OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by John W. Green and Gordon L. Walker

CONTENTS

MEETINGS
Calendar of Meetings ................................................. 712
Program of the November Meeting in Los Angeles, California ...... 713
 Abstracts for the Meeting - pages 741-747
Program of the November Meeting in Athens, Georgia ............. 716
 Abstracts for the Meeting - pages 748-758
Program of the November Meeting in Evanston, Illinois .......... 720
 Abstracts for the Meeting - pages 759-766

PRELIMINARY ANNOUNCEMENT OF MEETING .......................... 723

NATIONAL ACADEMY OF SCIENCES - NATIONAL RESEARCH COUNCIL ...... 726

VISITING FOREIGN MATHEMATICIANS ................................. 728

NEW AMS PUBLICATIONS ............................................... 732

PERSONAL ITEMS ..................................................... 733

MEMORANDA TO MEMBERS

Corporate Members .................................................... 737
 Retired Mathematicians ............................................. 737
 Summer Employment Opportunities ................................ 737
 The Employment Register ........................................... 737

NEWS ITEMS AND ANNOUNCEMENTS .................................... 738, 740

SUPPLEMENTARY PROGRAM - Number 28 .............................. 739

ABSTRACTS OF CONTRIBUTED PAPERS ................................. 741

INDEX TO ABSTRACTS - Volume 11 ................................ 775

INDEX - Volume 11 ..................................................... 801

INDEX TO ADVERTISERS ................................................ 810

RESERVATION FORM .................................................. 811

ADVANCED REGISTRATION FORM ....................................... 811
## MEETINGS

### Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>619</td>
<td>January 26-30, 1965</td>
<td>Denver, Colorado</td>
<td>Dec. 2</td>
</tr>
<tr>
<td>620</td>
<td>February 27, 1965</td>
<td>New York, New York</td>
<td>Jan. 14</td>
</tr>
<tr>
<td></td>
<td>April 24, 1965</td>
<td>Stanford, California</td>
<td>Feb. 24</td>
</tr>
<tr>
<td></td>
<td>June 19, 1965</td>
<td>Eugene, Oregon</td>
<td>Feb. 24</td>
</tr>
<tr>
<td></td>
<td>August 30 - September 3, 1965</td>
<td>Ithaca, New York</td>
<td></td>
</tr>
<tr>
<td></td>
<td>November 12-13, 1965</td>
<td>Lexington, Kentucky</td>
<td></td>
</tr>
<tr>
<td></td>
<td>January 24-28, 1966</td>
<td>Chicago, Illinois</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August 29 - September 2, 1966</td>
<td>New Brunswick, New Jersey</td>
<td></td>
</tr>
<tr>
<td></td>
<td>January 24-28, 1967</td>
<td>Houston, Texas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August 28 - September 1, 1967</td>
<td>Toronto, Ontario, Canada</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August 26-30, 1968</td>
<td>Madison, Wisconsin</td>
<td></td>
</tr>
</tbody>
</table>

* The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadline dates for by title abstracts are November 25, 1964, and January 14, 1965.
Six Hundred Sixteenth Meeting  
University of Southern California  
Los Angeles, California  
November 14, 1964

PROGRAM

The six hundred sixteenth meeting of the American Mathematical Society will be held on Saturday, November 14, 1964 at the University of Southern California in Los Angeles, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, an hour address will be presented by Professor Charles W. Curtis of the University of Oregon. The title of the talk by Professor Curtis is "Abstract groups of Lie type". This address will be given at 11:00 A.M. in Room 133 Founders Hall. There will be sessions for contributed papers at 9:30 A.M. and at 2:00 P.M. in Rooms 208 and 221 Founders Hall. Abstracts of the papers to be presented at these sessions appear on pages 741-747 of these Notices. There are cross references to the abstracts in the program. For example the title of paper (1) in the program is followed by (616-5) indicating that the abstract can be found under the designation 616-5 among the published abstracts. Late papers may be added to the program. Information concerning late papers will be available at the Registration Desk.

Registration for the meeting will begin at 9:00 A.M. The Registration Desk will be located outside Room 133 Founders Hall.

There are numerous hotels and motels in the Los Angeles area. The nearest ones to the campus are the Vagabond Motor Hotel, 3101 South Figueroa Street, telephone (area 213) 746-1531, and the Coliseum Hotel, 457 West Santa Barbara Avenue.

Luncheon will be available at the Commons Cafeteria. A section will be reserved for people attending the meeting. The Commons building is on Childs Way, just west of University Avenue.

The University of Southern California can be reached by automobile, taking the Harbor Freeway to Exposition Boulevard. Parking will be available on campus for a fee of fifty cents. Cars should enter at the main entrance which is located at the corner of Exposition Boulevard and Hoover Street. The guard at the gate will direct motorists to the proper parking area.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:30 A.M.

General Session, Room 208, Founders Hall
9:30 - 9:40
(1) Two notes on locally Macaulay rings
Professor L. J. Ratliff, Jr., University of California, Riverside (616-5)

713
9:45 - 9:55
(2) Intrinsic characterization of tangent spaces
Professor H. A. Osborn, University of Illinois (616-8)

10:00 - 10:10
(3) An extension of a theorem of Fenchel
Professor J. R. Reay, Western Washington State College (616-14)

10:15 - 10:25
(4) Paths in polyhedra
Professor V. L. Klee, University of Washington and Boeing Scientific
Research Laboratories, Seattle, Washington (616-16)

SATURDAY, 9:30 A.M.

Session on Analysis and Applied Mathematics, Room 221, Founders Hall
9:30 - 9:40
(5) Invariant imbedding formulation for radiative transfer on general media with
internal sources
Dr. R. W. Preisendorfer, University of California, La Jolla (616-10)

9:45 - 9:55
Professor J. L. Cooper, California Institute of Technology (616-18)

10:00 - 10:10
(7) Integral transformations
Mr. Ben Johnson, 618 West Foothill Boulevard, Monrovia, California
(616-4)

10:15 - 10:25
(8) Measurability theorems in the transformation theory for measure space
Dr. R. W. Chaney, Western Washington State College (616-15)

10:30 - 10:40
(9) Some absolutely continuous operators. I. Preliminary report
Professor P. A. Rejto, New York University (616-1)

SATURDAY, 11:00 A.M.

Invited Address, Room 133, Founders Hall
Abstract groups of Lie type
Professor Charles W. Curtis, University of Oregon

SATURDAY, 2:00 P.M.

Session on Algebra and Theory of Numbers, Room 208, Founders Hall
2:00 - 2:10
(10) Artiads characterized
Mrs. Emma Lehmer, 1180 Miller Avenue, Berkeley, California (616-17)
(Introduced by Professor D. H. Lehmer)

2:15 - 2:25
(11) Cyclotomic polynomials and Wedderburn's theorem
Dr. Kenneth Rogers, University of Hawaii (616-11)

2:30 - 2:40
(12) On a theorem concerning the greatest s-decomposition of a semigroup
Professor Takayuki Tamura, University of California, Davis (616-20)

2:45 - 2:55
(13) A normality relation for lattices
Professor R. A. Dean, California Institute of Technology, and Dr. R. L.
Kruse*, Sandia Corporation, Albuquerque, New Mexico (616-12)

*For papers with more than one author, an asterisk follows the name of the author who
plans to present the paper at the meeting.
3:00 - 3:10
(14) On the structure of ideals in certain universal algebras
Professor Adil Yaqub, University of California, Santa Barbara (616-6)

3:15 - 3:25
(15) The word problem and residually finite groups
Dr. V. H. Dyson, Hughes Aircraft Company, Fullerton, California (616-7)

SATURDAY, 2:00 P.M.

Session on Topology, Room 221, Founders Hall

2:00 - 2:10
(16) On a homotopy converse to the Lefschetz fixed point theorem
Professor R. F. Brown, University of California, Los Angeles (616-9)

2:15 - 2:25
(17) Some constructions of suitable spaces
Professor G. S. McCarty, Jr., Harvey Mudd College (616-19)

2:30 - 2:40
(18) On a conjecture of R. J. Koch
Professor L. E. Ward, Jr., University of Oregon (616-2)

2:45 - 2:55
(19) On spaces with point-countable bases
Professor R. W. Heath, Arizona State University (616-13)

R. S. Pierce
Seattle, Washington
Associate Secretary
The six hundred and seventeenth meeting of the American Mathematical Society will be held at the University of Georgia on November 20 and 21, 1964. All sessions will be in the Center for Continuing Education.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor John R. Isbell of Tulane University of Louisiana will speak on Structure of Categories at 2:00 P.M., Friday, November 20.

There will be sessions for contributed papers at 3:30 P.M. on Friday, November 20 and at 10:00 A.M. on Saturday, November 21. Places of sessions will be available at the Registration Desk.

The Registration Desk will be in the main lobby of the Center for Continuing Education; there will be a registration fee of $2.00. Rooms will be available in the Center at the rate of $7.00 single and $10.00 double; requests for reservations should be addressed to the Registration Desk, Center for Continuing Education, University of Georgia, Athens, Georgia. Since the number of single rooms is limited, it will be appreciated if double rooms will be utilized whenever possible.

There will be a beer party Friday night. Tickets are $1.50, and may be purchased at the time of registration.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that the listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

FRIDAY, 2:00 P.M.

Invited Address. Center for Continuing Education

Structure of Categories
Professor John R. Isbell of Tulane University of Louisiana.

FRIDAY, 3:30 P.M.

Session on Topology.
3:30 - 3:40
(1) A decomposition of semigroups
Professor A. D. Wallace, University of Florida (617-3)
3:45 - 3:55
(2) On chain-equivalent relations. Preliminary report
Professor A. R. Bednarek, University of Florida (617-6)
4:00 - 4:10
(3) Open and image-open relations
Professor S. P. Franklin, University of Florida (617-7)

4:15 - 4:25
(4) A monotone relation which preserves pseudocircles
Dr. J. M. Day, University of Florida (617-10)

4:30 - 4:40
(5) Homogeneous compact ordered spaces. Preliminary report
Professor M. A. Maurice, University of Florida (617-28)
(Introduced by Professor A. D. Wallace)

4:45 - 4:55
(6) The product of cluster spaces
Dr. Gerhard Grimeisen, University of Florida (617-32)
(Introduced by Professor A. D. Wallace)

5:00 - 5:10
(7) Euclidean geometry via intervals
Professor R. G. Vinson, Huntingdon College (617-20)

FRIDAY, 3:30 P.M.

Session on Analysis.
3:30 - 3:40
(8) The gamma function with varying difference interval
Professor Tomlinson Fort, Emory University (617-14)

3:45 - 3:55
(9) A solution-giving projection for partial differential equations. Preliminary report
Professor J. W. Neuberger, Emory University (617-12)

4:00 - 4:10
(10) On harmonic matrices
Professor A. J. Zettl, Louisiana State University at Baton Rouge (617-9)

4:15 - 4:25
(11) Centralizers of H*-algebras
Professor C. N. Kellogg, University of Kentucky (617-15)

4:30 - 4:40
(12) Bounded Hadamard products of functions belonging to Hardy p-classes. Preliminary report
Mr. D. J. Caveny, University of Kentucky (617-17)

4:45 - 4:55
(13) Power-series and Hausdorff matrices. Preliminary report
Mr. P. C. Tonne, University of North Carolina (617-27)

FRIDAY, 3:30 P.M.

Session on Algebra and Graph Theory.
3:30 - 3:40
(14) Identities, finite embeddability and residual finiteness
Professor Trevor Evans, Emory University (617-24)

3:45 - 3:55
(15) Substitution algebras and near-rings. I
Mrs. M. F. Neff* and Trevor Evans, Emory University (617-30)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
4:00 - 4:10
(16) Semigroups with a maximal semigroup with zero homomorphic image
Professor Robert Plemons, Auburn University (617-11)

4:15 - 4:25
(17) Some results on small modules. Preliminary report
Professor W. W. Leonard, University of South Carolina (617-1)

4:30 - 4:40
(18) On homomorphisms and characters of inverse semigroups
Professor R. O. Fulp*, Georgia State College and Professor Paul Hill,
Emory University (617-22)

SATURDAY, 10:00 A.M.

Session on Topology.
10:00 - 10:10
(19) On a theorem of Klee
Professor R. D. Anderson, Louisiana State University at Baton Rouge 
(617-34)

10:15 - 10:25
(20) A criterion for cellularity at the boundary (CAB) of a manifold
Professor C. A. Greathouse, Vanderbilt University (617-23)

10:30 - 10:40
(21) Imbedding collections of compact totally disconnected subsets of $E^2$ in continuous collections of mutually exclusive arcs
Mrs. Jo Ford, Athens, Georgia (617-5)

10:45 - 10:55
(22) On the dimensions of certain spaces of homeomorphisms
Professor B. L. Brechner, Louisiana State University in New Orleans 
(617-13)

11:00 - 11:10
(23) A note on projections and compactness
Mr. S. B. Nadler, Jr., University of Georgia (617-19)

11:15 - 11:25
(24) Concerning nonplanar circle-like continua
Professor W. T. Ingram, University of Houston (617-21)

11:30 - 11:40
(25) Factoring pointlike simplicial mappings of the 3-sphere
Mr. C. L. Wiginton, University of Tennessee (617-31)

SATURDAY, 10:00 A.M.

Session on Analysis and Applied Mathematics.
10:00 - 10:10
(26) Heights of convex polytopes
Professor Victor Klee, University of Washington and Boeing Scientific Research Laboratories (617-16)

10:15 - 10:25
(27) Remarks on the theory of integration
Professor Jan Mikusinski, University of Florida (617-33)

10:30 - 10:40
(28) A canonical form for a boundary value problem involving a quasi-differential operator of Euler type
Professor J. S. Bradley, University of Tennessee (617-25)

10:45 - 10:55
(29) Existence and construction of solutions for a class of problems in linear controls
Professor M. Z. Nashed, Georgia Institute of Technology (617-26)
11:00 - 11:10
(30) Continued fraction approximation to functions
Professor T. L. Hayden, University of Kentucky (617-18)

SATURDAY, 10:00 A.M.

Session on Algebra and Graph Theory
10:00 - 10:10
(31) The representation of a graph by set intersections
Professor Paul Erdös, Budapest, Hungary, Professor A. W. Goodman*,
University of South Florida and Mr. Ladislas Posa, Michael Fazekas
High School, Budapest, Hungary (617-4)

10:15 - 10:25
(32) Regular D-classes whose idempotents obey certain conditions. I
Professor R. J. Warne, West Virginia University (617-2)

10:30 - 10:40
(33) Products of permutation groups and their graphs
Professor R. L. Hemminger, Vanderbilt University (617-8)

10:45 - 10:55
(34) Finite wreath product characters and representations
Professor T. D. Phillips, Emory University (617-29)
(Introduced by Professor Trevor Evans)

Morton L. Curtis
Associate Secretary.

Houston, Texas
Six Hundred Eighteenth Meeting
Northwestern University
Evanston, Illinois
November 27-28, 1964

PROGRAM

The six hundred eighteenth meeting of the American Mathematical Society will be held at Northwestern University on Friday and Saturday, November 27-28, 1964. Meeting headquarters will be at the North Shore Hotel (1611 Chicago, Evanston) though registrations and all the scientific sessions of the meeting will be held at Northwestern University.

The hotel has guaranteed single rooms to the Society at $8.00 and doubles at $11.00. Consequently, those who wish to stay where most of the members of the Society will be housed and wish to avail themselves of these special rates should use the reservation form on page 707 of the October, 1964 issue of the Notices so as to make themselves eligible for these special rates.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, Professor James Serrin of the University of Minnesota will address the Society on "Multiple integral problems in the calculus of variations". Professor Serrin will speak at 2:00 P.M., Friday in Lecture Room 3 of the Technological Institute.

There will be a series of 20 minute papers on "Recent developments in ring theory" arranged by Professor I. N. Herstein. The speakers will be J. L. Alperin, S. A. Amitsur, P. M. Cohn, Carl Faith, Murray Gerstenhaber and E. Sasaki. Professor Amitsur's talk will be given on Friday at 3:15 in Lecture Room 2. The other talks in this series will be given on Saturday starting at 9:30 A.M. in Lecture Room 2.

Sessions for the presentation of contributed papers will be held at 3:15 P.M. on Friday and at 9:30 A.M. on Saturday.

Rooms 1396, 1400, 1405, 1667, and 1792 will be available for discussions. The registration desk will be located in the front lobby of the Northwestern Technological Institute Building located on Sheridan Road at Noyes Street. Those who attend the meetings are requested to register at any time from 9:30 A.M. to 5:00 P.M. on Friday and from 9:30 A.M. to noon Saturday.

There will be a tea at 5:00 P.M. on Friday in the Tech faculty lounge.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain the schedule, the time limit will be strictly enforced. All sessions will be held in the Northwestern Technological Institute Building.

FRIDAY, 2:00 P.M.

Invited address, Lecture Room 3

Multiple integral problems in the calculus of variations
Professor James Serrin, University of Minnesota
FRIDAY, 3:15 P.M.

Session on Analysis, Lecture Room 3
3:15 - 3:25
(1) Convexity with respect to Euler-Lagrange differential operators
    Professor J. C. Kegley, Iowa State University (618-6)
3:30 - 3:40
(2) Convergence of series whose terms are defined recursively
    Professor M. K. Fort (Posthumously), University of Georgia, and Professor Seymour Schuster*, University of Minnesota (618-19)
3:45 - 3:55
(3) On the existence of a weighted Stieltjes mean sigma integral, I
    Professor F. M. Wright, Iowa State University (618-12)
4:00 - 4:10
(4) The power-boundedness of Fourier series
    Professor G. W. Hedstrom, University of Michigan (618-17)
4:15 - 4:25
(5) Left-amenable semigroups and cancellation
    Professor J. R. Sorenson, Valparaiso University (618-16)
4:30 - 4:40
(6) Perturbation of ordinary differential operators
    Professor R. E. L. Turner, The University of Wisconsin (618-15)
4:45 - 4:55
(7) Classical and relativistic energetodynamics
    Professor M. Z. v. Krzywoblocki, Michigan State University (618-20)

FRIDAY, 3:15 P.M.

Session on algebra, Lecture Room 2
3:15 - 3:35
(8) The use of ultra-products in algebra
    Professor S. A. Amitsur, University of Chicago (618-22)
3:45 - 3:55
(9) Jordan isomorphisms of the symmetric elements of a simple ring with involution. Preliminary report
    Professor W. S. Martindale, III, University of Massachusetts (618-10)
4:00 - 4:10
(10) A characterization of rings in which all subrings are ideals
    Mr. R. L. Kruse, Sandia Corporation, Albuquerque, New Mexico (618-4)
4:15 - 4:25
(11) Autonomous categories and duality of functors
    Professor F. E. J. Linton, The University of Chicago (618-13)
4:30 - 4:40
(12) Fixed point subgroups which contain centralizers of involutions. Preliminary report
    Mr. George Glauberman, University of Wisconsin (618-5)
4:45 - 4:55
(13) Regular D-classes whose idempotents obey certain conditions, II
    Professor R. J. Warne, West Virginia University (618-2)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
General Session, Lecture Room 3
9:30 - 9:40
(14) The number of plane trees with a given partition
Professor Frank Harary*, University of Michigan and Professor W. T. Tutte, University of Waterloo, Canada (618-7)

9:45 - 9:55
(15) The geometry of the complex inner product
Professor John DeCicco and Professor John Synowiecz*, Illinois Institute of Technology (618-1)

10:00 - 10:10
(16) Extended topology: Carathéodory's theorem on convex hulls
Professor P. C. Hammer, University of Wisconsin (618-3)

10:15 - 10:25
(17) On Soddy's theorem and its generalization to n dimensions
Professor Daniel Pedoe, Purdue University and The University of Minnesota (618-8)

10:30 - 10:40
(18) Sublattices of lattices of real-valued continuous functions. Preliminary report
Professor G. A. Jensen, University of Florida (618-18)

10:45 - 10:55
(19) Noncontinuous functions which act like continuous functions for connected sets
Professor D. E. Sanderson, Iowa State University (618-21)

11:00 - 11:10
(20) A k-complex is tame in $E^n (n \geq 3k + 1)$ if each of its simplices is tame
Mr. John Cobb, University of Wisconsin (618-11)

11:15 - 11:25
(21) A note on compact transformation groups with a fixed end point
Professor Hsin Chu, University of Alabama Research Institute (618-9)

Special Session on Recent Developments in Ring Theory, Lecture Room 2
9:30 - 9:50
(22) Towers of fields, nil algebras and Burnside groups: A survey of Golod-Safarevič Theorems
Professor J. L. Alperin, University of Chicago (618-24)

10:00 - 10:20
(23) Free ideal rings
Professor P. M. Cohn, University of Chicago and London University (618-23)

10:30 - 10:50
(24) Characterizations of quasi-Frobenius rings
Professor Carl Faith*, Rutgers, The State University and Professor E. A. Walker, New Mexico State University (618-25)

11:00 - 11:20
(25) Deformation of rings
Professor Murray Gerstenhaber, University of Pennsylvania

11:30 - 11:50
(26) Simple radical rings
Professor E. Sasiada, University of Chicago

S. Sherman
Associate Secretary
PRELIMINARY ANNOUNCEMENT OF MEETING

Seventy-First Annual Meeting
Denver, Colorado
January 26-30, 1965

The seventy-first Annual Meeting of the American Mathematical Society will be held at the Denver-Hilton Hotel in Denver, Colorado. This meeting will be held in conjunction with the Annual Meeting of the Mathematical Association of America, a meeting of the Association for Symbolic Logic and a regional meeting of the National Council for Teachers of Mathematics. The Society will meet from Tuesday, January 26 through Friday, January 29. The Mathematical Association of America will meet from Thursday through Saturday, January 28-30; the Association for Symbolic Logic will meet on Wednesday, January 27; and the N.C.T.M. meeting will be held on Saturday, January 30.

The thirty-eighth Josiah Willard Gibbs Lecture will be delivered by Professor D. H. Lehmer of the University of California at Berkeley in the Grand Ballroom of the Denver-Hilton at 8:00 P.M. on Tuesday, January 26, 1964.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, addresses will be given by Professor Eugenio Calabi of the University of Pennsylvania, and by Professor David Mumford of Harvard University. Professor Calabi will speak at 2:00 P.M. on Thursday, January 28, and Professor Mumford's talk will be given at 2:00 P.M. on Friday, January 29. Both of these addresses will be presented in the Grand Ballroom.

The Cole Memorial Prize will be awarded in the Grand Ballroom at 2:00 P.M. on Tuesday, January 26.

As at the past two Annual Meetings of the Society, there will be five sessions of selected twenty-minute papers. The papers presented at these sessions will be partly by invitation, and partly drawn from the ten-minute papers submitted for the meeting, the authors of which will be permitted to expand their presentations to twenty minutes. The topics and chairmen of these sessions are planned as follows:

- Differential Geometry: James Eells
- Differential Topology: William Browder
- Function Algebras: John Wermer
- Ordinary Differential Equations: Stephen P. Diliberto
- Operator Algebras and Group Representations: Richard Kadison

Those persons contributing abstracts to the Annual Meeting who feel that their papers would be appropriate for presentation at one of these special sessions should submit their abstracts by November 25; that is, one week earlier than the regular deadline.

There will be regular sessions for contributed ten-minute papers. However, there is a limit of two hundred on the number of such papers which will be accepted for presentation at this meeting. Papers will be accepted for the meeting until either two hundred have been received or until the deadline date of December 2 is passed. There will be no sessions for late papers.

The Business Meeting of the Society will be held at 11:00 A.M. on Tuesday, January 26 in the Grand Ballroom. The Council has recommended that the membership be presented at this meeting with several proposals to change the By-laws relating to dues and privileges. One has the effect of raising annual dues from fourteen to twenty dollars, beginning in 1966, with the Notice and the BULLETIN
as privileges of membership. A second change removes the restriction that bills for dues may not be sent out until October 1; this is to make it possible to distribute the office work load more evenly through the year. The third change, which would go into effect in January, 1966, has to do with the method of setting dues, and proposes to remove the statement of the exact amount of dues from the By-laws, and to authorize the Council, with the approval of the Trustees, to set the dues.

The Council of the Society will meet at 4:00 P.M. on Monday, January 25 in the Silver Room on the Mezzanine Floor.

REGISTRATION

The Registration Headquarters for this meeting will be in the Convention Lobby of the Denver-Hilton Hotel. The Registration Desk will be open from 2:00 P.M. to 8:00 P.M. on Monday, January 25, and from 9:00 A.M. to 5:00 P.M. on Tuesday through Friday. As a new convenience to members, the Society is accepting advance registration for the Denver Meeting. An Advance Registration Form will be found on the back cover of these Notices. It is requested that this form be completed and sent with the proper registration fee to the Society office in Providence no later than January 10, 1965. A schedule of registration fees is also given on the back cover of these Notices. Those persons who have sent in their Advance Registration can pick up badges at the Registration Desk. Others must go through the usual registration process. It is requested that everyone attending the meeting register or pick up his badge if registered in advance, as soon as possible after arrival.

The Employment Register will be maintained from 9:00 A.M. to 5:00 P.M. on Wednesday, Thursday, and Friday in the Silver Room on the Mezzanine Floor. Book and other exhibits will be in the Convention Lobby on the Main Floor. The exhibits will be open from Tuesday through Thursday.

A new service to members will be available at the Denver Meeting. A Telephone Message Center will be located adjacent to the Registration Desk, providing a central location where calls can be directed. All calls for members at the meeting will be taken and filed at this booth. These messages can be picked up from the operator at the booth. The number of the Message Center is Area Code 303-534-7691.

All mail and telegrams for those attending the meeting should be addressed in care of the American Mathematical Society, Denver-Hilton Hotel, Denver, Colorado.

ACCOMMODATIONS

Accommodations for the meeting will be handled by the Denver Convention and Visitors Bureau. The reservation form on the back cover of these Notices is to be used in requesting accommodations. This form should be completed and sent to the Housing Bureau, Mathematics Meetings, 225 West Colfax Avenue, Denver, Colorado 80202. Requests for reservations must be sent soon enough to arrive in Denver no later than January 10, 1965. The Housing Bureau will make reservations as nearly as possible in accordance with the request given on the reservation form. A deposit on room reservation is not required. The map shown below gives the location of the various hotels that have reserved rooms for the meeting, including the headquarters hotel, the Denver-Hilton.

GENERAL INFORMATION

Denver is served by Braniff, Central Continental, Frontier, Trans World, and United Airlines; by the Burlington, Colorado and Southern, Missouri-Pacific, Rio Grande, Rock Island, Santa Fe, and Union Pacific Railroads; and by the Continental Trailways and Greyhound Bus Lines. Airport limousine service is available from the airport to downtown Denver for $1.25 per person. These limousines leave the airport every 15 minutes during a 16-hour period each day. Taxicab service from the airport costs about $2.50 per person. The taxicab fare from Denver Union Railway Terminal to the Denver downtown hotel area is approximately eighty cents per person.

A number of recreational events are planned for the meetings. These will
be announced in the Program of the Meet-
ing, which will appear in the January
NOTICES.

The average noon temperature for
Danver in January is 43 degrees, whereas
the average 8:00 P.M. temperature is 19
degrees. Precipitation for the month of
January averages .40 inches.

R. S. Pierce
Seattle, Washington
Associate Secretary
Abstract of Annual Report for 1963-1964

This year committees of the Division advised on the selection of candidates for fellowships and other awards as follows:

<table>
<thead>
<tr>
<th>Fellowship Type</th>
<th>Applications</th>
<th>Awards</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSF Postdoctoral (Regular) Fellowships</td>
<td>81</td>
<td>21</td>
</tr>
<tr>
<td>NAS-NRC Postdoctoral Research Fellowships</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>NATO Postdoctoral Fellowships</td>
<td>25</td>
<td>3</td>
</tr>
<tr>
<td>O.E.C.D. Senior Visiting Fellowships</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>NSF Senior Postdoctoral Fellowships</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>NSF Graduate Fellowships</td>
<td>1,311</td>
<td>372</td>
</tr>
<tr>
<td>NSF Cooperative Graduate Fellowships</td>
<td>833</td>
<td>236</td>
</tr>
<tr>
<td>NSF Summer Fellowships for Graduate Teaching Assistants</td>
<td>361</td>
<td>175</td>
</tr>
<tr>
<td>Postdoctoral Resident Research Associateships</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Fulbright Fellowships</td>
<td>34</td>
<td>11</td>
</tr>
<tr>
<td>ONR Postdoctoral Research Associateships</td>
<td>48</td>
<td>7</td>
</tr>
</tbody>
</table>

The United States Commission on Mathematical Instruction (USCMI) has begun preliminary planning for a display of mathematical textbooks at the next International Congress of Mathematicians (Moscow, 1966). The USCMI has also recommended to the International Commission on Mathematical Instruction (ICMI) that an international journal should be established which would contain announcements of new curriculum projects, critical reviews of materials produced by such projects, and articles on mathematics education. The USCMI believes that such a journal would be of very great value to all countries (including the U.S.A.) working to improve their mathematics programs.

The Committee on Applications of Mathematics has responded to the request that it "draw up suggestions as to how departments of mathematics should be organized to give proper attention to the applications of mathematics" made by the Division at its annual meeting in 1963. Several interesting and novel suggestions have been put forth. First a department can, if it wishes, develop its own center of applied mathematics, and the report indicates the first steps to be taken to this end. The release of staff members (who wish to prepare themselves for teaching courses in the applications) from some of their teaching duties is proposed. To finance this, a system of federal training grants is suggested. These grants would go directly to departments planning to strengthen their work in the applications. In addition, such funds could be used for summer salaries for faculty members who wish to develop courses in applied mathematics, support visits by faculty members to established centers of applied mathematics, or to assist the department in securing the services of a man already versed in the applications. The need for good textbooks in the applications is recognized and attention is drawn to certain areas of greatest need. Finally, the committee applauded the efforts of Gail S. Young, Chairman of the CUPM Teacher Training Panel in his efforts to introduce a course in the "Applications of Mathematics for the Secondary School Teacher."

The Editorial Committee for "Mathematics of Computation" reported that the number of pages will increase from 514
in 1963 to about 700 in 1964. The business management of the journal is presently in the hands of the American Mathematical Society and the Society is now in the process of determining the extent of its responsibility to the journal after May 31, 1965.

The Committee on Revision of Mathematical Tables reports that the "Handbook of Mathematical Functions" has been published by the U. S. Government Printing Office and will appear as No. 55 in the Applied Mathematics Series of the National Bureau of Standards. The volume, in addition to tables, offers comparatively simple methods for obtaining values of functions outside the tabulated range.

The Committee on Travel Grants has advised the National Science Foundation on a small number (eight) of applications for support of foreign travel.

The Committee on Statistics is available to stimulate and encourage worthwhile publications in statistics and has also provided advice on statistical personnel and problems.

The Special Committee on Publication of Selected Information on the Theory of Traffic Flow, a joint committee of the Division of Engineering and Industrial Research and the Division of Mathematics, has completed its report. Entitled "An Introduction to Traffic Flow Theory," it has appeared as Publication 1121 of the National Academy of Sciences--National Research Council.

The Committee on Uses of Computers has completed its report, which is now undergoing final editorial revisions and will appear as one of the National Academy of Sciences--National Research Council publications later in the year.

* * *

Copies of the complete annual report of the Division of Mathematics may be obtained by writing to:

Division of Mathematics
National academy of Sciences
National Research Council
2101 Constitution Avenue
Washington, D. C. 20418.
The following list contains the names of foreign mathematicians who are visiting at various institutions in the United States this year. The list is compiled from responses received on or before October 9, to requests sent out by the Society to academic institutions.

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atkin, M. A. (Australia)</td>
<td>Virginia Polytechnic Institute</td>
<td>Statistics</td>
<td>3 years</td>
</tr>
<tr>
<td>Al-Hussaini, A. (Iraq)</td>
<td>Michigan State University</td>
<td>Probability</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Amitsur, A. S. (Israel)</td>
<td>University of Chicago</td>
<td>Rings</td>
<td>6/64-6/65</td>
</tr>
<tr>
<td>Andre, My. (Switzerland)</td>
<td>Cornell University</td>
<td>Topology</td>
<td>7/64-6/65</td>
</tr>
<tr>
<td>Atiyah, Michael (England)</td>
<td>Harvard University</td>
<td></td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Avila, Geraldo (Brazil)</td>
<td>Mathematics Research Center, U. S. Army, University of Wisconsin</td>
<td>Differential Equations</td>
<td>9/63-6/65</td>
</tr>
<tr>
<td>Azumaya, Goro (Japan)</td>
<td>University of Massachusetts</td>
<td>Algebra</td>
<td>9/61-6/65</td>
</tr>
<tr>
<td>Barden, Dennis (England)</td>
<td>Cornell University</td>
<td>Differential Topology</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Barlotti, Adriano (Italy)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Bartholomew, D. J. (Wales)</td>
<td>Harvard University</td>
<td></td>
<td>9/64-7/65</td>
</tr>
<tr>
<td>Basu, Debabrata (India)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Bather, John (England)</td>
<td>Stanford University</td>
<td>Statistics</td>
<td>9/64-9/65</td>
</tr>
<tr>
<td>Besala, Piotr (Poland)</td>
<td>University of Minnesota</td>
<td>Partial Differential Equations</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Bhapkar, B. P. (India)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Bosanquet, L. S. (England)</td>
<td>University of Utah</td>
<td>Summability</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Brelot, Marcel (France)</td>
<td>Stanford University</td>
<td>Potential Theory</td>
<td>4/65-6/65</td>
</tr>
<tr>
<td>Burlak, J. (Scotland)</td>
<td>Duke University</td>
<td>Partial Differential Equations</td>
<td>7/64-9/64</td>
</tr>
<tr>
<td>Chakravarti, L. M. (India)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964-1967</td>
</tr>
<tr>
<td>Cohn, P. M. (United Kingdom)</td>
<td>University of Chicago</td>
<td>Rings</td>
<td>7/64-12/64</td>
</tr>
<tr>
<td>Cooper, Lionel (Wales)</td>
<td>California Institute of Technology</td>
<td>Abstract Analysis</td>
<td>9/64-9/65</td>
</tr>
<tr>
<td>Cormack, R. M. (Scotland)</td>
<td>University of Washington</td>
<td>Applied Statistics</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Coxeter, H. S. M. (Canada)</td>
<td>Ohio State University</td>
<td>Geometry</td>
<td>9/64-12/64</td>
</tr>
<tr>
<td>Darroch, J. (England)</td>
<td>University of Michigan</td>
<td>Number Theory</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Davenport, H. (England)</td>
<td>Ohio State University</td>
<td>Elasticity</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Dean, W. R. (England)</td>
<td>University of Arizona</td>
<td>Modern Analysis</td>
<td>3/63-6/64</td>
</tr>
<tr>
<td>Diego, Antonio (Argentina)</td>
<td>University of Rochester</td>
<td>Probability and Functional Analysis</td>
<td>7/64-10/64</td>
</tr>
<tr>
<td>Dvoretzky, Aryeh (Israel)</td>
<td>Columbia University</td>
<td>Functional Analysis</td>
<td>8/64-5/65</td>
</tr>
<tr>
<td>Ellis, Hubert (Canada)</td>
<td>California Institute of Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name and Home Country</td>
<td>Host Institution</td>
<td>Field of Special Interest</td>
<td>Period of Visit</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------------------</td>
<td>---------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Fejes Tóth, L. (Hungary)</td>
<td>Ohio State University</td>
<td>Geometry</td>
<td>9/64-5/65</td>
</tr>
<tr>
<td>Galier, Dieter (Germany)</td>
<td>California Institute of Technology</td>
<td>Numerical Analysis</td>
<td>9/64-12/64</td>
</tr>
<tr>
<td>Gamkrelidze, R. V. (U.S.S.R.)</td>
<td>University of Michigan</td>
<td>Functional Analysis</td>
<td>7/64-6/65</td>
</tr>
<tr>
<td>Granirer, E. E. (Israel)</td>
<td>Cornell University</td>
<td>Analysis (Integral Transforms)</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Green, H. S. (Australia)</td>
<td>University of Florida</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Griffith, J. L. (Australia)</td>
<td>University of Kansas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grimeisen, G. (Germany)</td>
<td>University of Florida</td>
<td>Topology</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Gromoll, Detlef (Germany)</td>
<td>Princeton University</td>
<td>Mathematical Programming</td>
<td>1/65-12/65</td>
</tr>
<tr>
<td>Hanson, M. A. (Australia)</td>
<td>Florida State University</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hartley, Brian (United Kingdom)</td>
<td>University of Chicago</td>
<td>Groups</td>
<td>10/64-6/65</td>
</tr>
<tr>
<td>Helwig, K. M. (Germany)</td>
<td>University of Minnesota</td>
<td>Automorphic Functions</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Heyde, C. C. (Australia)</td>
<td>Michigan State University</td>
<td>Probability</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Hijikata, Hiro-aki (Japan)</td>
<td>Yale University</td>
<td>Algebra</td>
<td>2/64-6/65</td>
</tr>
<tr>
<td>Holford, Richard (United Kingdom)</td>
<td>Stanford University</td>
<td>Applied Mathematics</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Holland, Finbarr (Wales)</td>
<td>California Institute of Technology</td>
<td>Functional Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Homma, Tatsuo (Japan)</td>
<td>University of Tennessee</td>
<td>Topology</td>
<td>3/65-12/65</td>
</tr>
<tr>
<td>Hornich, Hans (Austria)</td>
<td>The Catholic University of America</td>
<td>Analysis</td>
<td>2/65-5/65</td>
</tr>
<tr>
<td>Howie, J. M. (Scotland)</td>
<td>Tulane University</td>
<td>Algebra</td>
<td>8/64-8/65</td>
</tr>
<tr>
<td>Hubner, Otto (Germany)</td>
<td>California Institute of Technology</td>
<td>Complex Variables</td>
<td>9/64-5/65</td>
</tr>
<tr>
<td>Huckemann, F. (Germany)</td>
<td>Michigan State University</td>
<td>Analysis</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Hulanick, A. (Poland)</td>
<td>University of Washington</td>
<td>Algebra and Harmonic Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Hunter, John (Scotland)</td>
<td>Swarthmore College</td>
<td>Theory of Numbers</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Huppert, Bertram (Germany)</td>
<td>California Institute of Technology</td>
<td>Group Theory</td>
<td>9/64-12/64</td>
</tr>
<tr>
<td>Jain, P. C. (India)</td>
<td>Mathematics Research Center, U. S. Army, University of Wisconsin</td>
<td>Fluid Dynamics</td>
<td>3/63-3/65</td>
</tr>
<tr>
<td>Jain, S. K. (India)</td>
<td>University of California, Riverside</td>
<td>Algebra</td>
<td>10/63-6/65</td>
</tr>
<tr>
<td>Kale, B. K. (India)</td>
<td>Iowa State University</td>
<td>Mathematical Statistics (Statistical Inference)</td>
<td>9/64-5/65</td>
</tr>
<tr>
<td>Kampé de Feriet, J. (France)</td>
<td>The Catholic University of America</td>
<td>Probability</td>
<td>10/64-5/65</td>
</tr>
<tr>
<td>Kaniel, Shmuel (Israel)</td>
<td>Stanford University</td>
<td>Partial Differential Equations</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Karamata, Jean (Switzerland)</td>
<td>Mathematics Research Center, U. S. Army, University of Wisconsin</td>
<td>Analysis</td>
<td>7/64-10/64</td>
</tr>
<tr>
<td>Katzenelson, Yitzhak (Israel)</td>
<td>Stanford University</td>
<td>Harmonic Analysis</td>
<td>6/64-8/65</td>
</tr>
<tr>
<td>Khatri, C. G. (India)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964 (Fall)</td>
</tr>
<tr>
<td>Kinoshita, S. (Japan)</td>
<td>Florida State University</td>
<td>Topology, Knot Theory</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Komatsu, Hikosaburo (Japan)</td>
<td>Stanford University</td>
<td>Functional Analysis</td>
<td>7/64-6/65</td>
</tr>
<tr>
<td>Köhle, H. H. (Germany)</td>
<td>University of Utah</td>
<td>Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Krickeberg, Klaus (Germany)</td>
<td>Columbia University</td>
<td>Probability Theory and Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Name and Home Country</td>
<td>Host Institution</td>
<td>Field of Special Interest</td>
<td>Period of Visit</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Krishnan, M. (Mrs. V. S.) (India)</td>
<td>SUNY at Buffalo</td>
<td>Statistics</td>
<td>9/64-8/66</td>
</tr>
<tr>
<td>Krishnan, V. S. (India)</td>
<td>SUNY at Buffalo</td>
<td>Algebraic Topology</td>
<td>9/64-8/66</td>
</tr>
<tr>
<td>Kuratowski, K. (Poland)</td>
<td>Ohio State University</td>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td>Lindenstraule, J. (Israel)</td>
<td>University of Washington</td>
<td>Functional Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Lortz, D. G. (Germany)</td>
<td>Massachusetts Institute of Technology</td>
<td>Applied Mathematics</td>
<td>2/64-1/65</td>
</tr>
<tr>
<td>McNameee, John (Canada)</td>
<td>University of Oklahoma</td>
<td>Numerical Analysis and Applied Mathematics</td>
<td>9/64-9/65</td>
</tr>
<tr>
<td>Maissen, B. (Switzerland)</td>
<td>University of Washington</td>
<td>Differential Geometry and Topology</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Manohar, Rampurkar (India)</td>
<td>Mathematics Research Center, U. S. Army, University of Wisconsin</td>
<td>Fluid Dynamics</td>
<td>9/62-8/65</td>
</tr>
<tr>
<td>Maurice, M. (Netherlands)</td>
<td>University of Florida</td>
<td></td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Michael, J. H. (Australia)</td>
<td>Purdue University</td>
<td>Functional Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Miesner, W. (Germany)</td>
<td>University of Utah</td>
<td>Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Mikusinski, J. (Poland)</td>
<td>University of Florida</td>
<td></td>
<td>7/64-12/64</td>
</tr>
<tr>
<td>Miller, D. R. (Canada)</td>
<td>University of Oklahoma</td>
<td>Differential Equations</td>
<td>7/64-8/64</td>
</tr>
<tr>
<td>Milne-Thomson, L. M. (England)</td>
<td>University of Arizona</td>
<td>Hydrodynamics</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Mishchenko, E. F. (U. S. S. R.)</td>
<td>University of Michigan</td>
<td></td>
<td>9/64-12/64</td>
</tr>
<tr>
<td>Mizoguti, Y. (Japan)</td>
<td>University of Washington</td>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td>Mitsuya, Mori (Japan)</td>
<td>Iowa State University</td>
<td>Algebra</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Morimoto, Akihiko (Japan)</td>
<td>University of California, San Diego</td>
<td>Lie Groups and Lie Algebras</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Murakami, Haruo (Japan)</td>
<td>University of Kansas</td>
<td>Partial Differential Equations</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Nagahara, Takasi (Japan)</td>
<td>Northwestern University</td>
<td>Algebra</td>
<td>9/63-8/65</td>
</tr>
<tr>
<td>Nagami, Keio (Japan)</td>
<td>Duke University</td>
<td>Dimension Theory</td>
<td>12/63-6/65</td>
</tr>
<tr>
<td>Nagata, Jun-iti (Japan)</td>
<td>Texas Christian University</td>
<td>Point-Set Topology</td>
<td>5/64-3/65</td>
</tr>
<tr>
<td>Nevanlinna, Rolf (Finland)</td>
<td>Stanford University</td>
<td>Analysis</td>
<td>1/65-3/65</td>
</tr>
<tr>
<td>Newbery, A. C. R. (Canada)</td>
<td>University of Oklahoma</td>
<td>Numerical Analysis</td>
<td>7/64-8/64</td>
</tr>
<tr>
<td>Niampally, S. A. (India)</td>
<td>Iowa State University</td>
<td>Topology</td>
<td>1964-1965</td>
</tr>
<tr>
<td>Nishiro, Kiyoshi (Japan)</td>
<td>University of California, Los Angeles, University of Minnesota</td>
<td>Complex Variables and Analysis</td>
<td>9/64-1/65</td>
</tr>
<tr>
<td>Okubo, Taniro (Japan)</td>
<td>Montana State College</td>
<td>Differential Geometry</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Papangelou, Fredos (Greece)</td>
<td>The Catholic University of America</td>
<td>Analysis</td>
<td>10/63-5/65</td>
</tr>
<tr>
<td>Parameswaran, M. R. (India)</td>
<td>Michigan State University</td>
<td></td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Perles, M. (Israel)</td>
<td>University of Washington</td>
<td>Functional Analysis</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Pompilij, Giuseppe (Italy)</td>
<td>University of Pittsburgh</td>
<td>Statistics</td>
<td>12/63-12/64</td>
</tr>
<tr>
<td>Prabhu, N. U. (Australia)</td>
<td>Michigan State University</td>
<td>Probability</td>
<td>9/64-8/65</td>
</tr>
<tr>
<td>Rao, V. V. (India)</td>
<td>University of Arizona</td>
<td>Theory of Numbers</td>
<td>9/64-6/65</td>
</tr>
<tr>
<td>Roquette, P. (Germany)</td>
<td>Ohio State University</td>
<td>Algebra</td>
<td></td>
</tr>
<tr>
<td>Rund, Hanno (South Africa)</td>
<td>University of Waterloo</td>
<td></td>
<td>9/64-12/64</td>
</tr>
<tr>
<td></td>
<td>University of Toronto</td>
<td></td>
<td>1/65-5/65</td>
</tr>
<tr>
<td>Name and Home Country</td>
<td>Host Institution</td>
<td>Field of Special Interest</td>
<td>Period of Visit</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------------------</td>
<td>--------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Rutsch, Martin (Germany)</td>
<td>University of Cincinnati</td>
<td>Algebra</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Rutter, John (United Kingdom)</td>
<td>Stanford University</td>
<td>Topology</td>
<td>8/64–8/65</td>
</tr>
<tr>
<td>Samuel, Ester (Israel)</td>
<td>Columbia University</td>
<td>Mathematical Statistics</td>
<td>8/64–10/64</td>
</tr>
<tr>
<td>Samuel, P. (France)</td>
<td>Brandeis University</td>
<td>Algebra</td>
<td>9/64–1/65</td>
</tr>
<tr>
<td>Sands, A. D. (Scotland)</td>
<td>University of Washington</td>
<td>Algebra</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Sarma, V. L. N. (India)</td>
<td>SUNY at Buffalo</td>
<td>Functional Analysis</td>
<td>1/65–1/67</td>
</tr>
<tr>
<td>Sasiada, E. (Poland)</td>
<td>University of Chicago</td>
<td>Rings</td>
<td>10/64–3/65</td>
</tr>
<tr>
<td>Schäffer, J. J. (Uruguay)</td>
<td>Carnegie Institute of Technology</td>
<td>Functional Analysis</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Scheja, Gunter (Germany)</td>
<td>Purdue University</td>
<td>Logic</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Schinzel, A. (Poland)</td>
<td>Ohio State University</td>
<td>Number Theory</td>
<td>9/64–12/64</td>
</tr>
<tr>
<td>Schröder, J. (Germany)</td>
<td>University of Washington</td>
<td>Numerical Analysis</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Serre, J.-P. (France)</td>
<td>Harvard University</td>
<td>Topology</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Sethuraman, J. (India)</td>
<td>Stanford University</td>
<td>Statistics</td>
<td>9/64–9/65</td>
</tr>
<tr>
<td>Shah, K. R. (India)</td>
<td>Michigan State University</td>
<td>Statistics</td>
<td>9/64–8/65</td>
</tr>
<tr>
<td>Simons, William (Canada)</td>
<td>University of California, Davis</td>
<td>Number Theory</td>
<td>9/64–9/65</td>
</tr>
<tr>
<td>Sos, V. (Turan) (Hungary)</td>
<td>Ohio State University</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stamm, Emil (Switzerland)</td>
<td>University of Toronto</td>
<td></td>
<td>10/64–5/65</td>
</tr>
<tr>
<td>Suryanarayanan, E. R. (India)</td>
<td>University of Rhode Island</td>
<td>Research in Fluid Flow</td>
<td></td>
</tr>
<tr>
<td>Sutherland, W. A. (United Kingdom)</td>
<td>Massachusetts Institute of Technology</td>
<td>Topology</td>
<td>7/63–8/65</td>
</tr>
<tr>
<td>Suzuki, Noboru (Japan)</td>
<td>University of Minnesota</td>
<td>Functional Analysis</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Svec, A. (Czechoslovakia)</td>
<td>Brandeis University</td>
<td>Topology and Differential Equations</td>
<td>9/64–9/65</td>
</tr>
<tr>
<td>Targonski, Gyorgy (Hungary)</td>
<td>Fordham University</td>
<td>Functional Equations</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Tashiro, Yoshihiro (Japan)</td>
<td>University of Oklahoma</td>
<td>Differential Geometry</td>
<td>7/64–8/64</td>
</tr>
<tr>
<td>Tatarkiewicz, Krzysztof (Poland)</td>
<td>Illinois Institute of Technology</td>
<td>Analysis</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>Thomas, Fr. L. (England)</td>
<td>Ohio State University</td>
<td>Logic</td>
<td></td>
</tr>
<tr>
<td>Todd, Leonard (United Kingdom)</td>
<td>Massachusetts Institute of Technology</td>
<td>Applied Mathematics</td>
<td>7/64–6/65</td>
</tr>
<tr>
<td>Tranquilli, G. B. (Italy)</td>
<td>University of North Carolina at Chapel Hill</td>
<td>Statistics</td>
<td>1964–1965</td>
</tr>
<tr>
<td>Turan, P. (Hungary)</td>
<td>Ohio State University</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ursell, F. (England)</td>
<td>University of Michigan</td>
<td></td>
<td>9/64–12/64</td>
</tr>
<tr>
<td>Vadlamudi, P. (India)</td>
<td>Wisconsin State University</td>
<td>Numerical Analysis</td>
<td>1964–1966</td>
</tr>
<tr>
<td>Vaidya, P. C. (India)</td>
<td>Washington State University</td>
<td>Relativity Theory and Analysis</td>
<td>9/64–6/65</td>
</tr>
<tr>
<td>VanDalen, Dirk (Netherlands)</td>
<td>Massachusetts Institute of Technology</td>
<td>Logic</td>
<td>7/64–6/65</td>
</tr>
<tr>
<td>Veldkamp, F. D. (Netherlands)</td>
<td>Yale University</td>
<td>Algebra</td>
<td>1964–1965</td>
</tr>
<tr>
<td>Venter, J. H. (South Africa)</td>
<td>Stanford University</td>
<td>Statistics</td>
<td>9/63–1/65</td>
</tr>
<tr>
<td>Verma, G. R. (India)</td>
<td>University of Rhode Island</td>
<td>Algebra</td>
<td></td>
</tr>
<tr>
<td>Wadhwa, Y. D. (India)</td>
<td>Iowa State University</td>
<td>Applied Mathematics</td>
<td>1964–1965</td>
</tr>
<tr>
<td>Wang, Ju–kwei (Taiwan)</td>
<td>Yale University</td>
<td>Analysis</td>
<td>1963–1965</td>
</tr>
</tbody>
</table>
Name and Home Country | Host Institution | Field of Special Interest | Period of Visit
--- | --- | --- | ---
Weissinger, Johannes (Germany) | Mathematics Research Center, U.S. Army, University of Wisconsin | Applied Mathematics | 3/64-2/65
Wielandt, H. (Germany) | Ohio State University | Algebra | 10/64-5/65
Witt, E. (Germany) | State University of New York at Stony Brook | Algebra | 1964-1965
Wloka, Joseph (Germany) | University of California, Los Angeles | Partial Differential Equations | 1964-1965
Wold, Herman (Sweden) | Mathematics Research Center, U.S. Army, University of Wisconsin | Economics and Statistics | 6/64-11/64
Wynn, Peter (United Kingdom) | Mathematics Research Center, U.S. Army, University of Wisconsin | Numerical Analysis | 9/64-9/65
Yamada, Miyuki (Japan) | Sacramento State College | Semigroups | 1964-1966
Yff, Peter (Israel) | University of Toronto | 9/64-4/65
Yoshizawa, Hisaaki (Japan) | Yale University | Functional Analysis | 1964-1965
Zubrzycki, S. (Poland) | University of Washington | Statistics | 9/64-6/65
Zygmund, A. (Poland) | Ohio State University | Algebra | 1964-1965

NEW AMS PUBLICATIONS

MEMOIRS

Number 50

ON LIE ALGEBRAS AND SOME SPECIAL FUNCTIONS OF MATHEMATICAL PHYSICS
By Willard Miller, Jr.

47 pages, List Price $1.60; Member Price $1.20

Special function theory is notable in mathematics for its apparent lack of unity and for the multiplicity of tricks and devices needed to obtain information about its subject matter. This Memoir shows, however, that the properties of a large class of special functions can be derived in a uniform manner from a study of the representation theory of certain Lie algebras. In particular, the commutation relations of these Lie algebras lead directly to recursion relations and generating functions for the hypergeometric, confluent hypergeometric, Bessel, and parabolic cylinder functions. The Lie algebraic approach greatly unifies the theory of these special functions.

Number 51

HIERARCHIES OF PREDICATES OF FINITE TYPES
By D. A. Clarke

95 pages, List Price $1.90; Member Price $1.43

This Memoir contains an investigation into the structure of certain hierarchies of "definable" predicates based on S. C. Kleene's definition of recursiveness for functions and predicates whose variables are of arbitrary finite types. A vast array of hierarchies is considered, based on the types of the free variables, the types of the quantified variables, and the length of the quantifier strings; these are extended into the transfinite. The study parallels to a large extent the earlier studies of hierarchies of number-theoretic predicates, but many of the earlier results do not carry over.

732
PERSONAL ITEMS

Associate Professor N. L. ALLING of Purdue University has been awarded a National Science Foundation Senior Postdoctoral Fellowship at the Massachusetts Institute of Technology.

Dr. RAM BALLABH of the University of Lucknow has been appointed a Visiting Lecturer at the Lake Forest College for the academic year 1964-1965.

Professor GILBERT BAUMSLAG of the Courant Institute of Mathematical Sciences, New York University has been appointed to a professorship at the Graduate Mathematics Center of the City University of New York.

Mr. B. R. BERNSTEIN of the Queensborough Community College has been appointed a Lecturer at Brooklyn College.

Professor R. H. BING of the University of Wisconsin has been named a Research Professor of Mathematics.

Assistant Professor D. M. BURTON of the University of New Hampshire has been appointed to a visiting assistant professorship at Yale University for the academic year 1964-1965.

Dr. A. C. CARVER of the New Mexico State University has accepted a position as Senior Scientist with Spindletop Research Incorporated.

Mr. G. T. CHARTRAND of Michigan State University has been appointed to an assistant professorship at Western Michigan University.

Mr. J. A. CIMA of Stanford University has been appointed to an assistant professorship at the University of North Carolina.

Dr. L. W. COHEN of the University of Maryland has been appointed Executive Secretary of the Division of Mathematics, National Academy of Sciences, National Research Council.

Mr. HOWARD COOK of Auburn University has been appointed to an assistant professorship at the University of North Carolina.

Mr. MYRON DANZIG of the Belfer Graduate School of Science, Yeshiva University has accepted a position as a Computer Programmer with the Bubble Chamber Group of the Brookhaven National Laboratory.

Mr. E. D. DAVIS of Yale University has been appointed to an assistant professorship at Purdue University.

Dr. J. T. DAY of the University of Wisconsin has been appointed to an assistant professorship at Michigan State University.

Mr. GIACOMO DELLA RICCIA of the Massachusetts Institute of Technology has been appointed to a visiting assistant professorship at Indiana University.

Associate Professor H. M. ELLIOTT of Washington University has been appointed to an acting professorship at the College of William and Mary.

Dr. JACK ENGELHARDT of the Grumman Aircraft Corporation has been appointed to an assistant professorship at Washington State University.

Dr. L. T. GARDNER of Columbia University has been appointed to an assistant professorship at Queens College.

Dr. R. W. GOODMAN of Harvard University has been appointed a Lecturer at the Massachusetts Institute of Technology.

Dr. L. D. GRABER of Iowa State University has been appointed to an assistant professorship at the Florida Presbyterian College.

Dr. J. K. HALE of RIAS has been appointed to a professorship with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Mr. R. W. HEATH of the University of Georgia has been appointed to a visiting associate professorship at Arizona State University for the academic year 1964-1965.

Dr. H. G. HERMES of the Martin Company has been appointed to an assistant professorship with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Mr. HIROAKI HIJIKATA of Tokyo University has been appointed a Research Associate at Yale University.
Mr. W. C. HOLLAND of the University of Chicago has been appointed to an assistant professorship at the University of Wisconsin.

Mr. R. E. ISAAC of Hunter College has been appointed to a visiting assistant professorship at Cornell University for the academic year 1964-1965.

Mr. SHMUEL KANTOROVITZ of the Institute for Advanced Study has been appointed to an assistant professorship at Yale University.

Mr. PAUL KATZ of the University of Washington has been appointed a Research Fellow at the Hebrew University.

Associate Professor R. N. KESAR-WANI of the Wayne State University has been appointed to an associate professorship at the University of Ottawa.

Professor S. C. KLEENE of the University of Wisconsin has been named a Cyrus C. MacDuffee Professor of Mathematics.

Mr. R. L. KRUSE of the California Institute of Technology has accepted a position as a Staff Member with the Sandia Corporation, Albuquerque, New Mexico.

Dr. J. P. LASALLE of RIAS has been appointed a Professor and Director of the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Professor SOLOMON LEFSCHETZ of Princeton University has been appointed to a visiting professorship with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Mr. JACQUES LEWIN of the Courant Institute of Mathematical Sciences, New York University has been appointed a Research Fellow at the California Institute of Technology.

Mr. M. C. McCORD of the University of Wisconsin has been appointed to an assistant professorship at the University of Georgia.

Dr. W. L. MISER has been appointed a Visiting Lecturer at Sweet Briar College.

Associate Professor W. O. J. MOSER of the University of Manitoba has been appointed to an associate professorship at McGill University.

Dr. S. A. NAIMPALLY of the Michigan State University has been appointed to an assistant professorship at the Iowa State University.

Dr. J. L. NASSAR of the Socony Mobil Oil Company, Incorporated has been appointed to an assistant professorship at the American University of Beirut, Beirut, Lebanon.

Professor DANIEL PEDOE of Purdue University has been appointed to a visiting professorship at the University of Minnesota and the Minnesota School Mathematics Center for three quarters.

Professor M. M. PEIXOTO of the Institute of Pure and Applied Mathematics, Rio de Janeiro has been appointed to an associate professorship with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Mr. D. H. POTTS of the University of California, Santa Barbara, has been awarded a National Science Foundation Science Faculty Fellowship at the University of California, Berkeley.

Assistant Professor A. L. RABENSTEIN, JR. of the Pennsylvania State University has been appointed to an assistant professorship at Macalester College.

Mr. D. S. RAY of the University of Tennessee has been appointed to an associate professorship at Bucknell University.

Dr. AZRIEL ROSENFELD of the Budd Company has been appointed a Research Associate Professor with the Computer Science Center at the University of Maryland.

Mr. L. E. ROSS of the University of California, Berkeley, has been appointed to an acting assistant professorship at the University of California, Los Angeles.

Dr. E. O. ROXIN of the University of Buenos Aires has been appointed a Post-doctoral Fellow with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Professor N. W. SAVAGE of Arizona State University has been awarded a National Science Foundation Faculty Fellowship at Stanford University for the academic year 1964-1965.

Mr. J. E. SHOCKLEY of the College of William and Mary has been appointed to an associate professorship at the University of Wyoming.

Mr. D. J. SIMMS of Glasgow University has been appointed a Lecturer at Trinity College, Dublin, Ireland.

Dr. M. B. SMITH of the University of
Wisconsin has been named Vice Provost of the University Center System.

Mr. R. E. SMITHSON of the U. S. Naval Ordnance Test Station has been appointed to an assistant professorship at the University of Florida.

Dr. D. P. SQUIER of the California Research Corporation has been appointed to an associate professorship at Colorado State University.

Mr. D. E. STAHL of the Computer Concepts, Incorporated has accepted a position as an Applications Analyst with the Control Data Corporation, Minneapolis, Minnesota.

Mr. E. B. STEAR of Lear Siegler Incorporated has been appointed to an assistant professorship with the Department of Engineering at the University of California, Los Angeles.

Mr. NOBORU SUZUKI of the Kanazawa University has been appointed to a visiting assistant professorship at the University of Minnesota for the academic year 1964-1965.

Professor HENRY VAN ENGEN, on leave from the University of Wisconsin will spend the 1964-1965 academic year in South America, setting up teacher training programs, largely in Chile.

Dr. LEONARD WEISS of RIAS has been appointed to an assistant professorship with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Dr. M. C. WEISS of the University of Chicago has been appointed to a professorship at DePaul University.

Dr. F. W. WILSON, JR. of the University of Maryland has been appointed a Postdoctoral Fellow with the new Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Mr. A. S. WINEMAN of Brown University has been appointed to an assistant professorship at the University of Michigan.

Assistant Professor Y. F. WONG of Atlanta University has been appointed to an assistant professorship at DePaul University.

Dr. L. N. ZACCARO of the Massachusetts Institute of Technology has been appointed to an associate professorship at the Worcester Polytechnic Institute.

Mr. ELIAS ZAKON of Assumption University has been appointed to a professorship at the University of Windsor, Windsor, Ontario, Canada.

The following promotions are announced:

R. G. BILYEU, North Texas State University, to an associate professorship.

MARTIN FOX, Michigan State University, to an associate professorship.

A. F. GILMAN III, Bowdoin College, to an assistant professorship.

R. H. HOMER, Iowa State University, to an associate professorship.

W. E. JENNER, University of North Carolina, to a professorship.

J. R. KING, Providence College, to an associate professorship.

C. W. KOHLS, Syracuse University, to an associate professorship.

R. J. LAMBERT, Iowa State University, to a professorship.

Dr. N. D. LANE, McMaster University, to a professorship.

H. S. LEONARD, Carnegie Institute of Technology, to an associate professorship.

R. C. MACCAMY, Carnegie Institute of Technology, to a professorship.

A. C. MEWBORN, University of North Carolina, to a professorship.

V. J. MIZEL, Carnegie Institute of Technology, to an associate professorship.

J. T. MOHAT, North Texas State University, to a professorship.

D. E. MYERS, University of Arizona, to an associate professorship.

J. A. NOHEL, University of Wisconsin, to a professorship.

R. L. POE, Kansas State Teachers College, to an associate professorship.

B. S. RANDOL, Yale University, to an assistant professorship.

D. E. SANDERSON, Iowa State University, to a professorship.

S. L. SEGAL, University of Rochester, to an assistant professorship.

GASTON SMITH, University of Southern Mississippi, to a professorship.

JOSEPH VERDINA, California State College at Long Beach, to an associate professorship.

ROBERT VERMES, McGill University, to an assistant professorship.

F. M. WRIGHT, Iowa State University, to a professorship.

735
The following appointments to Instructor­ships are announced.

Cornell University: W. G. FARIS; Dartmouth College: E. M. BROWN; Knox College: R. S. BORDEN; North Texas State University: J. V. SHAW; Olympic College: D. J. SEAMAN; Polytechnic Institute of Brooklyn: HOWARD ALLEN; Providence College: S. W. SCHULTZ; Stanford University: R. J. SIBNER; Yale University: P. S. GREEN, BOAZ NATZITZ, N. T. PECK, J. D. STAFNEY, and E. L. STOUT.

Deaths:

Dr. J. W. CALKIN of the Brookhaven National Laboratory died on August 4, 1964 at the age of 54. He was a member of the Society for 27 years.
Professor Emeritus R. G. PUTNAM of New York University died on July 14, 1964 at the age of 77. He was a member of the Society for 39 years.
MEMORANDA TO MEMBERS

CORPORATE MEMBERS

The Society acknowledges with gratitude the support rendered by the following corporations who held Corporate Membership in the Society as of June 1, 1964.

Academic Press, Incorporated
Aerospace Corporation
Bell Telephone Laboratories, Incorporated
The Boeing Company
E. I. Du Pont de Nemours and Company, Incorporated
Eastman Kodak Company
Ford Motor Company
General Motors Corporation
Hughes Aircraft Company
International Business Machines Corporation
Lockheed Missiles and Space Company
Procter and Gamble Company
Radio Corporation of America
Remington-Rand UNIVAC
Shell Development Company
Socony Mobil Oil Company, Incorporated
Space Technology Laboratories, Incorporated
Standard Oil Company Incorporated in New Jersey
United Gas Corporation

RETIRED MATHEMATICIANS

The next issue of the List of Retired Mathematicians Available for Employment will be published in February, 1965, by the Mathematical Sciences Employment Register. The List is distributed to academic and industrial employers who request it from the Register. Retired Mathematicians who are interested in being included in the List are invited to send to the Register the following information: name, date of birth, highest degree earned and where it was obtained, most recent employment, present address, date available, and preferences, including preference for academic or industrial employment.

The address of the Mathematical Sciences Employment Register is 190 Hope Street, Providence, Rhode Island 02906.

SUMMER EMPLOYMENT OPPORTUNITIES

The annual list of Opportunities for Summer Employment for Mathematicians and College Mathematics Students is being compiled presently and will be issued early in January, 1965. The list will be available free of charge at the Annual Meeting in Denver in January, 1964, and on request from the Mathematical Sciences Employment Register.

Institutions which have summer job openings and would welcome applications from mathematicians and students of mathematics may request forms for the listing of summer employment from the Mathematical Sciences Employment Register, 190 Hope Street, Providence, Rhode Island 02906.

THE EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register, established by the American Mathematical Society, The Mathematical Association of America, and The Society for Industrial and Applied Mathematics, will be maintained at the Annual Meeting at the Denver Hilton Hotel, Denver, Colorado on January 27-28-29, 1965. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on each of these three days.

There is no charge for registration, either to job applicants or to employers, except when the late registration fee for employers is applicable. Provision will be made for anonymity of applicants upon request and upon payment of $3.00 to defray the cost involved in handling anonymous listings.

Job applicants and employers who wish to be listed will please write to the Employment Register, 190 Hope Street,
NEWS ITEMS AND ANNOUNCEMENTS

THE AUSTRALIAN NATIONAL UNIVERSITY

Conference on the Theory of Groups

An international conference on the Theory of Groups will be held at the Australian National University August 10-20, 1965. It is being sponsored by the International Mathematical Union and the Australian Academy of Science. The Organizing Committee has some funds at its disposal for assistance with the living expenses of participants in the conference, especially for junior group theorists. If you would like to have further information, and in particular if you might wish to attend the conference, please send your name and address to the Secretary of the Organizing Committee: Dr. L. G. Kovács, Department of Mathematics, Institute of Advanced Studies, Australian National University, Box 4, G.P.O., Canberra, A.C.T., Australia.

TOPOLOGICAL SPACES - By H. J. KOWALSKY

Academic Press Edition

Academic Press Incorporated regrets to announce that it has discovered a number of typographical errors in its English edition of Topological Spaces by H. J. Kowalsky. A corrected edition is now in preparation and will be ready for distribution within a few months. Copies of the original edition may be returned for exchange after the corrected edition has been announced.
SUPPLEMENTARY PROGRAM—Number 28

During the interval from September 4, 1964 through September 23, 1964 the papers listed below were accepted by the American Mathematical Society for presentation by title. After each title on this program there is an identifying number. The abstracts of the papers will be found following the same number in the section on Abstracts of Contributed papers in this issue of these NOTICES.

One abstract presented by title may be accepted per person per issue of the NOTICES. Joint authors are treated as a separate category; thus in addition to abstracts from two individually, one joint abstract by them may be accepted for a particular issue.

1. Higher-order spline interpolation
   Dr. J. H. Ahlberg, United Aircraft Corporation, East Hartford, Connecticut; Dr. E. N. Nilson, Pratt and Whitney Aircraft Corporation, East Hartford, Connecticut and Professor J. L. Walsh, Harvard University (64T-494)

2. An automorphism theorem for matrices
   Professor C. E. Aull, Kent State University (64T-507)

3. 2-Manifolds as maximal ideal spaces
   Mr. A. G. Brandstein, Brown University (64T-501)

4. Permanent of the direct product of matrices
   Mr. R. A. Brualdi, National Bureau of Standards, Washington, D.C. (64T-503)

5. Some multiple sums and binomial identities
   Professor Leonard Carlitz, Duke University (64T-492)

6. Weak solutions of conservation equations in general relativity
   Professor Nathaniel Coburn, University of Michigan (64T-509)

7. The number of zeroid elements (mod n). II
   Professor Eckford Cohen, The University of Tennessee (64T-491)

8. The A-B-E sequences and their relationship with Cramer's conjecture
   Dr. F. B. Correia, Rhode Island College (64T-504)

9. Measurable cardinals and constructible sets
   Dr. Haim Gaifman, The Hebrew University, Jerusalem, Israel (64T-64T-505)

10. Coarser topologies with the same class of homeomorphisms
    Professor Y.-L. Lee, University of Connecticut (64T-502)

11. Measurable cardinals and the continuum hypothesis. Preliminary report
    Professor Azriel Levy, Hebrew University, Jerusalem, Israel (64T-500)

12. Equivalent statements of generalized Haar theorem
    Mr. H. L. Loeb, Aerospace Corporation, Northridge, California (64T-498)

13. On powers of non-negative matrices
    Dr. M. S. Lynn and Dr. B. R. Heap, National Physical Laboratory, Teddington, England (64T-493)

14. On connecting semi-Fredholm operators, Preliminary report
    Dr. Gerhard Neubauer, Universität Heidelberg, Germany (64T-506)

15. A number-theoretic estimate
    Professor B. S. Randol, Yale University (64T-497)

16. The integral representation ring of a finite group
    Professor Irving Reiner, University of Illinois (64T-499)

17. Convergence of spline functions
    Professor Ambikeshwar Sharma and Mr. Amram Meir, University of Alberta, Canada (64T-496)

18. On the law of the iterated logarithm for continued fractions
    Professor O. P. Stackelberg, Duke University (64T-508)

19. A necessary condition for well-posed Cauchy problems
    Professor G. W. Strang, Massachu-
M.I.T. — Parts of connected analytic varieties and analytic polyhedra. Preliminary report
Mr. D. R. Wilken, Tulane University (64T-495)

Dr. J. M. Worrell, Jr., Sandia Corporation, Albuquerque, New Mexico (64T-511)

NEWS ITEMS AND ANNOUNCEMENTS

BENJAMIN PEIRCE INSTRUCTORSHIP

Applications for the Benjamin Peirce Instructorship at Harvard University are invited. The teaching commitment is six hours a week; the salary is $7,200 for the academic year with $300 annual increments. Appointments are annual but carry the presumption of two renewals. It is possible to earn summer income through teaching or research contracts. Additional information and application forms can be obtained by writing the Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge, Massachusetts 02138.

C.L.E. MOORE INSTRUCTORSHIPS AT M.I.T.

The M.I.T. Department of Mathematics wishes to announce the availability of C.L.E. Moore Instructorships in Mathematics for 1965-1966, open to young mathematicians with doctorates who show definite promise in research. The base salary for these instructorships will be at least $8300 and the teaching load will be six hours per week. The salary can be supplemented by summer work on a research grant, sponsored by the Air Force Office of Scientific Research, or by teaching in the summer session. The appointments are annual but are renewable for one additional year.

Applications should be filed not later than January 4, 1965, on forms obtained from the Department.

INSTITUTE FOR ADVANCED STUDY MEMBERSHIPS, 1965-1966

The School of Mathematics of the Institute for Advanced Study, Princeton, New Jersey, will grant a limited number of memberships, in some cases with financial support, for research in mathematics at the Institute during the academic year 1965-1966. Candidates must have given evidence of ability in research comparable at least with that expected for the Ph.D. degree. Application blanks may be obtained from the Secretary of the School of Mathematics, and should be returned by January 15 (whether or not funds are expected from some other source).

MATHEMATICAL PREPARATION FOR COMPUTER SCIENCE

The Committee on the Undergraduate Program in Mathematics (CUPM) Panel on Mathematics for the Physical Sciences and Engineering is concerned with mathematics as it relates to students in these fields. The Panel has recently outlined a program for the undergraduate mathematics preparation for students planning careers in the field of Computer Science. Anyone interested in examining the document can obtain a copy by writing to: CUPM, P. O. Box 1024, Berkeley, California 94701, and asking for the Computing Recommendations.

Let $A_0$ and $P$ be self-adjoint operators in the Hilbert space $H$ and set $A_1 = A_0 + P$, $R_1(z) = (z - A_1)^{-1}$. Suppose that there is a dense set $S$ in $H$ and a norm on it such that on $S$, the completion, we have for $\epsilon \to +0$ and $\xi \in [\xi_1, \xi_2]$, Condition 1. The forms $R_1(\xi + i\epsilon)$ converge weakly, Condition 2. The operator $PR_0(\xi \pm i\epsilon)$ converge in norm to compact operators, Condition 3. The nullspaces of $1 - PR_0(\xi \pm i\epsilon)$ are trivial. Then the second resolvent equation shows that the forms $R_1(\xi \pm i\epsilon)$ converge weakly; uniformly in $\xi$ if the conditions are. Thus the resolvent loop integral formula shows that the part of $A_1$ over $[\xi_1, \xi_2]$ is absolutely continuous. It is illustrated that this can be established for the operator $-D^2 + M(q)$ on $[0, \infty)$, if the continuous function $q$ is bounded and integrable with weight function $w(x)$ for $x \to \infty$. This yields a special case of a theorem of Titchmarsh (Eigenfunction expansions ..., Vol. I, p. 97). (Received June 29, 1964.)


Let $X$ be a Hausdorff space and let $\tau$ be a quasi-order on $X$ with a closed graph. For each $x \in X$ we write $L(x) = \{y : y \tau x\}$ and $E(x) = \{y : y \tau x$ and $x \tau y\}$. A subset $W$ of $X$ is said to have no local minima provided, for each $x \in W$ and each neighborhood $V$ of $x$, there exists $y \in W \cap V \cap L(x) - E(x)$. R. J. Koch has conjectured that if $X$ is compact, if each set $E(x)$ is totally disconnected, and if $W$ is an open set having no local minima, then each point of $W$ lies in an arc which meets $X - W$. It is shown here that Koch's conjecture is correct. As a corollary it follows that if $X$ is compact, if each set $E(x)$ is totally disconnected, if each set $L(x)$ is connected and if $X$ has a zero, then $X$ is arcwise connected. (Received September 3, 1964.)

616-3. WITHDRAWN.
A number of inversion integrals for transformations of the form \( \int_{X} K(t/x) g(t) dt = f(x) \) are found with the aid of Mellin Transforms. A typical transformation involves Gauss' hypergeometric function, and the inversion formula is an integral involving Gauss' hypergeometric function and a differential operator. The transformations are contained in a class of transformations whose kernels have the Mellin-Barnes integral representation

\[
\mathcal{F}_n(z) = \frac{(1/2\pi i) \prod_{k=1}^{n} \left( \frac{\Gamma(a_k - s)/\Gamma(1 - c_k + s)}{s} \right)}{z^t} \text{ds}
\]

(z > 1) where \( b_n - n - \sum_{k=1}^{n} (a_k + c_k) > -1 \) and \( Re s > Re S, k = 1, \ldots, n \). (Received September 18, 1964).

A sequence \( (a_1, \ldots, a_n) \) of nonunits in a Noetherian ring \( R \) is a prime sequence in case \( a_1 \) is not a zero divisor, \( (a_1, \ldots, a_i)R : a_{i+1} R = (a_1, \ldots, a_i)R \) (i = 1, \ldots, n - 1), and \( (a_1, \ldots, a_n)R \neq R \). A local ring \( R \) is a Macaulay local ring in case there is a system of parameters in \( R \) which form a prime sequence. A Noetherian ring \( R \) is a locally Macaulay ring in case \( RM \) is a Macaulay local ring, for every maximal ideal \( M \) in \( R \). \textbf{Theorem 1.} Let \( R \) be a locally Macaulay ring, and let \( (a_1, \ldots, a_n) \) be a prime sequence in \( R \). Then \( R' = R[a_2/a_1, \ldots, a_n/a_1] \) is a locally Macaulay ring, and \( (a_1, a_2/a_1, \ldots, a_n/a_1) \) is a prime sequence in \( R' \). The converse of Theorem 1 is not in general true. \textbf{Theorem 2.} Let \( t \) be transcendental over a locally Macaulay ring \( R \), and let \( (a_1, \ldots, a_n) \) be a prime sequence in \( R \). Then \( R^* = R[ta_1, \ldots, ta_n, 1/t] \) is a locally Macaulay ring, and \( (1/t, ta_1, \ldots, ta_n) \) is a prime sequence in \( R^* \). \textbf{Theorem 3.} A partial converse of Th. 2 is true. \textbf{Theorem 4.} Let \( R \) be a Noetherian ring, and let \( t \) be transcendental over \( R \). If there is an ideal \( A = (a_1, \ldots, a_n)R \) in \( R \) such that the ring \( R^* \) in Theorem 2 is a locally Macaulay ring, then for every non-zero-divisor \( a \in R \), \( R[a_1/a, \ldots, a_n/a] \) and \( R \) are locally Macaulay rings. (Received September 18, 1964.)

An algebra \( A \) is primal in the small if for every finite subset \( N \) of \( A \) and every mapping of a finite cartesian power of \( N \) into \( A \) some polynomial represents this mapping. \( n \)-algebras of the same species are independent in the small if for each \( n \)-sequence of polynomials and \( n \)-sequence of finite subsets, one from each algebra, there is a single polynomial which represents each of the polynomials in the corresponding subset. The identities of algebra \( A \) subextend to algebra \( B \) if there exists an intersection-preserving mapping \( f \) of the finite subsets of \( A \) to subsets of \( B \) such that every element of \( B \) is in some image and, moreover, \( f \) preserves the validity of identities. \textbf{Theorem.} Let \( A_1, \ldots, A_n \) be independent in the small and primal in the small algebras. Let \( B \) be any algebra (of same species as \( A_1, \ldots, A_n \)) such that the identities common to \( A_1, \ldots, A_n \) subextend to \( B \). Then each nonunit ideal of \( B \) is the intersection of its maximal ideal divisors. This generalizes some theorems of Foster [Math, Z, 65 (1956), 70-75] and Stone [Trans. Amer. Math. Soc. 40 (1936), 37-111]. (Received September 21, 1964.)
616-7. V. H. DYSON, Hughes Aircraft Company, P.O. Box 3310 Building 604, M.S. E-132, Fullerton, California. The word problem and residually finite groups.

Let \( T, T_f, T_{rf} \) respectively stand for the elementary theories of groups, finite groups and residually finite groups; and let \( C, F_C, R, \langle R \rangle, \Sigma(A) \) denote in order: a set of symbols, the free group generated by \( C \), a subset of \( F_C \), the normal closure of \( R \) in \( F_C \), the conjunction of the equations \( r = 1 \) for \( r \in A \). If \( C \) and \( R \) are both finite, then \( \langle R \rangle \) is a recursively enumerable subset of \( F_C \) and the intersection \( M(C,R) \) of all normal subgroups of \( F_C \) of finite index containing \( R \) is the \( F_C \)-complement of a recursively enumerable subset of \( F_C \). Thus, a finitely presentable residually finite group has a solvable word problem. If, by the \( R \)-conditional word problem, \( R \)-cwp (the conditional word problem, cwp) for a theory is meant its decision problem for the set of all sentences of the form \( \Sigma(A) \rightarrow r = 1 \), \( A \) a finite subset of \( R \) (\( A \) any finite set of words), then the \( R \)-cwp for \( T_f \) and for \( T_{rf} \), and the word problem for \( F_C/R \) are equivalent. Conjecture: The \( R \)-cwp for \( T_f \) is unsolvable, i.e., there exists a finitely presentable group \( G \) whose largest residually finite factor group, \( G/M(G) \), has an unsolvable word problem. A finitely generated, however infinitely related, group \( G \) with solvable word problem is constructed, for which \( G/M(G) \) has an unsolvable word problem. (Received September 21, 1964.)


For any \( k \geq 1 \) let \( L \) be a derivation of the \( C^k \) functions on a real \( C^k \) manifold \( M \) into the continuous functions on \( M \); then \( L \) is a vector field on \( M \) in the usual sense with continuous coefficients. Consequently the tangent space at any \( P \in M \) consists of those \( P \)-derivations of the germs of \( C^k \) functions at \( P \) which can be factored through a derivation into the germs of continuous functions at \( P \). (The space of all \( P \)-derivations of the germs of \( C^k \) functions at \( P \) is known to have the dimension of the continuum when \( k \) is finite.) (Received September 22, 1964.)

616-9. R. F. BROWN, University of California at Los Angeles 24, California. On a homotopy converse to the Lefschetz fixed point theorem.

Let \( \alpha \) be a homotopy class of maps of \( X \), a compact metric ANR, into itself and let \( L_\alpha \) denote the Lefschetz number of \( \alpha \). A converse to the Lefschetz fixed point theorem is: If \( L_\alpha = 0 \) then \( \alpha \) contains a fixed point free map. The converse is true if \( X \) is a compact connected simply-connected topological \( n \)-manifold (Fadell) or if \( X \) is a compact connected topological \( n \)-manifold, with or without boundary, and \( \alpha \) contains the identity map (Brown-Fadell). The theorem below generalizes both conditions when \( n \geq 3 \). Fixed points \( x \) and \( x' \) of \( f: X \rightarrow X \) are said to be in the same fixed point class of \( f \) if there is a path \( w \) in \( X \) from \( x \) to \( x' \) such that \( [w \cdot (f)^{-1}] = 0 \in \pi_n(X,x) \). If \( X \) is a connected compact metric ANR, then the number \( \mu \) of fixed point classes of \( f \) of nonzero index is an invariant of the homotopy class \( \alpha \) of \( f \). Every \( g \in \alpha \) has at least \( \mu \) fixed points [F. Browder, Summa Brasil,
Math. 4 (1960), 253-293. Theorem. If \( X \) is a compact connected topological \( n \)-manifold, \( n \geq 3 \), with or without boundary, then there exists \( g \in a \) with exactly \( \mu \) fixed points. This result is known for triangulable manifolds [F. Wecken, Math. Ann. 118 (1941-1943), 544-577]. (Received September 22, 1964.)

616-10. R. W. PREISENDORFER, University of California, Scripps Institute of Oceanography, Visibility Laboratory, San Diego 52, California, Invariant imbedding formulation for radiative transfer on general media with internal sources.

It is shown how the problem of arbitrarily distributed internal sources in scattering-absorbing media of arbitrary geometric structure can be solved by means of a systematic imbedding procedure of lower dimensional spaces into higher dimensional spaces. Explicit integral representations of the solutions have been formulated. (Received September 23, 1964.)

616-11. KENNETH ROGERS, University of Hawaii, Honolulu 14, Hawaii, Cyclotomic polynomials and Wedderburn's Theorem.

Witt's proof of Wedderburn's Theorem is freed from an implicit dependence on the complex plane or valuation theory. This is done by giving a number-theoretic proof that \( \phi_n(q) \geq q \) for \( n > 1, q > 1 \). At the same time, a new development of \( \phi_n(x) \) is given. (Received September 24, 1964.)


A reflexive relation \( \trianglelefteq \) on a lattice \( L \) is called a normality relation if for \( a, b, c, d \in L \):

1. \( a \trianglelefteq b \) implies \( a \leq b \);
2. \( a \trianglelefteq b, c \trianglelefteq d \) implies \( a \cap c \trianglelefteq b \cap d \);
3. \( a \trianglelefteq b, a \trianglelefteq c \) implies \( a \trianglelefteq b \cup c \);
4. \( a \trianglelefteq b, c \trianglelefteq d \) implies \( a \cup c \trianglelefteq a \cup c \cup (b \cap d) \);
5. \( a \leq b \) and either \( a \trianglelefteq a \cup c \) or \( c \trianglelefteq a \cup c \) implies \( a \cup (b \cap c) = b \cap (a \cup c) \).

Conditions (1) - (5) are independent, imply the normality relations for lattices suggested by Kurash and Zassenhaus, and generalize many results valid in a lattice \( L(G) \) of subgroups of a group. Define \( a \trianglelefteq u b \) if there exist \( a_i \in L \) such that \( a = a_n \trianglelefteq a_{n-1} \trianglelefteq \ldots \trianglelefteq a_0 = b \). We prove:

(I) If \( a_2 \trianglelefteq a_1 \trianglelefteq u b \trianglelefteq u \) then \( a \cup b \trianglelefteq u \); (II) If \( b \trianglelefteq u \) and \( a_3 \trianglelefteq a_2 \trianglelefteq a_1 \trianglelefteq a_0 = u \) where \( a_i \) covers \( a_{i+1} \) \((i = 0, 1, 2)\) then \( a_3 \trianglelefteq u b \trianglelefteq u \). We show that (I) and (II) cannot be extended by giving an example derived from one due to Zassenhaus. We call \( a \) and \( b \) a modular pair under \( u \) if \( a \leq u, b \leq u, a \equiv c \trianglelefteq u \) implies \( c \cap (a \cup b) = a \cup (b \cap c) \). Adding condition (6): if \( x \trianglelefteq u, y \trianglelefteq u, x \trianglelefteq x \cap y \) then \( x \cup y \trianglelefteq u \), we obtain:

(III) If \( (L, \trianglelefteq) \) satisfies (1) - (6), and if \( a \trianglelefteq u, b \trianglelefteq u \) are a modular pair under \( u \), then \( a \cup b \trianglelefteq u \). Condition (6) always holds in \( L(G) \). Result (III) extends certain results proved by Wielandt for subnormal subgroups. (Received September 25, 1964.)

A. Miščenko [Spaces with point-countable bases, Soviet Math, Dokl. 3 (1962), 855-858] has shown that every bicoompactum with a point-countable base is metrizable but that there exist a paracompactum and a Hausdorff (nonregular) hereditarily Lindelöf space each of which has a point-countable base but is nonmetrizable. The author has shown that there is a Moore space which has a point-countable base but not a uniform base (i.e. is not pointwise paracompact) [Abstract 605-22, these Notices 11 (1963), 649]. It is shown in this paper that: Theorem. A semi-metric space with a point-countable base is developable. From this it follows, in answer to a question raised by Jack Ceder in [Some generalizations of metric spaces, Pacific J. Math. 11 (1961), 105-125], that:

Corollary. An M1-space with a σ-closure preserving base B = U∞n=1 Bn, where Bn is point-countable, is metrizable—in fact any M3-space (hence any Nagata space) with a point-countable base is metrizable. (Received September 28, 1964.)


In an n-dimensional real linear space E, let X be a subset symmetric with respect to a group Γ of orthogonal transformations of E. (See Abstract 614-86, these Notices 11 (1964), 553, for definitions and terminology.) Let F be the linear subspace of E which is pointwise invariant under Γ. Let e be the dimension of F. Suppose Y ⊆ F ∩ intΓ con X and the cardinality of set Y is j ≥ 2. Then there exists a subset U of X such that Y ⊆ intΓ con U and the cardinality of U is at most max(ej,2d) if 0 ≤ d < e, at most (e + d - 1)j if e ≤ d < n, and at most ej + (n - e) if d = n. In certain cases, stronger but more complicated upper bounds are given for the cardinality of U. This extends a result of Fenchel (see reference) which considered the special case of j = 1. (Received September 28, 1964.)


Reichelderfer has developed a transformation theory for a function T whose domain is a measure space (S,M,u) and whose range is a measure space (S',M',u') (see Abstract 61T-267, these Notices 8 (1961), 518). A subfamily D of M whose members are denoted by D plays a key role in the theory. A weight function W' is a non-negative extended real-valued function defined for each D and each s' in S' which is inner continuous and under additive, which vanishes when s' is not in TD, and for which each function W'(*,D) is M'-measurable. Let S be generic for those sets each of which is the intersection of all D's containing some point. If w is a non-negative extended real-valued function defined for each S, let W(s',D) be the sum of the terms wS for which S ⊆ D and TS = s'. Assume that B is a σ-field such that D ⊆ B ⊆ M, TB ⊆ M', and T−1 B' ⊆ B. It is shown under certain standard hypotheses that if the point function on S induced by w is B-measurable then W* is a weight function for T. Moreover if W* is a weight function for T then w can effectively be taken so that its induced point function is B-measurable. Under certain conditions it is shown that if W* is a weight function then W*(*,E) is defined and M'-measurable for every E in a σ-field E with D ⊆ E ⊆ M. (Received September 28, 1964.)

(1) For each (convex) polyhedron $P$, let $\lambda(P)$ [resp. $\kappa(P)$] denote the length of the longest simple path [resp. circuit] formed from successively adjacent vertices of $P$. Various results are obtained concerning the maxima of $\lambda(P)$ and $\kappa(P)$ as $P$ ranges over certain important classes of polytopes (those of given dimension and given number of vertices or facets). An important tool is the fact that $P$ admits a Hamiltonian circuit if $P$ is a cyclic polytope or is combinatorially dual to such a polytope. (2) A path $(x_0, x_1, \ldots, x_k)$ in $P$ is called a $W_v$-path provided whenever $1 < j < k$ and a facet of $P$ includes both $x_i$ and $x_k$, then $P$ includes $x_j$ also. A conjecture of Philip Wolfe and the author is that any two vertices of $P$ can be joined by a $W_v$ path in $P$. This implies a well-known conjecture of Warren Hirsch, for if $P$ is a $d$-polyhedron having $n$ facets, then no $W_v$ path in $P$ can have length $> n - d$. The Wolfe-Klee conjecture is proved for $3$-polyhedra, and a similar result is established for certain restricted classes of cell-complexes. (Received September 28, 1964.)

616-17. EMMA LEHMER, 1180 Miller Avenue, Berkeley 8, California. Artiads characterized.

Lloyd Tanner noted in 1886 that there are two distinct classes of primes $p = 5k + 1$ with respect to the decomposition of $p$ into the so-called reciprocal factors in the field generated by $\epsilon = \exp(2\pi i / 5)$, namely $p = (q_1 + q_2 \epsilon^2 + q_3 \epsilon^3 + q_4 \epsilon^4)$, where $q_1 = \pm 1$. About 20% of such primes $p$, whose coefficients $q_i$ are all congruent modulo 5 were called "artiads," while the remaining primes whose coefficients are all incongruent modulo 5 were called "perissads." This implies that $w = (q_1 - q_2 - q_3 + q_4)/5$ is a multiple of 5 if and only if $16p = x^2 + 50u^2 + 50v^2 + 125w^2$ (with $xw = v^2 - u^2 - 4uv$) is an artiad. It is proved here that the Fibonacci root $\theta = (1 \pm \sqrt{5})/2$ is a quintic residue of $p = 5k + 1$ if and only if $w$ is divisible by 5, or in other words if and only if $p$ is an artiad. This explains the observed 20% density of artiads. Similar criteria hold for the quadratic and cubic character of $\theta$ in terms of the partitions of $p = a^2 + b^2$ and $p = x^2 + 15y^2$. From this it follows that the $k$th term of the Fibonacci series $U_0 = 0, U_1 = 1, U_{n+1} = U_n + U_{n-1}$ is divisible by $p$ if and only if $p$ is an artiad. (Received September 29, 1964.)


If the points of the phase space of a thermodynamic system are related by conditions of accessibility and inaccessibility, it can be shown that there is a real-valued function, increase in which corresponds to accessibility, provided the accessibility relation satisfies continuity conditions. Conditions in order that equilibrium of systems leads to a definition of empirical temperature and for the existence of an entropy function will be discussed. (Received September 30, 1964.)


Let $X$ be in a certain class of spaces (which includes manifolds) and let $s : X \to G(X)$ be a cross-section for $X$ in its group of homeomorphisms. Then $X$ has been called suitable; it is equi-
valent to say $G(X)$ is a product bundle over $X$. **Theorem.** Every covering space of $X$ has a unique (modulo translations) suitable structure with which the covering map is a suitable morphism.

**Theorem.** Let $Y$ be a suitable space and let $D$ be a subspace satisfying certain normality conditions. Then the orbit space $Y/D$ is well defined; if the quotient map is a bundle map then there is a unique suitable structure on $Y/D$ such that the quotient map is a suitable morphism. **Corollary.** Similar statements hold for topological loops. **Example.** Letting $Y = S^7$ and $D = \{\pm 1\}$, one obtains a topological loop structure for $P^7$. (Received September 30, 1964.)

616-20. TAKAYUKI TAMURA, University of California, Davis, California. **On a theorem concerning the greatest $s$-decomposition of a semigroup.**

Let $\rho$ be a congruence relation on a semigroup $S$ such that $S/\rho$ is a semilattice. The decomposition of $S$ due to such $\rho$ is called an $s$-decomposition of $S$. It is well known that there is the greatest $s$-decomposition of any semigroup. In the author's earlier paper [Osaka Math. J. 8 (1956), 243-261], he proved a theorem: In the greatest $s$-decomposition of a semigroup $S$, each congruence class $S_\alpha$ is $s$-indecomposable. Let $F_0$ be the free semigroup generated by $n$ distinct letters $a_1,\ldots,a_n$; and $F$ be the subsemigroup of $F_0$ which is composed of all words including all of $a_1,\ldots,a_n$. Using the above theorem, it can be shown that $F$ is $s$-indecomposable. In this paper, the author proves directly that $F$ is $s$-indecomposable, and then proves the above theorem by using it. (Received September 30, 1964.)
617-1. W. W. LEONARD, University of South Carolina, Columbia, South Carolina. Some results on small modules. Preliminary report.

Definition. A submodule $E$ of a left $A$-module $F$ is said to be small in $F$ if $E + E' = F$ for any submodule $E'$ of $F$ implies $E' = F$. Definition. A left $A$-module $E$ is said to be a small module if $E$ is a small submodule of some module. Theorem. A left $A$-module $F$ is small if and only if $F$ is small in its injective envelope. A finite direct sum of small modules is a small module, the quotient of a small module is a small module, and an infinite direct sum and product of small modules is not necessarily a small module. Theorem. If $A$ is a left hereditary ring then a module $F$ is small if and only if $F$ has no nontrivial injective quotients. Theorem. An abelian group $G$ is small if and only if (i) the primary parts of $T(G)$ (torsion subgroup) are bounded and (ii) $\dim \left[ \frac{G/T(G)}{p(G(T(G))_p} \right] = \text{rk}(G/T(G))_p$ for every prime $p$. Theorem. An abelian group $G$ is small if and only if every subgroup of $G$ is contained in a maximal subgroup of $G$. (Received July 10, 1964.)

617-2. R. J. WARNE, West Virginia University, Morgantown, West Virginia. Regular $D$-classes whose idempotents obey certain conditions, I.

For definitions and notations, see Notices Amer. Math. Soc. (1964) Abstract 609-11. Let $S$ be a semigroup, $S_R^*$ denote the set of right regular elements of $S$ contained in the $R$-class $R$ and $S_R^* = S_R - \bigcup \{ H_e : e \in E_R \}$. Several sets of equivalent conditions on a $D$-class of $S$ are given: For example, (A) $D$ is regular, $E_D^2 \subseteq D$, and $E_D$ is directed from above. (B) $D$ is a regular bisimple subsemigroup of $S$ for which $E_D$ is directed from above. (C) $D$ is the union of a set of regular bisimple subsemigroups of $S$ with identity, the set being directed from above with respect to inclusion. If $D$ is regular, (A') $E_D$ is a chain of rectangular bands, (B') if $e, f \in E_D$, $e f = e$ or $f e = f$. (C') $E_D$ is a band and $D$ is a disjoint union of subsemigroups each of which is contained in an $R$-class or an $L$-class of $D$. (A'') $D$ is regular and $E_D$ is componentwise ordered (if $e, f \in E_D$, then $e \preceq f$, $f \preceq e$ or $R_e \cap L_f$ and $R_f \cap L_e$ are groups). (B'') $D$ is a biregular, bisimple componentwise inversive semigroup. (C'') $D$ may be expressed as $D = \bigcup \{ e De : e \in E_D \}$ where $e De$ is a biregular, bisimple, componentwise inversive semigroup with identity $e$ such that $e, f \in E_D$ imply $e De \subseteq f De$ or $f De \subseteq e De$ or $R_e \cap L_f$ and $R_f \cap L_e$ are groups. (D'') $D$ is a completely simple semigroup or $E_D$ is componentwise commutative and $D$ has the following decomposition into groups, right cancellative semigroups without idempotent, and left cancellative semigroups without idempotent: $D = \bigcup \{ H_e : e \in E_D \} \cup (\bigcup S_R^*: R \in \mathscr{R}) \cup (\bigcup S_L^*: L \in \mathscr{L})$. (Received August 3, 1964.)


A semigroup is a nonvoid Hausdorff space together with a continuous associative multiplication (denoted by juxtaposition) and in what follows $S$ will denote one such, for convenience of exposition
assumed compact. If \( A, B \subseteq S\), \( A^{[-1]}B \) will denote all of those members \( x \) of \( S \) such that \( Ax \subseteq B \).

For a fixed compact subset \( T \) of \( S \) one is assured of the existence of closed sets \( A \) maximal relative to \( A^{[-1]}T \) being nonvoid, and these are characterized in various ways. Such sets \( A^{[-1]}T \) form a disjointed collection with useful properties. Somewhat analogous results are obtained for those closed sets \( A \) minimal relative to the nonvacuity of \( T^{[-1]}A \). (Received September 3, 1964.)

617-4. PAUL ERDŐS, Nemetvolgyi Ut 72c, Budapest II, Hungary, A. W. GOODMAN, University of Southern Florida, Tampa, Florida and LADISLAS POSA, Michael Fazekas High School, Budapest, Hungary. The representation of a graph by set intersections.

Let \( G^{(n)} \) be a graph with \( n \) vertices. With each vertex \( x_a \) associate a set \( S_a \) such that the vertices \( x_a \) and \( x_{\beta} \) are joined by an edge in \( G^{(n)} \) if and only if \( S_a \cap S_{\beta} \) is not empty. It is well known that this can always be done. What is the minimum number of elements necessary for the set \( \bigcup_{a=1}^{n} S_a \) in order that every \( G^{(n)} \) can be represented by such a collection of sets. The authors prove that the minimum is \( \lceil n^2/4 \rceil \). The problem is related to the representation of a graph as a sum of complete graphs, and several theorems of this nature are proved. (Received September 10, 1964.)

617-5. JO FORD, 121 Cherokee Avenue, Athens, Georgia. Imbedding collections of compact totally disconnected subsets of \( \mathbb{E}^2 \) in continuous collections of mutually exclusive arcs.

If \( G \) and \( H \) are collections of sets, then \( G \) is said to be imbedded in \( H \) if there is a 1-1 correspondence \( \phi \) from \( G \) onto \( H \) such that \( g \subseteq \phi(g) \) for each \( g \) in \( G \). M.-E. Hamstrom [Proc. Amer. Math. Soc. 4 (1953), 240-243] investigated the imbedding of upper semicontinuous collections of continuous curves in continuous collections of continuous curves. Suppose \( G \) is an upper semicontinuous collection of mutually exclusive compact, totally disconnected subsets of \( \mathbb{E}^2 \). It is shown that if \( G^* \) is compact and totally disconnected, and \( G \) with respect to its elements is a subset of an arc, then there is a continuous and equicontinuous collection of mutually exclusive arcs in \( \mathbb{E}^2 \) in which \( G \) is imbedded. If \( G \) is continuous and is an arc with respect to its elements, then a sufficient condition is found for the existence of a continuous collection of mutually exclusive arcs in \( \mathbb{E}^3 \) in which \( G \) is imbedded that is satisfied, in particular, if each element of \( G \) is finite; and a sufficient and almost necessary condition is found for the existence of a continuous and equicontinuous collection of mutually exclusive arcs in \( \mathbb{E}^3 \) in which \( G \) is imbedded. (Received September 16, 1964.)


Let \( R \) be a binary relation on \( X \). A set \( C \subseteq X \) is called an \( R \)-chain iff \((C \times C) \setminus \Delta \subseteq R \cup R^{-1} \), where \( \Delta \) is the diagonal of \( X \times X \). If \( R \) and \( S \) are binary relations on \( X \), then \( R \) and \( S \) are said to be chain-equivalent iff \((R \cup R^{-1}) \setminus \Delta = (S \cup S^{-1}) \setminus \Delta \). This is equivalent to the assertion that the family of maximal \( R \)-chains and the family of maximal \( S \)-chains are identical. Theorem. If \( Q \) is a quasi-order on \( X \) and \( P \) is a partial order contained in \( Q \), then \( P \) and \( Q \) are chain-equivalent iff \( P \) is a maximal partial order in \( Q \). A consequence of this result is the equivalence of Dilworth's decomposition theorem for partially ordered sets (Ann. of Math. (2) 51 (1950), 161-166) and the same proposition with partial order replaced by quasi-order. (Received September 17, 1964.)
Let $X$ and $Y$ be topological spaces and $T \subseteq X \times Y$ a relation such that $T(x) = \{ y \in Y | (x,y) \in T \} \neq \emptyset$. $T$ is open iff it is an open subset of $X \times Y$ and image-open iff for each $x \in X$, $T(x)$ is open in $Y$. $T$ is upper- (lower)-semicontinuous iff for each closed (open) subset $F(U)$ of $Y$, $T^{-1}(F) = \{ x | T(x) \cap F \neq \emptyset \}$ ($T^{-1}(U)$) is closed (open) in $X$. Prop. 1: If $T$ is open and upper-semicontinuous, then $T$ is constant on each component of $X$. Define a new relation $T' \subseteq X \times Y$ by $T'(x) = \overline{T(x)}$. Prop. 2: If $T$ is upper- and lower-semicontinuous and image-open, then $T'$ is constant on each component of $X$. A nonempty open subset $A$ of $X$ is a neighborhood of constancy of $T$ iff $T(x) = T(x')$ for all $x, x' \in A$. Prop. 3: If $X$ is locally countably compact and regular, and $T$ is image-open and upper-semicontinuous, then $X = E \cup F$ where $E$ is a union of neighborhoods of constancy of $T$ and $F$ is nowhere dense in $X$. (Details to appear in Colloquium Mathematicum 12 (1964).)

Products of permutation groups and their graphs.

We say that a permutation group $P$ has a directed (undirected) graph $X$ if $P = G(X)$, the group of automorphisms of $X$. Theorem 1: If $P_1, P_2, ..., P_k$ are permutation groups with pairwise disjoint support sets then $P_1P_2...P_k$ has a directed (undirected) graph if and only if $P_i$ has a directed (undirected) graph for $i = 1, 2, ..., k$. Theorem 2: If $P_1$ and $P_2$ are transitive permutation groups on disjoint support sets and $P_i$ has a fixed point free undirected graph for $i = 1, 2$, then $P_1P_2$ has a fixed point free undirected graph if and only if $P_1$ and $P_2$ are isomorphic permutation groups and all of their fixed point free undirected graphs are isomorphic. Theorem 2 is not vacuous since the dihedral group of order 10 is a permutation group which has all of its fixed point free undirected graphs isomorphic. (Received September 25, 1964.)

On harmonic matrices.

Let $F = (f_{ij})$ denote a $k \times k$ matrix of continuous complex-valued functions on an interval $[a,b]$ such that $f_{ij}(t) = 0$ for $t \in [a,b]$ if $i + j$ is even. Let $H = (H_{k+j-1}^{k-i+1})$ and let $M = (m_{ij})$ and $N = (n_{ij})$ be the functions such that $M(t,u) = I + \int_u^t H(s)N(s,\bar{u})ds$ respectively. Theorem A. $m_{ij}(t,u) = (-1)^{i+j}k^k H_{k+j-1}(u,t)$ for $t, u \in [a,b], i, j = 1, ..., k$. Theorem B. $m_{ij}(t,u) = \text{conjugate of the minor of } n_{k+j-1}^{k-i+1}(t,\bar{u})$ in $N(t,\bar{u})$ for $t, u \in [a,b], i, j = 1, ..., k$. Theorem C. $L = (\delta_{i,k+1-j})$ and $N^*(t,u) = (n_{ij}(t,u))$ for $t, u \in [a,b]$. The determinant of any sub-matrix of order $r$, $1 \leq r < k$ of $M(t,u)$ is equal to the determinant of its complementary submatrix (of order $k - r$) in $N^*(t,u)$ for any $t, u \in [a,b]$. (Received September 25, 1964.)

A pseudocircle is a nondegenerate continuum which is disconnected by the omission of any two of its points. R is a relation from X to Y iff R is the graph of a multi-function from X to Y: precisely, \( R \subseteq X \times Y \) and for \( A \subseteq X \) and \( B \subseteq Y \), \( R = \{ (a,b) \in R \mid a \in A \text{ and } b \in B \} \). **Theorem.** Let X be a pseudocircle, let Y be Hausdorff and let R be a relation from X to Y such that (i) \( R \) is upper semicontinuous, (ii) \( xR \) is compact and connected for each \( x \in X \), (iii) \( R \) is monotone (\( Ry \) is connected for each \( y \in Y \)), (iv) \( R \) is non-inclusive (\( y \neq y' \) in \( XR \) implies \( Ry \nsubseteq Ry' \)), (v) there are \( x_1, x_2 \in X \) such that \( x_1R \) is a point, \( x_2R \) is a point, and \( x_1R \neq x_2R \). Then \( XR \) is a pseudocircle and if X is metric then \( XR \) is also metric. This generalizes the well-known fact that if X is a simple closed curve, Y is Hausdorff and \( f: X \to Y \) is monotone and continuous, then \( f(X) \) is a simple closed curve. (Received September 24, 1964.)

617-11. ROBERT PLEMMONS, Auburn University, Auburn, Alabama. Semigroups with a maximal semigroup with zero homomorphic image.

A semigroup \( S \) has a maximal semigroup with zero homomorphic image if there exists a semigroup \( S' \) with zero, such that \( S' \) is a homomorphic image of \( S \) and any other semigroup with zero homomorphic image of \( S \) is also a homomorphic image of \( S' \). An ideal \( T \) of a semigroup \( S \) is a simple ideal of \( S \) if \( T \) does not properly contain an ideal of itself. **Theorem.** If a semigroup \( S \) has a simple ideal \( T \) then \( S \) has a maximal semigroup with zero homomorphic image, \( S/T \). **Theorem.** Let \( S \) be a semigroup with a maximal semigroup with zero homomorphic image \( S' \). Let \( \theta \) be the zero of \( S' \), let \( \theta \) be a homomorphism of \( S \) onto \( S' \) and let \( T = 0'\theta^{-1} = \{ a \in S | a\theta = 0' \} \). If \( T \) is not the simple ideal of \( S \), then \( S \) does not satisfy the ascending chain condition for ideals; specifically, there exists an infinite properly ascending chain of ideals of \( S \) such that \( S/J_n \) is a maximal semigroup with zero homomorphic image of \( S \) for each positive integer \( n \). (Received September 23, 1964.)


Suppose (1) \( p \) is in \( E_2 \), \( S_n \) is the set of all real symmetric \( n \)-linear forms on \( E_2 \) and \( c_n \) is in \( S_n \), \( n = 0,1,..., k \), (2) \( f \) is a function from a subset of \( S_k \times S_{k-1} \times \cdots \times S_0 \times E_2 \) to a number set which (a) is analytic at \( (c_k,...,c_0)p) = M \), (b) is such that \( f(M) = 0 \), and (c) is such that \( f_1(M) \neq 0 \) the null transformation on \( S_k \), (f_1, first place partial Fréchet derivative), (3) \( A \) is the set of all real analytic functions \( u \) such that (a) \( u(i)(p) = c_i \cdot (u(i), ith Fréchet derivative), i = 0,1,..., k \), (b) the domain of \( u \) is a connected open set containing \( p \) and (c) the domain of \( u \) is maximal with respect to (a) and (b), (4) \( B \) is the set of all \( u \) in \( A \) such that \( f(u(k)(q),..., u(0)(q),q) = 0 \) for all \( q \) in some open subset of \( E_2 \) which contains \( p \). **Theorem.** There is a transformation \( T \) from \( A \) onto \( B \) such that (i) \( T^2 = T \), (ii) \( T \) is continuous in the sense that if \( u_1, u_2,... \) is a sequence of elements of \( A \) converging analytically to the element \( u \) of \( A \), then \( Tu_1, Tu_2,... \) converges analytically to \( Tu \left( v_1,v_2,..., \text{ in } A \text{ converges analytically to } v \text{ in } A' \right) \text{ means } \sum_{n=0}^{\infty} (1/n!) \left| (v_1 - v)(x - p) \right| \to 0 \text{ as } x \to \infty \text{ uniformly for all } x \text{ in some open set containing } p \), and (iii) if \( u \) is in \( B \), then \( T^{-1}u \) is the translation of a linear manifold of analytic functions. (Received September 21, 1964.)
The following results are obtained in this paper. **Theorem 1.** The group of all homeomorphisms of a locally setwise homogeneous continuum (in a technical sense) is nonzero-dimensional. **Corollary 1.1.** The groups of all homeomorphisms of the universal plane curve and the universal curve are nonzero-dimensional. **Corollary (of proof) 1.2.** The group of those homeomorphisms of the n-sphere, $S^n$, $n > 1$, which carry a fixed, countable, dense subset of $S^n$ onto itself is nonzero-dimensional. **Theorem 2.** The group of all homeomorphisms, $G$, of a regular curve, $X$, is either 0- or $\infty$-dimensional. $G$ is $\infty$-dimensional if $X$ contains a free arc; it is 0-dimensional otherwise. **Theorem 3.** For each positive integer, $n$, there exists a continuum $M_n$, such that $G(M_n)$ (the group of all homeomorphisms of $M_n$) is totally disconnected, abelian, and homeomorphic to the product of $n$ 1-dimensional groups. $M_n$ may be constructed as a rational curve. In particular, for $n = 1$, $G(M_1)$ is exactly one-dimensional. (Received September 21, 1964.)

The author solves the difference equation, $\left[y(z + h(z)) - y(z)\right] / h(z) = \ln z$ where certain restrictions are placed on $h(z)$. The particular solution studied is the one which reduces to $\ln I(z)$ in case $h(z) = 1$. A generalization of the Euler-Maclaurin sum formula is made. This gives an existence theorem when $z = x$ is real and also an asymptotic form for the solution. A solution of equation $y(z + h(z)) - y(z) = \ln z$ follows as a corollary. Additional restrictions are placed on $h(z)$. The author points out how the methods employed will solve many linear difference equations of the first order with varying difference interval. (Received September 28, 1964.)

**Centralizers of $H^*$-algebras.**

A mapping $T$ from a Banach algebra $X$ into itself will be called a **centralizer** if $x(Ty) = (Tx)y$ for all $x$ and $y$ in $X$. A bounded linear operator on $X$ will be called a **right (left) centralizer** if $T(xy) = (Tx)y$ ($T(xy) = x(Ty)$). It is shown, in the case that $X$ is an $H^*$-algebra, that the algebra of right (left) centralizers is the $W^*$-algebra generated by the set of left (right) multiplication operators and that the commutant of the right (left) centralizers is the left (right) centralizers. **Theorem.** If $X$ is a commutative $H^*$-algebra, then the right and left centralizers agree with the algebra of centralizers which is isometric $^*$-algebra isomorphic with the space of all bounded functions on the discrete maximal ideal space of $X$. It is then possible to characterize the projection centralizers and the compact centralizers. (Received September 28, 1964.)

**Heights of convex polytopes.**

Though phrased in geometric terms, this report is concerned with the comparative efficiency of various pivot rules for the simplex method of linear programming, in which one seeks to maximize
a linear objective function $\phi$ by moving along the edges of the feasible region. The following three rules are considered: (1) From the vertex $v$, survey the adjacent vertices until a vertex $w$ is found for which $\phi(v) < \phi(w)$; then move to $w$ and continue the process; (2) From the vertex $v$, survey all of the adjacent vertices in order to find one, say $w$, at which $\phi$ has the greatest value; then move to $w$ and continue the process; (3) From the vertex $v$, survey all of the adjacent vertices in order to find one, say $w$, for which the slope $(\phi(w) - \phi(v))/\|w - v\|$ has the greatest value; then move to $w$ and continue the process. In each case, the concern is with the maximum number of moves which may be required when the feasible region is subjected to various restrictions on its structure, its dimension, the number of its vertices, or the number of its facets. The results obtained are complete for the case in which $d \leq 3$, but many unsolved problems remain in the higher-dimensional cases. The paper will appear in the Journal of Mathematical Analysis and Applications. (Received September 28, 1964.)


Denote by $B^p$ the set of all functions $q(z) = \sum a_n z^n$ analytic in $|z| < 1$ which have the property that the Hadamard product $(f \star g)(z) = \sum b_n z^n$ is a bounded analytic function in $|z| < 1$ whenever $f(z) = \sum b_n z^n$ belongs to the Hardy class $H^p$. Theorem. If $1 < p < \infty$ then $B^p = H^q$ where $1/p + 1/q = 1$. The proof is based on a well-known integral representation for $f \circ g$ and involves an application of the bounded projection theorem of M. Riesz. Examples are given to show that the conclusion does not hold when $p = 1$ or $p = \infty$. This result is an extension of the main theorem in a paper by R. A. Whiteman [Duke Math. J. 31 (1964), 321-324]. (Received September 28, 1964.)


The following problems are considered. Determine if a continued fraction expansion for a specified function is equal to that function. Find the rate of convergence of the continued fraction and error bounds for the approximants. Using methods due to R. E. Lane, we obtain the following typical result. Theorem. Suppose that for $n \geq 1$, $f(z) = T_n(w_n) = b_0 + a_1/b_1 + \ldots + a_n/(b_n + w_n)$, and $h_n$ is a number such that $T_n(h_n) = \infty$. Then if $T_n(0) \to L$ as $n \to \infty$, $f(z)$ is $L$ provided some infinite subset of the sequence $\{w_n\}$ is bounded away from the cluster points of the sequence $\{h_n\}$. Some estimates of the rate of convergence are obtained by a combination of value region techniques and contraction mappings. Error bounds are given which depend on the relationship of the numbers $h_n$ to the "remainders" $w_n$. (Received September 28, 1964.)


Necessary and sufficient conditions are given for the projection function on the cartesian product of certain types of spaces to be closed. The following two theorems are proved: Theorem 1. Let $X_1$ and $X_2$ be nondiscrete $T_1$-spaces satisfying the first axiom of countability. Then $X_1$ is sequentially compact if and only if the projection function $\pi_2$, from $X_1 \times X_2$ onto $X_2$, is closed.
Theorem 2. Let $X_1$ and $X_2$ be $T_1$-spaces, each having a countable base, such that neither $X_1$ nor $X_2$ is discrete. Then $X_1$ is compact if and only if the projection function $\pi_2$ is closed. (Received September 28, 1964.)

617-20. R. G. VINSON, P. O. Box 71, Huntingdon College, Montgomery, Alabama. Euclidean Geometry via intervals.

Intervals provide an approach to Euclidean Geometry that has some definite advantages over the usual axiomatic development using lines as undefined concepts. If we begin our development with point as an undefined concept and then define intervals as sets of points having certain desired properties, we can derive all the needed and usual properties of lines. To do this, we consider the concepts of order and continuity as basic ones. Then we postulate the existence of intervals as being closed, ordered, continuous sets of points. Restrictions are placed on these sets of points so that two points determine a unique interval. Lines are then defined in terms of intervals. We also are able to derive the properties of angle measure and distance in terms of intervals, once we have developed a congruence relation on intervals. Thus, the metric properties of the geometry are all derived synthetically from the basic concept of interval. (Received September 28, 1964.)


Theorem. If $M$ is a circle-like continuum and there exists a sequence $D_1, D_2, D_3, \ldots$ of circular chains defining $M$ such that, for each positive integer $n$, $D_{n+1}$ circles in $D_n$ zero times, then $M$ is chainable. Theorem. If $M$ is a circle-like continuum which cannot be embedded in the plane, there exists a continuous transformation throwing $M$ onto a solenoid which cannot be embedded in the plane. Since each solenoid which cannot be embedded in the plane is not a continuous image of any plane continuum [M. C. McCord, Inverse limit systems, Doctoral Dissertation, Yale University, 1963; M. K. Fort, Images of plane continua, Amer. J. Math. 81 (1959), 541-546], each circle-like continuum which cannot be embedded in the plane is not a continuous image of any plane continuum. (Received September 28, 1964.)

617-22. R. O. FULP, Georgia State College, Atlanta, Georgia and PAUL HILL, Emory University, Atlanta, Georgia. On homomorphisms and characters of inverse semigroups.

Suppose that $S$ and $T$ are commutative inverse semigroups. Let $E$ be the idempotents of $S$ and let $F$ be the idempotents of $T$. Theorem 1. $\text{Hom}(S, T)$ is the union of groups $G_\pi$ where $\pi$ ranges over the semilattice $\text{Hom}(E, F)$. Moreover, $G_\pi$ is a limit of the collection of groups $\text{Hom}(S(e), T(\pi(e'))) \quad e \geq e'$ and $S(e)$ is the maximal subgroup of $S$ containing the idempotent $e$. Theorem 2. $S^* = \bigcup_{\pi \in E^*} \lim_{\pi(e) \neq 0} S^*_e \quad$ where $S^*$ is the character semigroup of $S$ and $S^*_e$ is the character group of $S^*_e$, the maximal subgroup of $S$ containing $e$. (Received September 28, 1964.)
A criterion for cellularity at the boundary (CAB) of a manifold.

The concept of CAB was defined in (Abstract 64T-269, these Notices 11 (1964), 446).

**Theorem.** Let X be a compact subset of a p.w.1. n-manifold M^n, n > 5, such that X and X ∩ Bd(M^n) = Y are absolute retracts. Then X is CAB of M^n if for each open set U of M^n containing X, there is an open set V of M^n such that X ⊂ V ⊂ U and: (1) each loop in V - X is homotopic in U - X to a loop in Bd(M^n) and (2) each loop in (V - X) ∩ Bd(M^n) is nullhomotopic in (U - X) ∩ Bd(M^n). The theorem holds for n = 5 replacing (2) by (2') requiring Y to be a cellular subset of Bd(M^n). A modified version also holds for 3-manifolds. Theorems concerning products are obtained and with some restrictions on dimensions, a subarc of a CAB arc is either CAB or cellular in the interior of the manifold.

(Received September 29, 1964.)

**617-24.** TREVOR EVANS, Emory University, Atlanta, Georgia 30322. Identities, finite embeddability and residual finiteness.

Some connections are described between residual properties of algebras in a variety, identities satisfied by the variety and finite embeddability of incomplete algebras. It is proved that the free algebras in a variety are residually finite if and only if any identity satisfied by the finite algebras in the variety is satisfied by all algebras in the variety. It is also shown that finite incomplete algebras in a variety which are embeddable are finitely embeddable if and only if the finitely presented algebras in the variety are residually finite. There are applications to many of the standard varieties of algebras. (Received September 29, 1964.)

**617-25.** J. S. BRADLEY, University of Tennessee, Knoxville, Tennessee. A canonical form for a boundary value problem involving a quasi-differential operator of Euler type.

Reid (Trans. Amer. Math. Soc. 85 (1957), 446-461) has given a definition for adjoint linear differential operators and characterized the domain and form of the adjoint operator. These results are extended to quasi-differential operators of Euler type. Using generalized derivatives to replace higher order derivatives it is shown that for a fairly large class of two-point boundary problems in which the boundary conditions involve the characteristic parameter linearly there exists a simultaneous canonical representation of the boundary conditions for a given problem and those of its adjoint. In particular, in the self-adjoint case this canonical representation has the form of boundary conditions and transversality conditions for a variational problem. (Received September 29, 1964.)

**617-26.** M. Z. NASHED, School of Mathematics, Georgia Institute of Technology, Atlanta, Georgia 30332. Existence and construction of solutions for a class of problems in linear controls.

Let S denote a given closed convex set in an Euclidean m-space E_m; C a closed convex set of (control) functions f(t) in H, the L_2-space of p-dimensional functions on [0, T]; and A a bounded linear operator from H to E_m. It is shown that there is an element u_0 of minimal norm in C and an x_0 in S such that ∥Au_0 - x_0∥ = inf ∥Au - x∥, where the infimum is taken over u in C and x in S. The elements x_0 and Au_0 are shown to be fixed points of certain operators; constructive algorithms for x_0 and u_0.
are also given. The setting where \( S \) is simply a fixed point in \( E_m \) was studied by A. V. Balakrishnan [J. SIAM Control 1 (1963), 109-127]. This setting contains an abstract formulation of some final value and mean square minimization problems in control theory. (Received September 30, 1964.)


For each function \( f \) in \( C[0,1] \), the class of continuous functions from \([0,1]\) to the complex plane, and each complex sequence \( c \), \( L(f,c) = \sum_{p=0}^{n} C_{n,p} \sum_{q=0}^{p} (-1)^q C_{n-p,q} c_{p+q} f(p/n) \). It is known [J. S. MacNerney, Hermitian moment sequences, Trans. Amer. Math. Soc. 103 (1962), 45-81] that if \( f \) is a polynomial and \( c \) is a complex sequence then \( L(f,c) \) converges. Also [J. S. MacNerney, Characterization of regular Hausdorff moment sequences, Proc. Amer. Math. Soc. 15 (1964), 366-368], a necessary and sufficient condition that the complex sequence \( c \) be a moment sequence of a function of bounded variation on \([0,1]\) is that \( L(f,c) \) converge for each \( f \) in \( C[0,1] \). \textbf{Theorem.} A necessary and sufficient condition that the complex sequence \( c \) be bounded is that, for each analytic \( f \) on the unit disc such that \( \sum_{p=0}^{\infty} |f(p)/(0)/p!| \) converges, \( L(f,c) \) converge. \textbf{Theorem.} If \( f \) is in \( C[0,1] \), then a necessary and sufficient condition that \( f \) be entire is that \( L(f,c) \) converges for each complex sequence \( c \) which is dominated by a geometric sequence. \textbf{Theorem.} If \( f \) is in \( C[0,1] \) and \( L(f,c) \) converges for each complex sequence \( c \), then \( f \) is a polynomial. (Received September 30, 1964.)


There exist at least three sequences, each of type \( \Omega \), the elements of which are compact totally ordered sets ("cors"), such that in the interval topology no two cors from different sequences and no two different cors from the same sequence are homeomorphic, whereas all occurring cors are homogeneous. (Received September 25, 1964.)

617-29. T. D. PHILLIPS, Emory University, Atlanta, Georgia 30322, Finite wreath product characters and representations.

The finite wreath product, \( W = A \ltimes B \), of two permutation groups, \( A \) and \( B \), is defined in the usual way. It is shown that the general term of \( W \) belongs to a class which corresponds to the partition \( \Pi_1 [(\sigma_1^0 \alpha_1^{0} (2 \sigma_2^0) \beta_2^{0} ...) \), where \( \rho_x = (1 \alpha_1 \beta_1 ... ) \), and \( \rho_x, \sigma_x \) are classes to which belong permutations in \( A \) and \( B \), respectively. The order of this class is given, and a method for giving the total number of classes of \( W \) is given for a few special cases. Let \( \chi^{(r)}(x), \chi^{(r)}(x) \) be the characteristics of group elements in \( A, B \). Then the character of the \( W \) is given by \( \sum_{r} [\chi^{(r)}(x) \Pi_{(r)} \chi^{(r)}(x) \rho_x] \) where the product \( \Pi_{(r)} \chi^{(r)}(x) \rho_x \) is taken over the number of possible separations of \( \sigma_x, \rho_x \), being equivalent to partitions corresponding to the cycles in \( \sigma_x \). It is summed for all possible separations of \((\sigma_1, \sigma_2, ..., \sigma_i)\) of partition \( \sigma \) and for all possible partitions \( \rho_x \). The method of obtaining the matrix representation is demonstrated by example. (Received September 25, 1964.)
617-30. M. P. NEPP and TREVOR EVANS, Emory University, Atlanta, Georgia 30322. **Substitution algebras and near-rings, I.**

A substitution algebra \((S, [\_])\) is a set \(S\) and an \((n + 1)\)-ary operation \([\_]\), which is an abstraction from a set of functions of \(n\) variables closed under composition. Some general results are obtained on the structure of such algebras and also on the structure of substitution algebras \((S, [\_], +)\) having an additional binary operation \(+\) such that \([a, x_1, \ldots, x_n] + [b, x_1, \ldots, x_n] = [a + b, x_1, \ldots, x_n]\). If the projection elements for \([\_]\) generate \((S, +)\) then \((S, +)\) is free in some variety. If \((S, +)\) is a group, a multiplication may be defined on \(S^n\) such that with the natural addition \((S^n, +, \cdot)\) is a near-ring. If \((G, +)\) is a group (or loop), various substitution algebras \((S, , +)\) of functions on \((G, +)\) give rise to near-rings of "generalized matrices." Results on the simplicity of these matrix near-rings are obtained. A class of near-rings generalizing linear algebras is studied and an embedding theorem proved. (Received September 30, 1964.)

617-31. C. L. WIGINTON, 1317 Wilshire Road, Knoxville, Tennessee 37919. **Factoring pointlike simplicial mappings of the 3-sphere.**

Ross Finney, in his paper *Pointlike, simplicial mappings of a 3-sphere* (Canad. J. Math. Vol. 15, pp. 591-604) asked the question: "Can every orientation-preserving, pointlike, simplicial mapping of the 3-sphere be factored into a product of simplicial mappings of the sphere onto itself each of which identifies exactly two vertices, those bounding a 1-simplex?" A triangulation of the 3-sphere and a pointlike, simplicial mapping of the 3-sphere onto itself is described which cannot be factored as indicated. (Received September 30, 1964.)

617-32. GERHARD GRIMEISEN, 1508 NW 4th Avenue, Gainesville, Florida. **The product of cluster spaces.**

For each set \(M\) let \(\Phi M\) be the class of all filters in \(M\), i.e., of all filters on subsets of \(M\). A pair \((E, c)\) of a set \(E\) and a map \(c\) on \(\Phi E\) into the class of all subsets of \(E\) is said to be a cluster space, if \(c(F) = \bigcap_{F \in S} \bigcup_{G \in \Phi C(F)} \text{for all } F \in \Phi E\). If, e.g., \((E, \tau)\) is a topological space (with the topology \(\tau\)) and \(c(F)\) means the set of all cluster points of the filter \(F\) in \(E\), then \((E, c)\) is a cluster space. The theory of cluster spaces is closely related to that of limit spaces, and both theories can be imbedded logically into a theory of "generalized cluster spaces." If \((E_d, c_d)\) is a family of cluster spaces, their product is defined to be the pair \(\left(\prod_{d \in D} E_d, \prod_{d \in D} c_d\right)\), where \(\prod_{d \in D} E_d, c_d\) is introduced by \(\prod_{d \in D} E_d\) meaning the Cartesian product of sets \(E_d\) \((d \in D))\) and \(\prod_{d \in D} c_d\) means the dth projection operator, \(\prod_{d \in D} c_d\) the Cartesian product of sets \(M_d\) \((d \in D))\). The theory of Bourbaki's product topology is contained in a theory of the "product of generalized cluster spaces." (Received September 30, 1964.)

617-33. JAN MIKUSINSKI, University of Florida, Gainesville, Florida. **Remarks on the theory of integration.**

Let \(U\) be a set of functions \(f\) from \(R^d\) to \(R\) such that \(f \in U\) implies \(|f| \in U\), and let \(\int f\) be a real-valued functional defined on \(U\). A function \(f\) is called a simple function iff (1) \(f\) vanishes outside an
open and bounded interval \( J (a_i < \xi_i < b_i, i = 1,\ldots,q) \) and its value in \( J \) is a constant real number \( c \). The relation \( f \approx \sum f_n \ (f_n \in U) \) means that \( \sum |f_n| < \infty \) and \( f(x) = \sum f_n(x) \) wherever the last series converges. One assumes that (B) condition (1) implies \( f \in U \) and \( \int f = c \cdot \text{Vol} \ J \ (\text{Vol} \ J = \prod (b_i - a_i)) \), and that (E) the relation \( f \approx \sum f_n \ (f_n \in U) \) implies \( f \in U \) and \( \int f = \sum \int f_n \). **Theorem.** The smallest set \( U \) satisfying (B) and (E) is \( L(\mathbb{R}^q) \); the functional \( \int f \) is then the Lebesgue integral. **Corollary.** \( f \in F(\mathbb{R}^q) \) iff \( f \approx f_n \), where \( f_n \) are simple functions; the equality \( \int f = \sum \int f_n \) defines the Lebesgue integral. This result can be generalized: Let \( U \) be a set of functions \( f \) from an arbitrary set \( K \) to \( \mathbb{R} \) and let \( \int f \) be a real-valued functional on \( U \). One assumes that: (H) \( f \in U \) implies \( \lambda f \in U \ (\lambda \text{ real number}) \) and \( \int \lambda f = \lambda \int f \); (M) \( f \in U \) implies \( |f| \in U \) and \( |\int f| \leq \int |f| \). One assumes also (E). Let \( F \) be the family of sets \( S \) whose characteristic functions \( \chi_S \) belong to \( U \) and let \( \mu(S) = \int \chi_S \). **Theorem.** \( F \) is a \( \sigma \)-algebra and \( \mu(S) \) is a countably additive measure on \( F \). The integral induced by the measure \( \mu(S) \) is identical with \( \int f \). (Received September 30, 1964.)

617-34. R. D. ANDERSON, Louisiana State University, Baton Rouge, Louisiana. On a theorem of Klee.

For any \( p \geq 1 \) and any integer \( i \), let \( \pi_i \) be the projection of the space \( \ell_p \) onto the \( i \)th coordinate space of \( \ell_p \) and let \( \tau_i \) be the projection onto the product of the first \( i \) coordinate spaces. A set \( K \subset \ell_p \) is said to be projectible provided (1) \( K \) is closed, (2) for \( q \in \ell_p \setminus K \), there exists \( n \) such that \( \tau_n(q) \notin \tau_n(K) \), and (3) for infinitely many \( i \), \( \tau_i(K) \) is either bounded below or bounded above. The class of projectible subsets of \( \ell_p \) includes all compact sets, all weakly compact sets, and many unbounded sets. **Theorem.** For any projectible subset \( K \) of \( \ell_p \), \( \ell_p \setminus K \) is homeomorphic to \( \ell_p \). This theorem is a variant of a theorem of V. L. Klee [Convex bodies and periodic homeomorphisms in Hilbert space, Trans. Amer. Math. Soc. 74 (1953), 10-43]. In contrast to Klee's original proof which uses apparatus from nonreflexive Banach spaces, the short proof of the present theorem uses only the definitions and elementary set-theoretic properties. (Received September 30, 1964.)
In a complex inner product space \( V \), the complex inner product has a representation \( (a, b) = i e^{i \theta} |a||b| \cos \phi \), where \( |a| > 0, |b| > 0 \), are the norms of the two vectors \( a \) and \( b \), \( \phi \), with \( 0 \leq \phi \leq \pi/2 \), is the isocline angle between the two vectors \( a \) and \( b \), and \( \theta \), with \( -\pi < \theta \leq \pi \), is the pseudo-angle between the two vectors \( a \) and \( b \). The isocline angle \( \phi \) is the angle between the vector \( a \) and its completely orthogonal projection on the rectilinear isocline determined by the vector \( b \). The pseudo-angle \( \theta \) is the angle between the vector \( a \) and the vector subspace \( V' \) of deficiency one of the vector space \( V \) determined by all vectors \( x \) of \( V \) which are ordinary orthogonal to the vector \( b \). Two vectors \( a \) and \( b \) are either ordinary orthogonal or pseudo-orthogonal if and only if either \( (a, b) + (b, a) = 0 \), or \( (a, b) - (b, a) = 0 \). If the complex vector space \( V_{2n} \) is of real dimension \( 2n \geq 2 \), then the ordinary inner product is given by \( a \cdot b = |a||b| \sum_{h=1}^{n} \cos \phi_h \cos (\theta_h - \theta_h), \) and the skew-symmetric inner product is \( a \times b = |a||b| \sum_{h=1}^{n} \cos \phi_h \sin (\theta_h - \theta_h). \) These are extended to Hilbert space. (Received July 15, 1964.)

Abstract 618-2. Let \( T \in \mathbb{D}(e \in E_D) \), if \( L \) is an \( L \)-class of \( D \), \( T \in (g \in E_0) = (g \in E_0) \), and if \( L \) is an \( L \)-class of \( D \), \( T \in (g \in E_0) \), and \( L \) is not in \( E_L \). Sets of equivalent conditions on a regular \( D \)-class are given. (A) \( e \) in \( D \) implies \( e \) has an inverse \( e' \) such that \( ee' \leq e' e \) or \( e'e \leq ee' \). \( (B') \) \( D \) may be expressed as the following union of groups, right cancellative semigroups without idempotent and left cancellative semigroups without idempotent. \( \mathbb{D} = \bigcup (H_e : e \in E_D) \) \( \cup (\bigcup (S_R e : e \in E_R)) \cup (\bigcup (eS_L^* : e \in E_L)) \cup (\bigcup (R e : e \in E_L) \cup (\bigcup (eS_L : e \in E_L)). \) \( (A') E_D \) is a chain of left zero bands. \( (B') \) if \( e, f \in E_D \), \( ef = e \) or \( fe = f \). \( (C') D \) is a left group or \( E_D \) is a band and \( D \) may be expressed as the following union of disjoint right cancellative semigroups with-identity and left cancellative semigroups without idempotent: \( D = \bigcup (S_R : R \in \mathbb{B}) \cup (eS_L^* \cap S_L^* : e \in E_D, L \in L, eB_L = e). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (S_L^* : R \in \mathbb{B}) \cup (\bigcup (eS_L^* : e \in E_D, L \in L, eB_L = e). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (S_L^* : R \in \mathbb{B}) \cup (\bigcup (eS_L^* : e \in E_D, L \in L, eB_L = e). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (R e : e \in E_L). \) Assuming \( A' \), we then give additional conditions for \( eS_L^* \) to have a decomposition into left cancellative semigroups without idempotent. (Received August 3, 1964.)

Abstract 618-3. Let \( T \in \mathbb{D}(e \in E_D) \), if \( L \) is an \( L \)-class of \( D \), \( T \in (g \in E_0) = (g \in E_0) \), and if \( L \) is an \( L \)-class of \( D \), \( T \in (g \in E_0) \), and \( L \) is not in \( E_L \). Sets of equivalent conditions on a regular \( D \)-class are given. (A) \( e \) in \( D \) implies \( e \) has an inverse \( e' \) such that \( ee' \leq e' e \) or \( e'e \leq ee' \). \( (B') \) \( D \) may be expressed as the following union of groups, right cancellative semigroups without idempotent and left cancellative semigroups without idempotent. \( \mathbb{D} = \bigcup (H_e : e \in E_D) \) \( \cup (\bigcup (S_R e : e \in E_R)) \cup (\bigcup (eS_L^* : e \in E_L)) \cup (\bigcup (R e : e \in E_L). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (S_L^* : R \in \mathbb{B}) \cup (\bigcup (eS_L^* : e \in E_D, L \in L, eB_L = e). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (S_L^* : R \in \mathbb{B}) \cup (\bigcup (eS_L^* : e \in E_D, L \in L, eB_L = e). \) \( eS_L^* \cap S_L^* \) \( = \bigcup (R e : e \in E_L). \) Assuming \( A' \), we then give additional conditions for \( eS_L^* \) to have a decomposition into left cancellative semigroups without idempotent. (Received August 3, 1964.)

In this paper I define hulls of three types for sets in the plane. These hulls arise from considering an infinite sector with vertex \( p \) and angle \( \theta \), \( 0 < \theta \leq \pi \), as a neighborhood of \( p \) and all such with fixed angle \( \theta \) and vertex \( p \) as a neighborhood base for \( p \). The three types arise according as
the sectors are open, half-closed or closed. The hull $u_X$ of $X$ is defined as follows: $p \in u_X$ if and only if $X$ intersects each neighborhood of $p$. It turns out that the open and half-closed sector neighborhoods lead to hull functions for each $\theta$ which are generated from a class of finite sets each bounded by $|4\pi/\theta|$ in cardinal. However the closed sector hulls generally require infinite sets. These results include and extend Carathéodory's Theorem for the planar case and lead to many unsolved problems. (Received September 8, 1964.)

618-4. R. L. KRUSE, Sandia Laboratory, Sandia Corporation, Box 5800, Albuquerque, New Mexico 87115. A characterization of rings in which all subrings are ideals.

Following the notation introduced by L. Redei, we call an associative ring in which every subring is a two-sided ideal a v-ring; a v-ring in which all elements of finite additive order have additive order a power of the prime $p$ is called a $p$-v-ring. This paper is a characterization of all v-rings. Using the known results and a direct sum decomposition theorem the problem is first reduced to the study of nil $p$-v-rings. Nil $p$-v-rings containing elements of infinite or of unbounded additive order are next characterized. It is shown in particular that in such rings the $p$th multiple of any product is zero, and modulo its annihilator ideal such a ring is generated by at most two elements. In any nil $p$-v-ring an arbitrary element $x$ satisfies one of the two conditions: (I) $x^2$ is a natural multiple of $x$; (II) $px^2$ is a natural multiple of $x$ although $x^2$ is not a natural multiple of $x$. This result makes it possible to study a nil $p$-v-ring possessing a bound on the additive orders of its elements by decomposing the ring into an additive group direct sum of cyclic subgroups. It is shown that aside from elements in the annihilator of the ring, there is a decomposition of the ring with at most two generators of type (I) and three of type (II). The possible defining relations for these nil $p$-v-rings are enumerated. (Received September 15, 1964.)

618-5. GEORGE GLAUBERMAN, University of Wisconsin, Madison, Wisconsin. Fixed point subgroups which contain centralizers of involutions. Preliminary report.

Suppose $A$ is a group of automorphisms of a finite group $G$ such that $A$ and $G$ have relatively prime orders. Let $T = C_G(A)$. Let $U$ be the largest normal subgroup of $G$ contained in $T$ and $N$ be the subgroup of $G$ generated by the elements $g^{-1}a^g$, $g \in G$, $a \in A$. Lemma. $N$ is a normal subgroup of $G$ and $N \subseteq C_G(U)$. Theorem. Suppose $T \supseteq C_G(\tau)$ for some element $\tau$ of order two in $G$. Then $\tau$ centralizes $T/U$ and, if 3 does not divide $|G|$, $N$ is a nilpotent group of odd order. Corollary. Suppose $G$ is a simple non-Abelian group and $T \supseteq C_G(\tau)$ for some automorphism $\tau$ of $G$ which centralizes $A$ and has order two. Then $T = G$, i.e., $A = 1$. (Received September 14, 1964.)


This paper is concerned with the problem of characterizing sub-(L) functions, where $L$ is the Euler-Lagrange operator for the functional $I_{cd}[y] = \int_c^d \left[ \sum_{j=0}^n p_j(D^jy)^2 \right]$, with $n$ a positive integer, $[c, d]$ a subinterval of a fixed interval $[a, b]$, and $p_0, p_1, \ldots, p_n$ continuous real-valued functions on $[a, b]$ with $p_n(x) > 0$ on this interval. Under hypotheses which guarantee that $L$ is nonoscillatory on $[a, b]$ and that $L$ can be written as a composition of first order real linear operators, the following result
is obtained. If \( f \) belongs to the domain of \( L \) on a subinterval \([c,d]\) of \([a,b]\), then the statement that \( f \) is sub-(\( L \)) on \([c,d]\) is equivalent to each of the following conditions: (i) \((-1)^n L f(x) \leq 0 \) on \([c,d]\); (ii) \( I^{cd}_{\text{cd}}[y] \geq I^{cd}_{\text{cd}}[f] \) whenever \( y \) is a function having continuous derivatives of the first \( n - 1 \) orders with \( D^{n-1} y \) having a piecewise continuous derivative on \([c,d]\) such that \( D^{n-1} y \) and \( D^{n-1} f \) have the same value at \( c \) and at \( d \) for \( j \) in \( \{1,...,n\} \), and \( y(x) - f(x) \geq 0 \) on \([c,d]\). It is also established that the Green's function for a certain related boundary-value problem has constant sign. This discussion is a direct generalization of the second order case given by W. T. Reid (Variational aspects of generalized convex functions, Pacific J. Math. 9 (1959)).

618-7. FRANK HARARY and W. T. TUTTE, University of Michigan, Ann Arbor, Michigan 48104. The number of plane trees with a given partition.

A plane tree is a tree embedded in the plane with no two of its edges crossing. Two plane trees are regarded as equal if one can be mapped on the other by an orientation preserving homeomorphism of the plane. The number of different plane trees was found first [Harary, Prins, and Tutte, Indag. Math. 26 (1964), 319-329]. The partition of a graph is the expression of twice the number of edges as the sum of the degrees (valencies) of its vertices. In a rooted plane tree, one of the vertices is distinguished and is called the root. A planted plane tree is rooted so that the root is an end-vertex. An explicit formula was next found for the number of planted plane trees with a given partition [Tutte, Amer. Math. Monthly 71 (1964), 272-277]. In the present work, formulas are derived for the number of plane trees with a given partition. (Received September 23, 1964.)

618-8. DANIEL PEDOE, 400 Ford Hall, University of Minnesota, Minneapolis, Minnesota 55455. On Soddy's theorem and its generalization to \( n \) dimensions.

If four circles in a plane are in mutual contact, Soddy's theorem gives the following relation between the curvatures \( a, \beta, \gamma, \delta \) (reciprocals of the radii) of the circles: \( 2(a^2 + \beta^2 + \gamma^2 + \delta^2) = (a + \beta + \gamma + \delta)^2 \). The only proof of this theorem in print involves trigonometry, is unpleasing, and does not suggest the generalization to \( n + 2 \) hyperspheres with mutual contact in real Euclidean space of \( n \) dimensions. The statement of the generalization is known, but no proof has been published. The method given in this paper is simple, and proves both Soddy's theorem and its generalization. Although Soddy's theorem, when stated, was accompanied by a poem (see Coxeter: Introduction to geometry, p. 15), and the stated generalization by Gossett (Nature 139 (1937), 62) adds an extra verse to Soddy's poem, no attempt to improve the poem is made in this paper. (Received September 24, 1964.)

618-9. HSIN CHU, P.O. Box 1247, University of Alabama Research Institute, Huntsville, Alabama. A note on compact transformation groups with a fixed end point.

In the paper A remark on transformation groups leaving fixed an end point (Proc. Amer. Math. Soc. 3 (1952), 548-549) H. C. Wang proved the following result: "Let \((G, X, \pi)\) be a transformation group where \( G \) is a compact group and \( X \) is an arcwise connected Hausdorff space with a fixed end point by \( G \). Then \( G \) has another fixed point." In this note, we show, with the same assumption, that \( G \) has at least countably many fixed points. (Received September 28, 1964.)
618-10. W. S. MARTINDALE, III, University of Massachusetts, 194 Lincoln Avenue, Amherst, Massachusetts. Jordan isomorphisms of the symmetric elements of a simple ring with involution. Preliminary report.

Theorem. Let R and R' be simple rings (of characteristic $\neq 2$) which possess involutions of the first kind, with S and S' denoting the respective sets of symmetric elements. Furthermore assume that R contains three nonzero orthogonal symmetric idempotents whose sum is the identity element. Then every Jordan isomorphism of S onto S' can be uniquely extended to an associative isomorphism of R onto R'. (Received September 28, 1964.)

618-11. JOHN COBB, University of Wisconsin, Madison, Wisconsin. A k-complex is tame in $E^n$ ($n \geq 3k + 1$) if each of its simplices is tame.

Let P be a finite k-complex topologically imbedded in $E^n$, with $n \geq 3k + 1$. Theorem. If there exists a locally tame k-complex Q in $E^n$ so that P is locally tame mod $P \cap Q$, then P is tame.

Corollary. P is tame if each i-simplex of P ($1 \leq i \leq k$) is locally tame at each point of its interior (hence if each i-simplex is tame). (Received September 28, 1964.)

618-12. F. M. WRIGHT, Iowa State University, Ames, Iowa. On the existence of a weighted Stieltjes mean sigma integral. I.

Let p be an integer such that $p \geq 2$, let $(w_1, w_2, \ldots, w_p)$ be an ordered p-tuple of real numbers such that $w_1 + w_2 + \ldots + w_p = 1$, and let f and g be real-valued functions on a closed interval $[a, b]$ of the real axis. For a partition $P = \{a = x_0 < x_1 < \ldots < x_n = b\}$ of $[a, b]$, choose, for $i = 1, 2, \ldots, n$, any partition $\{x_{i-1} = r_{i-1} < r_i \leq r_i, z < \ldots < r_{i,p} = x_i\}$ of $[x_{i-1}, x_i]$ consisting of p points, and let $S(P) = \sum_{i=1}^{n} \left( \sum_{j=1}^{p} w_j f(r_i, j) \right) \left( g(x_i) - g(x_{i-1}) \right)$. If the sums $S(P)$ have a finite refinement limit, this limit is denoted by $\int_{a}^{b} f(x) dg(x)$. Necessary conditions are obtained for the existence of this integral which deal with the behaviour of f(x) and g(x) as x approaches from the right some point c of $[a, b]$; analogous results hold relative to left-hand limits. Sufficient conditions are obtained for the existence of this integral in case g is a saltus function and f is bounded by first considering the case where g is a step function; the proof for the latter case is patterned after a proof given by R. E. Lane (Proc. Amer. Math. Soc. 5 (1954), 59-66) for the Stieltjes mean sigma integral. For g of bounded variation, for s a saltus function and $\psi$ a continuous function of bounded variation on $[a, b]$ such that $g(x) = s(x) + \psi(x)$, and for f bounded, it follows that the existence of this integral implies the existence of $\int_{a}^{b} f(x) ds(x)$. (Received September 28, 1964.)


Adopt the terminology and notations of Švarc, Duality of functors, Soviet Math. Dokl. 4 (1963), 89-92. Let $\mathfrak{A}$ be a D-category. An $\mathfrak{A}$-based category is a category $\mathcal{L}$ equipped with a lifted hom functor $H_{\mathcal{L}}: \mathcal{L} \times \mathcal{L} \to \mathfrak{A}$ for which (a) there is a natural equivalence $H_{\mathcal{L}}(X,Y) \cong \text{Hom}_{\mathcal{L}}(X,Y)$, and (b) the composition rule of $\mathcal{L}$ comes from an $\mathfrak{A}$-morphism $H_{\mathcal{L}}(X,Y) \circ H_{\mathcal{L}}(Y,Z) \to H_{\mathcal{L}}(X,Z)$. A functor $F: \mathcal{L} \to \mathfrak{A}$ is admissible or strong if each function $F_{XY}: \text{Hom}_{\mathcal{L}}(X,Y) \to \text{Hom}(FX,FY)$ comes from an
\( \mathfrak{A} \)-morphism \( H_L(X,Y) \rightarrow H(FX,FY) \). Proposition. Assume \( \mathfrak{A} \) has a cogenerator. Let \( F: L \rightarrow \mathfrak{A} \) be a strong functor, and let \( G: L \rightarrow \mathfrak{A} \) be a functor (necessarily strong) having a strong right adjoint (say \( w: \mathfrak{A} \rightarrow L \) satisfying \( H_L(X,wA) \cong H(GX,A) \)). Then the natural transformations from \( F \) to \( G \) constitute a set. Indeed, if \( C \) is a cogenerator, these transformations are canonically embedded in the set of \( \mathfrak{A} \)-morphisms from \( F(wC) \) to \( C \). This result, startling even for group-valued additive functors on an abelian category, permits complete generalization (to the case of strong functors from \( L \) to \( \mathfrak{A} \)) of all the duality results announced by Švarc in the above-mentioned work. (Received September 28, 1964.)

618-14. WITHDRAWN.


By perturbing \((d/dx)^n\) it is shown that under a wide class of boundary conditions

\[
(d/dx)^n + C_{n-1}(d/dx)^{n-1} + B_{n-2}(d/dx)^{n-2} + \ldots + B_0 \text{ determines a spectral operator in } L_2[0,1] \text{ with a complete set of generalized eigenvectors, provided } C_{n-1} \text{ is a compact operator and the } B_j (j = 0,1, \ldots, n-2) \text{ are bounded operators. A similar result is proved for } C_{n-1} = 0 \text{ in } [\text{Dunford and Schwartz, Linear operators, Vol. III, to appear}] \text{ and for arbitrary bounded } C_{n-1} \text{ and } B_j \text{ with sufficiently small norms, in the author's thesis.} \text{ (Received September 28, 1964.)}
\]

618-16. J. R. SORENSON, Valparaiso University, Valparaiso, Indiana. Left-amenable semigroups and cancellation.


Theorem: A left-amenable semigroup \( S \) with right cancellation has left cancellation (and is therefore embeddable in an amenable group). The proof is based on showing that if \( x \neq y \) there exists a subset \( V \) of \( S \) so that \( xV \cap yV = \emptyset \) and \( \mu(x,y) > 0 \) for \( \mu \in L(S) \) (\( L(S) \) is the set of left-invariant means). Now let \( S \) be any left-amenable semigroup and define \( x R y \) if there exists \( z \) such that \( xz = yz \). Then \( R \) is a two-sided stable equivalence relation and the factor semigroup \( S/(R) \) is left-amenable with right cancellation. \( S/(R) \) is useful in the study of \( S \) and \( L(S) \). For example, \( S/(R) \) is finite iff every extreme point of \( L(S) \) is a finite average of multiplicative linear functionals. The construction of \( S/(R) \) also indicates how to embed any semigroup with left cancellation in a left-amenable semigroup with left cancellation. (Received September 28, 1964.)

618-17. G. W. HEDSTROM, University of Michigan, Ann Arbor, Michigan 48104. The power-boundedness of Fourier series.

Let \( f \) be a function on the circle with absolutely convergent Fourier series, \( f(t) = \sum c_k e^{ikt} \). Define \( \|f\| = \sum |c_k|^k \). Let \( |f(t)| \leq 1 \) and let there be only finitely many points \( t_k \) on the circle such
Let \( f(t) = \exp \sum_{k=1}^{\infty} \frac{\alpha_k(t - t_k)}{k!} \), where \( \alpha_k \) and \( \beta_k \) are real, and \( \gamma_k \neq 0 \). Then there is a \( M \) such that \( \| f^n \| < M, n = 1, 2, \ldots \), if and only if \( \text{Re} \gamma_k < 0 \) for each \( k \). The result is new when \( \text{Re} \gamma_k = 0 \). Precise estimates of \( \| f^n \| \) are made with the use of saddle point estimates. (Received September 29, 1964.)


Let \( C(X) \) be the lattice of continuous functions from a completely regular space \( X \) into the reals \( R \), and let \( \bar{R} \) be the extended reals. If lattices \( L \) and \( K \) contain (copies of) \( R \), then a homomorphism \( \phi \) from \( L \) into \( K \) is an \( R \)-homomorphism in case \( \phi(r) = r \) for all \( r \in R \). If \( L \) is a sublattice, containing \( R \), of \( C(X) \) for some space \( X \), and if \( \phi \) is an \( R \)-homomorphism of \( L \) into \( \bar{R} \), then \( \phi \) is fixed in \( X \) in case there is an \( x \in X \) such that \( \phi(f) = f(x) \) for all \( f \in L \). Sufficient conditions are given for a lattice \( L \) to be isomorphic to a sublattice of \( C(X) \) for some realcompact space \( X \). If \( L \) is a sublattice of \( C(X) \) for some space \( X \), then sufficient conditions are also obtained so that every \( R \)-homomorphism of \( L \) into \( \bar{R} \) is fixed in \( \beta X \) and every \( R \)-homomorphism of \( L \) into \( R \) is fixed in \( vX \). These latter conditions are sufficient for \( L \) to characterize \( \beta X \) and \( vX \). (Received September 29, 1964.)

618-19. M. K. FORT (Posthumously), University of Georgia and SEYMOUR SCHUSTER, Minnemath Center, University of Minnesota, Minneapolis, Minnesota 55455. Convergence of series whose terms are defined recursively.

Let \( f \) be a real function such that (1) \( f \) is differentiable on \([0, a]\) for some \( a > 0 \), (2) \( 0 < f(x) < x \) for \( 0 < x \leq a \), (3) there exists \( c > 0 \) such that \( f'(x) \geq c \) for \( 0 \leq x \leq a \), and (4) if \( 0 < x_1 < x_2 \leq a \), then \( f(x_1)/x_1 \geq f(x_2)/x_2 \). Conditions are derived for the convergence of \( \sum_{n=0}^{\infty} u_n \), where \( u_0 = a \) and \( u_{n+1} = f(u_n) \). Theorem 1. \( a + c \int_0^a (x/(x - f(x)))dx \leq \sum_{n=0}^{\infty} u_n \leq \int_0^a (x/(x - f(x)))dx \), and hence \( \sum_{n=0}^{\infty} u_n \) converges if and only if \( \int_0^a (x/(x - f(x)))dx \) converges. From this result follows the simply-stated Corollary. If \( f'(0) < 1 \), then \( \sum u_n \) converges. Theorem 2. If \( f'(0) = 1 \) and \( f'' \) exists and is bounded below on \([0, a]\), then \( \sum u_n \) diverges. Theorem 3. If \( s > 0 \), \( \sum_{n=0}^{\infty} u_n^s \) converges if and only if \( \int_0^a (x^s/(x - f(x)))dx \) converges. Theorem 4. If \( f(x) = x - cx^p + R(x) \), where \( c > 0 \) and \( R(x) = o(x^p) \) as \( x \to 0 \), then \( \sum_{n=0}^{\infty} u_n^s \) converges if \( s > p - 1 \) and diverges for \( 0 < s \leq p - 1 \). (Received September 30, 1964.)


There are known from the past two fundamental groups of laws in the theoretical mechanics: (1) conservation laws (Newton); (2) symmetry laws (Einstein). The author proposes a third group of laws as one of the fundamental groups: (3) optimum principle laws. Based on this, the author discusses the fundamental aspects of the classical and relativistic energodynamics. Every dynamic system we deal with should be a stable one or at least a quasi-stable one. This may be achieved by requiring that the potential (this serves only as an illustrative example) energy of the system in

764
question should be a minimum. This leads to a variational principle. The importance of the energetodynamics in complex dynamic systems is indicated. Two examples are cited: electro-magneto-hydrodynamic multi-fluid and the structure of the universe. (Received September 30, 1964.)


Several published results illustrate the title by showing that connected, connectivity and peripherally continuous functions often satisfy such properties as (1) $f(K) \subset f(K)$ if $K$ is connected and (2) the inverse of a closed set has closed components. Relations between these and other such properties are obtained, E.g., (1) and (2) are equivalent always although frequently their validity for certain functions has been divided into two theorems. A weak regularity condition, $R_0$, of A. S. Davis (Amer. Math. Monthly 68 (1961), 886-893) is useful. For instance, (1) and (2) are satisfied by every connected function into an $R_0$-space and every connectivity function on an $R_0$-space (the latter satisfies two successively stronger such properties as well). The properties (3) $f^{-1}(K) \supset f^{-1}(K)$ if $K$ is connected and (4) the inverse of a closed connected set is closed are also equivalent and imply a fifth property which K. Fun and R. A. Struble (Indag. Math. 16 (1954), 161-164) require of a connected function to obtain what they call a connectedness-preserving function. For connected functions into an $R_0$-space all three are equivalent and any function satisfying (3) or (4) whose range is semi-locally-connected must be continuous. Theorems of Fan, Struble and T. Tanaka (J. Math. Soc. Japan 7 (1955), 391-393) are corollaries. (Received September 30, 1964.)

ABSTRACTS FOR SPECIAL SESSION


Let $Q = \prod_{a \in A} R_a$ be a complete direct product of rings $R_a$, with an index set $A$. Let $\mathcal{F} = \{S\}$ be an ultra-filter of subsets of $A$, then define a congruence relating in $Q$ by setting: $f = g (\text{mod } \mathcal{F})$ if $\{a \mid f(a) = g(a) \in \mathcal{F}\}$. The congruence classes form a homomorphism image of $Q$, known as the ultra-product $Q/\mathcal{F}$, and which inherits elementary properties from the rings $R_a$. Thus, if all $R_a$ are division rings, ordered rings, or primitive rings then $Q/\mathcal{F}$ has the same property. The ultra-product is very useful in establishing isomorphisms between formed systems and their functional interpretation. Combining this tool with known results in algebra and a suitable choice of the filter $\mathcal{F}$, one obtains imbedding theorems, e.g., (1) the imbedding of a free ring generated by an (ordered) division ring and a set of indeterminates $\{x_i\}$ in an (ordered) division ring, (2) the imbedding of a prime ring $R$ in a primitive ring possessing some of the properties of $R$, or (3) preserving of rational identities of division algebras under transcendental extension of their center. (Received October 5, 1964.)

This is an exposition of results on free ideal rings (firs) which appeared in J. Algebra 1 (1964), 47-69. Further, a description of firs is discussed which is categorically invariant, i.e., which is shared by all rings with category of modules isomorphic to the category of modules over firs. Such rings are necessarily matrix rings over firs. (Received October 5, 1964.)


The main theorem of Golod and Šafarevič gives a criterion for certain quotient algebras of a free algebra to be infinite-dimensional. From this they construct infinite-dimensional and finitely generated nil algebras and finitely generated infinite periodic groups. In an entirely different direction their results apply to the cohomology of finite p-groups and give a negative solution to the class field tower problem. (Received October 5, 1964.)

618-25. CARL FAITH, 48 Cuyler Road, Princeton, New Jersey 08540 and E. A. WALKER, New Mexico State University, University Park, New Mexico. Characterizations of Quasi-Frobenius rings.

\( \mathcal{M}_R \) denotes the category of all right \( R \)-modules; \( \hat{\mathcal{M}} \) the injective hull of any \( M \in \mathcal{M}_R \).

**Theorem 1.** The following three conditions are equivalent: (1) A direct sum of countably many copies of \( \hat{R} \) is injective in \( \mathcal{M}_R \); (2) A direct sum of arbitrary many copies of \( \hat{R} \) is injective; (3) \( R \) satisfies the a.c.c. on annihilator right ideals. **Theorem 2.** If \( R = \hat{R} \), and if (3), then \( R \) is Quasi-Frobenius (QF). **Theorem 3.** The following two conditions are each equivalent to (1) when \( R = \hat{R} \): (i) The ring \( R_{\omega} \) of all row-finite matrices over \( R \) is right self-injective; (ii) \( R \) is QF (i.e., \( R \) is right or left artinian or noetherian). **Theorem 4.** The following three conditions are equivalent: (a) Each projective module in \( \mathcal{M}_R \) is injective; (b) Each injective module in \( \mathcal{M}_R \) is projective; (c) \( R \) is QF. Note that (b) is equivalent to the condition that each module in \( \mathcal{M}_R \) is contained in a projective module. **Theorem 5.** The following three conditions are equal: (I) Each cyclic module in \( \mathcal{M}_R \) is contained in a projective module; (II) Each right ideal of \( R \) is the right annihilator of a finite subset of \( R \); (III) Each right ideal of \( R \) is a right annihilator, and \( R \) is right artinian. A basic lemma in proofs is: \( R \) satisfies the a.c.c. on left annihilator ideals if and only if each right annihilator ideal is the annihilator of a finite subset of \( R \). Results of Bass, Chase, Eilenberg, and Nakayama also are used. This paper will appear in a special volume of Nagoya Math. J. commemorating the late T. Nakayama. (Received September 28, 1964.)
ABSTRACTS PRESENTED BY TITLE

64T-491  ECKFORD COHEN, University of Tennessee, Knoxville, Tennessee.  The number of zeroid elements (mod n), II.

Let n be an arbitrary positive integer, \( \phi(n) \) the Euler totient, and \( c(a,n) \) the Ramanujan sum (mod n). It is proved that \( \sum_{u(a, n)} c(u^2, n) = n\phi(r) \) or 0, according as n is a square \( n = r^2 \) or n is not a square. This result is applied to evaluate the number \( z_k(n) \) of solutions of the congruence, 

\[
u^2 = x_1 y_1 + \ldots + x_k y_k \pmod{n}.
\]

In case \( k = 0 \), \( z_k(n) \) becomes the number of "zeroid" elements (mod n). The average order of \( z_k(n) \) is also considered. (Received June 24, 1964.)

64T-492. LEONARD CARLITZ, Duke University, Durham, North Carolina.  Some multiple sums and binomial identities.

The sums considered are of the type \( s_n^{(k)}(x_1, \ldots, x_k) = \sum_{r=1}^{n-k} \frac{C_{n-r} x_1^{n-r} y_1^{n-r} \ldots y_k^{n-r}}{C_{n-r} x_1^{n-r} y_1^{n-r} \ldots y_k^{n-r}} \).

A generating function for \( s_n^{(k)} \) is obtained. The special cases \( x_1 = \ldots = x_k \) and \( (k = 2r); x_j + r = x_j (1 \leq j \leq r) \) are of particular interest. For example we have 

\[
s_n^{(k)}(x, x, \ldots, x) = \frac{a_1 \cdots a_{n-k} - \beta a_1 \cdots a_{n-k}}{a_1 - \beta},
\]

where \( a, \beta \) are the roots of \( z - xz - x = 0 \). As an application it is proved that the characteristic values of the matrix \( [C_{x,n-s}x^s] \) \((r, s = 0, 1, \ldots, n)\) are the numbers \( a^n, a^{n-1} \beta, \ldots, a\beta^{n-1}, \beta^n \). (Received June 29, 1964.)


Let \( A \geq 0 \) be an \( n \times n \) matrix. There exist least integers \( p \) and \( \gamma \), the period and index of convergence of \( A \), such that, if \( A^r = (a_{ij}^{(r)}) \), then for all \( r \geq \gamma, a_{ij}^{(r+p)} \neq 0 \) iff \( a_{ij}^{(r)} \neq 0 \) (see our Abstract 64T-353, these Notices 11 (1964), 580). Define \( w(A) \) to be the number of nonzero elements of \( A^\gamma \) and let \( w(A) = \max w(A) \) \((r = 1, 2, 3, \ldots)\); furthermore, following Pullman (Abstract 590-23, these Notices 9 (1962), 125), define index \( A \) to be the least value of \( r \) for which \( w(A) = w(A) \). It can be shown that, if \( A \) is irreducible, \( \gamma \leq \text{index}(A) \leq \gamma + p - 1 \); this result also holds for reducible matrices under a minor restriction (a sufficient condition being that, in the associated digraph, \( G(A) \), of \( A \), no two strong components are connected by an arc if each consists only of a single vertex without a loop). Again if \( A \) is irreducible, \( w(A) = \sum_{r=1}^{p} r^2 \), where \( r_i \) is the number of vertices in the \( i \)th cyclic component of the cyclically \( p \)-partite digraph, \( G(A) \). (Received August 12, 1964.)


By relaxing the continuity requirements on the conventional spline approximation of degree \( 2n + 1 \) and requiring higher-order interpolation in their place, piecewise polynomial functions are obtained which possess extremal properties similar to those for ordinary splines. Given \( f \in C^{n+1}[0,1] \)
and a partition $\Delta$: $0 = x_0 < x_1 < \ldots < x_N = 1$ of the unit interval, let $S_{i,j}^{(j)}$ be the piecewise polynomial function which is of degree $2n + 1$ in $x_{j-1} \leq x \leq x_j$ ($j = 1, \ldots, N$), $\in C^{2k-p}[0,1]$ and for which, at each $x_j$, $S_{i,j}^{(j)} = f^{(j)}$, $j = 0, 1, \ldots, n$ at $x = 0$ and $x = 1$, $S_{i,j}^{(j)}$ minimizes $\|f - S\|^2 = \int_0^1 \left(f^{(n+1)} - S^{(n+1)}\right)^2 \, dx$. All of the common extremal properties of the regular spline ($p = 0$) related to best approximation and minimum norm carry over to this case of higher-order interpolation for standard end conditions. Extensions to the generalized spline in one dimension and to multidimensional splines are immediate. (Received September 2, 1964.)

64T-495. G. W. STRANG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. A necessary condition for well-posed Cauchy problems.

Let $L = \sum |a| \leq m G_a(x)D^a$ be a partial differential operator with matrix coefficients depending on $x_1, \ldots, x_d$. Let $P(x_0)$ be its principal part at $x_0$: $P = \sum |a| = m G_a(x_0)D^a$. We suppose $G_a$ bounded on compact sets for $|a| < m$ and continuous for $|a| = m$; both $L$ and $P$ are to be closed operators on $L^2$, with dense domain. **Theorem.** If $\|L^{1/t}\| \leq M$ for $t \geq 0$, then for each $x_0$, $\|P^{1/t}\| \leq M$ for $t \geq 0$.

The converse is false, even for $m = 1$ and $C^\infty$ coefficients. Analogous results hold for partial difference operators. (Received September 8, 1964.)

64T-496. AMBIKESHWAR SHARMA and AMRAM MEIR, University of Alberta, Calgary, Alberta, Canada. Convergence of spline functions.

Ahlberg and Nilson ([*]), Soc. Indust. Appl. Math., 11 (1963), 95-104) have proved the uniform convergence of the cubic spline with the sequence of function points $x_i^{(k)}$ ($i = 1, \ldots, n$) ($L_i^{(k)} = x_i^{(k)} - x_{i-1}^{(k)}$) under the conditions that $L_i^{(k)} = 0$ uniformly in $i$ as $k \to \infty$. Later Walsh, Ahlberg and Nilson (Abstract 63T-103, these Notices 10 (1963), 202) report the same result for higher-order splines again under the condition of uniformly spaced mesh points. We have the following extension: **Theorem.** If $f \in C^2[0,1]$ and is periodic with period 1 and $\phi_k(x)$ denotes the interpolatory cubic spline to $f$ at the function points $x_i^{(k)}$ ($i = 1, \ldots, n; k = 1, 2, \ldots$), then if $\max_i L_i^{(k)} \to 0$ as $k \to \infty$, we have uniformly in $[0,1]$, $\lim_{k \to \infty} \phi_k(x) = f(x)$ ($\nu = 0, 1, 2$). The restriction on uniformity of mesh in Theorem 2 of [*] in the study of local convergence can be similarly removed. (Received September 9, 1964.)

64T-497. B. S. RANDOL, Yale University, New Haven, Connecticut. A number-theoretic estimate.

Let $k$ and $n$ be fixed positive integers. For each non-negative integer $m$, let $r_{kn}(m)$ denote the number of ways in which $m$ can be represented as a sum of $n$ $2k$th powers of (possibly negative) integers. For $x > 0$, set $N_{kn}(x) = \sum_{m} m x^{r_{kn}(m)}$, and denote by $V$ the volume of the set in $E^n$ defined by the inequality $y_1^{2k} + \ldots + y_n^{2k} \leq x$. **Theorem.** $N_{kn}(x) = Vx^{n/2k} + O(n(n-1)(2k-1)x^{n(2k-2k^2)}}$. (Received September 11, 1964.)
One is referred for notation to the recent abstract of the author titled Generalized Haar theorem. This theorem remains true if one restricts $k \leq n + m + 1$ instead of $k \leq n + m + 2$. Let $v(r) + 1$ be the order of maximal Chebyshev system obtained from set $F = \{g_0, \ldots, g_n, r_0, \ldots, r_m\}$. Let $u(r)$ be maximal number of zeros (with a double zero counted twice) obtained from any function of form: $\sum_{i=0}^{n} a_i g_i(x) + \sum_{i=0}^{m} b_i r_i(x)$. Then, the Chebyshev approximation to each $f \in C[0,1]$ is unique (if it exists) iff $u(r) = v(r)$ for each $r \in R$. Now let $w(r)$ be the order of a maximal linear independent subset of $F$. Then the Chebyshev approximation is unique for each $f \in C[0,1]$ iff $w(r) = v(r) + 1$ for each $r \in R$; that is, a maximal linear independent subset of $F$ is a Chebyshev system (and hence all maximal linear independent subsets). Results of Cheney and the author are used in proving these theorems. (Received September 14, 1964.)

Let $G$ be a finite group, $R$ a discrete valuation ring of characteristic zero, with maximal ideal $P$. Consider left $RG$-modules having finite $R$-bases, and assume that the Krull-Schmidt theorem holds for $RG$-modules. This is so if $R$ is complete, or if $G$ is a $p$-group and $p \in P$. The integral representation ring $A(RG)$ is the additive abelian group generated by symbols $\{M\}$, one for each isomorphism class of $RG$-modules, with relations $\{M\} = \{M'\} + \{M''\}$ whenever $M \cong M' \otimes M''$, and where multiplication is defined by $\{M\}\{M'\} = \{M \otimes R M'\}$ (tensor product of $G$-modules). The existence of nilpotent elements in $A(RG)$ is studied. Main Theorem. Let $G$ be cyclic of order $n$. Assume $n \in P^2$; if $2 \in P$, assume further that $n \equiv 0 \pmod{4}$. Then $A(RG)$ contains at least one nonzero nilpotent element. Corollary. Let $G$ be cyclic of order $p^e$, $e > 1$. Let $Z_p = \{a/b: a, b$ rational integers, $(b, p) = 1\}$. Then $A(Z_p G)$ contains nonzero nilpotent elements. The multiplication table of $A(Z_p G)$ is given explicitly, for $G$ cyclic of order 4. The basic tool in the proof of the theorem, and in the specific calculations, is the following: If $X$ is an $RG$-module satisfying $PG \subseteq X \subseteq RG$, and $Y$ is any $RG$-module, the calculation of $Y \otimes_R X$ can be reduced to a problem involving only tensor products of certain $(R/P)G$-modules. (Received September 14, 1964.)

Let $M$ be a denumerable complete model of the Zermelo-Fraenkel set theory with the axiom of choice ($ZF^*$) in which there is a measurable cardinal $\mu$ (i.e., on the power-set of $\mu$ there is a nonprincipal prime ideal which is $<\mu$-additive, i.e., which contains the union of any $<\mu$ of its members), $\mu > \omega$. For every ordinal $\lambda < \mu$ there is a complete extension $N$ of $M$ with the same ordinals as $M$ such that: (1) $N$ is a model of $ZF^*$, (2) $\mu$ is a measurable cardinal in $N$, and (3) in $N$, $\kappa_{\alpha+}^\lambda = \kappa_{\alpha+}^\lambda$ for every regular $\kappa_\alpha$ with $\alpha < \lambda$. $N$ is constructed from $M$ in a fashion similar to Easton's variant of Cohen's forcing method (Abstract 609-1, these Notices 11 (1964), 205), the difference being that one destroys cardinals of $M$ rather than add subsets to the cardinals of $M$. The $<\mu$-additive nonprincipal prime ideal on the power-set of $\mu$ in $N$ is the ideal generated by the co-

760
responding ideal in M. Additional relative consistency results can be obtained by extending N, e.g.,
for every \(0 < \sigma < \mu\) which is not confinal with \(\omega\) in N one can extend N to a model \(N^*\) which satisfies
(1) and (2) above in which \(\varepsilon_{N^*} = \varepsilon_{N}\). (Received September 14, 1964.)

64T-501. A. G. BRANDSTEIN, Brown University, Providence, Rhode Island 02912.

\(2\)-manifolds as maximal ideal spaces.

Let M be a compact \(2\)-manifold. Then M can be obtained from a disk by identifying suitable
pairs of arcs on a circle. Using this representation, it is shown that: Theorem. Every compact
\(2\)-manifold is the maximal ideal space of an antisymmetric algebra which is a Dirichlet algebra on
its Šilov boundary. (An algebra is antisymmetric if it contains no nonconstant real functions.) For
the \(2\)-sphere and the projective plane, this is known. For definitions of terms used, see J. Wermer,
Banach algebras and analytic functions, Vol. 1, Fasc. 1, Advances in Mathematics, Academic Press,
New York, 1961. (Received September 15, 1964.)

with the same class of homeomorphisms.

Let \((X, \tau)\) be a topological space. We denote the class of all homeomorphisms of \((X, \tau)\) onto
itself by \(H(X, \tau)\) and set \(H(A) = \{f(x): f \in H(X, \tau), x \in A\}\). A topology \(\tau\) on X is called a C-topology
of \(X\) if (1) if \(V \in \tau\) then \(f(V) \in \tau\) for all \(f \in H(X, \tau)\); (2) \(U \in \tau\) iff \(U \cup V \in \tau\) for all nonempty \(V\)
in \(\tau\). Theorem. The smallest topology containing a family of C-topologies of \(\tau\) is a C-topology
of \(X\). Theorem. The intersection of a finite family of C-topologies of \(\tau\) is a C-topology of \(\tau\).

Theorem. If \(\tau\) is a C-topology of \(\tau\) then \(\tau \subseteq \tau\) and \(H(X, \tau) = H(X, \tau)\). Theorem. Let \(H(A)\) be a
closed subset of a locally compact space \((X, \tau)\). Let \(P(V)\) mean that \(V \in \tau\) and \(H(A) - V = B \cup C\)
where \(B\) is a closed compact set and \(C\) is a closed and nowhere dense set and \(\text{Card}(C) \leq \alpha\) for some
infinite cardinal number \(\alpha\). Then \(\tau = \{U: U = \emptyset\) or \(P(U)\}\) is a C-topology of \(\tau\). Theorem. Let \(H(A)\)
be a closed subset of a first countable Hausdorff space \((X, \tau)\) and suppose the set of all isolated points
of \((X, \tau)\) is closed. If \(\tau = \{U \subseteq \tau: U = \emptyset\) or \(\text{Cl}(U \cap H(A)) = H(A)\}\), then \(\tau\) is a C-topology of \(\tau\).

Theorem. Let \((X, \tau)\) be an \(n\)-manifold and \(\tau\) a topology on X. If \(H(X, \tau) = H(X, \tau)\) and \(\tau \subseteq \tau\)
then \(\tau\) is a C-topology of \(\tau\). (Received September 15, 1964.)

64T-503. R. A. BRUALDI, National Bureau of Standards, Numerical Analysis Section,
Far West Building, Room 220, Washington 25, D. C. Permanent of the direct product of matrices.

Let \(A = [a_{ij}]\) be a non-negative matrix of order \(m\). The permanent of \(A\) is given by
\(\text{per}(A) = \sum a_{i_1} a_{i_2} \cdots a_{i_m} \) where the summation extends over all permutations \((i_1, i_2, \ldots, i_m)\) of
the integers \(1, 2, \ldots, \, m\). The term \(a_{i_1} a_{i_2} \cdots a_{i_m}\) is called a permutation product of \(A\). It is shown
that \(A\) has precisely one nonzero permutation product if and only if the rows and columns can be per­
muted to give a triangular matrix with zeros above the main diagonal and nonzero numbers on the
main diagonal. Let \(B\) be a non-negative matrix of order \(n\) and let \(A \times B\) denote the direct product of
\(A\) with \(B\). The preceding fact along with others is used to show that \(\text{per}(A \times B) \geq (\text{per}(A))^n (\text{per}(B))^m\)
with equality if and only if \(A\) or \(B\) has at most one nonzero permutation product. It is also shown that
every permutation product of \(A \times B\) is the product of \(n\) permutation products of \(A\) and \(m\) permutation

770
products of B, and conversely. This implies the existence of a minimal positive number $K_{m,n}$ such that $\text{per}(A \times B) \leq K_{m,n}(\text{per}(A))^m(\text{per}(B))^n$ for all non-negative A and B of orders m and n respectively. (Received September 16, 1964.)

64T-504. F. B. CORREIA, Rhode Island College, Providence 8, Rhode Island. The A-B-E sequences and their relationship with Cramer's conjecture.

This paper includes the results given in the Abstract 62T-100 of the Notices April 1962, the paper for which was lost, but presents the results in a more complete development. In the author's A theory of primes (Library of Congress Card No. MIC 61-789) the following result concerning the difference of consecutive primes was obtained: $d_n = p_{n+1} - p_n - 2(m_n - m_n^* + O(1))\log^2 p_n$ where $m_n$ and $m_n^*$ were functions defined on the set of primes and introduced in the above reference. This result is equivalent to the Prime Number Theorem. The properties of the three related A, B, and E sequences namely: $(m_{n+1} - m_n^*)$, $(m_{n+1} - m_n)$ and $(m_n - m_n^*)$ respectively are determined. The A sequence only has two values 0 or 1, its limit is 1 and its average order is 1. The B sequence has an average order of $1/2$. The most important E sequence contains every integer $k \geq 0$ and the probability that the random choice of a term has value $k$ is $p_k = 1/k(k + 1)(k + 2)$ for $k \geq 1$ and $p_0 = 3/4$. The exact order of the terms in the E sequence is $m_n - m_n^* \sim n^{1/3}$ With the first mentioned result this implies that the exact order of $d_n$ is: $d_n \sim 2n^{1/3}\log^2 p_n$. This proves Cramer's conjecture false; that there exists a prime between the squares of consecutive integers for $n > n_0$; and the same for the cubes for all $n$. (Received September 18, 1964.)

64T-505. HAIM GAIFMAN, Hebrew University, Jerusalem, Israel. Measurable cardinals and constructible sets.

We use Gödel's notation concerning constructible sets. $F'^a$ is the a-th constructible set and $F'^{a'} = \{F'^\beta|\beta < a\}$. A cardinal $\mu$ is measurable if there exists a nontrivial $\kappa_0$-additive two-valued measure defined on all its subsets, with value 0 for every single point. $\kappa = \text{cardinality of } x$.

Theorem I: If there is a measurable cardinal one can define a class $C$ of ordinals, so that: (i) for every infinite ordinal $\beta$, there is an ordinal $a > \beta$, $a = \beta$ and $a \in C$, (ii) $C$ is closed under limits, (hence every cardinal $> \kappa_0$ belongs to $C$), (iii) if $a, \beta \in C$ and $a < \beta$ then $F'^a$ is an elementary submodel of $F'^\beta$ and both are models of Zermelo-Fraenkel. (By the model $F'^a$ we mean $(F'^a, E | F'^a)$.)

Using this one can give a truth definition for the universe of all constructible sets, L, and prove that, for all $a \in C$, $F'^a$ is an elementary submodel of L. It follows that, for every infinite constructible set $x$, the number of constructible subsets of $x$ is $\kappa$. For every set $z$ let $L(z)$ be the universe of all sets constructible from $z$. Theorem II: If there is a measurable cardinal then there is a set $z$, consisting of natural numbers only, such that the class $C$ can be defined and Theorem I holds within the universe $L(z)$. Consequently arbitrary high cardinals of L cease to be cardinals in L($z$).

(Received September 21, 1964.)
Let $\Phi$ denote the set of all semi-Fredholm operators from $X$ into $Y$, $X$ separable ($X$ and $Y$ Banach spaces) (for definitions see Cordes-Labrousse, J. Math. Mech. 12 (1963), 693-720). **Theorem.** Let $A_1$ and $A_2$ be two operators in $\Phi$ with same finite index. Then there is a continuous arc in $\Phi$ connecting $A_1$ and $A_2$. Main tool for the proof of this theorem is the following **Lemma.** Let $X$ be separable, $B$ the normed space of bounded linear transformations from $Y$ into $X$. Let $B$ be the subset of all operators having an inverse (not necessarily bounded) and with range dense in $X$. Then $B$ is arcwise connected. The results indicated hold for real as well as complex Banach spaces. (Received September 21, 1964.)

**64T-507. C. E. AULL, Kent State University, Kent, Ohio 44240.** An automorphism theorem for matrices.

We define the $g$-rank of an $n \times n$ matrix, $a$, as the rank of $a^n$ if $a^n$ is a group member; the $g$-rank of an $S$-class of matrices is the highest $g$-rank of any of its members. [See Some theorems on the rank of a matrix, these Notices 11 (1964), 542.] For $n \times n$ matrices with elements in the field of real or complex numbers, this is equivalent to the rank of $a^2$ where $a$ is member of the $S$-class. Two $S$-classes of a semigroup $S$ are defined to be automorphic, if there is an automorphism of $S$ such that the two $S$-classes are in 1:1 correspondence. It is proved that for $n \times n$ matrices with elements in the field of real or complex numbers that (1) if matrices $a$ and $b$ correspond under a similarity transform, then their $S$-classes are automorphic; (2) two $S$-classes are automorphic iff one has the same rank and the same $g$-rank as the other; (3) every $S$-class is automorphic to one of a finite number of non-automorphic $S$-classes where this number is $(n + 2)^2/4$ for $n$ even and is $(n + 3)(n + 1)/4$ for $n$ odd. (Received September 21, 1964.)

**64T-508. O. P. STACKELBERG, Duke University, Durham, North Carolina.** On the law of the iterated logarithm for continued fractions.

A new proof for the law of the iterated logarithm for the partial quotients of simple continued fractions is given. The theorem in (On a limit theorem by Khintchine for continued fractions, Abstract 614-26, these Notices 11 (1964), 534) is strengthened, with $\leq$ substituted for $\leq$, using methods in Stackelberg (On the law of the iterated logarithm, Nederl. Akad. Wetensch. Proc. Ser. A 67 (1964), 48-67). This theorem was previously announced by W. Doblin in (Remarques sur la théorie métrique des fractions continues, Compositio Math. 7 (1940), 353-371), where he states that the theorem can be proved using a generalization of methods and results of his thesis (Sur les propriétés asymptotiques de mouvement régis par certains types de chaînes simples, Bull. Soc. Roumaine Sc. 39 (1937), 57-115). The methods of this paper are different from those of Doblin. (Received September 21, 1964.)
Two topics are discussed: (1) invariant formulation of the "quasi-linear system of hyperbolic partial differential equations" (denoted by "q.l.s.h.p.d.e." in the future) associated with a generalized system of conservation laws in general relativity; (2) definition and basic properties of weak solutions of these conservation laws. It is shown that the "q.l.s.h.p.d.e." cannot be characterized by a "fully invariant" scheme in Riemannian space. However, weak solutions of the conservation laws may be defined in an invariant manner and similar to that in Euclidean space, so that:

(1) \( C^1 \) solutions of the "q.l.s.h.p.d.e." are weak solutions of the conservation equations;
(2) the weak solutions are characterized by a generalized type of Rankine-Hugoniot relations. The proofs are given by properly modifying those of Courant and Hilbert ([Methods of mathematical physics, Vol. II, Partial differential equations], by R. Courant, Interscience, New York, 1962) and Jeffrey and Taniuti ([Nonlinear wave propagation], Academic Press, New York, 1964). The fact that the above theory is nonvacuous is shown by giving an example which is a modification of one given by Jeffrey and Taniuti. (Received September 23, 1964.)
Theorem 4. If $F$ is lower semicontinuous and the elements of $F$ are compact, then $D$ has a base of countable order. There are some analogous theorems having hypotheses relaxing considerably the requirement that the space be regular. Some applications of the above results are afforded by theorems stated in Abstract 64T-401 and the theorem of Arhangel'skiĭ referred to there. (Received September 23, 1964.)
INDEX OF ABSTRACTS
Volume 11, 1964

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Title and Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aanderaa, Stal.</td>
<td>See Dreben, Burton.</td>
</tr>
<tr>
<td>Abbott, J. C. and Small, James</td>
<td>Erratum to &quot;Isomorphism theorems in implication algebra,&quot; 152.</td>
</tr>
<tr>
<td>Abian, Alexander</td>
<td>On theories of sets and the independence of set-theoretical axioms, 377; On uniform structures, 675.</td>
</tr>
<tr>
<td>Abian, Alexander and LaMacchia, S. E.</td>
<td>Some consequences of the axiom of Power Set in finite theories of sets, 379.</td>
</tr>
<tr>
<td>Aczel, J.</td>
<td>Homotopism problems treated with the aid of functional equations, 105.</td>
</tr>
<tr>
<td>Adler, A. W.</td>
<td>Universal Laplacians, 113; Laplacians of Kähler manifolds, 537.</td>
</tr>
<tr>
<td>Ahlberg, J. H. and Nilson, E. N.</td>
<td>Convergence properties of generalized splines, 691.</td>
</tr>
<tr>
<td>Ahlberg, J. H.; Nilson, E. N. and Walsh, J. L.</td>
<td>Orthogonality properties of the spline function, 468; Extremal, orthogonality and convergence properties of multi-dimensional splines, 468; Fundamental properties of generalized splines, 680; Higher-order spline interpolation, 767.</td>
</tr>
<tr>
<td>Ahlborn, T. J.</td>
<td>See Bhargave, T. N.</td>
</tr>
<tr>
<td>Akutowicz, E. J.</td>
<td>Spectral approximation, 343.</td>
</tr>
<tr>
<td>Al-Salam, W. A.</td>
<td>A characterization of a certain class of orthogonal polynomials, 565.</td>
</tr>
<tr>
<td>Amir, Dan</td>
<td>Projections of norm &gt; 3 on continuous function spaces, 568.</td>
</tr>
<tr>
<td>Anastasio, Salvatore</td>
<td>Maximal abelian subalgebras in hyperfinite factors, 383.</td>
</tr>
<tr>
<td>Anderson, R. D.</td>
<td>The Hilbert cube as a product of dendrons, 572; On a theorem of Klee, 758.</td>
</tr>
<tr>
<td>Anderson, R. V.</td>
<td>The geometry of triangles in the Minkowski relativistic plane, 143; The nine point hyperbolic circle C_9 of a triangle T in the Minkowski plane M_2, 448; The circumscribed and tangent hyperbolic circles of a triangle in the Minkowski plane, 449.</td>
</tr>
<tr>
<td>Appel, K. I. and McLaughlin, T. G.</td>
<td>On complementary regressive sets, 139.</td>
</tr>
<tr>
<td>Appling, W. D. L.</td>
<td>A convergence theorem, Preliminary report, 131; Refinement-unbounded interval functions and absolute continuity, Preliminary report, 144; Characterizations of some absolutely continuous additive set functions, Preliminary report, 240; Nondecreasing functions, interval functions and real Hilbert spaces, Preliminary report, 385; Set functions and an extremal problem in absolute continuity, Preliminary report, 459; A note on nonnegative-valued set functions, Preliminary report, 594; Set functions and zero sets, Preliminary report, 677.</td>
</tr>
<tr>
<td>Armentrout, Steve</td>
<td>A property of a decomposition space described by Bing, 369; Point-like decompositions of E^3 with a compact 0-dimensional set of nondegenerate elements, 540.</td>
</tr>
<tr>
<td>Armstrong, J. W.</td>
<td>On the indecomposability of torsion free groups, 55.</td>
</tr>
<tr>
<td>Aronszajn, Nachman</td>
<td>The Banach algebra of multiplier on a functional Banach space, 98.</td>
</tr>
<tr>
<td>Arsove, M. G. and Johnson, Guy</td>
<td>A conformal mapping technique for infinitely connected regions, 569.</td>
</tr>
<tr>
<td>Artemiadis, N.</td>
<td>On a class of holomorphic functions, 547.</td>
</tr>
<tr>
<td>Asenjo, F. G.</td>
<td>Theory of multiplicities, 206; A calculus of antinomies, 663.</td>
</tr>
<tr>
<td>Askey, Richard, and Wainger, Stephen</td>
<td>Mean convergence of Laguerre series, 597; A trans-</td>
</tr>
</tbody>
</table>
plantation theorem for ultraspherical series, 669.

Aull, C. E. Identity theorems for generalized variations, 389; Some theorems on the rank of a matrix, 542; Nilpotent matrices, 692; An automorphism theorem for matrices, 772.

Auslander, Louis. Algebraic groups attached to complete compact locally affine spaces, 123.

Auslander, Maurice and Brumer, Armund. Brauer groups of fields with discrete rank one valuations, 119.


Axt, Paul. Bounded recursion and iterated recursion, 224; Classes of primitive recursive functions by restricting the number of variables, 234; Iteration of relative primitive recursion, 340.

Aziz, A. K. On a nonlinear integral equation, 223.


Azpeitia, A. G. and Dickinson, D. J. A decision rule in the simplex method that avoids cycling, 308.

Babcock, W. W. Concerning linear homogeneous continua, 368.

Bacon, Philip. Coincidence of real valued maps from the n-torus, 370.

Bade, W. G. and Curtis, P. C., Jr. Imbedding theorems for Banach spaces and commutative Banach algebras, 337.


Bailey, P. B. Bohr's quantization rule. Preliminary report, 96; Eigenvalues of Schrödinger's equation via a phase function, 564.


Ballard, D. J. Abstract recursive function theory, 245.

Banchoff, T. F. Minimally embedded 2-dimensional polyhedral manifolds, 125.

Barback, Joseph. Recursive functions and regressive isols. Preliminary report, 146; Three notes on regressive isols, 574; Double series of isols. Preliminary report, 582.


Barnhill, R. E. Convergence of quadratures on complex contours, 366.


Bass, Hyman. Grothendieck determinants, 120.

Baxter, G. E. An ergodic theorem with weighted averages, 247; A general ergodic theorem with weighted averages, 464.

Baxter, G. E. and Joichi, J. T. On functions that commute with full functions, 142.

Baxter, W. E. Concerning the commutator subgroup of a simple ring, 543.

Bazinet, Jacques. Fatigue Finsler geometry, 579.

Bean, R. J. Decompositions of $E^3$ with a null sequence of starlike equivalent nondegenerate elements are $E^3$, 558.

Bear, H. S. A geometric characterization of Gleason parts, 309; Integral representation of functions on parts of a locally compact space, 561.


Beckenbach, E. F. Isoperimetric inequalities for related conformal maps, 390; Superadditivity inequalities, 390.


Beideman, J. C. A radical for near-ring modules, 237.

Beineke, L. W. and Harary, Frank. The genus of the n-cube, 80; On the thickness of the complete graph, 80.

Belluce, L. P. and Jain, S. K. Power commutative rings, 308; Prime rings with a one-sided ideal satisfying a polynomial identity, 554.

Beltrami, E. J. and Wohlers, M. R. On a converse to a theorem of Fatou, 663.

Ben-Israel, Adi and Charnes, A. Polarity, duality and cones with general ordered scalars, 230.

Bennett, Ralph. Embedding products of chainable continua, 352.

Benson, G. M. A hierarchy in the theory of implicit definability, 689.

Berens, H. See Butzer, P. L.


Berger, Robert. The undecidability of the domino problem, 537.

Bernardi, S. D. Circular regions covered by schlicht functions, 128.


Beyer, W. A. The pseudo-sparseness of the inverse sequence of a sparse matrix sequence, 342.

Bhargava, T. N. Some results in theory of digraphs, 230.

Bhargave, T. N. and Ahlborn, T. J. Directed graphs and point-set topology, 341.

Bhatia, N. P. On a theorem of Malkin, 571.

Biggs, R. G. See Tamura, Takayuki.


Bing, Kurt. A special case of the axiom choice, 320.

Bing, R. H. and Borsuk, K. Some remarks concerning topologically homogeneous spaces, 237.

Birkhoff, Garrett. Error bounds for spline fits, 455.


Blakley, G. R. Some positive symmetric quadratic transformations have a convergent sequence of iterates, 363.

Blefko, R. L. See Mr6wka, S. G.

Blefko, R. L. and Mr6wka, Stanislaw. On E-completely regular transformations, 556.


Blum, E. K. Enumeration of recursive sets by Turing machine, 218; A direct proof that partial recursiveness implies partial computability, 246; Minimization of functionals with equality constraints, 589; Finitely generated free subsemigroups of a free semigroup, 686.


Boehm, B. W. Functions whose best rational Tchebycheff approximations are polynomials, 143; The degree of convergence of best rational Tchebycheff approximations, 144; Convergence of best rational Tchebycheff approximations, 144; Existence of best rational Tchebycheff approximations, 147.

Boehme, T. K. The positive cone in the space of locally integrable functions, 533.


Bogdanowicz, Witold and Krauze, Tad. On a functional-integral equation, 145.

deBoor, Carl and Lynch, R. E. General spline functions and their minimum properties, 681.

Borosh, I. and Fraenkel, A. S. Fractional dimension of a set of transcendental numbers, 555.

Borsuk, K. See Bing, R. H.

Bose, R. C. See Bruck, R. H.

Bose, R. C. and Bruck, R. H. Linear representations of projective planes in projective spaces, 565.

Bourne, S. G. On Pontryagin-van Kampen duality for a locally compact abelian semigroup, 322.

Brace, J. W. and Nielsen, R. M. Completions, 106.

Bradley, J. S. A canonical form for a boundary value problem involving a quasi-differential operator of Euler type, 755.

Bragg, L. R. A Rodrigue's formula for the Laguerre polynomials, 664.


Bramble, J. H. and Hubbard, B. E. A finite difference analog of the Neumann problem for Poisson's equation, 95.

Brandstein, A. G. 2-manifolds as maximal ideal spaces, 770.

Branner, N. P. On symmetry in a summability theory, 319.

Brechner, B. L. On the dimensions of certain spaces of homeomorphisms, 752.


Brickell, Frederick. Differentiable manifolds with an area measure, 364.

Brothers, J. E. Integral geometry in homogeneous spaces, 585.

Brown, D. R. Topological semilattices on the 2-cell, 365.

Brown, E. H., Jr. and Peterson, F. P. Relations among mod p characteristic classes, 359.

Brown, H. I. See Cowling, V. F.

Brown, K. R. and Wang, Hao. Short definitions of the class of ordinals, 455; Finite sets and axioms of limitation, 456.


Brown, R. F. On a homotopy converse to the Lefschetz fixed point theorem, 743.


Brualdi, R. A. Permanent of the direct product of matrices, 770.


Bruck, R. H. and Bose, R. C. The construction of translation planes from projective space, 122.

Brumer, Armund. See Auslander, Maurice.

Brunk, H. D. Integral inequalities for functions with nondecreasing increments, 78.

Buchsbaum, D. A. and Rim, D. S. Cramer's rule, complexes and multiplicity, 119.

Buck, R. C. The solution of systems of functional equations, 104.

Buckholtz, J. D. Power series whose sections have zeros of large modules, 86; Zeros of partial sums of power series for certain entire functions, 454.

Bucy, R. S. Stability and positive supermartingales, 381.

Bumcrot, R. J. Linearity geometry, 358.

Burgess, C. E. Pairs of 3-cells with intersecting boundaries in $E^3$, 93; 2-spheres with a tame Cantor set of wild points, 310.

Burlak, J. See Gergen, J. J.

Burnell, D. G. See Tamura, Takayuki.

Buschman, R. G. Dual integral equations, Bessel kernels, 137; Substitution formulas for Laplace transformations, 247.

Busemann, Herbert. Length preserving maps, 124.

Bush, K. A. The nonexistence of Hadamard circulants, 541.


Butler, Terence and Carroll, R. W. Some remarks on vector analysis, 239.

Butzer, P. L. and Berens, H. Semi-group operators and Dirichlet's problem, 566.

Cairns, S. S. Skewness, embeddings, and isotopies, 338.

Cameron, R. H. Error estimates for approximations to Feynman integrals. Preliminary report, 231; Error estimates for approximations to Feynman integrals, 532.

Cannon, J. R. An a priori estimate for the continuation of the solution of the heat equation in the space variable, 351; Numerical continuation backwards in time for solutions of the heat equation, 541.

Cantor, D. G. and Mills, W. H. Determination of a subset from certain combinatorial properties, 224.

Cantrell, J. C. Some relations between the annulus conjecture and union of flat cells theorems. Preliminary report, 241; Some theorems on the union of flat cells, 360; Topological embeddings of spheres, 560.


Cargo, G. T. See Shisha, Oved.

Carlitz, Leonard. Rings of arithmetic functions, 117; An extension of the congruences of Bauer and Lubelski, 148; The distribution of irreducible polynomials in several indeterminates, 148; Recurrences for the Bernoulli and Euler polynomials. II, 148; Generalized Dedekind sums, 148; A theorem on generalized Dedekind sums, 565; A note on multiple exponential sums, 675; Some multiple sums and binomial identities, 767.

Carroll, F. W. Functions whose differences belong to $I[P^{0,1}, 233$.

Carroll, R. W. Problems in linked operators. II. Preliminary report, 468.

———. See Butler, Terence.

Caveny, D. J. Bounded Hadamard products of functions belonging to Hardy p-classes. Preliminary report, 753.
Chacon, R. V. Ordinary means imply recurrent means, 556.

Chandler, R. E. Cellular subcomplexes of piecewise linear manifolds, 93.

Chaney, R. W. Measurability theorems in the transformation theory for measure space, 745.

Chang, C. C. On the formula "there exists x such that f(x) for all f in F", 587; Two refinements of Morley's method on omitting types of elements, 679.

Chang, S. S. An extension of Ascoli's theorem and its applications to the theory of optimal control, 112.

Chao, C. Y. Graphs and groups, 340.

Charnes, Abraham. See Ben-Israel, Adi.


Chatterji, S. D. Certain induced measures and the fractional dimensions of their "supports", 745.

Chen, Y. M. Some special results on the asymptotic behavior of the solution of the initial-boundary value problem of the two-dimensional wave equations for a large time, 63.

Cheney, E. W. Approximation by generalized rational functions, 461.

Cheney, E. W. and Goldstein, A. A. Existence theorems for Tchebycheff approximation, I, 139; Characterization theorems for Tchebycheff approximation, 140.

Chihara, T. S. On recursively defined orthogonal polynomials, 79.

Chilton, B. L. The stellated forms of the sixteen-cell, 662.


Chrislock, J. L. See Tamura, Takayuki.

Chrislock, J. L.' and Tamura, Takayuki. Semi-groups whose right regular representation is a group, 549.

Christy, J. H., Jr. Expansive transformation groups, 387.

Chu, Hsin. A note on compact transformation groups with a fixed end point, 761.

Chu, S. C. and Diaz, J. B. On a mixed boundary value problem for linear hyperbolic equations in two independent variables, 235.

Church, Alonzo. A further generalization of Laplace's transformation, Preliminary report, 249; A generalization of Laplace's theorem, Preliminary report, 249.

Church, P. T. Factorization of differentiable maps with small branch set dimension, 62.

Churchill, R. V. Integral transforms associated with boundary conditions of the third type, 353.

Clark, A. and Hazon, Dov. A matrix split for PR matrices using arbitrary constants, 584.

Cobb, John. A k-complex is tame in $E^0(n \geq 3k + 1)$ if each of its simplices is tame, 762.


Cochran, J. A. Remarks on the zeros of certain combinations of Bessel functions, 98.

Cohen, D. S. Zeros of Bessel functions and eigenvalues of non-self-adjoint boundary value problems, 104.

Cohen, Eckford. Remark on a set of integers, 141; The relative mean value of the Euler function, 675; The number of zeroid elements $\pmod{n}$, II, 767.

Cohen, H. B. The k-normal completion of function lattices, 381.

Cohn, Harvey and Knopp, M. L. A note on automorphic forms with real period polynomials, 53.

Cohn, J. A. and Livingstone, Donald. Group rings of finite groups. II. Preliminary report, 122.

Cohn, P. M. Free ideal rings, 766.

Cole, S. N. Real time computation by n-dimensional iterative arrays of finite automata, 318.

Coleman, B. D.; Duffin, R. J. and Mizel, V. J. Instability and uniqueness results for a third order PDE on a strip, 348.


Collins, H. S. Affine images of certain sets of measures, I, 596; Affine images of certain sets of measures, II, 668.

Comfort, W. W. Retractions and other continuous maps from $\mathbb{R}^n$ onto $\mathbb{R}^n - X$, 557.

Comfort, W. W. and Negrepontis, Stelios. The ring $C(X)$ determines the category of $X$, 683.

Comfort, W. W. and Ross, K. A. Pseudocompactness and uniform continuity in topological groups, 322.

Conlon, L. W. On the topology of the exceptional symmetric spaces III and IV, 379; Root systems for symmetric triads, 380; On the manifolds of $K/K_T$, 380.
Conlon, L. W. and Whitman, A. P. Lower bounds to holonomy, 685.

Connor, P. E. and Floyd, E. E. Fibering within a cobordism class, 242; Fibering within a cobordism class, II, 374.

Cook, Howard. An application of a theorem of F. B. Jones, 357.


Coppage, W. E. and Moran, D. A. A class of permutations associated with finite binary sequences, 376.


Courter, R. C. The maximal corational extension by a module. Preliminary report, 343.

Cowling, V. F. and Brown, H. I. On \( f \)-methods of summation, 575.

Coxeter, H. S. M. Accessibility with respect to a polarity, 228.


Crowe, D. W. A model for finite hyperbolic planes in \( GF(q^2) \), q odd, 685.

Csőrgő, Miklós. Some Smirnov type limit theorems of probability theory, 72.

Cullen, C. G. An integral theorem for analytic intrinsic functions on quaternions, 89.

Cullen, H. F. Generalized separability for product spaces, 552.


Curtis, M. W. On some commutative sets in a free monoid, 250.

Curtis, P. C., Jr. The degree of approximation by linear operators, 337.

_______. See Bade, W. G.

Curtiss, J. H. On the coefficients of the Faber polynomials, 557.

Cusick, T. W. On certain convex functions, 447.

Czarnecki, A. Z. See DeCicco, John.

Danese, A. E. Some properties of Jacobi polynomials, 105.

Darwin, J. T., Jr. A representation theorem for linear operators, 105.

Dauns, John and Widder, D. V. Convolution transforms whose inversion functions have complex roots, 673.


Davis, K. J. A generalization of the Dirichlet product of arithmetical functions, 139.


Dawson, D. A. The local diffusion associated with a harmonic sheaf, 543.

Dawson, D. F. Some theorems on linear summability methods, 227; On a note of E. Stepanić, 584; On a result of D. Gallarati concerning semigroups, 670.

Day, J. M. A monotone relation which preserves pseudocircles, 751.

Dean, D. W. Subspaces of \( C(H) \) which are direct factors of \( C(H) \), 344.


DeCicco, John and Anderson, R. V. Some differential geometric theorems of the Minkowski plane \( M_2 \), 388; Some properties of isothermal families of curves in the Minkowski plane \( M_2 \), 577.

DeCicco, John and Czarnecki, A. Z. Fundamentals of Finsler geometry, 676.

DeCicco, John and Krajciwicz, P. M. Physical systems in a bicentral positional field of force in the Euclidean plane, 136.

DeCicco, John and Ramachandran, S. T. The initial rates of departure in a directional field of force, 141.

DeCicco, John and Sullivan, Michael. Physical systems \( S_k \) of the curvature type in the Euclidean plane, 390.

DeCicco John and Synowiec, J. A. Theory of isoclines in a complex inner product space, 138; Some properties of isoclines of two dimensions, 382; Theory of isoclines in pseudo-conformal space, 383; Pseudo-orthogonal nets of multi-orthogonal families, 676; The geometry of the complex inner product, 759.

Deliyannis, P. C. Noncommutative imprimitivity
DeMar, R. F. A uniqueness theorem, 91.

Derdérian, J.-C. Some applications of residuated mappings, 225.

Derzko, N. A. and Pfeffer, A. M. Bounds for the spectral radius of a matrix, 595.

DeVore, R. H. A best possible inequality for the lowest points of the fundamental domain for the Hilbert modular group for $\mathbb{R} (\sqrt{5})$. Preliminary report, 309.

Deysach, L. G. and Sell, G. R. A necessary and sufficient condition for the existence of almost periodic motions, 559; A note on almost periodic solutions of ordinary differential equations, 670.

Diaz, J. B. See Chu, S. C.

Diaz, J. B. and Metcalf, F. T. Stronger forms of inequalities of Kantorovich and Strang for operators in a Hilbert space, 92.

Dickinson, D. J. See Azpeitia, A. G.

Dickson, D. G. Zeros of exponential sums, 68.

Dickson, S. E. A torsion theory for Abelian categories, 97.


Di Paola, R. A. Pseudo-complements: their extent, creative sets, and effectively inseparable sets, 317; Pseudo-complements and ordinal logics based on consistency statements, 372.

Distler, R. J. The domain of univalence of certain classes of meromorphic functions, 67.


Douglas, R. G. Contractive projections on an $L_1$-space, 61; Idempotents in the group algebra of a compact group, 140; On extreme points and subspace density. Preliminary report, 142.

Downing, J. A. See Young, D. M., Jr.

Doyle, P. H. A sufficient condition that an arc be cellular, 247.


Drescher, Melvin. Games with random variables as payoffs, 541.

Dressel, F. G. See Gergen, J. J.

Dubois, D. W. Applications of analytic number theory to the study of type sets of torsion free Abelian groups. I, 456.

Duda, Edwin. A locally compact separable metric space is almost invariant under a closed mapping, 52.

Duffin, R. J. See Coleman, B. D.

Duffin, R. J.; Hazony, D. and Morrison, N. Network synthesis through hybrid matrices, 240.

Duke, R. A. Extensions of open mappings on graphs, 316.

Dulmage, A. L. See Mendelsohn, N. S.


Dyson, V. H. The word problem and residually finite groups, 743.

Easton, W. B. Proper classes of generic sets. Preliminary report, 205.

Eberlein, W. F. Abstract charge conjugation, 329.

Edmonds, Jack. Extreme partitions of matroids, 552.


———. See Cantrell, J. C.

Edwards, J. E. On the existence of solutions of the steady-state Navier-Stokes equations for a class of nonsmooth boundary data, 347.

Effros, E. G. Transformation groups and Banach algebras, 208.

Eggleston, H. G. and Zirakzadeh, A. A property of two chords which divide a convex curve into four arcs of equal length, 62.

Ehrenpreis, Leon. The Cauchy problem for overdetermined systems, 124.

Eisenberg, Edmund and Wong, Eugene. On piecewise linear separability, 100.

Eisenberger, Isidore. Tests of hypotheses using quantiles, 117.

Elgot, C. C. and Rabin, M. O. Decision problems of extensions of second order theory of successor, 131; On the first order theory of generalized successor, 132.

Ellentuck, Erik. The power structure of Dedekind infinite cardinals, 599; Combinatorial functions of Dedekind infinite cardinals, 674.

Elliott, R. J. A result in spectral synthesis, 670.

Engeler, Erwin. Ultrastructures in first-order model theory, 465, 693.

Epstein, Bernard. See Schiffer, M. M.
Erdös, Paul and Goodman, A. W. The representation of a graph by set intersections, 749.

Ernest, J. A. A new group algebra for locally compact groups, 82; A new group algebra for locally compact groups. II, 341; The lattice of quasi-equivalence classes of group representations, Preliminary report, 553.

Evans, Trevor. Identities, finite embeddability and residual finiteness, 755.

Everett, C. J. An inequality on doubly stochastic matrices, 134.


Fadell, Edward. See Brown, R. F.


Faith, Carl and Walker, E. A. Characterizations of Quasi-Probenius rings, 766.

Feit, Walter and Higman, Graham. On the nonexistence of certain generalized polygons, 120.


Feller, E. H. Right noetherian injective rings, 585.

Fenichel, R. R. On the nonexistence of a type 1 grammar for FAP, 248.

Fiedler, Miroslav. Hankel matrices and 2-apolarity, 367.

Fife, P. C. A problem in boundary layer approximation, 247; Toward the validity of the Prandtl approximation in a boundary layer, 349.

Fine, A. I. An equivalent form of the axiom of choice, 546.


Finney, R. L. A product theorem for compact 2-manifolds, 131.

Firey, W. J. Convex bodies with preassigned sum of principal radii of curvature, Preliminary report, 443.

Fischer, P. C. Provable degrees of unsolvability, 239.


Fitzpatrick, Ben, Jr. On dense subspaces of Moore spaces, 531.


Flatto, Leopold. Partial differential equations and difference equations, 68.

Ford, E. E. See Conner, P. E.

Ford, Jo. Imbedding collections of compact totally disconnected subsets of $E^2$ in continuous collections of mutually exclusive arcs, 749.


Fort, M. K., Jr. and Schuster, Seymour. Convergence of series whose terms are defined recursively, 764.

Fort, Tomlinson. The gamma function with varying difference intervals, 752.


Fraenkel, A. S. See Borosh, I.

France, Michel Mendes. A set of nonnormal numbers, 681.

Frank, Stanley. See Hutcherson, W. R.

Franklin, S. P. An isomorphism theorem, 111; Open and image-open relations, 750.

Freyd, Peter. Abstract theorems and concrete problems, 120.

Fridy, J. A. Divisor summability methods, 211.


Frisch, H. L. See Morrison, J. A.

Fugate, J. B. On decomposable chainable compact continua, 539.

Fulp, R. O. Reflexive semigroups, 571.

Fulp, R. O. and Hill, Paul. On homomorphisms and characters of inverse semigroups, 754.

Fusaro, B. A. Spherical means in harmonic spaces, 540.

Gaifman, Haim. Measurable cardinals and constructible sets, 771.

Galmarino, A. R. and Panzone, Rafael. $L^p$-spaces with mixed norm, for $P$ a sequence, 212.

Ganea, Tudor. A generalization of the EHP-sequence, 108; On some functors generalizing $\Omega$, 336.

Gangolli, R. A. A Tauberian theorem for certain semi-simple Lie groups, 333.

Garland, Howard. On the cohomology of lattices
in solvable Lie groups. Preliminary report, 362.


Gilbert, R. C. Perturbation of a Sturm-Liouville operator by a finite function, 463.

Gilbert, R. P. Concerning the location of singularities of solutions to certain classes of elliptic partial differential equations in four variables, 112; On a class of elliptic partial differential equations in four-variables, 374; Multivalued harmonic functions in four variables, 458; On generalized axially symmetric potentials whose associates are distributions, 689.

Gilbert, R. P. and Howard, H. C. On solutions of the generalized axially symmetric wave equation represented by Bergman operators, 457; On a class of elliptic partial differential equations, 457; On solutions of the generalized axially symmetric Helmholtz equation generated by integral operators, 588.

Gillette, R. M. Isotopies which remove closed sets of rank 1 critical points of \( \infty \)-smooth homeomorphisms of the 2-sphere, 109.

Gillman, David. Taming 2-manifolds in hyperplanes of \( E^4 \), 591.

Gilmour, R. W., Jr. On valuation ideals, 384; Eleven nonequivalent conditions on a commutative ring, 687.

Gindler, H. A. An operational calculus for meromorphic functions, 93.


Gladstone, M. D. Propositional calculi with decision problems of any required recursively enumerable degree of unsolvability, 454.

Glaser, L. C. Contractible complexes in the n-sphere, 538; Contractible noncollapsible \((n - 1)\)-complexes in the n-sphere from images of n-manifolds with boundary, 581.

Glasner, Moses. See Sario, Leo.

Glauberman, George. On relatively prime automorphism groups, 128; Correspondence of characters in relatively prime automorphism groups, 129; On loops of odd order, 353; Fixed point subgroups which contain centralizers of involutions. Preliminary report, 760.


Gluck, H. R. Homogeneity of certain manifolds, 313.

Goes, G. W. Boundedly divergent sequences in FK-spaces, 444; Classes of conjugate gap Fourier series, 466, 600; On Fourier-Stieltjes sine-series, 688.

Goldberg, Michael. Rotors of variable regular polygons, 536.

Goldman, Lawrence. Differential geometry of a curve and Picard-Vessiot, 337.

Goldstein, A. A. Minimizing functionals on Hilbert space, 220; Stability of generalized rational approximation, 232.

Goldstein, A. A. and Kripke, B. R. Constrained minimization via unconstrained minimization, 220.


Gonshor, Harry. Density types of sets of integers, 206.

Gonzalez, M. O. A generalization of the Riemann-Stieltjes integral, 351.

Goodman, A. W. A generalization of the Riemann-Stieltjes integral, 351.

Goodman, A. W. Set equations, 527.

Goodman, R. W. Prediction theory for unitary operators, 221.

Goodwin, B. E. On the realization of the eigenvalues of integral equations whose kernels are entire or meromorphic in the eigenvalue parameter, 91.

Gordon, Basil. Projective homomorphisms of division rings, 52.
Gordon, Basil and Motzkin, T. S. The number of zeros of a polynomial in a division ring, 566.

Gorenstein, Daniel and Walter, D. H. The classification of finite groups with dihedral Sylow 2-subgroups, 123.


Granis, Andrzej. Stability properties of solutions of certain nonlinear equations, 211.

Granirer, E. E. On the invariant mean on topological semigroups, 234.

Grassl, R. M. See Stroot, M. T.

Gratzer, G. A. On the class of subdirect powers of a finite algebra, 67.

Gray, Alfred and Shah, S. M. Holomorphic functions with gap power series, 88; On entire functions and a conjecture of Erdős, 88.

Greathouse, Charles. Locally flat strings, 359; Cellularity at the boundary of a manifold, Preliminary report, 446; A criterion for cellularity at the boundary (CAB) of a manifold, 755.

Greechie, R. J. A class of orthomodular nonmodular lattices, 219.

Green, L. W. Geodesic inequalities, 123.

Greenberg, M. J. Perfect closure of a scheme, 100.

Greenleaf, F. P. Characterization of group algebras in terms of their translations, Preliminary report, 71.

Grenier, J. W. On some associated Pellian equations, 564.

Grimeisen, Gerhard. The product of cluster spaces, 757.


Guggenheimer, H. W. Unsymmetric Minkowski geometry, II. Hill's equation, 236.


Gulick, S. L. The bidual of a locally multiplicatively-convex algebra, 86.

Gundersen, R. M. Quasi-one-dimensional magnetohydrodynamic flow with heat addition, II. Oblique field, 65; General theory of simple wave flows in one-dimensional magnetohydrodynamics, 349.

Guterman, M. Operational calculus for functions of n variables, 376.

Hagan, M. R. Equivalence of connectivity maps and peripherally continuous transformations, 674.

Hahn, K. T. The pythagorean theorem with respect to the Bergman metric over a hypersphere in $C^2$, 311; A generalization of the Harnack inequality to the theory of functions of n complex variables, 577.

Haimo, D. T. Inversion for Hankel convolutions, 79; Representation for Hankel convolutions, 140.

Hajnal, Andras. On the topological product of the discrete spaces, 578.


Halfin, Shlomo. First integrals of prime differential ideals, 102.

Hall, Marshall, Jr. and Knuth, D. H. Groups of exponent 4, 120.

Halpern, J. D. and Lévy, Azriel. The ordering theorem does not imply the axiom of choice. Preliminary report, 56.

Hammer, P. C. Extended topology: connected sets and Wallace separations, 111; Extended topology: Caratheodory's Theorem on convex hulls, 759.


Hanna, M. S. and Smith, K. T. The Dirichlet problem in piecewise smooth domains, 335.

Hanson, R. J. A formal reduction theorem for a system of differential equations with a turning point, 467; Reduction and classification for second order systems of differential equations with a turning point, 540.

Harary, Frank. See Beineke, L. W.

Harary, Frank and Tutte, W. T. The number of plane trees with a given partition, 761.

Harris, W. A., Jr. and Sibuya, Yasutaka. Asymptotic solutions of systems of nonlinear difference equations. II, 82; General solution of nonlinear difference equations, 105.

Hartnett, W. E. Graph topology, 535; Total topological spaces, 580.

Hasumi, Morisuke and Srinivasan, T. P. Invariant subspaces of continuous functions, 108.

Hattemer, J. R. Boundary behaviour of temperatures. I. Preliminary report, 151; Boundary behaviour of temperatures. II. Preliminary report, 151.

Hackman, M. M. Exponentiation of measurable operators, 345.
Hay, L. S. Effective inseparability of sets of indices of partial recursive functions, 387.

Hayashi, H. S. Computer investigation of difference sets, 598.

Hayashi, Yoshio. The Dirichlet and Neumann problems for the wave equation and slotted coaxial cylindrical boundary, 99.

Hayden, T. L. Continued fraction approximation to functions, 753.

Hayden, T. L. and Merkes, E. P. Chain sequences and univalence, 83.

Hayes, D. R. Improvement of the error term in an asymptotic formula, 116; A polynomial generalized Gauss sum, 539.

Hazony, Dov. See Clark, A.

Hedstrom, G. W. A generalization of Holmgren's uniqueness theorem, 666; The power-boundedness of fourier series, 763.

Hellerstein, Simon. On the distribution of values of a meromorphic function and a theorem of H. S. Wilf, 573.


Higman, Graham. See Feit, Walter.


Hill, P. D. See Fulp, R. O.

Hill, P. D. and Megibben, C. K. Minimal pure subgroups in primary Abelian groups, 102.

Hinman, P. G. Some theorems on relative recursiveness of higher type objects, 677.

Hinrichs, L. A. Nonideal topologies on the ring of integers, 117.

Hirschman, I. I., Jr. Toeplitz sections on groups, 678.

Hobby, C. R. Methods of obtaining doubly stochastic matrices from positive matrices. Preliminary report, 234; The \( H_p \) subgroup of a finite \( p \)-group. Preliminary report, 569.

Hochstadt, Harry and Hessel, Alexander. Plane wave scattering by a doubly corrugated unit cell, 593.

Hodges, J. H. A bilinear matrix equation over a finite field, 239.

Hoffman, W. C. Solution of the initial value problem for the Riccati equation, 229.

Hooper, Robert. Exact convexity, 536.


Horadam, A. F. Groups leaving certain loci in \([5]\) invariant, 329.

Horadam, E. M. Ramanujan's sum for generalized integers, 357.

Horn, W. A. A generalization of Browder's fixed point theorem, 325.

Horowitz, Maurice. Mean random path across a square, 55.

Hosay, Norman. Erratum to "The sum of a cube and a crumpled cube is \( S^3 \), 152; Some sufficient conditions for a continuum on a 2-sphere to lie on a tame 2-sphere, 370.
Houh, C. S. The integrability of a structure on a differentiable manifold, 226.

Howard, H. C. Oscillation criteria for even order differential equations, 673.


Hubbard, B. E. See Bramble, J. H.

Huckleman, F. C. On extremal decompositions of a quadrilateral, 84.

Hudson, S. N. Lie loops with invariant uniformities, 340; Lie loops with invariant uniformities, II, 575.

Hunt, R. W. and Silber, Robert. A boundary-value control problem with guidance theory applications, 94.

Husain, Taqdir. B(3')-spaces and the closed graph theorem, II, 150.

Hutcherson, W. R. and Frank, Stanley. A quartic surface image on a hypersurface of thirty eight dimensions, 530.

Ihrig, A. H. Remarks on Post normal systems, 128; Post-lineal theorems for r.e.d.u., 456; Coincidence of partial propositional calculi, 457; Certain Thue systems and r.e.d.u.'s, 597; Classes of partial propositional calculi and r.e.d.u.'s, 669.


Jain, S. K. See Belluce, L. P.
Jurkat, W. B. On Cauchy's functional equation, 240.

Kabruoh, I. An application of the generalized universal lifting property, 579.


Kane, Julius. The quadrature of some exponential transforms of Bessel functions, 58.

Kantorovitz, Shmuel. Classification of operators by means of the operational calculus, 118.

Kappos, D. A. and Papangelou, F. An o-topological lattice which cannot be regularly extended to a σ-complete o-topological one, 96.

Kass, Seymour. A property of the set of coefficients of a multinomial expansion, 531.


Kegley, J. C. Convexity with respect to Euler-Lagrange differential operators, 760.

Kelloggs, J. A. Two new characterizations for plane quasiconformal mappings, 109.

Kellogg, C. N. Centralizers of H*n-algebras, 752.

Kellogg, R. B. Alternating direction methods for operator equations, 81.

Kelly, G. M. On the faithfulness of homology functors, 61.


Kent, C. F. Reduction of ordinal recursion, 94; Consistency of number theory, 542.

Keown, E. R. Wigner representations by means of Slater irreducible matrices, 551.


Kezlan, T. P. Higher commutators of bounded nilpotence, 529.

 Kimber, J. E., Jr. An extended Bolzano-Welerstrass theorem, 593.


Klee, Victor. Diameters of polyhedral graphs, 354; Paths in polyhedra, 746; Heights of convex polytopes, 752.

Kleinfeld, Erwin. Middle nucleus = center in a simple Jordan ring, 90.

Kleppner, Adam. Representations induced from compact subgroups, 342.


Knopp, M. I. See Cohn, Harvey.


Koch, R. J. Threads in compact semigroups, 671.

Koethe, G. M. An open mapping theorem, 100.


Kondö, Motoki. On some applications of P. J. Cohen's method in mathematical analysis, 463.

Korevaar, Jacob and Hellerstein, Simon. The real values of an entire function, 365.

Korfhage, R. R. On a sequence of prime numbers, 376.


Kosinski, A. A. On the inertia group of π-manifolds, 245.

Krabbe, G. L. All symmetric operators have a weakly-continuous functional calculus on (BV), 133; Heaviside operational calculus; algebraic derivative of an operator, 552.

Krajciewicz, P. M. Some elementary properties of analytic polygenic functions, 136.

Krauze, Tad. See Bogdanowicz, Witold.


Krishnamurthy, V. Conjugate locally convex spaces, 557.
Krishnan, V. S. Ordered products and sub-products of ordered sets, groups, modules with operators. Preliminary report, 548.


Kruse, R. L. A characterization of rings in which all subrings are ideals, 760.

_______ See Dean, R. A.

v. Krzywoblocki, M. Z. Integral operators in ordinary differential equations, 58; The last geometric theorem of Poincaré, 356; Classical and relativistic energodynamics, 764.


Kurepa, Svetozar. The Cauchy functional equation and scalar product in vector spaces, 452; Quadratic and sesquilinear functionals, 454.


Kuyk, W. An algebraic application of the wreath product, 85.

Kwun, K. W. See Curtis, M. L.

Kyner, W. T. Orbits about an oblate planet, II, 99.

LeBach, W. A. On very skew curves, 579.

Lacey, H. E. Strictly singular operators in locally convex spaces, 66.

Laffer, W. B. and Mann, H. B. Decomposition of sets of group elements, 70.


Lakshmikantham, V. Parabolic differential equations and Lyapunov like functions, 72.

LaMacchia, S. E. See Abian, Alexander.

Lambek, Joachim. A homological formulation of Goursat's theorem, 334.

Lappan, P. A., Jr. Identity and uniqueness theorems for automorphic functions, 137; On continua which are cluster sets, 237.


Lax, P. D. and Phillips, Ralph. Scattering theory, 121.

Laxton, R. R. On simple distributively generated near-rings, 579.

Label, J. E. On a simple integral and its equivalence to that of Riemann, 223.


Lee, Y.-L. Characterizing the topology by the class of homeomorphisms, 571; Homeomorphisms on manifolds, 691; Coarser topologies with the same class of homeomorphisms, 770.

Lehmer, Emma. Artiads characterized, 746.

Leicht, J. B. Axiomatic theory of additive relations, 662.

Leitner, Alfred. See Turner, W. W.


Levin, G. L. A condition for regularity in local rings, 131.

Levin, S. A. See Martin, M. H.


Levinger, B. W. and Varga, R. S. Minimal Gerschgorin sets, II, 332.

Levinson, Norman. One-sided inequalities for elliptic differential operators, 592.


_______ See Halpern, J. D.


Libera, R. J. Meromorphic close-to-convex functions, 593.

Lick, D. R. Sets of convergence of Dirichlet series, 70.


Lieberstein, H. M. Determination of the tension-stretch relation for a point in the aorta from measurement in vivo of pressure at three equally spaced points, 347.

Lima, E. L. The Rank of $S^3$, 94.

Lin, B. -L. On pseudo-reflexive Banach spaces, 590.

Lindberg, J. A., Jr. See Birtil, F. T.

Lindenstrauss, Joram. On operators which attain their norm, 135; On uniformly continuous maps between Banach spaces, 135; On non-
linear projections in Banach spaces, 213.
Linderholm, C. E. Extension of unitary opera-
tors, 674.
Lindsey, W. C. Infinite integrals containing
Bessel functions, 61.
Ling, C. H. See Yao, J. Z.
Lindinger, L. L. Some results on crumpled
cubes, 73; The union of a crumpled cube and
a 3-cell is topologically $S^3$, 127; The sum of
two crumpled cubes is $S^3$ if it is a 3-manifold,
678.
Linton, F. E. J. Autonomy in equational cat-
etories. Preliminary report, 110, 251; Commutativity
and orthogonality. Preliminary report, 147;
Autonomous categories and duality of func-
tors, 762.
Lipschutz, Seymour. The root problem in one-
eighth groups, 84.
Lissner, D. B. A generalization of a theorem
of Hermite, 310.
Livingstone, Donald. See Cohn, J. A.
Lloyd, S. P. On extreme Banach limits. Pre-
liminary report, 116; Ergodic structure of
the Banach limits, 539.
Lodato, M. W. On topologically induced gen-
eralized proximity relations. II, 66.
Looeb, H. L. Rational Chebyshev approximation,
335; Characterization of rational trigono-
metric Chebyshev approximation, 462; Gen-
eralized Haar theorem, 689; Equivalent state-
ments of generalized Haar theorem, 769.
Loeb, P. A. Erratum to "A new proof of the
Tychonoff theorem", 152.
Lorentz, G. G. Inequalities and the saturation
classes of Bernstein polynomials, 73.
Lotkin, M. M. Characteristic values of circu-
lant matrices, 65.
Loveland, D. W. Recursively random sequences.
Preliminary report, 381.
Loveland, L. D. A characterization of tame
surfaces in $E^3$, 313.
Luh, Jiang. Quasi-ideals of a regular ring, 527.
Lundell, A. T. A Samelson product in $SO(2n)$,
113.
Luxemburg, W. A. J. A remark about Sikorski's
extension theorem for homomorphisms in the
theory of Boolean algebras, 220; Ultrapowers
of normed linear spaces, 229.
Lynch, R. E. See de Boor, Carl.
Lynch, R. E.; Rice, J. R. and Thomas, D. H.
Direct solution of partial difference equations
by tensor product methods, 226; General
analysis of alternating direction implicit me-
thodes by tensor products, 232.
Lynn, M. S. and Heap, B. R. On the index of
primitivity of a non-negative matrix, 244;
A graph-theoretic algorithm for the solution of
a linear diophantine problem of Frobenius,
389; The index of convergence of a non-
negative matrix, 580; Further remarks on a
linear Diophantine problem of Frobenius, 682;
On powers of non-negative matrices, 767.
McAuley, L. F. More pointlike decompositions of
$E^3$, 460.
McCarty, G. S., Jr. Some constructions of
suitable spaces, 746.
McClurg, Arline. The sum of the degrees of the
irreducible representations of the symmetric
group, 589.
McCord, M. C. Universal P-like compacta, 593.
_______ See Fort, M. K., Jr.
McFarland, R. L. and Mann, H. B. On multipliers
difference sets, 367.
Mchaffey, R. A. The rim of a locally nilpotent
group, 664.
McKay, D. O. and Olson, F. R. Coefficients of the
Nörlund polynomials, 331.
McKelvey, Robert. Spectral measures, generalized
resolvents, and functions of positive type,
582.
McKinnan, M. A. A fatigue geometry, 391;
Fatigue geometry and "tired light", 688.
McKnight, J. D., Jr. Remarks on Kleene-
regularity, 248.
McLaughlin, T. G. A theorem of indecomposable
sets, 136; Strong reducibility on hypersimple
sets, 355; Regressive sets, supercohesion,
and permutations, 676.
_______ See Appel, K. I.
McMillan, D. R., Jr. Taming Cantor sets in $E^n$
460.
_______ See Hempel, John.
MacCamy, R. C. and Mizel, V. J. Results for a
quasi-linear hyperbolic equation, 348.
Mack, J. E. See Johnson, D. G.
MacNerney, J. S. A nonlinear integral operation,
54.
Macrae, R. E. Cardinality and cohomology, 243.
Magill, K. D., Jr. Homomorphisms of semirings
of continuous functions, 76; Regular homo-
morphisms of semirings of continuous func-
tions. Preliminary report, 373; Extending
isomorphisms of semirings of continuous functions. Preliminary report, 373; Isomor-
phisms of semigroups of continuous functions,
466; Some homomorphism theorems for a class of semigroups, 662.

Magliveras, S. S. See Moore, J. T.

Maharam, Dorothy. On orbits under ergodic measure-preserving transformations, 378.

Mahoney, R. T. J. On Howarth endomorphisms, 76.

Mahowald, M. E. Some Whitehead products in $S^n$, 306.

Maloney, J. P. See Aziz, A. K.


Manaster, A. B. A theorem on the decomposition of linear forms for the isols, 577.

Mandrekar, V. See Kallianpur, G.

Mann, H. B. See Laffer, W. B.

Mann, L. N. A note on counting isotropy subgroups, 106.

Marden, Albert. Weakly reproducing differentials on open Riemann surfaces, 77.

Marshall, A. W. and Olkin, Ingram. Inclusion theorems for eigenvalues from probability inequalities, 241; Norms and condition numbers, 527.

Martin, A. D. and Mizel, V. J. Representation of a nonlinear functional, 567.

Martin, D. A. Density, Turing degrees, and extendability relations of hyperhypersimple sets, 375.

Martin, M. H. See Levin, S. A.

Martin, M. H. and Levin, S. A. Nonlinearity and uniqueness, 214.

Martin, N. F. G. Continuity and algebraic properties of entropy, 326.

Martindale, W. S. III. Lie derivations of primitive rings, 211; Jordan isomorphisms of the symmetric elements of a simple ring with involution. Preliminary report, 762.

Martinek, Johann and Thielman, H. P. Circle theorem for the biharmonic equation. Interior problem, 85; Circle theorem for the biharmonic equation. II. Interior problem, 218; Circle and sphere theorems for the biharmonic equation. Interior and exterior problems, 221; Circle theorem related to pure bending of a circular elastically supported thin plate, 238; Laurent type expansion and general radiation condition for the reduced wave equation in three dimensions, 563.


Mayer-Kalkschmidt, Jorg and Steiner, E. F. Some theorems in set theory and consequences in partially ordered sets, 355.

Mayoh, B. H. Useful properties of Markov algorithms, 243.

Mayoh, V. G. A canonical form for some elementary functions and numbers, 242.

Megibben, C. K. On high subgroups. Preliminary report, 59; On a certain class of primary abelian groups. Preliminary report, 384; Subsocles which do not support pure subgroups, 462.

Meir, Amram. See Sharma, Ambikeshwar.

Mendelsohn, N. S. An algorithm for the solution of a word problem, 137.

Mendelsohn, N. S. and Dulmage, A. L. The exponent set of primitive matrices, 54.

Merkel, R. B. Finite semigroups with zero which are the union of their 0-minimal left ideals, 315.

Merkes, E. P. See Hayden, T. L.


Meyers, P. R. Extensions of Banach's contraction theorem. Preliminary report, 588.


Metcaif, F. T. See Diaz, J. B.

Michaels, Sister E. L. A note on simple groups, 384.


Miller, R. K. The asymptotic behavior of asymptotically almost periodic differential equations, 457.

Mills, W. H. See Cantor, D. G.

Minc, Henryk and Sathe, Leroy. Some inequalities involving the rth root of $r^r$, 537.

Minty, G. J. A theorem on maximal monotonic sets in Hilbert space, 328.

Mitchell, Josephine. Bounds for solutions of a
system of partial differential equations in a
domain with Bergman-Silov boundary surface,
690.

Mizel, V. J. See Coleman, B. D.

Moeller, J. W. A norm bound for the inverse shift operator, 83.

Mond, Bertram and Shisha, Oved. Fractional order differences of the coefficients of polynomials in several variables, 332.

Mooney, Michael. See Porcelli, Pasquale.


Morrison, J. A. Averaging to apply the method of planar orbit problems, 529.


Morrison, N. See Duffin, R. J.

Mosher, R. E. Whitney formulas for secondary characteristic classes, 368.

Motzkin, T. S. The evenness of the number of edges of a convex polyhedron, 242; Approximation in the sense of a deviator integral, 451.

Mrówka, S. G. and Blefko, R. L. Rings of rational-valued continuous functions, 331.

Mullin, A. A. Some new analogues to classical number-theoretic functions. Preliminary report, 680.


Muskat, J. B. Some representations of primes as sums of squares, 566.

Murray, C. B. On the mean integral and a mean double integral, 663.


Nadow, S. B., Jr. A note on projections and compactness, 753.

Nagahara, Takasi. A note on Galois theory of commutative rings, 569.


Nasser, J. I. and Wheeling, R. W. The convexity
of a certain function, 206.

Navot, Israel. On the summation of inverse circular, hyperbolic and elliptic functions, 591.


Neggers, Joseph. Derivations on \( p \)-adic fields, 64.


Nemitz, W. C. Implicative semi-lattices, 75; On the lattice of filters of an implicative semi-lattice, 672.

Nerode, Anil. A metatheorem on addition of \( R. \ E. T. \)'s, 207; Combinatorial series and recursive equivalence types. I, 322; Combinatorial series and recursive equivalence types. II, 377; Nonstandard models in isols, 535.


Neubauer, Gerhard. On connecting semi-Fredholm operators, Preliminary report, 772.


Neustadter, S. F. A trunking problem for three channels, 444.

Newberger, S. M. The \( \sigma \)-symbol of Calderón and Zygmund, 372.

Newhouse, Albert. Finding eigenvectors by Gaussian elimination, 543.

Newman, M. F. Outer automorphisms of nilpotent \( p \)-groups, 451.

Nielsen, R. M. See Brace, J. W.

Nilson, E. N. See Ahlberg, J. H.


Nix, E. D. A possible strong restriction to Einstein's unified field theory involving a preferred affine group of coordinate systems, 586; Definition of the order types of the arc and the real line without reference to separability, 690.

Nomizu, Katsumi and Yano, Kentaro. On tensor fields whose covariant differentials are recurrent, 324; A simple proof of a theorem on the affine holonomy group, 324.

Nordgren, E. A. An irreducibility condition for analytic Toeplitz operators, 361.

Nunnally, Ellard, Stable homeomorphisms of \( S_n \). Preliminary report, 215.

Oddson, J. K. Two theorems of the Phragmen-Lindelöf type for a sector, 236; A theorem of the Phragmen-Lindelöf type for a strip, 236; A theorem of the Phragmen-Lindelöf type, 675.

Olive, Gloria. Generalized Stirling numbers defined by generalized powers, 65.

Olkin, Ingram. See Marshall, A. W.

Olson, F. R. See McKay, D. O.


Osborn, H. A. Intrinsic characterization of tangent spaces, 743.


Ososky, B. L. On ring properties of injective hulls, 151; Cyclic injective modules of full linear rings. Preliminary report, 547.

Ostrom, T. G. Nets with critical deficiency, 356.


Owen, Guillermo. On a class of discriminatory solutions to simple games, 51; Combinatorially symmetric games. 211.

Painter, R. J. Extensions of theorems of Ostrowski on the zeros of polynomials, 354.


Pall, Gordon. Representation of \( -1, 2, \) or \( -2 \) by certain binary forms, 366.

Pan, T. K. Bundle of complementary frames and almost geodesic mappings, 559.

Panzone, Rafael. See Galmarino, A. R.

Papangelou, F. A natural lattice-theoretic concept of convergence in abelian \( 1 \)-groups, 96.

Peinado, R. E. Ancestral rings and left-idealizers, 110.

Pedoe, Daniel. On Soddy’s theorem and its generalization to \( n \) dimensions, 761.

Penadó, R. E. Ancestral rings and left-idealizers, 110.

Penner, Sidney. Bi- and tri-operational algebras of functions, 205.

Perello, C. See Hale, J. K.


Pernavs, Nora. A uniqueness theorem for the n-dimensional Holmoltz equation, 684.

Perry, C. L. See Baker, G. A., Jr.

Persson, Arne. Summation methods on locally compact spaces, 81.

Peterson, F. P. See Brown, E. H., Jr.

Petrich, Mario. The maximal matrix decomposition of a semigroup, 223.


Pfaltzgraff, J. A. Extremal problems and coefficient regions for analytic functions represented by a Stieltjes integral, 363.

Pfeffer, A. M. See Derzko, N. A.

Phelps, R. R. Extreme points in function algebras, 538.

Phillipp, W. V. Some metrical theorems on uniform distribution, 53; A note on a problem of Littlewood about diophantine approximation, 309.

Phillips, Ralph. See Lax, P. D.


Pierce, R. S. Modules over regular rings. Preliminary report, 556.


Plemons, Robert. Semigroups with a maximal semigroup with zero homomorphic image, 751.

Pollack, B. W. See Selfridge, J. L.

Porcelli, Pasquale and Mooney, Michael. Maximal ideals in spaces of analytic type functions, 370.


Porter, G. J. Generalized higher order Whitehead products, 210; Whitehead product order, 559.

Posey, E. E. A simple algorithm for computing the multiplication tables for transformation semigroups, 212.


Potts, D. H. See Preisendorfer, R. W.

Pour-El, M. B. and Howard, W. A. Structural criterion for recursive enumeration without repetition, I, 60; Structural criterion for recursive enumeration without repetition, 129.

Preisendorfer, R. W. Invariant imbedding formulation for radiative transfer on general media with internal sources, 744.


Price, J. J. and Zink, R. E. On sets of completeness for families of Haar functions, 678.

Price, Tom. Equivalence of embeddings of k-complexes in Euclidean n-spaces for n ≥ 2k + 1, 564.

Quintas, L. V. On the space of homeomorphisms of a multiply punctured surface of genus g, 449; A note on semi-homogeneous functions, 452.


Rabin, M. O. See Elgot, C. C.


Radlow, James and Chen, Y. M. Time decay in plane pulse diffraction by a smooth convex cylinder, 214.

Rainich, G. Y. Discreteness of electric charges and mappings of spheres, 364; The field of a photon, 572.

Rajagopalan, M. I-s-spaces of semigroups, 235; Characters of locally compact abelian groups. Preliminary report, 443.


Ramanujan, M. S. See Jakimovski, Amnon.

Ramchandran, S. T. See DeCicco John.

Randol, B. S. A lattice-point estimate, 449; A number-theoretic estimate, 768.

Rankin, Bayard. Quantum mechanical time, 545.


Rao, M. M. Radon-Nikodym theorem for vector measures, 216; Global representation of bounded linear functionals on Orlicz spaces, 452.

Ratliff, L. J., Jr. Two notes on locally Macaulay rings, 742.

Ratti, J. S. A Watson transform, 691.

Rav, Yehuda. On the representation of rational
numbers as a sum of a fixed number of unit fractions, 545, 693.

Reay, J. R. The solution of a problem of Bonnice-Klee, 55; Generalizations of a theorem on convex hulls, 308; A new proof of the Bonnice-Klee theorem, 442; A generalization of a theorem of Fenchel, 553; An extension of a theorem of Fenchel, 745.

Reddy, W. L. Upper equicontinuity in measure, 126; Sufficient conditions that a group not act expansively, 126; Remark on expansive automorphisms, 127; Illustration of the role of local connectedness in topological dynamics, 334.

Reich, Edgar and Walczak, H. R. On the behavior of quasiconformal mappings at a point, 233.

Reddy, W. L. Upper equicontinuity in measure, 126; Sufficient conditions that a group not act expansively, 126; Remark on expansive automorphisms, 127; Illustration of the role of local connectedness in topological dynamics, 334.

Reich, Edgar and Walczak, H. R. On the behavior of quasiconformal mappings at a point, 233.

Reichaw, Meir. Extension of homeomorphisms in lacunar spaces, 324.

Reiher, J. The integral representation ring of a finite group, 769.


Retherford, J. R. $w^*$-bases of subspaces, 114.

Rice, J. R. The degree of convergence for entire functions, 683.

Rice, P. M. The product of polyhedral homotopy manifolds is a homotopy manifold, 96; Homeomorphism groups of manifolds, 528.


Richmond, D. E. Elementary calculus in the large, 213.

Richter, M. K. Representation of consumer choice, 682.


Rider, D. G. Function algebras on groups and spheres, 544.

Riegler, G. J. On the number of those integers below a positive bound and in a prime residue class which can be represented as the sum of two squares, 130; On the number of integers of the form $p_1^2 + p_2^2$ and below a positive bound, 150; On linked representations of pairs of integers by a binary form and a binary polynomial, 238; Successive numbers as sums of two squares, 583.

Rim, D. S. See Buchsbaum, D. A.

Rinehart, Daryl. See Abian, Alexander.

Ritchie, R. W. Grzegorczyk's primitive recursive classes $\varepsilon^R$ and Ackermann's functions, 77.

Roberts, H. M. Erratum to "On the Pontrajagin classes of manifolds", 152.

Roberts, J. B. Relations between the digits of numbers and equal sums of like powers, 246.

Roberts, J. H. See Nagami, Kiyosi.


Robertson, J. M. Some topological properties of certain spaces of differentiable homeomorphisms, 311.

Robertson, M. S. An extremal problem for functions with positive real part, 673.

Robinson, Abraham. See Bernstein, A. R.

Robinson, D. W. On matrix commutators of higher order. Preliminary report, 231; On matrix commutators of higher order, 561.

Robinson, T. T. Propositional calculi which are classical in implication and minimal in negation, I, 592; Propositional calculi which are classical in implication and minimal in negation, II, 668.

Rockafellar, R. T. See Kripke, B. R.

Rodabaugh, D. J. Algebras with all commutators in the center. Preliminary report, 133; Some new results on simple algebras, 351; Anti-flexible algebras which are not power-associative, 690.

Rodin, Burton. Extremal length of weak homology classes on Riemann surfaces, 228.

Rogak, E. D. Mixed problems for the wave equation in a time dependent domain. Preliminary report, 80.

Rogers, Kenneth. Cyclotomic polynomials and Wedderburn's Theorem, 744.

Rose, N. J. On the eigenvalues of a matrix which commutes with its derivative, 327.

Rosen, R. H. The five-dimensional polyhedral Schoenflies theorem, 379.

Rosenberg, A. L. On n-tape finite state acceptors, 469.


Rosenblatt, Murray. Almost periodic transition operators acting on continuous functions on a compact space, 324.

Rosenbloom, P. C. Bounds on functions of matrices, 225.

Rosenkrantz, W. E. Probability and the (C,r)
sums of Fourier series, 60.

Ross, K. A. See Comfort, W. W.


Roxin, E. O. On generalized dynamical systems defined by contingent equations, 208; The connectedness of attainable sets of generalized dynamical systems, 243.

Rubinstein, Zalman. Extension of two results in the theory of rational functions to certain classes of analytic functions, 205.

Rucke, William. The sum of a boundedly complete sequence of closed subspaces, 530.

Rudin, Walter. Sets which carry no functions with small Fourier coefficients, 360.

Rumsey, H. C., Jr. See Posner, E. C.

Ryff, J. V. Orbits of L-functions under doubly stochastic transformations, 114.

Sacker, R. J. Bifurcation of a torus from a periodic solution. Preliminary report, 212.

Sacks, G. W. A maximal set which is not complete, 54.

Sade, A. J. V. Le groupe d'anti-autotopie, 149; Quasigroup abélien avec une autotopie principale à composantes inverses, 226.

Saffern, W. W. Invariant subspaces of subnormal operators, 333.

Sagle, A. A note on anti-commutative algebras and analytic loops, 546.

Sally, P. J., Jr. Uniformly bounded representations of the universal covering group of the two-by-two real unimodular group, 573.

Salz, N. P. An improved bound to the possible range of the next unknown prime, 325; The solution of a partial differential equation of mathematical statistics, 355; The solution of exact first order differential forms, 538; An improved bound to the possible range of the next unknown prime, 600.


Sandberg, R. T. On the compatibility of the uniform integral, II, 63.

Sanderson, D. E. Noncontinuous functions which act like continuous functions for connected sets, 765.


Sansone, F. J. On a class of sums of regressive isols, 74.


Sario, Leo; Schiffer, Menahem and Glasner, Moses. The span and principal functions in Riemannian spaces, 681.


Sass, Charles. A system of co-ordination as applied for mind-studies, 319.

Sathe, Leroy. See Minc, Henryk.

Sather, Duane. A maximum property of Cauchy's problem in n-dimensional space-time, 77.

Sawitz, P. H. An extension of algebra, 531.

Saworotnow, P. P. Characterization of a complemented algebra, 209; A realization of a special type of a two-sided $H^*$-algebra, 550.


Scarritt, N. S., Jr. See Lick, D. R.

Schaufele, C. B. On the genus of links, 110.


Schiffer, Menahem. See Sario, Leo.

Schneider, Hans. See Nirschl, N. E.

Schori, R. M. A universal snake-like continuum, 351.

Schreiner, E. A. Modular pairs in an orthomodular lattice, 461.

Schuster, Seymour. See Fort, M. K.

Schwerdtfeger, H. W. E. Geometry in the one-dimensional affine group, 216.

Seeley, R. T. Nonexistence of the resolvent for a Dirichlet problem, 570.

Segal, S. L. On Ingham's summation method, 667.

Seidman, T. I. See Goldstein, A. A.

Selfridge, J. L. Congruences covering consecutive integers, 547.

Selfridge, J. L. and Pollack, B. W. Fermat's last theorem is true for any exponent up to 25,000, 97.

Sell, G. R. See Deysach, L. G.

Shah, S. M. See Gray, Alfred.

Shamir, Eliahu. Multiplier transforms in half spaces, 684; Asymptotic expansions for mixed
Sharma, Ambikeshwar and Meir, Amram. Convergence of spline functions, 768.
———. See Motzkin, T. S.
Sherman, Seymour. Fluctuation and periodicity, 74, 470; Product property and cluster property equivalence, 327.
Shields, P. C. Hilbert spaces of continuous functions, 325.
Shisha, Oved. See Mond, Bertram.
Shisha, Oved and Cargo, G. T. A criterion for the comparability of means, 95.
Shore, S. D. See Mrówka, S. G.
Sibuya, Yasutaka. See Harris, W. A., Jr.
Siegel, Aaron. Summability on the (k - 1)-dimensional hypersphere, 87.
Sikkema, C. D. Characterization of nearly flat 2-manifolds in 3-manifolds and of pseudo-half spaces, 670.
Silber, Robert. See Hunt, R. W.
Sims, C. C. On the group (2,3,7;9), 687.
Sinclair, Annette. |f(x)|-closeness of approximation, 312.
Singletary, W. E. Examples of completely closed but not closed partial and partial implicational propositional calculi, 375.
Sinkhorn, R. D. Factor sets for doubly stochastic operators on a Hilbert space, 65.
Sklar, Abe. Uniform Stieltjes integrals, 342.
Small, James. See Abbott, J. C.
Smart, J. R. Basis for modular cusp forms, 224.
Smith, D. A. A theorem on automorphisms of classical Lie algebras, 82.
Smith, H. A. Tensor products of complete locally m-convex algebras, 567; Representations of tensor products of symmetric Banach algebras, 664; Proper tensor products of commutative Banach algebras, 668.
Smith, K. T. Some formulas to represent functions by means of derivatives, 117.
———. See Hanna, M. S.
Smithson, R. E. Fixed points for connected multi-valued functions, 310.
Snapper, Ernst. Spectral sequences and Frobenius groups, 97.
Solomon, Gustave and Stiffler, J. J. New bound on (n,k) binary group codes, 64.
Sonner, Johann. Canonical categories, 215.
Sorenson, J. R. Left-amenable semigroups and cancellation, 763.
Spira, R. S. Properties of the Riemann zeta function, 378; Zero-free regions of \( f^{(k)}(s) \), 684.
Spitzer, F. L. A recurrence criterion for random walk, 126.
Srinivasan, T. P. See Hasumi, Morisuke.
Stackelberg, O. P. On a limit theorem by Khintchine for continued fractions, 534; On the law of the iterated logarithm for continued fractions, 772.
Staley, D. H. Some theorems paralleling a theorem by Kuratowski, 100.
Stallings, J. R. Homology and central series of groups, 466.
Stampacchia, Guido. An interpolation theorem, 132.
Stark, J. M. A class of compressible fluid flow patterns generated by Bergman's integral operator, 311; Determination of associate functions for Bergman integral operators yielding transonic flow patterns of certain types, 444.
Stein, S. K. Filling n-space with crosses, 682.
Steiner, E. F. See Mayer-Kalkschmidt, Jorg.
Stellmacher, K. L. See Lagnese, J. E.
Stengle, G. A. The solutions of a second order differential equation containing a parameter on a region surrounding four turning points, 62; A divergence theorem for Gaussian stochastic process expectations, 329.
Stevens, H. R. Linear homogeneous equations over finite rings, 57.
Stewart, A. D. On a system of nonlinear integral equations in Hilbert space with perturbed initial conditions, 575.
Stiffler, J. J. See Solomon, Gustave.
Stoker, J. J. Generalizations of Christoffel's
theorem concerning the uniqueness of closed convex surfaces, 213.


Stone, Charles. Local limit theorems for asymptotically stable distribution functions, 465.

Stone, W. M. On the output of certain nonlinear devices in communication theory, 314.

Stout, E. L. Interpolation on finite Riemann surfaces, 534.

Strang, G. W. A necessary condition for well-posed Cauchy problems, 768.

Stratton, H. H. On certain ring-like continua, 671.


Strauss, E. G. See Motzkin, T. S.

Strooker, J. R. Faithfully projective modules and clean algebras, 567.

Su, J. C. A note on the bordism algebra of involutions, 686.

Subbarao, M. V. Some remarks on partitions, 385.

Sulinski, A. See Divinsky, N. J.


Suzuki, Michio. A characterization of 3-dimensional unitary groups, 124.

Swan, Richard. Minimal resolutions for finite groups, 119.

Synowiec, J. A. See DeCicco, John.

Szeptyczyk, Pawel. On restrictions of functions of the spaces $P^a,\mathcal{P}$ and $B^a,\mathcal{P}$, 78; A remark on Mihlin's theorem about multipliers of Fourier transforms, 558


Taft, E. J. Uniqueness of invariant Cartan sub-algebras of solvable Lie algebras, 69; Abelian automorphism groups of algebras over a perfect field, 529.

Tamura, Takayuki. Indecomposability of congruent classes in the greatest decomposition of a semigroup, 548; On a theorem concerning the greatest s-decomposition of a semigroup, 747.

Tamura, Takayuki; Biggs, R. G. and Burnell, D. G. Automorphism group of groupoids and determination of groupoids of order 3 and 4, 314.


Tarski, Alfred. On the ordering of certain sets of cardinals, 207; The comparability of cardinals and the axiom of choice, 578.

Tauber, Selmo. Existence theorems for difference and q-difference equations, 443.

Taulbee, O. E. Properties of classification matrices, 568.


Taylor, J. C. Prime ends and normal tubes, 592.

Taylor, Joseph. The maximal ideal space of the convolution measure algebra on a compact abelian group, 228.

Teller, J. R. On the extensions of lattice-ordered groups, 78.


Thatcher, J. W. Decision problems for first-order theories of generalized arithmetic, 582.

Thielman, H. P. See Martinex, Johann.

Thomas, D. H. See Lynch, R. E.

Thompson, R. C. On a classical result in linear algebra. Preliminary report, 243.


Thor, E. O. The range as range space for compact operators, II, 238.

Thor, E. O. and Walden, W. E. A winning bet in Nevada baccarat, 86.


Tierney, Martin; Waitman, P. E. and Wing, G. M. On some problems in the optimal design of shields and reflectors in particle physics, 92.

Topping, D. M. States and ideals in Jordan operator algebras, 73; Homomorphic images of Jordan operator algebras, 360; Maximal states of Jordan operator algebras, 686.

Townes, S. B. A geometric proof of Hadamard's inequality, 74.

Trampus, Anthony. A canonical basis for the matrix transformation X to AX + XB, 594.

Traub, J. F. Generalized sequences with applications to the discrete calculus, 561; On Lagrange-Hermite interpolation, 666.

Traylor, D. R. Some cardinality conditions for metrization of locally compact, normal Moore spaces, 84; Concerning metrizability and completeness in normal Moore spaces, Preliminary report, 543.

Truenfels, P. N. On error bounds for matrix inversion, 537.


Tucker, D. H. A representation theorem for a continuous linear transformation on a space of continuous functions, 92, 251.

Tucker, Patricia. Endomorphism ring of an induced module, 221.


Turner, R. E. L. Perturbation of ordinary differential operators, 763.


Tutte, W. T. See Harary, Frank.


Uppuluri, V. R. R. On a stronger version of Wallis' formula, 152.

Van Vleck, F. S. See Sonneborn, L. M.

Varga, R. S. Minimal Gerschgorin sets, 56.

Vaught, R. L. Axiomatizability by a schema, 590.

Vincent-Smith, G. A generalization of the Hahn-Banach theorem, 591.

Vinson, R. G. Euclidean geometry via intervals, 754.


Wagner, Stephen. See Askey, Richard.

Walczak, H. R. See Reich, Edgar.

Walden, W. E. See Thorp, E. O.

Walker, C. L. Concordant and harmonic functors, 597.

Walker, C. L. and Walker, E. A. Quotient categories of modules, 598.

Walker, E. A. Some classes of modules, 598.

Walker, C. L. Axiomatizability by a schema, 590.

Wallace, A. D. Applications of a result of Capel and Strother, 115; A decomposition of semigroups, 748.

Wallen, L. J. Finite convolution operators, 343.

Walsh, J. L. Padé approximants as limits of rational functions of best approximation, 132; Hyperbolic capacity and interpolating rational functions, 388.


Waltman, Paul. An oscillation criterion for a second order differential equation, 564.

Waltman, P. E. See Tierney, Martin.

Wang, Hao. A theorem on definitions of the Zermelo-Neumann ordinals, 137.

Warner, R. J. A characterization of certain regular D-classes in semigroups, 208; Regular D-classes whose idempotents obey certain conditions. I, 748; Regular D-classes whose idempotents obey certain conditions. II, 759.

Wasow, Wolfgang. Asymptotic decomposition of systems of linear differential equations with poles with respect to a parameter, 559.
Weill, G. G. See Sarlo, Leo.

Weinstein, J. M. Direct products and Horn sentences, 391.

Weiss, M. L. A limitation of uniform cluster operators, 71.

Well, James. Bounded continuous vector-valued functions on a locally compact space, 577.

Wermer, John. Approximation on a disc, 388; Polynomially convex discs, 462.


Weston, K. W. ZA groups satisfying an Engel condition, 87.

Widder, D. V. See Dauns, John.


Wiginton, C. L. Factoring pointlike simplicial mappings of the 3-sphere, 757.

Wilansky, Albert. See Rajagopalan, M.


Williams, G. K. Topological methods in functions of two complex variables, 316.

Williams, R. F. Examples of free p-adic group actions, 306.

Williamson, J. A. A relation between a class of limit laws and a renewal theorem, 562.

Williamson, R. E. Piecewise linear cobordism groups, 222.

Wilson, J. C. A second-order nonlinear differential equation which has a finite number of periodic solutions, 357.

Wing, G. M. See Tierney, Martin.

Wohlers, M. R. See Beltrami, E. J.

Wolf, C. E. Standard recursively enumerable classes. I, 386; Standard recursively enumerable classes. II, and partial recursive operators, 386.


Wolf, J. A. On the classification of hermitian symmetric spaces, 149; Translation-invariant function algebras on compact groups, 149.

Wong, Eugene. See Eisenberg, Edmund.

Wong, J. S. W. A generalization to the converse of contraction mapping principle, 385.

Wong, Y. F. A theorem on manifolds of the same homotopy type. Preliminary report, 141; A theorem on almost parallelizable manifold, 382; Homotopy type of almost differentiable manifold, 459.

Woodward, D. A. Continuous transformations and stochastic differential equations, 100.

Worrell, J. M., Jr. Upper semicontinuous decompositions of developable spaces, 250; Isolated sets of points in nonmetrizable spaces, 250; Continuous decompositions of paracompact spaces, 455; Upper semicontinuous decompositions of metacompact spaces, 455; Upper semicontinuous decompositions of paracompact spaces, 467; Decompositions of spaces having bases of countable order, 773.


Wright, F. M. On the existence of a weighted Stieltjes mean Sigma integral, I, 762.

Wyler, Oswald. Clans, 363; Weakly exact categories, 458.
Yano, Kentaro. See Nomizu, Katsumi.

Yao, J. Z. Representation of functions of n variables by superposition of functions of 1 variable and addition, 469.


Yates, C. E. M. A minimal pair of recursively enumerable degrees, 590.

Young, D. M., Jr. On the use of the successive overrelaxation method with one or more relaxation factors, 326.

Young, D. M., Jr.; Wheeler, M. F. and Downing, J. A. On the use of the successive overrelaxation method with several relaxation factors, 688.

Young P. R. A recursively enumerable splinter which is not a cylinder, 68.

Zachmanoglou, E. C. An example of slow decay of the solution of the initial-boundary value problem for the wave equation in unbounded regions, 332.

Zadek, Mishael. Continuity and location of zeros of linear combinations of polynomials, 85.

Zemanian, A. H. Necessary and sufficient conditions for a matrix distribution to have a positive-real Laplace transform, 56.

Zettl, A. J. On harmonic matrices, 750.


Zink, R. E. On semicontinuous functions and Baire functions, 447.

Zirakzadeh, A. See Eggleston, H. G.

Zitron, N. R. Multiple scattering of elastic waves by cylindrical cavities, 112.

INDEX
Volume 11, 1964

ABSTRACTS OF CONTRIBUTED PAPERS, 51, 205, 306, 442, 527, 662, 741
Index of Abstracts in Volume 11, 775
Errata, 152, 251, 470, 600, 693
ACTIVITIES OF OTHER ASSOCIATIONS, 34, 189, 441, 518
ADVERTISERS, INDEX TO, 175, 259, 403, 475, 627, 707, 812
DOCTORATES CONFERRED IN 1963, 415, 507
FEATURE ARTICLES
Annual Salary Survey, 643
Government Agencies Forecast Limited Budget, 291.
Manpower Problems in the Training of Mathematicians, 32
National Academy of Sciences-National Research Council, 726
Starting Salaries for Mathematicians with a Ph.D., 646
LETTERS TO THE EDITOR
R. D. M. Accola, H. Bass, H. M. Edwards,
S. Eilenberg, Herbert Federer, David Gale,
E. R. Kolchin, Norman Levinson, R. E.
MacRae, Katsumi Nomizu, Wendell Fleming,
Saunders MacLane, 40
G. W. Mackey, 199
Armand Siegel, 41
G. S. Young, 433
MEETINGS OF THE AMERICAN MATHEMATICAL SOCIETY
Abstracts of Contributed Papers, 51, 205, 306, 442, 527, 662, 741
Calendar of Meetings, 4, 180, 264, 408, 480, 632, 712
Preliminary Announcements
February: New York, 29
April: New York, 29, 186; Reno, 185;
Chicago, 188
June: Pullman, 287
August: Amherst, 288, 411
October: Garden City, 504
November: Los Angeles, 636; Athens, 636;
Evanston, 637
January: Denver, 637, 723
Program of Meetings
January: Miami, Florida, 5
February: New York, New York, 181
April: Reno, Nevada, 265; New York,
New York, 269, Chicago, Illinois, 280
June: Pullman, Washington, 409
August: Amherst, Massachusetts, 481
October: Garden City, New York, 633
November: Los Angeles, California, 713;
Athens, Georgia, 716; Evanston, Illinois, 720
Supplementary Programs, 47, 200, 303, 438, 523, 658, 739
MEMORANDA TO MEMBERS
Announcement of Changes in the Combined Membership List, 414
Backlog of Mathematical Research Journals, 191, 505
By Title Abstracts, 294
Chairmen, 41
Corporate Members, 190, 737
Employment of Retired Mathematicians, 190, 414, 506, 737
Employment Register, 43, 414, 737
London Mathematical Society, 43
National Register of Scientific and Technical Personnel, 41
Panel of Consultants on Graduate Instruction, 41
Postal Rates, 661
Reciprocity Agreement with the Turkish Society for Pure and Applied Mathematics, 294
Summer Employment Opportunities, 43, 737
NEW AMS PUBLICATIONS, 42, 194, 294, 520, 655, 732
NEWS ITEMS AND ANNOUNCEMENTS, 37, 46, 50, 184, 192, 198, 199, 298, 437, 503, 506, 514, 519, 526, 638, 641, 738, 740
PERSONAL ITEMS, 44, 195, 295, 435, 508, 647, 733
SYMPOSIA AND INSTITUTES
Summer Research Institute on Algebraic Geometry, 31, 290, 639
VISITING FOREIGN MATHEMATICIANS, 728
WHAT'S NEW IN POLISH MATHEMATICS?

Recently published:

H. Rasiewa and R. Sikorski
MATHEMATIC OF METAMATHEMATICS

W. Slebodzinski
FORMES EXTERIEURES ET LEURS APPLICATIONS

To be published soon:

W. Sierpinski
THE ELEMENTARY THEORY OF NUMBERS
An elementary presentation of the current state of the theory of numbers, in English, by the eminent Polish mathematician, Prof. Waclaw Sierpinski. The scope of ideas in this work will interest many mathematicians outside of its specialized field. It is revised and condensed from the author's two-volume "Theory of Numbers," published in Polish in 1960, which aroused considerable international interest. The work will appear as Vol. 42 of the series, Monographs in Mathematics. State Science Publishers, 450 pp. $12.00

Jozef Marcinkiewicz
COLLECTED PAPERS
All known papers of the prominent Polish mathematician, the late Jozef Marcinkiewicz. His work, though cut short by his premature death, was highly influential in the subsequent progress of mathematical analysis. To be published in English during 1964. Approximately $24.00

In the United States, order from:

STECHERT-HAFFNER, Inc.
31 East 10th Street, New York, N. Y. 10003

Sole exporter:
ARS POLONA
Krakowskie Przedmiescie, Warsaw, Poland

802
Newly Published

CALCULUS WITH ANALYTIC GEOMETRY, THIRD EDITION
Richard E. Johnson, University of Rochester
Fred L. Kiekemeister, Mt. Holyoke College
798 pp. List $11.50
"... the best Calculus on the market. It is extremely modern, beautifully written and, of course, accurate."—Cletus Oakley, Haverford College.

A SURVEY OF GEOMETRY, VOLUME I
Howard Eves, University of Maine
489 pp. List $9.95
This first of a two-volume work introduces the college student to the entire area of geometry.

A SURVEY OF MATRIX THEORY AND MATRIX INEQUALITIES
Marvin Marcus and Henryk Minc, both of the University of California, Santa Barbara
180 pp. List $8.75
This book is part of the Allyn and Bacon Series in Advanced Mathematics under the Consulting Editorship of Irving Kaplansky of the University of Chicago.

Other Outstanding Titles

INTRODUCTION TO COMPLEX ANALYSIS
Zeev Nehari, Carnegie Institute of Technology
258 pp. List $7.50
"... a splendid text in complex variable theory that is suitable for a 'one-semester course for seniors and first-year students in mathematics, engineering and the sciences.'" The American Mathematical Monthly

INTRODUCTION TO MODERN ALGEBRA
Neal H. McCoy, Smith College
304 pp. List $7.95
A popular text that presents the basic ideas of abstract algebra and features a clear exposition of the concepts of modern algebra. Adopted by approximately 250 colleges.

INTRODUCTION TO TOPOLOGY
Bert Mendelson, Smith College
217 pp. List $8.50
"An excellent text, perhaps the best introduction to general topology now available." John Dyer-Bennet, Carleton College

SET THEORY AND THE STRUCTURE OF ARITHMETIC
Joseph Landin and Norman T. Hamilton, both of the University of Illinois
264 pp. List $7.75
The purpose of this book is to provide first, a deep understanding of elementary arithmetic, and second, a foundation for the further study of algebra, geometry, and analysis.

VECTOR GEOMETRY
Gilbert de B. Robinson, University of Toronto
176 pp. List $6.95
Examines modern algebra and geometry with the assumption that each is more complete with the other.

For examination copies, write to: ARTHUR B. CONANT, Dept. G
ALLYN AND BACON COLLEGE DIVISION
150 Tremont Street, Boston, Massachusetts 02111
A TEXTBOOK ON ANALYTICAL GEOMETRY
This work provides a clear and easily comprehensible introduction to analytical geometry for scientists and engineers who use mathematics in their work. The relation between the geometric properties and the parameters of an equation are emphasized, and the geometric content of algebraic manipulation is stressed in working with lines, circles, and conics. New concepts are developed thoroughly, and curve tracing is covered in detail. Comprehension is aided by the use of concepts not traditionally employed in mathematics.

LINEAR REPRESENTATIONS OF THE LORENTZ GROUP
International Series of Monographs on Pure and Applied Mathematics, Volume 63
By M. A. Naimark, Moscow. Translation edited by H. K. Farahat.
Before the publication of this book, the materials included were covered only in specialized journals. The author has succeeded in producing the first self-contained exposition of recent work in the field. Background material from algebra and analysis and from representation theory, particularly representation theory of the group of rotations of three-dimensional space, is included. The work is intended primarily as an introduction to the general theory of representations. However, part of the book is devoted to a study of equations invariant under Lorentz transformations, a subject of great interest to theoretical physicists.

NON-LINEAR DIFFERENTIAL EQUATIONS
International Series of Monographs on Pure and Applied Mathematics
By G. Sansone and R. Conti, University of Florence. Translated by A. H. Diamond, Stevens Institute of Technology.
The authors offer a detailed analysis of the phase portrait of two-dimensional autonomous systems, a survey of the qualitative methods for the discovery of periodic solutions of periodic systems, and a study of asymptotic properties, especially stability properties, of the solution of n-dimensional systems. They also furnish an extensive analysis of singular points of two-dimensional autonomous systems, including perturbed systems. The equations of Vander Pol and Lienard of oscillations of relaxation are considered, including questions of existence and uniqueness of periodic solutions and asymptotic evaluation of the period.

A HANDBOOK FOR ENGINEERS
Volume 1, Mathematics and Physics
Edited by K. P. Yakovlev
This is the first of three volumes which will provide indispensable reference data for physicists and engineers in industry, universities, and colleges of technology. Volume 1 gives particular attention to mathematics, beginning with a summary covering the elementary stages and continuing with processes which are necessary for a proper understanding of more advanced material. Topics covered include determinants and simultaneous linear equations, analytical geometry, differential calculus, differential geometry, infinite series, vector calculus, and approximate methods of analysis.

STATICS OF GRANULAR MEDIA, Third Revised Edition
By V. V. Sokolovskii, Academy of Sciences of the U.S.S.R. Translation edited by A. W. T. Daniel.
This is an entirely new fundamental approach to the classical Rankine-Coulomb theories. A strictly mathematical analysis of typical retaining wall or allied problems, the work shows that there are anomalies in the Rankine-Coulomb results, and further, that mathematical solutions of many problems, after allowing for all given conditions, would not appear to exist. This book gives a new approach to such solutions, depending on the variation of a stress function along the slip planes.

THE MATHEMATICAL THEORY OF OPTIMAL PROCESSES
International Series of Monographs on Pure and Applied Mathematics, Volume 55
By L. S. Pontryagin, V. G. Bol'tanskii, R. S. Gamkrelidze, and E. F., Mischenko, Moscow.
This comprehensive and informative volume furnishes a detailed account of optimum processes, which are based on the maximum principle. The latter permits the solution of problems of a mathematical and applied nature which are variational, but which do not fit into the classical arrangement of variational calculus.
NEW PUBLICATIONS

L. W. KANTOROWITSCH/G. P. Akilow
Funktionsanalyse in normierten Räumen
(Analysis of functions in normalized spaces)
Translation from the Russian
Edited by Prof. Dr. P. Heinz Müller
1964, XVI, 624 pages, 7 illustrations,
size 8vo
Price: US $15.80

The first part of the book treats of the theory of linear operators and functionals (incl. classical fundamentals of the spectral theory, integral operators, Sobolev imbedding laws). The second part deals with linear and nonlinear functional equations, the general existential principles for the solutions (fixed point principles of Banach, Brouwer, Schauder) and the most important approximation methods for the practical treatment of such equations (for instance, the method of steepest descend, Newton method). The general theories discussed are always supplemented by interesting examples of application, while a comprehensive and detailed literary index refers to other, supplementary and additional publications.

A. S. PETROW
Einstein-Räume
(Einstein-Space)
Translation from the Russian
German Edition: Dr. Hans-Jürgen Treder
1964, X, 394 pages, Size 8vo,
Price: US $13.90

A comprehensive and detailed monography on the geometrical properties of four-dimensional diversities of Lorentz-Minkowski signature, whose metrics are determined by the vacuum equations $R_{\mu\nu} = \chi^2 \mu\nu$ of Einstein. The book represents the first treatise dealing with all known geometrical peculiarities of this space-time world corresponding to the free fields of gravitation in connection with differential-geometrical points of view.

Please order through your bookdealers!
AKADEMIE-VERLAG · BERLIN

---Latest Addition to the---
Athena Series...
Brief but brilliant studies covering specialized areas of mathematics not found in conventional texts.

The
Gamma
Function

Emil Artin,
late of the University of Hamburg
Translated by Michael Butler

A first-rate translation of a hard-to-find mathematical classic, incorporating corrections by the author.

June, 1964  64 pp.
$1.75

New and Notable
DIFFERENTIAL EQUATIONS

A Modern Approach
Harry Hochstadt, Professor of Mathematics,
Polytechnic Institute of Brooklyn
"...satisfies a real need for a modern approach to intermediate differential equations. I particularly like the author's development of the matrix method of solution of systems of linear equations."

John F. Lavelle, Gettysburg College
$6.50

HOLT, RINEHART
and WINSTON, inc.
383 Madison Avenue, New York, N.Y. 10017
NEW TEXTS
IN MATHEMATICS
FROM McGRAW-HILL

ELEMENTS OF ORDINARY DIFFERENTIAL EQUATIONS, Second Edition
By MICHAEL GOLOMB and MERRILL SHANKS, Purdue University. International Series in Pure and Applied Mathematics. Off Press.
A greatly improved revision of a highly successful class-tested text. Provides a thorough, systematic, but not formalistic treatment of everything in ordinary differential equations that is mathematically significant, useful for applications, and accessible to the student of mathematics, science, and engineering at the junior level.

HANDBOOK OF MATHEMATICAL TABLES AND FORMULAS, Fourth Edition
By RICHARD S. BURLINGTON, Chief Mathematician, Bureau of Naval Weapons, Navy Department, Washington, D. C. 384 pages, $4.50 (Text Edition also available).
A major revision of an outstanding handbook designed to meet the needs of students and workers in mathematics, engineering, physics, chemistry, science, and other fields.

ADVANCED CALCULUS, Second Edition
A careful revision of an extremely successful text for the standard junior-senior course. Presents a modern treatment of the fundamentals of analysis in a form which emphasizes both structure and technique, and which will help students in mathematics and the sciences to achieve mathematical maturity.

SIMILITUDE AND APPROXIMATION THEORY
By STEPHEN J. KLINE, Stanford University. Available in January.
Discusses various analytic techniques for deriving information about a scientific or engineering problem when the complete equations are either unknown or unsolvable. Attempts to organize, unify, and extend the materials on similitude and model theory to the research level. A work which every researcher, particularly beginners, whether engineer or scientist, should read.

Examination copies on request

McGRAW-HILL BOOK COMPANY
330 West 42nd Street/New York 10036

CUSHING-MALLOY, INC.
1350 N. Main St., P. O. Box 1187
Ann Arbor, Michigan

LITHOPRINTERS
Printers of the NOTICES

Known for
QUALITY — ECONOMY — SERVICE
Let us quote on your next printing
N. H. Abel
Oeuvres Complètes
(Nouvelle Edition)
Available Fall 1964
Cloth bound in 2 vols. $22.50
Oslo 1881, 964 pp., 1964 Reprint
Niels Henrik Abel, one of the great men of the Norwegian mathematical school, was born in 1802 and died in 1829, only 27 years old. Few scholars have had so brilliant a career in so short a space of time, nor can it be predicted to what summits this spirit endowed with a genius for pure mathematics might have soared if he had been granted a longer existence.

G. W. Hill
Collected Mathematical Works
Available Fall 1964
Published by the Carnegie Institution of Washington as their Publication No. 9
Cloth bound in 4 vols. $125.00
Washington 1905-1907, 1,788 pp., 1964 Reprint
“[He] knew all parts of celestial mechanics, yet the core of his work, that which made him immortal, is his Lunar Theory. He was, in this, not only a skilful artist and an inquisitive researcher, but, truly, an original inventor.”—Henri Poincaré

L. V. Lorenz
Oeuvres Scientifiques
Available Fall 1964
Cloth bound in 2 vols. $25.00
Copenhagen, 1896-1908, 1,112 pp., Reprint 1964
“His work is nearly all mathematical as well as physical; at times it is the mathematical and at other times the physical side which predominates, but his ideas on physics were almost always worked out mathematically. His main talent was his brilliant aptitude for the invention of methods, combined with a very fine sense of the value of approximate results, even when he was unable to put any limit to the errors committed.”—From the Preface

THE MATHEMATICAL WORKS OF ISAAC NEWTON
Vol. I. Assembled with an introduction by Dr. Derek T. Whiteside
Now Available
Cloth bound ....................... $14.50
Vol. II... In preparation
The first volume of Newton’s Mathematical Works is concerned with the three works on which Newton’s renown as an inventor in calculus has historically depended. Their texts are largely self-explanatory and, though often dense in argument and subject matter, are rarely obscure.
Mathematics and IDA

Washington is the decision-making center of the free world. In that center, IDA functions as a scientific adviser to the Department of Defense. Our working environment is the gray area of those major national problems where too little is known and too much is at risk to hazard an intuitive decision. IDA provides responsible DOD decision makers with the scientific/technical input required to eliminate or lessen the areas of uncertainty.

Mathematics is applied in two principal fields at IDA: in the study of the technical feasibility of weapons systems and in operations research to find the optimum choice among competing weapons systems.

In a world in which the complexities and exigencies of our nation's defense and foreign policy continue to increase and grow more critical, IDA's programs must also continue to expand. We invite qualified mathematicians to investigate both short term (two to three years) and permanent appointments in our Weapons Systems Evaluation Division and our Research and Engineering Support Division. Respondents should preferably have an advanced degree and be knowledgeable in at least one of the following: electromagnetic wave theory, information theory, stochastic processes, automatic control theory, statistics, queuing theory, numerical analysis, probability theory or the development of computer routines for problem solving or simulation.

A career at IDA can present a challenge of satisfying proportions and provide a reward of substance. Write us; we may have mutual interests.

Institute for Defense Analyses, 1666 Connecticut Avenue, N.W., Washington 9, D. C. An equal opportunity employer.
PROFILE OF A CNA PROFESSIONAL

A CNA analyst is a professional of superior competence. He may be a mathematician, a physical scientist, an economist, or a research engineer. He is a member of the Center for Naval Analyses of The Franklin Institute. CNA is a private scientific organization engaged in operations research, systems evaluation, and broad-based studies for the United States Navy.

CNA professionals work on current operational problems with the Operations Evaluation Group; on problems of cost effectiveness and force requirements of the mid-range future with the Naval Warfare Analysis Group; on studies of naval problems of the long-range future with the Institute of Naval Studies; or on parametric studies or development of new methodologies in CNA’s Research Group.

The CNA analyst has unusual analytical ability. His imagination is tempered by reality. He is capable of independent effort, but is amenable to inter-disciplinary research. He wants to apply his talents and knowledge to the nation’s security.

A few CNA staff appointments are available. For additional information, write:

Director
CENTER FOR NAVAL ANALYSES
Dept. AM

Achievement at Booz-Allen Applied Research is rarely accidental . . . whether it be in astronautics, communications, computer systems, feasibility analysis, operations analysis, reliability or any of our other technical areas.

The company’s conspicuous success in solving complex, non-routine problems reflects the hard work of a highly talented professional staff and has led to new and greater responsibilities. As a consequence, excellent career positions now await creative professionals of many scientific and engineering disciplines.

Prerequisites include the ability to apply critical perception to unusual problems, as well as a strong interest in meanings and relationships and an eye for both the theoretical and the practical. Please write Mr. Robert Flint, Director of Professional Appointments.

BOOZ • ALLEN
APPLIED RESEARCH Inc.
4815 Rugby Avenue
Bethesda, Maryland 20014
Washington • Cleveland
Chicago • Los Angeles
An equal opportunity employer
INDEX
TO ADVERTISERS

Name                              Page
Allyn and Bacon, Inc             803
American Mathematical Society    806
Ars Polona                      802
Booz · Allen Applied Research Inc 809
Center for Naval Analyses       809
Cushing-Malloy, Inc             806
Deutscher Buch-Export für Akademie-Verlag 805
Holt, Rinehart and Winston, Inc 805
Houghton Mifflin Company         810
Institute for Defense Analyses   808
Johnson Reprint Corporation      807
McGraw-Hill Book Company        806
The Macmillan Company           804
Robert College of Istanbul, Turkey 810

STATEMENT OF OWNERSHIP, MANAGEMENT
AND CIRCULATION
(Act of October 23, 1962; Section 4369, Title 39, United States Code)
1. Date of filing: September 22, 1964
2. Title of publication: Notices of the American Mathematical Society
3. Frequency of issue: Seven issues per year
4. Location of known offices of publication: 190 Hope Street, Providence, R.I. 02906
5. Location of the headquarters or general business offices of the publishers: 190 Hope Street, Providence, R.I. 02906
6. Names and addresses of publisher, editor, and managing editor. Publisher: American Mathematical Society, 190 Hope Street, Providence, R. I. Editors: John W. Green and Gordon L. Walker, 190 Hope Street, Providence, R. I. 02906
7. Owner: None
8. Known bondholders, mortgagees, and other security holders owning or holding 1 percent or more of total amount of bonds, mortgages or other securities: None
9. Paragraphs 7 and 8 include, in cases where the stockholder or security holder appears upon the books of the company as trustee or in any other fiduciary relation, the name of the person or corporation for whom such trustee is acting, also the statements in the two paragraphs show the officer's full knowledge and belief as to the circumstances and conditions under which stockholders and security holders who do not appear upon the books of the company as trustees, hold stock and securities in a capacity other than that of a bona fide owner. Names and addresses of individuals who are stockholders of a corporation which itself is a stockholder or holder of bonds, mortgages or other securities of the publishing corporation have been included in paragraphs 7 and 8 when the interests of such individuals are equivalent to 1 percent or more of the total amount of the stock or securities of the publishing corporation.

I certify that the statements made by me above are correct and complete. Gordon L. Walker

COLLEGE MATH
TEXTBOOK EDITOR

To develop a complete list of titles for undergraduates. Emphasis on pure math.

Job includes projecting list of books desired, securing editorial advisers, making author contact, editing manuscripts and seeing them through press. Considerable traveling. A.B. degree with math major a minimum requirement. Publishing or teaching experience desirable.

Send complete resume to:
Personnel
Houghton Mifflin Company
2 Park Street
Boston, Massachusetts 02107

OVERSEAS

Robert College, in Istanbul, Turkey—the oldest American-sponsored college abroad—presents a challenge in education where East meets West. An opportunity to contribute significantly to the development of a young republic is available to specialists in engineering, business administration and economics, the sciences, the humanities, and English as a foreign language. Graduate degrees required.

Address inquiries to Mrs. Judith W. Abrams, Recruitment Secretary, Robert College of Istanbul, Turkey, 548 Fifth Avenue, New York, New York 10036.
RESERVATION FORM

Denver, Colorado Meeting
January 26-30, 1965

TO BE SUBMITTED NOT LATER THAN 1/10/65

Please complete this housing application form and return it immediately to the Housing Bureau, Mathematics Meetings, 225 West Colfax Avenue, Denver, Colorado, 80202. Do not make reservations direct with hotels; no deposit necessary. All reservations will be confirmed by the Housing Bureau. Should cancellation be necessary, please advise the Housing Bureau at least 10 days in advance. Reservations will be held until 6:00 P.M. of the arrival date, unless a later hour is specified below.

Mail the Reservation Form to the Housing Bureau, Mathematics Meetings, 255 West Colfax Avenue, Denver, Colorado 80202.

(Please Print)
Hotel: 1st choice ______________________________________
2nd choice ______________________________________
3rd choice ______________________________________

Reserve ______ Single(s) at $______
______ Double Bed(s) at $______
______ Twin Bed(s) at $______
______ Suite(s) at $______

I will arrive ______ at ______ A.M.
(date) (hour)

I will (will not) share room.

I will depart ______ at ______ P.M.
(date) (hour)

Occupants: (List all names below and type of room requested)
1. __________________________
2. __________________________
3. __________________________

Send confirmation to:
Name __________________________
Institution __________________________
Street Address __________________________
City and State __________________________

ADVANCED REGISTRATION FORM

Denver, Colorado Meeting
January 26-30, 1965

Please fill out the form below and return with your payment no later than January 10, 1965 to:

Meeting Arrangements Section
American Mathematical Society
190 Hope Street
Providence, Rhode Island

Registration Fee:
1. Member of one or more organizations listed below: $2.00
2. Member when accompanied by wife or family: $2.50
3. Non-Member: $5.00
4. Student: No charge

MAKE CHECK PAYABLE TO:
American Mathematical Society

(Checks received after the deadline of January 10, 1965 will not be processed)

(Please Print)
NAME __________________________
(Last name first)
EMPLOYING INSTITUTION __________________________

Accompanied by: ( ) wife/husband __________________________
( ) children __________________________
(names)

RESIDENCE DURING MEETING __________________________

(If not known at this time, please notify registration desk upon arrival.)

MEMBER OF:

AMERICAN MATHEMATICAL SOCIETY ( )
MATHEMATICAL ASSOCIATION OF AMERICA ( )
ASSOCIATION FOR SYMBOLIC LOGIC ( )
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS ( )

I am a student at __________________________

Check in the amount of $__________enclosed.
Interest in the theory of stochastic processes takes its rise from the beginning of the twentieth century with the classic articles of Bachelier (1900) on the “Brownian motion” of the stock market and with Einstein’s immediately subsequent work on the same sort of motion for actual particles. As the century has progressed, the shift of interest from classical determinism to stochastic description of phenomena has become more and more pronounced, largely as a consequence of the continuing challenges to the mathematician to guide research in engineering, economics, biology, medicine and operations research, where a combination of complexity and uncertainty has forced the more frequent use of probabilistic concepts.

The fifteen papers in this book show how the theory of stochastic processes can be applied to a wide range of problems in such diverse fields as modern control theory, physics and astronomy, dynamic programming, and the theory of learning processes in mathematical psychology. Some of the papers also give a careful, detailed explanation of the abstract foundations of the theory and of its inner-mathematical applications to such fundamentally important topics as the distribution of solutions of a stochastic differential equation.