THE AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by John W. Green and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings ........................................ 540
Program of the October Meeting in Brooklyn, New York ........ 541
Abstracts for the Meeting on pages 567-573

PRELIMINARY ANNOUNCEMENTS OF MEETINGS ..................... 544

REPORT ON THE AFFAIRS OF THE SOCIETY ........................ 548

ACTIVITIES OF OTHER ASSOCIATIONS ............................. 550

NEW AMS PUBLICATIONS ....................................... 551

PERSONAL ITEMS ............................................. 552

NEWS ITEMS AND ANNOUNCEMENTS ............................... 547, 549, 556

MEMORANDA TO MEMBERS

Summer Meeting - 1964 ........................................ 556

LETTERS TO THE EDITOR ...................................... 557

THE ANNUAL SALARY SURVEY .................................. 559

STARTING SALARIES FOR MATHEMATICIANS WITH A Ph.D. ........ 562

SUPPLEMENTARY PROGRAM - No. 20 ............................ 563

ABSTRACTS OF CONTRIBUTED PAPERS ............................ 566

ERRATA ................................................... 595

INDEX TO ADVERTISERS ....................................... 603

RESERVATION FORM ......................................... 603
### MEETINGS

#### Calendar of Meetings

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
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<tr>
<td>605</td>
<td>November 15-16, 1963</td>
<td>Atlanta, Georgia</td>
<td>Oct. 1</td>
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<tr>
<td>606</td>
<td>November 21-23, 1963</td>
<td>Pasadena, California</td>
<td>Oct. 1</td>
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<td>607</td>
<td>November 30, 1963</td>
<td>Madison, Wisconsin</td>
<td>Oct. 1</td>
</tr>
<tr>
<td>608</td>
<td>January 23-27, 1964</td>
<td>(70th Annual Meeting)</td>
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<td></td>
<td>Miami, Florida</td>
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<tr>
<td>609</td>
<td>February 29, 1964</td>
<td>New York, New York</td>
<td>Nov. 26</td>
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<tr>
<td>610</td>
<td>April 18, 1964</td>
<td>Reno, Nevada</td>
<td>Jan. 16</td>
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<td>611</td>
<td>April 20-23, 1964</td>
<td>New York, New York</td>
<td>Mar. 5</td>
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<td>April 24-25, 1964</td>
<td>Chicago, Illinois</td>
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<td>613</td>
<td>June 20, 1964</td>
<td>Pullman, Washington</td>
<td>May 7</td>
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<tr>
<td>614</td>
<td>August 24-28, 1964</td>
<td>(69th Summer Meeting)</td>
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<tr>
<td></td>
<td>January 25-29, 1965</td>
<td>Amherst, Massachusetts</td>
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<tr>
<td></td>
<td>(71st Annual Meeting)</td>
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<tr>
<td></td>
<td>August 30-September 3</td>
<td>Denver, Colorado</td>
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<tr>
<td></td>
<td>1965</td>
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<td></td>
<td>(70th Summer Meeting)</td>
<td>Ithaca, New York</td>
<td></td>
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<tr>
<td></td>
<td>August 1966</td>
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<tr>
<td></td>
<td>(71st Summer Meeting)</td>
<td>New Brunswick, New Jersey</td>
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<tr>
<td></td>
<td>August 1967</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(72nd Summer Meeting)</td>
<td>Toronto, Canada</td>
<td></td>
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</tbody>
</table>

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are November 19, 1963 and January 9, 1964.

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The NOTICES of the American Mathematical Society is published by the Society in January, February, April, June, August, October and November. Price per annual volume is $7.00. Price per copy $2.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island.

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540
The six hundred fourth meeting of the American Mathematical Society will be held at the Polytechnic Institute of Brooklyn on Saturday, October 26, 1963. All sessions will be held in Rogers Hall. NEW YORK IS ON DAYLIGHT SAVING TIME.

The Committee to Select Hour Hour Speakers for Eastern Sectional Meetings has arranged two invited addresses.

Professor Jean Pierre Kahane of the University of Paris and the Institute for Advanced Study will speak at 11:00 A.M. on "Lacunary Taylor and Fourier Series."

Professor Andre Haefliger of the University of Geneva and Columbia University will address the Society at 2:00 P.M. on "Smoothing Combinational Immersions."

There will be sessions for contributed papers both morning and afternoon. Abstracts of the papers to be presented at these sessions appear on pages 567-573 of these NOTICES. There are cross references to the abstracts in the program. For example, the title of paper (1) in the program is followed by (604 - 11) indicating that the abstract can be found under the designation 604 - 11 among the published abstracts. It will be possible to schedule a limited number of late papers.

The registration desk will be located in the main lobby of Rogers Hall. It will open at 9:00 A.M.

Luncheon will be available, cafeteria style, from 12:00 noon till 2:00 P.M. at the cafeteria in the Administration Building, which adjoins Rogers Hall and is directly connected to Rogers Hall. There are restaurants on Fulton and Lawrence Streets near Rogers Hall.

By courtesy of the Institute, coffee and doughnuts will be served from 8:45 A.M. till 11:00 A.M. in the cafeteria, which will serve as a social meeting room.

Parking space is available near Rogers Hall. Anyone expecting to travel by automobile should write to the Department of Mathematics, Polytechnic Institute of Brooklyn, Brooklyn 1, New York requesting a parking chit and directions to the appropriate parking lot. A sheet of travel instructions is available to anyone writing to the same address. The information below is extracted from it.

The Polytechnic Institute is at 333 Jay Street, Brooklyn. The Institute may be reached as follows:

From New York City and Westchester by car:

East River Drive or West Side Highway through Brooklyn Battery Tunnel ... Exit at Atlantic Avenue to Smith Street... Turn left onto Smith Street (which becomes Jay Street) 6 blocks to school. Across Manhattan Bridge into Flatbush Avenue, Extension to Myrtle Avenue ... Right turn on Myrtle Avenue, 4 blocks to school.

From Grand Central Terminal, Pennsylvania Station and other points in Midtown Manhattan by Subway:

IRT to Borough Hall Station. BMT to Borough Hall Station. IND to Jay Street, Borough Hall Station.
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between different sessions. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 10:00 A.M.

Session on Algebra, Rogers Hall 215

10:00 - 10:10
(1) Purity in simple systems. Preliminary report
Mr. L. E. De Noya, Ohio Wesleyan University and The Ohio State University (604-11)

10:15 - 10:25
(2) On the presentation of a group
Professor E. S. Rapaport, Polytechnic Institute of Brooklyn (604-17)

10:30 - 10:40
(3) Homomorphisms of d-simple inverse semigroups with identity. I
Professor R. J. Warne, Virginia Polytechnic Institute (604-1)

10:45 - 10:55
(4) Group rings of finite groups. I
Mr. J. A. Cohn and Professor Donald Livingstone*, University of Michigan (604-8)

SATURDAY, 10:00 A.M.

Session on Analysis, Rogers Hall 319

10:00 - 10:10
(5) On stability of sets of dynamical systems. Preliminary report
Dr. N. P. Bhatia, RIAS, Baltimore, Maryland (604-10)

10:15 - 10:25
(6) Lyapunov stability and periodic solutions
Professor J. C. Scanlon, Polytechnic Institute of Brooklyn (604-9)

10:30 - 10:40
(7) An analyst's fixed-point theorem (with an application to differential equations)
Mr. G. R. Morris, RIAS, Baltimore, Maryland (604-19)

10:45 - 10:55
(8) Asymptotic solution of a class of second order differential equations containing a parameter
Mr. G. A. Stengle, New York University (604-18)

SATURDAY, 11:00 A.M.

Invited Address, Rogers Hall 116

Lacunary Taylor and Fourier series
Professor Jean Pierre Kahane, University of Paris and the Institute for Advanced Study

SATURDAY, 2:00 P.M.

Invited Address, Rogers Hall 116

Smoothing combinatorial immersions
Professor André Haefliger, University of Geneva and Columbia University

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
SATURDAY 3:15 P.M.

Session on Algebra and the Theory of Numbers, Rogers Hall 215
3:15 - 3:25
(9) Cohomology and extensions of algebras
Professor Murray Gerstenhaber, University of Pennsylvania (604-4)
3:30 - 3:40
(10) Maximal commutative algebras with unlawfully small dimension. Preliminary report
Professor R. C. Courter, Rutgers, The State University (604-3)
3:45 - 3:55
(11) Remarks on simple extended Lie algebras
Professor A. A. Sagle, Syracuse University (604-5)
4:00 - 4:10
(12) Jordan algebras and bounded symmetric domains
Mr. Ulrich Hirzebruch, Massachusetts Institute of Technology (604-16)
4:15 - 4:25
(13) Commutators in groups of homotopy classes
Dr. M. A. Arkowitz, Princeton University and Professor C. R. Curjel*, Institute for Advanced Study and Cornell University (604-12)
4:30 - 4:40
(14) The fundamental theorem of arithmetic and the theorem of Schnirelmann
Mr. A. A. Mullin, University of Illinois (604-20)

SATURDAY, 3:15 P.M.

General Session, Rogers Hall 319
3:15 - 3:25
(15) Lattice-homomorphisms of lattices of continuous functions
Professor S. G. Mrowka, Pennsylvania State University (604-14)
3:30 - 3:40
(16) A general arc sine law and its application to diffusion processes
Professor S. M. Berman, Columbia University (604-2)
3:45 - 3:55
(17) Highly critical sets
Professor Arthur Sard, Queens College (604-15)
4:00 - 4:10
(18) Eigenvalues of a Bessel difference system of order zero
Professor J. J. Gergen and Professor F. G. Dressel, Duke University and Mr. G. B. Parrish*, U. S. Army Research Office, Durham, North Carolina (604-6)
4:15 - 4:25
(19) Dirichlet problem for constant-coefficient homogeneouse operators
Professor Shmuel Kaniel, University of Chicago (604-13)
(Introduced by Professor A. P. Calderón)
4:30 - 4:40
(20) The number of trees in a certain network
Mr. P. V. O'Neill, Rensselaer Polytechnic Institute (604-7)
(Introduced by Professor Paul Slepian)

Bethlehem, Pennsylvania

Everett Pitcher
Associate Secretary
Six Hundred Fifth Meeting
Georgia Institute of Technology
Atlanta, Georgia
November 15-16, 1963

The six hundred and fifth meeting of the American Mathematical Society will be held at the Georgia Institute of Technology on November 15-16, 1963. Sessions for contributed papers will be held in the Price Gilbert Library and in the adjoining Classroom Building. The invited address will be held in the auditorium of the A. French Textile School.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor Fred B. Wright of the Tulane University will speak on "Invertible elements in Banach Algebras" at 2:00 P.M., Friday, November 15.

There will be sessions for contributed papers beginning at 10:00 A.M. on Friday, November 15 and at 10:00 A.M. on Saturday, November 16. Abstracts of papers should be sent to the American Mathematical Society, Providence 6, Rhode Island, prior to the deadline of October 1, 1963.

The registration desk will be in the lower lobby of the Price Gilbert Library.

ACCOMMODATIONS

While neither the Georgia Institute of Technology nor the American Mathematical Society can assume responsibility for hotel accommodations, the Convention Housing Bureau (a non-profit, civic agency) has graciously offered its facilities in this regard. For your convenience in making reservations for the coming Regional Meeting at Georgia Tech on November 15-16, 1963, please use the reservation form at the back of these NOTICES. While none of the available hotels are within close walking distance of the campus, all are readily accessible by cab or public transportation. Since there are other meetings and conferences in Atlanta at that time, all requests for accommodations should be mailed to: Convention Housing Bureau, American Mathematical Society, 1102 Commerce Building, Atlanta, Georgia, 30303. All requests will receive prompt confirmation.

Hotel or Motel

<table>
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<tr>
<th>Hotel or Motel</th>
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TRAVEL


M. L. Curtis
Associate Secretary

Tallahassee, Florida
The six hundred sixth meeting of the American Mathematical Society will be held on Thursday, Friday, and Saturday, November 21-23, 1963 at the California Institute of Technology in Pasadena, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, and with the financial support of the National Science Foundation, a Symposium on Recent Developments in the Theory of Numbers will be held in conjunction with this meeting. The Program Committee for the Symposium consists of Professors Leonard Carlitz, D. H. Lehmer, W. J. LeVeque, and A. L. Whiteman, Chairman. The Symposium will include four sessions devoted to the following topics: First Session--Developments in diophantine analysis, class-field theory, and exponential and character sum estimations; Second Session--Research in the algebraic theory of numbers, the impact of the computer on number theory; Third Session--Number theoretic techniques in combinatorial mathematics; Fourth Session--General session to include recent research in analytic number theory and the theory of quadratic forms.

The principal addresses at the Symposium will be given by Professor B. J. Birch, The University, Manchester, England, Professor Leonard Carlitz, Duke University, and Professor Kenkichi Iwasawa, Massachusetts Institute of Technology. Those who have been invited to give 15 minute talks are: N. C. Ankeny, Tom M. Apostol, Paul T. Bateman, Sarvadaman Chowla, Eckford Cohen, Harvey Cohn, Bernard M. Dwork, Marshall Hall, Joseph Lehner, D. J. Lewis, William H. Mills, Louis J. Mordell, Leo Moser, Morris Newman, Ivan Niven, Gordon Pall, E. T. Parker, E. G. Straus, John T. Tate, Olga Taussky, D. H. Lehmer, William J. LeVeque, and Albert L. Whiteman.

In addition to the Symposium, there will be a regular sectional meeting of the Society on Saturday, including sessions for contributed papers. The Committee to Select Hour Speakers for Far Western Sectional Meetings has invited Professor Kai Lai Chung to present an hour address on Saturday. The title of Professor Chung's talk is "Boundary theory for Markov chains".

The registration desk for the meeting will be located in the Sloan Laboratory of Mathematics and Physics. All sessions of the Symposium will be held in Room 151 Sloan.

The following hotels and motels in Pasadena are reasonably near the California Institute of Technology campus. Reservations for rooms should be made directly with the chosen hotel or motel.

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<td>Huntington-Sheraton</td>
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At noon on Saturday, November 23, a luncheon will be held at the Athenaeum for persons attending the meetings. There will also be a tea at the Athenaeum on Saturday afternoon after the last session of the meeting.
The California Institute of Technology is located on California Street, between Hill Avenue and Wilson Avenue. It is easily reached by automobile from downtown Los Angeles, taking the Pasadena Freeway (Arroyo Parkway) into Pasadena, then turning east on California Street for approximately one mile. There is a parking lot on the south side of California Street, directly opposite the Sloan Laboratory of Mathematics and Physics. Frequent limousine service is available from the Los Angeles International Airport to Pasadena, and there are buses from the Los Angeles railroad terminal to Pasadena.

R. S. Pierce
Associate Secretary
Seattle, Washington

Six Hundred Seventh Meeting
University of Wisconsin
Madison, Wisconsin
November 30, 1963

The six hundred seventh meeting of the American Mathematical Society will be held at the University of Wisconsin on Saturday, November 30. All sessions will be in Van Vleck Hall.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor Louis de Branges of Purdue University will address the Society. He will speak on "New and old problems for entire functions". In addition, several sessions are planned for the presentation of contributed papers.

Madison may be reached by car, bus (Greyhound), rail (Milwaukee Road) and air (North Central, North West and Ozark Air Lines). For complete details members should consult a travel agent.

The following hotel and motel information may be useful in planning the trip:

### Hotels and Motels

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<td>ALpine 6-0231</td>
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</table>
All of these hotels and motels are within a mile of Van Vleck Hall.

There are a number of good restaurants in Madison. Details will be available at the meeting.

J.W.T. Youngs
Associate Secretary
Princeton, New Jersey

NEWS ITEMS

PROGRAM OF VISITING LECTURERS IN STATISTICS


The presidents of the above mentioned statistical organizations initiated the present program because of the rapid development of modern statistics and the scarcity of statisticians in colleges and universities today. These factors make it even more necessary in statistics than in the older and better known scientific areas, to give insight and to correct misimpressions regarding the nature of the subject and the opportunities in the area.

The Organizing Committee for these lectures consists of R. L. Anderson, D. J. Bogue, R. A. Bradley, G. J. Lieberman, and J. Kiefer, Chairman. Brochures describing the program’s operation were mailed to colleges throughout the U. S. in August. Institutions not receiving the brochure or requiring further information should write to Professor J. Kiefer, Department of Mathematics, Cornell University, Ithaca, New York.

INSTITUTE OF MATHEMATICAL SCIENCE, MADRAS

The Institute of Mathematical Sciences, Madras, India, sponsors a visiting scientists program under which theoretical physicists and mathematicians from Europe, the United States and Japan are invited to spend a few weeks or months at the Institute and work in association with the members of the permanent staff. During their stay, they are considered members of the Institute and are free to do their own research, give lectures, hold seminars on topics of their choice, and collaborate with the members of the Institute. On behalf of the Board of Governors, invitations are extended periodically by the Director, Professor Alladi Ramakrishnan. The Institute pays for travel expenses by air and provides living allowances.

Visits of scientists who wish to pursue post-doctoral studies at Madras are also sponsored by the Institute either by invitation or on request. Lectures by eminent scientists who visit Madras are arranged and honoraria are paid by the Institute. The lectures delivered by visiting scientists and by members of the Institute are being published periodically as MATSCIENCE REPORTS and circulated to various centers of learning. Among the reports published so far are "Lectures on Weak Interactions" by R. E. Marshak, "Lectures on Foundations of Quantum Mechanics and Field Theory" by E. C. G. Sudarshan, and "Lectures on Differential Equations" by Einar Hille. Visitors to the Institute in the year 1963 include S. Abhyankar, Johns Hopkins University; A. T. Bharucha-Reid, Wayne State University; Marshall Stone, University of Chicago; and Richard Bellman, The Rand Corporation, Santa Monica, California.
At this time I would like to report on some recent activities, future plans, and present problems of the Society. First let me say that this is the largest summer meeting in the history of the Society. Registration is 1188*, as compared with the highest previous registration of 972, achieved at Massachusetts Institute of Technology in 1958.

The 69th Summer Meeting will be held at the University of Massachusetts in Amherst during the week of August 24 to August 28, 1964.

Following are a number of recent actions and decisions by the Council. There will be in 1964 a Summer Institute in Algebraic Geometry, under the chairmanship of Professor Oscar Zariski. The 1964 Spring Symposium in Applied Mathematics will be on Mathematical Physics in April in New York, under the chairmanship of Professor Robert Finn. There will be a new book in the Survey Series—"Uniform Spaces" by John R. Isbell. The Society will shortly publish (in an inexpensive paper-back edition) a Chinese-English dictionary of mathematical terms.

It should be mentioned that an increase in the Society expenses in a number of areas—expenses of meetings, postal costs, etc., will probably make it necessary to examine before long the question of dues which have been the same for 12 years.

As usual in recent years, our most serious and continuing problem has to do with the financial support of Mathematical Reviews. At the joint meeting in Providence last May 27-28, the Executive Committee and Board of Trustees, in discussing the MR. problem directed the President to appoint a committee to investigate this matter from the broadest point of view, and instructed the Secretary to prepare a report to be printed in the Notices, on the anticipated crisis and measures now being taken to prevent it. I can make no better report than to read from the letter which I wrote briefing the new committee on its duties.

Professors E. F. Beckenbach
Andrew Gleason
N. Kazarinoff
Marston Morse
D. C. Spencer
John Wehausen

"Dear Colleagues:

As you no doubt know, Mathematical Reviews has been in a state of financial instability for a number of years, and there is some reason to suppose that this matter may reach the crisis state next spring. Briefly and bluntly, this situation is this: At its present level of operation, Mathematical Reviews incurs a deficit of nearly $200,000 a year out of a total budget of about $360,000. There seems to be little chance of either reducing the deficit to something in the neighborhood of zero, if the present kind of operation is retained, or increasing the income from the mathematical community through increased subscriptions, price, or dues. Recently, the deficit has been covered by the National Science Foundation, to which we are immensely grateful. The present grant expires at the end of May, 1964, and there is some doubt whether or not the N.S.F. will be in a position to renew the grant. If it is not and if no other large scale subsidization is found, and if the operation is continued on any scale remotely resembling its present one, Mathematical Reviews can probably continue until the end of 1964 and will then go out of existence.

The Trustees and Executive Committee, meeting jointly in Providence on last May 26, moved that a select committee be appointed to examine ways and

* Final Attendance Report: Number of Registrants - 1305; Estimated Number of Additional Persons (wives and children) - 572.
means of saving Mathematical Reviews in its present form, and to consider alternatives if it cannot be saved. Professor Doob is asking you to form this committee, under the chairmanship of Professor E. F. Beckenbach.

Mathematical Reviews has continued for twenty-three years in a certain pattern. It is an excellent pattern, and although there have been ups-and-downs, Mathematical Reviews is a flourishing enterprise, scientifically speaking. There has not been, up to now, any reason for the Editors to wish to change the pattern by which Mathematical Reviews has been operated. However, financial pressure may well be such that we will have to change things drastically. For example, one proposal frequently brought up is that great use of authors' abstracts be made. This could conceivably yield a considerable saving. In order to examine this possibility, it has been suggested that for a period of time, Society publications and perhaps certain cooperating journals require authors' abstracts. These could then be collected and examined for suitability as Mathematical Review articles. Your committee might possibly wish to suggest some such experiment. Or, it might wish to suggest that Mathematical Reviews be replaced by any one of the various indexing services that have been proposed. Some people suggest that we publish only a list of articles, or that we agree to supply an author's abstract on demand, or various other possibilities. Or, it might wish to advise that anything short of the present operation is not worth the bother and to recommend that we abandon the whole field to the Germans and Russians.

To summarize, and perhaps simultaneously to overstate the case, Mathematical Reviews has been run on the theory that it should be as perfect as possible, and the cost was not the primary concern. This has been a perfectly good theory so far, but it does not seem feasible to continue with it. What is needed now is a look at what we can do, if anything, with the resources likely to be available.

I hope that all of you will be able to accept this charge. The situation is a grave one. Members of the Committee are known for their dedication to mathematics and the welfare of the Society, and by taking serious interest in the work of this committee, will do great service to the Society.

Sincerely yours

John W. Green

NEWS ITEMS AND ANNOUNCEMENTS

YESHIVA UNIVERSITY
October 14-15, 1963

A conference on "Some Recent Developments in the Basic Sciences," to be sponsored by the Belfer Graduate School of Science of Yeshiva University, will be held on October 14 and 15, 1963. The annual award of the Belfer Graduate School for distinguished service to science will be presented at the conference.

The schedule of the program will be:

Monday, October 14 - Physics
Maurice Goldhaber 9:30 - 10:30 A.M.
Julian Schwinger 11:00 - 12:00 A.M.
Henry Primakoff Chairman for morning session

Tuesday, October 15 - Mathematics
S. S. Chern 9:30 - 10:30 A.M.
Oscar Zariski 11:00 - 12:00 A.M.
Adrian A. Albert Chairman for morning session

C. N. Yang 2:00 - 3:00 P.M.
T. Gold 3:30 - 4:30 P.M.
Elliott W. Montroll Chairman for afternoon session

Arne Beurling 2:00 - 3:00 P.M.
Donald Spencer 3:30 - 4:30 P.M.
M. Marston Morse Chairman for afternoon session
MAA ANNOUNCES COOPERATIVE SUMMER SEMINAR

The Mathematical Association of America, through a grant from the International Business Machines Corporation, will sponsor a Cooperative Summer Seminar for college teachers of mathematics during the summer of 1964. This seminar can accommodate approximately thirty participants and will be held during the period June 22-August 14, at Cornell University, Ithaca, New York.

Activities will include daily lectures on Probability by Professor Marc Kac of the Rockefeller Institute and on Geometry by Professor Ernst Snapper of Dartmouth College, occasional guest lectures, and participation in seminars.

Participants will be limited to those mathematicians who teach in a college or university which offers a major program in mathematics leading to the B.A., or possibly the M.A., but which does not offer the Ph.D. in mathematics. Moreover, it will be necessary for the home institutions of applicants to guarantee, during the term following the summer seminar, a reduced teaching load for participants so they may conduct a seminar for the benefit of their colleagues and so they may have some free time to pursue mathematical activity initiated during the summer. In this way, the participants' institutions cooperate in the program, which is designed to stimulate increased mathematical activity on the part of undergraduate teachers.

Each participant will receive a stipend of $2,000 plus necessary travel expenses. Living accommodations will be provided at low cost to participants, but no extra allowance is provided for dependents, and no family housing arrangements are envisioned. It is expected that the participants will live and work together during the session in order to benefit most fully.

In charge of arrangements is the MAA Committee on Cooperative Summer Seminars:

Professor E. A. Cameron
University of North Carolina

Professor Samuel Goldberg
Oberlin College

Professor Marc Kac
The Rockefeller Institute

Professor Ernst Snapper
Dartmouth College

Dr. D. L. Thomsen
International Business Machines Corporation

Professor Robert J. Wisner
New Mexico State University
Chairman

Further information and requests for application forms, which must be completed and in the hands of the Committee before November 25, should be directed to Professor Wisner, Department of Mathematics, New Mexico State University, Las Cruces, New Mexico.

THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS

Annual Spring Meeting in 1964

Special sessions in honor of Theodore Von Kármán are being planned for the annual Spring Meeting in 1964 of the Society for Industrial and Applied Mathematics. Presentations at this meeting will reflect von Kármán's mathematical impact on science and technology. A special committee to plan this event has been appointed.
by the Society: W. R. Sears, Chairman, F. E. Marble, C. C. Lin, and D. L. Thomsen. The Committee will be pleased to receive for its consideration any abstracts of papers which might be suitable for presentation at the meeting and also, particularly in the near future, any appropriate suggestions of a general nature which would be helpful in planning this very special event.

THE SOCIETY FOR NATURAL PHILOSOPHY
Pittsburgh, November 11-12

The Society for Natural Philosophy will hold a meeting on the subject of Fluid Mechanics at Mellon Institute, Pittsburgh, Pennsylvania on November 11 and 12. The principal lectures will deal with the Navier-Stokes equation, viscoelastic fluids, and the structure of galaxies. The speakers will include C. Truesdell, J. Serrin, R. Finn, W. Noll, C. C. Lin, and H. Markovitz. For more information write to Dr. B. D. Coleman, Mellon Institute, Pittsburgh, Pennsylvania, 15213.

NEW AMS PUBLICATIONS

SELECTED TRANSLATIONS SERIES II

Volume 31
88 pages; List Price $3.00; 25% discount to members

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume IV

THE DISPERSION METHOD IN BINARY ADDITIVE PROBLEMS
by
Ju. V. Linnik

196 pages; List Price $12.30; 25% discount to members
In this work the author makes further applications of the method he so brilliantly used for proving the Hardy-Littlewood conjecture confirming an asymptotic formula for the number of representations of a large number as the sum of two squares and a prime.
PERSONAL ITEMS

Dean A. A. ALBERT of the University of Chicago has been elected to a corresponding membership in the National Academy of Sciences of Buenos Aires, Argentina.

Associate Professor F. G. ASENJO of the Southern Illinois University has been appointed to an associate professorship at the University of Pittsburgh.

Dr. R. A. ASKEY of the University of Chicago has been appointed to an assistant professorship at the University of Wisconsin, Madison.

Professor LOUIS AUSLANDER of Purdue University has been appointed to a visiting professorship at the University of California, Berkeley for the academic year 1963-1964.

Assistant Professor ADI BEN-ISRAEL of the Carnegie Institute of Technology has been appointed Senior Lecturer at the Israel Institute of Technology, Haifa, Israel.

Assistant Professor G. E. BREDON of the University of California, Berkeley will be on sabbatical leave for the fall semester 1963 with residence in Berkeley.

Dr. T. F. BRIDGLAND, Jr. of the Martin Company has been appointed to an associate professorship at the University of South Carolina.

Associate Professor W. E. BRIGGS of the University of Colorado has been appointed Acting Dean of the College of Arts and Sciences of the University of Colorado for the academic year 1963-1964.

Professor F. E. BROWDER of Yale University has been appointed to a professorship at the University of Chicago. He will spend the next academic year on leave at the University of California, Berkeley and at the Institute for Advanced Study before taking up residence at the University of Chicago, July, 1964.

Mr. D. R. BROWN of Louisiana State University has been appointed to an assistant professorship at the University of Tennessee.

Assistant Professor R. E. BRYAN of Rutgers, The State University has been appointed to an associate professorship at Knox College.

Mr. ALPHONSE BUCCINO of the University of Chicago has been appointed to an assistant professorship and acting Chairman at DePaul University.

Dr. J. R. BUCHI of the University of Michigan has been appointed to a professorship in Mathematics and Computer Sciences at Purdue University.

Associate Professor L. L. CAMPBELL of the University of Windsor, Canada has been appointed to an associate professorship at Queen's University, Kingston, Canada.

Dr. R. C. CARSON of Goodyear Aerospace Corporation has accepted a position as Coordinator of Research at the University of Akron.

Mr. S. R. CAVIOR of Duke University has been appointed to an assistant professorship at the State University of New York, Buffalo.

Mr. YUNG MING CHEN of New York University has been appointed to an assistant professorship at Purdue University.

Professor S. S. CHERN of the University of California, Berkeley has been appointed Miller Research Professor for the academic year 1963-1964.

Associate Professor C. K. CHU of New York University will be at Columbia University for the year 1963-1964 as a Research Associate in the Plasma Physics Laboratory.

Assistant Professor AUBERT DAIGNEAULT of the University of Montreal has been appointed to a visiting assistant professorship at the University of California, Berkeley for the academic year 1963-1964.

Professor S. P. DILIBERTO of the University of California, Berkeley will be on sabbatical leave for the academic year 1963-1964.

Dr. S. I. DROBNIES of General Dynamics Corporation, Fort Worth, has been appointed to an assistant professorship at San Diego State College.

Associate Professor L. E. DUBINS of
the University of California, Berkeley will be on leave for the academic year 1963-1964. He will spend eleven weeks in Israel in the late fall, and during the remainder of the year will be in residence in Berkeley.

Dr. ROBERT ELLIS of the University of Pennsylvania has been appointed to a visiting professorship at Wesleyan University.

Assistant Professor CHESTER FELDMAN of Monmouth College has been appointed to an assistant professorship at Kent State University.

Dr. L. R. FORD, Jr. of CEIR, Incorporated has accepted a position as Director of the Mathematics Group at the Defense Research Corporation, Santa Barbara, California.

Professor BERNARD FRIEDMAN of the University of California, Berkeley will be on leave for the academic year 1963-1964. During the fall term he will participate in the work of the Committee on School Mathematics at the University of Illinois and for the spring term he will be on sabbatical leave, with residence in Berkeley.

Dr. WALTER GAUTSCHI of the Oak Ridge National Laboratories has been appointed Professor of Mathematics at the Computer Sciences Center, Purdue University.

Assistant Professor R. C. GILBERT of the University of California at Riverside and of the Mathematical Research Center, University of Wisconsin, has been appointed to an associate professorship at Orange State College.

Professor W. H. GOTTSCALTH of the University of Pennsylvania has been appointed to a professorship at Wesleyan University.

Dr. E. A. GRAHAM, Jr. of the University of Turin, Turin, Italy has accepted a position as Staff Consultant with the Space-General Corporation, El Monte, California.

Mr. C. A. GREATHOUSE of Florida State University has been appointed Acting Assistant Professor and Research Associate at the University of Tennessee.

Associate Professor LEON GREENBERG of Brown University will be a Visiting Member of the Courant Institute of Mathematical Sciences, New York University for the academic year 1963-1964.

Mr. R. M. GUINERSEN of the Illinois Institute of Technology has been appointed to a professorship at the University of Wisconsin, Milwaukee.

Professor P. C. HAMMER of the University of Wisconsin has returned to his position as Chairman and Professor of the Numerical Analysis Department after a year's leave of absence at the University of California, San Diego in La Jolla.

Associate Professor V. R. HANCOCK of Virginia Polytechnic Institute has been appointed to a professorship at Emory and Henry College.

Dr. K. A. HARDIE of the University of Cape Town, South Africa, who has been a Visiting Professor at Wayne State University, has been appointed to a visiting associate professorship at the University of California, Berkeley for the fall semester.

Professor M. H. HEINS of the University of Illinois has been appointed to a visiting professorship at the University of California, Berkeley for the academic year 1963-1964.

Associate Professor M. W. HIRSCH of the University of California, Berkeley will be on leave for the academic year 1963-1964. He has accepted an invitation to take part in a symposium on topology at Cambridge University.

Associate Professor P. F. HULTQUIST of the University of Colorado has accepted a position as a Senior Staff Scientist with the Ball Brothers Research Corporation, Boulder, Colorado.

Professor H. D. HUSKEY of the University of California, Berkeley will be on leave for the academic year 1963-1964, to take part in the Kampur Indo-American Project, of which the University of California is a joint sponsor. He will spend the year in Kampur, India.

Dr. E. C. JOHNSEN of the National Bureau of Standards, Washington, D. C., has been appointed a Lecturer at the University of California, Santa Barbara.

Mr. TSUNEKO KANO of the Tohoku University, Sendai, Japan has been appointed to a visiting assistant professorship and Assistant Research Mathematician at the University of California, Berkeley for the academic year 1963-1964.

Associate Professor J. W. KENELLY, Jr. of the University of Southwestern
Louisiana has been appointed to an associate professorship at Clemson College.

Professor P. B. KENNEDY of the University College, Cork, Ireland has been appointed to a professorship at the University of York, York, England.

Assistant Professor ADAM KORÁNYI of the University of California, Berkeley has been appointed to a visiting assistant professorship at the University of Chicago during the Autumn Quarter 1963.

Assistant Professor A. A. KOSINSKI of the University of California, Berkeley will continue as a member of the Institute for Advanced Study during the fall semester of 1963.

Dr. L. G. KOVACS of the Manchester College of Science and Technology, England, has been appointed to a Fellowship at the Institute of Advanced Studies, Australian National University, Canberra.

Dr. LEIF KRISTENSEN of the Matematisk Institute, Aarhus, Denmark has been appointed to a visiting assistant professorship at the University of California, Berkeley for the academic year 1963-1964.

Dr. BOR-LUH LIN of Northwestern University has been appointed to an assistant professorship at the State University of Iowa.

Mr. F. E. J. LINTON of Columbia University has been appointed to an assistant professorship at Wesleyan University.

Dr. MORTON LOWENGRUB of The University, Glasgow, Scotland has been appointed to an assistant professorship at Wesleyan University.

Associate Professor M. J. MANSFIELD of Washington and Jefferson College has been appointed to an associate professorship at Purdue University.

Mr. C. K. MEGIBBEN of Auburn University has been appointed to an assistant professorship at Texas Technological College.

Dr. L. E. MEHLENBACHER of the University of Detroit has been named Coordinator of Sponsored Institutes and Special Programs.

Associate Professor HENRYK MINC of the University of Florida has been appointed to a professorship at the University of California, Santa Barbara.

Dr. G. R. MORRIS of Brisbane, Australia, has joined the mathematics center at Martin Company's Research Institute for Advanced Studies here as a one year Visiting Member of the staff.

Professor Emeritus MARSTON MORSE of the Institute for Advanced Study has been elected Foreign Correspondent of the Polish Academy of Sciences.

Assistant Professor TADASHI NAGANO of the University of Tokyo, Tokyo, Japan has been appointed Visiting Assistant Professor and Assistant Research Mathematician at the University of California, Berkeley, for the academic year 1963-1964.

Dr. MORRIS NEWMAN of the U. S. Commerce Department's National Bureau of Standards has been appointed Chief of the Numerical Analysis Section in the Applied Mathematics Division.

Assistant Professor HAJIMU OGAWA of the University of California, Riverside will spend the academic year 1963-1964 at the University of California, Berkeley.

Dr. S. V. PARTER of Stanford University has been appointed to an associate professorship in the Numerical Analysis Department and the Mathematics Department at the University of Wisconsin.

Dr. R. E. PEINADO of the University of Nebraska has been appointed to an assistant professorship at the State University of Iowa.

Associate Professor W. M. PEREL of Randolph-Macon Woman's College has been appointed to a professorship at Charlotte College.

Assistant Professor BARTH POLLAK of Syracuse University has been appointed to an associate professorship at the University of Notre Dame.

Professor GEORGE PÓLYA was recently honored by Stanford University at its dedication of the new Computation Center when one of the two buildings was named Pólya Hall.

Dr. G. B. PRESTON of the Royal Military College of Science, Swindon, England has been appointed to a professorship at the Monash University, Victoria, Australia.

Associate Professor W. H. REID of Brown University has been appointed to an associate professorship in both the Departments of Geophysical Sciences and Mathematics at the University of Chicago.

Mr. M. A. RIEFFEL of Columbia University has been appointed a Lecturer at
the University of California, Berkeley. Professor R. M. ROBINSON of the University of California, Berkeley has been appointed Miller Research Professor for the fall term 1963, and will be on sabbatical leave during the spring semester, with residence in Berkeley.

Mr. R. L. ROTH of the University of California, Berkeley has been appointed to an acting assistant professorship at the University of Colorado.

Mr. J. L. ROVNYAK of Yale University has been appointed to an assistant professorship at Purdue University.

Mr. A. H. SCHAINBLATT of the University of California, Berkeley has accepted a position as Philosopher with the Logistics Department of the Rand Corporation, Santa Monica, California.

Professor ABRAHAM SEIDENBERG of the University of California, Berkeley has been appointed a Visiting Lecturer at Harvard University for the academic year 1963-1964.

Assistant Professor DAVID SHALE on leave from the University of California, Berkeley has been appointed a Member of the Institute for Advanced Study for the academic year 1963-1964.

Dr. ELIAHU SHAMIR of the Hebrew University has been appointed to an assistant professorship at the University of California, Berkeley.

Mr. E. M. STONE of Clarkson College of Technology has been appointed to an assistant professorship at Long Beach State College.

Associate Professor R. L. VAUGHT of the University of California, Berkeley has been awarded a National Science Foundation Postdoctoral Fellowship at the University of California, Los Angeles for the academic year 1963-1964.

Dr. A. D. WALLACE of Tulane University has been appointed to a professorship at the University of Florida.

Mr. B. C. WHEATON of the State University of Iowa has been appointed to an assistant professorship at Western Illinois University.

Professor FRANTISEK WOLF of the University of Berkeley has been appointed Miller Research Professor for the academic year 1963-1964.

Mr. P. R. YOUNG of Massachusetts Institute of Technology has been appointed to an assistant professorship at Reed College.

The following promotions are announced:

P. J. ARPAIA, Clarkson College of Technology, to an assistant professorship.
H. O. CORDES, University of California, Berkeley, to a professorship.
R. M. DUDLEY, University of California, Berkeley, to an assistant professorship.
D. S. GREENSTEIN, Northwestern University, to an associate professorship.
J. C. HICKMAN, State University of Iowa, to an associate professorship.
SHOSICHI KOBAYASHI, University of California, Berkeley, to an associate professorship.
P. A. LAPPAN, Jr., Lehigh University, to an assistant professorship.
T. H. MACGREGOR, Lafayette College, to an associate professorship.
CLIFFORD MARSHALL, Polytechnic Institute of Brooklyn, to an associate professorship.
KATSUMI NOMIZU, Brown University, to a professorship.
G. P. PATIL, McGill University, Montreal, Canada, to an associate professorship.
R. B. REISEL, Loyola University, to an associate professorship.
G. B. ROBINSON, State University College at New Paltz, to a professorship.
J. V. TALACKO, Marquette University, to a professorship.

SELMO TAUBER, Portland State College, to a professorship.

The following appointments to instructorships are announced:

Deaths:

Dr. R. E. FULLERTON of the University of Maryland died on May 21, 1963 at the age of 47.

Professor RAPHAEL SALEM of the University of Paris, Paris, France died on June 20, 1963 at the age of 65.

Dr. MORGAN WARD of the California Institute of Technology died on June 26, 1963 at the age of 62. He had been a member of the Society for 38 years.

Dr. F. G. WILLIAMS of Long Island University died on May 20, 1963 at the age of 70.

Dr. ROSCOE WOODS of the California Institute of Technology died on June 20, 1963 at the age of 73.

NEWS ITEM

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

The Institute of Mathematical Statistics and the American Mathematical Society announce the publication of two volumes in the series of SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY. Volume III, containing 17 papers, was published in July and Volume IV, containing 35 papers, in September.

The Translations, made under a grant from the National Science Foundation, are published under the direction of a Joint AMS-IMS Committee on Translations from Russian and other foreign languages.

MEMO TO MEMBERS

SUMMER MEETING - 1964

New dates, August 24 to August 28, 1964, have been set for the 1964 Summer Meeting in Amherst, Massachusetts. These dates are one week earlier than those given in the Calendar of Meetings published in previous issues of these NOTICES.
LETTERS TO THE EDITOR

Editor, The NOTICES

In his letter (No. 67, April, 1963, p. 242), A. D. Wallace paints a forbidding picture of the "scandal" of first year teaching graduate students in Mathematics and English. As one of those recently in such a position (Mathematics), I would like to comment.

He states "Neither profits too much from such teaching, the graduate student getting about $1900 and tuition for six hours per week and the university getting, in some cases, mediocre, if enthusiastic, teaching." I disagree!

Remark 1. Financial aid for many students is an absolute necessity. True, the 4.0 student usually has little financial problem; but most graduate students, unfortunately, are not in this position. Then too, the best scholars are not necessarily the best teachers. Their interest often lies solely in the subject; and, not in the teaching of it.

Remark 2. Who can doubt that the dual role of a student and teacher is enormously beneficial to the individual, especially to the future college or high school teacher. One is allowed to take the best that one finds in his professors and to apply this in his own teaching.

Remark 3. In regards to the university, the teaching of the first year graduate students (usually required Mathematics or remedial English courses) frees the often over-burdened professor so that he may be creative. Professors invariably dislike teaching such courses.

Mr. Wallace states three facts: (A) the best of the graduate students receive fellowships, (B) the preparation in our colleges and universities is poor, and (C) freshmen are better prepared, personality, these do not seem to be the "serious" "concatenation of ... forces" that he sees. Remark 1 is applicable to fact (A). Concerning (B), I believe its validity is at the least questionable. Fact (C) seems paradoxical, if one considers (B) and the fact that the majority of high school teachers are directly from our colleges and universities.

Mr. Wallace does present a valuable suggestion when he mentions the possibility of a "few weeks of preliminary training."

In conclusion, graduate student teaching has at least three important functions: the financial one, a creative one — in the freeing of the professor so that he may create —, and an intangible one — in the result of the dual role of student and teacher that the teaching graduate student finds himself.

Royce D. Purinton III

Editor, The NOTICES

In his letter to the Editor (April, 1963, Page 242) A. D. Wallace discussed the problem concerning the teaching of undergraduate mathematics courses by inadequately trained graduate students. I should like to discuss the following methods as possible solutions, or at least partial solutions, of this problem. Certainly these ideas have been thought about and tried by various mathematics departments, but I should like to present some of my personal thoughts regarding these matters.

(1) The Department of Mathematics and/or the Department of Education may offer a special course or seminar in the principles of college teaching.

It may consist of a series of lectures by various members of the Departments of Mathematics and Education, to be attended by all graduate students during their first year of teaching. The lectures may be supplemented by a program of "outside reading", which would include articles and books by prominent mathematicians, psychologists, and educators. If the graduate instructor is aware that many great mathematicians have given serious consideration to problems of mathematical education, he may be in-
spired to at least examine his own attitude and philosophy toward the subject matter and the students.

(2) Each semester, the Department of Mathematics may conduct a seminar concerning specific problems commonly encountered by instructors in elementary college math courses.

Each seminar may closely follow a particular course being taught by a number of graduate instructors during that semester. Instructors who are teaching that particular course for the first time may be required to attend, participate in the discussions, and perhaps even to demonstrate their own techniques of teaching certain concepts. Also, examinations constructed by various instructors may be compared, and later the results of the examinations may be discussed.

(3) All graduate instructors may occasionally be visited in their classrooms by regular members of the faculty. The professor would sit quietly in back of the room to observe the day's lesson. His aim would not only be to evaluate, but also to offer helpful suggestions to the graduate instructor. I feel this would be very helpful even if each graduate instructor were visited only once per semester.

(4) Some elementary survey-type mathematics courses may be taught in large lecture sections by experienced members of the faculty, supplemented by work in small sections with graduate instructors.

I am not enthusiastic about this idea, but I feel this method of instruction may be worthwhile if the professor lecturing to the large section is an especially talented and inspiring teacher. This method offers the possibility to provide guidance to the graduate instructors as they work closely with an experienced member of the faculty. This method also seems to provide an opportunity to use a wider variety of teaching aids. (For example: films, slides, exhibits, special lecture notes, visiting lecturers.)

I feel the department may gain some extra advantages from any attempts to improve the teaching skills of its graduate instructors, since each member of the department would become more conscious of his own teaching methods and abilities.

Hyman Gabai

Editor, The NOTICES

For good and obvious reasons it has always been the policy of Mathematical journals not to disclose the identity of the referee of an article to its author. Similar considerations, it seems to me, suggest the desirability of instituting the reciprocal policy and, in so far as possible, keeping the identity of the author secret from the person refereeing his manuscript.

I think most of us will admit that no matter how objective we try to be in refereeing we cannot help being influenced to some degree by knowing who the author of a manuscript is. Even if it doesn't affect our final decision on whether to publish (and it well may), it surely influences the care with which we look for errors, our readiness to suggest revisions, the number of weeks (months) we let the manuscript lie on our desk before getting to work on it.

Would it not be feasible for editors to request that contributors submit unsigned manuscripts for refereeing, and that they further not give themselves away indirectly by writing "In [1] I proved that ..."? I have yet to discuss this proposal with a mathematician who did not agree that he would prefer to referee and be refereed on a basis of mutual anonymity and that this might lead to more careful, critical, and even prompter refereeing.

David Gale
THE ANNUAL SALARY SURVEY

The 1963 Salary Survey is the seventh in an annual series of surveys of academic institutions initiated in May 1957 by the Society's Committee on the Economic Status of Teachers. This year usable returns were received from 282 institutions reporting on 2831 academic positions in 1962-1963 and 3176 positions at the same institutions in 1963-1964. There has been a 12.2% average increase in the staffs of the institutions reporting, with the greatest percentage of increase, 15%, occurring in the staffs of those institutional members in group I, as defined below. The corresponding increases last year were 8.6% and 10.7% respectively.

The system of tabulation used in this report is based on institutional membership in the Society, since this provides a basis for classifying institutions according to their mathematical activity.

This survey thus classifies institutions into two major types: Institutional Members and Non-Institutional Members. Members are in turn divided into two groups: (1) those which in the three-year period from 1959 to 1961 sponsored 37 1/2 or more pages in journals published or subsidized by the Society, and (2) those which contributed less than 37 1/2 pages in the three-year period.

The salaries covered by the survey are based on an academic year (9 to 10 months). It was indicated in the questionnaire that by 'salary' is meant the payment by the school to the individual for a full-time appointment. Grants and contracts are included, but sabbatical payments and other part-time salaries are excluded. All salary figures are given in hundreds of dollars.

The salary information obtained from each institution is the minimum, medium, and maximum salary for each academic rank. In the report, intervals are specified which contain the middle 50% of data received from each group. For example, among the 61 institutions in Group I, the following report states that the minimum salary of instructors with a Ph.D. in 1963-1964 is less than $6600 at 25% of the institutions and greater than $7400 at 25% of the institutions.
INSTITUTIONAL MEMBERS OF THE SOCIETY "I"

Number of usable returns: 61

Total number on the staffs working full time on the campus

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<tr>
<td>Instructor</td>
<td></td>
<td></td>
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<tr>
<td>(only those holding Ph.D,)</td>
<td>103</td>
<td>102</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>401</td>
<td>482</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>340</td>
<td>388</td>
</tr>
<tr>
<td>Professor</td>
<td>456</td>
<td>523</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>1300</strong></td>
<td><strong>1495</strong></td>
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Salary Survey

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<tbody>
<tr>
<td>Instructor</td>
<td>Minimum</td>
<td>Median</td>
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<tr>
<td>(only those holding Ph.D,)</td>
<td>60-67</td>
<td>65-70</td>
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<tr>
<td>Assistant Professor</td>
<td>70-78</td>
<td>76-83</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>85-95</td>
<td>92-105</td>
</tr>
<tr>
<td>Professor</td>
<td>106-124</td>
<td>125-150</td>
</tr>
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INSTITUTIONAL MEMBERS OF THE SOCIETY "II"

Number of usable returns: 91

Total number on the staffs working full time on the campus

<table>
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</thead>
<tbody>
<tr>
<td>Instructor</td>
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</tr>
<tr>
<td>(only those holding Ph.D,)</td>
<td>13</td>
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<tr>
<td>Assistant Professor</td>
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<tr>
<td>Associate Professor</td>
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<tr>
<td>Professor</td>
<td>245</td>
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<td><strong>TOTAL</strong></td>
<td><strong>838</strong></td>
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Salary Survey

<table>
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<tr>
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<tbody>
<tr>
<td>Instructor</td>
<td>Minimum</td>
<td>Median</td>
</tr>
<tr>
<td>(only those holding Ph.D,)</td>
<td>60-63</td>
<td>60-65</td>
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<tr>
<td>Assistant Professor</td>
<td>65-74</td>
<td>71-80</td>
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<tr>
<td>Associate Professor</td>
<td>77-90</td>
<td>84-94</td>
</tr>
<tr>
<td>Professor</td>
<td>90-110</td>
<td>100-123</td>
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</table>

560
### NON-INSTITUTIONAL MEMBERS

Number of usable returns: 130

**Total number on the staffs working full time on the campus**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>(only those holding Ph.D.)</td>
<td>12</td>
<td>7</td>
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<tr>
<td>Assistant Professor</td>
<td>287</td>
<td>332</td>
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<tr>
<td>Associate Professor</td>
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<td>Professor</td>
<td>213</td>
<td>219</td>
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<td><strong>TOTAL</strong></td>
<td>693</td>
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### Salary Survey

<table>
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<tbody>
<tr>
<td>Instructor</td>
<td>50-61</td>
<td>54-66</td>
<td>60-73</td>
<td>50-64</td>
<td>60-69</td>
<td>65-75</td>
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<tr>
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<tr>
<td>Assistant Professor</td>
<td>59-72</td>
<td>65-76</td>
<td>70-84</td>
<td>62-74</td>
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<td>74-88</td>
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<tr>
<td>Associate Professor</td>
<td>69-88</td>
<td>76-91</td>
<td>80-98</td>
<td>73-88</td>
<td>80-96</td>
<td>85-107</td>
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<tr>
<td>Professor</td>
<td>83-105</td>
<td>90-111</td>
<td>93-130</td>
<td>88-106</td>
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</table>

### SUMMARY OF ALL INSTITUTIONS SURVEYED

Number of usable returns: 282

**Total number on the staffs working full time on the campus**

<table>
<thead>
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<tr>
<td>(only those holding Ph.D.)</td>
<td>128</td>
<td>122</td>
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<tr>
<td>Assistant Professor</td>
<td>1011</td>
<td>1189</td>
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<tr>
<td>Associate Professor</td>
<td>778</td>
<td>840</td>
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<tr>
<td>Professor</td>
<td>914</td>
<td>1025</td>
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<td><strong>TOTAL</strong></td>
<td>2831</td>
<td>3176</td>
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### Salary Survey

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<th>Maximum</th>
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<td>Instructor</td>
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<td>65-73</td>
<td>57-70</td>
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<td>67-76</td>
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<tr>
<td>Assistant Professor</td>
<td>63-75</td>
<td>70-80</td>
<td>75-88</td>
<td>66-78</td>
<td>73-85</td>
<td>80-93</td>
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<tr>
<td>Associate Professor</td>
<td>74-90</td>
<td>82-98</td>
<td>87-109</td>
<td>79-97</td>
<td>87-105</td>
<td>92-116</td>
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<tr>
<td>Professor</td>
<td>89-115</td>
<td>95-125</td>
<td>105-145</td>
<td>94-120</td>
<td>103-136</td>
<td>110-168</td>
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STARTING SALARIES FOR MATHEMATICIANS WITH A Ph.D

This is the fourth annual Survey of Starting Salaries of graduates with the Ph.D. degree in mathematics. The figures are based on 216 usable replies to 320 questionnaires sent to individuals receiving the Ph.D. degree in mathematics during the past year. Last year 308 questionnaires were sent and 228 usable replies were received.

Of the 216 Ph.D.'s reporting, 185 took employment in academic institutions: 163 in teaching, research, or both in universities, colleges and technical institutes; 9 in research institutes; 13 receiving fellowship grants. Fellowship grants are reported in the Survey for the first time. Industry and government employed 26 and 5 respectively of the new Ph.D.'s. Last year there were 45 employed by industry.

Thirty-one reported less than 1/2 year previous professional work experience; 30 reported between 1/2 and 1 year; and 150 reported more than 1 year of work experience before receiving the Ph.D.

Geographically, the heaviest concentration of new appointments of mathematicians is again in the North East, with 39.9 per cent; the Mid West has 24.4 per cent; the Far West 22.4 per cent; the South 10.7 per cent; and 1.8 per cent were employed abroad.

UNIVERSITIES, COLLEGES AND TECHNICAL INSTITUTES (Nine Month Salary)

<table>
<thead>
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<th>Year</th>
<th>Minimum</th>
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<th>Maximum</th>
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<tr>
<td>1959</td>
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<td>1960</td>
<td>4,900</td>
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<tr>
<td>1963</td>
<td>4,500</td>
<td>7,200</td>
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TEACHING

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<td>$7,700</td>
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<tr>
<td>5,200</td>
<td>6,500</td>
<td>8,000</td>
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<tr>
<td>4,800</td>
<td>6,500</td>
<td>9,000</td>
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<tr>
<td>4,500</td>
<td>6,500</td>
<td>9,000</td>
</tr>
<tr>
<td>4,500</td>
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RESEARCH

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<tr>
<td>$4,500</td>
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FELLOWSHIP (Yearly Stipend)

<table>
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<tr>
<th>Year</th>
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<td>1963</td>
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INDUSTRY (Twelve Month Salary)

<table>
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<th>Year</th>
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<td>11,000</td>
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<td>1961</td>
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RESEARCH INSTITUTES (Twelve Month Salary)

<table>
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<tr>
<th>Year</th>
<th>Minimum Salary</th>
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<th>Maximum Salary</th>
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<tr>
<td>1963</td>
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<td>11,700</td>
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GOVERNMENT (Twelve Month Salary)

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<th>Maximum Salary</th>
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<tr>
<td>1959</td>
<td>$8,800</td>
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<tr>
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<td>10,700</td>
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<tr>
<td>1963</td>
<td>10,135</td>
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SUPPLEMENTARY PROGRAM-No. 20

During the interval from June 30, 1963 through September 4, 1963 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed Papers in this issue of these NOTICES.

(1) Wiener integrals and singular self-adjoint differential operators
Professor D. G. Babbitt, University of California, Los Angeles (63T-343)

(2) WITHDRAWN.

(3) Bounds for a class of functions of two complex variables
Professor Stefan Bergman, Stanford University (63T-327)

(4) A ring of analytic functions
Mr. R. M. Brooks, Louisiana State University (63T-325)

(5) Functions and polynomials (mod p^m)
Professor Leonard Carlitz, Duke University (63T-328)

(6) Some inversion formulas
Professor Leonard Carlitz, Duke University (63T-329)

(7) Extended Bernoulli and Eulerian numbers
Professor Leonard Carlitz, Duke University (63T-330)

(8) Eigenvalue criteria for A when AH is Hermitian
Mr. David Carlson, Oregon State University (63T-379)

(9) A class of linear transformations
Professor R. V. Chacon, Brown University (63T-358)

(10) The centroid of a homogeneous wire
Mr. G. D. Chakerian and Professor S. K. Stein, University of California, Davis (63T-372)

(11) A class of simple semigroups of non-negative matrices. Preliminary report
Mr. W. E. Clark, Tulane University (63T-352)
(Introduced by Dr. G. B. Preston)

(12) General theory of simple waves in relaxation hydrodynamics
Professor Nathaniel Coburn, The University of Michigan (63T-357)

(13) A new Riemann function
Mr. R. M. Cooper, Illinois Institute of Technology (63T-385)

(14) On the distribution of eigenvalues of generalized Toeplitz forms
Mr. J. R. Davis, Washington University (63T-354)

(15) On the extension of some results of Lane and Wall
Professor D. F. Dawson, North Texas State University (63T-381)

(16) Martingale theory in vector lattices. Preliminary report
Professor R. A. DeMarr, University of Washington (63T-384)

(17) Three solvable cases
Professor B. S. Dreben and Professor J. S. Denton, Jr., Harvard University (63T-380)

(18) On the Marx conjecture for starlike functions
Professor P. L. Duren, University of Michigan (63T-371)

(19) Exact numerical solution of linear equations with rational coefficients. Preliminary report
Dr. A. S. Fraenkel, Weizmann Institute of Science, Rehovot, Israel (63T-362)

(20) Multiplication rings as rings in which ideals with prime radical are primary
Dr. R. W. Gilmer, Jr., University of Wisconsin and Professor J. L. Mott, University of Kansas (63T-346)

(21) The space of Baire functions and the bidual of the space of continuous functions
Professor Hugh Gordon, University of Pennsylvania (63T-378)

(22) Note on linear difference equations
Professor W. A. Harris, Jr., and Professor Yasutaka Sibuya, University of Minnesota (63T-326)

(23) The Perron-Wiener method
Professor L. L. Helms, University of Illinois (63T-334)

(24) Some small multi-tape universal Turing machines
Mr. P. K. Hooper, Harvard University (63T-359)
(Introduced by Professor Hao Wang)

(25) Finite dimensional analogues to boolean algebras
Mr. C. M. Howard, University of California, Berkeley (63T-351)

(26) On a kind of extended Fejér-Hermite interpolation polynomial
Mr. L. C. Hsu, Jilin University, Changchun, China (63T-382)
(Introduced by Dr. Sydney H. Gould)

(27) A kind of stochastic approximating polynomial
Mr. L. C. Hsu, Jilin University, Changchun, China (63T-383)
(Introduced by Dr. Sydney H. Gould)

(28) Extremal doubly stochastic operators and measure preserving transformations. Preliminary report
Mr. R. E. Jaffa, University of California, Berkeley (63T-386)
(Introduced by Professor Jacob Feldman)

(29) The dilation of some standard mappings. Preliminary report
Professor T. S. Klotz, University of California, Los Angeles (63T-365)

(30) On the determination of conformal imbeddings. Preliminary report
Professor T. S. Klotz, University of California, Los Angeles (63T-366)

(31) On the harmonic immersion of surfaces in E^3. Preliminary report
Professor T. S. Klotz, University of California, Los Angeles (63T-367)

(32) Concerning a mean integral. Preliminary report
Professor C. W. Leininger, Arlington State College (63T-392)

(33) Unboundedness point sets of pointwise bounded collections of analytic functions
Dr. K. O. Leland, Louisiana State University, Baton Rouge (63T-333)

(34) Topological analysis of analytic functions
Dr. K. O. Leland, Louisiana State University, Baton Rouge (63T-332)

(35) Independence results in set theory by Cohen's method, I
Dr. Azriel Lévy, Hebrew University, Jerusalem, Israel (63T-388)

(36) Independence results in set theory by Cohen's method, II
Dr. Azriel Lévy, Hebrew University, Jerusalem, Israel and Professor Solomon Feferman, Stanford University (63T-389)

(37) Independence results in set theory by Cohen's method, III
Dr. Azriel Lévy, Hebrew University, Jerusalem, Israel (63T-390)

(38) Independence results in set theory by Cohen's method, IV
Dr. Azriel Lévy, Hebrew University, Jerusalem, Israel (63T-391)

(39) Answers to two questions of Myhill
Mr. T. G. McLaughlin, University of Illinois (63T-387)

(40) The singular points of a topological embedding
Dr. D. R. McMillan, Jr., Florida State University and Institute for Advanced Study (63T-373)

(41) An integration-by-parts formula
Professor J. S. MacNerney, University of North Carolina (63T-347)

(42) Incompressible transformations
Professor Dorothy Maharam, University of Rochester (63T-360)

(43) Derivatives of incompressible transformations. Preliminary report
Professor Dorothy Maharam, University of Rochester (63T-361)

(44) Equilibrium points of bimatrix games
Dr. O. L. Mangasarian, Shell Development Company, Emeryville, California (63T-339)

(45) On a type of integral operators
Mr. V. S. Marić, Mathematics Research Center, University of Wisconsin (63T-350)

(46) "Monotonicity" methods in Banach spaces
Professor G. J. Minty, University of Michigan (63T-374)

(47) A corollary on rapidly increasing kernels
Dr. R. N. van Norton, Brookhaven National Laboratory, Upton, New York (63T-369)

(48) Rings all of whose finitely generated modules are injective
Mrs. B. L. Osofsky, Rutgers, The State University (63T-344)

On Witt's theorem for nonalternating symmetric bilinear forms over a field of characteristic 2
Dr. V. S. Pless, Air Force Cambridge Research Laboratory, Bedford, Massachusetts (63T-340)

An open cell of even dimension admits an expansive autohomeomorphism
Mr. William Reddy, Syracuse University (63T-348)

On the existence of best Tchebycheff approximations by general rational functions
Dr. J. R. Rice, General Motors Research Laboratories, Warren, Michigan (63T-331)

A Fourier series method for entire functions
Professor L. A. Rubel, University of Illinois (63T-336)

Transcendental entire function which together with all its higher derivatives assumes algebraic values at all algebraic points
Mr. Daihachiro Sato, University of Saskatchewan, Regina, Saskatchewan, Canada (63T-341)

On estimating partial differential operators. V
Professor Martin Schechter, New York University (63T-338)

On a method of construction of a class of finite Bolyai-Lobochevsky planes
Professor Esther Seiden, Michigan State University (63T-335)

A Tricomi problem for an equation of abruptly changing type. Preliminary report
Mrs. Lesley Sibner, New York University (63T-370)

Independence results in the theory of cardinals. I. Preliminary report
Mr. Robert Solovay, Princeton University (63T-395)

Independence results in the theory of cardinals. II. Preliminary report
Mr. Robert Solovay, Princeton University (63T-396)

A hyponormal operator with spectrum on an arc is normal
Professor J. G. Stampfli, New York University (63T-353)

Some congruence properties of the Hermite polynomials
Dr. H. R. Stevens, Pennsylvania State University (63T-393)

Remark on partial orders under which differentiation is stable
Professor Walter Strodt, Columbia University (63T-377)

On a result of Heineken. Preliminary report
Mr. D. W. Walkup, University of Wisconsin (63T-337)

The convergence of sequences of rational functions of best approximation. II
Professor J. L. Walsh, Harvard University (63T-376)

A universal axiom of conditional set existence
Professor Hao Wang, Harvard University (63T-375)

Natural hulls and set existence
Professor Hao Wang, Harvard University (63T-394)

On symmetric differentiation
Dr. M. C. Weiss, University of Chicago (63T-342)

Notes on the structure of r.e. sets
Professor P. R. Young, Reed College (63T-368)
The rectangular matrix equation $AY + B =YC$. 

An explicit $m \times n$ matrix solution $Y$ is found for the linear matrix equation $AY + B = YC$, where $A$, $B$, and $C$ are given $m \times m$, $m \times n$, and $n \times n$ constant matrices, respectively, such that no eigenvalue $\lambda_j$ of $A$ is also an eigenvalue of $C$. Let $m_j$ be the multiplicity of $\lambda_j$ as a root of the minimal equation of $A$, and let $A_j$ ($j = 1, \ldots, s$) be the idempotent matrices for $A$ such that $\sum A_j = I_m$ (the $m \times m$ unit matrix), $A_j A_k = A_j \delta_{jk}$, and $(A - \lambda_j I_m)^{m_j} A_j = 0$. Then the unique solution $Y$ of the equation $AY + B = YC$ is 

$$Y = \sum_{j,k} (A - \lambda_j I_m)^{k-1} A_j B (C - \lambda_j I_n)^{-k}.$$ 

The number of terms in this sum is the degree $\sum m_j$ of the minimal equation of $A$. Clearly the roles of $A$ and $C$ can be interchanged, or the factors $(C - \lambda_j I_n)^{-k}$ can be expanded using the constituent matrices for $C$. (Received September 19, 1963.)

Use Clifford's structure theorem (Amer. J. Math. 75 (1953), 547-556). Theorem 1. Let $S$ and $S^*$ be d-simple inverse semigroups with identity and let $P$ and $P^*$ be their right unit subsemigroups. Let $N$ be a sl-homomorphism of $P$ into $P^*$ (i.e. $N$ preserves the semilattice structure of the $L$-classes of $P$ and $P^*$), and let $k$ be an element of $P^*$. For each element $(a,b)$ of $S$, define $(a,b)M = [(aN)k,(bN)k]$, the square brackets indicating an element of $S^*$. Then $M$ is a homomorphism of $S$ into $S^*$. Conversely, every homomorphism of $S$ into $S^*$ is obtained in this fashion.

Theorem 2. Let $S,P,S^*$, and $P^*$ be as in Theorem 1. Let $\Omega$ be the set of isomorphisms of $P$ onto $P^*$. Define $(m,n)M_N = [mN,nN]$ for $N$ in $\Omega$. Then $\{M_N : N \in \Omega\}$ is the complete set of isomorphisms of $S$ onto $S^*$. Hence, $\Omega \rightarrow M_N$ is a one-to-one correspondence between the isomorphisms of $P$ onto $P^*$ and those of $S$ onto $S^*$ and $S$ is isomorphic to $S^*$ iff $P$ is isomorphic to $P^*$. The group of automorphisms of $P$ is isomorphic to the group of automorphisms of $S$. (Received June 24, 1963.)


Let $\{X_n, n \geq 1\}$ and $\{Y_n, n \geq 1\}$ be two independent sequences of mutually independent, non-negative random variables such that the $X$'s have the common distribution function $F(x)$ and the $Y$'s have the common distribution function $G(y)$, with respective Laplace-Stieltjes transforms $f(s)$ and $g(s)$. Define: $W_0 = 0$, $W_n = \sum_{i=1}^{n} (X_i + Y_i)$, $W_n = W_n + X_{n+1}$, $n = 0, 1, \ldots$; for $t > 0$, let $V(t)$ denote the indicator of the union of events $\bigcup_{n \geq 0} \{W_n < t \leq W_{n+1}\}$, and let $\beta(t)$ be the random variable $\beta(t) = \int_{0}^{t} V(s)ds$. Theorem. There exists a proper, nondegenerate d.f. $H(x)$ such that

$$\lim_{t \rightarrow \infty} \frac{\beta(t)}{t} = H(x) \text{ at all continuity points of } H \text{ if and only if } (i) \text{ there exists a positive constant } \delta \text{ such that } \lim_{s \rightarrow 0} \frac{[1 - g(s)]/[1 - f(s)] = \delta};$$

(ii) either $F$ or $G$ is in the domain of attraction of the positive stable law of index $\alpha$, for some $\alpha$ satisfying $0 < \alpha < 1$. Under these conditions, $H(x)$ is necessarily the distribution function of a random variable of the form $\xi/(\xi + \eta^{1/\alpha})$, where $\xi$ and $\eta$ are independent with a common positive stable distribution of index $\alpha$. This theorem generalizes some earlier results of Lamperti [Trans. Amer. Math. Soc. 88 (1958), 380-387] and Takács [Trans. Amer. Math. Soc. 93 (1959), 531-540]. The theorem is used to prove a "generalized arc sine law" for recurrent diffusion processes on the real line. (Received July 12, 1963.)


Let $K$ be an arbitrary field. The inequality "dimension $R \geq n$" has held heretofore in partial...
classifications by several authors of maximal commutative subalgebras \( R \) of the algebra \( K_n \) of \( n \) by \( n \) matrices over \( K \). The contrary phenomenon is presented here: an example of a 13-dimensional commutative subalgebra of \( K_{14} \) which is proved maximal commutative. The proof is made by applying to the representation space \( M \) of the algebra the theorem: Let \( R \) be a commutative ring with unit element. Let \( M \) be a unital \( R \)-module which is the sum of two submodules \( M_1 \) and \( M_2 \), both invariant under \( \text{Hom}_R(M, M) \), such that the annihilating ideal \((0: M_1 \cap M_2)\) equals \((0: M_1) + (0: M_2)\). If for \( i = 1,2 \), \( \text{Hom}_R(M_i, M_i) \) is the action on \( M_i \) of \( R \), then \( \text{Hom}_R(M, M) \) is the action on \( M \) of \( R \).

Using tensor products over \( K \) one easily obtains the following consequence of the example: given \( \epsilon > 0 \), there exists a positive integer \( n = n(\epsilon) \) and a subalgebra \( R \) maximal commutative in \( K_n \) with dimension less than \( \epsilon n \). (Received July 26, 1963.)


An \( n \)-fold generalized extension by an algebra \( A \) of an \( A \)-bimodule \( P \) is an exact sequence
\[
0 \to P \to P_{n-1} \to \ldots \to P_1 \to P_0 \to 0
\]
with an algebra extension \( 0 \to P_0 \to B \to A \to 0 \). Under an addition and equivalence analogous to Yoneda's, the \( n \)-fold generalized extensions, \( n \geq 1 \), form an additive group \( H^{n+1}(A,P) \) with \( H^2(A,P) \) the group of extensions by \( A \) of \( P \). Suitable restriction of the type of extension allowed automatically defines a corresponding cohomology theory. The theory is essentially normalized by the definition of \( H^2(A,P) \). Among the theories obtainable in this way are the classical Hochschild theory, the Harrison theory for commutative algebras, and the MacLane-Shukla theory in which algebra extensions are not necessarily additively split. The same formalism is valid for the cohomology of groups, Lie algebras (yielding both the Chevalley-Eilenberg and Dixmier theories), and various other classes of algebras. The present groups can also be computed from generalized projective resolutions. A natural interpretation is given for \( H^3(A,P) \) as an obstruction group. (Received September 9, 1963.)

604-5. A. A. SAGLE, 15 Smith Hall, Syracuse University, Syracuse 10, New York. Remarks on simple extended Lie algebras.

In this paper the discussion of finite dimensional simple extended Lie algebras \( A \) over an algebraically closed field of characteristic zero with nondegenerate form \( \langle x,y \rangle = \text{trace } R_x R_y \) where \( R_x: A \to A: a \to ax \) is continued (Abstract 597-68, Notices Amer. Math. Soc., January, 1963). The main result is that the usual Lie methods will probably yield no new algebras; more specifically, the following results occur. First, if \( A \) is an extended Lie algebra as above and \( A \) has a diagonalizable Cartan subalgebra \( N \) which upon decomposing \( A = \sum_n A(N, n) \) into its weight spaces relative to \( N \), for each weight \( a \) there exists a weight \( \beta \) so that \( A(N, a) \subset A(N, \beta) \) or \( A(N, a)^2 = 0 \), then \( A \) is a Lie or Malcev algebra. Secondly, if \( A \) is an extended Lie algebra as in the first sentence, then \( A \) is not Lie or Malcev iff there exists a nonzero element \( a \in A \) such that for every derivation \( D \) of \( A \) there is \( aD = 0 \). (Received August 15, 1963.)

The authors consider the system of difference equations (A):

\[\begin{align*}
Y_0 &= 1, \quad 4(Y_1 - Y_0)/h^2 + \lambda Y_0 = 0, \\
(Y_{j+1} + Y_{j-1} - 2Y_j)/h^2 + (Y_{j+1} - Y_{j-1})/(2jh^2) + \lambda Y_j &= 0, \quad j = 1, \ldots, n - 1, \\
Y_n &= 0,
\end{align*}\]

where \(n\) is a positive integer and \(h = 1/n\). This system corresponds to the Bessel differential system (B):

\[y'' + y'/x + \lambda y = 0,\]
\[y(0) = 1, \quad y(1) = 0,\]

of order zero. R. H. Boyer (Discrete Bessel functions, J. Math. Anal. Appl. 2 (1961), 509-524) has shown that the \(n\)th degree polynomial solution of system (A) can be represented in terms of Legendre functions of the first and second kind. He has also shown that system (A) has \(n\) real eigenvalues \(0 < \lambda_1 < \lambda_2 < \ldots < \lambda_n\), the first \(n - 1\) of which can be written \(\lambda_j = 4n^2 \sin^2 (\omega_j/2)\), \(0 < \omega_j < \pi\), and the largest of which satisfies \(4n^2 < \lambda_n < 5.2n^2\). It is classical that the positive real eigenvalues of the system (B) are \(0 < \lambda_1 < \lambda_2 < \ldots\) where \(\sqrt{\lambda_j} = (j - 1/4)\pi + \delta_j\), \(0 < \delta_j < \pi/8\), is the \(j\)th positive real zero of \(J_0(x)\). In the present paper it is shown that for \(1 \leq n\) and \(j = 1, 2, \ldots, n\), there is

\[0 < \lambda_j - \lambda_j \leq 3\lambda_2^2/2n^2.\]

(Received August 13, 1963.)

604-7. P. V. O'NEIL, Rensselaer Polytechnic Institute, Troy, New York. **The number of trees in a certain network.**

Trent (Proc. Nat. Acad. Sci. U.S.A. 40 (1954), 1004-1007) has defined a matrix \(T\) such that det(\(T\)) gives the number of trees in a connected network, L. Weinberg (Proc. I.R.E. 42 (1954), 427-437) has evaluated \(\det(T)\) in two special cases. The following is a generalization of these results: Let \(K\) be a complete network with \(N\) vertices. Let \(P\) be a set of sets of branches of \(K\). For each \(A \in P\) let \(s(A)\) be the number of branches in \(A\). Assume \(P\) satisfies the following two conditions: (1) if \(A \in P\) and \(B \in P\) and \(A \neq B\), then no branch of \(A\) has a vertex in common with any branch of \(B\); (2) if \(A \in P\) and \(s(A) \geq 2\), then all branches of \(A\) have exactly one vertex in common. Let \(r = \sum_{A \in P} s(A)\). Let \(t\) equal the number of sets in \(P\). Form the network \(K'\) by removing from \(K\) every branch in the union of all sets in \(P\). Then the number of trees in \(K'\) is equal to \(N^{N-t-t}(N - 1)^{t-t} \prod_{A \in P} (N - 1 - s(A))\). The first of Weinberg's formulas corresponds to the case \(s(A) = 1\) for every \(A \in P\). The second corresponds to the case \(t = 1\). If \(P\) is empty, then \(K = K'\), and the formula reduces to that of a complete graph, with appropriate notational conventions. (Received August 21, 1963.)


Let \(G\) be a finite group and \(R\) a ring of algebraic integers in the algebraic number field \(k\). If \(RH \subseteq RG\) implies \(H \subseteq G\) for all finite groups \(H, G\) is said to be \(R\)-isolated. Theorem 1. \(G\) is \(R\)-isolated for all \(R\) if \(G\) is (a) abelian, (b) Hamiltonian, (c) a group in which every Sylow subgroup is cyclic, (d) a \(p\)-group of order \(\leq p^4\), (e) a \(p\)-group with a cyclic normal subgroup of index \(\leq p^2\), (f) a symmetric group. When \(R = Z\), Berman (Dokl. Akad. Nauk SSSR (N.S.) 91 (1953), 7-9) showed (a), (b); and Passman (Abstract 63T-236, Notices Amer. Math. Soc. 10 (1963), 490) showed (d). Theorem 2. Let \(K = GL_p(p^a)\). If \(G = K\), or a Sylow subgroup of \(K\), or the normalizer of a Sylow subgroup, then \(G\) is \(R\)-isolated if \(|R'|/|F| \leq p^a\) where \(R'\) is the full ring of algebraic integers in \(k\) and \(F\) a prime ideal dividing \(p\) in \(R'\). (Received August 22, 1963.)
Lyapunov stability and periodic solutions.

For a 2-dimensional system \( (*) \), \( \dot{x} = F(x) \), the concept of strong stability of the point at infinity relative to \( (*) \) is defined. It is shown that if \( (*) \) has just one critical point \( x_0 \) and \( x_0 \) is asymptotically stable and if the point at infinity is strongly stable, then \( (*) \) has a periodic solution. Also if the point at infinity is strongly stable relative to \( (*) \) and if the function \( G(x,t) \) has period \( T \) in \( t \), then 
\[
\dot{x} = F(x) + \mu G(x,t)
\]
has a solution of period \( T \) for all sufficiently small \( |\mu| \) where \( \mu \) is a real parameter.

Practical sufficient conditions for strong stability of the point at infinity, i.e., conditions which can be easily verified for particular systems of ordinary differential equations, are derived. (Received August 26, 1963.)

On stability of sets of dynamical systems. Preliminary report.

Let \( (X, \mathbb{R}, \pi) \) be a dynamical system, where \( X \) is a locally compact, separable metric space with metric \( d \), \( \mathbb{R} \) is the real line and \( \pi(x, \mathbb{R}) \) denotes the trajectory through \( x \in X \). Let \( M \subset X \) be a nonempty closed set. Definition 1. \( M \) is stable if for any \( \varepsilon > 0 \) one can find a \( \delta > 0 \) such that \( \pi(x, \mathbb{R}^+) \subset S(M, \varepsilon) \), whenever \( x \in S(M, \delta) \). Definition 2. \( M \) is said to be semi-stable if given any \( \varepsilon > 0 \) and any compact set \( K \subset X \), one can find a \( \delta = \delta(\varepsilon, K) \) such that \( x \in K \cap S(M, \delta) \) implies \( \pi(x, \mathbb{R}^+) \subset S(M, \varepsilon) \). Theorem 1. \( M \) is stable if there exists a continuous scalar function \( V(x) \) with the properties: (i) \( a(d(x, M)) \leq V(x) \leq b(d(x, M)) \) and (ii) \( V(x) \leq V(\pi(x, t)) \) for \( t \geq 0 \), where \( a(r) \) and \( b(r) \) are two monotonic increasing functions defined on \( 0 \leq r \leq r_0 \) (\( r_0 > 0 \)) and \( a(0) = b(0) = 0 \). Theorem 2. \( M \) is semi-stable if there exists a continuous scalar function \( V(x) \) with the properties: (i) \( a(d(x, M)) \leq V(x) \) and (ii) \( V(x) \leq V(\pi(x, t)) \) for \( t \geq 0 \). Remark. The two forms of stability introduced are not distinguishable for compact sets.

Purity in simple systems. Preliminary report.

The systems considered here are sets with a binary relation and whose algebraic structures are less than that for a group, e.g., semigroups, loops and groupoids. Analogies between the study of purity in sub-simple systems and in the purity of subgroups are considered. Necessary and sufficient conditions are given to ensure the purity of certain sub-simple systems. (Received August 29, 1963.)

Commutators in groups of homotopy classes.

Let \( \pi(CX, Y) \) denote the group of homotopy classes of maps of the suspension \( CX \) of \( X \) into the space \( Y \) and let \( \mathbb{Q} \) denote the rationals. Theorem. If \( X \) is a finite polyhedron such that (i) cup products of \( k \) positive dimensional elements in the cohomology ring \( H^*(X; \mathbb{Q}) \) occur only in dimensions \( n_1, \ldots, n_r \) and (ii) all \( k \)-fold Whitehead products in \( \pi_{n_1+1}(Y) \) have finite order (\( i = 1, \ldots, r \)), then all \( k \)-fold com-
mutators of the group $\pi(X,Y)$ are of finite order. The theorem is a consequence of the following result, Proposition. If $Y$ is 1-connected then all $k$-fold Whitehead products in $\pi_{n+1}(Y)$ have finite order if and only if $\phi_k: \Omega^k Y \to \Omega^{n+1} Y$, where $\phi_k: \Omega^k Y = \Omega Y \times \cdots \times \Omega Y \to \Omega Y$ is the $k$-fold commutator map of the loop space $\Omega Y$. (Received September 3, 1963.)


In the special case where the operator is homogeneous, has constant coefficients and the domain is the (unit) ball, the Dirichlet problem can be investigated by the following method: If $Au = f$ and $f$ is sufficiently smooth, then $u$ is sufficiently smooth and can be expressed in the form $u = (r^2 - 1)^m v$, where $r$ denotes the radius vector, $2m$ is the order of $A$, and $v$ is a smooth function. Hence, instead of investigating the operator $A$, one can investigate the operator $A(r^2 - 1)^m$. If $v$ is a polynomial of degree $k$, then $A(r^2 - 1)^m v$ is a polynomial having the same degree. Using the index theorems for elliptic operators the following theorems are obtained: (I) If the coefficients of $A$ are algebraically independent over the rational field, then there exists a solution to the Dirichlet-problem, (II) If $Au = f$ and $f$ is a polynomial, then $u$ is too, (III) If $f$ is analytic in a (complex) domain containing the complex $n$-dimensional unit ball, then $u$ is analytic in a smaller domain; in particular, if $f$ is entire, then $u$ is too. In the special case of two dimensions additional properties may be obtained. (Received September 9, 1963.)


Let $X$ be a completely regular Hausdorff topological space and $C(X)$ be the set of all continuous real-valued functions defined on $X$. A lattice homomorphism of $C(X)$ is a map $\phi$ of $C(X)$ into the reals $R$ satisfying $\phi(f \vee g) = \phi(f) \vee \phi(g)$ and $\phi(f \wedge g) = \phi(f) \wedge \phi(g)$ for every $f$ and $g$ in $C(X)$ ($\vee$ and $\wedge$ stand for maximum and minimum, respectively). The simplest method of producing lattice-homomorphisms of $C(X)$ is as follows: Take a point $p_0 \in X$ and a nondecreasing function $a: R \to R$ and set

$$\phi(f) = a(f(p_0))$$

for every $f$ in $C(X)$. Theorem. Every lattice-homomorphism $\phi$ of $C(X)$ can be written in the form (1) iff the space $X$ is finite. (Received September 9, 1963.)


Consider a map $f$ of an open set $R \subset E^m$ into $E^n$, of class $C^q$, $q \geq 1$. Let $A_x$ denote the set of critical points of $f$ of rank $\leq r$, $r = 0, 1, \ldots, \min(m,n) - 1$, that is, the set of points in $R$ at which the Jacobian matrix is of rank at most $r$. In the cases $r = 0$ and $r = \min(m,n) - 1$, definitive theorems about the measure of the image $f(A_x)$ have been known since 1942. For other values of $r$ the following theorem is new, although not best possible. Say that a set is $\rho$-null if its $\rho$-dimensional Hausdorff measure is zero. For each $\rho > 0$, the image $f(A_x)$ is $[r + (m - r)/\rho]$-null if $q > m\rho - m + 1$. It follows that $f(A_x)$ is $(r + 1)$-null if $q > m(m - r - 1)/2$; also (A. Ya. Dubovicki1, Izv., Akad. Nauk SSSR Ser. Mat., 26 (1962), 489-494) that $f(A_x)$ is $(r + \epsilon)$-null for all $\epsilon > 0$, if $q = \infty$. An elegant assertion, not at present known to be true, would be that $f(A_x)$ is $[r + (m - r)/\rho]$-null if $q \geq \rho$. (Received September 10, 1963.)

Let \( X \) be a vector space of finite dimension \( n \) over the reals \( \mathbb{R} \). It is known that a formally real Jordan algebra \( J : X \times X \rightarrow X \) over \( \mathbb{R} \) gives rise to a domain of positivity \( Y \) in \( X \) and that the tube over \( Y \) in the complexification \( X(i) \) of \( X \) is holomorphically equivalent to a bounded symmetric domain \( \tilde{Z} \) in \( X(i) \). Let \( \sigma \) be an involutive automorphism of the Jordan algebra \( J \) and \( X = X^+ + X^- \) the decomposition of the vector space \( X \) into the eigenspaces of \( 1 \) and \( -1 \) relative to \( \sigma \). Theorem. \( \tilde{Z} \cap X^-(i) \) is a bounded symmetric domain in \( X^-(i) \); every irreducible bounded symmetric domain which is not holomorphically equivalent to a tube over a domain of positivity can be obtained in the way described above. (Received September 10, 1963.)

604-17. E. S. RAPAPORT, Box 175, 333 Jay Street, Brooklyn 1, New York. On the presentation of a group.

Let \( G = F/N = \langle x_1, \ldots, x_n; r_1, \ldots, r_k \rangle \), \( F \) the free group on \( x_1, \ldots, x_n \), \( N \) its normal subgroup generated by \( r_1, \ldots, r_k \). Let \( y_1, \ldots, y_{nt} \) be new symbols, \( F_{nt} = F^* \) the free group on \( x_1, \ldots, x_n, y_1, \ldots, y_{nt} \), and \( G = F^*/(N, y_1, \ldots, y_{nt}) = F^*/M \). Is it possible that \( M = \{ s_1, \ldots, s_c \} \) in \( F^* \) with \( c < k + t \) even if \( k \) is minimal for \( N \)? An affirmative answer is given. The proof uses standard techniques for generators and defining relations and a non-Hopfian group of Baumslag and Solitar. It is a corollary to the proof that \( k > 1 \), i.e., that this never happens if the presentation \( F/N \) of \( G \) has one defining relation. Another corollary is that if \( G = F_{n-1} \) then \( r_1 \) (the single relator of \( G \)) is simple (a free generator of \( F_n \)). (Received September 10, 1963.)


Consider the equation (*) \( p^2 w'' = a(t, \rho)w \) where \( t, \rho \) are real and \( a \) is \( C^{\infty} \). A Newton polygon is used to classify those equations which possess formal solutions of a kind obtained by adjoining the roots of \( \lambda^2 - a(t, \rho) = 0 \) to the usual resources. If this polygon has more than one side, (*) has a turning point at \( t = 0 \). In the special case of a single side the conditions reduce to \( a(t,0) \neq 0 \). The asymptotic character of these formal solutions is established and used to determine relations among various classical asymptotic solutions involving formal power series. (Received September 11, 1963.)


Suppose \( U \) is a topological transformation of the plane onto itself. If there exists a curve \( C \), not through the origin \( O \), such that (i) any ray through \( O \) meets \( C \) at most once and (ii) for any point \( P \) of \( C \), \( U P \) lies on the ray \( OP \), one says \( C \) is star-like for \( U \). It is easy to read from a figure or to prove from the properties of continuous functions that if \( U \) is area-preserving and \( C \) is a simple closed curve which is star-like for \( U \) then there are at least two fixed points of \( U \) on \( C \). The simplest
application is that, if \( p(t) \) has least period \( 2\pi \) and \( m \) is a positive integer, the differential equation \( x + 2x^3 = p(t) \) has an infinity of periodic solutions with least period \( 2m\pi \). (Received September 11, 1963.)


Let \( \psi \) be E.C. number-theoretic function defined by letting \( \psi(n) \) be the simple sum of the primes alone appearing in the mosaic of \( n \); with \( \psi(1) = 0 \). Then recall that the Soviet mathematician, L. Schnirelmann, proved (circa 1930) the valuable result: (*) Every natural number \( n > 1 \) is a sum of a finite number of primes. Theorem 1. (*) if, and only if, \( \psi(N^*) \) is onto \( N^* \) where \( N^* = \{ n \in N : n \geq 2 \} \) and \( N \) is the set of natural numbers. Theorem 2. \( \{ (1/i* : i \in N} \) is an infinite recursive set of Euclidean computable functions. Theorem 3. \( \psi^* \) has a conjugate in the sense of these Notices, 10 (1963), 357. Clearly if \( \psi^* \) is defined by \( \psi^*(n) \) is the simple sum of the primes alone in the classical Euclidean model of \( \text{FTA} \) for \( n \), then \( \psi^* = \psi^*(\psi^*) \), where \( \psi \) is defined, these Notices 10 (1963), 287. Clearly \( \psi^* \) can be restricted so as to produce an assertion logically equivalent to Goldbach's Conjecture. (Received September 17, 1963.)
ABSTRACTS PRESENTED BY TITLE

63T-325. R. M. BROOKS, Louisiana State University, Baton Rouge, Louisiana. A ring of analytic functions.

Let $R$ denote the topological ring of analytic functions on the unit disc with the compact-open topology, usual addition and scalar multiplication, and the Hadamard product for ring multiplication. The complex number field $\mathbb{C}$ is embedded in $R$ via $e(z) = e^z$, where $e$ is the identity of $R$. For each $f \in R$, $F(z) = \sum f_n z^n$, define $\tilde{f}: X \to \mathbb{C} (X \text{ non-negative integers})$ by $\tilde{f}(n) = |f_n|^{1/n}$. Then (1) the maximal ideal space of $R$ with the hull-kernel topology is homeomorphic to $\beta X$, the Stone-Čech compactification of $X$ ($p \mapsto M_p$), and $M_p$ is closed iff $p \in X$. (2) If $p \in X$, then $M_p = \{f: f_p = 0\}$; if $p \in \beta X - X$, then $M_p = \{f: f_\beta(p) < 1\}$, where $f_\beta$ is the continuous extension of $f$ to $\beta X$. (3) $R^*$ is isomorphic to $\bigcap \{M_p: p \in \beta X - X\}$; those functions in $R$ whose radius of convergence exceeds one. (4) If $\phi_p: R \to R/M_p$ is the natural map; then (a) $p \in X$ implies $\phi_p(\mathbb{C}) = R/M_p$, (b) $p \in \beta X - X$ implies $R/M_p$ is an algebraically closed extension of $\phi_p(\mathbb{C})$ having transcendence degree $c$; and $R/M_p$ is isomorphic to $\mathbb{C}$. (5) The closure of every principal ideal is principal and every closed ideal is principal. (6) Prime (primary) ideals are contained in unique maximal ideals, and closed prime (primary) ideals are maximal. (Received July 1, 1963.)

63T-326. W. A. HARRIS, Jr. and YASUTAKA SIBUYA, Institute of Technology, University of Minnesota, Minneapolis, Minnesota. Note on linear difference equations.

Consider a system of linear difference equations written in vector form $y(x + 1) = A(x)y(x)$ in which the elements of $A(x)$ are analytic in a neighborhood of $x = \infty$ with at most a pole of order $\mu$ there; $A(x) = \sum_{k=0}^{\infty} A_k x^{-k}$, $|x| > \rho$, $A_0 \neq 0$. Let the matrix $A_0$ have block diagonal form $A_0 = \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_k)$ where the eigenvalues $\lambda_{ij}$ of $\lambda_i$ satisfy $|\lambda_{ij}| = |\lambda_{i'j'}|, i \neq j$. Theorem. There exists a matrix $T(x)$ with elements analytic for $\lim x \to \infty$ if $R$ is sufficiently large for which $T^{-1}(x + 1)A(x)T(x) = B(x) = \text{diag} (B_1, B_2, \ldots, B_k)$. Further $T(x)$ has the asymptotic representation $T(x) \sim I + \sum_{k=1}^{\infty} T_k x^{-k}$ for $|\text{Im } x| > R > 0$. (Received July 1, 1963.)

63T-327. STEFAN BERGMAN, Stanford University, Stanford, California. Bounds for a class of functions of two complex variables.

Let $P^2(a)$ be the segment $[z_2 = az_1, |z_1| \leq r]$ of the analytic plane $z_2 = az_1$, and let $D^4$ be the domain $\bigcup_{|z_1| \leq 8} P^2(a)$ in the space of two complex variables. The boundary of $D^4$ is a sum of two analytic hypersurfaces $d^3 = \bigcup_{|z_1| < p} [P^2(se^{iX})]$ and of $b^3$. Let $f(z_1, z_2), f(z_1, z_2) = a_0 + a_0 z_1 + a_0 z_2 + \ldots$, be a function of two complex variables, holomorphic in $D^4$. It is assumed that $f$ omits in every lamina $[P^2(se^{iX})]$ of $d^3$ two values $A_1(x)$ and $A_2(x), A_1(x) \neq A_2(x)$. Then $\log |f(z_1, z_2)| + \log P_1 \leq (1/2\pi) \int_0^{2\pi} (|z_1^0 + z_1^0|/|z_1^0 - z_1^0|) [\log^+ a_00 + A_1(x)/|Q(x)| + 7] + \log^+ |Q(x)| + \log^+ A_1(x) + \log 2 P_2 dx, \quad P_1 = \frac{|(se^{iX})^2 - n_x(z_2 = z_1 = s_x(z_1))|}{[se^{iX}]^2 - n_x(z_2 - n_x(z_1))}, \quad Q(x)$
= A_2(x) - A_1(x), P_2 = \left[|sz_1|^2 + |rz_2|^2\right] \left[|sz_1|^2 - 2rsr|z_2|^2 \cos(x - \text{arg} z_2)\right]. \text{ Here } n_r(z) \text{ are the zero points of } f(z_1z_2) \text{ in } \{|z_2| \leq (s/r)|z_1|, |z_1| = z_1^0\}. \text{ A similar upper bound in terms of } a_{00}, a_{10}, a_{01} \text{ can be obtained for functions } f \text{ which, instead of omitting two values, are schlicht in every lamina of } a^3. \text{ (Received July 1, 1963.)}

63T-328. LEONARD CARLITZ, Duke University, Durham, North Carolina, Functions and polynomials (mod } p^n). \text{ Let } \mathbb{Z}_n \text{ denote the ring of integers (mod } p^n) \text{ where } p \text{ is a prime and } n \geq 1. \text{ By a function over } \mathbb{Z}_n \text{ is meant a mapping of } \mathbb{Z}_n \text{ into itself; two functions } f, g \text{ are equal provided } f(a) \equiv g(a) \text{ (mod } p^n) \text{ for all } a \in \mathbb{Z}_n. \text{ A polynomial function is one of the type } P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_jx^j \in \mathbb{Z}_n. \text{ When } n = 1 \text{ every function over } \mathbb{Z}_n \text{ can be represented by a polynomial, When } n > 1 \text{ this is no longer true. The present paper contains criteria that a function over } \mathbb{Z}_n \text{ be representable by a polynomial. For example } f(x) \text{ can be represented by a polynomial if and only if } \sum_{s=0}^n (-1)^s C_{m+s,s} v_{m+n-2r,r} \equiv 0 \text{ (mod } p^n) \text{ for } 0 \leq r < p^n, \text{ where } \sigma(n) = \min(n, \mu(r)) \text{ and } p^\mu(r) \text{ is the highest power of } p \text{ that divides } r!. \text{ Another criterion for polynomial representation is } f(x + kp) \equiv f_0(x) + kpf_1(x) + \ldots + (kp)^{n-1}f_{n-1}(x) \text{ (mod } p^n), \text{ where } k \text{ is an arbitrary integer and the } f_j(x) \text{ are functions over } \mathbb{Z}_n. \text{ (Received July 3, 1963.)}

63T-329. LEONARD CARLITZ, Duke University, Durham, North Carolina, Some inversion formulas. \text{ It is proved that the formulas } u_{mn} = \sum_r C_{m+n-2r,m-r} v_{m+n-2r,r} \text{ and } v_{mn} = \sum_{s=0}^n (-1)^s q^{s(s-1)/2} (1 - q^{m+2s})(1 - q^{m+s})^{-1} C_{m+s,s} v_{m+n-2r,r} \text{ are equivalent. More generally } u_{mn} = \sum_r C_{m+n-2r,m-r} v_{m+n-2r,r} \text{ and } v_{mn} = \sum_{s=0}^n (-1)^s q^{s(s-1)/2} (1 - q^r)(1 - q^{r-s})^{-1} C_{r,s} u_{r-2s+1} \text{ are equivalent. In the last two formulas } C_{m,n} \text{ denotes a } q\text{-binomial coefficient. The following simpler result may be noted. The formula } u_r = \sum_s C_{r,s} v_{r-2s} \text{ is equivalent to } v_r = \sum_s (-1)^s q^{s(s-1)/2} (1 - q^r)(1 - q^{r-s})^{-1} C_{r,s} u_{r-2s} \text{ (Received July 3, 1963.)}

63T-330. LEONARD CARLITZ, Duke University, Durham, North Carolina, Extended Bernoulli and Eulerian numbers. \text{ Extended Eulerian numbers are defined by means of } (1 - \lambda)/(\zeta(s) - \lambda) = \sum_0^\infty H(n, \lambda) n^{-s}, \lambda \neq 1; \text{ extended Bernoulli numbers are defined by means of } \log \zeta(s)/(\zeta(s) - 1) = \sum_0^\infty \beta(n) n^{-s}. \text{ For } n = p_1 p_2 \ldots p_r, \text{ where the } p_j \text{ are distinct primes, } H(n, \lambda) = H_r(\lambda), \beta(n) = B_r, \text{ where } H_r(\lambda), B_r \text{ are the ordinary Eulerian and Bernoulli numbers. The object of the paper is the study of algebraic properties of these numbers and certain related polynomials. These properties for the most part are generalizations of the corresponding properties of the ordinary Bernoulli and Eulerian numbers. (Received July 5, 1963.)
Let \( f(x), \phi_i(x), i = 1, 2, \ldots, n \) and \( \psi_j(x), i = 1, 2, \ldots, m \), be functions continuous on \([0, 1]\). Set \( N(A, x) = \sum a_i \phi_i(x) \), \( D(B, x) = \sum b_i \psi_i(x) \) and \( R(C, x) = N(A, x)/D(B, x) \). **Definition.** The approximating function \( R'(C, x) \) is said to be the **parameter closure of** \( R(C, x) \) on \([0, 1]\) if given a sequence \( \{R(C_k, x)D(B_k, x)\} \) which converges on \([0, 1]\) to \( g(x) \) which is continuous except for a finite number of removable discontinuities, then \( g(x) \in \{R'(C, x)\} \). Those functions of \( \{R'(C, x)\} \) which differ only on a finite point set are identified with the appropriate continuous function. **Theorem.** Let the number of zeros in \([0, 1]\) of \( D(B, x) \) be bounded and let \( R'(C, x) \) be the parameter closure of \( R(C, x) \) on \([0, 1]\). Then there exists \( C^* \) such that \( \max |f(x) - R'(C^*, x)| = \inf_C \max |f(x) - R(C, x)| \leq \max |f(x) - R'(C, x)| \) for all \( C \). (Received July 5, 1963.)

The approach is that of Whyburn's **Topological analysis.** Making no use of the notion of rectifiability, a topological analogue of the line integral is developed, and by use of "local" inverses of the exponential function Whyburn's topological index is obtained. Making use of anti-derivatives of differentiable functions, close analogues of the classical line integral and the Cauchy Integral Formula are obtained. Adapting a standard proof of the Riemann Mapping Theorem, a new proof of the power series expansion of an analytic function independent of those of Porcelli and Connell is obtained; also the Laurent expansion. Removable singularity problems are handled in the following setting. Let \( A \) be a closed nowhere dense subset of the closure \( D \) of the unit disk \( U \), and \( f \) a continuous function on \( D \) differentiable on \( U - A \). The principle results consist of two sets of sufficient conditions for \( f \) to be differentiable on \( U \). Examples of Denjoy show that conditions need be imposed even when \( A \) is an arc. One consequence is a new proof for the case when \( A \) is a rectifiable arc. (Received July 5, 1963.)

Let \( E \subseteq M \subseteq S \) be subsets of the complex plane \( K \) and \( F \) a collection of maps of \( S \) into \( K \). \( E \) is called the exceptional set \( E(F, M) \) of \( F \) with respect to \( M \) if \( E = \{x \in M | \exists f \in F (V \cap S) \text{ is unbounded for all neighborhoods } V \text{ of } x\} \). Let \( D \) denote the closure of the unit disk \( U \), and let \( E \subseteq D \). Then the following are equivalent: (1) \( E = E(F, D) \) for some pointwise bounded collection \( F \) of open maps of a set \( S \) containing \( D \) into \( K \); (2) \( E \) is such that: \( E \) is compact; each component of \( E \) intersects \( D - U \); there exists a sequence of open sets \( S_1 \supseteq S_2 \supseteq \ldots \) such that \( \bigcap_n S_n = \emptyset \), and \( E - S_n \) is equal to the boundary of the unbounded component of \( K - (E - S_n) \) for all \( n \); (3) there exists a sequence of polynomials \( P_1, P_2, \ldots \), such that \( \lim_{n \to \infty} P_n(z) = 0 \) for \( z \in D \), \( p = 0, 1, \ldots \), and such that \( E = E(P, D) \). Extensions are made to the cases where \( D \) is replaced by \( U \) or \( K \). The topology of exceptional sets is studied. In particular, conditions are formulated which exclude the universal plane curve from consideration because of an excess of "fine" structure. (Received July 5, 1963.)

576
Let $C(X)$ be the Banach space of real bounded continuous functions (relative to the supremum norm) on the locally compact separable metric space $X$ and let $\mathcal{W}$ be the class of nonempty open sets $W \subset X$ with compact closure $\overline{W}$ and nonempty boundary $W'$. For a fixed $W \in \mathcal{W}$ let $P \subset C(\overline{W})$ possess the following properties: (i) $P$ contains the constant functions, (ii) $P - P$ is dense in $C(\overline{W})$, (iii) $P$ is a wedge in $C(\overline{W})$, and (iv) if $U \in \mathcal{W}$, $U \subset W$, $f \in P$, then $f \leq \sup_{U} f$ on $\overline{U}$, $U \subset W$, $U \subset \mathcal{W}$, is called a regular set if the Choquet boundary of $U$ relative to $P$ is $U'$. It is assumed that the regular sets form a basis for the relative topology on $W$. If $Q$ is a maximal element, relative to the set inclusion order relation, of the class of wedges in $C(\overline{W})$ containing $P$ and possessing the same properties as $P$, if $U$ is a regular set, and if $x \in C(U')$, then $\varphi_U(x)$ and $\rho_U(x)$ are defined as $\sup \{ f : f \leq x \text{ on } U', f \in Q \}$ and $\inf \{ f : f \geq x \text{ on } U', f \in - Q \}$, respectively, Theorem. If $U$ is a regular set, $x \in C(U')$, and $\varphi_U(x)$ and $\rho_U(x)$ are continuous on $\overline{U}$, then $\varphi_U(x) = \rho_U(x)$ on $\overline{U}$. (Received July 8, 1963.)
The following improvement on a theorem of Heineken (Endomorphismenringe und engelsche
elemente, Arch. Math. 13 (1962), 29) is obtained: Theorem. Let \( A \) be an abelian normal subgroup
of the group \( G \). Suppose \( A \) contains no elements of order \( 2, 3 \) or \( 5 \) and that the repeated commutator
\( (((a, g), g), g) \) is the identity for all \( a \) in \( A \) and \( g \) in \( G \). Then \( A \) lies in \( Z_9 \), the ninth term of the upper
central series for \( G \). In his original result Heineken shows \( A \subset Z_k \), where \( k = 3^9 \), provided order \( 7 \)
is also excluded. The proof rests on the improved lemma: Lemma. Let \( L \) be a Lie ring embedded
in an associative ring having no elements of additive order \( 2, 3 \) or \( 5 \). Suppose the associative third
power of each Lie element is zero. Then \( L \) is nilpotent of class at most \( 6 \) and the associative product
of any \( 9 \) Lie elements is zero. By a computer calculation it has been shown that the \( 6 \) of this lemma
cannot be reduced to \( 5 \) nor the \( 9 \) reduced to \( 7 \). (Received July 8, 1963.)

Let \( P \) be a polynomial in \( \xi_1, \ldots, \xi_n \) and set \( \zeta = (\xi_1, \ldots, \xi_n - 1) \), \( \eta = \xi_n \). Assume that all derivatives
of \( P \) are bounded by \( K_1 |P(\xi, \eta)| \) for \( |\xi| \leq \xi_1^2 + \ldots + \xi_{n-1}^2 \) large. If \( m \) is the largest exponent of \( \eta \) in \( P \), let \( \tau_1(\xi), \ldots, \tau_m(\xi) \) be the roots of \( P(\xi, \eta) = 0 \). If \( n \geq 3 \), the numbers \( r \) of the \( \tau_k \) with positive imaginary
parts is constant for \( |\xi| \) large. For \( n = 2 \) this is assumed. Rearrange the \( \tau_k \) so that \( \text{Im} \tau_k > 0 \) for \( 1 \leq k \leq r \), \( \text{Im} \tau_k < 0 \) for \( r < k \leq m \). Set \( P_+ = \prod_{1}^{r} (\xi - \tau_k) \), \( P_- = P/P_+ \). Let \( Q_1, \ldots, Q_r \) be polynomials
of \( \xi, \eta \) of degree \( < \min \xi \). Resolve each \( Q_j \) into partial fractions: \( Q_j = Q_j^+/P_+ + Q_j^-/P_- \) and set
\( \phi_{ij} = \int_{-\infty}^{\infty} Q_j^+/P_+ d\eta \). Let \( A_{\pm} \) be the Hermitian matrix \( (\phi_{ij}) \) and assume \( A_{\pm} A_{\pm}^* A_{\pm} \leq K_2 A_{\pm} \) for
\( |\xi| \) large. The coefficient of \( \eta^m \) in \( P \) is to be bounded away from \( 0 \) for all \( \xi \). Theorem. Under
the above hypotheses, \( \| R(D)u \| \leq C(\| P(D)u \| + \sum \| Q_j(D)u \| + \| u \|) \) for all \( u \in C_0^\infty (x_n \geq 0) \), where \( R(D) \)
is any operator weaker than \( P(D) \), \( \| \cdot \| \) is the \( L^2(x_n > 0) \) norm and the \( \langle \cdot, \cdot \rangle \) are appropriate norms on
\( x_n = 0 \). This and related inequalities allow one to prove regularity up to the boundary for solutions of equations
with variable coefficients. They give new information even when \( P(D) \) is elliptic. (Received July 10, 1963.)

Consider a bimatrix game (i.e., a two-person, nonzero-sum game with a finite number of pure
strategies) with the \( n \)-by-\( m \) payoff matrices \( A \) and \( B \) to players I and II, respectively. Let the \( n \)-by-\( 1 \)
probability vector \( x \) be the mixed strategy of I, the \( m \)-by-\( 1 \) probability vector \( y \) the mixed strategy
of II, the scalar \( a \) the expected payoff to I and the scalar \( \beta \) the expected payoff to II. Let \( e \) and \( \ell \) be
\( n \)-by-\( 1 \) and \( m \)-by-\( 1 \) vectors of ones, respectively. Definition. Extreme equilibrium points are equilib­
rium points such that all other equilibrium points are convex combinations of extreme equilibrium
points. Theorem. The set of extreme equilibrium points of the game is finite and is given by the
vertices of the convex polyhedral sets \{ \( x, \beta \) \mid \( \{ x, \beta \} |B^*x - \beta \ell \leq 0, e'x = 1, x \geq 0 \} \} and \{ \( y, a \) \mid \( \{ y, a \} A y = ae \leq 0, \ell'y = 1, y \geq 0 \} \) that satisfy the equalities \( x'Ay - a \leq 0 \) and \( x'By = \beta = 0 \). This result
seems to be simpler still than Kuhn's simplification of Vorob'ev's results [Proc. Nat. Acad. Sci., U.S.A. 47 (1961), 1657-1662]. (Received July 15, 1963.)

63T-340. V. S. PLESS, Air Force Cambridge Research Laboratory, Bedford, Massachusetts, On Witt's Theorem for nonalternating symmetric bilinear forms over a field of characteristic 2.

Let $V(V')$ be a vector space over a field of characteristic 2 on which a nondegenerate, nonalternating symmetric bilinear form $f(f')$ is defined such that $f(x,x)(F'(x',x'))$ takes its values in a perfect subfield. Theorem 1. $V$ has an orthonormal basis, and hence any two spaces $V$ and $V'$ of the same dimension over the same field are isometric. Theorem 2. There exists a unique vector $h$ in $V$ such that $f(h,x) = \sqrt{f(x,x)}$ for all $x$ in $V$. The vector $h$ is the sum of any orthonormal basis. Theorem 3 (Analogue to Witt's Theorem). Let $V$ and $V'$ be of the same dimension and defined over the same field $K$. Let $\rho$ be the isometry that exists between $V$ and $V'$. Let $\sigma$ be an isometry of a subspace $U$ of $V$ into $V'$. Then $\sigma$ can be extended to an isometry on all of $V$ if and only if $\rho(h)$ is in $U$ and in case $h$ is in $U$, $\rho(h) = \rho(h)$. Corollary 3. Any isotropic space is contained in an isotropic space of maximal dimension $\nu$. If $n$ is odd, $\nu = (n - 1)/2$. If $n$ is even, $\nu = n/2$. Theorem 4. A subspace $U$ of $V$ has exactly four invariants under the orthogonal group. They are $\dim U$, $\dim U \cap U^*$, $\dim U \cap \{h\}^*$, $\dim U \cap \{h\}$. (Received July 16, 1963.)

63T-341. DAIIHACHIRO SATO, 26 College Court, Regina, Saskatchewan, Canada, Transcendental entire function which together with all its higher derivatives assumes algebraic values at all algebraic points.

Let $\{z_1\} = \{z_1, z_2, z_3, \ldots\}$ be an enumeration of all algebraic numbers. Construct a sequence $\{t_j\} = \{t_1, t_2, t_3, \ldots\}$ so that all of the algebraic numbers appear an infinite number of times in $\{t_j\}$. Then, for algebraic numbers $a_n$ with $0 < |a_n| < (n! \cdot \prod_{j=1}^{n} (1 + |t_j|))^{-1}$, the function $f(z) = \sum_{n=0}^{\infty} a_n \frac{z^n}{n!} (z - t_j)$ is an entire function having the said property. Since $|z - t_j| \leq 1 + |t_j|$ for $|z| \leq 1$ and $|z - t_j| \leq |z| \cdot (1 + |t_j/z|) < |z| \cdot (1 + |t_j|)$ for $|z| > 1$, the series for $f(z)$ converges absolutely in $|z| < \infty$ and $|f(z)| \leq \max \{e, e^{|z|}\}$. Since $f^{(n)}(t_j)$ is a polynomial of $t_j$ with algebraic coefficients $a_n$ and $\{t_j\}$ contains all algebraic numbers infinitely many times, $f^{(n)}(t_j)$ must be an algebraic number for any algebraic number $t_j$. Algebraic numbers here can be replaced by any dense denumerable number ring such as rationals. A similar (but longer) method can be applied in the case of discrete number ring such as integers; however, the problem in the case of nondenumerable numbers remains unsettled. Problem. Does there exist a transcendental entire function which (together with all its derivatives) has algebraic values at every algebraic point and has transcendental values at every transcendental point? (Received July 16, 1963.)

(1) Suppose that \( f(x) \in L^p \), \( 1 \leq p < \infty \), and that for each point \( x \) of a set \( E \) the function has a \( k \)th symmetric derivative in the metric \( L^p \), that is, there exists a polynomial \( P(t) = P_x(t) \) of degree \( \leq k \) such that
\[
|h^{-1} \int_0^h f(x + t) + (-1)^k f(x - t) / 2 - P(t)|^p dt^{1/p} = o(h^k).
\]
Then at almost all points of \( E \) the function has an unsymmetric \( k \)th derivative in \( L^p \), that is, there is a polynomial \( Q(t) = Q_x(t) \) of degree \( \leq k \) such that
\[
|h^{-1} \int_0^h |f(x + t) - P(t)|^p dt^{1/p} = o(h^k).
\]

(2) Suppose that a trigonometric series
\[
\sum_{n=0}^{+\infty} c_n e^{inx}
\]
is summable \((C, k - 1)(k = 1, 2, \ldots)\) to sum \( s(x) \) at each point of a set \( E \), and let \( F(x) = c_0 x^k / k! + \sum c_n (i n)^{-k} e^{inx} \). Then at almost all points of \( E \) the function \( F \) has a \( k \)th unsymmetric derivative equal to \( s(x) \). If \( S \) is of power series type the conclusion holds everywhere in \( E \). (Received July 16, 1963.)


Theorem. Let \( \mathbb{E}^n \) be Euclidean \( n \)-space and \( \Delta \) the Laplacian on \( \mathbb{E}^n \). Let \( V(x) \) be a real valued function continuous except for a set of capacity zero. Let \( \mathcal{D}(s) \) and \( \mathcal{D}(V) \) be the natural domains of \( \Delta, V \) considered as operators on \( L_2(\mathbb{E}^n) \). Let \( D = \mathcal{D}(s) \cap \mathcal{D}(V) \) and suppose \( \Delta - V \) is a semi-bounded from above, symmetric operator on \( D \). If \( E(\cdot |x(0) = x) \) denotes the Wiener integral over continuous paths \( x(\cdot) \) on \([0, \infty)\) such that \( x(0) = x \), then for \( \psi \in L_2(\mathbb{E}^n) \), \( T_t \psi(x) = E(\exp[- \int_0^t V(x(t)) dt] \psi(x(t)) | x(0) = x) \) exists and defines a strongly continuous self-adjoint semi-group of operators on \( L_2(\mathbb{E}^n) \) whose infinitesimal generator is a self-adjoint extension of \( \Delta - V \). Proof. Functional integration and spectral-theory are used in the proof. Remark. For \( V \) bounded below this result is well known [see e.g., Getoor, Pacific, J. Math. 7 (1957), 1577-1591]. These results can be applied to the Feynman integral and perturbation of singular elliptic operators. (Received July 16, 1963.)

63T-344. B. L. OSOFSKY, 255 Matilda Avenue, Somerset, New Jersey. Rings all of whose finitely generated modules are injective.

Theorem. Let \( R \) be a ring with identity such that every finitely generated right \( R \) module is injective. Then \( R \) is semi-simple Artin. Corollary. A ring \( R \) is right self injective and right hereditary if and only if it is semi-simple Artin. A ring with the properties of the theorem must be a right self injective regular (in the sense of von Neumann) ring, and in any such ring with an infinite set of orthogonal idempotents the author constructs a right ideal \( I \) such that \( R - I \) is not injective. The corollary follows immediately since all quotients of an injective module over a hereditary ring are injective. These results form a part of the author's Ph.D. thesis being written at Rutgers University under the direction of Professor Carl Faith. (After this abstract was written, it was called to the author's attention that L. A. Skornjakov has recently published this theorem; however, there is an error in his proof of an essential lemma → Lemma 9 → invalidating his proof of the theorem.) (Received July 17, 1963.)

63T-345. WITHDRAWN

580
An ideal $B$ of a commutative ring $R$ is an AM-ideal if each ideal of $R$ properly contained in $B$ is of the form $BC$ for some ideal $C$ of $R$. $B$ is a multiplication ideal if $B$ is an AM-ideal and if $RB = B$. $R$ is an AM-ring (respectively, multiplication ring) if each ideal of $R$ is an AM-ideal (respectively, multiplication ideal). $R$ satisfies Condition (*) if every ideal of $R$ with prime radical is primary. In the January and February 1963 issues of these Notices, Gilmer announced results classifying rings satisfying (*). These results are used to prove the following theorems: Theorem. Ring $R$ satisfies (*) if and only if every ideal of $R$ is the intersection of its isolated primary components. Theorem. If every prime ideal of the ring $R$ is an AM-ideal, $R$ is an AM-ring. Theorem. Ring $R$ is an AM-ring if and only if the following conditions hold: (i) $R$ satisfies (*), (ii) primary ideals of $R$ are prime powers, (iii) if $P$ is a proper prime ideal of $R$ and if the ideal $A$ and positive integer $n$ are such that $A \subseteq P^n$, $A \not\subseteq P^{n+1}$, then there exists $y \in R - P$ such that $P^n = A:y$.

(Received July 22, 1963.)


Suppose each of $F$ and $G$ is a function from the real line to the complete normed ring $N$. If each of $F$ and $G$ is of bounded variation on the interval $[a,b]$ then the Young integral $\int_a^b F \cdot dG$ is known to exist and the interior integral $(I) \int_a^b dF \cdot G$ is known to exist (see [Bull. Amer. Math. Soc. 69 (1963), 314-329] for references). Hence, the latter integral is known to exist under the condition that $F$ is of bounded variation on $[a,b]$ and $G$ is quasi-continuous, i.e., each of $G(x - )$ and $G(x +)$ exists for each number $x$. Here is a new connection between these integrals, which includes a new existence theorem for the former one. Theorem. If $F$ is of bounded variation on $[a,b]$ and $G$ is quasi-continuous then $(Y) \int_a^b F \cdot dG + (I) \int_a^b dF \cdot G = F(b)G(b) - F(a)G(a)$. (Received July 22, 1963.)

63T-348. WILLIAM REDDY, 15 Smith Hall, Syracuse University, Syracuse 10, New York. An open cell of even dimension admits an expansive autohomeomorphism. Preliminary report.

In the group of iterates of an autohomeomorphism $f$ of a metric space $X$ with metric $d$, the function $f$ is called expansive with expansive constant $c > 0$ if, to each pair $(x,y)$ of distinct points of $X$, there exists an integer $n$ such that $d[f^n(x), f^n(y)] > c$. An example of an expansive autohomeomorphism of the open 2-cell is constructed. Since an open cell of even dimension can be written as the product of open 2-cells, this suffices to establish the title statement. (Received July 22, 1963.)

63T-349. WITHDRAWN


There exists a Bergman's operator $P(g_n)$ transforming functions $g_n(X,Z)$ into solutions of
\[ L(\psi) \equiv \Delta \psi + F \psi \equiv \psi_{XX} + \psi_{ZZ*} + F(Z,Z*) = 0, \]
where \( X = x, Z = (z + iy)/2, Z^* = (z - iy)/2, \) and \( F \) is an entire function in \( Z \) and \( Z^* \) (see Bergman, J. Math. Mech. 7 (1958), 87-102). The associate function \( g_n(X,Z) \) and the corresponding solution \( \psi \) are both polynomials of \( n \)th degree in \( X \). The author considers the case of \( L(\psi) \) when \( F = F(Z^*) \) and generalizing the above results, gives the sufficient conditions for the coefficients of the series expansion \( \sum_{n=0}^{\infty} a_n g_n(X,Z) = g(X,Z) \) in order that \( P\{g(X,Z)\} \) is a solution of \( L(\psi) = 0 \). In this case the associate function \( g(X,Z) \) and the corresponding solutions are not necessarily polynomials in \( X \). (Received July 10, 1963.)

63T-351. C. M. HOWARD, University of California, Berkeley, California. Finite dimensional analogues to boolean algebras.

A transformation algebra of degree \( n \) (\( T[n] \) algebra) is an algebra \( \langle A,+,\cdot,\cdot,-,S,E \rangle \) which satisfies the (equational) axioms, \( E_q \) for polyadic algebras of degree \( n \) with equality \( E \) that do not involve quantifiers. The algebra is concrete when for some set \( B,+,\cdot,\cdot,- \) are union, intersection, compliment with respect to \( B^n, 0 \neq A \subseteq P(B^n) \), and for each \( a \) in \( A, f \) in \( B^n \), and \( i, j < n, S_f(a) = \{g \in B^n/gf \in a\} \), \( E(i,j) = \{g \in B^n/g(i) = g(j)\} \). The main result is that for \( n > 2 \) the class of isomorphic images of concrete \( T[n] \) algebras is an axiomatizable \( U\lambda\) class, but not \( U\lambda\). Also, with the obvious necessary modifications, all the results stated in Abstract 62T-140 (these Notices, June 1962) after deleting (4) and (5) parts (c) remain true for \( T[n] \) algebras. It follows that \( E_q \) is complete for equations holding in all concrete \( T[n] \) algebras, and that all \( T[n] \) algebras are representable as subdirect products of concrete algebras. It follows from the main result that there is no second order universal sentence which characterizes the concrete algebras for \( n > 2 \). The problem of a good algebraic characterization of the concrete algebras remains. (Received July 23, 1963.)


A semigroup \( S \) of real matrices of order \( n \) is said to be a P-semigroup if there exists a partition \( \{J,K\} \) of \( N \times N \), where \( N \) denotes the first \( n \) natural numbers, such that \( S \) consists of all \( n \times n \) matrices \( (a_{ij}) \) where \( a_{ij} > 0 \) if \( (i,j) \subseteq J \) and \( a_{ij} = 0 \) if \( (i,j) \subseteq K \). The following result is obtained: Theorem. A P-semigroup \( S \) is simple iif every element is of the same rank. If this is the case, \( S \) can be upper-triangulated (zeros above the main diagonal) by a series of row-column permutations, That a simple P-semigroup without idempotents contains no minimal one-sided ideals follows from the more general result: Theorem. Any simple semigroup of \( n \times n \) matrices over a field which contains a minimal left ideal is completely simple. (Received July 23, 1963.)

63T-353. J. G. STAMPFLI, New York University, University Heights, Bronx 53, New York. A hyponormal operator with spectrum on an arc is normal.

An operator \( T \) on a Hilbert space \( H \) is hyponormal if \( T*T - TT* \succeq 0 \). An arc is a simple, closed, rectifiable curve which has a continuous second derivative with respect to arc length, Theorem 1. If \( T \) is hyponormal and the spectrum of \( T \) lies on an arc, then \( T \) is normal. Corollary 1. If \( T \) is hyponormal and the spectrum of \( T \) is real, then \( T \) is self-adjoint. Corollary 2. If \( T \) is hypon-
normal and the spectrum of $T$ lies on the unit circle, then $T$ is unitary. **Theorem 2.** If $T$ is hypo-
normal (spectrum arbitrary) then the convex hull of the spectrum of $T$ equals the closure of the
numerical range of $T$. (Received July 24, 1963.)

63T-354. J. R. DAVIS, Washington University, St. Louis 30, Missouri. **On the distribution of
eigenvalues of generalized Toeplitz forms.**

Let $W_v(n,x)$ be the ultraspherical polynomial of index $v$ and order $n$ normalized so that

$$\int_1^{-1} W_v(n,x) W_v(m,x) \Omega_v(dx) = (\omega_v(n))^{-1} \delta_{n,m}$$

where $\Omega_v(dx) = (1 - x^2)^{-1/2} \Gamma(v+1/2) \Gamma(v+1/2)v!$. Let $c(j,n,k)$ be defined by $W_v(j,x)W_v(n,x) = \sum_{k=0}^{\infty} c(j,n,k)W_v(k,x)$. Let $f(r,x) = \sum_{j=0}^{\infty} b(r,j)W_v(j,x)$ where $b(r,j)$ is real and continuous on $0 \leq r \leq 1$
for each $j = 0,1,2,...$ and there exists a constant $M$ such that $\sum_{j=0}^{\infty} \max_{0 \leq r \leq 1} |b(r,j)| \leq M$.

The matrix $A_N(f) = (a(n,k,N))$, $0 \leq n, k \leq N$ is defined by $a(n,k,N) = \int_1^{-1} \omega_v(k)W_v(k,x)W_v(n,x)$
$-f((n+k)/(2N+2), x) \Omega_v(dx) = \sum_{j=0}^{\infty} \omega_v(k)c(k,n,j)b(k+n)/(2N+2), j)$. Let $\lambda(1,N) \leq \lambda(2,N) \leq ...
\leq \lambda(N+1, N)$ be the eigenvalues of $A_N(f)$. Finally let $D_N(b) = (N+1)^{-1} \lambda_{no}$, of $(k, N) < b_1$, $S(b) = \{f(r, \theta)|f(r, \cos \theta) < b_1$ and $D(b) = \pi^{-1} \int S(b) d\theta$.

**Theorem.** With the above definitions and notation

$$\lim_{N \to \infty} D_N(b) = D(b)$$

at every point of continuity of $D(b)$. This result is analogous to a result of Ker, Murdock, and Szegő (J. Rational Mech. Anal. 2(1953), 767-800) where $f(r,x) = \sum_{j=0}^{\infty} b(r,n)e^{inx}$.

(Received July 18, 1963.)

63T-355 and 63T-356. WITHDRAWN

63T-357. NATHANIEL COBURN, 347 West Engineering, University of Michigan, Ann Arbor, Michigan. **General theory of simple waves in relaxation hydrodynamics.**

In the case where the relaxation parameter, $K$, is a constant, sufficient condition for simple
waves to exist can be obtained from the results of a previous paper (The limiting speeds of character-
istics in relaxation hydrodynamics, N. Coburn, J. Math. Anal. Appl. 5 (1962), 269-286). However,
when $K$ is a function of class $C^1$ of (1) the density, the entropy, the relaxation variable (Type I), (2)
the space and time coordinates (Type II), the theory is more complex. First, necessary conditions
are obtained for a discontinuity theory problem and the corresponding Cauchy problem. It is shown
that neither of these systems furnish sufficient conditions to solve these problems. For $K$ of Type I,
the necessary conditions plus the following two additional conditions will form sufficient conditions:
(1) a regularity condition; (2) a limit condition on the characteristic partial differential equation,
that as $K$ approaches zero, the speed of propagation approaches a limit speed $c_\infty$.
In $K$ of Type II,
only the second condition is needed. Simple waves of Type I are shown to be families of parallel
lines; simple waves of Type II are shown to be families of nonparallel lines. The existence and
properties of both families are discussed. (Received July 30, 1963.)

63T-358. R. V. CHACON, Brown University, Providence, Rhode Island. **A class of linear
transformations.**

A class of positive and invertible isometrics of $L_1(0,1)$ is constructed such that for each trans-
formation $T$ of the class the limit $\lim_{n \to \infty} 1/n \sum_{k=0}^{n-1} |Tk|_1$ fails to exist almost everywhere for $f > 0$ in
$L_1$, thus answering a question raised by A. and C. Ionescu-Tulcea and S. Kakutani. The adjoint class

583
consists of measurable and invertible transformations of \((0,1)\) which clearly have no \(\sigma\)-finite invariant measure equivalent to Lebesgue measure, and thus another and simpler class of counter examples of this sort is incidentally obtained (see D. S. Ornstein, On invariant measures, Bull. Amer. Math. Soc, 66 (1960), 297-300). A small modification of the construction yields that there exists a subclass (depending on \(\delta\)) having in addition that \(\|T\|_\infty \leq 1 + \delta\) for each \(\delta > 0\). (Received August 1, 1963.)

63T-359. P. K. HOOPER, Computation Laboratory, Harvard University, Cambridge 38, Massachusetts, Some small multi-tape universal Turing machines.

The standard way of assigning complexity to a one-tape T.M. is by the state-symbol product, a measure clearly inadequate for machines with more than one tape. Letting an \((m,n,p)\)-machine denote a Turing machine with \(m\) states, \(n\) symbols (including any end markers) and \(p\) tapes, the number \(m \cdot n^p\) gives the maximum number of operating rules for an \((m,n,p)\)-machine and serves as a reasonable complexity criterion, reducing to the state-symbol product when \(p = 1\). A \((2,3,2)\)-machine has been designed which simulates an arbitrary tag system with deletion number 2, and therefore is universal (Minsky, A. I. Memo No. 33, Cambridge, Mass., 1962). Similarly a \((1,2,4)\)-machine which is universal, since it can simulate an arbitrary "B machine" (Wang, J. Assoc. Comput. Mach, 4, No. 1 (1957)), has been designed, having a fixed loop for one of its four tapes. Under the above criterion, these machines have complexities of 18 and 16, respectively, both less than any reported state-symbol product. Moreover, any \((m,n,p)\)-machine may be simulated, step-by-step, by a \((1,2,p')\)-machine, where \(p' = p \log(n) + \lceil \log^2(m) \rceil\). If the logarithms are integral, this transformation is realized with no increase in complexity. (Received August 2, 1963.)

63T-360. DOROTHY MAHARAM, University of Rochester, Rochester 27, New York.
Incompressible transformations.

Let \(T\) be a 1-1 measurable nonsingular transformation of a \(\sigma\)-finite measure space \((X, \mu)\) onto itself. It is first shown that one can extend \((X, \mu)\) to a larger measure space \((X^*, \mu^*)\), and (in a sense) extend \(T\) to a measure-preserving transformation \(T^*\) of \((X^*, \mu^*)\), in such a way that \(T^*\) is incompressible if \(T\) is. Variants of ergodic theorems of Hopf, Halmos and Hurewicz are deduced, and the behavior of the sequence \(\{\omega_n\}\) of Radon–Nikodym derivatives of the powers of \(T\) is investigated. It is shown, for instance, that the set where \(\omega_n(x) \to 0\) is dissipative; as a corollary, if \(\mu X < \infty\), then \(\sum \omega_n(x)\) converges almost everywhere where \(\omega_n(x) \to 0\). A suitable generalization is found when \(\mu X = \infty\). (Received August 5, 1963.)

63T-361. DOROTHY MAHARAM, University of Rochester, Rochester 27, New York.
Derivatives of incompressible transformations. Preliminary report.

In the notation of the author's previous abstract, suppose \(T\) is incompressible and \(\mu X = 1\).

Let \(f_n(x,c)\) denote the number of integers \(i\) such that \(0 \leq i < n\) and \(\omega_i(x) \geq c\). Theorem. There is a fixed null set \(N \subseteq X\) such that, for all \(x \in X - N\), and for all \(a,b\) such that \(0 \leq a \leq b\),

\[ \lim f_n(x,b)/f_n(x,a) \text{ exists as } n \to \infty. \]

The proof uses the construction of the accompanying abstract, and the following lemma. If \(T\) is measure-preserving and incompressible (here \(\mu X\) may be infinite),
and \( g \) is an everywhere positive function on \( X \) such that, for each \( c > 0 \), the set \( H_c = \{ x | g(x) \geq c \} \) has finite measure, then there exists a null set \( Z \subset X \) such that, for all \( x \in X - Z \), and for all \( a, b \) as above, 
\[
\lim_{n \to \infty} \sum (xH_b)(T^i x)/\sum (xH_a)(T^i x) \text{ exists as } n \to \infty,
\]
the summations being with respect to \( i = 0, 1, \ldots, n - 1 \).

(Received August 5, 1963.)


The main problem is that since division cannot be used, numbers grow explodingly during the elimination process, although the solution itself may be small. (See, for example, J. B. Rosser, J. Res. Nat. Bur. Standards 49 (1952), 349-358.) In the method proposed here, the system of equations is converted to a system of congruences modulo a number of small primes. At the \( i \)th step, the solution of the system of congruences mod \( p \) is combined with the previous solutions by the Chinese Remainder Theorem. The process is continued until the solution produced in the \( i \)th and \((i + 1)\)th steps are identical. At this point, a substitution check is made, and usually the process terminates there, although the check may reveal that more primes are needed. The process always converges, and the size of numbers throughout the computation is bounded by a number of the same order of magnitude as the solution. (Received August 5, 1963.)

63T-363 and 63T-364. WITHDRAWN


Let \( \Omega = a dx^2 + 2b dx dy + c dy^2 \) and \( \hat{\Omega} = a \hat{dx}^2 + 2b \hat{dx} \hat{dy} + c \hat{dy}^2 \) be smooth positive definite quadratic forms on the oriented surfaces \( S \) and \( \hat{S} \), determining Riemann surfaces \( R \) and \( \hat{R} \) on \( S \) and \( \hat{S} \), respectively. Where \( f: S \to \hat{S} \) has positive Jacobian, and dilatation \( K \) of \( f: R \to \hat{R} \) is given by 
\[
(\mathcal{N} - \mathcal{N}' )/(\mathcal{N} + \mathcal{N}') \text{ where (as suggested when } S = \hat{S}, I = \Omega, \Pi = \hat{\Omega} \text{ and } f(p) = p) \quad 2\mathcal{K} = \left( \left[ a + c - 2b \right]/\left[ ac - b^2 \right] \right), \quad \text{while } (\mathcal{N}' )^2 - 4\mathcal{K} \mathcal{N}' < 0 \text{ and } \mathcal{N}' = (a \hat{c} - b \hat{b})/(ac - b^2). \quad \text{With conformal structure determined by } I, \Pi \text{ or } \Pi' \text{ (where positive definite) on surfaces smoothly immersed in } E^3, \quad K \text{ is computed for any } f \text{ with positive Jacobian which preserves normals and a net of lines-of-curvature (and thus for the Gauss map, the standard map between parallel surfaces, and the identity map). As expected, the expressions obtained for } K \text{ (as a function of principal curvatures at corresponding points) are exactly those which (as shown previously) must be constant when } f \text{ is a Teichmüller mapping. (Received August 7, 1963.)}


Let \( \Lambda \) be a smooth quadratic form on Riemann surface \( R \) with \( \Lambda = A dx^2 + 2Bdxdy + Cdy^2 \) for \( z = x + iy \) on \( R \). Except where \( AC - B^2 = 0 \) or \( \Lambda = -C \) with \( \Lambda - iB \text{ analytic on } R, NASC \) are given on A, B and C for the existence (locally) and uniqueness (up to motions of \( E^3 \)) of a conformal imbedding of \( R \) in \( E^3 \) with \( \Lambda = II \). This is done by finding \( \lambda \) (assuming \( I = \lambda(x,y) \{ dx^2 + dy^2 \} \)) in terms of \( A, B, C \) and their partial derivatives. Then the known NASC for \( I \) and \( \Lambda = II \) to yield a surface \( S \) in \( E^3 \)
are rewritten eliminating reference to $\lambda$. Similarly, if $AC - B^2 > 0$, NASC are given for the existence of a (unique, local) "embedding of $R$ in $E^3"$ with $A = I$, where $R$ is realized by $R_2$ structure on $S$ (if $K > 0$ for $A = I$) or $R_1$ structure on $S$ (if $K < 0$ for $A = I$ while $\{A - C - iB\} \neq 0$). Here, coefficients of $II$ are found in terms of $A$, $B$, $C$ and their partial derivatives. Thus when working on $R_1$, $R_2$ or $R_2$ structure on $S$ in $E^3$, one fundamental form (if $HK \neq 0$) determines the other. (Received August 7, 1963.)


The following is noted. Given an arbitrary Riemann surface $R$ defined on an oriented surface $S$ with Riemannian metric, the smooth immersion of $S$ in $E^3$ can be harmonic on $R$ only if $K \leq 0$ on $S$, while points where $K = 0$ must be flat points of such an immersion. For an $S$ with $K < 0$ immersed smoothly in $E^3$, the form $II'$ defined by $\Omega^2 = (E - G - 2IF)dz^2$ is holomorphic on $R_2$. An elementary fact states that minimal surfaces (characterized wherever $K \neq 0$ by $\Omega \equiv 0$ on $R_2$) have such immersions. Presented here is an immersed nonminimal surface with $\Omega \neq 0$ holomorphic on $R_2$. The nature of such nonminimal $R_2$-harmonic immersions is studied. (Received August 7, 1963.)

63T-368. P. R. YOUNG. Reed College, Portland 2, Oregon, Notes on the structure of r.e. sets.

I. Def. (Kleene-Post): $J(A,B) = \{2x|x \subseteq A\} \cup \{2x + 1|x \subseteq B\}$, $A \subseteq_1 J(A,B)$ and $B \subseteq_1 J(A,B)$, For def. of pseudo-creative see 601-1 (these Notices, April 1963). If $A$ is pseudo-simple and $B \subset A$ is an infinite r.e. set such that $C \subset A$ and $C$ r.e. implies $B - C$ is finite, $B$ is called a r.e. center for $A$ (Dekker). Def. $D_1$ is the class of all hypersimple sets; $D_2$: all simple sets not in $D_1$; $D_3$: all pseudo-simple sets with recursive centers; $D_4$: all pseudo-simple sets with nonrecursive r.e. centers; $D_5$: all pseudo-simple sets without r.e. centers; $D_6$: all pseudo-creative sets; $D_7$: all creative sets. Results: $A \in D_i$, $B \in D_j$, and $1 \leq i \leq j \leq 7$ implies there exists $B \subset D_j$ such that $A \subseteq_1 B$, $A \in D_i$, $B \in D_j$, and $A \subseteq_1 B$ implies $i \leq j$, $A \in D_i$, $B \in D_j$, $A \subseteq_2 B$ and $i \neq 3$ or 6 implies $i \leq j$, $A \in D_3$ implies there exists $B \subset D_2$ such that $A \subset_1 B$. There exists $A \in D_3$ such that for $1 \leq j \leq 7$ there exists $B \subset D_j$ such that $A \subset_1 B$. There exists $A \in D_6$ such that for $1 \leq j \leq 7$ there exists $B \subset D_j$ such that $A \subset_2 B$. There exists $A \in D_6$ such that for all $j < 6$ there does not exist $B \subset D_j$ such that $A \subset_1 B$. With the exception of the last assertion, proofs are generally straightforward. The last assertion follows from results announced in 603-19 (these Notices, August 1963). II. Any two maximal sets are either many-one equivalent or are many-one incomparable. (Received August 12, 1963.)


In applications the following corollary to H. Widom's theorem \[ \text{Proc, Amer, Math, Soc, vol. 14 } \]
Corollary. Assume \( \gamma(x,y) = \log k(x,y) \) is real and symmetric, belongs to \( C^2 \), and satisfies (i) \( \gamma_y \geq 0 \), (ii) \( \lim_{x,y \to \infty} y \gamma_y(x,y) = \infty \), (iii) \( \gamma_y = o(\gamma^2) \) and \( \gamma_x = o(\gamma^2) \) and \( \gamma_y = o(\gamma^2) \) as \( x,y \to \infty \). Then the largest eigenvalue of \( k(x,y) \) on \((0,t)\) is asymptotic to \( k(t,t)^2/k'(t,t) \) as \( t \to \infty \). The eigenvalue is simple and the corresponding eigenfunction is asymptotic to \( k(x,t)/(\gamma_y(x,y) + \gamma_y(t,t)) \). All other eigenvalues are \( o(k(t,t)^2/k'(t,t)) \). The proof follows from Widom's work in two steps. (i) \( \int_0^t [k(x,z)k(z,t)/(\gamma_y(z,t) + \gamma_y(t,t))]dz \sim (k(t,t)^2/k'(t,t)) k(x,t)/(\gamma_y(x,t) + \gamma_y(t,t)) \) and (ii) if this form for the eigenfunction and the former form \( k(x,x)^{1/2} \) are normalized, then the two are asymptotic to each other in the \( L^2(0,t) \) sense as \( t \to \infty \). (Received August 13, 1963.)
theorems concerning centroids of curvature and area [T. Hayashi, Rend. Circ. Mat. Palermo 50 (1926); R. C. Bose and S. N. Roy, Bull. Calcutta Math. Soc. 27 (1935); H. Tietze, Elem. d. Math. 3 (1948)], depends on a comparison of the torques of two convex arcs, one inside the other. (Received August 13, 1963.)

63T-373. D. R. McMillan, Jr., Institute for Advanced Study, Princeton, New Jersey. The singular points of a topological embedding. Throughout this paragraph, let \( \Sigma \) be an \((n - 1)\)-sphere topologically embedded in \( S^n \), and let \( K \) be a component of \( S^n - \Sigma \). Theorem 1. If \( n \neq 4 \) and \( K \) is locally 1-connected at each point of \( \Sigma \), then \( K \) is an open \( n \)-cell. Now suppose that \( K \) is locally 1-connected at each point of \( \Sigma - X \), where \( X \subset \Sigma \) is compact, Theorem 2. Let \( n \geq 4 \). If the point \( x \in X \) has an open neighborhood \( U \) in \( \Sigma \) such that \( X \cap U \) is compact and countable, then \( K \) is locally 1-connected at \( x \). Corollary. If the nonlocally flat points of \( \Sigma \) form a countable set and \( n \geq 5 \), then each component of \( S^n - \Sigma \) is an open \( n \)-cell. (Received August 13, 1963.)

63T-374. G. J. Minty, 347 West Engineering, University of Michigan, Ann Arbor, Michigan. "Monotonicity" methods in Banach spaces. Let \( Y \) be a Banach space and \( X \) the conjugate space. Let \( B \) be the unit ball in \( X \) and \( C \) be an open set containing \( B \). Let \( f: C \rightarrow Y \) be monotonic: \( \text{Re}\langle x_1 - x_2, fx_1 - fx_2 \rangle \geq 0 \), where, for \( x \in X \) and \( y \in Y \), \( \langle x, y \rangle \) means \( x(y) \). Suppose also that \( x \in \partial B \) implies \( \langle x, fx \rangle \geq 0 \), and that \( f \) is continuous (or hemicontinuous in the sense of F. E. Browder). Then the equation \( f(x) = 0 \) has a solution in \( B \). Generalizations of this theorem have also been proved, but require too many definitions to be appropriate for an abstract. (Received August 13, 1963.)

63T-375. Hao Wang, 33 Oxford Street, Cambridge 38, Massachusetts. A universal axiom of conditional set existence. The basic axioms of the system \( ZF \) of set theory are, besides the axiom of extensionality, the axioms of (1) pairs, (2) sum set (union), (3) power set, and (4) replacement: Let \( A''x \) stand for the collection of values \( u \), \( \langle \forall v \rangle (v \in x \land Auv) \) when \( Auv \) defines a one-many correlation. Then the axioms (1) to (4) are equivalent to the single axiom (schema) saying that for any set \( x \) and one-many correlation \( A \), there is a set \( y \), \( y = U''x \forall v \) where \( U \) stands for union, \( P \) for power set. Take \( v = \{u\} \) as \( Auv \) to get (2), \( u = \{v\} \) as \( Auv \) to get (3), and \( \langle \forall z \rangle (\forall w) (Hzwv = \{z\}Av = \{w\}) \) as \( Auv \) to get \( H''x \) for (4). Moreover, (1) follows from (3), and (4) in familiar manner: Take \( PP0 \) as \( x \), \( (u = yVv = 0) \) \( \langle u = z \land v = \{0\} \rangle \) as \( Auv \) to get \( \{y, z\} \). (Received August 13, 1963.)

63T-376. J. L. Walsh, Harvard University, 474 Widener Library, Cambridge 38, Massachusetts. The convergence of sequences of rational functions of best approximation, II. Let \( E \) be a closed bounded set whose complement is connected and possesses a Green's function \( G(z) \) with pole at infinity. Let \( E_\rho \) denote generically the locus \( G(z) = \log \rho \) \((\rho > 0) \). Suppose \( f(z) \) is analytic on \( E \), meromorphic with precisely \( \rho \) poles \( a_j \) interior to \( E_\rho \); and that \( \rho \) \((1 < \rho \leq \omega) \) is the largest number for which this is true. Let \( R_n(z) \) of form \( (a_0z^n + \ldots + a_n)/(b_0z^\rho + \ldots + b_\rho) \), 588
\[ \sum |b_j| \neq 0, \text{ minimize the (uniform) Tchebycheff norm } \| f(z) - R_{n^p}(z) \| \text{ on } E. \]  

\[ \lim \sup_{n \to \infty} \| f(z) - R_{n^p}(z) \|^{1/n} = 1/p. \]  

Further, for \( n \) sufficiently large \( R_{n^p}(z) \) has precisely \( v \) finite poles, which approach respectively the \( a_j \), and \( R_{n^p}(z) \) approaches \( f(z) \) uniformly on every closed set interior to \( E_p \), containing no \( a_j \); degree of convergence is determined. This result applies also to the least \( p \)th power norm \( (p > 0) \) if \( E \) has rectifiable boundary and but a finite number of components.

(Received August 15, 1963.)

63T-377. WALTER STRODT, Hamilton Hall, Columbia University, New York 27, New York, Remark on partial orders under which differentiation is stable.

Let \( (K_0, <, U, C_0) \) be a graduated field (Trans. Amer. Math. Soc. 105 (1962), 230-231). Let \( D \) be an additive mapping from \( K_0 \) into \( K_0 \) such that \( D(uv) = uDv + vDu \), \( DC_0 = \{0\} \), and \( D \) is stable at every element \( g \) of \( U - \{1\} \) (i.e. \( [h < g] \) implies \( [Dh < DG] \)). Let \( (x_0, x_1, ...) \) be a sequence of elements of \( U \) such that \( 1 < x_0 \) and \( x_0 < x_1 \cdots x_{j-1} Df < 1 \) \((j = 0, 1, ...)\). Then \( f < 1 \), \( x_0 x_1 \cdots x_{j-1} Df < 1 \) \((j = 0, 1, ...)\). As a corollary, the very strong partial order \( < \) introduced by the author (Memoirs Amer. Math. Soc. no. 13 (1954), 11) is, among all partial orders \( < \) for which \( a < b \) implies \( a/b \to 0 \), the weakest such that \( d/dx \) is stable at every nonconstant logarithmic monomial. (Received August 15, 1963.)

63T-378. HUGH GORDON, University of Pennsylvania, Philadelphia, Pennsylvania. The space of Baire functions and the bidual of the space of continuous functions.

Let \( X \) be a compact Hausdorff space. By "function" will be meant "real-valued function." Let \( \mathcal{B} \) be the ring of bounded Baire functions on \( X \). As is well-known, even in more general circumstances, there is a compact Hausdorff space \( \hat{X} \), of which \( X \) is a dense subset (but not a subspace), such that each \( f \in \mathcal{B} \) admits a continuous extension \( \hat{f} \) to \( \hat{X} \) and each continuous function on \( \hat{X} \) is \( \hat{f} \) for some \( f \in B \). Let \( \mathcal{M} \) be the space of Radon measures on \( X \). For each positive \( \mu \in \mathcal{M} \), define \( X_\mu \subset \hat{X} \) by \( X_\mu = \{ f \in \mathcal{B} \mid f \geq 0, f(p) \neq 0 \implies \int_X f \, d\mu \neq 0 \} \). The main result is that every continuous linear functional on \( \mathcal{M} \), i.e., every element of the bidual of the space of continuous functions on \( X \), can be obtained in the following manner: Let \( \phi \) be any bounded function on \( \hat{X} \) whose restriction to each \( X_\mu \) is continuous. Choose, for each positive \( \mu \in \mathcal{M} \), an \( f \in \mathcal{B} \) such that \( \hat{f} \) and \( \phi \) agree on \( X_\mu \) and define \( \Phi_\mu = \int_X f \, d\mu \). \( \Phi \) can be extended to be a continuous linear functional on \( \mathcal{M} \). (Received August 19, 1963.)

63T-379. DAVID CARLSON, Oregon State University, Corvallis, Oregon. Eigenvalue criteria for \( A \) when \( AH \) is Hermitian.

The bounds discussed in a recent paper of the author (Abstract 603-150, Notices Amer. Math. Soc. 10 (1963), 477) give the following generalization of a theorem by Drazin and Haynsworth (Criteria for the reality of matrix eigenvalues, Math. Zeit. 78 (1962), 449-452): If for any \( A \) and any Hermitian \( H \) there is \( AH \) Hermitian, then \( |x(H) - v(H)| < \rho(A) \), the number of elementary divisors of \( A \) associated with real eigenvalues. It is also proved: \( A \) necessary and sufficient condition for the existence of a Hermitian \( H \), with \( \ln H = (x, v, \rho) \), for which \( AH \) is Hermitian positive semi-definite, of rank \( x + v \), is that \( A \) have at least \( x \) elementary divisors associated with positive eigenvalues and \( x \)
elementary divisors associated with negative eigenvalues. As in the Drazin-Haynsworth paper, similar theorems may be proved regarding the number of elementary divisors of $A$ associated with imaginary eigenvalues, and also the number associated with eigenvalues of modulus one. For Hermitian $H$, there is defined $\ln H = (\tau, \nu, \delta)$ if $H$ has $\tau$ positive, $\nu$ negative, and $\delta$ zero eigenvalues.

(Received August 19, 1963.)

63T-380. B. S. DREBEN and J. S. DENTON, JR., Emerson Hall, Harvard University, Cambridge, Massachusetts. Three solvable cases.

The following theorems extend results respectively of Drebenn (Ann. Harvard Compt. Lab. 31 (1962), 32-47), of Drebenn, Kahr, and Wang (Bull. Amer. Math. Soc. 68 (1962), 528-532), and of Ackermann (Math. Ann. 112 (1936), 418-432). Theorem 1. The class of prenex schemata of the form $Ez_1 ... Ez_i Ax_1 Ax_2 Ax_3 Ey_1 ... Ey_j M$ where no letter-atom occurring in $M$ contains both members of any of the pairs $(x_1 x_3), (x_1 x_4), (x_2 x_3), (x_2 x_4), (y_h x_4),$ $1 \leq h \leq j,$ is solvable for satisfiability.

Theorem 2. The class of prenex schemata of the form $Ax_1 Ey_1 ... Ey_j Ax_2 ... Ax_k M$ where all letter-atoms occurring in $M$ are either monadic or of the forms $Fx_a x_a', Fy_b y_b', Fx_1 y_1', Fy_1 x_1$, $Fy_i x_h, 1 \leq a, h \leq k, 1 \leq b, g \leq j,$ is solvable for satisfiability. This class contains axioms of infinity. Theorem 3. The class of schemata of the form $Ax Ey Gxy$ $& Az_1 ... Az_4 M$ where besides monadic letters the matrix $M$ contains only the single dyadic letter $G$ is solvable for satisfiability in finite domains. The question of satisfiability in infinite domains is open. If $M$ contains two dyadic letters, the class is a reduction class. (Notices Amer. Math. Soc. 10 (1963), Abstract 63T-7). (Received August 23, 1963.)

63T-381. D. F. DAWSON, North Texas State University, Denton, Texas. On the extension of some results of Lane and Wall.

Let $f(a)$ denote the continued fraction $1/1 + a_1/1 + a_2/1 + ...$, with $p$th approximant $f_p$. Let $A$ denote the set of all continued fractions $f(a)$ such that no $a_i = 0$ and no $f_i = \infty$. If $f(a) \in A$, let $t_p(u) = 1/(1 + a_p u)$, $T_p(u) = t_1 t_2 ... t_p(u)$, and $\{h_p\}_{p=1}^{\infty}$ be the complex sequence such that $T_p(h_p) = \infty$. A complex sequence $\{z_p\}_{p=1}^{\infty}$ will be said to satisfy condition $(H')$ if and only if $|z_p|$ satisfies one of the following conditions: (1) $\sum |z_{2p}|$ diverges, (2) $\sum |z_{2p+1}|$ converges.

Theorem. If $f(a) \in A$, then in order for $f(a)$ to converge, it is necessary that $\{h_p\}$ satisfy condition $(H')$. Theorem. If $f(a) \in A$, the odd part of $f(a)$ converges absolutely, and the even part of $f(a)$ converges, then $f(a)$ converges if and only if $\{h_p\}$ satisfies condition $(H')$. Similar theorems hold if the roles of even and odd indices are interchanged. These theorems extend some results of Lane and Wall (Trans. Amer. Math. Soc. 67 (1949), 368-380). (Received August 26, 1963.)

63T-382. L. C. HSU, Jilin University, Changchun, China. On a kind of extended Fejér-Hermite interpolation polynomials.

Let $F_n^m(f(t);x) \equiv \frac{\exp^{m+1}(x)}{(m+1)!}$ denote the Fejér-Hermite interpolation polynomial for $f(t)$, of degree at most $2n - 1$ in $t$. Denote $\exp(\log^m(n)) = \exp^m(x), \log(\log^m(n)) = \log^{m+1}(n)$. Theorem 1. For any continuous function $f(x)$ defined on $(-\infty, \infty)$ and satisfying the order condition $f(x) = O(\exp^m(|x|))$ $(|x| \to \infty), \lim_{n \to \infty} F_n^m(f(t \cdot \log^{m+1}(n)); x/\log^{m+1}(n)) = f(x)$ $(-\infty < x < \infty)$. Moreover, the limit relation holds uniformly on every finite interval, Theorem 2. Let $\{\phi_k\}$ be a sequence of interpolation
polynomial operators defined by \( \Phi_k(t;x) \equiv F_n(t(e^k t); e^{-k} x) \) (\( k = 1, 2, \ldots \)), where \( n \) is given by the integral part \( n = n_k = \lfloor \exp^{k+2}(k) \rfloor \). Then for every continuous function \( f(x) \) with any type of growth like \( f(x) = O(\exp^m(|x|)) \), \( m \) being arbitrary, the sequence \( \{\Phi_k(t;x)\} \) does always converge to \( f(x) \) almost uniformly in \(( -\infty, \infty )\). (Received August 16, 1963.)

63T-383. L. C. HSU, Jilin University, Changchun, China. A kind of stochastic approximating polynomials.

Let \( \{n\} \) be a sequence of positive integers: \( n_1, n_2, \ldots, n_{\nu}, \ldots \). Say that a sequence \( \{\xi_k\} \) of random numbers (points) is strongly uniformly distributed on the interval \([0,1]\) with respect to \( \{n\} \), if there is a number \( \lambda \) \((1/2 < \lambda < 1)\) such that, for every subinterval \([a_n, b_n]\) of \([0,1]\) with \( b_n - a_n = n^{-\lambda} \), \( N_n(a_n, b_n) \sim n(b_n - a_n) \) (\( n = n_{\nu} \to \infty \)) where \( N_n(a, b) \) denotes the number of \( \xi_k \)s (\( 1 \leq k \leq n \)) that occur just inside the interval \([a, b]\). **Theorem.** Let \( P_n(t;x) = (1/n)^{1/2} \sum_{k=0}^n f(\xi_k) [1 - (\xi_k - x)^2]^n \), where \( \{\xi_k\} \) is strongly uniformly distributed on \([0,1]\) with respect to \( \{n\} \). Then, for every continuous function \( f(x) \) defined on \((0,1)\), \( P_n(t;x) \to f(x) \) (\( n = n_{\nu} \to \infty \)). Proof is somewhat similar to the verification of convergence of the Landau type polynomials (cf. Hsu, Studia Math. 18 (1959), 43-47), but it requires more refined estimates for some separated summations. (Received August 16, 1963.)


Let \( X \) be a vector lattice. A sequence \( \{n, P_n\} \) is called a martingale if \( P_n(x_k) = x_n \) for \( k \geq n \). Here each \( x_n \in X \) and each \( P_n: X \to X \) is a positive linear projection. If \( P_n(x_k) \geq x_n \) for \( k \geq n \), then the sequence is called a semi-martingale. Using order convergence plus other reasonable conditions, one can prove a martingale convergence theorem. The first martingale convergence theorem was proved by J. L. Doob and used a lemma concerning the number of up-crossings of sample functions. The results give a new proof of Doob's theorem without using the lemma on up-crossings. (Received August 26, 1963.)


Since there are only several partial differential equations for which corresponding Riemann functions are known, it is desirable to extend this knowledge by obtaining Riemann functions for equations other than these. In this article a closed form representation of the Riemann function for an entire class of partial differential equations is given. Indeed, the Riemann function for \( U_{xy} + F(x)G(y)U = 0 \) is \( R(x,y; x', y') = J_0(2 \sqrt{W}) \) with \( W = \int_0^y \int_0^{x'} F(\xi + x') G(\lambda + y') d\xi d\lambda \). This was obtained by applying a Laplace Transform procedure to the characteristic boundary value problem \( R_{xy} + F(x)R = 0 \), \( R(x', y) = 1 \), \( R(x, y') = 1 \), \( R(x', y') = 1 \), so as to obtain its solution as a contour integral. The related series of residues was then summed and the resulting expression generalized. Hence, the closed form solution of a wide variety of Cauchy initial value problems for the associated non-homogeneous equation \( U_{xy} + F(x)G(y)U = H(x,y) \) is now made possible. The choices \( F(x) = A \exp kx \) and \( G(y) = \exp ky \) give \( J_0(2/\sqrt{Z}) \) with \( Z = A(\exp kx - \exp kx')(\exp ky - \exp ky') \) as the Riemann function for \( U_{xy} + A \exp k(x + y)U = 0 \), a result obtained earlier by Cohn (Duke Math. J. 14 (1947), 302) by employing confluence of singularities. (Received August 28, 1963.)
Let $\mu$ be Lebesgue measure on the unit interval. A linear map $T$ of $L_\infty(\mu)$ into itself is said to be doubly stochastic if $T \geq 0$, $T(1) = 1$ and for all $f \in L_\infty$, $\int Tf = \int f$. Note that if $T$ is d.s., $L_\infty$ is invariant under the transpose $T^*$ of $T$, and $T^*|_{L_\infty}$ is d.s. Now if $P(x,S)$ is a Markov transition function on the unit interval with $\mu$ as a stable distribution, there is a d.s. operator $T_P$ associated with $P$ by the relation $[T_P f](x) = \int fP(x, \cdot)$. A d.s. operator $T$ is said to arise from a measure-preserving transformation $U$ if $T$ is associated with a transition function arising from $U$. Now if $\Gamma$ is the class of all d.s. operators, $\Gamma$ is convex and all members of $\Gamma$ arising from measure-preserving transformations are extremal, together with their restricted transposes. In this paper, it is shown that the converse is not true as follows: Let $S$ be a measurable set. If $S \subseteq [0,1/3]$, let $T(Is) = (1/2)I_{2S+1/3}$. If $S \subseteq [1/3,1]$ let $T(Is) = I((1/2)(S-1/3)) - (1/2)Is$. For all other measurable sets, define $T(Is)$ by additivity. $T$ can be extended to an extremal member of $\Gamma$, and it is shown that neither $T$ nor $T^*$ arises from a measure-preserving transformation. (Received August 30, 1963.)
"set" of all definable (in set theory) n.t.f.'s is not a basis for $\Pi_2^1$. P. Cohen proves, by means of another model, that if ZF is consistent then it stays consistent after addition of a.c., g.c.h. and "there are only denumerably many constructible real numbers." Using the same model, it can be seen that also the further addition of the schema "every definable well-ordering of real numbers is denumerable" does not destroy consistency. (Received September 3, 1963.)

63T-389. SOLOMON FEFERMAN, Stanford University, Stanford, California and AZRIEL LEVY, Hebrew University, Jerusalem, Israel. Independence results in set theory by Cohen's method. II.

Use the notation of the preceding abstract. Using a model of Cohen's type, one can prove that if ZF is consistent then it stays consistent after addition of the following axioms: (a) the set of real numbers is a denumerable union of denumerable sets, (b) every well-ordering of real numbers is finite or denumerable, (c) $\omega_1$ is confinal with $\omega_0$, (d) there is a $\Pi_2^1$ predicate $P(n,f)$ (where $f,g$ denote variables ranging over n.t.f.'s) such that $(\forall n)(\exists f)P(n,f)$ but there is no sequence $\langle n(fn)\rangle_{n<\omega_1}$ of n.t.f.'s such that $(\forall n)P(n,f_n)$, (e) $\omega_1$ is the $\omega$th "constructible cardinal." (Received September 3, 1963.)

63T-390. AZRIEL LEVY, Hebrew University, Jerusalem, Israel. Independence results in set theory by Cohen's method. III.

Use the notation of the two preceding abstracts. Starting with a denumerable complete model $M$ of ZF with the axiom of constructibility and with an ordinal $\tau \geq 1$ of this model which is not confinal with $\omega$ (in the model), P. Cohen constructed, in his article cited above, a model $N$ of ZF with the same ordinals and cardinals as those of $M$ in which a.c. holds and $2^{\aleph_0} = \kappa_\tau$. In the same model one gets also the following: (a) Every infinite set of real numbers which is definable from any real number is of one of the cardinalities $\kappa_0$, $\kappa_1$ or $\kappa_\tau$. (b) There exists a $\Pi_2^1$ predicate $P(f,g)$ such that $(\forall f)(\exists g)P(f,g)$, but there is no function $q$ definable from a real number such that $(\forall f)P(f,q(f))$, i.e., the uniformization problem for CPCA-sets (i.e., $\Pi_2^1[\kappa_\tau]$-sets) has no reasonable solution. (Received September 3, 1963.)

63T-391. AZRIEL LEVY, Hebrew University, Jerusalem, Israel. Independence results in set theory by Cohen's method. IV.

Use the notation of Abstracts 63T-388, 63T-389 and 63T-390. Let $M$ be a denumerable complete model of ZF and the axiom of constructibility in which there is an ordinal $\delta$ which is inaccessible (in $M$). Let $\delta + \rho$ be an ordinal in $M$ which is not confinal in $M$ with any limit ordinal $\lambda < \delta$. There is a model $N$ of ZF and a.c. with the same ordinals as $M$ in which the following hold: (a) $2^{\aleph_0} = \kappa_\tau$, where $\tau = 1 + \rho$ and if $\tau = 1$ then g.c.h. holds, (b) every well-ordering of real numbers definable from some real number is finite or denumerable, (c) every infinite set of real numbers which is definable from some real number is of one of the cardinalities $\kappa_0$ or $\kappa_\tau$. (Received September 3, 1963.)

If \( f \) is a function bounded on \([a,b]\) and \( a \leq r < s \leq b \), let \( U[f,(r,s)] = \text{lub}\{f(t): r < t < s\} \). If \( a < x \leq b \), \( \overline{f}(x) = \text{glb}\{U[f,(p,x)]: a \leq p < x\} \); if \( a \leq x < b \), \( \overline{f}(x+) = \text{glb}\{U[f,(x,q)]: x < q \leq b\} \). If \( D \) is a partition of \([a,b]\), \([u,v]\) is a subinterval of \([a,b]\) formed by \( D \) and \( g \) is bounded on \([a,b]\), the upper integral \( f[a \to b]g \) is the limit under refinement of sums \( f(a) + \frac{f(a+) + f(b)}{2} + \sum_{[u,v] \in D \setminus (a,b)} \frac{g(u-) - g(u+)}{2} \). With \( f[a \to b]g \) similarly defined, \( f[a \to b]g = \frac{f(a) + f(b)}{2} + \frac{f(b-)+ f(b)}{2} \). The 'integration by parts' formula holds, the integral is homogeneous and under restrictions, linear and additive. If \( g \) is nondecreasing and \( f \);; > \( h \), \( f[a \to b]g \);; \( f[a \to b]hg \);; \( f[a \to b]fg \). Let \( f(a) = f(a) \), \( f(b) = f(b) \) and if \( a < x < b \), \( f(x) = \text{lub}\{f(x-), f(x), f(x+)\} \). Define \( f \) similarly. If each \( off \) and \( !f \) is quasicontinuous, \( f \) is q-continuous. Theorem. If \( g \) is of bounded variation on \([a,b]\), \( a < c < d < b \), and there is a collection \( K \) of pairs \( (p,x),(x,q) \) of subsegments of \((a,b)\) such that \( K \leq \sum_{[u,v] \in D \setminus (a,b)} g(u-) - g(u+), t_0 = e_0 r + [n/2], t_1 = (e_1 - 1)r + r_1 + \mu(r_1), \) and \( p_t^{(r_1)} \) is the highest power of \( p_t \) dividing \( r_1 \). If \( m \) is an odd prime and \( r \) is even, then the exponent in the modulus of this Kummer type congruence can be increased, for every \( n \geq 0 \), if and only if \( \mu(t) \geq 2\mu(r_1) \). (Received September 3, 1963.)


For arbitrary positive integers \( m,r \), let \( m \) have as its prime factorization \( 2^{e_0} e_1 \ldots e_k \) \( (e_0 \geq 0, e_1 > 0, i = 0) \) and put \( r_1 = [(r + 1)/2] \). Theorem. For all \( n \geq 0 \) the Hermite polynomials \( H_n(x) \) satisfy \( \sum_{s=0}^{r} \binom{r}{s} C_{r,s}(2x)(r-s)m^{H_n+sm(x)} \equiv 0 \) \( \text{mod}\ 2^{t_0} p_1^{t_1} \ldots p_k^{t_k} \), where \( t_0 = e_0 r + [n/2], t_1 = (e_1 - 1)r + r_1 + \mu(r_1), \) and \( p_t^{(r_1)} \) is the highest power of \( p_t \) dividing \( r_1 \). If \( m \) is an odd prime and \( r \) is even, then the exponent in the modulus of this Kummer type congruence can be increased, for every \( n \geq 0 \), if and only if \( \mu(t) > 2\mu(r_1) \). (Received September 3, 1963.)

63T-394. HAO WANG, 33 Oxford Street, Cambridge 38, Massachusetts. Natural hulls and set existence.

A partial hull is a transitive set closed with respect to power sets, i.e., \( PH(x) \) iff \((1)\) \( Ux \subseteq x \) and \((2)\) \( y \subseteq x \subseteq Py \subseteq x \). A natural hull is a partial hull closed with respect to sum sets, i.e., \( NH(x) \) iff \( PH(x) \) and \((3)\) \( y \subseteq x \subseteq Uy \subseteq x \). The natural hull \( \eta_a \) (partial hull \( \eta_a \)) of a set \( a \) is the intersection of all natural (partial) hulls \( x \) such that \((4)\) \( a \subseteq x \). Theorem I. In the familiar system \( ZF \), \( \eta \) \( \eta_a \) can be shown to exist for each \( a \), and to satisfy the conditions \((1)\) - \((4)\), as well as that of being the minimum; similarly for \( \eta \) \( \eta_a \) with the conditions \((1)\), \((2)\), \((4)\). Let \( SE \) be obtained from the usual axiom of replacement by substituting for the given set \( x, \eta_a \) (or \( \eta_a \)) \( SE \) \( (EY)(y = H''\eta x) \). Let \( UE \) be obtained from \( SE \) by adding uniqueness, i.e., by substituting \( (EY) \) \( y \) \( (EY)(y = H''\eta x) \). Both \( SE \) and \( UE \) can be expressed in the primitive notation of \( ZF \). Theorem II. In the predicate calculus with equality, \( SE \) yields all axioms of existence in \( ZF \) \( \) (power set, sum set, infinity, replacement, pairs), \( UE \) is equivalent to all these plus the axiom of extensionality. It is also possible to take natural hulls as ordinals and relate them to the natural models of von Neumann and Bernays. (Received September 4, 1963.)
Let ZF be Zermelo-Fraenkel set theory together with the axioms of choice and foundation. Let (M, C_M) denote a fixed countable model for ZF* in which V = L is valid. If \( \kappa \) is a cardinal number, \( P(\kappa) \) is the cardinality of its power set. We study the possible values of \( P(\kappa) \) in extensions of M. A model (N, C_N) for ZF* is an excellent extension of M if: (1) M is a complete submodel of N; that is, \( M \subseteq N \), and for \( x \in M \), \( y \in N \) there is \( y \in C_N x \) iff \( (y \in M \) and \( y \in C_M x) \). (2) M is improper; that is, the ordinals of N are exactly the ordinals of M. (3) The notion of cardinal number is absolute; that is, the cardinals of N are exactly the cardinals of M. Let \( cf(\kappa) \) be the least \( \kappa' \) such that \( \kappa \) is the sum of \( \kappa' \) smaller cardinals. (4) If \( \kappa \) is a cardinal of M, \( cf(\kappa) \) denotes the same cardinal in M and in N. 

Using the techniques of Cohen (The independence of the axiom of choice, to appear in Proc. Nat. Acad. Sci., U.S.A.), the following theorems are proved. (The first was discovered independently by Cohen.) 

**Theorem I.** Let \( \kappa \) be a cardinal of M such that \( cf(\kappa) > \kappa_0 \). Then there is an excellent extension N of M in which \( P(\kappa_0) = \kappa \). (Received September 3, 1963.)

**Theorem II.** Let \( \kappa, \kappa' \) be infinite cardinals such that \( \kappa \) is regular and \( cf(\kappa') > \kappa \). In a suitable excellent extension N of M there is \( P(\kappa) = \kappa \) and if \( \kappa_0 < \kappa \), \( P(\kappa_0) = \kappa_{\alpha+1} \). Now identify the ordinary integers in the obvious way with an initial segment of the integers of M. Let \( n_0 \leq n_1 \leq n_2 \ldots \leq n_k \) be a finite sequence of ordinary integers such that \( j < n_j \) for \( 0 \leq j \leq k \). 

**Theorem III.** There is an excellent extension N of M in which \( P(\kappa_i) = \kappa_i \) for \( 0 \leq i \leq k \). 

**Corollary.** If ZF* is consistent, it remains consistent if the axioms \( P(\kappa_0) = \kappa_2, P(\kappa_1) = \kappa_5, P(\kappa_2) = \kappa_7 \) are adjoined. Similar independence results follow from Theorems I and II. (Received September 3, 1963.)
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INDEX TO ADVERTISERS

<table>
<thead>
<tr>
<th>Name</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Mathematical Society.</td>
<td>604</td>
</tr>
<tr>
<td>Booz-Allen Applied Research, Inc.</td>
<td>601</td>
</tr>
<tr>
<td>Center for Naval Analyses</td>
<td>600</td>
</tr>
<tr>
<td>Cushing-Malloy, Inc.</td>
<td>598</td>
</tr>
<tr>
<td>General Electric Knolls Atomic Power Laboratory</td>
<td>600</td>
</tr>
<tr>
<td>Holt, Rinehart &amp; Winston, Inc.</td>
<td>598</td>
</tr>
<tr>
<td>Hughes Aircraft Company Aerospace Division</td>
<td>556</td>
</tr>
<tr>
<td>ITT Federal Electric Corporation.</td>
<td>599</td>
</tr>
<tr>
<td>The Johns Hopkins University – The Applied Physics Laboratory</td>
<td>596</td>
</tr>
<tr>
<td>McGraw-Hill Book Company</td>
<td>598</td>
</tr>
<tr>
<td>Melpar, Inc.</td>
<td>597</td>
</tr>
<tr>
<td>National Institute for Research in Nuclear Science</td>
<td></td>
</tr>
<tr>
<td>Atlas Computer Laboratory</td>
<td>597</td>
</tr>
<tr>
<td>Space Technology Laboratories, Inc.</td>
<td>602</td>
</tr>
<tr>
<td>U. S. Naval Civil Engineering Laboratory</td>
<td>600</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th>Singles</th>
<th>Doubles</th>
<th>Twins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta Americana</td>
<td>11.00-14.00</td>
<td>14.00-18.00</td>
<td>16.00-20.00</td>
</tr>
<tr>
<td>Motor Hotel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atlanta Cabana Motel</td>
<td>11.00-13.00</td>
<td>14.00-16.00</td>
<td>16.00-17.00</td>
</tr>
<tr>
<td>Atlanta Hotel</td>
<td>6.00-8.50</td>
<td>8.50-10.00</td>
<td>10.00-11.50</td>
</tr>
<tr>
<td>Dinkler Plaza Hotel</td>
<td>7.00-15.00</td>
<td>10.00-15.00</td>
<td>14.00-18.00</td>
</tr>
<tr>
<td>Howard Johnson Motor Lodge, N.W.</td>
<td>9.00-12.00</td>
<td>11.00-13.00</td>
<td>12.00-15.00</td>
</tr>
<tr>
<td>Peachtree Manor Hotel</td>
<td>6.00-8.00</td>
<td>9.00-11.00</td>
<td>9.00-12.00</td>
</tr>
<tr>
<td>Piedmont Hotel</td>
<td>6.50-10.50</td>
<td>10.50-13.50</td>
<td>12.50-16.00</td>
</tr>
</tbody>
</table>

603
MATHMATICS OF COMPUTATION

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TABLE OF CONTENTS

October 1963

The Euler-Maclaurin Functional For Functions with a Quasi-Step Discontinuity
I. NAVOT

The Stability Properties of a Coupled Pair of Non-Linear Partial Difference Equations
B. J. DALY

Optimal Numerical Integration of a Sphere
A. D. McLAREN

Quadrature Formulas with Simple Gaussian Nodes and Multiple Fixed Nodes
D. D. STANCU & A. H. STROUD

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Y. L. LUKE & J. WIMP

Second Order Correct Boundary Conditions for the Numerical Solution of the Mixed Boundary Problem for Parabolic Equations
G. W. BATTEN, JR.

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R. SANKAR & V. MALINI

The Definite Integral of the Product of Linear Functions
R. E. VON HOLDT & R. J. HOWERTON

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