denote the union of the conjugates of \( H \) and \( M = G - U \cup \{ e \}. \) For finite groups, using group characters, Frobenius proved \( M \) is a normal subgroup of \( G \) and recently J. Thompson proved \( M \) is nilpotent. In this paper it is proved for algebraic groups, \( M \) is a normal algebraic subgroup and \( G = H \rtimes M \) (semi-direct product). If \( H \) is not finite, \( M = G_{0u} \) is a connected unipotent group, hence nilpotent, and \( H_0 = T \) is a maximal torus of \( G \) (necessarily 1-dimensional). If \( H \) is finite, \( M_0 = G_0 \) is nilpotent and \((p \) denotes the characteristic of the ground field) either \( G_0 \) is a torus of dimension \( d = 0 \) (mod \( p - 1 \))
and $H$ is a $p$-group or $G_0$ is unipotent and the order of $H$ is prime to $p$. The result is essentially equivalent to the following theorems: A connected algebraic group admitting a rational automorphism of prime period with no fixed-points (other than $e$) is nilpotent. A rational automorphism of prime period of a semi-simple algebraic group has a fixed-point (other then $e$). (Received July 14, 1960.)

571-128. E. S. O'Keefe: A restriction on the primitive operations of primal algebras which implies strict independence.

The definitions are as in A. L. Foster, The identities of – and unique factorization within – classes of universal algebras, Math. Z. vol. 62 (1955) pp. 171 - 188. The following theorem is proved: Let $\{\mathcal{A}_i\}$ be a set of primal algebras and let no two of the algebras be isomorphic. If each primitive operation of each algebra is a transformation onto the algebra, then $\{\mathcal{A}_i\}$ is a primal cluster. (Received July 14, 1960.)

571-129. G. W. Patterson: A generalization of the ring of formal polynomials.

Consider a sequence of integers $\{\alpha_n\}$, and a sequence of sets of integers $\{R_n\}$ having the property that $R_n$ is a complete set of residue-class representatives mod $\beta_n$. Sequences belonging to $\bigotimes_{n=0}^{\infty} R_n$ are vectors in a unitary module over the ring of integers, $I$. The compositions are defined by a "méthode de balayage" that forces the $n$th element of the composition sequence to belong to $R_n$. Let $\overline{R_n}$ be the natural mapping from $I$ onto the residue-class $R_n$ and $r$ and $s$ be sequences belonging to $\bigotimes_{n=0}^{\infty} R_n$. Then the vector sum of the sequences $r$ and $s$, $\text{sesum}$, is defined recursively with the aid of the auxiliary sequence $\text{sesumca}$ (sequence sum carry): $\text{sesumca}_0(r,s) = 0$, $\text{sesumca}_{n+1}(r,s) = 0$, if $\beta_n = 0$, or $(1/\beta_n)(\sigma_n - \overline{R_n}(\sigma_n))$, if $\beta_n \neq 0$; $\text{sesum}_n(r,s) = R_n(\sigma_n)$, where $\sigma_n = r_n + s_n + \text{sesumca}_n(r,s)$. Similar definitions for the scalar product of an integer $g$ and a sequence $s$, and for the sequence product ($\text{sepro}$) of two sequences $r,s$, have been formulated. The compositions $\text{sesum}$ and $\text{sepro}$ constitute a ring containing a subring that is a homomorphic image of $I$. Under rather weak conditions the image is isomorphic. If $\alpha_n = 0$ uniformly we obtain the ring of formal polynomials over $I$. The motivation for studying these structures is that the subring is a model for computer algorithms on number-representation systems for $I$. (Received July 14, 1960.)
Using methods similar to those used in proving the corresponding result for the 3-sphere, $M_0$ (Abstract 555-26, Notices Amer. Math. Soc. vol. 6 (1959) p. 157), the following theorem is proved: If $M_n$ is the 3-sphere with $n \geq 0$ handles (i.e., the duplication of the solid torus of genus $n$) and $T$ and $T'$ are tori of genus $n$ regularly embedded in $M_n$ (i.e., $M_n - T$ and $M_n - T'$ both consist of two components whose closures are solid tori of genus $n$), then there is an isotopy of $M_n$ onto itself which deforms $T'$ onto $T$. Everything is considered from a semilinear point of view. This theorem extends somewhat further the solution of the problem of uniqueness (up to homeomorphism or isotopy) of representations of 3-manifolds via Heegaard Diagrams (see Papakyriakopoulos, Bull. Amer. Math. Soc. vol. 64 (1958) p. 329). (Received July 14, 1960.)

If $f(x)$ is a periodic function of bounded variation its Fourier series converges boundedly to $\left[\frac{f(x + 0) + f(x - 0)}{2}\right]$ and the convergence is uniform in any closed interval of continuity. This classical theorem is shown to follow simply and directly from the special case $g(0) = 0$, $g(x) = (\pi - x)/2$, $(0 < x < 2\pi)$; without the use of Bonnet's Theorem, the Riemann-Lebesgue Lemma, etc. One writes the partial sum as a Stieltjes integral and integrates by parts: $s_n(x) = f(x) - \int_0^\pi \frac{\pi}{2} h_n(t) d f(x + t)$ where $h_n(t)$ is the integral of the Dirichlet kernel. Since $h_n(t)$ is also $t/2$ plus the partial sum for $g(t)$, the general theorem follows from the easily proved special case. The overtly inductive nature of the proof is compared with similarly overtly inductive proofs of other leading theorems of analysis and the role of such an overtization in obtaining simplified proofs is discussed. (Received July 14, 1960.)

For notation, see Proc. Amer. Math. Soc. vol. 8 (1957) pp. 638-641. Sanov (Izv. Akad. Nauk SSSR, Ser. Mat. vol. 15 (1951) pp. 477-502) proved that for $u,v$, any elements of a free group, $F$, $(u,v,ap^a - 1)^{p^{\delta - a}} \in F(\delta)F_{ap^a + 1}$, $p$ a prime, $a,\delta$ positive integers. Sanov's proof involved an investigation of ideals in Lie Rings. In this paper, Sanov's relation is proved (and slightly generalized) for $a = 1,2$; $\delta$ arbitrary using P. Hall's Collection Process. The
method used produces other formulas, e.g. \((u,v,\mathbf{p}^2 - 1)\mathbf{p}^{a-1} \in \mathbb{F}_{2p^2-p}(\mathbf{p}^a)\); 
\((u,v,\mathbf{p}^3 - 1)\mathbf{p}^{a-1} \in \mathbb{F}_{2p^3-p^2}(\mathbf{p}^a)\). (Received July 14, 1960.)

571-133. W. C. Taylor: Gaussian probability measure defined by a generalized density.

Let \(\mathcal{S}\) be a hilbert space, \(\mathcal{S}'\) the normed conjugate space, and \(\mathcal{S}\) the conjugate of \(\mathcal{S}'\), consisting of linear functionals, not necessarily bounded, defined on \(\mathcal{S}'\). A generalized probability density \(\phi(u) = \exp - 2^{-1}(u,u), u \in \mathcal{S}\) was introduced by the author [Abstract 559-77, Notices Amer. Math. Soc. vol. 6 (1959)] as describing a sort of quasi probability distribution in \(\mathcal{S}\). It describes in fact an actual probability distribution in \(\mathcal{S}\) over the borel field of sets generated by half spaces, \(\{u \in \mathcal{S}: u(u') < k\} u' \in \mathcal{S}', k \) real. This probability measure space is faithfully represented by the measure similarly defined on the space of linear functionals defined on any complete subset \(\mathcal{V}'\) of \(\mathcal{S}'\). For separable spaces the measure is thus equivalent to that defined, with \(\mathcal{V}'\) an orthonormal sequence, by Friedrichs and Shapiro [Proc. Nat. Acad. Sci. vol. 43 (1957) pp. 336-338; also Integration of functionals, NYU Notes, 1957]. In discussion of Wiener's integral [N. Wiener, Acta Math. vol. 55 (1930) p. 117] the space is a function space and \(\mathcal{V}'\) consists of the values of the function. This material will appear first as Ballistic Research Laboratories Report No. 1109. (Received July 14, 1960.)


The aim is to establish a framework within which the various iteration methods for linear systems can be brought to bear on linear programming problems with notation \(\sum a_{1a}x_a = b_1, f = \sum a_{a}x_a\). This framework is an elimination method differing from the simplex method chiefly in the lack of artificial variables and in the phasing of the work, first as to cost, and second as to feasibility. The row column selection of the variable to be eliminated is based on a cost-quantity product. Duality considerations are prominent in the variable selection, in phasing the elimination, and in the test for incorrectly set problems. Indications point toward remarkable reliability in variable selection and electronic computer runs are to be used to study reliability on large scale problems. To replace the elimination feature by an iterative scheme the chief problems are a method of selecting an initial set of nonzero variables with initial values.
and a way to establish the total rate of change of the cost function in the direction of each zero variable. With notation $a_{ij}$ for the selected block and $A_{ij}$ for its inverse, the vectors $\sum_j a_{ij} b_j$ and $\sum_i A_{ij} c_i$ are computed by iteration (another illustration of duality) and $\sum_i A_{ij} c_i$ is scalared into remaining columns for more profitable rates of change. (Received July 14, 1960.)


Let $P_n$ be the number of minimal lattice paths from $(0,0)$ to $(ns, nr)$ which may touch but never rise above the line $y = rx/s$, where $r, s$ are positive integers, and let $Q_n$ be the number of such paths which touch only at $(ns, nr)$. Denote the generating functions by $P(x) = \sum_{n=1}^{\infty} P_n x^n$, $Q(x) = \sum_{n=1}^{\infty} Q_n x^n$. M. T. L. Bizley (J. Inst. Actuar. vol. 80 (1954) pp. 55-62) showed that $Q = P/(1 + P)$ and obtained expressions for $P$ and $Q$ in case $s = 1$. In general, $P = \prod_{i=1}^{s} (1 - u_i)^{-1/s} - 1$, where $u_i$ ranges through the $s$ solutions of $x = u^s(1 - u)^r$ closest to 0. This is a consequence for the case $a = 0$, $b = 0$ of the formula $\sum_{n=1}^{\infty} \binom{rn + sn + a + b, sn + a} x^n = \frac{1}{(s - ru_i - su_i)} u_i^a (1 - u_i)^b$ if $a, b$ are positive integers and $a < s$, $b < r$, and $\binom{n, m}$ is the binomial coefficient. This formula also helps to determine the expected number of returns to the origin during a random walk on the line when the probability of a step of length $r$ to the right is $p$ and to the left of length $s$ is $1 - p$, by taking $x = p^s (1 - p)^r$. (Received July 14, 1960.)


Let $C$ be the family of lattice-ordered rings each of which is isomorphic to the ring $C(X)$ of all continuous real-valued functions on some compact space $X$. A ring homomorphism $t: C(X) \rightarrow C(Y)$ is called complete if it is a complete lattice homomorphism (i.e. sup $f_i = f$ implies sup $t(f_i) = t(f)$). Let $\alpha$ be an infinite cardinal. Theorem 1: Given $C$ in $\mathcal{C}$ there exists $C_\alpha$ in $\mathcal{C}$, unique up to isomorphism, satisfying: (a) $C_\alpha$ is conditionally-$\alpha$-complete, (b) there is a complete isomorphism $t$ of $C$ into $C_\alpha$; and (c) if $E$ in $\mathcal{C}$ is conditionally-$\alpha$-complete and $s$ is a complete isomorphism of $t[C]$ into $E$, then $s$ can be extended to a complete isomorphism of $C_\alpha$ into $E$. Moreover, $C_\alpha$ is a sublattice of the Dedekind-completion of $C$. Theorem 2: Let $C$ be an element of $\mathcal{C}$. $C$ is $(\alpha, \alpha)$-distributive if and only if $C_\alpha$ is $(\alpha, \alpha)$-distributive. (For definition and related result see Pierce, Pacific J. Math. vol. 8 (1958) pp. 133-140.) Theorem 3: If $X$ is countably
paracompact and normal, then the Dedekind completion of \( C(X) \) is isomorphic to \( C(Y) \) for some \( Y \). Dilworth showed this in case \( X \) is compact (Trans. Amer. Math. Soc. vol. 68 (1950) pp. 427-438). (Received July 14, 1960.)

571-137. Volodmyr Bohun-Chudyniv: On loops represented by triplets.

The loops of \( 2^k \) order \((k > 3, k\text{-}nion \text{ loops})\), the basal units of which \( u_0, u_1, \ldots, u_{2^k} \) are satisfying the condition \( u_a^2 = -u_0, \ a = 1, \ldots, 2^k \) (1), can be represented by triplets, as in the case of Cayley loops of the 16th order. Question is whether there exist loops of other orders that can also be represented by triplets. The aims of this paper are: (Ia) To prove that if \( \lambda \), the order of a loop is equal only to \( \lambda = 2(3q + r) \) (2), where \( q \) is an arbitrary positive integer and \( r \) equals 1 or 2 and the order of all basal units equals 2 or 4, then the loops can be represented by triplets. (Ib) To show that if \( 3q + r = 2^\gamma (\gamma = 2) \), then one obtains \( k\text{-}nion \) loops. (II) To determine all types of loops of the 10th order \((q = 3, r = 2)\), and construct examples for \( \lambda = 14, 20 \) and others. (III) To show that if all basal units are of the order 4, and the number of units \( \lambda = 2(3q + r) \), we obtain generalized \( k\text{-}nion \) algebras. (Received July 15, 1960.)


Let \( H \) be the space of functions continuous in a (possibly multiply connected region) \( \Omega \) which are uniform limits of solutions of the equation \( \partial^2 f/\partial x \partial y = 0 \). The problem of approximating an arbitrary continuous function \( F \) on a compact set \( K \subset \Omega \) by members of \( H \) is reduced to a geometric question concerning the existence of certain closed polygonal paths in \( K \). This in turn leads to a systematic treatment of the approximate solution of functional equations of the form \( G(\phi(x)) = g(x) \). (Received July 15, 1960.)

571-139. C. J. Clark: Adjointness as a functor.

A new formulation of the functor concept is presented and in this formulation it is demonstrated that the notion of adjointness is a contravariant functor. This is accomplished by defining an adjoint function * with domain the linear space category and range the set category as follows. For any linear system \( Y, a(Y) \) is defined as the set of all functions \( y \) with domain the set associated with \( Y \) and with range contained in the field associated with \( Y \). If \( X \) and \( Y \) are linear spaces over the same field, then for any transformation \( f:X \to Y \), \((f:X \to Y)^*\)
\[ g: \alpha(Y) \rightarrow \alpha(X) \text{ where for all } y \in \alpha(Y), \ g(y) = y \circ f, \text{ the composition of } y \text{ and } f. \]

If for any system \( X \), \( s(X) \) is the set associated with \( X \) and \( i_s(X) \) is the identity function on \( s(X) \), then it is shown that \( (i_s(X)X \rightarrow X)* = i_s(\alpha(X)) : \alpha(X) \rightarrow \alpha(X) \) and \( (f:X \rightarrow Y)\circ(g:Y \rightarrow Z)* = (g \circ f:X \rightarrow Z)*. \) (Received July 15, 1960.)

571-140. D. J. Eustice: Collections of summability methods applied to orthogonal series.

The Riesz summability methods, \( R(\lambda, 1) \), are defined in terms of a positive, increasing function \( \lambda(n) \). The \( R(\lambda, 1) \) summability almost everywhere of orthogonal expansions of square integrable functions was investigated by Zygmund (Bull. de l'Academie Pol. Cracovie (1927) pp. 295-308) who showed a necessary and sufficient condition for the \( R(\lambda, 1) \) summability a.e. of the sequence of partial sums of an orthogonal expansion was the convergence a.e. of a subsequence of the sequence of partial sums determined by the function \( \lambda(n) \). This result is used to prove the following: Theorem. Given a countable collection of Riesz methods, there exists another Riesz method which sums every orthogonal series summable by a method in the collection. Corollary. Given a countable collection of orthonormal systems, there exists a Riesz method effective for the expansion of square integrable functions in systems of this collection. The corresponding results for Nörlund summability are also investigated. (Received July 15, 1960.)

571-141. John Greever and M. F. Tinsley: Stationary points for solvable transformation groups.

If \( G \) is a group of homeomorphisms operating on a space \( X \), then the set \( F_G \) of stationary points for \( G \) consists of those points in \( X \) which are left fixed by every element of \( G \). The following results are extensions of earlier ones of Greever (Duke Math. J. vol. 27 (1960) pp. 163-170): Theorem. Let \( X \) be a finite-dimensional compact Hausdorff space with trivial integral Čech cohomology groups and let \( G \) be a solvable group of homeomorphisms operating on \( X \) in such a fashion that \( F_H \) has finitely generated integral Čech cohomology groups whenever \( H \) is a subgroup of \( G \). Then, for \( F_G \) to be empty, the order of \( G \) must be either 72 or at least 108. Corollary. Let \( X \) be the polyhedron over a finite simplicial complex with trivial integral homology groups and let \( G \) be a solvable group of homeomorphisms operating simplicially on \( X \). Then, for \( F_G \) to be empty, either the order of \( G \) must be at least 108 of \( G \) must be a group of
order 72 isomorphic to the direct product of an alternating group with a symmetric group. (Received July 15, 1960.)

571-142. Nickolas Heerema: Derivations and embeddings in power series rings.

Let $R$ be an integral domain having nonzero characteristic $p$ and let $R_F$ be its quotient field. If $S = \{s_\alpha\}_{\alpha \in \mathcal{J}}$ is a $p$-basis for $R_F$ chosen from $R$ where $\mathcal{J}$ is an appropriate indexing set and $f$ is an arbitrary function whose domain is the Cartesian product of $\mathcal{J}$ and the positive integers $\mathcal{J}$ and whose range is $R_F$ then $f$ determines a sequence of derivation $\{\tau_n\}_{n \in \mathcal{J}}$ of $R$ into $R_F$ and an embedding sequence $\{\pi_n\}_{n \in \mathcal{J}}$ of $R$ into $R_F$ by the condition $\tau_n(s_\alpha) = \pi_n(s_\alpha) = f(\alpha, n)$. (See Derivations and embeddings of a field in its power series ring, Proc. Amer. Math. Soc., vol. 11 (1960) pp. 188-194). This provides a biunique correspondence between all sequences of derivations of $R$ into $R_F$ and all embedding sequences of $R$ into $R_F$. This in turn establishes a connection between derivations of $R$ into $R_F$ and embeddings of $R$ in $R_F[[x]]$. If $R^p(S) = R$ then the embeddings of $R$ in $R[[x]]$ are biuniquely determined by sequences of derivations of $R$ into $R$. These results can be extended to the case in which $R_F[[x]]$ and $R[[x]]$ are replaced by $R_F[[x_1, \ldots, x_n]]$ and $R[[x_1, \ldots, x_n]]$.

(Received July 15, 1960.)

571-143. J. M. Irwin: On high subgroups of abelian torsion groups.

Let $G^1$ be the subgroup of elements of infinite height in an Abelian group $G$. A subgroup $H$ of $G$ maximal with respect to disjointness from $G^1$ will be called a high subgroup of $G$. A group $E$ minimal divisible among those groups containing $G$ will be called a divisible hull of $G$. In a recent paper (see Abstract 566-19 Notices Amer. Math. Soc., vol. 7 (1960) p. 249) it has been shown that high subgroups of Abelian torsion groups are pure. Further results about high subgroups of Abelian torsion groups are Theorem 1. Let $E$ be a divisible hull of a torsion group $G$ with $D$ a divisible hull of $G^1$ in $E$. Then $H$ is a high subgroup of $G$ iff $H = A \cap G$ where $A \subseteq D = E$. Theorem 2. Let $H$ and $K$ be high subgroups of an Abelian primary group $G$. Then $H$ and $K$ have the same Ulm invariants. (Received July 15, 1960.)

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For the Riccati differential equation $zw' = zA(z) + B(z)w + C(z)w^2$, in which $A(z)$, $B(z)$, $C(z)$ are regular in a neighborhood of $z = 0$, there is a solution $w(z)$, regular in a neighborhood of $z = 0$, with $w(0) = 0$, when $B(0)$ is not an integer. A step-by-step process is given for computing the C-fraction expansion of this solution, and convergence of the C-fraction to the solution is discussed. In particular, the continued fraction of Gauss, and its confluenes, are derived as solutions of special Riccati equations. Restrictions on the coefficients $A(z)$, $B(z)$, $C(z)$ are found so that the cited solution of the given Riccati equation has a representation as a Stieltjes transform. This is the case, for example, when the power series coefficients of $A$, $B$, $C$ are positive and $0 < B(0) < 1$. (Received July 15, 1960.)

571-145. J. P. Roth: A combinatorial cubical homology theory.

This paper gives a constructive definition of a cubical theory of homology. It is akin to Alexander's simplicial theory in that it is combinatorial (as contrasted with Serre's "singular" cubical theory). Face-and-coface-maps are defined on the set $K$, which is the cubical complex. These inducing homomorphisms of the chain and cochain groups of $K$ are used to define the boundary and coboundary operators, $\partial, S$ and such that $\partial^2 = 0$, $S^2 = 0$, which in turn define the homology and cohomology groups $H(K)$, $H^*(K)$. Definitions (very simple) of cup and cap products on the set $K$ define corresponding operators in $H^*(K), H(K)$. Subdivision is easily constructed. This is the first phase of an approach toward the construction of a framework suitable for the treatment of certain problems of classical physics, e.g., the equations of Maxwell. (Received July 15, 1960.)

571-146. D. A. Storvick: Cluster sets of pseudo-meromorphic functions.

Theorems concerning the relationship between the cluster set at a point and the behaviour of the function under various modes of approach to neighboring boundary points are proved. It is proved that if $w = f(z)$ is a bounded pseudoanalytic function in $|z| < 1$, the cluster set of $f(z)$ at $e^{i\theta}$ is contained in the closure of the convex hull of such radial limits as exist on any arc of $|z| = 1$ containing $e^{i\theta}$ as an interior point. This extends a result of Carathéodory

571-147. Minoru Urabe: Potential forces which yield periodic motions of a fixed period.

Consider the equation (1) $\ddot{x} + g(x) = 0$ where $g(x) \in C^1_x$. A necessary and sufficient condition has been determined in order that (1) has oscillations around $x = \dot{x} = 0$ of constant period $\omega > 0$. The nec- and suff- condition states that $g(x)$ must be of the form $g(x) = (2\pi/\omega)X[1 + S(X) + T(X)]^{-1}$ where $X^2 = \int g(u)du$, $xX \equiv 0$, where $S(X)$ is an arbitrary odd function with $S(0) = 0$, $S(X) \in C^1_x$, $xS(x) \in C^1_x$, and where $T(X)$ is an arbitrary even function with $T(0) = 0$, $T(X) \in C^1_x$, $xT(X) \in C^1_x$, and $\int_0^{2\pi} T(R \cos u) du = 0$ for all $R > 0$. If $g(x)$ is analytic, then $T \equiv 0$. (Received July 15, 1960.)

571-148. E. A. Walker: Direct summands of direct products of Abelian groups.

In a recent paper (Direct summands of unrestricted direct sums of Abelian groups, Archiv. der Math. vol. X Fasc. 6 (1959)) Baumslag and Blackburn investigated direct summands of direct products of Abelian groups, but left open the question of whether or not the direct product of cyclic groups of orders $p, p^2, p^3, \ldots$, has a nonzero torsion free direct summand. We show that it does have such a summand, and indeed a maximum one. This result is generalized to direct products of cotorsion groups, and several corollaries are noted. (A group $A$ is cotorsion if $\text{Ext}(X,A) = 0$ for all torsion free $X$.) The methods used are homological. (Received July 15, 1960.)

571-149. Esther Seiden: On the maximum number of points no three on one line in a projective plane of order 10.

It is shown that a plane of order 10 could not include an oval consisting of 12 points. The proof consists in showing that if such an oval would exist then the remaining 99 points would have to form a PBIB which is shown not to exist. It seems conceivable that this fact implies the nonexistence of a plane.
of order 10. On the other hand, if such a plane nevertheless does exist, then the nonexistence of an oval consisting of the maximum possible number of points may prove useful in the investigation of non-Desarguessian planes.

(Received July 15, 1960.)

571-150. C. M. Petty: A geometrical approach to the second order linear differential equation.

Consider the real solutions of $\ddot{u} + R(t)u = 0$ where $R(t)$ is continuous for all $t$. Let $u_1(t), u_2(t)$, be solutions with initial conditions at $t_0$ given by $u_1(t_0) = 0, \dot{u}_1(t_0) = 1, u_2(t_0) = 1, \dot{u}_2(t_0) = 0$. If the equation is oscillatory as $t \to +\infty$, $\lambda(t), m(t), a(t)$ are defined by $\lambda(t_0) = t_1 - t_0, m(t_0) = t_2 - t_0, a(t_0) = u_1(t_2)$, where $t_1$ is the smallest zero of $u_1(t)$ exceeding $t_0$. A number of identities involving these functions and their derivatives can be derived, i.e., $1 + \dot{m}(t) = a^2(t)R(t)$. A geometrical characterization is given for all equations for which either $\lambda(t)$ or $a(t)$ is a constant. In particular if $\lambda(t)$ is constant and $R(t)$ is non-negative, then the solutions (except for normalization) are Minkowskian sine and cosine functions in a related Minkowski plane. If $a(t)$ is constant, then in addition to the above, the Minkowski unit circle is a Radon or self-conjugate curve.

(Received July 11, 1960.)
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Time __________________________
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<th>Age</th>
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