THE AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by John W. Green and Gordon L. Walker

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MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tbody>
<tr>
<td>598</td>
<td>February 23, 1963</td>
<td>New York, New York</td>
<td>Jan, 9</td>
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<tr>
<td>599</td>
<td>April 19-20, 1963</td>
<td>Chicago, Illinois</td>
<td>Mar, 5</td>
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<td>600</td>
<td>April 26-27, 1963</td>
<td>University Park, New Mexico</td>
<td>Mar, 5</td>
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<td>601</td>
<td>April 29 - May 2, 1963</td>
<td>New York, New York</td>
<td>Mar, 5</td>
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<td>602</td>
<td>June 22, 1963</td>
<td>Bellingham, Washington</td>
<td>May 8</td>
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<tr>
<td>603</td>
<td>August 26-30, 1963</td>
<td>Boulder, Colorado</td>
<td>July 5</td>
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<tr>
<td></td>
<td>(68th Summer Meeting)</td>
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<td></td>
<td>January 20-24, 1964</td>
<td>Miami, Florida</td>
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<tr>
<td></td>
<td>(70th Annual Meeting)</td>
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<tr>
<td></td>
<td>August 30 - September 4, 1964</td>
<td>Amherst, Massachusetts</td>
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<tr>
<td></td>
<td>(69th Summer Meeting)</td>
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<tr>
<td></td>
<td>January, 1965</td>
<td>Denver, Colorado</td>
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<tr>
<td></td>
<td>(71st Annual Meeting)</td>
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<tr>
<td></td>
<td>August 30 - September 3, 1965</td>
<td>Ithaca, New York</td>
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<tr>
<td></td>
<td>(70th Summer Meeting)</td>
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<tr>
<td></td>
<td>August, 1966</td>
<td>New Brunswick, New Jersey</td>
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</table>

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are January 2, and February 26, 1963.

The NOTICES of the American Mathematical Society is published by the Society in January, February, April, June, August, October and November. Price per annual volume is $7.00. Price per copy, $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (not available before 1958), and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island.

Second-class postage paid at Providence, Rhode Island, and additional mailing offices. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (Sec. 34, 21, P., L., and R.). Accepted for mailing at the special rate of postage provided for in section 34.40, paragraph (d).
The sixty-ninth Annual meeting of the American Mathematical Society will be held at the University of California, Berkeley, in conjunction with the Annual meetings of the Mathematical Association of America and the Association for Symbolic Logic. The Society will meet from Thursday, January 24, 1963 until Sunday, January 27, 1963. The Mathematical Association of America will meet on Saturday through Monday, January 26-28, and the Association for Symbolic Logic will meet on Saturday, January 26. All sessions of these meetings will be held in Wheeler Hall and Dwinelle Hall on the Berkeley Campus of the University of California.

The thirty-sixth Josiah Willard Gibbs Lecture will be delivered by Professor Claude E. Shannon of the Massachusetts Institute of Technology at 8:00 P.M. on Thursday, January 24 in the Auditorium of Wheeler Hall. The title of Professor Shannon's talk is "Information theory".

Professor E. J. McShane of the University of Virginia will deliver the Presidential Address at 9:00 A.M. on Friday, January 25 in Wheeler Hall Auditorium. He will speak on "Integrals devised for special purposes".

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, hour addresses will be given by Professor E. M. Stein of the University of Chicago and the Institute for Advanced Study, and by Professor John Thompson of the University of Chicago. Professor Stein will speak at 9:00 A.M. Thursday, January 24 on "Harmonic functions and Fourier analysis in several variables". Professor Thompson's talk, entitled "Some results about finite groups", will be given at 2:00 P.M. on Friday, January 25. Both of these addresses will be presented in the Auditorium of Wheeler Hall.

A special feature of this meeting will be five sessions of selected twenty minute papers. The first Special Session, devoted to Algebra, will be held at 3:00 P.M. Thursday in Room 155 Dwinelle Hall. This session was arranged by Professor L. J. Paige, and it consists of papers by I. N. Herstein, D. K. Harrison, S. U. Chase and Alex Rosenberg, Nathan Jacobson, and A. A. Sage. The Special Session on Algebraic Topology will begin at 10:15 A.M. on Friday in Room 155 Dwinelle Hall, Professor E. H. Spanier has arranged the program of this Session, consisting of papers by F. A. Raymond, F. P. Peterson and E. H. Brown, Jr., A. I. Weinzweig, William Browder, and P. E. Thomas. The Special Session on Functions of One or Several Complex Variables will also be in Room 155 Dwinelle Hall, beginning at 3:15 P.M. on Friday. Professor Halsey Royden has arranged a program of papers by A. E. Hurd, Nachman Aronszajn, Simon Hellerstein and L. A. Rubel, M. L. Weiss and Leo Sario. The last two Special Sessions of twenty minute papers will be on Saturday at 2:00 P.M. The Session on General Topology, arranged by Professor R. H. Bing will be in Room 155 Dwinelle Hall. Papers by R. F. Williams, D. S. Gillman and J. M. Martin, James Kister, Prabir Roy, and Melvin Henriksen and Meyer Jerison are featured in this session. The Session on Asymptotic Methods and Singular Perturbations in Analysis will be held in Room 145 Dwinelle Hall. Professor Arthur Erdélyi has arranged a program of papers by F. E. Browder, Y. Sibuya,
There will be sessions for contributed ten minute papers on Thursday at 10:15 A.M. and 3:00 P.M., on Friday at 10:15 A.M. and 3:15 P.M., on Saturday at 2:00 P.M., and on Sunday at 2:00 P.M. All of these sessions will be in Rooms 145, 155, 15, 23, and 33 Dwinelle Hall.

The Business Meeting of the Society will be held at 1:30 P.M. on Thursday, January 24 in Wheeler Hall Auditorium. The Council of the Society will meet at 4:00 P.M. on Wednesday, January 23 in Room 406-8 (the Tan Oak Room) of the Student Union Building.

The Mathematical Sciences Employment Register will be maintained from 9:00 A.M. to 5:00 P.M. on Friday, Saturday and Sunday in the East Ballroom of the Student Union Building.

ENTERTAINMENT AND RECREATION

There will be a tea sponsored by the University of California on Saturday from 4:30 P.M. to 6:30 P.M. in the Pauley Ballroom of the Student Union Building. On Saturday evening at 8:15 P.M. all persons attending the meetings, and their families, are cordially invited to a recital by the distinguished pianist Irene Schreier in Hertz Hall on the Berkeley Campus. This recital is sponsored by a group of local mathematicians.

Everyone attending the meetings is invited for wine tasting on Friday from 4:30 P.M. to 6:30 P.M. at the Claremont Hotel.

REGISTRATION

The Registration Headquarters for this meeting will be in the Student Union Building. The Registration Desk, located on the main floor of Student Union, will be open from 2:00 P.M. to 8:00 P.M. on Wednesday, January 23, from 9:00 A.M. to 5:00 P.M. on Thursday through Sunday, and from 9:00 A.M. to 2:00 P.M. on Monday. Members attending the meetings are requested to register as soon as possible after arrival. The registration fee is $2.00 for members of participating organizations. Another 50 cents is charged for the first nonmember of a member's family. Additional nonmembers in a member's family pay no registration fee. Students are also exempt from the registration fee. For nonmembers who are not in any of these categories the fee is $5.00.

Book and teaching machine exhibits will be located in the main lobby of the Student Union Building.

ROOMS AND MEALS

Accommodations for the meeting will be handled by the Housing Bureau of the Berkeley Chamber of Commerce. The reservation form on the back cover of these NOTICES should be used in requesting accommodations. The Housing Bureau will make reservations as nearly as possible in accordance with the member's request at one of the motels or hotels in the list below. A deposit fee of $10.00 for each room desired should be sent with the reservation form to the Housing Bureau, Berkeley Chamber of Commerce. Note that two of the hotels listed below are in San Francisco and one is in Oakland; persons who wish to stay in San Francisco or Oakland should indicate this on the reservation form.

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<th>Motel/Hotel</th>
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<td></td>
<td>Dble + Sgle w/bath</td>
<td>9.00</td>
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<td></td>
<td>2 Dbles w/bath</td>
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<td>1761 University Berkeley</td>
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<td>Twins</td>
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<td>Family cottages</td>
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<td>Kitchen Apartments</td>
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<td>Twin + Dble w/conn, bath</td>
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<td>Suites</td>
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<td>Dble + Dble w/2 baths</td>
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<td>Hotel Leamington</td>
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<td>19th and Franklin</td>
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<td>Oakland</td>
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<tr>
<td>San Francisco</td>
<td>Twins</td>
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The Dining Commons of the University of California (adjacent to the Student Union) will be open on Thursday, Friday, and Monday for meals as follows: from 6:45 A.M. to noon for breakfast, from 11:30 A.M. to 1:00 P.M. for lunch, and from 4:30 P.M. to 7:00 P.M. for dinner. On Saturday, January 26, the Dining Commons will be open only for lunch (from 11:45 A.M. to 1:15 P.M.) and for dinner (from 6:00 P.M. to 7:30 P.M.); on Sunday, January 27, it will be open for lunch only (from 11:45 A.M. until 1:15 P.M.).

**TRAVEL INFORMATION**

Railways serving Berkeley and San Francisco include the Santa Fe, the Southern Pacific, and the Western Pacific Railroads. Most of the major transcontinental airlines (for example, American, Delta, TWA, and United) have flights to the San Francisco International Airport. The Oakland International Airport is somewhat closer to Berkeley than the San Francisco Airport, but it is served by fewer airlines. There is bus, limousine and helicopter service from the San Francisco and Oakland airports to Berkeley and downtown San Francisco. Reservations on helicopter flights should be made along with the initial flight reservations, since by doing so passengers on the major U.S. Airlines can obtain a considerable fare reduction on the helicopter ticket. Individual airlines or travel agents should be consulted for specific information.

Arrangements have been made to run special busses from the Hotel Canterbury in San Francisco to the Student Union Building on the University of California Campus each morning, with return trips in the afternoon. It is expected that special busses will also run between the campus and the Claremont Hotel.

There is public bus service between Berkeley and San Francisco throughout the day, beginning at about 6:00 A.M. and ending at approximately 2:00 A.M. To reach Berkeley from San Francisco, catch the A-C Transit "F" bus at the Transbay Transit Terminal, and get off at Shattuck and University Avenues in Berkeley. The return trip can also be made on the "F" bus. To reach the campus from Oakland, take either an A-C Bus No. 51 on Broadway Avenue, or an A-C Bus No. 40 on Telegraph Avenue, and get off at Bancroft Avenue in Berkeley.

It will be possible for people attending the meetings to park in any of the following University parking lots for 25 cents per entry: Lot 2 at Fulton and Bancroft (entrance on Bancroft), Structure C at Channing and Ellsworth, Lot 4 on Channing between Telegraph and Bowditch, and Structure D at Channing and College. On Sunday it is possible to park without charge at these lots or on the campus in any places not reserved for A stickers only.

**COMMUNICATIONS**

Mail and telegrams for persons attending the meetings should be addressed in care of the American Mathematical Society, University of California, Berkeley 4, California.

**Committee on Arrangements**

R. M. Robinson, Chairman
H. L. Alder R. S. Pierce
W. G. Bade H. L. Royden
L. A. Henkin G. L. Walker
Mrs. Emma Lehmer E. H. Whitmore
## TIME TABLE
(Pacific Standard Time)

<table>
<thead>
<tr>
<th>WEDNESDAY</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
<th>Association for Symbolic Logic</th>
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<tr>
<td>January 23</td>
<td>2:00 P.M. - 8:00 P.M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
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<td>Council Meeting: 406-8, Student Union Building</td>
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<td>January 24</td>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
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<td>9:00 A.M.</td>
<td>Invited Address. E. M. Stein Wheeler Hall Auditorium</td>
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<td>10:15 A.M.</td>
<td>Session on Analysis 145 Dwinelle Hall</td>
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<td>Session on Analysis 155 Dwinelle Hall</td>
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<td>Session on Theory of Numbers 15 Dwinelle Hall</td>
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<td>Session on Topology 23 Dwinelle Hall</td>
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<td>Session on Geometry 33 Dwinelle Hall</td>
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<td>1:30 P.M.</td>
<td>Business Meeting Wheeler Hall Auditorium</td>
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<tr>
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<td>3:00 P.M.</td>
<td>Special Session on Algebra. Herstein, Harrison, Chase, Jacobson, Sagle 155 Dwinelle Hall</td>
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<td>3:00 P.M.</td>
<td>Session on Logic 145 Dwinelle Hall</td>
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<td>Session on Topology 15 Dwinelle Hall</td>
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<td>Session on Analysis 23 Dwinelle Hall</td>
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<td>Session on Analysis 33 Dwinelle Hall</td>
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<tr>
<td></td>
<td>8:00 P.M.</td>
<td>Josiah Willard Gibbs Lecture. Claude Shannon Wheeler Hall Auditorium</td>
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<td>January 25</td>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
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<td>EMPLOYMENT REGISTER--EAST BALLROOM, STUDENT UNION BUILDING</td>
<td>EXHIBITS--LOBBY, STUDENT UNION BUILDING</td>
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<td>9:00 A.M.</td>
<td>Presidential Address. E. J. McShane Wheeler Hall Auditorium</td>
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<tr>
<td></td>
<td>10:15 A.M.</td>
<td>Governor's Meeting: 406-8 Student Union Building</td>
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**TIME TABLE**
(Pacific Standard Time)

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<th>Mathematical Association of America</th>
<th>Association for Symbolic Logic</th>
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<tbody>
<tr>
<td>10:15 A.M.</td>
<td>Special Session on Algebraic Topology. Raymond, Peterson, Weinzweig, Browder, Thomas</td>
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<tr>
<td>10:15 A.M.</td>
<td>Session on Algebra</td>
<td>145 Dwinelle Hall</td>
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<tr>
<td>10:30 A.M.</td>
<td>Session on Applied Mathematics</td>
<td>15 Dwinelle Hall</td>
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<tr>
<td>10:15 A.M.</td>
<td>Session on Analysis</td>
<td>23 Dwinelle Hall</td>
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<tr>
<td>2:00 P.M.</td>
<td>Invited Address. John Thompson</td>
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<td>Wheeler Hall Auditorium</td>
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<tr>
<td>3:15 P.M.</td>
<td>Special Session on Functions of Complex Variables. Hurd, Aronszajn, Hellerstein, Weiss, Sario</td>
<td>155 Dwinelle Hall</td>
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<tr>
<td>3:15 P.M.</td>
<td>Session on Probability and Statistics</td>
<td>145 Dwinelle Hall</td>
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<td>3:15 P.M.</td>
<td>Session on Algebra</td>
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<td>Session on Algebra</td>
<td>23 Dwinelle Hall</td>
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<tr>
<td>4:30 P.M.</td>
<td>WINE TASTING--CLAREMONT HOTEL</td>
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<thead>
<tr>
<th>SATURDAY</th>
<th>January 26</th>
<th>Society</th>
<th>Association</th>
<th>ASL</th>
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<tbody>
<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
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<tr>
<td>9:00 A.M.</td>
<td>EMPLOYMENT REGISTER--EAST BALLROOM, STUDENT UNION BUILDING</td>
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<tr>
<td>9:00 A.M.</td>
<td>EXHIBITS--LOBBY, STUDENT UNION BUILDING</td>
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<tr>
<td>9:00 A.M.</td>
<td>Contributed Papers</td>
<td>155 Dwinelle Hall</td>
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<tr>
<td>10:30 A.M.</td>
<td>Invited Address Andrzei Ehrenfeucht</td>
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## TIME TABLE
(Pacific Standard Time)

<table>
<thead>
<tr>
<th>SATURDAY</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
<th>Association for Symbolic Logic</th>
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<tbody>
<tr>
<td>January 26</td>
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<tr>
<td>11:30 A.M.</td>
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<td>Council meeting. Dining Commons</td>
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<tr>
<td>1:00 P.M.</td>
<td></td>
<td>Nicolet Animated Geometry Films</td>
<td>Presentation of Award by the United States Steel Foundation 11 Wheeler Hall</td>
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<tr>
<td>1:50 P.M.</td>
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<td>Invited Address. Solomon Feferman 11 Wheeler Hall</td>
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<tr>
<td>2:00 P.M.</td>
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<tr>
<td>2:00 P.M.</td>
<td>Special Session on General Topology. Williams, Gillman, Kister, Roy, Henriksen. 155 Dwinelle Hall</td>
<td>Special Session on Asymptotic Methods and Singular Perturbations in Analysis. Browder, Sibuya, Wasow, Wyman, Fulks 145 Dwinelle Hall</td>
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<tr>
<td>2:00 P.M.</td>
<td>Session on Analysis 15 Dwinelle Hall</td>
<td>Session on Applied Mathematics 23 Dwinelle Hall</td>
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<tr>
<td>3:15 P.M.</td>
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<td>Contributed Papers 11 Wheeler Hall</td>
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<tr>
<td>4:30 P.M.</td>
<td>TEA--PAULEY BALLROOM, STUDENT UNION BUILDING</td>
<td>Film: The Kakeya Problem Wheeler Hall Auditorium</td>
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<tr>
<td>7:00 P.M.</td>
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<tr>
<td>8:15 P.M.</td>
<td>RECITAL, IRENE SCHREIR, HERTZ HALL</td>
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<th>January 27</th>
<th>Society</th>
<th>Association</th>
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<tbody>
<tr>
<td>9:00 A.M. - 5:00 P.M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
<td>Employment Register--East Ballroom, Student Union Building</td>
<td>Exhibits--Lobby, Student Union Building</td>
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<tr>
<td>9:10 A.M.</td>
<td>Address: Saunders MacLane Wheeler Hall Auditorium</td>
<td>Business Meeting; Distinguished Service Award, Chauvenet Prize Wheeler Hall Auditorium</td>
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<tr>
<td>10:10 A.M.</td>
<td>Business Meeting; Distinguished Service Award, Chauvenet Prize Wheeler Hall Auditorium</td>
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<td>1:00 P. M.</td>
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<td>Address: George Pólya Wheeler Hall Auditorium</td>
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<tr>
<td>1:00 P. M.</td>
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<td>Film: Mathematical Induction Wheeler Hall Auditorium</td>
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<td>2:00 P. M.</td>
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<td>Meeting, Council of the Conference Board of the Mathematical Sciences— Stephens Room</td>
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<tr>
<td>2:00 P. M.</td>
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<td>Session on Analysis 145 Dwinelle Hall</td>
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<td>Session on Differential Geometry and Topology 155 Dwinelle Hall</td>
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<td>Session on Analysis 15 Dwinelle Hall</td>
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<tr>
<td>7:00 P. M.</td>
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<td>Film: What is an integral? Wheeler Hall Auditorium</td>
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<td>8:15 P. M.</td>
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<td>Film: Theory of limits. Wheeler Hall Auditorium</td>
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<th>January 28</th>
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<tr>
<td>9:00 A. M. - 2:00 P. M.</td>
<td>REGISTRATION--MAIN FLOOR, STUDENT UNION BUILDING</td>
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<tr>
<td>9:10 A. M.</td>
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<td>Mathematics in non-physical sciences. J. G. Kemeny Wheeler Hall Auditorium</td>
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<tr>
<td>10:10 A. M.</td>
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<td>Mathematics in non-physical sciences. Patrick Suppes Wheeler Hall Auditorium</td>
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<td>11:10 A. M.</td>
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<td>Mathematics in non-physical sciences. G. B. Dantzig Wheeler Hall Auditorium</td>
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<tr>
<td>2:10 P. M.</td>
<td></td>
<td>Combinatorial Analysis. H. W. Kuhn Wheeler Hall Auditorium</td>
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<td>3:10 P. M.</td>
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<td>Combinatorial Analysis. D. R. Fulkerson Wheeler Hall Auditorium</td>
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<td>4:10 P. M.</td>
<td></td>
<td>Combinatorial Analysis. A. J. Hoffman Wheeler Hall Auditorium</td>
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PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the ordinary sessions is ten minutes. The papers are scheduled at 15 minute intervals so that listeners can circulate between different sessions. To maintain the schedule, the time limit will be strictly enforced.

THURSDAY, 9:00 A.M.

Invited Address, Wheeler Hall Auditorium

Harmonic functions and Fourier analysis in several variables
Professor E. M. Stein, University of Chicago and the Institute for Advanced Study

THURSDAY, 10:15 A.M.

Session on Analysis, Room 145, Dwinelle Hall
10:15 - 10:25
(1) Bounds for the invariant B-areas of a family of surfaces in the space of two complex variables
Mr. K. T. Hahn, Stanford University (597-205)
10:30 - 10:40
(2) Three circle theorems for harmonic continuation
Mr. Keith Miller, Rice University (597-174)
(Introduced by Professor Guy Johnson, Jr.)
10:45 - 10:55
(3) The order of certain real-valued functions defined in the unit disk
Professor D. C. Rung, Pennsylvania State University (597-15)
11:00 - 11:10
(4) A quasi-analyticity condition. Preliminary report
Professor J. W. Neuberger, University of Tennessee (597-3)
11:15 - 11:25
(5) A representation theorem for the Laplace transform
Professor J. T. White, University of Kansas (597-175)
11:30 - 11:40
(6) Orthogonal polynomials: Variable-signed weight functions
Professor G. W. Struble, University of Oregon (597-63)
11:45 - 11:55
(7) Some generalizations of the Cantor Lebesgue theorem
Mr. T. E. Mott, State University College (597-18)
12:00 - 12:10
(8) On an inequality for finite sums, and generalizations of an inequality of Kantorovich
Professor J. B. Diaz*, University of Maryland and Dr. F. T. Metcalf, U. S. Naval Ordnance Laboratory, White Oak, Maryland (597-117)

THURSDAY, 10:15 A.M.

Session on Analysis, Room 155, Dwinelle Hall
10:15 - 10:25
(9) Trace formulas for powers of a Sturm-Liouville operator
Professor R. C. Gilbert, University of California, Riverside and Professor V. A. Kramer*, Mathematics Research Center, University of Wisconsin (597-188)

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
10:30 - 10:40
(10) On bounded integral transformations with positive kernel
Professor Emilio Gagliardo, University of Kansas (597-35)

10:45 - 10:55
(11) An existence theorem for a nonlinear functional equation in Hilbert space,
Preliminary report
Professor G. J. Minty, University of Michigan (597-10)

11:00 - 11:10
(12) On existence and uniqueness of almost periodic solutions for nonlinear differential equations
Professor Witold Bogdanowicz, Georgetown University (597-105)

11:15 - 11:25
(13) Numerical range and spectral sets
Professor Morris Schreiber, Cornell University and New York University (597-161)

11:30 - 11:40
(14) Spectral permanence of scalar operators
Professor G. L. Krabbe, Purdue University (597-137)

11:45 - 11:55
(15) On the characterization of spectral operators, II
Dr. Shmuel Kantorovitz, Princeton University (597-80)

12:00 - 12:10
(16) On the representation of doubly stochastic operators
Mr. J. V. Ryff, Harvard University (597-111)

THURSDAY, 10:15 A.M.

Session on the Theory of Numbers, Room 15, Dwinelle Hall

10:15 - 10:25
(17) On a conjecture of Erdos
Dr. Ronald Graham, Bell Telephone Laboratories, Murray Hill, New Jersey (597-7)

10:30 - 10:40
(18) Prime-like sequences generated by a sieve process, II
Professor W. E. Briggs, University of Colorado (597-185)

10:45 - 10:55
(19) Approximate functional equations for a class of Dirichlet series
Professor Abe Sklar and Mrs. Marjorie Katz*, Illinois Institute of Technology (597-192)

11:00 - 11:10
(20) On Cohen's trigonometric sums
Professor M. V. Subbarao, University of Missouri (597-206)

11:15 - 11:25
(21) Integral criteria for prime power residuacity
Professor J. B. Muskat, Computing and Data Processing Center, University of Pittsburgh (597-128)

11:30 - 11:40
(22) A metric problem on transcendental numbers
Professor B. W. Volkmann, University of Utah and University of Mainz (597-127)

11:45 - 11:55
(23) Matrices associated with perfect difference sets, Preliminary report
Dr. B. W. Jones, University of Colorado (597-102)

12:00 - 12:10
(24) Number theory and integral linear programming, Preliminary report
Professor M. S. Cheema, University of Arizona (597-69)
THURSDAY, 10:15 A.M.

Session on Topology, Room 23, Dwinelle Hall
10:15 - 10:25
(25) Extended topology: Moore-Smith convergence
Professor D. V. Thampuran, University of Florida (597-23)
(Introduced by A. R. Amir-Moëz)

10:30 - 10:40
(26) Extended topology: Structure of isotonic functions
Professor P. C. Hammer, Physics Department, University of California, San Diego (597-204)

10:45 - 10:55
(27) Metrizability, compactness and paracompactness in Moore spaces. Preliminary report
Professor R. W. Heath, University of Georgia (597-159)

11:00 - 11:10
(28) Categories of certain minimal topological spaces
Professor M. P. Berri, Tulane University (597-55)

11:15 - 11:25
(29) Generalized metric spaces
Professor R. E. DeMarr*, University of Washington and Dr. Isidore Fleischer, University of Oslo, Norway (597-189)

11:30 - 11:40
(30) A-metrizability: A peculiar generalization of semi-metrizability
Professor D. E. Sanderson, Michigan State University and Mr. B. T. Sims*, San Jose State College (597-19)

11:45 - 11:55
(31) On metrizability
Professor H. H. Corson and Professor E. A. Michael*, University of Washington (597-70)

12:00 - 12:10
(32) Perivarieties
Professor Jack Segal, University of Washington (597-60)

THURSDAY, 10:15 A.M.

Session on Geometry, Room 33, Dwinelle Hall
10:15 - 10:25
(33) Coordinate systems of some semi-translation planes
Mr. D. L. Morgan and Dr. T. G. Ostrom*, Washington State University (597-50)

10:30 - 10:40
(34) Completions of quadrangles in projective planes
Professor R. B. Killgrove, San Diego State College (597-5)

10:45 - 10:55
(35) A generating function for trees and connected graphs, I
Professor Herman Rubin, Michigan State University (597-114)

11:00 - 11:10
(36) On the surface duality of linear graphs
Mr. Jack Edmonds, Princeton University and the National Bureau of Standards (597-148)

11:15 - 11:25
(37) The total length of the edges of a non-Euclidean polyhedron with triangular faces
Professor H. S. M. Coxeter*, University of Toronto and Professor L. Fejes Tóth, University of Veszprém, Hungary (597-81)
11:30 - 11:40
(38) A characterization of certain means of convex bodies
Professor W. J. Firey, Oregon State University (597-168)

THURSDAY, 1:30 P.M.

Business Meeting, Wheeler Hall Auditorium

THURSDAY, 3:00 P.M.

Special Session on Algebra, Room 155, Dwinelle Hall
3:00 - 3:20
Associative rings; a survey of some of the latest developments and possible
directions of research
Professor I. N. Herstein, University of Chicago

3:30 - 3:50
On Kummer fields
Professor D. K. Harrison, New Mexico State University and the Univer-
sity of Pennsylvania

4:00 - 4:20
On quasi-Frobenius algebras and complete cohomology of maximal orders
Professor S. U. Chase and Professor Alex Rosenberg, Cornell Univer-
sity (597-40)

4:30 - 4:50
Generic minimal polynomials of strictly power associative algebras
Professor Nathan Jacobson, Yale University

5:00 - 5:20
On simple extended Lie algebras over fields of characteristic zero
Dr. A. A. Sagle, Syracuse University (597-68)

THURSDAY, 3:00 P.M.

Session on Logic and Foundations, Room 145, Dwinelle Hall
3:00 - 3:10
(39) A simple set which is not effectively simple
Professor G. E. Sacks, Cornell University (597-26)

3:15 - 3:25
(40) Elementary properties of provable recursive sets
Professor P. C. Fischer, Computation Laboratory, Harvard University
(597-207)

3:30 - 3:40
(41) Functions which preserve recursiveness of sets
Professor C. F. Kent, Case Institute of Technology (597-16)

3:45 - 3:55
(42) General recursive functions by restricted ordinal recursion. Preliminary
report
Mr. R. J. Fabian* and Professor C. F. Kent, Case Institute of Technology
(597-54)

4:00 - 4:10
(43) Iteration of primitive recursion
Professor Paul Axt, Michigan State University (597-182)

4:15 - 4:25
(44) Two theorems on computability by two-way automata
Professor D. L. Kreider, Dartmouth College and Professor R. W.
Ritchie*, University of Washington (597-142)
4:30 - 4:40
(45) Quotients of context free languages
Dr. Seymour Ginsburg*, System Development Corporation, Santa Monica, California and Professor E. H. Spanier, University of California, Berkeley (597-8)

4:45 - 4:55
(46) Some remarks about sequences in context free languages
Dr. Seymour Ginsburg, System Development Corporation, Santa Monica, California and Dr. Joseph Ullian*, University of Chicago (597-186)

5:00 - 5:10
(47) The arithmetic of the term-relation number theory
Professor F. G. Asenjo, Southern Illinois University (597-115)

5:15 - 5:25
(48) The intuitionistic continuum
Dr. R. E. Vesley, University of Wisconsin-Milwaukee (597-84)

5:30 - 5:40
(49) A notion of random sequence, Preliminary report
Professor A. H. Kruse, New Mexico State University (597-109)

THURSDAY, 3:00 P.M.

Session on Topology, Room 15, Dwinelle Hall

3:00 - 3:10
(50) The Minkowski units of ribbon knots
Dr. J. J. Andrews*, Florida State University and Mr. F. E. Dristy, Florida Presbyterian College (597-187)

3:15 - 3:25
(51) A characterization of the pseudo-arc
Professor E. E. Grace, University of Georgia (597-98)

3:30 - 3:40
(52) Upper semicontinuous decompositions of $E^3$ with at most countably many non-degenerate elements
Professor Steve Armentrout, State University of Iowa (597-158)

3:45 - 3:55
(53) $C^n$ curves in Euclidean n-space, Preliminary report
Professor Beauregard Stubblefield, Michigan State University Oakland (597-24)

4:00 - 4:10
(54) Lifting disks and certain light open mappings, Preliminary report
Professor L. F. McAuley, University of Virginia (597-208)

4:15 - 4:25
(55) Order structures for certain acyclic topological spaces
Professor J. K. Harris, Portland State College (597-45)

4:30 - 4:40
(56) A fixed point theorem in linear topological spaces
Professor Andrzej Granas, Mathematical Institute of Polish Academy of Sciences and University of Georgia (597-92)
(Introduced by Professor R. S. Pierce)

4:45 - 4:55
(57) Invariant attractors in transformation groups
Dr. J. P. Clay, UNIVAC, Division of Sperry Rand Corporation, Philadelphia, Pennsylvania (597-17)
THURSDAY, 3:00 P.M.

Session on Analysis, Room 23, Dwinelle Hall

3:00 - 3:10  
(58) Bessel potentials in a domain  
Professors R. D. Adams*, Nachman Aronszajn, University of Kansas  
and Professor K. T. Smith, University of Wisconsin (597-34)

3:15 - 3:25  
(59) Spaces of potentials connected with $L^p$-classes  
Professor Nachman Aronszajn, Mr. F. J. Mulla and Dr. Pawel Szeptycki*,  
University of Kansas (597-36)

3:30 - 3:40  
(60) On the conjugate of $L_1$  
Professor E. O. Thorp, New Mexico State University (597-107)

3:45 - 3:55  
(61) A characterization of reflexivity. Preliminary report  
Professor B. L. Sanders, Texas Christian University (597-58)

4:00 - 4:10  
(62) Functional characterization of retracts  
Professor Andrew Sobczyk, University of Miami (597-76)

4:15 - 4:25  
(63) Cebyšev subspaces of finite codimension in $C(X)$  
Professor R. R. Phelps, University of Washington (597-83)

4:30 - 4:40  
(64) Concerning completely convex sets. Preliminary report  
Mr. J. R. Calder, University of Texas (597-167)  
(Introduced by Professor H. S. Wall)

4:45 - 4:55  
(65) On the coincidence of natural order and the order defined by a basis on a  
linear space  
Professor R. E. Fullerton, University of Maryland (597-155)

5:00 - 5:10  
(66) Measures and tensors  
Mr. Jesus Gil de Lamadrid, University of Minnesota (597-57)

5:15 - 5:25  
(67) Reflexivity and summability  
Dr. Togo Nishiura and Dr. Daniel Waterman*, Wayne State University  
(597-209)

THURSDAY, 3:00 P.M.

Session on Analysis, Room 33, Dwinelle Hall

3:00 - 3:10  
(68) Properties of solutions of abstract differential inequalities  
Dr. V. Lakshmikantham, RIAS, Baltimore, Maryland (597-135)

3:15 - 3:25  
(69) On a problem of unitary equivalence of ordinary differential operators of even  
order. Preliminary report,  
Professor J. B. Butler, Jr., Portland State College (597-2)

3:30 - 3:40  
(70) On correct partial differential operators and stable finite difference operators  
Professor D. G. Aronson, Institute of Technology, University of Minnes-  
ota (597-153)

3:45 - 3:55  
(71) Eventual boundedness of solutions of a perturbed system  
Professor Taro Yoshizawa, Iowa State University (597-103)
(72) A theorem on the existence of \( n \) stable limit cycles for Liénard's equation
Professor R. P. deFigueiredo, Purdue University (597-181)

(73) Isochronous oscillations in certain plane autonomous systems
Professor W. S. Loud, University of Minnesota (597-112)

(74) Boundary-value, separation, and oscillation properties of ordinary self-adjoint
differential equations of arbitrary even order
Professor R. W. Hunt, Southern Illinois University (597-47)

(75) Monotonicity of the differences of zeros of Bessel functions as a function of order
Professor Lee Lorch*, University of Alberta, and Professor P. A. Szegő,
Engineering School, University of Santa Clara (597-132)

(76) Note on a nonlinear Volterra equation
Dr. J. J. Levin, Lincoln Laboratory, Massachusetts Institute of Technology
and Professor J. A. Nohel*, University of Wisconsin (597-51)

(77) The asymptotic behavior of the solution of a Volterra equation
Dr. J. J. Levin, Lincoln Laboratory, Massachusetts Institute of Technology (597-61)

THURSDAY, 8:00 P.M.

Josiah Willard Gibbs Lecture, Wheeler Hall Auditorium
Information theory
Professor Claude Shannon, Massachusetts Institute of Technology

FRIDAY, 9:00 A.M.

Presidential Address, Wheeler Hall Auditorium
Integrals devised for special purposes
Professor E. J. McShane, University of Virginia

FRIDAY, 10:15 A.M.

Special Session on Algebraic Topology, Room 155, Dwinelle Hall

10:15 - 10:35
Local triviality for Hurewicz fiberings of manifolds
Professor P. A. Raymond, University of California, Berkeley and the University of Michigan (597-146)

10:45 - 11:05
All the relations among the Stiefel-Whitney classes of manifolds
Professor E. H. Brown, Jr., Brandeis University and Professor F. P. Peterson*, Massachusetts Institute of Technology

11:15 - 11:35
Homotopy classification of fibre bundles
Professor A. I. Weinzeig, Northwestern University (597-193)

11:45 - 12:05
\( H \)-spaces and fibrations of spheres
Professor William Browder, Cornell University

12:15 - 12:35
Classification of homotopy-abelian groups
Professor P. E. Thomas, University of California, Berkeley
Session on Algebra, Room 145, Dwinelle Hall
10:15 - 10:25
(78) The Riemann-Roch theorem for algebraic curves
Professor A. P. Mattuck*, Massachusetts Institute of Technology and Professor A. P. Mayer, University of Pisa, Italy (597-197)

10:30 - 10:40
(79) Multiplicativity of the local Hilbert symbol
Professor Ronald Jacobowitz, University of Arizona (597-77)

10:45 - 10:55
(80) A new approach to Witt vectors, Preliminary report
Professor W. J. Howe, University of Florida (597-202)

10:00 - 11:10
(81) Splitting of commutative algebras with respect to prime ideals
Dr. J. N. Mordeson and Professor Bernard Vinograde*, Iowa State University (597-134)

11:15 - 11:25
(82) The recent development of Frobenius-extensions
Professor Friedrich Kasch, Pennsylvania State University (597-149)

11:30 - 11:40
(83) Distinguished rings of linear transformations
Professor R. E. Johnson, University of Rochester (597-172)

11:45 - 11:55
(84) On algebras of bounded representation type
Professor C. W. Curtis, University of Wisconsin (597-141)

12:00 - 12:10
(85) The Lie algebras with a quotient, trace form
Professor Richard Block, California Institute of Technology (597-177)

12:15 - 12:25
(86) On forms of degree n permitting composition
Professor R. D. Schafer, Massachusetts Institute of Technology (597-9)

Session on Applied Mathematics, Room 15, Dwinelle Hall
10:15 - 10:25
(87) Positive real resolvents and linear passive Hilbert systems
Professor C. L. Dolph, University of Michigan (597-96)

10:30 - 10:40
(88) The numerical solution of singular biharmonic difference equations
Dr. S. D. Conte, Aerospace Corporation, Los Angeles, California and Professor Milton Lees*, California Institute of Technology (597-64)

10:45 - 10:55
(89) On the limit behavior of steepest descent, Preliminary report
Dr. R. J. Arms, National Bureau of Standards, Washington, D. C. (597-87)

11:00 - 11:10
(90) An energy inequality for the reflection coefficient matrix of a pair of nonuniform coupled transmission lines
Professor C. H. Wilcox, Mathematics Research Center, University of Wisconsin (597-203)

11:15 - 11:25
(91) The temperature distribution of a heat source between infinite parallel boundaries
Professor M. A. Dengler, General Electric Company, Santa Barbara, California (597-1)
11:30 - 11:40
(92) Asymptotic behavior of infinite fluid jets under gravity
Professor D. S. Carter, Oregon State University (597-163)

11:45 - 11:55
(93) Relativistic kinetic theory of a simple gas
Dr. Werner Israel, University of Alberta (597-183)

12:00 - 12:10
(94) Rigid motion in a gravitational field. Preliminary report
Dr. F. A. E. Pirani, Kings College, University of London, England and
Dr. Gareth Williams*, University of Florida (597-39)
(Introduced by Dr. J. E. Maxfield)

FRIDAY, 10:15 A.M.

Session on Analysis, Room 23, Dwinelle Hall
10:15 - 10:25
(95) On properties of derivatives
Mr. C. E. Weil, Purdue University (597-44)

10:30 - 10:40
(96) Outer measures on linear lattices
Dr. Leon Brown* and Dr. Hidegoro Nakano, Wayne State University
(597-110)

10:45 - 10:55
(97) Note on Daniell integration. Preliminary report
Professor E. C. Schlesinger, Connecticut College (597-210)

11:00 - 11:10
(98) Fundamental geometric properties of sets of lines
Professor A. S. Besicovitch, Dartmouth College (597-72)

11:15 - 11:25
(99) Rearrangements of series of vectors
Professor V. L. Klee, University of Washington (597-73)

11:30 - 11:40
(100) Vector-valued analytic functions
Mr. D. O. Etter, Jr., North American Aviation, Incorporated, Los Angeles, California (597-119)

11:45 - 11:55
(101) Inequalities
Dr. Nicolas Artemiadis, University of Wisconsin-Milwaukee (597-79)

12:00 - 12:10
(102) Some orthogonal expansions of distributions
Dr. G. G. Walter, University of Wisconsin-Milwaukee (597-52)

12:15 - 12:25
(103) A condition for weak mixing of transformations
Professor Louis Sucheston, University of Wisconsin-Milwaukee (597-78)

FRIDAY, 2:00 P.M.

Invited Address, Wheeler Hall Auditorium
Some results about finite groups
Professor John Thompson, University of Chicago
Special Session on Functions of One or Several Complex Variables, Room 155, Dwinelle Hall

3:15 - 3:35
Counterexample to a proof of Shilov
Mr. A. E. Hurd, Massachusetts Institute of Technology (597-160)
(Introduced by Professor H. L. Royden)

3:45 - 4:05
Extension of a theorem of Hartogs to analytic functions of n real variables
Professor Nachman A:onszajn, University of Kansas (597-33)

4:15 - 4:35
A family of subfields algebraically closed in the field of all meromorphic functions
Dr. Simon Hellerstein*, Stanford University and Professor L. A. Rubel, University of Illinois (597-184)

4:45 - 5:05
Cluster sets of bounded analytic functions
Dr. M. L. Weiss, University of Washington (597-133)

5:15 - 5:35
Complex analytic mappings
Professor Leo Sario, University of California, Los Angeles

Session on Probability and Statistics, Room 145, Dwinelle Hall

3:15 - 3:25
The convex hull of plane Brownian motion
Dr. J. R. Kinney, Lincoln Laboratory, Massachusetts Institute of Technology (597-104)

3:30 - 3:40
A ratio limit theorem for Markov chains
Professor Steven Orey, University of Minnesota (597-75)

3:45 - 3:55
A renewal theorem for independent random variables
Mr. J. A. Williamson, University of Minnesota (597-53)

4:00 - 4:10
Some asymptotic properties of sums of random variables. Preliminary report
Dr. S. C. Port, The Rand Corporation, Santa Monica, California (597-89)

4:15 - 4:25
Note on distributions of functions of interchangeable random variables. Preliminary report
Professor H. D. Brunk, University of California, Riverside (597-145)

4:30 - 4:40
Inequalities for distributions with monotone hazard rate, I. Preliminary report
Dr. R. E. Barlow, San Jose State College and Dr. A. W. Marshall*, Boeing Scientific Research Laboratories, Seattle, Washington (597-28)

4:45 - 4:55
Inequalities for distributions with monotone hazard rate. II. Preliminary report
Dr. R. E. Barlow*, San Jose State College and Dr. A. W. Marshall, Boeing Scientific Research Laboratories, Seattle, Washington (597-29)

5:00 - 5:10
A combinatorial lemma for sequences of partial sums
Dr. C. R. Hobby* and Professor Ronald Pyke, University of Washington (597-145)
5:15 - 5:25
(112) The relevance of observation in simple dichotomies
Professor H. E. Reinhardt, Mathematics Research Center, University of Wisconsin and Montana State University (597-131)

FRIDAY, 3:15 P.M.

Session on Algebra, Room 15, Dwinelle Hall
3:15 - 3:25
(113) On a matrix equation
Dr. D. E. Myers, University of Arizona (597-49)

3:30 - 3:40
(114) Compound matrix equations
Professor Marvin Marcus and Professor A. M. Yaqub*, University of California, Santa Barbara, California (597-38)

3:45 - 3:55
(115) On matrices with positive inverses
Professor Henryk Minc, University of Florida (597-67)

4:00 - 4:10
(116) Double direct decompositions over Dedekind domains, Preliminary report
Professor L. S. Levy, University of Wisconsin (597-191)

4:15 - 4:25
(117) The Jordan normal form: decomposition theorem for modules
Dr. J. L. Brenner, Stanford Research Institute, Menlo Park, California (597-176)

4:30 - 4:40
(118) On maximal decompositions of linear maps
Professor Peter Wilker, University of Kansas (597-59)
(Introduced by Professor G. B. Price)

4:45 - 4:55
(119) A generalized Schwarz inequality
Dr. F. T. Metcalf, U. S. Naval Ordnance Laboratory, White Oak, Maryland (597-90)

5:00 - 5:10
(120) Transitivity in the commutator subgroup of the orthogonal group
Professor Barth Pollak, Syracuse University (597-154)

5:15 - 5:25
(121) Determinantal ideals with application to differential algebra
Professor A. P. Hillman*, Mr. D. W. Forslund and Mr. G. J. Giaccai, University of Santa Clara (597-157)

FRIDAY, 3:15 P.M.

Session on Algebra, Room 23, Dwinelle Hall
3:15 - 3:25
(122) On strict independence in the weak sense
Dr. E. S. O'Keefe, The Boeing Company, Seattle, Washington (597-143)

3:30 - 3:40
(123) Essentially cancellative rings, Preliminary report
Professor A. T. Butson, University of Miami (597-162)

3:45 - 3:55
(124) Decomposition of linear forms in cardinal algebra
Mr. R. E. Bradford, University of California, Berkeley (597-211)
(Introduced by Professor Dana Scott)
4:00 - 4:10
(125) On the numbers of one-sided zeros and one-sided identities of semigroups
Mr. L. D. Heacock, University of California, Davis (597-201)
(Introduced by Professor Takayuki Tamura)

4:15 - 4:25
(126) Commutative divisible semigroups
Professor Takayuki Tamura, University of California, Davis (597-106)

4:30 - 4:40
(127) A class of commutative semigroups having a group-like property
Dr. E. J. Tully, Jr., University of California, Los Angeles (597-13)

4:45 - 4:55
(128) The structure of finite semigroups and the associated system of left ideals
Mr. R. B. Merkel, University of California, Davis (597-198)

SATURDAY, 2:00 P.M.

Special Session on General Topology, Room 155, Dwinelle Hall
2:00 - 2:20
Anamolous group actions and dimensionally deficient spaces
Professor R. F. Williams, University of Chicago (599-27)

2:30 - 2:50
Countable decompositions of $E^3$ into points and pointlike arcs
Mr. D. S. Gillman* and Mr. J. M. Martin, Institute for Advanced Study (597-65)

3:00 - 3:20
Taming complexes in hyperplanes
Professor J. M. Kister, University of Michigan and Institute for Advanced Study

3:30 - 3:50
Failure of equivalence of dimension concepts for metric spaces
Dr. Prabir Roy, University of Illinois (597-86)

4:00 - 4:20
Minimal projective extensions of compact spaces
Professor Melvin Henriksen* and Professor Meyer Jerison, Purdue University (597-46)

SATURDAY, 2:00 P.M.

Special Session on Asymptotic Methods and Singular Perturbations in Analysis, Room 145, Dwinelle Hall
2:00 - 2:20
The asymptotic distribution of eigenvalues for elliptic boundary value problems
Professor F. E. Browder, Yale University

2:30 - 2:50
Simplification of a linear ordinary differential equation of the nth order at a turning point
Professor Yasutaka Sibuya, University of Minnesota (597-150)

3:00 - 3:20
A singular perturbation problem for systems of two analytic differential equations
Professor W. R. Wasow, University of Wisconsin (597-42)

3:30 - 3:50
The asymptotics of certain integral representations
Professor Max Wyman, University of Alberta

4:00 - 4:20
Debye's series in the transitional region
Professor W. B. Fulks, Oregon State University (597-62)
**Session on Analysis, Room 15, Dwinelle Hall**

2:00 - 2:10  
(129) Notes on spectral synthesis  
Professor B. R. Gelbaum, University of Minnesota (597-120)

2:15 - 2:25  
(130) Commutative Banach algebras which are direct summands  
Professor R. F. Arens and Professor P. C. Curtis, Jr.*, University of California, Los Angeles (597-196)

2:30 - 2:40  
(131) Some uncomplemented function algebras  
Professor I. L. Glicksberg, University of Washington (597-122)

2:45 - 2:55  
(132) The topological algebra of infinitely differentiable functions, Preliminary report  
Professor Adam Kleppner, University of Maryland (597-156)

3:00 - 3:10  
(133) A uniqueness property of invariant means  
Mr. R. G. Douglas, University of Michigan (597-94)  
(Introduced by Professor G. J. Minty)

3:15 - 3:25  
(134) Spectral representations for some unbounded normal operators  
Dr. G. J. Maltese, Massachusetts Institute of Technology (597-136)

3:30 - 3:40  
(135) On function pairs related by a Fourier-Stieltjes transform  
Professor J. H. Wells, University of Kentucky (597-139)

3:45 - 3:55  
(136) Absolutely continuous measures on semigroups. Preliminary report  
Professor K. R. Stromberg, University of Oregon (597-173)

4:00 - 4:10  
(137) A theorem on amenable semigroups  
Dr. Edmond Granirer, University of Illinois (597-71)  
(Introduced by Professor M. M. Day)

4:15 - 4:25  
(138) On induced representations, III  
Professor R. J. Blattner, University of California, Los Angeles (597-116)

4:30 - 4:40  
(139) On matrices over the ring of continuous complex-valued functions on a Stonian space  
Mr. Don Deckard and Mr. C. M. Pearcy*, Humble Oil and Refining Company, Houston, Texas (597-91)

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**Session on Applied Mathematics, Room 23, Dwinelle Hall**

2:00 - 2:10  
(140) On some fundamental theorems of perceptron theory and their geometry  
Professor Abraham Charnes, Technological Institute, Northwestern University (597-21)

2:15 - 2:25  
(141) Simple separable switching sets  
Professor W. L. Strother, University of Florida (597-126)

2:30 - 2:40  
(142) A matrix reduction method for Maxwell's equations in anisotropic media  
Dr. A. J. Penico, Stanford Research Institute, Menlo Park, California (597-125)
2:45 - 2:55
(143) A least-squares method for computing the generalized inverse of an arbitrary matrix
Dr. Adi Ben-Israel*, Carnegie Institute of Technology and Mr. S. J. Wersan, Northwestern University (597-14)

3:00 - 3:10
(144) Two-level difference scheme
Dr. T. I. Seidman, Boeing Scientific Research Laboratories, Seattle, Washington (597-113)

3:15 - 3:25
(145) A set of numerical methods for solving simultaneous equations
Dr. W. M. Kincaid, University of Michigan (597-66)

(146) On the numerical solution of problems allowing mixed boundary conditions
Professor Donald Greenspan, Mathematics Research Center, University of Wisconsin (597-121)

3:45 - 3:55
(147) On error reducing multi-step methods
Dr. Walter Gautschi, Oak Ridge National Laboratory, Union Carbide Nuclear Company, Oak Ridge, Tennessee (597-130)

4:00 - 4:10
(148) On the numerical solution of finite difference approximations which are not of positive type

4:15 - 4:25
(149) Finite difference analogues for elliptic boundary value problems and the maximum principle
Professor J. H. Bramble and Professor B. E. Hubbard*, University of Maryland (597-170)

4:30 - 4:40
(150) On the convergence of iterative methods for the solution of systems of non-positive type
Professor J. H. Bramble* and Professor B. E. Hubbard, University of Maryland (597-171)

SUNDAY, 2:00 P.M.

Session on Analysis, Room 145, Dwinelle Hall
2:00 - 2:10
(151) Sets of non-uniform convergence of Taylor series
Professor D. R. Lick, Purdue University (597-4)

2:15 - 2:25
(152) Boundary behavior of pseudo-meromorphic functions
Dr. M. L. Weiss and Professor W. B. Woolf*, University of Washington (597-99)

2:30 - 2:40
(153) Harmonic interpolation in Fejér points with Faber polynomials
Professor J. H. Curtiss, University of Miami (597-108)

2:45 - 2:55
(154) On conformal maps of regions of infinite connectivity
Professor V. C. Williams, Reed College (597-147)

3:00 - 3:10
(155) Sharpened distortion theorems for quasiconformal mappings
Professor Edgar Reich, Institute of Technology, University of Minnesota (597-85)
3:15 - 3:25
(156) Distortion in certain conformal mappings of an annulus
  Professor P. L. Duren, University of Michigan (597-56)

3:30 - 3:40
(157) The Grunsky coefficients of a schlicht function
  Professor J. A. Hummel, University of Maryland (597-88)

3:45 - 3:55
(158) Uniqueness classes for difference functionals
  Dr. R. F. DeMar, Miami University and National Bureau of Standards, Washington, D. C. (597-118)

4:00 - 4:10
(159) A class of entire functions and a conjecture of Erdős
  Mr. Alfred Gray, University of California, Los Angeles and Professor S. M. Shah*, University of Kansas (597-74)

4:15 - 4:25
(160) Limits of polynomials with zeros in a radial set
  Professor J. E. Lange, St. John's University (597-30)

SUNDAY, 2:00 P.M.

Session on Differential Geometry and Topology, Room 155, Dwinelle Hall
2:00 - 2:10
(161) On complete minimal surfaces
  Professor Robert Osserman, Stanford University (597-179)

2:15 - 2:25
(162) Geodesic circles on complete $C^0$ two dimensional Riemannian manifolds
  Dr. G. G. Weill, Yale University (597-199)

2:30 - 2:40
(163) Submanifolds in a Riemannian manifold with general connections
  Mr. C. S. Houh, University of Florida (597-43)
  (Introduced by Professor W. L. Strother)

2:45 - 2:55
(164) Isotropic manifolds of indefinite metric
  Professor J. A. Wolf, University of California, Berkeley (597-11)

3:00 - 3:10
(165) Bracket and exponential for a new type of vector field
  Professor H. H. Johnson, University of Washington (597-129)

3:15 - 3:25
(166) Vector bundles on lens spaces
  Professor David Lissner*, Syracuse University and Professor R. H. Szczarba, Yale University (597-212)

3:30 - 3:40
(167) On compact complex coset spaces of reductive Lie groups
  Professor Jun-ichi Hano, Washington University (597-195)

3:45 - 3:55
(168) The first cohomology group of a minimal set
  Dr. Hsin Chu and Dr. M. A. Geraghty*, University of Alabama Research Institute (597-82)

4:00 - 4:10
(169) Some immersions of projective space in euclidean space
  Dr. Michael Ginsburg, University of Chicago (597-97)

4:15 - 4:25
(170) On points of Jacobian rank k
  Professor P. T. Church, Institute for Defense Analyses and Syracuse University (597-95)
4:30 - 4:40  
(171) A relative Samelson product which yields a generalized Whitehead product  
Mr. G. S. McCarty, Jr., University of Chicago (597-138)

4:45 - 4:55  
(172) The J functor and the non-stable homotopy groups of the unitary groups  
Dr. Melvin Rothenberg, University of Chicago (597-213)

SUNDAY, 2:00 P.M.

Session on Analysis, Room 15, Dwinelle Hall  
2:00 - 2:10  
(173) The existence of periodic solutions to nonlinear differential-difference equations \( \dot{x}(t) = g_1(x(t))g_2(x(t - 1)) \)  
Mr. T. A. Brown, The Rand Corporation, Santa Monica, California (597-164)

2:15 - 2:25  
(174) On periodic solutions of nonlinear second-order differential equations  
Professor H. W. Knobloch, University of Michigan and Universitat Munich, Germany (597-180)  
(Introduced by Professor Edward Halperin)

2:30 - 2:40  
(175) Bergman's method in linear ordinary differential equation of second order with arbitrary coefficients  
Professor M. Z. v. Krzywoblocki, Michigan State University (597-20)

2:45 - 2:55  
(176) On weak and strong solutions of boundary value problems  
Mr. Leonard Sarason, Courant Institute of Mathematical Sciences, New York University (597-152)

3:00 - 3:10  
(177) A class of improper boundary value problems with 'damped' Cauchy data  
Mr. J. M. Zimmerman, Rocketdyne, Canoga Park, California and University of Southern California, Los Angeles (597-12)

3:15 - 3:25  
(178) On a mixed boundary value for nonlinear hyperbolic equations  
Professor A. K. Aziz* and Professor Witold Bogdanowicz, Georgetown University (597-151)

3:30 - 3:40  
(179) The singular Cauchy problem for a quasi-linear hyperbolic equation  
Professor Hajimu Ogawa, University of California, Riverside (597-100)

3:45 - 3:55  
(180) Steady-state diffusion thru a cylinder into a reservoir: An exact solution  
Dr. R. B. Kelman, UNIVAC Division, Sperry Rand Corporation, Washington, D. C. (597-124)

4:00 - 4:10  
(181) Regular points for elliptic equations with discontinuous coefficients  
Professor H. F. Weinberger*, Professor Walter Littman, Institute of Technology, University of Minnesota and Professor Guido Stampacchia, University of Minnesota, New York University and University of Pisa, Italy (597-140)

4:15 - 4:25  
(182) Uniqueness of the integral value problem for the Helmholtz equation  
Professor O. G. Owens, Wayne State University (597-123)

4:30 - 4:40  
(183) Radiative singularities and wave propagation  
Professor Michael Papadopoulos, Mathematics Research Center, University of Wisconsin (597-165)
4:45 - 4:55
(184) On the existence of certain quantum fields, II
Dr. R. T. Prosser, Lincoln Laboratory, Massachusetts Institute of Technology (597-101)

SUNDAY, 2:00 P.M.

Session on Algebra, Room 23, Dwinelle Hall

2:00 - 2:10
(185) On some set-theoretic maximality principles
Mr. A. R. Bednarek, University of Akron (597-41)

2:15 - 2:25
(186) On equations in a free group
Professor Arthur Steinberg, Fairleigh Dickinson University (597-93)
(Introduced by Professor Wilhelm Magnus)

2:30 - 2:40
(187) Extensions of Boolean algebras
Professor F. M. Yaqub, University of California, Davis (597-31)

2:45 - 2:55
(188) Horizontal sums of orthocomplemented lattices
Mr. M. D. MacLaren, Boeing Scientific Research Laboratories, Seattle, Washington (597-32)

3:00 - 3:10
(189) The type set of a torsion free group of finite rank. Preliminary report
Dr. John Koehler, University of Washington and Alma College (597-48)

3:15 - 3:25
(190) p-heights of elements of Ext.
Professor J. M. Irwin, Miss Carol Peercy and Professor E. A. Walker*, New Mexico State University (597-190)

3:30 - 3:40
(191) On the structure of Tor
Professor R. J. Nunke, University of Washington (597-144)

3:45 - 3:55
(192) Isomorphic direct summands of abelian groups
Professor R. A. Beaumont* and Professor R. S. Pierce, University of Washington (597-166)

4:00 - 4:10
(193) The construction of relative injective modules
Dr. P. J. Freyd, University of Pennsylvania (597-22)

4:15 - 4:25
(194) Valuations on meromorphic function fields. Preliminary report
Professor Norman Alling, Massachusetts Institute of Technology and Purdue University (597-37)

Seattle, Washington

R. S. Pierce
Associate Secretary
The five hundred ninety-eighth meeting of the American Mathematical Society will be held at City College on Saturday, February 23, 1963. All sessions will probably be in Shepard Hall, except for the invited address.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Masatake Kuranishi of Columbia University will address the Society, with the title "On the theory of infinite Lie groups." The lecture will be in Townsend Harris Hall at 2:00 P.M.

There will be sessions for contributed papers on Saturday, both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive prior to the deadline of January 9, 1963.

The registration desk will be located in Shepard Hall at Convent Avenue and 139th Street. It will be open from 9:00 A.M. till 3:30 P.M.

Lunch will be available in a College cafeteria. This may be almost the only place in the immediate vicinity for lunch.

Shepard Hall is two blocks east of the 137th Street Station of the IRT subway (the Broadway-7th Avenue Line, not the 7th Avenue Line). Also it is one block west and five blocks south of the 145th Street station of the IND subway (8th Avenue Line "A" train or 6th Avenue Line "D" train from mid-town New York).

Buses marked "Broadway-230th Street" or "Fort George" may be taken from the 125th Street station of the New Haven or New York Central R.R. Riders should get off at 138th Street and walk east one block to Convent Avenue.

Persons who expect to travel by automobile may write to the Department of Mathematics, City College, New York 31, New York before the meeting date, requesting a one day campus parking permit and directions to the parking area.

Everett Pitcher
Associate Secretary
Bethlehem, Pennsylvania

THE AMS SUMMER INSTITUTE ON DIFFERENTIAL AND ALGEBRAIC TOPOLOGY

University of Washington
Seattle, Washington
July 15 through August 17, 1963

The Tenth American Mathematical Society Summer Institute will be held at the University of Washington, Seattle, Washington from July 15 through August 17, 1963. The topic of the Institute is Differential and Algebraic Topology.

The work of the Institute will be concentrated in differential topology, and in those parts of algebraic topology which are immediately relevant to differential topology. There will be a series of lectures covering the present knowledge of differential topology as well as individual lectures on current developments.

The organizing committee consists of Professor R. Bott of Harvard, Professor
E. Dyer of the University of Chicago, and Professors J. Milnor and N. Steenrod (chairman) of Princeton University.

The number of participants will be limited so as to obtain an informal, relaxed atmosphere conducive to the stimulation of ideas. However a few qualified individuals will be admitted in addition to those invited. Interested persons should write to N. Steenrod, Mathematics Department, University of Chicago, Chicago 37, Illinois. A graduate student should ask his research advisor to send a letter of recommendation. Funds for partial support may be available.

N. E. Steenrod
Chairman

A SYMPOSIUM IN APPLIED MATHEMATICS
ON STOCHASTIC PROCESSES IN
MATHEMATICAL PHYSICS AND ENGINEERING

The 601st Meeting of the American Mathematical Society will be held at the Hotel New Yorker in New York City from April 29 to May 2, 1963.

On the afternoon of April 30 and on May 1 and 2, there will be a Symposium on Stochastic Processes in Mathematical Physics and Engineering. This is one of the continuing series of Symposia in Applied Mathematics. The topic was chosen by the Committee on Applied Mathematics, consisting of V. Bargmann, G. E. Forsythe, T. R. Garabedian, R. C. Prim, J. J. Stoker, and D. M. Young, Chairman.

The Invitation and Organizing Committee, responsible for the program, consists of Richard Bellman, A. Bharucha-Reid, M. Kac, J. M. Richardson, J. Keller, Lofti Zadeh, and David Slepian.

The sponsorship of the Symposium is shared with The Society for Industrial and Applied Mathematics and financial support has been provided by the Air Force Office of Scientific Research and the U. S. Army Research Office (Durham).

Speakers, in addition to some of the members of the Invitation and Organizing Committee, will probably include G. Adomian, G.-C. Rota, W. C. Hoffman, V. Tversky, K. Gray, Jr., E. Montroll, E. Parzen, W. Root, E. Wong, D. Blackwell, C. Derman, R. Kalaba, and H. Robbins.

Proceedings of the Symposium will probably be published.

Everett Pitcher
Associate Secretary
Bethlehem, Pennsylvania

MEMORANDA TO MEMBERS

THE NATIONAL REGISTER OF
SCIENTIFIC AND TECHNICAL
PERSONNEL

The Mathematical and Statistical Sciences Section of the Register will maintain a desk during the Annual Meeting at the University of California at Berkeley, on January 25, 26, and 27. The National Register Desk will be located in the East Ball Room of the Student Union. The attendants will be pleased to assist with registrations and to supply information. The National Register as a whole is a responsibility of the National Science Foundation. The Mathematical and Statistical Sciences Section is operated by the American Mathematical Society with the cooperation of the Association for Computing Machinery, the Association for Symbolic Logic, the Econometric Society, the Industrial Mathematical Society, the Institute of Mathematical Statistics, the Mathematical Association of America, the Operations Research Society of America, the Society for Industrial and Applied Mathematics, the Society of Actuaries, and the American Statistical Association.
The annual meeting of the Association for Symbolic Logic will be held at the University of California, Berkeley, California, on Saturday, January 26, 1963. It will be in conjunction with annual meetings of the American Mathematical Society (January 24-27) and the Mathematical Association of America (January 26-28).

A luncheon meeting of the Council of the Association for Symbolic Logic will be held in a private room in the Dining Commons from 11:40 to 1:40 on Saturday.

**PROGRAM**

**Morning Session**, Room 155 Dwinelle Hall, Chairman: Professor Leon Henkin,

9:00 - 10:20 Contributed Papers

9:00 - 9:15 The axiom of comprehension in the Łukasiewicz infinite valued logic. Professor C. C. Chang, University of California, Los Angeles, and the Institute for Advanced Study.

9:20 - 9:35 Separation principles in the hierarchy theory of pure first order logic. Professor M. R. Crow, University of California, Davis.

9:40 - 9:55 Sets of absolutely indiscernible elements in models of theories categorical in power. Mr. Jack Silver, University of California, Berkeley.

10:00 - 10:15 A decision method for validity of sentences in two variables. Professor Dana Scott, University of California, Berkeley.

10:30 - 11:30 Invited Address: Some theorems concerning the possibility of proving theorems by machine. Professor Andrzej Ehrenfeucht, University of Warsaw, and University of California, Berkeley.

**Afternoon Session**, Room 11 Wheeler Hall, Chairman: Professor Benson Mates,

1:50 - 2:00 Presentation of award by United States Steel Foundation.

2:00 - 3:00 Invited Address: Systems of predicative analysis. Professor Solomon Feferman, Stanford University

3:10 - 4:30 Contributed Papers

3:10 - 3:15 The decision problem for identities in the theory of cylindric algebras. Professors Leon Henkin and Alfred Tarski, University of California, Berkeley.

3:20 - 4:30 Late Papers.
The forty-sixth Annual Meeting of the Mathematical Association of America will be held at the University of California, Berkeley, from Saturday to Monday, January 26-28, 1963.

FIRST SESSION, SATURDAY - 9:10 A.M.
ON TRAINING NON-TEACHING MATHEMATICIANS

Auditorium, Wheeler Hall

9:10 - 10:00 Mathematical Trends in Engineering Sciences
Dr. H. O. Pollak, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey

10:10 - 11:00 Mathematics in the Aero-Space Industry
Dr. J. P. Nash, Vice-President, Research and Engineering, Lockheed Missiles and Space Company, Sunnyvale, California

11:10 - 12:00 A Mathematician is a Mathematician is a Mathematician
Dr. R. E. Bellman, RAND Corporation, Santa Monica, California

SECOND SESSION, SUNDAY - 9:10 A.M.

Auditorium, Wheeler Hall

9:10 - 10:00 Algebra: from Groups to Categories
Professor Saunders MacLane, University of Chicago

10:10 - 11:00 Business Meeting of the Association; the Association's Second Award for Distinguished Service to Mathematics, and the Award of the 1963 Chauvenet Prize

11:10 - 12:00 On Learning, Teaching, and Learning Teaching
Professor George Polya, Stanford University

THIRD SESSION, MONDAY - 9:10 A.M.
MATHEMATICS IN THE NON-PHYSICAL SCIENCES

Auditorium, Wheeler Hall

9:10 - 10:00 Mathematical Models in the Social Sciences
Professor J. G. Kemeny, Dartmouth College

10:10 - 11:00 Mathematical Models of Learning Processes
Professor Patrick Suppes, Stanford University

11:10 - 12:00 On New Mathematical Methods for Problems in the Life Sciences
Professor G. B. Dantzig, University of California, Berkeley
FOURTH SESSION, MONDAY - 2:10 P.M.

COMBINATORIAL ANALYSIS

Auditorium, Wheeler Hall

2:10 - 3:00  Combinatorial Problems: Old and New
Professor H. W. Kuhn, Princeton University

3:10 - 4:00  Flow Networks and Combinatorial Operations Research
Dr. D. R. Fulkerson, RAND Corporation, Santa Monica, California

4:10 - 5:00  Matrix Concepts in Graph Theory
Dr. A. J. Hoffman, I.B.M. Research Center, Yorktown Heights, New York

FILM PROGRAM
Wheeler Hall, Auditorium

Saturday, 1:00 p.m. - 2:00 p.m.  Nicolet Animated Geometry Films, with an introduction
by Professor George Polya of Stanford University

Saturday, 7:00 p.m. - 8:00 p.m.  "The Kakeya Problem" (in color and with animation) ...
by Professor A. S. Besicovitch

Sunday, 1:00 p.m. - 2:00 p.m.  "Mathematical Induction" (in color) ... by Professor
Leon Henkin

Sunday, 7:00 p.m. - 8:00 p.m.  "What is an Integral?" (a kinescope) ... by Professor
Edwin Hewitt

Sunday, 8:15 p.m. 9:15 p.m.  "Theory of Limits" ... by Professor E. J. McShane

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CANADIAN MATHEMATICAL CONGRESS
BIENNIAL SEMINAR

Preliminary Announcement

The ninth biennial seminar of the
Canadian Mathematical Congress will take
place at the University of Saskatchewan,
Saskatoon, Canada, from August 12 to
August 30, 1963.

The main theme of the seminar
will be combinatorial mathematics, but
two of the series of lectures will be non-
combinatorial. There will be four series
of six research lectures and four series
of fifteen instructional lectures. While
the program is not yet complete the
following is a partial list which includes
all lecturers except one instructional
lecturer who has not yet been chosen.

Research Lecturers

R. H. Bruck, U.S.A., Existence Problems for classes of Finite Planes
G. Pickert, Germany, Projective Planes
E. M. Wright, Great Britain, Partitions
K. Kuratowski, Poland, Semi-Continuity in Topology

Instructional Lecturers (all from Canada)

W. J. Tutte, Graph Theory
A. L. Dulmage, Combinatorial Problems Related to Graph Theory and Linear
Programming
A. P. Guinand, Fourier Transforms and Summation Formulae
Besides the lecture series there will be a colloquium which will meet once or twice a week, at which one-half hour and one hour papers on combinatorial topics will be presented.

Dormitory accommodations for both single and married, will be available at the University of Saskatchewan residences.

Inquiries should be addressed to the Executive Director of the Canadian Mathematical Congress, Professor L. F. S. Ritcey, 985 Sherbrooke Avenue West, Montreal, Canada.

THE 1963 MEETING OF SIAM

The 1963 Spring meeting of the Society for Industrial and Applied Mathematics will be held at the Stanford Research Institute, Menlo Park, California on Friday and Saturday, April 26 and 27. Abstracts of contributed papers should be submitted to R. E. Gaskell, Department of Mathematics, Oregon State University, Corvallis, Oregon, prior to March 1. Additional details concerning the program will be communicated to members of SIAM.

THE LAWRENCE RADIATION LABORATORY
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U. S. Citizenship Required An Equal Opportunity Employer
A NEW TRANSLATION PROGRAM

Four volumes are now being produced as part of a new translation program begun by the Society under a grant from the National Science Foundation. The program is designed to recognize, select, and quickly translate significant mathematical books appearing in the USSR, at the same time maintaining highest scholarly standards, so that American mathematicians can be kept abreast of Soviet developments.

Several changes are being inaugurated with this program. The Society has increased its payment to translators. The new program will make it possible for more than one translator to work on a particular volume. Not only does this change shorten the length of time spent on a book, but it also permits leading research workers in a field to contribute a small amount of their time to translating a portion of a book. Among the translators contributing partial translations of the four volumes are: Ralph Boas, John M. Danskin, F. M. Goodspeed, Edwin Hewitt, Adam Koranyi, Jacob Korevaar, William LeVeque, Allen Shields, and Henry P. Thielman. The grant also provides for the additional attention of scientific editors to correlate and unify the separate contributions of the translators.

Since the primary objective of the program is to place translations in the hands of research workers and students as rapidly as possible, arrangements will be made to have the volumes produced by the quickest possible means. This implies that books may be typeset by commercial printers rather than varityped in the Providence Office. As a consequence, some of these volumes will be slightly more expensive than other AMS translations. To facilitate rapid distribution at the lowest possible cost, the Society will arrange to accept prepublication orders at reduced prices.

The four volumes now in progress will constitute volumes 3 to 6 of the AMS series Translations of Mathematical Monographs.

Volume 3, Semigroups, by E. S. Lyapin, 592 pages, is scheduled for publication in July, 1963. This book offers a complete introduction to the subject of semigroups and will be widely used as both a text and a reference. Orders will be accepted at prepublication prices of $19.10 (list price) and $14.33 (members' price) on or before May 1, 1963. After that, the list price will not be less than $21.20.

Volume 4, The dispersion method in binary additive problems, by Ju. V. Linnik, 208 pages, is also scheduled for a July publication. In this work the author makes further applications of the method he so brilliantly used for proving the Hardy-Littlewood conjecture confirming an asymptotic formula for the number of representations of a large number as the sum of two squares and a prime. Orders will be accepted at prepublication prices of $9.40 (list price) and $7.05 (members' price) on or before May 1, 1963. After that, the list price will not be less than $10.40.

Volume 5, Distribution of zeros of entire functions, by B. Ja. Levin, 632 pages, is scheduled for publication in September, 1963. The first half of this book gives an organized account of material that has appeared elsewhere; the second half contains interesting and important material that has not previously appeared. Orders will be accepted at prepublication prices of $20.30 (list price) and $15.23 (members' price) on or before June 1, 1963. After that, the list price will not be less than $22.50.

Volume 6, Harmonic analysis of functions of several complex variables in classical regions, by K. L. Hua, 163 pages, is scheduled for publication in October, 1963. This work is an exposition and a systematization of results of the author published in numerous articles over the past 15 years. Orders will be accepted at prepublication prices of $6.00 (list price) and $4.50 (members' price) on or before September 1, 1963. After that, the list price will not be less than $6.60.

To receive copies at the prepublication prices, orders must be received at the Headquarters Office, 190 Hope Street, Providence 6, Rhode Island, before the dates listed.
VISITING FOREIGN MATHEMATICIANS

The foreign mathematicians listed here are in addition to those who were listed on page 446 of the November, 1962 issue of these NOTICES.

<table>
<thead>
<tr>
<th>Name</th>
<th>Home Country</th>
<th>Host Institution</th>
<th>Period of Visit</th>
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<tbody>
<tr>
<td>Arlt, Dictmar</td>
<td>Germany</td>
<td>Brandeis University</td>
<td>Sept, 1962-Sept, 1963</td>
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<tr>
<td>Banerjee, Kali Shankar</td>
<td>India</td>
<td>Cornell University</td>
<td>Feb, 1962-Jan, 1963</td>
</tr>
<tr>
<td>Bhanot, Indira Vatsraj</td>
<td>India</td>
<td>Iowa State University</td>
<td>Sept, 1962-June 1963</td>
</tr>
<tr>
<td>Bhatia, N. P.</td>
<td>India</td>
<td>RIAS</td>
<td>Sept, 1962-June 1963</td>
</tr>
<tr>
<td>Hoyland, Arnljot</td>
<td>Norway</td>
<td>University of California, Berkeley</td>
<td>Sept, 1962-June 1963</td>
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<tr>
<td>Huzurbazar, Vasant Shankar</td>
<td>India</td>
<td>Iowa State University</td>
<td>Sept, 1962-May 1963</td>
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<tr>
<td>Miles, Roger Edmund</td>
<td>United Kingdom</td>
<td>Princeton University, University of California, Berkeley</td>
<td>Sept, 1961-June 1962</td>
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<tr>
<td>Ohtsuka, Makato</td>
<td>Japan</td>
<td>Washington University</td>
<td>Sept, 1962-June 1963</td>
</tr>
<tr>
<td>Pollachek, Felix</td>
<td>France</td>
<td>University of Maryland</td>
<td>Jan, 1963-June 1963</td>
</tr>
<tr>
<td>Smith, R. T. C.</td>
<td>Australia</td>
<td>University of Maryland</td>
<td>Sept, 1962-June 1963</td>
</tr>
<tr>
<td>Strassen, Volker</td>
<td>Germany</td>
<td>University of California, Berkeley</td>
<td>Sept, 1962-Sept, 1963</td>
</tr>
<tr>
<td>Straszak, A.</td>
<td>Poland</td>
<td>RIAS</td>
<td>Dec, 1962-June 1963</td>
</tr>
<tr>
<td>Tiago de Oliveira, Jose</td>
<td>Portugal</td>
<td>Columbia University</td>
<td>March 1963-June 1963</td>
</tr>
<tr>
<td>Tillmann, Heinz G.</td>
<td>Germany</td>
<td>University of Maryland</td>
<td>Sept, 1962-July 1963</td>
</tr>
<tr>
<td>Welter, Cornelis Petrus</td>
<td>United Kingdom</td>
<td>Stanford University</td>
<td>Sept, 1962-May 1963</td>
</tr>
</tbody>
</table>

NEWS ITEMS AND ANNOUNCEMENTS

THE DIVISION OF BIOSTATISTICS, SCHOOL OF PUBLIC HEALTH, UNIVERSITY OF CALIFORNIA, BERKELEY, is now accepting applications for awards in Biostatistics for the academic year 1963-1964. The awards are for study leading to the M. A. and Ph. D. degrees in Biostatistics and are given to qualified graduates in the fields of mathematics, statistics, or biomedical sciences. They begin at $250 per month for a person with a B. A. degree and $300 per month for a person with an M. A. degree (plus $30 per month for each dependent). Inquiries are invited.
A proof procedure of R. Stanley for the first order predicate calculus is examined to determine whether the procedure gives a method of decision for certain decidable classes of formulas. Results of K. J. J. Hintikka concerning perfect distributive normal forms in the predicate calculus are generalized to certain classes of non-perfect distributive normal forms of (quantificational) degree two, giving a set of necessary and sufficient conditions for the non-satisfiability of these non-perfect normal forms. These latter conditions are then applied to show that Stanley's procedure, when strengthened along lines suggested by W. Ackermann, gives a method of decision for the class of all closed formulas of degree two.
The completely revised edition presents a largely self-contained treatment of the foundations of continuity, or point set-theoretic, analysis situs (topology). The appended bibliography of the revised edition is much more extensive than that of the first edition.

LECTURES IN APPLIED MATHEMATICS
Proceedings of the Summer Seminar, Boulder, Colorado, 1960

Volume I

LECTURES IN STATISTICAL MECHANICS
by G. E. Uhlenbeck and G. W. Ford with E. W. Montroll
181 pages; List Price $7.40; 25% discount to members

These lectures were presented at the American Mathematical Society's Summer Seminar in Theoretical Physics. The purpose of the Summer Seminar was to acquaint mathematicians with some of the basic problems in present-day theoretical physics, hoping that this would stimulate a more intense collaboration. The authors believe that such a collaboration would be especially valuable in statistical mechanics since many of the unsolved problems can be formulated precisely and are of a technical mathematical nature. In the authors' opinion a rigorous mathematical treatment is especially needed in statistical mechanics and to provide it is the real challenge of the subject. The logical structure of the theory is stressed as much as possible and the mathematical gaps remaining in the argument have been indicated. The book can be used as a short but self-contained introduction to the subject. Textbook-like chapters alternate with chapters describing work still in progress.

Volume II

MATHEMATICAL PROBLEMS OF RELATIVISTIC PHYSICS
by Irving E. Segal with George W. Mackey
131 pages; List Price $6.00; 25% discount to members

This book gives the approximate text of a course of eight lectures from combined rigorous mathematical and physically conceptual viewpoints, supplemented by two more purely mathematical lectures, providing an up-to-date introduction to the central mathematical features of fundamental relativistic physics for the mathematically trained reader.

MEMORANDA TO MEMBERS

CHAIRMEN

The Annual List of Chairmen of Departments has been compiled. Copies may be obtained free of charge by writing to the Headquarters Offices, 190 Hope Street, Providence 6, Rhode Island.
THE MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH reports on two Conferences held last summer. On June 21-24, a Seminar was held under the chairmanship of Professor R. Baer with the assistance of R. H. Bing, W. E. Deskins, and M. Suzuki. The following lectures were presented:

R. H. Bing: Die Fundamentalgruppe der Vereinigung zweier Kreisscheiben.
H. Salzmann: Eben projektive Ebenen.
P. Dembowsky: Eine Klasse von Kollineationsgruppen.
O. H. Kegel: Eine Methode von Ito.
B. Fischer: Distributive Gruppen endlicher Ordnung.
H. Mertes: Zur Struktur des Idealverbandes bei Dorroh'schen Ringerweiterungen.

A Conference on Discrete Geometry was held July 23-29, under the chairmanship of Professor Laszlo Fejes Toth, Budapest, with the following participants:

England: H. T. Croft, C. A. Rogers;
Israel: B. Grünbaum;
Italy: B. Segre;
Canada: R. Blum, H. S. M. Coxeter;
Austria: A. Florian, H. Florian;
Switzerland: H. Bieri, B. L. van der Waerden;
Czechoslovakia: V. Polak;
Hungary: P. E. Erdős, L. Fejes Toth, J. Molnar;
United States: H. Groemer, V. Klee, H. Zassenhaus;
Germany: L. Danzer, W. Meretz, K. Schütte.

GENERAL INFORMATION ABOUT TRAVEL TO THE USSR AND EASTERN EUROPE. The Department of State has sent the Society a list of national scientific and technical conferences to be scheduled in the USSR and Eastern Europe from October 15, 1962, and on.

It is the policy of the Department of State to encourage American attendance at Soviet Bloc scientific and technical conferences. In a letter, Frank G. Siscoe, Director, Soviet and Eastern European Exchanges Staff, states that private financial assistance is sometimes available for American specialists to attend such meetings; also, the National Science Foundation gives grants in a limited number of cases. While the State Department has no funds for such purposes, the Exchanges Office will accept inquiries and may be able to suggest possible sources of financial aid. However, the Department can give no assurance that invitations to the various conferences will be forthcoming.

General information about travel to the USSR and Eastern Europe is available from the Governmental Affairs Institute's Information Center for Travelers to the Soviet Union, Room 230, 345 East 46th Street, New York 17, New York. Additional information about attendance and preparation for scientific and technical conferences in these countries is available from the National Academy of Sciences, 2101 Constitution Avenue, N.W., Washington 25, D.C., or from the Exchanges Office.

One scheduled conference is of particular interest to mathematicians. The Joint USA-USSR Symposium on Partial Differential Equations will be held in Novosibirsk, USSR, for two weeks in March, 1963. The Symposium was proposed by the Academy of Sciences, USSR. The invitation states that a delegation of up to 15 mathematicians from the United States (to be chosen by the U. S. National Academy of Sciences) will be welcome. Suggested Symposium topics include: (1) General questions on the theory of partial differential equations; (2) Boundary value problems, questions concerning uniqueness, stability and others; (3) The spectral theory of linear operators connected with differential equations; (4) Problems of equations of a special type; (5) Equations of an elliptical type on two-dimensional multidimensional manifolds; (6) Quasiconformal mapping of plane and multidimensional regions; (7) Approximate and numerical methods.
BENJAMIN PIERCE INSTRUCTORSHIP: Applications for the Benjamin Pierce Instructorship at Harvard University are invited. The teaching commitment is six hours and the salary is $7,000 for the academic year. There is also the possibility of summer income through teaching or research contracts. Appointments are annual but carry the presumption of two renewals. Additional information and application forms are available from the Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

THE HARVARD MEDICAL SCHOOL announced the formation of a Division of Mathematical Biology on July 1, 1962. Partially supported by grants from the National Institutes of Health, this new division of the Department of Medicine of the Medical School and the Peter Bent Brigham Hospital has for its aims the encouragement of research and the development of an educational and training program in the mathematical aspects of the biological, chemical and medical sciences. The Biomathematics Laboratory of the Division contains an IBM 1620 Computer System available to the staff for research and training. A program aimed at defining the role of the digital computer in medical and biological research will begin on January 1, 1963.

Through this division and in cooperation with other departments at Harvard University and the Harvard Medical School, arrangements can be made to undertake graduate work leading to a doctoral degree in mathematical biology with concentration in some specific area of the biological sciences. Such graduate students would have to meet the requirements of the other departments involved, as well. A few predoctoral and postdoctoral fellowships are available for the academic year 1963-1964. Also open are two academic appointments to the senior staff of the division, to be filled by mathematicians with some experience in, or at least a strong interest in one of the branches of the biological sciences and medical sciences. Further information and application forms may be obtained by writing directly to Dr. Anthony F. Bartholomay, Director of the new division.

THE AIR FORCE OFFICE OF SCIENTIFIC RESEARCH wishes to announce the availability of special grants for the academic year 1963-1964, for the support of research sabbaticals, and for the support of certain book-writing projects.

Proposals for research sabbaticals should follow the outlines of a conventional research proposal, including a statement of the research problems and methods of attack, a biographical sketch, the proposer's bibliography, and a budget. Support may be requested for a nine or twelve month period beginning October 1, 1963. It is helpful if the proposer's University can provide partial support, but not essential. The project officer in charge of this program is Major Oliver A. Shaw.

The goal of the book-writing project is to help bridge the gap between the creative professional mathematician and the user of mathematics, in areas of current and potential Air Force interest. By definition, these areas are analysis and functional analysis, in particular the solution of ordinary and partial differential equations, probability theory and statistics. Select portions of other areas of mathematics are also appropriate: e.g., combinatorial analysis, "useful" topology.

Again, proposals for book-writing projects should follow the outlines of a conventional research proposal, as appropriate. They should include a proposed table of contents, a sample chapter if available, and mention any unique features of the book. An indication of publisher interest is helpful. Support may be for a nine or twelve month period beginning October 1, 1963. The project officer in charge of this program is Capt. John F. Gander.

Proposals and inquiries should be directed to the appropriate project officer, in the Mathematics Division, Air Force Office of Scientific Research, Washington 25, D. C. It is hoped that all decisions can be made by late spring of 1963.
PERSONAL ITEMS

Mr. M. J. ANTONE of Salem State College has been appointed Director of Mathematics at the Medford Public Schools, Medford, Massachusetts.

Mr. C. W. AUSTIN of the University of Washington has been appointed to an assistant professorship at Northeastern University.

Mrs. A. S. BALSER of The Mitre Corporation has been appointed to an assistant professorship at San Jose State College.

Dr. R. W. BASS of the Aeronca Manufacturing Corporation has accepted a position as Staff Scientist with the Hughes Aircraft Company, Washington, D. C.

Mr. J. E. BELYEA of Sylvania Corporation has accepted a position as an Associate Research Physicist with the Conducktron Corporation, Ann Arbor, Michigan.

Mr. M. N. BLEICHER of the University of California, Berkeley has been appointed to an assistant professorship at the University of Wisconsin.

Dr. A. D. BOOTH, Head of the Department of Numerical Automation and Reader in Computational Methods at Birkbeck College, University of London has been elected to the Chair of Electrical Engineering at the University of Saskatchewan, Saskatchewan, Canada.

Dr. F. W. BROWN of the National Bureau of Standards has accepted a position as Science Attaché with the Foreign Service of the United States, Buenos Aires, Argentina.

Mr. J. M. VAN BUSKIRK of the University of Wisconsin has been appointed to an assistant professorship at the University of Oregon.

Dr. D. R. CECIL of Oklahoma State University has been appointed to an assistant professorship at North Texas State University.

Mr. J. L. CHAMBERLIN of Chief of Ordnance, U. S. Army has accepted a position as Special Assistant to the Commanding General with the Material Command, Washington, D. C.

Dr. J. P. CLAY has accepted the position of Researcher, in the Programming Research Department of the UNIVAC Division of Sperry Rand Corporation, Philadelphia, Pennsylvania.

Assistant Professor W. R. COOPER of Knoxville College has been appointed to an associate professorship at Grambling College.

Mr. T. A. COOTZ of the Douglas Aircraft Corporation has accepted a position as Programming Analyst with the System Development Corporation, Santa Monica, California.

Mr. W. C. CUTLER of the Stanford Research Institute has accepted a position as Data Processing Supervisor with the Boeing Company, Seattle, Washington.

Mr. R. H. DALE of New Mexico State University has accepted a position as Mathematician at the White Sands Missile Range, New Mexico.

Associate Professor E. A. DAVIS of the University of Utah has returned from a one year's leave of absence with the National Science Foundation in Washington, D. C.

Professor RENE DEHEUVELS of the University of Lille, Lille, France has been appointed to a professorship at the University of Paris, Paris, France.

Professor D. G. DEWEY on leave from the College of the Holy Cross has received a National Science Foundation Faculty Fellowship at Brown University for the academic year 1962-1963.

Miss S. E. DINGA of the North American Aviation Corporation has accepted a position as Assistant Research Mathematician with Conducktron Corporation, Ann Arbor, Michigan.

Mr. B. P. DONOHUE of the University of Maine has accepted a position as a Member of the Technical Staff with the Bell Telephone Laboratories, Whippany, New Jersey.

Dr. R. D. DRIVER of the Mathematics Research Center has accepted a position
as a Staff Member of the Sandia Corporation, Albuquerque, New Mexico.

Professor PHILIP DWINGER of Purdue University has been appointed to a professorship at the Technological University, Delft, Netherlands.

Mr. S. B. ELK of Brookline, Massachusetts has accepted a position as Research Specialist at the Lockheed Missile and Space Company, Sunnyvale, California. Associate Professor W. F. FREIBERGER of Brown University has been awarded a Guggenheim Fellowship and will spend the academic year 1962-1963 at the University of Stockholm, Stockholm, Sweden.

Professor DAVID GALE of Brown University has been awarded a Guggenheim Fellowship and will spend the academic year 1962-1963 at the University of Osaka, Osaka, Japan.

Assistant Professor T. M. GALLIE on leave from Duke University will spend the academic year 1962-1963 at the Institut fur Angewandte Mathematik, Zurich, Switzerland.

Associate Professor SEYMOUR GOLDBERG of New Mexico State University has been appointed to a visiting associate professorship at the University of Maryland.

Dr. J. M. GONZALEZ-FERNANDEZ of the University of Wisconsin has accepted a position as Mathematician with the National Institutes of Health, Bethesda, Maryland.

Dr. G. W. GRAVES of the University of Michigan has accepted a position as a Member of the Technical Staff with the Aerospace Corporation, El Segundo, California.

Associate Professor J. R. HANNA of the University of Wyoming has been appointed to an associate professorship at the University of Colorado.

Mr. L. A. HINRICHS of the University of Oregon has been appointed to an assistant professorship at Duke University.

Dr. E. C. JOHNSEN of Ohio State University has received a National Research Council Research Associateship at the National Bureau of Standards, Washington, D. C., for the year 1962-1963.

Mr. S. E. J. JOHNSEN of Purdue University has accepted a position as Senior Mathematician with General Motors Corporation, Indianapolis, Indiana.

Dr. R. E. KALMAN of RIAS, Baltimore, Maryland was named by the Maryland Academy of Sciences as the State's Outstanding Young Scientist of 1962.

Mr. D. G. KENDALL of Magdalen College, Oxford, England has been appointed to a professorship at Churchill College, Cambridge, England.

Mr. J. W. KITCHENS of Harvard University has been appointed to an assistant professorship at Duke University.

Dr. R. W. KLOPFENSTEIN of the Radio Corporation of America has been appointed to a professorship and Director of the Computation Center at Iowa State University.

Dr. ALI KYRALA of Motorola Incorporated has been appointed to a professorship at the U.S. Naval Postgraduate School, Monterey, California.

Dr. E. H. LEHMAN, JR. of the University of Florida has accepted a position as Staff Mathematical Engineer at the Westwood Division of Houston Fearless Corporation, Los Angeles, California.

Dr. F. M. LESLIE of the Massachusetts Institute of Technology has been appointed Lecturer at King's College, Newcastle-Upon-Tyne, England.

Professor S. C. LOWELL of Adelphi College has been appointed to a professorship in the Mathematics Department and Mathematician at the Computing Center at the Washington State University.

Dr. C. P. LUEHR of the University of California, Berkeley has accepted a position as a Member of the Technical Staff with the General Electric Company, Santa Barbara, California.

Dr. J. G. MACCARTHY of Naval Weapons Laboratories, Dahlgren, Virginia has been appointed to an associate professorship at St. Johns University.

Assistant Professor A. D. MARTIN of Carnegie Institute of Technology has accepted a position as a Visiting Member of RIAS, Baltimore, Maryland.

Dr. M. A. MEDICK of the Avco Manufacturing Corporation has been appointed to a professorship at Michigan State University.

Dr. JAMES MICHELOW of the University of Washington has accepted a position as Research Mathematician with California Research Corporation, Richmond, California.
Professor R. S. MISHRA of Gorakhpur University, Gorakhpur, India has been appointed to a professorship and Head of the Department of Mathematics at the University of Allahabad, Allahabad, India.

Mr. A. H. MOORE of Johns Hopkins University has been appointed to an associate professorship at the Air Force Institute of Technology, Wright-Patterson Air Force Base, Dayton, Ohio.

Professor T. S. MOTZKIN of the University of California, Los Angeles will be on leave during the fall semester of 1962 at the Hebrew University, Jerusalem, Israel.

Mr. A. C. MUNSTER of Philco Corporation has accepted a position as Associate Program Manager with the Conductron Corporation, Ann Arbor, Michigan.

Miss MILDRED NELSON of the Systems Development Corporation has accepted a position as Senior Data Processing Analyst with the International Telephone and Telegraph Company, Belleville, New Jersey.

Professor F. C. OGG, JR. of Bowling Green State University has been appointed to an associate professorship at Johns Hopkins University.

Mr. MARTIN ORR of Long Island University has accepted a position as Mathematician with the Vitro Laboratories, West Orange, New Jersey.

Mr. J. M. PEEK of Ohio State University has accepted a position as a Staff Member with the Sandia Corporation, Albuquerque, New Mexico.

Dr. A. M. PEISER of The M. W. Kellogg Company has accepted a position as Senior Associate with the Socony Mobil Oil Company, New York, New York.

Professor HANS RADEMACHER of the University of Pennsylvania has been appointed to a visiting professorship at the Courant Institute of Mathematical Sciences, New York University.

Associate Professor JAMES RADLOW of Adelphi College has been appointed to an associate professorship at Purdue University.

Assistant Professor R. W. RICHARDSON, JR. on leave from the University of Washington has received a National Science Foundation Faculty Fellowship at the Institute for Advanced Study for the academic year 1962-1963.

Professor HELMUT RÖHRL of the University of Minnesota has been appointed a Visiting Lecturer at Harvard University for the academic year 1962-1963.

Assistant Professor W. G. ROSEN of the University of Maryland has accepted a position as Associate Program Director, Developmental Program, Division of Scientific Personnel and Education, National Science Foundation, Washington, D. C.

Mr. M. S. ROWIN of The Boeing Company has been appointed a Teaching Assistant at the University of Washington.

Professor L. K. SCHMETTERER of the Universitat Wien, Wien, Austria has been appointed to a visiting professorship at the Catholic University.

Associate Professor S. H. SCHOT on leave from the American University has received a National Science Foundation Faculty Fellowship at the University of Gottingen, Gottingen, Germany for the academic year 1962-1963.

Professor LAURENT SCHWARTZ of the University of Paris, Paris, France has been appointed to a visiting professorship at the Courant Institute of Mathematical Sciences, New York University.

Dr. F. F. SELIG of the Socony Mobil Oil Company has been appointed a Lecturer at the University of Dallas.

Mr. W. H. SHIELDS of the System Development Corporation has accepted a position as Senior Technical Specialist with the North American Aviation Corporation, Anaheim, California.

Dr. YASUTAKA SIBUYA of the University of Tokyo, Tokyo, Japan has been appointed to a visiting associate professorship at the University of Minnesota for the academic year 1962-1963.

Dr. J. A. SIMMONS of the University of California, Berkeley, has accepted a position as Mathematical Physicist with the U. S. Department of Commerce, National Bureau of Standards, Washington, D. C.

Mr. R. D. SINKHORN of The Boeing Company has been appointed to an assistant professorship at the University of Houston.

Mr. G. J. SMITH, JR. on leave from the University of California, Berkeley has received a National Science Foundation Faculty Fellowship at the University of Paris, Paris, France for the academic year 1962-1963.
Mr. S. E. SMITH of Textran Corporation has accepted a position as Director of Operations Research with Tracor Incorporated, Austin, Texas.

Dr. NORMAN STEIN of Columbia University has been appointed to an associate professorship at the State University of New York, Stony Brook, New York.

Associate Professor J. P. TULL of Ohio State University has been appointed a Visiting Senior Lecturer at the University of Adelaide, Adelaide, Australia.

Dr. R. N. VAN NORTON of New York University has accepted a position as Mathematician with the Brookhaven National Laboratories, Upton, New York.

Dr. F. J. WALL of the Sperry Rand Corporation has accepted a position as Senior Research Mathematician with the Dikewood Corporation, Albuquerque, New Mexico.

Mr. M. L. WEISS on leave from the University of Washington has received a National Science Foundation Faculty Fellowship at the Institute for Advanced Study for the academic year 1962-1963.

Mr. R. H. WESSNER of the University of California has accepted a position as Mathematician with the Hughes Aircraft Company, Culver City, California.

Mr. N. F. WILLIAMSON, JR. of Louisiana State University has been appointed to an assistant professorship at the College of Charleston.

Associate Professor A. H. ZEMANIAN of New York University has been appointed to a professorship at the State University of New York, Stony Brook, New York.

Dr. N. R. ZITRON of the University of Wisconsin has been appointed to an associate professorship at Purdue University.

The following promotions are announced:

ALFRED AEPPLI, University of Minnesota, to an associate professorship.

D. G. ARONSON, University of Minnesota, to an associate professorship.

GEORGE BACHMAN, Polytechnic Institute of Brooklyn, to an associate professorship.

CHARLES BALLANTINE, Oregon State University, to an assistant professorship.

V. DOMINIC, St. Mary's College of California, to a professorship.


R. E. INGRAM, University College, Dublin, Ireland, to a Lecturer.

A. F. IVerson, North Park College, to an assistant professorship.

J. R. JOHNSON, JR., Wake Forest College, to an associate professorship.

R. F. KING, Argonne National Laboratory, Argonne, Illinois, to an Associate Mathematician.

B. W. LINDGREN, University of Minnesota, to an associate professorship.

J. P. LINE, Georgia Institute of Technology, to an associate professorship.

LEE LORCH, University of Alberta, Canada, to a professorship.

J. H. OPPENHEIM, Rutgers, The State University, to an assistant professorship.

M. H. PEARL, University of Maryland, to an associate professorship.

JACOB SHERMAN, Rutgers, The State University, to an associate professorship.

MICHAEL SKALSKY, Southern Illinois University, to an associate professorship.

A. E. VENTRIGLIA, Manhattan College, to an associate professorship.

STEPHEN WEINGRAM, Rutgers, The State University, to an assistant professorship.

E. S. WOLK, University of Connecticut, to an associate professorship.

A. D. ZIEBUR, Harpur College, to a professorship.

WILLIAM ZLOT, Yeshiva University, to an associate professorship.

The following appointments to instructorships are announced:

Columbia University: HAIM GAIFMAN; Georgia Institute of Technology: D. H. HUTCHINSON; University of Illinois: W. J. LEAHEY; Lehigh University: P. A. LAPPAN, JR.; Princeton University: S. F. BAUMAN; Stanford University: TA SUN WU.

Deaths:

Associate Professor AARON BAKST of New York University died on October 18, 1962 at the age of 62.
Professor Emeritus H. K. Fulmer of Georgia Institute of Technology died on October 30, 1962 at the age of 68.

Mr. J. B. Geiser of Brown University and Brandeis University died on September 26, 1962 at the age of 29.

Errata

The following are corrections of announcements in the October issue of the Notices.

Associate Professor H. M. Schaef of Washington University has been appointed to a visiting professorship at the Mathematics Research Center, U.S. Army, University of Wisconsin.

Assistant Professor S. F. Tuan of Brown University has been appointed to an associate professorship at Purdue University.

The following corrects an announcement in the November issue of the Notices.

BARTH POLLAK, Syracuse University is an Assistant Professor.

3 IMPORTANT REPRINTS

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During the interval from September 26, 1962 through November 17, 1962, the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program is an identifying number. The abstract of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these NOTICES.

(1) Interval functions and continuity. Preliminary report
   Dr. W. D. L. Appling, Duke University (63T-24)

(2) A summation procedure for certain Feynman integrals. II
   Professor D. G. Babbitt, University of California, Los Angeles (63T-47)

(3) Every simple closed curve in $E^3$ is unknotted in $E^4$
   Professor R. H. Bing, Institute for Advanced Study and University of Wisconsin and Professor V. L. Klee, University of Washington (63T-32)

(4) The subgroups of SL(3,q), Part I. Preliminary report
   Professor D. M. Bloom, University of Massachusetts (63T-18)

(5) A new Euclidean representation for the generalized inverse of an arbitrary matrix
   Dr. Abraham Charnes, Northwestern University and Dr. Adi Ben-Israel, Carnegie Institute of Technology (63T-43)

(6) Finite dimensional affine semigroups. Preliminary report
   Mr. Edwin Clark, Tulane University (63T-25)

(7) An analogue of the totient function
   Professor Eckford Cohen, University of Tennessee (63T-52)

(8) A reduction class with a single binary predicate
   Mr. J. S. Denton, Jr., Harvard University (63T-7)

(9) A false "decision procedure" for the halting problem
   Mr. J. S. Denton, Jr., Harvard University (63T-8)

(10) On the nonlinear theory of multistream electron devices. Preliminary report
    Professor C. L. Dolph and Mr. R. J. Lomax, University of Michigan (63T-57)

(11) A note on direct sums of cotorsion groups
    Professor D. W. Dubois, University of New Mexico (63T-37)

(12) Generalized free products of Boolean algebras with an amalgamated subalgebra
    Professor Philip Dwinger, Technische University, Delft, Netherlands and Professor F. M. Yaqub, University of California, Davis (63T-41)

(13) A representation theorem
    Mr. D. A. Ford, University of Utah (63T-15)

(14) Composition products
    Mr. H. H. Gershenson, Princeton University (63T-44)

(15) Integral domains which are almost Dedekind
    Dr. R. W. Gilmer, Jr., University of Wisconsin (63T-26)

(16) Rings in which every semi-primary ideal is primary
    Dr. R. W. Gilmer, Jr., University of Wisconsin (63T-27)

(17) A property of Fibonacci numbers
    Dr. Ronald Graham, Bell Telephone Laboratories, Murray Hill, New Jersey (63T-4)

(18) A theorem on partitions
    Dr. Ronald Graham, Bell Telephone Laboratories, Murray Hill, New Jersey (63T-5)

(19) On polyhedral graphs
    Dr. Branko Grünbaum, University of Washington and Hebrew University, Israel, and Professor T. S. Motzkin, University of California, Los Angeles (63T-38)

(20) Linearly solvable difference methods for quasilinear parabolic differential
equations
Mr. J. E. Gunn and Professor Milton Lees, California Institute of Technology (63T-39)

(21) Submanifolds
Dr. N. J. Hicks, University of Michigan (63T-49)

(22) On almost continuous mappings
Professor Taqdir Husain, University of Ottawa (63T-51)

(23) Intersections of PCA classes
Professor H. J. Keisler, University of Wisconsin (63T-48)

(24) More on the geometry of Teichmuller mappings
Professor T. S. Klotz, University of California, Los Angeles (63T-6)

(25) Spectral multiplicity theory for a class of singular integral operators
Professor Walter Koppelman, University of Pennsylvania (63T-14)

(28) An extension of Greendlinger's results on the word problem
Professor Seymour Lipschutz, Polytechnic Institute of Brooklyn (63T-17)

(29) A semicreative weak decomposition for certain r.e. sets
Mr. T. G. McLaughlin, University of California, Los Angeles (63T-33)

(30) An error-free digital algorithm for systems of linear equations
Dr. H. D. Mills, RCA Laboratories, Princeton, New Jersey (63T-9)

(31) On the resolvent of an antitonic operator in Hilbert space
Professor G. J. Minty, University of Michigan (63T-10)

(32) On the additive completion of sets of integers
Professor Leo Moser, University of Alberta (63T-56)

(33) Coverings of sets by linear combinations of boundary points
Professor T. S. Motzkin and Professor E. G. Straus, University of California, Los Angeles (63T-21)

(34) Coverings of sets by sums of transforms of boundary points
Professor T. S. Motzkin and Professor E. G. Straus, University of California, Los Angeles (63T-22)

(35) Topics on mutation, V. Antiideals of semigroups
Mr. A. A. Mullin, University of Illinois (63T-20)

(36) Topics on mutation, VI. Structure of antiideals of semigroups
Mr. A. A. Mullin, University of Illinois (63T-29)

(37) WITHDRAWN.

(38) The Poincaré conjecture
Dr. E. S. Rapaport, Polytechnic Institute of Brooklyn (63T-16)

(39) On the Pontrjagin classes of a manifold
Mr. H. M. Roberts, University of Connecticut (63T-23)

(40) Some radius of convexity problems
Professor M. S. Robertson, Rutgers, The State University (63T-42)

(41) Neighborhoods of trees in triangulated manifolds
Professor R. H. Rosen, University of Michigan (63T-28)

(42) A note on matrix quadratic residues
Dr. Azriel Rosenfeld, Yeshiva University (63T-2)

(43) A note on two special types of rings
Dr. Azriel Rosenfeld, Yeshiva University (63T-3)

(44) Idempotents in group algebras
Professor Walter Rudin, University of Wisconsin (63T-50)

(45) Semi-automorphismes de groupoides et de quasigroupes
Professor Albert Sade, Marseille, France (63T-11)

(46) A converse of the elimination theorem
Mr. L. E. Sanchis, Pennsylvania State University (63T-30)
(Introduced by Dr. H. B. Curry)

(47) A functional calculus for uniform operators on a reflexive Banach space
Mr. L. J. Senechalle, Illinois Institute of Technology (63T-35)

(48) Projections, retracts, and C*-embedding
Professor Andrew Sobczyk, University
sity of Miami (63T-40)
(49) Discrete full sets
Professor S. K. Stein, University of California, Davis (63T-45)

(50) On free nilpotent quotient images of single defining relation groups
Professor Arthur Steinberg, Fairleigh Dickinson University (63T-54)
(Introduced by Professor Wilhelm Magnus)

(51) Indescrivable cardinals
Professor R. L. Vaught, University of California, Berkeley (63T-12)

(52) Elementary classes of models closed under descending intersection
Professor R. L. Vaught, University of California, Berkeley (63T-13)

(53) Multiplicative groups of row-and/or column-finite infinite matrices
Dr. Paul Verme, Birkbeck College, England (63T-1)

(54) An inverse limit of Banach algebras of continuous functions reducing to the constants
Professor J. J. Westman, University of California, Los Angeles (63T-55)

(55) Invariant imbedding and the phase shift problem
Dr. G. M. Wing, Sandia Corporation, Albuquerque, New Mexico (63T-19)

(56) Complete maps and differentiable coverings
Dr. J. A. Wolf and Mr. P. A. Griffiths, University of California, Berkeley (63T-31)

MEMORANDA TO MEMBERS

THE EMPLOYMENT REGISTER

The following item is repeated from the November, 1962 issue of the NOTICES and gives a more detailed time schedule.

The Mathematical Sciences Employment Register, established by the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Annual Meeting at the University of California at Berkeley, on January 25, 26 and 27, 1963. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on each of these three days.

There is no charge for registration, either to job applicants or to employers, except when the late registration fee for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $3.00 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms or for position description forms. These forms must be completed and returned to Providence not later than January 4, 1963, in order to be included free of charge in the listings at the Annual Meeting at Berkeley. The printed listings will be available for distribution both during and after the meeting. Forms which arrive after this closing date, but before January 14, will be included in the Register at the meeting for a late registration fee of $3.00.

It is essential that applicants and employers register at the Employment Register Desk promptly upon arrival at the meeting to facilitate the arrangement of appointments.

SUMMER EMPLOYMENT OPPORTUNITIES

The Mathematical Sciences Employment Register, 190 Hope Street, Providence 6, Rhode Island, has compiled a list of opportunities for summer employment for mathematicians and college mathematics students. Members who are interested in summer employment and would like to obtain a copy of this list should write to the Mathematical Sciences Employment Register. There is no charge for this list.

Copies will be available at the Annual Meeting in the East Ball Room of the Student Union of the University of California at Berkeley.
ABSTRACTS OF CONTRIBUTED PAPERS

The October Meeting in Hanover, New Hampshire
October 27, 1962


On a covering group defined by C. W. Curtis. Preliminary report.

Let $L$ be a Lie algebra of classical type over an algebraically closed field $K$ of characteristic $p$, $7 < p$; $L = H + \sum K \cdot e(\pi)$ the sum extending over all nonzero roots of $L$ with respect to $H$. Let $G$ be the group of automorphisms of $L$ generated by the elements $\Delta(\pi;k) = \exp(k \cdot \text{ad} e(\pi))$, $k \in K$, $\pi$ any root of $L$, and $D$ the covering group of $G$ constructed by C. W. Curtis (J. Math. Mech. 9 (1960), 307-326). Using Curtis’ methods it is shown that $D$ is an irreducible (linear) algebraic group. An extension of Curtis’ procedure for identifying $D$ is given which together with the work of Ree (Trans. Amer. Math. Soc. 84 (1957), 392-400) yields: if $L$ is the orthogonal Lie algebra with respect to a nondegenerate, symmetric, bilinear form $(x,y)$ on $V$, an $m$-dimensional vector space over $K$, $m = 2n + 1$, $2 < n$, or $m = 2n$, $3 < n$, then in both cases $D$ is isomorphic to the reduced Clifford group defined by the quadratic form $Q(x) = (x,x)$ on $V$. $G$ is isomorphic to the projective commutator subgroup of the orthogonal group defined by $Q$. (Received September 10, 1962.)

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SETS RELATIONS AND FUNCTIONS: JAMES F. GRAY, St. Mary’s University

The outgrowth of a successful television series, Dr. Gray’s book concentrates on instilling confidence by use of exercises that progress in slow stages, leaving the reader to accelerate at his own pace. The result is an extremely useful text, for undergraduates, graduate students lacking this background, and adults seeking acquaintance with modern mathematics. 1962, 155 pages, $2.50 paper

ATHENA SERIES — Brief but brilliant studies covering specialized area of mathematics not found in conventional texts, edited by Edwin Hewitt.

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OPERATIONAL CALCULUS AND GENERALIZED FUNCTIONS, Arthur Erdelyi
INFINITE SERIES, I. I. Hirschman, Jr.
LEBESGUE INTEGRATION
John H. Williamson
LOGIC: THE THEORY OF FORMAL INFERENCE, Alice Ambrose and Morris Lazerowitz

A BRIEF INTRODUCTION TO THETA FUNCTIONS, Richard Bellman
THE SIMPLEX METHOD OF LINEAR PROGRAMMING, Frederick A. Ficken
THE BOOK ON GAMES OF CHANCE
Geralamo Cardano, translated by Sydney Henry Gould
SPECIAL FUNCTIONS OF MATHEMATICAL PHYSICS, Harry Hochstadt
ANALYTIC INEQUALITIES
Nicholas Kazarinoff

383 Madison Avenue, New York 17
On a method of Courant for minimizing functionals.

R. Courant has given a method of handling certain problems in the calculus of variations in which a functional of a path is to be minimized, subject to a single boundary condition, or side condition. The method can be exemplified by the problem of finding functions $y$ and $z$ of the independent variable $x$ which will minimize the integral $\int_a^b f(x,y,y',z,z') \, dx$, subject to the side condition $g(x,y,y',z,z') = 0$, where $f$ and $g$ are specified functions. The method of Courant consists in finding functions $y_K$ and $z_K$ which minimize $\int_a^b (f + Kg^2) \, dx$, and then taking the limits of $y_K$ and $z_K$ as $K$ tends to infinity. It was proved, in mimeographed notes of lectures by Courant, that the method yields a solution to the problem provided the minimizing functions $y_K$ and $z_K$ exist and approach limits. In the present paper, this result is extended to problems involving any finite number of side conditions, and related questions are investigated. The work was motivated by a desire to justify numerical calculations being used in the optimization of flight paths for aircraft. (Received October 2, 1962.)

Comparison of zero frequency and mean in estimation for uniparameter families.

Maximum likelihood estimates are cumbersome for a number of distributions. Ad hoc methods of estimation based on simple sample statistics have been studied as alternatives by a number of authors including Fisher and Evans. The principal advantage in this line of inquiry is that it may lead to relatively simple estimates of the parameters. A difficulty in using these methods is that none of these methods have high efficiency throughout the parameter space and hence none of them can be recommended for all populations. It is observed that in the methods that have been constructed, there is a strong tendency to give weight to the earlier frequencies and moments. As a preliminary study, the effectiveness of zero frequency in contrast with the mean in connection with the Poisson and the Logarithmic distributions has been evaluated, assuming that in any practical case it is possible to pick the better method to obtain an estimate with minimum expected mean square. (Received October 8, 1962.)
The November Meeting in Los Angeles, California
November 17, 1962


For the two-sample problem: \( F = G \) vs. \( F(x) = G(x - \Theta) \), Chernoff, Savage (1958), Capon (1961) and others have essentially considered statistic \( T' = \frac{1}{n} \sum E(\xi(R(x_i))) - \frac{1}{m} \sum E(\xi(R(y_j))) \) based on random samples \( \{x_i\} \) from \( F \) and \( \{y_j\} \) from \( G \), where \( R(w) \) is the rank of \( w \) in the combined sample, and \( \xi(r) \) is the \( r \)th order statistic of a random sample of \( N = n + m \) from cpf \( J \). Such statistics are restricted most powerful rank tests; have approximately the cpf \( J_D \) of the difference of means of samples from \( J \), under \( H_0 \); and are asymptotically normal under \( H_1 \). For the randomized statistic \( S' \), in which each \( E(\xi(r)) \) is replaced by \( \xi(r) \) one finds that Theorem 1. (a) \( S' \) is distribution-free; and under \( H_0 \), (b) \( S' \) has exactly the cpf \( J_D \), and (c) \( E(S') = 0 \) and \( \text{Var}(S') = N \sigma^2 j/2m \) if appropriate moments exist. Theorem 2. Under the Chernoff-Savage regularity conditions (a) \( E(S' - T') = 0 \); (b) \( N^2 \text{Var}(S' - T') \) tends to 0 for all \( \delta < 1 \); (c) \( N^2(S' - T') \) tends to 0 in probability for all \( \epsilon < 1/2 \); and (d) the asymptotic relative efficiency of \( S' \) with respect to \( T' \) is 1 for translation alternatives. (Received October 3, 1962.)


The integral matrices \( T \) for which \( T' AT = eA \) for some number \( e \), \( A \) being a primitive symmetric matrix of order 2, and for which \( \det T \) and \( e \) have the same sign, constitute (along with the zero matrix) an integral domain if the discriminant \( d \) of \( A \) is not a square. A similar theorem involving a commutative ring holds if \( d \) is a nonzero square. A more general theorem involving an additive group holds for the solutions \( T \) of \( T' AT = eB \) (A and B given), with the sign of \( e \cdot \det T \) restricted. The solutions \( T \) of the latter equation are associated one-to-one with the representations of \( e \) by one of two forms, which are determined by the sign of \( \det T \) and by composition of two forms derived from A and B. They are also, when \( d \) is not a square, related one-to-one with the factorizations of a certain number in a quadratic ideal as a product of numbers in that ideal with specified norms. (Received October 10, 1962.)

A point source $H$ generating heat at the rate $q$ is placed midway between two infinite parallel boundaries $B_1$ and $B_2$. The temperature variations along $B_1$ and $B_2$ are prescribed. Configurations are restricted to those which are symmetric with respect to coordinates $x$ and $y$ normal and parallel to the boundaries. The analysis determines exact solutions for steady state temperature distributions of the field compatible with the conditions imposed. Results are derived by the method of sources and sinks and the use of orthogonal functions. (Received June 28, 1962.)


Let $(i) L^0 = \sum_{j=0}^{h} P_j(x) (d/dx)^j$ be an ordinary differential operator of order $h$ whose coefficients are $(\gamma, \eta)$ matrices on the interval $0 \leq x < \infty$, $h = n = 2\nu$, and let boundary conditions at $x = 0$ associated with $L^0$ be $(ii) [\psi_{0j}]^T(0) = 0$, $j = 1, \ldots, \nu$. It is assumed that $q$ is a bounded, positive, symmetric operator, $e$ a real parameter, and that $L^e = L^0 + e q$ determines a self adjoint operator $H^e$ on the space $L^2(0,\infty)$ with spectral measure $E^e(\Delta)$ given by $(E(\Delta)u, v) = \int_\Delta (s_j^e, v)(s_k^e, u) d\rho_{jk}^e(\ell)$, $\Delta$ any real interval, $j, k = 1, \ldots, n$. Conditions have been given (Canad. J. Math. 14 (1962), 359) involving $L^0$ and $q$ which are sufficient to insure that $E^e(\Delta)$ is an analytic operator for sufficiently small $e$. Under these same conditions it is now further shown that a unimodular matrix $V_{jk}(\ell)$ exists, analytic in $e$, such that $(E^{e}(\Delta)u, v) = \int_\Delta (s_j^e, v)(s_k^e, u) V_{jk1}(\ell) V_{kk2}(\ell) d\rho_{jk}^e(\ell)$. In case $\Delta$ contains the spectrum of $H^e$ the matrix $V_{jk}$ determines a unitary transformation $U$ such that $U^* H^e U = H^0$. (Received July 30, 1962.)


Suppose that $K$ is the set of all continuous real valued functions $f$ on $(0,1)$ such that if $x$ is in $(0,1)$, there is an open interval $S$ containing $x$ such that if $u$ and $v$ are in $S$, then for some number $M < 3$ and some number $L$, (*) $\sum_{j=0}^{n} |(\frac{1}{j!} f(u + (j/n)(v - u)))| \leq M \cdot n$, $n = 1, 2, 3, \ldots$.

Theorem. $K$ is a quasi-analytic collection in the sense that if each of $f$ and $g$ is in $K$ and $f(x) = g(x)$ for all $x$ in some nondegenerate subinterval of $(0,1)$, then $f = g$. In a certain sense this result can not be improved upon since for any continuous function $f$ defined on a subinterval $[u,v]$ of $(0,1)$, (*) holds if $M = 3$ and $L$ is an upper bound for $f$ on $[u,v]$. (Received August 27, 1962.)

Let \( \sum_n a_n z^n \) be a Taylor series with radius of convergence one, \( \lim_n a_n = 0 \), and \( \sum_n |a_n| = +\infty \). A point \( z \) on the unit circle \( C \) (\( |z| = 1 \) ) will be called a point of local uniform convergence if the Taylor series converges uniformly on some neighborhood of \( z \) (that is, some open arc of \( C \) containing \( z \)); the set of all such points will be called the set of local uniform convergence. The complement of the set of local uniform convergence with respect to the set of convergence will be called the set of non-uniform convergence. The following results are shown: (1) If \( F \) is any closed set on \( C \), then there exists a function \( f(z) \), continuous in \( |z| = 1 \), whose Taylor series converges everywhere on \( C \) and has \( F \) as its set of non-uniform convergence; (2) If \( M \) is any set of type \( F_0 \) on \( C \), then there exists a Taylor series that has \( M \) as its set of non-uniform convergence; (3) Not every set of non-uniform convergence is of type \( F_0 \) but is of type \( F_{0\infty} \); and (4) If \( C = L \cup N \cup D \) where \( L, N, \) and \( D \) are mutually disjoint and \( L \) and \( D \) are open; then there exists a Taylor series which has (i) \( L \) as its set of local uniform convergence, (ii) \( N \) as its set of non-uniform convergence, and (iii) \( D \) as its set of divergence. (Received September 17, 1962.)

597-5. R. B. KILLGROVE, Box 423, San Diego State College, San Diego 15, California, Completions of quadrangles in projective planes.

Suppose one takes four distinct points as a partial plane, and then forms the free plane (due to Professor Hall) by a sequence of free extensions. The first four partial planes can be easily displayed, the last with seven points and nine lines. The next three partial planes add six, twenty-four, two hundred and eighty-two new elements respectively. Applying this procedure to the Fano configuration, the fourth partial plane contains no longer three new lines as in the free extension case, but instead it contains only one new line. Another approach to obtain the same result is to make identifications between new elements of the fourth partial plane. A plane is said to be singly-generated provided that there exists four points such that every element of the plane can be expressed in terms of these four points, joins, and intersections. Every non-singly-generated plane of order less than twelve is Desarguesian. There does exist a non-singly-generated non-Desarguesian plane. The dual of a singly-generated plane is singly-generated. The techniques used can be programmed for a digital computer. (Received September 17, 1962.)

597-6. WITHDRAWN
On a conjecture of Erdős.

A sequence \( S = (s_1, s_2, s_3, \ldots) \) of real numbers is said to be complete if every sufficiently large integer can be expressed in the form \( \sum_{k=1}^{n} s_k \) where \( i_1 < i_2 < \ldots < i_n \) and \( n \) is some positive integer. Let \( S_t(a) \) be the sequence of integers whose \( n \)th term is \( \lfloor a^n \rfloor \) (where \( \lfloor \cdot \rfloor \) denotes the greatest integer function). It was conjectured by Erdős that \( S_t(a) \) is complete for \( 1 < a < 2 \) and \( t > 0 \). In this paper, the set \( W \) of all pairs \( (t, a) \) for \( 0 < t < 1, 1 < a < 2, \) for which \( S_t(a) \) is complete is determined. In particular, for a positive it is shown that \( S_t(a) \) is complete iff \( 1 < a < 3/5 \). The set \( W \) has a certain amount of structure as indicated by the fact, for example, that for a positive, \( S_{53/100}(a) \) is complete iff \( a \in (1, (5r)^{1/4}) \cup [(13r)^{1/5}, (28r)^{1/6}] \cup [(37r)^{1/7}, (61r)^{1/7}] \cup [(88r)^{1/4}, (129r)^{1/8}] \) where \( r = 100/53 \). (Received September 28, 1962.)

597-8. SEYMOUR GINSBURG, System Development Corporation, Santa Monica, California, and E. H. SPANIER, University of California, Berkeley 4, California. Quotients of context free languages.

The following results on the quotient of context free languages \( CFL \) are shown:

1. It is recursively unsolvable to determine for arbitrary \( CFL \) whether the quotient of one by another is a \( CFL \).
2. If either set is regular and the other is a \( CFL \), then the quotient is a \( CFL \). (Received October 1, 1962.)


Let \( A \) be a finite-dimensional nonassociative algebra with 1 over a field \( F \) of characteristic 0 or \( p > n \). It is proved that, if \( N(x) \) is a nondegenerate form of degree \( n \) which permits composition \( (N(xy) = N(x)N(y)) \), then \( A \) is a separable algebra \( A = A_1 \oplus \ldots \oplus A_r \) with \( N(x) = N_1(x_1) \cdots N_r(x_r) \) where \( x = x_1 + \ldots + x_r \), \( x_i \) in the simple ideal \( A_i \), and \( N_i(x_i) \) is a nondegenerate form of degree \( n_i \) permitting composition, \( n = n_1 + \ldots + n_r \). This reduces determination of the structure of \( A \) and the nature of \( N(x) \) to consideration of simple algebras. If also \( F \) has characteristic \( \not= 5 \) (an additional hypothesis only in case \( n = 3 \) or 4), it is proved for arbitrary \( n \) that \( A \) is alternative, so the \( A_i \) are either associative or Cayley algebras over their centers. Extending known results for quadratic and cubic forms, it is proved that there is a nondegenerate quartic form \( N(x) \) on \( A \) (containing 1, and of characteristic 0 or \( p \not= 5 \)) if and only if \( A \) is alternative and \( N(x) = \prod_{i=1}^{r} [n_i(x_i)]^{f_i} \) where \( n_i(x_i) \) is the generic norm for \( A_i \) and \( f_i \) is a positive integer (\( i = 1, \ldots, r \)). Thus \( A \) has dimension 1,2,3,4,5,6,8,9,10, 12 or 16 in case \( n = 4 \). (Received October 2, 1962.)


Let \( H \) be a Hilbert space, with real or complex scalars, and zero-vector \( \theta \). Let \( f: H \rightarrow H \) be a (in general nonlinear) map having a "positive semidefinite" or "dissipative" Fréchet differential: \( \langle \Delta x, f'(x; \Delta x) \rangle \geq 0 \), for all \( x, \Delta x \) in \( H \). Then a sufficient condition for the equation \( f(x) = \theta \) to have a solution is that there exist \( A \subset H \) with the properties: (1) \( \) the convex hull of \( f(A) \) contains
some open sphere about $\theta$, and $(\mathbb{R}^2)$ the set $\{ \text{Re } \langle x, f(x) \rangle : x \in A \}$ is bounded. This result is closely related to the variational technique for the case where the Fréchet differential is symmetric (E. H. Rothe, Bull. Amer. Math. Soc. 59 (1953), 5-19), to the finite-dimensional result (Minty, Michigan Math. J. 8, 135-137), and to a result on the equation $x + f(x) = \theta$ (Minty, Duke Math. J. (to appear in 1962)).

The main tools are Alaoglu's theorem and a "combinatorial" theorem of the author (Proc. Amer. Math. Soc. 13 (1962), 11-12). (Received October 3, 1962.)

597-11. J. A. WOLF, University of California, Berkeley 4, California. Isotropic manifolds of indefinite metric.

A pseudo-Riemannian [nondegenerate but possibly indefinite metric] manifold $M$ is (locally) isotropic if, given nonzero tangent vectors $X$ and $Y$ at the same point of $M$ and with the same norm, there is a (local) isometry of $M$ sending $X$ to $Y$. Complete connected locally isotropic pseudo-Riemannian manifolds are characterized as the manifolds whose universal pseudo-Riemannian covering manifolds belong to an explicitly described class of 'model spaces' which are pseudo-Riemannian analogues of the Riemannian symmetric spaces of rank 1. This leads to a global classification of the complete connected homogeneous locally isotropic pseudo-Riemannian manifolds, except in the flat case. The starting point is the proof that a locally isotropic pseudo-Riemannian manifold is locally isometric to a 'model space'; that proof is based on classification theorems in Lie groups, and yields this generalization of a result of S. Helgason: If $M$ is a locally isotropic pseudo-Riemannian manifold whose $ds^2$ has either an odd number of - signs or an odd number of + signs, then $M$ has constant sectional curvature if the $ds^2$ is indefinite. (Received October 3, 1962.)


In this paper is studied strong solutions to the Cauchy problem for a certain class of nonlinear partial differential equations. The problems treated are, in general, improper in the sense of Hadamard (e.g., if the equation is elliptic). There has been demonstrated previously the existence of unique solutions to problems of this type which depend Lipschitz continuously on the initial data under the assumption that the latter possess Fourier transforms with compact support [J. Math. Mech. 11 (1962), No. 2, 183-196]. In the present work the same results are obtained under a weaker condition which restricts the growth at infinity of the Fourier transforms of the Cauchy data. The proof is based on appropriately representing the differential equation as a system and then applying the method of successive approximations. This approach is applied in particular to timelike initial value problems for hyperbolic equations, and it is shown that in this special case it is sufficient to restrict the growth of the Fourier transforms with respect to the space variables only. For this problem energy inequality techniques are employed to demonstrate the existence of unique solutions which depend Lipschitz continuously on the Cauchy data. (Received October 8, 1962.)
A class of commutative semigroups having a group-like property.

Let \( S \) be a commutative semigroup with maximum condition on principal ideals. Let \( b \in S \) be such that \( \{ x : xp b \} \neq \{ x : x \sigma b \} \) whenever \( \rho \) and \( \sigma \neq \rho \) are congruence relations on \( S \). Then \( S \) must be of one of the following types. I. Let \( b \) be the identity of any abelian group \( G \), \( n \) a positive integer indexed by \( \{ i : i \in I, n \text{ an integer}, 0 \leq n < n_i, (i, n) \neq (0,0) \} \) with an optional zero element adjoined. Define multiplication by 

\[
(i, n)g = g \quad \text{if} \quad g \in G, \quad (i, n)(j, m) = (i, n + m) \quad \text{if} \quad i = j \quad \text{and} \quad n + m < n_i, \quad (i, n)(j, m) = b \quad \text{otherwise.}
\]

II. Let \( b \) be any element of an abelian group \( G \), \( S = G \cup \{ c \} \), with an optional zero element. Define \( cg = bg \) if \( g \in G \), \( c^2 = b^2 \). III. \( S \) is either \( \{ 0, 1, \ldots, b + 1 \} \) or \( \{ 1, \ldots, b + 1 \} \), \( mn = \min(m + n, b + 1) \). IV. \( S = \{ 0, 1, \ldots, b + 1 \} \cup \{ c \} \), \( oc = b, xc = b + 1 \) if \( x \neq 0 \), \( mn = \min(m + n, b + 1) \).

Corollary. If \( S \) is as in the first sentence, and all the congruence relations on \( S \) commute, then \( S \) is of one of the following types.

V. Let \( G_0, G_1, \ldots, G_n \) be abelian groups, \( \pi_i \) (for \( 1 \leq i \leq n \)) a homomorphism of \( G_{i-1} \) onto \( G_i \), \( S = \bigcup G_i \), with a zero element adjoined. If \( x \in G_i \) and \( y \in G_j \), let \( x \cdot y = (x \pi_i+1 \pi_{i+2} \cdots \pi_{i+j}) (y \pi_{j+1} \pi_{j+2} \cdots \pi_{j+k}) \) if \( i + j \leq n \), \( x \cdot y = 0 \) otherwise. VI. \( S = \{ 1, 2, \ldots, n \} \), \( ij = \min(i + j, n) \). VII. Let \( S \) be any abelian group. (Received October 9, 1962.)

From the (equivalent) definition of \( A^\dagger \), the generalized inverse (Penrose, Proc. Cambridge Philos. Soc., 51, 3 (1955), 406-413) of an arbitrary \( m \times n \) complex matrix, as the least norm (using \( \| X \| = \sqrt{\text{trace } X^*X} \) solution \( X \) of \( A^* AX = A^* \), \( A^* \) the conjugate transpose of \( A \)), a new representation for \( A^\dagger \) is established: Theorem. \( A^\dagger = (I - D(D^*D)^{-1}D^*)EA^* \) where \( E \) is a product of elementary (row operator) matrices such that \( E(A^*A) \) is a matrix whose last \( (n - r) \) rows are zero and whose first \( r \) rows may be written \( [I_{r\times r} \Delta], \quad D^* = [\Delta^* : -I_{n-r}] \), and \( r \) is the rank of \( A^*A \). This result permits the development of new computational methods for \( A^\dagger \) using the extant methods (and numerical analysis) of matrix inversion. One such method for which a Fortran code has been written and tested is presented here together with applications and computational experience. (Received October 8, 1962.)

The order of certain real-valued functions defined in the unit disk. Let \( D(z, r) \) denote the open disk with center \( z \) and radius \( r \). Set \( D(0, 1) = D \). Let \( U(z) \) be any real-valued, non-negative, measurable function in \( D \) such that the double integral of \( U(z) \) over \( D \) is finite. Employing a result of the author (Results of the order of holomorphic functions defined in the unit disk, J. Math. Soc. Japan, to appear) the following is proved. Suppose either that \( U(z) \) is sequentially subharmonic, that is, for each sequence \( \{ z_n \} \) in \( D \), which tends to a point \( e^{i\theta} \), there is a constant \( t, 0 < t < 1 \), and a constant \( K, K > 0 \), (the constants may vary with \( \{ z_n \} \)) such that 

\[
U(z_n)(1 - |z_n|^2)^2 2 K \leq \int \int D(z_n, 1 - |z_n|^2) U(z)|dx|dy, \quad n = 1, 2, \ldots; \text{or that there exists a pair of con-}
\]
constants \((\mu, L)\) such that, if the non-Euclidean (hyperbolic) distance between two points in \(D, z_1,\) and \(z_2,\)
is less than \(\mu,\) then \(|U(z_1) - U(z_2)| < L.\) (This definition is analogous to the notion of a uniformly normal function defined by P. Lappan [written communication].) Then in either case \(U(z)(1 - |z|)\) tends to 0 as \(z\) tends nontangentially to \(e^{i\theta},\) for almost all \(\theta \in [0, 2\pi).\) Similar results are obtained in the case that the exceptional set is a set of \(h\)-measure \(\theta.\) (Received October 8, 1962.)

597-16. C. F. KENT, 2987 Berkshire Road, Cleveland Heights 18, Ohio. Functions which preserve recursiveness of sets.

Let \(f\) be a recursive function and \(\pi\) a recursive permutation, \(A\) a recursive set, and \(B\) a recursively enumerable (r.e.) set. Then \(\pi(A), \pi^{-1}(A), f^{-1}(A)\) are recursive sets and \(f(A), f(B), \pi(B), \pi^{-1}(B)\) are r.e. sets. The following questions suggest themselves. Can the recursive functions be characterized as that set of number theoretic functions whose inverses take recursive sets into recursive sets, or r.e. sets into r.e. sets? Can recursive permutations be characterized as those permutations which take recursive sets into recursive sets, or r.e. sets into r.e. sets? If the answer to the second is no, then so is the answer to the first, and both are answered in the negative by the following set theoretic result. Let \(N\) denote the set of natural numbers and \(P(N)\) its Boolean algebra of subsets. A subalgebra \(Q\) of \(P(N)\) is preserved by a function \(f,\) if \(f\) maps \(Q\) into itself. Theorem. Let \(Q\) be a countable Boolean subalgebra of \(P(N)\) containing all finite subsets of \(N.\) Then uncountably many permutations of \(N\) preserve \(Q.\) The proof makes use of an infinite set \(Z \subseteq N\) which (like the complement of Friedberg's "maximal" r.e. set, J. Symb. Logic 23 (1958)) intersects each member of \(Q,\) or its complement, in a finite subset. The uncountably many permutations are all the permutations of \(Z,\) extended as the identity on \(N - Z.\) This result seems an obstacle to a set theoretic characterization of recursive functions. (Received October 11, 1962.)

597-17. J. P. CLAY, Programming Research Department, Univac Division, 3625 Walnut Street, Philadelphia 4, Pennsylvania. Invariant attractors in transformation groups.

Let \((X, T)\) be a transformation group whose phase space is a uniform space and whose phase
group \(T\) is arbitrary. As a general reference, consult the colloquium volume of Gottschalk and
Hedlund (Top. Dy.). Let \(D\) be an invariant subset of \(X,\) and let \(x \in X.\) \(x\) is said to be \(\{\text{simply}\}\)
\{syndetically\} attracted to \(D\) under \((X, T),\) denoted by \(\{x \in P((X, T); D)\} \{x \in L((X, T); D)\}\) provided if \(a\) is an index of \(X,\) then there exists \(\{t \in T\} \{a \text{ syndetic subset of } T\}\) such that \(\{xt \in Da\}\)
\(\{xA \subseteq Da\}.\) \(x\) is said to be \(\text{regionally simply}\) \{syndetically\} attracted to \(D\) under \((X, T),\) denoted by \(\{x \in Q((X, T); D)\} \{x \in M((X, T); D)\}\) provided that if \(a\) is an index of \(X\) and if \(U\) is a neighborhood of \(x,\) then there exists \(y \in U\) and \(\{t \in T\} \{a \text{ syndetic subset of } A \text{ of } T\}\) such that \(\{yt \in Da\} \{yA \subseteq Da\}.\)
If \(R = P\) or \(Q\) or \(L\) or \(M,\) then \(R((X \times X, T); \Delta_X \times X) = R(X, T).\) Most of the results proven in two earlier papers by the author (Proximity relations in transformation groups, to appear in Trans. Amer. Math. Soc., see January 1962 Notices, and Variations on equicontinuity to appear in Duke Mathematical Journal, see February 1962 Notices) are generalized for the case when \(X\) is compact. New results are also obtained. (Received October 11, 1962.)

The object of this paper is to obtain necessary conditions and also sufficient conditions on a function \( f(x) \), defined for all real \( x \), such that the sequence \( \{f(\lambda_n x + t_0)\} \) (\( t_0 \) real) converges to \( p \) (real or complex) on a set of positive measure for no real sequence \( \{\lambda_n\} \). The more difficult problem of \( k \) function \( f_1(x), \ldots, f_k(x) \) is also considered. Namely, conditions on \( f_1(x), \ldots, f_k(x) \) such that the sequence \( \{\sum_{i=1}^{k} a_{i,n} f_i(\lambda_{i,n} x)\} \) does not converge to zero on a set of positive measure for any \( 2k \)-tuples \( \{\lambda_{1,n}\}, \ldots, \{\lambda_{k,n}\}, \{a_{1,n}\}, \ldots, \{a_{k,n}\} \) of sequence in some class \( C \). Finally the case with \( k \) functions \( f_1(x_1, \ldots, x_n), \ldots, f_k(x_1, \ldots, x_k) \) in \( \mathbb{R}^k \) space is considered. (Received October 15, 1962.)


Definition. A topological space \( \langle S, \mathcal{J} \rangle \) is a-metrizable if and only if there exists a metric \( \rho \), defined on \( S \times S \), and a real number \( a \geq 0 \) such that for each \( p \in S \), the collection \( \{S_p(p; r) \mid r > a\} \) of \( \rho \)-spheres about \( p \) constitutes a local base at \( p \). \( \rho \) is called an a-metric for \( \langle S, \mathcal{J} \rangle \). The following theorems are proved: Theorem 1. A-metrizability is a topological property. Theorem 2. A necessary and sufficient condition that a topological space be semi-metrizable is that it be an a-metrizable \( T_0 \)-space. Theorem 3. A \( T_0 \)-space \( \langle S, \mathcal{J} \rangle \) is a-metrizable if and only if there exists a sequence \( \{\rho_k\} \) of metrics, defined on \( S \times S \), and a nonincreasing sequence \( \{a_k\} \) of non-negative numbers with \( \lim_{k \to \infty} a_k = 0 \) such that: (i) for each \( k \in \mathbb{N} \), \( \rho_k \) generates the discrete topology on \( S \); (ii) for each \( k \in \mathbb{N} \) and each \( p \in S \), the collection \( \{S_{pk}(p; r) \mid r > a_k\} \) is a local base at \( p \); (iii) \( \rho = \lim_{k \to \infty} \rho_k \) exists and is a pseudo-metric which generates the indiscrete topology on \( S \). Corollary. For a semi-metrizable space, a sequence \( \{d_k\} \) of bounded, admissible semi-metrics can be constructed such that the deficiency in the triangle inequality with respect to \( d_k \) becomes arbitrarily small as \( k \to \infty \). (Received October 15, 1962.)


Bergman's method of linear integral operators was proposed for the homogeneous linear partial differential equations of the second order with variable coefficients. There are two characteristic aspects of this method: the recursion formulas for the variable coefficient functions appearing in the infinite series expressing the \( n \)th coefficient function in terms of \( (n - 1) \)th coefficient, and the integration operations. Of a particular interest are the solutions expressed in terms of spherical coordinates. Some of them are valid in the large, actually for any finite values of the radius. The author shows that actually some of those methods may be useful in seeking solutions of linear ordinary differential equations with variable coefficients of arbitrary form. The author demonstrates this on a particular example. (Received October 18, 1962.)

The fundamental theorems of the perceptron-learning theory of Rosenblatt, Block, Joseph, et al (cf. H. D. Block, Rev. Mod. Phys. 34 (1962), 123) are developed (loc. cit.) by ingenious "hard" analytic constructions and bounding techniques whose motivation and restrictiveness (vis-a-vis theorems) is unclear. In this paper new proofs, via an intuitive geometric approach, are given together with extensions which include a necessary and sufficient condition for possibility. (Received November 5, 1962.)


Let E be any class of short exact sequences in the category of finitely generated modules over a Noetherian ring, satisfying the usual axioms needed for relative homological algebra. By composing a number of recent representation theorems for functors an immediate proof is obtained for: E may be faithfully extended to a relative class on all modules which enjoys enough injectives. The finiteness conditions on the ring and modules may be replaced by a necessary "continuity" condition on E. (Received October 18, 1962.)


A theory of convergence is developed in terms of a system \((f, \sigma)\) which has little structure. It is possible, for instance, to dispense with the condition that the intersection of every two members of \(\sigma\) must contain a member of \(\sigma\). The fact that so many of the standard theorems on convergence are unaltered in form indicates that the assumptions for the theorems as usually stated are too strong. (Received October 18, 1962.)


R. L. Moore [Proc. Nat. Acad. Sci. U.S.A. 14 (1928), 85-88] has proved that there does not exist, in the plane, an uncountable set of mutually exclusive simple triods. Moore's result has been generalized to spaces of higher dimensions by G. S. Young [Bull. Amer. Math. Soc. 50 (1944), 714] using a \(T_n\)-set which is a simple triod for \(n = 1\). Since, for \(n \geq 2\), \(E^n\) does not contain uncountably many mutually disjoint \(T_{n-1}\)-sets, it follows immediately that \(T_{n-1} \times I\) cannot be imbedded in \(E^n\). The purpose of this paper is to show that for each \(n > 1\) although \(T_{n-1} \times I\) cannot be imbedded in \(E^n\), there are "minimal" subsets \(C_{n-1}\) of \(T_{n-1}\) which for \(m \geq 0\), though \(C_{n-1} \times I^m\) cannot be imbedded in \(E^{n+m-1}\), \(E^n\) does contain uncountably many mutually exclusive \(C_{n-1}\)-sets. The author [Trans. Amer. Math. Soc. 103 (1962), 403-420] has proved Theorem 1: If \(C\) is a Claytor curve, then for \(m \geq 2\), \(C \times I^{m-2}\) cannot be imbedded in an \(m\)-manifold. It is now proved Theorem 2: \(E^2\) does contain uncountably many mutually exclusive Claytor curves. These two theorems are generalized to
C_n-curves defined as subsets of T_n-sets in such a way that a C_2-curve is a Claytor curve. (Received October 19, 1962.)

597-25. WITHDRAWN.


Post (Bull. Amer. Math. Soc. 50 (1944), 284-315) calls a recursively enumerable set simple if its complement is infinite but does not contain any infinite recursively enumerable set. R. Smullyan calls a recursively enumerable set W effectively simple if the complement of W is infinite and if there is a partial recursive function f such that if \( \omega_e \) (the eth recursively enumerable set) is contained in the complement of W, then f(e) is defined and is greater than the cardinality of \( \omega_e \). The simple set S constructed by Post (op. cit.) is effectively simple. Theorem. There exists a simple set which is not effectively simple. The proof makes simultaneous use of the recursion theorem of Kleene and the priority method of Friedberg and Muchnik. (Received October 22, 1962.)


Examples X_{np} are constructed, one for each n = 2, 3, ..., and each prime p, such that \( \dim X_{np} = n \) and \( \dim(X_{np} \times X_{nq}) = n + 1 \) if p \( \neq \) q and 2n if p = q. For n = 2 such examples were given by Pontrjagin [C. R. Acad. Paris 190 (1930), 1105] and for n = 3 by V. Kuz'minow [Soviet Math. Z (1961), 1457 (translated from Russian)]. The X_{np} are orbit spaces Y_{np}/G_{np} where G_{np} is the inverse limit of finite p-groups and acts freely upon Y_{np}, which is 1-dimensional. Also given are one dimensional spaces Y_p and free actions of G_p (as above) on Y_p such that \( \dim(Y_p/G_p) = \infty \). It follows that Y_p/G_p has cohomology dimension 1 over any field of characteristic different from p. The construction uses mathematical induction and functorial techniques, avoiding most of the geometrical complications of previous examples. (Received October 25, 1962.)
A probability distribution $F$ with $F(0^-) = 0$ is said to have increasing (decreasing) hazard rate -denoted IHR(DHR) - if $\log [1 - F(x)]$ is concave where finite (is convex on $[0, \infty)$). If $t > 0$ and $\zeta$ is a strictly increasing or decreasing function on $[0, \infty)$ such that $\int_0^\infty \zeta(x) dF(x) < \infty$, then sharp upper and lower bounds on $1 - F(t)$ are obtained in both the IHR and DHR case. Specializations are: Theorem 1. If $F$ is IHR, $r \equiv 1$ and $\int_0^\infty x^r dF(x) = \mu_r$, then $1 - F(t^-) \leq \exp \{- t[\mu_r / \Gamma(r + 1)]^{-1/r}\}$ for $t \leq \mu_1^{1/r}$. Theorem 2. If $F$ is IHR, $r > 0$, then $1 - F(t^-) \leq w_0$ for $t \leq \mu_1^{1/r}$, where $w_0$ uniquely satisfies $\mu_r = r^r \int_0^1 x^r - w_0 dx$. Theorem 3. If $F$ is DHR, $r \equiv 0$ and $\mu_r < \infty$, then $0 \leq 1 - F(t) \leq r^r e^{-r \mu_r / r^r \Gamma(r + 1)}$ for $t \leq r[\mu_r / \Gamma(r + 1)]^{1/r}$. Other specializations are obtained with $\zeta(x) = e^{-sx}$. (Received October 25, 1962.)

Continue the notation of the preceding abstract. Sharp upper and lower bounds for $1 - F(t)$ are characterized when $t > 0$, $F$ is IHR or DHR and both $\mu_1$ and $\mu_2$ are given. The extremal distributions are piecewise exponentials. Bounds do not have explicit analytic expressions, but have been machine calculated. (Received October 25, 1962.)

Let $R$ be an arbitrary closed radial set ($z \in R$ implies $\beta z \in R$ for all $\beta \geq 0$). Let $C(R)$ be all entire functions $\not\equiv 0$ which can be obtained as the limit, uniformly in every bounded domain, of a sequence of polynomials whose zeros belong to $R$. Define the order, $w(R)$, of the set $R$ as the l.u.b. of the orders of the functions in $C(R)$. If $w(R) < \infty$, then $w(R)$ is a positive integer $N$, and attained by a zero free function. For each radial set $R$ of order $N$ there exist closed convex radial sets $S_j = S_j(R)$ such that $\exp(\sum a_j z^j) \in C(R)$ if and only if $a_j \in S_j$. The sets $S_j$, $j < N$, are either the plane, a half plane, or a line; $S_N$ is always an angle $\leq \pi$ (including just a ray). A complete characterization of the class $C(R)$ is also given. The method of proof depends on the geometry of the set $R$ and of subsequent sets introduced. (Received October 25, 1962.)

An $\alpha$-complete Boolean algebra $B^*$ is called an $\alpha$-regular extension of the Boolean algebra $B$ if $B^*$ is $\alpha$-generated by an $\alpha$-regular subalgebra $B_0$ isomorphic to $B$. If, in addition, every $\alpha$-homomorphism of $B_0$ into an $\alpha$-complete Boolean algebra $C$ can be extended to an $\alpha$-homomorphism of $B^*$ into $C$, then $B^*$ is called a free $\alpha$-regular extension of $B$. $\mathcal{M}_0$-regular extensions of Boolean algebras were
investigated by R. Sikorski (Fund. Math. 37 (1950), 25-54). In this paper some of Sikorski's results are extended to arbitrary cardinal numbers \( a \). Let \( B \) be a Boolean algebra, \( B_a \) the free \( a \)-extension of \( B \) (see Abstract 581-30, Notices Amer. Math. Soc. 8 (1961), 255), and \( I_a \) the \( a \)-ideal of \( B_a \) generated by all elements \( u = \prod_{x \in E} x \), where \( |E| \leq a \), each \( x \in B \), and \( \prod_1^{j=1} x = 0 \) (\( \prod \) and \( \prod' \) denote the g,l.b, in \( B_a \) and \( B \) respectively). Theorem 1. For every Boolean algebra and every infinite cardinal number \( a \), the free \( a \)-regular extension of \( B \) exists and is unique up to isomorphisms. Theorem 2. An \( a \)-complete Boolean algebra \( B^* \) is an \( a \)-regular extension of \( B \) if and only if \( B^* \) is isomorphic to \( B_a/I_a \), where \( I_a \) is an \( a \)-ideal of \( B_a \) with the properties: (i) \( I_a \supseteq I_a \), (ii) \( I_a \cap B = (0) \). (Received October 26, 1962.)


Let \( L_1 \) and \( L_2 \) be orthocomplemented lattices with no elements in common. Let \( S \) be the union of \( L_1 \) and \( L_2 \) with the two unit elements and the two zero elements identified. Then with the partial ordering inherited from \( L_1 \) and \( L_2 \), \( S \) is an orthocomplemented lattice. Call \( S \) the horizontal sum of \( L_1 \) and \( L_2 \). If \( L_1 \) and \( L_2 \) are complete, \( S \) is complete. If \( L_1 \) and \( L_2 \) are weakly modular, \( S \) is weakly modular. In studying orthocomplemented lattices, this construction is useful for providing counter examples. This use is illustrated by clearing up a point left open in the paper of Loomis on dimension theory [Memoirs Amer. Math. Soc. 18]. The connection between the horizontal sum construction and semi-modularity will also be discussed. (Received October 29, 1962.)

597-33. NACHMAN ARONSZAJN, University of Kansas, Lawrence, Kansas. Extension of a theorem of Hartogs to analytic functions of \( n \) real variables.

Let \( D \) be a connected domain in the space \( R^n \) of \( n \) real variables. Consider a fixed decomposition of the \( n \) variables \( x = (x_1,\ldots,x_n) \) into \( p \) disjoint, nonempty systems \( t_1,\ldots,t_p \). Theorem. Let \( f \) be a complex valued function defined everywhere in \( D \) and analytic in \( D \) in every of the systems \( t_k \), separately, otherwise arbitrary. Then \( f \) is analytic in \( x \) in a connected domain \( D_f \subset D \) which is dense in \( D \). The set \( D - D_f \) — the singular set of \( f \) in \( D \) — has the property that the projection of each of its compact subsets on any of the hyperplanes \( t_k = 0 \) \( (k = 1,\ldots,p) \) is nowhere dense in the hyperplane. Corollary. If, in addition, \( f \) has the property that there exists a positive \( \rho \), such that at every point of \( D \) the Taylor series of \( f \) in every system \( t_k \) — with the possible exception of one — has a radius of convergence \( \leq \rho \), then \( f \) is analytic in \( D \). (Received October 29, 1962.)

597-34. R. D. ADAMS, NACHMAN ARONSZAJN, University of Kansas, Lawrence, Kansas, and K. T. SMITH, University of Wisconsin, Madison, Wisconsin. Bessel potentials in a domain.

Consider the spaces \( \tilde{P}^\alpha(D) \) where \( D \) is a domain in \( R^n \). The space \( \tilde{P}^\alpha(D) \) is the perfect functional completion of \( C^\infty(D) \) with respect to the norm \( |u|_{a,D} \) where \( |u|_{a,D}^2 = \int_D |u|^2 dx \); for \( 0 \leq a < 1 \), \( |u|_{a,D}^2 = C(n,a) \int_D \max(0,|u| - |y|)^{-2a} dx \); and for \( a = 1 \), \( |u|_{1,D}^2 = \sum_{k=0}^m \sum_{\lvert x \rvert < \lvert \nu \rvert} \lvert x \rvert^2 \lvert a - m, D \rvert^2 \) where \( m = \lceil a \rceil \). Note that \( \tilde{P}^\alpha(R^n) = P^\alpha \) is the space of Bessel potentials of order \( \alpha \) of \( L^2 \) functions. One of the primary problems studied is to characterize intrinsically the domains \( D \) for which each function in the class \( \tilde{P}^\alpha(D) \) has an extension to a function in \( P^\alpha \). Several
methods of constructing domains with the extension property are given. In particular, it is shown that if the $\partial D$ is locally the graph of a Lipschitzian function (with minor additional hypotheses if $D$ is unbounded) and $p$ is an arbitrary positive integer, then there is a simultaneous, linear extension of functions $u \in \tilde{P}^a(D)$ to $P^a$ for all $a \leq p$. Among other problems considered are the continuity in $a$ of the norm $|u|_{a,D}$ and the density in $\tilde{P}^a(D)$ of restrictions of functions in $C_0^\infty (R^n)$. (Received October 29, 1962.)

597-35. EMILIO GAGLIARDO, University of Kansas, Lawrence, Kansas. On bounded integral transformations with positive kernel.

Let $(X, \mu)$, $(Y, \nu)$ be $\sigma$-finite measure spaces and $K(x,y)$ a non-negative measurable function on $X \times Y$. A necessary and sufficient condition in order that $T u = \int_X K(x,y)u(x)d\mu$ be a bounded transformation from $L^p(X, \mu)$ to $L^p(Y, \nu)$, ($1 < p < \infty$), with $\|T\| \leq c$ is the following: for every $\varepsilon > 0$, there exist two measurable functions: $\phi(x)$ and $\psi(y)$ such that (i) $\phi(x) > 0$, $\psi(y) > 0$, a.e. in $X$ or $Y$ respectively, (ii) $\|\phi\| \leq (c + \varepsilon)^{-p'/p}$, $\|\psi\| \leq (c + \varepsilon)^{-p'/p'}$, a.e. $(1/p' + 1/p = 1)$, where $\phi(x) = \int_y K(x,y)\psi(y)d\nu(y)$. The functions $\phi$ and $\psi$ may be assumed to be in $L^{p'}(X, \mu)$ and $L^1(Y, \nu)$ respectively. The sufficiency of the conditions (i) and (ii) were communicated to the author by N. Aronszajn. (Received October 29, 1962.)

597-36. NACHMAN ARONSON, F. J. MULLA and POWEL SZEPTYCKI, University of Kansas, Lawrence, Kansas. Spaces of potentials connected with $L^p$-classes.

The theory of Bessel potentials of $L^2$ functions suggests three ways of generalizing the potentials to an arbitrary exponent $p$, $1 \leq p \leq \infty$, which give rise to the spaces denoted by $P^a_p$, $\tilde{P}^a_p$ and $B^a_p$. The space $P^a_p$ (denoted by $L^p_0$) was studied by Calderon; the spaces $\tilde{P}^a_p$, $B^a_p$ with different definitions of norms were considered by Slobodecki, Gagliardo, Stein and others. In this paper a systematic theory of the spaces $\tilde{P}^a_p$ and $B^a_p$ is developed from the point of view of the theory of functional spaces and functional completion. Two basic tools are: (i) certain integral representation formulas; (ii) the theory of regular integral transformations. Known results concerning the spaces $\tilde{P}^a_p$ and $B^a_p$ (in particular about restrictions and extensions to and from hyperplanes) are obtained in a precise form. Also, several new results are proved. Among these are mentioned the representation of $\tilde{P}^a_p$ and $B^a_p$ as spaces of Bessel potentials of certain classes of tempered distributions, projection formulas of some $L^p$ spaces onto $\tilde{P}^a_p$ and $B^a_p$, and the determination of conjugate spaces for $\tilde{P}^a_p$ and $B^a_p$. (Received October 29, 1962.)


Let $X$ be a connected open Riemann surface, let $A$ be the algebra of analytic functions on $X$, and let $F$ be the field of meromorphic functions on $X$. The maximal ideals of $A$ are in one-to-one correspondence with the discrete ultrafilters on $X$: i.e., the ultrafilters in $\{\text{discrete subsets of } X\} \cup \{X\}$. If $M$ is a maximal free ideal in $A$ then $A/M$ is an algebraically closed extension of $C$, the complex number field, of transcendence degree $2^{\aleph_0}$, which has a natural valuation over $C$ onto $C$. 

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which is 1-maximal and whose value group is a divisible $\mathcal{G}_1$-group. The local ring $A_M$ is a valuation
ring of $F$ whose valuation is 1-maximal and whose value group is a nondivisible near $\mathcal{G}_1$-group. The
composite place $p_M$ is a place of $F$ over $C$ onto $C$ which is 1-maximal and which has a nondivisible
$\mathcal{G}_1$ value group. The set $S$ of all places of $F$ over $C$ onto $C$ is compact, when topologized with the
weak topology defined by $F$. The closure $T$ of $X$ in $S$ is not necessarily all of $S$. There is a canonical
projection $\sigma$ of $\beta X$ onto $T$ which is open and continuous; when restricted to $\delta X$, the limits of discrete
ultrafilters on $X$, $\tau$ is a homeomorphism onto $\{p_M: M$ is a maximal ideal of $A_f\}$. (Received October
30, 1962.)

597-38. MARVIN MARCUS and A. M. YAQUB, University of California, Santa Barbara, University, California. Compound matrix equations.

Let $C_r(A)$ and $C^{n-r}(A)$ designate the rth compound and $(n - r)$th supplementary compound
matrices of the n-square matrix $A$. The paper is concerned with finding all real solutions $A$ to

(1) $C_r(A) = C^{n-r}(A^T)$. Both sides of (1) vanish in case the rank of $A$ is less than $\min (r, n - r)$; such
solutions are called trivial. Theorem 1. $A$ is a nontrivial singular solution to (1) if and only if

$n 

2r$, $r$ is even, rank of $A = r$ and $A$ is similar over the reals to $J = \sum 1^E$ where $E$ is the 2-square
matrix whose only nonzero entry is a 1 in the (1,2) position. Theorem 2. If $A$ is a nonsingular
solution to (1) then $A$ is similar over the reals to $J = \sum 1^E$ where $E$ is the 2-square skew-symmetric
real matrix with 1 in the (1,2) position. Conversely if $A$ is similar to $B$, or $A$ is similar to $C$ and $n$ is even, then $A$ satisfies (1). These results and others extend theorems of H. Schwerdtfeger (Portugal Math. 20 (1961), 39-41). (Received October 30, 1962.)

GARETH WILLIAMS, Walker Hall, University of Florida, Gainesville, Florida. Rigid motion in a
gravitational field. Preliminary report.

Rigid motion, as was defined by Max Born (Ann. Physik 30 (1909), 1) for flat space-time, and
extended to Riemannian space-times by Salzman and Taub, (Phys. Rev. 95 (1954), 1659) is investigated.

The integrability conditions for the equations of rigid motion are derived, and certain consequences
Physik 31 (1910), 919) which states that in flat space-time every rotating rigid motion is isometric
is given. By a transformation which preserves rigidity it is shown that this is not in general true
in an arbitrary Riemannian space-time. (Received November 1, 1962.)

On quasi-Frobenius algebras and complete cohomology of maximal orders.

Let $A$ be a finitely generated projective $R$-algebra, $R$ an arbitrary commutative ring. Call $A$ a
quasi-Frobenius algebra if there exists an automorphism $T$ of the category of left $A$-modules such
that $T(A)$ is isomorphic to $\text{Hom}_R(A, R)$ as an $A$-bimodule. In case $R$ is a field this coincides with the
usual notion. If $A$ is quasi-Frobenius, $A$ has a complete resolution as $A$-bimodule. The positive
cohomology groups are the usual ones, while the negative ones, after the usual shift in dimension, are the usual homology groups with a modified coefficient module. This generalizes results of Nakayama (Osaka Math. J. 9 (1957), 165-187 and Nagoya Math. J. 13 (1958), 115-121). A maximal order in a separable algebra over the quotient field of a Dedekind ring is a quasi-Frobenius algebra. Compute explicitly these complete cohomology groups in case the Dedekind ring has all its residue class fields finite. This cohomology has period two. This extends and completes results of Nakayama (Abh. Math. Sem. Univ. Hamburg 23 (1959), 174-179). (Received November 1, 1962.)


Let \((X; R, R^*)\) denote a nonempty set \(X\) with binary relations \(R\) and \(R^*\) defined on \(X\). If \(x \in X\), let \(R(x) = \{y \in X \mid yRx\}\), \(R^*(x) = \{y \in X \mid yR^*x\}\), and \(N(x) = R(x) \cap R^*(x)\). For \(S \subseteq X\), define \(R(S) = \bigcup_{x \in S} R(x)\), \(R^*(S) = \bigcup_{x \in S} R^*(x)\), and \(N(S) = \bigcup_{x \in S} N(x)\). An element \(x\) is termed an \(R^*\)-dominant element for the nonempty set \(S \subseteq X\) if \(x \not\in S \cap R^*(x)\). A set \(S \subseteq X\) is called \(R\)-scattered if for every pair of distinct \(x, y \in S\), \(x\) and \(y\) are \(R\)-incomparable; that is, \(x\) non \(Ry\) and \(y\) non \(Rx\). E. J. Mickle and T. Rado (On covering theorems, Fund. Math. 45 (1957), 325-331) derive the following equivalent to Zorn's lemma: (MR) Given \((X; R, R^*)\), \(R\) is reflexive and symmetric, and every nonempty set \(E \subseteq X\) contains an \(R^*\)-dominant element. Then there exists an \(R\)-scattered set \(S \subseteq X\) such that \(X = N(S)\). The following equivalents to (MR) and, therefore, to the axiom of choice are derived.

Theorem 1. Under the hypotheses of (MR), there exists an \(R\)-scattered set \(S \subseteq X\) such that: (i) \(S\) is a maximal \(R\)-scattered set, and (ii) \(X - N(S - x) \subseteq N(x)\) for every \(x \in S\). Theorem 2. Under the hypotheses of (MR), there exists an \(R\)-scattered set \(S \subseteq X\) such that: (i) \(S\) is a maximal \(R\)-scattered set, and (ii) \(X - N(S - x) \subseteq R^*(x)\) for every \(x \in S\). These results are realized by a characterization of the \(R\)-scattered set in the conclusion of (MR). (Received November 1, 1962.)


It is shown that analytic systems of differential equations of the form \(\varepsilon dy_1/dx = f_1(x, y_1, y_2, \varepsilon)\), \(dy_2/dx = f_2(x, y_1, y_2, \varepsilon)\) can be simplified into \(\varepsilon dz_1/dt = -z_1 + z_2 H(t, z_2, \varepsilon)\), \(dz_2/dt = 0\) by means of transformations involving asymptotic series in powers of \(\varepsilon\). From this it follows that the solutions of initial value problem at \(x = 0\) can in a certain sector be written as convergent power series of the form \(y_j(x, \varepsilon) = \sum_{r=0}^{\infty} T_{jr}(x, \varepsilon) \exp[-rq(x)/\varepsilon]\), \(j = 1, 2\). Here \(q(x)\) is a function such that \(q(0) = 0\) and \(\Re q(x) > 0\) in the region of convergence. The functions \(T_{jr}(x, \varepsilon)\) possess asymptotic series in powers of \(\varepsilon\). A decisive assumption is that the reduced system \(0 = f_1(x, u_1, u_2, 0), du_2/dx = f_2(x, u_1, u_2, 0)\) possess a particular solution for which \(\partial f_2(x, u_1, u_2, 0)/\partial u_1\) does not vanish at \(x = 0\). (Received November 2, 1962.)
Submanifolds in a Riemannian manifold with general connections.

A general connection of a Riemannian manifold \( M_n \) with a metric \( g_{ij} \) in the sense of Ōtsuki (Math. J. Okayama University 1960, 1961; Proc. Japan Acad. 1961; Kōdai Math. Sem. Rep. 1961, 1962) contains a tensor \( P^i_j \) and coefficients \( r^i_{jk} \). When \( P^i_j = \delta^i_j \), it is a connection in the common sense.

Consider a regular submanifold \( M_m \) in \( M_n \) we have the following results. (1) \((P^i_j, r^i_{jk})\) can be induced in \( M_m \). (2) Covariant differentiation for mixed tensors can be defined. (3) For a kind of adapted submanifolds some theorems in the common connection theory of submanifolds can be generalized. For an adapted submanifold \( M_m \), the following theorems are proved. (4) With respect to a general connection a curve in \( M_m \) has at least one development in a subspace \( A_m \) of a pseudo-affine space \( A_n \). (5) If \((P^i_j, r^i_{jk})\) is regular (it means \( |P^i_j| \neq 0 \)) and metric (it means \( Dg_{ij} = 0 \)) then the induced connection is also metric. (6) A formula about curvature tensor with respect to a general connection of \( M_n \) and its induced connection of \( M_m \) can be generalized. (Received November 5, 1962.)

On properties of derivatives.

If \( f \), from a closed interval \( I \) into \( R \), is a derivative, then it is known that \( f \) has property (A): for every open interval \( (a,b) \), \( f^{-1}((a,b)) \) is either empty or has positive measure. In this paper the following theorem is proved: **Theorem 1.** If \( f: I \to R \) is of Baire class one, and if for each subinterval \( J \) of \( I \) on which \( f \) is either bounded above or below, \( f \) restricted to \( J \) satisfies (A), then \( f \) satisfies (A).

Theorem 1 implies that approximate derivatives and nth exact Peano derivatives satisfy (A). The latter was first proved by Oliver (The exact Peano derivative, Trans. Amer. Math. Soc. 76 (1954), 444-456). Zahorski proved that derivatives also have the stronger property (B): for every open interval \( (a,b) \), if \( x \in f^{-1}((a,b)) \) and if \( I_n \to x \) with \( m(I_n \cap f^{-1}((a,b))) = 0 \), then \( \lim_{n \to \infty} \frac{m(I_n)}{d(x, I_n)} = 0 \) where \( I_n \to x \) means every neighborhood of \( X \) contains all but a finite number of the closed intervals \( I_n \) and \( d(x, I_n) \) is the distance of \( x \) from \( I_n \). (Sur la première dérivée, Trans. Amer. Math. Soc. 69 (1950), 1-54.) Corresponding results are obtained here for approximate and nth exact Peano derivatives. **Theorem 2.** Every nth exact Peano derivative satisfies (B). **Theorem 3.** Every approximate derivative satisfies (B). (Received November 5, 1962.)

Order structures for certain acyclic topological spaces.

If a space \( X \) is acyclic and arcwise connected then a partial order \( \preceq \) may be defined on \( X \) by choosing a point \( g \) in \( X \) and saying \( x \preceq y \) provided \( x \) is an element of the unique arc with \( g \) and \( y \) as endpoints [see L. E. Ward, Jr., A fixed point theorem for multi-valued functions, Proc. Amer. Math. Soc. 8 (1958), 921-927]. This paper treats the properties of this order on certain classes of topological spaces. If \( \preceq \) is a partial order on a set \( X \), one denotes by \( L(x) \) (resp. \( M(x) \)) the set of \( y \) in \( X \) such that \( y \preceq x \) (resp. \( y \succeq x \)) and by \([x,y]\) the set \( M(x) \cap L(y) \). In 1946 G. S. Young [see The introduction of local connectivity by change of topology, Amer. J. Math. 68 (1946), 479-494] studied spaces in which the union of any nest of arcs is contained in an arc. It is shown that a compact metric space \( X \) has this property if and only if \( X \) admits a partial order \( \preceq \) satisfying (1) \( \preceq \) is order dense,
(2) M(e) = X for some e in X, (3) [x,y] is closed and linearly ordered for each x and y in X, (4) if x and y are elements of an arc A contained in X, then [x,y] ⊆ A, and (5) each linearly ordered subset of X has an upper bound. By using the above result two fixed point theorems are proved which generalize results of Ward and Young in the papers cited above. (Received November 5, 1962.)

597-46. MELVIN HENRIKSEN and MEYER JERISON. Purdue University, Lafayette, Indiana, Minimal projective extensions of compact spaces.

Gleason [Illinois J. Math, 2 (1958), 482-489] proved that for each compact Hausdorff space X there exist a unique extremally disconnected compact space G(X) and a mapping \( \pi_X \) of G(X) onto X such that \( \pi_X \) maps no proper closed subspace of G(X) onto X. G(X) will be called the minimal projective extension of X. It may be realized as the Stone space of the Boolean algebra of all regular closed subsets (i.e., sets that are the closures of their interiors) of X. If \( \tau \) is a mapping of X onto a space Y, then the "projective" property of G(X) implies that there exists a mapping \( \overline{\tau} \): G(X) → G(Y) such that \( \overline{\pi_X} \circ \overline{\tau} = \tau \circ \pi_X \). Minimality of \( \overline{\pi_Y} \): G(Y) → Y implies that \( \overline{\tau} \) is a surjection. This paper treats the problem of when the mapping \( \overline{\tau} \) is uniquely determined by \( \tau \). Theorem. \( \overline{\tau} \) is unique if and only if \( \text{cl} \overline{\tau}^{-1}(\text{int} \beta) = \text{cl}(\text{int} \overline{\tau}^{-1}(\beta)) \) for every regular closed set \( \beta \) in Y. In the general case, the following relation is valid for all \( \beta \): \( \pi_X^{-1}[\text{cl}(\text{int} \overline{\tau}^{-1}(\beta))] \subseteq \overline{\pi_Y}^{-1}(\beta) \subseteq \pi_X^{-1}[\text{cl}(\text{int} \overline{\tau}^{-1}(\beta))]. \) (Received November 5, 1962.)


Solutions of ordinary self-adjoint differential equations of arbitrary even order of the form (1) \( \left[ r(x) y^{(n)}(x) \right]^{(n)} + p(x)y = 0 \) with \( r(x) > 0 \), \( p(x) \neq 0 \), and \( r(x) \) and \( p(x) \) continuous on \([0,\infty)\) are investigated with regards to the general boundary conditions: \( y(a) = y'(a) = \ldots = y^{(n-1)}(a) = 0 \), and \( y(b) = y'(b) = \ldots = y^{(n-1)}(b) = 0 \), where \( y_1(x) \equiv r(x)y^{(n)}(x) \), \( 1 \leq i \leq n \), and \( b > a \). For \( i = n \), the problem is of the type investigated by J. H. Barrett, H. C. Howard, W. Leighton, Z. Nehari, H. M. Sternberg, and R. L. Sternberg for the case \( n = 2 \) and for general \( n \) by W. T. Reid, H. Kaufman, R. L. Sternberg, and the author. For \( i \neq n \), the same methods do not apply, and thus new techniques of investigation are developed. Theorem. If \( i \) is an even (odd) positive integer and \( y(x) \) is a nontrivial solution of (1) with \( p(x) \geq 0 \) \((p(x) < 0)\) on \([a,\infty)\) and \( y(x) \) has a zero of order \( \geq 2n - i \) at \( x = a \), then \( y(x) \) cannot have a zero of order \( \geq i + 1 \) on \((a,\infty)\). In other words, the above boundary problem has no nontrivial solution under these conditions. For \( p(x) > 0 \) on \([a,\infty)\), some separation and oscillation properties are then established. (Received November 5, 1962.)


Let A be a torsion free group of rank n. Use \( A^* \) to denote the minimal divisible group containing A. Let \( T(A) = \{t(x): x \in A\} \), where \( t(x) \) denotes the type of \( x \) in \( A \), and where we define \( t(0) = t_0^* \), a type greater than all other types. Let \( C(A) = T(A) \cup \{\text{all finite intersections of members of } T(A)\} \).
For any type \( t \), \( \mathcal{A}_t = \{ x \in A : t(x) = t \} \) is a pure subgroup of \( A \). \( P(A) = \{ \mathcal{A}_t : t \in \mathcal{C}(A) \} \) and \( P^*(A) = \{ \mathcal{A}_t^* : t \in \mathcal{C}(A) \} \) are isomorphic lattices, in which lattice meet is set intersection, and are dually isomorphic to the lattice \( \mathcal{C}(A) \). If \( T(A) \) is finite, then \( T(A) = \mathcal{C}(A) \); and, for \( t \in T(A) \), \( \mathcal{A}_t \) has a maximal independent set whose members all have the same type \( t \) in \( A \). Conversely, let \( T = \{ t_0, t_1, \ldots, t_n \} \) be a set of distinct types forming a lattice in which lattice meet is type intersection. Then there is a dually isomorphic lattice \( L^* = \{ 0, \mathcal{A}_0^*, \mathcal{A}_1^*, \ldots, \mathcal{A}_N^* \} \) of subspaces of a rational \( n \)-dimensional vector space in which lattice meet is set intersection. Then there is a torsion free group \( A \) such that \( T(A) = T \) and \( P^*(A) = L^* \). The construction of this group \( A \) suggests defining a class of groups for which quasi-isomorphism invariants are then found in terms of \( T(A) \) and \( P^*(A) \), together with other properties. (Received November 5, 1962.)


In certain problems involving incidence matrices the equation \( * AA^T = aA \) occurs. Goldberg (Amer. Math. Monthly 67 (1960), 367) established the necessary and sufficient conditions for \( A \) to satisfy \( * \) is that under a unitary transformation it has a certain direct sum decomposition, where \( A \) consists of only zeros and ones. Brenner (Amer. Math. Monthly 68 (1961), 895) extended this, requiring only that \( A \) have non-negative elements. The author has considered the more general problem \( AA^* = sA \), \( A^* \) the Hermitian of \( A \) and \( s \) complex. Several special cases are considered and finally since \( AA^* \) is Hermitian, there exists a unitary matrix \( U \) such that \( UAU^* \) has a direct sum decomposition. Several other related equations are solved also. (Received November 5, 1962.)


The affine planes constructed by Ostrom admit coordinate systems \( T \) of the following type: \( T \) is of order \( q^2 \) and includes a subfield \( F \) of order \( q \). \( T \) is a vector space of dimension two over \( F \). The equations of lines take the form \( y = xc + d \) or \( y = (x - a)m + b \) where \( a, b, \) and \( c \) are in \( F \) but \( m \) is not in \( F \). \( T \) admits a group of automorphisms of order \( q \) which leaves \( F \) elementwise fixed. We say that the partial left distributive law holds if \( u(v + a) = uv + ua \) for all \( u, v \) in \( T \) and all \( a \) in \( F \). The class of planes such that the partial left distributive law holds includes those which are self dual and those which admit collineations displacing the line at infinity. For this class, multiplication can be given in a relatively specific form. Multiplication is nonassociative. The translation group is exactly of order \( q^2 \) unless \( q \) is even. These planes can all be obtained by a chain of (reversible) constructions from other planes having linear coordinate systems which admit both partial distributive laws. (Received November 5, 1962.)

597-51. J. J. LEVIN, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington 73, Massachusetts, and J. A. NOHEL, University of Wisconsin, Madison, Wisconsin. Note on a nonlinear Volterra equation.

The equation considered is \( (1) \frac{dx}{dt} = - \int_{0}^{t} a(t - \tau)g(x(\tau))d\tau \), where \( (2) a(t) \) is completely monotonic on \( 0 \leq t < \infty \), \( a(t) \neq a(0) \), and \( (3) g(x) \in C(-\infty, \infty), xg(x) > 0(x \neq 0), \int_{0}^{\infty} g(\xi)d\xi \rightarrow \infty (|x| \rightarrow \infty) \).
It is shown that \( x^{(k)}(t) \to 0 \) as \( t \to \infty \) for every solution \( x(t) \) of (1). Equation (1), under conditions (2) and (3), occurs in certain problems of reactor dynamics. While the result is weaker than that of Levin (see abstract No. 597-61), the present approach draws together such different notions of positivity as Liapunov functions, completely monotonic functions, and kernels of positive type. It also provides another Liapunov function for (1). (Received November 5, 1962.)


Let \( \{g_n, \lambda_n\} \) be the characteristic pairs of a regular self-adjoint boundary value problem on the finite interval \((a,b)\). A finite order Schwartz distribution \( \phi \) on \((a,b)\), integrable from \( a^+ \) to \( b^- \) has an expansion with respect to the orthonormal sequence \( \{g_n\} \). This expansion converges to \( \phi \). The expansion coefficients \( a_n \) of \( \phi \) obey the inequality \( |a_n| \leq M|\lambda_n|^m \) for some positive constants \( M \) and \( m, \lambda_n \) not zero. On the other hand, any series of \( g_n \) whose coefficients obey such an inequality converges to a distribution. Any series converging to 0 must have coefficients given by linear combinations of the values of the derivatives of the \( g_n \) at \( a \) and \( b \). One arrives at the above conclusions by using properties of the iterates of the Green's function together with the fundamental sequence approach to distributions. On certain compact differentiable manifolds, distributions are always integrable and hence have an expansion. Any distribution on the surface of the sphere, for example, has a Laplace series which converges to that distribution. Here the conclusion is based on the fact that its Laplace series uniquely determines a distribution. (Received November 5, 1962.)


Let \( \{X_n\} \) be a sequence of positive integer valued random variables which are mutually independent but not necessarily identically distributed. Set \( S_n = \sum_{k=1}^{n} X_k \), and \( u_n = \sum_{k=1}^{n} P(S_k = n) \). Lemma. If there exists a subsequence \( \{X_{n_k}\} \) of \( \{X_n\} \) and a real number \( M > 0 \) such that the following conditions hold, then \( \lim_{n \to \infty} (u_n - u_{n-1}) = 0 \). (A) There exists \( \nu > 0 \) such that \( P(X_{n_k} \geq M) \geq \nu \) for all \( k \). (B) Let \( Y_k \) have the probability distribution \( P(Y_k = j) = P(X_{n_k} = j | X_{n_k} \leq M) \) and let \( \phi_k(t) = E(e^{it Y_k}) \). For any \( a > 0 \) there exists \( \beta > 0 \) depending only on \( a \) such that \( \sup_{q} |\phi_k(t)| \leq 1 - \beta \) for all \( t \) such that \( a \leq |t| \leq \pi \). Under additional restrictions \( \lim_{n \to \infty} u_n \) exists and can be evaluated. Let \( A_{n,k} = \bigcap_{j=0}^{\infty} [S_{j} < n] \cup [S_{j} \geq n + k] \). Theorem. \( \lim_{n \to \infty} u_n = 1/a \) provided (i) \( \lim_{n \to \infty} (u_n - u_{n-1}) = 0 \), (ii) \( (1/k) \sum_{n=1}^{k} \chi_{q+n} \) converges in probability to \( \alpha > 0 \) as \( k \to \infty \) uniformly in \( q \), and \( \lim_{k \to \infty} P(A_{n,k}) = 0 \) uniformly in \( n \). Let \( G(x) = \inf_{k \geq 1} P(X_k < x) \). If \( \lim_{k \to \infty} (1/k) \sum_{n=1}^{k} E(X_n) = m \) and if \( G(x) \) is a p.d.f. with \( \int_{0}^{\infty} xdG(x) < \infty \) then condition (ii) of the theorem is satisfied with \( \alpha = m \). Examples exist where \( \alpha \) is not the Cesaro average of the means of the summands. Similar results can be proven in the continuous case. For related results see W. Smith, Berkeley Symposium in Probability and Statistics, 1960, 467-514. (Received November 7, 1962.)

It is known that any general recursive function can be obtained by composition of an ordinal recursive function, defined over a well-ordering of order type $\omega$ and a primitive recursive function, where the well-ordering is primitive recursive (J. Myhill, Abstract, J. Symb. Logic 18 (1953), 190-191.) It has been shown that the well-ordering may be induced by a primitive recursive permutation (a well-ordering is induced by a permutation if permuting the natural numbers yields the well-ordering). The proof method is similar to that of S. Lui, Proc. Amer. Math. Soc. 11 (1960), 184-187. This result is not equivalent to the known result in the sense that: (I) Not every primitive recursive well-ordering of order type $\omega$ can be induced by a primitive recursive permutation, (II) Not every well-ordering induced by a primitive recursive permutation is primitive recursive. It has been further shown that any function ordinal recursive over a well-ordering induced by a primitive recursive permutation which has a primitive recursive inverse is primitive recursive. (Received November 7, 1962.)


Investigations of minimal topological spaces as given in papers by Bourbaki, Ramanathan, Sorgenfrey and Berri are continued in determining the categories of various types of minimal topological spaces. Theorem 1. Every countably infinite minimal Fréchet space is of first category; every uncountably infinite minimal Fréchet space is of second category. Theorem 2. Every countably infinite minimal Hausdorff space contains an isolated point. Corollary. Every countably infinite minimal Hausdorff space is of second category. Theorem 3. Every countably infinite minimal regular space contains an isolated point. Theorem 4. Every minimal regular space is of second category. (Received November 9, 1962.)


Let $\mathcal{F}_0$ denote the class of functions $f(z)$ analytic and univalent in the annulus $R: r < |z| < 1$, and satisfying $0 < |f'(z)| < 1$ for $z \in R$, $|f(z)| = 1$ for $|z| = 1$. For fixed $b$, $r < b < 1$, it is required to find the maximum and minimum of $|f'(b)|$ for all $f \in \mathcal{F}_0$. These two extremum problems are attacked by the variational method developed by M. Schiffer and the author (Arch. Rational Mech. Anal, 9 (1962), 260-272). For both problems, the extremal function is found to be a mapping of $R$ onto the full unit disk slit along a radial segment, except for the following phenomenon. As $b \rightarrow r$, there is a certain value $b^*$ at which the solution to the maximum problem changes character: the radial slit sprouts a fork at its end. The specific dependence of $b^*$ on $r$ is given through elliptic functions. Finally, an actual representation of the extremal function by means of Jacobian elliptic functions leads to the estimate $|f'(z)| \approx (C/|z|) \log(|z|/r)$ for all $f \in \mathcal{F}_0$ and all $z \in R$. Explicit formulas give the constant C in terms of r. (Received November 9, 1962.)
Measures and tensors.

Let $E$ and $F$ be Banach spaces, and $S$ a compact Hausdorff space. Denote by $C(S,F)$ the space of all continuous $f:S \to F$, and write $C(S) = C(S,F)$, when $F$ is the scalar field. Primes denote dual objects. Use the tensor product notations and conventions of Schatten. Theorem 1. $[C(S) \otimes F]' = (\text{Grothendieck}) C'(S,F) = M(S,F')$, where the last space is the one which consists of all $F'$-measures of bounded variation with the total variation norm. Theorem 2. $C'(S) \otimes F \subseteq M(S,F)$, isometrically. The embedding is effected by assigning to the tensor $T = \sum_{i=1}^{\infty} \mu_i \otimes y_i$, $\mu_i \in C'(S)$, $y_i \in F$, the vector measure $m(A) = \sum_{i=1}^{\infty} \mu_i(A)y_i$, where $A$ is a Borel set. The variation $\mu_m$ of every $m \in M(S,F)$ is a Radon measure, with respect to which $m$ is absolutely continuous (Dinculeanu). But in general there is no Radon-Nikodym-type theorem connecting the two. Theorem 3. Let $m \in M(S,F)$. Then $m \in C'(S) \otimes F$ if and only if $m$ and $\mu_m$ are connected by a Radon-Nikodym-type theorem, Theorem 3 yields a fairly elementary proof of Theorem 4 (Grothendieck) $(E \otimes F)' = E' \otimes F'$ if either $E'$ or $F'$ is reflexive or separable and if either $E'$ or $F'$ satisfies the condition of approximation. (Received November 9, 1962.)

A characterization of reflexivity. Preliminary report.

A sequence $\{M_i\}$ of subspaces of the Banach space $X$ is a subspace basis provided for each $x \in X$ there exists a unique sequence $\{x_i\}$ such that $x_i \in M_i$ and the sequence $\sum_{i=1}^{n} x_i$ converges to $x$. Each subspace basis gives rise to a sequence of projections. The subspace basis $\{M_i\}$ is called: continuous provided the corresponding projections are continuous; boundedly complete provided for each sequence $\{x_i\}$ such that $x_i \in M_i$ and $\sum_{i=1}^{n} x_i$ is bounded it follows that $\sum_{i=1}^{n} x_i$ converges; and shrinking provided for each $f \in X^*$, $p_n(f) \to 0$ as $n \to \infty$, (where $p_n(f) = \sup \{|f(x)|: \|x\| \leq 1$ and $x \in \sum_{i=1}^{n} M_i\}$). Theorem. A Banach space with a continuous subspace basis is reflexive if and only if each of these subspaces is reflexive and the subspace basis is both shrinking and boundedly complete. This generalizes James' Theorem 1 (Ann. of Math. 52 (1950), 519). Further, a shrinking continuous subspace basis of a Banach space induces a continuous subspace basis of the conjugate space. Thus, if a reflexive Banach space has a continuous subspace basis, each successive conjugate space has a shrinking continuous subspace basis. (Received November 9, 1962.)

On maximal decompositions of linear maps.

Let $A$ be a linear map of a finite-dimensional vector space $R$ over a field $K$ into itself, and let $m(x)$ be its minimum polynomial. A maximal decomposition of $R$ with respect to $A$ is a decomposition into $A$-invariant, indecomposable subspaces, where the underlying field $K$ has been extended so as to include all the roots of $m(x)$ (e.g., to an algebraically closed field). Some statements concerning maximal decompositions are presented, which can be proved by rational operations within the original field $K$. $A$ is maximal diagonalizable iff $\det m'(A) \neq 0$, where $m'(x)$ is the derivative of $m(x)$. An expression is given for $\det m'(A)$ in terms of discriminants and resultants of the invariant factors.
The number $n_k$ of invariant subspaces of given dimension $k$ in any maximal decomposition can be calculated by means of a rational algorithm involving $m(x)$ and $m'(x)$. The algorithm fails in some fields of characteristic $p \neq 0$, but can be extended, making use of the polynomials $q_j(x)$, so as to include this case. (Received November 9, 1962.)


Two metric spaces $A$ and $B$ are defined to be perimorphic if, for each $e > 0$, there exist a monotone $e$-mapping, $\phi_e$, of $A$ onto $B$ and a monotone $e$-mapping, $\psi_e$, of $B$ onto $A$. A mapping $f_1: X_1 \to Y$ is said to be periequivalent to a mapping $f_2: X_2 \to Y$ iff $X_1$ and $X_2$ are perimorphic with respect to $\{\phi_e\}$ and $\{\psi_e\}$ and $\bar{d}(f_1, f_2, \phi_e) < e$ and $\bar{d}(f_1, f_2, \psi_e) < e$ for each $e > 0$ (where $\bar{d}(f,g) = \sup \{|f(x), g(x)| : x \in X\}$). Periequivalence is an equivalence relation on the class of mappings from metric spaces into $Y$. Each equivalence class, $[f]$, will be called a perivariety. The representation problem for perivarieties is: Given one representation for a perivariety find all of its representations. So perivarieties are defined analogously to Fréchet varieties and apparently constitute a larger class of spaces. However, we prove that a perisurface (i.e., a perivariety whose associated range spaces are perimorphic to a 2-manifold) is actually a Fréchet surface. (All manifolds considered are compact and connected.) The representation problem for Fréchet surfaces has been solved (Youngs, The representation problem for Fréchet surfaces, Mem, Amer, Math, Soc, No. 8). So we are able to obtain an analogous solution in the case of perisurfaces. We show that if $X_1$ is a 2-manifold, then $f_1: X_1 \to Y$ and $f_2: X_2 \to Y$ are periequivalent iff they are Fréchet equivalent. (Received November 8, 1962.)

597-61. J. J. LEVIN, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington 73, Massachusetts. The asymptotic behavior of the solution of a Volterra equation.

The equation considered is (1) $dx/dt = -\int_0^T a(t - \tau)g(x(\tau))d\tau$, where $a(t) \in C[0, \infty)$, $(-1)^k a^{(k)}(t) \equiv 0 (0 < t < \infty; k = 0, 1, 2, 3)$, $a(t) \notin a(0)$ and $g(x) \in C(-\infty, \infty)$, $xg(x) > 0 (x \neq 0)$, $\int_0^\infty g(x)dx \to \infty (|x| \to \infty)$. It is shown that $x^{(k)}(t) \to 0$ as $t \to \infty (k = 0, 1, 2)$ for every solution $x(t)$ of (1). The proof depends upon a non-negative (Liapounov) functional of the solutions of (1). (Received November 8, 1962.)


P. Debye in 1910 gave certain series describing the asymptotic behavior of Bessel functions when the order $\nu$ and variable $z$ are both large and $\nu - z$ and $z$ are of comparable magnitude. Meissel's series describe the behavior when $\nu - z = O(e^{1/3})$. The range when $\nu - z$ is small compared with $z$ and large compared with $z^{1/3}$ is called by Watson (see his book on Bessel functions) the transitional region. By a reexamination of Watson's discussion of the development of Debye's series with a closer than usual study of the contribution of the neighborhood of the saddle point one sees that Debye's series remain valid in the transitional region (under the same hypotheses on arg $z$), the condition of validity being $z^{1/3} = O(\nu - z)$. (Received November 8, 1962.)
Let \( w(x) \) be a continuous function changing sign in \([a,b]\). Polynomials \( p_m(x) \) of degree \( \leq m \) satisfying \( \int_a^b w(x)p_m(x)x^i dx = 0 \) for \( i = 0,1,\ldots,m-1 \) exist for all \( m \geq 0 \). If \( \overline{p}_m(x) \) denotes a polynomial of minimum degree fulfilling these conditions, \( \overline{p}_m(x) \) is either exactly of degree \( m \) or is equal to some \( p_n(x) \), \( n < m \). If \( P_1(x) \) is of degree \( d_1 \), we have recursion relationships \( P_0(x) = 1 \), \( P_1(x) = (A_1 x^{d_1} + \sum_{h=0}^{d_1-1} B_{1,h} x^h)P_0(x) \), \( P_n(x) = (A_n x^{d_n} - \sum_{h=0}^{d_n-1} C_n P_{n-1}(x) - \sum_{h=0}^{d_n-2} C_n P_{n-2}(x), n = 2,3,\ldots \), where \( A_n \neq 0, C_n \neq 0 \).

Furthermore, for any set of integers \( 0 = d_0 < d_1 < \ldots \) and polynomials defined with arbitrary coefficients in the above recursions up to any given \( n \), a weight function \( w(x) \) can be constructed with respect to which these are the first \( n \) orthogonal polynomials of the sequence \( \{P_i(x)\} \). As a consequence, the zeros of the \( P_i(x) \) can be anywhere in the complex plane. These polynomials aid in defining numerical quadrature formulas analogous to Gaussian quadrature, weighted by \( w(x) \). (Received November 8, 1962.)

A problem in large deflection theory of elasticity leads to the following biharmonic boundary value problem: \( \Delta^2 \varphi = F \) in the unit square with \( \partial \varphi / \partial y = \partial \varphi / \partial y = 0 \) (\( \gamma \) = interior normal) on the boundary. A formal \( O(h^2) \) difference scheme is constructed that leads to a linear system of equations \( Au = f \). Because \( \varphi \) can be determined only up to an additive constant, it turns out that the matrix \( A \) is singular. It is proved that there is a symmetric positive-semidefinite matrix \( B \), with nullspace spanned by \( e = (1, 1,\ldots, 1) \), and a symmetric positive-definite matrix \( D \) such that \( A = (D^{-1} B)^2 \). From this result \( f \) is determined so that \( f - F = O(h^2) \) and such that \( Au = f \) has a solution \( f \) is orthogonal to the nullspace of the transpose of \( A \). If \( \varphi \) has mean value zero and \( u \) has discrete mean value zero, then the discretization error \( \varphi - u \) is shown to be \( O(h^2) \), uniformly. Finally, an alternating direction iterative method with a geometric parameter cycle is employed to solve the linear system \( Au = f \), and the iterates converge to \( u + e \), where \( Au = f \) and \( Ae = 0 \). (Received November 5, 1962.)

It has been shown by Bing that (i) there exists a decomposition of \( E^3 \) into points and countably many pointlike continua which is not topologically \( E^3 \), and (ii) any decomposition of \( E^3 \) into points and countably many tame arcs is topologically \( E^3 \). The first result may be sharpened in the following way: Theorem. There exists a decomposition \( D' \) of \( E^3 \) into points and countably many pointlike arcs which is not topologically \( E^3 \). The arcs used in this construction fail to be locally tame at each point. One might have hoped to use a simpler type of pointlike arc, perhaps of the Fox-Artin type. This, however, is impossible, as is shown by the following sharpening of (ii): Theorem. Any decomposition of \( E^3 \) into points and countably many pointlike arcs each of which is locally tame except perhaps at a
finite number of points, is topologically $E^3$. Finally, it is shown that the Cartesian product of the
decomposition $D'$ and a line is topologically $E^4$, by means of the more general result: Theorem.
If X is an upper semicontinuous decomposition of $E^n$ into a countable number of arcs (not necessarily
pointlike), then the Cartesian product of X with a line is topologically $E^{n+1}$. (Received November 13,
1962.)

597-66. W. M. KINCAID, University of Michigan, 3220 Angell Hall, Ann Arbor, Michigan.
A set of numerical methods for solving simultaneous equations.

A recent paper (Quart. Appl. Math. 18, 313-324) described an iterative method for the numerical
solution of systems of algebraic or transcendental equations. Several more sophisticated methods
related to this one are now being investigated. Each of these includes an effective starting procedure
and self-modifying features that come into play if successive iterative cycles fail to produce improved
values. Trials on an automatic computer have yielded encouraging results. (Received November 13,
1962.)

597-67. HENRYK MINC, University of Florida, Gainesville, Florida. On matrices with positive
inverses.

This paper corrects and generalizes a theorem by T. Kaczorek (Zastos. Mat. 5 (1960), 141-148;
MR, 23 (1962), no. A 2429). A necessary condition for a matrix to have a positive inverse is that it
be fully indecomposable. Any of the following three conditions is sufficient for an indecomposable
$n$-square matrix $A = (a_{ij})$ with nonpositive off-diagonal entries to have a positive inverse:
(i) $\sum_{i=1}^{n} a_{ij} \geq 0$, $i = 1, \ldots, n$, with strict inequality for at least one $i$; (ii) $a_{ii} \geq P_i^2 Q_i^{1-a}$, $i = 1, \ldots, n$,
where $0 \leq a \leq 1$, $P_i = - \sum_{j \neq i} a_{ij}$, $Q_i = - \sum_{j \neq i} a_{ji}$ and the inequality is strict for at least one $i$;
(iii) $a_{ii} a_{jj} > (P_i P_j)^{a} (Q_i Q_j)^{1-a}$, $i, j = 1, \ldots, n$, $i \neq j$, where $a, P_i$ and $Q_i$ are defined as in (ii). (Received
November 13, 1962.)

597-68. A. A. SAGLE, Syracuse University, Syracuse 10, New York. On simple extended Lie
algebras over fields of characteristic zero.

Investigated in this paper are algebras which generalize Lie algebras, Malcev algebras,
(Malcev algebras, Trans. Amer. Math. Soc., December 1961) and binary-Lie algebras (every two
elements generate a Lie subalgebra). Such an algebra $\mathcal{A}$ is called an extended Lie algebra and is
defined by $xy = -yx$ and $J(x, y, xy) = 0$ for all $x, y$ in $\mathcal{A}$, where $J(x, y, z) = (xy)z + (yz)x + (zx)y$.
Generalizing the techniques used for simple Lie algebras results in the Theorem. Let $\mathcal{A}$ be a simple
finite dimensional extended Lie algebra over an algebraically closed field of characteristic zero,
then $\mathcal{A}$ is a Malcev algebra if and only if the trace form, $f(x, y) = trace R_x R_y$, is a nondegenerate
invariant form, where $R_x$ denotes right multiplication in $\mathcal{A}$ by $x$. (Received November 13, 1962.)

Applications of geometry of numbers and Diophantine equations like Minkowski's theorem on linear forms, transference theorems, local and global solvability are considered for integral linear programming problems. By generating binary vectors certain problems like the loading problem can be solved by direct enumeration on a high speed computer. (Received November 13, 1962.)


**Theorem.** Let X be a normal space which the union of countably many metrizable subsets $M_i$, each of which is a dense subset of an open set. Then X is metrizable if it satisfies any of the following conditions: (1) X is separable (in particular, if each $M_i$ is separable), (2) X is locally compact, (3) X is $o$-compact, (4) Any open set in X is an $F_\sigma$. (5) There exist $F_\sigma$-sets $A_i \subseteq M_i$ which cover X. (6) There are only finitely many $M_i$, and there exist $G_\delta$-sets (in particular, completely metrizable sets) $A_i \subseteq M_i$ which cover X. The following examples show why such conditions are needed:

**Example 1.** There exists a non-metrizable regular Lindelöf space, which is the union of two dense, metrizable subsets (one of which is open), and which is also the union of countably many dense, completely metrizable ($G_\delta$-) subsets. **Example 2.** There exists a non-metrizable Tychonoff space which is the union of two dense, open, metrizable subsets. (Received November 26, 1962.)

**597-71.** EDMOND GRANIRER, University of Illinois, Urbana, Illinois. A theorem on amenable semigroups.

Let $G$ be a semigroup, $\mathcal{L}_1(G)$ its semigroup algebra and $\mathcal{L}_1(G)^* = m(G)$ the bounded real valued functions on $G$ with the sup, norm and let $Q$: $\mathcal{L}_1(G) \rightarrow m(G)^*$ be the natural embedding mapping. Let $M\mathcal{L}(G)$, $[Mr(G)] \subseteq m(G)^*$ denote the set of left [right] invariant means. It was proved by I. S. Luthar in Illinois J. Math. 1959 that: If $G$ is a commutative semigroup then $\dim M\mathcal{L}(G) = 1$ (i.e., $G$ has a unique invariant mean) if and only if $G$ contains a finite ideal. The unique invariant mean has to belong to $Q\mathcal{L}_1(G)$. The following result is proved here: If $G$ is a semigroup with $\dim M\mathcal{L}(G) = n$ and $\dim Mr(G) = m$ where $0 < m, n < \infty$ then $m = n = 1$, $M\mathcal{L}(G) = Mr(G) \subseteq Q\mathcal{L}_1(G)$ and $G$ contains a finite group which is a two sided ideal. It is easily shown that if $G$ contains a finite group which is a two sided ideal then $\dim M\mathcal{L}(G) = \dim Mr(G) = 1$, $Mr(G) = M\mathcal{L}(G) \subseteq Q\mathcal{L}_1(G)$. This result implies for commutative semigroups $G$ that only two cases may occur: either $\dim M\mathcal{L}(G) = 1$ or $\dim M\mathcal{L}(G) = \infty$ and also implies Luthar's result. The above result also implies that the radical of the second conjugate algebra $m(G)^*$ for commutative semigroups $G$ is infinite dimensional if $G$ has no finite ideals. If $G$ is an infinite commutative group (or even semigroup with cancellation) then as well known, $\mathcal{L}_1(G)$ is semisimple (i.e., has zero radical) and is $w^*$ dense in $m(G)^*$. Nevertheless $m(G)^*$ has infinite dimensional radical (which is a conjecture of P. Civin and B. Yood). (Received November 13, 1962.)
Sets of lines in a plane are studied. Corresponding to any set $\mathbb{E}$ of lines is considered the set $X$ of their poles with respect to a fixed conic. The linear measure of the set $\mathbb{E}$ is defined by the linear measure of the set $X$. The sets of finite linear measure are studied. If the set $X$ is regular (irregular) we say that the set $\mathbb{E}$ is regular (irregular). Many results are obtained by dualing the results on point sets. Thus to the existence of the tangent at points of a regular set corresponds the theorem that almost all lines touch an enumerable set of rectifiable arcs. Again to the theorem that the projection of an irregular set from almost all directions is of measure zero corresponds the theorem that the two-dimensional measure of an irregular set is zero. The two-dimensional measure of regular sets is studied directly and it is proved that it is always infinite. (Received November 13, 1962.)

Rearrangements of series of vectors.

When $\sum x_n$ is a conditionally convergent series of vectors in the space $E = \mathbb{R}^n$, the set $S$ of all sums of convergent rearrangements of the series is called its Steinitz subspace; $S$ is known to be an affine subspace of $E$ (Steinitz, 1914). In 1942, Hadwiger introduced a more general notion of convergence (here called $G_\varphi$ convergence) under which a rearrangement of the series may have as its $G_\varphi$-limit a subset of $S$. He showed that for a subset $X$ of $S$ to be obtainable as the $G_\varphi$-limit of some rearrangement of the series, it is sufficient for $X$ to be closed and convex. It is shown here that it is necessary and sufficient that $X$ is closed and for each $\varepsilon > 0$ the $\varepsilon$-neighborhood of $X$ (in the Euclidean metric $\varphi$) is connected. Fairly complete results are obtained for similar problems in the completion of $E$ with respect to any metric $\mu$ for $E$ such that the natural mapping of $(E, \varphi)$ onto $(E, \mu)$ is a uniformly continuous homeomorphism. From the application of one of these results to a metric corresponding to the one-point compactification of $E$, it follows that a subset $X$ of $S$ is obtainable as the set of all cluster points of the sequence of partial sums of some rearrangement of the series if and only if $X$ is connected or $\dim S \geq 2$ and every component of $S$ is unbounded. The paper will appear in Mathematische Zeitschrift. (Received November 13, 1962.)

A class of entire functions and a conjecture of Erdös.

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function. Write $h(z) = \sum_{n=1}^{\infty} |a_n| z^n / \sqrt{n}$, and suppose that $h(\alpha |z|^\omega) = o(\sqrt{n} M(r, f(\alpha z)))$; then $\lim_{r \to \infty} \mu(r, f(\alpha z)) / M(r, f(\alpha z)) = 0 \,*$. Here $\alpha \neq 0$ is a constant. If in particular, $a_n \equiv 0$, $a > 0$, then (*). Let $(N_k)$ denote the range of the rank $\gamma(r) = \gamma(r, f)$. If $\lim \inf_{k \to \infty} (N_{k+1} - N_k) < \infty$, then $\lim_{r \to \infty} \mu(r, f) / M(r, f) = 0$. (Received November 13, 1962.)
597-75. STEVEN OREY, University of Minnesota, Minneapolis, Minnesota. A ratio limit theorem for Markov chains.

Consider a Markov chain with state space \( S = \{0, 1, \ldots\} \) and stationary transition probabilities. Take time to be discrete (continuous) and let \( p_{ij}^{(n)}(t) \) be the \( n \)-step transition probabilities (transition probabilities at time \( t \)). Let \( p_{ij}^{(0)} = \delta_{ij} \) (\( p_{ij}^{(0)} = \lim_{t \to \infty} p_{ij}(t) = \delta_{ij} \)). Let the chain be recurrent, aperiodic, and irreducible, with invariant measure \( \pi \). Theorem. If there exists a positive integer \( k \) and \( s > 0 \) such that \( \sum_{n=0}^{k} p_{ii}^{(n)} > s \) for all \( i \) in \( S \) then for every integer \( m \) and every \( a, b, c, d \) in \( S \),

\[
p_{ab}(t+m)/p_{cd}(t) \to \pi_b/\pi_d \quad \text{as} \quad t \to \infty.
\]

This generalizes a result of Chung and Erdös [Mem. Amer. Math. Soc. No. 6 (1951), 1-19]. For the continuous time case one obtains a conjecture of J. F. C. Kingman as a Corollary. If there exists an \( M \) such that \( -\infty < M < p_{ii}^{(0)} \) for all \( i \) in \( S \) then for every real number \( h \) and every \( a, b, c, d \) in \( S \),

\[
p_{ab}(t+h)/p_{cd}(t) \to \pi_b/\pi_d \quad \text{as} \quad t \to \infty.
\]

(Received November 13, 1962.)

597-76. ANDREW SOBCZYK, Box 8052, University of Miami, Coral Gables 46, Florida. Functional characterization of retracts.

Denote by \( C(X, \mathcal{J}) = C(X) \) the Banach space of all real bounded continuous functions on a topological space \( (X, \mathcal{J}) \). For a subset \( R \) of \( (S, \mathcal{C}) \), if \( \mathcal{R} \) is the relative topology, let \( C_1(R) \) be the subspace of \( C(R, \mathcal{P}) \) which is the range of the restriction operator \( p_z = z|R = w, z \in C(S) \). Subset \( R \) is \( C^* \)-embedded if \( C_1(R) = C(R) \). For a decomposition \( \mathcal{E} \) of which \( R \) is a representative subset, if \( \tau \) is the corresponding projection, call \( x(s) = w(\tau s) \) the \( \mathcal{E} \)-extension on \( S \) of a function \( w \) on \( R \). Subset \( R \) is \( \mathcal{E} \)-embedded if there is a decomposition \( \mathcal{E} \), such that for each \( w \in C_1(R) \), the \( \mathcal{E} \)-extension of \( w \) is continuous. Theorems. A necessary condition for this is that \( C_1(R) \) be complete. If \( R \) is a retract of \( S \), then \( R \) is both \( C^* \)- and \( \mathcal{E} \)-embedded. If \( S \) is Hausdorff, and if a completely regular \( R \) is \( C^* \)- and \( \mathcal{E} \)-embedded, then \( R \) is a retract of \( S \). The following theorem generalizes a familiar property of Hausdorff images of compact spaces and the Banach-Stone theorem: If \( (X, \mathcal{J}) \) is completely regular, and \( (X, \mathcal{K}) \) is Hausdorff, with \( \mathcal{J} \supset \mathcal{K} \), \( C(X, \mathcal{K}) = C(X, \mathcal{J}) \), then \( \mathcal{K} = \mathcal{J} \).

(Received November 13, 1962.)

597-77. RONALD JACOBOWITZ, University of Arizona, Tucson, Arizona. Multiplicativity of the local Hilbert symbol.

Let \( F \) be a local field of characteristic not \( 2 \), \( a \) and \( b \) nonzero elements of \( F \); the Hilbert symbol \( (a, b) \) is defined to be \( +1 \) or \( -1 \) according as the equation \( ax^2 + by^2 = 1 \) has or has not a solution in \( F \). The object of this paper is to give a direct proof of the multiplicativity property \( (a,b)(a,c) = (a,bc) \).

The first step is a classification of the various types of quadratic extensions \( E \) of \( F \) with respect to properties such as ramification, and construction of a prime element for \( E \) in each case. Then, by refining suitable congruences modulo powers of a prime element in \( F \), the indices of various subgroups of the group \( u \) of \( F \)-units are computed, and there follows the Theorem. (\( u: N_{E/F}u \) is 1 if \( E/F \) is unramified, \( \geq 2 \) if ramified, where \( U \) denotes the group of \( E \)-units. Corollary ("2nd inequality of local class field theory"). \( (\star): N_{E/F}E^* \neq 2 \). Noting that \( (a,b) = +1 \) if and only if \( b \) is a norm from \( F(a^{1/2}) \), the multiplicative property follows immediately. (Received November 13, 1962.)
597-78. LOUIS SUCHESTON, 3702 N. Murray, Shorewood 11, Wisconsin. A condition for weak mixing of transformations.

A condition is given for strong Cesaro convergence of almost convergent sequences, from which one obtains: Theorem. An ergodic and invariant transformation T on a probability space \((\Omega, \mathcal{F}, P)\) is weakly mixing if and only if for each pair of measurable sets A, B and each \(\varepsilon > 0\) there exists a set \(J\) of positive integers, of density zero and such that if \(n\) is a positive integer not in \(J\), then \(|P(T^n A \cap B) - P(T^{(n-1)} A \cap B)| < \varepsilon\). (See Abstracts 584-2, 62T-107, Notices Amer. Math. Soc. Paper to appear in Duke Math. J.) (Received November 13, 1962.)


Let \(f \in L^1\) and \(\phi(a) = \int_{-\infty}^{\infty} f(x) e^{i a x} dx\) the Fourier transform of \(f\). Put \(A(x) = 1 + \cos(2 \arg \int_{0}^{\infty} f(y)dy\). Denote by \(Tz, \text{Re } z\) the imaginary and real parts of \(z\). Theorem 1. Hypothesis: \(f \in L^1\); \(f(x) = 0\) for \(x < 0\); \(f(x) = \text{real}\); \(\text{Re } \phi(a) \geq 0\). Conclusion: \(0 \leq \int_{0}^{\infty} f(y)dy \leq \int_{0}^{\infty} \phi(y)dy\) for every \(x\).

Theorem 2. Hypothesis: \(f \in L^1\); \(f(x) = 0\) for \(x < 0\); \(a T_{\phi(a) - \phi(-a)} \leq 0\); \(\text{Re } f(y)dy = 0(\tau^\varepsilon)\), \((\varepsilon \to 0, \tau > 0)\). Conclusion: \(\int_{0}^{\infty} f(y)dy |A(x)| \leq 2 \text{ Re } \int_{0}^{\infty} f(y)dy\) for every \(x\). The proof of Theorem 1 is simple. To prove Theorem 2, a previous theorem of the author is used, namely: Hypothesis: \(f \in L^1\); \(\text{Re } \phi(a) \geq 0\); there is a number \(c\) such that \(\text{Re } c > 0\) and \(\int_{0}^{1} f(t) + f(-t) - c|dt|/t < 0\). Conclusion: \(|f(x)| + |f(-x)| \cos \{\arg f(x) + \arg f(-x)\} \leq \text{ Re } c, \text{ a.e.}\) (See Ann. École Norm. Sup. (3) LXXIV, Fasc. 4). Both Theorems 1 and 2 have applications in the theory of summability. (Received November 13, 1962.)

597-80. SHMUEL KANTOROVITZ, Princeton University, Princeton, New Jersey. On the characterization of spectral operators. II.

A bounded linear operator \(S\) on a Banach space \(X\) into itself is pseudo-hermitian (p.h.) if it is spectral of scalar type and its spectrum lies on the real line \(R\) (cf. Dunford, Bull. Amer. Math. Soc. 64 (1958), 217). Theorems. I. If \(X\) is a Hilbert space, then \(S\) is p.h. (i.e., similar to a hermitian operator) iff the group \(e^{itS}\) (\(t\) in \(R\)) is uniformly bounded. II. If \(X\) is a reflexive Banach space, then the following statements are equivalent: (a) \(S\) is p.h.; (b) for every \(f\) in \(L^1_{\text{loc}}(R)\), the inequality \(\|f(t)e^{itS}dt\| \leq M\|f\|_\infty\) holds (\(f^*\) is the Fourier transform of \(f\)); (c) for every real vector \((t_1, ..., t_n)\) and every complex vector \((c_1, ..., c_n)\) one has: \(\|\sum_{k=1}^{n} c_k \exp(\text{it}_k S)\| \leq M \sup_u |\sum_{k=1}^{n} c_k \exp(\text{it}_k u)|, u\) real; (d) for every unit vector \(x\) and \(x^*\) in \(X\) and its adjoint respectively, the integral \(\int_{R} |x^* (R_{t-i\sigma} - R_{t+i\sigma}) x| dt\) is uniformly bounded as \(\sigma\) approaches \(0^+\) (\(R_{z}\) is the resolvent of \(S\)). Using (c), one gets the Corollary. Sums and products of commuting spectral operators with real spectrum on a reflexive Banach space are spectral. (Received November 13, 1962.)
597-81. H. S. M. COXETER, University of Toronto, Toronto 5, Ontario, Canada and L. FEJES TÓTH, University of Veszprém, Budapest II, Hungary. The total length of the edges of a non-Euclidean polyhedron with triangular faces.

The second author conjectured in 1960 that, of all convex triangle-faced polyhedra with a given inradius (in Euclidean space), the regular tetrahedron and octahedron have the smallest total edge-length. Although no proof of this Euclidean conjecture is in sight, a special trick makes it possible to solve the analogous problem in spherical space when the given inradius is arcsin 1/4, and in hyperbolic space when it is 0.364054 ... or 0.828375 ... . The extremal polyhedra in these three cases are (i) the tetrahedron \{3,3\} and trigonal dihedron \{3,2\}, (ii) the octahedron \{3,4\}, (iii) the icosahedron \{3,5\}. (Received November 13, 1962.)

597-82. HSIN CHU and M. A. GERAGHTY, University of Alabama Research Institute, P. O. Box 860, Huntsville, Alabama. The first cohomology group of a minimal set.

A question of long standing is whether an n-sphere cannot be a minimal set under a continuous flow, for n greater than one. More generally, must a compact manifold which is minimal under a continuous flow have a nontrivial fundamental group. Or more generally yet, must a compact Hausdorff space which is minimal under a continuous flow have a nontrivial first cohomology group (in the sense of Alexander-Spanier-Wallace). In this paper it is first shown that if a compact Hausdorff space with trivial first cohomology group, which is locally pathwise connected, is a minimal set then it must be totally minimal. The question then reduces to the existence of totally minimal flows. It is next shown that for compact Hausdorff spaces, locally almost periodic contradicts totally minimal. So on a compact, Hausdorff, locally pathwise-connected space with trivial first cohomology group (or, for manifolds, homology or homotopy) group one cannot have a locally almost periodic minimal continuous flow. In particular if an n-sphere, for n greater than one, is a minimal set under a continuous flow then it is totally minimal and so it is not locally almost periodic. (Received November 14, 1962.)


A Čebyšev [Haar] subspace of C(X) (the sup-normed space of real-valued continuous functions on the compact Hausdorff space X) is a closed subspace in which there exists, for each f in C(X), precisely one [at least one] nearest point to f. A well-known theorem of Haar characterizes the finite dimensional Čebyšev subspaces of C(X); this paper considers those of finite codimension.

**Theorems.** If X contains no isolated points, a Haar subspace M of finite codimension is a Čebyšev subspace if and only if each nontrivial measure \(M^0\) has closed support \(S(\mu)\) equal to all of X. If X contains n or more isolated points, a Haar subspace M of codimension n is a Čebyšev subspace if and only if X \(\sim S(\mu)\) contains at least n + 1 points, for each nontrivial \(\mu\) in \(M^0\). If C(X) contains a Čebyšev subspace of codimension n > 1, then X is totally disconnected and has at most countably many isolated points. There exist examples (in each of which X is extremely disconnected) illustrating the various possibilities. No characterization of the Haar subspaces of finite codimension in C(X) is known. (Received November 14, 1962.)
597-84. R. E. VESLEY, University of Wisconsin-Milwaukee, Milwaukee 11, Wisconsin.
The intuitionistic continuum.

Brouwer's theory of the continuum is developed in a formal system for "intuitionistic
analysis" set up by Kleene (to appear). Certain formulas abbreviated \( a \in \mathbb{R} \) and \( a \in \mathbb{R}' \) represent
the species (spread) of real number generators (r.n.g.). Other formulas represent the equality \( (\&) \),
apartness, pseudo- and virtual-ordering predicates for r.n.g., and the usual theorems are proved,
e.g., \( a, \beta \in \mathbb{R} \overset{?}{\Rightarrow} \gamma \), \( \alpha \), \( (3 \mathbb{R} \Leftrightarrow \langle \beta \, \gamma \rangle \) and \( \gamma \).
Using the single nonclassical postulate of the system, theorems
are proved corresponding to certain of Brouwer's counterexamples to classical theorems, e.g.,
\[ \forall a \in \mathbb{R} \overset{?}{\Rightarrow} \gamma \beta (\beta \in \mathbb{R} \Leftrightarrow \langle \beta \, \gamma \rangle \) and a form of the uniform continuity theorem is established.
Less well-known properties of the intuitionistic continuum are proved, following Brouwer, Die
Struktur des Kontinuums, 1928. The "sharp difference" predicate of this last paper is shown formally
equivalent to the familiar apartness predicate. (Received November 14, 1962.)

597-85. EDGAR REICH, Institute of Technology, University of Minnesota, Minneapolis 14,
Minnesota. Sharpened distortion theorems for quasiconformal mappings.

If \( w(z) \) is a quasiconformal mapping then it is known that the specification of the complex
dilatation \( \mathcal{K}(z) = \frac{w_z}{w_z} \) at almost all \( z \) determines the mapping within composition by a conformal
mapping. Since \( \mathcal{K}(z) \) thus contains more information about \( w(z) \) than the ordinary dilatation \( D(z) = \frac{1 + |\mathcal{K}(z)|}{1 - |\mathcal{K}(z)|} \) it is possible to sharpen the classical distortion theorems considerably by
taking \( \arg \mathcal{K}(z) \) into account. A basic result useful in this connection is the following. Lemma.
Suppose the oriented rectangle \( R \) with height \( a \) and length \( b \) is mapped quasiconformally onto a
quadrilateral \( Q' \). Consider the modulus of \( R, M(R) = \frac{a}{b} \), and let \( M(Q') \) be the corresponding
modulus of \( Q' \). Put \( \rho = \frac{M(Q')}{M(R)} \). Then \[ \left\{ (ab)^{-1} \int \frac{1}{|1 + \rho^2(x)|} \, dx \right\} \]
\[ \leq \rho \leq \left( ab \right)^{-1} \int \frac{1}{|1 + \rho^2(x)|} \, dx \]. As an application, one obtains, for instance, a sharpened form of
the Teichmüller-Wittich distortion theorem which gives a sufficient condition for \( \lim_{z \to \infty} \frac{|w(z)|}{|z|} \)
to exist. The condition thus obtained actually turns out to be both necessary and sufficient for the
class of mappings of the type \( w = a(r)e^{i\theta} \), \( z = re^{i\theta} \) to have the property that \( \lim_{r \to \infty} a(r)/r \)
exists, in contrast with the Teichmüller-Wittich theorem which only yields a sufficient condition for this case.
(Received November 14, 1962.)

597-86. PRABIR ROY, University of Illinois, Urbana, Illinois. Failure of equivalence of
dimension concepts for metric spaces.

That \( \text{Ind}(S) = \text{dim}(S) \) for arbitrary metric space \( S \) was established by Katětov. Here it is shown
that there is a complete metric space \( \Delta \) such that \( \text{ind}(\Delta) = 0 \) but \( \text{dim}(\Delta) = 1 \). Demonstration of the
above assertion includes an application of a metrization theorem due to R. L. Moore [A set of axioms
for plane analysis situs, Fund. Math. 25 (1935), 13-28] which states that a topological space \( S \) is
metrizable if (i) \( S \) is a Hausdorff space, and (ii) there is a decreasing sequence \( \{ H_1 \supset H_2 \supset H_3 \supset \ldots \} \)
of open coverings of \( S \) such that for every point \( p \) and every open set \( U \) containing \( p \) there is a positive
integer \( N \) such that if \( h_1 \) and \( h_2 \) belong to \( H_N, p \in h_1 \), and \( h_1 \cap h_2 \neq \emptyset \), then \( h_1 \cup h_2 \subset U \).
(Received November 14, 1962.)

On the limit behavior of steepest descent. Preliminary report.

Discussed is the steepest descent algorithm: \[ x_{n+1} = x_n - r_n/t_n, \quad r_n = A x_n, \quad t_n = (A x_n, r_n)/(r_n, r_n), \]
where \( A \) is a self adjoint operator satisfying \( 0 < \lambda_1 \leq (A x, x)/(x, x) \leq \lambda_2 < \infty \) for some \( \lambda_1, \lambda_2 \). This algorithm is for minimizing \( (A x, x) \). Previously shown and reported in 1955 to the ACM by the author is a theorem conjectured by G. E. Forsythe that the error vector \( r_n \) is in the limit a linear combination of vectors of the least \( (\lambda_1) \) and largest \( (\lambda_2) \) eigenvalues of \( A \). The result reported was limited to the case of finite dimensions. The new result is a generalization to an arbitrary Hilbert space with a self adjoint operator \( A \). In the newer theorem the notion of eigenvectors is replaced, with aid of the spectral theorem for self adjoint operators, by spectral mass of the error vector. It is shown that the spectral mass tends entirely to the end values \( \lambda_1 \) and \( \lambda_2 \). (Received November 15, 1962.)

597-88. J. A. HUMMEL, University of Maryland, College Park, Maryland. The Grunsky coefficients of a schlicht function.

In 1939 Grunsky gave necessary and sufficient conditions for a meromorphic function to be schlicht. Results due to Schiffer, together with the observation that if \( f(z) = z + a_2 z^2 + \ldots \) is schlicht in \( |z| < 1 \) then for every positive integer \( p \) so is \( f(z^p) \), Grunsky's result is that if we set
\[ \log \left( \frac{[\log(z^p)]}{[\log(z)]} \right) \]
and if \( f(z) \) is regular and schlicht in \( |z| < 1 \), then for every positive integer \( p \) and every sequence \( \{a_n\} \) of complex numbers, \[ \sum_n r_n a_n^{(p)} \leq \sum_n |x_n|^{1/2}. \]
In this paper, a formula for the Grunsky coefficients \( C^{(p)} \) in terms of the coefficients \( a_n \) of \( f(z) \) is developed. This formula is suitable for computational use. Some simple applications are given. (Received November 15, 1962.)

597-89. S. C. PORT, The Rand Corporation, 1700 Main Street, Santa Monica, California.

Some asymptotic properties of sums of random variables. Preliminary report.

Let \( S_n \) be the partial sums of independent random variables with a common distribution \( K(x) \). Let \( \Phi(\lambda) \) be the characteristic function of \( S_n \) and let \( Y_n \) be the position of the lst maximum of \( (0, S_1, \ldots, S_n) \). If \( P[S_n \leq \alpha n^{1/a}] \rightarrow V_\alpha(x)(1 < \alpha \leq 2) \) as \( n \rightarrow \infty \) then \( E[S_n/\alpha n] \rightarrow 0 \). If \( \{P(S_k \leq 0) = \infty \text{ and } \{P(S_k > 0) \leq \infty \} \text{ are (C,1) convergent to } p \), then the iterates, \( F_{n+1}(x) = \int_0^x K(x - y) dF_n(y) \) converge to a solution (unique modulo a constant) of the equation \( \int_0^x K(x - y) dF(y) = F(x) \) in the following way: \( \lim_{n \rightarrow \infty} P(S_n) = F(x) \). Further \( \int_0^x e^{-\lambda x} dF(x) = \exp(\lambda - x) \). If \( 1 - \Phi(\lambda) = \lambda |\lambda|^a \) and \( \Phi(\lambda) \) is symmetric then \( F(x) = Q x^{1/2} \). If \( K(x) \) is a stable law of index \( a \) and parameter \( \beta \neq \pm 1 \) then \( F(x) \sim x^{2p} \). These are extensions of some results of Spitzer. Let \( \{S_n\} \) be integer valued. Assume \( \Phi(\lambda) \neq 0 \) if \( 0 < |\lambda| < \pi \) and that \( 1 - \Phi(\lambda) \sim |\lambda|^a \) for \( -\pi < a \leq 2 \). Then if \( N_n, M_n \) are respectively the number of distinct points and number of points hit \( r \) times by step \( n \), then \( N_n / EN_n / M_n(r)/EM_r(r) \) converge in probability to 1. Asymptotic expressions for \( EM_n(r) \) and \( EN_n \) are also found. (Received November 15, 1962.)
Let $L$ be an inner-product space. Denote by $U = [u_1 \ldots u_n]^T$ of elements $u_1, \ldots, u_n$ of $L$. Likewise, let $V = [v_1 \ldots v_m]^T$. The Gram matrix of $U$ and $V$ is defined to be the $n \times m$ matrix whose $(i,j)$ element is given by $(u_i, v_j)$ and is denoted by $G(U,V)$. If $U = V$, then $G(U)$ is written for $G(U,U)$. The usual Schwarz inequality $|(u,v)|^2 \leq (u,u)(v,v)$, is generalized in the following theorem. **Theorem 1.** Let the elements of $U$ be linearly independent. Then, for arbitrary $V$, $\det [G(U,V)G^{-1}(U)G(U,V)] \leq \det G(V)$, where equality occurs if and only if $V = AU$ for some matrix $A$ or the elements of $V$ are linearly dependent. If $m = n$, then this inequality takes the form $|\det G(U,V)|^2 \leq \det G(U) \cdot \det G(V)$. If $m = 1$ and the elements of $U$ are orthonormal, then the first inequality becomes Bessel's inequality for a finite orthonormal sequence. A stronger result is the following: **Theorem 2.** Let $H = \begin{bmatrix} A & B \\ B^* & C \end{bmatrix}$ be a positive semidefinite Hermitian matrix, where $A$ and $C$ are square with $A$ non-singular. Then $0 \leq |\det H|^{1/p} \leq |\det A|^{1/p} \cdot |\det C|^{1/p} - |\det (B^*A^{-1}B)|^{1/p} \leq |\det A \cdot \det C|^{1/p}$, where $p$ is the order of $C$. (Received November 16, 1962.)

Let $X$ be an extremely disconnected, compact Hausdorff space, and let $M_n$ be the full ring of $n \times n$ complex matrices under the operator norm. Denote by $M_n(X)$ the $\ast$-algebra of continuous functions from $X$ to $M_n$. Any finite von Neumann algebra of type $I$ is a direct sum of algebras each of which has a faithful representation onto some $M_n(X)$, and thus information about such von Neumann algebras can be obtained by studying $M_n(X)$. **Theorems.** (1) Every invertible element of $M_n(X)$ has a logarithm, and thus roots of all orders, in $M_n(X)$. (2) Every element in $M_n(X)$ with an identically vanishing trace is a commutator in $M_n(X)$. (3) If $f$ is an entire function which assumes every finite value in a schlicht fashion, then $f$ maps $M_n(X)$ onto itself. (Received November 16, 1962.)

Let $E$ be a linear topological Hausdorff space. A map $F:X \to E$, ($X \subseteq E$) is said to be compact iff the closure $\overline{F(X)}$ of the range of $F$ is compact; $F$ is finite dimensional iff it is compact and $F(X) \subseteq E^k$, where $E^k$ is a finite dimensional subspace of $E$. Say that $X \subseteq E$ has the fixed point property (f.p.p.) in the narrow sense iff for each compact map $F:X \to X$ there is at least one fixed point for $F$. If $X$ has the f.p.p. in the narrow sense, then so does every retract of $X$. A closed convex set $X \subseteq E$ is called admissible provided any compact map of $X$ into itself can be uniformly approximated on $X$ by finite dimensional maps with range in $X$. **Theorem.** If a closed convex set $X \subseteq E$ is admissible, then $X$ has the f.p.p. in the narrow sense. It follows that the unit ball in some (not locally convex) linear $F$-spaces has the f.p.p. in the narrow sense. The Tychonoff fixed point theorem is an easy consequence of the above theorem. (Received November 16, 1962.)
On equations in a free group.

Schützenberger has shown: If a, b, c, generate a free group $F$ and $a^k b^l c^m = 1$ where 
$k, l, m, \neq 2$ then $F$ is infinite cyclic. This suggests the question: If a group $G$ is presented on $n$
generators $x_1, \ldots, x_n$, and a single defining relation $R = R(x_1, \ldots, x_n) = 1$, does $G$ have a free
quotient group of rank $n - 1$. For, if $F$ is the free group on the free generators $x_1, \ldots, x_n$, and an element $P$
called primitive if $P$ can be included in a set of free generators of $F$, it is shown: Theorem. The
above question is equivalent to deciding whether $R$ lies in the normal subgroup of $F$ generated by a
primitive $P$; in the case $n = 3$, it is equivalent to deciding which equations $R(a, b, c) = 1$ can hold in a
noncyclic free group. Using methods developed by Magnus in his Freiheitssatz paper the question is
answered for a large class of groups satisfying the condition: The exponent sum $\neq 0$ on some
$x_1, i = 1, \ldots, n$, in $R(x_1, \ldots, x_n)$. For example, Schützenberger's result is obtained. Moreover, under
the above hypothesis it is shown: Theorem. If $G$ has a free quotient group $H$ of rank $n - 1$, then the
kernel of the mapping $G \to H$ is unique. (Received November 16, 1962.)

A uniqueness property of invariant means.

Let $G$ be an abelian group, $m(G)$ the bounded complex-valued functions on $G$ and $m(G)^*$ the
conjugate space of $m(G)$; then it follows from Banach that there exists an invariant mean in $m(G)^*$. Moreover there exists one invariant mean if $G$ is finite (Day, Illinois J. Math. 1 (1957), 509-544). The
author establishes, however, that although the invariant mean is not unique, its $L_p$ space structure is,
(The $L_p$ space structure is that relative to the measure $\mu_L$ defined on the set algebra $S_G$ of all subsets of $G$ which corresponds to the invariant mean $L$ via the Hildebrandt-Kantorovich-Fichtenholz theorem.) Using a theorem on abstract character groups due to Kakutani (Proc. Imp. Acad, Tokyo 19 (1943),
366-372), it is shown that if $k$ is the cardinality of $G$ and $k$ is infinite, then the dimension of
$L_2(\mu_L, G, S_G)$ is $2^k$. It is then shown that $L_2(\mu_L, G, S_G)$ is homogeneous in the sense of Maharam
be an abelian group and each of $L_1$ and $L_2$ an invariant mean in $m(G)^*$; then there exists a positive
isomorphism-isometry from $L_1(\mu_{L_1}, G, S_G)$ onto $L_1(\mu_{L_2}, G, S_G)$ which preserves the $p$-norm for
$1 \leq p \leq \infty$. (Received November 16, 1962.)

On points of Jacobian rank $k$.

Let $h: M^p \to N^p$ be $C^k$, where $M^p$ and $N^p$ are $C^n$ manifolds of dimensions $n$ and $p$, respectively.
Let $R_k$ be the set of points in $M^p$ at which the Jacobian matrix of $h$ has rank at most $k$. If $i*$ mapping
$\tau_m^{(N^p - f(R_k), x_0)}$ into $\tau_m^{(N^p, x_0)}$ is the homomorphism on the $m$th homotopy groups induced by the
inclusion map $i$, then $i*$ is an isomorphism (onto) for $k = 0, 1, \ldots, p - 2$ and $m = 0, 1, \ldots, p - k - 2$; and
is onto for $k = 0, 1, \ldots, p - 1$ and $m = 0, 1, \ldots, p - k - 1$. For example, if $f: E^2 \to E^5$, $f$ a $C^2$
map (not necessarily an immersion), then $\tau_1(E^5 - f(E^2), x_0) = 0$ for all $x_0$ in $E^5 - f(E^2)$. (Received
November 16, 1962.)
C. L. DOLPH, 347 West Engineering, University of Michigan, Ann Arbor, Michigan, Positive real resolvents and linear passive Hilbert systems.

Let \( \rho(\lambda) \) be a one parameter family of bounded operators with a dense range in a Hilbert space with an inner product \((f,g)\), which admits a Cauchy integral resolvent representation in the half-plane \( \text{Re} \lambda > 0 \) with a generalized resolution of the identity. If there exists one \( \lambda_0 \) with \( \text{Re} \lambda_0 > 0 \) such that \( \rho(\lambda_0)f = 0 \) implies \( f = 0 \), then: A necessary and sufficient condition that \( \rho(\lambda) \) satisfy the Hilbert resolvent equations. Such operators have the property that \( (\text{Re} \lambda \rho(\lambda)f,f) \geq 0 \) for \( \text{Re} \lambda > 0 \) and arbitrary \( f \), and will be called positive real resolvents. The operator having \( \rho(\lambda) \) as its resolvent can be associated with a "linear passive Hilbert system", an abstract evolution equation which is linear, translation invariant with respect to time, causal, and passive in the sense of Meixner (Zeit. Phys. 139 (1954), 30) and represent a generalization of the theory of so-called positive real functions. The results will appear in the Annals of the Finish Academy's Proceedings of the Helsinki Colloquium of 1962. (Received November 16, 1962.)

MICHAEL GINSBURG, University of Chicago, Chicago 37, Illinois. Some immersions of projective space in Euclidean space.

Let \( P_r \) be \( r \)-dimensional real projective space and \( E_n \) Euclidean \( n \)-space, Assume \( r > 7 \). We prove the Theorem. (a) If \( r \equiv 7 \mod 8 \), \( P_r \) is immersible in \( E_{2r-7} \); (b) If \( r \equiv 3 \mod 8 \), \( P_r \) is immersible in \( E_{2r-3} \). Since \( P_r \subset P_{r+1} \), results for the other residue classes modulo 8 are implicit in this theorem. The method of proof is to find an appropriately sized inverse to the tangent bundle of \( P_r \). (Received November 16, 1962.)

E. E. GRACE, University of Georgia, Athens, Georgia. A characterization of the pseudo-arc.

Theorem. A nondegenerate, snakelike (i.e., chainable) continuum \( M \) is a pseudo-arc iff for each pair \((x,y)\) of points and each positive number \( \epsilon \) there is a chain \( E = \{e_1, \ldots, e_n\} \) covering \( M \) for which there are natural numbers \( i < r < s < j \) such that \( x \in e_i, y \in e_j \), the diameters of \( e_r \) and \( e_s \) are less than \( \epsilon \), and the distances from \( x \) to \( e_s \) and \( y \) to \( e_r \) are less than \( \epsilon \). (Received November 16, 1962.)


Let \( D \) be the unit disk in the complex plane, and \( C \) the unit circle. Let \( f \) be a pseudo-meromorphic in \( D \). Let \( E \) be a subset of \( C \). Let \( A = \{\lambda_\theta\}, 0 \neq \theta \neq 2\pi \), be a family of mutually disjoint simple arcs \( \lambda_\theta \) each of which lies in \( D \) except for the endpoint \( e^{i\theta} \). For each \( P = e^{i\theta} \in C \), let \( C(f,P) \) be the cluster set of \( f \) at \( P \) with respect to \( D \), \( R(f,P) \) the set of values assumed in every neighborhood of \( P \) by \( f \), and \( C_{\lambda_\theta-E}(f,P) = \bigcap \bigcup C_{\lambda_\theta}(f,e^{i\theta}) \) where \( C_{\lambda_\theta}(f,e^{i\theta}) \) is the cluster set of \( f \) at \( e^{i\theta} \) with respect to \( \lambda_\theta \), the union is over all \( e^{i\theta} \notin E \) satisfying \( 0 < |\theta - \theta_0| < \gamma \), and the intersection is over all \( \gamma > 0 \). Let \( C^*_A(f,P) = \bigcap C_{\lambda_\theta-E}(f,P) \), where the intersection is over all sets \( E \subset C \) of capacity zero.
Theorem. The set $C(F,P) - C^*_A(f,P)$ is open; if $S$ is any component of this set then $S - R(f,P)$ contains at most two points. This generalizes results of Woolf, Storvich and Noshiro. (Received November 16, 1962.)

597-100. HAJIMU OGAWA, University of California, Riverside, California. The singular Cauchy problem for a quasi-linear hyperbolic equation.

Consider the equation (1) $r^2(x,y)u_{xx} - u_{yy} + f(x,y,u,u_x,u_y) = 0$ with initial conditions (2) $u(x,0) = 0$, $u_y(x,0) = g(x)$, $x \in I$, a finite interval. Assume $m$ is a positive constant, $r$ is positive and $r$, $f$ and $g$ are twice differentiable with Lipschitz continuous second derivatives. Assume also that there is a constant $a$ such that $g(x) \leq a > 0$, $x \in I$. Choose $A$ so that $0 < A < a$ and denote by $\Gamma_1$ and $\Gamma_2$ the solutions of $dx/dy = r(x,y)(Ay)^m$ and $dx/dy = -r(x,y)(Ay)^m$ passing through the left and right endpoints of $I$, respectively. For any $\delta > 0$, let $D_\delta$ be the open region bounded by $I$, $\Gamma_1$, $\Gamma_2$ and the line $y = \delta$. Theorem. If $f_p(x,y,u,p,q) = O(y^m)$ as $y \to 0$, then there exists a $\delta > 0$ and a function $u$, twice differentiable with Lipschitz continuous second derivatives on $D_\delta$, which is a solution of (1) on $D_\delta$ satisfying the initial conditions (2). (Received November 16, 1962.)

597-101. R. T. PROSSER, Lincoln Laboratory, Room L-122, Massachusetts Institute of Technology, Lexington 73, Massachusetts. On the existence of certain quantum fields, II.

In Part I of this series the existence of the Heisenberg quantum fields associated with a modified form of the Dirac-Klein-Gordon system was established (International Congress of Mathematicians, Stockholm, 1962). In Part II the perturbation expansions for these fields in terms of free fields and interaction terms are shown to be convergent for small values of the coupling parameter. (Received November 16, 1962.)


The set of integers: $d_1, d_2, \ldots, d_s$ is called a perfect difference set if the differences: $d_i - d_j, i \neq j$, form a complete residue system modulo $q = s^2 - s + 1$. With each perfect difference set can be associated a matrix $A = (a_{ij})$ with $a_{ij} = d_i - d_j$. For $q$ a prime various properties of the matrix $A$ are shown, two of them being: (1) The minimum function of $A$ is $x^3$. (2) The sum of the squares of the elements in each row is different from that in every other row. (Received November 16, 1962.)

597-103. TARO YOSHIZAWA, Iowa State University, Ames, Iowa. Eventual boundedness of solutions of a perturbed system.

Consider (1) $x' = F(t,x)$ ($x$: $n$-vector), where $F(t,x)$ is defined and continuous on $I \times \mathbb{R}^n$ ($I: 0 \leq t < \infty$, $\mathbb{R}^n$: Euclidean $n$-space). Let $M$ be a set in $\mathbb{R}^n$. Denote by $M(\sigma)$ the set $M \cap \pi_{\sigma}(\pi_{\sigma}:$ hyperplane $t = \sigma$). Assume that there exists a compact set $Q$ in $\mathbb{R}^n$ such that $M(t) \subseteq Q$ for all $t \in I$. $M$ is said to be quasi-uniform-asymptotically stable set of (1) in the large, if for each $\epsilon > 0$, $\gamma > 0$, there exists a $T(\epsilon, \gamma)$ such that if $d(x_0, M(t_0)) \leq \gamma$, $d(x(t; x_0, t_0), M(t)) < \epsilon$ for all $t \geq t_0 + T(\epsilon, \gamma)$, where
d(x,A) is the distance between a point x and a set A and x(t; xo, t_0) is a solution of (1) through (t_0, xo).

The solutions of (1) are said to be eventually uniform-bounded if for any a > 0, there exist S(a) > 0 and \( \beta(a) > 0 \) such that if \( t_0 \leq S(a) \), \( \|x(t; xo, t_0)\| \leq \beta(a) \) for all \( t \geq t_0 \). Under some conditions for M, assume that M is a quasi-uniform-asymptotically stable set of (1) in the large and that the solutions of (1) are uniform-bounded. Corresponding to (1), consider a perturbed system (2) \( x' = F(t, x) + G_1(t, x) + G_2(t, x) \), where \( G_1, G_2 \) are continuous on \( I \times \mathbb{R}^n \), \( G_1(t, x(t)) \rightarrow 0 \) as \( t \rightarrow \infty \) and \( \int_0^\infty \|G_2(t, x(t))\|dt < \infty \). Then it can be seen that the solutions of (2) are eventually uniformly bounded. (Received November 16, 1962.)

597-104. J. R. KINNEY, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington 73, Massachusetts. The convex hull of plane Brownian motion.

Let \( K(\omega) \) be the boundary of the closed convex hull of a sample path of the plane Brownian motion. It is shown that the set \( T(\omega) \) of \( K(\omega) \) which supports the change of direction of the tangent of \( K(\omega) \) is of logarithmic dimension not greater than one. (Received November 16, 1962.)


Let Y be a complex Banach space and \( \mathcal{H} \) the space of real numbers. Let A be a square matrix of complex numbers such that real parts of all its characteristic roots are different from zero. Let \( f(x, y) \) be defined on the space \( \mathcal{H} \times Y^m \) with values in \( Y^m \) and let for every fixed \( y_0 \in Y^m \) the function \( f(x, y_0) \) be almost periodic and satisfy Lipschitz's condition with respect to \( y \) with a constant \( L \). There exists a constant \( C \) depending only on \( A \) such that if \( L < C \) then the differential equation \( y' + Ay = f(x, y) \) has one and only one almost periodic solution. If function \( f(x, y, \mu) \) satisfies in \( x, y \) previous conditions, and if it is uniformly continuous in \( \mu \) on a compact set \( \Omega \) then the solution of the differential equation is uniformly continuous in \( \mu \) on the set \( \Omega \). The proof makes use of Banach fixed point theorem and some topological construction which establishes an isomorphism and an isometry of the space of almost periodic function with values in \( Y^m \) and a certain space of continuous functions on a compact group having values in \( Y^m \). (Received November 19, 1962.)

597-106. TAKA YUKI TAMURA, University of California, Davis, California. Commutative divisible semigroups.

A semigroup \( S \) is said to be divisible if for any \( a \in S \) and for any positive integer \( n \), there is \( b \in S \) such that \( b^n = a \). In this paper the author discusses the embedding problem and the structure of commutative divisible semigroups. (1) A commutative semigroup \( S \) is divisible iff \( S \) is a homomorphic image of the direct product of the additive semigroups of all positive rational numbers, (2) Any commutative semigroup \( T \) can be embedded into a commutative divisible semigroup \( S \) and there is the smallest \( D(T) \) of \( S \) in the sense that \( D(T) \) can be embedded into \( S \), (3) Let \( S \) be a commutative divisible semigroup and \( T \) be a subsemigroup of \( S \). Then the minimal divisible subsemigroups containing \( T \) in the sense of inclusion are isomorphic to \( D(T) \). (4) As the simplest case, the structure of cyclic divisible semigroups is determined. (5) The structure of commutative divisible semigroups is dis-
cussed. In general a commutative semigroup is a semilattice of archimedean commutative nonpotent or unipotent semigroups. Accordingly the problem of the structure of commutative divisible semigroups is reduced to that of commutative divisible archimedean nonpotent or unipotent semigroups. (Received November 19, 1962.)

597-107. E. O. THORP, New Mexico State University, University Park, New Mexico. On the conjugate of $L_1$.

Unexplained definitions and terminology are those of Linear Operators by Dunford and Schwartz.

**Definition:** The canonical map $J : L_\infty(S,\Sigma,\mu) \rightarrow L_1^*(S,\Sigma,\mu)$ relates vectors $x^* = Jg$ in $L_1^*(S,\Sigma,\mu)$ to vectors $g$ in $L_\infty(S,\Sigma,\mu)$ by the identity $[^*] x^*f = \int g(s)f(s)\mu(ds)$. **Lemma.** The map $J$ is defined on all of $L_\infty(S,\Sigma,\mu)$ and $|Jg|_\infty \leq |g|_\infty$. **Theorem.** Let $(S,\Sigma,\mu)$ be a positive measure space. If $S$ is the union of a (possibly uncountable) collection of disjoint subsets $\{S_j\}$ in $\Sigma$, each of which is $\sigma$-finite and such that each $E$ of finite measure is covered, except for a subset of a set of $\mu$ measure zero, then for each $x^*$ there is a $g$ such that $Jg = x^*$ and $|g|_\infty = |x^*|$. In particular $J$ is onto. Further, $J$ is an isometry iff whenever $K$ is a subset of $S$ such that $KS_a$ is a subset of a null set for each $a$, then $K$ is a subset of a null set. (Received November 19, 1962.)


Let $C$ be an analytic Jordan curve in the complex $z$-plane and let $u(z)$ be a continuous real-valued function given on $C$. Let $z = \phi(w)$ be a schlicht function which maps the region exterior to $C$ onto $|w| > 1$ so that $\infty = \phi(\infty)$. The nth set $S_n$ of Fejér points on $C$ are the image under $z = \phi(w)$ of the nth roots of unity in the $w$-plane. It is shown that for all $n$ sufficiently large a harmonic polynomial $H_n(u;z)$ of degree at most $n$ can be found by interpolation to $u(z)$ in the set $S_{2n+1}$, and that as $n \to \infty$, $H_n(u;z)$ converges for $z$ interior to $C$ to the solution of the Dirichlet problem for $C$ and $u$. The proof is given by expressing $H_n(u;z)$ as the real part of a linear combination of the first $n$ Faber polynomials belonging to $C$. A detailed analysis of the coefficient structure of the Faber polynomials is required. Walsh (J. Math. Mech. 9 (1960), 193-196) previously obtained the result when $C$ is an ellipse by pointing out that $H_n(u;z)$ for $z$ on the ellipse is simply an nth degree trigonometric interpolation polynomial. (Received November 19, 1962.)

597-109. A. H. KRUSE, Box 756, New Mexico State University, University Park, New Mexico. A notion of Random sequence. Preliminary report.

Let $\Sigma$ be the system of set theory of Gödel's, Consistency of the continuum hypothesis, including an axiom of choice for sets. We shall use nameable in the sense of Myhill, Proc. Nat. Acad. Sci. U.S.A. 38 (1952), 979-981. Let $d \in \omega_1$ let $S$ be the set of all sequences of members of $d(thus S = d^\omega)$, let $\mu$ be a pointwise positive probability measure on the power set $P(d)$ of $d$, and let $\triangleright$ be the resulting product measure with domain $B \subseteq P(S)$. A statistical criterion is any $X \in B$ such that for all $f, g \in S$ differing in only finitely many terms, $f \in X$ iff $g \notin X$. By the zero-one law, for each statistical criterion $X$, $\triangleright(X)$ is 0 or 1. A sequence $f \in S$ is random iff $f \in X$ for each nameable statistical criterion $X$ with
\(\gamma(x) = 1\). Theorem 1. No random sequence is nameable. Theorem 2. If there is a class of all pairs [x, y] such that \(x \in \omega, y\) is a set, and \(\phi x = y\), where \(\phi\) is the naming operation of Myhill, then \(\gamma\)-almost every \(d \in S\) is random. Theorem 3. If \(\mu\) is nameable for each \(f \in S\), \(f\) is random iff \(f \in X\) for each nameable \(X \in B\) with \(\gamma(X) = 1\). These theorems may be proved in \(\Sigma\). Theorem 1 and Myhill's result on the consistency of all classes being nameable yield: Theorem 4. If \(\Sigma\) is consistent, there is no proof in \(\Sigma\) of the existence of a random sequence. (Received November 19, 1962.)

597-110. LEON BROWN and HIDEGORO NAKANO, Wayne State University, Detroit 2, Michigan.

Outer measures on linear lattices.

In this paper is defined and discussed the theory of abstract outer measures on a \(\sigma\)-complete linear lattice \(S\). This is a generalization of the usual concept of outer measures on a function space.

It is discovered that the theories of H. Nakano (Über Erweiterungen von allgemein teilweisegeordneten Moduln, II, Proc. Imp. Acad. Tokyo 19 (1943), 138-143) and M. H. Stone (Notes on integration, I, Proc. Nat. Acad. Sci. U.S.A. 34 (1948), 336-342) can be unified. A main result of this paper is: If \(P^*\) is an outer measure, let \(N_{P^*} = \{x | P^*(|x|) = 0\}\) and \(\mathcal{S} = \{x | P^*(|x|) < +\infty\}\) then if \(S\) is full (\(a_n \wedge a_m = 0\) for \(n \neq m \Rightarrow \sum_{n=1}^{\infty} a_n\) exists), it can be concluded that \(\mathcal{S}/N_{P^*}(\|x\|) = P^*(|x|)\) is monotone complete and thus a Banach space. Conversely, if \(\mathcal{S}/N_{P^*}\) is complete for every outer measure \(P^*\), then \(S\) is full. (Received November 19, 1962.)

597-111. J. V. RYFF, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

On the representation of doubly stochastic operators.

For a real \(n\)-vector \(x = (x_1, \ldots, x_n)\), let \(x^* = (x_1^*, \ldots, x_n^*)\) be the vector obtained from \(x\) by rearranging the components of \(x\) into nonincreasing order. If \(T\) is a doubly stochastic matrix, and \(y = Tx\), then it is well known that \(y_1^* + \ldots + y_k^* \leq x_1^* + \ldots + x_k^*, k = 1, \ldots, n\), with equality holding when \(k = n\). Denoting by \(f^*\) the decreasing rearrangement of the function \(f\), those bounded linear operators which have the property that \(\mathcal{J}_1^s(Tf) \leq \mathcal{J}_1^s f^*, 0 \leq s < 1\), and \(\mathcal{J}_1^0 Tf = \mathcal{J}_1^0 f\) are studied. A concrete representation of all such operators is given, and this class is shown to coincide with the set of doubly stochastic operators defined by Rota (Bull. Amer. Math. Soc. 68 (1962), 95-102). (Received November 19, 1962.)

597-112. W. S. LOUD, University of Minnesota, Minneapolis, Minnesota.

Isochronous oscillations in certain plane autonomous systems.

The real plane autonomous system (1) \(x' = -y + Ax^2 + Bxy + Cy^2, y' = x + Dx^2 + Exy + Fy^2\), is completely understood so far as the distinction between a center and a focus at the singular point \((0,0)\) is concerned. Results of Frommer and Saharnikov give a complete catalogue of those cases in which (1) has a center at \((0,0)\). In the present paper these various cases of a center are examined to determine under what circumstances all sufficiently small orbits about \((0,0)\) have the same period. One such case is the obvious linear case with \(A = B = C = D = E = F = 0\). Using implicit function techniques the question in nonlinear cases is reduced to a study of (2) \(x' = -y + xy, y' = x + Dx^2 + Fy^2\). Using further implicit function techniques and also results of Urabe, Levin, and Shatz on isochronous
oscillation of solutions of the scalar equation \( x'^{11} + g(x) = 0 \), necessary conditions on \( D \) and \( F \) that (2) be isochronous are developed. There are found just seven possibilities, of which four can be shown to give isochronous orbits. (Received November 19, 1962.)


It is a common rule of thumb that if a two-level difference scheme is constructed for an initial-value problem it will always be stable if the "space" differing is taken (implicitly) entirely at the "new" level. An example is given (parabolic, constant coefficients) for which such a difference scheme can be shown, by the von Neumann criterion, to be unstable for every value of the mesh-ratio. (Received November 19, 1962.)

597-114. HERMAN RUBIN, Department of Statistics, Michigan State University, East Lansing, Michigan. A generating function for trees and connected graphs. I.

Let \( G \) be an arbitrary connected labeled graph on the elements \( 1, \ldots, n \). Then, associate uniquely a tree to \( G \) in the following way: Define the functions \( \phi \) and \( \psi \) so that \( \phi(1) = 1 \), and given \( \phi(1), \ldots, \phi(k), k < n \), \( \psi(k) = i \) if and only if there is an element \( x \) not in \( \{\phi(1), \ldots, \phi(k)\} \) so that \( (x, \phi(i)) \in G \) and \( (x, \phi(j)) \notin G \) for any \( j < i \), and \( i \) is the largest such number, and \( \phi(k + 1) \) is the smallest element establishing the value of \( \psi(k) \). An analogous construction can be made with largest replaced by smallest. \( T(G) \) is then the tree whose lines are \( (\phi(k + 1), \psi(k)) \). The relabeling for \( T(G) \) is seen to be the relabeling for \( G \), and any given tree is associated with \( 2^h \) graphs, where the index \( h = \Sigma(i - \psi(i)) \). If \( A_{nm} \) is the number of labeled trees of \( n \) elements, with index \( h \), and \( m \) elements adjacent to \( 1 \), and if \( f(a,b,c) = \Sigma A_{nm} a^n b^m c^m / (n - 1)! \), the generating function \( f \) satisfies the equation \( f(a,b,c) = a + f(a,b,1) \int_0^c f(ab,b,x)dx / b \). The domain of \( f \) contains \( \{(a,b,c) : |a| \leq e^{-1}, |b| \leq 1, |c| \leq 1\} \). (Received November 19, 1962.)


The term-relation number theory results from formalizing internal relations as a kind of variable instead of as a kind of two-place predicate (F. G. Asenjo, Relations irreducible to classes, Notre Dame J. Formal Logic 4 (1963)). The present paper introduces negative term-relation numbers and studies rings of term-relation numbers as partially ordered sets. These rings are proved to be modular lattices. Prime term-relation numbers are also defined; divisibility is studied, and the factorization theorem is proved. Other aspects of the term-relation number theory are examined, such as the introduction of rational term-relation numbers and a comparison of different systems of term-relation numbers with hypercomplex numbers, principally with respect to the validity of the formal laws of arithmetic. In connection with the last point, the relevance of Weierstrass' final theorem of arithmetic is considered. (Received November 19, 1962.)
Using positive definite operator-valued measures on a locally compact group, the notion of infinitesimally induced representation (Amer. J. Math. 83 (1961), 499-512) is extended to include all of the nondegenerate principal series of representations of SL(n,R) of Gel'fand and Graev (Izv. Akad. Nauk SSSR, Ser. Mat. 17 (1953), 189-248). An intertwining number theorem is proved for the extended type of representation, which however is not sharp enough to imply the irreducibility of almost all of the Gel'fand-Graev representations. (Received November 19, 1962.)

On an inequality for finite sums, and generalizations of an inequality of Kantorovich.

A "Blitzbeweis" is given of the following inequality(*):

\[ \sum_{k=1}^{n} (a_k + b_k - m_1 M_1 - m_2 M_2)^2 \leq n(M_1 M_2 - m_1 m_2)^2 \]

where each summation is taken from \( k = 1 \) to \( k = n \), and the real numbers \( a_k \) and \( b_k \) are subject to the inequalities:

\[ 0 \leq m_1 \leq a_k \leq M_1, \quad 0 \leq m_2 \leq b_k \leq M_2, \]

for \( k = 1, 2, ..., n \). Since \( 2(m_1 M_1 m_2 M_2)^{1/2} (\sum_{k=1}^{n} a_k^{1/2} b_k^{1/2}) \leq m_1 M_1 \sum_{k=1}^{n} a_k^{1/2} + m_2 M_2 \sum_{k=1}^{n} b_k^{1/2} \), the left hand inequality of (*) is actually sharper than an inequality which goes back to P. Schweitzer (see G. Pólya and G. Szegő, Aufgaben und Lehrsätze aus der Analysis, I, Dover, New York, 1945, pp. 57 and 213).

The same simple idea employed in proving (*), suitably interpreted, yields immediately inequalities which are sharper than the "generalized Kantorovich inequality in Hilbert space" of W. Greub and W. Rheinboldt, Proc. Amer. Math. Soc. 10 (1959), 409. Still other inequalities of the same general nature follow readily. (Received November 19, 1962.)

Uniqueness classes for difference functionals.

Let \( C \) denote the class of all entire functions \( f \) for which there exist numbers \( A \) and \( c \) with \( c < \pi \) such that \( |f(\pi y)| \leq A \exp(c|y|) \) for all real \( y \). For the sequence \( \{\Delta^m f(x_n)\} \) of generalized difference functionals, \( C \) is a uniqueness class if \( \beta = 0 \). Buck [Trans. Amer. Math. Soc. 64 (1948), 283-298] showed that for \( \beta > 0 \), \( C \) is not a uniqueness class. Since there is symmetry in \( \beta \) about \( \beta = -1/2 \), \( C \) is a uniqueness class if \( \beta = -1 \), and is not if \( \beta < -1 \). It is shown that \( C \) is a uniqueness class for \( -1 < \beta < 0 \) and the question is asked: For what sequences \( \{x_n\} \) is \( C \) a uniqueness class for \( \Delta^m f(x_n) \)? Examples of results: 1. If \( \{x_n\} \) is a periodic sequence of rational integers of period \( p \) and if for some \( m \leq p \) such that \( x_n < x_{n+m} \) for all \( n < m \), then \( C \) is a uniqueness class. 2. If \( \{x_n\} \) is a periodic sequence of real numbers of period \( p \) with each period arithmetic with difference \( d \), then \( d > 1/(p - 1) \) implies \( C \) is not a uniqueness class. (Received November 19, 1962.)

Vector-valued analytic functions.

Let \( E \) be a complete, metrizable (perhaps not locally convex) topological linear space (e.g., \( L_p \), \( 0 < p \leq \infty \)). Let \( G \) be a domain in the complex plane, Definition 1. \( A_0(G,E) \) is the set of different-
tiable functions $f:G \to E$ such that $f(G)$ spans a finite-dimensional subspace. **Definition 2.** $A(G,E)$ is the set of functions $f:G \to E$ such that there exists a sequence $f_n$ in $A_0(G,E)$ such that for every compact $K \subseteq G$, the closed convex hull of $(f - f_n)(K)$ converges to $0$ in $E$. **Theorem 1.** The Cauchy integral formulas are valid for functions in $A(G,E)$. There exist differentiable $L_p$-valued functions ($0 < p < 1/2$) for which the Cauchy integral formulas fail. **Theorem 2.** A function $f:G \to E$ is in $A(G,E)$ if and only if it can be locally factored into the composition of an analytic Banach-space-valued function and a continuous linear operator. **Theorem 3.** Let $\rho$ be a metric for $E$, and let $\psi(\cdot) = \rho(\cdot, 0)$. If $\psi$ is plurisubharmonic then for every $f$ in $A(G,E)$ there holds $\sup \{ f(x) : x \in K \} = \sup \{ f(\xi) : \xi \in \partial K \}$ for every compact $K \subseteq G$. If $\psi$ is not plurisubharmonic, or if $f$ is differentiable but not in $A(G,E)$, then the maximum modulus principle may fail. (Received November 19, 1962.)

597-120, B. R. GELBAUM, University of Minnesota, Minneapolis 14, Minnesota. Notes on spectral synthesis.

Let $A$ be a commutative regular self-adjoint Banach algebra such that for every closed ideal $I$ in $A$, $I = k_h(I)$ (in particular $A$ is semisimple). Then if $x \in A$ and $h(x) = h(I)$, then $I = I_x \neq \text{smallest closed ideal containing } x$. **Corollaries.** (a) If $G$ is a separable locally compact abelian group and $A = L^1(G)$, then every regular maximal ideal in $A$ is of the form $I_x$ for a suitable $x$. (b) If $G$ is compact and separable, every closed ideal $I$ in $L^1(G)$ is of the form $I_x$ for a suitable $x$. (c) A simplification in some cases of Malliavin's proof of the absence of general spectral synthesis for arbitrary $L^1(G)$, $G$ locally compact, not compact and abelian would stem from the exhibition of a closed ideal $I$ in $L^1(G)$ for which $I \neq I_x$ for all $x$. (Received November 19, 1962.)

597-121, DONALD GREENSPAN, Mathematics Research Center, U. S. Army, University of Wisconsin, Madison, Wisconsin. On the numerical solution of problems allowing mixed boundary conditions.

In domain $G$, if points $(x,y)$, $(x - d,y)$, $(x - d, y + h)$, $(x - d - h,y)$, $(x - d, y - g)$, $(x - d - h, y + h)$, $d \neq h, g \neq h$, are numbered 0-5, respectively, and $\alpha$ is the angle of the outwardly directed normal at 0 of a boundary curve through 0, then $\partial u/\partial n|_0$ is approximated to second order by $\sum_{i=0}^{5} a_i u_i$, where $a_0 = (\cos \alpha)(h + 2d)/d(h + d)$, $a_1 = -[(hd + g(h + d) \cos \alpha + (hdg - h^2d + d^2g) \sin \alpha)]/h^2dg$, $a_2 = [(dh + gh + dg) \sin \alpha]/h^2(g + h)$, $a_3 = [dh \cos \alpha + d(d + h) \sin \alpha]/h^2(d + h)$, $a_4 = -h \sin \alpha/g(g + h)$, $a_5 = -d \sin \alpha/h^2$. Corresponding formulas for other arrangements of points follow by symmetry. A purely "internal" numerical method is then described for approximating solutions of elliptic problems with mixed boundary conditions. Theoretical results favor application of the usual first order approximation for $\partial u/\partial n|_0$ since, for example, use of the second order method may result in a linear algebraic system whose coefficient matrix is singular, but numerical results clearly support considering the second order method for its relatively high degree of accuracy. (Received November 19, 1962.)

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Let $A$ be a closed subalgebra of $C_0(X)$, $X$ locally compact and the Šilov boundary of $A$. Let $B \subset F \subset C_0(X)$ be closed subspaces, with $B$ an $A$-module with a nowhere dense set of common zeroes on $X$. Then any bounded operator $T : F \to C_0(X)$ which acts as the identity on $B$ satisfies $\|T - I\| = \|T\| - 1$, where $I$ is the identity operator. As a consequence of this result the technique of Rudin (Proc. Amer. Math. Soc. 13 (1962), 429) can be applied to yield the Theorem. If $G$ is a compact group or a locally compact abelian group, and $A_1 \subset A_2$ are closed (doubly) invariant subalgebras of $C_0(G)$ with the conjugates $\overline{A_1} \not\subset A_2$, then any closed subspace $F$ of $C_0(G)$ between $A_1$ and $A_2$ is uncomplemented in $C_0(G)$. The proofs utilize several results of Bishop. (Received November 19, 1962.)

597-12.3. O. G. OWENS, Wayne State University, Detroit 2, Michigan. Uniqueness of the integral value problem for the Helmholtz equation.

Another proof is given of the following uniqueness theorem of Hartman and Wilcox [Math. Zeitschr. 75 (1961), 228-255] which generalized a theorem of Owens [Trans. Amer. Math. Soc. 88 (1958), 388-399]. Theorem. If $u(x_1, x_2)$ is an everywhere defined $C^2$ solution of the 2-dimensional Helmholtz equation $\Delta u + u = 0$, if $\int_0^1 u(r \cos \theta, r \sin \theta) dr < M$ (M a positive constant independent of $R$ and $\theta$) and if $\lim_{r \to 1} \sqrt{\int_0^{2\pi} (u(r \cos \theta, r \sin \theta) dr \theta} = 0$, then $u(x_1, x_2)$ vanishes everywhere. The method of proof is analogous to that used by F. Rellich [Jber. Deutsch. Math. Verein. 53 (1943), 57-65] in establishing the uniqueness of the solution of the radiation problem for the 3-dimensional Helmholtz equation. More precisely, under the assumptions of the theorem one shows that all derivatives of $u(x_1, x_2)$ vanish at the origin. Thus, as any $C^2$ solution of the Helmholtz equation is analytic, $u(x_1, x_2)$ vanishes identically. (Received November 19, 1962.)


The problem is to find a continuous solution $u(r, x)$ for the equation $u_{xx} + u_{rr} + (u_r)/r = 0$ in the region $(\lambda$ a non-negative constant) $\{0 < r < 1; -\lambda < x \leq 0\} \cup \{r > 0; x > 0\}$ subject to $\int_0^1 r u(r, -\lambda^+) - H(r) dr = 0$, $u = 0$ when $r + x = \infty$, $\partial u/\partial n = 0$ on the 3 line segments $\{r = 0; x > 0\}$, $\{r = 1; -\lambda < x < 0\}$, $\{r > 1; x = 0\}$, where $H(r)$ is given and $r^{1/2} H \in L^2(0, 1)$. Changing to oblate spheroidal coordinates in the reservoir suggests finding square summable vectors $j = \{j_n\}$ and $p = \{p_n\}$ such that $u(r, 0^+) = J_0^{1/2} + \sum_{n=0}^{\infty} J_0(a_n r)/J_0(a_n)$ and $u(r, 0^+) = \sum_{n=0}^{\infty} p_n (4n + 1)^{-1/2} P_{2n}$. Using this, formal solutions are written down for cylinder and reservoir. Setting $u(r, 0^-) = u(r, 0^+)$ and $u_x(r, 0^-) = u_x(r, 0^+)$ gives 2 linear algebraic equations $p = Bj$ and $j = Ap + h$ (h known). Setting $k_n = a_n j_n$ yields an equation $(I - D)k = h'$ where $h'$ is known, $d_{kn}(k, n = 0, 1, \ldots)$ is simply expressible in closed form, and $\|D\| < 0.646$ uniformly in $\lambda$. A series expansion for the flux into the reservoir is derived in which the lead term in the case $H = constant$ yields the flux with a relative error $< 6\%$ uniformly in $\lambda$. This problem arises in studying the biological phenomenon of diffusion through pores. (Received November 19, 1962.)

Maxwell's equations in an electromagnetically (linear) anisotropic medium can be cast in the matrix equation form $AX = 0$, where $A$ and $X$ are, respectively, $3 \times 3$, and $3 \times 1$ matrices whose elements are taken from a noncommutative ring with zero-divisors. Specifically, $A$ is an "operator" matrix whose elements are linear partial differential operators having spatially dependent coefficients whose form is determined by one's choice of coordinate system. The elements of $X$ are merely functions of the space coordinates. The author will show how, using elementary matrix techniques, one may reduce the above matrix equation $AX = 0$ to a form $BX = 0$, where $B$ is another $3 \times 3$ operator matrix which is in triangular form. The form $BX = 0$ thus obtained is often more convenient for use in the solution of boundary-value problems. Limitations on the applicability of the method of reduction from $AX = 0$ to $BX = 0$ will be discussed. (Received July 9, 1962.)


A subset $S$ of the vertices $T$ of an $n$-dimensional cube $I^n$ is called a switching set. $S$ is called separable if there exists a hyperplane which separates $S$ from $T - S$. $S$ is called simply separable if there exists a hyperplane which is determined by midpoints of edges of $I^n$ and separates $S$ from $T - S$. Only for small values of $n$ is it known which (or how many) sets $S$ contained in $I^n$ are separable. These known cases are also simply separable. (Received November 19, 1962.)

597-127. B. W. VOLKMANN, University of Utah, Salt Lake City, Utah. A metric problem on transcendental numbers.

In Mahler's classification of transcendental numbers there is assigned to each $\xi$ a sequence of parameters $\vartheta_n(\xi)$ ($n = 1, 2, \ldots$) which measure how well the number zero can be approximated by polynomials of degree $n$, with integer coefficients, at the point $\xi$. (See, for instance, Th. Schneider, Einführung in die transzendenten Zahlen, Springer, Berlin, 1957, Chapter 3.) It was conjectured by Mahler that for almost all real or complex $\xi, \vartheta_n(\xi)$ has the smallest possible value, i.e., $1$ or $(n - 1)/(2n)$; respectively. Though this conjecture is still unproved, a number of partial results were obtained by several authors, the most recent one, by the speaker, being that $\vartheta_n(\xi)$ is a.e. at most $4/3$ or $2/3 - 1/(2n)$ in the real and complex cases, respectively. The improvement over previous results was accomplished mainly by introducing a classification of polynomials according to their distribution of zeros and by proving, by means of resultants, that there are not "too many" irreducible polynomials of given degree and height which have two or more roots "very close" to each other. (Received November 19, 1962.)


Let $r$ be an odd prime. Let $p \equiv 1 \pmod{r}$ and $q \not\equiv r$ be primes. Let $\chi$ be a primitive $r$th power character, modulo $p$. Let $\tau(\chi)$ denote the generalized Gaussian sum. Ankeny [Pacific J. Math. 10
proved that \( q \) is an \( r \)-th power, modulo \( p \), if and only if \( \tau(\chi)^r \) is an \( r \)-th power residue in the field formed by adjoining an \( r \)-th root of unity to the field of integers modulo \( q \). He gave a rational integral formulation for the case \( q \equiv 1 \pmod{r} \). If \( q \equiv -1 \pmod{r} \), \( q \) is an \( r \)-th power, modulo \( p \), if and only if the trace \( \tau(\chi)^r + \tau(\chi^q)^r \) is in certain residue classes, modulo \( q \), depending upon \( p \pmod{q} \). It is shown that for a given \( q \) and \( r \), a list of only \( 1 + (q + 1)/r \) residue classes suffices. An algorithm for computing these residue classes efficiently has been developed, Let \( \tau(\chi)^r = \sum_{j=1}^{r-1} b_j \zeta_r^j \), where \( \zeta_r = e^{2\pi i/r} \neq 1 \). If \( r \) and \( q \) are both very small, one could examine all combinations of the \( b_j \), modulo \( q \), which might be \( r \)-th power Gaussian sums. A typical result is that \( 5 \) is a seventh power residue, modulo \( p \), if and only if either \( b_1 \equiv b_2 \equiv b_4 \pmod{5} \) and \( b_3 \equiv b_5 \equiv b_6 \equiv 0 \pmod{5} \). (Received November 19, 1962.)

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If \( M \) and \( N \) are two \( C^\infty \) manifolds, \( J^k(N,M) \) the manifold of \( k \)-jets \( j^k(f) \) of maps \( f:N \rightarrow M \), a \( k \)-vector field on \( C^\infty(N,M) \) is a map \( \theta : J^k(N,M) \rightarrow T(M) \) so that \( \theta(j^k(f)) \in M_{f(x)} \) (R. Hermann, Formal tangency of vector fields in function spaces, Notes, Univ. of Calif., 1961). The prolongation of \( \theta \), \( P^r \theta : J^{r+k}(N,M) \rightarrow T(J^r(N,M)) \) is defined and using this a bracket \( [\theta, \psi] = P^h \theta - \psi - T^k \psi \cdot \theta \) of the \( k \)-vector field \( \theta \) with the \( h \)-vector field \( \psi \) are defined and justified. An exponential formula is derived. The concept of invariance of partial differential equations under \( k \)-vector fields is discussed and compared with invariance under pseudo groups. (Received November 19, 1962.)

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Consider a \( k \)-step method \( \sum_{s=0}^{k} a_s y_{n+1-s} = h \sum_{s=0}^{k} \beta_s y'_{n+1-s} (a_0 = 1) \) for the numerical integration of ordinary differential equations. Associate with it the linear functional \( L[w(t)] = \sum_{s=0}^{k} [w(k-s) - (k-s)w'(k-s)] \), and let \( L_0 = L [r^k/r!] \). The \( k \)-step method is said to have order \( p \) if \( L_0 = 0 \) for \( r = 0, 1, \ldots, p \). It is said to be stable (in the sense of Dahlquist) if all zeros \( z_1 \) of the polynomial \( a(r) = r^k + a_1 r^{k-1} + \ldots + a_k \) are located on the unit disc \( |z| \leq 1 \), and those on the circumference \( |z| = 1 \) are simple. The local truncation error of a method of order \( p \) may be measured by the error constant \( C = L_{kp}^+ / \sum_{s=0}^{k} \beta_s \) \( \{P. Henrici, Discrete variable methods in ordinary differential equations, John Wiley, New York, 1962; p. 223\} \). Define \( \sigma_p \) as the infimum of \( |C| \) extended over all stable \( k \)-step methods of order \( p \). Then, for \( k \equiv 3 \) and \( k \) odd, we have \( \sigma_{k+1} = 2^k |c_{k+1}| \), where the \( c_{2m} \) are defined by \( z [L_n(1 + z)/(1 - z)]^{-1} = c_0 + c_2 z^2 + c_4 z^4 + \ldots \). Moreover, the infimum \( \sigma_{k+1} \) is never assumed, except when \( k = 3 \), in which case every \( 3 \)-step method having zeros \( z_1 = 1, z_2 = -1, -1 < z_3 < 1 \) has minimum error constant \( C = 1/180 \). The growth parameter associated with \( z_2 = -1 \) has the value \( \lambda = -1/3 \) for all \( z_3 \). For analogous results, when \( k \) is even, see P. Henrici, loc. cit., problem 37, p. 286. (Received November 19, 1962.)

For testing a simple hypothesis against a simple alternative, the set of admissible error rate pairs \((a,\beta)\) is a convex curve with end points \((0,1)\) and \((1,0)\). Let \(C_n\) be the curve of error rates for \(n\) independent repetitions of a particular experiment. Then \(C_{n+1}\) cannot lie above \(C_n\). H. Raiffa (Statistical decision theory approach to item selection for dichotomous test and criterion variable, Studies in Item analysis and Prediction, Herbert Solomon, editor, Stanford, 1961) has given an example where \(C_n \cap C_{n+1}\) contains points other than end points. The following results characterize such examples: If the measures \(\mu_0\) and \(\mu_1\) of the hypothesis and alternate are absolutely continuous with respect to each other and if \(C_1\) does not consist of one or two line segments then \(C_n \cap C_{n+1} = \{(0,1), (1,0)\}\). If \(C_1\) consists of two line segments then, for every positive integer \(n\), \(C_n \cap C_{n+1}\) is exactly the set of extreme points of \(C_n\). The first theorem follows from the Neyman-Pearson lemma, the second from an elementary probability argument. (Received November 19, 1962.)

597-132. LEE LORCH, University of Alberta, Edmonton, Alberta, Canada, and P. A. SZEGO, Engineering School, University of Santa Clara, Santa Clara, California. Monotonicity of the differences of zeros of Bessel functions as a function of order.

Let \(c_{ym}, \gamma_{yk}\) denote, respectively, the \(m\)th and \(k\)th positive zeros of any pair (distinct or not) of real Bessel functions of order \(\nu\), arranged so that \(c_{ym} > \gamma_{yk}\). Each such zero increases with \(\nu\) [G. N. Watson, Bessel functions, 2nd ed., p. 508]. It has been shown [Lee Lorch and Peter Szego, Higher monotonicity properties of certain Sturm-Liouville functions, Acta Math. (to appear), §3] that 
\((- 1)_n^\nu \Delta^n c_{yk} > 0, n = 1,2,\ldots, \) for \(\nu > 1/2\). The principal purpose of this note (to appear in Proc. Amer. Math. Soc.) is to prove that (i) the positive quantity \((- 1)_n^\nu \Delta^n c_{yk}\) increases with \(\nu\) for each fixed \(k\), \(n = 1,2,\ldots\), when \(\nu > 1/2\), and that (ii) the difference \(c_{ym} - \gamma_{yk}\) increases with \(\nu\) when \(\nu \equiv 0\). (Received November 19, 1962.)


Let \(D\) be the open unit disc, \(C\) its boundary, and let \(J\) be a proper \(\sigma\)-ideal of subsets of \(C\). Let \(H_\infty\) denote the Banach algebra of bounded analytic functions in \(D\). For \(f\) in \(H_\infty\), \(P\) on \(C\) and \(M\) in \(J\) let \(K_M(f,P)\) denote either the boundary or radial boundary cluster set modulo \(P\) at \(f\) at \(P\). Define \(K(f,P) = \bigcap K_M(f,P)\) where the intersection extends over all \(M\) in \(J\). Theorem. For each \(f\) in \(H_\infty\), \(P\) on \(C\) there is a \(M\) in \(J\) such that \(K(f,P) = K_M(f,P)\). Theorem. For each \(P\) on \(C\) there is a compact subset \(E_P\) of the fiber of the maximal ideal space of \(H_\infty\) which lies above \(P\) such that \(K(f,P) = \hat{f}\{E_P\}\) for each \(f\) in \(H_\infty\), where \(\hat{f}\) is the Gel'fand representation of \(f\). In particular when \(J\) is the ideal of sets of Lebesgue measure zero, the set \(E_P\) may be taken to be that part of the Šilov boundary for \(H_\infty\) which lies in the fiber above \(P\). As corollaries, refinements are obtained of theorems of Lohwater, Noshiro, and Woolf concerning the boundary behavior of bounded analytic functions. (Received November 19, 1962.)
597-134. J. N. MORDESON and BERNARD VINOGRADE, Iowa State University, Ames, Iowa. 

Splitting of commutative algebras with respect to prime ideals.

A commutative algebra $A$ with identity containing the base field $K$ is said to split with respect to a prime ideal $N$ if $A = I + N$ (group direct), where $I$ contains $K$. The residue algebra $A/N$ may be regarded nonuniquely as an integral domain composite of two integral domains over $K$, say $F_0, F_1$. When $A$ contains ($K$-isomorphic) images of $F_0, F_1$, then $A$ is said to have a commutative diagram with respect to $N$ if $f = auh$ where $f$ is the canonical homomorphism of $F_0 \times F_1$ into $A$, $u$ is the natural homomorphism of $A$ onto $A/N$, and $a$ is an automorphism of $A/N$. The splitting of $A$ is equivalent to the existence of a commutative diagram plus the relative splitting of $F_0 \times F_1$ modulo an ideal in $\ker h$. In particular, conditions are found under which unicity of composite, or $A/N$ a free join, or $A$ algebraic over $K$ implies splitting. The product $A \times F$ is also analyzed, where $F$ is an integral domain in $A/N$. All the results apply in particular to the case where $A$ is a quasi-local algebra over $K$ (hence $I$ a coefficient field). (Received November 19, 1962.)

597-135. V. LAKSHMIKANTHAM, RIAS, 7212 Bellona Avenue, Baltimore, Maryland. 

Properties of solutions of abstract differential inequalities.

Many properties concerning the behavior of solution of ordinary differential equations can be made to depend on a scalar differential equation. Comparison principle of this type has been widely used. Using a similar approach, consider some properties of solutions of abstract differential inequalities of the type $|x' - A(t)x - f(t,x)| \leq W_1(t, |x|)$ where $A(t)$ is a given function whose values are linear operators acting in a Banach space. Estimates are obtained of the norm of solutions, and also consider stability and boundedness results. A general uniqueness theorem for abstract differential equations is also included. (Received November 19, 1962.)


Spectral representations for some unbounded normal operators.

Let $Z$ be a locally compact Abelian group and let $z \mapsto U_z$ be a mapping of $Z$ into the set of unbounded normal operators on a Hilbert space $H$. Suppose that (for all $s, t \in Z$) the functional equation $aU_s + a(t) + bU_s + b(t) \subseteq U_s g(t) U_b(t)$ is satisfied for $a,b$ complex; $o, o', \phi$ homeomorphisms of $Z$ onto $Z$; and $g$ a (nonzero) continuous function. A character is a (nonzero) continuous function $\chi$ so that $a \chi(s + a(t)) + b \chi(s + o(t)) = g(t) \chi(s) \chi(o(t))$. Assuming mild continuity conditions the following representation is valid (for $z \in Z; x, y \in H$): $(U_z x | y) = \int_E \chi(x) d\mu_x, y(\chi)$ where $E$ is the Stone-Čech compactification of the space of all characters $E$ and $\mathcal{F} = (\mu_{x,y})$ is a spectral family concentrated on $E$. For the case $Z = \text{reals}$ and for suitable choices of the parameters an immediate corollary is the representation $(U_z x | y) = \int \cos h \lambda d\nu_{x,y}(\lambda)$ for $s$ real, $x, y \in \text{domain of } U_s$ and $y \in H$. (The above representation, more easily stated for groups, is valid for any locally compact space $Z$ having an algebra of measures as in C. Ionescu Tulcea and A. Simon, Proc. Nat. Acad. Sci. U.S.A. 45 (1959).) The proof is obtained by considering certain bounded restrictions for which a spectral representation is available (G. Maltese, Comp. Math. 15 (1962)) and then applying results of Ionescu Tulcea on spectral families and direct sums thereof. (Received November 20, 1962.)
597-137. G. L. KRABBE, Purdue University, Lafayette, Indiana. Spectral permanence of scalar operators.

Let \((S, \mathcal{G}, \mu)\) be an arbitrary measure-space, and denote by \([L(p)]\) the Banach algebra of all bounded linear mappings of \((S, \mathcal{G}, \mu)\)-simple functions into the normed space \(L^p(S, \mathcal{G}, \mu)\). Take \(1 < p_1 < \infty\) and let \(T\) be a self-adjoint operator with \(T \in [L(2)]\) whose spectral resolution \(\{E_\lambda : -\infty < \lambda < \infty\}\) is a bounded subset of the Banach space \([L(p_1)]\). Conclusion. If \(p\) is any number between \(p_1\) and \(2\), then \(T \in [L(p)]\) and the spectrum of \(T\) coincides with the spectrum of \(T\) relative to \([L(p)]\). The conclusion can be extended by replacing \(T\) by any operator of the form \(f(T)\), where \(f\) is any continuous function of bounded variation on a finite interval containing the spectrum of \(T\). This result can be extended in various directions; in particular, one may require only of \(T\) that it be of scalar type (in the sense of Dunford) -- the conclusion will then hold whenever the spectral resolution is a bounded subset of the Banach space \([L(p_1)]\). (Received November 20, 1962.)


The notion of an \(H^*\)-space (G. W. Whitehead, Comment. Math. Helv. 28 (1954), 320-328) is made relative by the definition of an \(H^*\)-pair \((G,H)\), where \(G\) and \(H\) are \(H^*\)-spaces, \(G \supset H\). A product \(\lambda: \pi_q(G,H) \otimes \pi_q(G,H) \to \pi_{q+s-1}(G,H)\) is defined and shown to have many properties analogous to those of the Samelson product in \(\pi_q(G)\). The product \(\lambda\) is generalized, via the boundary operator \(d: \pi_q(G,H) \to \pi_{q-1}(G,H)\), to a product \(\lambda: \pi_q(H) \otimes \pi_q(G,H) \to \pi_{q+s}(G,H)\) which determines, for each \(a \in \pi_q(H)\), an endomorphism of the homotopy sequence of \((G,H)\). If \(H \to G \to X\) is a fibration, identification of \(\pi_q(G,H)\) with \(\pi_{q+s}(G,H)\) gives a product \(\lambda:\pi_q(H) \otimes \pi_q(X) \to \pi_{q+s}(X)\) which generalizes the Whitehead product over \(X: a, \beta \in \pi_q(X) \Rightarrow [a, \beta] = (-1)^{\dim(da)}(da \otimes \beta)\). The group of all homeomorphisms of a manifold \(X\) is a universal space for the actions on \(X\) (by \(\phi\)) of each subgroup \(H\) of a group \(G\) fibered over \(X = G/H\). One obvious byproduct is the Theorem. Let \(a \in \pi_q(X)\), and let there exist an \(H^*\)-pair \((G,H)\) with fibration \(H \to G \to X\) \(\exists da = 0\). Then for all \(\beta \in \pi_q(X)\) the Whitehead products \([a, \beta] = 0\). (Received November 20, 1962.)


Let \(M_1(M_2)\) denote the convolution algebra of bounded Baire measures \(\mu\) on the line (half-line) with Fourier-Stieltjes (Laplace-Stieltjes) transform \(\hat{\mu}\). Let \(L^1\) denote the Lebesgue summable functions on the line. Theorem. Let \(u\) and \(v\) be functions defined on the line. A necessary and sufficient condition that \(u = \hat{v}\) for some \(\mu\) in \(M_1\) is that, for each \(f\) in \(L^1\), \(\hat{fu} = \hat{gv}\) for some \(g\) in \(L^1\). The proof involves an adaptation to a quotient space setting of the known proof (see Henry Helson, Isomorphisms of abelian group algebras, Ark. Mat. 2 (1953), 475-487) for the case \(v \equiv 1\). Theorem. In \(M_2\) the maximal ideals \(m_a = \{\mu | \hat{\mu}(a) = 0\}\), \(Re a > 0\), are principal. Corollary (T. Carleman). If \(Re a_i \leq 0\) and \(\sum 1/|a_i| < \infty\), then there exists a nontrivial \(\mu\) in \(M_2\) such that \(\hat{\mu}\) vanishes at each \(a_i\). (Received November 20, 1962.)
Regular points for elliptic equations with discontinuous coefficients.

The Dirichlet problem for the uniformly elliptic equation $Lu = (\sum a_{ij}u_{x_i}x_j) = 0$ with bounded measurable coefficients in a bounded domain $\Omega$ and for arbitrarily assigned continuous boundary values is considered. The boundary of $\Omega$ is said to be regular with respect to $L$ if this problem has a unique solution $u$ in the class of functions continuous in $\bar{\Omega}$ with locally square integrable first derivatives for any continuous boundary values. It is shown that the boundary of $\Omega$ is regular with respect to $L$ if and only if it is regular with respect to the Laplace operator. More generally the regular boundary points of $\Omega$ are the same for all uniformly elliptic operators. (Received November 21, 1962.)

On algebras of bounded representation type.

A ring $R$ is said to be of finite representation type if the number of isomorphism classes of indecomposable left $R$-modules is finite. A completely reducible module is called anti-homogeneous if each of its homogeneous components is irreducible. The following theorem is proved. Let $R$ be a finite dimensional algebra over a field $K$, which satisfies the following hypothesis: every indecomposable left (or right) module has an anti-homogeneous socle and is isomorphic to a left (or right, respectively) ideal in $R$. Then $R$ is of finite representation type. This theorem shows that in certain cases the number of isomorphism classes of left ideals in a finite dimensional algebra is finite. The fact that this number is in general infinite is shown by the group algebra over an infinite field of characteristic $p$ of the direct product of two cyclic groups of order $p$. (Received November 21, 1962.)

Two theorems on computability by two-way automata.

Let a two-way automaton $A$ be a Turing machine $Z$ together with a non-negative integer $k$. The automaton $A$ is said to compute the partial recursive function $g(x_1, \ldots, x_n)$ if (i) $Z$ computes $g(x_1, \ldots, x_n)$ (in binary notation), and (ii) given any input $(x_1, \ldots, x_n)$, $Z$ uses at most $k + \ell(x_1, \ldots, x_n)$ tape squares where $\ell(x_1, \ldots, x_n)$ is the number of tape squares used to hold the input $(x_1, \ldots, x_n)$. Let $T$ be the class of partial recursive functions computed by two-way automata. Theorem 1. An indexing of $T$ can be given for which there is a partial function $U(x,y)$ such that $U(x,y) = g_x(y)$ if the $x$th member $g_x$ of $T$ is defined on the input $y$. Theorem 2. The class $T$ is not closed under identification of variables. Theorem 1 is proved by a direct construction of a universal two-way automaton using an indexing similar to that in [Ritchie, Predictably computable functions, Trans. Amer. Math. Soc. (to appear)]. Theorem 2 results from Theorem 1, a diagonalization argument, and a lemma which states that if $T$ were closed under identification of variables then every partial function in $T$ would be the restriction of some total function in $T$. (Received November 21, 1962.)
In certain universal algebras, called primal/sm, every function on a finite portion of the domain is representable by some combination of the primitive operations of the algebra. If several functions in several algebras are always simultaneously thus representable, the algebras are strictly independent in the weak sense. This independence occurs among sets of distinct primal/sm finite algebras if the primitive operations have the whole algebra in their range. It also occurs in infinite algebras in the same circumstances with the added condition that each constant is representable throughout the algebra by a single expression. (Received November 21, 1962.)

If A and B are abelian groups, p is a prime, and α is an ordinal number, then $p^\alpha \text{Tor}(A,B) = \text{Tor}(p^\alpha A, p^\alpha B)$. Also $\text{Tor}(A,B)[n] = \text{Tor}(A[n], B[n])$. Let $f_\alpha(A)$ be the $\alpha$th Ulm invariant of the $p$-group A and let $r_\alpha(A)$ be the dimension of $(p^\alpha A)[p]$ over the field with $p$ elements. Then the $\alpha$th Ulm invariant of $\text{Tor}(A,B)$ is $f_\alpha(A)r_\alpha(B) + r_\alpha(A)f_\alpha(B) + f_\alpha(A)f_\alpha(B)$. If $A'$ and $B'$ are basic subgroups of $A$ and $B$ respectively, then $\text{Tor}(A',B) + \text{Tor}(A,B')$ is a basic subgroup of $\text{Tor}(A,B)$. (Received November 21, 1962.)

A combinatorial lemma for sequences of partial sums.

Let $c = (c_1, ..., c_n)$ be a sequence of $n$ real numbers, and denote by $\mathcal{F}_c$ the set of all distinct sequences obtainable from $c$ by permuting the $c_i$. The sequence $c$ is said to have property $E$ if the $n$th partial sum $s_n(c)$ is zero, and if for every $x \in \mathcal{F}_c$ it follows from $s_1(x) < 0, s_i+1(x) \geq 0$ that $s_i+1(x) = 0$. Define $u(x)$ to be the number of partial sums $s_i(x)$ for which $s_i(x) > 0$ and $x_i > 0$. Let $v(k)$ denote the number of sequences $x \in \mathcal{F}_c$ for which $u(x) = k$. If $s_j(x)$ is the first maximum among $s_0(x), ..., s_n(x)$, denote by $w(x)$ the number of positive $x_i$ for $i \neq j$. Let $z(k)$ be the number of $x \in \mathcal{F}_c$ for which $w(x) = k$. Theorem. If $c$ possesses property $E$ and $m$ of the $c_i$ are positive, then $v(i) = z(j)$ for $0 \leq i, j \leq m$. This theorem can be applied to problems in comparing two empirical distribution functions. In particular, generalizations are obtained of Theorems 1 and 2 of Gnedenko and Mikhailovich, Dokl. Akad. Nauk. SSSR 85 (1952), 25-27. (Received November 21, 1962.)

Local triviality for Hurewicz fiberings of manifolds.

The following seems plausible: Let $p: E \to B$ be a fibering in the sense of Hurewicz (covering homotopy property for all spaces) of a connected $n$-manifold $E$ onto a locally contractible base $B$, then the fibering is locally trivial, i.e., a bundle. It is proved that this conjecture is true if some fiber has dimension $\leq 2$ and there is a fiber with a compact component. In general, it is established that each fiber is a generalized $k$-manifold and the base $B$ is a generalized $(n - k)$-manifold, i.e., Poincaré
duality holds locally and globally for base and fiber). Hence if \( k \neq 2 \), each fiber is locally Euclidean. Also, since all the fibers are of the same homotopy type they are homeomorphic if some component is compact. In addition, the mapping \( p \) has sufficiently nice local homotopy properties. These facts enable one to apply a result of Hamstrom and Dyer which implies the theorem. There are several applications such as the nonexistence of compact Hurewicz fiberings of contractible manifolds.

(Received November 21, 1962.)

597-147. V. C. WILLIAMS, Reed College, Portland 2, Oregon. On conformal maps of regions of infinite connectivity.

Let \( D \) be a region of the extended \( z \)-plane for which the Green's function exists. Suppose that the boundary points of \( D \) are separated into two disjoint, nonempty, closed point sets \( B \) and \( C \).

**Theorem.** There exists a conformal map of \( D \) onto a region \( \Delta \) of the extended \( Z \)-plane, where \( \Delta \) is defined by \( 1 < T(Z) < e^{1/t} \); \( T(Z) = A \exp\left(\int \log|Z - a|\,d\mu(a) - d\psi(a)\right) \cdot \prod_{i,j} |Z - a_i|^{M_i} / \prod_{i,j} |Z - b_j|^{N_j} \), with \( \tau, A, M_i, N_j > 0 \) and \( \Sigma_i M_i, \Sigma_j N_j \neq 1 \). The measures \( \mu \) and \( \psi \) are regular Borel measures such that: (1) \( \mu(\Delta) = \psi(\Delta) = 0 \), (2) \( \mu(Z\text{-plane}) = 1 - \Sigma_i M_i \); \( \psi(Z\text{-plane}) = 1 - \Sigma_j N_j \), (3) the \( \mu \)- and \( \psi \)-measure of each component of \( \Delta' \) is zero, (4) the \( \mu \)- and \( \psi \)-measure of the exterior of \( \Delta \) is zero.

The function \( T(Z) \) is continuous in \( \Delta \), with the values \( 1, e^{1/t} \) on the boundary of \( \Delta \). The locus \( T(Z) = 1 \) corresponds to the point set \( B \) and separates the points \( a_i \) from \( \Delta \). The locus \( T(Z) = e^{1/t} \) corresponds to \( C \) and separates the \( b_j \) from \( \Delta \). \( T(Z) \) is also continuous on each component of \( \Delta' \) (except at the \( b_j \)). The proof involves approximating \( D \) by finitely connected subregions (for which the theorem has already been proved by J. L. Walsh, H. Grunsky, J. A. Jenkins, H. J. Landau) and examining limits. There is also a similar theorem generalizing the Riemann mapping theorem. In this case, the image region \( \Delta \) is defined by \( T(Z) < 1 \). (Received November 21, 1962.)

597-148. JACK EDMONDS, Fine Hall, Princeton University, Princeton, New Jersey

On the surface duality of linear graphs.

With any embedding of a graph \( G \) in a closed surface \( S \) such that the components of \( S - G \) are disks, there is associated a unique dual graph. **Theorem.** A 1-1 correspondence between the edges of two graphs is a duality with respect to some closed-surface embedding if and only if for each vertex \( v \) of each graph, the edges which meet \( v \) correspond to a subgraph which is connected and which has an even number of edges to each of its vertices. (If an edge meets \( v \) at both ends, its image is counted twice in the corresponding subgraph.) Using the Euler formula, the characteristic of the surface is determined by the two graphs. Thus the theorem generalizes a version of the H. Whitney condition for a graph to be planar. (Received November 21, 1962.)

597-149. FRIEDRICH KASCH, Pennsylvania State University, University Park, Pennsylvania.

The recent development of Frobenius-extensions.

In 1954 the author introduced the notion of a Frobenius-extension as a generalization of a Frobenius-algebra. This has been studied in recent years particularly from the point of view of homological algebra in papers by T. Nakayama, T. Tsuzuku, B. Pareigis, B. Müller and the author.
Some of these papers are not yet published. A survey about today's situation in this subject will be
given and the question will be discussed whether also the notion of a quasi-Frobenius-algebra can be
generalised in a satisfying way for ring extensions. (Received November 21, 1962.)

597-150. YASUTAKA SIBUYA, Institute of Technology, University of Minnesota, Minneapolis 14,
Minnesota. Simplification of a linear ordinary differential equation of the nth order at a turning point.

Consider an equation of the form: \( \mathcal{E}^{n-m} L_n(y) + L_m(y) = 0 \), where \( L_n \) and \( L_m \) are respectively
linear ordinary differential operators of the nth and the mth order. Assume \( n - 2 \geq m \geq 2 \). The
operator \( L_m \) is reduced to a form \( z y^{(m)} + ... \), when the small parameter \( \varepsilon \) is reduced to 0, where \( z \)
is the independent variable and \( ... \), denotes terms of order lower than \( m \). Then \( z = 0 \) is usually a turn­
ning point. The author of the present report constructed in one of his previous works (Funkcialaj
Ekvacioj 4 (1962), 115-139) formal linear transformations which reduce the equation given above to
certain standard forms. In this report, the existence of analytic transformations of this kind, whose
asymptotic forms are given by the formal transformations, will be proved. (Received November 21,
1962.)

597-151. A. K. AZIZ and WITOLD BOGDANOWICZ, Georgetown University, Washington 7, D.C.
On a mixed boundary value for nonlinear hyperbolic equations.

Consider the differential equation (1) \( u_{xy}(x,y) = f(x,y,u,u_x,u_y) \), with the boundary conditions
(2) \( \alpha_0(x)u(x,y) + \alpha_1(x)u_x(x,y) + \alpha_2(x)u_y(x,y) = \sigma(x), \beta_0^1(y)u(x,y) + \beta_1^1(y)u_x(x,y) + \beta_2^1(y)u_y(x,y) = \tau(y) \), on
the curves \( y = \Gamma_1^1(x) \) and \( x = \Gamma_2^1(y) \) respectively, and \( u(0,0) = c \), in the rectangle \( R = [0,x_0] \times [0,y_0] \).
Among other smoothness assumptions concerning the functions involved, the function \( f \) satisfies
(3) \( |f(x,y,u,u_x,u_y) - f(x,y,u,u_x,u_y)| < \omega(|u - u| + |u_x - u_x| + |u_y - u_y|) \), where \( \omega(t) \) is nondecreasing and
continuous with \( \omega(0) = 0 \) and \( \omega(\varepsilon) > 0 \) for \( \varepsilon > 0 \). Under the above hypotheses there exists a unique
solution of the given boundary value problem in \( R \). The proof is based on the reduction of the given
problem to an equivalent problem for a certain compact operator in the space \( C(R) \), which satisfies
the conditions of Schauder's fixed point theorem. (Received November 21, 1962.)

597-152. LEONARD SARASON, Courant Institute of Mathematical Sciences, New York Univer-

Let \( L \) be a first order linear partial differential operator with matrix coefficients in \( C_2 \), acting
in a region \( G \subseteq \mathbb{R}^n \) with boundary \( S \). Weak and strong solutions of boundary value problems (*)
\( Lu = f \) in \( G \), \( u = g \) on \( S \), or (***) \( Lu = f \) in \( G \), \( Pu = g \) on \( S \), are taken as elements of the space
\( H = L^2_2(G) \times L^2_2(S) \), with test functions in \( \mathcal{C}_0^\infty(\mathbb{R}^n) \). If \( L \) is symmetric, \( L^2_2(S) \) is modified by a weight
function \( m = + (B^2)^{1/2} \) where \( B \) is the coefficient of \( \partial \partial n \). \( P \) is taken as a projector such that the
forms \( \int_S u \cdot B(1 - 2P)u \, dx \) and \( \int_S u \cdot m \, u \, dx \) are equivalent. \( L \) is called positive if \( L + L^* \) is locally
symmetric positive. **Theorem.** Let \( L \) be positive. Then \( \{f,g\} \subseteq H \rightarrow (***) \) has a weak solution.

**Theorem.** Suppose \( n = 2 \). Let \( S \) be piecewise smooth, with tangent bounded away from nonsimple
characteristic directions, and such that at corners \( p \) of \( G \) where \( L \) changes type, all characteristic
directions cross \( S \). Then weak solutions of (*) or (***) are strong. **Theorem.** Let \( L + t \) be positive

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for some $t > 0$. Suppose the hyperplanes $x = \text{constant}$ are uniformly non-characteristic. Then there exists $c > 0$ such that if $S$ is given by $x = h(x_1, \ldots, x_n)$, where $h \in \text{Lip } 1$, with Lipschitz constant $< c$, weak solutions of (**) are strong. (Received November 21, 1962.)

597-153. D. G. ARONSON, Institute of Technology, University of Minnesota, Minneapolis 14, Minnesota. On correct partial differential operators and stable finite difference operators.

Let $P(\xi)$ be an $N \times N$ matrix whose elements are polynomials of degree $\leq m$ in $\xi = (\xi_1, \ldots, \xi_n)$ and let $L = P(D) - E(\partial/\partial t)$, where $D = (\partial/\partial x_1, \ldots, \partial/\partial x_n)$ and $E$ is the $N \times N$ identity matrix. $L$ is said to be correct in the sense of Petrovskii if there exists a constant $K \geq 0$ such that $\max_j \text{Re} \lambda_j(\xi) \leq K$ where the $\lambda_j$ are the eigenvalues of $P(\xi)$. Theorem. $L$ is correct in the sense of Petrovskii if and only if there exists at least one family of two-level finite difference operators which is consistent with $L$ and which satisfies the von Neumann condition for stability. In addition, the finite difference analogues of certain estimates for the solution of the initial value problem with correct $L$ are derived. (Received November 21, 1962.)

597-154. BARTH POLLAK, Syracuse University, Syracuse 10, New York. Transitivity in the commutator subgroup of the orthogonal group.

If $V$ is a nonsingular quadratic space of finite dimension over a field $K$ of characteristic $\neq 2$, $A \in V$ and $L(A)$ denotes the set of all vectors in $V$ having the same length as $A$ then, by Witt's theorem, the orthogonal group $O(V)$ acts transitively on $L(A)$. If $H$ is a subgroup of $O(V)$ then $H$ partitions $L(A)$ in an obvious manner into a certain number $n(H)$ of equivalence classes. We derive an expression for $n(H)$ when $H = O'(V)$, the spinorial kernel. Under a special assumption we show that $n(\Omega(V)) = n(O'(V)) \cdot (O'(V); \Omega(V))$ where $\Omega(V)$ is the commutator subgroup of $O(V)$. If $K$ is a field for which a representation theory for quadratic forms is known (i.e., finite fields, real numbers, local fields, global fields) we obtain explicit formulas for $n(O'(V))$ and $n(\Omega(V))$. As a corollary we obtain necessary and sufficient conditions for $O'(V)$ and $\Omega(V)$ to operate transitively on $L(A)$. Sample result: if $K$ is a global field, then $\Omega(V)$ operates transitively on $L(A)$ if and only if the Witt index of $V_\rho \neq 1$ for all real infinite $\rho$ and either $\dim V \geq 5$ or $\dim V = 4$, the Witt index of $V \neq 1$ and $V_\rho$ is isotropic at all finite nondyadic $\rho$. (Received November 21, 1962.)

597-155. R. E. FULLERTON, University of Maryland, College Park, Maryland. On the coincidence of natural order and the order defined by a basis on a linear space.

Let $X$ be a locally convex linear topological space. A cone $K \subset X$ is called an absolute basis cone if (1) $K$ spans $X$ and is the closure of the convex set determined by its set $\{r_a\}$ of extreme rays (2) All closed linear subspaces $L_\theta \subset X$ generated by the $\{r_a\}$, $\theta \neq \emptyset$ are hyperplanes each not containing $r_\emptyset$. (3) For all $x \in X$, the set $K \cap (x - K)$ is compact. In this case, it is known that a set $\{x_a\}$, $x_a \in r_a$ for each $a$, $x_a \neq 0$, are the elements of a generalized unconditional Schauder basis for $X$. It is shown that if $X$ is a separable abstract $L$ space in which the positive cone is an absolute basis cone then $X = \ell^1$. If $X$ is a separable abstract $M$ space with this property, then $X = c_0$. (Received November 21, 1962.)
Let G be a σ-compact locally compact group. For each \( f \in C(G) \) (continuous complex valued functions on G) set \( \tilde{f}(x) = f(x^{-1}) \) and for each linear form \( \varphi \) on \( C(G) \) set \( \varphi \_f(x) = \int (f(yx))d\varphi(y) \). A subspace \( \mathcal{C} \) of \( C(G) \) is a space of smooth functions on G if: (1) U a neighborhood of \( e \in G \rightarrow \exists 0 \neq f \in \mathcal{C} \) such that \( \text{supp}(f) \subseteq U \) and \( f(e) = 1 \), (2) \( f, g \in \mathcal{C} \Rightarrow fg \in \mathcal{C} \), (3) \( \mathcal{C} \) a nuclear Fréchet topology stronger than compact convergence such that (a) \( f, g \rightarrow fg \) is continuous \( e \xrightarrow{} e \), (b) \( \varphi(f) \rightarrow \varphi(f) \) is hypocontinuous \( \mathcal{C}_b \times \mathcal{C} \rightarrow \mathcal{C} \), (c) V a neighborhood of \( 0 \in \mathcal{C} \Rightarrow \exists K \) compact in G such that if \( f \in \mathcal{C} \) and \( f|_K = 0 \) then \( f \in V \). Let \( N \) be the closure in \( \mathcal{C} \) of all \( f \in \mathcal{C} \) which vanish on a neighborhood of \( e \in G \). Theorem. If G is a separable Lie group, the only space \( \mathcal{C} \) of smooth functions on G such that \( N^0 \subset \mathcal{C}_b \) has the strongest locally convex topology is the space of infinitely differentiable functions on G. (Received November 21, 1962.)

This sharpens and generalizes the results on ideals of Wronskians in Abstract 62T-187 and extends them to other determinantal ideals. Let \( y_{ij} (i = 1, \ldots, n; j = 0, 1, \ldots) \) be independent indeterminates over a field \( F \) of characteristic zero. Let \( b_1, \ldots, b_n \) be distinct non-negative integers and let \( B = \begin{bmatrix} b_1 & \cdots & b_n \end{bmatrix} \) be the determinant whose hth row is \( y_{1j}, \ldots, y_{nj} \) with \( j = b_h \). The weight of \( B \) is \( w = b_1 + \cdots + b_n \). Let \( a_i \) be a linear combination, with nonzero coefficients from \( F \), of all the \( B \)'s of weight \( a + 1 \). A constructive test for a product \( P = B_1 \cdots B_d \) of determinants \( B_i \) to be in the ideal \( I = (A_0, A_1, \ldots) \) is obtained. This leads to a function \( f(d) \) such that \( P \notin I \) whenever the total weight of \( P \) is less than \( f(d) \) and for which there is at least one \( P \) of weight \( f(d) \) not in \( I \). For \( d \geq a \) let \( \beta \) be defined by \( (\beta - 1)\beta < 2d \leq \beta(\beta + 1) \); then \( f(d) = da + C_{d+a+1,2} - C_{d+2,3} \). For \( d > a + n - 2 \) let \( q \) and \( r \) be defined by \( d - a + n + 2 = q(a + 1) + r \), \( 0 \leq r < n - 1 \); then \( f(d) = (2d - a + n - 1)a + C_{n,2} \). Since \( (d/d) \) is monotonic and unbounded, a function \( g(w) \) may be defined as the least \( d \) with \( f(d)/d \geq w \). Then \( B^P \in I \) for \( p \geq g(w) \), where \( w \) is the weight of \( B \). It is shown that \( g(w) \) is best for \( n = 2 \). For \( n > 2 \), \( g(w) \) is best in some cases and can be improved in others. (Received November 21, 1962.)

Suppose that \( G \) is an upper semicontinuous decomposition of \( \mathbb{E}^3 \) with at most countably many nondegenerate elements. Let \( H \) be the set of nondegenerate elements of \( G \), and let \( P \) be the projection mapping from \( \mathbb{E}^3 \) onto \( G \). Theorem 1. Suppose that \( H \) is countable and the decomposition space \( G \) is homeomorphic to \( \mathbb{E}^3 \). If \( S \) is a tame 2-sphere in \( G \) such that \( S \) contains no point of \( H \), then \( P^{-1}[S] \) is a tame 2-sphere in \( \mathbb{E}^3 \). Theorem 2. Suppose that \( H \) is countable and the decomposition space \( G \) is a 3-manifold. Then \( G \) is homeomorphic to \( \mathbb{E}^3 \). (Received November 21, 1962.)
Let $S$ be a Moore space. The following two conditions on $S$ are shown to be equivalent,

(A) There is a development $G_1, G_2, \ldots$ for $S$ such that, if $H$ and $K$ are mutually exclusive closed compact sets, then there is an $n$ such that no member of $G_n$ intersects both $H$ and $K$. (B) There is a development $G_1', G_2', \ldots$ such that if $p \in S$, $q \in S$ and $p \neq q$, then there is an $n$ such that if $H$, $K$ and $J$ are in $G_n$, $p \in H$, $H \cap K \neq \emptyset$ and $K \cap J \neq \emptyset$, then $q \notin J$. It is shown that, if $S$ satisfies (A) or (B), then the closure of a (conditionally) compact set in $S$ is compact, and, if $S$ is also locally connected and locally peripherally compact, then $S$ is metrizable. It is also shown that in a countably paracompact $T_1$-space the closure of a (conditionally) compact set is (conditionally) compact and that there exists a normal countably paracompact space which is not perfectly normal. (Received November 21, 1962.)

In a recent paper (Uspehi Mat. Nauk 3 (93) (1960), 181-183) G. Shilov announced a proof of the following: (a) If a continuous function $f$ on the maximal ideal space of a Banach algebra $A$ can be locally expanded in power series in a finite number of functions in $\hat{A}$ (the Gel'fand representation) then $f$ is in $\hat{A}$. His proof depended on the following assertion: (b) If $S$ is the joint spectrum in $C^\Omega$ of a finite number of functions in $\hat{A}$ and $f$ is a continuous function on $S$ which is locally the restriction to $S$ of analytic functions, then $f$ is globally the restriction to $S$ of an analytic function defined in some neighborhood of $S$. An example is given which shows (b) is incorrect. However the example does not disprove (a). (Received November 21, 1962.)

The closure of the numerical range $W(T) = \{(Tx, x): \|x\| = 1\}$ of a bounded operator $T$ on a Hilbert space $H$ is shown to be a spectral set (in the sense of von Neumann) for $T$ if and only if there exists a normal operator $N$ on $K \supset H$ with $PNx = T^n x$, $x \in H$, $n = 0, 1, 2, \ldots$, $P$ the projection onto $H$, such that $W(T) = \overline{W(N)}$, and a sufficient condition for this is that the convex hull of the spectrum of $T$ be spectral for $T$. This follows from a theorem of Foias (Studii si Cercelari Math. 1959) independently discovered by C. A. Berger and A. Lebow (unpublished). (Received November 21, 1962.)

In this note is investigated a commutative ring $A$ which contains a unit element and in which $ab = ac \neq 0$ implies $b = uc$ where $u$ is a unit. In an obvious manner an essentially Gaussian ring is defined and then studied. It is noted that $I/(m)$ is essentially cancellative if and only if $m = p^k$ where $p$ is a prime. (Received November 21, 1962.)
597-163. D. S. CARTER, Oregon State University, Corvallis, Oregon. Asymptotic behavior of infinite fluid jets under gravity.

Consider a steady plane irrational flow of an incompressible inviscid fluid under gravity, in an infinite jet with a sink at infinity, bounded on one side by a "fixed boundary" which is analytic at infinity and on the other side by a "free boundary" on which the flow velocity approaches infinity. Under these conditions it is shown: (1) That the velocity approaches infinity throughout the jet, which becomes asymptotic to the fixed boundary and eventually makes an acute angle with the direction of gravity; (2) That the inverse complex velocity potential \( z = f(w) \) and conjugate velocity \( u = 1/f'(w) \) have infinite asymptotic expansions of the form \( z \sim w^{2/3} P(w) \), \( u \sim w^{1/3} Q(w) \) where \( P \) and \( Q \) are formal series in powers of \( w^{-1/3} \) and \( w^{-1/3} \log w \) (and of simpler form if the fixed boundary is straight or asymptotically vertical); (3) That these expansions are uniquely determined, except for inessential reflections, translations, and rotations, by the fixed boundary shape, the acceleration of gravity and its direction relative to the jet, the source-sink strength, and the free boundary constant \( |u|^2 - 2gh \). The results support a conjecture of Garabedian, Proc. Roy. Soc. 241 (1957), 423-431 for the case of a straight vertical boundary. (Received November 21, 1962.)

597-164. T. A. BROWN, The Rand Corporation, 1700 Main Street, Santa Monica, California. The existence of periodic solutions to nonlinear differential-difference equations \( \dot{x}(t) = g_1(x(t))g_2(x(t - 1)) \).

Using the Birkhoff-Kellogg fixed point theorem and a method first used by G. S. Jones, one is able to prove the existence of periodic solutions to nonlinear differential-difference equations \( \dot{x}(t) = g_1(x(t))g_2(x(t - 1)) \) under fairly weak conditions. The following theorem is typical: Theorem. Given an equation of the form in question, and numbers \( m > 0 \), \( a > 0 \), \( k > 0 \), and \( c_1 < c < c_2 \) such that (1) \( g_1(c_1) = g_1(c_2) = 0 \), \( g_1(x) > 0 \) for \( c_1 < x < c_2 \); (2) \( g_2(c) = 0 \), \( \|g_2(x)\| \leq k \min(a,|x-c|) \) for \( c_1 < x < c_2 \); (3) \( g_2(x) \) is monotone decreasing for \( |x-c| < a/4 \); (4) \( 4m > g_1(x) > m > 4/k \) for \( |x-c| \leq a \); (5) \( g_1 \) satisfies a Lipschitz condition and \( g_2 \) is continuous for \( c_1 \leq x \leq c_2 \); then there exists a periodic solution \( x(t) \) with period greater than 2, such that \( x(t) = c \) twice in a period and \( \max(x(t) - c) \leq \alpha \), \( \min(x(t) - c) \geq -\alpha \). (Received November 21, 1962.)


A new integral representation has been found as an elementary solution of the wave equation in the situation where this solution is homogeneous in space and time variables and of degree - 1. Thus in the two-dimensional case, the solution of Hadamard, in the form \( f = (c^2t^2 - r^2)^{-1/2} \) for \( ct > r = (x^2 + y^2)^{1/2} > 0 \) satisfies the equation \((\nabla^2 - (1/c^2)(\partial^2/\partial t^2))f = 0 \) and the equivalent form is \( 2\pi t = \int_{-\infty}^{\infty} dp/(1 - p^2)^{1/2} \{ct - px - (1 - p^2)^{1/2} y \text{ sgn } y \} \). This is an integral of singular plane waves with respect to one wave parameter. For the three-dimensional situation, the corresponding solution is an integral of singular plane waves with respect to two parameters. Given such a representation of a point source field as the super-position of a distribution of real and complex plane waves, there are a number of problems involving homogeneous boundary conditions at plane boundaries where the scattered field associated with a primary point source field can be easily determined.
The point here is that although formal integral transform solutions may be already well known the problem of inversion seems only to have been completely solved when symmetry properties are present; no such restriction is implied in the new method. (Received November 21, 1962.)


An abelian group \( G \) is an ID-group if \( G \) has an isomorphic proper direct summand, and \( G \) is an IP-group if \( G \) has an isomorphic proper pure subgroup. \( G \) is an ID-group if and only if there exist \( \phi, \psi \) in the endomorphism ring of \( G \) such that \( \phi \psi = 1 \) and \( \phi \neq 1 \). The problem of characterizing IP-groups and ID-groups has the familiar reduction to the divisible and reduced cases, and for torsion groups, to the primary case. A group of finite rank is not an IP-group. A divisible group \( G \) is an IP-group (ID-group) if and only if \( G \) has infinite rank. A reduced \( p \)-group is an ID-group if and only if the \( n \)th Ulm invariant of \( G \) is infinite for some non-negative integer \( n \). An ID-system is a triple \( \langle G; \phi, \psi \rangle \) where \( G \) is an abelian group and \( \phi \) and \( \psi \) are endomorphisms of \( G \) such that \( \phi \psi = 1 \). The study of ID-systems is equivalent to the study of modules over the ring \( \Delta \) which is freely generated over \( Z \) by \( x \) and \( y \), subject to the relation \( xy = 1 \). Every \( \Delta \)-module is an extension of a \( \Delta \)-module \( K \) in which all elements have infinite \( y \)-height by a \( \Delta \)-module \( T \) in which there are no nonzero elements of infinite \( y \)-height. (Received November 21, 1962.)


Let \( S \) be a Banach space, and for each point pair \( P, Q \) in \( S \), let \( [P, Q] \) (called the complete interval, in contrast to the ordinary interval \([P, Q]\)) be \( \{ R : \| P - R \| + \| R - Q \| = \| P - Q \| \} \). Let completely convex mean convex with respect to complete intervals. \( S \) has property (A) means that if \( c > 0 \) then there exists a number \( d > 0 \) such that if \( \| P \| = \| Q \| = 1 \) and \( \| P - Q \| < c \), then there exists a point \( R \) in \( [P, Q] \) such that \( \| R \| < 1 - d \). Theorem (1). The following two statements are equivalent: (a) there exist points \( P \) and \( Q \) such that \( [P, Q] \subset [P, Q] \); (b) there exist three points \( P', Q', \) and \( R' \) such that \( \| P' \| = \| Q' \| = \| R' \| \) and \( R' \) is in \( [P', Q'] \). Theorem (2). If \( S \) has property (A), \( M \) is closed and completely convex, and \( P \) is a point not in \( M \), then there exists a point \( Q \) of \( M \) such that \( \| Q \| < \| P \| \). Theorem (3). There exists a Banach space \( S_1 \) such that (a) \( S_1 \) has property (A), and (b) there exists a closed convex set \( M \) such that if \( P \) is in \( M \), there exists a \( Q \) in \( M \) such that \( \| Q \| < \| P \| \). Theorem (4). If \( S \) has property (A), \( M \) is closed, \( M' \) is the smallest closed completely convex set containing \( M \), and \( M \) and \( M' \) are the same distance from the origin, then \( M \) contains a point \( Q \) nearest the origin. (Received November 21, 1962.)


A family \( t \rightarrow K(t), 0 \leq t \leq 1, \) of convex bodies in Euclidean \( n \)-space \( R_n \) sharing the origin as an interior point is bounded and measurable if, for each \( x \) in \( R_n \), the distance function \( F(x, t) \) is measurable and \( 0 < b \leq F(x, t) \leq B \) for \( (x, x) = 1 \). The support function \( H(x, t) \) has these properties if
K(t) is a bounded, measurable family. Means $M_p(K(t))$, $\hat{M}_p(K(t))$ are defined for $0 \neq |p| < \infty$ by:

$$M_p(K(t)) = \bigcap_{(u,u) = h(u)} h(u)$$

where $h(u)$ is the half-space: $(x,u) \notin M_p(H(u,t))$; $\hat{M}_p(K(t))$ is the convex closure of the set of $x$ such that $M_p^t(F(x,t)) \notin 1$. Here $M_p^t(f(t))$ is the power mean of index $p$ of the function $f$.

$M_p$ and $\hat{M}_p$ satisfy analogues of conditions characterizing power means $M_p^t(f(t))$ and one further:

if $0 < b \neq \lambda(t) \neq B$ is measurable over $0 \leq t \leq 1$, the family $K(t) = \lambda(t)K_0$, $K_0$ fixed, has means homothetic to $K_0$. Other means satisfying these conditions can be constructed and indexed as power means $N_p(K(t))$ by examining means of families of spheres $(x,x) \notin \lambda(t)$. Theorem. If $N_p$ satisfies the conditions for a power mean of index $p$, then for any bounded, measurable family $K(t)$:

$$M_p(K(t)) \subseteq N_p(K(t)) \subseteq M_p(K(t)).$$

(Received November 21, 1962.)


Bramble and Hubbard [Num. Math., in press] have proposed a finite difference analogue of the Dirichlet problem for Poisson's equation with an $O(h^4)$ estimate for the truncation error. However, the approximation at irregular boundary points is not of positive type. This paper treats the numerical solution of the resulting system of linear equations: $Ax = b$. Here $A = I - L - U$ where $L$ and $U$ are strictly lower and upper triangular matrices respectively. Define $S(H)$ to be the spectral radius of $H$. Let $x_0 = [I - \omega U]^{-1}[\omega U - (\omega - 1)]$ be the matrix defining the method of successive overrelaxation as developed by Young [Trans. Amer. Math. Soc. 76 (1954), 92-111]. $L + U$ is neither symmetric nor non-negative, however $S(L + U) < 1$. It is shown that an ordering can be found such that $x_0$ and $L$ are non-negative. For this ordering it is shown that $S(x_0) < S(L + U)$. Further, there exists an $\omega > 1$ such that for $1 < \omega < \omega_0$, $S(x_0) < S(x_1) - [\omega - 1][1 - S(x_1)]$. These results generalize parts of the results of Stein and Rosenberg [J. Lond. Math. Soc. 23 (1948), 111-118] and Kahan [Ph. D. Thesis, Univ. of Toronto, 1958]. (Received November 23, 1962.)


An $O(h^4)$ finite difference analogue is given for the problem $\Delta u = f$ in a region $R$ in two dimensions and $u = g$ on the boundary $C$. At regular interior points no equation is of "non-negative" type, nor does it possess the property of "diagonal dominance", c f. [Forsythe and Wasow; John Wiley and Sons, 1960, 193, Eq. 20,55]. It is shown however that this formulation does satisfy a maximum principle, i.e., the elements of the inverse matrix are non-negative. As a result the discretization error is shown to be $O(h^4)$. Extensions to more general problems are considered. (Received November 23, 1962.)


Let $Ax = b$ be the linear system defined in the above paper Finite difference analogues for elliptic boundary value problems and the maximum principle. Let $A = I - B = I - L - U$ where $B$ is the point Jacobi matrix and $L$ and $U$ are respectively lower and upper triangular matrices. The
matrix A is not in general symmetric. It is shown that for sufficiently small mesh width h the point Jacobi iterative method diverges while a "forward-backward" Gauss-Seidel method converges, for any starting vector $x^0$ provided the equations are ordered in a certain manner. This method is given by $X^{n+1} = LX^{n+1} + UX^{n+1/2} + b$ and $X^{n+1/2} = UX^{n+1/2} + LX^n + b$. This method was proposed by Aitken [Proc. Roy. Soc. Edinburgh, Sect. A, 63 (1950), 52-60] and considered for the case of positive definite matrices. (Received November 23, 1962.)


Let $D_M$ be a vector space over a division ring $D$, $E$ be the ring of all endomorphisms of $D_M$, $L$ be the lattice of all subspaces of $D_M$, $J$ be a sublattice of $L$, and $R(J) = \{a \in E | Na \subset N \text{ for every } N \in J\}$. Call $J$ an $E$-sublattice of $L$ and $R = R(J)$ a distinguished ring of linear transformations of $D_M$ iff $J = LR$, the lattice of $R$-submodules of $M$. This paper is concerned with characterizing each distinguished ring $R(J)$ for which $J$ is finite and distributive. If $\text{char } D = 0$, then the distributivity of $J$ follows from its finiteness. The case in which $J$ is a chain was recently discussed by K. G. Wolfson (Math. Zeit. 75 (1961), 328-332). While $R(J)$ is not in general a Baer ring (as it is if $J$ is a chain), it does have analogous properties. (Received November 23, 1962.)


Let $S$ be a locally compact commutative semigroup and let $M$ be the algebra of all complex regular Borel measures on $S$. For $\mu \in M$ and $x \in S$ define $\mu_x(E) = \mu(\{y \in S : xy \in E\})$ for every Borel set $E \subset S$. Call $\mu$ absolutely continuous and write $\mu \in M_a$ if $\|\mu_x - \mu_a\| \to 0$ as $x \to a$ for each $a \in S$. It is well known that if $S$ is a group, then $\mu \in M_a$ iff $\mu$ is absolutely continuous relative to Haar measure. It is obvious that $M_a = M$ in case $S$ is discrete. The set $M_a$ is a closed ideal in $M$. By $S_0$, the core of $S$, we mean the closure of the union of the supports of the members of $M_a$. A semicharacter of $S_0$ is a homomorphism $X : S_0 \to \mathbb{C}$ of $S_0$ into the closed unit disk. Theorem. The multiplicative linear functionals of the algebra $M_a$ are just the functions, $\mathcal{J}(\mu) = \int_{S_0} Xd\mu$ for $X$ a continuous semicharacter of $S_0$. The proof of this theorem uses the generalized characters of Sreider. Theorem. The algebra $M_a$ is semisimple iff there are enough continuous semicharacters of $S_0$ to separate points. (Received November 23, 1962.)

597-174. KEITH MILLER, Rice University, Houston, Texas. Three circle theorems for harmonic continuation.

Three circle theorems (analogous to Hadamard's three circle theorem for analytic functions on an annulus) are found for harmonic functions on a disc. Let $\| \cdot \|_r$ and $\| \cdot \|_\infty$ denote the uniform and the $L_2$ norms on the circle of radius $r$. For $a < 1$ and $m < 1$, let $\Gamma$ denote the class of functions $u$ harmonic on the unit disc and satisfying $\|u\|_\infty \leq m$, $\|u\|_1 \leq 1$. Then for $a < r < 1$, $(1/2)Cm^\beta \leq \sup_{r} |u|_r \leq Cm^\beta$, where $\beta = \log r/\log a$, and $C$ is a somewhat complicated expression only slightly less than $\sum_{n=0}^{a} (a/r)^{2n}|1/2 + [1 - r^2]^{-1/2}$. Here $a = \log m/\log a$. This and similar results lead to numerical
methods for harmonic continuation. If \( u \) is harmonic and \( \| u \|_1 \leq 1 \), and data \( g(\theta) \) is given on the circle of radius \( a \) such that \( \| u - g \|_a \leq m \), then \( \| u - g \|_a < Cm^\rho \), where \( g_a(r,\theta) \) denotes the \( \rho \)th order truncated Fourier expansion of \( g \). If data is given only at a finite number of interior points, the method involves the solution of a set of linear inequalities. Completely analogous results may be found for analytic continuation, backward solution of the heat equation, and the Cauchy problem for Laplace's equation. (Received November 23, 1962.)

597-175. J. T. WHITE, University of Kansas, Lawrence, Kansas. A representation theorem for the Laplace transform.

This paper is concerned with the development of a representation theorem for the Laplace transform. In particular, it is shown that a necessary and sufficient condition that a function \( f(s) \) be the Laplace transform of a function \( \phi(t) \), which is nondecreasing and attains its maximum value at \( t_0 \), where \( 0 < t_0 \leq b \), is that \((-1)^m(a^n/ds^n)(exp(bs)\phi^{(m)}(s)) \geq 0 \), for \( s > 0 \), and that \( f(s) \) be continuous in \( 0 \leq s \leq \infty \). Furthermore, if \( c = \inf f[b(-1)^m(d^n/ds^n)(exp(bs)\phi^{(m)}(s)) \geq 0 \), for \( s > 0 \)\), then \( c \) is the smallest number such that \( \phi(t) \) is constant for \( t > c \). Throughout this paper the Laplace transform is defined as \( f(s) = \int_0^\infty \exp(-st)\phi(t)dt \). (Received November 23, 1962.)


The canonical form for a linear transformation \( T \) over a vector space is derived in two ways. The first derivation assumes the vector space to have finite dimension over an algebraically closed field. The second derivation is free of both these assumptions, and uses only the polynomial equation satisfied by \( T \). The proof is probably the shortest and most direct one known. Its ideas appear in an early paper of the author [Bull. Amer. Math. Soc. 47 (1941), 116-117]. (Received November 23, 1962.)

597-177. RICHARD BLOCK, California Institute of Technology, Pasadena, California. The Lie algebras with a quotient trace form.

This paper determines all Lie algebras \( L \), over an algebraically closed field \( F \) of characteristic \( p \neq 2, 3 \), of the form \( L = \hat{L}/\hat{L}^+ \) where \( \hat{L}^+ \) is the annihilator of the trace form \( f(a,b) = \text{tr}(U(a)U(b)) \) on a Lie algebra \( \hat{L} \) for some representation \( U \) of \( L \). Such an algebra is called an algebra with a quotient trace form (q.t.f.) -- cf. R. Block (Canad. J. Math., 1962), where the simple q.t.f. algebras are shown to be the algebras of classical type, i.e., the analogues over \( F \) of the complex simple algebras; of H. Zassenhaus (Proc. Glasgow Math. Assoc., 1959), where the structure of q.t.f. algebras is examined; q.t.f. algebras are in particular direct sums of indecomposable q.t.f. algebras. Block and Zassenhaus (The Lie algebras with a nondegenerate trace form, Proc. Int. Cong. Math., Stockholm, 1962) have shown that if \( \hat{L}^+ = 0 \), then \( L = \hat{L} \) is a direct sum of algebras which are either abelian, simple of classical type, or total matrix algebras of degree a multiple of \( p \). Here it is proved that when \( \hat{L}^+ \neq 0 \) the only other possible direct summands in \( L \) are the members of a new class of indecomposable nonsimple q.t.f. algebras which are explicitly constructed by taking \( \hat{L} \) such that \( M^2 \subset \hat{L} \subset M \), with \( M \) a direct sum of total matrix algebras each of degree a multiple of \( p \). (Received November 23, 1962.)
597-179. ROBERT OSSERMAN, Stanford University, Stanford, California. On complete minimal surfaces.

This is a continuation of earlier work (Comment. Math. Helv. 35 (1961), 65-76) on global properties of minimal surfaces. The results include the following. 1. At an isolated boundary component of a complete minimal surface, if the normal directions omit a set of positive capacity then they must tend to a single limit. 2. If a complete minimal surface has finite total curvature, then its normals can omit at most two directions. Concerning the first of these results, it is not necessary to have a complete minimal surface, but merely to assume that all curves tending to the particular boundary component have infinite length. A special case is the theorem of Bers, that for a minimal surface \( z = f(x,y) \) in \( x^2 + y^2 > R^2 \), the gradient of \( f \) tends to a limit at infinity. It is further noted that the following result: 3. "The Gauss curvature cannot be bounded away from zero on a complete minimal surface," has a very simple proof for a large class of surfaces including minimal surfaces. It is a standing conjecture that this result holds for all surfaces of negative curvature embedded in three-space. (Received November 23, 1962.)


Solutions \( \xi(t) \) of a differential equation \( \ddot{x} = f(x,\dot{x},t) \) are sought which satisfy the boundary condition \( \xi(0) = \xi(T), \dot{\xi}(0) = \dot{\xi}(T) \) (called periodic solutions, for shortness). Several existence theorems for periodic solutions will be presented, a typical one is this: Let \( \alpha(t), \beta(t) \) be two functions, twice differentiable, periodic with period \( T \) and satisfying the conditions \(-\ddot{\alpha} + f(\alpha,\dot{\alpha},t) \not\equiv 0, -\ddot{\beta} + f(\beta,\dot{\beta},t) \not\equiv 0, \alpha(t) \not\equiv \beta(t), \) for all \( t \in [0,T] \). If \( |f(x,y,t)| \) does not increase too strongly for \( |y| \to \infty \), then there exists a periodic solution \( \xi(t) \) with \( \alpha(t) \not\equiv \xi(t) \not\equiv \beta(t) \). A general approach of L. Cesari to nonlinear existence problems is used in connection with a method, which roughly can be described as a change of the differential equation without changing its periodic solutions. (Received November 23, 1962.)
597-181, R. P. de FIGUEIREDO, Purdue University, Lafayette, Indiana. A theorem on the existence of \(n\) stable limit cycles for Liénard's equation.

Let \(S\) denote the system (Liénard's equation) \(\dot{y} = v, \dot{v} = -f(y)v - y\), \((*=\frac{d}{dt})\), where \(f \in C^1\).

Define \(F(y) = \int_0^y f(s)ds\). Theorem. The system \(S\) has at least \(n\) stable limit cycles \((n \geq 2)\) if
\[
|F(y)| \geq M_0 y, \quad (y < 0), \text{ and provided either (soft oscillations): (I) } f(0) < 0, \text{ and (II) } (-1)^k F(y) \geq -B_k + M_{k+1} y, \quad (y_k < y < y_{k+1}, k = 0, 1, \ldots, 2n - 2); \text{ or (hard oscillations): (I) } f(0) > 0, \text{ and (II) } (-1)^k F(y) \geq B_k - M_{k+1} y, \quad (y_k < y < y_{k+1}, k = 0, 1, \ldots, 2n),
\]
where: \(M_i, i = 0, 1, \ldots, 2n\), are positive constants satisfying \(M_0 < 2, M_i > M_0, i = 1, 2, \ldots, 2n; y_i, i = 0, 1, \ldots, 2n\), are constants such that \(0 = y_0 < y_1 < \ldots < y_{2n}\) and each \(y_i, i \neq 0\), is sufficiently large; \(B_k = (-1)^k \sum_{i=0}^{k} (-1)^j \{A_j + (M_j + M_{j+1})y_{j+1}\}, k = 0, 1, \ldots, 2n - 1\), where \(A_j, j = 0, 1, \ldots, 2n - 1\), are positive constants. (Received November 16, 1962.)

597-182, PAUL AXT, Michigan State University, East Lansing, Michigan. Iteration of primitive recursion.

In [Grzegorczyk, Rozprawy Matematyczne 4 (1953)], the primitive recursive functions are classified by a hierarchy of length \(\omega \{C^2\}_\omega\), based upon an operation of bounded primitive recursion. In Abstract 62T-124, this hierarchy beginning with \(C^4\) is established by enumeration and diagonalization. Here a hierarchy of the primitive recursive functions is given by yet another means, namely by the number of iterations of the schema of primitive recursion. For \(n = 0, 1, 2, \ldots\), let \(K_n\) be the smallest class of functions containing the constant, identity and successor functions, closed under substitution, and in case \(n > 0\), closed under primitive recursion with given functions in \(K_{n-1}\). It is immediate that \(K_n \neq K_{n+1}\) and that \(\cup K_n\) is the class of primitive recursive functions. For the functions \(f_n\) used in the definition of \(C^n, f_n \in K_n\). And \(K_0 \subseteq C^0, K_1 \subseteq C^1\), and if \(n > 1, K_n \subseteq C^{n+1}\) but \(K_n \not\subseteq C^n\). Using these results, \(K_n \neq K_{n+1}\). For all \(n\) there is an \(m\) such that \(C^m \subseteq K_m\). (Received November 23, 1962.)

597-183, WERNER ISRAEL, University of Alberta, Edmonton, Alberta, Canada. Relativistic kinetic theory of a simple gas.

A consistent relativistic theory of transport processes in a simple gas is developed. The approach is the four-dimensional geometric one due to Synge (The relativistic gas, North-Holland Publishing Company, Amsterdam, 1957). Scalar and vector eigenfunctions of the linearized collision operator are derived when the scattering cross-section is a simple separable function of scattering angle and relative velocity (Maxwell-type gas). Fourier's coefficient of heat conduction is computed explicitly for this case. (Received November 23, 1962.)

597-184, SIMON HELLERSTEIN, Stanford University, Stanford, California and L. A. RUBEL, University of Illinois, Urbana, Illinois. A family of subfields algebraically closed in the field of all meromorphic functions.

It is well known that if \(f = f_1/f_2\) is entire, where \(f_1\) and \(f_2\) are of exponential type, then \(f\) is also of exponential type. This result is extended here to the general situation of meromorphic functions \(f\).
which are solutions of algebraic equations with meromorphic coefficients. Let \( \lambda(r) \) be a positive, continuous, nondecreasing function of \( r \) defined for \( 0 \leq r < R(R \equiv + \infty) \). The class \( \mathcal{S} \) defined by 

\[ f \in \mathcal{S} \text{ if (i) } f(z) \text{ is meromorphic in } |z| < R, (\text{ii) } T(r,f) = o(\lambda(r)) \text{ as } r \to R, \text{ forms a field.} \]  

(T(r,f) denotes the Nevanlinna characteristic of \( f \).) Theorem. \( \mathcal{S} \) is algebraically closed in the field of all functions meromorphic in \( |z| < R \). Corollary. The ring of all entire functions of order \( \phi \) and mean type is algebraically closed in the ring of all entire functions. Indeed, the property of algebraic closure in the ring of entire functions holds for a large class of subrings defined by means of growth restrictions on the maximum modulus in place of the Nevanlinna characteristic. (Received November 23, 1962.)


Let \( A^{(1)} \) be the sequence \( \{a_k^{(1)}\} \) where \( a_k^{(1)} = k + 1 \), so that \( A^{(1)} = \{2, 3, 4, \ldots\} \). A sequence \( A^{(n+1)} = \{a_k^{(n+1)}\} \) is obtained from \( A^{(n)} \) by choosing a positive integer \( r_n \) and then deleting from \( A^{(n)} \) the \( r_n \)th member following \( a_k^{(n)} = a_n \) and every \( a_n \)th one thereafter. The sequence \( \{a_n\} \) is investigated for various choices of \( r_n \). Previous results are extended to the cases where \( r_n \) is nondecreasing and \( o(n/\log n) \), \( cn/\log n \), \( cn \), and \( ca_n \). In each of these cases it is shown that \( a_n = n \log n + (1/2)n(\log \log n)^2 + c(r)[1 + o(1)]n \log \log n \), where \( c(r) \) is a parameter depending on the manner of choosing the \( r_n \). (Received November 23, 1962.)

597-186. SEYMOUR GINSBURG, System Development Corporation, Santa Monica, California and JOSEPH ULLIAN, University of Chicago, Chicago, Illinois. Some remarks about sequences in context free languages.

The following conjecture is considered: (*) It is unsolvable whether a language (= context free language) contains a sequence. While this conjecture is left unresolved, a number of results pertaining to it are obtained. For example, the unsolvability of whether a language contains the set \( \{aba^nba^3 \ldots ba^n / n \geq 1\} \), or whether a language contains \( \{a^ib^ia/i \geq 1\} \) implies (*). It is shown that (*) is equivalent to the unsolvability of whether a language contains a chain of a special form. Several facts about whether a language contains a specific sequence are also demonstrated. In particular, it is shown that whether a language contains a given sequence is unsolvable, but whether a language contains a given ultimately periodic sequence is solvable. (Received November 23, 1962.)


A ribbon knot has been defined by R. H. Fox [A quick trip through knot theory, and Topology of 3-manifolds and related topics]. The authors show that if \( k \) is a ribbon knot, then the Minkowski unit \( C_p(k) = 1 \) for each prime \( p \). Hence we see that the granny knot can not be a ribbon knot. (Received November 23, 1962.)
Let \( p \) be a real valued, \( 2m - 2 \) times continuously differentiable function defined on the interval \([0, \pi]\) such that \( p^{(j)}(0) = p^{(j)}(\pi) = 0 \) for \( j \) odd and less than \( 2m - 4 \). Let \( H_1 \) be the \( m \)-th power of the operator defined in \( L^2[0, \pi] \) by \(-d^2/dx^2 + p(x)\) and the boundary conditions \( u(0) = u(\pi) = 0\). Let \( H_0 \) be the same operator with \( p(x) \equiv 0 \). Then \( H_1 \) and \( H_0 \) are self-adjoint operators with a common domain, \( \mathcal{D}(H_1) = \mathcal{D}(H_0) \).

Let \( V = H_1 - H_0 \). Let \( \lambda_1 < \lambda_2 < \ldots \) be the eigenvalues of \( H_1 \), \( \mu_1 \leq \mu_2 \leq \ldots \) those of \( H_0 \). For sufficiently small \( \epsilon \), the operator \( H_0 + \epsilon V \) will have eigenvalues \( \gamma_n(\epsilon) = \mu_n + \epsilon \mu_n^{(1)} + \epsilon^2 \mu_n^{(2)} + \ldots \).

**Theorem.** If \( s < 2m \), \( \sum_{n=1}^{\infty} \{ \lambda_n - \mu_n - \mu_n^{(1)} - \ldots - \mu_n^{(s)} \} = 0 \). 

**Proof.** Using a modification of methods of Kato, we obtain

\[
\int_{\kappa(k)} S[z R_1(z)]^{-1} dz.
\]

Here \( R_1(z) \) is the resolvent of \( H_0 \), \( S \) stands for trace, \( \kappa(k) \) is the circle with center at the origin and radius \((k + 1/2)^{2m}\). From bounds on the Schmidt norm of \( VR_0(z) \) and the norms of \( R_1(z) \) and \( VR_0(z) \), we obtain that the contour integral is \( O(k^{-r}) \) for some positive \( r \).

Other methods can be used to prove the theorem for smaller values of \( s \), provided \( p \) satisfies additional conditions. (Received November 23, 1962.)
Theorem 2. If $0 \rightarrow A \rightarrow X \rightarrow B \rightarrow 0$ represents an element of $p^\otimes\text{Ext}(B, A)$, then $A \cap p^\otimes X = p^\otimes A$ for all $\beta \subseteq a$. If $B$ is a $p$-group and $a = \beta + n$ with $\beta$ a limit ordinal and $n < \omega$, then (a) $p^\otimes(X/A) = (p^\otimes X + A)/A$ for $\beta < \beta$, and (b) $(p^\otimes(X/A))[p^\otimes X] = (p^\otimes X + A)/A[p^\otimes X]$. (Received November 23, 1962.)


Let $R$ be a Dedekind domain. The main theorem proved is: If $R M$ is isomorphic to a (possibly infinite) direct sum of ideals $\{K_i\}$ of $R$ and $f$ a homomorphism of $M$ onto $M = \sum_{i=1}^{\infty} R M_i$, where $M$ is torsion, then the decomposition of $M$ can be "lifted" to a decomposition $M = \sum_i R M_i$ where $f(M_i) = R M_i$ ($i \leq n$), $f(M_i) = 0$ ($i > n$), and $M_i \cong K_i$ (all $i$). If $M$ is a direct sum of cyclic torsion modules it can happen that no decomposition of $M$ can be lifted to $M$. The necessary and sufficient condition that a particular decomposition can be lifted turns out to be a purely combinatorial one involving comparison of certain integers. Some consequences: (1) If $R N \subseteq R M$ are finitely generated, torsion-free, and of rank $n$, then there are decompositions $M = \sum_{i=1}^{n} R M_i$ and $N = \sum_{i=1}^{n} R E_i M_i$ where $M_1, \ldots, M_{n-1}$ can be prescribed up to isomorphism, and the ideals $E_i$ are subject only to the restriction $\sum_{i=1}^{n} E_i \cong M/N$. (2) If $M_1, M_2$ are finitely generated and torsion-free, $M_1/N_1 \cong M_2/N_2$, and $N_1 \cong N_2$, there is an isomorphism of $M_1$ onto $M_2$ which carries $N_1$ onto $N_2$. (3) If $R$ is the ring of all algebraic integers in an algebraic number field, and if $R$ is not a principal ideal domain, then there exist square matrices $A$ of all ranks such that $P AQ$ is never a diagonal matrix for unimodular $P$ and $Q$. (Received November 23, 1962.)


The work of Sklar is extended (see Abstract No. 638, Bull, Amer. Math. Soc. 62 (1956), 559, and Report of the institute in the theory of numbers, Boulder, Colorado, 1959) concerning meromorphic functions which can be represented by absolutely convergent Dirichlet series in a suitable half-plane. In particular, approximate functional equations are obtained for a much larger class of functions than heretofore. (Received November 23, 1962.)


To every associative $h$-space with a homotopy inverse, is associated a universal principal fibre space (in the sense of: Fibre spaces and fibre homotopy equivalence, Colloquium on algebraic topology 1962, Aarhus, Denmark) and hence a universal classifying space. This is used to show that the homotopy equivalence classes of fibre bundles with fibre $F$ structural group $G$ and base a polyhedron $B$ are in 1-1 correspondence with the homotopy classes of maps of $B \rightarrow B_H$ where $B_H$ is the classifying space of an associated, associative $h$-space with homotopy inverse. (Received November 23, 1962.)
597-195. JUN-ICHI HANO, Washington University, St. Louis 30, Missouri. On compact complex coset spaces of reductive Lie groups.

Theorem 1. Let $G/B$ be a compact connected complex coset space of a connected complex Lie group $G$ by a closed complex Lie subgroup $B$. Let $U$ be the normalizer of the identity connected component $B_0$ of $B$. If $G$ is a reductive complex Lie group, then the fibre of the holomorphic fibre bundle $(G/B, p, G/U)$ is a compact complex connected coset space of a reductive complex Lie group $U/B_0$ by the discrete subgroup $B/B_0$. Theorem 2. Let $M$ be a connected compact complex coset space, and let $A(M)$ be the identity connected component of the complex Lie group of holomorphic homeomorphisms of $M$ onto itself. If a connected reductive Lie subgroup in $A(M)$ is transitive on $M$, then $A(M)$ is a complex reductive Lie group. (Received November 23, 1962.)

597-196. R. F. ARENS and P. C. CURTIS, JR, University of California, Los Angeles 24, California. Commutative Banach algebras which are direct summands.

Let $X$ be a compact Hausdorff space, $C(X)$ the algebra of continuous complex functions on $X$, and $A$ a closed subalgebra of $C(X)$ containing the constants. Let $Y$ be the continuous image of $X$ obtained by identifying the sets of constancy for $A$. Theorem 1. If $A$ is a direct summand in $C(X)$, then $A$ is isomorphic to $C(Y)$. If $X$ is a metric space, the converse statement holds. The latter assertion is a special case of the following Theorem 2. If $C(X)$ is isomorphic to a closed subspace $E$ of a separable Banach space $F$, then $E$ is a direct summand in $F$. (Received November 23, 1962.)

597-197. A. P. MATTUCK, Massachusetts Institute of Technology, Cambridge, 38 Massachusetts, and A. P. MAYER, Istituto Torelli, University of Pisa, Pisa, Italy. The Riemann-Roch theorem for algebraic curves.

Once the Jacobian of an algebraic curve has been constructed by Chow's method, the Riemann-Roch theorem for the curve can be given an intuitive geometrical formulation and proof. To get the Jacobian, one only uses the Riemann part of the theorem, i.e., the Hilbert postulation formula for curves. This formulation of the theorem has analogues in higher dimensions, but in general yields only inequalities. (Received November 23, 1962.)
597-198. R. B. MERKEL, University of California, Davis, California. The structure of finite semigroups and the associated system of left ideals.

Let $P(S)$ be the power set of a finite set $S$. For which subsets $L$ (properly restricted by union and intersection) of $P(S)$ is it possible to define a semigroup $(S, \cdot)$ such that $L$ is the system of left ideals of $(S, \cdot)$? For $L$ such that $S$ of $(S, \cdot)$ is the union of the minimal elements, $(S, \cdot)$ is completely simple. For $L$ which contain a smallest element and such that $S$ of $(S, \cdot)$ is the union of the atoms, $(S, \cdot)$ is homomorphic to an elementary semilattice all of whose components except one are completely simple. (Received November 23, 1962.)

597-199. G. G. WEILL, Yale University, New Haven, Connecticut. Geodesic circles on complete $C^\infty$ two dimensional Riemannian manifolds.

Let $M$ be a complete $C^\infty$ two dimensional Riemannian manifold. The circle $C(m, R)$ of center $m \in M$ and radius $R$ is defined as: $C(m, R) = \{ \mu: \mu \in M, d(m, \mu) = R \}$ where the distance function $d$ is obtained from the Riemannian metric. The topological nature of the $C(m, R)$ is investigated. It is shown that apart from a set of radii of measure zero, a (nonvoid) geodesic circle can be described as a topological graph. (Received November 23, 1962.)

597-200. WITHDRAWN

597-201. L. D. HEACOCK, c/o T. Tamura, University of California, Davis, California. On the numbers of one-sided zeros and one-sided identities of semigroups.

Some symbols are defined as follows: $x$ corresponding to left zeros; $y$ corresponding to right zeros; $z$ corresponding to left identities; $u$ corresponding to right identities, where each of $x, y, z$ and $u$ represents one of 0, 1, and 2. For example "$x = 0$" means "A semigroup has no left zero"; "$x = 1$" means "having a unique left zero"; "$x = 2$" means "having more than one left zero". Then it holds that $x + y = a$ or 2 and $0 \leq z + u \leq 2$, and conversely for such arbitrary $x, y, z, u$, there is a semigroup with the indicated properties. (Received November 23, 1962.)


Let $R$ be a ring of characteristic 0 with unit and $p$ a prime integer. Let $x$ be a sequence $\{x_i\}, x_i \in R$, define $f_n(x) = \sum_{i=0}^{n} x_ip^{n-i}$. If * is any of the operations $+, -, \cdot$, $\exists$ sequences $u^* = \{u^*_i\}$ such
that $u^*_{j}$ is an integral polynomial in $x_j y_j$, $0 \leq j \leq n$, and $f_n(x) f_n(y) \equiv f_n(u^*) \mod p^{n+1}$. Let $[f_n(x)]_{p^m}$ denote the residue class of $f_n(x) \mod p^m$. Then $O_n(R) \equiv [f_{n-1}(x)]_{p^m}$ is a ring. If $\mathbb{R}/p$ is an integral domain let $W_n(\mathbb{R}/p) = \{x = (x_0, \ldots, x_n-1); x_i \in \mathbb{R}/p\}$ and define * on $W_n(\mathbb{R}/p)$ as follows: if $[\xi]_p = x_0$, $[\eta]_p = y_1$, then $x * y = \bar{z} * \bar{g}$ where $g_i = [u^*_{i}(\xi_0, y_0, \ldots, \xi_{i-1}, y_{i-1})]_p$ is the polynomial in $x_0, y_0, \ldots, x_{i-1}, y_{i-1}$ with coefficients the residue classes mod $p$ of the coefficients of $u^*_{i}$.

\begin{align*}
O_n(R) &\nsubseteq W_n(\mathbb{R}/p). \quad W_n(k) \text{ can be defined for any integral domain } k \text{ of characteristic } p \text{ using the } g_i^* \text{ to define *}. \quad \text{Let } W(k) = \left\{X = (x_0, x_1, \ldots); x_i \in k\right\} \text{ where } X \times Y = (g^*_{i0}, g^*_{i1}, \ldots). \quad \text{Then } W(k) = \lim_{\substack{\rightarrow \quad \{W_n(k)\}}} \text{ with the natural homomorphisms}. \quad W(k) \text{ is complete under the nonarchimedean rank } 1 \text{ valuation given by: } v(0) = 0 \text{ and } v(0, \ldots, 0, x_{\ell}, \ldots) = p^{-\ell}. \quad \text{Let } Q(k) \text{ be the field of quotients of } W(k). \quad \text{Then (1) The ring of integral elements of } Q(k) \text{ is } U_n W(k^{p-n}). \quad (2) Q(k) = \left\{\left[\prod_{i=0}^{j-1} (x_i y_i)^{p-1}, x_j y_j \cdots \right]; x_i \in k, x_0 \neq 0 \text{ unless all } x_i = 0^j. \quad (3) \text{If } k \text{ is not perfect then } Q(k) \text{ is not complete.} \quad (4) Q(k^{p^{-\infty}}) \text{ is the completion of } Q(k). \quad \text{(Received November 19, 1962.)}
\end{align*}

597-203. C. H. WILCOX, Mathematics Research Center, U. S. Army, University of Wisconsin, Madison 6, Wisconsin. \text{An energy inequality for the reflection coefficient matrix of a pair of nonuniform coupled transmission lines.}

The analysis of electric wave propagation on a pair of nonuniform coupled transmission lines leads to the following problem. Find $2 \times 2$ matrix-valued functions $I(x)$ and $E(x)$ and $2 \times 2$ constant matrices $R$ and $T$ (reflection and transmission coefficients) satisfying the matrix equations

1. $dI/dx + sC(x)E = 0, dE/dx + sL(x)I = 0 \text{ on } 0 \leq x \leq \bar{I} \text{ and boundary conditions} \quad 2. I(0) = U + R, E(0) = Z(U - R), I(\bar{I}) = T, E(\bar{I}) = ZT. \quad \text{Here } C(x) \text{ (the capacitance) and } L(x) \text{ (the inductance) are real, symmetric positive definite matrices, and } s \text{ is a Laplace transform parameter (Re } s \geq 0). \quad U = ((\delta_{jk})) \text{ is the unit } 2 \times 2 \text{ matrix and } Z = ((\delta_{jk} Z_{jk})) \text{ is a known constant impedance. Theorem. Problem (1), (2) has a unique solution of the form } \text{I}(x) = [I + R(x)]/I(x), E(x) = Z[I - R(x)]/I(x) \text{ where } R(x) \text{ and } J(x) \text{ are determined by } \text{dR}/\text{dx} + s(\text{BR} - \text{AR} - \text{RA} + B) = 0, \text{ dJ}/\text{dx} + s(\text{A} - \text{BR})J = 0, \text{ R}(\bar{I}) = 0 \text{ and } J(0) = U. \quad \text{Here } A = (CZ + Z^{-1}L)/2 \text{ and } B = (CZ - Z^{-1}L)/2. \quad \text{The existence of } R(x) \text{ on the whole interval } 0 \leq x \leq \bar{I} \text{ is guaranteed by the theorem. The solution } R(x) \text{ satisfies } Z_1 |R_{11}(x)|^2 + Z_2 |R_{22}(x)|^2 \leq Z_2 \text{ on any interval } x_0 \leq x \leq \bar{I} \text{ on which it exists.} \quad \text{(Received November 19, 1962.)}

597-204. P. C. HAMMER, Physics Department, University of California, San Diego, California. \text{Extended topology: Structure of isotonic functions.}

Let $\mathcal{H}$ be the class of all subsets of a space $M$ with null set $N$. A function $f$ mapping $\mathcal{H}$ into itself is \textit{isotonic} provided it is inclusion preserving. A subclass $\mathcal{C}$ of $\mathcal{H}$ is a union base for $f$ provided $fX \equiv \bigcup \{fY; Y \subseteq X, Y \in \mathcal{C}\}$. If $f$ has a union base comprised of finite sets $f$ is domain finite. A function $u$ is a union generator of $f$ provided $fX \equiv \bigcup \{uY; Y \subseteq X\}$. Two functions $u, v$ are equivalent union generators provided they are union generators of the same isotonic function. For an isotonic function $f$ define $(\Delta f)X \equiv fX - U \bigcup \{fY; Y \subseteq X\}$ and $f_0X \equiv U \bigcup \{\Delta fY; Y \subseteq X\}$. Properties of union bases and union generators are derived. Remarks. This theory is analogous to that of integration of derivatives of monotonic real-valued functions. When $f \neq f_0$ an "integration" theory based on equivalent union generators is given and then differences coarser than $\Delta f$ must be used. The theory extends simply into functions from one class to another. \text{(Received November 19, 1962.)}
597-Z05. K. T. HAHN, Stanford University, Stanford, California. Bounds for the invariant B-areas of a family of surfaces in the space of two complex variables.

The author considers a bounded domain $B^4$ with four piecewise smooth boundary components $b^k$, $k = 1, 2, 3, 4$, in the space of two complex variables and such that $B^4$ is topologically equivalent to the product $[R_1 \subseteq |z_1| \leq 1] \times [R_2 \subseteq |z_2| \leq 1]$ of two ring domains. $B^4$ possesses the distinguished boundary $D^2 = \bigcup_{k=1}^{4} D^2_k$, where $D^2_k$ can be deformed into a surface $S^2_1 = [z_1 = r_1] \times [|z_2| = r_2]$ which is assumed to lie in the complement of $B^4$. It is assumed that the surface $z_1 = 0$ as well as $z_2 = 0$ lies in the complement of $B^4$. Let $F(\theta)$ be the family of surfaces $S^2 \in B^4$ with the following properties: (a) each $S^2 \in F(\theta)$ can be deformed into $D^2_1$ in $B^4$; (b) at every point on $S^2 \in F(\theta)$ the Euclidean B-area element $db$ (see Bergman, Atti Ac. Lincei Rend. 19 (1934), p. 474) and the Euclidean area element $da$ are defined and furthermore, there exists a positive number $\delta$ such that $db/da \leq \delta$. Generalizing his previous results, the author determines two positive constants $c$ and $e$ such that the Bergman (invariant) B-area $B(S^2)$ $\leq c$, for every $S^2 \in F(\theta)$, and that there exists a surface $S^2 \in F(\theta)$ with $B(S^2) \geq c + e$. (Received November 23, 1962.)


The paper considers some new properties of two trigonometric sums introduced by Eckford Cohen: $C^*(m,n) = \sum \exp(2\pi ima/n)$, summation being over the integers $a$ of a semi-reduced residue system (mod $n$), and $C^*_b(m,n)$ (Mathematische Zeit. 74 (1960), 66-80; Duke Math. J. 28 (1961), 475-485 and 16 (1949), 85-90). For instance, it is shown that if $f(n) = n$ when $n$ is the power of a prime, and $f(n) = (-1)^w(n)$ if $w(n) > 1$, where $w(n)$ is the number of distinct prime factors of $n$, then for $n > 1$, $\sum (1/l) C^*(l,n) = - f(n)$. This is analogous to a classical result of Ramanujan involving his well known trigonometric sum $C(m,n)$ (Collected papers, Cambridge, 1927, 179-199). (Received November 23, 1962.)


Reference is made to an abstract of Rogers, Bull. Amer. Math. Soc. 63 (1957), p. 140, and to Abstract 590-33, these Notices, April, 1962, p. 128. A recursive set is provably recursive (p-recursive) with respect to an axiomatic system $S$ if its characteristic function is a p-function. A p-recursive set is provably infinite if the statement that its characteristic function assumes the value 1 infinitely many times is a theorem of $S$. The following results are easily proved: (1) If $S$ is sound, there is a recursive set which is not p-recursive, (2) A set is p-recursive and p-infinite if and only if it is the range of some p-increasing p-function, (3) If $S$ is sound, then a set is p-recursive and nonempty if and only if it is the range of some p-non-decreasing p-function, (4) If $S$ is sound, then there exist p-recursive sets $A_1$, $A_2$, $A_3$ each of which is infinite and has an infinite complement, such that (i) $A_1$ is p-infinite and $\bar{A}_1$ is p-infinite; (ii) $A_2$ is p-infinite and $\bar{A}_2$ is not p-infinite; (iii) $A_3$ is not p-infinite and $\bar{A}_3$ is not p-infinite. The construction of $A_3$ uses an extension of methods of Kreisel and of Kent. (Received November 23, 1962.)

One result of this paper is the following: Theorem. Suppose that $f$ is a light open mapping (continuous) of a compact metric continuum $X$ onto a disk $Y$. Furthermore, $J$ denotes a simple closed curve in $X$ such that $f|J$ is a homeomorphism, $f(J) = Bd Y$, and $f(X - J) = \text{Int } Y$. If $f$ possesses a certain Property $P$, then there exists a disk $D$ in $X$ whose boundary is $J$ such that $f|D$ is a homeomorphism. The Property $P$ is as follows: For each $\varepsilon > 0$ and each arc (simple) $a_1 b_1$ in $f^{-1}(ab)$ such that $ab$ is an arc which spans $Y$ and $f|a_1 b_1$ is a homeomorphism, there exists an open set $T \supset a_1 b_1$ such that if $xy$ is an arc from $C_a$ (the component of $T \cap J$ containing $a$) to $C_b$ (the component of $T \cap J$ containing $b$) where $f|xy$ is a homeomorphism, then $N_\varepsilon(xy) \supset a_1 b_1$ and $N_\varepsilon(a_1 b_1) \supset xy$. Furthermore, there exists a $\varepsilon > 0$ such that for each arc $pq$ in $N_\varepsilon(ab)$ which spans $Y$, there exists an arc $p_1 q_1$ in $f^{-1}(pq)$ which lies in $T$ such that $f|p_1 q_1$ maps $p_1 q_1$ homeomorphically onto $pq$. (Received November 23, 1962.)

597-209. TOGO NISHIURA and DANIEL WATERMAN, Wayne State University, Detroit 2, Michigan. Reflexivity and summability.

A theorem of Banach and Saks asserts that a bounded sequence in $L_p(0,1)$ or $L_p$, $p > 1$, has a subsequence whose $(C,1)$ means converge strongly. Kakutani showed that for weakly convergent sequences in a uniformly convex Banach space the same conclusion holds. Weakly convergent can now be replaced by bounded since uniformly convex spaces are reflexive. We show that: (1) there are reflexive spaces which are not uniformly convex and for which the Banach-Saks theorem holds; (2) reflexivity is equivalent to a summability property, namely, for every bounded sequence there is a regular method $T = (c_{mn})$ with $\sum_n |c_{mn}| \to 1$ as $m \to \infty$, such that the $T$-means of the sequence converge strongly (or weakly). As a corollary of (2) we have, if the $(C,1)$ means of a bounded sequence converge strongly, the space is reflexive. It is shown further that a Schauder base in a Banach space is boundedly complete if the above property holds weakly. (Received November 23, 1962.)


The notation and terminology of Chapter III of L. H. Loomis, Abstract harmonic analysis is adopted. Together with the elementary integral $I$ on $L$, consider an upper integral, $J$, that extends $I$. Explicitly, $J$ is a functional defined on the extended-real valued functions on $X$ that satisfies the requirements: (i) $f \equiv 0 \implies J(f) \equiv 0$; (ii) $J(f + g) \equiv J(f) + J(g)$; (iii) $a > 0$ and a real $\implies J(af) = aJ(f)$; and (iv) $f \in L \implies J(f) = I(f)$. An extended-real valued function $f$ on $X$ is called solvable, provided $J(f) = -J(-f)$, and provided this common value is finite. The inclusion relations between the class of solvable functions and $L_1$ (cf. Loomis) are investigated under suitable continuity assumptions on $J$. This work generalizes results of the author on the Perron method of solving the Dirichlet problem of potential theory (Amer. J. Math. 84 (1962), 317-323). (Received November 23, 1962.)
597-211. R. E. BRADFORD, University of California, Berkeley 4, California. Decomposition of linear forms in cardinal algebras.

Using the terminology of Tarski's Cardinal algebras (New York, 1949) let \( \langle A, +, \sum \rangle \) be a cardinal algebra; let \( \mathbf{a}_0, \mathbf{a}_1, \ldots, \mathbf{a}_i, \ldots \) and \( m_0, m_1, \ldots, m_i, \ldots \) and \( n \) be finite non-negative integers, and \( a_0, a_1, \ldots, a_i, \ldots \) be members of \( A \). Under the assumption, \( \sum_{i \leq n} \mathbf{a}_i = \sum_{i \leq n} m_i \cdot a_i \), the following conclusions hold: (1) There is an \( 0 \leq \bar{n} < \infty \), a double sequence \( k_{i,j} \) of non-negative integers (\( \leq \infty \)), and a sequence \( b_j \) of members of \( A \) such that, \( a_i = \sum_{j < \infty} \mathbf{k}_{i,j} \cdot b_j \) for \( i < n \) and \( \sum_{i \leq n} \mathbf{k}_{i,j} = \sum_{i \leq n} m_i \cdot k_{i,j} \) for \( j < \infty \). (2) There is an \( \bar{n} \leq \infty \) and a double sequence \( c_{i,j} \) in \( A \) such that \( a_i = \sum_{j < \infty} c_{i,j} \) for \( i < n \), and \( c_{i,j} \) and \( c_{i,j} \) are comparable for all \( i_0, i_1, \ldots, i_n \) and \( j < \infty \). (3) There are two sequences \( d_i \) and \( e_i \) of members of \( A \) such that \( a_i = \sum_{i \leq n} d_i \cdot d_i \cdot e_i = \sum_{i \leq n} m_i \cdot d_i \cdot e_i \) for every \( i < n \), \( d_0 \neq d_1 \), and \( e_0 \neq e_1 \). (2) and (3) can easily be derived from (1). (3) provides a partial solution to a problem raised on p. 242 f. op. cit. (1), (2), (3) can be established in the arithmetic of cardinals without applying the axiom of choice. In this regard (1) and (2) seem to be new. (Received November 23, 1962.)

597-212. DAVID LISSNER, Syracuse University, Syracuse, New York and R. H. SZCZARBA, Yale University, New Haven, Connecticut. Vector bundles on lens spaces.

Let \( R = \mathbb{Z}_p[x]/(x^{n+1}) \) and \( G \subset R \) be the multiplicative group of polynomials with constant term 1. Theorem. If \( p \leq n \) the group \( G \) is generated by the elements \( 1 + qx \), \( 0 < q < p \). If \( L(p,n) \) is a lens space of the form \( S^{2n+1}/\mathbb{Z}_p \), this result can be used to determine the ring of (real or complex) vector bundles over \( L(p,n) \) for \( p \leq n \). (Received November 23, 1962.)

597-213. MELVIN ROTHENBERG, University of Chicago, Chicago, Illinois. The \( J \) functor and the nonstable homotopy groups of the unitary groups.

The set of equivalence classes of stable \( J \) equivalent vector bundles over a finite \( C, W \), complex \( X \) form a finite Abelian group, \( J(X) \). Take \( X \) to be \( \mathbb{C}P_k \), the complex projective space of \( k \) complex dimensions, and \( E \) to be the canonical complex line bundle. Denote by \( m_k \) the order of \( E \) in \( J(X) \). The integers \( m_k \) are now known due to the work of a number of British topologists. Given \( n \), define \( m_k^n \) to be the order of \( n \) in \( \mathbb{Z}/m_k \mathbb{Z} \). Let \( V_{n+k,k} \) be the Stiefel manifold of unitary \( k \) frames in \( n + k \) space. Define \( l^n_k \) to be the minimum positive integer for which there exists a map of degree \( S^{2n+1} \rightarrow S^{2n+r+1} \) which factors through a map into the \( r \)th suspension of \( V_{n+k,k} \). Theorem I. \( l^n_k \) and \( m_k^n \) have the same prime factors. The main application is the following. Corollary I. Consider the well known fibration \( S^{2n+1} \rightarrow BU_n \rightarrow BU_{n+1} \). Let \( 1 \leq r < 2k \neq 2n \). Then the kernel of \( \prod_{2n+1} (S^{2n+1}) \rightarrow \prod_{2n+1} (BU_n) \) consists only of classes \( x \in \prod_{2n+1} (S^{2n+1}) \) such that for some \( q \), \( (m_k^n)^q = 0 \) (mod order of \( x \)). (Received November 23, 1962.)


Let \( X_1, \ldots, X_n \) be interchangeable random variables, and let \( S_k = \sum_{i=1}^k X_i \), \( S_0 = 0 \). The greatest convex minorant of the set \( \{ (k, S_k), k = 0, 1, \ldots, n \} \) of points in the Cartesian plane may be described
in terms of arithmetic averages \( \frac{S_k - S_j}{(k - j)} \), which impose a partial ordering "\( \preceq \)" on ordered k-tuples \( (0 \leq k \leq n) \) of \( \Omega = \{1, 2, ..., n\} \); e.g., \( (2,1,6) \preceq (3,1) \) if \( \frac{x_2 + x_1 + x_6}{3} \preceq \frac{x_3 + x_1}{2} \). Distributions of random variables defined in terms of the greatest convex minorant have been studied by Sparre Andersen (Math. Scand. 2 (1954), 195-223), Spitzer (Trans. Amer. Math. Soc. 82 (1956), 323-339) and others. A method due to Bohnenblust is used to extend these results via the generalized mean values introduced by Brøns. A key concept is that of a 1-1 correspondence, determined by a partial ordering of ordered k-tuples \( (0 \leq k \leq n) \) of elements of \( \Omega \), between the \( n! \) linear orderings of \( \Omega \) and the \( n! \) permutations of \( \Omega \) as represented by their sets of cycles. Specialization yields some results of Bohnenblust (Amer. Math. Monthly 69 (1962), Abstract, p. 586), of Barton and Mallows (J. Roy. Stat. Soc., 23 (1961), 423-433), and of the author (Math. Scand. 8 (1960), 305-326), as well as those mentioned above of Sparre Andersen and Spitzer. (Received November 23, 1962.)
Abstracts Presented by Title


Multiplicative groups of row-and/or column-finite infinite matrices.

An infinite matrix of complex elements will be called nonsingular of a certain property if it has a unique two-sided reciprocal with the same property. A diagonal string is the matrix: diag (B_1, B_2,...), where the 'beads' B_n are nonsingular square matrices not necessarily of the same size. The following structural results are obtained for elements of some multiplicative groups:

Theorem 1. Every permutator matrix is the product of at most two strings. Theorem 2. Every nonsingular row-finite upper-normal matrix is the product of at most two upper-normal strings.

Theorem 3. Every nonsingular row-finite matrix is the product of at most one normal matrix and four strings (2 permutator strings and 2 upper-normal strings). Theorem 4. Every nonsingular both row- and column-finite matrix is the product of at most six strings (2 normal, 2 permutator and 2 upper-normal strings). Theorem 3 gives information about the bases in the space of finite sequences and about basic sets of polynomials. (Received September 26, 1962.)


Two n by n matrices of integers will be called congruent mod k if their corresponding elements are all pairwise congruent mod k. A natural question connected with this type of congruence is the characterization of those matrices which are congruent to squares of matrices mod k. The case n = 2, k = p an odd prime is considered. Typical results for this case: (1) A matrix has more than four incongruent matrix square roots if and only if it is scalar; exactly four, if and only if it has incongruent nonzero eigenvalues which are quadratic residues mod p; and none or two, otherwise, (2) The number of incongruent matrix squares is (3p^4 - p^3 + p^2 + p + 4)/8. (Received September 27, 1962.)


Main results: (1) A ring with identity which has only finitely many subrings is finite. (2) A ring with identity whose subrings are totally inclusion-ordered is commutative, has prime power characteristic p and has a sole prime ideal (consisting of the integer multiples of p) whose residue ring is the union of a (finite or infinite) tower of finite fields whose degrees over the prime field are powers of some prime. Corresponding results for groups are recalled (2) becomes "is a subgroup of a group of type p^{\infty}\); some observations are made on the corresponding questions for fields. (Received September 27, 1962.)
A sequence \( S = (s_1, s_2, \ldots) \) of real numbers is said to be complete if every sufficiently large integer can be represented in the form \( \sum_{k=1}^{n} \alpha_k \). Suppose \( s_n = F_n - (-1)^n \) where \( F_n \) is the \( n \)th Fibonacci number, i.e., \( F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n \). Then the sequence \( S = (s_1, s_2, \ldots) \) has the following properties: (1) If any finite subsequence is deleted from \( S \) then the resulting sequence is complete. (2) If any infinite subsequence is deleted from \( S \) then the resulting sequence is not complete. (Received September 28, 1962.)

Let \( a \) be a positive rational and let \( m \) be a positive integer. Then there exists an integer \( N(a,m) \) such that \( n > N(a,m) \) implies that there exist integers \( k, n_1, n_2, \ldots, n_k \), such that:

(1) \( m < n_1 < n_2 < \ldots < n_k \).
(2) \( n_1 + n_2 + \ldots + n_k = n \).
(3) \( a = \frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_k} \).

In particular, it is shown that \( N(1,0) \) can be taken to be 77. Recent work by D. H. Lehmer has established the fact that \( N(1,0) \leq 77 \). (Received September 28, 1962.)

Let \( R_1 \) be the usual Riemann surface determined on an oriented surface \( S \) immersed in \( \mathbb{R}^3 \) by the first fundamental form \( \gamma_1 \). If \( K, H > 0 \) on \( S \), let \( R_2 \) be the Riemann surface determined on \( S \) by the positive definite second fundamental form \( \gamma_2 \). If \( K < 0 \) on \( S \), let \( R'_2 \) be the Riemann surface determined on \( S \) by the positive definite form \( \gamma'_2 \) given by \( \gamma'_2 H' = H \gamma_1 H - K \gamma_1 \). Results in this paper characterize geometrically cases in which standard differential geometric mappings \( S \rightarrow S \) (for example, the spherical image map, or the defining map between parallel surfaces) yield Teichmüller mappings between \( R'_2 \) and \( R_2 \) or \( R'_2 \) and \( R_2 \). All results are neatly analogous to those (previously announced) describing cases in which Teichmüller mappings occur between \( R_1 \) and \( R_1 \) or \( R'_1 \) and \( R_2 \) or \( R'_2 \) and \( R_2 \). (Received September 28, 1962.)

The class of expressions of the form \( (Ax)(Ey)((F_{x+y} \leftrightarrow F_{y+y}) \& F_{x+y}) \& (Az_1 \ldots (Az_g) \alpha(M; z_1, \ldots, z_g) \) where \( M \) is a truth-functional matrix containing only the binary predicate letter \( F \) and the individual variables \( z_1, \ldots, z_g \) is a reduction class for satisfiability. The proof uses and modifies the methods of Kahr, Moore, and Wang (Proc. Nat. Acad. Sci., 48 (1962), 365-377), and Kahr (Abstract 590-37, Notices Amer. Math. Soc., 9 (1962), 129). This result improves upon theorems of Kalmár and Surányi (J. Symb. Logic. 4 (1939), 1-9; 15 (1950), 161-173). The procedure is not "conservative" in the sense of Kahr and Wang (Abstract 590-38, Notices Amer. Math. Soc., loc. cit.) since it leads only to formulas satisfiable in no finite domain. However, a modification of the method gives that the
problem of satisfiability in some finite domain for expressions of the above form is equivalent to the halting problem for Turing machines. (Received October 8, 1962.)


I. I. Gégalkine (Zegalkin) published in Ucenye Zapiski, Mosk. Gos. Univ., No. 100, I (1946), 155-212, a purported recursive procedure for deciding whether an expression of the form (Ex)(Ay) \( \land F \) is valid in all finite domains, where \( M \) is a truth-functional matrix containing only the dyadic letter \( F \), the monadic or dyadic letter \( G_1, G_2, \ldots, G_r \), and the individual variables \( z_1, z_2, \ldots, z_b \). That this is impossible follows from the result of Kahr and Wang (Abstract 590-38, Notices Amer. Math. Soc 9 (1962), 130), that such a procedure would solve the halting problem for Turing machines. The crucial error in Gégalkine's paper is in the proof of Theorem XXXVI, p. 192, at the end of the first paragraph. By the result of Kahr and Wang, we can restrict to the case where \( s = 3 \), all but one of the \( G_i \) are monadic, and \( M \) is of the form \( Fz_1z_2 \& M'(G_1, \ldots, G_r; z_1, z_2, z_3) \). Alternatively, we can restrict to the case where \( r = 1 \), and \( M \) is of the form \( Fz_1z_2 \& M'(G_1; z_1, \ldots, z_b) \) (see the preceding abstract). The problem for the case where the \( G_i \) do not occur is known to be solvable (Ackermann, Math. Ann. 112 (1936), 419-432). The case where all of the \( G_i \) are monadic remains open. (Received October 8, 1962.)


An algorithm is given for solving systems of linear equations which remains automatically in a digital number system without roundoff errors. It employs an integer program algorithm of R. E. Gomory (All-integer integer programming algorithm, RC-189, IBM, Yorktown Heights, New York, (1960)) as a subroutine; an \( n \) by \( n \) linear system leads to generalized elimination procedures in a sequence of matrices \((3n + 4)\) by \((n + 3)\). A solution, if such exists, may be obtained to any prescribed accuracy, although the computation is carried out in a fixed number of digits, independent of the number of digits of accuracy required. The algorithm determines successive vectors of a Cauchy sequence whose limit is some rational solution of the system; each successive vector is represented in a digital number system containing an additional significant digit. In contrast to typical methods, where classical algebraic algorithms are adapted to digital processes by roundoff procedures, and which have definite computation time and indefinite accuracy, this error-free algorithm has definite accuracy and indefinite (though finite) computation time. (Received October 9, 1062.)


Let \( H \) be a Hilbert space with complex scalars. Let \( f: H \to H \) be a monotonic operator, in the sense that \( \Re \langle \Delta x, f'(x, \Delta x) \rangle \geq 0 \) for all \( x \) and \( \Delta x \), where \( f' \) is the Fréchet differential of \( f \). Let \( \Re (\lambda) > 0 \). Then the equation \( \lambda x + f(x) = u \) has, for any \( u \), a unique solution for \( x \) which depends
continuously on \(u\). It follows that the "resolvent" of an antitonic operator (the negative of a monotonic operator) is defined everywhere in the right half-plane. The proof is the same as that of (Minty, Monotone (nonlinear) operators in Hilbert space, Duke Math. J., September, 1962) except that the map \(\phi\) of that paper is replaced by \(\phi(x,y) = (\lambda x + y, \lambda x - y)\). Most of the auxiliary results of that paper remain true in this situation. (Received October 15, 1962.)

63T-11. ALBERT SADE, Marseille (4) B.-du-Rh. ou 86, Cours, Pertuis, Vaucluse, France. Semi-automorphismes de groupoides et de quasigroupes.

Le groupe de semi-automorphisme (SA) d'un groupoïde \(G = E(x)\) coïncide avec le produit de Mann \(GP_{12} \setminus G\) du conjoint de \(G GP_{12} = E(*), x \times y = y \star x\) par \(G\). Le groupe SA de \(C_n\) coïncide avec son automorphe si \(n = 2k + 1\) ou \(\infty\); il se compose de \((x \rightarrow ax)\) et \((x \rightarrow ax + k)\), \((a, n) = 1\), si \(n = 2k\).

Pour que le groupe d'anti-automorphisme de \(G\) soit \(\subseteq G\)-SA il faut et il suffit que \(G\) soit flexible. Les groupoides \(M\) sur un ensemble \(E\) de cardinal \(> 3\) qui ont \(\mathcal{E}_E\) pour automorphe sont les semi-groupes de Thillirin, avec, en plus, \(xy = -x - y\) sur \(Z/3\) et \(xy = x + 1\) ou \(y + 1\) sur \(Z/2\). Pour que \(G\), sur \(E\) de cardinal \(> 3\) ait \(\mathcal{E}_E\) pour \(G\)-SA il faut et il suffit que \(G\) soit demi-symétrique, ou qu'il satisfasse \((yx)y = y\); si \(G\) est un quasigroupe, la première condition seule subsiste. Il n'y a aucun groupoïde, sur une ensemble \(E\) de cardinal \(> 4\) ayant \(\mathcal{U}_E\) pour \(G\)-SA. Pour que \(M = GP_{12} \setminus G\) soit de la forme \(xy = xF\), \((F \in \mathcal{E}_E)\) il faut et il suffit que \(G\) soit isotope de son transposé \(GP_{123} (xy = z \Longleftrightarrow z \cdot x = y)\) par la distorsion \((1, 1, F^{-1})\); l'automorphe de \(G\) est le centralisateur de \(F\) dans \(\mathcal{E}_E\). (Received October 18, 1962.)

63T-12. R. L. VAUGHT, University of California, Berkeley 4, California. Indescribable cardinals.

For notation see abstract 61T-240 of Hanf and Scott. An affirmative answer is given to the question raised there whether the first measurable cardinal \(\mu\) (assumed to exist) is strictly greater than the first \(\mathfrak{p}^{+\frac{1}{2}}\)-indescribable inaccessible cardinal. Indeed, a result is obtained which implies, in particular, that \(\mu\) is greater than the first \(n^{th}\) order \(- (i.e., \mathfrak{p}^{n+1}_{0-})\) indescribable inaccessible cardinal, for each \(n\). (Received October 19, 1962.)

63T-13. R. L. VAUGHT, University of California, Berkeley 4, California. Elementary classes of models closed under descending intersection.

An elementary class is a class \(K\) of similar relational systems consisting of all models of some set of elementary, or first order, sentences. \(K\) is closed under (descending) intersection if whenever \(\mathcal{A} \in K\) and \(L \subseteq K\) is a nonempty family of submodels of \(\mathcal{A}\) (which is ordered by \(\supseteq\)) then \(\bigcap \{ L \in L\} \in K\). A. Robinson and C. C. Chang (cf. Chang, Proc. Amer. Math. Soc. 10 (1959), 120-127) have shown that any elementary class closed under intersection is closed under ascending union, and hence, as is known, is an \(\varphi\)-class, i.e., is the class of all models of some set of \(\varphi\)-sentences. The following conjecture of M. Rabin has been proved: Any elementary class closed under descending intersection is an \(\varphi\)-class. This result, like Chang's, extends to pseudo-elementary (PC) classes. (Received October 19, 1962.)

Using an observation by J. Pincus about Hypothesis 6.1 of a previous paper (Trans. Amer. Math. Soc. 97 (1960), 35-63), the author has obtained a complete spectral representation for the operator

\[(Lx)(\lambda) = \int a(k(\lambda)k(\mu))^{1/2} x(\lambda) \, d\mu \text{ defined on } L^2(a,b) \text{ under the hypothesis that the real functions } f,k \text{ are of class } C^2 \text{ on } [a,b], \text{ with } k > 0 \text{ almost everywhere, such that } f' + k' \text{ have only finitely many zeros at which the corresponding second derivatives do not vanish. In particular, for almost all real } \lambda, \text{ the spectral multiplicity is the number of connected components of the open set } \{ \lambda \in (a,b) | (f(\lambda) - \xi - k(\lambda)) (f(\lambda) - \xi + k(\lambda))^{-1} < 0 \}. \] (Received October 24, 1962.)


Let \( S_1 \) and \( S_2 \) be Banach spaces, \( B[S_1,S_2] \) the space of bounded linear transformations from \( S_1 \) into \( S_2 \), and \( L[S_1] \) the Banach space of Bochner integrable functions from the real interval \([0,1]\) into \( S_1 \). The following extension of a result of D. H. Tucker [Abstract 587-16, Notices Amer. Math. Soc. 8 (1961), 572] is proved: Theorem. \( T \) is a bounded linear transformation from \( L[S_1] \) into \( S_2 \) if and only if there exists a function \( K \) from \((0,1], \) satisfying a Lipschitz condition \( \|K(x) - K(y)\| \leq M|x - y| \), and such that \( T(f) = \int_0^1 K(x) f(x) \) for each \( f \) in \( L[S_1] \). \( \|T\| \) is equal to the greatest lower bound of the Lipschitz constants for \( M \). (Received September 24, 1962.)

63T-16. E. S. RAPAPORT, Polytechnic Institute of Brooklyn, 333 Jay Street, Brooklyn 1, New York. The Poincaré conjecture.

Theorem. The group \( G \) is torsion free: \( G = (F_2g; R_1, R_2) \) is factor group \( \pi/R_2 \) of \( \pi = (F_2g; R_1) \) \( a_1, b_1, \ldots, a_g, b_g; a_1 b_1^{-1} \ldots a_g b_g^{-1}, \) with \( R_2 = a_1 b_1 W a_g^{-1} W^{-1} b_g^{-1}, \) and \( W \) an element of the commutator subgroup of the free group \( F_2g = \langle a_1, \ldots, b_g \rangle. \) The proof is in two steps:

Theorem 1. \( R_2 \) is not of the form \( V^k \) with \( K \neq 1 \) for any element \( V \) of \( \pi. \) Theorem 2. If \( Rc\pi \) is not a power \( V^K, K \neq 1, \) then \( \pi/R \) is torsion free. The proof of Theorem 1 is combinatorial, that of Theorem 2 uses a method due to Magnus and Theorem 1 of Karrass, Magnus, Solitar, Comm. Pure Appl. Math., 1960. Cf. Papakyriakopoulos, Bull. Amer. Math. Soc. 68 (1962), 365 (5,1). (Received October 24, 1962.)

63T-17. SEYMOUR LIPSCHUTZ, Polytechnic Institute of Brooklyn, Brooklyn 1, New York. An extension of Greenlinger’s results on the word problem.

Greenlinger (Dehn’s algorithm for the word problem, Comm. Pure Appl. Math., 13 (1960), 67-83) solved the word problem for a set \( \mathcal{G} \) of groups. Let \( G \) be the generalized free product of groups \( G_i \) in \( \mathcal{G} \) with the subgroups \( H_i \) of \( G_i \) amalgamated. Then \( G \) also admits a solution to its word problem if (a) \( H_i \) is infinite cyclic; or if (b) \( H_i \) is generated by elements \( A_1, \ldots, A_n \) of \( G_i \) with the property that any word in the elements \( A_1, \ldots, A_n \) contains at most half of a defining relator of \( G_i \). As a necessary step in the proof, it is shown that there are only a finite number of words equivalent in \( G_i \) containing at most half of a defining relator of \( G_i \). (Received October 25, 1962.)
Let $p$ be an odd prime; let $q = p^a$ be a power of $p$ with $q \equiv -1 \pmod{4}$, $q \not\equiv 1 \pmod{3}$. Let $G$ be a subgroup of $SL(3,q)$. Then one of the following seven cases occurs: (1) $G$ is solvable and has an abelian normal subgroup of index $\neq 6$; (2) $G$ has a normal elementary-abelian $p$-subgroup $Q$ such that $G/Q$ is isomorphic to a subgroup of $GL(2,q)$; (3) $G$ is conjugate to the subgroup of $SL(3,q)$ obtained by restricting all matrix coefficients to a fixed subfield; (4) $G$ is simple of order 60 or 168; (5) $G$ is isomorphic to $PSL(2,p^b)$ where $b$ divides $a$ and $p^b \not\equiv 3$; (6) $G$ has a subgroup of index 2 of type (4) or (5); (7) $p = 3$ and $G$ has a nonsolvable normal subgroup of odd order. Assuming a recent result of W. Feit and J. G. Thompson (Solvability of groups of odd order), case (7) cannot occur.

In case (5) there is exactly one conjugacy class of subgroups of $SL(3,q)$ corresponding to each choice of $b$; a representative of this class is given explicitly. In certain other cases slightly more detail can be given. (Received October 29, 1962.)

Invariant imbedding and the phase shift problem.

Given (*) $y'' + y = f(t)y$, $f(t)$ continuous on $[0,\infty)$, $\int_{0}^{\infty} |f(t)| dt < \infty$. For some $x \equiv 0$ require

$y(x) = \cos \theta$, $y'(x) = \sin \theta$, $-\pi/2 < \theta \leq \pi/2$. Then for $t \geq x$ the solution to (*) exists and

$y(t;x,\theta) = A(x,\theta) \cos(t - x - \theta - \Psi(x,\theta)) + o_t(1)$ for large $t$. $\Psi(x,\theta)$ is the phase shift. For $\Delta > 0$, one may write $y(x + \Delta; x,\theta) = \Psi \cos \theta t$, $y'(x + \Delta; x,\theta) = \Psi \sin \theta t$. Then $y(t;x,\theta) = \Psi y(t;x + \Delta,\theta')$ and so $A(x,\theta) \cos(t - x - \theta - \Psi(x,\theta)) = \Psi A(x + \Delta,\theta') \cos(t - x - \theta - \Psi(x + \Delta,\theta')) + o_t(1)$. Since $\Psi$ and $\theta'$ may be computed easily to order $\Delta$, partial differential equations for $\Psi$ and for $A$, the amplitude, can be found. It is shown that one may choose $\lim_{x \to \infty} A(x,\theta) = 1$, $\lim_{x \to \infty} \Psi(x,\theta) = 0$, and the equations for $A$ and $\Psi$ are then solved by the method of characteristics. Results generalize and extend those of S. Franchetti, Nuovo Cimento, 6 (1957), 601-613. The ideas employed are applicable to the study of asymptotic behavior of solutions to systems more complicated than (*). (Received October 29, 1962.)

This paper generalizes upon the concepts of an antiideal, useful for investigating the structure of semigroups, as given by K. Iseki (Proc. Japan Acad. 38 (1962), 316-317) and by S. Schwarz (Czech. Math. J. 3 (78) (1953), 139-153 and 365-383). In addition a clear connection with the theory of mutant sets is shown. Definition. Let $(A,\cdot)$ be a semigroup. By a left $(\lambda,T,\mu)$-antiideal of $(A,\cdot)$ is meant a subset $B$ of $A$ for which $A \ast B \subseteq (T \cap \overline{B})^\mu$, where $T$ together with $\ast$ is a subsemigroup of $(A,\cdot)$ and $\overline{B}$ is the set-theoretical complement of $B$ relative to $A$. The condition for a right $(\lambda,T,\mu)$-antiideal is that $B^\mu \ast A \subseteq (T \cap \overline{B})^\mu$. Upon putting $A = \mu = 1$ and $T = A$ one obtains the theorems associated with Schwarz' antiideals and if $(\overline{B})^\mu \subseteq (B^\sigma)$ for some $\sigma$ then upon putting $T = A$ one obtains the theorems associated with Iseki's antiideals. In addition, every left or right $(\lambda,T,\mu)$-antiideal of a semigroup is a $(\lambda + 1, T, \mu)$-mutant set of the semigroup in the sense of the author's abstract (Notices Amer. Math. Soc. 9 (1962), 37). (Received October 29, 1962.)

Let $S$ be a bounded set in $E_k$, $k \geq 2$, and $S'$ its outer boundary. An $n$-tuple of real numbers $(a_1, \ldots, a_n)$ is admissible for $S$ if for every $s \in S$ there exist $s_1, \ldots, s_n \in S'$ with $s = \sum a_j s_j$. An $n$-tuple is admissible for all $S$ if and only if $\sum a_j = 1$ and $\max |a_j| \leq \sum |a_j|$. Similar and related results are obtained for individual sets and for general linear spaces. The paper will appear in Proceedings of Symposia in Pure Mathematics, Vol. 7. (Received October 30, 1962.)


Let $B$ be a real Banach space, $2 \leq \dim B < \aleph_0$, and $B'$ the conjugate space. If $A_1, \ldots, A_n$ are linear operators in $B$ such that (i) $EA_j$ is the identity and (ii) $\max \|x' A_j\| \leq \sum \|x' A_j\|$ for all $x' \in B'$ then for every bounded set $S$ in $B$ and point $s \in S$ there exist $s_1, \ldots, s_n \in \partial S$ such that $s = \sum s_j A_j$. If the $A_j$ are similarities and (ii) is replaced by $\max \|A_j\| \leq \sum \|A_j\|$ the condition becomes necessary and sufficient. This includes the generalization of a previous result for the case in which the $A_j$ are real scalars (see the preceding abstract). The proof is based on an inequality for the directional width of that part of the vector sum of bounded connected sets which is not contained in the vector sum of their boundaries. (Received October 30, 1962.)


Let $M^n$ be a compact, oriented, connected, differentiable, $n$-dimensional manifold. Let $C^k, h(\sigma_1, \ldots, \sigma_m)$ be the polynomial which expresses the symmetric function $\sum 1 \leq i_1 < \cdots < i_k \leq m \cdot x_{i_1}^{h_1} \cdots x_{i_k}^{h_k}$ as a polynomial in the elementary symmetric functions $\sigma_j = \sum 1 \leq i_1 < \cdots < i_j \leq m \cdot x_{i_1} \cdots x_{i_j}$ $p$ an odd prime, $P_i$ and $\overline{P}_i$ are the Pontrjagin class and dual Pontrjagin class respectively of dimension $4i$ modulo $p$. Using some results of Borel and Serre (see Groupes de Lie et puissances réduites de Steenrod, Amer. J. Math. 25 (1953), 409-448) the following generalization of a theorem due to William Massey (see On the Stiefel-Whitney classes of a manifold, Amer. J. Math. 82 (1960), 92-102) is obtained. Theorem. Let $q$ be an integer such that $0 < q < n$ and $n - q \equiv 0 \pmod{2(p - 1)}$. In $H^*(M^n, Z_p)$ if $C(n - q)/2(p - 1), (p - 1)/2(\overline{P}_1, \ldots, \overline{P}_{n/2}) = 0$, then there exist integers $h_1, \ldots, h_q$ such that $h_1 \geq h_2 \geq \ldots \geq h_q \equiv 0$ and $n = p^{h_1} + p^{h_2} + \ldots + p^{h_q}$. Corollary. Let $a(n)$ denote the sum of the digits in the $p$-adic expansion of $n$. If $n - q \equiv 0 \pmod{2(p - 1)}$ and $q \neq a(n) \pmod{p - 1}$, then $C(n-q)/2(p-1), (p-1)/2(\overline{P}_1, \ldots, \overline{P}_{n/2}) = 0$. If $n - q \equiv 0 \pmod{2(p - 1)}$ and $0 \leq q < a(n)$, then $C(n-q)/2(p-1), (p-1)/2(\overline{P}_1, \ldots, \overline{P}_{n/2}) = 0$. (Received October 31, 1962.)


Integrals discussed are Hellinger type limits of the appropriate sums. Theorem. If $F$ is a real valued function on the rectangular interval $[a, b; c, d]$, then $F$ is continuous if and only if it is true that
if \([u, v]\) is a number interval, each of \(H\) and \(K\) is a function of subintervals of \([u, v]\) into \([a, b]\) and \([c, d]\) respectively, and \(g\) is a real valued function of bounded variation on \([u, v]\) such that each of

\[
\int_{[u, v]} H(\lambda) dg \quad \text{and} \quad \int_{[u, v]} K(\lambda) dg
\]

exists, then \(\int_{[u, v]} H(\lambda) K(\lambda) dg\) exists. (Received November 2, 1962.)

63T-25. EDWIN CLARK, Tulane University, New Orleans 18, Louisiana. Finite dimensional affine semigroups. Preliminary report.

Let \(S\) be a finite dimensional affine semigroup such that \(S = M(S)\), where \(M(S)\) denotes the linear manifold generated by \(S\). By adjoining in a natural manner a zero and unit to \(S\), the following theorem is obtained: **Theorem.** If the dimension of \(S\) is \(n\), then \(S\) is equivalent to a semigroup of \(n + 2\) by \(n + 2\) real matrices. (It may be shown that the plane affine semigroup with multiplication \((x, y)(a, b) = (a, y)\) is not equivalent to a semigroup of \(3 \times 3\) real matrices.) **Corollary.** If \(S\) contains a zero (unit), there is an open set about zero (the unit) which contains no other idempotents. **Corollary.** \(S\) contains a primitive idempotent iff \(S\) contains a nonzero idempotent. **Theorem.** Some power of each element of \(S\) lies in a subgroup of \(S\). In particular, \(S\) always contains an idempotent. **Corollary.** If \(S\) is simple, then \(S\) is a union of groups. Let \(E = \{e \in S : e^2 = e\}\), and for \(e \in E\) let \(H(e)\) denote the maximal subgroup of \(S\) containing \(e\). **Theorem.** If \(E = \{e\}\), then \(H(e) = Se = es\). **Corollary.** If \(E = \{e\}\) and \(e\) is a left unit, then \(S\) is a group. **Theorem.** If \(E\) is compact, then \(E\) is finite. **Theorem.** If \(S\) is simple, then \(E\) is arcwise connected. **Corollary.** If \(S\) is simple and \(E\) is compact, then \(S\) is a group. (Received November 2, 1962.)

63T-26. R. W. GILMER, JR., Box 45, North Hall, University of Wisconsin, Madison 6, Wisconsin. Integral domains which are almost Dedekind.

An integral domain \(J\) is said to be almost Dedekind if \(J_P\) is a Dedekind domain for each proper prime ideal \(P\) of \(J\). **Theorem.** An integral domain \(J\) is almost Dedekind if and only if \(J\) is one dimensional and each primary ideal of \(J\) is a power of its radical. This theorem yields an elementary proof of the following result: If \(K\) is the quotient field of the Dedekind domain \(J\) and if \(J \subseteq J' \subseteq K\), then \(J'\) is a Dedekind domain. (Received November 2, 1962.)

63T-27. R. W. GILMER, JR., Box 45, North Hall, University of Wisconsin, Madison 6, Wisconsin. Rings in which every semi-primary ideal is primary.

This paper is a study of commutative rings with unit satisfying condition \((\#)\): an ideal of the ring is primary if its radical is a prime ideal. **Theorem 1.** If ring \(R\) satisfies \((\#)\) and if the prime ideal \(P\) of \(R\) contains a regular element, then \(P\) is a maximal ideal. **Corollary.** An integral domain \(J\) satisfies \((\#)\) if and only if the dimension of \(J\) is less than two. **Theorem 2.** Ring \(R\) satisfies \((\#)\) if and only if \(R\) has dimension less than two and if \(A\) is an ideal of \(R\) such that \(A\) is a nonmaximal prime ideal of \(R\), then \(A = \sqrt{A}\). **Theorem 3.** A Noetherian ring satisfying \((\#)\) is a finite direct sum of Dedekind domains and primary rings. (Received November 2, 1962.)

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In what is to follow M shall be a triangulated n-manifold. Theorem 1. Let K be a tree (a finite acyclic 1-complex) which is a subcomplex of M, and let N be the closed second barycentric neighborhood of K in M. The double of N is homeomorphic to S^n. Corollary. N × I is homeomorphic to I^{n+1}. The theorem generalizes the author's previous result (Notices Amer. Math. Soc. 9 (1962), 410) in the case when K is a 1-simplex. Let s be a simplex of M and C, the join of s with its link. Theorem 2. The cone over C, the suspension over C and C × I are each homeomorphic with I^{n+1}. This is actually a rather straightforward observation which can be established in a manner analogous to the proof of the previous corollary. (Received November 5, 1962.)


This paper applies the results of an earlier paper (Notre Dame J. Formal Logic, 3, No. 3 (1962)) to the theory of antiideals. E.g., Theorem. Let $\phi$ be a homomorphism from the semigroup $(A, \cdot)$ onto the semigroup $(B, \cdot)$. Let $C$ be a $(\lambda, T, \mu)$-antiideal of $(A, \cdot)$. If $\phi(T \cap \overline{C}) \subseteq S \cap \overline{\phi(C)}$ then $\phi(C)$ is a $(\lambda, S, \mu)$-antiideal of $(B, \cdot)$. Under no hypothesis the inverse image of a $(\lambda, Y, 1)$-antiideal of $(B, \cdot)$ is a $(\lambda, A, 1)$-antiideal of $(A, \cdot)$. Let $D$ be a maximal $(\lambda, A, 1)$-antiideal of $(A, \cdot)$. If $\phi(D) \subseteq T \cap \overline{\phi(D)}$ then $\phi(D)$ is a maximal $(\lambda, T, 1)$-antiideal of $(B, \cdot)$. Theorem. Let $(A, \cdot)$ and $(B, \cdot)$ be semigroups. Let $E$ be a $(\lambda, T, 1)$-antiideal of $(B, \cdot)$. Then $E$ is an isomorphic to $A$ and a fortiori $E$ is anisomorphic to $A$. (Received November 5, 1962.)


A Gentzen-type formalization of first order logic (classical and intuitionistic) is considered. Letters $M, N, M_1, \ldots$ denote finite (possibly empty) sequences of well formed formulas. If $M$ is such a sequence, $|M|$ denotes the set of all propositional and predicate variables occurring in some formula of $M$. The expression $M \subseteq N_1 N_2$ means that every formula of the sequence $M$ is also a formula of one of the sequences $N_1$ or $N_2$. The following converse of the elimination theorem is proved; Suppose $M \rightarrow N$ is derivable, $M \subseteq M_1 M_2$, $N \subseteq N_1 N_2$ and set $U = |M_1 N_1| \cap |M_2 N_2|$. Then the following hold: (i) If $U = \emptyset$ then either $M_1 \rightarrow N_1$ is derivable or $M_2 \rightarrow N_2$ is derivable; (ii) If $U \neq \emptyset$ there is a formula $B$ such that $|B| \subseteq U$ and both $M_1 \rightarrow N_1 B$ and $M_2 B \rightarrow N_2$ are derivable. As a corollary is obtained Craig's interpolation theorem for classical and intuitionistic logic. (Received November 5, 1962.)


Definition. Let $f: M \rightarrow N$ be a smooth mapping of Riemannian n-manifolds. Then $f$ is complete if there exists a positive continuous function $\lambda$ on $N$ such that, if $x \notin M$ and $X$ is tangent to $M$ at $x$,
Furthermore, \( f \) is uniformly complete if it is complete and \( \lambda \) is bounded above zero on bounded subsets of \( N \). Theorem. Let \( f: M \rightarrow N \) be a differentiable mapping of connected Riemannian \( n \)-manifolds, where both \( f \) and \( M \) are complete. Then: (1) \( f(M) = N \); (2) \( f: M \rightarrow N \) is a differentiable covering; and (3) \( N \) is complete if and only if \( f \) is uniformly complete. This theorem is along the lines of a recent theorem of Nijenhuis and Richardson (Michigan Math. J. 9 (1962)); however, in order to include nonproper mappings, it is in general necessary to keep \( f \) from 'blowing down' subvarieties of \( M \). By putting a complete Riemannian metric on \( M \), it follows that a proper differentiable mapping with nonzero Jacobian is onto, which is a special case of the theorem of Nijenhuis and Richardson. (Received November 7, 1962.)

63T-32. R. H. BING, Institute for Advanced Study, Fuld Hall, Princeton, New Jersey and V. L. KLEE, University of Washington, Seattle 5, Washington. Every simple closed curve in \( E^3 \) is unknotted in \( E^4 \).

With respect to the standard embedding of \( E^n \) in \( E^{n+1} \), the following problem is of interest: For \( n \geq 3 \) and \( 1 \leq k \leq n - 1 \), determine the smallest integer \( j = J(n,k) \) such that every \( k \)-sphere in \( E^n \) is unknotted in \( E^{n+1} \). From a recent result of Stallings in conjunction with a lemma of Klee, it follows that \( J(n,k) \leq k \) for \( n \geq 4 \) and \( k = 1 \) as well as for \( n \geq 3 \) and \( 2 \leq k \leq n - 1 \). However, these results show only that \( J(3,1) \leq 2 \). The present paper gives an elementary proof that \( J(3,1) = 1 \); that is, every simple closed curve in \( E^3 \) is unknotted in \( E^4 \). It is also proved that every topological 1-complex \( C \) in \( E^3 \) is tame in \( E^4 \), and in fact \( C \) can be carried onto a polyhedral 1-complex in \( E^3 \) by means of a self-homeomorphism of \( E^4 \) which moves no point more than \( \varepsilon \) and is the identity outside the \( \varepsilon \)-neighborhood of \( C \). (Received November 7, 1962.)

63T-33. T. G. McLAUGHLIN, University of California, Los Angeles 24, California. A semi­creative weak decomposition for certain r.e. sets.

Theorem. Let \( \beta \) be any infinite r.e. set which is not creative. Then, there exist two semi­creative but not creative r.e. sets, \( \beta_0 \), \( \beta_1 \), such that \( \beta_0 \cap \beta_1 \) is also semicreative noncreative, and \( \beta_0 \cup \beta_1 = \beta \). (The corresponding fact (similar to a lemma used by Myhill) that any infinite r.e. set is the union of two creative sets with creative intersection, is very easy to prove and doubtless well­known.) (Received November 7, 1962.)
63T-34. WITHDRAWN.

63T-35. L. J. SENECHALLE, 2240 West 113th Street, Chicago 43, Illinois. A functional calculus for uniform operators on a reflexive Banach space.

Suppose that $T$ is a closed linear transformation from a dense subset of a reflexive Banach space $B$ into $B$ such that the spectrum of $T$ is sparse (i.e., each continuous and complex-valued function on $\sigma(T)$ is uniformly approximable by rational functions). Suppose further that there is a number $K$ such that $|f(T)| \leq K \cdot |f|$ for each rational function $f$ which is nonsingular on $\sigma(T)$, where $|f| = 1, u, v, \in \sigma(T)$. $T$ is said to be a uniform operator on $B$. Then there exists a functional calculus for $T$ which is uniform and defined for all modulated functions on $\sigma(T)$ (bounded complex-valued functions which are uniformly approximable by rectangular step functions). I.e., there is a correspondence $\alpha: f \mapsto f(T)$ which associates with each modulated function on $\sigma(T)$ a bounded operator on $B$ in such a way that (i) $\alpha$ is linear and multiplicative; (ii) $f(T)$ has its ordinary meaning in case $f$ is rational; (iii) $|f(T)| \leq K \cdot |f|$. Techniques developed by E. R. Lorch (Trans. Amer. Math. Soc. 45 (1939), 217-234) are then applicable and the functional calculus is extensible in a uniform way to larger classes of functions. (Received November 9, 1962.)

63T-36. JORAM LINDENSTRAUSS, Yale University, New Haven, Connecticut. Extension of operators with range in a C(K) space.

The following theorem is proved: Let $X$ be a C(K) space, then the following three statements are equivalent: (i) For every two Banach spaces $Z \supseteq Y$ with dim $Z/Y = 1$ and for every operator $T$ from $Y$ into $X$ with a separable range, there is an extension $T$ from $Z$ into $X$ with $\|T\| = \|T\|$. (ii) With $Z, Y, T$ as above there is, for every $e > 0$ an extension $T$ with $\|T\| \leq (1 + e)\|T\|$. (iii) For every two Banach spaces $Z \supseteq Y$ with dim $Z = 3$, dim $Y = 2$ and every $T$ from $Y$ into $X$ there is an extension $T$ from $Z$ into $X$ with $\|T\| = \|T\|$. This theorem does not hold if $X$ is not assumed to be a C(K) space. (E.g., $c_0$ satisfies (iii) but not (ii).) Aronszajn and Panitchpakdi (Pacific J. Math. 1956) studied an extension property for cells in a space $X$ which can be shown to be equivalent to (i). One of their results can be reformulated (cf. also Henriksen, Pacific J. Math. 1957) to state that (i) is equivalent to (iv). Every two disjoint open $F_\sigma$ sets in $K$ are completely separated. (Received November 9, 1962.)

63T-37. D. W. DUBOIS, University of New Mexico, Albuquerque, New Mexico. A note on direct sums of cotorsion groups.

The following theorem is proved: Let $A_i$ for each $i$ in $I$, be a torsionfree abelian group with $\text{Ext} (\mathbb{Q}, A_i) = 0$. Then $\text{Ext} (\mathbb{Q}, \sum_{i} A_i) = 0$ if and only if almost all $A_i$ are divisible. The proof uses properties of Hom, Ext and the obvious monomorphism $\delta(\mathbb{Q}, A): \sum_i \text{Hom} (\mathbb{Q}, A_i) \rightarrow \text{Hom} (\mathbb{Q}, \sum_i A_i)$. (Q is the additive group of the rationals.) (Received November 9, 1962.)
63T-38. BRANKO GRÜNBAUM, Hebrew University, Jerusalem, Israel, and T. S. MOTZKIN, University of California, Los Angeles, California. On polyhedral graphs.

A graph G is k-polyhedral if it is the graph of the vertices and edges of a k-dimensional convex polyhedron (a realization of G). Every k-polyhedral graph contains k + 1 nodes all connected by pairwise disjoint paths. There exist graphs G that are dimensionally (strongly; weakly) ambiguous, i.e., possessing realizations of different dimension (of same dimension but combinatorially different; combinatorially equivalent but such that some set of nodes of G determines a face in one realization and not in the other); none of these graphs is k-polyhedral for k ≤ 3. Procedures for the formation of polyhedral graphs are discussed. Among various open problems is mentioned: Prove that every k-polyhedral graph (k ≥ 4) is 4-polyhedral. The paper will appear in Proceedings of Symposia in Pure Mathematics, Vol. 7. (Received October 30, 1962.)


Linearly solvable difference schemes are investigated for the numerical solution of the first boundary value problem for parabolic equations of the form (*) (\(\partial^2 u / \partial x^2\))(a(x,t)(\partial u / \partial x)) + b_1(x,t,u, (\partial u / \partial x)) \partial u / \partial t = b_2(x,t,u, (\partial u / \partial x)) or (**) (\(\partial / \partial x\))(a(x,t,u)(\partial u / \partial x)) - \partial u / \partial t = b(x,t,u). The difference schemes are called linearly solvable because their solutions are obtained simply by solving a linear tridiagonal system of simultaneous equations at each time step. To obtain linearly solvable difference schemes for (*) and (**), three-level difference equations are utilized. Under suitable conditions, it is proved that these difference schemes are unconditionally stable and convergent, and that the discretization error is \(O(k^2 + h^2)\). (Received November 5, 1962.)

63T-40. ANDREW SOBCZYK, Box 8052, University of Miami, Coral Gables 46, Florida. Projections, retracts, and C*-embedding.

For a subset R of a topological space S, let \(C_R(S)\) denote the subspace of \(C(S)\) of bounded real continuous functions which vanish on R, and \(C_1(R)\) the subspace of \(C(R)\) of those functions w which are restrictions to R, \(\rho w = w\), of functions \(z \in C(S)\). For each \(w \in C_1(R)\), there is an extension \(x \in C(S)\) with the same upper and lower bounds as w; the isometry \(\gamma w = x\) is linear by the Ulam-Mazur theorem if its range is linear. A transformation \(\gamma\) of \(C_1(R)\) onto \(M \subseteq C(S)\) is extending if for \(x = \gamma w, x(s)\) is an extension on \(S\) of \(w(r)\) on R. Theorems: A (not necessarily closed) linear subspace \(M \subseteq C(S)\) is a complement of \(C_R(S)\) iff M is the range of an extending linear transformation \(\gamma\) on \(C_1(R)\). In the case of R closed and S compact metric, there is a linear isometry \(\gamma\) of \(C_1(R) = C(R)\) onto \(M \subseteq C(S)\). Therefore in that case M is a closed complement, and the projection \(P = \gamma \circ \rho\) of \(C(S)\) onto M, which has \(C_R(S)\) as null-space, has bound 1. In general if \(C_1(R)\) is complete and if there is an M such that \(\gamma = \rho^{-1}\) on \(C_1(R)\) to M is bounded, then the projection through \(C_R(S)\) onto M is continuous. Subspace \(C_1(R)\) is complete, with \(C_1(R)\) a proper subspace of \(C(R)\), in case R is \(A\)-embedded but not \(C^*\)-embedded. Other cases are examined, and examples are given. (Received November 2, 1962.)
Let $B_y, y \in \Gamma$, be a family of Boolean algebras and let $A$ be a Boolean algebra such that for every $y \in \Gamma$ there exists a monomorphism $h_y: A \rightarrow B_y$. The generalized free product of the family $B_y, y \in \Gamma$, with the amalgamated subalgebra $A$ is a Boolean algebra $C$ satisfying the following conditions: (1) For every $y \in \Gamma$ there exists a monomorphism $f_y: B_y \rightarrow C$ such that for every pair $y', y' \in \Gamma$, $f_y h_y = f_{y'} h_{y'}$. (2) $\bigcup_{y \in \Gamma} f_y(B_y)$ generates $C$. (3) If $D$ is a Boolean algebra and if $g_y, y \in \Gamma$ is a family of homomorphisms $g_y: B_y \rightarrow D$ such that for every pair $y', y' \in \Gamma$, $g_{y'} h_{y'} = g_{y''} h_{y''}$, then there exists a homomorphism $g: C \rightarrow D$ such that $g_{y'} = g_{y''}$ for every $y \in \Gamma$. It follows from this definition that for every $y$, $y \in \Gamma$, $f_y(B_y \cap y(B_y))$ is isomorphic to $A$. The generalized free product of groups with an amalgamated subgroup was discussed in (H. Neumann, Amer. J. Math. 70 (1948), 590-625). It is proved that the generalized free product of Boolean algebras with an amalgamated subalgebra exists and is unique up to isomorphisms. The topological dual of this result is also of interest and a brief topological proof has been obtained. (Received November 13, 1962.)

63T-42. M. S. ROBERTSON, 361 Lenox Avenue, Middletown, New Jersey. Some radius of convexity problems.

If $g(z) = z^{-1} + \sum_{n=0}^{\infty} a_n z^n$ is regular in $0 < |z| < 1$ and if $\Re \{-zg'(z)/g(z)\} > 0$, $0 \leq \beta < 1$, $|z| < 1$, say that $g(z)$ is starlike of order $\beta$ with respect to the unit circle. For a given $\beta$ let $\sum(\phi)$ denote the class of all such functions $g(z)$. The radius of convexity for the class $\sum(\phi)$ is the number $R(\beta)$ defined as $R(\beta) = \inf g \rho = \max \rho$ where $w = g(z)$ maps $0 < |z| \neq \rho$ onto a schlicht domain whose complement is a convex region. The author shows that $R(0) = 3^{-1/2}$, $R(1/2) = (2(3)^{1/2} - 3)^{1/2}$. The proofs depend on a more general theorem, obtained by variational methods, that $\min_p \min_{|z| = r} \Re F(P(z), zP'(z))$ for a given analytic function $F(u, v)$ and where $\Re P(z) > 0$ in $|z| < 1$, $P(0) = 1$, is always attained by an extremal function $P(z)$ described by the equation $(P(z) - 1) (P(z) + 1)^{-1} = (bz - z^2)(1 - \bar{b}z)^{-1}, b = \cos \phi + \alpha \sin \phi, -1 \leq \alpha \leq 1$. (Received November 14, 1962.)


Representations of $A^+$, the generalized inverse of an arbitrary rectangular complex matrix $A$, have been given by various authors, (e.g., R. Penrose, Proc. Cambridge Philos. Soc. 52., 1 (1956), 17-19; A. Ben-Israel and S. J. Wersan ONR Report 61, Northwestern University, June 1962) using the least squares properties of $A^+$. Call such representations Euclidean. The new Euclidean representation $A^+ = P \begin{bmatrix} I_r & \Delta \end{bmatrix} \begin{bmatrix} I_r + \Delta^* A^* \end{bmatrix}^{-1} E A^*$ -- where $P$ is a permutation matrix, $E$ is a product of elementary (row operator) matrices such that $E A^* A P$ is a matrix whose last $(n - r)$ rows are zero and whose first $r$ rows may be written $\begin{bmatrix} 1_r \Delta \end{bmatrix}$. It appears to have computational advantages over existing representations. (Received October 18, 1962.)
Toda's toric construction resembles a homotopy theoretic version of the Massey triple product, W. S. Massey (International Symposium on Algebraic Topology, Mexico City, 1956) generalized the triple product by means of specific formulas and as differentials in a spectral sequence. The formulas, translated in terms of homotopies, do give longer products, for which the following sort of theorem holds (only the version for products of length three is given here). **Theorem.** Let $W, X, Y, Z$ be locally compact Hausdorff spaces with base points; $X^W, \ldots$, the function spaces of base-point preserving maps. There is a spectral sequence such that

$$E^1_{k+i+m} = E^{0, k+l+m+1}_{1} \Rightarrow H_k(W) \otimes H_{l}(Y^X) \otimes H_{m}(Z^Y),$$

$$E^2_{k+i+m+1} = E^{0, k+l+m+1}_{1}.$$  

Suppose $a \in \pi(S^k W, X)$, $\rho \in \pi(S^l X, Y)$, $\gamma \in \pi(S^m Y, Z)$, and $a \ast S^l \rho = \gamma \ast S^m \beta = 0$. Let $a, b, c$, be the Hurewicz images of $a, \rho, \gamma$ in $H_k(W), H_l(Y^X), H_m(Z^Y)$ resp. Then, $d_1(a \ast b \ast c) = 0$, and $d_2(a \ast b \ast c)$ is, up to sign, the Hurewicz image of $\{\gamma, S^m \beta, S^l \alpha\}$ in $E^{0, k+l+m+1}_{2}$. (Received November 14, 1962.)

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**63T-45.** S. J. STEIN, University of California, Davis, California.

**Discrete full sets.**

A set $A$ is full if each element of $A$ is also a subset of $A$. A set is discrete if any two distinct elements of $A$ have a void intersection. **Theorem 1.** There is precisely one discrete full set of cardinality $n$, where $n$ is finite. **Theorem 2.** There is only one infinite discrete full set, and it is denumerable. The proofs depend primarily on the Axiom of Regularity. (Received October 18, 1962.)

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**63T-46.** WITHDRAWN.

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**63T-47.** D. G. BABBITT, University of California, Los Angeles 24, California. **A summation procedure for certain Feynman integrals.**

This is an extension of an earlier note (A summation procedure for certain Feynman integrals, Notices Amer. Math. Soc. 9 (1962), 402). **Theorem.** Let $\Omega$ be a subset of $E^m$ with piecewise smooth boundary. Let $V(x)$ be a continuous real-valued function on $\Omega$, bounded on bounded subsets of $\Omega$ and bounded below everywhere in $\Omega$. Then for $t > 0$, $\Re(\sigma) > 0$, $\forall(x) \in L^2(\Omega)$, the $L^2$ double limit:

$$l.i.m. \sigma \rightarrow 1, l.i.m. n \rightarrow \infty (2\pi \sigma t/n)^{-m/2} \int_{\Omega} \cdots \int_{\Omega} \forall(x_0) \prod_{j=1}^{n} \exp\left[ - (x_j - x_{j-1})^2 (2\sigma t/n)^{-1} - \sigma t/n V(x_{j-1}) \right] \, dx_0 \cdots dx_{n-1},$$

exists and equals $\forall(x,t)$ and $\|\forall(x,t)\|_2 = \|\forall(x)\|_2$. For $\forall(x) \in \mathcal{D}\{(1/2)\Delta - V\}$, $\forall(x,t)$ is a solution of Schrödinger's equation $i[(1/2)\Delta - V]\forall(x,t) = \partial \forall(x,t) / \partial t$. Similarly for $\sigma \rightarrow - 1$.

**Remark.** If the double limit, in reverse order, exists and is weakly continuous in $t$, then it equals $\forall(x,t)$. The proof of the theorem is a modification of the proof of the earlier result. (Received November 15, 1962.)

For notation see Tarski, Indag. Math. 16, 572–588. The definitions of ultraproduct and limit ultrapower may be found in abstracts 550-7 (Vol. 5, p. 673) and 574-35 (Vol. 7, p. 878), respectively. Let $K$, $L$ range over arbitrary classes of relational systems of a fixed similarity type. Define $K^* = \bigcap \{ L: K \subseteq L \in \mathrm{PC}_\Delta \}$. Theorem. $A \in K^*$ if and only if $A$ is isomorphic to a limit ultrapower of an ultraproduct of some sequence of members of $K$. Corollaries. (1) $K$ is an intersection of $\mathrm{PC}_\Delta$ classes if and only if $K$ is closed under isomorphisms, ultraproducts, and limit ultrapowers. (2) $K^*$ is the union of some ascending chain of $\mathrm{PC}_\Delta$ classes. (3) If $K$ is a set, then $K^* \in \mathrm{PC}_\Delta$. (4) In weak second order logic (see Abstract 550-6, Vol. 5, p. 673), let $\mathrm{PC}_w$ be the notion corresponding to $\mathrm{PC}_\Delta$. Then $K$ is an intersection of $\mathrm{PC}_w$ classes if and only if $K = I(K)$. Moreover, if $K$ is a set, then $I(K) \in \mathrm{PC}_w$. (Received November 15, 1962.)

63T-49. N. J. HICKS, University of Michigan, Ann Arbor, Michigan. Submanifolds.

Let $\bar{M}$ be a semi-Riemannian manifold with Riemannian connexion $\bar{D}$. Let $M$ be a submanifold of $\bar{M}$ which is either space-like or time-like. For fields $X$ and $Y$ on $M$, define $D_XY$ and $V(X, Y)$ by decomposing $D_XY$ into tangential and normal components, $D_XY = D^T_XY + V(X, Y)$. Then $D$ is the Riemannian connexion on $M$ and $V$ is the second fundamental form tensor. Apply and use this formulation to generalize the fundamental equations of classical differential geometry, and prove a generalization of the fundamental theorem of surface theory concerning the local existence of a $k$ dimensional submanifold of $\mathbb{R}^n$ with prescribed first and second fundamental forms. (Received November 15, 1962.)

63T-50. WALTER RUDIN, University of Wisconsin, Madison 6, Wisconsin. Idempotents in group algebras.

Let $I(G)$ be the set of all idempotents in the group algebra of a group $G$, i.e., the set of all complex functions $f$ on $G$ such that $\|f\| = \sum |f(y)| < \infty$ and $(tx) = \sum f(xy^-1)f(y)$ for all $x$ in $G$. (The sums extend over all $y$ in $G$.) The support group of $f$ is the smallest subgroup of $G$ which contains the support of $f$. If $G$ is abelian and $f \in I(G)$, it is known that the support group of $f$ is finite; also, $\|f\| \geq (5/4)^{1/2}$ if $\|f\| > 1$. The present note shows that these results do not extend to noncommutative groups. For example, if $G$ is generated by elements $a$ and $b$, subject to the relations $a^3 = 1$, $aba = b$, and if $r > 1$, there exists $f \in I(G)$ with $\|f\| = r$ and infinite support; there also exists $f \in I(G)$, with $\|f\| = r$, which has finite support but infinite support group. However, if $G$ is any group, if $f \in I(G)$, and if $\|f\| = 1$, then $f$ has finite support group $H$, of order $h$, and $f(xy) = h f(x)f(y)$ for all $x, y$ in $H$. (Received November 15, 1962.)

63T-51. TAQDIR HUSAIN, University of Ottawa, Ottawa 2, Ontario, Canada. On almost continuous mappings.

Let $E$ and $F$ be two topological spaces and $f$ a mapping of $E$ into $F$. $f$ is said to be almost
continuous at } \mathbf{x} \in E \text{ if, for each neighborhood } V \text{ of } f(\mathbf{x}) \in F, \text{ Cl } f^{-1}(V) \text{ (Cl = closure) is a neighborhood of } \mathbf{x}. \text{ Besides other results concerning the comparison between almost continuity, approximate continuity and quasi-continuity, the following theorems have been proved: Theorem. Neither almost continuity of a mapping } f \text{ implies that the graph of } f \text{ is closed, nor the closed graph of } f \text{ implies that } f \text{ is almost continuous. Theorem. A locally convex Hausdorff linear (l.c.) space } E \text{ is barrelled if and only if, for every } B_{\varepsilon}\text{-complete l.c. space } F, \text{ the following statement is true: Every linear mapping of } E \text{ into } F \text{ is almost continuous. Theorem. Let } f \text{ be a linear one-to-one mapping of an l.c. space } E \text{ onto an l.c. space } F_{w}, \text{ where } w \text{ is the finest locally convex topology on } F. \text{ Then } f \text{ is almost continuous if and only if } E \text{ is barrelled. For linear spaces the latter two theorems generalize the theorem due to Bradford and Goffman (Proc. Amer. Math. Soc. 11 (1960), 667-670). (Received November 16, 1962.)}

63T-52. ECKFORD COHEN, University of Tennessee, Knoxville, Tennessee. An analogue of the totient function.

Let } v(n) \text{ denote the number of zeroid elements of the multiplicative semigroup of the integers } \mod n; \text{ that is, the number of residue classes } \mod n \text{ whose square is the zero class. On the basis of its multiplicative properties, } v(n) \text{ is evaluated in terms of the Euler } \phi \text{-function. Using this evaluation, it is shown that } V(x) \sim 3x \log x/\pi^2, \text{ where } V(x) = \sum_{n \leq x} v(n). \text{ A refinement of this estimate is also proved. A second proof of the evaluation of } v(n), \text{ valid for the case } \Omega \text{ odd, is included. (Received November 16, 1962.)}

63T-53. P. G. KUMPEL, JR., Brown University, Providence, Rhode Island. On the homotopy groups of the exceptional groups.

Let } G_2, F_4, E_6 \text{ denote compact forms of these exceptional Lie groups. It is shown that for all } i \text{ the sequence } 0 \rightarrow \pi_1(F_4) \rightarrow \pi_1(E_6) \rightarrow \pi_1(E_6/F_4) \rightarrow 0 \text{ is exact and split modulo } \mathfrak{C}_2, \text{ the class of 2-primary abelian groups. The splitting is given by the map } q: E_6/F_4 \rightarrow E_6, q(xF_4) = xo(x)^{-1}, x \sigma \text{ the involutive automorphism of } E_6 \text{ having } F_4 \text{ as fixed point set. This result, together with results of B. Harris (Ann. of Math. 76 (1962)), enables us to state the Theorem. If } G/K \text{ is a compact irreducible Riemannian globally symmetric space, and if } K \text{ is totally nonhomologous to zero in } G \text{ with real coefficients, then for all } i \text{ the sequence } 0 \rightarrow \pi_i(K) \rightarrow \pi_i(G) \rightarrow \pi_i(G/K) \rightarrow 0 \text{ is exact and split modulo the class } \mathfrak{C}_2. \text{ It is also shown that for all } i \text{ the sequence } 0 \rightarrow \pi_i(F_4/G_2) \rightarrow \pi_i(F_4/Spin(8)) \rightarrow \pi_{i-1}(Spin(8)/G_2) \rightarrow 0 \text{ is exact and split modulo } \mathfrak{C}_3, \text{ the class of 3-primary abelian groups. The splitting is given by an analogue } E \text{ of the Bott suspension map, } E: \pi_{i-1}(Spin(8)/G_2) \rightarrow \pi_i(F_4/Spin(8)). \text{ (Received November 16, 1962.)}

63T-54. ARTHUR STEINBERG, 177 Stanton Street, New York 2, New York. On free nilpotent quotient images of single defining relation groups.

For groups } G \text{ presented on } n \text{ generators and a single defining relation the problem is posed: What is the maximal rank of a free quotient group of } G. \text{ Some partial results are obtained by considering possible ranks of free nilpotent quotient groups. Using theorems of G. Baumslag on "free nilpotent D_r groups" it is shown: Theorem. If } G \text{ is presented on the generators } a_1, b_1, \ldots, a_k, b_k,
and the relation \( \prod_{i=1}^{k} [a_i, b_i] = 1 \) then \( G \) has a free nilpotent quotient group of rank \( r \) and class 2 if and only if \( r \geq k \). (It is well known that \( G \) is the fundamental group of a closed orientable two-dimensional manifold of genus \( k \).) More generally, Theorem. A procedure is described for determining in a finite number of steps the maximal rank of a free nilpotent quotient group of class 2 of \( G \) if all the generators have exponent sum 0 in the relator. Also, one has: Theorem. If at least one generator has exponent sum \( \neq 0 \) in the relator one can decide whether \( G \) has a free nilpotent quotient group of rank \( n - 1 \) and prescribed class \( c \). (Received November 16, 1962.)

63T-55, J. J. WESTMAN, 1668 Greenfield Avenue, Los Angeles 25, California, An inverse limit of Banach algebras of continuous functions reducing to the constants.

A nontrivial example is given where the inverse limit of an (onto) inverse mapping system of Banach algebras over \( \mathcal{C} \), is just \( \mathcal{C} \). We use a Hausdorff space \( X \) satisfying the first axiom of countability, for which \( \mathcal{C}(X, \mathcal{C}) = \mathcal{C} \), (say as constructed by Urysohn, \( \text{"Uber die M"achtigkeit zusammenangender Mengen} \), Math. Ann. 94 (1925), 262-295). Let \( \{ X_a \} = D \) be the family of all compact subsets of \( X \), directed by containment, and let \( \lim \mathcal{C}(X_a, \mathcal{C}) \); \( \lim \mathcal{C}(X_a, \mathcal{C}) \) = \( \mathcal{C}(X, \mathcal{C}) = \mathcal{C} \). A slight modification of this approach yields an (onto) inverse mapping system of Banach algebras whose inverse limit is \( = [0] \). A further change yields an (onto) inverse mapping system of complete metric spaces whose inverse limit is empty. (Received November 17, 1962.)

63T-56, LEO MOSER, University of Alberta, Edmonton, Alberta, Canada, On the additive completion of sets of integers.

Let \( a_1 < a_2 < \ldots < a_k \) and \( b_1 < b_2 < \ldots < b_l \) be sets of integers such that every integer in the interval \([1, n]\) can be represented in the form \( a_i + b_j \). Trivially this implies \( kl \geq n \). In this paper four distinct proofs are presented that if the \( a \)'s are the squares not exceeding \( n \) then \( kl \geq n(1 + \varepsilon) \) for some fixed \( \varepsilon > 0 \). This settles a conjecture of P. Erdős. Two of the proofs are strictly elementary and two make use of exponential sums, One point of giving four distinct proofs is that each of these may be generalized in a different way to give corresponding results for other sets of numbers \( \{ a_i \} \). (Received November 17, 1962.)


Techniques similar to those used in neutron transport calculations may be adapted to a variety of problems in micro-wave and plasma devices. Here they are applied to the travelling wave tube in the one-dimensional approximation based on the transmission line equations of Brillouin (J. Appl. Phys. 20 (1949), 1196) and the Landau-Vlasov equations. The distribution function is expanded in Hermite functions in velocity using the theory of Pansions (Korevaar, Trans. Amer. Math. Soc, 91 (1959), 53). This permits quantities of physical interest such as density, current, etc, to be evaluated as in the method of characteristic functions of statistics. The equation for the determination of the coefficients in the expansion of the distribution function involves two Jacobi matrices which define essentially self-adjoint transformation in a Hilbert space. In any \( n \times n \) truncation, the system
is readily reduced to the standard form of a system of hyperbolic equations in two independent variables and the over-all approximate system is amenable to machine computation. This method has the advantage of requiring computation only with respect to the variables in $x$ and $t$ without at the same time sacrificing the velocity dependence which is essential to the physical problem.

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