OF THE
AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

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# MEETINGS

## Calendar of Meetings

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>666</td>
<td>April 26, 1969</td>
<td>Santa Cruz, California</td>
<td>Feb. 14, 1969</td>
</tr>
<tr>
<td></td>
<td>August 25-29, 1969</td>
<td>Eugene, Oregon</td>
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<tr>
<td></td>
<td>January 22-26, 1970</td>
<td>Miami, Florida</td>
<td></td>
</tr>
<tr>
<td></td>
<td>August 24-28, 1970</td>
<td>Laramie, Wyoming</td>
<td></td>
</tr>
<tr>
<td></td>
<td>January 21-25, 1971</td>
<td>Atlantic City, New Jersey</td>
<td></td>
</tr>
</tbody>
</table>

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next deadline for by-title abstracts will be February 11, 1969.*

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The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Second-class postage paid at Providence, Rhode Island, and additional mailing offices.

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Printed in the United States of America
Seventy-Fifth Annual Meeting  
New Orleans, Louisiana  
January 23-26, 1969

The seventy-fifth annual meeting of the American Mathematical Society will be held at the Jung Hotel in New Orleans, Louisiana, in conjunction with the annual meeting of the Association for Symbolic Logic, the annual meeting of the Mathematical Association of America, and a meeting of the National Council of Teachers of Mathematics. Some sessions are to be held at Governor House and Howard Johnson's Motels which are within walking distance of the headquarters of the meeting at the Jung Hotel. The Society will meet from Thursday, January 23, through Sunday, January 26. The Association for Symbolic Logic will meet on Wednesday, January 22, and Thursday, January 23. The Mathematical Association of America will meet on Saturday, January 25, in the morning; on Sunday, January 26, in the morning; and all day on Monday, January 27. The meetings on Saturday, January 25, and on Sunday, January 26, of the Mathematical Association of America will be joint sessions with the National Council of Teachers of Mathematics.

The forty-second Josiah Willard Gibbs Lecture will be delivered by Professor R. L. Wilder of the University of Michigan at 8:00 p.m. on Thursday, January 23, in the Grand Ballroom of the Jung Hotel. The title of his address will be "Stability theory and invariant manifolds for dynamical systems." Professor Calvin C. Moore of the University of California, Berkeley, will present an address entitled "Geometric ergodic theory" at 11:00 a.m. on Friday, January 24. Both of these invited addresses will be given in the Grand Ballroom of the Jung Hotel.

The Bôcher Memorial Prize will be awarded on Thursday, January 23, at 1:30 p.m. in the Grand Ballroom of the Jung Hotel.

There will be two special sessions of selected twenty-minute papers. Both sessions will be held on Friday, January 24, from 8:45 a.m. to 10:45 a.m. and from 2:45 p.m. to 4:45 p.m. The special session on "Geometry in Mathematics" will be under the chairmanship of Professor Preston C. Hammer of Pennsylvania State University and will be held in the Plantation Oak Room of the Governor House Motel. The special session on "Complex Analysis," which will be under the chairmanship of Professor W. H. J. Fuchs of Cornell University, will be held in the Presidential Salon of the Jung Hotel.

There will be regular sessions for contributed ten-minute papers at 8:45-10:45 a.m. and 2:45-5:45 p.m. on Thursday, January 23; at 8:45-10:45 a.m. and 2:45-5:45 p.m. on Friday, January 24; at 2:45-5:45 p.m. on Saturday, January 25; and at 2:45-5:45 p.m. on Sunday, January 26.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet at 2:00 p.m. on Wednesday, January 22, in Terrace Suite 5 of the Jung Hotel. The Business Meeting of the Society will be held on Saturday, January 25, at 1:15 p.m. in the Grand Ballroom of the Jung Hotel.
REGISTRATION

The Registration Desk for this meeting will be located in the Convention Lobby on the Ground Floor of the Jung Hotel. The Desk will be open from 9:00 a.m. to 8:00 p.m. on Wednesday, January 22; from 8:00 a.m. to 5:00 p.m. on Thursday, January 23; from 9:00 a.m. to 5:00 p.m. on Friday through Sunday, January 24-26; and from 9:00 a.m. to 3:00 p.m. on Monday, January 27. It will be helpful if persons attending the meetings will register as soon as possible after their arrival.

The registration fees for the meeting are as follows:

<table>
<thead>
<tr>
<th>Category</th>
<th>Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member</td>
<td>$3.00</td>
</tr>
<tr>
<td>Member's family</td>
<td>0.50</td>
</tr>
<tr>
<td>Students</td>
<td>no fee</td>
</tr>
<tr>
<td>Nonmember</td>
<td>7.50</td>
</tr>
<tr>
<td>Nonmember's family</td>
<td>0.50</td>
</tr>
</tbody>
</table>

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 5:00 p.m. on Friday, Saturday, Sunday, and Monday, in the Tulane Room of the Jung Hotel.

EXHIBITS

The book and educational media exhibits will be displayed in the Exhibition Hall (Hall of the Americas) in the Jung Hotel on Friday, Saturday, and Sunday. The exhibits will be open from 9:00 a.m. to 5:00 p.m. on each of the three days.

BOOK SALE

Books published by the Society will be sold for cash prices somewhat below the usual prices when these same books are sold by mail on invoice.

ACCOMMODATIONS

Accommodations for the meeting will be handled by the New Orleans Tourist and Convention Commission. A form for requesting accommodations will be found on page 1060 of the November issue of these Notices. Persons desiring accommodations should complete this reserva-

<table>
<thead>
<tr>
<th>Hotel Name</th>
<th>Accommodation Type</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPRI MOTEL</td>
<td>Singles</td>
<td>$13.00</td>
</tr>
<tr>
<td></td>
<td>Doubles</td>
<td>15.00</td>
</tr>
<tr>
<td></td>
<td>Extra person</td>
<td>3.00</td>
</tr>
<tr>
<td>FONTAINEBLEAU MOTOR HOTEL</td>
<td>Singles</td>
<td>$16.00</td>
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<tr>
<td></td>
<td>Doubles</td>
<td>20.00 to $23.00</td>
</tr>
<tr>
<td></td>
<td>Twins</td>
<td>20.00 to 23.00</td>
</tr>
<tr>
<td>GOVERNOR HOUSE MOTOR HOTEL</td>
<td>Singles</td>
<td>$14.00 to $17.00</td>
</tr>
<tr>
<td></td>
<td>Doubles</td>
<td>18.00 to 20.00</td>
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<tr>
<td></td>
<td>Twins</td>
<td>24.00</td>
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<tr>
<td>HOWARD JOHNSON'S (CIVIC CENTER)</td>
<td>Singles</td>
<td>$16.00 to $18.00</td>
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<td></td>
<td>Studio parlors</td>
<td>25.00 to 29.00</td>
</tr>
<tr>
<td></td>
<td>Twins</td>
<td>23.00 to 27.00</td>
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<tr>
<td></td>
<td>Twin doubles</td>
<td>25.00 to 29.00</td>
</tr>
<tr>
<td></td>
<td>Suites</td>
<td>48.00 to 53.00</td>
</tr>
<tr>
<td></td>
<td>1st extra person</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>2nd extra person</td>
<td>3.00</td>
</tr>
<tr>
<td>JUNG HOTEL (headquarters hotel)</td>
<td>Singles</td>
<td>$13.00 to $21.00</td>
</tr>
<tr>
<td></td>
<td>Doubles</td>
<td>19.00 to 26.00</td>
</tr>
<tr>
<td></td>
<td>Twins</td>
<td>19.00 to 26.00</td>
</tr>
<tr>
<td></td>
<td>Suites</td>
<td>40.00 to 85.00</td>
</tr>
<tr>
<td></td>
<td>(parlor and 1 bedroom)</td>
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</tr>
<tr>
<td>HOTEL LASALLE</td>
<td>Singles</td>
<td>$8.00 to $8.50</td>
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<tr>
<td></td>
<td>Doubles</td>
<td>10.50 to 11.00</td>
</tr>
<tr>
<td></td>
<td>Twins</td>
<td>13.00</td>
</tr>
<tr>
<td></td>
<td>Family Rooms</td>
<td>14.50 to 16.00</td>
</tr>
<tr>
<td></td>
<td>(3 or 4 persons)</td>
<td></td>
</tr>
</tbody>
</table>
1. Jung Hotel
2. Fontainebleau Motor Hotel
3. Le Petit Motel
4. Patio Motel
5. Capri Motel
6. Ramada Inn
7. Thunderbird Motel
8. Tamanaca Downtown Motel
9. Sheraton-Delta Motor Hotel
10. Governor House Motor Hotel

11. Hotel LaSalle
12. Continental Trailways Bus Terminal
13. The Warwick
14. Maison de Ville
15. Sheraton-Charles Hotel
16. Howard Johnson's Motor Lodge
17. Union Passenger Terminal
18. YMCA
19. Pontchartrain Hotel
20. Greyhound Bus Terminal
LE PETIT MOTEL
Singles $12.00 to $14.00
Doubles 16.00
Twins 16.00 to 20.00
Twin doubles 18.00 to 20.00
(2 persons)
Additional persons 2.00
MAISON DE VILLE
Singles $15.00 to $16.50
Doubles 18.50 to 25.00
Twins 22.00 to 26.00
PATIO MOTEL ("Best Western")
Doubles $12.00 to $14.00
Twin Doubles 14.00 to 16.00
Extra person 2.00
Will not be held beyond 5 p.m. without one night's deposit.
PONTCHARTRAIN HOTEL
Singles $19.00 to $22.00
Twins 20.00 to 30.00
Suites 38.00 to 55.00
RAMADA INN
Singles $12.50
Doubles 15.50
Twin Doubles 20.00
Additional person 3.00
SHERATON-CHARLES HOTEL
Singles $13.50 to $15.50
Doubles 17.00 to 19.00
Twins 17.50 to 19.50
Suites 25.00 to 45.00
SHERATON-DELTA MOTOR HOTEL
Singles $12.50
Doubles 16.50
Twins 16.50
Suites 27.00
Extra person 3.00
TAMANACA DOWNTOWN MOTEL
Doubles $14.00 to $16.00
Twins 17.00 to 22.00
Additional person 2.50
THUNDERBIRD MOTEL
Doubles $15.00
Twin Doubles 22.00
Deluxe (3 persons) 19.00
Additional persons 3.00
THE WARWICK
Singles $12.00
Doubles 15.00
YMCA (Men only)
Singles $ 4.25 to $3.75
(A $5.00 deposit is required)
NATIONAL SCIENCE FOUNDATION
INFORMATION CENTER
NSF staff members will be available to provide counsel and information on all NSF programs of interest to mathematicians from 10:00 a.m. to 4:00 p.m. on January 23, 24, and 25, 1969, in Room 233 of the Jung Hotel.

ENTERTAINMENT
There are many things to see and to do in the New Orleans area. Brochures describing various tours around the city will be available at the hospitality desk in the Registration area. These will include walking and automobile trips, bus and limousine tours, and boat tours on the Mississippi River. There will also be leaflets in the Registration area describing some of the major attractions of New Orleans, such as the French Quarter (Vieux Carré), City and Audubon Parks, the Mississippi River, and the various colleges and universities.

New Orleans has several museums and art galleries. At night, there is entertainment available to suit all tastes, from jazz and nightclubs to legitimate theater and concerts of various sorts.

Some of the finest restaurants in the nation are located in New Orleans. There will be available at the hospitality desk a guide to some of the better restaurants in the city, as well as information about other nearby eating places.

MAIL AND MESSAGE CENTER
All mail and telegrams for persons attending the meeting should be addressed in care of Mathematical Meetings, Jung Hotel, 1500 Canal Street, New Orleans, Louisiana 70140. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located in the lounge area just outside the Grand Ballroom on the Upper Mezzanine Floor of the hotel.

A message center will be located in the same area to receive incoming calls for all members in attendance. The center
will be open from January 23 through January 27 between 9:00 a.m. and 5:00 p.m. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message has been picked up at the message center. Members are advised to leave the following number with anyone who might want to reach them at the meeting: (504) 523-4471, Ext. 1138 or 1139.

TRAVEL AND LOCAL INFORMATION

Airlines serving New Orleans include Braniff, Continental, Delta, Eastern, National, Southern, Trans-Texas, and various international carriers. There is limousine service from the New Orleans International Airport to most downtown hotels for a $2.00 charge.

Railroad service to New Orleans is provided by Illinois Central, Kansas City Southern, Louisville & Nashville, Texas & Pacific, Southern, and Southern Pacific. The Union Passenger Terminal is near the heart of the city, with adequate taxi service available.

New Orleans is also served by buses of both Continental Trailways and Greyhound Lines. Both terminals are located within a few blocks of the Jung Hotel.

Those who come to New Orleans by car will find that some of the listed hotels offer free parking to their guests. However, this does not include the Jung Hotel.

The average maximum January temperature in New Orleans is 60°; the average minimum is 43°. January rainfall averages about one inch per week, so that an umbrella may be helpful; generally, showers are heavy but brief.

In January, New Orleans is subject to early morning fogs which might cause delays of from thirty minutes to three hours in the landing of planes. It might be wise for participants to plan their flights to avoid critical delays.

COMMITTEE ON ARRANGEMENTS

## TIMETABLE

**(Central Standard Time)**

<table>
<thead>
<tr>
<th>WEDNESDAY, January 22</th>
<th>American Mathematical Society</th>
<th>Association for Symbolic Logic</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 a.m. - 8:00 p.m.</td>
<td>REGISTRATION - Convention Lobby (J)</td>
<td>Terrace Suites 1-2 (J)</td>
</tr>
<tr>
<td>9:00 a.m. - 11:50 a.m.</td>
<td></td>
<td>Contributed Papers</td>
</tr>
<tr>
<td>2:00 p.m. - 3:00 p.m.</td>
<td></td>
<td>Chairman: Abraham Robinson</td>
</tr>
<tr>
<td>2:00 p.m.</td>
<td>Council Meeting</td>
<td>Invited Address: On the uses of saturated and special models C. C. Chang</td>
</tr>
<tr>
<td>3:10 p.m. - 5:10 p.m.</td>
<td>Terrace Suite 5 (J)</td>
<td></td>
</tr>
<tr>
<td>5:30 p.m. - 7:00 p.m.</td>
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<tr>
<td>7:30 p.m. - 11:00 p.m.</td>
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<table>
<thead>
<tr>
<th>THURSDAY, January 23</th>
<th>AMS</th>
<th>ASL</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Convention Lobby (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Algebra I</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Terrace Suite 5 (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Session on Algebra II</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Meeting Room 1 (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Topological Dynamics</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Meeting Room 2 (J)</td>
<td></td>
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<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Session on Analysis I</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Meeting Room 4-5 (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis II</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Meeting Room 9 (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis III</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Meeting Room 10 (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis IV</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Presidential Salon (J)</td>
<td></td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Statistics and Probability I</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Pavilion Room (J)</td>
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<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Session on Geometry I</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Meeting Room 3 (H)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Topology I</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Plantation Oak Room (G)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Topology II</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Pine Room (G)</td>
<td></td>
</tr>
<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
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</tr>
<tr>
<td>10:10 a.m. - 11:45 a.m.</td>
<td>Invited Address:</td>
<td></td>
</tr>
<tr>
<td>11:00 a.m. - Noon</td>
<td>Stability theory and invariant</td>
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<td></td>
<td>models for dynamical systems</td>
<td></td>
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<td></td>
<td>Jürgen K. Moser</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grand Ballroom (J)</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>American Mathematical Society</td>
<td>Association for Symbolic Logic</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td>1:30 p.m.</td>
<td>Award of the Bocher Memorial Prize - Grand Ballroom (J)</td>
<td>Terrace Suites 1-2 (J)</td>
</tr>
<tr>
<td>2:00 p.m. - 3:00 p.m.</td>
<td></td>
<td>Invited Address: Definability and decidability in second-order theories Michael Rabin</td>
</tr>
<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra III - Terrace Suite 5 (J)</td>
<td></td>
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<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Algebra IV - Meeting Room 1 (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra V - Meeting Room 2 (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 6:00 p.m.</td>
<td>Session on Analysis V - Meeting Room 4-5 (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis VI - Meeting Room 9 (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Analysis VII - Meeting Room 10 (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Analysis VIII - Presidential Salon (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Applied Mathematics I - Pavilion Room (J)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Graph Theory I - Meeting Room 3 (H)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Topology III - Plantation Oak Room (G)</td>
<td></td>
</tr>
<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Topology IV - Pine Room (G)</td>
<td></td>
</tr>
<tr>
<td>3:10 p.m. - 5:35 p.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8:00 p.m.</td>
<td>Josiah Willard Gibbs Lecture: Trends and social implications of research - R. L. Wilder</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grand Ballroom (J)</td>
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**FRIDAY, January 24**

<table>
<thead>
<tr>
<th>Time</th>
<th>AMS</th>
<th>Mathematical Association of America</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Convention Lobby (J)</td>
<td></td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Exhibition Hall (J)</td>
<td></td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Tulane Room (J)</td>
<td></td>
</tr>
<tr>
<td>8:45 a.m. - 10:20 a.m.</td>
<td>Special Session: Geometry in mathematics - Preston C. Hammer, Chairman Plantation Oak Room (G)</td>
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<tr>
<td>8:45 a.m. - 10:45 a.m.</td>
<td>Special Session: Complex analysis - W. H. J. Fuchs, Chairman Presidential Salon (J)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Topology V - Terrace Suite 1 (J)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Topology VI - Terrace Suite 3 (J)</td>
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<tr>
<td>8:45 a.m. - 10:10 a.m.</td>
<td>Session on Topological Groups and Lie Theory - Terrace Suite 5 (J)</td>
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<tr>
<td>Time</td>
<td>American Mathematical Society</td>
<td>Mathematical Association of America</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis IX</td>
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<td>Meeting Room 1 (J)</td>
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<tr>
<td>8:45 a.m. - 10:25 a.m.</td>
<td>Session on Analysis X</td>
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<td>Meeting Room 2 (J)</td>
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<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis XI</td>
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<td>Meeting Room 4-5 (J)</td>
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<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Analysis XII</td>
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<td>Meeting Room 9 (J)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Algebra VI</td>
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<td>Meeting Room 10 (J)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Algebra VII</td>
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<td>Pavilion Room (J)</td>
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<tr>
<td>8:45 a.m. - 10:40 a.m.</td>
<td>Session on Statistics and Probability II</td>
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<td>Pine Room (G)</td>
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<tr>
<td>9:00 a.m. - 4:00 p.m.</td>
<td>Invited Address: Geometric ergodic theory</td>
<td>Board of Governors</td>
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<tr>
<td></td>
<td>C. C. Moore</td>
<td>Meeting Room 1 (H)</td>
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<td></td>
<td>Grand Ballroom (J)</td>
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<tr>
<td>11:00 a.m. - Noon</td>
<td>Presidential Address: Differentiability theorems for weak solutions of differential equations</td>
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<td></td>
<td>C. B. Morrey, Jr.</td>
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<td>Grand Ballroom (J)</td>
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<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Special Session: Geometry in mathematics</td>
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<td></td>
<td>Preston C. Hammer, Chairman</td>
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<td></td>
<td>Plantation Oak Room (G)</td>
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<tr>
<td>2:45 p.m. - 4:20 p.m.</td>
<td>Special Session: Complex analysis</td>
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<td></td>
<td>W. H. J. Fuchs, Chairman</td>
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<td>Presidential Salon (J)</td>
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<tr>
<td>2:45 p.m. - 4:45 p.m.</td>
<td>Special Session: Geometry in mathematics</td>
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<td>Preston C. Hammer, Chairman</td>
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<td>Plantation Oak Room (G)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra VIII</td>
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<td>Terrace Suite 1 (J)</td>
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<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra IX</td>
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<td>Terrace Suite 3 (J)</td>
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<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra X</td>
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<td>Terrace Suite 5 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XIII</td>
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<td>Meeting Room 1 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XIV</td>
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<td>Meeting Room 2 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XV</td>
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<td>Meeting Room 4-5 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XVI</td>
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<td>Meeting Room 9 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Applied Mathematics II</td>
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<td>Meeting Room 10 (J)</td>
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<tr>
<td>2:45 p.m. - 4:40 p.m.</td>
<td>Session on Graph Theory II</td>
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<td>Pavilion Room (J)</td>
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<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Topology VII</td>
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<td>Meeting Room 3 (H)</td>
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<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Topology VIII</td>
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<td>Pine Room (G)</td>
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<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>7:30 p.m. - 7:38 p.m.</td>
<td>Tulane Room (J)</td>
<td>FILMS OF THE COLLEGE GEOMETRY PROJECT OF THE UNIVERSITY OF MINNESOTA (in color)</td>
</tr>
<tr>
<td>7:39 p.m. - 7:50 p.m.</td>
<td>CAROMS by Chandler Davis</td>
<td>EQUIDECOMPOSABLE POLYGONS by J. D. E. Konhauser</td>
</tr>
<tr>
<td>7:51 p.m. - 8:04 p.m.</td>
<td>PROJECTIVE GENERATION OF CONICS by S. Schuster</td>
<td>CEM ANIMATED CALCULUS FILMS (in color)</td>
</tr>
<tr>
<td>8:15 p.m. - 8:23 p.m.</td>
<td>I MAXIMIZE by Chandler Davis</td>
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<tr>
<td>8:24 p.m. - 8:32 p.m.</td>
<td>CONTINUITY OF MAPPINGS by Albert Fadell</td>
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<tr>
<td>8:33 p.m. - 8:45 p.m.</td>
<td>THE THEOREM OF THE MEAN by Felix P. Welch</td>
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<tr>
<td>8:46 p.m. - 9:06 p.m.</td>
<td>NEWTON'S METHOD by Herbert Wilf</td>
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<tr>
<td>9:07 p.m. - 9:17 p.m.</td>
<td>LIMIT by Robert C. Fisher</td>
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**SATURDAY, January 25**

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<tr>
<th>Time</th>
<th>AMS</th>
<th>MAA NCTM</th>
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</thead>
<tbody>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Convention Lobby (J)</td>
<td>Report of the Joint Committee of the American Statistical Association and NCTM</td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Exhibition Hall (J)</td>
<td>Frederick Mosteller</td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Tulane Room (J)</td>
<td>Reactions to the Report</td>
</tr>
<tr>
<td>9:00 a.m. - 9:30 a.m.</td>
<td>Grand Ballroom (J)</td>
<td>G. R. Rising</td>
</tr>
<tr>
<td>9:30 a.m. - 9:50 a.m.</td>
<td>Panel Discussion: Secondary School Preparation of Students for Freshman Calculus</td>
<td>Panel Discussion: Secondary School Preparation of Students for Freshman Calculus</td>
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<tr>
<td>10:10 a.m. - 11:50 a.m.</td>
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<td>Moderator: Dorothy L. Bernstein</td>
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<tr>
<td>1:15 p.m.</td>
<td>Business Meeting</td>
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<tr>
<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Algebra XI</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra XII</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Algebra XIII</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Geometry II</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XVII</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XVIII</td>
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</table>
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<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America National Council of Teachers of Mathematics</th>
</tr>
</thead>
</table>
| 2:45 p.m. - 5:55 p.m. | Session on Analysis XIX  
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| 2:45 p.m. - 5:55 p.m. | Session on Analysis XX  
Meeting Room 10 (J)        |                                                                                |
| 2:45 p.m. - 5:40 p.m. | Session on Applied Mathematics III  
Presidential Salon (J)  |
| 2:45 p.m. - 4:40 p.m. | Session on Logic and Categories  
Pavilion Room (J) |
| 2:45 p.m. - 5:25 p.m. | Session on Applied Mathematics IV  
Meeting Room 3 (H)       |                                                                                |
| 2:45 p.m. - 5:40 p.m. | Session on Topology IX  
Plantation Oak Room (G) |
| 2:45 p.m. - 5:25 p.m. | Session on Topology X  
Pine Room (G)         |                                                                                |
| 7:30 p.m. - 7:45 p.m. | Film Program  
Grand Ballroom (J) |
| 7:55 p.m. - 8:56 p.m. | SURFACE AREA WITH BLOCKS, a first grade taught by Phyllis Klein  
(A film of the University of Illinois Arithmetic Project) |
| 9:10 p.m. - 9:43 p.m. | LET US TEACH GUESSING: A DEMONSTRATION WITH GEORGE POLYA  
(A CEM Individual Lectures Film in Color) |

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<tr>
<th>Time</th>
<th>AMS</th>
<th>MAA NCTM</th>
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</table>
| 9:00 a.m. - 5:00 p.m. | REGISTRATION - Convention Lobby (J)  
Grand Ballroom (J) |                                                                                |
| 9:00 a.m. - 5:00 p.m. | EXHIBITS - Exhibition Hall (J) |                                                                                |
| 9:00 a.m. - 5:00 p.m. | EMPLOYMENT REGISTER - Tulane Room (J)  
CEO Center (J) |
| 9:00 a.m. - 9:50 a.m. | A Further Look at Teacher Training  
Clarence Ethel Hardgrove |
| 10:00 a.m. - 10:50 a.m. | Business Meeting; the Association's  
Eighth Award for Distinguished Service to Mathematics |
| 11:00 a.m. - 11:50 a.m. | Computing and the Mathematics Teacher  
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| 2:45 p.m. - 5:40 p.m. | Session on Algebra XIV  
Terrace Suite 1 (J) |
| 2:45 p.m. - 5:40 p.m. | Session on Algebra XV  
Terrace Suite 3 (J) |
| 2:45 p.m. - 5:40 p.m. | Session on Algebra XVI  
Terrace Suite 5 (J) |
| 2:45 p.m. - 5:40 p.m. | Session on Analysis XXI  
Meeting Room 1 (J)  |
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XXII</td>
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<td>Meeting Room 2 (J)</td>
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<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Analysis XXIII</td>
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<td>Meeting Room 4-5 (J)</td>
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<td>2:45 p.m. - 5:55 p.m.</td>
<td>Session on Analysis XXIV</td>
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<td>Meeting Room 9 (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Geometry III</td>
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<td>Meeting Room 10 (J)</td>
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<tr>
<td>2:45 p.m. - 5:10 p.m.</td>
<td>Session on Logic and Foundations</td>
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<td>Pavilion Room (J)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Applied Mathematics</td>
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<td>Meeting Room 3 (H)</td>
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<tr>
<td>2:45 p.m. - 5:40 p.m.</td>
<td>Session on Topology XI</td>
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<td>Plantation Oak Room (G)</td>
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<tr>
<td>2:45 p.m. - 5:25 p.m.</td>
<td>Session on Statistics and Probability III</td>
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<td>Pine Oak Room (G)</td>
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<tr>
<td>7:00 p.m.</td>
<td>CONFERENCE BOARD Council Meeting - Meeting Room 3 (J)</td>
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<td>Grand Ballroom (J)</td>
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<td>Film Program</td>
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<tr>
<td>7:30 p.m. - 8:05 p.m.</td>
<td>INTRODUCTION TO COMPOSITION</td>
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<td>WITH JUMPING RULES, a fifth grade taught by Marie Herman</td>
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<td>(A film of the University of Illinois Arithmetic Project)</td>
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<tr>
<td>8:15 p.m. - 9:10 p.m.</td>
<td>CHALLENGE IN THE CLASSROOM: THE METHODS OF R. L. MOORE</td>
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<td>(A CEM Individual Lectures Film in color)</td>
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<tr>
<td>9:20 p.m. - 10:07 p.m.</td>
<td>CAN YOU HEAR THE SHAPE OF A DRUM? by Mark Kac (A CEM</td>
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<td>Individual Lectures Film in color)</td>
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<td>9:00 a.m. - 3:00 p.m.</td>
<td>REGISTRATION - Convention Lobby (J)</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>EMPLOYMENT REGISTER - Tulane Room (J)</td>
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<tr>
<td>9:00 a.m. - 9:50 a.m.</td>
<td>Session on the Theory of Distributions</td>
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<td>10:00 a.m. - 10:50 a.m.</td>
<td>Connections between Distributions and Boundary Values of Analytic Functions with Applications</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
<td>Applications of Distributions and Other Generalized Functions to Partial Differential Equations</td>
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<tr>
<td>1:30 p.m. - 2:20 p.m.</td>
<td>Introductory Multivariate Calculus A. P. Mattuck</td>
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<td>2:30 p.m. - 3:20 p.m.</td>
<td>Advanced Multivariate Calculus W. H. Fleming</td>
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<td>3:30 p.m. - 4:20 p.m.</td>
<td>Applications of Differential Forms in Multivariate Calculus Harley Flanders</td>
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Clark, C. E. #570
Clarke-Carroll, F. M. #395
Clay, J. R. #2
Clayton, D. D. #475
Clements, G. F. #194
Cohen, J. S. #500
Cohen, M. E. #153
Cohn, D. K. #512
Coleman, D. B. #97
Conal, P. C. #136
Conway, W. E. #690
Cooper, J. B. #352
Dyer, R. #382
Coppen, C. A. #161
Coury, J. E. #338
Coxeter, H. S. M. p. 39
Crittenden, R. S. #588
Croom, F. H. #198
Cryer, C. W. #532
Casick, T. W. #321
Cutlip, W. F. #539
Dacey, J. C. #401
Dancis, J. #228
Danes, A. E. #168
Darwin, J. T., Jr. #159
Davis, R. D. #115
Davison, W. F. #236
Dawson, D. F. #364
Debnath, L. #698
DeKleine, H. A. #178
DeLeau, A. #579
Della Riccia, G. #19
DeMar, R. F. #651
DeMarr, R. E. #555
DeMeyer, F. R. #90
Derderian, J. C. #602
Derr, L. J. #548
Derrick, W. R. #163
DeVun, E. #574
Dhillon, R. S. #691
Di Antonio, G. #506
Diaz, J. B. #641
Diestel, J. #429
Dinges, H. #719
Dixon, J. D. #335
Dolan, J. M. #46
Donaldson, J. A. #377
Dotson, W. G. Jr. #360
Douglas, R. J. #397
Drake, D. A. #486
Drobnies, S. L. #176
Duggal, K. L. #68
Duncan, L. D. #160
Duren, P. L. p. 33
Duskin, J. #540
Dwight, S. H. #255
Dykes, N. #214
Eaton, W. T. #429
Eberhart, C. #575
Edmonston, D. E. #632
Eenigenburg, P. #521
Eggan, L. C. #595
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

THURSDAY, 8:45 A.M.

Session on Algebra 1 (Near Rings), Terrace Suite 5, Jung Hotel
8:45-8:55
(1) Boolean near-rings
Professor J. R. Clay and Professor D. A. Lawver*, University of Arizona (663-44)

9:00-9:10
(2) Using a digital computer in research for near-rings
Dr. J. R. Clay, University of Arizona (663-102)

9:15-9:25
(3) Completely simple semirings
Dr. P. A. Grillet, Tulane University, and Dr. M. P. Grillet*, Louisiana State University (663-51)

9:30-9:40
(4) Nil semisimple hemirings with descending chain condition
Professor J. R. Mosher, Texas A & M University (663-341)
(Introduced by Professor J. J. Malone, Jr.)

9:45-9:55
(5) Ideals in near-rings
Professor J. J. Malone, Jr., Texas A & M University (663-340)

10:00-10:10
(6) On distributively generated near-rings
Mr. Steve Ligh, Texas A & M University (663-339)

10:15-10:25
(7) Right ideals of transformation near-rings
Professor H. E. Heatherly, University of Southwestern Louisiana (663-318)

10:30-10:40
(8) The Hilbert basis theorem for halfrings. Preliminary report
Mr. H. E. Stone, Texas Christian University (663-556)

THURSDAY, 8:45 A.M.

Session on Algebra 2 (Semigroups), Meeting Room 1, Jung Hotel
8:45-8:55
(9) On free commutative semigroups
Dr. P. A. Grillet, Tulane University (663-10)

9:00-9:10
(10) Congruences on ω^n-bisimple semigroups
Professor R. J. Warne, West Virginia University (663-111)

9:15-9:25
(11) Idempotent generated Rees matrix semigroups
Professor J. B. Kim, West Virginia University (663-176)

9:30-9:40
(12) A structure theory for finite regular semigroups
Mr. Dennis Allen, Jr., Bell Telephone Laboratories, Holmdel, New Jersey (663-258)

9:45-9:55
(13) On radicals in a certain class of semigroups
Professor J. E. Kuczkowski, Purdue University, Indianapolis Campus (663-238)
10:00-10:10  
(14) Subsemigroups of cycle semigroups  
Professor J. C. Higgins, Brigham Young University (663-396)

10:15-10:25  
(15) Factorizable semigroups  
Professor K. W. Tolo, University of Tennessee (663-509)

10:30-10:40  
(16) Coordinatization of orthomodular posets  
Professor S. P. Gudder, University of Wisconsin (663-361)

THURSDAY, 8:45 A.M.

Session on Topological Dynamics, Meeting Room 2, Jung Hotel  
8:45-8:55  
(17) Orbits of the stabilizer subgroup of an ordered permutation group  
Professor S. H. McCleary, University of Georgia (663-79)

9:00-9:10  
(18) An orthomodular poset with a full set of states not embeddable in Hilbert space  
Professor R. J. Greechie, Kansas State University (663-394)

9:15-9:25  
(19) Minimal sets of equicontinuous flows on locally compact metric spaces  
Professor Giacomo Della Riccia, Indiana University (663-174)

9:30-9:40  
(20) The equivalence of attainability and trajectories in abstract polysystems  
Professor H. D. Sullivan, Eastern Washington State College (663-273)

9:45-9:55  
(21) Locally setwise homogeneous continua and their homeomorphism groups  
Professor B. L. Brechner, University of Florida (663-231)

10:00-10:10  
(22) A topologically strongly mixing symbolic minimal set  
Mr. K. E. Petersen, Yale University (663-328)

10:15-10:25  
(23) The limit sets of orbits of a plane autonomous dynamical system  
Professor Ronald Tannenwald, Southeastern Massachusetts Technological Institute (663-464)

THURSDAY, 8:45 A.M.

Session on Analysis 1 (Functional Analysis), Meeting Room 4-5, Jung Hotel  
8:45-8:55  
(24) Convergence of monotone nets in ordered topological vector spaces  
Professor C. W. McArthur, Florida State University (663-432)

9:00-9:10  
(25) Continuous decompositions  
Professor D. J. Fleming, Clarkson College of Technology, and Professor W. H. Ruckle*, Lehigh University (663-184)

9:15-9:25  
(26) Tensor product of biorthogonal systems  
Professor H. F. Joiner, II, University of Massachusetts (663-234)

9:30-9:40  
(27) Generalized coordinate spaces  
Professor D. J. Fleming, Clarkson College of Technology (663-392)

9:45-9:55  
(28) On the coefficient spaces of $C[0,1]$  
Mr. J. R. Holub* and Professor J. R. Retherford, Louisiana State University (663-385)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
10:00-10:10
(29) Schauder bases in spaces of differentiable functions
Mr. S. A. Schonefeld, Purdue University and University of California, Irvine (663-381)

10:15-10:25
(30) The weak basis theorem for (F)- and (LF)-spaces
Professor G. Bennett*, St. John's College, Cambridge, England, and Lehigh University, and Professor J. B. Cooper, Clare College, Cambridge, England (663-585)
(Introduced by Professor Everett Pitcher)

10:30-10:40
(31) The comparison of an unconditionally converging operator
Professor J. L. Howard, Oklahoma State University (663-540)

THURSDAY, 8:45 A.M.

Session on Analysis 2 (Complex Functions), Meeting Room 9, Jung Hotel
8:45-8:55
(32) An inclusion theorem for generalized Lototsky summability methods. Preliminary report
Professor H. B. Skerry, Lehigh University (663-457)

9:00-9:10
(33) Expansion of analytic functions in series of some recursively generated polynomials
Professor J. W. Jayne, United States Naval Academy (663-442)

9:15-9:25
(34) Analytic functions with quasi-analytic boundary values
Professor W. A. Groening, Eastern Michigan University (663-589)

9:30-9:40
(35) Fourier coefficients of wave forms of dimension-1
Professor V. V. Rao, University of Saskatchewan, Regina Campus (663-516)

9:45-9:55
(36) On Fatou values of normal harmonic functions
Professor J. L. Meek, University of Arkansas (663-151)

10:00-10:10
(37) Variations with constraints for classes of analytic functions
Professor J. A. Pfaltzgraff, University of North Carolina (663-171)

10:15-10:25
(38) A uniqueness theorem for holomorphic functions
Professor J. A. Siddiqi, University of Sherbrooke (663-192)

THURSDAY, 8:45 A.M.

Session on Analysis 3 (Differential Equations), Meeting Room 10, Jung Hotel
8:45-8:55
(39) Oscillation theorems for functional-differential equations
Professor W. J. Coles, Professor L. J. Grimm and Professor Klaus Schmitt*, University of Utah (663-521)

9:00-9:10
(40) A canonical representation for solutions of a class of nonlinear problems
Dr. Jagdish Chandra, Maggs Research Center, Watervliet, New York, and Professor B. A. Fleishman*, Rensselaer Polytechnic Institute (663-187)

9:15-9:25
(41) An existence and uniqueness theorem for a nonlinear partial differential equation
Dr. M. N. Manougian, University of South Florida (663-194)
9:30-9:40
(42) Discreteness of the spectrum of the Schrödinger operator
Professor Erik Balslev, State University of New York at Buffalo (663-749)

9:45-9:55
(43) Disconjugacy criteria for selfadjoint differential systems
Professor C. D. Ahlbrandt, University of Missouri (663-19)

10:00-10:10
(44) Boundary value problems with interior point boundary conditions
Professor A. M. Krall, Pennsylvania State University (663-66)

10:15-10:25
(45) Linear differential-difference operators and their adjoints
Professor D. K. Hughes, Abilene Christian College (663-617)

10:30-10:40
(46) On the relationship between the oscillatory behavior of a linear third-order equation and its adjoint
Mr. J. M. Dolan, Oak Ridge National Laboratory, Oak Ridge, Tennessee (663-747)

THURSDAY, 8:45 A.M.

Session on Analysis 4 (Applied Mathematics), Presidential Salon, Jung Hotel
8:45-8:55
(47) Multiplier rules for variational problems with finite constraints by algebraic methods
Professor Robert Silber, North Carolina State University, (663-205)

9:00-9:10
(48) A Poisson's integral formula for elliptic equations with constant coefficients
Dr. A. K. Bose, Clemson University (663-465)

9:15-9:25
(49) Poisson kernels for degenerating elliptic boundary value problems
Professor P. C. Fife, University of Arizona (663-602)

9:30-9:40
(50) On inclusion relations between parabolic Lipschitz and Lebesgue spaces
Professor C. H. Sampson, Texas A & M University (663-524)

9:45-9:55
(51) Asymptotic behavior of solutions of a second order autonomous differential system which describes the onset of an explosion
Mr. Norman Gerri and Dr. W. C. Taylor*, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland (663-671)

10:00-10:10
(52) On an integral transform. II
Professor T. N. Srivastava, Loyola College of Montreal (663-695)
(Introduced by Professor A. J. Prillo)

10:15-10:25
(53) On the general H transform
Dr. V. C. Nair, Reginal College of Education, Ajmer, India, and Ohio State University (663-698)
(Introduced by Professor Ranko Bojanic)

10:30-10:40
(54) A generalization of extensors
Professor P. S. Morey, Jr., Texas A & I University (663-452)

THURSDAY, 8:45 A.M.

Session on Statistics and Probability 1, Pavilion, Jung Hotel
8:45-8:55
(55) On distance between zeros of a covariance stationary Gaussian process
Mr. E. H. Lezak, Indiana University Northwest (663-210)
9:00-9:10
(56) On a martingale related to a strictly stationary random process
Professor Z. R. Pop-Stojanovic, University of Florida (663-583)
(Introduced by Professor A. R. Bednarek)

9:15-9:25
(57) A lower bound for the Hőlder condition of Gaussian processes with stationary increments
Professor M. B. Marcus, Northwestern University (663-299)

9:30-9:40
(58) Limit theorems for last passage times
Professor J. G. Wendel*, University of Michigan, Professor H. Robbins, Columbia University, and Professor D. Siegmund, Stanford University (663-528)

9:45-9:55
(59) Stochastic processes with sample path functions of bounded variation
Mr. R. H. Shachtman, University of North Carolina at Chapel Hill (663-534)

10:00-10:10
(60) Gaussian processes with bounded paths
Mr. E. R. Rodemich* and Dr. H. C. Rumsey, Jr., Jet Propulsion Laboratories, California Institute of Technology (663-663)

10:15-10:25
(61) Random variables similar to Student's $t$
Professor Z. W. Birnbaum, University of Washington (663-533)

10:30-10:40
(62) Estimators of the parameters of an extreme-value distribution using quantiles
Mr. Isidore Eisenberger, Jet Propulsion Laboratory, California Institute of Technology (663-65)

THURSDAY, 8:45 A.M.

Session on Geometry 1, Meeting Room 3, Howard Johnson's
8:45-8:55
(63) Geometry of $f$-Kaehlerian manifolds
Professor D. E. Blair, Michigan State University (663-440)

9:00-9:10
(64) Commensurability classes of compact Clifford-Klein forms of symmetric spaces. Preliminary report
Professor F. W. Kamber, University of Illinois (663-447)

9:15-9:25
(65) On the upper bound conjecture for convex polytopes
Dr. Peter McMullen, Western Washington State College (663-463)
(Introduced by Professor J. R. Reay)

9:30-9:40
(66) Ovals in flat projective planes
Professor Hansjoachim Groh, Kansas State University (663-467)
(Introduced by Professor J. E. Maxfield)

9:45-9:55
(67) A new series of line involutions in $S_3$
Professor G. M. Stratopoulos*, Weber State College, and Professor C. R. Wylie, Jr., University of Utah (663-374)

10:00-10:10
(68) Singular Riemannian structures compatible with $\mathfrak{P}$-structures
Mr. K. L. Duggal, University of Windsor (663-387)
(Introduced by Professor H. A. Eliopoulos)
The missing seventh circle
Professor Daniel Pedoe, University of Minnesota (663-397)

THURSDAY, 9:45 A.M.

Session on Topology 1 (Embeddings; knots, isotopies), Plantation Oak Room, Governor House
8:45-8:55
(70) A characterization of $S^n$
Professor Richard Osborne* and Mr. J. R. Campbell, University of Idaho (663-72)

9:00-9:10
(71) Covering manifolds with open cells
Professor R. P. Osborne, University of Idaho, and Professor J. L. Stern*, University of Hawaii (663-141)

9:15-9:25
(72) Reidemeister homotopy chains and the second homotopy group of 2-spheres in 4-space
Professor S. J. Lomonaco, Jr., Florida State University (663-278)

9:30-9:40
(73) Factoring connected sums of tori embedded in codimension one
Professor D. R. Anderson, Northwestern University (663-365)

9:45-9:55
(74) On higher-dimensional fibered knots
Professor J. J. Andrews* and Professor D. W. Sumners, Florida State University (663-507)

10:00-10:10
(75) Extending isotopies of $S^{n-1}$ in $S^n$
Professor T. P. Wright, Florida State University (663-504)

10:15-10:25
(76) Small extensions of small homeomorphisms
Mr. William Barit, Louisiana State University (663-715)

10:30-10:40
(77) On continua quasi-embeddable in a torus
Professor R. B. Bennett, Auburn University (663-390)

THURSDAY, 8:45 A.M.

Session on Topology 2 (Peano Spaces), Pine Room, Governor House
8:45-8:55
(78) Closed mappings and local dimension
Dr. J. E. Keesling, University of Florida (663-186)

9:00-9:10
(79) Acyclicity of compact connected means
Professor K. N. Sigmon, University of Florida (663-200)

9:15-9:25
(80) Concerning a property of certain points of compact continua
Mr. M. H. Proffitt, State University College at New Paltz (663-296)

(Introduced by Professor R. L. Moore)

9:30-9:40
(81) Open maps of the universal curve onto continuous curves
Mr. D. C. Wilson, Rutgers University (663-477)

(Introduced by Professor L. F. McAuley)

9:45-9:55
(82) A note on spaces of certain nonalternating mappings onto an interval
Professor L. F. McAuley, Rutgers University (663-420)
### Session on Universal Tree-like Continua

**10:00-10:10**

(83) On universal tree-like continua  
Mr. J. W. Rogers, Jr., Emory University (663-413)

### Approximating Continua from Within

**10:15-10:25**

(84) Approximating continua from within  
Professor C. A. Eberhart and Professor J. B. Fugate*, University of Kentucky (663-607)

### Separation of Flat Spaces by Peano Continua

**10:30-10:40**

(85) Separation of flat spaces by Peano continua  
Professor J. W. Green, University of Oklahoma (663-553)

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###-invited address-

### Stability Theory and Invariant Manifolds for Dynamical Systems

**THURSDAY, 11:00 A.M.**

Invited Address: Grand Ballroom, Jung Hotel  
Stability theory and invariant manifolds for dynamical systems  
Professor Jurgen K. Moser, Courant Institute of Mathematical Sciences, New York University

### Bocher Memorial Prize

**THURSDAY, 1:30 P.M.**

Bocher Memorial Prize, Grand Ballroom, Jung Hotel

### Session on Algebra 3 (Associative Algebras and Rings)

**THURSDAY, 2:45 P.M.**

**2:45-2:55**

(86) L-primes in noncommutative rings  
Professor H. G. Rutherford, Montana State University (663-714)

**3:00-3:10**

(87) Countable Boolean algebras  
Professor R. S. Pierce, University of Washington (663-600)

**3:15-3:25**

(88) On the construction of the Kurosh lower radical class for associative rings. Preliminary report  
Mr. Daryl Kreiling* and Professor T. L. Jenkins, University of Wyoming (663-508)

**3:30-3:40**

(89) A Galois theory for separable algebras. Preliminary report  
Professor H. F. Kreimer, Florida State University (663-97)

**3:45-3:55**

(90) The Galois group of a commutative ring. Preliminary report  
Professor F. R. DeMeyer, Colorado State University (663-377)

**4:00-4:10**

(91) On the subring structure of finite nilpotent rings  
Dr. R. L. Kruse*, Sandia Corporation, Albuquerque, New Mexico, and Mr. D. T. Price, University of Chicago (663-351)

**4:15-4:25**

(92) Artinian q-torsion rings with p-groups  
Professor K. E. Eldridge, Ohio University (663-348)

**4:30-4:40**

(93) Global dimension of residue class rings  
Dr. G. A. Helzer, University of Maryland (663-305)

**4:45-4:55**

(94) Structure theorems for noncommutative complete local rings  
Mr. J. L. Fisher, California Institute of Technology (663-456)

**5:00-5:10**

(95) On the embedding of topological rings  
Professor J. K. Luedeman, Clemson University (663-691)
5:15-5:25
(96) Towber rings
Professor D. B. Lissner and Mr. A. V. Geramita*, Syracuse University (663-359)

5:30-5:40
(97) The tensor product of semisimple algebras
Professor D. B. Coleman, University of Kentucky (663-58)

THURSDAY, 2:45 P.M.

Session on Algebra 4 (Nonassociative Algebras), Meeting Room 1, Jung Hotel
2:45-2:55
(98) On factorization of Cayley numbers
Professor J. T. Hardy, University of Georgia (663-98)

3:00-3:10
(99) Euclidean Lie algebras
Professor R. V. Moody, University of Saskatchewan (663-291)

3:15-3:25
(100) On the Frattini subalgebra of a Lie algebra
Mr. E. L. Stitzinger, University of Pittsburgh (663-267)
(Introduced by Professor C. Y. Chao)

3:30-3:40
(101) Contributions to genetic algebras
Professor Harry Gonshor, Rutgers University (663-263)

3:45-3:55
(102) The Lavitzki radical in Jordan rings
Professor C. E. Tsai, Michigan State University (663-451)

4:00-4:10
(103) On noncommutative, nonassociative algebras. Preliminary report
Professor R. G. Hathway, Illinois State University (663-582)

4:15-4:25
(104) Lie and Jordan structures of simple graded rings. Preliminary report
Professor R. L. Speers, Skidmore College (663-547)

4:30-4:40
(105) (-1,1) rings
Professor I. R. Hentzel, Iowa State University (663-288)

4:45-4:55
(106) The Grothendieck groups of modules of finite rank over a valuation ring
Mr. S. M. Fakhruddin, Queen's University (663-693)
(Introduced by Professor Paulo Ribenboim)

5:00-5:10
(107) On M-loops
Mrs. Hala Pflugfelder, Temple University (663-753)

5:15-5:25
(108) Lie derivations of derived rings of simple rings
Professor R. A. Howland, Franklin and Marshall College (663-167)
(Introduced by Professor D. W. Western)

5:30-5:40
(109) Point algebras. Preliminary report
Professor Marshall Saade, University of Georgia (663-27)

5:45-5:55
(110) The cohomology theory of transitive filtered modules
Mr. Charles Freifeld, Northeastern University (663-696)
(Introduced by Professor Arshag Hajian)
THURSDAY, 2:45 P.M.

Session on Algebra 5 (Algebraic Geometry and Valuation Rings), Meeting Room 2, Jung Hotel

2:45-2:55
(111) Resolutions of singularities in characteristic p for almost all p
   Dr. P. C. Eklof, Yale University (663-624)

3:00-3:10
(112) A p-adic fixed point formula
   Professor Daniel Reich, Johns Hopkins University (663-555)

3:15-3:25
(113) Algebraic varieties over real closed fields
   Dr. C.-D. Geyer, Queen's University (663-664)
   (Introduced by Professor Paulo Ribenboim)

3:30-3:40
(114) The solution of algebraic equations in the positive orthant
   Professor Morris Weisfeld, Duke University (663-448)

3:45-3:55
(115) Inertial automorphisms of wildly ramified v-rings with ramification 2p
   Mr. R. D. Davis, Florida State University (663-628)

4:00-4:10
(116) Higher derivations of wildly ramified v-rings
   Dr. M. N. Heinzer, Florida State University (663-629)

4:15-4:25
(117) Ramification theory for extensions of degree p. Preliminary report
   Professor Susan Williamson, Regis College (663-575)

4:30-4:40
(118) Hahn valuations and (locally) compact rings
   Professor T. M. Viswanathan, University of Western Ontario (663-676)

4:45-4:55
(119) Nonconservative function fields of genus one
   Mr. C. S. Queen, Ohio State University (663-689)

5:00-5:10
(120) Explicit reciprocity for a local field
   Professor L. D. Geissinger and Professor W. H. Graves*, University of North Carolina at Chapel Hill (663-107)

5:15-5:25
(121) A construction of the local norm residue map
   Professor W. F. Brady, Connecticut College (663-709)

5:30-5:40
(122) On Dirichlet series associated with rational algebras
   Professor W. E. Jenner, University of North Carolina at Chapel Hill (663-59)

THURSDAY, 2:45 P.M.

Session on Analysis 5 (Functional Analysis), Meeting Room 4-5, Jung Hotel

2:45-2:55
(123) Operators on Banach lattices. Preliminary report
   Mr. P. C. Shields, Wayne State University (663-552)

3:00-3:10
(124) Partial solution to Mackey's problem about modular pairs and completeness
   Professor S. S. Holland, Jr., University of Massachusetts (663-576)

3:15-3:25
(125) The part metric in a convex set
   Professor Heinz Bauer and Professor H. S. Bear*, New Mexico State University (663-592)
3:30-3:40
(126) On the geometry of Banach algebras
Professor A. C. Thompson, Dalhousie University (663-639)

3:45-3:55
(127) On preordering a topological vector space over the reals. Preliminary report
Professor J. I. Nassar*, Muhlenberg College, and Dr. A. C. Williams,
Socony Mobil, Princeton, New Jersey (663-256)

4:00-4:10
(128) Local convergence in locally convex Riesz spaces
Professor L. C. Moore, Jr., Duke University (663-573)

4:15-4:25
(129) Extreme positive operators
Professor M. S. Espelie, University of Cincinnati (663-191)

4:30-4:40
(130) A dense set of extremal subadditive functions
Professor Richard Laatsch, Miami University, Oxford, Ohio (663-90)

4:45-4:55
(131) Meromorphic functions of elements of a commutative Banach algebra
Professor B. W. Glickfeld, University of Washington (663-757)

5:00-5:10
(132) The approximate continuity of $H^0$ boundary functions
Professor T. K. Boehme, Professor Melvin Rosenfeld and Professor
M. L. Weiss*, University of California, Santa Barbara (663-410)

5:15-5:25
(133) Multipliers of $H^P$ spaces
Professor J. H. Hedlund, University of Massachusetts (663-135)

5:30-5:40
(134) Quasiconformal mappings and Royden algebras in space
Mr. L. G. Lewis, Indiana University (663-182)

THURSDAY, 2:45 P.M.

Session on Analysis 6 (Differential and Functional Equations), Meeting Room 9, Jung Hotel
2:45-2:55
(135) Generalized prologational limit sets
Mr. D. H. Carlson, Case Western Reserve University (663-677)

3:00-3:10
(136) A new asymptotic expansion and approximation for the ratio of two gamma functions
Professor P. C. Consul, University of Calgary (663-132)

3:15-3:25
(137) On the distribution of Fekete points
Professor Thomas Kövari*, Imperial College of Science, London, and Uni­
versity of Waterloo, and Dr. Christian Pommerenke, Imperial College of
Science, London (663-568)

3:30-3:40
(138) Common fixed points for equicontinuous semigroups of mappings. Prelimi­
inary report
Professor Theodore Mitchell, Temple University (663-101)

3:45-3:55
(139) A theorem on nonlinear difference equations
Professor J. W. Tolle, University of North Carolina at Chapel Hill (663-109)

4:00-4:10
(140) A continued fraction convergence theorem
Professor Robert Heller, Mississippi State University (663-127)
4:15-4:25
(141) Functions defined by continued fractions over a vector space. Preliminary report
Professor F. A. Roach, University of Georgia (663-143)

4:30-4:40
(142) Elliptic operators and topological invariants
Professor M. M. LaSalle, University of Southwestern Louisiana (663-87)

4:45-4:55
(143) Some remarks on the theory of decomposable operators. Preliminary report
Professor S. M. Plafker, Tulane University (663-398)

5:00-5:10
(144) Fallacies and misconceptions about continuity
Professor A. C. Sugar, California State Polytechnic College (663-52)

5:15-5:25
(145) The distribution of zeros of extremal solutions of a fourth order differential
equation for the nth conjugate point
Professor Allan Peterson, University of Nebraska (663-667)
(Introduced by Dr. Gene A. Klaasen)

5:30-5:40
(146) Differential-difference properties of certain polynomials
Mr. Jet Wimp, Midwest Research Institute, Kansas City, Missouri (663-708)

THURSDAY, 2:45 P.M.

Session on Analysis 7 (Series, Fourier Analysis, etc.), Meeting Room 10, Jung Hotel
2:45-2:55
(147) Some subspaces of a Hilbert space of analytic functions
Dr. A. J. Froderberg, Western Washington State College (663-623)

3:00-3:10
(148) Solution of a general functional equation with five unknown functions having
different numbers of variables
Professor J. D. Aczel, University of Waterloo (663-110)

3:15-3:25
(149) The bases which are possible for +b bases infinite exponentials
Professor L. P. Maher, Jr., North Texas State University (663-683)

3:30-3:40
(150) Chain sequence preserving linear transformations
Professor H. M. Haddad, University of Colorado (663-368)

3:45-3:55
(151) Limitation theorem for Cesàro summable series
Dr. S. Mukhoti, Fort William, Ontario, Canada (663-240)

4:00-4:10
(152) On cosine and sine functional equations
Professor Palaniappan Kannappan, University of Waterloo (663-334)

4:15-4:25
(153) A new unified class of polynomials. Preliminary report
Dr. M. E. Cohen, Michigan Technological University (663-550)
(Introduced by Professor H. L. Hunzeker)

4:30-4:40
(154) Mean convergence of Hermite and Laguerre series
Professor Benjamin Muckenhoupt, Institute for Advanced Study (663-515)

4:45-4:55
(155) The uniqueness of Hermite series under Poisson-Abel summability
Professor L. S. Kroll, University of California, Davis (663-4)

5:00-5:10
(156) On the convex sum of certain convex functions. Preliminary report
Mr. S. Y. Trimble, University of Kentucky (663-96)
5:15-5:25
(157) Permissible bounds on the coefficients of approximating polynomials in $C[0,1]$
Professor J. A. Roulier, Michigan State University (663-159)

5:30-5:40
(158) Continuous functions with zero derivative almost everywhere
Professor S. W. Young, University of Utah (663-175)

5:45-5:55
(159) On Fourier coefficients
Professor J. T. Darwin, Jr., Auburn University (663-713)

THURSDAY, 2:45 P.M.

Session on Analysis 8 (Integration Theorem), Presidential Salon, Jung Hotel
2:45-2:55
(160) The hull of a channel
Dr. L. D. Duncan* and Mr. W. B. Miller, U. S. Army Atmospheric Science Office, White Sands Missile Range, New Mexico (663-38)

3:00-3:10
(161) A property of an integral defined on a dense set
Professor C. A. Coppin*, University of Dallas, and Professor J. F. Vance, St. Mary's University of San Antonio (663-8)

3:15-3:25
(162) A dominated convergence theorem
Professor S. G. Wayment, Utah State University (663-112)

3:30-3:40
(163) Inequalities concerning the module of curve families
Professor W. R. Derrick, University of Utah (663-148)

3:45-3:55
(164) Integration by substitution in Denjoy-Khintchine and Denjoy-Perron integrals
Mr. K. G. Johnson, Northeast Louisiana State College (663-295)

4:00-4:10
(165) The space of log-summable functions
Dr. D. O. Etter, Jr., Institute for Defense Analyses, Arlington, Virginia (663-269)

4:15-4:25
(166) On Baire systems generated by collections of integrable functions
Mr. R. D. Mauldin, University of Texas (663-213)

4:30-4:40
(167) Results relating the behavior of Fourier transforms near the origin and at infinity
Professor Cyril Nasim, University of Calgary (663-496)
(Introduced by Professor R. S. Dhaliwal)

4:45-4:55
(168) The Riemann transform
Professor A. E. Danese, State University of New York, College at Fredonia (663-362)

5:00-5:10
(169) Functions of bounded convexity
Mr. A. W. Roberts*, Macalester College, and Professor D. E. Varberg, Hamline University (663-449)

5:15-5:25
(170) On substitution for weighted integrals
Professor F. M. Wright and Mr. J. D. Baker*, Iowa State University (663-416)

5:30-5:40
(171) Substitution for a Lebesgue-Stieltjes type integral
Mr. M. L. Klasi* and Professor F. M. Wright, Iowa State University (663-601)
Completion of a noncomplete Lebesgue integral to a complete one
Professor Witold Bogdanowicz, Catholic University of America, and Dr. Martha Mattama***, Howard University (663-596)

THURSDAY, 2:45 P.M.

Session on Applied Mathematics (Differential Equations), Pavilion, Jung Hotel
2:45-2:55
(173) Stable difference schemes with uneven mesh spacings
Mr. Melvyn Ciment, New York University (663-114)
(Introduced by Professor P. D. Lax)

3:00-3:10
(174) Asymptotic solution of a nonlinear stability problem; a time dependent bifurcation theory
Professor B. J. Matkowsky, Rensselaer Polytechnic Institute (663-46)

3:15-3:25
(175) Uniform asymptotic theory of edge diffraction by a wedge-like-object
Mr. D. S. Ahluwalia, Courant Institute, New York University (663-91)

3:30-3:40
(176) On a method of Bellman and Richardson in perturbation theory
Professor S. I. Drobies, San Diego State College (663-14)

3:45-3:55
(177) General selfadjoint differential equations
Dr. M.S.T. Namboodiri Southern Illinois University (663-33)

4:00-4:10
(178) Boundedness and asymptotic behavior of a second order nonlinear equation
Professor H. A. DeKleine, State University of New York at Buffalo (663-280)

4:15-4:25
(179) On a free interface problem for linear ordinary differential equations. Preliminary report
Mr. G. H. Meyer, Mobil Research and Development Corporation, Dallas, Texas (663-603)

4:30-4:40
(180) The solution of ordinary simultaneous equations
Professor Stephen Kulik, California State College at Long Beach (663-564)

4:45-4:55
(181) Dual series and harmonic mixed boundary value problems
Professor J. R. Whiteman, University of Texas (663-130)
(Introduced by Mr. G. W. Stewart, III)

5:00-5:10
(182) On a distribution problem in a half space of a linear partial differential operator
Professor N. L. Maria, Stanislaus State College (663-462)

5:15-5:25
(183) Solution of the reduced wave equation in an infinite domain
Dr. B. S. Kleinman**, Brookhaven National Laboratories, Upton, New York, and Dr. K. T. Tang, Pacific Lutheran University (663-614)

5:30-5:40
(184) Asymptotic properties of the perturbed Klein-Gordon equation. II
Professor J. M. Chadam, Indiana University (663-157)

THURSDAY, 2:45 P.M.

Session on Graph Theory 1, Meeting Room 3, Howard Johnson's
2:45-2:55
(185) Some theorems on tensor products of graphs
Professor M. F. Capobianco, St. John's University (663-177)
3:00-3:10  (186) The cycle index of the exponentiation group
Professor E. M. Palmer, Michigan State University, and Professor R. W. Robinson*, University of California, Berkeley (663-77)

3:15-3:25  (187) Orbit complexes of graphs
Professor L. V. Quintas*, Pace College, and Professor Moses Richardson, City University of New York, Brooklyn College (663-37)

3:30-3:40  (188) A special class of hamiltonian graphs
Professor G. T. Chartrand, Western Michigan University, and Professor H. V. Kronk*, State University of New York at Binghamton (663-3)

3:45-3:55  (189) The diameters of the graphs of semigroups and semirings
Professors J. S. Ratti* and Y.-F. Lin, University of South Florida (663-195)

4:00-4:10  (190) The structure of edge-minimal graphs
Mr. M. M. Krieger, University of California, Los Angeles (663-237)

4:15-4:25  (191) On powers of graphs and Hamiltonian graphs
Professor G. T. Chartrand* and Professor S. F. Kapoor, Western Michigan University (663-222)

4:30-4:40  (192) On the parity of the number of maximal trees of a planar graph
Mr. H. S. Shank, Cornell University (663-399)

4:45-4:55  (193) A generalization of Hamiltonian connected graphs
Professor D. R. Lick, Western Michigan University (663-370)

5:00-5:10  (194) A generalization of a combinatorial theory of Macaulay
Professor G. F. Clements*, University of Colorado, and Dr. Bernt Lindström, University of Stockholm, Sweden (663-297)

5:15-5:25  (195) Ramsey's theorem for n-dimensional arrays
Mr. R. L. Graham*, Bell Telephone Laboratories, Murray Hill, New Jersey, and Mr. B. L. Rothschild, Massachusetts Institute of Technology (663-219)

5:30-5:40  (196) The composition of two tournaments
Professor Myron Goldberg and Professor J. W. Moon*, University of Alberta (663-214)

5:45-5:55  (197) Graphs, matroids and geometric lattices. Preliminary report
Professor David Sachs, Wright State University (663-42)

THURSDAY, 2:45 P.M.

Session on Topology 3 (Algebraic Topology), Plantation Oak Room, Governor House

2:45-2:55  (198) Homotopy type of loop spaces
Professor F. H. Croom, University of Kentucky (663-82)

3:00-3:10  (199) Diffeomorphisms of a product of spheres. Preliminary report
Mr. E. C. Turner, University of California, Los Angeles (663-85)
(Introduced by Professor R. C. Kirby)
<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Presenter</th>
<th>Institution/University</th>
<th>Phone Number</th>
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<tbody>
<tr>
<td>3:15-3:25</td>
<td>Two characterizations of Pontrjagin duality</td>
<td>Mr. D. W. Roeder, Dartmouth College</td>
<td>(663-259)</td>
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<tr>
<td>3:30-3:40</td>
<td>Invariant subspaces and projective representations</td>
<td>Professor Keith Yale, University of Montana</td>
<td>(663-408)</td>
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<tr>
<td>3:45-3:55</td>
<td>$A_n$-actions on fibre spaces</td>
<td>Professor R. A. Nowlan, St. Mary's College and University of Notre Dame</td>
<td>(663-379)</td>
<td></td>
</tr>
<tr>
<td>4:00-4:10</td>
<td>The deRham theorem for the group of diffeomorphisms</td>
<td>Professor F. J. Flaherty, Oregon State University</td>
<td>(663-308)</td>
<td></td>
</tr>
<tr>
<td>4:15-4:25</td>
<td>Stable obstructions mod $p$ and their multiplicative properties. Preliminary report</td>
<td>Professor R. E. Mosher, California State College at Long Beach</td>
<td>(663-492)</td>
<td></td>
</tr>
<tr>
<td>4:30-4:40</td>
<td>The Wang sequence and some calculations in K-theory</td>
<td>Professor E. C. Boes, New Mexico State University</td>
<td>(663-485)</td>
<td></td>
</tr>
<tr>
<td>4:45-4:55</td>
<td>M-S coverings on $n$-manifolds</td>
<td>Professor W. L. Reddy*, Wesleyan University, and Professor Erik Hemmingsen, Syracuse University</td>
<td>(663-581)</td>
<td></td>
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<tr>
<td>5:00-5:10</td>
<td>On mod $p$ connected K-theory</td>
<td>Professor J. C. Alexander, Johns Hopkins University</td>
<td>(663-529)</td>
<td></td>
</tr>
<tr>
<td>5:15-5:25</td>
<td>On the normal degree of immersions</td>
<td>Professor J. H. White, University of California, Los Angeles</td>
<td>(553-679)</td>
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<tr>
<td>5:30-5:40</td>
<td>Steenrod operations in the cohomology of modules over Hopf algebras. Preliminary report</td>
<td>Dr. A. Zachariou, Oklahoma State University</td>
<td>(663-726)</td>
<td></td>
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<tr>
<td>5:45-5:55</td>
<td>Retracts of semicomplexes</td>
<td>Professor R. B. Thompson, University of Arizona</td>
<td>(663-47)</td>
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**THURSDAY, 2:45 P.M.**

**Session on Topology 4 (General Topology), Pine Room, Governor House**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Presenter</th>
<th>Institution/University</th>
<th>Phone Number</th>
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<tbody>
<tr>
<td>2:45-2:55</td>
<td>Compact extensions of topological spaces. Preliminary report</td>
<td>Mr. E. D. Shirley, University of California, Riverside</td>
<td>(663-375)</td>
<td></td>
</tr>
<tr>
<td>3:00-3:10</td>
<td>Compactification of mappings</td>
<td>Professor G. L. Cain, Jr., Georgia Institute of Technology</td>
<td>(663-651)</td>
<td></td>
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<tr>
<td>3:15-3:25</td>
<td>Round $z$-filters and round subsets of $\mathbb{R}^X$</td>
<td>Professor Mark Mandelker, University of Kansas</td>
<td>(663-481)</td>
<td></td>
</tr>
<tr>
<td>3:30-3:40</td>
<td>Mappings and realcompact spaces</td>
<td>Professor Nancy Dykes, Kent State University</td>
<td>(663-322)</td>
<td></td>
</tr>
<tr>
<td>3:45-3:55</td>
<td>Compactifications of Hausdorff spaces</td>
<td>Professor P. A. Loeb, University of Illinois</td>
<td>(663-239)</td>
<td></td>
</tr>
</tbody>
</table>
4:00-4:10  
(216) A factorization theorem for weights of coverings and the property of Souslin  
Professor H. M. Schaerf, McGill University (663-725)

4:15-4:25  
(217) Metrization of uniform spaces in ordered groups  
Professor T. W. Shook, Ohio State University (663-605)

4:30-4:40  
(218) Generalized uniformities and set functions  
Professor G. C. Gastl, University of Wyoming (663-345)

4:45-4:55  
(219) A characterization of locally totally bounded uniformities  
Dr. Kazumi Nakano, Loyola University, New Orleans (663-275)

5:00-5:10  
(220) Continuous uniformities  
Professor R. A. Alo and Professor H. L. Shapiro*, Carnegie-Mellon University (663-166)

5:15-5:25  
(221) A generalized contraction principle  
Professor R. E. Chandler, North Carolina State University (663-94)

5:30-5:40  
(222) Semiproximities and preorder  
Professor M. D. Green, George Washington University (663-74)

THURSDAY, 8:00 P.M.

Gibbs Lecture, Grand Ballroom, Jung Hotel

Trends and social implications of research  
Professor R. L. Wilder, University of Michigan

FRIDAY, 8:45 A.M.

Special Session on Geometry in Mathematics, Plantation Oak Room, Governor House

8:45-9:05  
Concerning the isolated character of solutions of Plateau's problem  
Professor J. C. C. Nitsche, University of Minnesota (663-228)

9:10-9:30  
On the role of geometry in contemporary mathematics  
Professor Branko Grünbaum, University of Washington (663-438)

9:35-9:50  
The Hilbert cube  
Professor R. D. Anderson, Louisiana State University (663-716)

10:00-10:20  
The logic of intuitive geometry  
Professor Patrick Suppes, Stanford University (663-494)

FRIDAY, 8:45 A.M.

Special Session on Complex Analysis, Presidential Salon, Jung Hotel

8:45-9:05  
Coefficient multipliers of $H^p$ and $B^p$ spaces  
Professor P. L. Duren*, Institute for Advanced Study and University of Michigan, and Professor A. L. Shields, University of Michigan (663-355)

9:10-9:30  
A proof of the Bieberbach conjecture for the sixth coefficient  
Professor R. N. Pederson, Carnegie-Mellon University and Stanford University (663-331)
9:35-9:55
Bounds on the number of deficient values of meromorphic functions
Professor A. W. Weitsman, Purdue University (663-181)

10:00-10:20
Theta constants moduli and compact Riemann surfaces
Professor H. M. Farkas, State University of New York at Stony Brook (663-441)

10:25-10:45
On the growth of Blaschke products
Professor G. R. MacLane, Purdue University, and Professor L. A. Rubel*,
University of Illinois (663-380)

FRIDAY, 8:45 A.M.

Session on Topology 5 (Embeddings), Terrace Suite 1, Jung Hotel
8:45-8:55
(223) Trivially extending decompositions of n-space
Mr. Joseph Zaks, University of Washington (663-15)

9:00-9:10
(224) Taming locally flat embeddings
Professor T. B. Rushing, University of Georgia (663-251)

9:15-9:25
(225) Some wild spheres and group actions
Professor R. C. Lacher, Florida State University (663-229)

9:30-9:40
(226) Smooth embeddings of homologically similar manifolds
Professor D. M. Roseman, Indiana University (663-630)

9:45-9:55
(227) P. L. approximations of embeddings of polyhedra in the metastable range. Preliminary report
Mr. H. W. Berkowitz, University of Georgia (663-627)

10:00-10:10
(228) P L embeddings of bounded manifolds
Professor Jerome Dancis, University of Maryland (663-615)

10:15-10:25
(229) Aligning functions
Mrs. J. W. Ford, Auburn University, and Professor E. S. Thomas, Jr.*, University of Michigan (663-673)

10:30-10:40
(230) Locally flat imbeddings of disks in topological manifolds. Preliminary report
Mr. G. P. Weller, University of Chicago (663-719)

FRIDAY, 8:45 A.M.

Session on Topology 6 (Topological Semilattices), Terrace Suite 3, Jung Hotel
8:45-8:55
(231) Chain conditions in topological semilattices
Mr. J. B. Rhodes, University of Texas (663-729)

9:00-9:10
(232) Semilattices on retracts of a two-cell
Mr. W. W. Williams, Louisiana State University (663-363)

9:15-9:25
(233) The lattice of complete Boolean algebras is complemented
Mr. M. C. Rayburn, University of Kentucky (663-220)

9:30-9:40
(234) The lattice of pseudotopologies on S
Professor A. M. Carstens, Washington State University (663-223)
9:45-9:55  
(235) Continuity of the operation of a semilattice  
Professor J. T. Borrego, University of Massachusetts (663-298)

10:00-10:10  
(236) Closure and orthocomplementation on complete lattices  
Professor W. F. Davison, San Fernando Valley State College (663-193)

10:15-10:25  
(237) A lattice theoretic characterization of topological categories  
Professor H. L. Bentley, University of New Mexico (663-122)

10:30-10:40  
(238) Pre-symmetry and completeness in quasi-uniform spaces  
Professor Peter Fletcher, Virginia Polytechnic Institute (663-24)

FRIDAY, 8:45 A.M.

Session on Topological Groups and Lie Theory, Terrace Suite 5, Jung Hotel
8:45-8:55  
(239) On counting connections on Lie groups  
Mr. R. E. Beck, Villanova University (663-34)

9:00-9:10  
(240) Projective topological groups, II  
Professor C. E. Hall, Virginia Polytechnic Institute (663-67)

9:15-9:25  
(241) Locally compact groups with compact conjugacy classes  
Professor R. T. Ramsay, North Carolina State University (663-149)  
(Introduced by Professor Hans Sagan)

9:30-9:40  
(242) The structure of Moore groups  
Professor Lewis Robertson, University of Washington (663-478)

9:45-9:55  
(243) On the structure of Lie p-algebras  
Professor D. J. Winter, University of Michigan (663-563)

10:00-10:10  
(244) Extensions of locally compact Abelian groups  
Dr. R. O. Fulp*, University of Houston, and Dr. P. A. Griffith, University of Chicago (663-707)

FRIDAY, 8:45 A.M.

Session on Analysis 9 (Functional Analysis), Meeting Room 1, Jung Hotel
8:45-8:55  
(245) Contractions and Korovkin's theorem  
Professor D. E. Wulbert, University of Washington (663-268)

9:00-9:10  
(246) Measurable functions in configurations of Banach spaces  
Professor Pawel Szeptycki, University of Kansas (663-260)

9:15-9:25  
(247) Interpolation of sublinear operations on generalized Orlicz and Hardy spaces  
Professor W. T. Kraynek, University of Pittsburgh (663-309)

9:30-9:40  
(248) A Fubini-Jessen theorem for the generalized Lebesgue-Bochner integral on Lp-spaces  
Professor Witold Bogdanowicz, Catholic University of America, and Professor Vernon Zander*, West Georgia College (663-56)

9:45-9:55  
(249) Representation of operators into Lebesgue-Bochner spaces  
Dr. Joseph Diestel, West Georgia College (663-443)  
(Introduced by Dr. V. E. Zander)
Bochner-Raikov theorem for a generalized positive definite function. Preliminary report
Professor P. P. Saworotnow, Catholic University of America (663-546)

A generalization of the vectorial Bartel integral
Dr. Witold Bogdanowicz, Catholic University of America, and Professor G. E. Haborak*, U. S. Naval Academy (663-545)

Solvability of nonlinear integral equations of Hammerstein type
Professor D. G. de Figueiredo, University of Illinois at Chicago Circle, and Professor C. P. Gupta*, University of Chicago (663-681)

FRIDAY, 8:45 A.M.

Session on Analysis 10 (Complex Analysis), Meeting Room 2, Jung Hotel
8:45-8:55
Characteristic function of a meromorphic function and its derivative
Professor S. K. Singh*, University of Missouri-Kansas City, and Professor V. N. Kulkarni, Karnatak University, India (663-142)
(Introduced by Professor P. W. Liebnitz)

9:00-9:10
Univalent functions with univalent derivatives
Professor S. M. Shah* and Mr. S. Y. Trimble, University of Kentucky (663-335)

9:15-9:25
Proximate orders and growth of meromorphic functions
Professor S. H. Dwivedi, University of Oklahoma (663-586)
(Introduced by Professor R. V. Andree)

9:30-9:40
Entire functions of bounded index
Dr. A. R. Reddy, University of Alberta (663-527)
(Introduced by Professor A. Sharma)

9:45-9:55
Holomorphic functions bounded on a spiral
Professor Joseph Warren, Pennsylvania State University (663-637)

10:00-10:10
A short proof of a lemma of G. R. MacLane
Professor K. F. Barth* and Professor W. J. Schneider, Syracuse University (663-616)

10:15-10:25
Distribution of values of meromorphic functions which approach an asymptotic value rapidly
Professor P. M. Gauthier, Université de Montreal (663-590)

10:30-10:40
Solvability of nonlinear integral equations of Hammerstein type
Professor D. G. de Figueiredo, University of Illinois at Chicago Circle, and Professor C. P. Gupta*, University of Chicago (663-681)

Session on Analysis 11 (Stability Theory), Meeting Room 4-5, Jung Hotel
8:45-8:55
On stability and $L^p$ solutions of ordinary differential equations
Professor T. G. Hallam, Florida State University (663-264)

9:00-9:10
Necessary and sufficient conditions for mixed stability
Professor A. A. Kayande* and Professor M. R. M. Rao, University of Rhode Island (663-209)
9:15-9:25
(263) Integro-differential equations and extension of Lyapunov's method
   Professor V. Lakshmikantham and Professor M. R. M. Rao*, University of Rhode Island (663-350)
9:30-9:40
(264) Eventual asymptotic stability of control systems
   Professor C. P. Tsokos, University of Rhode Island (663-349)
9:45-9:55
(265) Identifying perturbations which preserve asymptotic stability
   Professor Aaron Strauss*, Mathematics Research Center, U. S. Army, University of Wisconsin, and Professor J. A. Yorke, University of Maryland (663-337)
10:00-10:10
(266) Conditionally invariant sets. Preliminary report
   Professor S. Leela, State University of New York, College at Geneseo (663-445)
10:15-10:30
(267) Prolongations in semidynamical systems
   Professor P. N. Bajaj, Wichita State University (663-631)
10:30-10:40
(268) Asymptotic behavior of solutions to some nth order linear differential equations
   Professor I. N. Katz, Washington University (663-118)

FRIDAY, 8:45 A.M.

Session on Analysis 12 (Measure Theory), Meeting Room 9, Jung Hotel
8:45-8:55
(269) Measures in topological spaces and B-compactness
   Professor R. B. Kirk, Southern Illinois University (663-329)
9:00-9:10
(270) Regularity of a product from regular conditional measures
   Professor W. W. Bledsoe and Mr. C. E. Wilks*, University of Texas (663-356)
9:15-9:25
(271) \( \mu \)-completion measures
   Professor W. W. Bledsoe, University of Texas (663-211)
9:30-9:40
(272) The norm infimum function
   Professor W. D. L. Appling, North Texas State University (663-54)
9:45-9:55
(273) A uniqueness lemma for measures on \( \mathbb{R}^{n} \). Preliminary report
   Professor D. R. Chalice, Western Washington State College (663-472)
10:00-10:10
(274) Linear functionals on a Banach function space
   Professor N. E. Gretsky, University of California, Riverside (663-292)
10:15-10:25
(275) On the derivatives of arbitrary real-valued set functions
   Dr. Harvel Wright* and Dr. W. S. Snyder, Oak Ridge National Laboratory, Oak Ridge, Tennessee (663-36)
10:30-10:40
(276) The selfadjoint operators of a von Neumann algebra form a conditionally complete lattice
   Mr. M. P. Olson, University of California, Davis (663-282)
FRIDAY, 8:45 A.M.

Session on Algebra 6 (Field Theory), Meeting Room 10, Jung Hotel
8:45-8:55
(277) Galois theory for \(\alpha\)-differential fields
Professor A. M. Sayied, Boston College (663-727)
(Introduced by Professor G. G. Bilodeau)

9:00-9:10
(278) Local classification of hermitian forms over algebraic number fields. Preliminary report
Professor L. J. Gerstein, University of California, Santa Barbara (663-170)

9:15-9:25
(279) S-units of Galois extensions
Professor J. H. Smith, Boston College (663-265)

9:30-9:40
(280) Corresponding residue systems in normal extensions of algebraic number fields
Professor L. R. McCulloh, University of Illinois, and Professor W. T. Stout, Jr.*, University of Hawaii (663-450)

9:45-9:55
(281) Galois theory
Professor S. S. Shatz, University of Pennsylvania (663-435)

10:00-10:10
(282) On the fundamental units of certain relatively biquadratic fields
Professor R. B. Lakein, State University of New York at Buffalo (663-609)

10:15-10:25
(283) On Pfister's theory of quadratic forms
Dr. W. W. Scharlau, Queen's University (663-541)
(Introduced by Professor Paula Ribenboim)

10:30-10:40
(284) A characterization of \(p\)-adic and finite algebraic number fields
Dr. Jurgen Neukirch, Queen's University (663-685)

FRIDAY, 8:45 A.M.

Session on Algebra 7 (Lattice Theory), Pavilion, Jung Hotel
8:45-8:55
(285) Distributive local Noether lattices
Professor K. P. Bogart, Dartmouth College (663-20)

9:00-9:10
(286) Chain conditions in distributive free products of lattices
Professor G. A. Grätzer and Professor H. Lakser*, University of Manitoba (663-50)

9:15-9:25
(287) Projectivity of prime quotients and simple lattices
Mr. F. A. Smith, Carnegie-Mellon University (663-140)

9:30-9:40
(288) Distributivity of the lattice of filters of a groupoid. Preliminary report
Professor Orrin Frink and Mr. R. S. Smith*, Pennsylvania State University (663-393)

9:45-9:55
(289) On the lattice of equational classes of one- and two-dimensional polyadic algebras
Professor J. D. Monk, University of Colorado (663-338)

10:00-10:10
(290) Characterization of property D in orthomodular lattices
Professor J. H. Bevis, Virginia Polytechnic Institute (663-333)
10:15-10:25
(291) Some results on Brouwerian lattices. Preliminary report
Professor D. P. Smith, University of Oklahoma (663-717)

10:30-10:40
(292) The commutator in an orthomodular lattice
Professor E. L. Marsden, Kansas State University (663-500)
(Introduced by Professor R. J. Greechie)

FRIDAY, 8:45 A.M.

Session on Statistics and Probability 2, Pine Room, Governor House
8:45-8:55
(293) A general strong law
Mr. S. D. Chatterji, University of Montreal (663-513)

9:00-9:10
(294) Random operator equations
Professor Arunava Mukherjea, Eastern Michigan University (663-104)

9:15-9:25
(295) An existence theorem for a random ordinary differential equation
Dr. J. L. Strand, Bellcomm, Incorporated, Washington, D. C. (663-323)

9:30-9:40
(296) Remarks on a generalized form of the Weyl criterion
Professor Lauwerens Kuipers, Southern Illinois University (663-128)

9:45-9:55
(297) On the continuity of the triangle function
Professor Berthold Schweizer, University of Arizona (663-217)

10:00-10:10
(298) Moment sequences and renewal sequences
Professor R. A. Horn, Johns Hopkins University (663-367)

10:15-10:25
(299) Quotient space of random normed spaces. Preliminary report
Professor J. W. Thomas* and Professor V. M. Sehgal, University of Wyoming (663-336)

10:30-10:40
(300) On distinct fields over finite probability space
Professor P. C. Wang, University of Iowa (663-710)

FRIDAY, 11:00 A.M.

Invited Address, Grand Ballroom, Jung Hotel
Geometric ergodic theory
Professor Calvin C. Moore, University of California, Berkeley

FRIDAY, 1:30 P.M.

Presidential Address, Grand Ballroom, Jung Hotel
Differentiability theorems for weak solutions of differential equations
Professor C. B. Morrey, Jr., University of California, Berkeley

FRIDAY, 2:45 P.M.

Special Session on Geometry in Mathematics, Plantation Oak Room, Governor House
2:45-3:05
Geometry in group theory
Professor Marshall Hall, Jr., California Institute of Technology (663-360)

3:10-3:30
Affinely regular polygons
Professor H. S. M. Coxeter, University of Toronto (663-287)
Local and nonlocal properties of differential equations
Professor K. O. Friedrichs, Courant Institute, New York University (663-760)

The varied foundations of geometry
Professor Garrett Birkhoff, Harvard University (663-453)

FRIDAY, 2:45 P.M.

Special Session on Complex Analysis, Presidential Salon, Jung Hotel
2:45-3:05
A fixed point theorem for holomorphic mappings on Banach spaces
Professor Clifford J. Earle, Harvard University, and Professor Richard S. Hamilton*, Cornell University (663-758)

3:10-3:30
A nonlinear Hodge-de Rham theorem
Professor R. J. Sibner*, Rutgers University, and Professor L. M. Sibner, Polytechnic Institute of Brooklyn (663-352)

3:35-3:55
An n-dimensional Casorati-Weierstrass theorem
Professor Hung-Hsi Wu, University of California, Berkeley (663-156)

4:00-4:20
On differential rings of entire functions
Professor A. H. Cayford*, University of British Columbia, and Professor E. G. Straus, University of California, Los Angeles (663-414)

4:25-4:45
On the ideal theory of some rings of analytic functions
Mr. J. J. Kelleher, Columbia University (663-565)

FRIDAY, 2:45 P.M.

Session on Algebra 8 (Commutative Rings), Terrace Suite 1, Jung Hotel
2:45-2:55
(301) Semigroup algebras whose ideals are spanned by semigroup ideals
Professor Eckhart Hotzel, McMaster University (663-741)
(Introduced by Professor B. J. Mueller)

3:00-3:10
(302) Finitely presented modules. Preliminary report
Professor R. B. Warfield, Jr., University of Washington (663-733)

3:15-3:25
(303) Noetherian and non-Noetherian commutative rings
Professor W. J. Heinzer*, Purdue University, and Professor Jack Ohm, Louisiana State University (663-606)

3:30-3:40
(304) Some remarks on Weierstrass preparation theorem
Professor N. Sankaran, Queen's University (663-649)

3:45-3:55
(305) Some conditions on commutative semiprime rings
Professor A. C. Mewborn, University of North Carolina at Chapel Hill (663-428)

4:00-4:10
(306) Noetherian tensor products
Dr. E. A. Magarian*, Stetson University, and Dr. J. L. Mott, Florida State University (663-436)

4:15-4:25
(307) On overrings of a domain
Professor H. S. Butts, Louisiana State University, and Professor N. H. Vaughan*, North Texas State University (663-300)
4:30-4:40
(308) Rings for which every proper homomorphic image is a multiplication ring
Professor C. A. Wood, Oklahoma State University (663-358)

4:45-4:55
(309) On a generalization of Krull domains
Professor Elbert Pirtle, University of Missouri at Kansas City (663-62)

5:00-5:10
(310) Structure of a certain class of rings
Professor Maurice Chacron, University of Windsor (663-86)

5:15-5:25
(311) The ideal transform and overrings of an integral domain. II. Preliminary report
Professor J. W. Brewer*, Virginia Polytechnic Institute, and Professor Robert Gilmer, Florida State University (663-354)

5:30-5:40
(312) Embedding a topological domain in a countably generated algebraic ring extension
Dr. J. O. Kiltinen, University of Minnesota (663-310)

FRIDAY, 2:45 P.M.

Session on Algebra 9 (Number Theory), Terrace Suite 3, Jung Hotel
2:45-2:55
(313) The number of subspaces of a vector space
Professor J. R. Goldman*, Harvard University, and Professor Gian-Carlo Rota, Massachusetts Institute of Technology (663-561)

3:00-3:10
(314) An elementary method for estimating number-theoretic sums. Preliminary report
Mr. S. A. Burr, Bell Telephone Laboratories, Incorporated, Whippany, New Jersey (663-433)

3:15-3:25
(315) Remainder estimates for squarefree integers in arithmetic progressions
Professor R. C. Orr, Syracuse University (663-566)

3:30-3:40
(316) van der Waerden's theorem on arithmetic progressions
Professor T. C. Brown, Simon Fraser University (663-544)

3:45-3:55
(317) Euler's numbers and the diophantine equation $x^i + y^i = cz^i$. III
Professor J. M. Gandhi, York University (663-511)

4:00-4:10
(318) Integral solution of $y^2 + 25 = x^3$
Mr. Hymie London, Montreal, Quebec, Canada (663-638)

4:15-4:25
(319) Study of some arithmetic functions
Professor Emil Grosswald, Temple University (663-384)

4:30-4:40
(320) Some modular properties of the Euler and Bernoulli polynomials
Professor J. D. Brillhart, University of Arizona (663-661)

4:45-4:55
(321) Littlewood's problem for series
Professor T. W. Cusick, State University of New York at Buffalo (663-11)

5:00-5:10
(322) A note on the theorem of Sylvester and Schur
Professor C. R. Hobby, University of Washington, and Professor J. L. Selfridge*, University of Illinois (663-736)
5:15-5:25
(323) Waring's problem in GF[q,x]
Mr. W. A. Webb, Pennsylvania State University (663-407)

5:30-5:40
(324) Sets of integers containing no n terms in geometric progression
Professor James Riddell, University of Victoria (663-69)

FRIDAY, 2:45 P.M.

Session on Algebra 10 (Groups), Terrace Suite 5, Jung Hotel

2:45-2:55
(325) Invariant connexions on coset spaces
Professor N. Hicks, University of Michigan (663-641)

3:00-3:10
(326) On the irreducible subrepresentations of induced representations of real simple Lie groups
Dr. Ernest Thieleker, Argonne National Laboratory, Argonne, Illinois (663-475)
(Introduced by Dr. J. M. Cook)

3:15-3:25
(327) Concerning the degrees of irreducible characters
Professor G. E. Keller*, University of Minnesota, and Professor Leonard Scott, University of Chicago (663-41)

3:30-3:40
(328) Nilpotent groups as fixed point free automorphism groups
Mr. T. R. Berger, University of Minnesota (663-45)

3:45-3:55
(329) Commuting pairs in a finite group. Preliminary report
Mr. Keith Joseph, University of California, Los Angeles (663-53)

4:00-4:10
(330) Power-translation invariance in discrete groups
Professor W. R. Emerson, Courant Institute, New York University (663-71)

4:15-4:25
(331) Dihedrator subgroups
Professor D. R. Cecil, North Texas State University (663-183)

4:30-4:40
(332) On ascending series of subgroups in infinite groups
Professor R. E. Phillips* and Mr. K. K. Hickin, University of Kansas (663-284)

4:45-4:55
(333) A generalization of Hall-complementation in finite supersolvable groups
Professor Homer Bechtell, University of New Hampshire, (663-207)

5:00-5:10
(334) A sufficient condition for the existence of a normal π-complement
Professor F. I. Gross, University of Utah (663-395)

5:15-5:25
(335) Criteria for nilpotence of certain subgroups
Professor J. D. Dixon, Carleton University (663-486)

5:30-5:40
(336) Orthogonal groups over global fields of characteristic 2
Professor Barth Pollak, University of Notre Dame (663-484)

FRIDAY, 2:45 P.M.

Session on Analysis 13 (Topological Algebras, Groups, etc.), Meeting Room 1, Jung Hotel

2:45-2:55
(337) Isometries of finite-dimensional Banach spaces
Professor Andrew Sobczyk*, Clemson University, and Professor R. Keown, University of Arkansas (663-241)
3:00-3:10
(338) Sets of uniqueness and multiplicity on the infinite product of compact topological groups. Preliminary report
Mr. J. E. Coury, University of Washington (663-25)

3:15-3:25
(339) On the simplex of completely monotonic functions on a commutative semigroup
Professor N. J. Fine and Professor P. H. Maserick*, Pennsylvania State University (663-283)

3:30-3:40
(340) On the torsion free, self dual, locally compact abelian groups
Professor Kwang-Chul Ha, Illinois State University (663-279)

3:45-3:55
(341) Uniqueness theorems for convolution type equations
Dr. J. S. Byrnes*, Naval Research Laboratory, Washington, D. C., and Professor D. J. Newman, Yeshiva University (663-204)

4:00-4:10
(342) Locally compact principal ideal domains
Professor S. L. Warner, Duke University (663-502)

4:15-4:25
(343) Closed subalgebras of group algebras. Preliminary report
Dr. Stephen Friedberg, Massachusetts Institute of Technology (663-491)

4:30-4:40
(344) Randomly continuous Fourier series on compact groups. Preliminary report
Professor Alessandro Figà-Talamanca, University of Genoa, Italy, and University of California, Berkeley (663-635)

4:45-4:55
(345) Differentiation on groups
Professor A. B. Simon, Northwestern University (663-598)

5:00-5:10
(346) Convolutions of operator valued functions on the dual of a compact group. Preliminary report
Professor Richard Iltis, University of North Carolina at Chapel Hill (663-751)

5:15-5:25
(347) A general theorem on the convergence of sequences of semigroups of linear operators
Dr. T. G. Kurtz, University of Wisconsin (663-647)

5:30-5:40
(348) Rings of analytic functions
Professor W. J. Bourque, Jr., University of North Dakota and Louisiana State University (663-682)

FRIDAY, 2:45 P.M.

Session on Analysis 14 (Linear Topological Space), Meeting Room 2, Jung Hotel

2:45-2:55
(349) Affine quotients of Choquet simplexes
Professor J. B. Bednar, Drexel Institute of Technology (663-311)

3:00-3:10
(350) A theorem on convex bodies of the Busemann-Brunn-Minkowski type
Professor Edward Silverman, Purdue University (663-400)

3:15-3:25
(351) Sections and subsets of infinite-dimensional simplexes
Professor A. J. Lazar, University of Washington (663-530)
3:30-3:40
(352) A generalization of Markušević's duality principle
Mr. J. B. Cooper, University of California, Santa Barbara, and Clare College, Cambridge, England (663-153)
(Introduced by Professor J. A. Ernest)

3:45-3:55
(353) On the normed semilinear space of convex sets
Professor Dagmar Henney, George Washington University (663-68)

4:00-4:10
(354) Packing and reflexivity in Banach spaces
Mr. C. A. Kottman, University of Iowa (663-469)

4:15-4:25
(355) Weak completeness in spaces of operators. Preliminary report
Mr. J. M. Baker, Florida State University (663-687)

4:30-4:40
(356) On the inheritance of properties of locally convex spaces by subspaces of countable codimension
Professor M. D. Levin* and Mr. S. A. Saxon, Florida State University (663-580)

4:45-4:55
(357) Quasi-reflexivity and dual norms
Mr. E. B. Roth, Florida State University (663-728)

5:00-5:10
(358) Two LF-space questions of Dieudonné and Schwartz
Mr. M. J. Kascic, Jr.*, and Mr. B. G. Roth, Dartmouth College (663-197)

5:15-5:25
(359) Two theorems on support points
Professor N. T. Peck, University of Illinois (663-139)

5:30-5:40
(360) Fixed points of quasi-nonexpansive mappings. Preliminary report
Professor W. G. Dotson, Jr., North Carolina State University (663-498)

FRIDAY, 2:45 P.M.
Session on Analysis 15 (Complex Functions), Meeting Rooms 4-5, Jung Hotel

2:45-2:55
(361) Some remarks on the string of beads
Professor D. R. Wilken, State University of New York at Albany (663-249)

3:00-3:10
(362) A local characterization of Hol (D)
Mr. R. L. Carpenter, University of Utah (663-654)

3:15-3:25
(363) On closed ideals in \( A^\infty \)
Professor B. A. Taylor*, Courant Institute, New York University, and Professor D. L. Williams, Syracuse University (663-203)

3:30-3:40
(364) A generalization of a theorem of Hans Hahn concerning matrix summability
Professor D. F. Dawson, North Texas State University (663-93)

3:45-3:55
(365) Inclusion theorems for Sonnenschein matrices
Professor F. W. Hartmann, Villanova University (663-43)

4:00-4:10
(366) The discrete analogue of a class of entire functions
Professor R. J. Duffin, Carnegie-Mellon University, and Professor E. J. Peterson*, University of Michigan (663-1)
4:15-4:25  
(367) Products of monodiffric functions  
Professor George Berzsenyi, Northeast Louisiana State College (663-488)

4:30-4:40  
(368) Function growth in ringed spaces  
Professor M. E. Shauck, Jr., Duke University (663-675)

4:45-4:55  
(369) Some properties of functions and cluster sets  
Mr. Shu-Chung Koo, Marquette University (663-427)  
(Introduced by Professor E. W. Swokowski)

5:00-5:10  
(370) The residue calculus in several complex variables. Preliminary report  
Professor G. L. Gordon, University of Illinois at Chicago Circle (663-262)

5:15-5:25  
(371) Algebras of integrable functions  
Professor K. O. Leland, Illinois Institute of Technology (663-303)

5:30-5:40  
(372) On the Hornich spaces  
Professor J. A. Cima, University of North Carolina at Chapel Hill (663-285)

FRIDAY, 2:45 P.M.

Session on Analysis 16 (Ordinary Differential Equations), Meeting Room 9, Jung Hotel

2:45-2:55  
(373) Modulus-phase relations for second order linear differential equations  
Mr. H. E. Fettis, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio (663-124)

3:00-3:10  
(374) Regular perturbation for a turning point problem  
Professor P.-F. Hsieh, Western Michigan University (663-178)

3:15-3:25  
(375) Compact inverses of ordinary differential operators  
Professor R. M. Kauffman, Western Washington State College (663-272)

3:30-3:40  
(376) Existence theorems for the initial value problem with infinite sum  
Professor R. B. Grafton, University of Missouri at Columbia (663-232)

3:45-3:55  
(377) Expansions for K_\nu(\xi) involving Airy's function  
Mr. J. A. Donaldson, University of Illinois at Chicago Circle (663-353)

4:00-4:10  
(378) A Tauberian theorem of Landau-Ingham type  
Professor S. L. Segal, University of Rochester (663-330)

4:15-4:25  
(379) On periodic solutions of Hamiltonian systems near a stationary point  
Professor M. S. Berger, University of Minneapolis (663-503)

4:30-4:40  
(380) The linear differential-difference equation with constant coefficients  
Professor C. H. Anderson, Ohio University (663-315)

4:45-4:55  
(381) Oscillation theorems for a linear delay equation of the second order  
Professor J. S. Bradley, University of Tennessee (663-499)

5:00-5:10  
(382) On the existence of periodic solutions for general two-dimensional differential systems  
Professor Roger Cooper, Chicago State College (663-495)

45
5:15-5:25
(383) Exterior problems for semilinear hyperbolic equations in time-dependent domains
Professor N. D. Kazarinoff*, University of Michigan, and Professor E. D. Rogak, University of Victoria (663-493)

5:30-5:40
(384) An implicit function theoretic approach to variation of solutions of an O.D.E.
Preliminary report
Professor A. J. Schwartz, University of Michigan (663-660)

FRIDAY, 2:45 P.M.

Session on Applied Mathematics 2 (Classical Analysis), Meeting Room 10, Jung Hotel
2:45-2:55
(385) The asymptotic approximation of certain integrals
Professor Frank Stenger, University of Michigan (663-31)

3:00-3:10
(386) Optimal approximate integration and interpolation for functions of two variables.
Preliminary report
Miss L. E. Mansfield, University of Utah (663-520)

3:15-3:25
(387) An integral identity due to Ramanujan which occurs in neutron transport theory
Mr. J. J. Dorning, Mr. B. Nicolaenko and Mr. J. K. Thurber*, Brookhaven National Laboratory, Upton, New York (663-512)

3:30-3:40
(388) A new functional for the Hausdorff moment problem
Dr. C. Y. Cho, Maggs Research Center, Watervliet, New York (663-212)
(Introduced by Professor Donald Greenspan)

3:45-3:55
(389) On the dynamo problem
Professor Shmuel Kaniel*, Northwestern University, and Professor A. Kovetz, Tel Aviv University, Israel (663-665)
(Introduced by Professor Avner Friedman)

4:00-4:10
(390) Closure approximation in stochastic differential equations
Professor George Adomian, University of Georgia (663-125)

4:15-4:25
(391) Absolute continuity of Hamiltonian operators with repulsive potential
Professor R. B. Lavine, Cornell University (663-562)

4:30-4:40
(392) Superpositions for nonlinear operators
Mr. Alfred Inselberg, IBM Scientific Center, Los Angeles, California (663-383)

4:45-4:55
(393) Some convexity theorems for the eigenvalues of certain Fredholm-type operators
Professor W. R. Boland, Drexel Institute of Technology (663-199)

5:00-5:10
(394) A completeness theorem for an integrodifferential operator
Dr. Jerome Eisenfeld, Rensselaer Polytechnic Institute (663-32)

5:15-5:25
(395) Optimization and confidence regions
Mrs. F. M. Clarke-Carroll, Raytheon Company, Sudbury, Massachusetts (663-570)

5:30-5:40
(396) Controllability in differential games
Professor Emilio Roxin, University of Rhode Island (663-595)
Session on Graph Theory 2, Pavilion, Jung Hotel
2:45-2:55
   (397) Upper bounds on the length of circuits of even spread in the d-cube
         Dr. R. J. Douglas, University of North Carolina at Chapel Hill (663-439)
3:00-3:10
   (398) On the cartesian product of groups
         Mr. M. Rosenfeld, University of Washington (663-730)
3:15-3:25
   (399) The color numbers of certain graphs
         Mr. Renu Laskar and Mr. W. R. Hare, Jr.*, Clemson University (663-718)
3:30-3:40
   (400) On the planarity of regular incidence sequences
         Mr. A. B. Owens, Naval Research Laboratory, Washington, D. C. (663-668)
3:45-3:55
   (401) The ortho-conditioned orthomodular product
         Professor J. C. Dacey, Kansas State University (663-538)
         (Introduced by Professor R. J. Greechie)
4:00-4:10
   (402) On two types of partial geometrics
         Dr. M. S. Aigner, University of North Carolina at Chapel Hill (663-539)
4:15-4:25
   (403) On packings of complete bipartite graphs
         Professor L. W. Beineke, Purdue University at Fort Wayne (663-653)
4:30-4:40
   (404) On the corona of two graphs
         Professor R. W. Frucht, University of Santa Maria, Valparaiso, Chile, and
         Professor Frank Harary*, University of Michigan (663-434)

Session on Topology 7 (General Topology), Meeting Room 3, Howard Johnson's
2:45-2:55
   (405) A note on contractive mappings
         Mr. K. L. Singh, Western Michigan University (663-554)
3:00-3:10
   (406) On sequence of contraction mappings. Preliminary report
         Professor S. P. Singh* and Mr. W. Russell, Memorial University of New­
         foundland (663-625)
3:15-3:25
   (407) Local contractions on a-metric spaces
         Professor B. T. Sims, Eastern Washington State College (663-319)
3:30-3:40
   (408) Asymptotic properties of expansive homeomorphisms
         Professor B. F. Bryant*, Vanderbilt University, and Professor P. Walters,
         University of Warwick, England (663-548)
3:45-3:55
   (409) A characterization of pointwise expansive homeomorphisms on compact metric
         spaces
         Miss E. S. Goldman, Vanderbilt University (663-611)
         (Introduced by Professor B. F. Bryant)
4:00-4:10
   (410) Ultra space representations
         Professor J. M. Cibulskis, Northeastern Illinois State College (663-113)
4:15-4:25
   (411) A note on stratifiable spaces
         Professor R. A. Stoltenberg, Washington State University (663-253)
4:30-4:40
(412) A note on p-spaces and Moore spaces
Mr. D. K. Burke* and Professor R. A. Stoltenberg, Washington State University (663-168)

4:45-4:55
(413) Metrizable subsets of Moore spaces
Mr. C. W. Proctor, University of Houston (663-30)

5:00-5:10
(414) \(\Sigma\)-spaces
Professor Keio Nagami, University of Pittsburgh (663-640)

5:15-5:25
(415) On fundamental open coverings
Professor W. M. Fleischman, State University of New York at Buffalo (663-672)

5:30-5:40
(416) Some generalizations of paracompactness. Preliminary report
Mr. S. B. Higgins, Texas Christian University (663-748)

5:45-5:55
(417) Concerning certain hyperspaces
Dr. J. M. Worrell, Jr., and Dr. H. H. Wicke*, Sandia Corporation, Albuquerque, New Mexico (663-744)

FRIDAY, 2:45 P.M.

Session on Topology 8 (3-dimensional), Pine Room, Governor House

2:45-2:55
(418) An alternative proof that Bing's dogbone space is not topologically \(E^3\)
Professor E. H. Anderson, Mississippi State University (663-75)

3:00-3:10
(419) Compact subsets of 3-manifolds definable by cubes-with-handles. Preliminary report
Mr. W. H. Row, Jr., University of Wisconsin (663-648)

3:15-3:25
(420) On knot groups
Professor P. M. Rice, University of Georgia (663-227)

3:30-3:40
(421) A 1-linked link whose longitudes lie in the second commutator subgroup
Professor H. W. Lambert, University of Iowa (663-206)

3:45-3:55
(422) Mapping cylinder neighborhoods
Professor Victor Nicholson, Kent State University (663-342)
(Introduced by Professor B. H. McCandless)

4:00-4:10
(423) Characterizations of sets which can be missed by side approximations of spheres
Mr. J. W. Cannon, University of Utah (663-306)

4:15-4:25
(424) A characterization of a translation in three space within a class of quasi-translations
Professor Shin'ichi Kinoshita, Florida State University (663-594)

4:30-4:40
(425) The Chen's group of a link
Professor Kunio Murasugi, University of Toronto (663-572)

4:45-4:55
(426) Manifolds in which the Poincare' conjecture is true
Mr. J. L. Gross, Princeton University (663-558)
5:00-5:10
(427) Some results on arcs in n-books in $\mathbb{E}^3$
Dr. G. H. Atneosen, Western Washington State College (663-549)

5:15-5:25
(428) Compact, acyclic ANR's in 3-space are AR's
Professor D. R. McMillan, Jr., University of Wisconsin (663-542)

5:30-5:40
(429) The existence of nontrivial universal crumpled cubes
Professor R. J. Daverman and Professor W. T. Eaton*, University of Tennessee (663-510)

5:45-5:55
(430) Cubes with knotted holes
Mr. R. H. Bing and Mr. J. M. Martin*, University of Wisconsin (663-731)

SATURDAY, 1:15 P.M.

Business Meeting, Grand Ballroom, Jung Hotel

SATURDAY, 2:45 P.M.

Session on Algebra II (Groups), Terrace Suite 1, Jung Hotel

2:45-2:55
(431) On the Brauer splitting theorem
Professor George Szeto, Bradley University (663-480)

3:00-3:10
(432) On the order of the automorphism group of a finite group
Mr. K. H. Hyde, University of Utah (663-430)
(Introduced by Professor W. R. Scott)

3:15-3:25
(433) On rank 4 extensions of finite permutation groups. Preliminary report
Professor J. S. Montague, University of Tennessee (663-652)

3:30-3:40
(434) On eighth-groups
Professor Seymour Lipschutz, Temple University (663-619)

3:45-3:55
(435) Coverings by powers and properties of groups
Professor L. -C. Kappe, State University of New York at Binghamton (663-567)

4:00-4:10
(436) On finite factor coverings of groups. Preliminary report
Professor O. A. Slotterbeck, University of Texas, Austin (663-557)

4:15-4:25
(437) A group topology for completely distributive lattice-ordered groups
Professor R. L. Madell, University of Tennessee (663-526)

4:30-4:40
(438) Decomposition of the alternating group into the product of conjugacy classes
Professor E. A. Bertram, University of Hawaii (663-525)

4:45-4:55
(439) Elements with trivial centralizer in wreath products
Professor W. P. Kappe, State University of New York at Binghamton (663-514)

5:00-5:10
(440) Quasi split translations
Mr. Joseph Altinger, Case Western Reserve University (663-688)

5:15-5:25
(441) A study of rank 4 permutation groups
Mr. J. T. Renfrow, California Institute of Technology (663-734)
5:30-5:40  (442) Centralizers of subgroups of permutation groups. Preliminary report
     Professor C. F. Wells, Case Western Reserve University (663-739)

5:45-5:55  (443) On hyperfinite factors of type III
     Mr. W. J. Krieger, Ohio State University (663-680)

SATURDAY, 2:45 P.M.

Session on Algebra 12 (Matrix Theory), Terrace Suite 3, Jung Hotel
2:45-2:55  (444) A generalization of the unitary group
     Professor Marvin Marcus, University of California, Santa Barbara, and
     Professor W. R. Gordon*, University of Victoria (663-105)

3:00-3:10  (445) Matrices of Schur functions
     Professor Marvin Marcus and Miss S. M. Katz*, University of California,
     Santa Barbara (663-293)

3:15-3:25  (446) Some properties of the Schur complement
     Professor E. V. Haynsworth, Auburn University (663-247)

3:30-3:40  (447) Criteria for positive definiteness of certain matrices
     Professor E. L. Allgower, Colorado State University (663-357)

3:45-3:55  (448) Bound norms
     Professor Hans Schneider, University of Wisconsin (663-346)

4:00-4:10  (449) On essentially positive matrices
     Professor R. B. Kellogg, University of Maryland (663-321)

4:15-4:25  (450) Laurent expansion for a nearly singular matrix
     Professor C. E. Langenhop, University of Kentucky (663-505)

4:30-4:40  (451) On ranks of pseudo-inverses
     Professor C. D. Meyer, North Carolina State University (663-459)

4:45-4:55  (452) Row, column and null-modules of a matrix module \( p^n \)
     Professor B. R. McDonald, University of Oklahoma (663-577)

5:00-5:10  (453) A generalized incidence equation
     Dr. S. E. Payne, Miami University, Oxford, Ohio (663-81)

5:15-5:25  (454) On the extension of linearly independent subsets of free modules to bases
     Professor B.-S. Chwe* and Professor Joseph Neggers, University of
     Alabama (663-215)

5:30-5:40  (455) Characterization and enumeration of linear classes of involutions over a
     finite field. Preliminary report
     Dr. J. D. Fulton, Clemson University (663-746)

SATURDAY, 2:45 P.M.

Session on Algebra 13 (Commutative Groups), Terrace Suite 5, Jung Hotel
2:45-2:55  (456) Radicals and torsion free groups
     Professor Ray Mines, New Mexico State University (663-286)
3:00-3:10  
(457) Partial solution of a splitting problem of Misina (Fuchs' Problem 34). Preliminary report  
Professor Adolf Mader, University of Hawaii (663-49)

3:15-3:25  
(458) Quasi-relations on torsion-free modules  
Professor E. W. Swokowski, Marquette University (663-146)

3:30-3:40  
(459) On the semi-radical of a topological abelian group  
Mr. R. A. Massagli, Central Washington State College (663-169)

3:45-3:55  
(460) The structure of large subgroups of primary abelian groups  
Mr. E. W. Poluinov, Professor B. J. Eisenstadt, Professor Khalid Benabdallah and Professor J. M. Irwin*, Wayne State University (663-254)

4:00-4:10  
(461) Endomorphism rings of periodic abelian groups  
Professor Phillip Schultz, University of Montana (663-401)

4:15-4:25  
(462) Duals of the divisible hull of an abelian group  
Professor P. L. Sperry, University of South Carolina (663-483)

4:30-4:40  
(463) Endomorphism rings of modules over complete discrete valuation rings  
Professor Wolfgang Liebert, New Mexico State University (663-471)

4:45-4:55  
(464) A duality for quasi-isomorphism classes of torsion free modules with finite rank over a discrete valuation ring  
Mr. D. M. Arnold, New Mexico State University (663-437)

5:00-5:10  
(465) Isomorphism invariants for quotient categories of abelian groups  
Professor D. E. Bertholf, Oklahoma State University (663-594)

5:15-5:25  
(466) Embedding in summands of infinite abelian groups  
Professor F. M. Williams, West Texas State University (663-702)

5:30-5:40  
(467) A congruence for congruences on a regular semigroup  
Professor H. E. Scheiblich, University of South Carolina (663-599)

SATURDAY, 2:45 P.M.

Session on Analysis 17 (Topological Algebras), Meeting Room 1, Jung Hotel

2:45-2:55  
(468) The spectrum of automorphisms of Banach algebras  
Professor H. M. Kamowitz, University of Massachusetts, Boston Campus and Mr. Stephen Scheinberg*, Massachusetts Institute of Technology (663)

3:00-3:10  
(469) Commutativity in locally compact rings  
Mr. J. B. Lucke, Duke University (663-115)

3:15-3:25  
(470) Functions of bounded variation on idempotent semigroups  
Professor S. E. Newman, University of Missouri at St. Louis (663-458)

3:30-3:40  
(471) Parts in the maximal ideal space of a convolution measure algebra. Preliminary report  
Mr. R. R. Miller, University of Utah (663-332)  
(Introduced by Professor J. L. Taylor)
3:45-3:55  
(472) On the homotopy type of the group of regular elements of semifinite von Neumann algebras  
Professor Manfred Breuer, University of Kansas (663-316)

4:00-4:10  
(473) The boundary of a semi-algebra  
Professor R. E. Worth, Georgia State College (663-312)

4:15-4:35  
(474) A real B*-algebra is C* iff it is Hermitian  
Professor T. W. Palmer, University of Kansas (663-468)

4:30-4:40  
(475) A local characterization of analytic structure in a Banach algebra. Preliminary report  
Mr. D. D. Clayton, University of Utah (663-656)

4:45-4:55  
(476) Two definitions of Arens multiplication  
Professor J. E. Simpson, University of Kentucky (663-621)

5:00-5:15  
(477) Some results on generalized H^p-spaces  
Mr. Nand Lal* and Professor Samuel Merrill, University of Rochester (663-743)

5:15-5:25  
(478) Transforming integrals on locally compact spaces  
Professor R. W. Chaney, University of California, Santa Barbara (663-172)

5:30-5:40  
(479) On the growth in norm of the powers of an operator  
Professor E. W. Packel, Reed College (663-571)

5:45-5:55  
(480) Ideals of analytic functions  
Professor B. V. Limaye, University of California, Irvine (663-425)  
(Introduced by Professor G. K. Kalisch)

SATURDAY, 2:45 P.M.

Session on Geometry 2, Meeting Room 2, Jung Hotel

2:45-2:55  
(481) Extreme points in convexity structures  
Professor Lino Gutierrez-Novoa, University of Miami (663-409)

3:00-3:10  
(482) On minimal digraphs with given girth and degree  
Mr. C. E. Wall, Olivet College (663-224)  
(Introduced by Professor G. T. Chartrand)

3:15-3:25  
(483) Derivable chains of planes  
Professor N. L. Johnson, Eastern Washington State College (663-235)

3:30-3:40  
(484) Analytic maps between tori  
Dr. H. G. Helfenstein, University of Ottawa (663-190)

3:45-3:55  
(485) Higher order connections and dissections  
Professor R. H. Bowman, Vanderbilt University (663-155)

4:00-4:10  
(486) On n-uniform Hjelmslev planes  
Professor D. A. Drake, University of Florida (663-133)
4:15-4:25
(487) Quasi-regular saddle polyhedra (QRSP)
Dr. A. H. Schoen, NASA-Electronics Research Center, Cambridge, Massachusetts (663-39)
(Introduced by Mr. J. C. Kotelly)

4:30-4:40
(488) The completion of finite incomplete Steiner triple systems
Mrs. Christine Treash, Mount Allison University (663-180)

SATURDAY, 2:45 P.M.

Session on Analysis 18 (Linear Topological Spaces), Meeting Rooms 4-5, Jung Hotel
2:45-2:55
(489) Reflexive modulared Banach spaces of generalized variation
Professor H. -H. W. Herda, Tufts University (663-378)

3:00-3:10
(490) A characterization of sequential norms
Dr. E. J. Kay, Cedar Crest College (663-325)

3:15-3:25
(491) A problem concerning lipschitzian homeomorphisms in Banach spaces
Professor Nachman Aronszajn, University of Kansas (663-271)

3:30-3:40
(492) Extensions of vector-valued modular functions
Professor J. E. Huneycutt, Jr., North Carolina State University (663-347)

3:45-3:55
(493) Convexity with respect to differential operators with flat characteristic cones
Professor E. C. Zachmanoglou, Purdue University (663-610)

4:00-4:10
(494) On farthest points of convex sets
Mr. J. H. Yates, Oklahoma State University (663-388)
(Introduced by Professor E. K. McLachlan)

4:15-4:25
(495) On uniqueness of fixed-points
Mr. R. E. Huff, University of North Carolina at Chapel Hill (663-536)

4:30-4:40
(496) On sequences of contractive mappings
Professor S. B. Nadler, Jr.*, and Professor R. B. Fraser, Jr., Louisiana State University (663-313)

4:45-4:55
(497) The expectation semigroup of a random evolution
Professor R. J. Griego* and Professor Reuben Hersh, University of New Mexico (663-489)

5:00-5:10
(498) Linear transformations on a sequence space
Professor G. U. Brauer, University of Minnesota (663-587)

5:15-5:25
(499) CL-products of CL-spaces. Preliminary report
Professor C. C. Braunschweiger, Marquette University (663-230)

5:30-5:40
(500) Some generalizations of Hilbert Schmidt and nuclear operators. Preliminary report
Mr. J. S. Cohen, University of Maryland (663-684)
(Introduced by Professor J. W. Brace)
SATURDAY, 2:45 P.M.

Session on Analysis 19 (Ordinary Differential Equations), Meeting Room 9, Jung Hotel

2:45-2:55
(501) LaGrange and Hermite interpolation in Banach spaces
Miss P. M. Prenter, Mathematics Research Center, U. S. Army, University of Wisconsin, and Colorado State University (663-244)

3:00-3:10
(502) Oscillation coefficients
Professor A. M. Fink*, Iowa State University, and Mr. D. F. St. Mary, University of Massachusetts (663-9)

3:15-3:25
(503) On an inequality of Nehari
Professor A. M. Fink, Iowa State University, and Mr. D. F. St. Mary*, University of Massachusetts (663-17)

3:30-3:40
(504) Qualitative properties of satellite orbits
Professor P. C. Loh, Purdue University (663-248)

3:45-3:55
(505) A boundedness theorem for a second order equation
Professor H. E. Gollwitzer, University of Tennessee (663-29)

4:00-4:10
(506) The alternate plane in oscillation theory. Preliminary report
Professor G. Di Antonio, Indiana University of Pennsylvania (663-48)

4:15-4:25
(507) On a result of Hartman and Wintner
Dr. R. R. Stevens, University of Montana (663-70)

4:30-4:40
(508) Continuous dependence of solutions to boundary value problems
Professor G. A. Klaassen, University of Nebraska (663-669)

4:45-4:55
(509) A linear differential system with interface conditions
Professor R. N. Bryan, Ithaca College (663-92)

5:00-5:10
(510) Green's function on large intervals. Preliminary report
Dr. H. E. Benzinger, Department of Defense (663-99)

5:15-5:25
(511) On the Riccati equation
Professor R. M. Koch and Professor Franklin Lowenthal*, University of Oregon (663-120)

5:30-5:40
(512) Nonexistence of a continuous right inverse for parabolic operators
Mr. D. K. Cohoon, Bell Telephone Laboratories, Whippany, New Jersey (663-608)

5:45-5:55
(513) Nonoscillation of the second order matrix systems
Professor E. C. Tomastik, University of Connecticut (663-121)

SATURDAY, 2:45 P.M.

Session on Analysis 20 (Complex Functions), Meeting Room 10, Jung Hotel

2:45-2:55
(514) Countability of factorization sets for \( H^p \) functions
Professor J. G. Caughran, University of Kentucky (663-391)

3:00-3:10
(515) How to build a picnic table for use on a mountain range of known period
Professor J. T. Rosenbaum, University of Pittsburgh (663-26)
3:15-3:25
(516) On the derivative of a polynomial
Mr. M. A. Malik, Sir George Williams University (663-697)
(Introduced by Professor Martin Harrow)

3:30-3:40
(517) Special types of annular functions
Dr. D. D. Bonar, Wayne State University (663-270)

3:45-3:55
(518) A more stringent criterion for k-polar polynomials
Dr. R. E. Goodman, Westinghouse Electric Corporation, Aerospace Division, Baltimore, Maryland (663-233)

4:00-4:10
(519) A Taylor series representation for a class of functions of a complex variable
Professor M. O. Gonzalez, University of Alabama (663-218)

4:15-4:25
(520) The valence of certain means
Professor A. W. Goodman, University of South Florida (663-21)

4:30-4:40
(521) On a class of analytic functions whose images are star-shaped Jordan domains
Mr. Paul Eenigenburg, University of Kentucky (663-371)

4:45-4:55
(522) On the radius of convexity and boundary distortion of Schlicht functions
Mr. D. E. Tepper, Temple University (663-482)
(Introduced by Professor Albert Schild)

5:00-5:10
(523) A coefficient problem for a class of meromorphic univalent functions
Mr. D. H. Schnack, University of Kentucky (663-446)

5:15-5:25
(524) An integral mean problem for bounded starlike functions
Professor J. B. Twomey, University of South Florida (663-620)
(Introduced by Professor J. S. Ratti)

5:30-5:40
(525) Distributional boundary values of functions analytic in an octant
Professor R. D. Carmichael, Virginia Polytechnic Institute (663-307)

5:45-5:55
(526) On quaternionic bicomplex analytic and wave-analytic (pseudo-analytic) functions
Professor Makoto Itoh, North Carolina State University (663-755)

SATURDAY, 2:45 P.M.

Session on Applied Mathematics 3 (Numerical Analysis), Presidential Salon, Jung Hotel
2:45-2:55
(527) Exact numerical analysis of axially symmetric flow between two rotating infinite porous disks
Professor M. S. Jawa, University of Missouri (663-740)
(Introduced by Professor Glen Haddock)

3:00-3:10
(528) Higher order accurate methods for boundary value problems for ordinary and elliptic differential equations
Professor R. B. Guenther and Professor E. L. Roetman*, Marathon Oil Company, Littleton, Colorado (663-64)

3:15-3:25
(529) The weak Newton method
Professor R. A. Tapia, Mathematics Research Center, U. S. Army, University of Wisconsin (663-522)
Numerical solution of weakly singular integral equations. Preliminary report
Professor R. K. Miller*, Brown University, and Professor Alan Feldstein,
University of Virginia (663-537)

Error formula for multidimensional Hermite interpolation
Professor Alan Feldstein, University of Virginia (663-645)

The approximate solution of free boundary problems
Professor C. W. Cryer, University of Wisconsin (663-344)

Jacobi and Gauss-Seidel methods for nonlinear network problems
Professor T. A. Porsching, Bettis Atomic Power Laboratory, West Mifflin,
Pennsylvania (663-650)
(Introduced by Dr. R. E. Carlson)

Improvement of Aparo's numerical method for Volterra integral equations
Professor Diran Sarafyan and Mr. J. O. Gettys*, Louisiana State University in New Orleans (663-470)

Error bounds for periodic quintic splines
Dr. C. A. Hall, Bettis Atomic Power Laboratory, Westinghouse Electric
Corporation, West Mifflin, Pennsylvania (663-22)

Runge-Kutta approximate continuous solutions for ordinary differential equations
Professor Diran Sarafyan, Louisiana State University in New Orleans (633-417)

Runge-Kutta techniques for nonlinear Volterra integro-differential equations
Professor Alan Feldstein, University of Virginia, and Mr. J. R. Sopka*,
Los Alamos Scientific Laboratories, University of California, Los Alamos (663-506)

A restricted Runge-Kutta method. Preliminary report
Mr. G. J. Landry*, Nicholls State College, and Dr. H. A. Luther, Texas A & M University (663-188)

SCHEDULED SESSIONS

Saturday, 2:45 P.M.

Session on Logic and Categories, Pavilion, Jung Hotel

Synchronizing sequences and central-definite events
Professor W. F. Cutlip, Central Washington State College (663-584)

Change of base by Grothendieck fibrations
Professor John Duskin, Case Western Reserve University (633-426)

A characterization of the category of relational systems of given type
Dr. Eric Mendelsohn, University of Montreal (663-634)

Foundation for a unified theory of sets and logic
Professor J. G. Maxwell, Kent State University (663-633)

Retraction of ultrapowers
Professor W. F. Taylor, University of Colorado (663-591)
4:00-4:10
(544) Semi-intuitionistic set theory
Dr. L. J. Pozsgay, St. Louis University (663-569)

4:15-4:25
(545) Cotorsion in an abelian category. Preliminary report
Mr. G. J. Wimbish, Oklahoma College of Liberal Arts (663-674)

4:30-4:40
(546) Extensions of the hyperanalytic hierarchy
Professor P. G. Hinman, University of Michigan (663-721)

SATURDAY, 2:45 P.M.

Session on Applied Mathematics 4 (Matrices and Polynomial Approximations), Meeting Room 3, Howard Johnson's
2:45-2:55
(547) Cubature error estimates. Preliminary report
Mr. F. G. Lether, University of Utah (663-694)
(Introduced by Professor R. E. Barnhill)

3:00-3:10
(548) Splines and pseudo-splines
Dr. L. J. Derr, Shell Oil Company, New Orleans, Louisiana (663-266)

3:15-3:25
(549) The generality of Bauer's abbreviated iteration for a zero of a polynomial
Professor G. W. Stewart, III, University of Texas (663-226)

3:30-3:40
(550) Piecewise polynomial approximation of smooth functions in rectangular polygons
Dr. R. E. Carlson* and Dr. C. A. Hall, Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, West Mifflin, Pennsylvania (663-23)

3:45-3:55
(551) Unsmoothing of data
Professor F. W. Stallman, University of Tennessee (663-574)

4:00-4:10
(552) On $K_U$-symmetric and $K_U$-p.d. matrices
Mr. Sanjo Zlobec and Professor Adi Ben-Israel*, Northwestern University (663-154)

4:15-4:25
(553) Bounded quadratic systems in the plane
Dr. R. J. Dickson, Lockheed Research Laboratories, Palo Alto, California, and Dr. L. M. Perko*, Northern Arizona University (663-754)

4:30-4:45
(554) Scaling of matrices to achieve specified row and column sums
Professor A. W. Marshall, Boeing Scientific Research Laboratories, Seattle, Washington, and Professor Ingram Olkin*, Stanford University (663-659)

4:45-4:55
(555) Bounds for eigenvalues of nonnegative matrices
Professor R. E. De Marr, University of New Mexico (633-255)

5:00-5:10
(556) Duality geometry of quadratic programs
Mr. T. D. Parsons* and Professor A. W. Tucker, Princeton University (663-364)

5:15-5:25
(557) Concerning nearly reducible matrices
Professor R. D. Sinkhorn and Mr. Mark Hedrick*, University of Houston (663-501)
SATURDAY, 2:45 P.M.

Session on Topology 9 (General Topology), Plantation Oak Room, Governor House

2:45-2:55
(558) Discrete sets and cardinal arithmetic; pointwise paracompact normal non-metrizable Moore spaces. Preliminary report
Mr. F. D. Tall, University of Wisconsin (663-699)

3:00-3:10
(559) A linearly ordered cofinal set of open covers almost implies metrizability and compactness
Professor G. L. Pfeifer, University of Arizona (6633-578)

3:15-3:25
(560) Minimal R(\(\omega_0\)) spaces. Preliminary report
Professor J. R. Porter, University of Kansas (663-454)

3:30-3:40
(561) On Souslin's problem
Professor Gary Miller, University of Victoria (663-466)
(Introduced by Professor B. J. Pearson)

3:45-3:55
(562) Covering properties of linearly ordered spaces and Souslin's problem
Mr. D. J. Lutzer*, University of Washington, and Dr. H. R. Bennett, Texas Technological College (663-386)

4:00-4:10
(563) Topologies with compact subtopologies
Mr. H. B. Reiter, Clemson University (663-246)

4:15-4:25
(564) Applications of a class of countably paracompact spaces
Professor P. L. Zenor, Auburn University (663-274)

4:30-4:40
Professor J. L. Hursch, Jr., University of Florida (663-137)

4:45-4:55
(566) A convergence characterization of Lindelöf
Dr. N. R. Howes, Texas Christian University (663-136)

5:00-5:10
(567) A class of pseudo-finite spaces
Professor M. R. Kirch, State University of New York at Buffalo (663-162)

5:15-5:25
(568) Minimal first countable topologies
Professor R. M. Stephenson, Jr., University of North Carolina at Chapel Hill (663-129)

5:30-5:40
(569) Separability and the point-countable property
Professor R. E. Hodel, Duke University (663-35)

SATURDAY, 2:45 P.M.

Session on Topology 10 (Topological Semigroups, Algebraic and Differential Topology), Pine Room, Governor House

2:45-2:55
(570) Representations of certain compact semigroups by L-semigroups
Mr. J. H. Carruth, University of Tennessee, and Mr. C. E. Clark*, University of Missouri (663-535)

3:00-3:25
(571) Noncut points in a nondegenerate continuum
Professor H. Subramanian, Case Western Reserve University (663-444)
3:15-3:25  
(572) Embedding of topological semigroups  
Mr. F. T. Christoph, Jr., Rutgers University (663-474)

3:30-3:40  
(573) On a class of locally compact metric semigroups  
Mr. L. F. Kinch, University of Kentucky (663-302)

3:45-3:55  
(574) Special semigroups on the two-cell  
Mr. Esmond DeVun, University of Massachusetts (663-411)  
(Introduced by Professor Haskell Cohen)

4:00-4:10  
(575) On the closure of the bicyclic semigroup  
Professor Carl Eberhart* and Professor John Selden, University of Kentucky (663-208)

4:15-4:25  
(576) Normal semifield metrics  
Professor Martin Kleiber, Villanova University, and Professor W. J. Pervin*, Drexel Institute of Technology (663-277)

4:30-4:40  
(577) A remark on periodic maps  
Professor Ludvik Janos, University of Florida (663-161)

4:45-4:55  
(578) Semigroup actions. Preliminary report  
Mr. C. F. Kelemen, University of California, Santa Barbara, and Pennsylvania State University (663-138)

5:00-5:10  
(579) On certain maps of compact spaces inducing isomorphisms for cohomology  
Professor Aristide Deleanu, Syracuse University (663-759)  
(Introduced by Professor D. E. Kibbey)

5:15-5:25  
(580) Infinite-dimensional manifolds are open subsets of Hilbert space  
Professor D. W. Henderson, Cornell University (663-612)

SUNDAY, 2:45 P.M.

Session on Algebra 14 (Associative Rings and Algebras), Terrace Suite 1, Jung Hotel

2:45-2:55  
(581) Annihilators and rationally closed right ideals. Preliminary report  
Mr. R. C. Shock, University of North Carolina at Chapel Hill (663-626)

3:00-3:10  
(582) On co-Noetherian rings  
Professor J. P. Jans, University of Washington (663-613)

3:15-3:25  
(583) Splitting hereditary torsion theories over semiperfect rings  
Professor R. L. Bernhardt, University of North Carolina at Greensboro (663-40)

3:30-3:40  
(584) Rings of quotients of Morita equivalent rings  
Professor D. R. Turnidge, Kent State University (663-100)

3:45-3:55  
(585) On commuting automorphisms on rings  
Professor Jian Luh, North Carolina State University (663-103)

4:00-4:10  
(586) Injective envelopes of semisimple modules. Preliminary report  
Mr. R. P. Kurshan, University of Washington (663-117)
4:15-4:25  
(587) Endomorphism rings of ideals in a commutative regular ring  
Professor R. A. Wiegand, University of Wisconsin (663-405)

4:30-4:40  
(588) The Jacobson radical of row-finite matrices  
Professor R. S. Crittenden, Virginia Polytechnic Institute (663-216)

4:45-4:55  
(589) A class of rings with all singular simple modules injective  
Professor J. S. Alin*, University of Utah, and Mr. E. P. Armendariz, University of Texas (663-131)

5:00-5:10  
(590) Some generalizations of QF-3 rings  
Professor C. I. Vinsonhaler, University of Connecticut (663-152)

5:15-5:25  
(591) On the intersection of the powers of the Jacobson radical  
Professor M. L. Larsen and Professor Ahmad Mirbagheri, University of Nebraska (663-281)

5:30-5:40  
(592) Admissible modules and reduced left Artinian rings  
Professor G. R. Krause, Washington State University (663-236)  
(Introduced by Professor Lorne Houten)

SUNDAY, 2:45 P.M.

Session on Algebra 15 (Universal and General Algebra, Algebraic Geometry), Terrace Suite 3, Jung Hotel

2:45-2:55  
(593) Two Mal'cev type theorems in universal algebra  
Professor G. A. Grätzer, University of Manitoba (663-134)

3:00-3:10  
(594) Equational compactness of unary algebras  
Dr. G. H. Wenzel, Queen's University (663-700)  
(Introduced by Professor G. A. Grätzer)

3:15-3:25  
(595) Cassini ovals and covering sets. Preliminary report  
Professor L. C. Eggan*, Illinois State University, and Professor E. A. Maier, University of Oregon (663-723)

3:30-3:40  
(596) Embedding a compact ring in a compact ring with unit  
Dr. K. R. Pearson, Pennsylvania State University (663-261)  
(Introduced by Professor C. W. Ayoub)

3:45-3:55  
(597) Internal structure of t-designs  
Dr. R. N. Lane, General Electric Company-Tempo, Santa Barbara, California (663-63)

4:00-4:10  
(598) Vanishing of the "symmetric polynomials". Preliminary report  
Professor Seymour Bachmuth, University of California, Santa Barbara (663-202)

4:15-4:25  
(599) On the reducibility of an object in the exact category  
Professor Kwangil Koh, North Carolina State University (663-144)

4:30-4:40  
(600) Free products and the dependence number of rings  
Professor R. E. Williams, Kansas State University (663-16)

4:45-4:55  
(601) On injective cogenerator rings. Preliminary report  
Professor F. L. Sandomierski, University of Wisconsin (663-711)
5:00-5:10  (602) Pair algebras and Galois connections
Professor J. C. Derderian, State University of New York at Buffalo (663-657)
(Introduced by Professor W. M. Fleischman)

5:15-5:25  (603) Group-like Manger algebras
Miss Helen Skala, Illinois Institute of Technology (663-712)

5:30-5:40  (604) Moduli of intermediate Jacobians
Professor D. I. Lieberman, Brandeis University (663-678)

SUNDAY, 2:45 P.M.

Session on Algebra 16 (Number Theory), Terrace Suite 5, Jung Hotel
2:45-2:55  (605) On a question related to diophantine approximation
Professor D. L. Goldsmith, Courant Institute, New York University (663-12)

3:00-3:10  (606) Asymptotic diophantine approximations to a basis of a real cubic number field
Professor W. W. Adams, University of California, Berkeley (663-13)

3:15-3:25  (607) Diophantine equations of degree n ≥ 2
Professor Leon Bernstein, Illinois Institute of Technology (663-579)

3:30-3:40  (608) An elementary approach to Diophantine equations of the second degree
Professor J. C. Owings, Jr., University of Maryland (663-19)

3:45-3:55  (609) On certain congruence properties of Fibonacci numbers
Mr. M. R. Turner, Regis College (663-95)
(Introduced by

4:00-4:10  (610) Fibonacci numbers which are perfect cubes
Mr. R. P. Finkelstein, Bowling Green, Ohio (663-643)
(Introduced by Hymie London)

4:15-4:25  (611) Incidence functions as generalized arithmetic functions. III
Professor D. A. Smith, Duke University (663-402)

4:30-4:40  (612) Iterated arithmetic functions
Dr. G. K. White, University of British Columbia (663-704)

4:45-4:55  (613) A new plane generalization of the partition function q(n)
Professor Basil Gordon, University of California, Los Angeles, and Professor Lorne Houten*, Washington State University (663-724)

5:00-5:10  (614) On the distribution of kth power residues and nonresidues mod n
Professor K. K. Norton, University of Colorado (663-185)

5:15-5:25  (615) A distribution of the primitive roots of a prime
Mr. Emanuel Vegh, Naval Research Laboratory, Washington, D. C. (663-221)

5:30-5:40  (616) A new property of partitions with applications to the Rogers-Ramanujan identities
Professor G. E. Andrews, Pennsylvania State University (663-243)
SUNDAY, 2:45 P.M.

Session on Analysis 21 (Functional Analysis), Meeting Room 1, Jung Hotel

2:45-2:55
(617) Convergence and applications of noncommutative continued fractions
Mr. W. G. Fair, Midwest Research Institute, Kansas City, Missouri (663-703)
(Introduced by Mr. Y. L. Luke)

3:00-3:10
(618) A generalization of the mean ergodic theorem
Professor Dany Leviatan, University of Illinois, and Professor M. S. Ramanujan*, University of Michigan (663-551)

3:15-3:25
(619) An isomorphic characterization of $L_p$ and $c_0$-spaces
Professor Lior Tzafriri, Northwestern University (663-666)

3:30-3:40
(620) Properties of absolute summability matrices
Professor J. A. Fridy, Kent State University (663-276)

3:45-3:55
(621) Moment sequences of operators in Banach spaces
Professor Dany Leviatan*, University of Illinois, and Professor M. S. Ramanujan, University of Michigan (663-642)

4:00-4:10
(622) Existence of a spectrum for nonlinear transformations
Professor J. W. Neuberger, Emory University (663-376)

4:15-4:25
(623) Nilpotency and the rate of growth condition
Professor K. I. Oberai, Queen's University (663-373)

4:30-4:40
(624) Generalized Szasz operators for the approximation in the complex domain
Professor Bruce Wood, University of Arizona (663-60)

4:45-4:55
(625) Algebraic kernels of planar sets
Professor Le Baron Ferguson, University of California, Riverside (663-73)

5:00-5:10
(626) Smoothness and rotundity in quotient spaces
Professor Jörg Blatter, University of Texas (663-242)
(Introduced by Professor E. W. Cheney)

5:15-5:25
(627) Polynomial approximation of a function and its derivative
Professor H. W. McLaughlin and Professor J. A. Voytuk*, Rensselaer Polytechnic Institute (663-326)

5:30-5:40
(628) The approximation of linear operators
Professor J. W. Brace*, University of Maryland, and Professor P. J. Richetta, Lehigh University (663-618)

SUNDAY, 2:45 P.M.

Session on Analysis 22 (Spaces C(X), etc.), Meeting Room 2, Jung Hotel

2:45-2:55
(629) Existence of uncomplemented subspaces of C(X) which are algebraically isomorphic to C(X). Preliminary report
Professor J. W. Baker, Florida State University (663-720)

3:00-3:10
(630) Uniform algebras with relatively maximal Gelfand transforms
Professor R. W. Honerlah, Marquette University (663-415)
3:15-3:25
(631) On a characterization of function algebras
Professor Ernst Binz, Queen's University (663-455)

3:30-3:40
(632) An extension theorem for regular Borel measures on compact spaces
Professor D. E. Edmondson, University of Texas (663-460)

3:45-3:55
(633) Maximal ideals of upper semicontinuous functions
Professor C. W. Sloyer, Jr.*, and Professor R. Nielsen, University of Delaware (663-461)

4:00-4:10
(634) Convolution of measures on locally compact spaces
Dr. W. W. Fairchild, Northwestern University (663-389)
(Introduced by Professor R. P. Boas)

4:15-4:25
(635) Subalgebras in a subspace of C(X)
Dr. S. P. Lloyd, Bell Telephone Laboratories, Murray Hill, New Jersey (663-80)

4:30-4:40
(636) Principal ideals and problems of division
Mr. B. G. Roth, Dartmouth College (663-198)

4:45-4:55
(637) Applications of Paley-Wiener-Zygmund integral
Professor Chull Park, Miami University (663-163)

5:00-5:10
(638) Approximation-theoretic characterization of Beurling test-functions
Professor Gőran Bjőrck, University of California, Riverside (663-76)

5:15-5:25
(639) A representation theorem for biequicontinuous completed tensor products of weighted spaces
Mr. W. H. Summers, Louisiana State University (663-2)

5:30-5:40
(640) About subrings of rings of continuous functions. Preliminary report
Professor L. P. Su, University of Oklahoma (663-750)
(Introduced by Professor W. T. Reid)

SUNDAY, 2:45 P.M.

Session on Analysis 23 (Partial Differential Equations and Others), Meeting Room 4-5, Jung Hotel

2:45-2:55
(641) The divergence need not possess the Darboux property
Professor J. B. Diaz* and Dr. J. R. McLaughlin, Rensselaer Polytechnic Institute (663-123)

3:00-3:10
(642) Basic sets of polynomials for the heat equation and its iterates. Preliminary report
Professor E. C. Young, Florida State University (663-147)

3:15-3:25
(643) A generalization of the Morse index theorem to a class of degenerate elliptic operators
Professor L. M. Sibner, Polytechnic Institute of Brooklyn (663-372)

3:30-3:40
(644) Optimization with hyperbolic partial differential equations. Preliminary report
Professor M. B. Suryanarana, University of Michigan (663-497)
(Introduced by Professor Lamberto Cesari)
3:45-3:55
(645) Probabilistic solutions of hyperbolic systems
   Professor Reuben Hersh* and Professor R. J. Griego, University of New
   Mexico (663-490)

4:00-4:10
(646) Partial differential equations of Sobolev-Galpern type
   Professor R. E. Showalter, University of Texas (663-419)

4:15-4:25
(647) Partial differential systems of generalized Wiener and Feynman integrals
   Professor D. L. Skoug, University of Nebraska (663-632)

4:30-4:40
(648) Green's function for singular boundary value problems. Preliminary report
   Mr. C. L. Irwin, Emory University (663-655)

4:45-4:55
(649) On uniform simplification of linear differential equation in a full neighborhood
   on a turning point
   Dr. R. Y. Lee, Sandia Corporation, Albuquerque, New Mexico (663-559)

5:00-5:10
(650) Moment sequences and the Bernstein polynomials
   Professor S. M. Eisenberg, University of Hartford (66y-84)

5:15-5:25
(651) Uniqueness classes for periodic-type functionals
   Professor R. F. DeMar, University of Cincinnati (663-327)

5:30-5:40
(652) On the kernel function for the intersection of two simply connected domains
   Professor Ted Suffridge, University of Kentucky (663-317)

SUNDAY, 2:45 P.M.

Session on Analysis 24 (Hilbert Spaces, etc.), Meeting Room 9, Jung Hotel
2:45-2:55
(653) Semi-Fredholm operators in von Neumann algebras
   Mr. M. J. O'Neill, University of Kansas (663-686)

3:00-3:10
(654) Dense embeddings of Hilbert spaces
   Professor J. S. Mac Nerney, University of Houston (663-662)

3:15-3:25
(655) On when an almost selfadjoint operator has no selfadjoint part
   Professor T. L. Kriete, University of Miami (663-518)

3:30-3:40
(656) Spectral partial isometries. Preliminary report
   Professor I. N. Erdelyi, Kansas State University (663-523)

3:45-3:55
(657) Outer factorization for vectorial Toeplitz operators. Preliminary report
   Mr. Berrien Moore, III, University of Virginia (663-560)

4:00-4:10
(658) Eigenfunction expansions and similarity for nonselfadjoint operators
   Dr. C. I. Goldstein, Brookhaven National Laboratory, Upton, New York
   (663-622)

4:15-4:25
(659) An example of nonuniqueness
   Professor M. M. Hackman, University of Washington (663-429)

4:30-4:40
(660) Selfadjointness in infinite tensor product spaces
   Mr. M. C. Reed, Princeton University (663-382)
4:45-4:55
(661) Boundary values of the numerical range. Preliminary report
Professor M. R. Embry, University of North Carolina at Charlotte (663-289)

5:00-5:10
(662) Some new results on Hilbert algebras
Professor J. H. Justice, University of Tulsa (663-290)

5:15-5:25
(663) Square roots of unitary operators
Professor L. J. Wallen, University of Hawaii (663-294)

5:30-5:40
(664) A two parameter perturbation estimate. Preliminary report
Professor W. M. Greenlee, Northwestern University (663-196)

5:45-5:55
(665) Critical images of maps of negative index, with a theory of cotypes
Professor Arthur Sard, City University of New York, Queens College (663-28)

SUNDAY, 2:45 P.M.

Session on Geometry 3, Meeting Room 10, Jung Hotel

2:45-2:55
(666) Semi-affine geometry on a set. Preliminary report
Professor S. C. Saxena, University of Akron (663-756)

3:00-3:10
(667) Differentiable structures and cohomology on locally compact groups
Professor Kenneth Whyburn, University of Washington (663-742)

3:15-3:25
(668) Approximation of Finsler spaces by Riemannian spaces
Professor E. M. Zaustinsky, State University of New York at Stony Brook (663-705)

3:30-3:40
(669) On sets which have finitely many points of local nonconvexity and a certain property Pm
Professor D. C. Kay*, University of Oklahoma, and Professor M. D. Guay, University of New Hampshire (663-690)

3:45-3:55
(670) Geometry of a metric space and isometries of Lipschitz spaces
Professor D. J. Patil*, University of Wisconsin, Milwaukee Campus, and Mr. M. H. Vashavada, University of Wisconsin (663-519)

4:00-4:10
(671) Submanifolds of a manifold with an f-structure
Professor G. D. Ludden, Michigan State University (663-543)

4:15-4:25
(672) The differentiable pinching problem for symmetric spaces of rank one
Professor Jeff Cheeger, University of Michigan (663-588)

4:30-4:40
(673) k-flags in a finite projective plane. Preliminary report
Dr. G. E. Martin, State University of New York at Albany (663-597)

4:45-4:55
(674) Transitivity and measures
Professor P. C. Hammer, Pennsylvania State University (663-658)

5:00-5:10
(675) Projections of F-vectors of 4-polytopes. II
Dr. J. R. Reay, Western Washington State College (663-418)
5:15-5:25  
(676) Surfaces with the spherical two-piece property  
Professor T. F. Banchoff, Brown University (663-421)

5:30-5:40  
(677) Quasi-piecewise flatness, differentiability and surface area  
Professor L. V. Toralballa, New York University (663-422)

SUNDAY, 2:45 P.M.

Session on Logic and Foundations, Pavilion, Jung Hotel
2:45-2:55  
(678) Elimination of the paradoxes in the set theory  
Professor Hidegoro Nakano, Wayne State University (663-5)

3:00-3:10  
(679) 2m = m and the axiom of choice  
Professor J. D. Halpern, University of Michigan (663-160)

3:15-3:25  
(680) Negationless analysis and recursive realizability  
Mr. J. K. Minichiello, Howard University (663-173)

3:30-3:40  
(681) Eliminability of terms which do not refer in first order theories. Preliminary report  
Professor R. C. Wherritt, Wichita State University (663-406)

3:45-3:55  
(682) On certain endomorphism semigroups in universal algebras. Preliminary report  
Mr. M. G. Stone, University of Colorado (663-404)

4:00-4:10  
(683) A generalization of the Kreisel-Shoenfield basis theorem  
Professor R. I. Soare, University of Illinois at Chicago Circle (663-403)

4:15-4:25  
(684) On congruence relations in abstract algebras  
Professor Mohammad Ishaq, Laval University (663-369)

4:30-4:40  
(685) On \( \pi \)-polyadic algebras  
Professor C. C. Pinter, Bucknell University (663-324)  
(Introduced by Professor J. S. Gold)

4:45-4:55  
(686) Models with universal properties  
Professor Robert Fittler, Rutgers University (663-476)  
(Introduced by Professor Frank Levin)

5:00-5:10  
(687) Predictably enumerable sets  
Professor W. E. Marsh, Talladega College, and Professor R. W. Ritchie*, University of Washington (663-732)

SUNDAY, 2:45 P.M.

Session on Applied Mathematics 5 (Mathematical Physics), Meeting Room 3, Howard Johnson's
2:45-2:55  
(688) Equilibrium deformations of harmonic materials. Preliminary report  
Professor B. Subramanian, Youngstown State University (663-604)  
(Introduced by Professor Everett Pitcher)

3:00-3:10  
(689) On optimal control of vibrating thin plates  
Professor Vadim Komkov, Florida State University (663-126)
3:15-3:25
(690) The interior flow of a vertical jet under gravity
Professor W. E. Conway, University of Arizona (663-116)

3:30-3:40
(691) Vibrations of nonhomogeneous anisotropic spherical shells
Professor R. S. Dhaliwal, University of Calgary (663-158)

3:45-3:55
(692) Vibrations of plates bounded by parts of elliptical and hyperbolic cylinders
Mr. E. N. Krishnappa, Howard University (663-366)
(Introduced by Professor W. R. Callahan)

4:00-4:10
(693) Normal modes of vibrations of a finite truncated circular sector plate
Professor W. R. Callahan, St. John's University (663-343)

4:15-4:25
(694) Influence of rotary inertia and shear on vibrations of a clamped circular ring
Professor J. S. Bakshi, State University of New York, College at Buffalo (663-424)

4:30-4:40
(695) On fluctuating flow of an elastico-viscous fluid past an infinite plate with variable suction
Mr. V. M. Soundalgekar, Indian Institute of Technology, Bombay, India, and Professor Pratap Puri*, Louisiana State University in New Orleans (663-532)
(Introduced by Dr. Diran Sarafyan)

4:45-4:55
(696) Coupled thermoelastic problem for an elastic half-space with an embedded spherical cavity
Professor P. K. Kulshrestha* and Professor Pratap Puri*, Louisiana State University in New Orleans (663-531)
(Introduced by Dr. Diran Sarafyan)

5:00-5:10
(697) Classification of electromagnetic and gravitational fields
Dr. G. A. Ludwig, University of Alberta (663-670)

5:15-5:25
(698) On transient development of internal waves in stratified ocean
Dr. Lokenath Debnath, East Carolina University (663-692)
(Introduced by Dr. T. J. Pignani)

5:30-5:40
(699) On continuum theories of fluids suspension
Dr. S. J. Allen* and Dr. K. A. Kline, Wayne State University (663-722)
(Introduced by Professor C. N. DeSilva)

SUNDAY, 2:45 P.M.

Session on Topology II (General Topology), Plantation Oak Room, Governor House

2:45-2:55
(700) Topologies for probabilistic metric spaces
Professor R. T. Fritsche, Northeast Louisiana State College (663-55)

3:00-3:10
(701) Invariants and dimension functions in topological spaces
Professor R. G. Lintz, McMaster University (663-78)

3:15-3:25
(702) A characterization of the Čech homology theory
Professor S. K. Kaul, University of Saskatchewan, Regina Campus (663-179)

3:30-3:40
(703) The Gauss realizability problem
Professor M. L. Marx, Vanderbilt University (663-189)
3:45-3:55
(704) On the cohomology of compact directed spaces
Professor L. E. Ward, Jr., University of Oregon (663-245)

4:00-4:10
(705) Quilted two manifolds
Professor J. M. Slye, University of Houston (663-320)

4:15-4:25
(706) Noncomparable homogeneous topologies with the same class of homeomorphisms
Professor Yu-Lee Lee, Kansas State University (663-301)

4:30-4:40
(707) Quotients of the space of irrationals
Professor E. A. Michael*, University of Washington, and Professor A. H. Stone, University of Rochester (663-517)

4:45-4:55
(708) On m-sequential spaces. Preliminary report
Professor S. L. Gulden, Lehigh University (663-706)

5:00-5:10
(709) Transfinite cardinal dimension. Preliminary report
Mr. S. W. Williams, Pennsylvania State University, Allentown Center (663-423)
(Introduced by Professor S. L. Gulden)

5:15-5:25
(710) Lebesgue characterizations of uniformity-dimension functions
Dr. J. C. Smith, Jr., Virginia Polytechnic Institute (663-145)

5:30-5:40
(711) A note on a theorem of J. Nagata
Professor J. E. Vaughan*, University of North Carolina at Chapel Hill, and Professor B. R. Wenner, University of Missouri (663-108)

SUNDAY, 2:45 P.M.

Session on Statistics and Probability 3 (Branching Process, Markov Processes and Entropy), Pine Oak Room, Governor House

2:45-2:55
(712) On the supercritical age dependent branching process
Mr. Krishna B. Athreya, Mathematics Research Center, U. S. Army, University of Wisconsin (663-646)
(Introduced by Dr. T. G. Kurtz)

3:00-3:10
(713) Diffusion processes. I: Existence and uniqueness
Mr. D. W. Stroock and Professor S. R. S. Varadhan*, Courant Institute, New York University (663-164)

3:15-3:25
(714) Diffusion processes. II: Stability
Mr. D. W. Stroock* and Professor S. R. S. Varadhan, Courant Institute, New York University (663-165)

3:30-3:40
(715) Some ratio limit theorems for a general state space Markov process
Professor M. L. Levitan, Drexel Institute of Technology (663-150)

3:45-3:55
(716) The explosion problem for branching Markov processes
Mr. T. H. Savits, Princeton University (663-701)

4:00-4:10
(717) A formula for semigroups with an application to branching diffusion processes
Professor S. A. Sawyer, Brown University (663-745)
4:15-4:25
(718) On the recurrence of sums of independent nonidentically distributed random variables
Professor J. C. Mineka*, City University of New York, Lehman College, and Mr. Steven Silverman, University of British Columbia (663-304)

4:30-4:40
(719) An order for conic distributions and the ergodic theorem
Professor Hermann Dinges* and Mr. H. Rost, Catholic University of America (663-752)
(Introduced by Professor Eugene Lukacs)

4:45-4:55
(720) Differential entropy and tiling
Dr. E. C. Posner* and Mr. E. R. Rodemich, Jet Propulsion Laboratories, California Institute of Technology (663-431)

5:00-5:10
(721) Conditional (P,Q) entropy
Professor R. W. Allen, Jr., University of Miami (663-473)

5:15-5:25
(722) Stochastic games
Professor G. J. Smith, University of California, Davis (663-314)

Tallahassee, Florida

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NEWS ITEM

POSTGRADUATE TRAINING IN THE UNITED KINGDOM

In November 1965, the Council of the Royal Society decided to set up an ad hoc committee to examine postgraduate training in science and technology in the United Kingdom. It was decided to investigate the content and quality of the training, especially for the Ph.D. degree. Six subject groups were set up, the studies in each to be made by groups of Fellows of the Royal Society and others; the subject groups were to investigate biology, chemistry, engineering, earth sciences, mathematics and physics, with a Fellow of the Royal Society as chairman of each group.

The chemistry report was published in March 1967, the physics report in February 1968, and the biology report in March 1968. The fourth report, issued from the mathematics subcommittee, is now on sale at $1.00 a copy, post-free, from the Executive Secretary, The Royal Society, 6 Carlton House Terrace, London, S. W. 1. Copies may also be obtained through any bookseller.

The report comprises 43 pages, with a foreword by the President of the Royal Society. It considers the problems of postgraduate education in the application of mathematics to science, engineering, technology, and the social sciences. There are sections on the increasing needs for applied mathematics in British university education, the importance of strong research schools in applied mathematics, and the sub-committee’s suggestions for intensifying association with practical work, together with a survey of research schools in applied mathematics in British university institutions.
PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Sixty-Fifth Meeting
Palmer House
Chicago, Illinois
April 18-19, 1969

The six hundred sixty-fifth meeting of the American Mathematical Society will be held at the Palmer House, Chicago, Illinois, on April 18-19, 1969. All sessions will be held either on the Club Floor or in the Red Lacquer Room of the Palmer House.

By invitation of the Committee to Select Hour Speakers for Western sectional Meetings there will be hour addresses by Professor William W. Boone of the University of Illinois, Professor Edward R. Fadell of the University of Wisconsin, Professor P. R. Masani of Indiana University, and Professor Francois Treves of Purdue University.

There will be sessions for contributed papers on both days of the meeting. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of February 14, 1969.

There will be two special sessions of twenty-minute papers, one on K-Theory and Cohomology Operations under the chairmanship of Professor A. L. Lillevisius of the University of Chicago, and one on Algebraic Geometry under the chairmanship of Professor Maxwell Rosenlicht of Northwestern University. Most of the papers to be presented at these sessions will be by invitation. However, anyone contributing an abstract for the meeting who feels that his paper would be particularly appropriate for one of these special sessions should indicate this clearly on his abstract and submit it three weeks earlier than the above deadline, namely by January 24, to allow time for additional handling.

Paul Bateman
Associate Secretary
Urbana, Illinois

Six Hundred Sixty-Sixth Meeting
Americana Hotel
New York City
April 2-5, 1969

The six hundred sixty-sixth meeting of the American Mathematical Society will be held at the Americana Hotel in New York, New York, from April 2 to April 5, 1969.

By invitation of the Committee to Select Hour Speakers for Eastern sectional Meetings the following four addresses will be presented:

Professor David Shale of the University of Pennsylvania will speak on "Some probabilistic ideas in mathematical physics" at 11:00 a.m. on Friday, April 4.

Professor Cathleen Morawetz of New York University will speak on "Energy flow: wave motion and geometrical optics" at 2:00 p.m. on Friday, April 4.

Professor Katsumi Nomizu of Brown University will speak on "Differential geometry of complex hypersurfaces" at 11:00 a.m. on Saturday, April 5.

Professor William Browder of Princeton University will speak at 2:00 p.m. on Saturday, April 5; his topic will be announced later.
A symposium on "Mathematical aspects of electrical network theory" will be held on Wednesday, April 2, and Thursday, April 3, under the sponsorship of the AMS-SIAM Committee on Applied Mathematics, with anticipated support from the Air Force Office of Scientific Research and the U.S. Army Research Office (Durham). The organizing Committee for this symposium consists of W. A. Blackwell, F. Branin, R. Brayton, F. Harary, and H. S. Wilf (chairman). The program will contain hour lectures by R. Brayton, R. J. Duffin, D. C. Youla, J. W. T. Youngs, and twelve half-hour talks.

There will be sessions for contributed ten-minute papers during both mornings and afternoons of Friday, April 4, and Saturday, April 5. The deadline for receipt of papers to be placed on the program is February 6, 1969. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Abstract blanks can be obtained by request from the same address. There will be no provision for late papers.

Travel instructions and hotel reservation blanks will appear in the February issue of these Notices.

Herbert Federer
Associate Secretary
Providence, Rhode Island

1969 Summer Institute on Number Theory

The American Mathematical Society will hold its Sixteenth Annual Summer Research Institute at the State University of New York at Stony Brook from July 7 through August 1. The topics for the institute will be Number Theory: Analytic Number Theory, Diophantine Problems, and Algebraic Number Theory. The institute will be supported under a grant from the New York State Science and Technology Foundation and a proposed grant from the National Science Foundation.

In this decade, number theory has enjoyed a renaissance, and a startling number of ancient and venerable problems have been resolved. To name but a few: complex quadratic fields of class number 1; simultaneous approximation to several real numbers; approximation to logarithm of real numbers; distribution of S-numbers; large sieve Artin's conjecture; effective methods for solving binary diophantine equations; integral points on a general cubic polynomial; zeta functions for varieties over finite fields; quantitative form of Hilbert's 17th problem; class tower problem. Both the quality and the quantity of research in number theory during this decade argue strongly that we should close the decade with an intensive conference, taking stock of what has been done and of the methods used in preparation for the future.

Approximately 65 participants are being invited and of this number 15 are from foreign countries. Qualified mathematicians who wish to participate in this institute are invited to write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

The Organizing Committee for the 1969 Summer Institute is composed of Professors James Ax, P. T. Bateman, Kenkichi Iwasawa, D. J. Lewis (Chairman), and Atle Selberg.
The sixth AMS summer seminar will be held on the campus of Rensselaer Polytechnic Institute from July 7 through August 1, 1969, and will be co-sponsored by the American Meteorological Society. The Organizing Committee consists of Hirsh G. Cohen, IBM, T. J. Watson Research Center; Richard C. DiPrima, Rensselaer Polytechnic Institute; Dave Fultz, University of Chicago; C. C. Lin, Massachusetts Institute of Technology; and William H. Reid (chairman), University of Chicago. It is expected that the members of this committee will take part as lecturers in the seminar and will report on their recent work. Proposals for support have been submitted to several granting agencies.

The AMS-SIAM Committee on Applied Mathematics, consisting of Hirsh G. Cohen, Joaquin B. Diaz, Jim Douglas, Jr., William H. Reid, Richard S. Varga, and Herbert S. Wilf (chairman), has proposed the topic for the seminar, Mathematical Problems in the Geophysical Sciences. Because there is an important interplay between the observation and experimental work on the one hand and the mathematical developments on the other, this topic is of widespread interest at the present time.

The seminar program is intended to be largely instructional in purpose and to provide graduate students and recent Ph.D.'s with the opportunity of becoming familiar with the recent advances in several areas of the geophysical sciences in which applied mathematics plays a central role. The program has, therefore, been designed to include topics in the atmospheric, oceanographic, and earth sciences. There will be a basic series of lectures in these three subjects plus several special lectures and seminars. As with previous AMS seminars, the lectures presented at the seminar will be published.

Dormitory accommodations with meals served cafeteria style will be available on the campus. A complete brochure of information will be sent to the participants in the course of the winter.

Individuals may apply for admission to the seminar. Application blanks for admission and financial assistance can be obtained from the Meeting Arrangements Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. An applicant will be asked to indicate his scientific background and interest. Graduate students should have their faculty advisers write to the committee concerning their ability and promise. Those who wish to apply for a grant-in-aid should so indicate.

NEWS ITEM

A NEW DEPARTMENT OF STATISTICS AND BIOMETRY AT EMORY UNIVERSITY

The Department of Statistics and Biometry in the Medical and Graduate Schools offers M. S. and Ph.D. degrees. Methods of making and testing quantitative models of scientific phenomena are emphasized. Applicants should have studied advanced calculus and have some scientific background. Fellowships are available throughout the year. The faculty of ten is engaged in research and teaching. The Department is closely associated with the university's scientific computation center which is equipped with an RCA Spectra 70/55, interfaced to an analog computer and to specialized data acquisition equipment.

Further information may be obtained from Malcolm E. Turner, Chairman of the Department.
The Mathematical Sciences: A Report*, the first of three volumes prepared by the Committee on Support of Research in the Mathematical Sciences (COSRIMS) of the National Academy of Sciences under the chairmanship of Lipman Bers, Professor of Mathematics, Columbia University, is the first comprehensive effort by U.S. mathematicians to identify the demands that are being made on the field of mathematics, to assess the capabilities of the mathematical community, to satisfy them and to suggest ways to increase these capabilities.

A main conclusion of the report is that "there is now a shortage of qualified college teachers of the mathematical sciences, and this situation is likely to get worse before it gets better." The changing and improving preparation of entering college students, the need for higher mathematical education on the part of those who are or will become secondary and elementary school teachers, and the need for increasingly sophisticated mathematical education on the part of those entering other disciplines have combined to produce massive new demands. At the same time, the last decade has brought an increase in the percentage of mathematics majors from 1.5% to 4.0% of all college majors. It is projected that by 1970, 41,000 students will be earning bachelor's degrees in mathematics and statistics, and an additional 796,800 students majoring in other subjects will be enrolled in mathematics courses.

Because of this massive demand for mathematical education, the report recommends continued and increased federal support for faculty improvement and urges two new programs to prevent loss of talented individuals at the early graduate level. The special programs proposed are an experimental program for federal support of graduate students in qualified institutions that do not offer a Ph.D. degree in mathematical sciences, and a program of special fellowships, or forgivable loans, for talented college graduates with weak or inadequate preparation in the mathematical sciences. The goal of national planning, the report says, should be to see that the U.S. does not squander its mathematical talent and that a sufficient number of trained personnel are produced to provide the mathematical skills necessary to the nation.

The report also recommends that federal support for mathematical research should, as a matter of national policy, keep pace with the increased number of qualified mathematical investigators. Federal Research assistantships, fellowships, and traineeships should provide support for at least one third of the full-time graduate enrollment in mathematical sciences, and opportunities for postdoctoral study should be expanded. New programs are recommended to provide postdoctoral research instructorships that will allow the holders to teach at smaller colleges and conduct research at nearby centers and to provide part-time graduate fellowships for women who have completed their master's degrees.

In discussing the levels of support necessary for the various areas of mathematics, the report urges continued support for pure mathematics research, citing the extraordinary rate at which classical mathematical problems have fallen before the onslaught of young mathematicians in recent years.

Although the new scientifically based technologies utilize the ideas of all areas of mathematics, the Committee concludes that the areas of applied, or physical, mathematics and computer science have not received the attention they demand.

Computer science has grown so rapidly that it now suffers from a critical undersupply of leaders for research and

*Publication 1681. Available at $6.00 from the Printing and Publishing Office, National Academy of Sciences, 2101 Constitution Avenue, N. W., Washington, D. C. 20418.
education. In addition, special problems are brought about by the high cost of computer science research and by the fact that it is both a scientific and an engineering discipline (which has made it difficult to fit into the academic structure). The project-grant type of support common in other areas of mathematics does not lend itself to meaningful research in computer science. The report recommends special support for developing and updating courses, expansion of numbers of research assistantships and traineeships, and grants to departments to cover costs of computer usage in research.

Physical mathematics does not need the massive support required by computer science, the report concludes, but it does need a more clearly recognized place in the academic structure. The applied mathematical sciences must be allowed to develop as a field in their own right, and not only as an auxiliary for other sciences.

In considering possible forms of support for mathematics research the report urges that the National Science Foundation and the mission-oriented agencies continue to support mathematics research through project grants. At the same time it recommends that greater use be made of department grants, both to sustain research activity and to assist departments to improve their quality.

A second volume of this report, The Mathematical Sciences: Undergraduate Education**, prepared for COSRIMS by its Panel on Undergraduate Education, discusses in greater detail means of making available a more high-quality teaching staff, means of enhancing the effectiveness of existing staff, and means of dealing with the special problems of underdeveloped colleges and underprepared students.

The third volume, The Mathematical Sciences: A Collection of Essays***, presents, in a manner accessible to the general scientifically oriented reader, twenty-two essays by distinguished experts covering selected subfields and areas of application of the mathematical sciences.

This three-part report emphasizes that "mathematization" has overtaken U.S. society. Because mathematical fields have grown in depth and diversity and mathematical methods and techniques have penetrated increasingly into other human activities, what must be recognized is that "the mathematization of our society brings with it an increasing need for people able to understand and use mathematics."

**Publication 1682. Available at $4.00 from the Printing and Publishing Office, National Academy of Sciences, 2101 Constitution Avenue, N.W., Washington, D.C. 20418.
***Available later from the Massachusetts Institute of Technology Press.

NEWS ITEM

UNIVERSITY OF NORTH DAKOTA RECEIVES NSF GRANT

The University of North Dakota, Grand Forks, North Dakota, is the recipient of a National Science Foundation Grant in support of the College Science Improvement Program. Areas of major emphasis under the grant are chemistry, geography, mathematics, and physics.

The mathematics portion of the $210,000 grant will be concerned with the development of curriculum in numerical analysis and computer-related mathematics, and the financial support for three staff members of the mathematics department in order that they may pursue further postgraduate work in mathematics at the University of their choice.
ACTIVITIES OF OTHER ASSOCIATIONS

THE MATHEMATICAL ASSOCIATION OF AMERICA
New Orleans, Louisiana
January 25-27, 1969

The fifty-second annual meeting of The Mathematical Association of America will be held in the Jung Hotel, New Orleans, Louisiana, in conjunction with the annual meeting of the Association for Symbolic Logic, the annual meeting of the American Mathematical Society, and a meeting of the National Council of Teachers of Mathematics. On Saturday and Sunday, the Association sessions will be held jointly with the National Council of Teachers of Mathematics.

A complete program of the meeting is included in the timetable in this issue of these Notices.

ASSOCIATION FOR SYMBOLIC LOGIC
New Orleans, Louisiana
January 22-23, 1969

A complete program of the sessions is included in the timetable in this issue of these Notices.

NEWS ITEMS AND ANNOUNCEMENTS

TWENTY-FIRST BRITISH MATHEMATICAL COLLOQUIUM

The twenty-first British Mathematical Colloquium will be held at the University of Birmingham in Birmingham, England, from March 25 through 29, 1969. The provisional program is as follows: M. F. Atiyah, A. F. Beardon will lecture on geometry; J. F. Adams, A. Fröhlich, M. H. Löb, and A. C. Offord on topology, number theory, logic, and analysis; A. O. L. Atkin, J. Duncan, C. P. Rourke on functional analysis, topology, and number theory; S. Smale on global stability in dynamical systems; R. A. Rankin, A. D. Sands, and J. G. Thompson on algebra; J. A. Dieudonné on Lie groups (classical, algebraic, and formal). There will also be meetings of splinter groups on various topics. Members are invited to contribute short papers to these groups, and there will be opportunity for discussion.

The membership for the colloquium is 10 shillings if the application is received by February 28, 1969. Application for membership should be made to Dr. H. C. Wilkie, Department of Pure Mathematics, The University, Edgbaston, Birmingham, 15.

SYMPOSIUM ON TOPOLOGICAL SEMIGROUPS

The Department of Mathematics of the University of Florida in Gainesville will sponsor a Symposium on Topological Semigroups, April 7-11, 1969. A connected sequence of four one-hour lectures will be given by K. H. Hofmann and P. S. Mostert, and there will be other such sequences by persons distinguished in this field to an expected total of 16 one-hour lectures. One day of the week of the symposium will probably be set aside for brief invited papers from advanced predoctoral and immediately postdoctoral researchers. While some support is expected to be available for nonlecturing participants, this will be mainly used for those in this region.

There will be a conference related to the projected periodical Colloquium on Semigroups, and it is expected that the proceedings of the symposium will be published in the early issues of this journal.

Information may be obtained by writing to Professor A. D. Wallace, Walker Hall, University of Florida, Gainesville, Florida 32601.
VISITING FOREIGN MATHEMATICIANS
(Supplementary List)

The following foreign mathematicians are among those visiting in the United States and Canada during 1968-1969. These mathematicians are in addition to those listed on pages 987-996 of the November 1968 issue of these Notices.

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berger, Robert W. (Germany)</td>
<td>Louisiana State University</td>
<td>Commutative Algebra</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Bhatia, Nam P. (India)</td>
<td>Case Western Reserve University</td>
<td>Ordinary Differential Equations</td>
<td>9/68-6/30/69</td>
</tr>
<tr>
<td>Ching, Wai-Mee (Formosa)</td>
<td>Louisiana State University</td>
<td>C*-Algebras</td>
<td>9/1/68-6/1/69</td>
</tr>
<tr>
<td>Dugdale, John Keith (Great Britain)</td>
<td>State University of New York at Albany</td>
<td>Ergodic Theory</td>
<td>9/68-9/69</td>
</tr>
<tr>
<td>Dwivedi, Shankar. H. (India)</td>
<td>University of Oklahoma</td>
<td>Complex variables</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Geertsema, Jan Cornelius (South Africa)</td>
<td>Florida State University</td>
<td>Statistics (probability)</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Ghosh, Busadev (India)</td>
<td>University of South Florida</td>
<td>Continuum mechanics</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Knowles, John David (Great Britain)</td>
<td>State University of New York at Albany</td>
<td>Topology, measure theory</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Kunz, Ernst (Germany)</td>
<td>Louisiana State University</td>
<td>Commutative algebra</td>
<td>9/1/68-6/1/69</td>
</tr>
<tr>
<td>Lorenz, Dan H. (Israel)</td>
<td>Louisiana State University</td>
<td>Applied Mathematics</td>
<td>9/1/68-6/1/69</td>
</tr>
<tr>
<td>Pitman, Edwin J. G. (Australia)</td>
<td>University of Chicago</td>
<td>Theoretical statistics</td>
<td>10/1/68-3/21/69</td>
</tr>
<tr>
<td>Rohrbach, Hans (Germany)</td>
<td>Wake Forest University</td>
<td>Algebra and number theory</td>
<td>4/69</td>
</tr>
<tr>
<td>Rotenberg, Michael Ashley (England)</td>
<td>Southwestern at Memphis</td>
<td>Relativity theory</td>
<td>7/68-</td>
</tr>
<tr>
<td>Su, Li Pi (China)</td>
<td>University of Oklahoma</td>
<td>Topology and functional analysis</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Twomey, Brian (Ireland)</td>
<td>University of South Florida</td>
<td>Complex analysis</td>
<td>9/68-6/69</td>
</tr>
<tr>
<td>Winer, Paul (Australia)</td>
<td>Simon Fraser University</td>
<td>Statistics</td>
<td>9/68-4/69</td>
</tr>
</tbody>
</table>

Errata

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avadhani, T. V. (India)</td>
<td>University of Michigan</td>
<td>Stochastic processes</td>
<td>2/69-6/69</td>
</tr>
<tr>
<td>Kövari, Thomas (United Kingdom)</td>
<td>University of Waterloo</td>
<td>Theory and Applications</td>
<td></td>
</tr>
</tbody>
</table>

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LETTERS TO THE EDITOR

Editor, the Notices.

The October issue of the Notices published the text of a letter which I wrote to the editors of SCIENCE concerning some remarks made by Dr. Hornig, President Johnson's scientific adviser. I want to emphasize that this letter was approved unanimously by the Council at its August meeting. In its issue of November 1, SCIENCE printed this letter with a few editorial changes (including the omission of all titles) and two substantive changes. These latter consisted in the replacement of each of the words "invidious" and "completely" in the second sentence by a set of dots. The letter was prefaced by a statement to the effect that I was requested by the Council to send the letter to the editors of SCIENCE.

Charles B. Morrey, Jr.

Editor, the Notices.

In their letter Professors Bass et al. (these Notices 15 (1968), 861-862) correctly perceive the fundamentally important principle of the "autonomy of the individual scientist in determining for himself how he works and what he works upon." Yet they fail to perceive how this is basically incompatible with responsible government support of mathematics and science.

Certainly Donald Hornig's remarks (same Notices, p. 855) about publicly supported vacationing mathematicians were inappropriate. Yet on the principle that the only possible justification for public support of mathematical research activity is that the public get some reasonably direct benefits from it, any responsible administrator of those public funds must direct them to those areas where he sees promise of the greatest returns. Thus mathematicians, if they accept the support of funds from responsible agencies of the government, find their research activities being directed toward those problems which the administrator has found most urgent. Obviously this is contrary to the aforementioned autonomy principle.

Although many research workers in mathematics and science may get some short-run benefits from the flow of federal funds to these areas, this support can only bring about effects inimical to the long-run vitality and significance of these fields.

James Turner

Editor, the Notices.

Dr. Turner is correct in implying that an installation of the Department of Defense may let a contract for the investigation of a specific problem or for work in a narrowly-defined area. However, the basic research in the physical (including mathematical) and environmental sciences that the Army supports is nearly all administered by the U.S. Army Research Office-Durham (ARO-D). In this work, unsolicited proposals are carefully evaluated, and accepted (as long as the money lasts) on the basis of scientific quality and probability of results (not necessarily immediate) of interest and use to the Army. No attempt is made to direct the research into certain channels, or to persuade the proposer to consider certain problems; he directs himself. Extra money is sometimes allotted to work in certain areas, for example Operations Research or Brittle Fracture of Materials, where basic research is expected to yield more immediate dividends, but again there is no direction of the investigator.

A. S. Galbraith

Editor, the Notices.

John Walsh's article (these Notices 15, 859) raises questions of political consideration entering into the continuance of grants but I find incompatible the idea of a person actively opposing the war in Vietnam and accepting grants from any of the military services.

Frank Flaherty

Editor, the Notices.

Serge Lang's provocative letter in the November Notices touches on some sensitive points which, I think, are less clear-cut than his letter might suggest.
Let me begin by citing my personal experience. I have been accepting DOD support for many years, and I can report that from the point of view of freedom from political interference (and, incidentally, also from every other point of view) the relationship has been exemplary. There has been every evidence that the perquisites of creative work have been recognized, and the occasional bureaucratic demands of paperwork have been minor and peripheral.

I am one of the signers of the declaration in question. I have also signed stronger declarations, and--Mr. Foster's memo notwithstanding--neither my political feelings nor my willingness to express them have changed significantly in the recent past. As yet I have received no letter questioning my activities. Should one arrive I would have to deal with it on its merits, but I rather hope that by now those who are responsible for such letters have thought better of the matter.

Of course, I do not wish to dispute Professor Lang's assertion that the present incident will have an intimidating effect. It certainly will. But is one to infer, as Mr. Lang seems to do, that our proper response is to turn down all DOD money. If so, then our next step must be to reject NSF support because of the obvious intimidation implicit in the Smale incident, and finally we must give up our positions, because our Universities, which are responsive to pressures from business and government, also view political protest with disfavor.

I am inclined to view the matter in a somewhat different light. For my part, I find it a ray of hope that an agency whose chief business is violence and destruction has found it possible to divert some of its assets to the support of humanist activity, the achievements of which are published openly and available to all who can survive our definitions. This far-sighted step certainly lies behind much of the great surge of scientific achievement in our country during the past two decades. Would it have been better for American scientists to take umbrage at the letterhead, and to return the money so that it could be used for the purpose originally designated? There is perhaps a case for this view, but it is by no means a clear-cut one. My own hope is that the present procedure will eventually develop into one of many expedient channels through which the enormous wealth of our nation can be diverted from its present destructive use into the solution of urgent social problems, and to the material support for a new age of unparalleled creative and cultural activity. This rose-colored view may appear belied by our everyday professional experience, in which we are beset by crises, retrenchments, and by demands for subordination and conformity. There are indeed great obstacles on the way to Utopia, and the ones we face as professionals are very real. My point is that these latter cannot be resolved by turning the Universities into ivory towers. They are an inevitable consequence of a deep and pervasive social disorder, and they will return to haunt us no matter how we twist and maneuver, until we are prepared to deal with the basic issues from which they arise.

Robert Finn

Editor, the Notices

The Chicago Convention Bureau estimates that conventions bring in 300 million dollars to the city of Chicago annually. Deprivation of revenue is a particularly effective weapon. In response to the atrocities committed by the Daley establishment during the Democratic convention, the American Sociological Association has imposed upon itself a ten year ban on Chicago conventions. The American Historical Association and the American Psychological Association have cancelled their Chicago meetings. I suggest that the American Mathematical Society do the same.

James P. Jones
NEW AMS PUBLICATIONS

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS
Volume 23
LINEAR AND QUASILINEAR EQUATIONS OF PARABOLIC TYPE
By O. A. Ladyženskaja, V. A. Solonnikov, and N. N. Ural'ceva.
660 Pages; List Price $34.20; Member Price $25.65

In this volume boundary value problems are studied from two points of view: solvability, unique or otherwise, and the effect of requiring various smoothness properties required for the given functions on the smoothness of the solutions.

There are seven chapters contained in this volume. Chapter one gives a statement of the new results and an historical sketch. Chapter two introduces the various function spaces typical of modern Russian-style functional analysis. Chapters three and four deal with linear equations, chapter six concerns itself with quasilinear equations, and chapter seven with systems of equations. These last four chapters can be read independently of one another.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY
Number 84
REPRESENTATION THEOREMS ON BANACH FUNCTION SPACES
By Neil E. Gretsky.
56 Pages; List Price $1.50; Member Price $1.13

This Memoir gives characterization theorems for bounded linear operators to and from certain types of function spaces, viz. Banach function spaces as developed by W. A. J. Luxemburg and A. C. Zaanen. The first result deals with operators from a Banach space \( Y \) to a Banach function space \( L^p \), and gives the representation in terms of \( \mathcal{Y} \)-valued set functions of bounded \( p \)-variation. The second result gives a characterization of bounded linear operators from \( M^p \), the closed span of the simple functions in \( L^p \), to a Banach space \( Y \) in terms of \( \mathcal{Y} \)-valued set functions of bounded \( p \)-variation.

The last half of the paper deals with \( L^p \), the bounded linear functionals on \( L^p \). It is here that the role of \( L^p \) as an \( AB \)-lattice comes into greater prominence. A decomposition of \( L^p \) into \( M^p \) and its lattice orthogonal complement provides the framework for the characterization. Let \( N^p = L^p/M^p \). Then \( N^p \) is representable in terms of purely finitely additive set functions with supports off \( M^p \). A correspondence between \( N^p \) and \( M^p \) gives the result for \( M^p \). A correspondence between \( M^p \) and the lattice orthogonal complement of \( M^p \) suggests that all remains to be found is a representation for \( M^p \). This is given by specializing the result obtained above for operators on \( M^p \). Another (more general but less useful) characterization of \( M^p \) is also presented.

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS
Volume 11
ENTIRE FUNCTIONS AND RELATED PARTS OF ANALYSIS
Edited by J. Korevaar.
560 Pages; List Price $12.00; Member Price $9.00

This volume contains a collection of papers from the Summer Institute held at the University of California from June 27 to July 22, 1966.

The topics discussed include complex analysis from the point of view of mapping of manifolds, Fourier methods, function spaces, and distribution of values.

There is also a section containing the results of the problem sessions held at the Institute. This section deals with both problems and solutions.

TRANSLATIONS—SERIES II
Volume 76
THIRTEEN PAPERS ON ALGEBRA AND ANALYSIS
By Şanin, Gun, Ismagilov, Klimyk, Iohvidov, Mitropol'skiı, Sneider, Lykova, Golin'skiı, Vladimirov, Kadec, Gurariı, and Olevskiı.
268 Pages; List Price $13.40; Member Price $10.05
PERSONAL ITEMS

Professor R. E. BLOCK of the University of Illinois has been appointed to a professorship at the University of California, Riverside.

Dr. J. L. BRENNER of the Stanford Research Institute has been appointed to a professorship at the University of Arizona.

Professor PHILIP CALABRESE of the Illinois Institute of Technology has been appointed to an assistant professorship at the Naval Postgraduate School, Monterey, California.

Dr. J. M. CUSHING of the University of Maryland has been appointed to an assistant professorship at the University of Arizona.

Professor P. C. FIFE of the University of Minnesota has been appointed to a professorship at the University of Arizona.

Mrs. F. K. HOLLEY of the University of New Mexico has been appointed a lecturer at Ithaca College.

Professor W. D. LINDSTROM of Kenyon College is on leave for the academic year 1968-1969, and is teaching mathematics at Robert College, Istanbul, Turkey.

Professor D. P. MAKI of Indiana University is on leave for the academic year 1968-1969, and is teaching mathematics at Robert College, Istanbul, Turkey.

Professor D. P. MAKI of Indiana University is on leave for the academic year 1968-1969, and is teaching mathematics at Robert College, Istanbul, Turkey.

Professor H. B. MANN of the University of Wisconsin, Mathematics Research Center, has been appointed to a visiting professorship for the academic year 1968-1969 at the University of Arizona.

Dr. G. W. MORRIS of McDonnell-Douglas, California, has been appointed to a professorship at the Naval Postgraduate School, Monterey, California.

Professor C. R. NICOLAYSEN of the United States Naval Academy has been appointed Director of the Computer Center at Coe College.

Professor M. L. PURI of the Courant Institute of Mathematical Sciences, New York University, has been appointed to a professorship at Indiana University, Bloomington Campus.

Professor C. D. ROBINSON of the University of Mississippi has been appointed to a professorship at Hardin-Simmons University.

Professor S. C. SAXENA of Northern Illinois University has been appointed to an associate professorship at the University of Akron.

Dr. L. A. SEGEL of Rensselaer Polytechnic Institute is on leave for the academic year 1968-1969. He has been appointed a visiting investigator in the Biomathematics Division of the Sloan-Kettering Institute and a visiting associate professor of Biomathematics in the Graduate School of Medical Science, Cornell University.

Professor C. S. SMITH of Lake Forest College is on sabbatical leave for the academic year 1968-1969, and is spending the year at the University of Arizona.

Mr. SUNDARAM SWETHARANYAM of the Joint Plant Committee, Calcutta, India, has been appointed to an assistant professorship in the Computer Center at McNeese State College for the academic year 1968-1969.

Professor R. J. TROYER of the University of North Carolina has been appointed to an associate professorship at Lake Forest College.

Professor R. M. WARTEN of the International Business Machines Corporation, Scientific Center, Palo Alto, California, has been appointed to an associate professorship at California State Polytechnic College.

Professor C. H. WILCOX of the Argonne National Laboratories, Argonne, Illinois, has been appointed to a professorship at the University of Arizona.

PROMOTIONS

To Professor, University of Arizona: R. K. BHATTACHARYA, M. S. CHEEMA, D. E. MYERS.

To Assistant Professor, Mount Angel College: R. W. COOKE.

INSTRUCTORSHIPS

Hamilton College: R. J. BALLENTINE; Lake Forest College: P. C. MCQUEEN; Naval Postgraduate School: A. M. SHORB.
DOCTORATES CONFERRED IN 1967-1968
(Supplementary List)

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1967-1968. This list is a continuation of the list found on pages 877-902 of the October 1968 issue of these Notices.

CALIFORNIA

UNIVERSITY OF SOUTHERN CALIFORNIA (4)

Beem, John Kelly
General G-spaces
Chao, Wu Chao
Minor: Philosophy
A geometric interpretation of Stiefel-Whitney classes for PL-microbundle
Markin, Jack Traver
Fixed point theorems for set valued contractions
Woo, Peter Yam-Poon
Doubly-timelike general G-spaces

FLORIDA

UNIVERSITY OF FLORIDA (1)

Shershin, Anthony Connors
Results concerning the Schutzenberger-Walace theorem

ILLINOIS

NORTHWESTERN UNIVERSITY (3)

Committee on Applied Mathematics
Agnew, Robert
Transformations of uniform and stationary point processes
Asner, Bernard A.
On an iterative technique for the Kalman-Yacubovich equation
Janz, Ronald F.
Diffraction by a circular arc reflector

OHIO

CASE WESTERN RESERVE (5)

Operations Research
Kumin, Hillel J.
The design of Markovian congestion systems

Mann, Stuart
A mathematical theory for the exploitation and control of biological population
Schoeman, Milton E. F.
Resource allocation for new product development
Silverman, Gary J.
Primal decomposition of mathematical programs by resource allocation
Singh, Vijendra
Queuing systems with balking heterogeneous servers

WISCONSIN

UNIVERSITY OF WISCONSIN (1)

Hawkins, Thomas W., Jr.
Lebesgue's theory of integration

CANADA

UNIVERSITY OF MONTREAL (2)

Carreau, François
Structures radicales et théorie générale des radicaux
Boucher, Claude
Automates et grammaires universels

Errata

CLARK UNIVERSITY (MASSACHUSETTS)

Kaput, James Joseph
Locally reflective subcategories

UNIVERSITY OF PENNSYLVANIA

Chernack, Paul Francis
Homotopy groups of algebraic arcs found by smooth approximations
Gordon, Paul
Homotopy properties of vector fields on the 3-sphere
NEWS ITEMS AND ANNOUNCEMENTS

AFOSR MATHEMATICS DIVISION RESEARCH PROGRAMS

The Mathematics Division of the Air Force Office of Scientific Research annually plans its research program during the period January 1 through June 30 of any given calendar year. Research proposals considered under this program may request support to begin no earlier than September 1 of the same year and no later than September 30 of the following year.

In addition to the Division's continuing research program in analysis, functional analysis, statistics and probability theory, special programs for the support of specifically-oriented research monographs, intensified research, conferences and symposia are emphasized in all areas of mathematics and statistics.

The research-monograph program is designed to bridge the gap between journal-level research mathematics and user-level understanding and applications. The intensified research program is designed to accelerate the development of mathematics, most often in conjunction with university leave. Both programs provide for a maximum of one-half time support for 9 months, plus full-time support for an adjoining 3 months on a nonrecurring basis. Proposals submitted for consideration under either of these programs follow the outlines of a conventional research proposal, emphasizing the unique features of the anticipated results. It is helpful if research-monograph proposals include a sample chapter, in addition to a table of contents and an indication of publisher interest.

All proposals to be considered under the Division's current planning cycle should be submitted before June 30, 1969. Decisions on all proposals submitted under the research-monograph and intensified-research programs requesting support for some portion of the 1969-1970 academic year will be made no later than July 31, 1969. Decisions on all other proposals will be made as soon as possible after July 1, 1969. Proposals should be submitted to the Mathematics Division, Air Force Office of Scientific Research, 1400 Wilson Boulevard, Arlington, Virginia 22209.

PROPOSED INTERNATIONAL SYMPOSIUM ON STATISTICAL ECOLOGY

An International Symposium on Statistical Ecology will be held from August 24-August 30, 1969 at Yale University and the U.S. Forest Service Research Laboratory, New Haven, Connecticut. The Symposium is being supported by the Ford Foundation, Yale University, Pennsylvania State University, the U.S. Forest Service, and the Canadian Department of Agriculture.

The primary objective of the proposed Symposium is to provide opportunity for an exchange of ideas and information between ecologists, mathematicians, statisticians, and systems analysts. Particular emphasis will be placed on approaches and techniques applicable to the solution of man-environment problems or significant components thereof.

The Symposium is expected to cover the following subject areas: growth and regulation of populations; interacting populations; systems analysis and ecological prediction; productivity and the energy relations of eco-systems; population diffusion and migration; classification and ordination of communities and discrimination problems; compiling and interpreting ecological maps; distribution and abundance of species and species diversity; spatial patterns; homogeneity in vegetation; model making in ecology; distributions in ecology; sampling biological populations; fundamentals and principles, and aggregation; meaning and measurement.

Further information may be available from any member of the organizing committee, including Professor G. P. Patil, Department of Statistics, 302 McAllister Building, Pennsylvania State University, University Park, Pennsylvania 16802, U.S.A.

MATHEMATICS TYPING FORM

A form for the preparation of mathematical expressions for typing is described in an article by Ernest M. Scheuer, which appeared in IEEE Transactions on Engineering Writing and Speech, Vol. EWS-11 (1968), pp. 17-19. The form introduced in this article may be purchased from the Codex Book Company, Inc., 74 Broadway, Norwood, Massachusetts 02062, as their catalog no. 39,414. The form is designed to allow an author to indicate precisely the spacing he wants among the symbols he uses in equations, and to have a typist, although inexperienced in the typing of mathematics, prepare the material flawlessly.

KANPUR CONFERENCE ON TOPOLOGY

The Kanpur Conference on Topology was sponsored by the Indian Institute of Technology, Kanpur, with the cooperation of the Indian Mathematical Society. It was held on the IIT-Kanpur campus from October 3 to October 12, 1968. There were fifty-five participants, fourteen of whom represented five countries other than India. Texts of individual talks may be obtained from the Mathematics Department, IIT-Kanpur, IIT Post Office, Kanpur, U. P., India.

VISITING LECTURESHIP IN OPERATIONS RESEARCH

The Operations Research Society of America is continuing its Visiting Lectureship Program for the 1968-1969 academic year with sponsorship from the National Science Foundation.

Lecturers presenting formal lectures will also be available to discuss teaching problems and curricula matters with members of the staff and to advise students on future opportunities in study and employment.

Persons wishing more information should write to Visiting Lectureship Series in Operations Research, c/o Professor J. R. Borsting, Department of Operations Analysis, Naval Postgraduate School, Monterey, California 93940.

INSTRUCTIONAL CONFERENCE ON FINITE SIMPLE GROUPS

The London Mathematical Society will be holding an instructional conference on finite simple groups from September 2 to September 20, 1969, at Oxford. Professors G. Glauberman and D. Gorenstein will each give a course on "Methods for characterizing finite simple groups" and Professor E. C. Dade will give a course on "Character theory pertaining to finite simple groups." An additional course on "Finite simple groups of Lie type" is planned. Portions of these courses, and other courses not yet arranged in detail, will be of an introductory nature. A number of formal seminars will be arranged, and there will be opportunities for informal seminars and discussions as well as for social activities.

It is estimated that the cost of accommodation, which has been arranged at St. Peter's College, for the full conference will be less than $120 for full board. A limited amount of financial support may be available for living and traveling expenses of applicants who cannot obtain support from other sources.

A further notice will be issued early in 1969. Further details may be obtained from Dr. D. E. Cohen, Queen Mary College, Mile End Road, London, E. 1, England.

JOURNAL OF DIFFERENTIAL GEOMETRY

In 1969, the Journal of Differential Geometry will increase in price to $18.00 with a special price of $9.00 for personal subscriptions. The Journal will also increase in size to about 500 pages (about 25% increase from the 1967 volume).
MICHIGAN STATE UNIVERSITY POSTDOCTORAL FELLOWSHIPS

As part of a University Development Program under a grant from the National Science Foundation, the Department of Mathematics at Michigan State University has established five postdoctoral research fellowships. These are to be awarded each year to promising young Ph.D.'s. The teaching duties of a postdoctoral research fellow consist of one course each term which may be an advanced graduate course or seminar. The stipend for 1969-1970 will be $10,000 for an academic year appointment. Research fellows are eligible for reappointment as a fellow for a second year or for appointment to a regular staff position.

Additional information and application forms may be obtained from the Department of Mathematics, Michigan State University, Wells Hall, East Lansing, Michigan 48823. Completed applications should be filed by February 14, 1969.

ASSISTANCE TO DEVELOPING COLLEGES

The Mathematical Association of America Committee on Assistance to Developing Colleges is continuing its program of providing liaison service between developing colleges and mathematicians interested in temporary or permanent appointments at these colleges. The committee would also like to encourage mathematicians to serve as consultants or short-term visiting faculty.

For the purpose of interviewing mathematicians and providing further information on the program, a table will be set up in the Employment Register at the annual meeting in New Orleans. The Register will be open from 9:00 a.m. to 5:00 p.m. on Friday through Monday, January 24-27, in the Tulane Room of the Jung Hotel. Persons interested in any phase of the work of the committee are invited to come to this table or to write to Professor George Springer, Department of Mathematics, Indiana University, Bloomington, Indiana 47401.

NATO POSTDOCTORAL FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State has announced the award of 40 North Atlantic Treaty Organization (NATO) Postdoctoral Fellowships in Science.

Of the awards announced, 11 are in the life sciences, 25 in the physical sciences, including mathematics and engineering, and 4 in the social sciences. Fellows will attend institutions in Belgium, Denmark, France, and the Federal Republic of Germany, Italy, the Netherlands, Norway, Switzerland, and the United Kingdom.

The United States citizens who are being offered awards were selected from among 471 applicants. All applicants were evaluated for NSF by panels of scientists appointed by the National Research Council. Final selections were made by the Foundation.

NATO Fellows will receive a stipend of $6,500 for 12 months, $4,875 for 9 months. In addition, dependency allowances and limited allowances for round-trip travel will be provided.

Of the 40 U. S. citizens being offered NATO Postdoctoral Fellowships in Science, 3 are in the field of mathematics: Richard A. Brualdi, in Algebra, presently from the University of Wisconsin will go to the University of Sheffield in England; Jack A. Lees, in Topology, from Rice University will go to the University of Durham in England; Michael I. Shub from Brandeis University will go to the University of Warwick in England and the Institut des Hautes Etudes Scientifiques in Paris.

G. C. EVANS INSTRUCTORSHIPS AT RICE UNIVERSITY

The Evans Instructorships are intended for promising young mathematicians with the doctorate, primarily for new Ph.D.'s. Appointments are for two years, at a salary of at least $10,000 for the regular academic year. Teaching duties will amount to six hours per week in one semester and three hours per week in the other, or the equivalent. Inquiries and applications should be addressed to Professor Morton L. Curtis, Chairman, Department of Mathematics, Rice University, Houston, Texas 77001.
AUSTRALIAN MATHEMATICAL SOCIETY
NINTH
SUMMER RESEARCH INSTITUTE

The ninth summer research institute will be held at the Australian National University, Canberra, from January 7 to February 14, 1969. The following is a list­ ing of the lecturers and their topics:
Saunders Mac Lane, Categories and adjoint functors, covering those parts of potential interest to mathematicians in all fields.
W. Gaschutz, New developments in the theory of finite soluble groups.
P. Mandl, Mathematical control theory and probability.
Further lectures, seminars and discussion groups will be arranged in addition to those listed.

ADVANCED SEMINAR ON
GRAPH THEORY AND
ITS APPLICATIONS

An advanced seminar on Graph Theory and its Applications will be held at the Mathematics Research Center, U. S. Army, on October 13-15, 1969. The pro­ ceedings will be published. Tentatively scheduled as participants are Professors W. T. Tutte, Frank Harary, George Minty, Alan Hoffman, Gian-Carlo Rota, D. K. Ray-Chaudhuri, and D. R. Fulkerson. For fur­ ther information please write to Professor Bernard Harris, Mathematics Research Center, U. S. Army, University of Wiscon­ sin, Madison, Wisconsin 53706.

MEMORANDA TO MEMBERS

TAX EXEMPTION FOR BRITISH CITIZENS

The following quotation from the Taxes Branch of the Inland Revenue in London may be of interest to American Mathematical Society members who are assessable to income tax in Great Britain:
"I have to inform you that the Commissioners of Inland Revenue have ap­ proved American Mathematical Society for the purposes of Section 16, Finance Act, 1958, and that the whole of the annual sub­ scription paid by a member who qualifies for relief under that Section will be allow­ able as a deduction from his emoluments assessable to income tax under Schedule E. If any material relevant change in the circumstances of the society should occur in the future you are requested to notify this office.
"I should be glad if you would in­ form your members as soon as possible of the approval of the society. ...
"A member of the society who is entitled to the relief should apply to his tax office as soon as possible for form P358 on which to make a claim for the relief due to him."
Mixed boundary value problems in strips.

Method of artificial interfaces is applied to solve Laplace's equation, \( u_{xx} = -u_{yy} \), in rectangular strips (finite, semi-infinite, or infinite). Let strip be given by \( \{(x,y): a < x < b; 0 < b < \pi/2\} \) where \(-\infty < a < b < \infty\). For \( x > 0 \) separable linear boundary conditions are given and likewise for \( x < 0 \). Fourier series solutions are developed for \( x > 0 \) and \( x < 0 \). Using Szasz's theorem and a result of Voyutk and MacCamey (Proc. Amer. Math. Soc. 16 (1965), 276) the absolute and uniform convergence of the series is shown. Numerical results are given. The analysis here generalizes the interesting results obtained by Whiteman (Quart. J. Mech. Appl. Math 21 (1968), 41) using dual series. (Received September 27, 1968.)

661-18. TAKAYUKI TAMURA, University of California, Davis, California, and W. A. ETTERBEEK, Sacramento State College, Sacramento, California. Commutative archimedean semi-groups satisfying the divisibility chain condition.

Let \( S \) be a commutative archimedean cancellative idempotent-free semigroup. Then by Tamura [1] \( S \cong N \times G \) where \( N = \{0,1,2,...\} \) and the multiplication is defined by an \( I \)-function from \( G \times G \) into \( N \). If \( S \) also satisfies the divisibility chain condition (d.c.c.), then it is shown that this \( I \)-function in addition satisfies the following two equivalent conditions: (i) \( \forall a \in G \), \( I(a, a^{-1}) = 0 \); (ii) \( \forall a, \beta \in G, a \neq \beta \), either \( I(a, a^{-1}\beta) = 0 \) and \( I(\beta, \beta^{-1}a) = 1 \) or \( I(a, a^{-1}\beta) = 1 \) and \( I(\beta, \beta^{-1}a) = 0 \). For a nil-semigroup \( S \) with d.c.c. we construct a totally ordered abelian group \( G \) and show that \( S \cong (N \times G)/K \) where \( K \) is an ideal of \( N \times G \). A total order is defined on \( N \times G \) and either \( K = \{x \in N \times G: x > a\} \) or \( K = \{x \in N \times G: x \leq a\} \) where \( a \in N \times G \). Reference. [1] Commutative nonpotent archimedean semigroup with cancellation law, J. Gakugei, Tokushima Univ. 8 (1957), 5-11. (Received September 30, 1968.)


Two (dual) ways of characterizing finite-dimensional polytopes in terms of simplexes are extended to define (in terms of Choquet simplexes) two distinct classes of infinite-dimensional compact convex polytopes: a compact convex set \( K \) is an \( \alpha \)-polytope if there exist a simplex \( S \) and a continuous affine map \( \varphi \) of \( S \) onto \( K \) such that \( \varphi^{-1}(x) \) is finite dimensional for each \( x \) in \( K \). (This is equivalent to an earlier definition of Alfsen [Math. Scand. 15 (1964), 97-110].) The set \( K \) is a \( \beta \)-polytope if it is affinely homeomorphic to the intersection of some simplex and a flat of finite codimension. The finite-dimensional members of both classes are polytopes in the usual sense, but with a few exceptions, neither class is closed under the operations which preserve the class of finite-dimensional polytopes. A set \( K \) in either class, however, inherits two important properties of simplexes: (1) Any continuous real affine function on a face of \( K \) has a continuous extension to \( K \). (2) Any closed \( G_\delta \) face of \( K \) is a set of strict maximum for some affine continuous real function on \( K \). For \( \alpha \)-polytopes, (2) was proved by Asimow [Directed Banach spaces of affine functions, Trans. Amer. Soc. (to appear)]. (This paper to appear in Math. Scand.) (Received November 1, 1968.)
The January Meeting in New Orleans, Louisiana
January 23-26, 1969

663-1. R. J. DUFFIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, and
E. J. PETERSON, University of Michigan, Ann Arbor, Michigan 48104. The discrete analogue of a
class of entire functions.

Discrete analytic functions are complex-valued functions defined at points of the $z$-plane with
integer coordinates. The real and imaginary parts of these functions are required to satisfy difference
equations analogous to the Cauchy-Riemann equations. The pseudo power $z^{(n)}$ is a discrete analytic
function that is asymptotic to the ordinary power $z^n$ for large $z$. The central topic of this paper
(to appear in J. Math. Anal. Appl.) is the correspondence between the pseudo power series $f = \sum c_n z^{(n)}$
and the ordinary power series $F = \sum c_n z^n$. The coefficients are restricted by the relation,
\[ \limsup |n! c_n|^{1/n} < 2, \]
which insures that both series converge at all points. The correspondence defines a linear transformation $T$ such that $f = TF$. It is shown that $T$ can be expressed as a contour
integral and that $T$ has a unique inverse. By virtue of the transformation, various operations on the
entire function class $(F)$ induce corresponding operations on the discrete function class $(f)$. In partic­
ular a ring of discrete analytic functions is formed by defining the product of the functions $f$ and $g$ as $T(FG)$. (Received December 14, 1967.)

663-2. W. H. SUMMERS, Louisiana State University, Baton Rouge, Louisiana 70803.
A representation theorem for biequicontinuous completed tensor products of weighted spaces.

Weighted spaces of continuous functions have played a role in mathematics dating back at least
to the Bernstein approximation problem. In this paper, a simple representation of the biequicon tin u­
ous completed tensor product of two weighted spaces as another weighted space via a topological
isomorphism is obtained. The technique for obtaining this result entails an investigation of weighted
spaces as locally convex spaces, and several results of independent interest are proved. These
include characterizations of their topological duals, of a base for the equicontinuous subsets of these
duals, and of the extremal points of the members of this base, as well as a general criterion for com­
pleteness of weighted spaces. This representation theorem, among its several corollaries, includes
Grothendieck's result for $C_0(X) \hat{\otimes} C_0(Y)$, establishes an analogous result for one of the more inter­
esting weighted spaces $(C_b(X),\beta)$, namely, $(C_b(X),\beta) \hat{\otimes} (C_b(Y),\beta) = (C_b(X \times Y),\beta)$, and leads to the
discovery of some interesting new spaces of continuous functions on $X \times Y$. (Received June 7, 1968.)

663-3. G. T. CHARTRAND, Western Michigan University, Kalamazoo, Michigan 49001, and
H. V. KRONK, State University of New York, Binghamton, New York 13901. A special class of
hamiltonian graphs.

A graph $G$ is randomly hamiltonian from a point $v$ if the following procedure always results in a
hamiltonian cycle. Begin at the point $v$ and proceed to any adjacent point. On arriving at a point,
select any adjacent point not previously encountered. When no new points remain, then a line exists between the final point chosen and \( v \), and the procedure terminates. Denote by \( C_p \) and \( K_p \) the cycle and complete graph, respectively, with \( p \) points. Furthermore, let \( K(m,n) \) represent the complete bipartite graph on \( m \) and \( n \) points. We express a graph \( G \) as \( H + v \) provided \( G \) contains a point \( v \) adjacent to all other points of \( G \), where \( H \) then is the subgraph of \( G \) obtained by removing \( v \). **Theorem.** A graph \( G \) is randomly hamiltonian from a point \( v \) if and only if \( G \) is one of the following: \( C_p, K_p, K(p/2,p/2), C_{p-1} + v, K((p-1)/2, (p-1)/2) + v \). (Received June 13, 1968.)

663-4. L. S. KROLL, University of California, Davis, California 95616. The uniqueness of Hermite series under Poisson-Abel summability.

Let \( \sum_{n=0}^{\infty} a_n \alpha_n(x) \) be an arbitrary series of Hermite functions, not necessarily convergent. We suppose \( \sum_{n=0}^{\infty} a_n \alpha_n(x)r^n \) converges, for \( 0 \leq r < 1 \), to \( f(x,r) \) and that \( |f(x,r)| = o(1/(1-r)) \) uniformly in \( x \) as \( r \to 1 \). A theorem is proved giving conditions under which the series \( \sum_{n=0}^{\infty} a_n \alpha_n(x) \) is Poisson summable almost everywhere and is the Hermite series of its Poisson sum. **Theorem.** If \( \lim_{r \to 1} f(x,r) = 0 \) for all \( x \), then \( a_n = 0 \) for all \( n \). These results are, in a certain sense, best possible. (Received June 14, 1968.)

663-5. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. Elimination of the paradoxes in the set theory.

The Cantor's set theory has some paradoxes in itself. B. Russell attempted to eliminate them by placing restriction on the logic. However, his "step logic" is too inconvenient in the mathematics. The axiomatic set theory also is another attempt. It forbids us to construct sets, because the paradoxes appear when we construct some special sets. However, it is incomplete, as pointed out by K. Godel. I succeeded to eliminate the paradoxes by changing the concept of sets. A Cantor's set is considered a regulation by which we can grasp each element individually. A regulation is called a space if we can grasp each element individually and independently from others. For instance, all of the Dedekind's natural numbers form a Cantor's set but not a space. To define the natural number space, we need an axiom system like the Peano's axiom system. If we consider spaces instead of Cantor's sets, then all the paradoxes disappear. The detail is found in the book: H. Nakano, *Set theory* (to appear). (Received June 24, 1968.)

663-6. WITHDRAWN.

663-7. WITHDRAWN.
663-8. C. A. COPPIN, University of Dallas, Dallas, Texas 75061, and J. F. VANCE, St. Mary's University, San Antonio, Texas 78228. A property of an integral defined on a dense set.

The phrase, $f$ is $g$-integrable on $M$, and $\Delta$ are as in Abstract 656-11, these Notices 15 (1968), 505. Suppose $f$ is a real-valued function whose domain includes $M$, then $f_M$ is the real-valued function whose domain is $M$ and $f_M(x) = f(x)$ for each $x$ in $M$. Theorem. Suppose each of $f$ and $g$ is a real-valued function with domain $[a,b]$ and $M$ is a member of $\Delta$. Then the following two statements are equivalent: (1) $f$ is $g$-integrable on $M$ and $f_M$ and $g_M$ have no common points of discontinuity, and (2) If $M'$ is a countable member of $\Delta$ and a subset of $M$, then $f$ is $g$-integrable on $M'$. (Received July 2, 1968.)


We prove that if $0 < j < k$, then solutions of $(ry')' + jpy = 0$ oscillate slower than those of $(ry')' + kpy = 0$. For $p \equiv 0$, this is a corollary of the Sturm Comparison Theorem. We require no such condition. It follows that $O(p) = \{k \mid k \equiv 0, (ry')' + kpy = 0$ oscillates on $(0,\infty)\}$ is either empty or a ray. Examples are given to show that $O(p)$ may be $(0,\infty), [a,\infty), (a,\infty), (a > 0),$ or $\emptyset$. Two necessary and sufficient conditions that $O(p)$ be open are given. (Received July 8, 1968.)


M.-L. Dubreil-Jacotin's characterization of free semigroups is extended to free commutative semigroups as follows: Theorem. A commutative semigroup $S$ is a free commutative semigroup if and only if (i) for all $x,y \in S$, $xy \neq x$; (ii) if $a \in S$, then $a$ has only finitely many divisors; (iii) if $a,b,c,d \in S$ are such that $ab = cd$, then there exist $u,v,w,z \in S$ such that $a = uw$, $b = vz$, $c = uv$, $d = wz$. A consequence of that theorem is that, if $\mathcal{D}$ is any family of endomorphisms of a free commutative semigroup $S$, then the set of all elements of $S$ which are invariant under every endomorphism of $\mathcal{D}$ is either empty or a free commutative semigroup. If now $S$ is a free commutative semigroup with identity, the latter result need not hold true, but does when $\mathcal{D}$ consists only of one idempotent endomorphism. It follows that a semigroup which is projective in the category of all commutative semigroups, or of all commutative semigroups with identity, is free, or free with identity. (Received July 11, 1968.)


Let $k$ be a given field and let $k[t]$ be the field of formal power series $\lambda = a_m t^m + a_{m-1} t^{m-1} + \ldots + a_0 + a_{-1} t^{-1} + \ldots$ with coefficients in $k$ and $m$ an arbitrary integer. Let $k[t]$ be the ring of polynomials in $t$ with coefficients in $k$. Define a valuation (absolute value) on $k[t]$ by specifying $|\lambda| = e^m$, provided $a_m \neq 0$. For any field $k$, there is an analogue, namely whether for each pair $\theta, \phi$ of elements of $k[t]$ there exist $q, r$ and nonzero $p$ in $k[t]$ such that $|p| \geq |p\theta - q| \geq |p\phi - r|$ is arbitrarily small, of Littlewood's well-known diophantine approximation problem. This analogue problem was solved by Davenport and Lewis (Michigan Math. J. 10 (1963), 157-160) in the case where $k$ is an
infinite field of constants. They proved that there exist \( \theta, \varphi \in k[t] \) such that \( |p| |p \theta - \varphi| |p \varphi - r| \geq e^{-2} \) for all nonzero \( p \in k[t] \). The paper summarized here proves the following (with the obvious definition of the element \( e^{a/t} \) of \( k(t) \)): Theorem. Given an integer \( n \geq 2 \) and distinct nonzero elements \( a_1, a_2, \ldots, a_n \) of \( k \). Then \( |p| \prod_{i=1}^{n} |p \exp(a_i/t) - q_i| \geq e^{-n} \) for all \( q_1, q_2, \ldots, q_n \) and nonzero \( p \in k[t] \). A. Baker (Michigan Math. J, 11(1964), 247-250) proved the theorem with the weaker constant \( \exp((-1/2)(n^3 + n)) \) in place of \( e^{-n} \) (which is clearly best possible). (Received July 11, 1968.)


In connection with a problem in simultaneous diophantine approximation, Davenport ["A note on diophantine approximation" in Studies in mathematical analysis and related topics, Stanford Univ. Press, Stanford, Calif., 1962, pp. 77-81] asked whether, given any finite set of lines and hyperplanes through 0 in \((k + 1)\)-dimensional space, there was a lattice which would have no point (except 0) in a collection of tubes and layers about the lines and hyperplanes, respectively. An affirmative answer is given by the following Theorem. Let \( r \) lines \( L_q (q = 1, \ldots, r) \), defined by the equations \( x_j - \lambda(j) x_0 = 0 \) \((j = 1, \ldots, k)\), and \( s \) planes \( P_t \), defined by \( x_0 + \mu(1)_t x_1 + \ldots + \mu(k)_t x_k = 0 \) \((t = 1, \ldots, s)\), be given in \((k + 1)\)-dimensional space. Then there exists a \((k + 1)\)-dimensional lattice \( \Lambda \) and a positive number \( \rho \) such that every point \((x_0, \ldots, x_k) \) of \( \Lambda \) other than the origin satisfies (1) \( \max_j |x_j - \lambda(j) x_0| > \rho \min_j |x_j|^{-1/k} \) \((q = 1, \ldots, r)\) and (2) \( |x_0 + \mu(1)_t x_1 + \ldots + \mu(k)_t x_k| > \rho(1 + \max_j |x_j|)^{-1} \) \((t = 1, \ldots, s)\). In fact, the set of all such lattices has the cardinal of the continuum. The proof combines the method of Davenport with the classical results of Minkowski and Mahler. (Received July 12, 1968.)

663-13. W. W. ADAMS, University of California, Berkeley, California 94720. Asymptotic diophantine approximations to a basis of a real cubic number field.

The following theorem is proved. Theorem. Let \( 1, \beta_1, \beta_2 \) be a basis of a real cubic number field. Let \( C > 0 \) be a given constant. Let \( \lambda(B) \) equal the number of solutions in integers \( q, p_1, p_2 \) of the inequalities \( 0 < q \beta_1 - p_1 < C/q^{1/2} \), \( 0 < q \beta_2 - p_2 < C/q^{1/2} \) and \( 1 \leq q \leq B \). Then either \( \lambda(B) = 0(1) \) or there is a constant \( C' > 0 \) such that \( \lambda(B) \sim C' \log B \) \((B \to \infty)\). The theorem is proved by first reducing the problem to counting units in the number field, and then showing that this can be done by applying the uniform distribution theory. (Received July 15, 1968.)


\[ (\lambda + Ay)x = y \]
by using the continued fraction
\[ \frac{1}{\lambda + Ay} = c_0 \frac{1}{1 + c_1 \lambda} + c_0 \frac{1}{1 + c_1 \lambda} + c_1 \lambda + c_2 \lambda + c_3 \lambda + \ldots \]
where \( \lambda = A^ny \) as a formal correspondence for \( x \). Brown [Analysis of a new formalism in perturbation theory, Proc. Nat. Acad. Sci. U.S.A. 50 (1963)] obtains a convergence property for this method assuming that \( A^n x = A^n x \) equal \( c_n \) for some index \( n \). The author shows that the above method is equivalent to the Neumann series, \(-1/\lambda \sum_{n=0}^{\infty} (A^\lambda)^n y\), formal correspondence to \( x \) in the sense that whenever the \( n \)th convergent of the continued fraction exists it is the same operator as the \( n \)th partial
sum of the series. Hence, the convergence properties of this method are now determined. (Received July 16, 1968.)


For any monotone decomposition G of $E^n$, let $G'$ denote the monotone decomposition of $E^{n+1} = E^n \times E^1$, obtained from G by adding all the points of $E^{n+1} - E^n$. Our main result is the following Theorem. If G is a monotone decomposition of $E^n$ such that $E^n/G$ is homeomorphic to $E^n$, then $E^n/G$ can be obtained by a pseudo-isotopy if and only if $E^{n+1}/G'$ is homeomorphic to $E^{n+1}$. The monotone decomposition B of $E^3$ to points, circles and figure-eights, due to R. H. Bing, has the property that $E^3/B$ is homeomorphic to $E^3$, but $E^3/B$ cannot be obtained by a pseudo-isotopy; hence: Corollary. $E^4/B'$ is not homeomorphic to $E^4$. Another proof is given to this corollary, which was also proved by M. M. Cohen (Corollary 10.3 in Simplicial structures and transverse cellularity, Ann. of Math, 85 (1967), 218-245). (Received July 16, 1968.)

663-16. R. E. WILLIAMS, Kansas State University, Manhattan, Kansas 66502. Free products and the dependence number of rings.

Let $(R_n)$ be a family of associative $K$-rings, $K$ a division ring, such that $R_n \cap R_\beta = K$ for $\alpha \neq \beta$. Using some definitions and techniques of [P. M. Cohn, Some remarks on the invariant basis property, Topology 5 (1966), 215-228; and P. M. Cohn, Free products of associative rings, III, J. Algebra 8 (1968), 376-383] it is possible to prove: Theorem 1. If $P$ is the free product of $(R_n)$ over $K$, then $\lambda(P) = \min \{\lambda(R_n)\}$. Also, with a rather different approach there is: Theorem 2. If each $R_n$ in $(R_n)$ is a ring with a weak algorithm, then $P$ also has the weak algorithm. These theorems have a number of corollaries, some of which shall be presented if time permits. (Received October 2, 1968.)

663-17. A. M. FINK, Iowa State University, Ames, Iowa 50010, and D. F. ST. MARY, University of Massachusetts, Amherst, Massachusetts 01002. On an inequality of Nehari.

Z. Nehari (Studies in mathematical analysis and related topics, Stanford Univ. Press, Stanford, Calif., 1962) states that if $y$ is a nontrivial solution of $y^{(n)} + p_n y^{(n-1)} + \ldots + p_1 y = 0$ having n zeros on $[a,b]$, then $\sum_{k=1}^{n} \frac{b}{2}(b-a)^{n-k} \int_{a}^{b} |p_{k}| > 2^{n+1}$. In a private communication with the authors Professor Nehari has indicated that the inequality is undecided since the argument given in Studies ... is invalid. It is shown here that in the case $n = 2$ Nehari's inequality is correct. In fact, we prove a stronger result. Theorem. Let $a$ and $b$ be successive zeros of a nontrivial solution of $y'' + gy' + fy = 0$, where $f$ and $g$ are integrable. Then $b - a$ $\int_{a}^{b} (x)dx - 4 \exp (-1/2) \int_{a}^{b} |g(x)|dx > 0$ and a fortiori $b - a$ $\int_{a}^{b} (x)dx + \int_{a}^{b} |g(x)|dx > 4$. (Received July 22, 1968.)

663-18. H. M. KAMOWITZ, University of Massachusetts, Boston, Massachusetts 02116, and STEPHEN SCHEINBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The spectrum of automorphisms of Banach algebras.

Theorem. Let $T$ be an automorphism of a semisimple commutative Banach algebra. Then
either (A) $T^N = I$ for some $N$, in which case the spectrum of $T$ is a finite union of subgroups of the circle, or (B) the spectrum of $T$ contains the unit circle. (Received July 25, 1968.)


In 1948 Hille (Trans. Amer. Math. Soc.) established criteria for nonoscillation for large $x$ of the scalar differential equation $y'' + f(x) y = 0$, $0 < x < \infty$, by means of a nonlinear (Riccati) integral equation. Subsequently, Sternberg (Duke Math. J. (1952)) extended certain of Hille's results to differential equations of the form $y' = G(x)z$, $z' = -F(x)y$, with $G(x)$ and $F(x)$ being $n \times n$ nonnegative definite matrices on $(0, \infty)$. Under the assumption of nonsingularity of $G$, Sternberg obtained criteria for nonoscillation for large $x$ by means of an analogous Riccati integral equation. It is shown in the present paper that a generalization of Hille's proof may be used to obtain nonoscillation (disconjugacy) criteria without the hypothesis of nonsingularity of $G$. (Received July 29, 1968.)


Noether lattices were introduced by R. P. Dilworth as abstractions of the lattices of ideals of Noetherian rings [Abstract commutative ideal theory, Pacific J. Math. 12 (1962), 481-498]. This paper describes the structure of distributive local Noether lattices. In Structure theorems for regular local Noether lattices [Michigan Math. J. 15 (1968), 167-176], the author showed that a distributive regular local Noether lattice of dimension $n$ is isomorphic to $RL_n$, the sublattice of the lattice of ideals of $F[x_1, \ldots, x_n]$ generated by the ideals $(x_1), \ldots, (x_n)$ under the operations of join and multiplication. In this paper it is shown that a Noether lattice $L$ is a distributive local Noether lattice if and only if it is isomorphic to $RL_n/\theta$ where $\theta$ is an equivalence relation which is an extension of an equivalence relation on the proper principal elements of $L$, and $\theta$ preserves join, multiplication, and cancellation of principal elements in nonzero products. Some examples illustrating this theorem are given. (Received July 26, 1968.)

663-21. A. W. GOODMAN, University of South Florida, Tampa, Florida 33620. The valence of certain means.

Let $V(1)$ be the set of all functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ that are regular and univalent in $E$, the unit disk. Let $\alpha, \beta$ be any two positive numbers such that $\alpha + \beta = 1$, and suppose further that $\alpha \beta < e^\pi/(1 + e^\pi) \approx 0.958$. Theorem 1. Under these conditions there exist $f(z)$ and $g(z)$ in $V(1)$ such that $F(z) = \alpha f(z) + \beta g(z)$ has infinite valence in $E$. Theorem 2. Under the same conditions there exist $f(z)$ and $g(z)$ in $V(1)$ such that $G(z) = f^\alpha(z)g^\beta(z)$ has infinite valence in $E$. (Received August 5, 1968.)


Let $q$ be the periodic quintic spline associated with a periodic function $f \in C^6[a, b]$ and the uniform mesh $\pi$. Then $\|q - f\|_E \leq \epsilon \|f - f(\pi)\|_h^b$, $0 \leq r \leq 5$, where $\|g\| = \max \{|g(x)| : a \leq x \leq b\}$, $\epsilon =$
Further, if \( f \in C^r[a,b] \), then at each mesh point \( x_i \),
\[
|q^{(1)}(x_i) - f^{(1)}(x_i)| \leq (0.04366) \|f^{(7)}\| h^6 = O(h^6)
\]
and
\[
|q^{(3)}(x_i) - f^{(3)}(x_i)| \leq (0.54961) \|f^{(7)}\| h^4 = O(h^4).
\]

(Received August 5, 1968.)


Let \( \mathcal{R} \) be a rectangular polygon and \( \pi \) be a rectangular mesh containing each corner of \( \mathcal{R} \) as a mesh point. If \( \mathcal{P}^n(\mathcal{R},\pi) \) is the space of piecewise polynomials \( p \) such that, in each rectangular element of \( \mathcal{R} \), \( p(x,y) \) is a polynomial of degree (at most) \( 2n-1 \) in both \( x \) and \( y \), then the smooth Hermite space \( H^n(\mathcal{R},\pi) = \mathcal{P}^n(\mathcal{R},\pi) \cap C^{n-1}[\mathcal{R}] \). Define the subspaces \( S^n_k(\mathcal{R},\pi) \) of \( H^n(\mathcal{R},\pi) \) by
\[
S^n_k(\mathcal{R},\pi) = H^n(\mathcal{R},\pi) \cap C^{n-1+k}[\mathcal{R}].
\]
The functions in each subspace are characterized by their interpolation properties as follows: Theorem. Assume the values \( (f^{(r,s)}): 0 \leq r, s \leq n-1 \) are given at each mesh point \((x_i, y_j) \in \pi\). For each \( k, 0 \leq k \leq n-1 \), there exists a unique \( p_k \in S^n_k(\mathcal{R},\pi) \) such that
\[
p_k(x_i, y_j) = f^{(r,s)}(x_i, y_j)
\]
where (i) \( 0 \leq r, s \leq n-1-k \) at each interior mesh point and re-entrant corner,
(ii) \( 0 \leq r \leq n-1, 0 \leq s \leq n-1-k \) at each (non-re-entrant corner) mesh point on a vertical boundary segment of \( \mathcal{R} \),
(iii) \( 0 \leq r \leq n-1-k, 0 \leq s \leq n-1 \) at each (non-re-entrant corner) mesh point on a horizontal boundary segment of \( \mathcal{R} \), and (iv) \( n-k \leq r, s \leq n-1 \) at four "suitably chosen" corners of \( \mathcal{R} \).

(Received August 5, 1968.)


Definition. Let \( \U \) be a quasi-uniformity on a set \( X \) and let \( x \in X \). Then \( \U \) is pre-symmetric at \( x \) provided if \( V \in \U \) then there exists \( W \in \U \) with \( W \circ W \subseteq V \) and \( p \in X \) such that \( x, z \in V(p) \) whenever \( W[x] \cap W[z] \neq \emptyset \). A quasi-uniformity \( \U \) on a set \( X \) is pre-symmetric if \( \U \) is pre-symmetric at each \( x \in X \). Theorem. Let \((X,\U)\) be a Hausdorff quasi-uniform space. If \( \U \) is complete and pre-symmetric, then \((X,\U)\) is regular. If \( \U \) is complete pre-symmetric and has a countable base, then \((X,\U)\) is homeomorphic to a metric space. (Received August 7, 1968.)


Let \( X = \Pi_{n=1}^\infty X_n \) be the product of compact metrizable topological groups (not necessarily Abelian), and denote by \( \mu \) the Haar measure on \( X \). For each \( n \), let \( f_n \) be a coordinate function of \( V_n \), where \( V_n \) is a continuous unitary irreducible representation of \( X_n \). For \( x = (x_1, x_2, \ldots) \in X \), define \( f_n(x) \) to be \( f_n(x_n) \). A subset \( C \) of \( X \) is called a set of uniqueness with respect to a regular method \( S \) of summability, or a \( U_S \)-set, if whenever a series \( \sum_{n=1}^\infty c_n f_n \) with \( c_n \in C \), is \( S \)-summable to zero on \( X \setminus C \) then each \( c_n = 0 \). Otherwise, \( C \) is a set of multiplicity. Let \( d_n \) be the dimension of the representation space of \( V_n \), and set \( M = \sup_n \{d_n : n \geq 1\} \). It is proved that if \( M < \infty \) and \( \mu(C) < 1 - 1/2M \), then \( C \) is a \( U_S \)-set. If \( M = \infty \), then every subset of \( X \) of measure zero is a \( U_S \)-set. It is also demonstrated that if \( M < \infty \) and the series \( \sum_{n=1}^\infty c_n f_n \) is \( S \)-summable to zero on a set of positive measure, then there exists a positive integer \( N \) such that \( c_n = 0 \) for every \( n \geq N \). (Received August 8, 1968.)
It is proved that \( \bigcap f[x - y : f(x) = f(y)] = [0, 1/2, 1/3, \ldots] \) where \( f \) varies over all real-valued continuous functions on the interval \([0, 1]\) with \( f(0) = f(1) \). This is then applied to give an elementary proof of the fact that a holomorphic function is not locally invertible at a point where its derivative vanishes. (Received August 8, 1968.)

Preliminary report.

Let \( S \) be a nonempty set, \( n \) an integer \( \geq 2 \) and \( j(1), j(2), \ldots, j(n) \) a sequence of (necessarily distinct) integers where \( 1 \leq j(i) \leq n \). A shuffling operation (s-operation), \( (\cdot) \), on \( S^n = S \times S \times \ldots \times S \) is defined as follows: \( (x_1, \ldots, x_n) \cdot (y_1, \ldots, y_n) = (x_{j(1)}, \ldots, x_{j(n)}) = (y_{j(1)}, \ldots, y_{j(n)}) \) for all \( (x_1, \ldots, x_n) \), \( (y_1, \ldots, y_n) \) in \( S^n \), where each \( i \) is a fixed element either equal to \( x_{j(i)} \) or \( y_{j(i)} \). The groupoid \((S^n, \cdot)\) is called a point algebra. An s-operation, \( (\cdot) \), is called a generating operation if for each other s-operation, \( (\cdot) \), on \( S^n \), there is a polynomial \( \theta \) in \( (x_1, \ldots, x_n), (y_1, \ldots, y_n) \) such that \( (x_1, \ldots, x_n) \cdot (y_1, \ldots, y_n) = \theta((x_1, \ldots, x_n), (y_1, \ldots, y_n)) \). For a prime, necessary and sufficient conditions are given in order for an s-operation to be a generating operation. Also some classes of point algebras are shown to be rectangular bands and some to be unions of disjoint constant semigroups. (Received August 13, 1968.)

Let \( N \) be a \( C^q \)-manifold modelled on a real Hilbert space \( Y \), \( q \geq 1 \). A theory of cotypes of subsets of \( N \) is constructed. If \( S \subseteq N \), cotype \( S \) is a nonnegative integer or \( \infty \). The larger cotype \( S \), the smaller is \( S \). If \( N \) is infinite dimensional and cotype \( S < \infty \), then \( S \) is still a large set. Let \( M \) be another \( C^q \)-manifold modelled on a real Banach space. Suppose that \( f \) is a \( C^q \) Fredholm map of \( M \) into \( N \) of constant and nonpositive index \(-j\). Let \( A \) be the set of points \( x \in M \) at which the tangent map \( Df(x) \) fails to be injective. Theorem. cotype \( fA \geq j + 1 \). And if one point \( x_0 \) exists at which \( Df(x_0) \) is injective, then cotype \( fM = j \). The following fundamental lemma is proved and used. If \( \gamma_x \) is a \( C^q \) diffeomorph of an \( r \)-coball in \( Y \) and if \( Z \) is a subspace of \( Y \) of codimension \( s \leq r \), then at least one cross section of \( \gamma_x \) normal to \( Z \) contains a diffeomorph of an \((s - r)\)-ball. (Received August 13, 1968.)

Consider the second order linear equation \( y'' + q(t)y = 0 \). Theorem. If \( q(t) \) is positive and nondecreasing on \([0, \infty)\), then every solution of \( y'' + q(t)y = 0 \) is bounded. The function \( q(t) \) is not necessarily continuous. This theorem generalizes the well-known result of Osgood and Biernacki. The theorem also admits several generalizations to nonlinear equations of a general nature. (Received August 16, 1968.)
A subset $M$ of a Moore space $S$ is said to be $m$-dense in $S$ with respect to a development $G_1, G_2, \ldots$ of $S$ if and only if $m$ is a positive integer with the property that for each point $P$ in $S$ and each positive integer $n$ there are regions $R_1, \ldots, R_m$ in $G_n$ such that $P$ is contained in $R_1$, $R_1$ intersects $R_{i+1}$ for $1 \leq i \leq m - 1$ whenever $m > 1$, and $R_m$ intersects $M$. J. N. Younglove [Fund. Math. 48 (1960), 15-25] proved that each complete Moore space contains a dense, metrizable, inner limiting subset.

**Theorem 1.** If $S$ is a Moore space, then for each development $G_1, G_2, \ldots$ for the space there is a metrizable, inner limiting set $M$ which is $3$-dense ($2$-dense) in $S$ with respect to the development. Moreover, $S$ has a development which satisfies Axiom C at each point of $M$ (if $S$ is normal).

B. Fitzpatrick [Fund. Math. 61 (1967), 91-92] has shown that a normal Moore space which is not a counterexample of Type D has a dense, metrizable subset.

**Theorem 2.** If $S$ is a locally connected, normal, Moore space which has a base $G$ with the property that $S$ is collectionwise normal with respect to each discrete subset that is contained in the boundary of some element of $G$, then $S$ contains a dense metrizable subset. (Received August 5, 1968.)

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**663-31.** FRANK STENGER, University of Michigan, Ann Arbor, Michigan 48104. The asymptotic approximation of certain integrals.

A workable definition employing approximation theory is given for obtaining an asymptotic approximation of certain integrals. This enables the use of quadrature methods for obtaining asymptotic approximations. One result is the following. Let the $r$th modulus of continuity of $f$ on the real axis $R$ satisfy $\omega_f^r(x; R; \delta) = O(\delta^n)$ for $0 < s \leq s_0$, where $a > 0$. Let $x(t) \neq 0$, and let $\int_R |f(kx(t))| dt < \infty$ for $k = 0, 1, \ldots, d$, where $d \leq r$. Let $t_j$ and $w_j$ ($j = 1, 2, \ldots, m$) be chosen such that $\sum_{j=1}^{m} w_j t_j = \int_R x(t) dt$ for $k = 0, 1, \ldots, n - 1$, where $n \leq r$. Then, for $\int_{R} f(t)x(t) \lambda(t) dt - \sum_{j=1}^{m} w_j f(t_j) / \lambda$ is $O(\lambda^{-n})$, $O(\lambda^{-\alpha} \log \lambda)$, $O(\lambda^{-\alpha})$ as $\lambda \to \infty$ when $n < \alpha$, $n = \alpha$ or $n > \alpha$ respectively. Error bounds can also be easily obtained. For example, if $f''(t)$ is bounded on $[0, \infty)$, then $\int_{0}^{\infty} e^{-\lambda t} f(t) dt = [f(1/\lambda) + \epsilon] / \lambda$ where $|\epsilon| \leq \sup \{|f''(t)| : 0 \leq t \leq \infty\}/(2 \lambda^2)$. (Received August 13, 1968.)

**663-32.** JEROME EISENFELD, Rensselaer Polytechnic Institute, Troy, New York 12181. A completeness theorem for an integrodifferential operator.

In order to show that the eigenvectors of an integrodifferential operator $T$ are complete we prove and apply the following result. **Theorem.** If the spectrum of the normal or symmetric operator $T$ in a Hilbert space $H$ is countable, then there is an orthonormal basis for $H$ consisting of eigenvectors of $T$. The completeness theorem is needed in the verification of Schwarzschild's criterion for the stability of gaseous masses. (Received August 19, 1968.)

**663-33.** M. S. T. NAMBOODIRI, Wisconsin State University, Superior, Wisconsin. General selfadjoint differential equations.

Some oscillatory properties of solutions of the differential equation $Ly = (p_n(x)y^{(n)}(x))^{(n)} + \sum_{i=0}^{n-1} (-1)^{n+i+1} p_i(x)y^{(i)}(x) = 0$, where $p_n(x) > 0$, and $p_i(x) \in C[0, \infty)$, for $i = 0, 1, \ldots, n - 1$, are con-
considered by transforming \( Ly = 0 \) to a matrix system \( Y' = E(x)Z \), \( Z' = -F(x)Y \). The operator \( L \) is nonoscillatory on \([0, \infty)\) if and only if \( Y(x) \) is nonsingular on \([0, \infty)\). Comparison theorems are also obtained using the techniques developed by J. H. Barrett [Canad. J. Math. 13 (1961), 625-638]. It is assumed here that the \( p_i(x) \equiv 0 \). Sufficient conditions for oscillation and nonoscillation involving the coefficient functions \( p_i(x) \) \( (i = 0, 1, \ldots, n - 1) \) are also established. These results generalize those of Don B. Hinton [Proc. Amer. Math. Soc. (3) 19 (1968)]. The author obtained these results before Hinton's paper was published. No restrictions on the signs of \( p_i(x) \) \( (i = 0, 1, 2, \ldots, n - 1) \) are made here. Finally, the connection between the existence of conjugate points in the sense of Barrett [Canad. J. Math. 13 (1961), 625-638] and the oscillatory behavior of \( L \) is established. (Received August 19, 1968.)

663-34. R. E. BECK, Villanova University, Villanova, Pennsylvania 19085. On counting connections on Lie groups.

There are always two left invariant connections on a Lie group which have (a) zero curvature and (b) maximal geodesics through the identity as one-parameter subgroups; these are the + and connections of Cartan. It is shown that in the case the Lie group is simple, these are the only two connections satisfying properties (a) and (b). The proof uses the computational representation theory of Lie algebras, including the Theorem. If \((p, V)\) is an irreducible representation of a complex simple Lie algebra \( g \) such that \( \dim V = \dim g \), then \( p \) is equivalent to the adjoint representation. Examples are given to show that the + and - connections are not unique on other kinds of Lie groups. (Received August 19, 1968.)


A topological space \( X \) has the point-countable property if every open cover of \( X \) has a point-countable open refinement. This paper is a study of the effect of certain separability conditions on spaces with the point-countable property. Main Theorem. Let \( X \) be a topological space with the point-countable property which is either locally separable or locally connected and locally peripherally separable. Then \( X \) has the star-countable property. If in addition \( X \) is \( T_3 \), then \( X \) has the star-finite property. Generalizations of several well-known theorems are derived from this result. Finally, an example is given of a normal space with the point-countable property which is neither pointwise paracompact nor screenable. (Received August 19, 1968.)


Let \( f \) be an arbitrary (not necessarily additive) function defined and finite on sets \( T \) from a family \( \mathcal{F} \) of Euclidean \( n \)-space such that \( |T|_1 = |\overline{T}|_1 \), where \( |T|_1 \) denotes Lebesgue measure and \( \overline{T} \) the closure of \( T \). Define the upper derivative \( \overline{D}(f, a, x) = \lim_{\text{diam}(T) \to 0; r(T) > a} f(T) / |T|_1 \), where \( r(T) \) is the parameter of regularity of \( T \). Replace \( \lim \) by \( \liminf \) to define \( \underline{D}(f, a, x) \) and let \( S_a \) be the set on which \( \overline{D} \) and \( \underline{D} \) are defined. The following theorem is proved. Theorem. Let \( E \) be a measurable with \( |E| < + \infty \) and assume \( |E \cdot S_a| = 0 \). A necessary and sufficient condition that a unique, finite derivative exists a.e. on \( E \) is that for any \( \varepsilon > 0 \) and \( \Delta > 0 \), there exist \( \eta > 0 \) and \( \zeta > 0 \) such that if \( \mathcal{D} \) and \( \mathcal{V} \) are finite
collections of disjoint sets $T$ from $\mathcal{S}$ such that $r(T) > a$, $\text{diam}(T) < \xi$, $|\hat{\delta} - E| < \eta$, $|\check{\nu} - E| < \eta$, and $|\check{\nu} \setminus \hat{\kappa}| < \eta$, then there exist subcollections $\mathcal{S}'$ and $\mathcal{K}'$ of $\mathcal{K}$ and $\mathcal{K}$ such that $|\check{\nu} \setminus \hat{\kappa}'| < \Delta$, $|\check{\nu} - \hat{\kappa}'| < \Delta$, and $|\sum_{T \in \mathcal{S}} f(T) - \sum_{T \in \mathcal{S}'} f(T)| < \epsilon$, where $\widetilde{\mathcal{U}}$ denotes $\cup_{T \in \mathcal{S}} T$. This result represents part of a general theory of the differentiability of set functions as contained in the doctoral thesis of the first author, completed at the University of Tennessee, 1967, under the direction of the second author. (Research sponsored by the U.S. Atomic Energy Commission under contract with Union Carbide Corporation.) (Received August 19, 1968.)


Let $\Gamma$ be a finite, loopless, undirected graph and $G$ any subgroup of the automorphism group of $\Gamma$. The orbit complex $\Gamma/G$ of $\Gamma$ with respect to $G$ is defined. Relationships between various graph theoretic properties of $\Gamma$ and $\Gamma/G$ are investigated. In particular, extremum problems concerning internal stability numbers, external stability numbers, cyclomatic numbers, and chromatic numbers are considered. (Received August 19, 1968.)


In this paper a channel is identified, in a natural way, with a subset (the fundamental measures) of the normed linear space of all totally finite signed measures on a given measurable space (the channel output space). With this consideration it is natural to define the hull of a channel to be the convex hull of the fundamental measures. The hull is said to be generated by the fundamental measures. It is shown that any channel which generates the hull has the same capacity as the hull itself. Finally, it is shown that the coding theorem and the strong converse hold for the hull if and only if they hold for any channel which generates the hull. This immediately proves the coding theorem and the converse for any channel whose hull is finitely generated. (Received August 19, 1968.)


Let a QRSP, $(p \cdot q)^n$, be any unbounded surface which has regular skew faces $[p]$ and $[q]$, each spanned by a minimal surface, with cyclic and equiangiular plane vertex figures [cf. (i) H. S. M. Coxeter, Regular polytopes $n = n$ = the number of $[p]$ and also of $[q]$ at each vertex. Of the 17 known examples of quasi-regular ["flat"] polyhedra $(p^* \cdot q^*)^n$ [(ii) H. S. M. Coxeter et al, Philos. Trans. Roy. Soc. London A 246 (1954), 401-450], 9 include equatorial faces (and are truncations of regular saddle polyhedra [(iii) Abstract 68T-D6, these Notices 15 (1968), 929)]. To each of the 8 other $(p^* \cdot q^*)^n$, there corresponds a $(p' \cdot q')^n$ with a similar vertex figure $-(3 \cdot 3)^2 : (\hat{6} \cdot \hat{6})^2 \equiv [\hat{6}, 4]_3 (iii)$; $(3 \cdot 4)^2 : (\hat{6} \cdot \hat{4})^2; (3 \cdot 5)^m : (\hat{6} \cdot \hat{0}/3)^m; (3 \cdot 5/2)^m : (\hat{6} \cdot \hat{10})^m; (5 \cdot 5/2)^m : ((\hat{10}/3) \cdot \hat{10})^m (m = 2, 3), The equatorial polygons (i) of $(\hat{6}, 4)^2$, $(\hat{6} \cdot \hat{10}/3)^2$, $(\hat{6}, \hat{10})^2$, and $(\hat{10}/3, \hat{10})^2$ are the regular compounds $(2) (3), (2) (5/2), (2) (5),$ and $(2) (3)$, respectively. Each edge of a $(p' \cdot q')^n$ joins 2 vertices lying in opposite faces of the corresponding $(p' \cdot q')^n$, and every face of a $[p] \cdot [q]$ passes through the center of the $(p' \cdot q')^n$. $(\hat{6}, \hat{4})^2$, the only $(p' \cdot q')^n$ for which $N_2$ (no. of faces) $\neq N_2$ in the corresponding $(p' \cdot q')^n$,

A torsion theory \((\mathcal{S}, \mathcal{Z})\) in the category of left \(R\)-modules over a ring \(R\) (with unit) is called

(i) hereditary provided \(\mathcal{S}\) is closed under submodules, (ii) splitting provided the torsion submodule is a direct summand of every module, and (iii) centrally splitting provided \(\mathcal{S} = \{M|1 - \epsilon M = M\}\) and \(\mathcal{Z} = \{N|\epsilon N = N\}\) for some central idempotent \(\epsilon\) of \(R\). If \(R\) is a semiperfect ring, \((\mathcal{S}, \mathcal{Z})\) is called principal provided \(Re \in \mathcal{S}\) (respectively \(\mathcal{Z}\)) if and only if \(Re/Je \in \mathcal{S}\) (respectively \(\mathcal{Z}\)) for every primitive idempotent \(e\) of \(R\).

Theorem I. Let \(R\) be a semiperfect ring, and let \((\mathcal{S}, \mathcal{Z})\) be a hereditary theory. If \(\mathcal{S}\) is closed under arbitrary direct products, then \((\mathcal{S}, \mathcal{Z})\) is principal if and only if \((\mathcal{S}, \mathcal{Z})\) is centrally splitting. Theorem II. If \(R\) is a quasi-Frobenius ring and if \((\mathcal{S}, \mathcal{Z})\) is a hereditary torsion theory, then the following are equivalent: (a) \(\mathcal{S}\) is closed under injective envelopes; (b) \((\mathcal{S}, \mathcal{Z})\) is principal; (c) \((\mathcal{S}, \mathcal{Z})\) is centrally splitting; (d) \((\mathcal{S}, \mathcal{Z})\) is splitting. (Received August 26, 1968.)

663-41. G. E. KELLER, University of Minnesota, Minneapolis, Minnesota 55455, and LEONARD SCOTT, University of Chicago, Chicago, Illinois. Concerning the degrees of irreducible characters.

Let \(G\) be a finite group with irreducible characters \(\chi_i\) for \(1 \leq i \leq \ell\). Let \(H\) and \(K\) be subgroups of \(G\) with linear characters \(\lambda\) and \(\mu\) respectively. Let \(\lambda|G = \sum_{i=1}^{\ell} d_i \chi_i\) and \(\mu|G = \sum_{i=1}^{\ell} e_i \chi_i\). Let \(y_1, \ldots, y_k\) be \([H, K]\) double coset representatives for those \([H, K]\)-double cosets which contain elements \(z\) for which \(\lambda_1^{z\mathbb{Z}_K} = \mu_1^{z\mathbb{Z}_K}\). If \(D_1\) is the diagonal matrix which has entries \(\chi_i(1)\) with multiplicity \(d_i\) for \(1 \leq i \leq \ell\) and \(D_2\) is the diagonal matrix with entries \([G:H]^{y_j\mathbb{Z}_K}\) for \(1 \leq j \leq k\), then the invariant factors of \(D_1\) divide the invariant factors of \(D_2\). In particular \(\prod_{j=1}^{k} [G:H]^{y_j \mathbb{Z}_K} / \prod_{i=1}^{\ell} \chi_i(1)^{e_i d_i}\) is an integer. (Received August 23, 1968.)

663-42. DAVID SACHS, Wright State University, Dayton, Ohio 45431. Graphs, matroids and geometric lattices. Preliminary report.

Hassler Whitney (Amer. J. Math. 54 (1932), 150-168) has shown that two triply-connected finite graphs are isomorphic if their circuit structures are isomorphic (in modern terminology, their associated matroids are isomorphic). By using the geometric lattices associated with the matroids, we give a new and rather simple proof of Whitney's result. The proof suggests a way of characterizing the geometric geometric lattices, and necessary and sufficient conditions that a geometric lattice be graphic are given. The results can be extended rather easily to infinite graphs as well. (Received August 22, 1968.)

663-43. F. W. HARTMANN, Villanova University, Villanova, Pennsylvania 19085. Inclusion theorems for Sonnenschein matrices.

Let \(f\) be a function which is analytic for \(z \in D_1 = \{z : |z| < R\}\), \(R > 1\), \(f(1) = 1\), and \(A(f) = (a_{nk})\) be defined by \([f(z)]^n = \sum_{k=0}^{\infty} a_{nk} z^k\), \(n = 1, 2, \ldots\). \(A(f)\) is called a Sonnenschein matrix and determines
a sequence to sequence transformation where if \( \{s_k\} \) is a sequence then the \( \Lambda(f) \)-transform of \( \{s_k\} \) is 
\[
\sigma_n = \sum_{k=0}^{\infty} a_{nk} s_k.
\]
Let \( \Delta(0,1) = \{ z : |z| = 1 \} \) and 
\[
c_{\Lambda(f)} = \{ x : \Lambda(f)x = \{ \sum_{k=0}^{\infty} a_{nk} x_k \} \in c \},
\]
where \( c \) is the set of all convergent sequences. Theorem 1. Suppose \( \Lambda(f) \), \( \Lambda(g) \) and \( \Lambda(f^{-1}) \) are Sonnenschein matrices with \( \Lambda(f) \) a regular, one-to-one transformation, then 
\[
c_{\Lambda(f)} \subseteq c_{\Lambda(g)} \text{ and } \Lambda(f)x \text{ and } \Lambda(g)x
\]
converge to the same limit if and only if \( D_g \geq f^{-1}(\Delta(0,1)) \) and 
\[
\Lambda(g)\Lambda(f^{-1}) = \Lambda(g \circ f^{-1}) \text{ is regular.}
\]
Immediate corollaries to Theorem 1 for the well-known Taylor, \( T(r) \), Laurent, \( S(q) \), Euler, \( E(p) \), and Karamata, \( K(\alpha, \beta) \), matrices will be discussed. Typical corollary. If \( 1/2 < p \leq 1 \) and \( 0 < r < 1 \), then \( E(p) \subseteq T(r) \) if and only if \( (1 - p)/(2 - p) < r < p/(2 - p) \). (Received August 26, 1968.)

663-44. J. R. CLAY and D. A. LAWVER, University of Arizona, Tucson, Arizona 85721.

Boolean near-rings.

Given any Boolean ring with identity, \( (B,+,\wedge,1) \), and an \( x \in B \), we define a near-ring \( (B,+,\cdot) \), called a special Boolean near-ring, by \( a \cdot b = (a \vee x) \wedge b \) for all \( a,b \in B \). Left ideals, ideals, ideals \( I \) making \( B/I \) Boolean, isomorphisms, and ideal direct summands are characterized. (Received August 26, 1968.)

663-45. T. R. BERGER, University of Minnesota, Minneapolis, Minnesota 55455. Nilpotent groups as fixed point free automorphism groups.

This announcement extends results reported earlier (Abstracts 68T-14, 68T-523, these Notices 15 (1968), 192, 658). Suppose \( A \) is a nilpotent group of automorphisms of a solvable group \( G \). Assume \( (|A|, |G|) = 1 \) and \( A \) is fixed point free on \( G \). Then, except for certain prime conditions, the Fitting length of \( G \) is bounded by the number of primes, counting multiplicities, dividing \( |A| \). (Received August 28, 1968.)


We study the stability of a fluid layer heated from below. For temperature gradients smaller than a critical value \( (R_C) \), heat is transported by conduction and the fluid remains at rest. As the temperature gradient exceeds this critical value, heat is transported by convection as well as conduction, and cellular patterns appear. For ease of exposition we consider a variant of a model equation introduced by Segel, rather than the system of equations actually governing the flow. Thus we consider 
\[
u_\xi = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \cdot \nu(R(\eta^2 + \partial^2 u)u = \left( u_k \right)_x \text{ for } t \equiv 0, 0 \leq t \equiv 1, -\infty < x,y < \infty, \text{ together with appropriate initial and boundary conditions. We wish to determine whether there exist initial perturbations with respect to which the system is unstable, and which will grow into steady state periodic solutions. Following Joseph, we can show by energy estimates that all disturbances decay to } u \equiv 0 \text{ for } R < R_C. \text{ We study the behavior of solutions which bifurcate from } u = 0 \text{ at } R = R_C, \text{ and trace their development in time from initial perturbation to a final steady state. We are interested in their dependence on } R \text{ in a neighborhood of } R = R_C, \text{ and derive a formal asymptotic representation for } u \text{ in powers of a small parameter } \epsilon, \text{ related to } R - R_C. \text{ (Received August 29, 1968.)} \]

Semicomplexes were first introduced by F. Browder and have recently been studied by the author [On the semicomplexes of F. Browder, Bull. Amer. Math. Soc. 73 (1967), 531-536]. It is known that the Lefschetz fixed point theorem holds for weak semicomplexes and that a local fixed point index may be defined on semicomplexes. The following results are established in the present paper. Theorem. Any retract of a weak semicomplex is a weak semicomplex. Theorem. Any retract of a metric semicomplex is a semicomplex. Theorem. Equivalent semicomplexes induce equivalent semicomplexes under a retraction. Conditions are also given under which the induced structures are simple, provided that the original structures have that property. This material, together with the fact that the product of two semicomplexes is again a semicomplex, provides a very short new proof that the category of compact metric ANR's and their maps admits a local fixed point index. (Received August 29, 1968.)


Transform the equation (1) \( \frac{d^2 x}{dt^2} + P(x)\left(\frac{dx}{dt}\right)^2 = Q(x)\left(\frac{dx}{dt}\right)^{n+1} \) into the phase plane by using \( \frac{dx}{dt} = v, \frac{d^2 x}{dt^2} = dv/dx \). This gives Bernoulli's equation (2) \( dv/dx + vP(x) = v^nQ(x) \), with solution given by (3) \( v^{1-n} = e^{(n-1)\int P(x)dx}\left[(1-n)\int Q(x)exp((1-n)\int P(x)dx)+c\right] \). Setting \( v = 0 \) in (3) gives an \((x,c)\) plane, called the alternate plane. The following result can now be established. Theorem. Let \( n \) be a negative odd integer. Equation (2) will have a closed curve solution, hence equation (1) has oscillatory solutions if the following conditions hold, (i) \( c + (1-n)\int Q(x)exp((1-n)\int P(x)dx) = 0 \) is continuous on an interval \( x_0 \leq x \leq x_1 \), (ii) \( C(x) \) has a min \([\text{max}]\) and 2 max \([\text{min}]\), \( x_0 \leq x \leq x_1 \). (iii) \( \nu(1-n)(x) > 0 \) in the convex part of the alternate plane bounded by the curve in (i), and the lines \( x = x_0, x = x_1 \). The proof will be outlined by the special case \( n = -1 \). The general case then follows easily. Note in particular that the existence of limit cycles depends on the integration constant \( c \) as well as \( n, P(x), Q(x) \). (Received September 3, 1968.)


M an abelian group, \( T \) its maximal torsion subgroup, \( \phi: M \rightarrow M/T \) the natural homomorphism, \( R: \text{Aut } M \rightarrow \text{Aut } T \) the restriction map, \( \text{Aut}_T M = \{ f \in \text{Aut } M : f(T) = T \} \), \( S: \text{Aut}_T M \rightarrow \text{Aut}(M/T) \) the map given by \( \phi(fS) = f\phi \) for \( f \in \text{Aut}_T M \). Problem. If \( R, S \) are both onto, does \( M \) split (over \( T \))? Let \( T \) be a reduced torsion group and \( K \) a torsion free group, let \( \mathbf{E} \in \text{Ext}(K,T) \), and let \( M = \mathbf{E}^{-1} \) be the mixed group associated with \( \mathbf{E} \). Then \( -1 \in \text{im } S \) iff \( 2\xi = 0 \). This simple result gives an affirmative answer to Misina's problem in various cases using results of Baer (Die Torsionsuntergruppe einer Abelschen Gruppe, Math. Ann. 135 (1958)) and Mader (The group of extensions of a torsion group by a torsion free group, Archiv, Math. (to appear)). If every 2-primary homomorphic image of \( K \) is bounded and if \( T \) is an unbounded 2-group, then \( \text{Ext}(K,T)[2] \neq 0 \), so that the above result does not answer the Problem. But the answer is still affirmative in this case for every mixed group \( M \mathbf{E}^{-1} \mathbf{E} \in \text{Ext}(K,T) \). On the other hand, there exist nonsplitting mixed groups such that \( S \) is onto. (Received August 30, 1968.)
A lattice $\mathcal{D}$ satisfies the $m$-chain condition (where $m$ is a cardinal $>\aleph_0$) iff every chain in $\mathcal{D}$ is of cardinality $<m$. Let $\mathcal{D}_i$, $i \in I$, be distributive lattices with 0 and 1, $\mathcal{D} = \bigoplus \mathcal{D}_i$ be the distributive free product with the 0 and the 1 identified (i.e., the free product in the category of distributive lattices with 0 and 1 preserving homomorphisms). 1. A characterization of $\mathcal{D}$ is given. 2. A solution to the word problem of $\mathcal{D}$ is described. Let $P(m)$ denote the property that if all $\mathcal{D}_i$, $i \in I$, satisfy the $m$-chain condition, so does $\mathcal{D}$. Theorem. $P(m)$ holds if and only if $m$ is regular. Identical results hold for arbitrary distributive lattices, and for Boolean algebras. Further, it is pointed out that the existence of these three kinds of free products does not depend on any form of the Axiom of Choice. However, the proof of the Theorem, even for the countable chain condition ($m = \aleph_0$), uses a generalized König lemma. (Received August 30, 1968.)

We call completely simple a semiring $R$ whose multiplicative semigroup is completely simple. We denote by $\overline{A}(p)$ the set of all inner left (right) translations of $R$ [Mireille Poignignon, Inner translations of a semiring, Abstract 68T-10, these Notices 15 (1968), 784]. Theorem. The following properties are equivalent for a semiring $R$: (i) $R$ is completely simple; (ii) $R$ is a bisimple semiring which contains a minimal left ideal and a minimal right ideal and such that $\overline{A}$ and $\overline{P}$ commute element by element; (iii) $R$ is the union of its minimal left ideals and of its minimal right ideals. Furthermore, under any of these conditions, any $H$-class $H$ of $R$ is a division semiring (i.e. $(H,*)$ is a group). Using a complete description of division semirings, one can give an explicit description of the semiring operations in any additive $\mathcal{H}$-class of the semiring. (Received August 30, 1968.)

It is a popular misconception that when a function suffers a jump in value it is always discontinuous. If we apply the definition of continuity meticulously, we find, surprisingly, that contrary to popular opinion this is false. The step function is frequently (and best) defined as follows: $S_k(t) = 0$ when $0 < t < k$, and $S_k(t) = 1$ when $t > k$. $S_k$ is continuous. More generally, let $F_n = \{(x,y) : x \in [x_1,x_n], \ y \in \mathbb{R}\}$ be a bounded continuous function over the interval $[x_1,x_n]$. Then $F_n = F - \{(x_i,y_i) \in F : i \in \{1,\ldots,n\}\}$ is continuous. Furthermore, a function $F_j$ obtained from $F_n$ by a finite translation (or jump) of any restriction of $F_n$ between a successive pair of points of deletion (or jumps of any set of such restrictions) will be continuous. More precisely, $F_j = \bigcup_{i=1}^{n} \{(x\cdot \xi_i + b_i) : x_1 < \xi_1 < x_{i+1}, \ (\xi_i, \eta_i) \in F, b_i \in \mathbb{R}\}$. $F_j$ eliminates the concept of sectional continuity. Obviously, if the $b_i = 0$, then $F_j = F_n$. Furthermore, we show that the operational calculus may generate a restrictive set of solutions for particular differential equations, e.g. $\ddot{y}(t) = S_k(t)$. Finally, we exhibit a differential equation arising from a dynamical problem for which the physical solution cannot be obtained by use of Laplace Transforms. (Received September 5, 1968.)
663-53. KEITH JOSEPH, University of California, Los Angeles, California 90024. Commuting pairs in a finite group. Preliminary report.

Let $G$ be a finite group. Let $\Pi(G)$ be the probability that two elements of $G$ commute. Then $\Pi(G) = k/|G|$ where $k$ is the class number of $G$. $\Pi$ has the following properties: (i) If $H \subseteq G$, then $\Pi(H) \leq \Pi(G)$, (ii) If $H \leq G$, then $\Pi(G/H) \leq \Pi(G)$. Call $G$ an almost abelian group if $|G'| = p$ where $p$ is a prime. There are two cases: (i) If $G' \cong Z(G)$, then $\Pi(G) = 1/p + (p - 1)/p^2 s = 1$, $G = G_1 \times G_2$ where $|G_1| = p^a$, $(p, |G_2|) = 1$, $G_2$ is abelian and $G_1$ is almost abelian. (ii) If $G' \cap Z(G) = 1$, then $\Pi(G) = 1/p + (p - 1)/m^2 p$ where $m|p - 1$. (Received September 5, 1968.)

663-54. W. D. L. APPLING, North Texas State University, Denton, Texas 76203. The norm infimum function.

Suppose $F$ is a field of subsets of a set $U$, $R^+$ is the set of all real-nonnegative-valued functions defined on $F$ and $R^+_A$ is the set of all finitely additive elements of $R^+$. All integrals considered are Hellinger-type limits of the appropriate sums. Suppose $m$ is in $R^+_A$ and, for each $I$ in $F$, $m*(I) = \inf(\max\{m(J) | J \in D \} | D$ a subdivision of $I)$. Theorem 1. $\int_U [m(I)^2 - m*(I)^2] = 0$. Now suppose that, for each $I$ in $F$, $a(I) = 0$ if $m(I) = m*(I)$, $a(I) = 1$ otherwise, and for each $V$ in $F$, $m**(V) = \int_V a(I)m(I)$. Theorem 2. If $g$ is in $R^+_A$, then $g$ is absolutely continuous with respect to $m**$ iff $g$ is absolutely continuous with respect to $m$, and for all $I$ in $F$, $g(I) = 0$ if $a(I) = 0$. Theorem 3. There is an $m**$-refinement-unbounded element of $R^+$. (Received September 6, 1968.)


Let $(S, \mathcal{F})$ be a PM-space, let $\phi$ be a one-place function on $[0, \infty)$, let $A \subseteq S$, and let $p \in S$. Then $p$ is a $\phi$-accumulation point of $A$ if for every $\epsilon$, $\lambda > 0$, there exists $q \in A$ ($q \neq p$) such that $\phi_{pq}(\epsilon) > \phi(\epsilon) - \lambda$. $A$ is $\phi$-closed if $\phi(A) \subseteq A$, where $\phi(A)$ is the set of $\phi$-accumulation points of $A$. $\phi$-closed sets are closed under arbitrary intersections and, thus, a generalized topology for $S$ is induced by $\phi$. This approach to topologies for PM-spaces includes that of Schweizer and Sklar (Pacific J. Math. 10 (1960), 313-334), whose topology is obtained by setting $\phi(x) = 1$ for all $x$ and also contains several other special cases of interest. In the general case, the lower semicontinuity of $\phi$ is established and, in one of the special cases, a metrization theorem is proved. Finally, the topologies obtained in this way are compared with those of E. Thorp (Fund. Math. 51 (1962), 9-21), which are defined using subsets of the positive quadrant, and a mild condition for the equivalence of the two is demonstrated. Note: PM-space = Probabilistic Metric space. (Received September 5, 1968.)


Let $W, Y, Z$ be Banach spaces and $u$ a bilinear continuous operator from $Y \times Z$ into $W$. Let $p, q > 1$ such that $1/p + 1/q = 1$. For a volume space $(X, V, \nu)$, let $L_p(v, Y)$ be the $L_p$-space of Bochner
measurable functions defined as in Bogdanowicz, Bull. Acad. Polon. Sci. 13 (1965), 793-800. Let $K_q(v,Z)$ be the space of vector-valued volumes as in the above paper. The triple $(X,V,v)$ is a probability volume space if $X \in V$ and $v(X) = 1$. If $(X_t,V_t,v_t)$ is a family of probability volume spaces $(t \in T)$ define $X_T = \bigvee_{t \in T} X_t$, $V_T = \{ A = \bigvee_{t \in T} A_t : A_t \in V_t \text{ and } A_t \neq X_t \text{ for at most a finite number of } t \in T \}$, $v_T(A) = \prod_{t \in T} v_t(A_t)$ for $A = \bigvee_{t \in T} A_t \in V_T$. For $f \in L_p(v_T,Y)$, the function $f(x_S, \cdot) \in L_p(v_S,Y) v_S$ almost everywhere, where $S' = T \setminus S$ and $x_S$ is the natural projection of $x \in X_T$ onto $X_S$. (See Bogdanowicz, Proc. Japan Acad. 42 (1966), 979-983.) For $S \subset T$, $f \in L_p(v_T,Y)$, $\mu \in K_q(v_T,Z)$, define $\mu_S(A) = \mu(A \times X_S')$ for all $A \in V_S$ and the operator $J_S$ by $(J_S(f))(x) = \int u(f(x_S,x_S'), \mu_S(dx_S))$ when meaningful and zero otherwise. Theorem. If $f \in L_p(v_T,Y)$ and $\mu \in K_q(v_T,Z)$, then the operator $J_S$ maps $L_p(v_T,Y)$ into $L_p(v_T,W)$ and $J_S f$ converges to the function $u(f, \mu(X))$ in the space $L_p(v_T,Y)$, where $S$ runs through all finite subsets of $T$ ordered by inclusion. (Received September 6, 1968.)

663-57. WITHDRAWN.


Let $A$ be an algebra over a field $K$. We say that $A$ is completely semisimple if, for each nonzero element $a \in A$, there is an irreducible $A$-module $M$ whose commuting algebra consists of scalars only and such that $a M \neq 0$. Clearly, a completely semisimple algebra is (Jacobson) semisimple. If $K$ is algebraically closed and if $A$ is a semisimple algebraic algebra, then $A$ is completely semisimple. Theorem. Suppose that $A$ is a completely semisimple algebra over $K$ and that $B$ is a semisimple algebra over $K$. Then $A \otimes_K B$ is semisimple. If $B$ is also completely semisimple, then so is $A \otimes_K B$. (Received September 9, 1968.)


Let $A$ be a finite-dimensional algebra with unit over the rational numbers $Q$. A subring $J$ of $A$ is called an order if it contains the unit element, is a finitely-generated $Z$-module, and $QJ = A$. It is shown that the series $\sum N a^{-s} b$, where $a$ ranges over the integral left-ideals of $J$, converges absolutely for $\text{Re}(s) > \dim A$. Much stronger information is, of course, available in the classical case where $A$ is semisimple and $J$ is maximal. The present result does not even require associativity of the algebra. (Received September 9, 1968.)


Let $g(z) = \sum_{n=0}^{\infty} a_n z^n$ be analytic in $|z| < R$, $R > 1$, and suppose $g(1) \neq 0$. Define the Appell polynomials $p_k(x,g)(k \geq 0)$ by $g(u)^{\nu x} = \sum_{k=0}^{\infty} p_k(x) u^k$. To each function $f(t)$ defined in $[0,\infty]$, associate the operators $P_n(f,x) = (e^{-nx} / g(1)) \sum_{k=0}^{\infty} p_k(xf(k/n))$, $n = 1,2,\ldots$. If $g(z) = 1$, then $P_n$ becomes the classical Szasz operator. Theorem 1. Assume $f$ and $n$ are such that $\sum_{\nu=0}^{\infty} P_{\nu}(nx)f(\nu/n)$
converges uniformly and absolutely in \([0,b]\) for any \(b > 0\). Suppose \(P_n\) is positive in \([0,\infty)\). Then \(P_n\) is variation diminishing. **Theorem 2.** Let \(P_n\) be positive in \([0,\infty)\) and assume \(f(z) = \sum_{\nu=0}^{\infty} a_{\nu} z^{\nu}\) is entire. Then (1) if \(f\) is of exponential type, then \(P_n f(z) \to f(z)\) almost uniformly in the finite plane; and (2) if \([P_n f(z)]\) is almost uniformly convergent in the finite plane and \(a_{\nu} \equiv 0, \nu = 0,1,\ldots\), then order \(f\) \(\sim 1.\) Let \(A(g,z) = (a_{nk}(z))\) be the sequence to sequence summability transform defined by \(a_{nk}(z) = p_k(nz)/g(1)e^{nz}\). If \(g(w) = 1\) then \(A\) is the classical Borel method. **Theorem 3.** Let \(x > 0\) and \(C[0,\infty]\) denote the space of all functions \(h\) such that \(h\) is continuous in \([0,x]\), defined in \([0,\infty)\) and \(\sup_{0 \leq y < \infty} |h(y)| = \sup_{0 \leq y \leq x} |h(y)|\). Suppose \([P_n h(x)]\) is almost convergent for all \(h \in C[0,x]\). Then \(A(g,x)\) is regular. (Received September 9, 1968.)

663-61. WITHDRAWN.

663-62. ELBERT PIRTL, University of Missouri, Kansas City, Missouri 64110. On a generalization of Krull domains.

Let \(R\) be an integral domain with quotient field \(L\) and let \(F\) be a family of valuations on \(L\) satisfying the following: (1) Each \(v \in F\) has rank one and is discrete. (2) \(R = \bigcap \{R_v | v \in F\}\). (3) \(R_v = R_{P(v)}\) for each \(v \in F\). If \(F\) is of finite character, it is well known that the semigroup \(\mathcal{O}(R)\) of fractional ideal classes constructed using the family \(F\) is a group and that \(\mathcal{O}(R) \approx \mathcal{B}(R)\), where \(\mathcal{B}(R)\) is the divisor group of \(R\) (see Bourbaki, Livre III. Topologie g\^{e}n\^{e}rale, Actualit\^{e}s Sci. Ind., No. 916, 1029, Hermann, Paris, 1942, 1947, Chapter 7). Furthermore if \(F\) is of finite character, the above reference shows that \(P(v)\) is divisoriel for each \(v \in F\). Let \(R, L, F\) be as above. The following are equivalent: (i) \(\mathcal{O}(R)\) is a group. (ii) \(\mathcal{O}(R) \approx \mathcal{B}(R)\). (iii) \(P(v)\) is divisoriel for each \(v \in F\). An example is given to show that \(F\) need not be of finite character for the above to hold.

Definition. \(R\) is called a \(K\)-domain if there is a family \(F\) of valuations on \(L\) satisfying (1), (2), (3) above, and (4) \(P(v)\) is divisoriel for each \(v \in F\). Various extensions of \(K\)-domains are studies. If \(M\) is a multiplicative system in \(R\), then \(R_M\) need not be a \(K\)-domain. It is shown that \(\{R_v | v \in F\}\) satisfies Gilmer's (S) condition. (Received September 11, 1968.)

663-63. R. N. LANE, General Electric Company-Tempo, Santa Barbara, California 93102. Internal structure of \(t\)-designs.

A \(t\)-design is a pair \((B, P)\), where each element of \(B\) is a \(k\)-subset of \(P\), and each \(t\)-subset of \(P\) occurs in exactly \(\lambda\) elements of \(B\) for some fixed integers \(k \leq \lambda\). Elements of \(P\) are called "points", elements of \(B\) are called "blocks". A design homomorphism from one \(t\)-design onto another is a mapping of points onto points and blocks onto blocks which preserves incidence of points on blocks. Via a special class of design homomorphisms, a theory of internal structure of \(t\)-designs can be developed. The theory is quite similar in results to the internal structure analysis of a group with respect to its normal subgroups and quotient groups. In particular, it is shown that any \(t\)-design can be decomposed in a natural fashion into a sequence of "simple" subdesigns. (Received September 11, 1968.)

Numerical solutions to the boundary value problems for second and fourth order ordinary differential equations and second order elliptic partial differential equations with constant coefficients are considered. Methods which are third, fourth, fifth, and sixth order accurate are derived and convergence theorems for them are proven. It is also indicated how to derive even higher order accurate methods and how to prove their convergence. The convergence proofs are based on a priori bounds on the solutions to the difference schemes in the discrete $L^2$ norm. (Received September 12, 1968.)

663-65. ISIDORE EISENBERGER, Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, California 91103, 238/420. Estimators of the parameters of an extreme-value distribution using quantiles.

Given $n$ independent sample values taken from a population with a distribution function of the form $G(x) = \exp(-\exp[-(x - \alpha)/\beta])$, $-\infty < x < \infty$, where $n$ is large and $\alpha$ and $\beta$ are unknown, asymptotically unbiased estimators of $\alpha$ and $\beta$ are constructed using up to ten sample quantiles. The estimators are optimum or near-optimum with respect to minimizing the variances of the estimators. The efficiencies are also given relative to the maximum-likelihood estimators of $\alpha$ and $\beta$. (Received September 12, 1968.)


Recently Neuberger, Zettl and Loud have reviewed interest in selfadjoint boundary value problems with interior point boundary conditions. Their results depend heavily upon symmetry in the Greens function. By using a method similar to the fundamental lemma of the calculus of variations, we are able to derive the adjoint problem under a more general setting and to produce a necessary and sufficient condition for selfadjointness, which is similar to the classical two point result. (Received September 12, 1968.)

663-67. C. E. HALL, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Projective topological groups. II.

In this paper the concept of a projective topological group [Hall, Proc. Amer. Math. Soc. 18 (1967), 425-431] is generalized by deleting the requirement that the topological groups considered be Abelian. Let $A$ and $B$ denote topological groups. Let $\mathcal{F}$ be the family of all exact sequences of the form $\cdots \rightarrow A \rightarrow B \rightarrow 0$ such that all free topological groups are projective relative to $\mathcal{F}$. A topological group is $F$-projective iff it is projective relative to $\mathcal{F}$. Theorem 1. If $(A \rightarrow B \rightarrow 0) \in \mathcal{F}$, then $f$ is an open mapping. Theorem 2. A topological group $P$ is $F$-projective iff there is a free topological group $F(X)$, a one-to-one continuous homomorphism $\alpha : P \rightarrow F(X)$, and a closed normal subgroup $H$ of $F(X)$, such that $\alpha[P] \cdot H = F(X)$, $\alpha[P] \cap H = \{e\}$, $U$ open in $P = \alpha[U] \cdot H$ is open in $F(X)$, and $\alpha[P]$ is a closed subset of $F(X)$. (Received September 12, 1968.)
Let $A$ be a nonempty, convex and compact set in a reflexive Banach space $Y$ and let $C(Y)$ denote the collection of all such sets. One defines two operations on $C(Y)$, namely (i) addition of sets by $A + B = \{a + b : a \in A \text{ and } b \in B\}$ and (ii) scalar multiplication of a set by $\alpha A = \{\alpha a : a \in A\}$. These operations define an algebraic structure on the class $C(Y)$ (see Abstract 648-13, these Notices 14 (1967), 632). Let $V$ represent a neighborhood of zero in any locally convex topology of the space $Y$. The family of sets $N = \{B : B \subset A + V \text{ and } A \subset B + V\}$ constitutes a base of neighborhoods for the set $A$ of the space $C(Y)$. This topology is said to be the strong, respectively the weak, topology of the space $C(Y)$ if it is generated by the strong, respectively the weak, topology of the space $Y$. Define $\|A\| = \sup\{\|y\| : y \in A\}$. The class $C(Y)$ forms a semilinear topological space with the above topology, and the operations of algebraic addition of sets and multiplication of a set by a scalar. A family of sets $\{A_t : t \in T\}$ is said to be summable to an element $A = \sum_{t \in T} A_t$ if the net $S_J = \sum_{t \in J} A_t$ is convergent to $A$. Properties of the space $C(Y)$ and conditions of summability in the space $C(Y)$ are given. The theorems obtained are essential to find the structure of additive set-valued functions defined on base-cones in Banach spaces. (See Abstract 658-6, these Notices 15 (1968), 720 and these Notices 11 (1964), 323 and 549.) (Received September 13, 1968.)

If $E$ is a set of nonnegative integers containing 0, let $Q(E)$ be the set of all integers $N$ of the form $N = \prod_{i=1}^{\infty} p_i^{a_i}$, where $p_i$ is the $i$th prime and each $a_i$ is chosen from $E$. We call $Q(E)$ the set developed from the exponent choice set $E$. If $E$ contains no $n$-terms arithmetic progression, then $Q(E)$ contains no $n$-terms geometric progression. For each $n \geq 4$, we either estimate or find the density of such a progression-free set $Q(E)$, after proving Lemma 1. For any exponent choice set $E$, $\lim_{x \to \infty} [Q(E)](x)/x$ exists; that is, $Q(E)$ has a density $D(Q(E))$. If $n$ is composite we construct a set $E_n$ containing no $n$-term A.P., such that $D(Q(E_n)) > \zeta(n)/[\zeta(n - 1)\zeta(hn)] - [1/\zeta(hn - 1) - 1/\zeta(hn - h)]$, where $h$ is the smallest prime divisor of $n$. If $n$ is prime we find $E_n$ such that $D(Q(E_n)) = \zeta(n)/[\zeta(n - 1)\zeta((n - 1)h)]$. These densities exceed those of similar sets constructed by R. A. Rankin in the paper "Sets of integers containing not more than a given number of terms in arithmetical progression," Proc. Roy. Soc. Edinburgh Sect. A 65 (1962), 332-344. Rankin, however, also constructed a set containing no 3 terms in geometric progression and we have found no other such set having a greater density. We also find, for each $n \geq 3$, an upper estimate for the possible density of this kind of set. (Received September 12, 1968.)

Among the many results concerning asymptotic properties as $t \to + \infty$ of solutions of the differential equation $x + f(t)x = 0$ found by P. Hartman and A. Wintner [Am. J. Math. 75 (1953), 717-730] is the following: (I) There exists a solution of (1) satisfying (i) $x \to 1$ if and only if there exists a solution satisfying (ii) $x(t) = o(t^{-1})$. A counterexample (showing that (ii) is not sufficient for (i)) leads to an oscillation criterion for (1) containing the hypothesis that (iii)
\[ \lim_{a \to \infty} \int_A f(s) \, ds \geq 0 \quad \text{for all large } a. \] Interest in (iii) derives from its similarity to the well-known Leighton condition \[ \int_0^\infty f(t) \, dt = +\infty. \] (Received September 13, 1968.)


If \( G \) is any finite group, and \( S \) any subset of \( G \), we define \( P(S) = \min \{ n = |S^n| = |S^{n+1}| \} \), \( \gamma(G) = \max \{ P(S) = S \subseteq G \} \), where \(|\cdot|\) denotes cardinality. Our main result is: The discrete non-Abelian group \( G \) satisfies \( \{ A_n\}|(aS)^n| = |S^n| \) for all \( a \in G \) and \( S \subseteq G \) if and only if \( G \) is a finite Hamiltonian group and \( \gamma(G) \geq n \). Furthermore, \( \gamma \) is explicitly evaluated in terms of structural invariants of \( G \). (Received September 13, 1968.)

663-72. RICHARD OSBORNE and J. R. CAMPBELL, University of Idaho, Moscow, Idaho 83843. A characterization of \( S^n \).

Bing (Ann. of Math. 68, 317-37) has shown that a compact 3-manifold in which every simple closed curve lies in an open 3-cell is \( S^3 \). The authors have observed that the generalized Antoine's necklaces of Blankinship (Ann. of Math. 53, 276-297) act, with respect to lying in open cells, like surfaces of codimension 2. Using this idea it is possible to "algebraically approximate" spheres in codimension 2 or more by Cantor sets. Thus the following is shown. Theorem. A compact n-manifold \( (n = 4) \) in which every simple closed curve lies in an open n-cell is \( S^n \). (Received September 13, 1968.)

663-73. LeBARON FERGUSON, University of California, Riverside, California 92502. Algebraic kernels of planar sets.

M. Fekete has defined the algebraic kernel \( J(X, I_L) \) of any compact subset \( X \) of the complex plane with respect to the ring of integers \( I_L \) of any imaginary quadratic field \( L \) to be the union of all complete sets of conjugates over \( L \) which are integral over \( L \) and are contained in \( X \). The purpose of this paper is the explicit determination of \( J(X, I_L) \). This seems to be difficult in general. We determine it for certain subsets of the closed unit disk. The determination of the algebraic kernel is crucial in the problem of uniform approximation by polynomials with integral coefficients. In the course of the determination of \( J \) for arcs of the unit circle we found it necessary to determine explicitly the Galois groups of the cyclotomic fields over their quadratic subfields. (Received September 13, 1968.)


Let \( \prec \) be a reflexive and transitive relation on a set \( X \). A semiproximity \( \Theta \) is said to be compatible with \( (X, \prec) \) when \( x \prec y \) if and only if \( (x,y) \in \Theta \) (where \( (x,y) \in \Theta \) abbreviates \( \{ (x), (y) \} \in \Theta \) ). Let \( \mathcal{J}(\Theta) \) be the topology on \( X \) defined by the closure operator \( c(A) = \{ x : (x,A) \in \Theta \} \). Theorem 1. If \( \Theta \) is any compatible semiproximity on \( (X, \prec) \) and \( \mathcal{J} = \text{l.u.b.}[\mathcal{J}(\Theta), \mathcal{J}(\Theta^{-1})] \), then \( \mathcal{J} \) is a locally convex Hausdorff topology on \( X \). Let \( \Theta^* \) be a proximity on \( X \). The triple \( (X, \prec, \Theta^*) \) is called a.
proximity preordered space if there is a compatible semiproximity \( \varrho \) on \((X, \prec)\) such that 
\[ \mathcal{S}(\varrho^*) = \text{u.b.}[\mathcal{S}(\varrho), \mathcal{S}(\varrho^{-1})]. \] Theorem 2. A proximity preordered space is a uniformizable preordered space if and only if, for each \( x \in X \) and each increasing neighborhood \( U \) and decreasing neighborhood \( V \) of \( x \), there exist continuous functions \( f \) and \( g \) from \( X \) into the closed unit interval \([0,1]\) such that 
\[ f(x) = g(x) = 1, f(y) = 0 \text{ on } X \setminus U, g(y) = 0 \text{ on } X \setminus V, f \text{ is increasing, and } g \text{ is decreasing.} \]

(Received September 16, 1968.)

663-75. E. H. ANDERSON, Mississippi State University, State College, Mississippi 39762. An alternative proof that Bing’s dogbone space is not topologically \( E^3 \).

In Ann. of Math. 65 (1957), 484-500, Bing presented an example of an USC decomposition of \( E^3 \) into points and tame arcs and showed that the example was not topologically \( E^3 \). In this paper, an example slightly simpler in construction than Bing’s is presented and shown to be topologically different from \( E^3 \). The argument may be easily modified to give an alternative proof that Bing’s example is not topologically \( E^3 \). Other constructions are presented where neither Bing’s argument or the argument of this paper can be easily modified to show whether or not the associated decomposition spaces are topologically \( E^3 \). (Received September 16, 1968.)

663-76. GÖRAN BJÖRCK, University of California, Riverside, California 92502. Approximation-theoretic characterization of Beurling test-functions.

Let \( \omega \) be a nonnegative concave function of a nonnegative real variable and such that \( \omega(0) = 0, \omega(0) = 0 \) and \( \int_1^\infty \omega(t) t^{-2} \, dt < \infty \). Let \( \mathcal{B}_\omega \) be the set of all \( \varphi \in C^\infty_0(\mathbb{R}^n) \) such that 
\[ \int_{|\xi|} \mathcal{E}(\xi) \exp(\lambda \omega(|\xi|)) \, d\xi < \infty \] for all real \( \lambda \). (Subscript zero indicates compact support, and \( \mathcal{E} \) is the Fourier transform.) Let \( \mathcal{E}_t \) be the set of all entire functions \( f \in L_1(\mathbb{R}^n) \) such that \( \hat{f} \in C^\infty_0 \) and \( \hat{f}(\xi) = 0 \) if \( |\xi| \geq t \). For each \( u \in \mathcal{E}_t \), we define \( A_t(u) = \inf \left\{ \|f - u\|_\infty \right\} \), where the inf is taken over those \( f \in \mathcal{E}_t \) for which \( \|f\|_\infty \leq 1 \). Theorem. Let \( u \in C^\infty_0(\mathbb{R}^n) \). Then \( u \in \mathcal{B}_\omega \) if and only if for each \( \lambda > 0 \) we have \( A_t(u) = O(e^{-\lambda \omega(t)}) \) when \( t \to + \infty \). A similar theorem characterizes \( \mathcal{B}_\omega(\Omega) \), where \( \Omega \) is an open set in \( \mathbb{R}^n \) and \( \mathcal{B}_\omega(\Omega) \) is the set of all \( C^\infty_0 \) functions \( u \in \Omega \) for which \( \varphi u \in \mathcal{B}_\omega \) for each \( \varphi \in \mathcal{B}_\omega \) with its support in \( \Omega \). Finally, this theorem is used to give a simple proof of the fact (Ark. Mat. 6 (1966), 351-407, Theorem 1.5.12) that \( u \in \mathcal{B}_\omega(\Omega) \) for all \( \omega \) if and only if \( u \) is in the Denjoy-Carleman class \( C[k \log k](\Omega) \). (Received September 16, 1968.)

663-77. E. M. PALMER, Michigan State University, East Lansing, Michigan 48823, and R. W. ROBINSON, University of California, Berkeley, California 94720. The cycle index of the exponentiation group.

The exponentiation group \([B]^A \) of two permutation groups \( A \) and \( B \) was introduced by Harary (Amer. Math. Monthly 66 (1959), 572-575) who also found a formula for the cycle index \( Z([S_n]^B) \) which was used to enumerate bicolored graphs (Pacific J. Math. 8 (1958), 743-755). An expression for \( Z([S_n]^B) \) was obtained by Slepian (Canad. J. Math. 5(1953), 185-193) and an algorithm for computing \( Z([B]^A) \) was conceived by Harrison and High (J. Combinatorial Theory 3 (1968), 1-23) and used to enumerate Post functions. We have found an explicit, general formula for \( Z([B]^A) \) in terms of \( Z(A) \) and \( Z(B) \). The result is readily obtained by substituting operators for the variables of \( Z(A) \) and letting them act on \( Z(B) \). (Received September 16, 1968.)
663-78. R. G. LINTZ, McMaster University, Hamilton, Ontario, Canada. Invariants and dimension functions in topological spaces.

In a former paper of ours (A generalization of the concept of continuous function and homeomorphism, Ann. Mat. Pura Appl. 67 (1965), 215-234) we introduced the idea of modeling function. Recently we obtained a generalization of this concept and together with it the concept of model. If a topological space $Y$ is a model of a space $X$ then $Y$ shares many interesting properties with $X$. In few words, a model is a generalization of homeomorphism. Among other things we think that if homology duality is true in $X$ it will also be true in $Y$. We have obtained a particular case of this, namely, Theorem. Let $\langle \Gamma_n \rangle$ be the cartesian multiplication of $n$ generalized arcs without extremities (a generalized arc is a connected, loc. connected, irreducible space between two points, its extremities) and let $Y$ be a compact subset of $\langle \Gamma_n \rangle$. Then $Y$ separates $\langle \Gamma_n \rangle$ if and only if $H_{n-1}(Y) \neq 0$ ($n > 0$). (Received September 16, 1968.)

663-79. S. H. McCLEARY, University of Georgia, Athens, Georgia 30601. Orbits of the stabilizer subgroup of an ordered permutation group.

In the lattice-ordered group of order preserving permutations of a totally ordered set ($0$-set) $\Omega$, let $G$ be a transitive $\Omega$-subgroup. The orbits of the stabilizer subgroup $G_\alpha$ ($\alpha \in \Omega$) are convex and themselves form an $0$-set, which is the main focus of investigation. This $0$-set is independent of $\alpha \in \Omega$. Every symmetric $0$-set is obtainable as the $0$-set of orbits of $G_\alpha$ for some $G$. If $G_\alpha$ has only a finite number of orbits, then $G$ can be embedded in a wreath product of $0$-$2$-transitive groups. In any $G$, each orbit $A$ is paired with the orbit $A^* = \{ag : a \in A\}$. If $\Gamma$ is a (convex) block of $G$ and is the union of orbits of $G_\alpha$, $\Gamma^*$ is a (convex) block. $G$ is said to be $0$-primitive if it has no nontrivial convex blocks. Charles Holland has shown that every $G$ can be embedded in a generalized wreath product of $0$-primitive groups. For $G$ $0$-primitive, but not $0$-$2$-transitive or regular, the $0$-set of nontrivial orbits of $G_\alpha$ is $0$-isomorphic to the integers; the fixed points of $G_\alpha$ are distributed in a very simple pattern; and $G$ is "periodic". (Received September 16, 1968.)

663-80. S. P. LLOYD, Bell Telephone Laboratories, Murray Hill, New Jersey 07974. Subalgebras in a subspace of $C(X)$.

With $X$ compact Hausdorff, let $L$ be a closed separating subspace of real $C(X)$ containing the constants. Let $\mathcal{W}$ be the $L$-harmonic functions [H. Bauer, Ann. Inst. Fourier (Grenoble) 11 (1961), 89-136]. Let $\mathcal{M}$ be the multipliers for $\mathcal{W}$ : $f \in \mathcal{M}$ iff $fg \in \mathcal{W}$ for all $g \in \mathcal{W}$. Theorem 1. $f \in \mathcal{M}$ iff for each $x \in X$, $f$ is constant on the union of the closed supports of all probabilities which represent $x$ wrt $L$. Theorem 2. Every subalgebra of $C(X)$ contained in $\mathcal{W}$ is contained in $\mathcal{M}$. Theorem 3. If the conjugate space $L^*$ is a lattice, then $\mathcal{W} = L$. Thus we have a characterization of the multipliers for $L$ itself in the case where $L^*$ is a lattice. (Received September 16, 1968.)


A theorem brought to our attention by Goldhaber (J. Algebra 7 (1967), 389-393) is applied to the rational matrix equation $A' \! A = c_0 \mathbb{I} + c_1 J + c_2 (\mathbb{I}_m \otimes J_n)$, where $A$ is of size $mn \times mn$, $m > 1$, to obtain
a nonexistence theorem. If \( A \) is a \((0,1)\)-matrix with \( c_2 = -c_1, c_0 > 0 \), this theorem says the following:

(i) if \( m \) is odd, then \( +1 = (c_0 - nc_1, (-1)^{(m-1)/2}(mn))_p \) (the Hilbert symbol); (ii) if \( m \equiv 2 \pmod{4} \) and \( n \) is even, then \( (c_0, -1)_p = +1 \), in each case the equality holding for all primes \( p \). If furthermore \( A \) consists of \( n \times n \) blocks \( A_{ij} \), then \( A \) is normal if and only if each block is a permutation matrix or zero. Several basic results for relative difference sets (Elliott and Butson, Illinois J. Math. 10 (1966), 517-531) are true more generally. Unfortunately our nontrivial examples of \((0,1)\) matrices satisfying the incidence equation are all incidence matrices of relative difference sets, though the examples given by Elliott and Butson can be mildly generalized. (Received September 18, 1968.)

663-82. F. H. CROOM, University of Kentucky, Lexington, Kentucky 40506. Homotopy type of loop spaces.

For a given topological space \( X \), let \( \Gamma(X) \) denote the space of (nonbased) loops in \( X \) with the compact-open topology. If \( X \) and \( Y \) have the same homotopy type, it follows easily that \( \Gamma(X) \) and \( \Gamma(Y) \) have the same homotopy type, but the converse is false. The author gives necessary and sufficient conditions that a homotopy equivalence between \( \Gamma(X) \) and \( \Gamma(Y) \) induce (1) a homotopy equivalence between \( X \) and \( Y \), and (2) a homotopy equivalence between the spaces \( \Omega X \) and \( \Omega Y \) of based loops in \( X \) and \( Y \) respectively. For H-spaces, \( \Gamma(X) \) is homotopically equivalent to \( X \times \Omega X \) and an H-isomorphism between \( \Gamma(X) \) and \( \Gamma(Y) \) satisfying a local null homotopy condition induces an H-isomorphism between \( X \) and \( Y \). (Received September 18, 1968.)

663-83. WITHDRAWN.

663-84. S. M. EISENBERG, University of Hartford, West Hartford, Connecticut 06117. Moment sequences and the Bernstein polynomials.

Let \( [a_j(x)] \) be a sequence of real-valued functions defined on \([0,1]\). Denote by \( (h_{nk}(x)) \) and \( (q_{nk}(x)) \) respectively the Hausdorff and quasi-Hausdorff matrices generated by \( [a_j(x)] \) (see G. H. Hardy, Divergent series, Oxford Press, London, 1949, Chapter 11). We associate linear operators with these matrices, for all functions \( f \) defined on \([0,1]\), by \( H_n(f,x) = \sum_{k=0}^{\infty} f(k/n)h_{nk}(x) \) and \( Q_n(f,x) = \sum_{k=0}^{\infty} f((k-n)/k)q_{nk}(x) \). For the choice \( a_j(x) = x^j \) (\( j = 0,1, \ldots \)), the operator \( H_n \) becomes the \( n \)th order Bernstein polynomial. When \( a_j(x) = (1-x)^j \) for \( j = -1,0,1, \ldots \), \( Q_n \) becomes the Bernstein power series. The following are established when \( [a_j(x)] \) is a totally monotone moment sequence for all \( x \in [0,1] \) (see H. S. Wall, Continued fractions, Chelsea, New York, 1967, Chapter 14).

Theorem 1. A necessary and sufficient condition that the sequence \( [H_n(f,x)] \) converge to \( f(x) \) uniformly on \([0,1]\), for each \( f \in C[0,1] \), is \( a_j(x) = x^j \) for \( j = 0,1,2 \) and all \( x \in [0,1] \). Theorem 2. Let \( B(x,t) \) be the function having \( [a_j(x)] \) as its moment sequence. Let \( a_{-1}(x) = \int_0^1 dB(x,t)/t \) and be finite. Let \( 0 \leq a \leq 1 \). Then a condition necessary and sufficient that \( [Q_n(f,x)] \) converge uniformly to \( f(x) \) on \([0,a]\), for each \( f \in C[0,1] \), is \( a_j(x) = (1-x)^j \) for \( j = -1,0,1, \ldots \), and \( x \in [0,a] \) (Received September 18, 1968.)

Let \( D_{k,1} = \text{Diff}(S^k \times S^1) \) and \( D^+_{k,1} = \{ f \in D_{k,1} | H_1(f) = \text{id} \forall 1 \} \). For \( a : S^k \to SO_{k+1} \) representing \( a \in \pi_k(SO_{k+1}) \), \( f_a(u,v) = (u,a(u)v) \); for \( b : S^1 \to SO_{k+1} \), representing \( b \in \pi_1(SO_{k+1}) \), \( f_b(u,v) = (b(v)u,v) \). By Cerf's results, \( \Gamma_{k+1} = \pi_0(\text{Diff}(S^{k+1})) \) is the fundamental group of \( \text{Diff}(S^{k+1}) \); if \( c : D^{k+1} \to D^{k+1} \) represents \( c \in \pi_0(\text{Diff}(D^{k+1}) \text{ rel } S^{k+1}) \) and \( D^{k+1} \) is a disc in \( S^k \times S^1 \), then \( f_c[D^{k+1}] = c \) and \( f_c[S^k \times S^1 - D^{k+1}] = \text{id} \).

(The \( f_a, f_b, f_c, f_y \) are well defined only up to isotopy.) Theorem. If \( k < 1 < 2k - 3 \), then \( \pi_0(D^+_{k,1}) \) is generated by the \( f_a, f_b, f_c, f_y \). In fact, \( \pi_0(D^+_{k,1}) \) can be written as a semi-direct product \( \pi_0(D^+_{k,1}) \cong (\pi_1(SO_{k+1}) \oplus \Gamma_{k+1}) \rtimes \pi_1(SO) \). The use of Heifling's unknotted theorem accounts for the dimension restrictions.) The main application is Theorem. If \( M^{m+n} \) is a simply connected manifold with the homology groups of \( S^n \times S^m \), with \( n < m - 1 < 2n - 3 \) and \( n = 3,5,6,7 \) (mod \( 8 \)), then \( M^{m+n} \cong S_B \rtimes \Sigma^{m+n} \), where \( S_B \) is the sphere bundle classified by \( B \in \pi_{m-1}(SO_{n+1}) \) and \( \Sigma^{m+n} \in \theta_{m+n}^* \). (Received September 18, 1968.)

663-86. MAURICE CHACRON, University of Windsor, Windsor, Ontario, Canada. Structure of a certain class of rings.

Each of the following rings \( R \) satisfy the property (P) that for every element \( x \) there exists an integer \( n = x(x) \) such that \( x^{n(x)} \in \mathcal{E} \) where \( \mathcal{E} \) is the subring of \( R \) generated by the set of the idempotent elements of \( R \); the ring of integers; the finite rings; the rings \( R \) in which \( x^{m(x)} = x^{m(x)} \) for every element \( x \) or \( R \); the ring of all matrices over a commutative \( x^{n(x)} = x^{m(x)} \) ring; the underlying ring of an algebra over a finite field. It seems likely to study the structure of the class of rings with property (P) after the works of Neal McCoy, I. N. Herstein, N. Jacobson, D. C. Rees and G. Thierrin on particular aspects of property (P). This is done in a paper to be published in the Journal of Algebra. However, it is assumed almost everywhere that the idempotent elements of \( R \) commute elementwise. (Received September 18, 1968.)


Recent developments in the theory of elliptic operators are presented. The relation of these operators to topological invariants and the extension of the Riemann-Roch theorem to general elliptic systems is emphasized. (Received September 19, 1968.)

663-88. WITHDRAWN.

663-89. WITHDRAWN.

663-90. RICHARD LAATSCH, Miami University, Oxford, Ohio 45056. A dense set of extremal subadditive functions.

The cone \( S \) of all real valued, continuous, nondecreasing, subadditive functions on \([0,1]\) which vanish at \( 0 \) has a set of extremal elements which is dense in \( S \). This is proved by showing that a
polygonal function in $S$ with only the slopes 0 and $m > 0$ is extremal and that each element of $S$ is the uniform limit of a sequence of such functions. The proof involves an interpolation theorem for nondecreasing subadditive functions. Variations of the conditions defining $S$ are possible. The elliptic conic sections of $S$ are topologically characterized as closed but not compact in the topology of uniform convergence, and it is noted that subadditivity may be imposed on the functions used to decompose a function of bounded variation. (Received September 19, 1968.)


A wave is a solution of the reduced wave equation $\Delta u + k^2 u = 0$ and is of the form $u(x) = \exp(ik s) \sum_{m=p}^{\infty} (ik)^{-m} z_m$ for large $k$ (high frequency). The phase function $s(x)$ is constant along the bicharacteristics (rays) of the reduced wave equation and the amplitude functions $z_m(x)$ satisfy first order ordinary differential equations which can be solved explicitly along the bicharacteristics. Here we find the total field $u(x)$ which satisfies (i) the reduced wave equation, (ii) the boundary condition $u = 0$ or $\partial u / \partial n = 0$ on the wedge-like-object $S$, (iii) $u$ has a finite limit at the edge, and (iv) $u - u_0$ is outgoing from $S$. Here $u_0$ is the prescribed incident field. Since the edge of $S$ is a caustic of the diffracted wave (corresponding to the diffracted rays which emanate from the edge), we introduce a boundary layer expansion near the edge. Then away from the edge we introduce an Ansatz which involves Fresnel integrals. After finding the initial conditions of the unknown functions by "matching principle", we prove that this formal asymptotic solution so constructed is uniform, i.e. regular across the shadow boundaries of the incident and reflected waves. Moreover, away from these shadow boundaries the solution reduces to the sum of the incident, reflected, and diffracted waves which verifies Keller's theory of geometrical diffraction. (Received September 20, 1968.)


The $n \times n$ matrix equation considered is $Y' = PY + H'Q$, with boundary conditions $AY(a) + BY(b) + \int_{a}^{b} KY = 0$ and $CY(a) + DY(b) = Q$ and interface conditions given in the condition that $Y - HQ$ be absolutely continuous on $[a, b]$. In this system $P$ is continuous, $H$ and $K$ are of bounded variation, and $A$, $B$, $C$, $D$, and $Q$ are constants; the differentiation takes place at the points of $[a, b]$ at which $H$ is differentiable. An adjoint system, which has the same form as the original system, is defined. Theorems on solvability and compatibility are given. A Green's matrix is defined, and its properties are exhibited. (Received September 20, 1968.)


If $z = \{z_p\}$ is a complex sequence, let $\Delta z_p = z_p - z_{p+1}$, $\Delta^2 z_p = \Delta z_p - \Delta z_{p+1}$, etc. Let $B$ denote the set of all bounded real sequences, and if $k$ is a positive integer, let $M_k = \{ x \in B : \Delta^k x_p \geq 0, p = 1, 2, 3, ..., \}$. Let $B^*_k$ denote the set of all bounded complex sequences $z$ such that

$$
\sum_{p=1}^{\infty} \left( \frac{p+k-2}{k-1} \right) |\Delta^k z_p| < \infty.
$$

Theorem. A complex matrix $A = \{a_{pq}\}$ sums every sequence in $B^*_k$ if and
only if the following three conditions hold: (i) A has convergent columns, (ii) \( \sum_{q=1}^{\infty} a_{pq} 1_{[n+k-2]} \) converges, and (iii) there exists a number \( L \) such that \( \sum_{k=1}^{n} \sum_{q=1}^{\infty} a_{pq} 1_{[n+k-2]} < L \), \( n, p = 1, 2, 3, \ldots \). Since \( BV_k \) is the complex linear space generated by \( M_k \), then for \( k = 1 \) the theorem is a result of Hahn (Monatsh. f. Math. Phys. 32 (1922), 3-88) concerning sequences of bounded variation, and for \( k = 2 \) the theorem is essentially a result previously announced by the author (Abstract 653-111, these Notices 15 (1968), 110) concerning convex sequences. (Received September 20, 1968.)


Let \((X, U)\) be a uniform space. A mapping \( f : X \to X \) will be called a generalized contraction provided there is a collection of symmetric sets \( \{V_n\}_{n \in Z} \) cofinal in \( U \) (with respect \( \leq \)), which satisfy (i) \( V_i \subseteq V_j \) if \( i \leq j \), \( \cap_{n \in Z} V_n = \Delta_X \), \( \bigcup_{n \in Z} V_n = X \times X \), (ii) for each \( n \in Z \) there is an integer \( p(n) > 0 \) such that \( \{p(n) n \in Z\} \) is bounded and \( V_{n-p(n)} \subseteq V_n \), (iii) if \((x, y) \in V_n\) then \((f(x), f(y)) \in V_{n-1} \). Theorem. Let \( f : X \to X \) be a generalized contraction where \((X, U)\) is a complete uniform space. \( f \) has a unique fixed point. Corollary 1. Banach’s fixed point theorem. Corollary 2 [Edelstein]. If \( f : X \to X \) is \((\varepsilon, a)\)-uniformly locally contractive where \((X, d)\) is a complete metric space and if for each \((x, y) \in X \times X\) there is an integer \( n > 0 \) such that \( d(f^n(x), f^n(y)) < \varepsilon \), then \( f \) has a unique fixed point. (Received September 24, 1968.)


Let \( F_n \) denote the \( n \)th Fibonacci number. This paper characterizes those Fibonacci numbers which terminate in the same last two digits as their indices. Theorem. \( F_n \equiv n \pmod{100} \) if and only if \( n \equiv 0 \pmod{300} \) or \( n \equiv 1, 5, 25, 29, 41, \) or \( 49 \pmod{60} \). Corollary. If \( p \) is a prime greater than 3, then \( F_{p^2} \equiv p^2 \pmod{100} \). The proofs of this theorem and corollary do not involve any techniques other than those found in elementary number theory. (Received September 25, 1968.)


In Hayman’s book, Research problems in function theory, p. 38, Problem 6.11, the following problem is posed: If \( f \) and \( g \) are convex functions in \( S \), is it true that for \( 0 < \lambda < 1 \), \( \lambda f + (1 - \lambda) g \) is starlike? It is shown that the answer is no. Theorem. Let \( f \) be a convex function in \( S \). Define \( h \) in the open unit disc by \( h(z) = \lambda f(z) + (1 - \lambda) z \), \( 0 < \lambda < 1 \). Then \( h \) is close-to-convex. If \( 2/3 \not\equiv \lambda \), then \( f \) is starlike. \( \lambda < 2/3 \), then \( h \) need not be starlike. (Received September 23, 1968.)


Let \( \Lambda \) be a separable algebra with identity element over a commutative ring \( R \), and let \( C \) be the center of \( \Lambda \). Let \( G \) be a finite group of automorphisms of the \( R \)-algebra \( \Lambda \), and let \( \Gamma \) be the sub-
algebra of $G$-invariant elements in $A$. Assume the $G$ is faithfully represented as a group of automorphisms of $C$ by restriction. Then $\Lambda = \Gamma \otimes \Gamma \cap C$; and, for any subalgebra $\Omega$ of $\Lambda$ such that $\Gamma \subseteq \Omega$, the following statements are equivalent: (1) $\Lambda$ is a Frobenius extension of $\Omega$, (2) there exists a finite group $H$ of automorphisms of $\Lambda$ such that $\Omega$ is the subring of $H$-invariant elements in $\Lambda$, (3) $\Omega \cap C$ is a separable $R$-algebra and $\Omega = \Gamma \otimes \Gamma \cap C$, (4) $\Omega$ is a separable $R$-algebra. Since there is no assumption about the existence of idempotent elements in $C$, this result generalizes parts of a theorem of Kanzaki and DeMeyer [F. R. DeMeyer, Osaka J. Math. 2 (1965), 117-127, Theorem 3]. (Received September 23, 1968.)

663-98. J. T. HARDY, University of Georgia, Athens, Georgia 30601. On factorization of Cayley numbers.

G. Pall and O. Taussky [Factorization of Cayley numbers (to appear)] have given a complete and useful solution to the problem of finding and counting the factorizations of a Cayley number $x$ of norm $\lambda m$ as a product of Cayley numbers of norms $\lambda$ and $m$: $x = \lambda \mu$, $N(\lambda) = \lambda$, $N(\mu) = m$. Using their methods, this paper extends the results to other rings of integral elements in the Cayley algebra. (Received September 23, 1968.)

663-99. H. E. BENZINGER, 13115 Larchdale Road, Laurel, Maryland 20810. Green's function on large intervals. Preliminary report.

We consider a class of two-point boundary value problems on a closed interval $[a,b]$, where the differential equation has constant coefficients. The resulting Green's function $G(a, b, x, t, \rho^n)$ is then considered as $a \to -\infty$ and $b \to +\infty$. It is shown that for $\rho$ restricted to various regions of the complex plane, and for $x$ and $t$ restricted to specified subsets of the real line, $G$ converges to the Green's function of a singular problem. This discussion includes many nonselfadjoint problems, and also problems which are not regular in the sense of Birkhoff. (Received September 23, 1968.)

663-100. D. R. TURNIDGE, Kent State University, Kent, Ohio 44240. Rings of quotients of Morita equivalent rings.

Let $R$ and $S$ be associative rings with 1 and denote the categories of unitary left modules over $R$ and $S$ by $R^\mathbb{M}$ and $S^\mathbb{M}$ respectively. Say $R$ and $S$ are Morita equivalent if there exist inverse category equivalences $G: R^\mathbb{M} \to S^\mathbb{M}$ and $H: S^\mathbb{M} \to R^\mathbb{M}$. A class $\mathcal{J}(R)$ of left $R$-modules closed under submodules, homomorphic images, extensions, and arbitrary direct sums which contains no nonzero left ideal of $R$ is called a faithful Serre class of $R^\mathbb{M}$. Theorem 1. Let $R$ and $S$ be Morita equivalent and let $\mathcal{J}(R)$ be a faithful Serre class of $R^\mathbb{M}$. Then $\mathcal{J}(S) = \{ M \in S^\mathbb{M} | H(M) \in \mathcal{J}(R) \}$ is a faithful Serre class of $S^\mathbb{M}$. Moreover, the rings $Z_1(\mathcal{J}(R))$ and $Z_1(\mathcal{J}(S))$ of left quotients of $R$ and $S$ relative to $\mathcal{J}(R)$ and $\mathcal{J}(S)$ respectively are Morita equivalent. Theorem 2. Morita equivalent rings have Morita equivalent maximal rings of left quotients. Corollary. Let $R$ and $S$ be Morita equivalent. Then $Z(R) = 0$ if and only if $Z(S) = 0$. Theorem 3. Let $R$ be commutative and let $S$ be Morita equivalent to $R$. Then $S$ is left (and right) Ore and the classical rings of quotients of $R$ and $S$ are Morita equivalent. (Received September 23, 1968.)
Common fixed points for equicontinuous semigroups of mappings. Preliminary report.

W. M. Boyce and J. P. Huneke (Abstract 67T-218, these Notices 14 (1967), 280; and Abstract 67T-231, these Notices 14 (1967), 284) have shown that if f and g are two commuting continuous self-maps of I, the closed unit interval, then I need not contain a common fixed point of f and g. However, if one imposes the additional condition that the semigroup of maps generated by f and g (under functional composition) forms an equicontinuous family of functions, then f and g must have a common fixed point in I. This follows as a special case of the Theorem. Let S be a semigroup of equicontinuous self-maps of I. If S is left reversible (i.e. any two right ideals of S have nonempty intersection), then I contains a common fixed point of S. (Received September 23, 1968.)


Let (G, +) be a group and End G the endomorphisms of G. Then f: G → End G defines a left distributive binary operation * on (G, +) by a * b = f(a)(b), and conversely. Then (G, +, *) is a near-ring if, and only if, f(a * b) = f(a) o f(b) for all a, b ∈ G. Placing this and perhaps other restrictions on f, one can reduce the problem of computing examples of near-rings definable on certain finite groups to something manageable by a general purpose digital computer. These examples have proven useful for testing conjectures about near-rings and for formulating more meaningful conjectures for a larger class of near-rings. Several interesting theorems have been discovered by studying the examples of near-ring constructed in this manner. (Received September 23, 1968.)

663-103. JIANG LUH, North Carolina State University, Raleigh, North Carolina 27607.
On commuting automorphisms on rings.

An automorphism T on an associative ring R is called a commuting automorphism if xTx = xxT for every x ∈ R. T is said to be nontrivial if T is not the identity automorphism.

Theorem 1. If R is a primitive ring and if R possesses a nontrivial commuting automorphism, then R is a field. Theorem 2. If R is a prime Goldie ring and if R possesses a nontrivial commuting automorphism, then R is a commutative integral domain. These are generalizations of a result given by N. Divinsky (Trans. Roy. Soc. Canada Ser. III 49 (1955), 19-22). (Received September 23, 1968.)

663-104. ARUNAVA MUKHERJEA, Eastern Michigan University, Ypsilanti, Michigan 48197.
Random operator equations.

In this note we are concerned with the existence and uniqueness of certain random operator (r.o.) equations and consider as a concrete example a random integral equation of Fredholm type with a random kernel. We consider the equation (*) p(ω)T(ω, x(ω)) + a(ω) = x(ω) where ω is in Ω and (Ω, a, φ) is some probability measure space, T is a.s. a linear bounded r.o. on X, a separable Banach space and p(ω) (≠ 0, a.s.) is a real random variable, a(ω) & x(ω) being the known and unknown X-valued random variables. We show that (*) has a unique solution for every a(ω) iff there exists a linear r.o. T0 such that T(ω, ·) + T0(ω, ·) = p(ω)T0(ω, T0(ω, ·)) and p(ω)T0(ω, ·) - 1 is a.s. onto iff (p(ω)T(ω, ·) - 1)−1 exists a.s. and p(ω)T(ω, ·) - 1 is onto a.s. We also show how we can exploit this result to tackle the above mentioned integral equation problem. (Received September 25, 1968.)
Let $f$ be a continuous real-valued function defined on the set of $n$-tuples of nonnegative reals. If $A$ is an $m \times n$ complex matrix and $A^*$ is its conjugate transpose, then the nonnegative square roots of the eigenvalues of the $n \times n$ matrix $A^*A$ are called the singular values of $A$ and will be denoted here by $s_1(A) \geq s_2(A) \geq \ldots \geq s_n(A)$. Define $\hat{f}$, a real-valued function of the matrix $A$, by $\hat{f}(A) = f(s_1(A), \ldots, s_n(A))$. For various choices of the function $f$ the following question is answered: What is the nature of a linear transformation $T$ which maps the space $M_{m,n}$ of all $m \times n$ complex matrices into itself and satisfies $\hat{f}(T(A)) = \hat{f}(A)$ for all $A$ in $M_{m,n}$? For example, if $f$ is concave, symmetric, strictly increasing in each coordinate and $f(0, \ldots, 0) = 0$, then $\hat{f}(T(A)) = \hat{f}(A)$ for all $A$ in $M_{m,n}$ if and only if there exist $m \times m$ unitary $U$ and $n \times n$ unitary $V$ such that (i) for $m \neq n$, $T(A) = UAV$ for all $A$, (ii) for $m = n$, $T(A) = UAV$ for all $A$ or $T(A) = UATV$ for all $A$ ($A^T$ is the transpose of $A$). (Received September 25, 1968.)

663-106. WITHDRAWN.


Choose an ultrafilter $D$ on the set of prime rational integers $P$ to contain the filter on $P$ generated by the family $\{D_n\}$ where for each positive integer $n$, $D_n$ is the infinite subset (Dirichlet's theorem) of primes $p \equiv 1 \pmod{n}$. Let $R$ denote the ultraproduct with respect to $D$ of the family of prime fields of $p$ elements, $p \in P$. By the choice of $D$, the abelian closure of the field of rational numbers is a subfield of the quasi-finite field $R$, and hence is a subfield of $F = R((t))$, the field of formal power series in one variable over $R$. For each positive integer $n$, let $F_n = F(t^{1/n})$, a cyclic extension of degree $n$ of the local field $F$. The composite $L$ of the $F_n$ is a totally ramified, abelian extension of $F$, and $\text{Gal}(L/F) \cong \mathbb{Z}$. The abelian closure $A$ of $F$ is the composite of $L$ with the nonramified closure of $F$, and $\text{Gal}(A/F) \cong \mathbb{Z} \times \mathbb{Z}$. In this situation, an explicit reciprocity homomorphism $\phi : F^* \to \text{Gal}(A/F)$ can be given. In fact, a norm residue symbol can be given for the system of finite extensions of $F$. It should be remarked that the ultraproduct with respect to $D$ of the rational numbers admits a valuation for which $F$ is the residue field. (Received September 25, 1968.)


In Compositio Math. 15 (1963), 227-237, J. Nagata proved an important theorem (Theorem 1) which characterized the dimension of a metric space in terms of a topology-preserving metric. Subsequently in that article he gave further properties of that metric (Corollaries 2 and 3). It can be shown, however, that the metric constructed in Theorem 1 need not possess the properties mentioned in Corollaries 2 and 3. Further, for connected metric spaces of dimension greater than zero, there is no topology-preserving metric which possesses the property of Corollary 2. It is not known if there exists a topology-preserving metric which possesses the property of Corollary 3. (Received September 25, 1968.)
Consider the system of nonlinear difference equations \( y(x + 1) = Ay(x) + f(x, y(x)) \) where \( x \) is a complex variable, \( y \) and \( f \) are \( n \)-dimensional vectors, and \( A \) is an \( n \times n \) matrix all of whose eigenvalues have modulus one. \( f(x, y) \) is assumed to be holomorphic for \( \|y\| < \delta \) and \( x \in S \), a sector containing a right half-plane. Moreover, \( f(x, y) \) is assumed to have an uniform asymptotic expansion in powers of \( x^{-1} \) as \( x \) tends to infinity in \( S \). It is shown that under suitable small divisor restrictions on the eigenvalues of \( A \) the system can be linearized by means of a holomorphic transformation which has an asymptotic expansion valid in a subsector of \( S \) containing a half-plane. This extends the results given previously for the case when the eigenvalues of \( A \) have modulus different from one [Abstract 642-133, these Notices 14 (1967), 102]. (Received September 25, 1968.)
Let \( \{f_n\} \) be a sequence of functions from a compact Hausdorff space \( H \) into a Banach space \( X \). Let the measure function \( K \) be a finitely additive operator-valued set function defined on a field \( E \) of subsets of \( H \). Suppose also that the sequence \( \{f_n\} \) of functions is integrable with respect to \( K \) as defined and used in recent works by D. J. Uherka, D. H. Tucker, and R. J. Easton. The concepts of convergence almost everywhere and convergence in measure are introduced and it is shown by example that pointwise convergence and almost everywhere convergence do not imply convergence in measure, that convergence in measure does not imply a subsequence converges almost everywhere, that a sequence can converge almost everywhere to a function \( g = 1 \) and converge in measure to a function \( f \equiv 0 \), and finally that a sequence \( \{f_n\} \) and a function \( f \) may satisfy \( \lim_{n \to \infty} \|f_n - f\|_1 = 0 \) but \( \{f_n\} \) need not converge to \( f \) in measure. It is shown that if one replaces convergence almost everywhere by convergence in measure in the classical dominated convergence theorem, then it remains a theorem in this setting. (Received September 26, 1968.)

We adopt here the notation of P. S. Schnare (Fund. Math. 62, 53-59) with the addition that we allow \( s(x, \mathcal{U}(x)) \) to indicate the discrete topology on \( X \). The following results are easy to obtain, but considerably shorten the proofs of and improve upon a number of results of A. K. Steiner (Trans. Amer. Math. Soc. 122 (1966), 379-398) and O. Fröhlich (Math. Ann. 156, 79-95). Lemma 1. \( t \in s(x, \mathcal{U}) \) if and only if \( \mathcal{U} \) converges to \( x \) with respect to the topology \( t \). Corollary 1. \( t \) is \( T_2 \) if and only if \( t \in s(x, \mathcal{U}) \) and \( t \in s(y, \mathcal{U}) \) implies \( x = y \). Corollary 2. \( t \) is compact if and only if for each ultrafilter \( \mathcal{U} \) on \( X \), there exists an \( x \in X \) such that \( t \in s(x, \mathcal{U}) \). The usual minimax property of a compact Hausdorff space follows immediately. Following A. K. Steiner, if \( t \) is a topology on \( X \), we define the relation \( G \) on \( X \) by \( x \sim y \) if and only if \( t \in s(x, \mathcal{U}(y)) \). Lemma 2. \( x \sim y \) if and only if \( [x] \cap [y] \neq \emptyset \), where the closures are taken with respect to \( t \). Corollary 3. \( t \) is \( T_0 \) if and only if \( G \) is a partial ordering of \( X \). Corollary 4. \( t \) is \( R_0 \) (A. S. Davis, Amer. Math. Monthly 68, 886-893) if and only if \( G \) is an equivalence relation on \( X \). (Received September 26, 1968.)

We consider finite difference approximations to the Cauchy problem for first-order hyperbolic and second-order parabolic partial differential equations using different mesh spacings in different portions of the domain. By reformulating our problem as a difference approximation to an initial-boundary value problem, we are able to use the theory of H. O. Kreiss and S. Osher to prove stability of our schemes. Stable interface conditions between general mesh patterns are developed by using interpolation. In particular, the alternating direction schemes of D. W. Peaceman and H. H. Rachford are extended in a stable manner to our mesh refinement schemes. We computed test problems for the schemes discussed in our theoretical treatment. Our numerical experiments indicate that our techniques are an efficient method for improving accuracy in regions of special interest, while preventing new inaccuracies at the interface. (Received September 9, 1968.)
A topological ring $A$ is called a J-ring if for each $x$ in $A$, $x$ belongs to the closure of \( \{ x^n : n \geq 2 \} \). Theorem. A locally compact ring $A$ is a J-ring if and only if $A$ is the topological direct sum of a discrete J-ring and a ring $B$ which is topologically isomorphic to the local direct sum of a family of discrete J-rings with respect to finite subfields. In particular, all such rings are commutative, a result which extends a well-known theorem of Jacobson for the discrete case. Analogues of two of Herstein's generalizations of Jacobson's theorem are given in the following theorem: all semisimple locally compact rings such that, for all $x$ and $y$, $xy - yx$ is in the closure of $\{ x^n y - yx^n : n \geq 2 \}$ are commutative. The proofs of the theorems rely on the structure of locally compact rings developed by Kaplansky. (Received September 26, 1968.)

The interior flow of a vertical jet under gravity.

The flow properties on the free streamlines that result from flow through a slit in the lower of two horizontal planes which bound a liquid have been determined in terms of a nonlinear integral equation. Complex analysis can be used to yield the interior flow in terms of this same integral equation. (Received September 26, 1968.)

Injective envelopes of semisimple modules. Preliminary report.

In his doctoral thesis H. Bass showed that if an arbitrary direct sum of injective left modules over a ring $R$ with identity is always injective, then it must follow that the ring is left noetherian (the converse was well known). This result is herein extended by demonstrating the equivalence of the following three conditions concerning the set $\{ S_a \}_{a \in A}$ of pairwise nonisomorphic simple left modules over a ring $R$ with identity, where $\hat{S}_a$ is the injective envelope of $S_a$: (1) $\prod_{a \in A} \hat{S}_a$ is isomorphic to a direct sum of nonzero left modules indexed on $A$; and for each $a \in A$ the product $\hat{S}_a \omega$ is isomorphic to a direct sum of nonzero left modules indexed on $\omega$; (2) $\bigoplus_{b \in B} \hat{T}_b$ is injective where $B$ is any set and each $T_b$ is isomorphic to some $S_a$; (3) $R$ is left noetherian. Each of these conditions implies that no cyclic left module can contain an infinite direct sum of simple left modules; this property, which is equivalent to every finitely generated left module being finite dimensional (containing no infinite direct sum of nonzero left modules), implies that $R$ is left noetherian in the presence of the following chain condition: there exists no infinite ascending chain of essential extensions of a simple left module, contained in any cyclic left module. (Received September 26, 1968.)

Asymptotic behavior of solutions to some nth order linear differential equations.

Let $L u = \sum_{j=0}^{n-2} a_j(t) u^{(j)}$, $a_j(t)$ continuous on $[t_0, \infty)$, and let $z_j(t)$, $j = 1, \ldots, n$, be linearly independent solutions to $L z = 0$. Let $W(t) = W(z_1(t), \ldots, z_n(t))$ be the Wronskian of $z_j(t)$ and let $W_j(t)$ be the determinant obtained by replacing all elements of the jth column of $W(t)$ by zeros except for the nth element which is replaced by 1. Consider solutions to $L u = \sum_{j=0}^{n-2} g_j(t) u^{(j)}$, $g_j(t)$ continuous on
[t_0, \infty). \text{ Let } y(t) = \max_{k=1, \ldots, n} \sum_{j=0}^{n-2} g_j(t)z_k^{(i)}(t)]. \text{ Theorem. Given arbitrary constants } A_j, j = 1, \ldots, n, \text{ there exists a solution to (1) in the form (2) } u(t) = \sum_{j=1}^{n} A_j(t)z_k(t), \text{ with (3) } \lim_{t \to \infty} A_j(t) = A_j < \infty \text{ iff (4) } \sum_{j=1}^{n} \int_{t_0}^{t} |y(t)| dt < \infty. \text{ Also, every solution to (1) can be written in the form (2) with } A_j(t) \text{ satisfying (3) iff (4) is satisfied. This generalizes a result of Trench, Proc. Amer. Math. Soc. 14 (1963), 12-14. (Received September 27, 1968.)}

663-119. J. C. OWINGS, JR., University of Maryland, College Park, Maryland 20742. An elementary approach to Diophantine equations of the second degree.

An easy method is introduced for finding all solutions in integers to certain Diophantine equations of the second degree. Using it, one can quickly solve any 2-variable equation

\[ ax^2 + bxy + cy^2 + dx + ey + f = 0, \]

not a perfect square, satisfying the conditions \(a \neq 0, b \neq 0, c \neq 0, a|b, a|d, c|b, c|e, \) by producing the solutions as a finite set of chains of the form ... \(x(-2), y(-2), x(-1), y(-1), x(0), y(0), x(1), y(1), x(2), y(2) \ldots \) in which \((x(n), y(n))\) and \((x(n), y(n - 1))\) are solutions for every \(n. \) For example, every solution to the equation \(x^2 + y^2 - 3xy - 2y - 1 = 0\) lies in the single chain ... \(-16, -7, -3, -2, -1, 0, 1, 5, 14, 39, 103, 272 \ldots \) in which all \(x\)-values have been underlined. The method is applied to the 3-variable equation \(x^2 + y^2 + z^2 - xy - xz - yz - x - 2y - 3z = 0\) with the result that every solution is shown to lie in one of two "planes" of solutions, a plane of solutions being an infinite planar triangular lattice of integers in which each atomic triangle constitutes a solution. One plane is generated by \((x, y, z) = (0, 0, 0),\) the other by \((x, y, z) = (-1, 1, 1).\) One finds there are 472 solutions \((x, y, z)\) with \(x < 100, y < 100, z < 100;\) all but 3 of them -- \((-1, 1, 1), (-1, 1, 2),\) and \((-1, 2, 2) -- \) have strictly positive entries. (Received September 27, 1968.)

663-120. R. M. KOCH and FRANKLIN LOWENTHAL, University of Oregon, Eugene, Oregon 97403. On the Riccati equation.

The nature of all solutions of the Riccati equation: \(dy/dz = a_0(z) + a_1(z)y + a_2(z)y^2\) with \(a_0, a_1, \) and \(a_2\) all analytic in a domain \(D\) of the complex plane and \(a_2 \neq 0\) is determined through the use of the function \(g = f_1 - f_2/f_3 \) where \(f_1, f_2\) and \(f_3\) are particular solutions. Necessary and sufficient conditions are formulated for one function, two functions, and three functions respectively to simultaneously satisfy a Riccati equation. The special cases where \(D\) is the complex plane and where \(D\) is the complex plane with a single point deleted are studied in detail. It is shown that either all solutions have a finite number of poles or all solutions except at most two have an infinite number of poles. In the former case, the nature of both the equation and the solutions is determined in detail. (Received September 27, 1968.)


Consider the second order matrix differential equation \(Y'' + P(x)Y = 0,\) where \(P(x)\) is an \(n \times n\) symmetric matrix of continuous, real-valued functions on \([a, \infty), a \neq 0.\) A necessary condition that (1) be nonoscillatory is that every one of a certain set of \(n\) independent scalar equations of the form (1) be nonoscillatory. For many necessary conditions for the nonoscillation of (1) in the scalar case, this fact will yield analogous necessary conditions for the nonoscillation of (1) in the general
case. The following are just two examples of the many theorems that can be proven using this technique. Theorem 1. If \( P(x) \) is positive semi-definite on \([a, \infty)\) and (1) is nonoscillatory, then \( \lim \sup \int_a^\infty \text{trace} \ P(x) \, dx \leq n \) and \( \lim \inf \int_a^\infty \text{trace} \ P(x) \, dx \geq n/4 \). Theorem 2. If (1) is nonoscillatory and \( \lim P(x) = 0 \), then either \( \lim x^{-1} \int_a^x \text{trace} \ P(x) \, dx = -\infty \) or \( \lim x^{-1} \int_a^x \text{trace} \ P(x) \, dx \) exists as a finite limit. In the scalar case, \( n = 1 \), Theorem 1 is due to Hille \[\text{Trans. Amer. Math. Soc. 64 (1948), 234-252}\] and Theorem 2 is due to Hartman \[\text{Amer. J. Math. 74 (1952), 389-400}\]. Extensions are given for \( [R(x)Y'] + P(x)Y = 0 \). (Received October 14, 1968.)

663-122. H. L. BENTLY, University of New Mexico, Albuquerque, New Mexico 87106. A lattice theoretic characterization of topological categories.

A topological category is a category which is isomorphic to the category of topological spaces and continuous maps. Topological categories can be characterized by lattice theoretic methods. The theorem which presents this characterization is roughly as follows: A topological category is a category which carries at the same time the structure of a complete lattice, which can be mapped functorially onto the category of sets and whose structure is determined by certain axioms relating these two properties. Besides being intrinsically interesting, this theorem provides a set of axioms of which all of the multitudinous axiomatizations of topological spaces are special cases. (Received September 27, 1968.)

663-123. J. B. DIAZ and J. R. MCLAUGHLIN, Rensselaer Polytechnic Institute, Troy, New York 12181. The divergence need not possess the Darboux property.

Consider the **Pseudo-lemma**. Suppose: (1) \( D \) is a nonempty, open, connected, bounded set in \( \mathbb{R}^n \) (real \( n \)-dimensional Euclidean space) with boundary \( B \); an element \( x \in \mathbb{R}^n \) will be denoted by \( x = (x_1, ..., x_n) \); (2) \( f = (f_1, ..., f_n) \) is a vector valued function, with real components, defined and continuous on \( D + B \); (3) the partial derivative \( \partial f_i / \partial x_i \) exists in \( D \), for \( i = 1, ..., n \); \( \text{div} \ f = \sum_{i=1}^n \partial f_i / \partial x_i \geq 0 \) in \( D \), while div \( f \neq 0 \) in \( D \). Then: \( f \neq \text{const.} \) on \( B \). For \( n = 1 \), this lemma may be proved directly from the elementary mean value theorem of the differential calculus. However, for \( n \geq 2 \), there are functions \( f \) which satisfy the hypotheses of the pseudo-lemma, but not the conclusion. These examples may be constructed starting from a classical example of G. Peano. Peano's example, suitably interpreted, already shows that for \( n \geq 2 \) there are functions \( f \), defined on a nonempty, connected, open set, such that the set of values of div \( f \) consists only of the two numbers 0 and 1; whereas, for \( n = 1 \), there cannot be any such function, in view of the Darboux intermediate value property of a derivative. (Received September 27, 1968.)


Let \( y = p e^{i\theta} \) be a solution of the general second order differential equation in normal form:

\[ y'' + R(x)y = 0. \]

If \( R(x) \) is real, it is shown that \( p \) satisfies a linear differential equation of third order, while if \( R(x) \) is complex it satisfies a linear equation of fourth order. (Received September 27, 1968.)
663-125. GEORGE ADOMIAN, University of Georgia, Athens, Georgia. Closure approximation in Stochastic differential equations.

In a number of applications, e.g., turbulence theory, scattering and propagation of waves, and many-body problems, stochastic equations arise. In commonly used averaging methods, the so-called hierarchy equations arise from the dynamical equations. Ensemble averages are obtained through a closure approximation which is generally not valid. The significance and error involved is defined at each level of approximation. (Received September 30, 1968.)

663-126. VADIM KOMKOV, Florida State University, Tallahassee, Florida 32306. On optimal control of vibrating thin plates.

Using the results of the author (SIAM J. Control (3) 6 (1968), and Abstract 650-11, these Notices) 14 (1967), 919), boundary conditions are specified for which the analogous formulae are applicable to thin vibrating plates. Some other cases are also discussed, where complicated boundary terms appear in the corresponding maximum principle for a fixed time interval. However, even in these cases it is possible to make additional assumptions concerning the flexural rigidity and density (for example \( \phi^4(D,w) = 0 \)) so that a fairly simple formulae emerge for the maximum principle. (Received September 30, 1968.)

663-127. ROBERT HELLER, Mississippi State University, State College, Mississippi 39762. A continued fraction convergence theorem.

Let \([t_1, t_2, t_3, \ldots] = \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \ldots}}} \)

**Theorem.** Let \( R \) denote the set to which a sequence \( a \) belongs if and only if for each positive integer \( n \), \( a_n > 0 \).

(a) If \( a \in R \), then the sequences \( f_1, f_4, f_5, f_8, f_9, \ldots \) and \( f_2, f_3, f_6, f_7, f_{10}, \ldots \) are absolutely convergent. (b) There exists a sequence \( a \) in \( R \) such that the sequence \( f_1, f_2, f_3, f_4, \ldots \) is divergent. (c) If \( a \in R \) and either the sequence \( \{a_{2p-1}^{-1}\} \) or the sequence \( \{1 + a_{2p-1}^{-1}\} \) has a bounded infinite subsequence, then the sequence \( f_1, f_2, f_3, f_4, \ldots \) is convergent.

(Received September 30, 1968.)


Let \( f(t) \) be a real Borel measurable function defined on \([0, \infty)\). Let \( (\xi) \) denote the fractional part of the real number \( \xi \). Define the following class of distribution functions: \( F_T(\xi) = \frac{1}{B(T)} \int_{[0,T]} \chi(\xi - f(t))dB(t) \) for all \( T \) with \( B(T) > 0 \), where \( F_T(\xi) = 0 \) (\( \xi \equiv 0 \)), \( F_T(\xi) = 1 \) (\( \xi > 1 \)). Let \( F(\xi) \) be a distribution function with \( F(\xi) = 0 \) (\( \xi \equiv 0 \)), \( F(\xi) = 1 \) (\( \xi > 1 \)), \( \Delta F(0) = \Delta F(1) \) (or, the increments of \( F \) at 0 and 1 being equal). Then, using probability theory methods, Professor A. J. Stam, Groningen, Netherlands, and author showed that \( F_T(\xi) \) converges completely to \( F(\xi) \) as \( T \to \infty \) if and only if for \( k = 1, 2, \ldots, \lim_{T \to \infty} \int_{[0,T]} \exp 2\pi ikf(t) dB(t) \)

\( = \int_0^1 \exp 2\pi ikx \ dF(x) \). This result implies the Weyl criterion and many of its generalizations and its analogues such as the Niven-Uchiyama criterion in the theory of uniform distribution modulo \( m \) (\( m \) being a positive integer \( \equiv 2 \)). (Received September 30, 1968.)
If \( P \) is a property of topologies, a \( P \)-space \((X, T)\) is called \( P \)-minimal provided that, for every \( P \)-topology \( T' \) on \( X \), \( T' \subset T \) implies that \( T' = T \). A \( P \)-space \((X, T)\) is said to be \( P \)-closed if it is closed in every \( P \)-space in which it can be embedded. In this paper we consider first countable- and \( P \)-minimal spaces and first countable- and \( P \)-closed spaces for various separation properties \( P \). Such spaces are characterized, and several product and embedding theorems are obtained. (Received September 30, 1968.)

Dual series and harmonic mixed boundary value problems.

Dual trigonometrical series occur frequently in the analysis of harmonic mixed boundary value problems; in particular those for which the region of definition is a semi-infinite strip. However, solution of series of this type involves complicated mathematical analysis, and many of the existing methods have involved an initial guess as to the form of the solution. In this paper we give new methods based on orthogonality relations for Legendre polynomials to overcome this difficulty. (Received September 30, 1968.)

A class of rings with all singular simple modules injective.

A ring \( R \) with unit is a \( T \)-ring if every nonzero \( R \)-module has nonzero socle. This paper is an investigation of \( T \)-rings with the property (*): Every nonprojective simple module is injective.

Theorem 1. A \( T \)-ring \( R \) satisfies (*) if and only if \( z(R) = 0 \) and \( \text{rad} (R/I) = 0 \) for every essential left ideal \( I \) of \( R \). Theorem 2. If a \( T \)-ring \( R \) satisfies (*), then every essential left ideal of \( R \) is idempotent and \( (\text{rad} R)^2 = 0 \). As a consequence, a commutative \( T \)-ring satisfies (*) if and only if \( R \) is regular, while a \( T \)-ring satisfying (*) is regular if and only if \( \text{rad} R = 0 \). Finally, Theorem 3. (a) If \( R \) is a \( T \)-ring, then \( R_n \) is a \( T \)-ring for all \( n \geq 1 \); (b) a \( T \)-ring \( R \) satisfies (*) if and only if \( R_n \) satisfies (*) for all \( n \geq 1 \). (Received September 30, 1968.)

A new asymptotic expansion and approximation for the ratio of two gamma functions.

Many problems in mathematical analysis, statistics and applied mathematics require a knowledge of an approximate value of the ratio \( \Gamma(n_1 z + a)/\Gamma(n_2 z + \beta) \) or its asymptotic behaviour for large values of \( |z| \), where \( n_1 \) and \( n_2 \) are integers and \( a \) and \( \beta \) are bounded. As Stirling's series does not give a good approximation, the problem has been studied by Frame (1949), Tricomi and Erdélyi (1951), and Fields (1966) for particular cases. Using Tricomi and Erdélyi's asymptotic expansion for \( \Gamma(z + a)/\Gamma(z + \beta) \) in terms of generalised Bernoulli polynomials, defined by Nörlund (1924), a new asymptotic expansion for the ratio \( \Gamma(n_1 z + a)/\Gamma(n_2 z + \beta) \) has been obtained in even powers of another quantity \( M = (1/2)[(n_1 + n_2)z + a + \beta - 1] \), in place of \( z \). It has been tested by a number of computations that the first two terms of the new asymptotic expansion give an approximation of 99.5% accuracy. (Received September 30, 1968.)

The notion of uniform Hjelmslev planes (H-planes) is generalized in two ways. An ordinary affine or projective plane is called 1-uniform. For an H-plane \( \pi \) to be n-uniform, the neighborhood structure of each point of \( \pi \) must be an \((n - 1)\)-uniform affine H-plane. If \( \pi \) is n-uniform, then the invariants \( s, t \) of \( \pi \) satisfy \( s = r^n, t = r^{n-1} \) for some integer \( r \). Distinct points of \( \pi \) are joined by \( r^i \) lines, \( i \leq n - 1 \); distinct intersecting lines meet in \( r^i \) points. We write \( Q(\sim i)P \) to mean \( Q \) is joined to \( P \) by at least \( r^i \) lines. Let \( P \in \pi \). The number of points \( Q \) on \( \pi \) satisfying \( Q(\sim i)P \) is \( r^{n-1} \) for \( 1 \leq i \leq n \); the number of lines \( k \) through \( P \) such that \( |k \cap \pi| = r^i \) is \( r^{n-1} \). From an n-uniform projective H-plane \( \pi \), form a structure \( \pi^1 \) by identifying points \( P, Q \) which satisfy \( P(\sim (n - 1))Q \); lines \( \pi, k \) which satisfy \( |\pi \cap k| \geq r^{n-1} \). For a projective H-plane \( \pi \) to be strongly n-uniform, we demand that \( \pi^1 \) be strongly \((n - 1)\)-uniform, the strongly 1-uniform H-planes being the ordinary projective planes. Let \( \sigma \) be an n-uniform projective H-plane with dual \( \sigma^* \). Then \( \sigma \) is strongly n-uniform if and only if \( \sigma^* \) is n-uniform if and only if \( \sigma \) satisfies: when \( P \in \pi \cap k, |\pi \cap k| = r^i, Q \in \pi \), \( P(\sim j)Q, P(\neq j - 1)Q, i + j < n \), then \( Q \) is joined to some point of \( k \) by \( r^{i+j} \) lines; but to no point of \( k \) by more than \( r^{i+j} \) lines. (Received September 30, 1968.)

663-134. G. A. GRATZER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Two Mal'cev type theorems in universal algebra.

A class \( K \) of algebras is (weakly) regular if for every \( \Pi \in K \) (there exists an \( o \in A \), \( \Theta, \Phi \) congruence relations of \( \Pi \), if \( \Theta \) and \( \Phi \) have one congruence class (containing \( o \) in common, then
\( \Theta = \Phi. \) Theorem. For every positive integer \( n \), a set of identities \( \Sigma_n (\Omega_n) \) is given such that an equational class \( K \) is (weakly) regular iff \( \Sigma_n (\Omega_n) \) holds for some \( n \). From the form of \( \Sigma_n \) and \( \Omega_n \) we conclude (1) An equational class \( K \) is (weakly) regular iff every algebra of \( K \) generated by three elements is (weakly) regular. (2) There is an algorithm to decide whether the equational class generated by a finite algebra of finite type is (weakly) regular. (3) A weakly regular and idempotent \( (f(x, x, \ldots) = x \) for all operations) equational class is regular. (Received September 30, 1968.)

663-135. J. H. HEDLUND, University of Massachusetts, Amherst, Massachusetts 01002. Multipliers of \( H^p \) spaces.

Let \( H^p \) denote the space of functions \( f(z) \) analytic in the unit disc for which the functions \( f(\theta) = f(re^{i\theta}) \) are bounded in \( L^p \) norm as \( r \to 1 \). A sequence of complex numbers \( \lambda = \{\lambda(n)\} \) is said to be a multiplier of \( H^p \) to \( H^q \) if \( \lambda(z) = \sum \lambda(n) a(n) z^n \) belongs to \( H^q \) for every \( f(z) = \sum a(n) z^n \) in \( H^p \). The multipliers from \( H^p \) to \( H^q \) are studied. The principal result is the sufficient condition that if \( 1 \leq p \leq 2 \) and \( q = 2p/(2 - p) \) then \( \lambda \) is a multiplier of \( H^p \) to \( H^2 \) if \( \sum \lambda(n) q^n = O(N) \). This condition is derived from the known cases \( p = 1 \) and \( p = 2 \) by a method of operator interpolation. (Received September 30, 1968.)
A convergence characterization of Lindelöf.

For definitions of well ordered sequence (WOS), quasi-convergence, and linear Lindelöf, see Abstract 654-33, these Notices 15 (1968), 346, where the following appeared: Theorem. Lindelöf, linear Lindelöf, and the property that each WOS with no countable WOSS quasi-converges are equivalent in a countably metacompact space. A net $\psi : D \rightarrow X$ is $\sigma$ complete if for each $\{d_a | a \in \Lambda\} \subseteq D$, where $\text{card}(\Lambda) = \sigma$, there exists a $d \in D$ with $d_a \leq d$ for each $a \in \Lambda$. $\psi$ quasi-converges to a point $p \in X$ if for each neighborhood $N$ of $p$ there exists a cofinal $C \sub D$ such that $\psi(C) \subseteq N$. Theorem. A space is Lindelöf iff each $\omega$ complete net in $X$ quasi-converges (where $\omega$ is used to denote $\aleph_0$). Necessary and sufficient conditions are also given in order that a linear Lindelöf space be Lindelöf. (Received September 30, 1968.)


Following and extending the terminology of J. de Groot and J. M. Aarts, Complete regularity as a separation axiom, to appear in Canad. J. Math., and J. de Groot, Connectedly generated spaces and their compactifications, in preparation, a space is said to be $T_3 \text{ sub c.g.}$ if the collection of all connected closed sets is a regular subbase. Theorem. Locally compact and $T_3 \text{ sub c.g.}$ imply locally connected. (Received September 30, 1968.)

Semigroup actions. Preliminary report.

All spaces referred to here are Hausdorff. Let $S$ be a topological semigroup and $X$ a topological space. $S$ is said to Act on $X$ if there exists a continuous onto function $\mu : S \times X \rightarrow X$ such that $\mu(s,\mu(t,x)) = \mu(st,x)$ for all $s, t \in S$ and all $x \in X$. $\mu(s,x)$ is denoted by $sx$. $S$ acts effectively if $sx = tx$ for all $x \in X$ implies $s = t$. Actions with various restrictions on $S, X$ and $\mu$ are considered. Theorem 1. Let $S$ be a compact simple semigroup Acting on $X$ such that $Sx = X$ for some $x \in X$, then $S$ acts transitively on $X$ (i.e., $Sx = X$ for all $x \in X$). Theorem 2. Let $S$ be a compact simple band Acting transitively and effectively on $X$, then $S$ must be a left zero semigroup, $X$ and $S$ are homeomorphic and the Action is equivalent to multiplication in $S$. Theorem 3. Let $S$ be a compact simple semigroup Acting effectively on $X$, (1) if $tx$ is a point for some $t \in S$, then $S$ is a band. (2) Conversely, if $S$ is a band that Acts transitively, then $tx$ is a point for all $t \in S$. Theorem 4. Let $S$ be a compact, connected semigroup Acting transitively on $X$ such that $tx$ is nondegenerate for all $t \in S$, then $X$ has no cut-points. Theorem 5. Let $S$ be a clan Acting effectively on a metric indecomposable continuum $Y$. Then $S$ is a group. (Received September 30, 1968.)

Two theorems on support points.

A support point of a convex set $C$ in a real topological linear space $E$ is a point $x$ of $C$ for which there is a nonzero continuous linear functional on $E$ which attains its supremum on $C$ at $x$. 

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An example is given of a bounded closed convex set $C$ in a Fréchet space with no support points, answering a question of Klee (Amer. J. Math. 85 (1963), 95-98). The Fréchet space can be taken to be any countable product of nonreflexive Banach spaces. The proof uses a theorem of James that on every nonreflexive Banach space $B$, there is a continuous linear functional which does not attain its supremum on the closed unit ball of $B$. It is also proved that a $C$ of the above sort always has points at which it can be supported by linear functionals on the containing Fréchet space which are continuous on $C$ and nonvanishing on $C$. The proof of this applied a support theorem of Bishop and Phelps to a suitably chosen Banach space. (Received September 30, 1968.)


A lattice $L$ is called antidistributive if it is not distributive and whenever $a > b > c > d$, either $\{x : c \leq x \leq a\} = \{c,a\}$ or $\{b,d\}$ is a nondistributive sublattice. We say $L$ is locally complemented if for every $0 \neq x \neq 1$ there is an $x' \in L$ such that $x \vee x' > x \wedge x'$. In Abstract 642-55, these (Notices 14 (1967), 77), we observed that a finite-dimensional antidistributive modular lattice is simple if and only if it is locally complemented. We can now show modularity can be weakened to semimodularity and the following: Theorem 1. If $L$ is a locally complemented antidistributive lattice of finite length which satisfies the Jordan-Dedekind chain condition, then $L$ is simple. Theorem 2. A finite-dimensional antidistributive lattice is simple if each finite subset is contained in a locally complemented interval sublattice which satisfies the Jordan-Dedekind chain condition. (Received September 30, 1968.)


The following results are consequences of the topological engulfing theorem of M.H.A. Newman (Ann. of Math. 84 (1966), 555-571). Theorem 1. If $M^n$ is a $k$-connected topological $n$-manifold (without boundary), and $q$ is the minimum of $k$ and $n - 3$, then $M^n$ can be covered by $p$ open cells if $p(q + 1) > n$. Furthermore, these cells may be chosen so that the intersections of any collection of these cells is $(q - 1)$ connected. Corollary. A contractible manifold, $n \geq 5$, is the union of two open cells whose intersection is a contractible open manifold. Theorem 2. If $M^n$ is a compact manifold with boundary and $M^n$ and $\partial M^n$ are $q$-connected, $q \leq n - 4$, then $M^n$ can be covered by $p$ closed cells if $p(q + 1) > n$. (Received October 1, 1968.)

663-142. S. K. SINGH, University of Missouri, Kansas City, Missouri 64110, and V. N. KULKARNI, Karnatak University, Dharwar, India. Characteristic function of a meromorphic function and its derivative.

Let $f(z)$ be a meromorphic function of order $\rho$ ($0 \leq \rho \leq \infty$). Let $n(r,a)$, $N(r,a)$, $m(r,a)$, $\overline{N}(r,a)$, $\phi(a) = \phi(a,f)$, $\Delta(a) = \Delta(a,f)$ and $T(r,f)$ have the usual meaning as in Nevanlinna theory. In this paper, we prove the following: Theorem 1. Let $f(z)$ be a meromorphic function of order $\rho$ ($0 < \rho < \infty$), then (i) $\lim_{r \to \infty} T(r,f')/T(r,f) < \infty$, (ii) $\lim_{r \to \infty} T(Kr,f)/T(r,f) < \infty$ where $K > 1$. Theorem 2. If $f(z)$ is a meromorphic function of finite order having $\{a_i\}$ and only $\{a_i\}$ as e.v. N (exceptional value
in the sense of Nevanlinna) where the $a_i$'s are distinct ($0 \leq |a_i| \leq \infty$) and if $\sum_{a_i \neq \infty} \delta(a_i) = \alpha$ and $\sum_{a_i} \delta(a_i) = 2$, then $T(r,f_t) \sim T(r,f)$. As corollaries we deduce that if $\sum_{a_i \neq \infty} \delta(a_i) = 1$, $\delta(\infty) = 1$, then $T(r,f_t) \sim T(r,f)$ and if $\sum_{a_i \neq \infty} \delta(a_i) = 2$, then $T(r,f_t) \sim 2T(r,f)$. Theorem 3. If $f(z)$ is a meromorphic function of finite order such that $T(r,f_t) \sim aT(r,f)$, $a ;< 1$, then $T(r,f') \sim T(r,f)$. As corollaries we deduce that if $\sum_{a_i \neq \infty} \delta(a_i) = 1$, $\delta(\infty) = 1$, then $T(r,f') \sim T(r,f)$ and if $\sum_{a_i \neq \infty} \delta(a_i) = 2$, then $T(r,f') \sim 2T(r,f)$. Theorem 4. (i) If $f(z)$ is a meromorphic function of finite order with $[a_i]_1$ as e.v. $N$ such that $\sum_{a_i \neq \infty} \delta(a_i) = \alpha$ and $\delta(\infty) = 2 - \alpha$, then $\delta(\infty) = 2$. (Received October 2, 1968.)

663-143. F. A. ROACH, University of Georgia, Athens, Georgia 30601. Functions defined by continued fractions over a vector space. Preliminary report.

Let $S$ be a real inner product space and $u$ be a point of $S$ with norm 1. If $x$ is a point of $S$ distinct from 0, the point $[2((x,u))u - x]/\|x\|^2$ is denoted by $1/x$. Making use of a type of continued fraction based on this "reciprocal" (see Abstract 656-64, these Notices 15 (1968), 521), certain classes of functions are described and some of their properties established. For example, the following provides a generalization of ordinary linear fractional transformations: with each finite sequence $b_0,b_1,...,b_n$ from $S$, a transformation $T_b$ is associated by means of the equation $T_b(x) = b_0 + \sum_{i=0}^{n} b_i + x$. (Received October 2, 1968.)

663-144. KWANGIL KOH, North Carolina State University, Raleigh, North Carolina 27607. On the reducibility of an object in the exact category.

Let $E$ be an exact category with unions and $U$ be an object in $E$. Let $S(U)$ be the family of subobjects of $U$. We say $N \in S(U)$ is superfluous in $U$ provided that $N \cup I = U$ for any $I \in S(U)$ implies that $I = U$ (see Abstract 653-75, these Notices 15 (1968), 99). We say $U$ is completely reducible if $U$ is the union of the minimal subobjects of $U$. We say $M \in E$ has a projective cover if there is a projective object $P$ and an epimorphism $f$ from $P$ onto $M$ such that the kernel of $f$ is superfluous.

Theorem A. If an object in $E$ has a projective cover then it is unique. Theorem B. Suppose $U$ is a projective object in $E$ and $J(U) =$ the union of all superfluous subobjects of $U$ exists (Abstract 653-75, ibid.). If every quotient object of $U$ has a projective cover, then $U/J(U)$ is completely reducible. (Received October 2, 1968.)


Let $(X,\rho)$ be a metric space, let $\dim (X)$ be the covering dimension of $X$, and let $d_0(X,\rho)$ be the metric dimension of $X$. Let $d_2$ and $d_3$ denote the metric-dependent dimension functions introduced by Nagami and Roberts and $d_6$ and $d_7$ be the metric-dependent dimension functions defined by J. C. Smith, V. I. Egorov, J. B. Wilkinson, and J. C. Smith have characterized these metric-dependent dimension functions in terms of Lebesgue covers. These results are described by the following theorem. Theorem. The following are equivalent for metric spaces $(X,\rho)$: (1) $d_2(X,\rho) \geq n$ iff every Lebesgue cover consisting of $n + 2$ members has an open refinement of order $\geq n + 1$. (2) $d_3(X,\rho) \geq n$ iff every finite Lebesgue cover has an open refinement of order $\geq n + 1$. (3) $d_6(X,\rho) \geq n$ iff every countable Lebesgue cover has an open refinement of order $\geq n + 1$. (4) $d_0(X,\rho) \geq n$ iff every locally finite Lebesgue cover has an open refinement of order $\geq n + 1$. (5) $d_7(X,\rho) \geq n$ iff every locally finite Lebesgue cover has an open refinement of order $\geq n + 1$. (6) $d_0(X,\rho) \geq n$ iff every...
Lebesgue cover has an open refinement of order $\leq n + 1$. L. Soniat has generalized the dimension functions $d_0$, $d_2$ and $d_3$ for uniform spaces and has obtained Lebesgue-type characterizations for $d_3$ and $d_0$. In this paper the author generalizes $d_6$ and $d_7$ for uniform spaces and proves the above theorem for uniform spaces. (Received October 2, 1968.)

663-146. E. W. Słowiński, Marquette University, Milwaukee, Wisconsin 53233. Quasi-relations on torsion-free modules.

Let $R$ denote a ring which contains a nonempty set $R'$ of regular elements and let $M$ denote a torsion-free right $R$-module. If $L$ and $N$ are submodules of $M$, then $L$ is quasi-contained in $N$ if for each $x \in L$ there exists $c \in R'$ such that $xc \in N$. Two submodules of $M$ are quasi-equal if each is quasi-contained in the other. Various properties are established for these relations. The concepts of quasi-isomorphism and quasi-endomorphism are defined as in the theory of torsion-free abelian groups. It is shown that if two submodules of $M$ are quasi-equal (quasi-isomorphic), then their quasi-endomorphism rings are equal (isomorphic). If $R$ has a right quotient ring, then two submodules of $M$ are quasi-equal (quasi-isomorphic) if and only if their quotient modules are equal (isomorphic). (Received October 2, 1968.)

663-147. E. C. Young, Florida State University, Tallahassee, Florida 32306. Basic sets of polynomials for the heat equation and its iterates. Preliminary report.

Let $L = \partial^2 / \partial t - \Delta$, where $\Delta$ denotes the Laplace operator in the variables $x_1, ..., x_m$. Basic sets of polynomials homogeneous in the variables $x_1, ..., x_m, t^{1/2}$ of degree $n$ are developed for the equations $L^k(u) = 0, k = 1, 2, ...$. (Received October 3, 1968.)

663-148. W. R. Derrick, University of Utah, Salt Lake City, Utah 84112. Inequalities concerning the module of curve families.

Let $A$ be the topological image of an $n$-cube in $n$-dimensional Euclidean space and $v, w$ be nonnegative $L_1$-integrable, Borel-measurable, real-valued functions defined on $A$. Theorem. $\int_A \int_A v w dH^n \leq (\inf c \int_c v dH^1)(\inf Q Q w dH^{n-1})$, where $c$ is an arc joining, and $Q$ is a surface separating, the images of a fixed pair of opposite sides of the $n$-cube, and $H^k$ is the $k$-dimensional Hausdorff measure in $n$-space. Corollary. $M(\Gamma) M(\Sigma) \leq 1$, where $M(\Gamma), M(\Sigma)$ are the modules of the families $\Gamma, \Sigma$ of curves $c$ and separating surfaces $Q$ in $A$, respectively. The theorem answers and extends a conjecture posed in 3-space by R. J. Duffin (J. Math. Anal. Appl. 5 (1962), 200-215) and the corollary shortens the proof of the identity $M(\Gamma) M(\Sigma) = 1$ due to F. W. Gehring (Michigan J. Math. 9 (1962), 137-150). Finally, an example is given for which $\prod_{i=1}^n M(\Gamma_i) > 1$, where $\Gamma_i$ is the family of curves joining the images of the $i$th pair of opposite faces of the cube, showing the author's theorem (Abstract 656-71, these Notices 15 (1968), 523) is best possible. (Received October 3, 1968.)

663-149. R. T. Ramsay, North Carolina State University, Raleigh, North Carolina 27606. Locally compact groups with compact conjugacy classes.

The locally compact groups $G$ such that $G/Z$ is compact, where $Z$ is the center of $G$, have the
structure \( V \times H \), where \( V \) is a vector group and \( H \) has a compact open normal subgroup (Grosser & Moskowitz, Bull. Amer. Math. Soc. 72 (1966), 826-830). Using this structure theorem and some results from the author's dissertation, a number of theorems for (discrete) groups can be generalized to locally compact groups, usually by replacing "finite" by "compact" in the statements of the theorems.

For example, it can be shown that if \( G \) is a locally compact group such that for every compact subset \( D \) of \( G \), the set \( \{g \cdot x : x \in D, g \in G\} \) has compact closure, then the commutator subgroup \( G' \) is periodic (every element of \( G' \) is contained in a compact subgroup of \( G \)). (Received October 3, 1968.)

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Consider a discrete time parameter Markov process with stationary probability functions, a general state space \( X \) and the Harris recurrence condition. This then implies the existence and essential uniqueness of a sigma-finite stationary measure \( \pi \). It is also assumed that the class of measurable sets \( \mathcal{B} \) contains single point sets. Let \( p^{(m)}(x,S) \) denote the \( m \)-step transition probability from \( x \) to \( S \in \mathcal{B} \) and \( p^{(m)}(x,\cdot) \), the component of \( p^{(m)}(x,\cdot) \) which is absolutely continuous with respect to \( \pi \). Let \( \mathcal{J} = \{ \mathcal{C} : \mathcal{C} \in \mathcal{B} \) for some \( n \}, \operatorname{inf}_{x,y \in \mathcal{C}} p^{(n)}(x,y) > 0 \} \) and \( \mathcal{J} = \{ n : \inf_{x,y \in \mathcal{C}} p^{(n)}(x,y) > 0, \mathcal{C} \in \mathcal{J} \} \).

**Theorem 1.** Let \( S \in \mathcal{J} \) with \( \pi(S) > 0 \), measurable \( B \subseteq S, \pi(B) > 0 \) and \( q \in B \) with \( \lim_{m \to \infty} \int_B p^{(m+k)}(x,A)/p^{(m)}(q,B) = \pi(A)/\pi(B) \) in measure \( \pi \) on \( S \). If \( \gcd(n,\pi'(X)) = 1 \) and \( \pi' << \pi \) with \( \pi'(X) < \infty \), then the above limit holds in measure \( \pi' \) on \( X \). Theorem 2. Let \( S \in \mathcal{J} \) with \( \pi(S) > 0 \). Let measurable \( A_1 \subseteq S \) with \( \pi(A_1) > 0, i = 1,2 \). If \( \rho_1 \) and \( \rho_2 \) are two probability measures on \( X \) with \( \rho_2(S) > 0 \), then \( \lim_{m \to \infty} \int_X \sum_{i=1}^{\rho_1} p^{(m)}(x,A_i)p_1(dx)/\sum_{i=1}^{\rho_1} p^{(m)}(x,A_i)p_2(dx) = \rho_2(S)\pi(A_1)/(\rho_2(S)\pi(A_2)) \). (Received October 3, 1968.)

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663-151. J. L. MEEK, University of Arkansas, Fayetteville, Arkansas 72701. On Fatou values of normal harmonic functions.

For \( q \) a point on the unit circle \( C \) and \( \delta \) a positive number, let \( H(q,\delta) \) denote the subdomain of the unit disk bounded by the hypercycles which are at constant hyperbolic distance \( \delta \) from the diameter through \(-q\) and \( q \). **Theorem.** Let \( u \) be a normal harmonic function defined on the unit disk. If there exist \( \delta > 0 \) and \( q \in C \) such that \( u \sim K \) as \( z \to q \) inside \( H(q,\delta) \), then \( K \) is a Fatou value of \( u \). An example is given to show that the theorem cannot be extended to include the class of normal subharmonic functions defined on the disk. (Received October 3, 1968.)

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663-152. C. I. VINSONHALER, 31 Foster Drive, Willimantic, Connecticut 06226. Some generalizations of \( QF-3 \) rings.

Let \( R \) be a ring with identity. \( R \) is (left) \( QF-3 \) if \( R \) has a faithful projective injective left module which is a direct summand of every faithful module. Any \( QF-3 \) ring satisfies (i) \( \operatorname{Hom}_R(M,R) = 0 \) implies \( \operatorname{Hom}_R(M',R) = 0 \) for all \( R \)-modules \( M' \subseteq M \), and (ii) the class of torsionless \( R \)-modules is closed under extension. [L. E. T. Wu, H. Y. Mochizuki, J. P. Jans, Nagoya Math. J. 27 (1966), 7-13.]

Rings satisfying (i) are called \( \text{SZD} \), and those satisfying (ii), \( \text{TCE} \). Finally, a ring \( R \) is called \( \text{CI} \) if
R contains an injective left ideal. Characterizations are obtained for each of these generalizations as well as some relationships between them. For example, if R is left Noetherian, R is TCE iff every exact sequence of R-modules, \(0 \to A \to B \to C \to 0\), with C torsionless and A a left ideal in R with no torsionless factors, must have B torsionless. If R is SZD and has zero singular ideal, then the injective hull of \(\mathfrak{r} R\) is torsionless, hence R is TCE. For semiprimary rings, R is CI iff R has a factor ring which is QF-3. (Received October 3, 1968.)

663-153. J. B. COOPER, University of California, Santa Barbara, California 93106. A generalization of Markuševič's duality principle.

Markuševič's duality theorem for systems of analytic functions is stated in terms of topological tensor products of locally convex spaces. This allows a much simpler and more natural proof to be given. As an application, Markuševič's results and its corollaries on complete systems of analytic functions are extended to vector-valued functions. (Received October 3, 1968.)


For any subspace \(U \subset C^n\), let \(K_U = \{L \in C^{n \times n} \colon L = L^*, N(L) = U, 0 \neq x \in U^+ = (Lx, x) > 0\}\). If for \(B \in C^{n \times n}\) exists an \(L \in K_U\) such that \(B^* L = LB\), then B is \(K_U\)-symmetric w.r.t. L. If also \(N(B) = U, 0 \neq x \in U^+ = (Bx, x) > 0\), then B is \(K_U\)-p.d. w.r.t. L. Sample results. (I) If B is \(K_U\)-symmetric w.r.t. L, \(N(B) = U, \text{rank } B = \text{rank } B^2\), then B has a complete set of eigenvectors \([u_i : i = 1, \ldots, n]\) satisfying \((Lu_i, u_j) = 0, i \neq j\). (II) \([B \text{ is } K_U\text{-p.d. w.r.t. } L]\) iff \([N(B) = U, B \text{ has a complete set of eigenvectors, nonnegative eigenvalues with exactly } \dim U^+ \text{ positive ones}]\) iff \([B = HL, H \text{ pos. def.}, L \in K_U]\). For \(U = \{0\}\) these matrices were studied by Petryshyn and Wigner. Applications to hyper-power iterative methods of generalized inversion are given. (Received October 3, 1968.)


It is well known that a connection on an n-dimensional \(C^\infty\) manifold M may be regarded as an n-dimensional \(C^\infty\) distribution on the tangent bundle TM of M which is invariant under the action of the group \(R^*\), the multiplicative group of nonzero real numbers and which splits TTM, the tangent bundle of TM. By taking \(T = \{A|A \in TTM, \pi_1 A = T\pi A\}\) where \(\pi, T\pi\) are the projections of TM and TTM respectively and giving TE a vector bundle structure over M, we may introduce second order connections. Definition. A second order connection on M is an n-dimensional \(C^\infty\) distribution on TE which is invariant under \(R^*\) and which splits TTE, the tangent bundle of TE. Using this definition we investigate covariant differentiation of an extended vector field, i.e., a cross section \(A : M \to \mathcal{T}\) with respect to an ordinary vector field, parallel displacement of an extended vector field along a curve of M, geodesics of TM, normal coordinates of TM, and the second order spray of a second order connection. Using these we prove the theorem. Theorem. Each accessible dissection of TM is induced by a unique second order spray. (Received October 3, 1968.)
n-dimensional Casorati-Weierstrass theorem.

While any meromorphic function \( f : \mathbb{C} \to \mathbb{P} \mathbb{C} (= \text{Riemann sphere}) \) must have an open dense image (Casorati-Weierstrass), a nondegenerate holomorphic mapping \( f : \mathbb{C}^2 \to \mathbb{P}^2 \mathbb{C} (= \text{two-dimensional complex projective space}) \) can omit an open set (Fatou-Bieberbach). This lends weight to the following Theorem. If \( f : \mathbb{C}^n \to \mathbb{P}^n \mathbb{C} \) is a quasiconformal holomorphic mapping with respect to the Fubini-Study metrics, then it does not omit a set of positive measure. Here, we regard \( \mathbb{C}^n \) as \( \mathbb{P}^n \mathbb{C} \) minus a hyperplane, and so \( \mathbb{C}^n \) inherits the Fubini-Study metric of \( \mathbb{P}^n \mathbb{C} \). By quasi-conformal, we mean that the image of the unit sphere in every tangent space of \( \mathbb{C}^n \) under \( df \) is a hyperellipsoid such that the ratio of its longest axis to its shortest axis is bounded above by an absolute constant.

(Received October 3, 1968.)

Asymptotic properties of the perturbed Klein-Gordon equation, II.

Suppose \( U_0(t) \) and \( U(t) \) are the one-parameter groups governing the time development of the equations \( \Box \phi = m^2 \phi \) and \( \Box \phi = m^2 \phi + V(x) \phi \) respectively. In previous work (Abstract 68T-223, these Notices 15 (1968), 366) the author showed that if the real-valued function \( V \) satisfied (i) \( V \in L^p(\mathbb{R}^3) \) for any \( 2 \leq p < 3 \) and (ii) the \( q \)-norm of the negative part of \( V \) is sufficiently small for any \( q \geq 3/2 \), then the wave operators, \( W_\pm = \lim_{t \to \pm \infty} U(-t)U_0(t) \) as \( t \to \pm \infty \), exist on the finite-energy and Lorentz-invariant solution spaces of \( \Box \phi = m^2 \phi \). If, in addition, (iii) \( V \in L^1(\mathbb{R}^3) \cap L^2(\mathbb{R}^3) \) or (iii)' \( V \in L^2(\mathbb{R}^3) \), \( V(x) \) is locally Hölder continuous with a finite number of singularities, and \( V(x) = o(|x|^{-2-\epsilon}) \), \( \epsilon > 0 \), as \( |x| \to \infty \), then the \( W_\pm \) are complete and isometric as operators on either of the above solution spaces. (Received October 3, 1968.)

Vibrations of nonhomogeneous anisotropic spherical shells.

The frequency equations for the radial vibrations of nonhomogeneous anisotropic spherical shells has been established when the elastic constants of the material vary as the \( n \)th power of the radial coordinate. Numerical values of the frequencies for various materials have been obtained by solving the frequency equation for various values of \( n \) and for different thicknesses of the shell. The case \( n = 2 \) has been discussed separately. (Received October 3, 1968.)

Permissible bounds on the coefficients of approximating polynomials in \( C[0,1] \).

Let \( f \) be continuous on the interval \([0,1]\), with \( |f(x)| \leq M \) for \( 0 \leq x \leq 1 \). The theory of Bernstein polynomials is used to show that there exists a sequence \( \{P_n\} \) of polynomials \( P_n(x) = a_{n0} + a_{n1}x + \ldots + a_{nn}x^n \) which converges uniformly to \( f \) on \([0,1]\) and such that \( |a_{n0}| + \ldots + |a_{nn}| \leq M(1 + \epsilon_n)n \) where \( \omega(f,n^{-1/2}) = O(\epsilon_n) \). \( \omega(f,n) \) is the modulus of continuity of \( f \) on \([0,1]\). Similar results are obtained for various other restrictions on \( f \) and on the coefficients of the approximating polynomials. (Received October 7, 1968.)
663-160. J. D. HALPERN, University of Michigan, Ann Arbor, Michigan 48104. \(2m = m\) and the axiom of choice.

Let \(E\) be the statement, "For all infinite cardinals \(m\), \(m + m = m\)." Both the independence of AC from \(E\) in set theory with regularity and in set theory without regularity remains open. Thus in the absence of a proof of \(E \rightarrow AC\) in either theory, given the available techniques for proving independence results, there are two questions to consider. Is there a Cohen model, is there a Fraenkel-Mostowski model in which \(E \rightarrow AC\) is false? We answer neither of these questions but contribute to the answer of the second as follows: Let \(E^*\) be the statement, "For all sets \(M\) there exists a function \(f\) on the power set of \(M\) such that if \(X \subseteq M\) is infinite, \(f(X)\) is an ordered pair of 1-1 functions on \(X\) whose ranges constitute a two partition of \(X\)." \(E^*\) is a generalization of \(E\) since \(E\) is equivalent to the assertion, "For every infinite set \(X\) there is a pair of 1-1 functions on \(X\) whose ranges constitute a two partition of \(X\)." We show that \(E^* \rightarrow AC\) holds in all Fraenkel-Mostowski models by showing that \(E^* \rightarrow YX \exists a[\exists X \subseteq 2^X] \). (Received October 7, 1968.)

663-161. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. A remark on periodic maps.

Let \(X\) be a compact Hausdorff space. We denote by \(S\) the set of all continuous maps \(f : X \rightarrow X\). \(S\) is a topological semigroup under the composition of maps and compact-open topology on \(S\). If \(f \in S\) we denote by \(O(f)\) the subsemigroup generated by \(f\) and by \(T(f)\) the closure of \(O(f)\) in \(S\). Theorem. Let \(f \in S\) and \(f(X) = X\). Then \(f\) is periodic if and only if \(T(f)\) is discrete in \(S\) and \(O(f)\) an evenly continuous family. This result uses some theorems of A. D. Wallace on semigroups and generalizes the Gerald Jungck result Periodic maps via equi-continuity (Amer. Math. Monthly, March 1968). (Received October 7, 1968.)


A topological space \((X,T)\) is said to be pseudo-finite if every compact subset is finite. Levine (Amer. Math. Monthly 75 (1968), 178-180) has recently shown that if, in addition, \((X,T)\) is reasonably nice (e.g. first countable \(T_1\)), then \((X,T)\) is discrete. Theorem. Every MI-space is pseudo-finite. (A topological space is MI if it possesses no isolated point and has the property that every dense subset is open.) If \((X,T)\) is any topological space with infinite dispersion character then there exists a filter of \(T\)-dense sets such that \(T \cup F\) generates an MI-topology for \(X\). Thus the preceding theorem implies that the class of nondiscrete pseudo-finite spaces is quite extensive. (Received October 7, 1968.)


Let \(C_N = C(Y_N)\) denote the \(N\)-dimensional Yeh-Wiener space, i.e. \(C_N\) is the set of all real-valued continuous functions \(f\) on \(Y_N = \prod_{j=1}^N [a_j, b_j]\) such that \(f(p_1, \ldots, p_N) = 0\) if \(p_j = a_j\) for some \(1 \leq j \leq N\). Thus \(C_N\) becomes the Wiener space \(C_W\) if \(Y_N = [0,1]\). For a \(C,O,N,\) set \(\{a_k\}\), each of which is of B. V. on \(Y_N\), and a function \(h \in L^2(Y_N)\), the Paley-Wiener-Zygmund integral \(\int_{Y_N} h d^*f\)
is defined by \( \lim_{n \to \infty} \int_{-N}^{N} (hf)_{n} \, df \) for \( f \in C_{W} \), where \((hf)_{n}\) is the nth partial sum of the Fourier series of \( hf \) with respect to the \( \{a_{k}\} \), and \( \int_{-N}^{N} (hf)_{n} \, df \) is the ordinary Riemann-Stieltjes integral. The author has previously established the almost everywhere existence of this integral and the essential consistency of it for a class of C.O.N. sets. The case when \( N = 1 \) is of special interest. 

Theorem. Let \( h(t) \) be absolutely continuous and \( h'(t) \in L^{2}[0,1] \). Then 
\[
\int_{0}^{1} h(t)(t) \, df(t) = \frac{1}{2} \int_{0}^{1} h(t) dt [f'²(t)]
\]
for almost all \( f \in C_{W} \).

The author has previously established the almost everywhere existence of this integral and the essential consistency of it for a class of C.O.N. sets. The case when \( N = 1 \) is of special interest. 

Theorem. Let \( h(t) \) be absolutely continuous and \( h'(t) \in L^{2}[0,1] \). Then 
\[
\int_{-1}^{1} f(t) h(t) \, df(t) = \frac{1}{2} \int_{-1}^{1} f(t) \, dt
\]
for almost all \( f \in C_{W} \).

(Received October 7, 1968.)


\( a(t,y) = \{a_{ij}(t,y)\} \) is a symmetric strictly positive definite \( d \times d \) matrix of bounded continuous coefficients defined for \( t \leq 0 \) and \( y \in \mathbb{R}^d \). \( b(t,y) = \{b_{i}(t,y)\} \) is a \( d \)-vector of bounded measurable coefficients, which are defined for \( t \geq 0 \) and \( y \in \mathbb{R}^d \). \( \Omega \) is the space of \( \mathbb{R}^d \)-valued continuous functions on \([0,\infty)\) and \( \omega \) is a point of \( \Omega \). \( x(t,\omega) \) is the value of the function \( \omega \) in \( \Omega \) at time \( t \geq 0 \). \( M^{B} \) is the \( \sigma \)-field generated by \( x(u,\omega) \) for \( s \leq u \leq t \). \( M^{S} \) denotes \( M^{B} \) with \( t = \infty \). A measure \( P \) on the space \( (\Omega,M^{B}) \) is a diffusion process corresponding to the given coefficients \( a(t,y), b(t,y) \) starting at time \( s \) from the point \( x \in \mathbb{R}^d \) if 
\[
P[w:x(s,w)=x] = 1 \quad \text{and} \quad \exp \left\{ \langle \theta, x(t,w)-x \rangle \right\} - \int_{0}^{t} \left\{ \langle \theta, b(u,x(u,\omega)) \rangle \right\} du
\]
are martingales with respect to \( (\Omega,M^{B},P,t \leq s) \), for each vector \( \theta \in \mathbb{R}^d \).

The existence and uniqueness of such a \( P \) is proved. Denoting \( P \) by \( P^{s,x} \) it is proved that \( \{P^{s,x}\} \) is a strong Markov process corresponding to the given coefficients. (Received October 7, 1968.)


Let \( \{P^{s,x}\} \) be as in Part I (Abstract 663-164 above), and let \( P(s,x,t,A) = P^{s,x}[x(t) \in A] \) be the transition probabilities. Then for each fixed \( t \), the family \( P(s,x,t,\cdot) \) is equicontinuous at each point \((s,x)\) with \( s < t \). Let \( a^{n}(t,x) \) and \( b^{n}(t,x) \) be a sequence of coefficients converging in a suitable manner to \( a(t,x) \) and \( b(t,x) \). Then the sequence \( \{P^{n}_{s,x}\} \) converges to \( \{P^{s,x}\} \) weakly. This can be strengthened considerably in the time homogeneous case. A result is established showing, under mild conditions, that Markov chains which reasonably resemble a diffusion process actually converge to that diffusion process. (Received October 7, 1968.)


A uniformity \( \mathcal{U} \) on a topological space \((X,\mathcal{F})\) is said to be continuous in case the topology \( \mathcal{F}(\mathcal{U}) \) generated by \( \mathcal{U} \) is a subcollection of the original topology \( \mathcal{F} \). A subspace \( S \) is \( P \)-embedded in \( X \) in case every continuous pseudometric on \( S \) can be extended to a continuous pseudometric on \( X \).

Theorem 1. \( S \) is \( P \)-embedded in \( X \) if and only if every admissible uniformity \( \mathcal{U} \) on \( S \) can be extended to a uniformity \( \mathcal{U}^{*} \) on \( X \) such that \( \mathcal{F}(\mathcal{U}^{*}) \subset \mathcal{F} \). Theorem 2. A completely regular space \( X \) is collection-
wise normal if and only if for every closed subset $F$ of $X$, every admissible uniformity on $F$ has a continuous extension to $X$. \textbf{Theorem 3.} A completely regular space is normal if and only if for every closed subset $F$ of $X$, every admissible precompact uniformity has a continuous precompact extension. \textbf{Theorem 4.} The Stone-Čech compactification $βX$ of a space $X$ is that unique compact Hausdorff space containing $X$ densely such that every admissible precompact uniformity on $X$ has a continuous extension. In a like manner the Hewitt realcompactification $ρX$ of $X$ is that unique realcompact Hausdorff space containing $X$ densely such that every admissible uniformity on $X$ generated by a collection of continuous real-valued functions has a continuous extension. (Received October 7, 1968.)

663-167. \textbf{R. A. HOWLAND,} Franklin and Marshall College, Lancaster, Pennsylvania 17604. \textit{Lie derivations of derived rings of simple rings.} Let $R$ be a simple ring with $1$, containing $3 \times 3$ matrix units; that is, there exist $e_{ij}$, $i,j = 1,2,3$, such that $e_{ij}e_{kl} = δ_{jk}e_{il}$ and the sum of the $e_{ii}$'s is $1$. A Lie derivation $L$ of the Lie ring $[R,R]$ is a mapping of $[R,R]$ into itself such that $L(x + y) = L(x) + L(y)$ and $L[x,y] = [L(x),y] + [x,L(y)]$.

\textbf{Theorem.} Let $R$ be as above and let $L$ be a Lie derivation of $[R,R]$. Then $L$ can be extended to an ordinary derivation of $R$. (Received October 7, 1968.)

663-168. \textbf{D. K. BURKE and R. A. STOLTENBERG,} Washington State University, Pullman, Washington 99163. \textit{A note on p-spaces and Moore spaces.} \textbf{Definition.} A completely regular space $X$ is a p-space if in its Stone-Čech compactification $βX$ there is a sequence $\{ν_n\}_{n=1}^{∞}$ of open covers of $X$ such that $∪_{n=1}^{∞} St(x,ν_n) ⊂ X$ for each $x ∈ X$. \textbf{Theorem 1.1.} For a completely regular space $X$ the following are equivalent: (a) $X$ has a development, (b) $X$ is a p-space with a $σ$-discrete net, (c) $X$ is a semi-metrizable p-space, (d) $X$ is a symmetric p-space. \textbf{Theorem 2.2.} A completely regular space $X$ is a strict p-space if and only if there is a sequence $\{G_n\}_{n=1}^{∞}$ of open covers of $X$ satisfying: (a) $P_n = ∪_{n=1}^{∞} St(x,G_n)$ is compact for each $x ∈ X$, (b) the family $\{St(x,G_n) : n ∈ Z^+\}$ is a neighborhood base for the set $Px$. \textbf{Theorem 2.7.} A pointwise paracompact p-space with a point countable base is a pointwise paracompact Moore space. (Received October 7, 1968.)

663-169. \textbf{R. A. MASSAGLI,} Central Washington State College, Ellensburg, Washington 98926. \textit{On the semi-radical of a topological abelian group.} The radical of a topological abelian group is treated by Wright (Amer. J. Math. 79 (1957), 477-496). The notion of a semi-open set in arbitrary topological spaces appears on several occasions in recent journals (Amer. Math. Monthly 70 (1963), 36-41). One may incorporate these ideas to define the concept of a semi-radical of a group and prove some immediate results involving it and the radical. The definitions of a "semi-radical" and a "semi-residual" subgroup (which will be discussed) are similar to the definitions given by Wright (cited above, pp. 481-482) with some modifications. \textbf{Theorem 1.} Let $G$ be a topological abelian group with radical and semi-radical $T$ and $T_S$ respectively. Then $T_S ≤ T$. \textbf{Theorem 2.} Let $G$, $T$, and $T_S$ be as in Theorem 1. Then there exists a maximal semi-radical subgroup $H$ in $G$. In particular, if $T_S$ is a semi-radical subgroup of $G$, then $T/H$ is homeomorphic to $(T/T_S)/(H/T_S)$. (Received October 7, 1968.)
Let \( \mathcal{D} \) be a Dedekind domain, and \( R = \mathcal{D} \times \mathcal{D} \). Let \( * \) denote the involution \((a, b)^* = (b, a)\) on \( R \).

An \( R \)-lattice \( L \) is a finitely generated \( R \)-module; and a hermitian form on \( L \) is a mapping \( h : L \times L \to R \) derived from a sesquilinear (with respect to \( * \)) form \( q : L \times L \to R \), by \( h(x) = q(x, x) \). A complete set of invariants for such hermitian lattices is obtained. In particular, if \( \mathcal{D} \) is the ring of integers of a local field, then, in the setting of Shimura (Arithmetic of unitary groups, Ann. of Math. 79 (1964), 369-409), this completes the local classification of hermitian forms over the integers of an algebraic number field. Further applications to the global theory of hermitian forms are given. The methods are similar to those used to investigate lattices on quadratic spaces in O'Meara, Introduction to quadratic forms (Academic Press, New York, 1963). (Received October 7, 1968.)


Let \( \mathcal{S}(g) \) denote the class of functions \( S(z) \) analytic in \(|z| < 1\) with Stieltjes integral representation \( S(z) = \int_0^{2\pi} g(z, t) dm(t), \) \( m(t) \) nondecreasing, \( m([0, 2\pi]) = 1, \) and \( g(z, t), g_z(z, t) \) analytic in \( z, \) Lipschitz continuous in \( t \) (uniformly on compact subsets of \(|z| < 1\)). Let \( \mathcal{S}(g, A) \) denote the subclass of \( S(z) \) in \( \mathcal{S}(g) \) such that \( \int_0^{2\pi} u_k(t) dm(t) = A_k, \) for fixed, real \( A_k \) and \( u_k(t) \in C^1([0, 2\pi]), k = 1, \ldots, n. \) Theorem. Let \( S(z) \in \mathcal{S}(g, A) \) with \( m(t) \) not a step function. Then there exist numbers \( \lambda_k \) such that \( S^*(z) = S(z) + \epsilon \int_0^{2\pi} g_z(z, t) + \lambda_k u_k(t)|m(t) - c| dt + O(\epsilon^2) \) belongs to \( \mathcal{S}(g, A) \) for all sufficiently small \( \epsilon \) (real) and open intervals \( e. \) Further, if \( t_1, t_2 \) are discontinuities of \( m(t) \), then \( S^*(z) = S(z) + \epsilon (H(z, t_1) - H(z, t_2)) + O(\epsilon^2) \) belongs to \( \mathcal{S}(g, A), \) \( (H(z, t) = g_z(z, t) + \Sigma_k \lambda_k u_k(t)). \) These variational formulas generalize results of Goluzin (Amer. Math. Soc, Transl. (2) 18 (1961), 1-14). The variations are useful in the solution of extremal problems with constraints in classes of meromorphic univalent and multivalent functions. (Received October 7, 1968.)

663-172. R. W. CHANEY, University of California, Santa Barbara, California 93106. Transforming integrals on locally compact spaces.

Let \( S \) and \( X \) be locally compact \( T_2 \) spaces, \( \{S, \mathcal{M}, \mu\} \) and \( \{X, \mathcal{T}, \nu\} \) be complete measure spaces, and \( \mu \) and \( \nu \) be regular. Assume \( E \in \mathcal{M} \) (resp., \( \mathcal{T} \)) iff \( E \cap F \in \mathcal{M} \) (resp., \( \mathcal{T} \)) for each compact \( F. \) Let \( T : S \to X \) be continuous and let \( \mathbf{U} \) be a family of open sets in \( S \) containing \( S \) and such that every open set in \( S \) is the disjoint union of sets in \( \mathbf{U}. \) Under these conditions a nonnegative function \( w' \) on \( X \times \mathbf{U} \) is a weight function if \( (i) \) each \( w'(-, U) \) is \( \mathcal{T} \)-measurable, \( (ii) \) whenever \( \{U_n\} \) is a disjoint sequence in \( \mathbf{U} \) such that \( U_n \subset U \in \mathbf{U}, \) then \( \sum w'(-, U_n) \leq w'(-, U) \) a.e. \( \nu, \) and \( (iii) \) if \( U \subset \mathbf{U}, \) \( V \subset X, \) and \( T(V) \subset \mathcal{T}, \) then \( w'(-, U) = 0 \) a.e. \( \nu \) off \( T(V). \) \( w' \) is ACT in case \( w'(-, S) \in L^1(\nu) \) and there exists \( f \) in \( L_1(\mu) \) so that \( \int_U f d\mu = \int_X w'(-, U) d\nu, \) \( U \subset \mathbf{U}. \) If \( w' \) is ACT and \( f \) is as above, then the formula \( \int_U (H \circ T)f d\mu = \int_X Hw'(-, U) d\nu \) holds as soon as either integral exists; here, \( H \) is complex-valued, \( \mathcal{T} \)-measurable. Criteria for a weight function \( w' \) to be ACT are given in terms of \( w' \) alone and in terms of \( \int w'(-, U) d\nu \) alone. (Received October 7, 1968.)
David Nelson [J, Symbolic Logic 31 (1966), 562-572] has given three formal systems of
negationless logic and arithmetic together with a realizability concept for each system. In this
paper we extend Nelson's systems to negationless intuitionistic analysis and provide realizability
concepts which are allied to Kleene's realizability for intuitionistic analysis [Kleene and Vesley,
The foundations of intuitionistic mathematics, North-Holland, 1965]. We prove that the rules of
inference of the extended systems preserve realizability and the axioms are realizable. In the
extension of Nelson's $P_2 - A_2$ system, we have carried out the development of all the negationless
analysis originally done by G. F. C. Griss. (Received October 7, 1968.)

Let $(X, T)$ be a flow where (1) $X$ is a locally compact metric space and (2) $T : X \to X$ is such
that the semigroup of iterates of $T$, $\mathcal{S} = \{T^n; n = 0, 1, 2, \ldots\}$ is an equicontinuous family of functions
on $X$. For $x \in X$, let $\omega(x)$ be the set of $\omega$-limit points of $x$, i.e. the set of limit points of
$0_x^+ = \{y \in X : y = T^n(x) \text{ for some } n \geq 0\}$. Theorem 1. If a set $\omega(x)$ is not empty, then it is minimal
under $T$, i.e. $a \in \omega(x) \Rightarrow 0_a^+ = \omega(x)$. Theorem 2. If $\omega$ is a minimal set, then $T$ restricted to $\omega$ is
a homeomorphism of $\omega$ onto itself and, therefore, $\omega$ is compact. These results have various applica-
tions, especially in the case where $T$ is an analytic mapping of several complex variables. (Received
October 7, 1968.)

Suppose that $f$ is a continuous real valued function of one real variable defined on the non-
degenerate connected domain $A$. $B$ denotes the range of $f$. Theorem 1. There exists an increasing
homeomorphism $h$ of $A$ onto $A$ such that $(fh)' = 0$ a.e. on $A$. Corollary. There exists a continuous
function $g$ of $A$ onto $B$ such that $g' = 0$ a.e. and for each $y \in B$, the sets $f^{-1}(y)$ and $g^{-1}(y)$ are homeo-
morphic. Theorem 2. There exists an increasing homeomorphism $h$ of $B$ onto $B$ such that
$(hf)' = 0$ a.e. on $A$. (Received October 7, 1968.)

Let $S$ be a semigroup. $S$ is said to be an idempotent generated semigroup if every element
of $S$ is expressible as a finite product of idempotents in $S$. A subset $B$ of $S$ is said to be an IG set
(idempotent generated set) if every element of $B$ is expressible as a finite product of idempotents in $S$.
Let $S = M^0(G; I, J; P)$ be a Rees matrix semigroup over a group with zero $G^0$ with a sandwich matrix
$P$. We define $(g)_{ij} = ((g)_{ij} : i \in I$ and $j \in J)$, where $g$ is a fixed element of the group $G$. Theorem,
$S = M^0(G; I, J; P)$ is an IG semigroup iff (i) $(g_0)_{ij}$ is an IG set and (ii) $P$ contains entries which generate
the group $G$, where $g_0$ is the identity of $G$. (Received October 7, 1968.)
663-177. M. F. CAPOBIANCO, St. John's University, Jamaica, New York 11432. Some theorems on tensor products of graphs.

A (undirected) graph which is isomorphic to a tensor product of two graphs is said to be a tensor product. A number of theorems giving various properties of tensor products are presented. Among these is the result that no tree is a tensor product. This does not hold for directed graphs (digraphs). An example is given of a tensor product of two digraphs that is an unoriented tree. The question of whether an oriented tree can be a tensor product remains open. (Received October 7, 1968.)

663-178. PO-FANG HSIEH, Western Michigan University, Kalamazoo, Michigan 49001. Regular perturbation for a turning point problem.

Consider a system of ordinary differential equations (1) \( \frac{dy}{dx} = [x^pA(x, \epsilon) + pB(x, \epsilon)]y \) where \( x \) is a complex variable in a disc \( D \) centered at the turning point \( x = 0 \), \( p \) and \( \epsilon \) are complex parameters in certain sectors, \( A(x, \epsilon) \) and \( B(x, \epsilon) \) are \( n \times n \) matrices admitting asymptotic expansions in powers of \( \epsilon \) uniformly in \( D \), and \( p \) is a positive integer. Putting \( \mu = p^{1/(p+1)} \) and \( x = \mu t \), (1) can be written as (2) \( \frac{dy}{dt} = [t^pA(x, \epsilon) + tB(x, \epsilon)]y \) where \( \beta = p^{1/(p+1)} \) which is considered a parameter independent of \( p \) and \( \epsilon \). Then there exists an \( n \times n \) nonsingular matrix \( Y(t, x, \epsilon, \mu, \beta) \) which is holomorphic with respect to its variables in suitable regions and (3) \( y = Y(t, x, \epsilon, \mu, \beta)u \) reduces (2) to be a block-diagonal system according to the Jordan canonical form of \( A(0,0) \). The asymptotic expansions of \( Y \) in powers of \( t^{-1} \) as well as that of \( \epsilon \) are also studied. (Received October 7, 1968.)

663-179. S. K. KAUL, University of Saskatchewan, Regina, Canada. A characterization of the Čech homology theory.

Let \( C \) be the category of metrizable compact pairs and their maps. Let \( H = [H, *, \delta] \) be a homology theory on \( C \) satisfying the Eilenberg-Steenrod axioms. We say that \( H \) has property \( P \) if whenever \( (X, A) \) is the inverse limit of an inverse sequence \( \{((X_n, A_n), \pi_n^m)\} \) of simplicial pairs and simplicial maps, then the natural transformation \( H(X, A) \rightarrow \text{Inv Lim} [H(X_n, A_n), \pi_n^m] \) is a natural equivalence. \( H \) has property \( Q \) if for any map \( f: (X, A) \rightarrow (Y, B) \) such that (1) \( f \) is onto, (2) \( f^{-1}(B) = A \), (3) \( f^{-1}(y) \) is acyclic for each \( y \) in \( Y, f_A : H(X, A) \rightarrow H(Y, B) \) is an onto isomorphism. Theorem. If \( H \) has properties \( P \) and \( Q \), then \( H \) is isomorphic to the Čech homology theory on \( C \) on the same coefficient group. (Received October 7, 1968.)

663-180. CHRISTINE TREASH, Mount Allison University, Sackville, New Brunswick, Canada. The completion of finite incomplete Steiner triple systems.

A set of triples \( T \) on set \( S \) is called a Steiner triple system if each distinct pair from \( S \) occurs in exactly one triple of \( T \). A system is called incomplete if each pair from \( S \) occurs in at most one triple of \( T \). It can be shown that any finite incomplete Steiner triple system is contained (isomorphically) in a finite (complete) Steiner triple system. This is equivalent to saying that any finite incomplete totally symmetric loop can be embedded in a finite totally symmetric loop, and hence totally symmetric loops are residually finite. (Received October 7, 1968.)
Bounds on the number of deficient values of meromorphic functions.

Let \( f(z) \) be a meromorphic function of lower order \( \mu < \infty \) and order \( \lambda \equiv \infty \), and let \( \delta(\tau, f) \) denote the deficiency, in the sense of Nevanlinna, of the value \( \tau \). The author shows that if \( \sum \delta(\tau, f) = 2 \) then the number of deficient values \( \tau \) of \( f(z) \) does not exceed the bound \( 2\mu \). This shows, in particular, that a longstanding conjecture of F. Nevanlinna is valid for \( \mu < 3/2 \); namely, the conditions \( \sum \delta(\tau, f) = 2 \) and \( \mu < 3/2 \) imply \( \mu = \lambda = 1 \) and \( \delta(\mu, f) = \delta(\tau_1, f) = \delta(\tau_2, f) = 1 \) for some values \( \tau_1 \) and \( \tau_2 \). The bound \( 2\mu \) on the number of deficient values is established by investigation of the derivative of functions \( f(z) \) satisfying \( \sum \delta(\tau, f) = 2 \) and \( \mu(\infty, f) = 0 \).

The author applies to \( 1/f' \) localized versions of the methods developed by Ahlfors and Carleman in their solutions of the Denjoy conjecture. (Received October 4, 1968.)

Quasiconformal mappings and Royden algebras in space.

G is a bounded open set in \( \mathbb{E}^n \). For \( 1 < p \leq n \) the Royden \( p \)-algebra \( M_p(G) \) denotes all continuous complex-valued functions \( f \) satisfying (1) \( f \in ACT(G) \), (2) \( \sup |f| < \infty \) and (3) \( D_p[f] = \sum |\nabla f|^p \, dL_n < \infty \), where \( L_n \) is \( n \)-dimensional Lebesgue measure. With the norm \( \|f\| = \sup |f| + D_p[f]^{1/p} \), \( M_p(G) \) is a Banach algebra. The \( p \)-capacity of a ring \( R \) contained in \( G \) is \( C(R) = \inf \{ D[p|u|] : u \leq 1 \} \). Let \( T \) be a homeomorphism of \( G \) onto \( G' \). Denote \( T \in Q_p(G,G') \) iff \( T \in Q_p(T(R))/C_p(R) < \infty \) and \( \sup C_p(T(R))/C_p(R) > 0 \) for all rings \( R \) in \( G \); denote \( T \in Q^C_p(G,G') \) iff \( C_p(T(R))/C_p(R) < \infty \) for all spherical rings in \( G \). \( I_p(G,G') \) denotes all algebraic isomorphisms of \( M_p(G) \) onto \( M_p(G') \). Theorem. If \( T \in Q_p(G,G') \) then \( \varphi_T \in I_p(G,G') \) and if \( \varphi_T \in I_p(G,G') \) then \( T \in Q_p^C(G,G') \). Theorem. If \( \psi \in I_p(G,G') \) then there exists some \( T \in Q^C_p(G,G') \) such that \( \psi = \varphi_T \). Since \( Q^C_p(G,G') = Q'_n(G,G') \) is the set of all quasiconformal mappings of \( G \) onto \( G' \), \( T \in \varphi_T \) is a 1-1 correspondence from \( Q^C_p(G,G') \) onto \( I_p(G,G') \) and Corollary. \( G \) and \( G' \) are quasiconformally equivalent iff \( M_n(G) \) and \( M_n(G') \) are algebraically isomorphic. Nakai proved these results [Nagoya Math. J. 16 (1960, 157-184) in the case that \( G \) and \( G' \) are Riemann surfaces and \( p = n = 2 \). (Received October 4, 1968.)

The dihedral subgroup of a group \( E \) is that subgroup generated by all dihedrators of \( E \), i.e. all \( a_1a_2a_1^{-1}a_2^{-1} \) with \( a_1, a_2 \in E \). Theorem 1. The dihedral subgroup of \( E \), denoted by \( D(E) \), is equal to \( \langle [a_1] : a \in E \rangle \) and is normal in \( E \). Suppose \( E \) is the dihedral product \( H \cdot K \), i.e. \( H \subseteq E \), \( H \cap K = \{ e \} \), and there exist sets of generators \( \{ h \} \) and \( \{ k \} \) for \( H \) and \( K \), respectively, such that \( h_k = k_h \) for all \( h, k \). Theorem 2. If \( E = H \cdot K \) and if \( H^C \) is the commutator subgroup of \( H \), then \( E^D = (H^C \cdot K^D) \cup (H^D \cdot K^D) \). If \( a = (H^n \cdot k^1)/(H^m \cdot k^1) \) in \( H \cdot K \), then define \( a \), the conjugate of \( a \), to be \( (H^n \cdot a_h^1)/(H^m \cdot a_k^1) \). We note that \( a = a \) and that \( ab = \overline{ab} E \) for all \( a, b \) in \( H \cdot K \). Theorem 3. If \( E = H \cdot K \), then \( E^D = \langle \{ a \} : a \in E \rangle \). (Received October 4, 1968.)
A generalized Schauder decomposition (G.S.D.) is defined to be a net \( \{ P_a : a \in A \} \) of projection on a linear topological space \( X \) such that \( \lim_{a} P_a x = x \) for \( x \in X \). It is shown that various facts about a space which has Schauder basis or decomposition can be extended to a space with a G.S.D. For instance: Let \( \{ P_a \} \) be a uniformly bounded G.S.D. for a Banach space \( X \). Then \( X \) is reflexive if and only if (a) for each bounded net \( \{ x_a : a \in A \} \), \( P_a x = x_a \) for each \( a \); (b) the range of \( P_a \) is reflexive for each \( a \); (c) \( \{ P_a \} \) is G.S.D. for \( X^* \) (Theorem 3.7). Another problem discussed is that of selecting a Schauder basis from a space which has a Schauder decomposition or a G.S.D. Every separable space having a G.S.D. has a Schauder decomposition (Theorem 5.1). Suppose \( \{ P_n : n = 1, 2, \ldots \} \) is a Schauder decomposition of a Banach space \( X \) such that \( P_n X \) is finite dimensional for each \( n \) and has a basis of constant \( k \) which can be extended to a basis of \( P_{n+1} X \) with a basis of constant \( k \). Then \( X \) has a Schauder basis of constant \( k \). Let \( n > 1 \) and \( k \) be positive integers, let \( C(n) \) be the coprime residue-class group mod \( n \), \( C_k(n) \) the subgroup of \( k \)th powers, \( \# = [C(n) : C_k(n)] \). First we obtain asymptotic formulas for the number of members of any given coset of \( C_k(n) \) in any interval of length \( \tau n^{3/8+\delta} \) for some \( \delta > 0 \) (and sometimes shorter). Now fix a coset, and let \( h_0 < h_1 < \ldots < h_a = n + h_0 \) (where \( a = \phi(n)/\nu \)) be the \( a + 1 \) smallest positive members of this coset. We find asymptotic formulas for \( h_j \), e.g. \( h_j = (\nu n/\phi(n)) (1 + \Theta_k,\delta(n^{3/8+\delta})) \) for any \( \delta > 0 \), provided \( n^{3/8+\delta} \) is large. Also, for each \( k \leq 2 \) and infinitely many \( n \), \( M \approx \exp(\log k \log n(1 + o(1))/\log \log n) \). For \( \nu > 1 \), the maximum number of consecutive members of \( C(n) \) in a fixed coset of \( C_k(n) \) is \( \Theta_k,\delta(n^{3/8+\delta}) \) (often \( \Theta_k,\delta(n^{1/4+\delta}) \)). Finally, we estimate the sum \( S(\beta) = \sum (h_j - h_{j-1})^\beta : 1 \leq j \leq a \) for real \( \beta \geq 1 \). For example, \( \sum \beta(n^{-1+\epsilon}) = \Theta_{\beta,\epsilon}(n^{2\beta-2}+\epsilon) \) for \( 1 \leq \beta \leq 2 \), \( \epsilon > 0 \). The upper estimate depends on a character-sum estimate communicated to me by Professor D. A. Burgess. (Received October 4, 1968.)
A canonical representation for solutions of a class of nonlinear problems.

This paper illustrates a technique for representing periodic solutions of a class of nonlinear problems in terms of corresponding solutions of a simpler nonlinear problem. Consider the equation (1) \[ \frac{d^2 x}{dt^2} + x + g_m(x) = a \sin wt, \]
where \( a \) and \( w \) are constants, and \( g_m(x) \) is an odd piecewise-linear (discontinuous) function with \( 2m \) branches. For any such \( g_m(x) \), it is shown, under certain conditions, that the simplest periodic solution \( x(t) \) of (1), of period \( 2\pi/w \) (or of period \( 2\pi/j \), \( j \) an odd integer), may be represented in the canonical form:

\[ x(t) = \frac{1}{2m} \left[ \frac{y(t - t_1) + y(t + t_1)}{2} \right] \]

where \( y(t) \) is the corresponding solution of the equation \[ \frac{d^2 x}{dt^2} + \text{sgn} x = b \sin wt, \]
for properly chosen \( b \). The phase shifts \( t_1, \ldots, t_m \) must satisfy a set of \( m \) simultaneous transcendental equations; once they are known, \( b \) is given explicitly in terms of \( a \). (Received October 4, 1968.)


The Runge-Kutta method applies to all functions \( f(x,y) \) of suitable differentiability. By restricting the class of functions to \( g(x) + c \cdot y \) where \( g(x) \) is an arbitrary function of \( x \) and \( c \) is an arbitrary constant, the \( n \)th order of this restricted Runge-Kutta method for the explicit case can be defined using exactly \( n \) functional evaluations. The nonlinear equation system which the parameters \( a_i, b_{ij} \) and \( R_i \) needed to define the \( n \)th order must satisfy assumes a form for which all possible solutions can be found. Also the nonlinear equation system generated in the context of a single ordinary differential equation of the type under consideration is precisely the same as the one generated in the context of a finite system of the same type. The usual assumption that \( a_i = \sum b_{ij} \) does not have to be made. Progress has been made on the general linear case, i.e., when \( f(x,y) = g(x) + c(x) \cdot y \). For example, the equation system for the \( a_i, b_{ij} \) and \( R_i \), while larger than that for the special case above, is still properly contained in that for the unspecialized functions \( f(x,y) \). (Received October 4, 1968.)

The Gauss realizability problem.

Suppose \( A = \{a_1, a_2, \ldots, a_{2n}\} \) is a set of points in \((0,1)\) with \( a_{i-1} < a_i, 2 \leq i \leq 2n \). Let \( * \) denote an involution of \( A \) with no fixed points. Gauss considered the question of when there exists a regular mapping \( f \) from \([0,1]\) into \( \mathbb{R}^2 \) such that \( f(0) = f(1), f(x) = f(x') \) if and only if \( x = a_i, x' = a_i^* \) for some \( i, 1 \leq i \leq 2n \), and \( f'(a_i) \) and \( f'(a_i^*) \) are linearly independent, \( 1 \leq i \leq 2n \). If such an \( f \) exists, then the pair \((A,*)\) is said to be realizable and \( f \) is called a realization. Nagy gave a proof of a necessary condition conjectured by Gauss; Titus found a stronger necessary condition. Treybig was the first to give necessary conditions that are also sufficient. We present somewhat simpler necessary and sufficient conditions for realizability. These conditions are obtained from an easy modification of Lefschetz' proof (Proc. Nat. Acad. Sci. 54 (1965), 1763-1765) of Mac Lane's condition for the planarity of a finite graph. (Received October 4, 1968.)

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A necessary and sufficient condition for the existence of an analytic covering map between two conformal types of tori is determined. It is expressed in terms of equivalence under the action of a certain subgroup of hyperbolic isometries on the points of the space $H/M$ ($H =$ hyperbolic plane, $M =$ modular group) representing the conformal types of tori. If it is satisfied all covering maps are determined. The conformal tori fall into two classes: For tori in the first one the analytic homotopy classes of maps are parametrized by the integers $Z$; for the second class they are parametrized by $Z^2$. In both cases a generalized Riemann-Hurwitz relation determines the number of sheets of the coverings as those integers which are representable by certain quadratic forms.

(Received October 4, 1968.)

Let $A, B$ denote Banach $*$-algebras with identity (of norm one) and isometric involution. By $L(A, B)$ denote the continuous linear operators from $A$ to $B$; and by $\bar{P}$, the cone of positive operators in $L(A, B)$. Let $P = \{T \in \bar{P} : \|T\| = 1\}$. If $B$ is semisimple and symmetric, then the positive multiplicative operators, $M$, are extreme points of $P$. Both conditions on $B$ are necessary. If in addition, $A$ is regular and for $T \in \bar{P}$, $\|T\| = \|Te\|$, then the extreme points of $P$ satisfy: $Te = Ta = Tb$ for all $a, b$ in $A$. With $A$ and $B$ as above, $B$ regular, and (what appears to be) a stronger norm condition on $\bar{P}$, we have the following: $T$ is an extreme element of $P$ if and only if $T$ is multiplicative. The results are extended to include algebras with bounded approximate identities. Let $B$ be the complex field, then for $T \in P$, there exists a unique positive Borel measure $\mu$ defined on $\Lambda$ such that $T(x) = \int_{\Lambda} m(x)d\mu(m)$, for all $x$ in $A$. This is an extension of the Raikov result and the method of proof is that employed by Bucy and Maltese in a representation theorem for positive functionals on involution algebras, Math. Ann. 162 (1966), 364-367. (Received October 9, 1968.)

Let $H(s)$ be a function holomorphic in the domain $\Delta_s$ of the plane $s = \sigma + it$ defined by $|t| < \cos^{-1}(p(\sigma)) < \pi/2$, $\sigma > 0$ ($\sigma_0 = \infty$) where $p(\sigma)$ is a nonnegative continuous function tending monotonically to zero and let $H(s)$ be continuous on $\Delta_s$. If $\log |H(s)| = O\{\exp (l + 2/\pi)|\sigma|^0p(u)du\}$ ($s \rightarrow \infty$), $H(s + i \cos^{-1}(p(\sigma))) = O(\exp (-Q(\sigma)))$ where $Q(\sigma) = O(\exp (l + 2/\pi)|\sigma|^0p(u)du)$. $s = \infty$, then $H(s) \equiv 0$. Applications of this theorem to the problems of generalized quasi-analyticity, uniqueness of moment problems and the Bernstein weighted approximation problem are given. (Received October 9, 1968.)

In two papers McKinsey and Tarski investigated topological closure operators on Boolean algebras (Ann. of Math. 45 (1944) and 47 (1946)). They found that the set of closed elements of a...
given operator is a Browverian algebra, and that the set of regular closed elements is a Boolean algebra. If the closure operator is not necessarily topological then much less can be said, which is the following: If the initial set is a complete, orthocomplemented lattice with a closure operator, then the set of closed elements is a complete lattice, and the set of regular closed elements is a complete, orthocomplemented lattice. For a pertinent counterexample there is exhibited a simple power set with a nontopological closure operator for which the lattice of closed elements is non-modular and noncomplemented, but necessarily complete, and for which the lattice of regular closed elements is nonmodular, but necessarily complete and orthocomplemented. (Received October 7, 1968.)


Let \( f(z, u) \) be a function defined a.e. (almost everywhere) on \( I = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\} \) and \( u(z) \) on \( S^1 \). Let \( \text{Du}(z) = \lim_{(h, k) \to (0,0)} [h \cdot k]^{-1} [u(x + h, y + k) - u(x, y + k) - u(x + h, y) + u(x, y)] \) for \( h \cdot k \neq 0 \) and \( x + h, y + k \) in \( I \). Theorem. (H1) \( f(z, u) \) is continuous in \( u \) for \( z \) a.e. on \( I \); (H2) \( f(z, u(z)) \) is a Perron integrable function in \( z \) on \( I \) for \( u(z) \) continuous in \( z \) on \( I \); (H3) \( |f(z, u(z)) - f(z, u^*(z))| \leq 
\nu(z)|u(z) - u^*(z)| \) a.e. on \( I \) and \( \nu(z) \) is Lebesgue integrable on \( I \); (H4) \( f(z, u(z)) \leq g(z) \) a.e. on \( I \) and \( g(z) \) is Perron integrable on \( I \). Under hypotheses (H1)-(H4), if \( u_0(z) = a(x) + b(y) - a(0), u_n(z) = u_0(z) + \sum_{t=0}^{n} \int_{t}^{f(t, u_{n-1}(t))} (t, u_{n-1}(t)) dt \) is a Picard sequence such that \( \left\{ \sum_{t=0}^{n} \int_{t}^{f(t, u_{n-1}(t))} (t, u_{n-1}(t)) dt \right\} \) is EAC (equi-absolutely continuous) on \( I \) for a set of continuous functions \( \{u_n(z)\} \), then there exists a unique solution to the differential equation \( \text{Du}(z) = f(z, u(z)) \) a.e. on \( I \) with \( u(x, 0) = a(x), u(0, y) = b(y) \) and \( a(0) = b(0) \) where \( a(x) + b(y) \) is continuous in \( z \) on \( I \). The proof is based on a limit theorem concerning a sequence of Perron integrals [see Abstract 655-121, these Notices 15 (1968), 501]. (Received October 8, 1968.)


Let \( S \) be a semigroup, \( J \) the set of all proper subsemigroups of \( S \). By the graph \( G(J) \) of \( S \), we mean the nondirected graph whose set of vertices is \( J \) and the vertices \( A \) and \( B \) are adjacent iff \( A \cap B \neq \emptyset \). A graph is connected provided there is a path between every pair of its vertices. The distance \( d(A, B) \) between two vertices \( A \) and \( B \) of a graph is the number of edges in a shortest path between these vertices. The diameter of a graph is the supremum of \( d(A, B) \), where \( (A, B) \) runs over all pair of vertices of the graph. Theorem. The diameter of the graph of a semigroup \( S \) with more than two elements does not exceed three. We also answer the analogous question for some special semirings. (Received October 8, 1968.)


Let \( V \subset V_0 \) be complex Hilbert spaces with the inclusion algebraic and topological and \( V \) dense in \( V_0 \). Let \( a(v, w) \) (resp. \( b(v, w) \)) be a bounded sesquilinear form on \( V \) (resp. \( V_0 \)). Assume there exists \( \varepsilon > 0 \) such that \( |b(v, v)| \geq \varepsilon |v|_V^2, v \in V_0 \), and that for \( 0 < \varepsilon \leq \varepsilon_0 \) there exist \( a(\varepsilon) > 0, a(\varepsilon) - 0 \) as \( \varepsilon \to 0 \), and \( \delta > 0 \) such that \( |a(\varepsilon)(v, v) + b(v, v)| \leq a(\varepsilon)|v|^2 + \delta |v|_{V_0}^2, v \in V \). Let \( v, w = c(v, w) \): \( V_0 \times V_0 - \mathbb{C} \) satisfy (i)
\( w \rightarrow c(v,w) \) is antilinear; (ii) \( c(0,v) = 0 \); and (iii) \( |c(v,x) - c(w,x)| \leq \gamma |v - w|_{0} |x|_{0} \), \( v, w, x \in V_{0} \). Also assume that for \( 0 < \epsilon \leq \epsilon_{0} \) and \( 0 < \mu \leq \mu_{0} \) there exist \( \nu(\epsilon) > 0 \), \( \nu(\epsilon) \rightarrow 0 \) as \( \epsilon \rightarrow 0 \) and \( \eta > 0 \) such that

\[
|\epsilon a(v,w) + c(\mu v,w) - c(v,w) + b(v,w) - b(v,w)| \leq \nu(\epsilon) |v|_{0}^{-2} + \eta |v|_{0}^{-2}, \quad v, w \in V.
\]

Let \( L \in V_{0}^{\ast} \), the antidual of \( V_{0} \), and let \( A \) be the linear operator in \( V_{0} \) defined by \( b(A(v),w) = a(v,w) \) on \( D(A) = \{ v \in V_{0} : w \rightarrow a(v,w) \) is continuous on \( V \) in the topology of \( V_{0} \} \). Denote \( D(A) \), provided with the graph norm, by \( V_{1} \). Let \( u_{0} \) be the solution in \( V_{1} \) of \( b(u_{0},v) = L(v) \) for all \( v \in V_{0} \) and for \( \epsilon \in (0, \epsilon_{0}] \) and \( \mu \in (0, \mu_{0}] \) let \( u \) be the solution in \( V \) of \( c(a(u) + c(\mu u,v) + b(u,v)) = L(v) \) for all \( v \in V \). Theorem. If \( u_{0} \in V_{1} \), the \( \tau \)th quadratic interpolation space between \( V_{1} \) and \( V_{0} \), then

\[ |u - u_{0}|_{10} = o(\epsilon^{\tau}) + O(\mu) \text{ as } \epsilon \downarrow 0, \mu \downarrow 0. \]

(Received October 9, 1968.)


An example is presented of a subspace of a reflexive LF-space which is almost closed but not LF. Thus it answers negatively one of the two fundamental questions concerning LF-spaces posed by Dieudonné and Schwartz. Although previous examples in this direction have been found by Grothendieck and Ehrenpreis, the present one is (to the authors' knowledge) the first to be found in a space \( D(\Omega) \) where \( \Omega \) is an open subset of \( \mathbb{R}^{n} \). Moreover, the subspace is the image of \( D(\Omega) \) under a partial differential operator with constant coefficients. (Received October 9, 1968.)


Let \( \delta_{m}(\Omega), \) for \( 0 \leq m \leq \infty \), denote the algebra of real-valued \( m \)-times continuously differentiable functions on an open subset \( \Omega \) of \( \mathbb{R}^{n} \). Theorem. For \( m < \infty \) and \( f \in \delta_{m}(\Omega) \), \( f \in \delta_{m}(\Omega) \) is closed in \( \delta_{m}(\Omega) \) if and only if in each component of \( \Omega \) either \( f \) never vanishes or \( f \) is identically zero. Corollary. For \( m < \infty \) and \( f \in \delta_{m}(\Omega) \), \( f \in \delta_{m}^{\ast}(\Omega) \) if and only if \( f \) is never zero. For \( m = \infty \), the situation is more complex. Malgrange [Ideals of differentiable functions, Oxford Univ. Press, London, 1966, p. 88] shows that for \( f \in \delta(\Omega) \), \( f \in \delta(\Omega) \) closed implies the inequality of Jtojasiewicz, which says that for each compact \( K \subset \Omega \) there exist constants \( C > 0 \) and \( a > 0 \) such that \( |f(x)| \leq C[d(x,Z)]^{a} \) for all \( x \in K \), where \( Z = \{ x \in \Omega : f(x) = 0 \} \). Theorem. For \( \Omega \subset \mathbb{R}^{1} \) and \( f \in \delta(\Omega) \), if \( f \) satisfies the Jtojasiewicz inequality, then \( f \in \delta(\Omega) \) is closed in \( \delta(\Omega) \). Corollary. For \( \Omega \subset \mathbb{R}^{1} \) and \( f \in \delta(\Omega) \), \( f \in \delta(\Omega) = \delta(\Omega) \) if and only if \( f \) satisfies the Jtojasiewicz inequality and \( f \) is not identically zero in any component of \( \Omega \). (Received October 9, 1968.)


The eigenvalues of integral equations of the type \( \lambda \varphi(x) = \int_{-a}^{a} K(x,y)\varphi(y)dy \), where \( K \) is real, symmetric, and in \( L_{2} \) on the square \(-a \leq x, y \leq a\), are considered as functions of the half-length \( a \). The behavior of these functions is studied; in this way it is sometimes possible to obtain quite good approximations of the eigenvalues. It is shown that under appropriate conditions on the kernel, certain functions related to the eigenvalues are convex. Kernels of the displacement type, \( K = K(|x - y|) \), and kernels which can be represented as integrals of other kernels are examined in detail. Several
examples which illustrate the results, together with some numerical work which tests the efficacy of the method, are included. (Received October 9, 1968.)


An n-mean on a space X is a continuous function \( \mu : X^n \to X \) satisfying (i) \( \mu(x, \ldots, x) = x \) for each \( x \) in \( X \) and (ii) \( \mu(x_1, x_2, \ldots, x_n) \) is symmetric in its argument. We use Alexander cohomology.

Theorem. Suppose \( R \) is a principal ideal domain and that \( X \) is a continuum such that \( H^p(X; R) \) is (as in \( R \)-module) torsion-free for each \( p \geq 0 \). If \( X \) admits an n-mean, then for each \( p \geq 1 \), \( H^p(X; R) \) is uniquely \( n \)-divisible (i.e., for \( h \) in \( H^p(X; R) \), there is a unique \( h' \) in \( H^p(X; R) \) such that \( nh' = h \)).

Corollary 1. If \( X \) is a continuum admitting an n-mean, then \( H^p(X; \mathbb{Z}_n) = 0 \) for all \( p \geq 1 \). Corollary 2. No cohomological p-sphere admits an n-mean. Corollary 3. A continuum which admits an n-mean is unicoherent. Corollary 4. If \( X \) is a locally connected continuum admitting an n-mean and having codimension \( (X; \mathbb{Z}_n) = 1 \), then \( X \) is a tree. (Received October 9, 1968.)

663-201. WITHDRAWN.

663-202. SEYMOUR BACHMUTH, University of California, Santa Barbara, California 93106. Vanishing of the "symmetric polynomials". Preliminary report.

Let \( G \) be a group of exponent \( p \), \( p \) a prime, \( \Sigma \) the augmentation ideal of the group ring of \( G \) over the integers modulo \( p \) factored by the ideal of "cyclotomic polynomials". If \( x_1, \ldots, x_{p-1} \) are elements of \( \Sigma \), we denote by \( S_{p-1}(x_1, \ldots, x_{p-1}) \) the sum of the \( (p - 1)! \) products formed by taking \( x_1, \ldots, x_{p-1} \) in all possible orders. These symmetric polynomials are said to be of weight \( n \) if each term in \( S_{p-1} \) is in \( \Sigma^n \). Then the following has thus far been verified: For small \( n \), an \( S_{p-1} \) of weight \( n \) is in \( \Sigma^{n+1} \). (Received October 9, 1968.)

663-203. B. A. TAYLOR, Courant Institute, New York University, New York, New York 10012, and D. L. WILLIAMS, Syracuse University, Syracuse, New York 13210. On closed ideals in \( A^{\infty} \).

Let \( A^{\infty} \) denote the algebra of functions analytic in the open unit disk \( D \) with \( f^{(n)} \) continuous in \( \overline{D} \), \( n = 0, 1, \ldots \). By restricting each \( f \in A^{\infty} \) to \( \partial D \), \( A^{\infty} \) is identified with a closed subalgebra of \( C^{\infty}(\partial D) \). Theorem 1. \( Z \subset \partial D \) is the zero set of an \( A^{\infty} \) function iff \( Z \) is a Carleson set (see P. Novinger, these Notices (1968), 816). Theorem 2. If \( f \in A^{\infty} \), then the outer part of \( f \) belongs to \( A^{\infty} \). Theorem 3. Let \( I \) be a closed ideal of \( A^{\infty} \) such that the intersection of the zeros of the functions in \( I \) is a finite set \( Z \subset \partial D \). Assume that for each \( a \in Z \), \( f^{(n)}(a) = 0 \), \( n = 0, 1, \ldots \). If \( F \) is the g.c.d. of the inner parts of the nonzero functions in \( I \), then \( I \) is precisely the set of functions of the form \( Fg \) where \( g \in A^{\infty} \) and \( g^{(n)}(a) = 0 \), \( n = 0, 1, \ldots \), for each \( a \in Z \). This theorem can also be proven for certain countable sets \( Z \). Several other results and some extensions to algebras of functions satisfying bounds of the form \( |f^{(n)}(z)| \leq A_n M_n \) are also given. (Received October 10, 1968.)
Our considerations involve the uniqueness of a solution $F$ of the equation (1) $\lambda F(x) - \int_{s} F(t)K(x - t)dt = H(x)$ for $x \in S$ in the following special sense: Conditions on $\lambda$ and $K$, together with class conditions on $K$ and $F$, are sought which will imply the uniqueness of the solution $F$ of (1) for all measurable sets $S$. Thus, we wish to decide when the following implication holds: (2) If, for each $x \in G$, either $F(x) = 0$ or $\lambda F(x) = \int_{s} F(t)K(x - t)dt$, then $F(x) = 0$, where $G$ is a locally compact abelian group with Haar measure $dt$. A complete answer is given only under very special circumstances, and in other cases we present partial results. (Received October 10, 1968.)

The multiplier rule for variational problems with finite constraints is obtained as a linear dependence relation between $n$-tuples of real valued functions considered as members of a free module over the ring $C[a,b]$. This derivation possesses an exact analogue in elementary constrained extremum problems. Moreover, the derivation brings into complete focus the interface between analysis and linear algebra in the treatment of these problems. (Received October 10, 1968.)

We give an example of a link $L$ of two polygonal simple closed curves in $S^3$ such that the longitudes of $L$ lie in the second commutator subgroup $G''$ of its link group $G = \pi_1(S^3 - L)$, but $L$ is 1-linked, that is, the two simple closed curves of $L$ do not bound disjoint orientable surfaces in $S^3$. The question of the existence of such a link was raised by Smythe in (Ann. of Math. Studies 60 (1966), 71) and one motivation for this question is the observation that the longitudes of any boundary link lie in the second commutator subgroup of its link group. Theorem. If $K = k_1 \cup k_2$ is a link of two components, then $K$ is a boundary link if and only if $k_1$ bounds an orientable surface $S_1$ in $S^3 - k_2$ such that the inclusion of $H_1(S_1)$ in $H_1(S^3 - k_2)$ is trivial. This theorem and our example motivate the following question. Does there exist a link $K = k_1 \cup \ldots \cup k_n, 2 < n$, such that each $k_i$ bounds an orientable surface $S_i$ in $S^3 - (K - k_i)$ and the inclusion of $H_1(S_i)$ in $H_1(S^3 - (K - k_i))$ is trivial but $K$ is 1-linked? (Received October 10, 1968.)

Finite supersolvable groups are characterized satisfying either $P$: $G$ splits over each normal subgroup $N \not\in \Phi(G)$, $\Phi(G)$ the Fratini subgroup of $G$, or $P^*$: For each normal subgroup $N \not\in \Phi(G)$, each reduced product of $G$ over $N$ is a semidirect product. If $\Phi(G) = 1$, both properties reduce to Hall-complementation. Assume $\Phi(G) \neq 1$. A nilpotent group $G \in P$ iff $G$ is a $p$-group satisfying $P$, i.e. iff $G$ is a $p$-group generated by two elements of order $p$. Assume also that $G$ is not expressible as a direct product and is nonnilpotent. If the Fitting subgroup $F(G)$ is abelian, $G \notin P$ iff $F(G)$ is a cyclic.
p-group for the largest prime p dividing |G| and G = \([F(G)]C\), C a cyclic group of square-free order acting faithfully on F(G). If F(G) is non-abelian, then G ∈ P iff for the largest prime p dividing |G|, (1) F(G) is a p-group satisfying P, (2) G = \([F(G)]C\), p divides |G|, such that C acts faithfully on F(G) and C is the direct product of two cyclic groups of square-free order, (3) G splits over each C-invariant maximal subgroup in F(G). The groups for which the properties are hereditary, as well as special classes of p-groups satisfying either P or P*, are also examined. (Received October 10, 1968.)

663-208. CARL EBERHART and JOHN SELDEN, University of Kentucky, Lexington, Kentucky 40506. On the closure of the bicyclic semigroup.

Let B denote the bicyclic semigroup (see Clifford and Preston, Algebraic theory of semigroups, pp. 43-45). Suppose that B is a subsemigroup of a Hausdorff topological semigroup S. Let T denote the closure of B in S. Proposition 1. B is a discrete open subspace of T and T \( \setminus \) B is an ideal of T if it is nonvoid. A topological inverse semigroup is a Hausdorff topological semigroup satisfying (i) for each x ∈ S there is a unique element \( x^{-1} \) satisfying the equations \( xx^{-1}x = x \) and \( x^{-1}xx^{-1} = x^{-1} \) and (ii) the function \( x \rightarrow x^{-1} \) is continuous. Proposition 2. There is one and only one locally compact topological inverse semigroup containing B as a proper dense subsemigroup. Corollary 3. Let θ be a homomorphism of B into a locally compact topological inverse semigroup S. Then one and only one of the following is true. (i) \( \Theta(B) \) is a finite cyclic group. (ii) \( \Theta(B) \) is the integers and either \( \Theta(B) \) is a closed discrete subspace of S or the closure of \( \Theta(B) \) is a compact group. (iii) \( \Theta \) is 1-1 and \( \Theta(B) \) is a closed discrete subspace of S or the closure of \( \Theta(B) \) is the semigroup described in the proof of Proposition 2. (Received October 10, 1968.)


Recently Verma and Lakshmikantham [Bull. Math. de la Soc. Sci. Math. de la R. S. de Roumanie, (2) 11 (59) (1967), 219-223] introduced the notion of mixed stability for ordinary differential systems and obtained sufficient conditions in terms of several Lyapunov functions. In this paper, the definitions of mixed stability in terms of some comparison functions belonging to the class of monotone functions [W. Hahn, Stability of motion, Springer-Verlag, Berlin, 1967] are given. Necessary and sufficient conditions in terms of Lyapunov functions are obtained such that these concepts hold. (Received October 10, 1968.)


Let X(t) be a covariance stationary process which is normal, and whose first derivative exists at \( t = 0 \). Let X(t) have the covariance function \( K(t) = 3g(t)/2 - g^3(t)/2 \), where \( g(t) \) has the following properties: (1) \( g(t) \) is everywhere differentiable; (2) \( g^2(t) \neq 0 \) for all \( t > 0 \); (3) \( g(t_1)/g(t_2) = g(t_1 - t_2) \); (4) \( g(t) \) is of exponential growth. Now let \( γ \) be the length of interval between successive zero crossings of X(t), and let F(t) be the probability distribution function of \( γ \), i.e. \( F(t) = P[γ ≤ t] \). Under the above assumptions X(t) can be represented in terms of a Brownian motion, and using a result due to McKean for Brownian motions F(t) is given an explicit form. (Received October 10, 1968.)
If \( \psi \) measures the \( \sigma \)-ring \( F \), the classical completion \( \psi^* \) of \( \psi \) can be defined by letting 
\[
\psi^*(A \triangle N) = \psi(A) \quad \text{where} \quad A \in F \text{ and } N \text{ is a subset of a } \psi \text{-null member of } F.
\]
and then defined on the \( \sigma \)-ring \( F \triangle J \), where \( J \) is the \( \sigma \)-ideal 
\[
J = \{ A \in F : A \triangle N = 0 \text{ for some } B \in F \}.
\]
Let \( J \) be any \( \sigma \)-ideal with respect to \( F \) and suppose that \( \mu \) measures \( J \),
and let \( \theta = \psi \triangle \mu \) be the \( \mu \)-completion of \( \psi \) defined on \( F \triangle J \) as follows:
\[
\theta(A \triangle J) = \psi(A) + \mu(N \triangle A) \quad \text{where} \quad A \in F \text{ and } N \in J.
\]
It is shown that \( \theta \) is \( \psi \)-measurable whenever \( A \in F \) and \( N \in J \). The notion of \( \mu \)-completeness is applied to certain product measures, and other results are given. (Received October 10, 1968.)


It is a well-known fact that the Hausdorff moment problem, using ordinary integrals as the given functionals, gives rise to classes of classical orthogonal polynomials. It is shown that the Hausdorff moment problem may be extended by means of a new functional, called \( q \)-integral, which yields new classes of orthogonal polynomials. An explicit representation of the polynomials as well as its generating equation is obtained for the set of moments, 
\[
c_s = \frac{1 - q}{1 - q^1 + s + m}, \quad s = 0, 1, 2, \ldots,
\]
where \( q \) and \( m \) are constants. The results are reduced to those obtained by the classical moment problem in the limit as \( q \to 1 \). (Received October 10, 1968.)

On Baire systems generated by collections of integrable functions.

Suppose \( g \) is a real-valued function on the interval \([a,b] \), \( I \) is the collection of all functions which are integrable with respect to \( g \) on \([a,b] \) in the Riemann-Stieltjes sense using refinements, and \( B(I) \) is the Baire system of functions generated by \( I \). Definition. The statement that the subset \( M \) of \([a,b] \) has \( g \) variation 0 means that if \( c > 0 \), then there is a subdivision \( D \) of \([a,b] \) such that if \( E \) is a refinement of \( D \), then 
\[
\sum \left| g(q) - g(p) \right| < c
\]
where the sum is taken over all intervals \([p,q] \) of \( E \) whose interiors intersect \( M \). Definition. The subset \( M \) of \([a,b] \) is scattered means that \( M \) does not contain any set which is dense in itself. Theorem. In order that every real-valued function on \([a,b] \) belong to \( B(I) \) it is necessary and sufficient that there be a scattered subset \( H \) of \([a,b] \) such that \( g \) is continuous on \([a,b] \) - \( H \) and \([a,b] \) - \( H \) is the sum of countably many sets, each one of which has \( g \) variation 0. Moreover, if every function is in \( B(I) \), then every function is the pointwise limit of a sequence of \( g \)-integrable functions. (Received October 10, 1968.)

The composition of two tournaments.

A (round-robin) tournament \( R \) consists of a finite set of nodes 1, 2, ..., \( n \) such that each pair of distinct nodes \( i \) and \( j \) is joined by exactly one of the oriented arcs \( ij \) or \( ji \) (if arc \( ij \) is in \( R \) we write \( i \rightarrow j \)). A tournament is transitive if its nodes can be labelled so that \( i \rightarrow j \) iff \( i > j \) for \( 1 \leq i, j \leq n \). If the tournaments \( R \) and \( S \) have \( r \) and \( s \) nodes, respectively, then the composition of \( R \) and \( S \) is the
tournament $R \circ S$ with $rs$ nodes $(i,k)$ where $1 \leq i \leq r$ and $1 \leq k \leq s$, such that $(i,k) \rightarrow (j,l)$ iff $i = j$ in $R$ or $i = j$ and $k > l$ in $S$. Let $R^1 = R$ and $R^{t+1} = R \circ R^t$ for $t = 1,2,\ldots$. Various algebraic properties of the composition operation are derived; in particular, it is shown that if $R \circ S = S \circ R$ for nontrivial tournaments $R$ and $S$, then $R$ and $S$ are both transitive or $R = H^U$ and $S = H^V$ for some tournament $H$. (Received October 10, 1968.)

663-215. B.-S. CHWE and JOSEPH NEGGERS, University of Alabama, University, Alabama 35486. On the extension of linearly independent subsets of free modules to bases.

Suppose $R$ is a ring with unique maximal ideal of nonunits $M$ satisfying the following condition: For each infinite matrix $T$, with entries in $M$, such that (1) $T_{ij} = 0$ if $i \geq j$, $i,j = 1,2,3,\ldots$ (2) for all $i$, $T_{ij} \neq 0$, only for finitely many $j$ there is an integral function $n(i)$ defined on the positive integers such that $(\sum T_{ij})_{ij} = 0$ if $j > n(i)$. Then each linearly independent subset of a free $R$-module can be extended to a basis by adjoining elements of a given basis. The converse is also true. (Received October 10, 1968.)

663-216. R. S. CRITTENDEN, Virginia Polytechnic Institute, Blacksburg, Virginia 24061.

The Jacobson radical of row-finite matrices.

The purpose of this paper is to give a necessary and sufficient condition for an element to be a member of the Jacobson radical of the ring $R_\omega$ of all row-finite matrices over a ring $R$. The Jacobson radical of $R$ will be denoted by $N(R)$. Let $\|a_{ij}\| \in [N(R)]_\omega$ and let $m$ be a positive integer. If each of $k$ and $p$ is a positive integer let $g^1_{k,m,p} = a_{kp}$ and let $d^1_m$ be the quasi-inverse of $a_{mm}$. If $n \leq m$, let $g^{n+1}_{k,m,p} = g^n_{k,m,p} + (1 - d^n_m) g^n_{m-n+1,m,p}$ if $n < m$, let $a^{n+1}_m$ be the quasi-inverse of $g^{n+1}_m$. Since $\|a_{ij}\|$ is row-finite, let $m_1 < m_2 < m_3 < \ldots$ be an increasing sequence of positive integers such that if $q$ is an integer greater than $m_k$ then $a_{kq} = 0$. The matrix $\|a_{ij}\|$ is quasi-nilpotent if there exists an increasing sequence of positive integers $n_1 < n_2 < n_3 < \ldots$ such that if $s$ is a positive integer then $n_1 < n_2 < n_3 < \ldots$ such that if $s$ is a positive integer then $n_s \leq m_s$ and if $1 \leq q \leq s$, $p > n_q$ then $a_{pq} = 0$. We shall show that $\|a_{ij}\| \in N(R_\omega)$ if and only if each element of the left ideal of $R_\omega$ generated by $\|a_{ij}\|$ is quasi-nilpotent. The matrix $\|a_{ij}\|$ is diagonalized provided that if $\{a_{ij_{1}}, a_{i2_{1}}, a_{i3_{1}}, \ldots\}$ is any set of entries of $\|a_{ij}\|$ such that $\{j_{1}, j_{2}, j_{3}, \ldots\}$ is an unbounded sequence of positive integers then, for some positive integer $k$, $a_{i1_{1}} a_{i2_{1}} \ldots a_{i_k_{1}} = 0$. If $R$ is a commutative ring and $\|a_{ij}\| \in [N(R)]_\omega$ then $\|a_{ij}\| \in N(R_\omega)$ if and only if $\|a_{ij}\|$ is diagonalized. (Received October 11, 1968.)

663-217. BERTHOLD SCHWEIZER, University of Arizona, Tucson, Arizona, 85721. On the continuity of the triangle function.

Let $\Delta$ denote the set of all one-dimensional probability distribution functions in the extended sense, i.e. the set of all left-continuous, nondecreasing functions from $[0,1]$ into $[0,1]$. Let $\Delta^+ = \{F \in \Delta | F(0) = 0\}$. Let $L$ denote the usual Lévy metric on $\Delta$; and let $\mathcal{L}$ denote the modified Lévy metric defined by D. A. Sibley [Abstract 68T F3, these Notices 14 (1968), 808]. Let $T$ be a continuous $t$-norm (i.e. topological semigroup on $[0,1]$) and let $\tau_T: \Delta \times \Delta \rightarrow \Delta$ be defined by $\tau_T(F,G; x) = \sup_{u+v=x} T(F(u), G(v))$. Then $\tau_T$ is a continuous function on $(\Delta, L)$ and $(\Delta^+, \mathcal{L})$ but not on $(\Delta, \mathcal{L})$. Since $(\Delta^+, \mathcal{L})$ is a compact metric space, it follows that $((\Delta^+, \mathcal{L}), \tau_T)$ is a compact topological semigroup. (Received October 11, 1968.)
663-218. M. O. GONZALEZ, University of Alabama, University, Alabama 35486. A Taylor series representation for a class of functions of a complex variable.

Let \( f(z) = u(x,y) + iv(x,y) \). If \( u \) and \( v \) are of class \( C^{(\infty)} \) in a neighborhood \( N_r(z) \) it is shown that the series expansion \( f(z + \Delta z) = f(z) + \sum_{n=1}^{\infty} \frac{1}{n!} \left( \frac{\partial^n f}{\partial z^n} \right)(z)(\Delta z)^n \), where the symbolic power is to be interpreted in the usual manner, is valid in \( N_r(z) \) under either of the following additional assumptions:

1. The successive partial derivatives of \( u \) and \( v \) are uniformly bounded in \( N_r(z) \).
2. Both \( u \) and \( v \) together with their partial derivatives are nonnegative in \( N_r(z) \). The usual Cauchy-Taylor expansion theorem for analytic functions is a special case of the result above. (Received October 11, 1968.)


Let \( A = \{a_1, \ldots, a_t\} \) be a finite set. In this paper certain distinguished subsets of \( A^n \) called \( j \)-parameter sets, \( 0 \leq j \leq n \), are introduced for which the following analogue to Ramsey's theorem holds: Theorem. Given positive integers \( k, i, r \), there exists an integer \( M(k, i, r) \) such that if \( m \geq M(k, i, r) \) and the \( k \)-parameter sets of an \( m \)-parameter set \( P_m \) of \( A^n \) are partitioned into \( r \) classes, then there exists an \( i \)-parameter set \( P_i \subseteq P_m \) such that all \( k \)-parameter subsets \( \subseteq P_1 \) belong to the same class. Among the immediate corollaries of this result are: (1) The van der Waerden theorem for arithmetic progressions, (2) Ramsey's theorem for \( 1 \)-dimensional subspaces of \( n \)-dimensional vector (affine) spaces over GF(q) (conjectured for the general case by Rota), (3) An analogue to Rado's theorem on the regularity of solutions of a system of homogeneous linear equations, (4) Let us call a subset of \( 2^l \) vertices of the \( n \)-cube \( C^n \subseteq E^n \) a \( j \)-space if the vertices span a euclidean subspace of dimension \( j \leq E^n \). Then given integers \( k, l, r \), there exists an integer \( N(k, l, r) \) such that if \( n \geq N(k, l, r) \) and the \( k \)-spaces of \( C^n \) are partitioned onto \( r \) classes, then some \( l \)-space of \( C^n \) has all \( k \)-spaces contained in it in one class. (Received October 11, 1968.)

663-220. M. C. RAYBURN, University of Kentucky, Lexington, Kentucky 40506. The lattice of complete Boolean algebras is complemented.

Let \( p, q \) be distinct points in set \( X \), \( \text{card}(X) \geq 3 \), and \( G(p, q) \) be the family of all subsets of \( X \) which either miss both \( p \) and \( q \), or contain both. The author has previously shown (Canad. J. Math., to appear) these to be maximal proper complete Boolean algebras, and that their intersections generate the lattice of complete Boolean algebras over \( X \). Using this characterization, it is shown that every proper complete Boolean algebra over \( X \) has multiple lattice complements. Hence the lattice is nondistributive. Corollary. The lattice of \( \sigma \)-algebras over a countable space is an anticomponently generated, (multiply) complemented, nondistributive lattice. (Received October 11, 1968.)


We continue the work that was presented in Abstract 653-176, these \( \text{Notices} \) 15 (1968), 130, concerning the distribution of the primitive roots of a prime. Theorem 1. Let each of \( s \) and \( k \) be a positive integer. There is an integer \( N = N(s,k) \) depending only on \( s \) and \( k \) such that if \( p > N \) is a
prime, \( p \equiv 1 \pmod{k} \), then at least one class of \( k \)th power nonresidues modulo \( p \) contains \( s \) distinct primitive roots modulo \( p \) in arithmetic progression. The common difference is in this class and is a primitive root as well. \textbf{Theorem 2.} Let each of \( s \) and \( t \) be a positive integer and let \( p_i \) denote the \( i \)th prime. There is an integer \( N = N(s, t) \) depending only on \( s \) and \( t \) such that if \( p > N \) is a prime of the form \( 1 + p_1^{a_1}p_2^{a_2} \ldots p_t^{a_t} \), \( a_i \equiv 0 \), then there is a sequence of at least \( s \) consecutive primitive roots modulo \( p \). (Received October 11, 1968.)

663-222. G. T. CHARTRAND and S. F. KAPOOR, Western Michigan University, Kalamazoo, Michigan 49001. \textbf{On powers of graphs and Hamiltonian graphs.}

The \( n \)th power \( G^n \) of a connected graph \( G \) is a graph which has as its point set that of \( G \), and two distinct points are adjacent in \( G^n \) if and only if their distance in \( G \) does not exceed \( n \). The graphs \( G^2 \) and \( G^3 \) are referred to as the square and cube, respectively. Results and conjectures involving the Hamiltonian properties of the square and cube are discussed, and the following result is presented. \textbf{Theorem.} If \( G \) is a connected graph with at least four points, then \( G^3 \) is Hamiltonian as in \( G^3 - v \) for every point \( v \) of \( G \). (Received October 11, 1968.)

663-223. A. M. CARSTENS, Washington State University, Pullman, Washington 99163. \textbf{The lattice of pseudotopologies on \( S \).}

Let \( (\Pi(S), \leq) \) be the lattice of pseudotopologies on \( S \), and let \( \mathcal{U}(S) \) denote the family of ultrafilters on \( S \). Let \( X = (\mathcal{U}(S) \times S) \setminus \Delta \), where \( \Delta = \{ (x, x) \mid x \in S \} \), and let \( \Theta(X) \) be the lattice of subsets of \( X \), ordered by set inclusion. \textbf{Theorem.} \( (\Pi(S), \leq) \) is order isomorphic to the dual of \( \Theta(X) \). \textbf{Corollary.} \( (\Pi(S), \leq) \) is a completely distributive, complete Boolean lattice. (Received October 11, 1968.)

663-224. C. E. WALL, Olivet College, Olivet, Michigan 49076. \textbf{On minimal digraphs with given girth and degree.}

A digraph (directed graph) \( D \) is \textbf{regular} of degree \( r \) if every point of \( D \) has indegree \( r \) and outdegree \( r \); the \textbf{girth} of \( D \) is the length of the smallest (directed) cycle of \( D \). We define \( g(n, r) \) to be the fewest number of points in any regular digraph of degree \( r \) having girth \( n \). It is shown for all \( n \geq 2, r \geq 1 \) that \( g(n, r) \neq r(n - 1) + 1 \). A few lower bounds for \( g(n, r) \) are also presented. The number \( g(n, r) \) is determined for several pairs \( (n, r) \) and in some cases the corresponding digraphs are shown to be unique. (Received October 11, 1968.)

663-225. WITHDRAWN.

663-226. G. W. STEWART, Ill., University of Texas, Austin, Texas. \textbf{The generality of Bauer's abbreviated iteration for a zero of a polynomial.}

Let \( f(z) = (z - r_1)(z - r_2) \ldots (z - r_k) \) be a polynomial of degree \( n \) with simple zeros over the field of complex numbers. Let \( \pi_{i}(z) = f(z)/(z - r_i) \). Given the polynomial \( g \) of degree not greater than \( n - 1 \), define the polynomial \( \pi(g) \) as the remainder after dividing \( [g(z)]^2 \) by \( f(z) \); \( \pi(g) = g^2 \pmod{f} \).
Bauer [Sitz. Ber. Bayer. Acad. Wiss. 1954] has shown that if \( g_1(z) = z \) and \(|r_1| > |r_i| (i > 1)\), then the sequence of polynomials defined by \( g_{k+1} = R(g_k) (k \geq 1) \), is in the limit proportional to \( \pi_1 \) and the convergence is quadratic. He has also shown [Z. Angew. Math. Phys. 7 (1956), 17-32] that if some \( g_k \) is nearly proportional to \( \pi_1 \) then again the sequence converges. The following theorem shows that in fact the iteration converges for almost any starting polynomial. **Theorem.** Let \(|g_1(r_1)| > |g_i(r_i)| (i > 1)\). Then if \( g_{k+1} = R(g_k) (k > 1) \), the sequence of polynomials \( g_k(z)/g_k(r_1) \) converges quadratically to \( \pi_1(z)/\pi_1(r_1) \). A similar, but weaker, theorem holds when the zeros of \( f \) are not simple. (Received October 11, 1968.)

663-227. P. M. Rice, University of Georgia, Athens, Georgia 30601. On knot groups.

The paper includes a characterization of those knots of \( S^1 \) in \( S^3 \) for which the knot group has finitely generated commutator subgroup, called Neuwirth knots, in terms of automorphisms of the fundamental groups of their minimal surfaces. (This characterization is implicit in the work of Neuwirth.) By analogy, it is suggested that the class of knots having a minimal surface whose complement is a solid torus may prove tractable enough to admit a similar characterization. One example shows that it is possible for the inclusion in one direction of a minimal surface into its complement (which is a solid torus) to induce a homology isomorphism and a homotopy monomorphism, yet not a homotopy epimorphism. (Received October 11, 1968.)

663-228. J. C. C. Nitsche, University of Minnesota, Minneapolis, Minnesota 55455. Concerning the isolated character of solutions of Plateau's problem.

For a given Jordan curve \( \Gamma \) let \( \mathfrak{S} \) be the set of vectors \( f(u,v) \in C^2(\mathcal{P}) \cap C^0(\overline{\mathcal{P}}) \) which are harmonic in \( \mathcal{P} = \{u,v; u^2 + v^2 < 1\} \), map \( \partial \mathcal{P} \) onto \( \Gamma \) monotonically, and satisfy a three point condition. \( \mathfrak{S} \) is a metric space with distance \( \|f - g\| = \max_{(u,v) \in \overline{\mathcal{P}}} |f(u,v) - g(u,v)| \). A solution of Plateau's problem is a surface \( \mathfrak{I} = \mathfrak{I}(u,v); (u,v) \in \mathcal{P} \) where \( \mathfrak{I} \in \mathfrak{S} \) and \( \mathfrak{I}_u = \mathfrak{I}_v = 0 \) in \( \mathcal{P} \). Denote by \( \mathfrak{M} \) the set of all such vectors. For \( M < \infty \) let \( \mathfrak{M}_M \) be the subset of vectors whose Dirichlet integral equals \( M \). Applying the apparatus of Morse theory to various existence problems, one encounters the concept of a critical set, or block, of minimal surfaces; a block being a component of any nonempty set \( \mathfrak{M}_M \). It has not yet been decided whether a block is made up of an isolated element or may consist of a genuine continuum. Including this is the question whether the solutions of Plateau's problem are isolated in \( \mathfrak{M} \). For either problem there exists in the literature today not even one example which could give a hint to a plausible answer. Here such an example is provided. For a special curve \( \Gamma \) and a special vector \( \mathfrak{I}^{(0)} \in \mathfrak{M} \) it is proved that there exists a number \( \varepsilon > 0 \) such that \( \|\mathfrak{I} - \mathfrak{I}^{(0)}\| \leq \varepsilon \) for all \( \mathfrak{I} \in \mathfrak{M} \). (Received October 10, 1968.)

663-229. R. C. Lacher, Florida State University, Tallahassee, Florida 32306. Some wild spheres and group actions.

**Theorem I.** Let \( n \neq 2 \) and \( k \neq 3 \). Then there exist uncountably many weakly flat embeddings of \( S^n \) into \( S^{n+k} \) no two of which are setwise equivalent. Each is the fixed point set of an involution on \( S^{n+k} \). **Theorem II.** Let \( G \) be a finite group. Then there exist uncountably many topologically distinct actions of \( G \) on \( S^n \) when \( n \) is sufficiently large. **Theorem III.** Let \( p \neq 2 \) and \( q \neq 2 \). Then
there are uncountably many topologically distinct actions of $SO(2)$ on $S^{2p+q}$, each rotating freely about a q-sphere of fixed points. (Received October 2, 1968.)


If $(X_1, \| \cdot \|_1)$ (i = 1, 2) are normed spaces then $X_1 \times X_2$ is the C-product $(X_1 \times X_2)_C$ (resp. L-product $(X_1 \times X_2)_L$) if its norm is $\| (x_1, x_2) \|_C = \max \{ \|x_1\|_1, \|x_2\|_2\}$ (resp. $\| (x_1, x_2) \|_L = \|x_1\|_1 + \|x_2\|_2$). $X_1 \times X_2$ is a CL-product $(X_1 \times X_2)_{CL}$ if it is either a C-product or an L-product. A normed space is itself a CL-product of one factor. $Z$ is a CL-product of n factors $X_i$, $Z = (X_1 \times X_2 \times \ldots \times X_n)_{CL}$, if $Z = (U \times V)_{CL}$ where $U$ is a CL-product of p distinct factors from among the $X_i$ and $V$ is a CL-product of the remaining $n - p$ factors ($1 \leq p \leq n - 1$). In no C-product (resp. L-product) is involved in forming $(X_1 \times X_2 \times \ldots \times X_n)_{CL}$, it is called a pure L-product (resp. pure C-product). A C-space (resp. L-space) is an abstract (M) space with unit (resp. abstract (L) space with F-unit). A CL-space is a Banach space whose unit ball $U = co[F \cup (-F)]$ for every maximal convex subset $F$ of its surface. C-spaces and L-spaces are CL-spaces. A CL-product of CL-spaces is a CL-space. A CL-product $Z$ of Banach lattices $X_i$ (1 $\leq i \leq n$) is a C-space (resp. L-space) in its natural ordering if and only if each $X_i$ is a C-space (resp. L-space) and $Z$ is the pure C-product (resp. pure L-product) of the $X_i$. (Received October 14, 1968.)

663-231. B. L. BRECHNER, University of Florida, Gainesville, Florida 32601. Locally setwise homogeneous continua and their homeomorphism groups.

In an earlier paper, On the dimensions of certain spaces of homeomorphisms, Trans. Amer. Math. Soc. 121 (1966), 516-548, the author defined locally setwise homogeneous (1-s-h) continua, and showed that their groups of homeomorphisms are nonzero dimensional. In this paper the minimal normal subgroups of 1-s-h continua are determined. In order to do this, the notion of near basis is defined. The union of the elements of a near basis is called the core of X. Theorem 1. Every 1-s-h continuum has a near basis. Theorem 2. Let X be a 1-s-h continuum, and let $\mathcal{D}$ be a near basis for X. Let $Q(\mathcal{D})$ be the group of homeomorphisms of X generated by those homeomorphisms which are supported on an element of $\mathcal{D}$. Then $Q(\mathcal{D})$ is a minimal normal subgroup of X. Corollary. If $\mathcal{D}_1$ and $\mathcal{D}_2$ are near bases for X, then $Q(\mathcal{D}_1) = Q(\mathcal{D}_2)$. Theorem 3. The core of X is a dense, open, connected subset of X, and is the union of the dense orbits of X. (Received October 14, 1968.)

663-232. R. B. GRAFTON, University of Missouri, Columbia, Missouri 65201. Existence theorems for the initial value problem with infinite sum.

We consider the initial value problem for ordinary differential equations with deviating argument, $\dot{x}(t) = A_0(t)x(t) + \sum_{i=1}^{\infty} A_i(t)x(a_i(t))$, $x(0) = x_0 \in \mathbb{R}^n$ given, and $a_i(t)$ are continuous functions on $R_+$ = $\{t, t \geq 0\}$ with $0 \leq a_i(t) \leq t$. Under the assumption that $A_i(t)$ are continuous on $R_+$, and on any compact subset of $R_+$, $\sum_{i=1}^{\infty} A_i(t)$ converges to a continuous function, the following theorems are proven. Theorem 1 (Existence). For any $x_0 \in \mathbb{R}^n$, there exists a solution of the initial value problem, $x(t,x_0)$, with the property that $\|x(t,x_0)\| \leq \|x_0\|\exp{\int_{0}^{t}\sum_{i=1}^{\infty} A_i(s)ds}$, $t \in R_+$. Theorem 2 (Stability). If the principal matrix solution, $X(t)$, of $\dot{X}(t) = A(t)X(t)$ has the properties $\|X(t)\| \leq K \exp(-\alpha t)$, $\alpha > 0$,
\( t \in \mathbb{R}_+, \text{ and } \|x(t)X^{-1}(s)\| \leq K \exp\{-a(t - s)\}, t \in \mathbb{R}_+ \), then providing that \( \lambda(t) = \sum_{i=1}^{\infty} |A_i(t)| \) satisfies \( K \int_0^t \exp\{-a(t - s)\} |\tilde{x}(s)| ds = m < 1, t \in \mathbb{R}_+ \), the solution of the initial value problem, \( x(t,x_0) \) is unique and satisfies \( \|x(t,x_0)\| \leq K \|x_0\|/(1 - m), t \in \mathbb{R}_+ \). With a stronger condition on \( \lambda(t) \), an asymptotic stability theorem is also proven. Proofs of the above theorems are applications of a theorem of Corduneanu on the functional integral equation, \( x(t) = h(t) + \int_0^t k(t,s)f(s,x_s)ds, t \in \mathbb{R}_+ \), where \( x_s \) is the restriction of the function \( x \) to \([0,s] \). Corduneanu's theorem is to appear in the Bulletin Mathematique (Bucharest). (Received October 14, 1968.)


A more stringent criterion for k-polar polynomials.

The present author previously published (Pacific J. Math. 12 (1962), 1277-1288) a generalization of Grace's theorem on apolar polynomials which gave conditions under which every circular domain which contains all the zeros of one of a pair of polynomials must contain at least \( k \) zeros of the other, where \( 1 \leq k \leq n \). These conditions consist of \( k^2 \) relations involving the coefficients of the two polynomials. (For values of \( k \) sufficiently close to \( n \), these conditions necessitated that the two polynomials have a common, repeated zero.) The present paper shows that the same conclusion (except for that regarding multiple zeros) can be reached by using only \( 2k - 1 \) relations among the coefficients. (Received October 14, 1968.)

663-234. H. F. JOINER, II, University of Massachusetts, Amherst, Massachusetts 01002.

Tensor product of biorthogonal systems.

The following theorem is proved concerning the tensor product of biorthogonal systems (for a definition of \([x_i,x_j] \otimes [y_i,y_j]\), see Pacific J. Math. 2 (1961), 1281-1286). Theorem. Let \( E \) and \( F \) be separated barreled spaces with respective biorthogonal systems \([x_i,x_i']\) and \([y_i,y_i']\). Consider the properties: (a) a Schauder basis; (b) a basis of type \( t \); (c) a basis of type \( P \); (d) a basis of type \( P^* \); and (e) an absolute basis. In order that \([x_i,x_i'] \otimes [y_i,y_i']\) have any of the above properties on \( E \otimes F \) with a topology between the projective tensor product topology and the topology of biequicontinuous convergence, it is necessary and sufficient that both \([x_i,x_i']\) and \([y_i,y_i']\) have the same property. Several corollaries are also obtained concerning the completion of \( E \otimes F \). (Received October 14, 1968.)


Derivable chains of planes.

Let \( D_1 \) be a derivable plane, \( D_2 \) the plane derived from \( D_1 \), and \( D_3 \) the affine restriction of the dual of the projective extension of \( D_2 \). If \( D_{1+2n} \) is derivable for each positive integer \( n \), repeat the above derive-dualize-derive process for each plane \( D_{1+2n} \). If \( D_1 \) is a dual translation plane of order \( q^2 \) for \( q \) a prime power, the additive group of the (Hall) coordinate system \((C,+,\cdot)\) of \( D_1 \) is a (right) 2-dimensional vector space over the (dual) kernel of \( D_1 \) (relative to \( x \cdot a, x \in C \), and \( a \in GF(q) \) as scalar product), and \( x \cdot a = a \cdot x \) for all \( x \in C \) and for all \( a \) in \( GF(q) \), then \( D_{1+2n} \) is derivable for each positive integer \( n \) and the chain of planes is periodic of length eight. That is,

\[
D_1 = D_2 = D_4 = D_8, \quad D_{10} = D_3 = D_6, \quad D_{11} = D_5, \quad \text{etc.}
\]

(Received October 14, 1968.)
Admissible modules and reduced left Artinian rings.

Let $R$ denote a ring with identity, $J$ its Jacobson radical, $N^*$ its generalized nil-radical. $R$ is called reduced if $R/J$ is a direct sum of division rings. A module $M$ is a unitary left-$R$-module, $E_R(M)$ its injective hull, $S_R(M)$ its socle and $C_R(M)$ its heart (cf. L. Lesieur and R. Croisot, Coeur d'un module, J. de Math. 42 (1963), 367-406). $M$ is called admissible if each prime ideal which is the left-annihilator of all nonzero submodules of a nonzero submodule $N$ of $M$ is also the left-annihilator of all nonzero elements of $N$. Theorem 1. The following properties of $R$ are equivalent: (1)(a) $R$ is left Noetherian, (b) each module is admissible, (c) $C_R(R/P) = S_R(R/P)$ for each prime ideal $P$. (2) Each module is isomorphic to a submodule of a direct sum of copies of $E_R(R/N^*)$. (3) $R$ is a reduced left Artinian ring. Theorem 2. The following properties of a reduced left Artinian ring $R$ are equivalent: (1) $S_R(M) = C_R(M)$ for all modules $M$. (2) Each left ideal $L$ of $R$ is contained in its tertiary radical ter $L$. (Received October 14, 1968.)

The structure of edge-minimal graphs.

A graph $G$ is a pair $(V, E)$ where $V$ (the vertices) is a set and $E$ (the edges) is a subset of $P_2(V)$, the set of 2-element subsets of $V$. $H = (V', E')$ is a subgraph of $G$ if $V' \subseteq V$ and $E' = E \cap P_2(V')$. $V' \subseteq V$ is in the class $\mathcal{J}$ if $E \cap P_2(V') = \{\}$, the empty set. We assume card $V < \infty$. $I(G) = \max\{\text{card } V'\}$ over all $V' \in \mathcal{J}$. If, for each $e \in E$, $i = I(G) < I(G')$ where $G' = (V, E \setminus \{e\}$, then $G \in \mathfrak{m}_i$, the class of i-minimal graphs. The class $\mathfrak{m}$ of edge-minimal graphs is defined as $\bigcup \mathfrak{m}_i$. $i = 1, 2, \ldots$. Theorem 1. If $G$ is connected and in $\mathfrak{m}$, then for some integer $k > 1$, $G$ has as a subgraph $H_{2k+1}$, $2k + 1$ vertices connected cyclically by as many edges. Corollary. Every connected graph in $\mathfrak{m}_2$ contains an $H_5$. $G \in \mathfrak{p}$, the class of primitive graphs, if $(\{a, x\} \in E \Rightarrow \{b, x\} \in E$ for all $x \neq a$ or $b$ in $V$) holds for no $\{a, b\} \in E$. Letting $\mathcal{J}$ be the partial ordering of $\mathfrak{m} \cap \mathfrak{p}$ by inclusion as subgraphs, we have Theorem II. At any height above its least element, $\mathcal{J}$ has a finite number of elements. (Received October 14, 1968.)

On radicals in a certain class of semigroups.

Let $S$ be a semigroup and consider the law $xyzx = yxxzx$ for all $x, y, z \in S$. The commutative semigroups and all subsemigroups of class 2 nilpotent groups satisfy this law. Accordingly, this law will be called the $C_2$ law and any semigroup which satisfies it will be referred to as a $C_2$ semigroup. R. Sulka (Mat. Fyz. Casopis Sloven Akad. Vied 13 (1963), 209-222) proved that if $S$ is a commutative semigroup and $J$ is an ideal of $S$, then the Clifford, McCoy, and completely prime radicals with respect to $J$ are equal to $N(J)$, the set of all nilpotent elements with respect to $J$. The purpose of this paper is to extend the above mentioned result to the class of $C_2$ semigroups. (Received October 14, 1968.)

Compactifications of Hausdorff spaces.

One describes methods of imbedding a Hausdorff space $X$ in a compact space $\overline{X}$ so that each
function in a given family of continuous functions on X has a continuous extension to X and the family of extensions separates the points of X - X. In particular, if X is completely regular but not locally compact, then there is a non-Hausdorff compactification which contains X as an open subset.


We consider the Cesàro summability, for integral orders of the series \( \sum_{\nu=0}^{\infty} a_\nu d_\nu \), and establish limitation theorem. **Theorem** (\( k = 0,1,2, \ldots \)). Suppose that \( d_n > 0 \), for \( n \geq 0 \), and (i) \( d_{n+1} = o(1) \) as \( n \to \infty \); (ii) \( | \Delta b_j | \leq K | \Delta^{j-1} b_j | \), \( j = 1,2, \ldots, k+1 \), where \( \Delta b_j = b_j - b_{j-1} \); (iii) \( (d_{n+1}/n)^k \sum_{\nu=0}^{n} \nu^k b_\nu = O(1) \), where \( b_\nu = 1/d_{\nu+k+1} \), and \( B^\nu_k = \binom{n+k}{k} \). Then \( A_n = o(n^k/d_{n+1}) \) whenever \( \sum_{\nu=0}^{\infty} a_\nu d_\nu \) is summable (C,k). (Received October 15, 1968.)

663-241. ANDREW SOBCZYK, Clemson University, Clemson, South Carolina 29631, and R. KEOWN, University of Arkansas, Fayetteville, Arkansas 72701. Isometries of finite-dimensional Banach spaces.

The groups \( \mathfrak{O}(n) \), \( \mathfrak{U}(n) \) of respectively all orthogonal, unitary transformations of \( n \)-dimensional Euclidean, unitary space, are compact; the unimodular groups are not compact. In this paper, it is proved that the groups of all isometries of an arbitrary finite-dimensional, real, complex Banach space \( X \), which leave the origin of \( X \) fixed, also are compact groups. For real \( X \), it may happen that there is a complexification \( Z \) of \( X \), and a Hermitian inner product \((,\) on \( Z \) such that the group \( G \) of isometries of \( X \) is isomorphic with a subgroup of \( \mathfrak{U}(n) \) for \( Z \) with \((,\) ). When this is the case, a necessary and sufficient condition is found, in terms of the nature of certain irreducible representations, in order that there will exist a "real" basis for \( Z \). It is shown that the existence of such a basis is equivalent to \( G \) being a subgroup of \( \mathfrak{O}(n) \) (where \( \mathfrak{O}(n) \) is with respect to the real inner product obtained by restriction to \( X \) of the given \((,\) ) on \( Z \)). (Received October 15, 1968.)

663-242. JÖRG BLATTER, University of Texas, Austin, Texas 78712. Smoothness and rotundity in quotient spaces.

Let \( X \) be a normed linear space and \( A \) a closed linear subspace of \( X \). \( \mathfrak{N} \) denotes the canonical projection of \( X \) onto the quotient space \( X/ A \), normed by \( \| \mathfrak{N} x \| = \inf \{ \| x - a \| : a \in A \} \). \( A \) is called an "existence subspace" of \( X \) if for each \( x \) in \( X \) there exists at least one best approximation in \( A \), i.e. an element \( a \) in \( A \) such that \( \| x - a \| = \| \mathfrak{N} x \| \). The results obtained imply the following (cf. M. M. Day, Normed linear spaces, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1958; and V. Klee, Some new results on smoothness and rotundity in normed linear spaces, Math. Ann. 139 (1959), 51-63, for related results and for the terminology). **Theorem.** If \( X \) is smooth (resp. rotund) then \( X/ A \) is smooth (resp. rotund) if and only if for each \( x \) in \( X \) that has no best approximation in \( A \), \( \| \mathfrak{N} x \|^{-1} \cdot \mathfrak{N} x \) is a smooth point (resp. extreme point) of the unit ball of \( X/ A \). **Corollary.** The quotient space of a smooth (resp. rotund) normed linear space by an existence subspace is smooth (resp. rotund). **Theorem.** Let \( 1 < p < \infty \). If \( (X_1: i \in I) \) is a family of at least two smooth and rotund nonreflexive normed linear spaces, then the smooth and rotund space \( \ell_p (X_1: i \in I) \) (cf. M. M. Day, loc. cit.) contains closed linear subspaces of finite and infinite codimension that are not existence subspaces, whereas the corresponding quotient spaces are smooth and rotund. (Received October 15, 1968.)
and is "bigger" than the Stone-Čech compactification of X. (Of course, every compactification of X is non-Hausdorff if X is not completely regular.) One also shows that the completion of a metric space M may be obtained as a subset of a compactification of M. (Received October 14, 1968.)


The following identity is proved. \((- xq;q)_N = \sum_{n=0}^{N} (- xq;q)_n \frac{x^n q^{n(3n-1)/2} (1 + xq^2n)}{(1 - xq^{2n})} \) \( [N,n] = xq^{n+N-1} q^{-n} \), where \((a;q)_n = \prod_{j=0}^{n-1} (1 - aq^j)\), \([N,n] = (q;q)_N (q;q)_n (q;q)_{N-n}\). This result is deduced by studying the order of a partition involving distinct parts. The order of a partition (relative to N) is defined to be the largest integer n for which the number of parts of the partition in the interval \([n,N + n - 1]\) is \(\equiv n\). The above identity may be utilized with Bailey's transform to prove the Rogers-Ramanujan identities, the Göllnitz-Gordon identities, Schur's partition theorem, and other results. (Received October 16, 1968.)

663-244. P. M. PREENTER, Colorado State University, Fort Collins, Colorado 80521. LaGrange and Hermite interpolation in Banach spaces.

Let X be a Banach space and let \(f: X \rightarrow X\) be a function. Then there exists a polynomial (polynomial operator) \(y(x)\) of degree \(n - 1\) which interpolates \(f\) at \(n\)-distinct points \(x_1, x_2, ..., x_n\). Furthermore \(y(x)\) has a LaGrange representation given by \(y(x) = \sum_{i=1}^{n} \left[ w_i'(x_i) \right] -1 w_i(x)/(x - x_i)f(x_i)\) where \(w_i(x) = L_i(x - x_1, x - x_2, ..., x - x_n)\), \(w_i'(x_i)\) is the first Fréchet derivative of \(w\) at \(x_i\) and each \(L_i\), \(i = 1, 2, ..., n\), is an appropriately chosen \(n\)-linear operator. In an analogous manner, a Hermite polynomial (polynomial operator) \(\bar{y}(x)\) of degree \((2n - 1)\) is derived which interpolates \(f\) and \(f'\) at \(n\)-distinct points \(x_1, x_2, ..., x_n\). Finally, if X is a Hilbert space, the polynomials \(y(x)\) and \(\bar{y}(x)\) are shown to have a simple representation in terms of inner product. (Received October 15, 1968.)


A directed space is a Hausdorff space X endowed with a partial order \(\Gamma\) satisfying (i) \(\Gamma\) is a closed subset of \(X \times X\) and (ii) if \(x, y \in X\) then \(\Gamma x \cap \Gamma y\) is nonempty. **Sample Theorem.** If X is a compact directed space and if there exists a unique \(x_1 \in X\) such that \(\Gamma x_1\) is not acyclic, then \(x_1\) is maximal and \(\Gamma x_1\) and X have the same cohomology. This is an extension of Wallace's theorem on acyclicity. Further extensions, of a fairly natural sort, can be obtained for all cases where \(\{x \in X: \Gamma x\) is not acyclic\} is assumed to be finite. (Received October 15, 1968.)

663-246. H. B. REITETER, Clemson University, Clemson, South Carolina 29631. Topologies with compact subtopologies.

**Definition.** A topological space X is called a \(\gamma\)-space if there is a compact Hausdorff space K and a continuous bijection f on X to K. Every locally compact Hausdorff space is a \(\gamma\)-space. The product of any family of \(\gamma\)-spaces is a \(\gamma\)-space. There are \(\gamma\)-spaces with closed subsets which are not \(\gamma\)-spaces. **Theorem.** Let X be a topological space. The following are equivalent: (a) X is a
Y-space. (b) The topology of $X$ contains a compact Hausdorff topology. (c) $X$ is homeomorphic with
the graph of a function $f$ on some compact Hausdorff space. (Received October 15, 1968.)

663-247. E. V. HAYNSWORTH, Auburn University, Auburn, Alabama 36830. Some properties
of the Schur complement.

Let $A = (A_{ij})$ be an $n \times n$ complex matrix with a nonsingular principal submatrix $B$. We define
the Schur complement of $B$ in $A$, denoted by $(A/B)$, as follows: By a simultaneous permutation of
rows and columns, obtain a matrix $\hat{A}$ with $B$ in the upper left corner and partition $\hat{A}$ into a $2 \times 2$
block matrix with $B = A_{11}$. Then $(A/B) = (\hat{A}/\hat{A}_{11}) = \hat{A}_{22} - \hat{A}_{21}\hat{A}_{11}^{-1}\hat{A}_{12}$. Schur proved $\det A = \det B \det (A/B)$. In a paper with D. Crabtree (submitted for publication) it was shown that if $B$ is
defined as above and $C$ is a nonsingular principal submatrix of $B$, then $(A/B) = ((A/C)/(B/C))$. In a
previous paper the author proved that the inertia of an Hermitian matrix $A$ can be obtained from the
inertia of any nonsingular principal submatrix of $A$ together with that of its Schur complement. Other
properties of the Schur complement are developed and applied to the computation of the eigenvalues
of a matrix and of the inverse of a partitioned matrix. (Received October 15, 1968.)

663-248. P. C. LOH, 1201 E. 38th Street, Indianapolis, Indiana 46205. Qualitative properties
of satellite orbits.

The potential at a point $(r, \theta, \phi)$ in space due to an oblate spheroid with rotational symmetry is
$-r^{-1} - J r^{-2} \cos \theta - U(r, \cos \theta)$. In a coordinate system where the origin is not at the center of mass,
$J$ is a constant different from zero. $U(r, \cos \theta)$ is taken as a perturbation term. The unperturbed
equations of motion can be solved explicitly. The perturbed equations induce a mapping (due to
Poincaré) which satisfies the conditions of a theorem by J. Moser (Nachr. Akad. Wiss. Göttingen
Math.-Phys. Kl. II (1962), 1-20). As a consequence, the existence of solutions for all time, and the
existence of periodic invariant manifolds and almost periodic solutions, are established. (Received
October 15, 1968.)

663-249. D. R. WILKEN, State University of New York, Albany, New York 12203. Some
remarks on the string of beads.

Let $D$ be the closed unit disc in the plane. Let $K$ be a closed subset of the interval $[-1,1]$. If $I$ is any
interval component of $[-1,1]$-$K$, remove from $D$ the open disc with $I$ as a diameter. Let $X$
be the compact set which remains when all such open discs have been deleted. In the case where
$K$ has no interior relative to $[-1,1]$, $X$ is referred to as the "string of beads" example. Let $R(X)$
denote the function algebra consisting of uniform limits on $X$ of rational functions with poles off $X$.
If $K$ has positive linear measure Hoffman (K. Hoffman, "Parts and analyticity" in Function algebras,
Scott, Foresman, pp. 1-5) has conjectured that there is always some point of $K$ in the same part as
int $K$. We partially answer this conjecture as follows. Let $\rho_n$ denote the radius of the $n$th deleted
disc. Theorem. If $\sum_{n=1}^{\infty} a_n < 1$ and there exist positive numbers $a_n$ and $b_n$ such that (i) $\rho_n = a_n b_n$
and (ii) both $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, then almost every point of $K$ lies in the same part as
int $X$ (the condition $\sum_{n=1}^{\infty} \rho_n < 1$ is equivalent to $K$ having positive linear measure, of course). To my
knowledge the general conjecture remains unanswered although it is, I think, universally believed to be true. This conjecture is only a special case of the much broader conjecture that the Gleason parts of R(X) are connected, for any compact plane set X. (Received October 15, 1968.)

663-250. WITHDRAWN.

663-251. T. B. RUSHING, University of Georgia, Athens, Georgia. Taming locally flat embeddings.

Theorem 1. Let \( f: M^k \to N^n \), \( n - k \geq 3 \), \( n \geq 5 \), be a proper embedding of the PL \( k \)-manifold \( M^k \) into the PL \( n \)-manifold \( N^n \) such that \( f|M \) is PL and \( f|\overline{M} \to \overline{N} \) is locally flat and let \( \varepsilon > 0 \) be given. Then there exists a neighborhood \( U \) of \( M \) in \( M \) and an \( \varepsilon \)-push \( e_\varepsilon \) of \( (N, f(M)) \) which is fixed on \( \overline{N} \) such that \( e_\varepsilon f|U : U \to N \) is PL. Definitions. If \( P \subseteq Q \) are polyhedra we say there is an elementary \( C(X) \)-construction from \( P \) to \( Q \), written \( P \overset{C(X)}{\to} Q \), if there exists a pair \((C(\gamma), \gamma) \overset{P}{\to} (C(X), X) \) \((C(X) = \text{cone over polyhedron } X)\) such that (1) \( Q = P \cup C(X) \), (2) \( X = P \cap C(X) \), and (3) \( P \) is link collapsible on \( X \) and \( P \to X \to P \). \( Q \) is said to be \( C(B) \)-constructible from \( P \), written \( P \overset{C(B)}{\to} Q \), if there is a sequence \( P \overset{C(B)}{\to} P_1 \overset{C(B)}{\to} P_2 \overset{C(B)}{\to} \ldots \overset{C(B)}{\to} Q \), where \( X_i \) is a PL-ball. Similarly, \( Q \) is \( C(S) \)-constructible if there is a sequence where each \( X_i \) is a PL-sphere. A \( P \)-nice polyhedron \( Q \) has the property that \( P \overset{C(B)}{\to} L \overset{C(S)}{\to} Q \). Theorem 2. Let \( Q \) be a \( P \)-nice polyhedron such that \( \dim Q = P \leq n - 3 \), \( n \geq 5 \), and let \( f: Q \to \overline{M}^n \) be an embedding of \( Q \) into the interior of the PL \( n \)-manifold \( M \) which is locally flat on the open simplexes of some triangulation \( T \) and is such that \( f|P: P \to M^n \) is tame. Then, \( f: Q \to M^n \) is tame and if \( f|P \) is PL, the taming isotopy is fixed on \( f(P) \). (Received October 14, 1968.)

663-252. WITHDRAWN.


Theorem 1. Let \( \tau \) be any cardinal number. For a stratifiable space \( X \) the following are equivalent: (a) There is a subset of cardinality \( \tau \) which is dense in \( X \). (b) Each open cover \( \mathcal{U} \) has a subcover of cardinality \( \tau \). (c) Every disjoint system of open sets has cardinality less than equal to \( \tau \). (d) \( X \) has a network of cardinality \( \tau \). Theorem 2. If \( X \) is a stratifiable space and \( f: X \to Y \) is a continuous closed surjection, then \( Y = Y' \cup Y'' \) where \( Y' \) is a \( \sigma \)-discrete subset of \( Y \) and \( f^{-1}(y) \) is compact for each \( y \in Y'' \). Moreover \( \aleph(Y') \leq \text{weight of } X \) where \( \aleph(Y') \) denotes the cardinal number of \( Y' \). Theorem 1 generalizes a result due to Ceder (Some generalizations of metric spaces, Pacific, J. Math. 11 (1961), 105-126). Theorem 2 generalizes a result of Lashnev (Continuous decompositions and closed mappings of metric spaces, Dokl. Akad. Nauk. SSSR 165 (1965), 756-578 = Soviet Math. Dokl. 6 (1965), 1504-1506). (Received October 14, 1968.)
The structure of large subgroups of primary abelian groups.

If \( L \) is a subgroup of \( G \), then \( L \) is a large subgroup of \( G \) iff \( L = \sum_{k=1}^{\infty} p \mathbf{G} \left[ \mathbf{G}^{k-n_k} \right] \) where

1. \( n_k \geq k \forall k \in \mathbb{Z}^+ \);
2. \( n_k \leq n_{k+1} \leq n_k + 1 \forall k \in \mathbb{Z}^+ \) and the sequence \( 1 - n_1, 2 - n_2, 3 - n_3, \ldots \) is unbounded.

This representation of large subgroups is of interest as the Ulm invariants of \( L \) can now be given in terms of those of \( G \), that is, \( f_L(n) = \sum f_G(k - 1) : k - n_k - 1 = m \forall m \in \mathbb{Z}^+ \). In general, a large subgroup of \( G \) is not quasi-isomorphic to \( G \); however, Theorem. If \( G \) is a \( p \)-group such that \( G/G_1 \) is a direct sum of cyclic groups, then all large subgroups of \( G \) are quasi-isomorphic iff either \( G = B \oplus D \) where \( B \) is bounded and \( D \) is divisible or there exists a sequence of positive integers \( n_1, n_2, n_3, \ldots \) and an infinite cardinal number \( m^* \) such that \( f_G(n_i) = m^* \) for all \( i \), \( f_G(n) = m^* \) if \( n \geq n_1 \) and there exists an integer \( d > 0 \) such that \( n_{i+1} - n_i \geq d \) for all \( i \). Theorem. Among the properties enjoyed both by a group and its large subgroups are finite length, direct sum of cyclic groups, direct sum of countable groups, totally projective, torsion-complete, and quasi-closed. (Received October 14, 1968.)

 Bounds for eigenvalues of nonnegative matrices.

By using methods in the theory of partially ordered linear algebras one can obtain bounds for the real eigenvalues of nonnegative matrices. For example, if \( 0 \leq x^2 = x \), where \( x \) is a matrix of any order, then the real eigenvalues of \( x \) must lie between 1 and \((1 - \sqrt{2})/2\). This also gives us some estimates for complex eigenvalues. (Received October 14, 1968.)

On preordering a topological vector space over the reals.

Preliminary report.

By a preorder on a set \( S \) is meant a binary relation which is reflexive and transitive and in which every two members are comparable. Theorem. Let \( X \) be a topological linear space over the reals. Let \( (\geq) \) be a preorder on \( X \) and suppose that (i) if \( x, y \in X \) and \( x (\geq) y \), then \( x + z (\geq) y + z \) for all \( z \in X \) and that (ii) \( \{ x \mid x (\geq) 0 \} \) is closed, then there exists \( A \in X' \), where \( X' \) is the conjugate space of \( X \), such that \( x (\geq) y \) if and only if \( A(x) \geq A(y) \). Furthermore, if there exists \( x_0 \in X \) such that \( x_0 > 0 \), i.e. \( 0 \leq x (\geq) 0 \), then \( x (> y) \) if and only if \( A(x) > A(y) \). If no such \( x_0 \) exists the proof of the theorem is easy to see after showing that \( X \) is a subset of \( X_0 = \{ x \mid x (\geq) 0 \text{ and } 0 (\equiv) x \} \) and \( A = 0 \). On the other hand if there exists \( x_0 (> 0) \), then it can be shown that \( X_0 \) is a proper subspace of \( X \) and that \( \dim X/X_0 = 1 \), and that if \( x \in X \), there exists a unique real number \( a \) such that \( x - ax_0 \in X_0 \). A is defined by \( A(x) = a \). (Received October 14, 1968.)

Withrawn.

A structure theory for finite regular semigroups.

Let \( S \) be an arbitrary finite regular semigroup. Then by generalizing the Rees theorem for
regular finite zero-simple semigroups, we inductively construct a regular semigroup $W$ and an epimorphism $\varphi: W \to S$ such that (i) if $I_1$ and $I_2$ are intersecting principal right (left) ideals of $W$, then either $I_1 \subseteq I_2$ or $I_2 \subseteq I_1$, (ii) $\varphi$ separates $D$-classes and is one-to-one on subgroups of $W$, and (iii) $W$ is functorial, so that in a sense $W$ is a "universal covering space" for $S$. These results have important applications to the problem of computing the group-complexity of a finite semigroup $S$.

(Received October 15, 1968.)


Let $\mathcal{A}$ be the category of locally compact abelian groups, with continuous homomorphisms as morphisms. Let $\chi: \mathcal{A} \to \mathcal{A}$ denote the contravariant functor which assigns to each object in $\mathcal{A}$ its character group and to each morphism its adjoint morphism. The Pontrjagin duality theorem is then the statement that $\chi \circ \chi$ is naturally equivalent to the identity functor in $\mathcal{A}$. A sequence of morphisms in $\mathcal{A}$ is called proper exact if it is exact in the algebraic sense and is composed of morphisms each of which is open considered as a function onto its image. Let $R$ denote the real numbers and $T$ the circle group. Let $\varphi: \mathcal{A} \to \mathcal{A}$ be an arbitrary contravariant functor. Theorem A. $\varphi$ is naturally equivalent to $\chi$ if and only if the following four statements are all true: (1) $\varphi(R)$ is isomorphic to $R$, (2) the map $\text{Hom}(R, T) \to \text{Hom}(\varphi(T), \varphi(R))$ induced by $\varphi$ is continuous, (3) $\varphi$ takes short proper exact sequences to short proper exact sequences, and (4) $\varphi$ takes inductive limits of discrete groups to projective limits and takes projective limits of compact groups to inductive limits. From this follows Theorem B. $\varphi$ is naturally equivalent to $\chi$ if and only if $\varphi$ is a category equivalence and the map $\text{Hom}(R, T) \to \text{Hom}(\varphi(T), \varphi(R))$ induced by $\varphi$ is continuous. (Received October 15, 1968.)

663-260. PAWEL SZEPTYCKI, University of Kansas, Lawrence, Kansas 66044. Measurable functions in configurations of Banach spaces.

Let $I_0 \subseteq R^1$ be an interval, denote by $\mathcal{J}$ the family of all compact subintervals of $I_0$ and by $\omega$ the usual distance on $\mathcal{J}$. Let $\{J_i\}_{i \in J}$ be a family of closed subspaces of a Banach space $\mathcal{X}$ such that $\mathcal{X}_i \subseteq \mathcal{X}_{i'}$ for $i \subseteq j$ and $\text{dist}(x, \mathcal{X}_i)/\text{dist}(x, \mathcal{X}_j)$ for $i \subseteq j$ and every $x \in \mathcal{X}$. Let $\mathcal{V}_i = \mathcal{X}/\mathcal{X}_i$; then the natural projections $P_{I}: \mathcal{X} \to \mathcal{V}_i$ for $I_1 \subseteq I$ is also in an obvious way considered as norm decreasing mappings $P_{I}: \mathcal{V}_i \to \mathcal{V}_i$ for $I_1 \subseteq I$. The disjoint union $\bigcup_{I \in \mathcal{J}} \mathcal{V}_i$ is provided with the metric $d(u_{I}, v_{I}) = \inf \{\|P_{I}u_{I} - P_{I}v_{I}\| + \omega(I, I') + \omega(I, I')' : I_1 \subseteq I \subseteq J\}$; it is an arcwise connected metric space. Proposition. A function $f: I_0 \to \bigcup_{I \in \mathcal{J}} \mathcal{V}_i$, $t \in I_0$, is (Lebesgue) measurable iff $P_{I}f_{I}: I \to \mathcal{V}_i$ is measurable for every $I \in \mathcal{J}$. Theorem. If $\mathcal{J}$ is separable or reflexive and round, then for every (Lebesgue) measurable function $f: I_0 \to \bigcup_{I \in \mathcal{J}} \mathcal{V}_i$, $t \in I_0$, there is a measurable function $\tilde{f}: I_0 \to \bigcup_{I \in \mathcal{J}} \mathcal{V}_i$ such that $P_{I}f_{I}(t) = \tilde{f}(t)$, $t \in I_0$. For every measurable $\varepsilon(f(t)) > 0$, $\tilde{f}(t)$ may be chosen so that $\|\tilde{f}(t)\| \leq \|f(t)\| + \varepsilon(t)$. (Received October 14, 1968.)


It has been known for some time that if a compact ring can be embedded in a compact ring with unit it must be totally disconnected, and hence 0-dimensional---see the increasingly stronger results.
of H. Anzai (Proc. Imp. Acad. Tokyo 19 (1943), 613-615), Y. Otobe (Proc. Imp. Acad. Tokyo 20 (1944), 278-282), I. Kaplansky (Amer. J. Math. 69 (1947), 153-183), and J. Selden (Proc. Amer. Math. Soc. 15 (1964), 862-886). We use well-known results for topological abelian groups and results due to the author (J. Australian Math. Soc. 8 (1968), 183-191) to prove the converse. This gives the following theorem: Theorem. A compact ring can be embedded in a compact ring with unit if and only if it is 0-dimensional. (Received October 9, 1968.)


Let $W$ be an $n$-dim complex manifold and $V$ a hypersurface of $W$, i.e. a complex analytic subvariety of codim one. A $p$-cycle $\gamma_p \in H(W - V)$ which bounds in $W$ is called a residue. Then a tubular neighborhood of $V$ in $W$ is defined so that Theorem. $\gamma_p$ is homologous in $W - V$ to a tube over some $(p - 1)$-cycle of $V$. (Received October 10, 1968.)

663-263. HARRY GONSHOR, Rutgers, The State University, New Brunswick, New Jersey 08903. Contributions to genetic algebras.

I. M. H. Etherington introduced the technique of using nonassociative algebras to study population genetics, and included the concepts of special train algebra and the operation of duplication. In this paper the author generalizes previous results by showing that the algebra obtained by combining polytopidy, multiple alleles, and mutations is a special train algebra. The author also introduces the operation of sex-linked duplication which is analogous to duplication and is useful for the study of sex-linked traits. (Received October 16, 1968.)


Consider the system of differential equations (*) $\frac{dx}{dt} = g(t,x)$, $t \geq t_0$, where $x,g$ are $n$-vectors and $g(t,0) \equiv 0$. The usual definition of stability of the zero solution of (*) is extended by introducing positive continuous control functions $\phi = \phi(t)$ and $\psi = \psi(t)$ in the following manner. Definition. The solution $x = 0$ of (*) will be called $(\phi, \psi)$-stable if given any $\epsilon > 0$ there exists a positive number $\delta_\phi = \delta_\phi(t, \epsilon)$ such that if $x_1$ satisfies the inequality $|x_1| < \delta_\phi(t, \epsilon)$ then $x(t; t_1, x_1)$ exists and satisfies the inequality $|x(t; t_1, x_1)| < \epsilon \psi(t)$ for $t \geq t_1$. Definitions and results are given for the corresponding boundedness, weak boundedness, and uniform properties of the zero solution of (*). A problem developed jointly here is that of determining when the solutions of certain linear and weakly nonlinear differential equations lie in a modified $L^p$-space. (Received October 16, 1968.)


Let $K/k$ be a Galois extension of number fields with Galois group $G$. Let $S$ be a finite set of primes of $K$, stable under $G$, and including the infinite primes. Let $\mathfrak{O}$ be a set of intermediate fields, stable under $G$. We investigate the question of when the $S$-units of the fields of $\mathfrak{O}$ generate a subgroup of finite index in the $S$-units of $K$. The answer is if and only if every irreducible character of $G$
occuring in some $\chi_H$ (the character induced by the trivial character on $H$), for $H$ a splitting group of a prime in $S$, also occurs in some $\chi_K$, for $K$ the group of some field in $\mathfrak{R}$. The proof uses Frobenius reciprocity. (Received October 16, 1968.)


Given the positive integer $m$ and a set of (real) tabular points $x_0 < x_1 < \ldots < x_n$ for the $(r - v)$ function $f(x)$, let $g_i(x)$ be a polynomial of degree at most $m$ agreeing with $f(x)$ at $x_{i-1}$ and $x_i$ and also the first $m - 1$ derivatives agreeing at $x_{i-1}$. The pseudo-spline $PS(x)$ is $g_i(x)$, $x_{i-1} \leq x < x_i$, $i = 1, 2, \ldots, n$. The spline approximation $SP(x)$ is initialized by making it agree with $PS(x)$ in $[x_0, x_1]$. The primary object of this paper is to give an error analysis when $SP(x)$ or $PS(x)$ are used to generate approximations to the derivatives of $f(x)$. Several theorems are given in both exact integral and mean value form. For example, the errors when using $SP(x)$ can be expressed by those of $PS(x)$ and the propagation of error, at a given $x_i$, in the derivatives of $SP(x)$. A final theorem on the error for $SP(x)$ introduces a special class of matrices whose coefficients depend solely on $m$. (Received October 16, 1968.)


Let $L$ be a finite-dimensional Lie algebra over an arbitrary field. This paper presents various properties of the Frattini subalgebra of $L$. We let $N(L)$ be the nil-radical of $L$, $Z(L)$ the center of $L$, $\Phi(L)$ the maximal ideal of $L$ contained in $\Phi(L)$. The major results are Theorem 1. A Lie algebra $L$ with characteristic ideal $K$ such that $L/K$ is not abelian and $\dim Z(L/K) = 1$ cannot be contained in the Frattini subalgebra of any Lie algebra. Theorem 2. A non-abelian Lie algebra whose derived algebra has codimension two cannot be contained in the Frattini subalgebra of any Lie algebra. Theorem 3. If $L$ is a Lie algebra over a field of characteristic $0$ and $\Phi(L) = 0$, then $L$ is the semidirect sum of an abelian ideal and a subalgebra $B$ where $B$ is the direct sum of a semisimple subalgebra and the center of $B$. Theorem 4. The following are equivalent: (i) $L$ is solvable, (ii) $L/\Phi(L)$ is solvable, (iii) $\psi(Q) \subset N(Q)$ for any nonzero quotient algebra $Q$ of $L$, (iv) $\psi(S) \subset N(S)$ for any nonzero subalgebra $S$ of $L$. (Received October 16, 1968.)


Theorem. Let $L_n$ be a sequence of norm one linear operators defined on $L^1[0,1]$. If $L_n^1$ converges to $L$ and $L_n p$ converges weakly to $p$ for the two functions $p = x$ and $p = x^2$, then $L_n f$ converges to $f$ for all $f$ in $L^1[0,1]$. Theorem. Let $M$ be a subspace of $C$, the continuous complex-valued functions on the complex unit circle (sphere resp.). Let $L_n$ be a sequence of norm one linear operators from $M$ into $C$. If $L_n p$ converges to $p$ for $p$ in $\{1, z, \bar{z}\} \{1, z, \bar{z}, z^2, \bar{z}^2\}$ resp., then $L_n f$ converges to $f$ for all $f$ in $M$. Theorem. The range of a norm one projection defined on $C_0(X)$ is itself a space of the type $C_0(Y)$. Theorem. If $E$ is a Banach space such that $E^* = L^1(\mu)$ for some measure space, then if $\text{ext } S(E^*)$ is $w(E^*, E)$-closed, $E = C_0(\mathcal{X})$ for some $\mathcal{G}$ and some compact Hausdorff space $X$.
The above results are derived from a lemma which extends the classical theorem of Korovkin, on convergence of positive operators on C[0,1], to convergence of norm one operators on a normed linear space. (Received October 16, 1968.)


Dunford and Schwartz [Linear operators, 1, Interscience, New York, 1964, pp. 534-535] define the linear space $L_0$ for functions $f$ on a finite measure space as consisting of those functions for which $\int \log |f(x)| \, d\mu(x) < \infty$. Let $M_0(f) = \int \log (1 + |f(x)|) \, d\mu(x)$ for such functions and set $\rho(f, g) = M_0(f - g)$ for $f, g$ in $L_0$. Then $(L_0, \rho)$ is a complete metric linear space if functions equal a.e. are identified. Consider in particular $L_0 = L_0([0,1])$, with Lebesgue measure. It has the following properties: (1) $L_0^* = [0]$; (2) no sphere is bounded; (3) $M_0$ is plurisubharmonic on $L_0$. It is the first identified space with this combination of properties. The corresponding Hardy space $H_0$ consists of functions $f$ analytic on $|z| < 1$ such that if $f_r(x) = f(r \exp(2\pi i x))$ and $m_0(r; f) = \infty$, then $m_0(r; f) < \infty$ as $r \to 1$. The implied topology of $H_0$ is not locally convex, and spheres are not bounded. However, evaluations at $z$ in $|z| < 1$ are continuous linear functionals. The "norm" $m_0(r; f)$ is usable as a measure of the affinity of $f$ for $\infty$. (Received October 16, 1968.)

663-270. D. D. DONAR, Wayne State University, Detroit, Michigan 48202. Special types of annular functions.

Definition 1. Let $f(z) = \sum a_n z^n (n \geq 0)$. $f$ is called dominated term strongly annular (DTSA) if and only if $\lim \sup |a_n|^{1/n} = 1$ and there exists (1) $\{n_k\}, k \geq 1$, (2) $\{r_k\}, k \geq 1$, and (3) a positive real constant $c$ such that (a) $0 < r_k < r_{k+1} - 1$, (b) $0 < c < 1$, (c) $\lim |a_n| r_k^{n_k} = \infty$, and (d) for all $k \geq 1$, $|a_n| r_k^{n_k} (n \neq n_k) / (|a_{n_k}| r_k^{n_k}) < c$. Definition 2. Let $g(z) = \sum b_n z^{n_k} (k \geq 1)$. $g$ is called saturatated dominated term strongly annular (SDTSA) if and only if $\lim \sup |b_n|^{1/n} = 1$ and there exists (1) $\{r_k\}, k \geq 1$, (2) $\lim \sup |b_n| r_k^{n_k} = \infty$, (c) $0 < c < 1$, and (d) for all $k \geq 1$, $(|b_n| r_k^{n_k} (q \neq k)) / (|b_{n_k}| r_k^{n_k}) < c$. Theorem 1. Suppose that $f(z) = \sum a_n z^n (n \geq 0)$ and that $\lim \sup |a_n|^{1/n} = 1$. A necessary and sufficient condition that there exists a sequence $\{n_k\}, k \geq 1$, such that $g(z) = \sum a_n z^{n_k} (k \geq 1)$ is SDTSA in that $\lim \sup |a_{n_k}| = \infty$. Theorem 2. If $\lim \sup |a_{n_k}|^{1/n} = 1$ and $\lim \sup |a_{n_k}| = \infty$, then there exists $\{n_k\}, k \geq 1$, such that $f(z) = \sum a_n z^{n_k} (k \geq 1)$ is DTSA and for each $\nu \geq 1$, the $\nu$th derivative of $f$ is DTSA with the property that if its first $\nu$ terms are dropped the resulting series is SDTSA.

(Received October 16, 1968.)

663-271. NACHMAN ARONSZAJN, University of Kansas, Lawrence, Kansas 66044. A problem concerning lipschitzian homeomorphisms in Banach spaces.

The problem arises in the theory of bordered lipschitzian Banach manifolds. Let $X$ and $Y$ be Banach spaces, $\varphi \in X^*, X_+ = \{x : \varphi(x) \geq 0\}$, and $D$ and $R$ closed neighborhoods of 0 in $X_+$ and $Y$ respectively, $T$ a mapping of $D$ onto $R$ which is a lipschitzian homeomorphism, i.e. such that for some constant $M \geq 1, M^{-1} \|x_1 - x_2\| \leq \|T(x_1) - T(x_2)\| \leq M \|x_1 - x_2\|$ for all $x_1, x_2$ in $D$. The conjecture is that $T(0) \neq 0$. This conjecture is proved if $M < 1.008$; this result is sufficient for the needs in the theory of
Compact inverses of ordinary differential operators.

Let $T$ be an ordinary differential expression on $[0,\infty)$. When does the associated minimal operator, considered as an operator from a subspace of $L^p[0,\infty)$ into $L^q[0,\infty)$, have an inverse which is compact? It is not required that the inverse be defined on all of $L^q[0,\infty)$.

Theorem. Let $T$ be classically selfadjoint, and $p = q = 2$. The property holds if and only if the essential spectrum of the minimal operator is void.

Let $T$ be classically selfadjoint, and let $p = q = 2$. Let $f_i$ be a bounded complex valued function on $[0,\infty)$ for all $i \geq 1$ and $< 2n$. Let $f_{2n}$ be the identically 1 function, and suppose the real part of $f_0(x)$ approaches infinity with $x$. Consider $V = \sum_{i=1}^{2n} f_i T_i$, where we assume enough differentiability on $T$ and on each $f_i$ to assure that $V$ is of the form $\sum_{i=1}^{2n} g_i T_i$ and has a classical adjoint. Then the minimal operator associated with $V$ has compact inverse. (Received October 17, 1968.)
neighborhood $U$ of $a$ is a set \( \{ y; \varphi(a) - \varphi(y) < \epsilon \text{ for all } \varphi \in \mathcal{U} \} \) for some $\mathcal{U} \in \mathfrak{U}$ and $\epsilon > 0$. Define $f(x) = \epsilon - \operatorname{Min.} \{ \epsilon, \sup_{\varphi(x) - \varphi(a)} \}$. Let $\mathfrak{J}$ be a set of all such $f$'s defined associated with neighborhoods whose closures are totally bounded. Then $\mathfrak{U} \times \mathfrak{J}$ and a support of $f$ of $\mathfrak{J}$ is totally bounded. It follows that Theorem. $\mathfrak{U} \mathfrak{J}$ is locally totally bounded iff there exists a simple unity $\mathfrak{J}$ such that $\mathfrak{U} \sim \mathfrak{J}$ on a support of each $f$ of $\mathfrak{J}$ and $S = \bigcup_{f \in \mathfrak{J}} \{ x; f(x) > 0 \}$. (Received October 17, 1968.)


Let $A$ denote the sequence-to-sequence transformation given by $(Ax)_n = \sum_{k} a_n x_k$. Let $A^p$ denote the set of complex number sequences $x$ such that $\sum |x|^p$ converges, and let $A^{\sim}$ denote the inverse image of $A$ under $A$. Then $A$ is called an $1$-$1$ matrix mapping if $A^{\sim}$ is contained in $A^{-1}$. Theorem. If $p > 1$ and $A$ is an $1$-$1$ matrix mapping such that $\lim \inf_{k} |\sum a_n x_k| > 0$, then $A^{\sim}$ cannot contain $\mathbb{P}$ [cf. Steinhaus, Prace Mat. Fiz. 22 (1911), 121-134]. Other results include criteria that imply that $A$ is equivalent to the identity mapping; e.g. Theorem. If $A$ is a triangular $1$-$1$ matrix mapping such that $\lim \inf_{k} |\sum a_n x_k| > 0$, then $A^{\sim} = A^{-1}$ [cf. Agnew, Tôhoku Math. J. 35 (1932), 244-252]. (Received October 17, 1968.)


Antonovskii, Boltianskii, and Sarymsakov (MR 25 #1461 and MR 28 #1583) introduced metric spaces over semifields. Every semifield can be isometrically embedded in a product of reals so we only consider metrics over such “Tichonov” semifields (complete spaces "of the first kind" in MR 34 #5718). In $\mathbb{R}^1$, we order by setting $x < y$ iff $\pi_1(x) \neq \pi_1(y)$ for all $i \in I$. A metric $d_1 : X \times X \rightarrow \mathbb{R}^1$ has properties $d_1(x,y) = 0$ iff $x = y$. Basic neighborhoods of a point $x \in X$ are taken to be $\{ y \in X; \pi_1 d_1(x,y) < \epsilon, i \in J \}$ for some $\epsilon > 0$ and some finite $J \subseteq I$. Metrizable spaces are exactly the completely regular (c.r.) spaces. $d_1$ is completely regular (c.r.) iff $d_1(x,F) \neq 0$ for every closed $F$ and $x \notin F$. Theorem. Every c.r. space possesses a c.r. semifield metric. Theorem. For every semifield metric there exists an equivalent c.r. semifield metric. $d_1$ is normal iff $d_1(F,G) \neq 0$ for every disjoint closed $F \in G$. Theorem. A semifield metric space is normal iff it possesses a normal semifield metric. Theorem. Every compact semifield metric space possesses an equivalent normal semifield metric. (Received October 17, 1968.)


Reidemeister homotopy chains are used to calculate the second homotopy group of 2-spheres in 4-space. In particular, the second homotopy group of spun knots is calculated by this method. (Received October 17, 1968.)
On the torsion free, self dual, locally compact abelian groups.

All notations and definitions can be found in [Hewitt and Ross, Abstract harmonic analysis, I, Springer-Verlag, New York]. Main Theorem. Let G be a torsion-free LCA group. Then G is self dual if and only if G is the direct product $\mathbb{R}^n \times D \times D^* \times H$, where $\mathbb{R}^n$ is the real n-dimensional Euclidean space and D is a discrete, torsion free divisible group and $D^*$ is its dual and H is the local direct product of $\mathcal{P}(p)$'s relative to $\mathcal{P}(p^*)$ where P is the set of all positive prime integers and $\mathcal{P}(p)$, $p \in P$, are cardinal numbers. Lemma. Let G be a radical torsion free LCA group which has a compact open subgroup H which is topologically isomorphic to $\times_{p \in P} \mathbb{A}^{n_{p}}$ where P is the set of all primes, and $\mathbb{A}^{n_{p}}$, $p \in P$, are cardinal numbers. Then G is uniquely topologically imbedded into $\times_{p \in P} \mathcal{P}(p)$, the minimal extension of $\times_{p \in P} \mathcal{P}(p)$.

Boundedness and asymptotic behavior of a second order nonlinear equations.

The boundedness and asymptotic behavior of the nonlinear second order equation

\[ u'' + \sum_{i=1}^{n} a_i(t)f_i(u) = 0 \]

is investigated. It is assumed that the $a_i(t)$ are positive functions belonging to $C'([0,\infty))$ and that the functions $f_i(u)$ are continuous on the interval $-\infty < u < \infty$ and satisfy $uf_i(u) > 0$ for $u \neq 0$. Let $F_i(u) = \int_0^u f_i(x)dx$. Further assume that $F_i(u) \to \infty$ as $|u| \to \infty$. A positive nondecreasing function $b(t)$ belonging to $C'([0,\infty))$ is said to be a bounding function for the coefficients $a_i(t)$ if $a_i''(t)b^{-1}(t) \leq b'(t)b^{-1}(t)$ for all $i$ and $b^{-1}(t)\sum_{i=1}^{n} a_i(t) \leq \epsilon$ for some $\epsilon > 0$. Theorem 1. If $b(t)$ is a bounding function, then every solution $u(t)$ of (1) is bounded. Theorem 2. If $2F_i(u) \leq u_f(u)$ and there exists a positive function $p(t)$ such that $\int_0^1 b^{-1}(t)dt = \infty$, $\lim_{t \to -\infty} p'(t)[p(t) \cdot b^{1/2}(t)]^{-1} \leq 0$, and $\lim_{t \to -\infty} a'(t)a^{-1}(t)p(t) > 0$, then every solution $u(t)$ of (1) satisfies $\lim_{t \to -\infty} u(t) = 0$. (Received October 17, 1968.)

On the intersection of the powers of the Jacobson radical.

Let $R$ be a ring with identity and Jacobson radical $J$. An ipri-ring is one in which every ideal is a principal right ideal [J, C. Robson, Proc. London Math. Soc. (3) 17 (1967), 600-616]. Theorem. If $R$ is a right Noetherian J-prime ipri-ring such that $J = aR = Rb$, then $J$ is the nilpotent radical or $\cap J^i = 0$ and $R$ is a prime ring with $J^k$ as its only proper ideals. Theorem. If $R$ is a right Noetherian ring with $J = aR$ and if $\cap J^i$ is a finitely-generated left ideal of $R$, then $\cap J^i = 0$. In both theorems, the left-hand restrictions are necessary. (Received October 17, 1968.)

The selfadjoint operators of a von Neumann algebra form a conditionally complete lattice.

Equivalence classes of real-valued measurable functions on a finite measure space can be regarded as order-preserving maps of the real line into the measure algebra. If $f$ is such an equivalence class, $T_f(r)$ is the element of the measure algebra defined by the requirement $f \in r$. $T_f$ is clearly an order-preserving map of the real line into the measure algebra. Subject to a few conditions, if the range of $T_f$ lies in a complete orthocomplemented lattice of projections on a Hilbert space, $T_f$ is called a resolution of the identity. The projections of a von Neumann algebra form a complete
orthocomplemented lattice. The bounded resolutions of the identity taking values in such a lattice form a conditionally complete lattice when equipped with the reverse of product order. This determines the order "\( \succ \)" on selfadjoint operators. It agrees with the usual order on commutative subsets, but it is not a vector ordering in general, i.e. \( A - B \succ 0 \) does not imply \( A \succ B \). An example due to Kadison is examined in detail. (Received October 18, 1968.)


H. Bauer [Convexity in topological linear spaces, lecture notes, University of Hamburg, 1963-1964] has recovered the classical Bernstein integral representation theorem for completely monotonic functions on a commutative semigroup \( A \), with identity, through the Krein-Milman theorem. The key to this approach is the identification (as exponentials) of the extreme points of the normalized completely monotonic functions. Alternate proofs of this identification are included here. One of them involves an elementary proof that \( f'(x) \equiv f(2x) \) for every normalized completely monotonic function \( f \), with equality holding only when \( f \) is an extreme point. It is also noted that the normalized completely monotonic functions form both a simplex and an affine semigroup. Some extension theorems are presented. For example if \( A \) admits enough exponentials to separate points, then \( A \) can be embedded in a "power closed" subsemigroup \( \tilde{A} \) of the semigroup \( \exp^2 A \), consisting of the exponentials on the exponentials of \( A \). In this event every completely monotonic function admits a completely monotonic extension to \( \exp^2 A \) and a unique completely monotonic extension to \( A \). (Received October 18, 1968.)

663-284. R. E.-PHILLIPS and K. K. HICKIN, University of Kansas, Lawrence, Kansas 66044, On ascending series of subgroups in infinite groups.

Definition. Let \( \alpha \geq 2 \) be a cardinal number. An \( \alpha \) series of \( G \) is a well ordered ascending series of subgroups \( E \subset G_1 \subset \ldots \subset G_{\tau} \subset \ldots \subset G_\alpha = G \) such that for all \( \tau < \alpha \), \( [G_{\tau+1}:G_\tau] < \alpha \), and if \( \tau \) is a limit ordinal, \( G_\tau = \bigcup \{ [G_\beta: \beta < \tau] \} \). The class \( F(\alpha) \) is defined as follows: \( G \in F(\alpha) \) iff \( G \) possesses an \( \alpha \) series. Theorem. If \( \alpha > 2 \), the class \( F(\alpha) \) is radical in the sense of Kuro5 (Radicals in the theory of groups, Dokl. Akad. Nauk SSSR 141 (1961), 789-791 = Soviet Math. Dokl. 2 (1961), 1546-1548). Theorem. Let \( \alpha \) be an infinite cardinal and let \( G \) be free of rank \( \alpha^+ \). Then \( G \) is in \( F(\alpha^+) \) but not in \( F(\alpha) \). Theorem. If \( (\alpha, \beta) \) is a pair of cardinal numbers with \( 2 < \alpha < \beta \) and \( (\alpha, \beta) \neq (4,5) \), then \( F(\alpha) \not\subset F(\beta) \). Theorem. Every \( F(\aleph_0) \) group is locally finite. Theorem. If \( \alpha \) is an infinite cardinal number, there exists a locally finite \( p \)-group \( G \) such that \( G \not\in F(\alpha) \). Theorem. Let \( G \) be a locally nilpotent group. Then \( G \in F(\aleph_0) \) iff \( G \) is an SN*-group. Theorem. If \( G \in F(\aleph_0) \) and \( A \) is a finite subgroup of \( G \), then there is an \( \aleph_0 \) series passing through \( A \). Theorem. Let \( G \in F(\aleph_0) \). \( G \) is locally supersolvable iff every \( \aleph_0 \) series of \( G \) can be refined to an \( \aleph_0 \) series \( E \subset G_1 \subset \ldots \subset G_{\tau} \subset \ldots \subset G_\alpha = G \) such that for all \( \tau < \alpha \), \( [G_{\tau+1}:G_\tau] \) is a prime. Theorem. Let \( \alpha \) be an infinite cardinal number. Then \( \text{Alt}(\alpha^+) \not\subset F(\alpha) \). (Received October 18, 1968.)


On the Hornich spaces.

Let \( f \) be holomorphic in the disc and assume \( f(0) = 0, f'(z) \neq 0, f'(0) = 1 \) and \( |\arg f'(z)| \leq M \). The set \( \mathcal{V} \) of such functions forms a real linear space under the operations \( (1) [f_n g_n](z) = \ldots \)
\[ \int f'(\zeta)g'(\zeta)\,d\zeta \text{ and } (2) \, (nf)(z) = \int [f'(l;)]^M \, d\zeta. \]

and is a Banach space with the norm \( \|f\| = \sup \{|f'(z') - f'(z'')|; \, z', \, z'' \in D\}. \) Theorem 1. \( f \) is in \( W \) if and only if there exists an inner function \( w(z), \, w(0) = 0 \) satisfying (3) \( f(z) = \int (1 + w(\zeta))/(1 - w(\zeta))^M \, d\zeta \) for \( M \) a positive constant.

Theorem 2. Each \( f \) in \( W \) has well defined radial limits a.e. on \( |z| = 1. \) (Received October 18, 1968.)

663-286. RAY MINES, New Mexico State University, Las Cruces, New Mexico 88001. Radicals and torsion free groups.

A subfunctor of the identity \( R \) is called a radical if \( R(G/RG) = 0 \) for all groups \( G. \) Theorem 1. If \( R \) is a radical and there exists a sequence of groups \( \{G_n\} \) with \( RG_{n+1} = G_n, \) then for all ordinals \( \alpha \) there exists a sequence of groups \( G(\alpha, n) \) with \( R^\alpha G(\alpha, n) = G_n. \) An application of this theorem to the radical defined by \( R(G) = \bigcap \{ \ker f | f \in \text{Hom}(G, Z) \} \) leads to a construction of a torsion free group \( G \) with \( R^0 G \neq R^0 G^{+1} G. \) This answers negatively a question posed by B. Charles at the colloquium on Abelian groups held at Montpellier, France. The above construction also leads to a group \( G \) which is \( K_0 \)-free and \( \text{Hom}(G, Z) = 0. \) (Received October 18, 1968.)

663-287. H. S. M. COXETER, University of Toronto, Toronto, 5, Ontario, Canada. Affinely regular polygons.

In an affine plane, a polygon (not necessarily closed) is said to be regular if its vertices \( A_0 A_1 A_2 \ldots \) form an orbit for an equi-affinity, that is, the product of two affine reflections. All the vertices are found to lie on a conic, whose nature depends on the value of the ratio \( T = A_0 A_3 / A_1 A_2. \) For instance, the conic is a parabola if \( T = 3, \) and a pair of parallel lines if \( T = -1. \) Every closed polygon has a value of \( T \) lying between \(-1 \) and \( 3; \) more precisely, a regular \( n \)-gon has \( T = 1 + 2 \cos (2\pi/n). \) The only values of \( n \) for which such a polygon can be constructed with a parallel-ruler are 2, 3, 4, 6. (Received October 18, 1968.)

663-288. I. R. HENTZEL, Iowa State University, Ames, Iowa 50010. (-1,1) rings.

(-1,1) rings are nonassociative rings which satisfy the identities \((a, b, c) + (a, c, b) = 0\) and \((a, b, c) + (b, c, a) + (c, a, b) = 0.\) They are special cases of Albert's \((\gamma, \delta)\) rings. An ideal \( I \) is trivial if \( I \neq 0 \) and \( I^2 = 0. \) Theorem 1. If \( A \) is a (-1,1) ring with an idempotent and no trivial ideals contained in the center and such that \( na = 0 \) implies \( a = 0 \) for \( n = 2,3, \) then \( A \) has a Pierce decomposition \( A = A_{11} + A_{10} + A_{01} + A_{00} \) with respect to the idempotent, and the multiplication between subspaces is the same as if \( A \) were associative. Furthermore, \( A_{10} A_{01} + A_{10} + A_{01} + A_{01} A_{10} \) is an ideal in the nucleus. To Theorem 1, Maneri's result that simple (-1,1) rings with idempotent are associative, and Sterling's result that prime (-1,1) rings with idempotent are associative, are corollaries.

Theorem 2. Nil finite dimensional (-1,1) algebras such that \( 2a = 0 \) implies \( a = 0 \) are nilpotent. (Received October 18, 1968.)


Let \( A \) be a continuous linear operator on a Hilbert space \( X, \) \( W(A) = \{\langle Ax, x\rangle | \|x\| = 1\} \) the numerical range of \( A \) and for \( z \) complex \( M_z = \{x | \langle Ax, x\rangle = z\|x\|^2\}. \) Theorem 1. \( M_z \) is linear if and
only if \( z \) is not in the interior of any line segment in \( W(A) \). (See J. G. Stampfli, Extreme points of the numerical range of a hyponormal operator, Michigan Math. J. 13 (1966), 87-89, for sufficiency.)

Let \( L \) be a maximal line segment (possibly degenerate) in the boundary of \( W(A) \). Let \( N = \bigcup \{ M_z | z \in L \} \).

**Theorem 2.** If \( z \in L \), either \( \sqrt{M_z} = N \) or (ii) \( M_z \subset [(A - z)N]^{\perp} \cap [(A^* - z*)N]^{\perp} \). If \( z \) is in the interior of \( L \cap W(A) \), case (i) occurs; if \( z \) is an extreme point of \( W(A) \), case (ii) occurs. The convexity of \( W(A) \) and standard Hilbert space techniques are used to prove Theorems 1 and 2 and the following:

**Corollary 1.** If \( W(A) \) is a convex body and \( \sqrt{M_z} = X \), \( z \) is in the interior of \( W(A) \).

**Corollary 2.** If \( A(N) \subset N \) and \( z \in L \), then \( \{ x | Ax = zx \} \) and \( \{ A^* x = z^* x \} \) is in \( \bigcap \{ \text{maximal linear subspaces of } M_z \} \).

(Received October 18, 1968.)


Let \( A \) be a semi-simple Hilbert algebra with the property that all the homomorphisms on \( A \) to the complex numbers have norm 1. The elements \( x \) in \( A \) such that \( \| x \| = \| x \| \) (where \( \hat{x} \) denotes the Gelfand transform of \( x \)) form a complete orthonormal set in \( A \). This result gives information about the topology of the maximal ideal space of \( A \) and the existence of an involution and approximate identity in \( A \). If \( A \) has an identity, then it is possible to calculate the dimension of \( A \), generalizing a result due to Lars Ingelstam (Bull. Amer. Math. Soc. 69 (1963), 794-796) and M. F. Smiley (Proc. Amer. Math. Soc. 15 (1965), 440-441). (Received October 18, 1968.)

663-291. R. V. MOODY, University of Saskatchewan, Saskatoon, Saskatchewan, Canada.

**Euclidean Lie algebras.**

Let \( (A_{ij}) \) be an \( 1 \times 1 \) integral matrix satisfying \( A_{ij} = 2 \), \( A_{ij} = 0 \) if \( i \neq j \), \( A_{ij} = 0 \) iff \( A_{ij} = 0 \), there exist nonzero rational numbers \( q_1, \ldots, q_s \) such that \( (A_{ij} q_j) \) is symmetric. Over any field \( \Phi \) it can be used to construct a Lie algebra \( L \) in a natural way. \( L \) is not simple (called Euclidean) iff \( (A_{ij}) \) is singular and removal of any row and corresponding column leaves a Cartan matrix. Euclidean algebras are infinite dimensional but for each \( \mu \in \Phi - \{ 0 \} \) there is a finite-dimensional simple quotient \( L(\mu) \) of \( L \). There are three distinct types of Euclidean Lie algebras. If \( L \) is of the first type then the \( L(\mu)'s \) are isomorphic and are split simple Lie algebras over \( \Phi \). There are five classes of the second type. If \( L \) is one of the three classes \( B_{1,2}, C_{1,2}, \) or \( F_{4,2} \) then \( L(\mu) \) is of type \( D_{1+1}, A_{21-1}, \) or \( B_5 \) respectively, there is a \( \mu \in \Phi \) for which \( L(\mu) \) is split (and in any case \( L(\mu) \) always splits in a suitable quadratic extension of \( \Phi \)), and \( L(\mu) \cong L(\nu) \) iff \( \mu \) and \( \nu \) are in the same square class in \( \Phi \). (Received October 18, 1968.)

663-292. N. E. GRETSKY, University of California, Riverside, California 92502. Linear functionals on a Banach function space.

Let \( L_\rho(\Omega, \Sigma, \mu) \) be a Banach function space on the \( \sigma \)-finite measure space \( (\Omega, \Sigma, \mu) \) with the function norm \( \rho \) having the weak Fatou property [see Luxemburg and Zaanen, Math. Ann. 149 (1963)]. Let \( M_\rho \) be the closed span of the simple functions. Define \( N_\rho = L_\rho / M_\rho \) equipped with the usual norm and order. Denote the canonical map as \( \lambda : L_\rho \rightarrow N_\rho \). We assume that the \( \lambda \)-image of the lattice closure of the unit ball of \( L_\rho \) is within the closed unit ball of \( N_\rho \). **Theorem.** \( L_\rho \) is the direct sum of
and its lattice orthogonal complement $(M_\rho^\perp)^{\perp,\rho}$. Moreover, $\|x^*\| = \|y^*\| + \|z^*\|$ where $x^* = y^* + z^*$ with $y^* \in M_\rho^\perp$ and $z^* \in (M_\rho^\perp)^{\perp,\rho}$. Note that $M_\rho^\perp \cong N_\rho^\perp$ and $(M_\rho^\perp)^{\perp,\rho} \cong M_\rho^\perp$. Thus to represent a linear functional on $L_\rho$ we will use representations of $N_\rho^\perp$ and $M_\rho^\perp$. There is a closed subspace, $P_\rho^\perp$, of the purely finitely additive set functions on $(\Sigma, \Sigma, \mu)$ such that $N_\rho^\perp \cong P_\rho^\perp$. There are two representations of $M_\rho^\perp$. One which assumes an averaging condition on $\rho$ gives $M_\rho^\perp \cong P_\rho^\perp$. The other representation, with no assumption, gives $M_\rho^\perp \cong$ the space of real additive set functions on $\Sigma_0$ with finite $\rho'$-variation [see Abstract 642-34, these Notices 14 (1967), 70].

Let $X$ be an $n \times n$ matrix. For $1 \leq p \leq n$, let $f(X)$ denote any of the following symmetric functions of the eigenvalues of $X$: (i) the $p$th elementary symmetric function, $E_p(X)$; (ii) the $p$th completely symmetric function, $h_p(X)$; (iii) the sum of the eigenvalues to the $p$th power, $(\text{tr}(X))^p$. The following result is proved, generalizing an earlier result of de Pillis [Abstract 648-23, these Notices 14 (1967), 636]. Let $H$ be an $m \times n$ matrix, and partition $H$ into $m \times n$ matrices $H_{st}$. Suppose that $H$ is positive semidefinite hermitian. Then the $m \times m$ matrices (i) $(E_p(H_{st}))$, (ii) $(h_p(H_{st}))$, (iii) $(\text{tr}(H_{st}))^p$ are also positive semidefinite hermitian. The general theorem of which the preceding is in fact a special case holds for $f(X)$ any Schur function of the eigenvalues of $X$. This paper will appear in the Duke Journal of Mathematics. (Received October 18, 1968.)
variation in the restricted sense [Saks], and \( F \circ \varphi \) is generalized absolutely continuous in the restricted sense, and \( \varphi' \) is replaced by \( \varphi' : J - R \). a function almost everywhere equal to the derivative of \( \varphi \) on \( J \), \( (D^*) \int_{t_1}^{t_2} (F \circ \varphi(t)) \varphi'(t) \, dt = (D^*) \int_{t_1}^{t_2} f(u) \, du \) where the integrals are both in the sense of Denjoy-Perron. (Received October 18, 1968.)


Concerning a property of certain points of compact continua.

Suppose \( M \) is a compact continuum in a space satisfying R, L, Moore's Axiom 0 and Axiom 1. A point \( P \) is said to have property E with respect to \( M \) if and only if \( P \) belongs to \( M \) and \( M \) is not the sum of two continua each distinct from \( M \) and having \( P \) in their common part. \( E_M \) denotes the set to which \( P \) belongs if and only if \( P \) has property E with respect to \( M \). Theorem. If \( E_M \) is a connected proper subset of \( M \) then \( M \) is the sum of two continua, \( g \) and \( h \), each distinct from \( M \) such that \( g \) is indecomposable and does not intersect every composant of \( h \), and \( h \) intersects every composant of \( g \). Theorem. If \( M \) is a nondegenerate compact continuum then (1) \( M \) has only one composant if and only if no point has property E with respect to \( M \) or \( E_M \) is a connected proper subset of \( M \), (2) \( M \) has only three composants if and only if \( E_M \) is not connected, and (3) \( M \) has uncountably many composants if and only if every point of \( M \) has property E with respect to \( M \). (Received October 18, 1968.)


Let \( k_1 \leq k_2 \leq \ldots \leq k_n \) be given positive integers, and let \( K \) be the set of points in \( \mathbb{E}_n \) satisfying \( 0 \leq a_i \leq k_i \), \( i = 1, 2, \ldots, n \), and ordered lexicographically. Define \( \Gamma(a_1, \ldots, a_n) \) to be the set \( \{ (a_1 - 1, a_2, \ldots, a_n), (a_1, a_2 - 1, a_3, \ldots, a_n), \ldots, (a_1, a_2, \ldots, a_n - 1, a_n - 1) \} \cap K \), and for \( 0 \leq t \leq k_1 + \ldots + k_n \), let \( E_t \) denote the subset of \( K \) with component sums \( a_1 + a_2 + \ldots + a_n \) equal to \( t \). For subsets \( H_1 \) of \( E_1 \), let \( C(H_1) \) denote the first \( |H_1| \) elements of \( E_1 \). Macaulay's theorem [Proc. London Math. Soc. 26 (1927), 531-555] shows that \( \Gamma(C(H_1)) \subset C(\Gamma(H_1)) \) if \( t \) is sufficiently large. In this paper, which will appear in the Journal of Combinatorial Theory, the restriction on \( t \) is removed. An application is given. (Received October 18, 1968.)

663-298. J. T. BORREGO, University of Massachusetts, Amherst, Massachusetts 01002.

Continuity of the operation of a semilattice.

Let \( X \) be a compact Hausdorff space with a semilattice operation \( \wedge \). Let \( P = \{ (x,y) \mid x \wedge y = x \} \) and \( p(-1) = \{ (y,x) \mid (x,y) \in P \} \). Theorem 1. If \( X \) is order dense, then \( \wedge \) is continuous iff (1) \( P \) is closed in \( X \times X \) and (2) \( (x,y) \in P \), \( [y_0] \) and \( [y'_0] \) are two nets converging to \( y \) implies there exists a net \( [x_n] \) converging to \( x \) such that for each \( n \), \( (x_n, y_0 \wedge y'_0) \in P \). Definition. \( P(-1) \) is continuous iff \( P \) is closed in \( X \times X \) and for all open subsets \( U \) of \( X \), the set \( \{ x \mid \text{there exist } w \in U \text{ such that } x \leq w \} \) is open in \( X \). Theorem 2. The operation \( \wedge \) is continuous iff (i) \( \wedge \) is continuous on the diagonal of \( X \times X \) and (ii) \( p(-1) \) is continuous. (Received October 18, 1968.)
Let $X(t)$ be a separable real-valued Gaussian process with stationary increments for which $E([X(t+h) - X(t)]^2) = \sigma^2(h)$ where $\sigma^2(h)$ is concave for $h \in [0, \delta]$ for some $\delta > 0$. Assume also that $\int_0^h \sigma^2(u)/udu$ exists and that $f(h) = 1/\sigma^2(h)\int_0^h \sigma^2(u)/udu$ increases monotonically as $h$ goes to zero from the right. Also let $f(h) \leq \beta \log 1/h$ for some $\beta > 0$. Then

$$P\{\lim \sup_{h \to 0} \frac{|X(t+h) - X(t)|}{f(h)} > \text{Const.}\} = 1.$$  

For those processes shown to be continuous by Fernique, $\int_0^h \sigma^2(u)/udu \to 0$ as $h \to 0$. On the other hand, $\int_0^h \sigma^2(u)/udu$ diverges for those processes that the author has shown to be discontinuous. (Received October 18, 1968.)

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Let $D$ denote an integral domain with $1 \neq 0$ and quotient field $K$. By an overring of $D$ we will mean a ring $J$ such that $D \subseteq J \subseteq K$. If $\pi$ is a general ring property, then we shall refer to an ideal $A$ of $D$ as a $\pi$-ideal provided there exists an overring $J$ of $D$ such that $J$ is a $\pi$-domain and $A = AJ \cap D$. Gilmer and Ohm (Trans. Amer. Math. Soc. 117 (1965), 237-250) have shown that if every ideal of $D$ is a valuation ideal, then $D$ is a valuation ring. We are mainly concerned with the following question. When does the statement (a) "there exists a collection $S$ of $\pi$-ideals of $D$" imply the statement (b) "$D$ is a $\pi$-domain"? Our main result is that (a) implies (b) when "$\pi$-domain" = "Krull domain" and $S$ is the collection of proper principal ideals of $D$, i.e., if every proper principal ideal of $D$ is a Krull ideal, then $D$ is a Krull domain. We show also that (a) implies (b) when $S$ is the collection of proper finitely generated ideals of $D$ and $\pi$ is any of the following ring properties: Prüfer, 1-dim Prüfer, almost Dedekind, or Dedekind. We remark that (a) does not always imply (b), even in the case $S$ is the set of all ideals of $D$ (e.g., if "$\pi$-domain" = "Bezout domain"). (Received October 18, 1968.)

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Let $H(X, \mathcal{U})$ be the class of all homeomorphisms from the topological space $(X, \mathcal{U})$ onto itself. If $\mathcal{U}$ and $\mathcal{V}$ are two topologies on $X$, we say that $\mathcal{U}$ and $\mathcal{V}$ are noncomparable if there does not exist a one-to-one onto function $\varphi$ from $X$ to itself such that $\mathcal{U} \subseteq \{\varphi(w) | w \in \mathcal{V}\}$ or $\mathcal{V} \subseteq \{\varphi(u) | u \in \mathcal{U}\}$.

Theorem. Let $(X, \mathcal{U})$ be an $n$-manifold and let $\mathcal{V} = \{u \in \mathcal{U} | X - u$ is compact$\}$, and let $\mathcal{N}$ be the topology on $X$ having $\mathcal{N} \setminus \bigcup_{i=1}^{\infty} \{p_i\}$ $\subseteq \mathcal{V}$ and $\{p_i\}$ converges to $p_0$ in $\mathcal{U}$ as a subbase. Then $H(X, \mathcal{U}) = H(X, \mathcal{N})$ and $\mathcal{U}$ and $\mathcal{N}$ are noncomparable if $(X, \mathcal{U})$ is not compact. (Received October 18, 1968.)

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Let $R$ denote the additive real numbers with the usual topology and let $R^n$ denote the $n$-fold product of $R$. Let $N$ denote the additive semigroup of nonnegative integers. If $S$ is a locally compact metric semigroup containing $R^n$ as an open and dense subgroup, is generated by $R^n$ and one additional element and has no idempotents outside $R^n$, then $S$ is algebraically isomorphic with $N \times R^n$. Let $\tau$ be
the topology on $\mathbb{N} \times \mathbb{R}^n$ induced by this isomorphism. If the operation in $S$ is open, there is a sequence $q$ in $\mathbb{R}^n$ which generates $\tau$ in the following way: If, for each positive integer $k$, $T_k = \{(1,0)\} \cup \{(0, q_i)\}$ and $B_k = \{(0, r)| \|r\| < k^{-1}\}$, then the collection $\{mT_k + (0,t) + B_k|(m,t) \in \mathbb{N} \times \mathbb{R}^n$ and $k = 1,2,3,...\}$ is a basis for $\tau$. A necessary and sufficient condition for a sequence in $\mathbb{R}^n$ to generate such a topology is given. (Received October 18, 1968.)


Morera's theorem in complex function theory raises the possibility that this theory can be based on integration rather than differentiation. Heffter [Begrundung der Functionen Theorie auf Alten und Neuen Wegen, Springer-Verlag, Berlin, 1955], and Macintyre and Wilbur [A proof of the power series expansion without differentiation theory, Proc. Amer. Math. Soc. 18 (1967), 419-424] have given such a development. We generalize to a context of operator valued functions. Let $E$ be a finite-dimensional real Euclidean space, $T$ a commutative algebra, with identity, of linear transformations of $E$ into $E$, $S$ a simply connected open set in $E$, and $f$ a continuous function on $S$ into $T$. $f$ is said to be integrable if $\int_C f(z)dz = 0$ for all closed contours $C$ lying in $S$. The integral $g$ of $f$ is a mapping of $S$ into $E$, and for $z \in S$, the (Fréchet) derivative of $f$ at $z$ is the operator $A = f(z)$ of $T$. The family of all such functions is a commutative algebra satisfying versions of the maximum modulus theorem. Theorem. There exists $N > 0$ such that $x \in \{t \in E; \|t\| \leq 1\}$ in domain $f$ implies $\|f(x)\| \leq N \sup \{\|f(t)\|; \|t\| = 1\}$, and $\|f(x)\|_g \leq \sup \{\|f(t)\|_g; \|t\| = 1\}$, where $\|A\|_g = \lim \sup_{n \to \infty} \|A^n\|^{1/n}$ for $A \in T$. Simple examples show that integrable functions need not be analytic. Conditions insuring analyticity are studied. (Received October 21, 1968.)


Let $\{X_i\}$ be a sequence of independent random variables, $S_n = X_1 + ... + X_n$. S. Orey has given conditions [J. Math. Mech. 15, 937-951] under which the sequence of partial sums $\{S_n\}$ is either recurrent or transient, in the sense that for all $z$, and $e > 0$, $P(\{|S_n - z| < e \text{ infinitely many } n\})$ is either identically zero or identically one. Using a standard renewal argument it is shown that if Orey's condition holds, then sufficient conditions for recurrence are that (1) $EX_i = 0$, all $i$, (2) $\lim \sigma_i^2 > 0$, $\sigma_i^2 = EX_i^2$, (3) $3M$ such that $\lim 1/\sigma_i^2 \int |X| < M x^2 dF_i(x) > 0$, and (4) if $B_{nk} = \sum_{k}^{n+k} \sigma_i^2$ then as $n \to \infty$, $1/B_{nk} \sum_{k}^{n+k} |X| > \epsilon B_{nk} x^2 dF_i(x) - 0$, uniformly in $k$, for all $\epsilon > 0$. If the $X_i$ are identically distributed, and in the domain of attraction of a stable law, the same renewal argument gives sufficient condition on an increasing sequence $\{n_i\}$ for the sequence of partial sums $\{S_{n_i}\}$ to be recurrent. A by-product of these results is a local limit theorem for nonlattice random variables. (Received October 21, 1968.)
Theorem. The residue class rings of a right perfect ring all have finite right global dimension if and only if the ring is itself a residue class ring of a right hereditary and right perfect ring. If P and Q are maximal two sided ideals of a right perfect ring we write P - Q if PQ ≠ P ∩ Q and thus construct an oriented graph which we call the skeleton of the ring. The theorem is proved by showing that each condition is equivalent to the absence of closed paths in the skeleton. (Received October 21, 1968.)
The deRham theorem for the group of diffeomorphisms.

Let $\text{Diff}(X)$ be the group of smooth ($C^\infty$) diffeomorphisms of a finite dimensional manifold, provided with the $C^\infty$-topology. The evaluation map $\text{ev}: \text{Diff}(X) \times X \to X$ defined by $\text{ev}(f,x) = f(x)$ is continuous thus generating a (generalized) differentiable structure on $\text{Diff}(X)$. As a result one obtains differential forms on $\text{Diff}(X)$. One verifies that the sheaves of germs of homogeneous forms are soft. Further, if $X$ is compact, $\text{Diff}(X)$ is locally differentiably contractible; hence locally closed forms are locally exact. Thus by standard sheaf theoretic arguments the generalized deRham cohomology is isomorphic to the singular cohomology with real coefficients. (Received October 21, 1968.)

Interpolation of sublinear operations on generalized Orlicz and Hardy spaces.

The Reisz-Thorin interpolation theorem is proved for sublinear operators on certain generalized Orlicz spaces, $L^\Psi$. The corresponding interpolation theorem for the Hardy-Orlicz spaces, $H^\Psi$, is also obtained. The interpolation theory above is extended to the case when there are certain factors, resembling the Radon-Nikodym derivatives of measures, included. This treatment includes the known work on the change of measures in the $L^p$-theory and generalizes the work to certain Orlicz spaces. Interpolation of multilinear operations with factors is also treated in the Orlicz spaces. Finally, the interpolation of $\{T_z\}$, a family of operators which depend on a smooth parameter, is obtained. (Received October 22, 1968.)

Embedding a topological domain in a countably generated algebraic extension.

Generalizing L. Hinrichs (The existence of topologies on field extensions, Trans. Amer. Math. Soc. 113 (1964), 404, Theorem 3.4), we prove the following Theorem. Let $I$ be an integral domain, and let $\mathcal{J}$ be a locally bounded ring topology on $I$ such that for every nonzero $a$ in $I$, the mapping $x \mapsto ax$ is open. Let $A$ be a commutative ring containing $I$ such that $A$ is generated as a ring by $I \cup B$, where $B$ is a countable set of elements of $A$, each of which is a root of a polynomial with coefficients in $I$. Then there is a ring topology $\mathcal{J}'$ on $A$ such that $\mathcal{J}'|_I = \mathcal{J}$, and such that $\mathcal{J}'$ is Hausdorff if $\mathcal{J}$ is. (In Hinrichs' result, both $I$ and $A$ were fields.) As a corollary, since there are Hausdorff topologies of the hypothesized type on any infinite domain, we can assert the existence of a nondiscrete, Hausdorff ring topology on any ring of the type described in the theorem over an infinite domain. (Received October 21, 1968.)

Affine quotients of Choquet simplexes.

As is well known, the ideas associated with quotients of topological spaces have played an important role in the development of modern mathematics. In this setting, if $h$ maps the compact Hausdorff space $X$ continuously onto the similar space $Y$, then $h(Y)$ may be "recovered" from $X$ as a quotient space. Now replace $X$ and $Y$ by compact convex subsets $K$ and $L$ of two locally convex
Hausdorff topological linear spaces. Conditions for \( h \) to carry extreme points of \( K \) to extreme points of \( h(K) \), as well as conditions for \( h(K) \) to be recovered from \( K \) by a "quotient process", are obtained. (Received October 21, 1968.)

663-312. R. E. WORTH, Georgia State College, Atlanta, Georgia 30303. The boundary of a semi-algebra.

See Abstract 650-25, these Notices 14 (1967), 923, for definitions of semilinear space, boundary \( B(S) \) of a semilinear space \( S \), etc. A semi-algebra is a semilinear space \( S \) on which is defined a (pointwise) multiplication such that \( (S, \cdot) \) is a semigroup, \( (ax) \cdot (by) = (a \beta)(x \cdot y) \), for any scalars \( a, \beta \), and \( x \cdot (y + z) = x \cdot y + x \cdot z \), \( (y + z) \cdot x = y \cdot x + z \cdot x \), for any \( x, y, z \) in \( S \). We have the natural definitions of topological, metric, and Banach semi-algebra. \( M \subset S \) is an ideal if \( M \) is a subsemialgebra and \( a \in S, m \in M \) imply \( am, ma \in M \). \( M \) is nonassimilating if there exists \( x \in S \) such that \( x + m \not\in M \) for any \( m \in M \). Theorem 1. Suppose \( M \) is a maximal nonassimilating ideal in a Banach semi-algebra with identity \( e \) such that \( d(e, 0) = 1 \), and suppose there exists \( x \) such that \( x + m = y + n \), for some \( m, n \in M \), implies \( y \not\in B(S) \), then \( B(S) = M \) and \( M \) is a group under addition. Theorem 2. If \( S \) is a Banach semi-algebra with identity \( e \) such that \( d(e, 0) = 1 \), then \( d(x, e) < 1 \) implies \( x \) is regular. F. F. Bonsall, in Proc. Int. Sympos. on Linear Spaces, Jerusalem, 1960, pp. 101-114, calls a uniformly closed semi-algebra \( S \) in \( C(X) \) of type \( n \) \( (n \geq 0) \) if \( \phi \in S \) implies \( f^n/(1 + f) \in S \). Theorem 3. The uniformly closed semi-algebra \( S \) in \( C(X) \) is of type 0 if and only if \( 1 \in S \) and \( B(S) \) is the set of singular elements of \( S \). (Received October 21, 1968.)


For each \( n = 0, 1, 2, \ldots \), let \( (X, \rho_n) \) be a metric space and \( A_n : (X, \rho_n) \to (X, \rho_n) \) a \( \rho_n \)-contractive mapping with fixed point \( a_n \). Suppose that \( \rho_n \) converges uniformly to \( \rho_0 \) and \( \rho_0 \)-converges pointwise to \( A_0 \) (1) If \( (X, \rho_0) \) is locally compact, \( \rho_n \) is topologically equivalent to \( \rho_0 \) for each \( n = 1, 2, \ldots \), and \( A_0 \) is a contraction, then \( \{a_n\} \) converges to \( a_0 \). (2) If \( \rho_n(A_n(x), A_n(y)) \leq \alpha \rho_n(x, y) \), where \( 0 \leq \alpha < 1 \) and \( \alpha \) is independent of \( n \), then \( \{a_n\} \rho_0 \)-converges to \( a_0 \). (3) If \( \rho_n \) is topologically equivalent to \( \rho_0 \) for each \( n = 1, 2, \ldots \), \( A_0 \) is a contraction, and \( \{a_n\} \) converges to \( x \), then \( x = a_0 \). These results extend those given by Nadler in Abstract 653-204, these Notices 15 (1968), 137. An example is given to show that pointwise convergence of \( \{\rho_n\} \) to \( \rho_0 \) is not sufficient to yield any of the above results, even if the space is compact. Furthermore, extensions of these results to analogous results on iterates of mappings being contractions cannot be obtained. (Received October 21, 1968.)

663-314. G. J. SMITH, University of California, Davis, California 95616. Stochastic games.

A stochastic game is a two-person game in which the players move from matrix to matrix probabilistically, depending on the previous state and plays of the players. The net payoff to player one is defined to be the lim sup of the average of the individual payoffs. Theorem. There exists a stochastic game with a finite number of finite matrices which has no solution. Also, several results on discovering and solving the solvable cases are given, extending a result of Blackwell and Ferguson. (Received October 21, 1968.)

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The linear differential-difference equation with constant coefficients.

Let \( m, n, \) and \( p \) be positive integers. Define \( h_0 = 0 \), and let \(-\infty < h_{-n} < \ldots < h_{-1} < h_0 < h_1 < \ldots < h_p < \infty \). Then consider the \( D \triangleq E \sum_{q=-n}^{p} \sum_{r=0}^{m} a_{qr} y(q)(x + h_r) = g(x) \), where the \( a_{qr} \) are real numbers, each of \( a_{0m} \), \( a_{-n,0} \), and \( a_{p0} \) are \( 0 \), and \( g \) is absolutely integrable over some finite interval \([a,b]\), where \( b - a > h_p + h_{-n} \).

Theorem. Every solution to the \( D \triangleq E \) can be written in a series of the form \( y(x) = \sum_{j=1}^{\infty} c_j \exp(s_j x) + y_p(x) \), convergent in \((a,b)\), where \( y_p \) is a particular solution and the \( s_j \) are roots of the associated characteristic equation. The particular solution arises as a natural part of the proof. The theorem is an improvement of a result of Verblunsky (Proc. London Math. Soc. (3) 6 (1956), 355-365) which is available in Bellman and Cooke (Differential-difference equations, Academic Press, New York, 1963, pp. 197-205). (Received October 21, 1968.)

On the homotopy type of the group of regular elements of semifinite von Neumann algebras.

Theorem. Let \( A \) be a countably decomposable von Neumann algebra of type \( (I_{\omega}, I_{\infty}) \). Then the group of regular elements of \( A \), equipped with its norm topology, is contractible to a point.

Corollary. Let \( E \) be a finite projection of \( A \). Let \( \Gamma_E \) be the Grassmann space of all projections \( F \in A, F \sim E \), equipped with the norm topology. Let \( G_E \) be the group of regular elements of the reduced algebra \( AE \) equipped with the norm topology. Then \( \pi_i \Gamma_E \) is canonically isomorphic to \( \pi_i-1G_E \) for \( i = 1, 2, 3, \ldots \). In particular, \( \Gamma_E \) is simply connected. (Received October 21, 1968.)

On the kernel function for the intersection of two simply connected domains.

Let \( D_1 \) and \( D_2 \) be bounded simply connected domains in the complex plane each containing the origin and let \( D \) be the component of \( D_1 \cap D_2 \) which contains the origin. Let \( \{W_n\}_{n=1}^{\infty} \) and \( \{V_n\}_{n=1}^{\infty} \) be complete orthonormal sets in the spaces \( L^2(D_1) \) and \( L^2(D_2) \) of functions analytic in \( D_1 \) and \( D_2 \) respectively. Then the set \( \{W_n\}_{n=1}^{\infty} \cup \{V_n\}_{n=1}^{\infty} \), with the domain of each function restricted to \( D \), spans \( L^2(D) \). (Received October 22, 1968.)

Right ideals of transformation near-rings.

Let \( (G, +) \) be a group and let \( T_0(G) \) be the set of all mappings from \( G \) into \( G \) such that zero goes into zero. \( T_0(G) \) together with the operations of pointwise addition, \( (x)[f + g] = (x)f + (x)g \), and composition, \( (x)[fg] = ((x)f)g \), is a (left) near-ring. The set \( P_x \) of all mappings which annihilate \( G - \{x\}, x \neq 0 \), is a minimal right ideal generated by an idempotent \( a_x \), where \( ta_x = 0 \) if \( t \neq x, xa_x = x \).

The mappings \( a_x \) are pairwise orthogonal. The sum of all the \( P_x, x \neq 0 \), is a direct sum and is equal to \( T_0(G) \) if \( G \) is finite, but is a proper right ideal of \( T_0(G) \) if \( G \) is infinite. The former was first shown by D. W. Blackett in 1950 who classified all right ideals of \( T_0(G) \) for \( G \) finite. Every minimal right ideal of \( T_0(G) \) is of the form \( P_x \). For any nonempty subset \( M \) of \( G \), the annihilating set \( A(M) = \{ f \in T_0(G): Mf = 0 \} \) is a right ideal. If \( M = G - \{x\}, x \neq 0 \), then \( A(M) \) is a maximal right ideal. The question is raised whether these are all the maximal right ideals. (Received October 21, 1968.)
Local contractions on a-metric spaces.

In Abstract 675-25, these Notice 15 (1968), 620, the author extended the Contraction Mapping Theorem of Banach as follows: Theorem 1. Every contraction mapping \( f \) on an a-metric space \( (S,d) \), with \( a > 0 \), has a unique fixed point. In this note, we obtain an analogous extension of Edelstein's Theorem on local contraction mappings (Theorem 5.2, An extension of Banach's contraction principle, Proc. Amer. Math. Soc. 12 (1961), 8). We preface our statement of this extension with the following definitions: Definition 1. \( (S,d) \) is an \((a+\epsilon)\)-chainable a-metric space iff \( (S,d) \) is an a-metric space such that \( \forall x,y \in S \exists x_0, x_1, \ldots, x_n \in S \) with \( x_0 = x, x_n = y \), and \( d(x_{i-1}, x_i) < a + \epsilon \) \( (i = 1, 2, \ldots, n) \). Definition 2. \( f \) is an \((a + \epsilon, \lambda)\)-uniform local contraction on the a-metric space \( (S,d) \) iff \( \forall x \in S \exists \epsilon > 0, 0 \leq \lambda < 1 \) (both independent of \( x \)) such that \( p, q \in S_d(x; \epsilon) \) implies \( d(f(p), f(q)) < \lambda d(p, q) \). Theorem 2. If \( f \) is an \((a + \epsilon, \lambda)\)-uniform local contraction on an \((a + \epsilon)\)-chainable a-metric space \( (S,d) \), then \( f \) has a unique fixed point. (Received October 21, 1968.)

663-320. J. M. SLYE, University of Houston, Houston, Texas 77004. Quilted two manifolds.

This result is part of a project to construct relative holes in an arbitrary complex. First conditions were established under which relative holes of the two skeleton may be used to construct relative holes of the complex. These conditions are not always verifiable, so further reduction was necessary. Next for a complex \( C \), a simplicial map of a complex \( K \) onto the barycentric subdivision of \( C \) and relative holes of \( K \) may be used to construct relative holes of \( C \). Also the cyclic elements of \( K \) that are not one-dimensional have no local or global cut points. The method of construction generalizes to two dimensions. The problem is to find a proper objective for such a transformation. The concept of a quilted two manifold is introduced. A quilted two manifold is to be a connected properly two-dimensional complex without local or global cut points. In addition, around each vertex it has the structure of a collection of planes and half-planes piled up and united along a number of simplex with \( P \) as a vertex. Theorem. If \( C \) is a properly two-dimensional connected complex without local or global cut point, there is a simplicial transformation of a quilted two-manifold onto the barycentric subdivision of \( C \). (Received October 21, 1968.)

663-321. R. B. KELLOGG, University of Maryland, College Park, Maryland 20740. On essentially positive matrices.

Let \( A \) be a real \((N + 1)\)-square matrix with eigenvalues \( \lambda_0, \ldots, \lambda_N \). Let \( \lambda_1, \ldots, \lambda_K \) be all the eigenvalues of \( A \) with positive imaginary parts ordered so that \( \text{Re} \lambda_{k-1} \leq \text{Re} \lambda_k \), \( 2 \leq k \leq K \). \( A \) is called strictly essentially positive (SEP) if its off diagonal entries are all positive. Theorem. A sufficient condition that \( A \) be similar to a SEP matrix is that \( \lambda_0 \) is real and \( \text{Re} \lambda_{k-1} > \text{Re} \lambda_k + \sqrt{3} \text{Im} \lambda_k, 1 \leq k \leq K \). A necessary condition that \( A \) be similar to a SEP matrix is that \( \lambda_0 \) is real and \( \lambda_0 > \text{Re} \lambda_k + (\tan \pi/(N + 1)) \text{Im} \lambda_k, 1 \leq k \leq K \). The same method of proof is used to generalize a theorem of Suleimanova giving sufficient conditions for a matrix with real eigenvalues to be similar to a positive matrix. The necessity follows directly from a theorem of Dmitriev and Dynkin. (Received October 21, 1968.)
663-322. NANCY DYKES, Kent State University, Kent, Ohio 44240. Mappings and realcompact spaces.

All spaces considered are completely regular. Theorem 1. Let X be a topologically complete space and \( \phi \) a closed map of X onto Y. If C is a compact subset of \( \Phi(X) \), then all compact subsets of \( \Phi(C) \cap Y \) are finite. Theorem 2. Realcompactness is preserved under closed maps if the range is normal, weak cb, k-space. Theorem 3. Realcompactness is preserved under perfect maps if the range is weak cb. Theorem 4. All closed maps whose domain is topologically complete are k-covering maps. Theorem 5. If X is a topologically complete, G\( \delta \) space and \( \phi \) is a closed map of X onto Y, then \( Y = \bigcup_{i=1}^{\infty} Y_i \) where each \( Y_i \) is closed and discrete if \( i \neq 1 \) and \( \phi^{-1}(y) \) is compact for all \( y \in Y_0 \). Theorem 6. Let \( \phi \) be a WZ-map of X onto a realcompact space Y. Then X is realcompact iff \( \text{cl}(\phi^{-1}(y)) = \phi^{-1}(y) \) for each \( y \in Y \). (Received October 21, 1968.)


Consider (*) \( x'(t,\omega) = f(t,x(t,\omega),\omega), x(a,\omega) \) given, where \( x' \) is a time derivative of \( x \), \( \omega \in \Omega \) (a probability space). Even in a simple case like \( x' = A(\omega)x \), \( x(0) = 1 \), the solution need not have finite moments even if A has all finite moments. We prove a general existence theorem guaranteeing the existence of \( p \neq 1 \) moments. Applied to the linear problem \( x' = A(t,\omega)x + P(t,\omega) \) where A is a matrix, and P a vector, of stochastic processes, this theorem gives a precise condition guaranteeing \( p \) moments on some interval \([a,b]\). The condition involves the time integrability of the Laplace transforms \( E(\exp(sa_j(t,\omega))) \) for \( s = p(b-a) \). The condition is the best possible in many important cases. (Received October 23, 1968.)

663-324. C. C. PINTER, Bucknell University, Lewisburg, Pennsylvania 17837. On \( \pi \)-polyadic algebras.

In this paper, a simplified set of axioms is presented for locally finite polyadic algebras. A \( \pi \)-polyadic algebra is defined to be a quadruple \( (A, I, S, \Xi) \) where \( (A, I, S) \) is a locally finite transformation algebra (LeBlanc, Canad. J. Math. 13 (1961), 602-613) and \( \Xi \) is a mapping from elements of I to quantifiers on A, satisfying (\( \pi \)-P1) \( S(i/j)\Xi_i = \Xi_j \), (\( \pi \)-P2) \( \Xi_i S(i/j) = S(i/j) \) if \( i \neq j \), (\( \pi \)-P3) \( \Xi_i \Xi_j = \Xi_j \Xi_i \) for all \( i, j \in I \). Theorem. Every \( \pi \)-polyadic algebra is a locally finite polyadic algebra, and conversely. We let \( B_i \) designate the range of \( S(i/k) \); note that \( B_i \) is independent of the choice of \( k \); note also that \( B_i \) coincides with the range of \( \Xi_i \). Theorem. Condition (\( \pi \)-P3) is equivalent to the following: The range of \( \Xi_i \Xi_j \) is \( B_i \cap B_j \). Theorem. A transformation algebra \( (A, I, S) \) is a \( \pi \)-polyadic algebra if and only if the following two conditions hold: (\( \pi \)-Q1) For some \( i \in I, B_i \) is a relatively complete subalgebra of \( A \). (\( \pi \)-Q2) If \( x \in B_i \) and \( y \in B_j \) and \( x \neq y \), then there is an element \( z \in B_i \cap B_j \) such that \( x \neq z \neq y \). (Received October 21, 1968.)
A sequential norm (s.n.), $N$, is an extended norm on $s$, the space of all sequences, such that (i) $N(e_i)$ is finite for all $i$ and (ii) $\sup_n (N(x(\sim n))) = N(x)$. Let $A \subseteq s$ be such that, for all positive integers $i$, there is an $i \in A$ with $a_i \neq 0$. The s.n. $P_A$ is given by $P_A(x) = \sup_n \{\sum_{i=1}^{n} a_i x_i : a \in A\}$.

Theorem. $N$ is an s.n. iff there is $A$ in $s$ such that $N = P_A$. In the above theorem, $A$ may be chosen to be either countable or a subset of $\phi$, the space of eventually zero sequences. (Received October 21, 1968.)

For $f(x)$ a real-valued continuously differentiable function $-1 \leq x \leq 1 (1 > 0)$, define the norms $\|f\| = \max [-1,1] |f(x)|$ and $\|f\|_1 = \max \{\|f\|, \|f'\|\}$ ($' = d/dx$). For a nonnegative integer, define the set $P_n = \{a_0 + a_1 x + \ldots + a_n x^n : a_i \text{ real}, i = 0,1, \ldots, n\}$. Define the polynomial $p(f,n,1) \in P_n$ and the real number $E(f,n,1)$ by $E(f,n,1) = \min_{q \in P_n} \|q - f\|$. Define a polynomial $p'(f',n,1) \in P_n$ and the real number $E'(f',n,1)$ by $E'(f',n,1) = \min_{q' \in P_n} \|q' - f'\|$. Define a polynomial $\tilde{p}(f,n,1) \in P_n$ and the real number $\tilde{E}(f,n,1)$ by $\tilde{E}(f,n,1) = \min_{q \in P_n} \|q - \tilde{f}\|$. The problems of expressing $\tilde{E}(f,n,1)$ as a function of $E(f,n,1)$ and $E'(f',n,1)$ and of expressing $\tilde{p}(f,n,1)$ as a function of $p(f,n,1)$ and $p'(f',n,1)$ are discussed. (The question of existence and uniqueness for $\tilde{p}(f,n,1)$ has been investigated by D. G. Moursund, Math. of Comp. 18 (1964), 382-289.) (Received October 21, 1968.)

Let $K[\Omega]$ denote the class of all entire functions of exponential type whose Laplace transforms are analytic on $\Omega^c$, the complement of the simply connected domain $\Omega$. Let $\{L_n\}$ be a sequence of linear functionals defined on $K[\Omega]$ by $L_n(f) = (2\pi)^{-1} \int_{\Gamma} g_n(\zeta) F(\zeta) d\zeta$ where $F$ is the Laplace transform of $f$, each $g_n$ is analytic on $\Omega$, $\Gamma \subset \Omega$ is a simple closed curve enclosing all singularities of $F$. The author showed earlier [Proc. Amer. Math. Soc. 16 (1965), 69-71] that for $g_n(\zeta) = [W(\zeta)]^n$ for some function $W$, a nasc that $K[\Omega]$ be a uniqueness class for $\{L_n\}$ is that $W$ be univalent on $\Omega$. It is shown here that the method used there can be generalized to obtain uniqueness classes for the case $g_{kn+k}(\zeta) = h_k(\zeta) [W(\zeta)]^{Pn}$; $k = 0,1,\ldots, p - 1$; $n = 0,1,2,\ldots$. (Received October 21, 1968.)

A bilateral sequence of 0's and 1's is constructed whose orbit closure in the sequence space under the shift transformation is minimal, topologically strongly mixing, uniquely ergodic, and not measure-theoretically strongly mixing. This dynamical system also has topological and metric entropy zero. (Received October 21, 1968.)

Let X be a completely-regular, Hausdorff space. A regular, finite Baire measure m is net-additive if for every downward directed family \( \{ Z_\tau \} \) of zero sets of X with \( Z_\tau \downarrow \emptyset \), it follows that \( m(Z_\tau) \downarrow 0 \). Say that X is B-compact if every regular, finite Baire measure on X is net-additive. In this paper, B-compactness is shown to be a topological condition, and the stability properties of this condition are studied. For instance, it is shown that a closed subspace of a B-compact space is B-compact and that the product of a compact space and a B-compact space is B-compact. However, it is also shown that, in general, products and intersections of B-compact spaces fail to be B-compact. Finally, the relation of B-compactness to other compactness conditions are indicated. For example, if a space is Lindelöf, then it is B-compact; and if a space is B-compact, then it is realcompact. (Received October 22, 1968.)


The following two theorems are proved. Theorem 1. Let \( f(x) \) be a positive nondecreasing function such that (*) \( \sum_{\nu \in X} f(x/\nu) = xg(x) + o(x^2g'(x)) \) where \( g(x) \) is a positive, twice continuously differentiable function defined in \([1,\infty)\) such that (i) \( g'(x) > 0 \) for all \( x \in (1,\infty) \); (ii) \( xg'(x) \) is nonincreasing from some point on; (iii) for some positive integer \( k \), \( h(x) = x(\log x)^kg'(x) \) is nondecreasing from some point on and \( \lim \inf_{x \to \infty} h(x) = \infty \). Then \( f(x) \sim x^2g'(x) \) as \( x \to \infty \). Theorem 2. With \( g(x) \) as in Theorem 1, if \( f(x) \) is any function bounded and integrable in every finite subinterval of \([1,\infty)\) which satisfies (*), then \( \int f(t)/t^2 \, dt = g(x) - \gamma xg'(x) + o(xg'(x)) \), where \( \gamma \) is Euler's constant. The proofs of these theorems involve the rather deep estimate from prime number theory that \( \sum_{\nu \in X} \mu(\nu)/\nu = o(1/(\log x)^k) \) for all \( k > 0 \) where \( \mu(\nu) \) is the Möbius function. The first theorems of the above type are due to Landau, and the special cases of Theorems 1 and 2, when \( g(x) = \log x + b \), were proved by Ingham. (Received October 21, 1968.)

663-331. R. N. PEDERSON, Stanford University, Stanford, California 94305. A proof of the Bieberbach conjecture for the sixth coefficient.

Theorem. If \( f(z) = z + a_2z^2 + \ldots + a_nz^n + \ldots \) is analytic and univalent in the unit disk, then \( |a_6| \leq 6 \). Equality holds only for the Koebe function: \( K(z) = z/(1 - az)^2 \), \( |a| = 1 \). The proof uses the formulas of Garabedian, Ross and Schiffer, J. Math. Mech. 14 (1965), together with an observation of the author, Arch. Rat'l. Mech. Anal. 29 (1968), that the (suitably normalized) Grunsky matrix of a slit mapping is unitary. (Received October 21, 1968.)

663-332. R. R. MILLER, University of Utah, Salt Lake City, Utah 84112. Parts in the maximal ideal space of a convolution measure algebra. Preliminary report.

Let \( M \) be a semisimple convolution measure algebra with structure semigroup \( S \). Then each complex homomorphism of \( M \) is given by integrating a semicharacter on \( S \). Gleason parts can be defined on \( \hat{S} \), the set of semicharacters of \( S \), by considering the function algebra obtained from the transforms of elements of \( M \). We give a partial characterization of the parts of \( \hat{S} \) utilizing only
the functional values of the elements of $S$. \textbf{Theorem.} If $f$ and $g$ are elements of $S$ that are in the same part, then $f(x) = g(x)$ whenever either $|f(x)| = 1$ or $|g(x)| = 1$. For the special case of nonnegative semicharacters satisfying the additional requirement that $f(x) = 0$ if and only if $g(x) = 0$, we show that the converse holds. (Received October 21, 1968.)

663-333. J. H. BEVIS, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. \textbf{Characterization of property D in orthomodular lattices.} The author has related associativity of matrix multiplication for matrices over an orthomodular lattice $L$ to a certain distributive rule on $L$ called property D [J. H. Bevis, \textit{Matrices over orthomodular lattices}, Glasgow Math. J., to appear]. In this talk the author gives a characterization of property D in terms of the commuting relation on $L$. For example, the following is given: \textbf{Theorem.} $[a_1, a_2]$ satisfy property D on $L$ if and only if $a_1$ and $a_2$ are central in $L$ and $L[a_1 \wedge a_2]$ is a Boolean lattice. (Received October 21, 1968.)

663-334. PALANIAPPAN KANNAPPAN, University of Waterloo, Waterloo, Ontario, Canada. \textbf{On cosine and sine functional equations.} Consider the functional equations \textbf{(A)} $f(xy) + f(xy^{-1}) = 2f(x)f(y)$, and \textbf{(C)} $f(xy)f(xy^{-1}) = f(x)^2 - f(y)^2$, where $f$ is a complex valued function on an arbitrary group $G$. Then it is evident that if $g$ is a homomorphism of $G$ into the multiplicative group of complex numbers, $K$, \textbf{(B)} $f(x) = (g(x) + g(x)^{-1})/2$, and \textbf{(D)} $f(x) = (g(x) - g(x)^{-1})/2$, are solutions of \textbf{(A)} and \textbf{(C)} respectively. \textbf{Theorem 1.} Let $I$ be any arbitrary index set. Let $(G_a)_{a \in I}$ be a family of groups such that for every $a$ in $I$, each solution of \textbf{(A)} on $G_a$ is of the form \textbf{(B)} and $G_a \subseteq G_B$, $a < B$, $a$, $B$ $\in I$. Then every solution of \textbf{(A)} on $G = \bigcup G_a$, $a$ in $I$, is of the form \textbf{(B)}, provided $f(xyz) = f(xzy)$ for all $x, y, z$ in $G$. \textbf{Application.} Consider the additive group of rationals $Q = \bigcup G_n$, where $G_n = [K/n! : k \in Z]$, $K$ any integer, $n$ any positive integer $\neq 1$. As it is known that every solution $f$ of \textbf{(A)} on $G_n$ (being cyclic) is of the form \textbf{(B)}, by Theorem 1, every solution of \textbf{(A)} on $Q$ has the form \textbf{(B)}. \textbf{Theorem 2.} Let $G$ be a cyclic group. Let $f$ be a complex-valued function on $G$ satisfying \textbf{(C)} with the properties that (1) $f$ is not identically zero and (2) $f$ is of the form \textbf{(D)}. Then we assert that there is one and only one homomorphism $g$ of $G$ into $K$ satisfying \textbf{(D)}. (Received October 21, 1968.)

663-335. S. M. SHAH and S. Y. TRIMBLE, University of Kentucky, Lexington, Kentucky 40506. \textbf{Univalent functions with univalent derivatives.} This paper is in continuation of our earlier papers [Abstracts 658-55 and 658-139, these Notices] 15 (1968), 734, 759]. \textbf{Theorem 1.} Let $f$ be holomorphic in $|z| < R$ and let $f^{(n)}$ be univalent in $|z| < \rho_n$. Then (i) $\lim \inf_{n \to \infty} n \rho_n \neq 4R$, (ii) $\lim \inf_{n \to \infty} n (\rho_0 \cdots \rho_n)^{1/n} \neq 4eR$ where $\rho_k^+ = \max(1, \rho_k)$, and (iii) if $f$ is not a polynomial, then $R \log 2 \equiv \lim \sup_{n \to \infty} n \rho_n^-$ \textbf{Theorem 2.} If $f$ is a transcendental entire function of order $\rho$ and lower order $\lambda$, then $1/\rho \neq \lim \inf_{n \to \infty} (\log \max(1, \rho_n))/\log n$ and $1/\lambda \neq \lim \sup_{n \to \infty} (\log n \rho_n)/\log n$. \textbf{Theorem 3.} There is a function $f$, analytic in $D$: $|z| < 1$ and an increasing sequence of integers, $(n_p)_{p \geq 1}$, such that $f$ and each $f^{(n_p)}$ map $D$ univalently onto convex domains, and $|z| = 1$ is the natural boundary of $f$. Here, $n_p \log n_p < n_{p+1} < n_p \log^2 n_p$ $p > p_0$. (Received October 22, 1968.)

Suppose \((L, \ell, \mu)\) is a random normed space as defined by A. N. Šerstnev, On the concept of a random normed space, Soviet Math. Dokl. 4 (1963), 388-391. Let \(M\) be a closed subspace of \(L\) and consider the quotient space \(L/M\). Functions \(g\) and \(\mu^*\) are defined that makes \((L/M, g, \mu^*)\) a linear topological space. Conditions are then imposed on \(\mu^*\) to make \((L/M, g, \mu^*)\) a random normed space. (Received October 22, 1968.)

663-337. AARON STRAUSS, Mathematical Research Center, U. S. Army, University of Wisconsin, Madison, Wisconsin 53706, and J. A. YORKE, Institute of Fluid Dynamics, University of Maryland, College Park, Maryland 20740. Identifying perturbations which preserve asymptotic stability.

Let \(x = 0\) be uniform-asymptotically stable (UAS) for \((1) x' = f(t,x)\). Let \(f\) be Lipschitz. Then it is known that \(x = 0\) is UAS for \((2) x' = f(t,x) + g(t,x)\) if \(g\) is sufficiently small. In the standard proof estimates on the allowable size of \(g\) are obtained in terms of a Liapunov function associated with \((1)\). We establish estimates on \(g\) in terms of the rate of approach to zero of the solutions of \((1)\). In so doing we also obtain an estimate on the rate of approach to zero of solutions of \((2)\) for such \(g\). This leads to a new proof and slight extension of Hahn's theorem: If \(f\) is homogeneous of degree \(k\), then uniform-asymptotic stability is preserved by \(g(t,x) = o(|x|^k)\). (See W. Hahn, Stability of motion, Springer-Verlag, Berlin, 1967, Sections 56 and 57.) (Received October 22, 1968.)


The lattice of equational classes of monadic algebras is a chain of length \(\omega + 1\), and a single explicit equation is given for each such class. There are \(\aleph_0\) equational classes of two-dimensional polyadic algebras; each class is finitely based and is determined by its finite members. For any class considered the decision problem for equations holding in each member of the class is solvable. The proofs give some insight into the structure of simple polyadic algebras of dimension two. (Received October 22, 1968.)

663-339. STEVE LIGH, Texas A & M University, College Station, Texas 77843. On distributively generated near-rings.

Let \(R\) be a distributively generated near-ring. Among the results obtained are the following: If \(e\) is a unique left (right) identity of \(R\), then \(e\) is also a right (left) identity. If \(R\) has more than one element, then \(R\) is a division ring if and only if, for each \(a \neq 0\) in \(R\), there exists a unique \(b\) in \(R\) such that \(aba = a\). If \(R\) is finite and has no nonzero divisors of zero, then \(R\) is a field. (Received October 22, 1968.)
Ideals in near-rings.

It is known that an idempotent element induces a decomposition of a near-ring \( R \). However, one of the two summands--although it is a right ideal in \( R \)---need not be a subnear-ring. If \( 0r = r0 = 0 \) for each \( r \) in \( R \), then each of the summands is a subnear-ring. In this note right ideals are studied without imposing this condition on \( R \). A right ideal of \( R \) is decomposed into a sum of right ideals of the summands (these need not be right ideals in \( R \)). It is shown which right ideals of the summands may be summed to give a right ideal of \( R \). (Received October 22, 1968.)

Nil semi-simple hemirings with descending chain condition.

The semisubtractive property, the nil semisimple property, and the descending chain condition for left \( k \)-ideals (DCC) are defined for hemirings. The class of hemirings to be considered is restricted to those that are semisubtractive and nil semisimple, and have DCC and zero as their zeroid. This class contains the rings considered by Artin, Nesbitt, and Thrall in their classical work (E. Artin, C. J. Nesbitt, and R. M. Thrall, Rings with minimum condition (Univ. of Michigan Publ. in Math., no. 1, 1944)). The results they obtained are generalized to this class of hemirings, the main result being that any hemiring in this class has only a finite number of simple \( k \)-ideals and is their direct sum. (Received October 22, 1968.)

Mapping cylinder neighborhoods.

Let \( C \) be a closed subset of a space \( X \). A subspace \( U \) of \( X \) is called a mapping cylinder neighborhood (MCN) of \( C \) if \( U = f(M \times I) \cup C \) where \( f \) is a map of a space \( M \times I \) into \( X \) such that \( f|M \times \{0,1\} \) is a homeomorphism into \( X - C \), \( f(M \times 1) = C \cap C_1(X - C) \) and \( f(M \times \{0,1\}) \cup C \) is open in \( X \). Regular neighborhoods are MCN's. If \( X \) is a 3-manifold and \( U \) is a MCN of a compact subset \( C \) of \( X \) then \( U \) is a compact 3-manifold with boundary and all MCN's of \( C \) are homeomorphic. A topological complex is a space homeomorphic to a locally finite simplical complex. Theorem 1. Suppose \( C \) is a topological complex which is a closed subset of a 3-manifold \( X \). Then \( C \) is tamely embedded in \( X \) iff \( C \) has a MCN. Theorem 2. Suppose \( C \) is a tame topological complex in a 3-manifold \( X \), \( g \) is a map of \( X \) into a 3-manifold \( Y \) such that \( g^{-1}g(C) = C \), \( g \) is a homeomorphism on \( X - C \) and \( g(C) \) is a topological complex. Then \( g(C) \) is tamely embedded in \( Y \). Theorem 3. Suppose \( N \) is a compact connected 3-manifold in a 4-manifold \( Y \). Suppose \( M \) is a 3-manifold and \( U = f(M \times I) \) is a MCN of \( N \) where the restriction of \( f \) to each component of \( M \times I \) is a cellular map. Then \( U \) is a bicollar for \( N \) in \( Y \). (Received October 16, 1968.)

Normal modes of vibrations of a finite truncated circular sector plate.

The theory advanced by Reissner, Mindlin, and Uflyang is extended to the actual finding of the normal modes of vibrations of a plate of thickness \( h \) in the form of a finite truncated circular
sector plate. The three partial differential equations governing the motion of plates are solved
and solutions satisfying the eight different types of boundary conditions entering into the theory are
chosen. By eliminating the arbitrary constants that occur, each problem reduces to the setting of a
twelfth order determinant equal to zero and solving the resulting transcendental equation for the
roots which will represent the different modes of vibration for each characteristic number \( m \).
Results obtained from high speed computers show close correlation between what the theory predicts
and actual tests made thereby giving evidence of the truth of the theory predicted. (Received
October 23, 1968.)

663-344. C. W. CRYER, Computer Sciences Department, University of Wisconsin, Madison,
Wisconsin 53706. The approximate solution of free boundary problems.

An algorithm for solving one-phase free boundary problems in two dimensions is described.
The algorithm uses the method of finite differences and is an automated version of methods due to
Southwell. An improved version of the algorithm is also described and the algorithms are analysed
in detail for a simple problem. The algorithms have been implemented as a general program
FREEBOUN which has been used to compute approximate solutions to a model problem, a problem
from astrophysics, and the problem of the flow of a water jet out of a pipe. (Received October 23, 1968.)

uniformities and set functions.

Let \( \Sigma \) be a Tukey-Smirnov uniformity on a set \( M \). Then the function \( T: P(M) \rightarrow P(M) \) given by
\[
T(A) = \{ a; a \in \Sigma \text{ implies } St_a a \cap A \neq \emptyset \}
\]
is a Kuratowski closure function. If \( \Phi \) is a Weil uniformity on \( M \), then \( t: P(M) \rightarrow P(M) \) given by \( t(A) = \{ p; \{ p \} \times A \text{ intersects every } V \in \Phi \} \) is also a Kuratowski
closure function. Theorem 1. If \( \Sigma \) and \( \Phi \) are associated, then \( T = t \). Theorem 2. If \( r: P(M) \rightarrow P(M) \) is
given by \( r(A) = \{ p; \text{there exists a } U \in \Phi \text{ for which } U[p] = A \} \), then \( r(A) = c(t(c(A))) \) so \( r \) is a topological
interior function. For weaker structures such as extended uniformities \( r \) need not equal \( t \) and
hence need not be a topological interior. (Received October 23, 1968.)

663-346. HANS SCHNEIDER, University of Wisconsin, Madison, Wisconsin 53711. Bound norms.

A matrix norm is a function \( \nu \) from the set of all complex (or real) matrices (of all orders
\((m \times n), m, n = 1,2,\ldots\)) into the nonnegative reals, which satisfies certain standard conditions.
A matrix norm \( \nu \) is a matrix bound norm if for every matrix \( A \), \( \nu(A) = \sup \{ \nu(Ax)/\nu(x); x \text{ is a nonzero }
\]
column vector and \( Ax \) is defined\}. Theorem 1. Let \( \nu \) be a matrix bound norm. Then there exist non-
singular \((n \times n)\) matrices \( P_n, n = 1,2,\ldots \), with \( P_1 = 1 \), such that for all \((m \times n)\) matrices \( A \),
\[
\| A \|_{\infty} \leq \nu(P_m^{-1} A P_n) \leq \| A \|_1, \quad \text{where } \| A \|_{\infty} = \max_{i,j} |a_{ij}|, \text{ and } \| A \|_1 = \sum_{i,j} |a_{ij}|. \hspace{1cm} \text{Theorem 2. There}
\]
exists a matrix bound norm \( \nu \) such that for all \((m \times n)\) there exist matrices \( A \) and \( B \) for which
\[
\| A \|_{\infty} = \nu(P_m^{-1} A P_n) = (mn)^{-1} \| A \|_1, \quad \text{mn}\|B\|_{\infty} = \nu(P_m^{-1} B P_n) = \| B \|_1. \hspace{1cm} \text{(Received October 23, 1968.)}
\]

Let \( L \) be a lattice of subsets of a topological space \( X \) with \( \varphi \in L \). Let \( \mu \) be a modular function on \( L \) with values in a Banach space \( E \) and with \( \mu(\varphi) = 0 \). We investigate the extension of \( \mu \) to a countably additive function on the generated \( \sigma \)-ring \( \Sigma(L) \) and with values in \( E \). The problem is broken into three parts: (1) for each \( \gamma \) in \( E^* \), we must be able to extend \( \gamma \circ \mu \) to a unique scalar valued, countably additive function \( (\gamma \circ \mu)^* \) on \( \Sigma(L) \); (2) considering \( E \) as a subset of \( E^{**} \), we must show that there is a unique countably additive extension \( \mu^* \) on \( \Sigma(L) \) but with values in \( E^{**} \); and (3) we must ensure that \( \mu^* \) actually take values in \( E \). We achieve such an extension for a special case: \( L \) is a \( \delta \)-lattice of closed compacta. We accomplish (1) by an assumption of regularity and a concept of bounded variation of \( \mu \). [See Abstract 653-307, these Notices 15 (1968), 169.] The Closed Graph Theorem helps ensure (2). Requirement (3) is produced by regularity and the assumption that the range of \( \mu \) be relatively weakly sequentially compact. (Received October 23, 1968.)

663-348. K. E. ELDRIDGE, Ohio University, Athens, Ohio 45701. Artinian \( q \)-torsion rings with \( p \)-groups.

A ring \( R \) is said to have \( q \)-torsion if every element of \( R \) has additive order a power of the prime \( q \). Denote by \( J \) the Jacobson radical of \( R \) and by \( O^R \) the quasi-regular group of \( R \). For Artinian \( q \)-torsion rings with \( O^R \) a \( p \)-group we establish the following: If \( p = q \) is an odd prime, then \( J = R \); if \( p = 2 = q \), then \( R/J \cong GF(2) \oplus \ldots \oplus GF(2) \) and \( O^R \cong O^J \); if \( p \neq q \), then \( R \) is a sum of finite fields which can be characterized in terms of the solutions to the diophantine equations \( q^n - 1 = p^m \).

(Received October 23, 1968.)

663-349. C. P. TSOKOS, University of Rhode Island, Kingston, Rhode Island 02881. Eventual asymptotic stability of control systems.

Consider the control system (*) \( x' = f(t,x,u) \) where \( u \) is a control vector. Assume that (i) the origin \( x = 0 \) of \( x' = f(t,x,u_0(t)) \) is eventually uniformly asymptotically stable (EVAS) for some control \( u_0(t) \); (ii) for \( a \leq \|x\| < \rho \), \( t \geq \Theta(a) \), \( \|f(t,x,u_1) - f(t,x,u_2)\| \leq L\|u_1 - u_2\| \), where \( \Theta(r) \) is nonincreasing in \( r \); and (iii) \( \lambda(t) \geq 0 \) is continuous for \( t \geq \Theta \) and satisfies \( \int_0^t \lambda(s)ds \to 0 \) as \( t \to \infty \). It is proved that under these conditions, the origin \( x = 0 \) of (*) is EVAS for every control \( u = u(t) \in \Omega \), where \( \Omega \) is given by \( \Omega = [u: \|u - u_0(t)\| \leq \lambda(t), t \geq 0] \). (Received October 23, 1968.)


In discussing the stability properties of solutions of integro-differential equations by means of Lyapunov functions, two situations arise, namely, (i) to choose appropriate minimal class of functions along which it is possible to estimate the derivative of Lyapunov function in terms of a scalar function, (ii) to reduce the study of system of integro-differential equations to the study of a scalar integro-differential equation. In the former case, the discussion reduces to the study of stability properties of a scalar differential equation; and in the latter situation, the existence of
maximal solution and the theory of integro-differential inequalities are to be developed since it plays a crucial role. The present work is devoted to extend Lyapunov method by the foregoing two approaches. The advantage of each method is illustrated by applications. (Received October 23, 1968.)


This paper obtains the nilpotent ring analogues of several well-known results for finite nilpotent groups. To study finite rings it is clearly sufficient to consider rings whose additive groups are p-groups, called p-rings. The paper begins with an analogue for finite nilpotent p-rings of the Burnside basis theorem. This is used to obtain some information on the automorphism groups of these rings and to establish Anzahl results. It is shown that the number of subrings, right ideals, and two-sided ideals of a given order in a finite nilpotent p-ring is congruent to 1 mod p. In addition, the paper characterizes the finite nilpotent p-rings R which contain only one subring S of a given order, 0 ≠ S ≠ R. When |S| = p or |S| = p², certain exceptional rings occur. Otherwise, either R has a cyclic additive group, or else |S| = |R|/p and R is generated by a single element. (Received October 23, 1968.)


Since a harmonic differential describes an incompressible fluid flow, the Hodge-de Rham theorem can be interpreted as follows: on a surface there exists a unique incompressible flow with prescribed periods (or, roughly speaking, with prescribed circulation around the handles). The flow of a (compressible) gas is described by a quasi-linear second order partial differential equation of divergence type. Bers has conjectured the existence of a compressible subsonic flow on a Riemannian manifold having prescribed periods. By reducing the above conjecture to an appropriate nonlinear variational problem an affirmative answer can be given to this question. (Received October 23, 1968.)


Let $u_1(x) = 3A_i(-x) + \sqrt[3]{5}B_i(-x)$ where $A_i(x)$ and $B_i(x)$ are the two standard linearly independent solutions of Airy's equation $y'' = xy$. Mursi [On the relation of the Airy and allied integrals to the Bessel functions, Proc. Math. Soc. Egypt (4) 3 (1948)] obtained the relation $x^{3/2n+1}J_{n+2/3}(\xi) = R(x)u_1(x) + S(x)u_1'(x)$ where $R(x)$ and $S(x)$ are polynomials in x and where $J_\nu(x)$ is the Bessel function of order ν. In this paper it is shown that the modified Bessel function $K_{\nu}(\xi)$ of arbitrary order may be expressed as the sum of two products where one involves Airy's function and the other product involves its first order derivative. (Received October 23, 1968.)

663-354. J. W. BREWER, Virginia Polytechnic Institute, Blacksburg, Virginia 24061, and ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. The ideal transform and overrings of an integral domain. II. Preliminary report.

Let D be an integral domain having quotient field K. By an overring of D we mean a domain $D_1$
between $D$ and $K$. If $A$ is an ideal of $D$, denoted $T(A)$, we mean the overring of $D$ by the transform of $A$, denoted $T(A)$, we mean the overring $B_n$, where $B_n = \{ \xi \in K | \xi A^n \subseteq D \}$. Theorem 1. The following are equivalent: (i) Every overring of $D$ is the transform of some finitely generated ideal of $D$. (ii) Every overring of $D$ is the transform of some principal ideal of $D$. (iii) Every overring of $D$ is of the form $D[a^{-1}]$ for some nonzero element $a \in D$. (iv) $D$ is a semi-quasi-local Prüfer domain in which the following condition holds: If $P$ is a proper prime ideal of $D$ and if $\{P_i\}$ is a strictly descending infinite sequence of prime ideals of $D$, then there exists an integer $i$ such that $P_i \subseteq P$. Theorem 2. Let $D$ be a Krull domain, then every overring of $D$ is the transform of some ideal of $D$ if and only if $D$ is a semi-quasi-local PID. (Received October 23, 1968.)


A function $f(z)$ analytic in $|z| < 1$ is of class $H^p (0 < p < \infty)$ if $M_p(r, f) = \{1/2 \pi \int_0^2 |f(re^{i\theta})|^p d\theta\}^{1/p}$ remains bounded as $r \to 1$. Hardy and Littlewood showed that if $p < 1$, $H^p$ is contained in the space $L^p$ of analytic functions such that $\int_0^1 (1 - r)^{p-1} M_p(r, f) dr < \infty$. A complex sequence $\{\lambda_n\}$ is called a multiplier of $H^p$ into the sequence space $l^q$ if $\langle \lambda_n, \alpha_n \rangle \in l^q$ whenever $\sum \alpha_n z^n \in H^p$. Theorem. $\{\lambda_n\}$ is a multiplier of $H^p (0 < p \leq 1)$ into $l^\infty$ if and only if $\lambda_n = O(n^{1-1/p})$. Theorem 2. $\{\lambda_n\}$ is a multiplier of $H^p (0 < p < 1)$ into $l^q$ if and only if $\sum \alpha_n q^n |\lambda_n|^q = O(N^q)$. Theorem 3. Suppose $(\nu + 1)^{-1} \leq p < \nu^{-1}, \nu = 1, 2, \ldots$. Then $\{\lambda_n\}$ is a multiplier of $H^p$ into $H^q (1 \leq q \leq \infty)$ if and only if $g(z) = \sum \lambda_n z^n$ has the property $M_q(1-x)^{1/p-1} = O(1-x)^{1/p-1}$. Theorem 4. $\{\lambda_n\}$ is a multiplier of $H^p$ into $L^q (0 < p < 1, 0 < q < 1)$ if and only if $M_1(1-x)^{1/p-1/2} = O(1-x)^{1/p-1/2}$. In all cases except $p \leq q < 1$ in Theorem 2, the given condition also characterizes the multipliers of $L^p$ into the space in question. (Received October 23, 1968.)

663-356. W. W. BLEDSOE and C. E. WILKS, University of Texas, Austin, Texas 78712. Regularity of a product from a regular conditional measures.

A finite Borel (outer) measure on a topological space is inner regular (i.r.) if open sets are approximable from within by closed sets, and is almost-Lindelöf (a.L.) if every open cover of the space can be reduced to a countable family which almost covers the space. It is not known whether the classical product measure $\phi = \mu \times \nu$ is i.r. when $\mu$ and $\nu$ are. It has been shown (e.g. Trans. Amer. Math. Soc. 79 (1955), 173-215) that a product measure $\theta$ can be defined which inherits from the component measures the properties of being Borel, i.r. and a.L. and which agrees with $\phi$ on measurable rectangles. Elliott generalized this result (Trans. Amer. Math. Soc. 123 (1967), 379-388) by defining a measure $\psi$ as a product of $\mu$ and a regular conditional measure $\nu_x$ and showed that if $\nu_x$ is continuous in $x$, the product again inherits the two properties. It is not known whether the result holds when $\nu_x$ is only $\mu$-summable. The authors replace the continuity condition of Elliott by introducing another measure $\nu'$ for which $\nu_x << \nu'$ for every $x$ and for which the Radon-Nikodym derivative $d\nu_x/d\nu'$ is $\phi$-measurable. They then obtain a "separation of variables" representation for $\nu_x$ in terms of $\mu$-measurable functions and i.r., a.L. measures. Further they show a representation for $\psi$ as sums of measures and show that $\psi$ inherits the properties of being Borel, a.L. and i.r. (Received October 24, 1968.)

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663-357. E. L. ALLGOWER, Colorado State University, Fort Collins, Colorado 80521.  
Criteria for positive definiteness of certain matrices.

Let \( n, j \) be positive integers, \( I_n \) the \( n \times n \) identity matrix and \( H_n = (h_{ik}) \) the \( n \times n \) matrix in which \( h_{i+1,k} = 1 \) and \( h_{ik} = 0 \) if \( k \neq i + 1 \). For fixed \( j, 2 \leq j \leq n \), and complex numbers \( \{a_p\}_{p=1}^{j} \), define \( A_n(j) = a_1 I_n + \sum_{p=1}^{j-1} a_p H_n^p + \bar{a}_j H_n^j \). Theorem. \( A_n(j) \) is positive definite (p.d.) for all \( n \) if and only if the system \( \sum_{k=0}^{j} z_k z_{i+k} = \alpha_{k+1} \) for \( k = 0, \ldots, j - 1 \) has a solution \( (z_1, \ldots, z_j) \neq (0, \ldots, 0) \).

From this theorem we may establish simple determinative criteria that a sequence of Hermitian or real symmetric matrices \( A_n(j) \) be p.d. for all \( n \). (Received October 24, 1968.)

663-358. C. A. WOOD, Oklahoma State University, Stillwater, Oklahoma 74074. Rings for which every proper homomorphic image is a multiplication ring.

Let \( R \) be a commutative ring with identity. \( R \) is called a multiplication ring if whenever \( A \) and \( B \) are ideals of \( R \) with \( A \) contained in \( B \), then there is an ideal \( C \) of \( R \) such that \( A = BC \). \( R \) satisfies (*) if each ideal of \( R \) with prime radical is primary, and \( R \) satisfies (**) if each ideal of \( R \) with prime radical is a prime power. If every proper homomorphic image of \( R \) is a multiplication ring, we say that \( R \) satisfies \( (H_m) \). If every proper homomorphic image of \( R \) satisfies (*) (satisfies (**)), we say that \( R \) satisfies \( (H^*) \) (satisfies \( (H^{**}) \).

Theorem 1. \( R \) satisfies \( (H^*) \) and each nonmaximal prime ideal is idempotent if and only if \( R \) satisfies (*). Theorem 2. If \( R \) is not a primary ring, \( R \) satisfies \( (H^{**}) \) if and only if \( R \) satisfies (**). Theorem 3. If \( R \) is not a primary ring, \( R \) satisfies \( (H_m) \) if and only if \( R \) is a multiplication ring. Theorem 4. If \( R \) is a primary ring with maximal ideal \( M \), \( R \) satisfies \( (H_m) \) if and only if \( R \) is either a special primary ring or \( R \) satisfies the following three conditions: (a) \( M^2 = (0) \); (b) \( M \) is generated by any two elements in \( M \) that do not compare, i.e., if \( x, y \in M \) such that \( (x) \nsubseteq (y) \) and \( (y) \nsubseteq (x) \), then \( M = (x, y) \); and (c) there exist ideals properly between \( M \) and \( M^2 = (0) \). An example is then given to show that a ring satisfying \( (H_m) \) need not be a multiplication ring. (Received October 24, 1968.)


Let \( R \) be a commutative integral domain and \( u \in \bigwedge PR_n \); \( u \) will be called a Plücker vector if it is decomposable over the quotient field of \( R \). \( R \) is said to have property \( T^n_p \) if all Plücker vectors in \( \bigwedge PR_n \) are actually decomposable over \( R \), and \( R \) is a Towber ring if it has property \( T^n_p \) for all \( n \) and \( p \). In the following theorems all rings are noetherian and all modules are finitely generated. \( (D(R) \) denotes the global dimension of \( R \).) Theorem 1. If \( D(R) \leq 2 \) and every projective \( R \)-module is the direct sum of a free module and an ideal, then \( R \) is a Towber ring. Theorem 2. If \( R \) is a Towber ring, then \( D(R) \leq 2 \) and every projective \( R \)-module of rank \( \neq 2 \) is the direct sum of a free module and an ideal. Theorem 3. \( R \) is a Towber ring if and only if \( R \) has property \( T^2_4 \). Theorem 4. A unique factorization domain \( R \) is a Towber ring if and only if \( D(R) \leq 2 \) and projective \( R \)-modules are free. Theorem 5. A projective \( R \)-module of rank \( k \) over a Towber ring can be generated by \( k + 1 \) elements. (Received October 24, 1968.)
This brief survey of the role of geometry in group theory stresses some of the more recent applications. The interplay between geometry and group theory deals with the groups of transformations leaving certain metrics or certain geometric structures invariant. It has been shown by Tits that the Suzuki groups $Sz(2^{2n+1})$ may be considered to be the transformations leaving an ovoid fixed in projective three space over a field $GF(q)$, $q = 2^{2n+1}$. The ovoid is a surface of $q^2 + 1$ points, which, like a nonruled quadric surface, meets any line in at most two points. It has also been shown by Luneburg that these groups can be considered as collineation groups in appropriate non-Desarguesian planes. Very recent development is the discovery by John Conway and John Thompson of several new finite simple groups associated with a sphere packing in 24 dimensions discovered by John Leech. (Received October 24, 1968.)

Let $S = \{x, y, z, \ldots \}$ be a set and let $R \subseteq S \times S$. $(S, R)$ is a partial semigroup if there is a map $(x, y) \rightarrow xy$ from $R$ into $S$ with the property that for any $x, y, z \in S$ such that $(x, y), (y, z) \in R$, then $(x, yz) \in R$ iff $(xy, z) \in R$ and in this case $(xy)z = x(yz)$. $(S, R)$ is a partial involution semigroup if there is a partial order relation $\leq$ on $S$ and a map $*$ from $S$ into $S$ such that $x \leq x^{**}$ and $x^{*y^*} \leq (yx)^*$ if these exist. A zero $0$ in $S$ satisfies (i) $x0 = 0$ for all $x \in S$; (ii) if $0 \leq x$, then $x = 0$; (iii) if $x \leq 0$, then $x^* = 0$; (iv) if $x \leq 0$, then $xy \leq 0$, $yx \leq 0$ for all $y \in S$, $e \in S$ is a projection if $e = ee = e^*$. A partial involution semigroup $S$ is a partial Baer*-semigroup if $0 \in S$ and if for any $x \in S$ there is a projection $x' \in S$ such that (1) if $(y, x') \in R$ and $yx' = y$, then $(y, x) \in R$ and $yx \leq 0$; (2) if $(y, x) \in R$ and $yx \leq 0$, then $(x', y^*) \in R$ and $x'y^* = y^*$. A projection is closed if $e = e''$. Let $P_c(S)$ be the set of closed projections in $S$. Theorem 1. If $S$ is a partial Baer*-semigroup, then $P_c(S)$ is an orthocomplemented poset. Theorem 2. If $L$ is an orthomodular poset, then there is a partial Baer*-semigroup $S$ such that $L$ is isomorphic to $P_c(S)$. (Received October 24, 1968.)

Let $f$ be Riemann integrable in $[a, b]$ and let $P$ be a partition of this interval. Let $S_n(f) = \sum (x_{k+1} - x_k)h(x_k)$ and $s_n(f) = \sum (x_{k+1} - x_k)h(x_{k+1})$. $S_n(f)$ is the sequence of upper Riemann sums and $s_n(f)$ is the sequence of lower Riemann sums for the function $f$. The function-to-sequence transformation $f \rightarrow S_n(f)$ is called the upper Riemann transform and the function-to-sequence transformation $f \rightarrow s_n(f)$ is called the lower Riemann transform. The present study determines certain properties of the function that are preserved under these transformations for given partitions. (Received October 24, 1968.)
Semitlattices on retracts of a two-cell.


Lemma. If p and q are each either cut points or end points of a retract X of a two-cell, then \( C(p, q) \) (the cyclic chain from p to q in X) admits the structure of a topological semilattice with zero p and identity q.

Theorem. Any retract X of a two-cell admits the structure of a topological semilattice with zero and identity. The proof uses a generalization of Eberhardt's technique of dendritic extension [Some classes of continua related to clan structures, Dissertation, Louisiana State University, 1966] to extend the structure of the lemma to a dense collection of cyclic chains in X. (Received October 24, 1968.)

Duality geometry of quadratic programs.

The maximization (minimization) of a concave (convex) quadratic function of nonnegative variables constrained by (consistent) linear equations can, by Gauss-Jordan elimination and affine transformation, be put in the canonical form: (I) maximize \( f = cx - x^TCx/2 \) for \( Ax \leq b \), \( x \geq 0 \). The Lagrangian dual (Dennis, Dorn) then becomes (II) minimize \( \phi = \eta b + \lambda^T/2 \) for \( \eta A + \lambda C \leq c \), \( \eta \geq 0 \). If \( \eta, b, \lambda \) are vacuous, (II) is a (Tucker) program for the least distance from the origin to the set \( \{ \lambda | \lambda C \leq c \} \). Let \( y = b - Ax \) and \( \xi = \eta A + \lambda C - c \). Then \( \phi - f = \xi x + \eta y + (\lambda - x^TCx) + \lambda^T/Cx/2 \).

Hence \( \phi \geq f \) if (1) \( x \geq 0 \), \( \eta \geq 0 \), \( \xi \geq 0 \), \( y \geq 0 \), and \( \phi = f \) if (2) \( \xi x + \eta y = 0 \), \( \lambda^T = Cx \).

(1) and (2) are necessary and sufficient (Kuhn-Tucker) conditions that (I) and/or (II) have optimal solutions. Eliminating \( \lambda \), (1) and (2) become conditions for an equilibrium point of a system \((\xi, y^T) = (x^T, \eta) M + q \) (Cottle-Dantzig, Lemke). To each complementary basic solution of this system there corresponds a pair of (Cottle) dual programs equivalent to (I) and (II), obtained by principal pivoting in M, and also a "foot of the perpendicular" trial solution of a least-distance subprogram of (II), obtained by setting some \( \eta^* = 0 \) after pivoting in A to replace \( \eta \) by \( \eta^* \). (Received October 24, 1968.)

Factoring connected sums of tori embedded in codimension one.

We consider pairs \((S^{n+1}, M^n)\) where \(S^{n+1}\) is the \((n + 1)\) sphere with some orientation and \(M^n\) is an oriented, connected, locally unknotted, p.l. submanifold of \(S^{n+1}\). Two such pairs are equivalent, written \((S^{n+1}, M_1^n) \sim (S^{n+1}, M_2^n)\), if there is an orientation preserving homeomorphism \( h: (S^{n+1}, M_1^n) \sim (S^{n+1}, M_2^n)\). The connected sum of such pairs is defined in the obvious way.

Theorem. Let \(M^n = \bigotimes_{i=1}^r (S^{p(i)} \times S^{q(i)})\) where \(2 \leq p(i) \neq q(i)\) and \(p(i) + q(i) = n \leq 5\). Then for any pair \((S^{n+1}, M^n)\) there are pairs \((S^{n+1}, S^{p(i)} \times S^{q(i)})\), \(i = 1, \ldots, r\), such that \((S^{n+1}, M^n) \sim \bigotimes_{i=1}^r (S^{n+1}, S^{p(i)} \times S^{q(i)})\). We also give examples of pairs \((S^{n+1}, M^n)\) with \(M^n = M_1^n \# M_2^n\) which are not equivalent to \((S^{n+1}, M_1^n) \# (S^{n+1}, M_2^n)\) for any choice of orientations. (Received October 24, 1968.)
The partial differential equations governing the motion of plates, when transverse shear and rotary inertia are considered, are solved for plates bounded by elliptical and hyperbolic cylinders. By assuming product solutions for the partial differential equations (in elliptical coordinates), Mathieu's equations and Mathieu's modified equations are obtained. The resulting solutions satisfy all of eight different boundary conditions entering into the theory. The elimination of the arbitrary constants in each boundary problem leads to the frequency equation for the normal modes of vibration. Recent calculations using electronic high speed computing machines have revealed close correlation between theory and experiment as well as relationships between the classical theory and the new theory advanced here. (Received October 24, 1968.)

D. Kendall has recently called attention to the following observation of T. Kaluza: If a positive bounded sequence \((U_n), U_0 = 1\), satisfies (*) \(U_n^2 \leq U_{n+1} U_{n-1}, n = 1, 2, \ldots\), then \([U_n]\) is a renewal sequence (it is even an infinitely divisible renewal sequence), i.e., the reciprocal sequence \([f_{n+1}]\) defined by \(\sum_{n=0}^{\infty} U_n x^n = (1 - \sum_{n=1}^{\infty} f_n x^n)^{-1}\) satisfies \(f_n \geq 0, \sum_{n=1}^{\infty} f_n < 1\). Since the inequality (*) is satisfied by every Stieltjes moment sequence, which is bounded if it is a Hausdorff moment sequence, one is led to consider the above formal sequence-to-sequence transformation \([U_n] \rightarrow [f_{n+1}]\) and to ask whether the moment structure of a sequence is preserved under these transformations. Using the Loewner theory of \(\mathcal{H}_\infty\) analytic self-mappings of the upper half-plane it is shown that (a) \([U_n]\) is a Hamburger (Stieltjes) moment sequence iff \([f_{n+1}]\) is a Hamburger (Stieltjes) moment sequence; (b) if \([U_n]\) is a Hausdorff moment sequence, then so is \([f_{n+1}]\); (c) if \([f_{n+1}]\) is a Hausdorff moment sequence such that \(f_1 < 1\) and \(\lim \sup (f_n)^{1/n} = 1 - f_1\), then \([U_n]\) is also a Hausdorff moment sequence. This last condition is sharp. Thus, it can happen that \([U_n]\) and \([f_{n+1}/f_1]\) are (infinitely divisible) renewal sequences simultaneously. (Received October 24, 1968.)

In this paper we consider linear transformations which preserve chain sequences. Sufficient conditions are developed and proved. One of the main results is the Theorem. Let \(T\) be an infinite matrix with entries \(a_{mn}\) such that for every positive integer \(m\), (i) \(a_{mn} = 0\) for \(n < m\) and \(a_{mn} \geq 0\) for \(n \geq m\), (ii) \(\sum_{n=1}^{\infty} a_{mn} \leq 1/2\) and (iii) either \(a_{mn} \leq a_{m+1,n+1}\) for \(n \geq m\) or \(a_{mn} \leq a_{m+1,n+1}\) for \(n < m\). Then \(Tx\) is a chain sequence for every chain sequence \(x\). The following two theorems are also proved and used. Theorem. If \(x = (x_n)\) is a sequence of nonnegative numbers such that for each positive integer \(n\), \(\sqrt{x_n} + \sqrt{x_{n+1}} \leq 1\), then \(x\) is a chain sequence. Theorem. If for each positive integer \(n\), \(x^n = \sum_{n=1}^{\infty} a_n x^n\) is a chain sequence (convergent chain sequence), and if \((a_n)\) is a sequence of nonnegative numbers with \(\sum a_n \leq 1/2\), then \([\sum_{n=1}^{\infty} a_n x^n]\) is a chain sequence (convergent chain sequence). (Received October 24, 1968.)
Let \([A,\Omega]\) onto \([B,\Omega]\) be two similar algebras and \(\theta\) a morphism of \([A,\Omega]\) into \([B,\Omega]\) with kernel \(\ker \theta\). If \(R_2\) is a congruence relation in \([B,\Omega]\), then the binary relation \(R_1\) defined by \(a \equiv a' \mod R_1\) iff \(\theta(a) \equiv \theta(a') \mod R_2\) with \(a, a' \in A\) is a congruence relation in \([A,\Omega]\) satisfying \(R_1 \supseteq \ker \theta\).

**Theorem 1.** Let \(\varphi\) be a mapping \(R_1 \rightarrow R_2\) of the set of all congruence relations \(\{R_1\}\) in \([A,\Omega]\) which contain \(\ker \theta\) onto the set of all congruence relations \(\{R_2\}\) in \([B,\Omega]\). Then the relation \(a_1 \equiv a'_1 \mod R_2\varphi^{-1}\) iff \(\theta(a_1) \equiv \theta(a'_1) \mod R_2\) \((a_1, a'_1 \in A)\) is a congruence relation in \([A,\Omega]\) with \(R_2\varphi^{-1} \supseteq \ker \theta\) and \(R_1 = (R_1 \varphi)\varphi^{-1}\).

**Theorem 2.** If \(R_1\) and \(R'_1\) are two congruence relations in \([A,\Omega]\) satisfying \(R_1 \supseteq \ker \theta\) and \(R'_1 \supseteq \ker \theta\), then \(R_1 \subseteq R'_1\) iff \(R_1 \subseteq R'_1\varphi\). (Received October 25, 1968.)

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A graph \(G\) is said to be Hamiltonian connected if every pair of distinct vertices of \(G\) is connected by a Hamiltonian path. The following sufficient condition for Hamiltonian connectedness is proved.

**Theorem.** If \(G\) is a graph with \(p \equiv 4\) vertices such that for every \(j, 2 \equiv j \equiv p/2\), the number of vertices of degree not exceeding \(j\) is less than \(j - 1\), then \(G\) is Hamiltonian connected. A graph \(G\) with \(p \equiv 4\) vertices is said to be \(n\)-Hamiltonian connected, \(0 \equiv n \equiv p - 4\), if the removal of any \(k\) vertices, \(0 \equiv k \equiv n\), results in a Hamiltonian connected graph. A number of sufficient conditions and a few necessary conditions for a graph \(G\) to be \(n\)-Hamiltonian connected are given. (Received October 25, 1968.)

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Let \(f(z) = z + \sum_{n=2}^{\infty} a_n z^n\) be analytic for \(|z| < 1\). We say that \(f(z)\) belongs to the class \(\mathcal{S}_a\), \(0 < a \leq 1\), if \(|zf'(z)/f'(z) - 1| < a\) for \(|z| < 1\). **Theorem I.** If \(f(z) \in \mathcal{S}_a\), then \(\sum_{n=2}^{\infty} |a_n| \equiv \pi(1 + a)e^a - 1\).

And so \(f(z)\) admits a continuous extension to \(|z| \equiv 1\), and the restriction of this extension to \(|z| = 1\) is an absolutely continuous function. **Theorem II.** Let \(E\) be the unit disc, and let \(f(z) \in \mathcal{S}_a\). Then the boundary of \(f(E)\) is a Jordan curve. The radius of convexity for the class \(\mathcal{S}_a\) is given. (Received October 25, 1968.)

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In a recent paper, J. J. Kohn and L. Nirenberg show that some of the fundamental properties of strongly elliptic operators also hold for a class of degenerate elliptic operators. A simple example which is typical of some of those they consider is the second order operator in the plane having principal part \(y^2 \partial^2 / \partial x^2 + \partial^2 / \partial y^2\) which is elliptic except on the real axis. Using the Kohn-Nirenberg estimates, one can obtain the classical Sturm-Liouville results of strongly elliptic theory. More specifically, the Morse index theorem is valid for an appropriate class of such degenerate problems. (Received October 25, 1968.)
663-373. K. K. OBERAI, Queen's University, Kingston, Ontario, Canada. Nilpotency and the rate of growth condition.

Let $T$ be a spectral operator on a space $E$ and let $N$ be its radical part. Let the resolvent of $T$ satisfy $m$th order rate of growth condition. It is known that if $E$ is a Hilbert space then $N^m = 0$ and that if $E$ is a Banach space then, in general, $N^{m+2} = 0$ and if $E$ is reflexive then $N^{m+1} = 0$. Also, if $E$ is an $L^p (1 < p < \infty)$ space then $N^m = 0$. In this paper we show that if $E$ is the space $\Omega^p (1 < p < \infty)$ (see author's paper Sum and product of commuting spectral operators, Pacific J. Math. 25 (1968), 129-146) then also $N^m = 0$. (Received October 25, 1968.)


In this paper we investigate a new series of line involutions in a projective space of three dimensions, $S_3$, over the field of complex numbers. These are defined by a simple involutorial transformation of the points in which a general line meets a nonsingular quadratic surface $Q$. The order, the invariant complex, the singular complex, and the loci of exceptional lines are determined for each involution in the series. Various special cases are considered, and it is shown that among them is the line involution defined by the well-known skew homology. (Received October 25, 1968.)


Let $M = \{x_\alpha : \alpha \in \Gamma \}$ be a set with $\mathcal{J} = \{\alpha_\alpha : \alpha \in \Lambda \}$ as its topology, and suppose $\Gamma \cap \Lambda = \emptyset$. Let $M_\alpha$ be the connected dyad for $\alpha \in \Lambda$, and let $M_\alpha$ be the trivial dyad for $\alpha \in \Gamma$, then $M$ is embeddable in $I(M) = \bigcup_{\alpha} M_\alpha$ by the map $e_M$ defined by $(e_M(x))_\alpha = 0$ if and only if $\alpha \in \Lambda$ and $x \in \alpha_\alpha$, or $\alpha \in \Gamma$ and $x = x_\alpha$. For any base $S$ of $M$ closed under finite unions and intersections a compact subspace $C_S(M)$ of $I(M)$ containing $e_M[M]$ is chosen. When $M$ is $T_0$, the $C_S(M)$ are precisely the compactifications of $L$. Rudolf [On compactifications of $T_0$ spaces, Colloq. Math. 17(1967), 41-50]. Various conditions for $e_M$ to be extended homeomorphically from a compactification $N$ containing $M$ onto some $C_S(M)$ are investigated. The following theorem is proved. If $N$ is compact and regular and $S$ is a base for $N$ closed under finite unions and intersections and $M$ is dense in $N$, then $e_M^S : e_M[M] \to N$ can be continuously extended to $C_S(M)$. (Received October 25, 1968.)

663-376. J. W. NEUBERGER, Emory University, Atlanta, Georgia 30322. Existence of a spectrum for nonlinear transformations.

Suppose that $S$ is a (nondegenerate) complex Banach space and $T$ is a transformation from a subset of $S$ to $S$ so that the domain of $T$ contains some open subset of $S$ on which $T$ is Lipschitz. A complex number $\lambda$ is said to be in the resolvent of $T$ if $(\lambda I - T)^{-1}$ exists, has domain $S$, is Fréchet differentiable at every point of $S$ and is locally Lipschitz. A complex number is said to be in the spectrum of $T$ if it is not in the resolvent of $T$. Theorem. T has a spectrum. (Received October 25, 1968.)
A contravariant functor from the category of commutative rings with identity to the category of compact topological groups is defined. To each ring \( R \) is associated the "Galois group" of the "separable closure" of \( R \). Properties of the "separable closure" are derived and an infinite Galois theory is discussed in this context. This generalizes the standard infinite Galois theory for fields and the Galois theory for rings with no indempotents other than 0 and 1 presented by G. J. Janusz in Trans. Amer. Math. Soc. 122 (1966), 461-479. (Received October 25, 1968.)
many rays through the origin. Then \( I(r) \) is bounded if and only if (*) holds. **Theorem.** There exists a \( B \) such that \( I(r) \) is bounded but such that \( I'(r) \) is unbounded, where \( I'(r) \) corresponds to \( B' \), which is the Blaschke product formed with zeros \( \{|z_n|\} \). So \( I(r) \) being bounded really depends on the angular distribution of the zeros as well as on their radial distribution. The principal method is an analysis of the Fourier coefficients of \( \log |B(re^{i\theta})| \). (Received October 25, 1968.)

663-381. S. A. SCHONEFELD, University of California, Irvine, California 92664. Schauder bases in spaces of differentiable functions.

Let \( I \) be the interval \([0,1]\) and \( C'(I \times I) \) be the Banach space of real-valued functions \( f = f(x,y) \), defined on \( I \times I \), which have continuous first partial derivatives. A norm for this space is the following:

\[
\|f\| = \sup |f(x,y)| + \sup |\partial/\partial x f(x,y)| + \sup |\partial/\partial y f(x,y)|.
\]

A sequence \( \{z_n\} \) of elements from the Banach space \( X \) is called a (Schauder) basis if for each \( x \in X \) there exist unique scalars \( a_i \) such that the series \( \sum a_i x_i \) converges in norm to \( x \). Banach [Théorie des opérations linéaires, Warsaw, 1932, p. 238] states that no basis is known for \( C'(I \times I) \). Z. Ciesielski [MR 28 #419] has shown that if \( \{\varphi_n\} \) is the Franklin basis for \( C(I) \), then the functions \( \{f_1 = 1, f_2(x) = \int_0^x \varphi_{n-1}(t) dt; n = 2,3,\ldots\} \) form a basis for both \( C(I) \) and \( C'(I) \) (with the usual norms). With small alterations the calculations of B. R. Gelbaum and J. Gil De Lamadrid [MR 26 #5394] show that, when properly enumerated, the functions \( \{f_m(x)\varphi_n(y)\} \) form a basis for \( C'(I \times I) \). The same functions also form a basis for \( C(I \times I) \). (Received October 25, 1968.)


Let \( \{H_k\}_{k=1}^{\infty} \) be separable Hilbert spaces, \( \{A_k\}_{k=1}^{\infty} \) operators, \( A_k \) selfadjoint on \( H_k \). Let \( H = \bigotimes_{k=1}^{\infty} H_k \) which may be written as a direct sum (uncountable) of separable Hilbert spaces \( H(\chi) \), generated by \( c_0 \)-vectors \( \chi \). \( A_k \) acts in a natural way on each \( H(\chi) \). The problem is the following:

On which \( H(\chi) \), if any, can \( \sum_{k=1}^{\infty} A_k \) be well defined and selfadjoint? Three methods are investigated:

1) If \( \prod_{k=1}^{\infty} e^{it_0 A_k} \) is reduced by \( H(\chi) \), \( \sum_{k=1}^{\infty} A_k \) can be defined as its infinitesimal generator. (2) If \( \sum_{k=1}^{\infty} A_k \) converges strongly on a \( c_0 \)-vector \( f = \prod_{k=1}^{\infty} f_k \in H(\chi) \), then \( f \) can be used to generate a domain on which \( \sum_{k=1}^{\infty} A_k \) converges strongly and is essentially selfadjoint. (3) If \( \sum_{k=1}^{\infty} (A_k f_k f_k^*) < \infty \), then the form \( \langle \sum A_k f_k g \rangle \) makes sense on a dense domain in \( H(\chi) \), and if it is semibounded, its Friedrichs' extension gives a definition of \( \sum_{k=1}^{\infty} A_k \) on \( H(\chi) \). The three methods are related to one another. In particular, if the \( A_k \) are all positive the methods are equivalent. (Received October 25, 1968.)

663-383. ALFRED INSELBERG, IBM Scientific Center, 1930 Century Park West, Los Angeles, California 90067. Superpositions for nonlinear operators.

Let \( T : D \to E \) be an operator, \( T \) has a superposition with respect to an element \( e \in E = \exists \) a binary operation \( * \) on \( D \) such that \( T(x * y) = e \) when \( T(x) = T(y) = e \). Several hierarchies of equivalence classes of operators, each class consisting of operators having a given superposition, exist with the linear operators being one such class. Canonical forms for operators having certain superpositions are provided. The algebraic structure induced by the superposition operation on the domain and range of the operator has in certain cases some unusual properties. Some well-known nonlinear differential operators, arising in the applications, having a superposition are given. (Received October 25, 1968.)
For \( n,b,x \in \mathbb{Z}^+ \), let \( n = \sum_{j=1}^{k} m_j b^j \) \((1 \leq m_j \leq b - 1, r_1 > r_2 > ... > r_k \geq 0)\) be the representation of \( n \) to the base \( b \), so that \( \varphi_b(n) = k \) \((1 \leq k \leq r_1 + 1)\) is the number of nonvanishing \( b \)-adic digits of \( n \).

The function \( \rho_b(x) = \sum_{n \leq x} \varphi_b(n) \) is relevant in combinatorial problems (see Wilf, Bull. Amer. Math. Soc. 74 (1968), 960-964). Bellman and Shapiro (Ann. of Math. 49 (1948), 330-340) have studied \( \rho_2(x) \) and some related functions, showing that \( \rho_2(x) - (\log 4)^{-1} x \log x \) is a natural lower bound. It turns out that it is more natural to study \( \psi_b(x) = \rho_b(x - 1) \).

Clearly, for \( x \leq b + 1 \), \( \psi_b(x) = x - 1 \), for \( x > b + 1 \), \( \psi_b(x) \equiv x \); also, \( \psi_b(2) = 1 \), \( \psi_b(3) = 2 \) for all \( b \geq 2 \). Generally, \( \psi_b(x) = b^{-1}(b - 1)r_1 x + x - b^{r_1} - T_b(x) \), where \( T_b(x) = \sum_{j=2}^{k} b^{r_j}(1 + m_j b^j) \), \( m_j = (1 - b^{-1})(r_1 - r_j) - j + 1 \). For \( k = 1 \), \( \psi_2(x) \log 4 = x \log x \), for \( k = r_1 + 1 \), \( \psi_2(x) \log 4 = (x - 1) \log(x + 1) \), for \( 2 \leq k \leq r_1 \), \( x \log x - (4/3)^{x-1} \log(4/3) \leq \psi_2(x) \log 4 \leq (x + 1) \log(2x/ e \log 2) - 2 \log(1 - x - 1/2) 1 \). As a consequence it follows that, for \( b \geq 3 \) and \( x \geq 4 \), \( \psi_2(x) \lneq \psi_b(x) \) with equality only for \( x = 5,6, b = 3 \); in general, \( \psi_2(b^2 + b) = \psi_{b+1}(b^2 + b) = 2b^2 - 1 \) and, for \( b_1 > b_2 \) and \( x > x_0(b_1,b_2) \), \( \psi_{b_1}(x) < \psi_{b_2}(x) \). The methods are elementary. Conjectures. (i) \( x_0(b_1,b_2) = b_2(b_2 + 1) \); (ii) \( \psi_{b_1}(x) \lneq \psi_{b_2}(x) \) for \( b_1 > b_2 \) and all \( x \). (Received October 25, 1968.)
C. J. Hsu (Tohoku Math. J. 12 (1960), 429-454) defined $\Gamma$-structure on a differentiable manifold $V_n$. By introducing the compatibility condition $JG = \lambda G$, where $G = (g_{ij})$ is a symmetric complex metric of rank $n - 1$, we obtain a singular Riemannian structure, briefly $\Gamma_p$-structure, on $\Gamma$-structure.

We introduce special bases $(e_i) = (e_1; \ldots; e_n)$ adapted to $\Gamma_p$-structure such that $(e_i)$ are orthonormal. The set $E_p(V_n)$ of the special adapted bases admits a natural structure of principal fibre bundle and consequently one is able to define infinitesimal connection, briefly $\Gamma_p$-connection, on $E_p(V_n)$.

Theorem 1. In an $\Gamma_p$-connection $\nabla g_{ij} = 0$. Theorem 2. A complex linear connection can be identified with an $\Gamma_p$-connection iff $\nabla g_{ij} = \nabla F_j = 0$. Theorem 3. $V_n$ has an $\Gamma_p$-structure iff there exists a complex linear connection whose holonomy group is a subgroup of the set of all the transformation matrices of any two special adapted bases. Theorem 4. The first characteristic form $\psi_1$ is zero for any $\Gamma_p$-connection. (Received October 21, 1968.)

J. H. YATES, Oklahoma State University, Stillwater, Oklahoma 74074. On farthest points of convex sets.

Let $E$ be a finite-dimensional, normed linear space and let $S$ be a subset of $E$. If $z \in S$, then $F(z,S) = \{x \in E : ||x - z|| = \sup \{||x - y|| : y \in S\}\}. Theorem. The space $E$ is an inner-product space if and only if for each set $S \subset E$ and each point $z \in S$ such that $F(z,S)$ is not empty, $F(z,S)$ is convex. This theorem is analogous to a theorem by Motzkin (Atti Acad. Naz, Lincei Rend. (6) 21 (1935), 773-779) and can be used to obtain a different proof of Motzkin's theorem. (Received October 21, 1968.)


Let $Z$ be a locally compact space and $m$ a measure on $Z$ with $\text{Supp}(m) = Z$. Let $\mathcal{K}(Z)$ be the vector space of continuous functions on $Z$ having compact support, and for $A \subset Z$ let $\mathcal{K}(Z,A)$ be the vector space of $f \in \mathcal{K}(Z)$ such that $\text{Supp}(f) \subset A$. Let $z \rightarrow z'$ be an involutive homeomorphism of $Z$ and, for each $f \in \mathcal{K}(Z)$, let $\bar{f}$ be the mapping $z \rightarrow f(z')$ of $Z$ into $C$. A bilinear mapping $(f,g) \rightarrow f \ast g$ of $\mathcal{K}(Z) \times \mathcal{K}(Z)$ into $\mathcal{K}(Z)$ is a convolution if (1) for each compact $A \subset Z$ there is a compact $K_A \subset Z$ such that $f \ast g \in \mathcal{K}(Z,K_A)$ whenever $f, g \in \mathcal{K}(Z,A)$; (2) we have $m(|f \ast g|) \leq m(|f|) \cdot m(|g|)$, $m(h) = m(h)$, $m((f \ast g)h) = m(g(f \ast h))$ for $f, g, h \in \mathcal{K}(Z)$; (3) for fixed $g \in \mathcal{K}(Z)$ and compact $K \subset Z$, $[f \ast g]f \in \mathcal{K}(Z,K)$, $m(|f|) \neq 1$ is equicontinuous; (4) for each $k \in \mathcal{K}(Z)$ the mapping $(f \rightarrow (f \ast k) \cdot m$ of $\mathcal{K}(Z)$ to $\mathcal{K}(Z)$ is bounded. Denote by $\mathcal{M}_c(Z)$ the vector space of all measures with compact support. Theorem. There exists a unique bilinear mapping $(\mu, k) \rightarrow \mu \ast k$ of $\mathcal{M}_c(Z) \times \mathcal{K}(Z)$ into $\mathcal{K}(Z)$ which extends the mapping $(f \rightarrow (f \ast k) \cdot m$ of $\mathcal{K}(Z)$ to $\mathcal{K}(Z)$, and such that for each $k \in \mathcal{K}(Z)$ the mapping $\mu \rightarrow (\mu \ast k) \cdot m$ of $\mathcal{M}_c(Z)$ into $\mathcal{K}(Z)$ is vaguely continuous. This theorem may be applied to define a convolution of measures on $\mathcal{M}_c(Z)$. (Received October 15, 1968.)
Theorem 1. A locally connected, nondegenerate continuum is torus-like if and only if it contains at most one simple closed curve. Theorem 2. A polyhedral continuum is quasi-embeddable in a torus if and only if embeddable in a torus. The method used to prove Theorem 2 can be used to show that a 1-dimensional polyhedron can be quasi-embedded in a 2-manifold $M$ if and only if it can be embedded in $M$. (Received October 28, 1968.)

A function in a Hardy class $H^p$ can be uniquely factored into the product of an "outer function", a Blaschke product, and a "singular inner factor". Theorem. If $f' \in H^p$ for some $p > 1$, then the set of constants $c$ for which $f + c$ has a singular inner part is countable. This theorem fails if one only assumes $f \in H^\infty$, as an example shows. (Received October 28, 1968.)

Let $E$ be a linear space and $\{P_a\}$ a total, commuting net of projections on $E$ which satisfy $P_a P_\beta = P_a$ for $a \neq \beta$. Let each $E_a = P_a E$ have a locally convex topology such that $E_a$ is a topological subspace of $E_\beta$ for $a \neq \beta$ and $P_a E_\beta : E_\beta \to E_a$ is continuous for each $a$. We further assume a condition that guarantees that $E$ is algebraically isomorphic to the projective limit of the $E_a$'s with respect to the maps $g_a \beta = P_a$ for $a \neq \beta$. A subspace $X \subset E$ such that $\cup_a E_a \subset X$ is called a generalized coordinate space (g.c.s.). The dual $X'$ of $X$ is all families $(g_a \lambda)$ such that $g_a \lambda$ extends $g_\beta$ for $a \neq \beta$ and $\lim a.g_a (P_a x) = 0$ for each $x \in X$. A topology on $X$ is induced by $X'$ with the duality $\langle x, (f_a \lambda) \rangle = \lim a.f_a (P_a x)$. We note each $P_a$ is continuous on $X$ and $P_a \to$ pointwise. Theorem. If $\{P_a\}$ is equicontinuous then $B' \subset X$ is bounded $= P_a B$ is bounded for each $a$ and $P_a x \to x$ uniformly for $x \in B$. A condition for compactness in terms of the $P_a$'s is obtained. A g.c.s. is called perfect if it is maximal with respect to its dual. Theorem. If $\{P_a\}$ is equicontinuous then $X$ is complete $= each E_a$ is complete and $X$ is perfect. (Received October 28, 1968.)

Let $(G, \cdot)$ be a groupoid. Definition. $F \subset G$ is a filter of $G$ iff $ab$ is equivalent in $F$ and $a$ is contained in $F$. Consider the lattice of all filters of a groupoid $G$, $\mathcal{F}(G)$. $\mathcal{F}(G)$ is an algebraic lattice and every algebraic lattice is isomorphic to $\mathcal{F}(G)$ for some groupoid $G$. It is shown that $\mathcal{F}(G)$ is distributive whenever it is a sublattice of the lattice of all subgroupoids of $G$; but this condition is not necessary. Generalizing an unpublished result of Grätzer and Schmidt, a necessary and sufficient condition for the distributivity of $\mathcal{F}(G)$ is obtained as follows. Let $F_a$ stand for the principal filter generated by $a$. Define $a \subset b$ to mean $F_a \subset F_b$ and $a \sim b$ to mean $a \subset b$ and $b \subset a$. Then $\sim$ is a congruence relation on $G$ and $G/\sim$ is isomorphic to the semilattice of all principal filters of $G$, and is the maximal homomorphic semilattice image of $G$. Theorem. Let $G$ be a groupoid. $\mathcal{F}(G)$ is distributive iff $G^1$
has the property that for $a, b, c$ in $G$ with $a \leq bc$, there exists $b' \leq b$, $c' \leq c$ such that $a \sim bc$. (Received October 28, 1968.)

663-394. R. J. GREECHIE, Kansas State University, Manhattan, Kansas 66502. An orthomodular poset with a full set of states not embeddable in Hilbert space.

By a state on an orthomodular poset $P$ we mean a mapping $\rho : P \to [0,1]$ such that $\rho(a) = 0$, $\rho(1) = 1$, and $x \leq y'$ implies $\rho(x \vee y) = \rho(x) + \rho(y)$. The set $S_P$ of all states on $P$ is said to be full in case $x \leq y$ if and only if $\rho(x) \leq \rho(y)$ for all $a \in S_P$. The set $S_P$ is strongly order determining in case $x \leq y$ if and only if $x \leq y$ for all $a \in S_P$, $\rho(a) = 1$ implies $\rho(a) = 1$. The distinction between these orderings is of interest in the study of the logic of quantum mechanics and, more generally, in the study of empirical logics. Each state of a physical system induces a probability measure $\rho$ (which we call a state). If "fullness" determines the order, then $x \leq y$ is interpreted to mean that, for every state $\rho$, the probability that the event $x$ occurs is less than or equal to the probability that the event $y$ occurs; if the states are strongly order determining then the interpretation is as follows: If $x$ occurs with certainty, then $y$ occurs with certainty. Theorem. There exists an orthomodular poset $P$ such that $S_P$ is full but not strongly order determining. Corollary. Not every orthomodular poset which admits a full set of states may be embedded in the lattice of all closed subspaces of a complex Hilbert space. (Received October 28, 1968.)

663-395. F. I. GROSS, University of Utah, Salt Lake City, Utah 84112. A sufficient condition for the existence of a normal $\pi$-complement.

A group $G$ is called $\pi$-serial for some set of primes $\pi$ if the composition factors of $G$ are either $\pi$- or $\pi'$-groups. Theorem. Let $H$ be a Hall $\pi$-subgroup of the finite group $G$ such that $N_G(H) = H \cap G$ and $H \cap x^{-1}H$ is cyclic for all $x \in G - N_G(H)$. Assume further that (*) either $H$ is nilpotent or $G$ is $\pi$-serial. Then $G$ has a normal $\pi$-complement. This theorem generalizes a result announced by Richards (Abstract 653-206, these Notices 15 (1968), 138). The example $G = A_5$, $H = A_4$ shows that the theorem is false if (*) is omitted from the hypothesis. (Received October 28, 1968.)

663-396. J. C. HIGGINS, Brigham Young University, Provo, Utah 84601. Subsemigroups of cycle semigroups.

Let $(I,\cdot)$ be the semigroup of additive positive integers. It is known that subsemigroups of $(I,\cdot)$ are finitely generated and such that if $(1,\ldots,i)$ is a generating set for a subsemigroup of $(I,\cdot)$ then this subsemigroup contains all integers greater than some fixed integer $k$ (where $k$ depends on $(1,\ldots,i)$). Theorem. Let $S$ be the cyclic semigroup of index $r$ and period $m$. Let $T$ be the subsemigroup of $S$ generated by $(s_1,\ldots,s_n)$. Then $T$ contains the periodic part of $S$ if and only if the greatest common divisor of $(s_1,\ldots,s_n,m)$ is one. Corollary. Let $P$ be the periodic part of $S$. Then $T$ is the union of $P$ and the intersection of $S$ with the subsemigroup of $(I,\cdot)$ generated by $(s_1,\ldots,s_n)$. (Received October 28, 1968.)

663-397. DANIEL PEDOE, University of Minnesota, Minneapolis, Minnesota 55455. The missing seventh circle.

The classical problem of Apollonius "To construct circles tangent to three given circles is...
$C_i (i = 1,2,3)^{''}$ can be shown to have not more than 8 distinct solutions if the number is finite. If $C_0$ is the circle orthogonal to each circle $C_i$, the eight contact circles divide into four pairs, each pair being inverse with respect to $C_0$. An algebraic treatment can be found in Pedoe, Circles, Pergamon, New York, p. 35. Specialization of the given circles can produce $n < 8$ distinct solutions. It is of interest to determine specializations which produce $n = 0,1,2,3,4,5,6$ and 7 distinct solutions. In the numerous papers on the Apollonius problem nobody appears to have remarked that there is no specialization which produces the case $n = 7$. Analyzing this case we look for the coincidence of a pair of contact circles, since any other coincidence decreases $n$ too much. If contact circles $C$ and $C'$ inverse with respect to $C_0$ coincide, $C = C'$ must be orthogonal with respect to $C_0$. We then prove that two of the $C_i$ must touch each other. But in this case $n = 6$ at most. The cases $n = 0,1,2,3,4,5$ can be realized quite simply. Other proofs that $n = 7$ is an impossible case are also possible. (Received October 28, 1968.)


For definition of spectral maximal space and decomposable operator see Foiaş [Archiv. der Math. 14 (1963), 341-349]; for definition of strongly decomposable operator see Apostol [Rev. Roumaine Math. Pures Appl. 13 (1968), 147-150]. The notation of these two papers will be used. Let $E$ be a Banach space and $n \geq 1$ an integer. $T \in \mathscr{L}(E)$ is n-decomposable (resp. strongly n-decomposable) if, for any open cover $(G_i)_{1 \leq i \leq n}$ of the complex plane consisting of $n$ sets, there exists a corresponding family $(D_i)_{1 \leq i \leq n}$ of spectral maximal spaces such that $\text{sp}(T|D_i) \subset G_i$ and $E = D_1 + \ldots + D_n$ [resp. $D = D \cap D_1 + \ldots + D \cap D_n$ for every spectral maximal space $D$]. $T$ is decomposable (resp. strongly decomposable) if $T$ is n-decomposable (resp. strongly n-decomposable) for every positive integer $n$. Theorem. If $T$ is strongly 2-decomposable, then, for any two open subsets $G, H$ of $C$, $X_T(G \cup H) = X_T(G) + X_T(H)$. Corollary. Every strongly 2-decomposable operator is strongly decomposable. (Received October 28, 1968.)

663-399. H. S. SHANK, 1553 Slaterville Road, Ithaca, New York. On the parity of the number of maximal trees of a planar graph.

Let $G$ be a finite connected embedded planar graph having $N$ maximal trees. Let $n(e)$ be the number of times edge $e$ occurs in one period of a path in which consecutive edges are alternately leftmost and rightmost. Theorem. $N$ is odd iff each $n(e) = 2$. In machine terms, this validates an extremely simple machine (that accepts embedded planar graphs as inputs) that identifies the parity of $N$. (Received October 28, 1968.)

663-400. EDWARD SILVERMAN, Purdue University, Lafayette, Indiana 47907. A theorem on convex bodies of the Busemann-Brunn-Minkowski type.

Let $V$ be a vector space with norm $Q$ and unit ball $J$. Let $k$ be a natural number, $k < \text{dim } V$, and let $a_k$ be the volume of the unit $k$-dim Euclidean ball. If $a_1, \ldots, a_k \in V$, let $\bigwedge a = a_1 \wedge \ldots \wedge a_k$. If $\bigwedge a \neq 0$, let $H$ be the $k$-plane containing $a_1, \ldots, a_k$. Let $M_k$ be defined on simple (decomposable) elements of $\bigwedge^k V$ by $M(0) = 0$ and $M_k(\bigwedge a) = a_k \bigwedge a \|\bigwedge a\| (\text{vol } J \cap H)^{-1}$ if $\bigwedge a \neq 0$. Then $M_k$ is well defined.
independently of the particular Euclidean norm imposed upon H. If \( V = R^{k+1} \), then \( M_k \) is defined on all of \( \Lambda^k V \) and the essential feature in the theorem referred to in the title is that \( M_k \) is a norm on \( \Lambda^k V \). In this paper a little more than the following is shown: If \( k < \dim V \), then there exists a norm \( N_k \) on \( \Lambda^k V \) which extends \( M_k \). (Received October 28, 1968.)

663-401. PHILLIP SCHULTZ, University of Montana, Missoula, Montana 59801. Endomorphism rings of periodic abelian groups.

For any abelian group \( G \), define a sequence of groups \( H_i = H_i(G), i = 0,1, ... \), by \( H_0 = G \) and \( H_{i+1} = \text{Hom}(H_i,G) \). \( G \) is called \((n,m)\)-periodic if \( m \) is the least integer such that \( H_m = H_n \) for some \( n < m \). \( G \) is a regular periodic group if a natural isomorphism \( e_i : H_i \rightarrow H_{i+2} \) is surjective for some \( i \).

1. There are periodic groups that are not regular. This answers a question of Grosse [Archiv. der Math. 16 (1965)].
2. Every regular \((0,1)\)- or \((0,2)\)-periodic group has commutative endomorphism ring.
3. Every endomorphism of every \((0,1)\)-periodic group \( G \) is a left multiplication in some ring with identity over \( G \) if and only if every \((0,1)\)- or \((0,2)\)-periodic group is regular.
4. Let \( R \) be a ring with 1, and let \( \mathcal{J}(R) \) be a set of subgroups of \( R \) for which \( \text{Hom}(G,H) \subseteq \mathcal{J}(R) \) for all \( G, H \in \mathcal{J}(R) \). Then every \( G \in \mathcal{J}(R) \) is a regular periodic group.

Examples. (a) \( R \) is the rationals, \( \mathcal{J}(R) \) the subgroups which contain 1. (b) \( R \) is the \( p \)-adic integers, \( \mathcal{J}(R) \) the \( p \)-pure subgroups. (c) \( R \) is the direct product \( \prod_{p \in S} C(p) \) for some set of primes \( S \), and \( \mathcal{J}(R) \) the \( S \)-pure subgroups of \( R \) which contain the torsion subgroup of \( R \). (Received October 28, 1968.)

663-402. D. A. SMITH, Duke University, Durham, North Carolina 27706. Incidence functions as generalized arithmetic functions. III.

We consider the various arithmetic function algebras as subalgebras of incidence algebras of suitable partially ordered sets. (See Duke Math. J. 34 (1967), 617-634, and Abstract 648-80, these Notices 14 (1967), 83.) Let \( (S, \preceq) \) be a lower semilattice in which each interval is the direct product of a finite number of finite chains. Let \( A \) be the incidence algebra of \( S \) over an arbitrary field \( K \) and \( \mu \) the Möbius function in \( A \). We establish a number of identities relating elements of \( A \) which have as special cases the classical arithmetic function identities of Brauer-Rademacher, Hölzer, and Landau, as well as various generalizations of these given by E. Cohen, Anderson-Apostol, and Subbarao, as well as unitary and Lucas analogues of all of these. A sample theorem. Let \( f \in A \) be factorable and have the property that whenever \( [x,y] \) is the chain \( x = x_0 < x_1 < ... < x_n = y \), \( f(x,y) = f(x,x_1) \). Let \( h \in A \) be factorable and such that \( h(x,y) = f(x,y) - 1 \) whenever \( y \) covers \( x \) (e.g. \( h = f - \mu \) has this property). Let \( \tau \in S \) be fixed. Then \( \sum f(x,z)\mu(z,y) \), where the sum is over \( z \in [x,y] \) such that \( \tau \land z = x \), is equal to \( \mu(x,y)\mu(y \land \tau,y)(y \land \tau,y) \). Very special case (cf. Subbarao, Amer. Math. Monthly 72 (1965), 135). For ordinary Möbius and Euler functions \( \mu \) and \( \varphi \) we have \( \sum_{d|\tau, (d,n)=1} \mu(r/d)\varphi(d) = \mu(r)\mu(r/(r,n))/\varphi(r/(r,n)). \) (Received October 28, 1968.)


The Kreisel-Shoenfield basis theorem (cf. J. Shoenfield, Degrees of models, J. Symbolic Logic 25 (1960), 233-237) states that the functions of degree \( \prec \) form a basis for the \( \Pi_1^0 \) predicates which have recursively bounded solutions. We generalize this by proving that the functions whose jump has

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degree \( \delta' \) form a basis for the above predicates. **Theorem.** Let \( R \) be a recursive predicate and let \( \beta \) be a recursive function. Let \( \sigma \) be the set of functions \( a \) such that \((x)R(\sigma(x))\) and \((x)[a(x) \equiv \beta(x)].\) If \( \sigma \) is nonempty, it contains a function \( a \) such that \( a' \) has degree \( \delta'. \)**

**Corollary.** Let \( T_1 \) be a consistent axiomatizable theory. Then there is a complete extension \( T_2 \) of \( T_1 \) which has the same symbols as \( T_1 \) and whose degree has jump \( \delta'. \) The corollary follows immediately by the argument of Schoenfield (op. cit. Theorem 4, p. 235). We are grateful to A. H. Lachlan for suggesting this problem and to C. G. Jockusch, Jr., for several corrections and simplifications. (Received October 28, 1968.)


**E \subseteq A^A** is m-algebraic, \( a_m(E) \), if \( E = \text{end } \mathbb{A} \) for an algebraic with universe \( A \) and operations of rank \( < m. \)**

To find when \( E \) is m-algebraic is nontrivial only for \(|A| > 2 \) and \( m \equiv 2. \) W. Lampe (Abstract 68T-417, these *Notices* 15 (1968), 625) characterizes algebraic (i.e. \( m = \omega \)) semigroups \( G \cup K \) for \( G \) a group of permutations and \( K \) a set of constant maps. **Theorem 1** characterizes m-algebraic semigroups \( G \cup K \) for \(|A| > 2 \) and \( 2 \equiv m \equiv \mathbb{N}_0. \) Let \( C_{G \cup K} = \{\varphi_a : a \in A, \varphi_a \in A^A, \forall x \in A, \varphi_a(x) = a, \forall b \neq \exists \sigma, \tau \in G \cup K, o = \tau a \text{ and } o b \neq \tau b\}. \) For \( X \subseteq A \) let \( D_{G \cup K}(X) = \{y \in A : \forall \sigma, \tau \in G \cup K, \exists X = \tau \uparrow X - \sigma y = \tau y\}. \)**

Write \( \gamma_m(G \cup K) \) iff \( C_{G \cup K} \subseteq K \) and \( \forall \varphi \in A^A, \forall X \subseteq A, |X| < m, \exists g \in G \cup K, \exists D_{G \cup K}(X) = \varphi \uparrow D_{G \cup K}(X) - \varphi \in G \cup K\). **Theorem 1.** \( a_m(G \cup K) = \gamma_m(G \cup K). \) The proof is similar to PJonka (Colloq. Math. 14 (1968), 5-8). Another generalization of Lampe's result: Let \( E \subseteq A^A \) be locally-solvable iff \( \forall n \in \omega, a, b \in E^A \), \( \tau \in E, [a = \tau b \neq \exists e \in E, \varphi(a) = b]. \)**

**Theorem 2.** If \( E \) is a locally solvable semigroup, and \( E \cup K \) is a semigroup with 1, then \( E \cup K \) algebraic iff (1) \( C_{E \cup K} \subseteq E \cup K \) and (2) \( E \cup K \) locally closed. Examples show this is a stronger theorem. Finally, a general necessary condition follows from Lampe's result: **Theorem 3.** If \( E \subseteq A^A \) is algebraic, then \( G \cup K \) is algebraic, where \( G \cup K \) consists of the constant maps and permutations of \( A \) in \( E. \) (Received October 28, 1968.)


Let \( J \) be an ideal of the commutative regular ring \( R, \) and let \( S = \text{Hom}_R(J, J). \) It is shown that \( S \) is a commutative regular ring, and that \( h, \text{ dim}_R(J) = h, \text{ dim}_S(J). \) Necessary and sufficient conditions on \( J \) are found for \( S \) to be \( R \)-projective, and it is shown that \( R \) is self-injective if and only if \( \text{Hom}_R(J, J) \) is \( R \)-projective for each ideal \( J. \) (Received October 28, 1968.)

663-406. R. C. WHERRITT, Wichita State University, Wichita, Kansas 67208. Eliminability of terms which do not refer in first order theories. Preliminary report.

Suppose \( \text{\mathcal{F}} \) is a first-order theory with equality in which the standard logical axioms and rules hold, except that \( \exists E, \Gamma, A(a/x) \vdash E, \exists a, a(x) \vdash E, \exists a, A(a/x) \vdash E, \) and \( \exists a, A(a/x) \vdash E, \) are restricted by: both \( x \) and \( a \) are variables (cf. D. Scott, "Existence and description in formal logic" in Bertrand Russell, Philosopher of the century, edited by R. Schoenman, Boston, 1967, pp. 181-200; and H. Leblanc and R. H. Thomason, Completeness theorems for some presupposition-free logics, Fund. Math. 62 (1968), 203.
125-164). **Theorem 1.** Suppose \(P(x,y)\) is a formula of \(\mathcal{L}(\mathcal{F})\) which satisfies the uniqueness condition: 
\[P(x,a), P(x,\beta) \vdash \alpha = \beta\] for all proper terms \(\alpha, \beta\). Also suppose \(f\) is a unary function letter of \(\mathcal{L}(\mathcal{F})\) such that \((*)\) \[\forall Q(\text{f}(x)) = \forall y(P(x,y) = Q(y))\] for every atomic predicate \(Q\). Then \[\vdash \forall y(y = \text{f}(x) \equiv P(x,y))\], furthermore, \(f\) is eliminable from \(\mathcal{F}\). **Theorem 2.** If \(P\) and \(f\) are as in Theorem 1 and (a) \(f\) does not occur in the proper axioms of \(\mathcal{F}\) other than \((*)\) and (b) \(\forall x \exists y P(x,y)\) is not a theorem of \(\mathcal{F}\), then \(\forall x \exists y(y = \text{f}(x))\) is not provable in \(\mathcal{F}\). There is such a theory \(\mathcal{F}\) with \(P\) and \(f\) as above and there is a model \(M\) of \(\mathcal{F}\) such that \(f(a) \notin M\) for some \(a \in M\). Thus we say \(f\) does not (necessarily) refer. (Received October 28, 1968.)

663-407. W. A. WEBB, Pennsylvania State University, University Park, Pennsylvania 16802. Waring's problem in GF\([q,x]\).

Let \(K\) and \(A_1\) denote polynomials of degrees \(nk\) and \(n\) respectively, over the field of \(q = p^m\) elements. By completing the field \(GF(q,x)\) and defining a Haar integral, we can use an analog of the Hardy-Littlewood method to prove. **Theorem.** If \(p > k\) and \(n\) is sufficiently large, there exists an \(s = s(k)\) such that \(K = a_1A_1^k + \cdots + a_8A_8^k\) always has a solution, where the \(A_i\) are monic and \(a_1 + \cdots + a_8 = \text{signum} K\). Using Weyl type estimates for trigonometric sums, it is found that \(s > k^2k^{-1}\) suffices. Using methods analogous to those of Vinogradov, we find that \(s\) of order \(k \log k\) suffices. However, combining these methods with special properties of certain exponential type sums over \(GF[q,x]\), it can be shown that \(s \geq 2k + 1\) suffices. This is a better result than has ever been obtained for similar problems concerning the rational integers. (Received October 28, 1968.)

663-408. KEITH YALE, University of Montana, Missoula, Montana 59801. Invariant subspaces and projective representations.

Let \(G\) be the Bohr compactification of a subgroup \(\Gamma\) of the real line and let \(\chi_\lambda\) be the character on \(G\) corresponding to \(\lambda\) in \(\Gamma\). The Hardy space \(H^2\) is the set of \(L^2\) functions on \(G\) whose Fourier transforms vanish for \(\lambda < 0\). A closed subspace \(M\) of \(L^2\) is simply invariant in case \(\chi_\lambda M \subseteq M\) iff \(\lambda \neq 0\). We prove that there exists a nontrivial simply invariant subspace of \(L^2\) which is not of the form \(\{\phi f : f \in H^2\}\) where \(\phi\) has modulus one a.e. whenever \(\Gamma\) is dense in the real line. This extends a result of Helson and Lowdenslager [Invariant subspaces, Proc. Internat. Sympos. Linear Spaces, Jerusalem, 1960, pp. 251-262]. Our proof uses the existence of nontrivial local projective representations on the two-dimensional torus [cf. Bargmann, Ann. of Math. 59 (1954), 1-46] to construct a cocycle which is not a coboundary. (Received October 28, 1968.)

663-409. LINO GUTIERREZ-NOVOA, University of Miami, Coral Gables, Florida 33124. Extreme points in convexity structures.

A general convexity structure \((S, (\cdot))\) is a set \(S\) with a closure operator \((\cdot)\) defined on \(2^S\) and which satisfies for every \(A_1 \subseteq S:\)
\[C_1, \langle A \rangle \supseteq A_1, C_2, \langle A \rangle = \langle A \rangle, C_3, \langle A_1 \cup A_2 \rangle = \bigcup a_1 \in \langle A_1 \rangle \{[a_1 a_2] \} \text{ and } C_4. \] If \(x_1 \notin \langle A \rangle, x_2 \notin \langle A \rangle,\) then \(x_1 \in \langle A \cup x_2 \rangle\) and \(x_2 \in \langle A \cup x_1 \rangle\) imply \(x_1 = x_2\). A convexity structure also satisfies: \(C_5, a \in \langle A \rangle\) iff \(a \in \langle K \rangle, K\) finite, \(K \subseteq A\). A flat is a set which contains with every pair of its points, \(a,b\); the line \(ab\), i.e. the segment \(\langle(ab)\rangle\) and its prolongations. \(F(A)\) is the minimal flat which contains \(A\). A geometric convexity structure is one
in which both the uniqueness of a line through two points and the usual separation properties of finite-dimensional subspaces by hyperplanes hold. A closure structure on $S$ is said to be related to $(S, \langle \cdot, \cdot \rangle)$ if for any $A \subseteq S$, $C1 A \subseteq F(A)$. Theorem. If $(S, \langle \cdot, \cdot \rangle)$ is a geometric convexity structure, which has a compact related structure, then $S$ has an extreme point $p$ ($\langle S - p \rangle = S - p$). This theorem is related, and in some cases implies, the well-known Krein-Milman theorem of linear spaces. (Received October 28, 1968.)

663-410. T. K. BOEHME, MELVIN ROSENFIELD, and M. L. WEISS, University of California, Santa Barbara, California 93106. The approximate continuity of $H^\infty$ boundary functions.

Let $H^\infty$ be the Banach algebra of all bounded analytic functions on the open unit disc $D$, let $C$ denote the unit circle. For $f \in H^\infty$, let $F$ be its Fatou boundary function and let $\hat{f}$ be its Gelfand representation. $F$ is [quasi-] approximately continuous at $P \in C$ with value $a$ if the [upper] density of $|F - a| < \varepsilon$ is 1 for every $\varepsilon > 0$. Theorem. $F$ is [quasi-] approximately continuous at $P \in C$ iff $\hat{f} = a$ on the support of [some] each measure which represents a homomorphism of $H^\infty$ which lies on the $w^*$-closure of the radius drawn to $P$. From this result we derive several theorems of Doob in (Trans. Amer. Math. Soc. 35 (1933), 418-451) including Theorem (Doob). If $F$ is appr. cont. [quasi-appr. cont.] at $P$ with value $a$, then $f$ tends to $a$ [has cluster value $a$] along the radius to $P$. If $a$ is a cluster value at $P$ of maximum modulus, then the converse of the last statement also holds. (Received October 28, 1968.)

663-411. ESMOND DE VUN, University of Massachusetts, Amherst, Massachusetts 01002. Special semigroups on the two-cell.

A semigroup $S$ has property (a) if (1) $S$ is topologically a two cell, (2) $S$ has no zero divisors, and (3) the boundary of $S$ is the union of two unit intervals with the usual multiplication. A characterization of semigroups having property (a) will be given. Let $(I, \cdot)$ denote the closed unit interval with the usual multiplication. Let $M$ be a closed ideal of $(I, \cdot) \times (I, \cdot)$ such that $M$ contains $(I \times \{0\}) \cup \{0\} \times I)$, and $M \cap ((I \times \{1\}) = ([0,1])$ or $M \cap ((I \times \{1\}) = \{(0,1)\}$.

For each $a, b \in (0,1)$ define a relation $R(a, b, M)$ on $(I, \cdot)$ by $(x, y)$ $\in$ $R(a, b, M)$ if (1) $x = y$ or (2) $(x, y)$ or (3) there exists an $s \in (0, \infty)$ such that $(x, y)$ are in the same component of $M \cap \{a^t, b^{s-t}0 \leq t \leq 1\}$. Lemma. The relation $R(a, b, M)$ is a closed congruence. Theorem. A semigroup $S$ has property (a) iff there exists $a, b, M$ such that $(I, \cdot) \times (I, \cdot) / R(a, b, M)$ is isomorphic to $S$. (Received October 28, 1968.)

663-412. WITHDTAWN.

663-413. J. W. ROGERS, JR., Emory University, Atlanta, Georgia 30322. On universal tree-like continua.

R. M. Schori has given an example of a universal chainable continuum, i.e., a chainable continuum in which every chainable continuum can be imbedded. He has conjectured that if $T$ is a tree, but not an arc, then there is no universal $T$-like continuum. We show that if $G$ is a finite collection of trees and there is a universal $G$-like continuum, then each element of $G$ is an arc. It then follows from a result of M. C. McCord [Universal $\varnothing$-like compacta, Michigan Math. J. 13 (1966), 72, Theorem 4] that if $G$ is a finite collection of one-dimensional (connected) polyhedra, and there is a universal $G$-like continuum, then each element of $G$ is an arc. (Received October 28, 1968.)
Consider the differential equation \( L(f^n) = P(f, f', \ldots, f^{(k)}) \) where \( L = c_0(z) + \cdots + c_n(z)D^m \) with \( c_i(z) \) entire functions in a differential ring \( R \), \( f \) an entire function, and \( P(x_0, x_1, \ldots, x_k) \) a polynomial in \( F[x_0, \ldots, x_k] \) where \( F \) is a differential field of meromorphic functions related to \( R \) in the manner described below; and \( \deg \ P < n \). We define a growth ring \( \mathfrak{m}_R \) of a ring of entire functions \( R \) to be a semiring (closed under addition and multiplication) of functions \( M(r) \) so that for each \( g \in R \) there exists an \( M(r) \in \mathfrak{m}_R \) such that \( M_g(r) = (\max|g(x)|; \ |x| = r) \leq M(r) \) for \( 0 \leq r < \infty \). We define a growth ring \( \mathfrak{m}_F \) for a field of meromorphic functions \( F \) to be a semiring of functions \( M(r) \) so that for every \( h \in F \) there exist entire functions \( h_1 \) and \( h_2 \) and a function \( M(r) \in \mathfrak{m}_F \) so that \( h = h_1/h_2 \) and \( \max[M_{h_1}(r), M_{h_2}(r)] \leq M(r), 0 \leq r < \infty \). We now assume that the field \( F \) above is related to the ring \( R \) by \( \mathfrak{m}_F = \{ \sum_{i=1}^N c_{i1} \exp(c_{i2}rM_i(r)) \mid M_i(r) \in \mathfrak{m}_R, c_{i1}, c_{i2} > 0 \} \). Our main result is the following: Theorem. If \( f \) satisfies the differential equation above then \( M_f(r) \) has an upper bound in \( \mathfrak{m}_F \). That is to say, the entire solutions of the differential equation cannot have a faster growth rate than the solutions of the homogeneous linear differential equations with coefficients in \( R \). (Received October 28, 1968.)

Let \( X \) be a compact Hausdorff space which contains perfect subsets. Consider a pair \( (A, \varphi) \) where \( A \) is a closed subalgebra of complex \( C(X) \) and \( \varphi \) is a complex homomorphism on \( A \) which cannot be extended to a larger closed subalgebra of \( C(X) \). Recent results of M. Tomlinson guarantee the existence of such pairs. The maximal ideal space of \( A \) is \( \Sigma(A) \), \( \hat{A} \) is the Gelfand transform of \( A \), and \( E \) is the essential set of \( A \). Theorem 1. The Silov boundary of \( A \) is \( X \), and \( E \) is perfect. Moreover, \( E \) is the unique nonsingleton maximal set of antisymmetry for \( A \), and is precisely the closed support of every representing measure for \( \varphi \). Theorem 2. \( \hat{A} \) is a relatively maximal subalgebra of \( C(\Sigma(A)) \). Glicksberg (Pacific J. Math. 14 (1964), 919-941) has defined a separating closed subalgebra \( A \) of \( C(X) \) to be relatively maximal if whenever \( A \) is properly contained in a proper closed subalgebra \( B \) of \( C(X) \), then the Silov boundary of \( A \) is relatively maximal if whenever \( A \) is properly contained in a proper closed subalgebra \( B \) of \( C(X) \), then the Silov boundary of \( A \) is properly contained in the Silov boundary of \( B \). (Received October 24, 1968.)

Let \( (w_1, w_2, w_3) \) be an ordered triple of real numbers such that \( w_1 + w_2 + w_3 = 1 \). Let \( g \) be a real-valued function of bounded variation on the closed interval \([a,b]\) of the real axis, let \( f \) be a bounded real-valued function on \([a,b]\) such that the weighted refinement integral \( \left[ F, (w_1, w_2, w_3) \right] \int_a^b f(x)dg(x) \), and let \( h(x) \) be a bounded real-valued function on \([a,b]\). If either \( w_2 \) or \( w_3 \) is not 0, suppose \( h(x^+) \) exists for all \( x \) in \([a,b]\) such that \( g(x^+) \neq g(x) \); if either \( w_1 \) or \( w_3 \) is not 0, suppose \( h(x^-) \) exists for all \( x \) in \([a,b]\) such that \( g(x^-) \neq g(x) \). Let \( p(x) = \left[ F, (w_1, w_2, w_3) \right] \int_a^b f(u)dg(u) \) for all \( x \) in \([a,b]\) and let \( p(a) = 0 \). The function \( p \) is of bounded variation on \([a,b]\). For each \( x \) in \([a,b]\), \( p(x^+) - p(x^-) = 0 \).
The weighted refinement integral \( \int_{a}^{b} h(x) dp(x) \) exists iff the weighted refinement integral \( \int_{a}^{b} h(x) dg(x) \) exists, and in this case \( \int_{a}^{b} h(x) dp(x) = \int_{a}^{b} h(x) dg(x) - w_1 \sum_{x \in (a,b)} [h(x^+) - h(x)] \cdot \left[ f(x^+) - f(x^-) \right] \cdot [g(x) - g(x^-)] \). The above formula is also established for the case where \( h \) and \( f \) are of bounded variation on \([a,b] \), and \( g \) is bounded. (Received October 29, 1968.)


Using an internal property of Runge-Kutta formulas, the author has established a new method for the approximate solution of ordinary differential equations. At every application, this method provides a polynomial as an approximation for the solution. Another advantage of the method resides in the fact that it possesses also the characteristics of the predictor-corrector process, i.e. it can predict points and correct them. The prediction does not require additional substitutions. Methods of order four, five and six will be specifically described. (Received October 29, 1968.)


Lemma. \((v,t) \in \pi(0,2) = \{([0,1]^2) \mid 4\text{-polytope with } f_i \text{faces}\} \) represents (A) a pyramid, (B) a cylinder, (C) a bipyramid, respectively, iff (A) \( 2v \leq t \leq 5v - 15, t + v = 1 \text{ (mod 2), and } v \leq 5 \), (B) \( (5v/4) + 4 \leq (7v/2) - 14, t + v = 1 \text{ (mod 3), and } v \leq 8 \text{ is even} \), (C) \( (7v/2) - 5 \leq t \leq 8v - 32, t + v = 1 \text{ (mod 3), and } v \leq 6 \), respectively. These sets and the cyclic 4-polytopes, with their iterates under cutting off vertices and pulling vertices from simplicial facets, supply representatives of all \((v,t) \in \pi(0,2) \) provided \( v > 19 \) and \( t > 29 \). It is shown that \((v,v^2 - 3v - 1) \notin \pi(0,2) \), so infinitely many exceptions occur within \( \pi(0,2) \). Using Gale diagrams and the above, representations of all but 11 values of \((v,t) \) within the known bounds of \( \pi(0,2) \) are found. This extends the results reported in Abstract 657-24, these Notices 15 (1968), 620. (Received October 29, 1968.)

663-419. R. E. SHOWALTER, University of Texas, Austin, Texas 78712. Partial differential equations of Sobolev-Galpern type.

A mixed initial and boundary value problem is considered for a partial differential equation of the form \( M u_t(x,t) + L u(x,t) = 0 \) where \( M \) and \( L \) are elliptic differential operators of orders \( 2m \) and \( 2l \), respectively, with \( m \leq l \). The solution sought is a function of the \( n \)-dimensional space variable and the time variable. This problem is generalized and studied in the Hilbert space \( H_0^1 \) of functions having square-integrable derivatives of order \( 1 \), all of whose derivatives through order \( 1 - 1 \) vanish on the boundary in the variational sense. Under very mild conditions on the smoothness of the boundary and coefficients, the existence and uniqueness of the solution to this generalized problem are studied by semigroup methods. (Received October 29, 1968.)
A note on spaces of certain nonalternating mappings onto an interval.

We consider the space of all nonalternating mappings of a compact metric continuum $X$ (in particular, a Peano continuum) onto the interval $[0,1]$. Such a space is topologically complete as well as the subspaces (1) of all open nonalternating mappings $ON(X)$ and (2) of all light open nonalternating mappings $LON(X)$. Easy examples show that $LON(X)$ is not locally connected even when $X$ is a Peano continuum. Conditions are given which imply that $LON(X)$ is locally contractible.

Another Theorem. Suppose that $f$ is a completely regular mapping of a compact metric continuum $X$ onto $[0,1]$. Furthermore, $K$ is a 1-dimensional continuum which is homeomorphic to $f^{-1}(x)$ for each $x$ in $X$ such that $LON(K)$ (the space of all light open nonalternating mappings of $K$ onto $I$) is locally connected. Then there is a light open mapping $g$ of $X$ onto $I \times I$, a 2-cell. (Received October 29, 1968.)

Surfaces with the spherical two-piece property.

A set $A$ in Euclidean space has the spherical two-piece property (STPP) if no sphere $S$ separates $A$ into more than two pieces. Examples include a sphere, a plane, an annulus, and a standard torus of revolution, as well as the flat torus $S^1 \times S^1$ in the 3-sphere in $E^4$. Motivation for introducing the STPP concept comes from the study of total absolute curvature, as developed by Chern and Lashof and by Kuiper—indeed a surface $M^2$ embedded in the 3-sphere in $E^4$ has minimal total absolute curvature in $E^4$ if and only if it is the image under inverse stereographic projection of a surface in $E^3$ with the STPP. We proceed to identify all such surfaces. An important but elementary lemma shows that the circle is the only simple closed curve with the STPP, and this leads to a complete classification of the closed STPP subsets of 2-spheres. A "support sphere" construction then shows that a smooth STPP surface in $E^3$ is either a sphere or a topological torus. A finer analysis shows that an STPP torus must be a Dupin cyclide, i.e. the image of a standard torus of revolution under inversion with respect to a sphere. This classification leads finally to a rigidity theorem: If two STPP surfaces are isometric, then they are congruent. (Received October 29, 1968.)

Quasi-piecewise flatness, differentiability and surface area.

The deviation of a face $T$ of a polyhedron inscribed on a nonparametric surface $S : z = f(E)$ is the LUB of the acute angles between the normal to $T$ and the normals to triangles inscribed on $S$ whose $xy$ projections are subsets of that of $T$. $S$ is a quasi-piecewise flat if for every $\alpha > 0$ and $g > 0$, there exists a polyhedron $\Pi$ inscribed on $S$ such that (1) for each of some of the faces (the regular faces) of $\Pi$ the deviation is less than $\alpha$ and (2) the sum of the areas of the other faces is less than $\beta$. A regular sequence $(\Pi_n)$ of inscribed polyhedra is one for which the corresponding sequences $(\alpha_n)$ and $(\beta_n)$ both converge to 0. The deviation $D(P)$ at $P \in E$ is the GLB of the deviations of the triangles inscribed on $S$ whose projections contain $P$. Chief results. If $S$ is qpf then for regular sequences of inscribed polyhedra, the corresponding sequences of polyhedra areas converge to a unique limit. $S$ is qpf if and only if for each $\epsilon > 0$, the sets of the points $P$ of $S$ for which $D(P) \geq \epsilon$ is of Jordan measure zero. (Received October 29, 1968.)
Let m be a fixed cardinal. If for any open cover $R$ of a topological space $X$ there exists an open point-$m$ refinement $S$ of $R$, then we say (1) $\dim X \leq m - 1$ whenever $m$ is finite, and (2) $\dim X = m$ whenever $m$ is infinite. Examples of spaces of arbitrarily large dimension are given and existence of 0-dimensional spaces containing subspaces of arbitrarily large dimension is shown.

Theorem 1. For infinite cardinals $m$, $\dim X \leq m$ and $m$-compactness imply compactness.

Theorem 2. For infinite dimensional spaces $X$, $\dim(X \times \mathbb{R}^n) = \dim X$.

Theorem 3. $\dim X = 0$ for any $\omega_\mu$-metric spaces $X$ with $\mu > 0$. (Received October 29, 1968.)

The influence of rotary inertia and shear on vibrations of a clamped circular ring.

At high frequencies the normal modes of vibrations are described poorly by Lagrange's equation of motion for the elastic plates given by $D \psi^4 \omega + \rho h \frac{\partial^2 \omega}{\partial t^2} = q(x,y,t)$. The partial differential equation governing the flexural motions of clamped circular ring is given by $\delta^2 \omega_i / \delta r^2 + (1/r) \delta \omega_i / \delta r + (1/r^3) \frac{\partial}{\partial \theta} \frac{\partial^2 \omega_i}{\partial \theta^2} + \delta^2 \omega_i = 0$ for $i = 1, 2, 3$, where $\delta_i$ includes the effects of shear and rotary inertia, with the line integral $\oint_{\gamma} (V_n M + W_n N + W) \xi ds = 0$ along the boundary.

The solution is obtained in terms of Bessel functions of 1st and 2nd kind. The resulting frequency spectrum for a limiting case with rotary inertia and shear deformation correction term is compared with the classical theory. (Received October 29, 1968.)

The ideals of analytic functions.

Let $R$ be a finite open Riemann surface with boundary $X$, $H^\infty$ the algebra of all bounded analytic functions on $R$, and $A$ the algebra of all continuous functions on $X$ that can be extended analytically to $R$. **Theorem 1.** Every function $f$ in $H^\infty$ has an essentially unique factorization $f = WF$, where $W$ is an inner function and $F$ is an outer function. Further, every inner function $W$ has an essentially unique factorization $W = BS$, where $B$ is a Blaschke product for $R$ and $S$ is a singular function. **Theorem 2.** Every nonzero closed ideal of $A$ has an essentially unique representation $I = WI(E)$, where $W$ is an inner function, $E$ is a closed subset of $X$, and $(I(E)$ is the ideal of all functions in $A$ that vanish on $E$. The interactions of these theorems help generalize almost all the results in the well-known case in which $R$ is the open unit disk and $X$ is the unit circle. No use of the universal covering space techniques is made. (Received October 21, 1968.)

The change of base by Grothendieck fibrations.

**Definition.** A functor $G: \mathcal{C} \to \mathcal{E}$ will be called a change of base for adjoint functors provided that given any functor $U: \mathcal{Z} \to \mathcal{C}$ which admits a co-adjoint $F: \mathcal{E} \to \mathcal{Z}$ the functor $\text{pr}_1: \mathcal{G} \times_U \mathcal{Z} \to \mathcal{C}$ admits a co-adjoint $F\#: \mathcal{C} \to \mathcal{G} \times_U \mathcal{Z}$ such that $\text{pr}_2 F\# = F G$. $G$ will be called a universal change of base for adjoint functors provided that given any fiber-product $\mathcal{G} \times_U \mathcal{Z}$ of $G$ with a functor $U$, the functor $\text{pr}_2: \mathcal{G} \times_U \mathcal{Z} \to \mathcal{Z}$ is a change of base for adjoint functors. With this definition, one has
the following proposition which characterizes cofibrations within (CAT). Proposition. \( \mathcal{G} \rightarrow \mathcal{B} \) is a Grothendieck cofibration if and only if \( \mathcal{G} \) is a universal change of base for adjoint functors. Moreover, if \( \mathcal{U} \) is tripleable, then so is \( \text{pr}_1 \) for any such change of base by a cofibration. (Received October 30, 1968.)

663-427. SHU-CHUNG KOO, Marquette University, Milwaukee, Wisconsin 53233. Some properties of functions and cluster sets.

Let \( \mathcal{F} = \{ f : D \rightarrow \Omega \} \) where \( D = \{ z \mid |z| < 1 \} \) and \( \Omega \) is the Riemann sphere. Let \((H, \rho)\) be the space of closed subsets of \( \Omega \) with the Hausdorff metric. For every fixed \( e^{i\theta} \), the global cluster set \( C(t) \) of \( f \in \mathcal{H} \) at \( e^{i\theta} \) is in \( \mathcal{H} \). It can be shown that \( \mathcal{H} = \{ C(t) \mid f \in \mathcal{F} \} \) and, if \( \mathcal{F} \) is given the sup norm metric \( d \), then the mapping \( f \mapsto C(t) \) from \((\mathcal{F}, d)\) to \((H, \rho)\) is continuous. Introduce an equivalence relation \( " \sim " \) on \( \mathcal{F} \) so that \( f \sim g \) if and only if \( C(t) = C(g) \). Let \([f]\) be the equivalence class to which \( f \) belongs. Theorem 1. Let \( \epsilon > 0 \). If \( p(C(t)C(g)) < \epsilon/2 \), then for every \( \phi \in [f] \), there exists \( \psi \in [g] \) such that \( d(\phi, \psi) < \epsilon \). Now define a topology \( T \) for \( \mathcal{F} = \{ [f] \mid f \in \mathcal{F} \} \) by requiring a neighborhood of \([f]\) to be the set \( U(\epsilon([f])) = \{ [g] \mid d(\phi, \psi) < \epsilon \text{ for some } \phi \in [f], \psi \in [g] \} \). Theorem 2. The topology \( T \) for \( \mathcal{F} \) is the quotient topology with respect to \( " \sim " \) and \((\mathcal{F}, d)\). Theorem 3. The mapping \([f] \mapsto C(t)\) from \((\mathcal{F}, T)\) to \((H, \rho)\) is a homeomorphism. (Received October 15, 1968.)

663-428. A. C. MEWBORN, University of North Carolina, Chapel Hill, North Carolina 27514. Some conditions on commutative semiprime rings.

Let \( \mathcal{R} \) be a commutative semiprime ring. The following are equivalent: (1) the complete ring of quotients \( \mathcal{Q} \) of \( \mathcal{R} \) is flat as an \( \mathcal{R} \)-module; (2) the space of minimal prime ideals of \( \mathcal{R} \) is compact; (3) \( M \cap \mathcal{R} \) is a minimal prime ideal of \( \mathcal{R} \) for each \( M \in \text{Spec } \mathcal{Q} \); (4) if \( a \in \mathcal{R} \) and \( U = \{ M \in \text{Spec } \mathcal{Q} : a \notin M \} \), then there is a finitely generated ideal \( I \) of \( \mathcal{R} \) such that \( \text{Spec } \mathcal{Q} \setminus U = \{ M \in \text{Spec } \mathcal{Q} : I \notin M \} \). The implication \( (2) \Rightarrow (4) \) is due to Henriksen and Jerison [Trans. Amer. Math. Soc. 115 (1965), 121]. (Received October 30, 1968.)


Theorem. Let \( H \) be a separable Hilbert space. There exist (unbounded) selfadjoint operators \( B \) and \( C \) in \( H \), a function \( A(\cdot) \) on \([0, 1]\) taking values in \([B, iC]\) and measurable in the obvious sense, and a function \( u(\cdot) \) from \([0, 1]\) to \( H \) having \( u(t) \) in the domain of \( A(t) \), \( A(\cdot)u(\cdot) \) measurable with \( \|A(\cdot)u(\cdot)\| \) integrable on \([0, 1]\), \( u(1) \neq 0 \), and \( u(t) = \int_0^t A(s)u(s)ds \) in the strong sense on \([0, 1]\). (Received October 30, 1968.)

663-430. K. H. HYDE, University of Utah, Salt Lake City, Utah 84112. On the order of the automorphism group of a finite group.

Let \( p \) be a prime, and let \( |G|_p \) be the highest order of \( p \) dividing the order, \( |G| \), of a finite group \( G \). It is shown that the least function \( g(h) \), such that \( |\text{Aut}(G)|_p \leq p^h \) whenever \( |G|_p \leq p^g(h) \), satisfies the inequality \( g(h) \leq 1/2 (h^2 - h + 2\sqrt{3h} + 2) \). This improves a result of Howarth [Proc. Glasgow Math.
Assoc. 4 (1960), 163-170] where he shows that \( g(h) \equiv 1/2(h^2 + 4) \) for \( h \equiv 6 \). It is also shown that if \( G \) is a \( p \)-group and \( |G| \equiv p^{1/2(h^2-h+4)} \), then \( p^h \equiv |\text{Aut}(G)|_p \). (Received October 30, 1968.)


This note defines the entropic packing constant \( C(S) \) for a compact convex symmetric set \( S \) with nonempty interior in Euclidean \( n \)-space \( E^n \). Call \( f \) on \( E^n \) strongly integrable of order \( \alpha \equiv 0 \) if \( f \) is in \( L_1(E^n) \) and the integral of \( f \) can be approximated to within \( O(\delta^\alpha) \) by Riemann sums over partitions of \( E^n \) of small mesh \( \delta \). Let \( \rho(x) \) be a continuous probability density function on \( E^n \) such that \( \rho \) and \( \rho \log 1/\rho \) are strongly integrable of order \( \alpha > 0 \). Let \( X \) be \( E^n \), but with metric induced by the norm \( N \) corresponding to \( S \), and probability induced by \( \rho \). The epsilon entropy \( H(\varepsilon(X)) \) is then defined as the infimum of the entropies of all partitions of \( X \) by Borel sets of diameter at most \( \varepsilon \) under the norm \( N \).

Let \( v_1 \) denote the Euclidean volume of \( S \). Theorem 1. There is a constant \( C(S) \), the entropic packing constant of \( S \), such that, as \( \varepsilon \) approaches 0, \( H(\varepsilon(X)) = n \log 2/\varepsilon + H(p) + \log 1/v_1 + C(S) + o(1) \), where \( H(p) = \int p \log 1/p \, dx \) is the differential entropy of \( p \).


The main question dealt with in this paper is "When does an order bounded net in an ordered topological vector space converge?" An answer to the question is provided by the following theorem. Let \( E \) be a locally convex Hausdorff space and \( \{x_i, f_i\} \) a biorthogonal system for \( E \) with \( \{f_i\} \) total. Let \( E \) be ordered by the cone \( K = \{x \in E : f_i(x) \equiv 0, i = 1,2,\ldots\} \). Then the following statements are equivalent: (i) For each \( x \in K \) and each bounded sequence \( \{b_i\} \) of nonnegative real numbers, the series \( \sum_{i=1}^\infty b_i f_i(x) \) converges. (ii) \( [\theta,x] \) is compact for each \( x \in K \). (iii) \( [\theta,x] \) is weakly compact for each \( x \in K \). (iv) Each order bounded increasing net in \( K \) converges weakly to an element of \( K \). (v) Each order bounded increasing sequence in \( K \) converges weakly to an element of \( K \). (vi) Each order bounded increasing net in \( K \) converges to an element of \( K \). (vii) Each order bounded increasing sequence in \( K \) converges to an element of \( K \). (viii) \( [\theta,x] \) is weakly sequentially complete and bounded for each \( x \in K \). (Received October 30, 1968.)


A simple elementary method is given for estimating a rather broad class of number-theoretic sums. A special case of the method is the following theorem. Let \( f(n) = \sum_d \sum_m g(d) \tau_k(n/d) \), where \( \tau_k(n) \) is the number of ways of representing \( n \) as a product of \( k \) factors. Then if the Dirichlet series \( G(s) = \sum_{n=1}^\infty g(n)n^{-s} \) converges absolutely for \( s > \alpha, \alpha < 1 \), then \( \sum_{n \leq x} f(n) = xP(\log x) + O(x^{0.7}), \) where \( \beta = \max(\alpha, 1 - 1/k) \) and \( P \) is a polynomial of degree \( k - 1 \). The coefficients of \( P \) are computable, and the implied constant in the error can be estimated; the leading coefficient of \( P \) is \( G(1)/(k - 1)! \). An advantage of the method is that it can often be applied to fairly complicated functions. Thus consider the sum, of a type often encountered in Selberg's method, \( \sum_{n \leq x} 2^{\nu(n)} \), where \( n \) is restricted.
to squarefree numbers relatively prime to some number \( N \). Estimation of this sum by the above theorem is not essentially more difficult than estimating the same sum unrestricted. (Received October 30, 1968.)

663-434. R. W. FRUCHT, University of Santa Maria, Valparaiso, Chile, and FRANK HARARY, University of Michigan, Ann Arbor, Michigan 48104. On the corona of two graphs.

Let \( A_i \) be the automorphism group of graph \( G_i \), \( i = 1, 2 \). The lexicographic product \( G_1 \circ G_2 \) of two graphs has been shown to have as its automorphism group the wreath product \( A_1 \wr A_2 \) of their groups, under certain rather complex conditions. Our present object is the construction of a new binary operation, called the corona \( G_1 \circ G_2 \) of the two given graphs such that, in general, the group of \( G_1 \circ G_2 \) is isomorphic with \( A_1 \wr A_2 \). The graph \( G = G_1 \circ G_2 \) is obtained by taking one copy of \( G_1 \) and \( p_1 \) (the number of points of \( G_1 \)) copies of \( G_2 \), and then joining the \( i \)th point of \( G_1 \) to every point in the \( i \)th copy of \( G_2 \). Then the group of \( G_1 \circ G_2 \) is isomorphic to \( A_1 \wr A_2 \) if and only if \( G_2 \) has no isolated points. (Received October 30, 1968.)


We give a formulation of Galois theory using automorphism group schemes which applies equally well to inseparable, separable, and purely inseparable cases. If \( K/k \) is a normal field extension, let \( \text{Aut}(K/k) \) be the Grothendieck automorphism scheme of \( K/k \). If \( H \) is a subgroupscheme of \( \text{Aut}(K/k) \), then \( \text{Fix}_H \), a subschemescheme of \( K/k \), is defined whose rational points, \( \text{Fix}_H(k) \), form a subfield \( L \) of \( K/k \). If \( L \) is a subfield of the layer \( K/k \), we define \( \text{Inv}_L \), a subgroupscheme of \( \text{Aut}(K/k) \), in a natural way and prove Theorem. The correspondences \( H \rightarrow \text{Fix}_H(k) \), \( L \rightarrow \text{Inv}_L \) and \( \sim \) lattice inverting correspondences between the set of equivalence classes of subgroupschemes of \( \text{Aut}(K/k) \) and the set of all subfields of \( K/k \). In each equivalence class of \( \text{Aut}(K/k) \) there is a unique maximal member, namely, \( \text{Inv}_L \). The other theorems of Galois theory are proven in this general context, as well. (Received October 30, 1968.)


All rings are commutative with identity. If \( A \) and \( A' \) are \( R \)-algebras, when is \( A \otimes_R A' \) Noetherian? Theorem 1. Let \( R \) be a ring and \( A \) and \( A' \) rings which are faithfully flat \( R \)-modules. If \( A \otimes_R A' \) is Noetherian, then \( A, A' \), and \( R \). Example. Let \( F \) be a field of characteristic \( p \neq 0 \). Let \( R = F(X), X \) transcendental over \( F \). If \( A = A' = F(Y^{1/p}, Y^{1/p^2}, \ldots) \), then \( A \otimes_R A' \) is not Noetherian.

Theorem 2. Let \( R \) be a Noetherian ring containing a field \( K \). Let \( M \) be an ideal of \( R \) such that \( \bigcap_{n=1}^\infty M^n = (0) \). Let \( K \) be an algebraic field extension of \( k \). Let \( R \otimes_k K \) be the completion of \( R \otimes_k K \) under the \( M(R \otimes_k K) \)-topology. If \( (R/M) \otimes_k K \) is Noetherian and if for every maximal ideal \( N \) of \( R \otimes_k K \), \( N(R \otimes_k K) \) is not \( \hat{R} \otimes_k K \), then \( R \otimes_k K \) is Noetherian. Theorem 3. The hypotheses of Theorem 2 are satisfied if \( K \) is an algebraic extension of \( k \) and if \( (i) (R, N_1, \ldots, N_n) \) is a semilocal ring, each \( R/N_i \) is finite over \( k \), and \( M = \bigcap_{i=1}^n N_i \); or \( (ii) (R, N_1, \ldots, N_n) \) is a semilocal ring, \( R/(\bigcap_{i=1}^n N_i) \) is finitely generated over \( k \), and \( M = \bigcap_{i=1}^n N_i \); or \( (iii) (R, M) \) is a local ring and \( R/M \) is linearly disjoint
from $K$ over $k$; or (iv) $R$ is Noetherian, $M$ is the Jacobson radical of $R$, $R/M$ is finitely generated over $k$, and $R/N$ is linearly disjoint from $K$ over $k$ for every maximal ideal $N$ of $R$. (Received October 30, 1968.)

663-437. D. M. ARNOLD, New Mexico State University, Las Cruces, New Mexico 88001. *A duality for quasi-isomorphism classes of torsion free modules with finite rank over a discrete valuation ring.*

Two t.f. modules $A$ and $B$ for $R$ a d.v.r. with unique prime $p$ are quasi-isomorphic ($A \sim B$) if $A$ is isomorphic to a submodule $A'$ of $B$, $B$ is isomorphic to a submodule $B'$ of $A$, and $B/A'$ and $A/B'$ are bounded. Let $C$ denote the quotient category $\mathcal{G}/\mathcal{B}$ where $\mathcal{G}$ is the category of $R$-modules and $\mathcal{B}$ the Serre class of bounded $R$-modules, and let $\mathcal{F}$ denote the full subcategory of $C$ consisting of all t.f. $R$-modules of finite rank. The $p$-rank of $A$ is $\dim R/pR \otimes R A$ as a vector space over $R/pR$. Theorem. There is a contravariant exact functor $F$ from $\mathcal{F}$ onto $\mathcal{F}$ with the following properties: (a) $F(F(A)) = A$ for all $A \in \text{obj } \mathcal{F}$. (b) If $A$ has $p$-rank $n$, rank $n + k$, then $F(A)$ has $p$-rank $k$, rank $n + k$. (c) $F(R) = K$ and $F(K) = R$, where $K$ is the quotient field of $R$. (d) If $A$ has $p$-rank $n$, rank $n + 1$, then $F(A) \sim \wedge^n A$, the $n$th exterior power of $A$. As a corollary we have the following unpublished result of M. C. R. Butler.

**Corollary.** Assume $A$, $B$ are t.f. $R$-modules with $p$-rank $n$, rank $n + 1$. Then $A \sim B$ iff $\wedge^n A \not\sim \wedge^n B$.

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The paper presents the author's views on the significance of geometry in modern mathematics (i) as an independent discipline; (ii) as a tool in other fields; (iii) as a pedagogical device; (iv) as a motive force for development of major new mathematical disciplines and trends. (Received October 30, 1968.)


A circuit code of dimension $d$ and spread $s$ is a simple circuit $C$ in the graph $I^d$ of the unit $d$-dimensional cube which satisfies the condition that $d(x,y) < s$ implies $d_c(x,y) = d(x,y)$ for all vertices $x$ and $y$ in $C$, where $d$ and $d_c$ are the graph-theoretic distance functions with respect to $I^d$ and $C$ respectively. Such circuits are useful for introducing an error-limiting feature into the design of certain analog-to-digital conversion systems, increased length corresponding to greater accuracy of the system and increased spread to greater error-limiting capability. New upper bounds are given for $K(d,s)$, the length of a longest $d$-circuit of spread $s$, for the case of even $s$. Theorem. $K(d,2) \leq 2^{d-1} - \left[\frac{(d^2 - 12)/(7d(d - 1)^2 + 2)}{d} \right]$ for $d \equiv 6$. For even $s \equiv 4$, the upper bound by Chien, Freiman, and Tang [Error correction and circuits on the $n$-cube, Proc. Second Annual Allerton Conference on Circuit and System Theory, September 29-30, 1964, Univ. of Illinois, Allerton House, 899-912] is reduced roughly by a factor of 2. (Received October 31, 1968.)

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K. Yano has introduced the notion of an $f$-structure, i.e. a tensor field $f$ of type $(1,1)$ and rank $2n$ with $f^3 + f = 0$. Almost complex structures and almost contact structures are, of course, examples. The purpose of this paper is to develop the Kaehler analogue of a manifold with an $f$-structure.

Let $(f, g)$ be an $f$-structure with a compatible metric and complemented frames $\xi_1, ..., \xi_s$ (dual 1-forms $\Omega^1, ..., \Omega^s$), i.e.

$$g(f\Omega, f\Omega') = g(\Omega, \Omega') - \sum \xi_i \cdot \eta_i(\Omega) \eta_i(\Omega').$$

$M^{2n+2}$ is $f$-Kaehlerian if it has such an $f$-structure which is normal and has closed fundamental 2-form, i.e.

$$[f, f] + \omega \equiv d\iota = 0, \quad d\omega = 0,
\quad F(X, Y) = g(X, fY).$$

Cases where $d\iota_1 = ... = d\iota_s = 0$ and $\omega = d1\iota_1 \wedge ... \wedge d1\iota_s$ are of special interest and are called $f$-cosymplectic and $f$-Sasakian respectively. We define $f$-sectional curvature, prove the flatness of the distribution determined by $f^2 + I$, show that the $f$-sectional curvature determines the curvature completely, and most importantly introduce a standard compact example playing the role of complex projective space in Kaehler geometry and the sphere in Sasakian geometry. (Received October 30, 1968.)

Let $S$ be a compact Riemann surface of genus $g$, $g \geq 2$. Denote the Riemann theta constants associated with $S$ by $\Theta$, the Schottky theta constants associated with $S$ by $\eta$, and the Riemann theta constants associated with $S$ (the smooth two sheeted cover of $S$ on which the Prym differentials with characteristic $\psi = \frac{1}{2} \omega$ and $\frac{1}{2} \omega'$ are ordinary abelian differentials) by $\bar{\Theta}$. Theorem. A sufficient condition for $\eta^2[\xi_1/\omega, \xi_0/\omega', \eta_1/\omega', \eta_0/\omega]$, to be independent of $[\xi_1, \xi_0, \eta_1, \eta_0]$ is that $\bar{\Theta}[1, 1, 0] = 0$ for the $2g-2$ $(2g-1)$ characteristics $[0, 0, 1, 1]$, where $[1, 1, 0, 0]$ ranges over all odd $g - 1$ characteristics. This result generalizes the result announced in the Proc. Nat. Acad. Sci. 59 (1968), 52-55. (Received October 25, 1968).

The polynomial sequence $\{p_n\}$ generated by $\phi_0 = 1$, $\phi_1 = z + b$, $\phi_{n+1} = (z + b)\phi_n - c\phi_{n-1}$ ($n \geq 1$, $b$ and $c$ arbitrary complex constants) is an interesting example of a "basic set" (J. M. Whittaker, Interpolatory function theory, Cambridge, 1935) because the expansion of an analytic function $f$ in a series of these polynomials contains as special cases the Taylor series for $f$ (when $c = 0$) and the Fourier series for $f$ (when $b$ is real and $c > 0$).--in this case the $\phi_n$ are orthogonal on $(-b - 2i/c, -b + 2i/c)$ with weight function $w(x) = ((4c - (x + b)^2)^{1/2})$. Theorem. Suppose an analytic function $f$ and its derivatives can be represented in some region by an infinite series $\sum a_n \phi_n$ and its appropriate derivatives. Then (formally, at this point) $a_n = \sum_{j=0}^{\infty} (z^{(2j+n)}(-b)(n+1)c/(1 + n + j)!)$, $n = 0, 1, 2, ...$. Theorem. Suppose $f$ can be expanded in a Taylor series with center at $z = -b$ and radius of convergence $R$. Suppose also that $c$ satisfies $0 < |c| < R^2/4$. Let $E$ denote the ellipse having focal points at $-b \pm 2i/c$ and semimajor axis of length $R$. Then the series $\sum a_n \phi_n$ converges absolutely for every $x$ interior to $E$ and diverges for every $x$ exterior to $E$. At each interior point the value of the series is $f(x)$. (Received October 30, 1968.)
Let \( Y, Z \) be fixed Banach spaces. Denote by \( L(Z; Y) \) the space of bounded linear operators from \( Z \) into \( Y \). Let \( (X, \mathcal{V}, \nu) \) be a volume space and generate the spaces \( L^p(\mathcal{V}, Y) \) for \( 1 \leq p < \infty \) as in Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 493-498, and Bogdanowicz, Bull. Acad. Polon. Sci. 13 (1965), 743-800. Proposition. If \( f, g \in L^p(\mathcal{V}, Y) \) and if for each \( A \in \mathcal{V}, \int_A f(x) d\nu(x) = \int_A g(x) d\nu(x) \), then \( f(x) = g(x) \) \( \nu \)-a.e. Theorem 1. Let \( T: Z \to L^p(\mathcal{V}, Y) \) be a linear mapping. Then \( T \) is continuous if and only if for each \( A \in \mathcal{V}, \) the operator \( T_A: Z \to Y \) defined by \( T_A z = \int_A (Tz)(x) d\nu(x) (z \in Z) \) is continuous. Theorem 2. Let \( T: Z \to L^p(\mathcal{V}, Y) \) be a continuous linear mapping. Then there exists a unique mapping \( \mu: \mathcal{V} \to L(Z, Y) \) for which \( \mu(\cdot)z \in M_p(\mathcal{V}, Y), \|\mu(\cdot)z\|_p = \|Tz\|_{p, \mathcal{V}} \) and \( \mu(A)z = \int_A (Tz)(x) d\nu(x) \) for each \( z \in Z \) and \( A \in \mathcal{V} \). (Received October 23, 1968.)

Noncut points in a nondegenerate continuum.

Let \( R \) denote a semisimple commutative ring with unit element, and \( \mathfrak{m} \) its maximal ideal space with hull-kernel topology. The following theorem is proved: If \( R \) has no nonzero idempotents then either \( R \) is a field or \( \mathfrak{m} \) has at least two noncut points. Since \( \mathfrak{m} \) is not necessarily Hausdorff, this generalizes a standard theorem in general topology, viz. every nondegenerate continuum contains at least two noncut points. (Received October 25, 1968.)

Conditionally invariant sets. Preliminary report.

Direct and converse theorems for stability and generalized exponential stability of conditionally invariant sets (see, for the definition of conditionally invariant sets, Kayande and Lakshmikantham, J. Math. Anal. Appl. 14 (1966), 285-293) are obtained in terms of single Lyapunov function and differential inequalities. Moreover, preservation of stability behaviour of conditionally invariant sets under certain perturbations is discussed. This development is parallel to Lyapunov theory and includes it as a special case. (Received October 11, 1968.)

A coefficient problem for a class of meromorphic univalent functions.

Let \( V_n \) denote the class of functions of the form \( F_n(z) = 1/z + \sum_{k=1}^n a_k z^k \) which are univalent in \( |z| < 1 \) and where \( a_k (k = 1, 2, \ldots, n) \) is real with \( |a_k| = 1/n \). This implies \( a_{n-1} = 0 \) [J. D. Brannan, Mathematika 14 (1967), 165-169]. Theorem 1. If \( F_n(z) \in V_n \), then \( a_1 \neq (n - 2)/n \) \( (n \geq 4) \) and the result is sharp. Theorem 2. If \( F_4(z) = 1/z + \sum_{k=1}^4 a_k z^k \in V_4 \), then \( -5/8 \leq a_1 \leq 1/2, -5/16 \leq a_2 \leq 5/16 \), and these results are sharp. Theorem 3. If \( F_5(z) = 1/z + \sum_{k=1}^5 a_k z^k \in V_5 \), then \( -3/5 \leq a_1 \leq 3/5, -2\sqrt{2}/5 \leq a_2 \leq 2\sqrt{2}/5, -1/5 \leq a_3 \leq 1/5 \), and these results are sharp. (Received October 25, 1968.)

Let $M = G/K$ be a 1-connected Riemannian symmetric space, $G = I(M)$ its Lie group of isometries. A. Borel (On the automorphisms of certain subgroups of semisimple Lie groups) proved the Theorem 1. If $M$ has no compact or euclidean factor, there exists an infinite sequence of finite coverings $\ldots \rightarrow M_{i+1} \rightarrow \ldots \rightarrow M_0$ of compact Clifford-Klein forms $M_i = \Gamma_i \backslash \mathcal{M}$, $\Gamma_i \subset G$, such that $\Gamma_i \not\equiv \Gamma_j$ for $i \neq j$. Two forms are called commensurable ($M_1 \sim M_2$) if they admit finite coverings which are isometric. Theorem 2. If $M$ is noncompact and has no euclidean factor, there exist infinitely many compact forms $M_i$, $i = 1, \ldots$, such that $M_i \not\sim M_j$ for $i \neq j$. If moreover $M$ has no noncompact factor of dimension 2, then any sequence of compact forms $M_i$, $i = 1, \ldots$, satisfying $M_i \not\sim M_j$, $i \neq j$, contains infinitely many mutually nonisomorphic groups among the $\Gamma_i$. Theorem 3. Assume that all irreducible factors of $M$ are noncompact, non-euclidean and of dimension $>2$ and let $\Gamma \backslash \mathcal{M}$ be a compact form. Then there is only a finite number of isometry classes of compact forms $\Gamma' \backslash \mathcal{M}$ such that $\Gamma' \equiv \Gamma$. The proofs rely on results of A. Borel, Harish-Chandra and A. Weil. (Received October 31, 1968.)

663-448. Morris Weisfeld, Duke University, Durham, North Carolina 27706. The solution of algebraic equations in the positive orthant.

A solution of the system of algebraic equations $P_i(s) = 0, i = 1, \ldots, m$, where $s = (s_1, \ldots, s_m)$ is an $m$-tuple of real numbers and the $P_i$ are polynomials in the components $s_j$ of $s$, is sought in the positive orthant $\{s | s_j > 0, j = 1, \ldots, m\}$. The critical points of this problem are the boundary points of the positive orthant and the points at which the Jacobian $\det(\partial P_i/\partial s_j)$ vanishes. An algorithm is constructed which is proved to converge either to a solution or to a critical point. (Received October 31, 1968.)


We say that $f: [a,b] \rightarrow \mathbb{R}$ is of bounded convexity on $[a,b]$ ($f \in BC[a,b]$) if $K_b^b(f) = \sup \sum_{j=1}^{n-1} \int f_{j+1} - f_j$ is finite. Here $P = [a = x_0 < x_1 < \ldots < x_n = b]$ is a partition of $[a,b]$ and $f_j = [f(x_j) - f(x_{j-1})]/(x_j - x_{j-1})$. We see immediately that a convex function $f \in BC[a,b]$ iff $f_t(a)$ and $f_t(b)$ are both finite and in this case $K_b^b(f) = \int f_t(b) - f_t(a)$. Theorem 1. $f \in BC[a,b]$ iff $f = g - h$ where $g$ and $h$ are convex and belong to $BC[a,b]$. Theorem 2. $f \in BC[a,b]$ iff $f(x) = \int g_t(x)dt$ for some $g$ of bounded variation. Theorem 3. $BC[a,b]$ with norm given by $\|f\| = K_b^b(f) + \|f_t\|_1(a) + \|f_t\|_1(b)$ is a Banach space. There is an obvious generalization of $K_b^b(f)$ for vector valued functions $f: [a,b] \rightarrow \mathbb{R}^m$. Theorem 4. If $C$ is a rectifiable curve which is parameterized with respect to arc length by $f: [0,L] \rightarrow \mathbb{R}^m$ and if $\nu$ exists and is integrable, then $K_b^b(f) = \int_0^L \kappa(s)ds$ where $\kappa(s)$ is the curvature of $C$ at $s$. This last result connects the notion of total convexity $K_b^b(f)$ with the so-called integral curvature of classical differential geometry. Finally we mention that the notion of bounded convexity can be generalized to functions $f: \mathbb{R}^m \rightarrow \mathbb{R}$. (Received October 31, 1968.)

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Let \( K_1 \) and \( K_2 \) be algebraic number fields. Let \( L \) denote the composite of \( K_1 \) and \( K_2 \). Let \( \mathfrak{p} \) be a prime ideal in \( K_1 \) and in \( K_2 \). Suppose that \( \mathfrak{p} \) divides the rational prime \( p \). Let \( K_1 \cap K_2 = F \), and suppose that \( [K_1 : F] = [K_2 : F] = p^r \). Suppose that the extensions \( K_1/F \) and \( K_2/F \) are normal extensions and that \( \mathfrak{p} \) is totally ramified in these extensions. Let the Hilbert sequence of subgroups of \( G(K_i/F) \) for \( \mathfrak{p} \) be given by \( G(K_i/F) = G_0(K_i/F) = \ldots = G_{v_i-1}(K_i/F) \neq G_{v_i}(K_i/F) \) for \( i = 1, 2 \). We define the constant \( v \) as follows: \( v = 0 \) if \( \mathfrak{p} \) is not totally ramified in \( L/F \). If \( \mathfrak{p} \) is totally ramified in the extension \( L/F \), let the Hilbert sequence for \( \mathfrak{p} \) be given by \( G(L/F) = G_0 = \ldots = G_{v-1} \neq G_v \). Let \( \bar{v} = \min \{ v_1, v_2 \} \). Then we have the following result: Theorem. The largest rational integer \( m \) for which \( K_1 \) and \( K_2 \) have corresponding residue systems mod \( \mathfrak{p}^m \) is given by \( m = \left\lfloor \frac{pv}{p^r} + v + 1 \right\rfloor \), provided one of the following conditions is satisfied: (i) \( \mathfrak{p} \) is unramified in \( L/K \), (ii) the extensions \( K_i/F \) are of degree \( p \) or cyclic of degree \( p^2 \). (See H. S. Butts, and H. B. Mann, Corresponding residue systems in algebraic number fields, Pacific J. Math 6 (1956), 211.) In the above theorem, \( \left\lfloor \cdot \right\rfloor \) denotes the greatest integer function. (Received October 31, 1968.)


It is proved by K. A. Zavlakov that in a Jordan ring \( A \), there exists a maximal locally nilpotent ideal, \( L(A) \), such that the factor ring \( A/L(A) \) is \( L \)-semisimple, i.e. \( A/L(A) \) does not contain nonzero locally nilpotent ideal. Let us call the ideal \( L(A) \), as in the case of associative rings, the Lavitzki radical of the ring \( A \). Theorem 1. Let \( A \) be a Jordan ring in which \( 2a = 0 \) implies \( a = 0 \) and, for any \( a \) in \( A \), \( 2x = a \) has a unique solution. Then the Lavitzki radical of \( A \) is the intersection of the family of prime ideals \( \langle P \rangle \) in \( A \) such that \( A/P \) is Lavitzki semisimple. Theorem 2. For any Jordan ring \( A \), the prime radical \( R(A) \) is contained in the Lavitzki radical \( L(A) \) of \( A \). \( R(A) = L(A) \) if the minimal condition on ideals holds in \( A \). (Received October 31, 1968.)

663-452. P. S. MOREY, Jr., Texas A & I University, Kingsville, Texas 78363. A generalization of extensors.

Examination of the transformation laws for \( V_{\alpha\alpha}(B) \) and \( (M^\alpha B)V_{\alpha\alpha}(M^{-1}B) \), \( V_{\alpha\alpha} \) and \( V_{\alpha\alpha} \) being absolute extensors of type indicated by the free indices and of class \( c^M \), provides the basis for the transformation laws of quantities called components of second degree extensors. Theorem. These quantities obey the algebraic laws of extensors, i.e. the addition, multiplication, contraction, quotient, and transitive laws. (Received October 31, 1968.)

663-453. GARRETT BIRKHOFF, Harvard University, Cambridge, Massachusetts 02138. The varied foundations of geometry.

Geometry is the oldest branch of Mathematical Physics. Euclid gave its first known axiomatic treatment, using many primitive terms whose interpretation was intuitive. Since then, many other
fruitful axiom systems for geometry have been studied, referring to properties of many entities: points as n-tuples, incidence (projective geometry), distance (metric space), rigid motions and "free mobility" (Riemann-Helmholtz), curvature (differential geometry), and reflection (Hjelmslev). A discussion will be given of the sense in which all these varied foundations of geometry are "equivalent" by cryptomorphism. (Received October 31, 1968.)

663-454. J. R. PORTER, University of Kansas, Lawrence, Kansas 66044. Minimal R(w0) spaces. Preliminary report.

Let (X,τ) be a topological space. For x in X, let τ(x) denote {U ∈ τ | x ∈ U}. For an ordinal a > 0, a topological space (X,τ) is said to be R(a) (resp. U(a)) provided that, for each pair of distinct points x and y in X, there are families {Ub ∈ τ(x) | b < a} and {Vb ∈ τ(y) | b < a} with the properties that U0 ∩ V0 = ∅ (resp. U0 ∩ V0 = ∅) and for b + 1 < a, Ub ⊃ U_b+1 and Vb ⊃ V_b+1. Clearly, a U(a) space is R(a), and a T3 (regular plus Hausdorff) space is R(w0) where w0 is the first infinite ordinal. Also, it is easy to show that for any ordinal a ≥ w0, each R(a) space is U(a). A topological property P is expansive provided that each topology finer than a topology with property P has property P. It is trivial to show that both R(a) and U(a) are expansive properties. For a topological property P, a space (X,τ) is minimal P provided τ is a minimal element in the partially ordered set of topologies on X with property P. Theorem. A space is minimal T3 if and only if it is minimal R(w0). (Received November 1, 1968.)

663-455. ERNST BINZ, Queen's University, Kingston, Ontario, Canada. On a characterization of function algebras.

(For notation and terminology see E. Binz, Bemerkungen zu limitierten Funktionenalgebren, Math. Ann. 175 (1968), 169-184.) Let A be an associative, commutative, algebra over the reals R, possessing a unity element and at least one real maximal ideal M (A/M = R), and furthermore, let the intersection of all real maximal ideals in A consist of the zero-element only. A is said to be a function algebra if it is unitarily isomorphic to C(X), the R-algebra of all real-valued continuous functions on some topological space X. The space HomA denotes the set of all real-valued unitary homomorphism on A together with the topology of the pointwise convergence. The weakest convergence structure on A making the monomorphism d: A → Cc(HomA) (defined by d(a)(h) = h(a) for all a ∈ A and all h ∈ HomA) continuous, is called the extremal continuous convergence structure (e.c.c.s). A together with this e.c.c.s is denoted by Ac. We say that A is a C-algebra if Ac is complete. Theorem. A is a function algebra iff it is a C-algebra. (Received October 3, 1968.)

663-456. J. L. FISHER, California Institute of Technology, Pasadena, California 91109. Structure theorems for noncommutative complete local rings.

Let R be a noncommutative local ring in the sense of Goldie (J. Algebra 5 (1967), 89-105). It is known that the completion of R, denoted R', is equal to (B)_(n) the full n x n matrix ring over B where B/N is a division ring, N the Jacobson radical of B. Theorem 1. If B is an algebra over F where B/N is a finite-dimensional normal extension of F and N is generated by n_1,...,n_k, then there exists automorphisms g_1,...,g_k of B/N such that B is a homomorphic image of B/N[[x_1,...,x_k; g_1,...,g_k]],
the power series ring over $B/N$ in noncommuting indeterminates $x_i$, where $x_i b = g_i(b)x_i$ for all $b \in B/N$. A similar theorem is obtained for noncommutative complete local rings which suitable commutative subrings. As a result of this theorem, we have Corollary. If $B/N$ is a finite field and $N$ is generated by $n_1, \ldots, n_k$, then there exist automorphisms $g_1, \ldots, g_k$ of a $v$-ring $V$ such that $B$ is a homomorphic image of $V[[x_1, \ldots, x_k; g_1, \ldots, g_k]]$. (Received November 4, 1968.)


Let $f$ be a complex polynomial, $(d_n)$ and $(d'_n)$ complex sequences, and $z_1$ and $z_2$ complex numbers. Suppose $(f, d_n, z_2)$ and $(z, d_n, z_1)$ to be generalized Lototsky summability methods. Theorem. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and let $a_{n+1} = f(z_2) + d_{n+1} + f_0$. For each integer $t$ with $0 \leq t \leq m$ and each non-negative integer $k$, define $\phi(k, t) = \sum_{j=t}^{m} (-1)^{j-t} \frac{1}{j!} (z_2/z_1)^{j} \sum_{(k+1)} d_1 d_2 \ldots d_{j-t}$, where the last summation is over all products of length $j - t$ whose subscripts satisfy $k + l + 1 \leq i_1 \leq i_2 \leq \ldots \leq k + t + 1$, and where $\sum_{d_1} \ldots d_j = 1 = \sum_{k+1} (z_1 + d_n)$. Let $N \geq 0$ be arbitrary and for each $n > N$ suppose that $\sigma_{n+1}^{-1}(\phi(k, 0) + d_{n+1}) = 0$ for $0 \leq k \leq m$ and that $\sigma_{n+1}^{-1}(\phi(k, t)) = 0$ for $1 \leq t \leq m$ and $0 \leq k \leq m$. Furthermore, assume that for each $k \geq 0$ there is an $n_k > 0$ such that $\sigma_{n+1}^{-1}(\phi(k, 0) + d_{n+1}) < 1$ for every $n \geq n_k$. Then $(f, d_n, z_2)$ is consistent with, and at least as strong as, $(z, d_n, z_1)$. This theorem generalizes one of A. Meir (Bull. Res. Council Israel Sec. F 10 (1961-1962.), 167). (Received October 31, 1968.)

663-458. S. E. NEWMAN, 8001 Natural Bridge, St. Louis, Missouri 63121. Functions of bounded variation on idempotent semigroups.

Let $S$ be an idempotent semigroup, $T$ a semigroup of semicharacters on $S$ containing the identity semicharacter, and $A$ the set algebra of subsets of $S$ generated by kernels of semicharacters in $T$. A convolution product of two finitely additive measures on $A$ is defined. A notion of a function of bounded variation on an idempotent semigroup with identity is given, together with a notion of the norm of a function of bounded variation. We prove the following theorems. Theorem 1. There is an isomorphism $\mu \rightarrow \hat{\mu}$ between all finitely additive measures on $A$, with convolution multiplication, and the algebra of all functions on $T$, with pointwise multiplication. The function $\hat{\mu}$ is given by $\hat{\mu}(f) = \int_S f d\mu$ for all $f$ in $T$. Furthermore, this map carries bounded (in total variation norm) measures to functions of bounded variation in a norm preserving manner. Theorem 2. Let $Q$ be an idempotent semigroup with identity. Then the pointwise product of two functions of bounded variation on $Q$ is again a function of bounded variation, and the algebra of all functions of bounded variation on $Q$, with bounded variation norm, is a commutative convolution measure algebra. (Received October 31, 1968.)


Let $A$ be an $m \times n$ complex matrix of rank $r$. By an $(i,j,k)$-inverse for $A$ we mean an $n \times m$ matrix satisfying the $i$th, $j$th, and $k$th equations of the four Penrose equations. We prove the existence of $(1,3)$- and $(1,4)$-inverses which have rank $r + q$ for each $q$ such that $0 \leq q \leq \min(m,n) - r$. This is accomplished by proving the existence of solutions $U_q$ to $AX = A$ and solutions $V_q$ to $XA = A$ such that...
rank(Uq) = r + q, 0 ≤ q ≤ n - r, and rank(Vq) = r + q, 0 ≤ q ≤ m - r. The desired result follows by using this together with the fact that there exist full rank (1,3)- and (1,4)-inverses for A. We also prove the existence of (2,3)- and (2,4)-inverses for A which have rank q for each q such that 0 ≤ q ≤ r. This is accomplished by obtaining hermitian (2)-inverses, S_q and S_q', for A^*A and AA^*, respectively, which have rank q, 0 ≤ q ≤ r. The products S_q A^* and A^* S_q produce the desired result.

(Received October 31, 1968.)


Let (X, T) be a compact topological space, T a subtopology of T, μ a regular Borel measure on S(T), the sigma algebra generated by T. It is proved that there exists a maximal pair (r, j: i) such that T is a topology between r and 1, j: i is a regular Borel measure on S(T), and j: i is an extension of μ. The proof is accomplished directly by applying the maximality principle in the lattice of topologies of X. Certain corollaries are immediate when one imposes separation conditions on T and T. (Received October 31, 1968.)

663-461. C. W. SLOVER, JR., and R. NIELSEN, University of Delaware, Newark, Delaware 19711. Maximal ideals of upper semicontinuous functions.

It is well known that two compact T spaces are homeomorphic iff their rings of continuous functions are isomorphic. The following theorem shows that analogous results can be obtained for any compact T space (X, T) using the algebraic structure of the nonnegative real-valued upper semicontinuous functions \( \mathcal{U}(X) \).

Theorem. Let (X, \( \mathcal{T}_X \)), (Y, \( \mathcal{T}_Y \)) be two compact T spaces. Then X and Y are homeomorphic iff \( \mathcal{U}(\mathcal{T}_X) \) and \( \mathcal{U}(\mathcal{T}_Y) \) are isomorphic. (Received October 31, 1968.)


Let \( P(x,D) = \sum_{|\alpha| \leq m} a_\alpha(x)D^\alpha \), \( a_\alpha(x) \in C^\infty(\mathbb{R}^{n+1}) \), be a linear partial differential operator of order m with coefficient of unity. Let \( P_0(x,D) = \sum_{|\alpha| = m} a_\alpha(x)D^\alpha \). Let \( \Gamma : x_0 = \beta(\bar{x}), \bar{x} = (x_1,\ldots,x_n) \), be an initial surface such that \( \beta(\bar{x}) \), defined for all \( \bar{x} \), is of class \( C^\infty \) with \( |\nabla \beta| \neq 0 \), and \( P_0(x, D(x_0 - \beta(\bar{x}))) > 0 \) (i.e. \( \Gamma \) is space-like). Let \( H_\Gamma \) be the half space \( x_0 \notin \beta(\bar{x}) \). Let \( f \in \mathcal{D}(H_\Gamma) \) and be a continuous function in a neighbourhood \( \mathcal{N} \) of \( \Gamma \). Let \( \phi^k(\bar{x}), k = 0,1,\ldots, m - 1 \), be sufficiently smooth functions on \( \Gamma \). Then the distribution problem over \( H_\Gamma \): \( P(x,D)u = f \) with \( \phi^k \in H_\Gamma \) and \( \phi^k \) and their derivatives, is a distribution with support on \( \Gamma \) well determined by \( \phi^k \) and their derivatives of order \( \leq m - 1 \). As an application, the Cauchy problem of the wave operator \( D^2 = \partial_x^2 - \sum_{j=1}^n \partial^2 / \partial x_j^2 \) is treated both for odd as well as even \( n \). Also, a technique of getting an elementary solution with support in a half space of the general linear partial differential operator with constant coefficients is given. (Received October 31, 1968.)
The upper bound conjecture (UBC) is concerned with maximizing the number $f_j$ of $j$-faces ($1 \leq j < d$) of a $d$-polytope while keeping the number $v = f_0$ of vertices fixed, and states that the maximum is attained for a cyclic polytope $C(v,d)$, whose vertices are $v$ distinct points on the moment curve \( \{(t, t^2, \ldots, t^d) \in \mathbb{E}^d \mid \ -\infty < t < \infty \} \). The problem can be reduced to considering simplicial polytopes, the numbers of whose faces are related by the Dehn-Sommerville equations. Using a method devised by I. G. Macdonald, the equations are solved in a new way, showing that in the expression for $f_{n+p}$ ($n = \lfloor d/2 \rfloor$, $0 \leq p < d - n$) in terms of $f_j$ ($j < n$) the coefficients alternate in sign, that of $f_{n-1}$ being positive. Adapting a procedure of V. Klee it is shown that the UBC holds for $(n + p)$-faces when $d = 2n$ and $v \leq n - p - 2 + n(n + 1)(p + 1)^{-1}$ or $d = 2n + 1$ and $v \leq n - p - 2 + (n + 1)(n + 2)(p + 1)^{-1}$. (Received October 23, 1968.)

The classical Poincaré-Bendixson theorem states that given a two-dimensional autonomous system defined in a bounded domain $D$ of $\mathbb{R}^2$ and having only finitely many critical points, then the limit set of any semi-orbit remaining in a closed subset of $D$ will be either (i) a single critical point, (ii) a periodic orbit, or (iii) a finite number of critical points and a set of orbits each of which tends to one of those critical points as $t \to \pm \infty$. Topologizing the set, $X$, of systems defined on $\mathbb{R}^2$ with the Whitney $C^r$-topology ($r \geq 1$), the following result is obtained: Theorem. For all systems belonging to the complement of a set of first category in $X$, the limit set of any semi-orbit of a system will be either (i) a single critical point of the system, (ii) a periodic orbit of the system, or (iii) empty. (Received November 1, 1968.)

In $\mathbb{E}^3$ let $B(P',R)$ be a ball (center $P'$, radius $R$) and $S(P',R)$ be its boundary. Let $f$ be a given function continuous on $S(P',R)$ and $\lambda > 0$. Theorem 1. The function $u(P) = (1/4\pi R) \int_{S(P',R)} f^*(Q) \exp(-\lambda r)(1 + \lambda r)(R^2 - \rho^2)/r^3 + \lambda \rho \, d\sigma(Q)$, $P \in B(P',R)$, is the unique solution of $\Delta u - \lambda^2 u = 0$ in $B(P',R)$ with $\lim_{P \to Q', Q' \in S(P',R)} u(P) = f(Q')$, where $\rho = |P - P'|$, $r = |P - Q|$, $f^*$ is the continuous sum function of the series $\sum_{n=0}^{\infty} (-1)^n f_n(Q') = f, f_n(Q') = (1/4\pi R) \int_{S(P',R)} \exp(-\lambda |Q - Q'|) f_n^{*}(Q) \, d\sigma(Q)$, $Q' \in S(P',R)$. Theorem 2. If $R < \pi/4\lambda$, then the function $u(P) = (1/4\pi R) \int_{S(P',R)} f^*(Q) \left[(\cos \lambda r + \sin \lambda r)(R^2 - \rho^2)/r^3 + \lambda \sin \lambda r \right] \, d\sigma(Q)$, is the unique solution of $\Delta u + \lambda^2 u = 0$ in $B(P',R)$ with $\lim_{P \to Q', Q' \in S(P',R)} u(P) = f(Q')$, as $P \to Q'$, where $f_0$ is the continuous sum functions of the series $\sum_{n=0}^{\infty} (-1)^n f_n(Q') = f, f_n(Q') = (1/4\pi R) \int_{S(P',R)} \exp(-\lambda |Q - Q'|) f_n^{*}(Q) \, d\sigma(Q)$. Remark 1. Similar integral formula can be obtained for equations of the type $\Delta u + 2 \sum_{i=1}^{3} a_i u x_i + cu = 0$, $a_i$'s and $c$ are constant, by splitting out an exponential factor. Remark 2. Each of the above integral formulas reduces to the usual Poisson's integral formula for harmonic functions if we set $\lambda = 0$. (Received November 1, 1968.)
A topological space is paraseparable if every collection of pairwise disjoint open subsets is countable. A Souslin space is a simply ordered set with the interval topology which is connected and paraseparable but not separable. Souslin's problem is then—is there such a space. Recently models of set theory have been constructed in which Souslin spaces exist. Certain of these models satisfy both the axiom of choice and the continuum hypothesis. Furthermore, models of set theory have been constructed in which there are no Souslin spaces. A tree is a Hausdorff continuum in which each pair of points is separated by a third point. An Eberhart continuum is a paraseparable tree such that each of its arcs is separable. The main theorem. The existence of a Souslin space is equivalent to the existence of a nonmetrizable Eberhart continuum. This resolves a problem raised by Carl Eberhart, Abstract 653-157, these Notices 15 (1968), 124. Corollary. Every Eberhart continuum in the Tychonoff cube $I^I$ is an absolute retract for the class of normal spaces if and only if there are no Souslin spaces. (Received November 1, 1968.)

HANSJOACHIM GROH, Kansas State University, Manhattan, Kansas 66502. Ovals in flat projective planes.

A flat projective plane is a topological projective plane $P, L$ (i.e. $P, L$ have topologies such that joining and intersecting are continuous) where $P$ is a topological 2-manifold. By H. Salzmann (Math. Ann. 145 (1962), 401-428) the group $\Gamma$ of all automorphisms, given the compact open topology, is a Lie group. It is shown that each nontrivial orbit (i.e. not a point, not a line) of any circle subgroup of $\Gamma$ is an oval. Each oval whose stabilizer is closed in $\Gamma$ and of positive dimension is a Jordan curve ($\cong S^1$) and separates $P$. All such ovals in the Desarguesian plane, the Moulton planes, and other planes are determined. (Received November 1, 1968.)

T. W. PALMER, University of Kansas, Lawrence, Kansas 66044. A real $B^*$-algebra is $C^*$ iff it is Hermitian.

The title answers a question raised by C. E. Rickart on page 181 of General theory of Banach algebras, Van Nostrand, 1960. (For all definitions see this book.) Obviously a non-hermitian real $B^*$-algebra (e.g. the complex numbers considered as a real Banach algebra with the identity map as involution) is not a $C^*$-algebra. However, Theorem 1. The following are equivalent for a real Banach $*$-algebra $A$: (1) $A$ is a real $C^*$-algebra, (2) $A$ is a hermitian $B^*$-algebra, (3) $A$ is a symmetric $B^*$-algebra, (4) for all $S, T \in A$, $\|S\|^2 \leq \|S*S + T*T\|$. The theorem is proved by imbedding the real algebras in complex algebras and using a result of the author in Bull. Amer. Math. Soc. 74 (1968), 538-540. The same method is used to give several similar characterizations of those real Banach $*$-algebras which are homeomorphically $*-$isomorphic to $C^*$-algebras. Some of these conditions do not immediately imply that the involution is continuous, and are in this respect formally much weaker than the conditions of Theorem 1. The proofs adapt several arguments from J. G. Glimm and R. V. Kadison, Pacific J. Math. 10 (1960), 547-556. (Received November 1, 1968.)
Let X be a Banach space, U(X) its closed unit ball, and \( \alpha \) a cardinal number. **Definition 1.** \[ P(\alpha, X) = \sup \{ r \mid \text{there exists a disjoint balls of radius r contained in } U(X) \} \]

**Theorem 1.** If X is an infinite-dimensional Banach space and \( \alpha \) is greater than 1 but less than or equal to the density characteristic of X, then \( 1/3 \leq P(\alpha, X) \leq 1/2 \). Two Banach spaces X and Y are called nearly isometric if \( \inf \| T \cdot T^{-1} \| = 1 \) where the infimum is taken over all isomorphisms T of X onto Y. **Theorem 3.** If for some cardinal number \( \alpha \), \( P(\alpha, X) \neq P(\alpha, Y) \), then X and Y are not nearly isometric.

**Theorem 4.** X is P-convex implies both X is B-convex and X is reflexive. Theorem 4 extends a theorem of James (Ann. of Math. (2) 80 (1964), 547) and it fails if the n in the definition of P-convex is allowed to assume infinite values. The property dual to P-convex is also examined. (Received November 1, 1968.)

**663-470.** DIRAN SARAFYAN and J. O. GETTYS, Louisiana State University, New Orleans, Louisiana 70122. Improvement of Aparo's numerical method for Volterra integral equations.

In 1959 Enzo Aparo developed a formula for the numerical solution of Volterra integral equations of second kind (Rend. Accad. Nazionale Lincei Ser. VIII (2) 26 (1959). Aparo's formula was similar to the well-known Runge-Kutta formula of third order. This method has been improved and a family of fourth order Runge-Kutta type formulas have been established by the authors for the numerical solution of Volterra's integral equations. Numerical examples show that the improved version is indeed superior to Aparo's original formula. (Received November 1, 1968.)

**663-471.** WOLFGANG LIEBERT, New Mexico State University, Las Cruces, New Mexico 88001. Endomorphism rings of modules over complete discrete valuation rings.

An ideal I of a ring R is called "nonradical" if I is not contained in \( J(R) \), the Jacobson radical of R. The \( J \)-adic topology of R is obtained by taking the powers of \( J(R) \) as neighborhoods of zero. \( E_R(M) \) denotes the R-endomorphism ring of the R-module M. **Theorem.** Let E be a ring and \( E_0 \) the sum of all its minimal nonradical right ideals. Then the following three properties I-III are equivalent. I. There exists a complete discrete valuation ring \( S \) and a divisible torsion \( S \)-module G such that E is isomorphic to \( E_R(G) \). II. There exists a complete discrete valuation ring \( R \) and a reduced complete torsion-free \( R \)-module H such that E is isomorphic to \( E_R(H) \). III. (i) E is Hausdorff and complete in its \( J \)-adic topology. (2) \( J(E) = pE \) where p is a central nonzero divisor in E. (3) \( 0 \neq E_0 \leq I \) for every nonradical two-sided ideal of E. (4) Let L be a closed left ideal with \( L \cap pE = pL \). If the right annihilator of L is zero then \( L \supseteq E_0 \). (5) The sum of two left annihilators whose intersection is zero is again a left annihilator. (6) E has an identity element. (Received November 1, 1968.)


Let S be a compact set in \( C^n \), n-dimensional complex space. For a complex (bounded) Borel
measure $\mu$ on $S$ define $\hat{\mu}(a_1,\ldots,a_n) = \int_{S}^{1}/(a_1 - x_1) \ldots (a_n - x_n) d\mu(x_1,\ldots,x_n)$. Let $m_{2n}$ denote $2n$-dimensional Lebesgue measure on $\mathbb{R}^n$. Lemma. $\hat{\mu}(a_1,\ldots,a_n)$ is defined a.e. $m_{2n}$ on $\mathbb{R}^n$ and if $\hat{\mu}(a_1,\ldots,a_n) = 0$ a.e. $m_{2n}$, then $\mu = 0$. For the case $n = 1$ the result has been proved by Bishop and is well known. Define $R(S)$ to be the uniform closure on $S$ of quotients $P(x_1,\ldots,x_n)/Q(x_1,\ldots,x_n)$ of polynomials in $n$-variables where $Q$ does not vanish on $S$. Corollary. If $\pi_j: \mathbb{R}^n \rightarrow S$ is the $j$th projection, and if $m_2(\pi_j(S)) = 0$ for $j = 1,\ldots,n$, then $R(S) = C(S)$. (Supported by the Bureau of Faculty Research, Western Washington State College.) (Received November 1, 1968.)


Let $(\Omega, \mathcal{F}, P)$ be a probability space. Continuing ideas of Daroczy in Über eine Verallgemeinerung des Entropiebegriffs der maßstreu Abdildungen, Publ. Math. Debrecen 12 (1965), 117-125, we define the following family of conditional entropy functions of a finite $\sigma$-subfield $\mathcal{F}$ of $\mathcal{G}$: Let $\pi(\mathcal{F})$ denote the atoms of $\mathcal{F}$, $\mathcal{B}$ be an arbitrary $\sigma$-subfield of $\mathcal{G}$, and $p$ and $q$ be real numbers such that either $q \geq 1$ and $p + q \geq 0$ or $0 \leq q < 1$ and $p + q \geq 1$. Then the conditional $(p,q)$ entropy of $\mathcal{F}$ given $\mathcal{B}$, denoted $H_{p,q}^B(\mathcal{F})$, is defined as $H_{p,q}^B(\mathcal{F}) = E[-\log \left(\sum_{\varphi \in \pi(\mathcal{F})} [P^B(\varphi)]^p [P^B(S)]^q \right)/P]$, $P \neq 0$, and $H_{0,0}^B(\mathcal{F}) = E[\log P^B(\mathcal{F})]$, where $E(\cdot)$ denotes expectation w.r.t. $P$ and $P^B(\mathcal{F})$ indicates the conditional probability of $F$ given $B$. Theorem 1. \(0 \leq H_{p,q}^B(\mathcal{F}) \leq \log N\) if $N = \text{card. of } \pi(\mathcal{F})$. Further $H_{p,q}^B(\mathcal{F}) = 0$ iff $\mathcal{F} \subset \mathcal{B}$. Theorem 2. Suppose $\mathcal{F} \subset \mathcal{B}$ and either $q \geq 1$ and $0 < p + q \leq 1$ or $0 < q \leq 1$ and $p + q \geq 1$. Then $H_{p,q}^B(\mathcal{F}) \leq H_{1,q}^B(\mathcal{F})$. One does not have the usual additivity property except for $p = 0$, $q = 1$. Theorem 3. Let $\mathcal{F}$ and $\mathcal{F}'$ be $\sigma$-subfields of $\mathcal{G}$ such that $\mathcal{F} \subset \mathcal{F}'$ and let $q = 1$, $-1 < p \leq 0$. Then $H_{p,1}^{\mathcal{G}}(\mathcal{F}) \geq H_{p,1}^{\mathcal{G}}(\mathcal{F}')$. Theorem 4. Let $\{\mathcal{F}_n\}$ be an increasing (decreasing) sequence of $\sigma$-subfields of $\mathcal{G}$ and $\lim_{n \to \infty} \mathcal{F}_n = \mathcal{B}$. Then if $q = 1$, $-1 < p$, then $\lim_{n \to \infty} H_{p,1}^{\mathcal{G}}(\mathcal{F}_n) = H_{p,1}^{\mathcal{G}}(\mathcal{B})$ and if $p \leq 0$, the convergence is monotone. (Received November 1, 1968.)


Rothman [Embedding of topological semigroups, Math. Ann. 139 (1960), 197-203] defines a concept called Property $F$ and shows that Property $F$ is a necessary and sufficient condition for embedding a commutative, cancellative topological semigroup in its group of quotients as an open subset. Rothman's result is generalized by defining a concept called Property $E$ and proving that a completely regular topological semigroup $S$ can be embedded in a topological group by a topological isomorphism if and only if $S$ can be embedded (algebraically) in a group and $S$ has Property $E$. It follows as a corollary that if $S$ is a commutative, cancellative topological semigroup and $S$ has Property $F$, then $S$ has Property $E$. (Received November 1, 1968.)


Let $G$ denote the group $SL(n,R)$ for $n > 2$. Consider those representations of $G$ that are induced by one-dimensional representations $\Delta$ of a minimal parabolic subgroup $NAM$. Denote this representation by $\mathcal{G}^\Delta$. Let $G_1$ be any parabolic subgroup containing $NAM$. Let $\mathcal{G}_1^\Delta$ denote the representation of $G_1$ induced by $\Delta$. It can be shown that if $\mathcal{F}$ is any subrepresentation of $\mathcal{G}^\Delta$ then $\mathcal{F}$ is equivalent
to a subrepresentation of $C_j^1$, the representation induced by a suitable subrepresentation $F_j^1$ of $C_i^A$. Moreover, if $F$ is an irreducible subrepresentation of $C^A$, then it can be shown that for each $G_i$ there exists an irreducible subrepresentation $F_j^1$ of $C_i^A$ such that $F$ is precisely the intersection $\bigcap F_j^1$ taken over all those parabolic subgroups $G_i \supset \text{NAM}$ which have a semisimple part isomorphic to $\text{SL}(2,\mathbb{R})$. Thus all the irreducible subspaces may be obtained by using classical results of Bargmann. The methods proceed by first obtaining analogous results for the algebraically irreducible Lie algebra representations obtained on the linear spaces of $K$-finite vectors. (Received November 1, 1968.)

663-476. ROBERT FITTLER, Rutgers University, New Brunswick, New Jersey 08903. Models with universal properties.

Let $T$ be a first order theory. $F$ is a set of formulas from $T$ which is closed under conjunction, disjunction, and existential quantification. An $F$-map between two models is a map which preserves all formulas of $F$. Theorem. A model $\varphi$ of $T$ admits precisely one $F$-map into each model of $T$ if and only if $\varphi$ is $F$-generated, i.e. the underlying set of $\varphi$ consists of equivalence classes of the determinant unary formulas of $F$, two such formulas being equivalent if this is provable in $T$ and the interpretation of the formulas of $F$ in $\varphi$ is determined by what can be proved to hold (in the theory $T$). (Received November 1, 1968.)


Theorem 1. There exists a monotone open map of the universal curve onto any continuous curve. This result was announced by R. D. Anderson in the Proc. Natl. Acad. Sci. U.S.A. (6) 42 (1956), but the proof was never published. Theorem 2. There exists a light open map of the universal curve onto any nondegenerate connected $n$-complex. The proofs of these theorems develop Anderson's techniques for constructing monotone open dimension raising mappings (Duke Math. J. 19 (1952)). The main tools are his characterization of the universal curve (Ann. of Math. (2) 67 (1958)) and R. H. Bing's theory of partitioning (Bull. Amer. Math. Soc. 58 (1952)). (Received November 1, 1968.)

663-478. LEWIS ROBERTSON, University of Washington, Seattle, Washington 98105. The structure of Moore groups.

Let $[\text{Moore}]$ denote the class of all locally compact groups such that every continuous irreducible unitary representation is finite dimensional. Compact groups and abelian groups are examples of Moore groups. Using a recent characterization due to C. C. Moore, it follows that $G \in [\text{Moore}]$ iff $G$ is built up in a certain specific fashion out of a compact group, an abelian group, and a finite group. Closed subgroups of Moore groups are Moore groups, and every Moore group $G$ contains a closed normal subgroup $V = \mathbb{R}^B$ and a closed subgroup $B$ with compact identity component such that $G$ is topologically the direct product of $V$ and $B$. In other words, $G$ is a semidirect product of a vector group and a Moore group which has compact identity component. The centralizer of $V$ must have finite index in $G$, hence $G$ is "almost" a direct product. (Received November 1, 1968.)

G. J. Janusz defined a ring R to be a splitting ring for a group G of order n invertible in R if the group algebra RG is the direct sum of central separable R-algebras, each equivalent to R in the Brauer group of R, that is, \( RG \cong \bigoplus_{i=1}^{S} \text{Hom}_R(P_i, P_i) \) where \( \{P_i\} \) are finitely generated projective faithful R-modules, \( i = 1, 2, \ldots, s \) (the number of different conjugate classes of G is equal to s) (see Separable algebra over commutative rings, Trans. Amer. Math. Soc. 122 (1966), 461-479). Theorem 1. If R is a commutative ring and the order of a finite group G is invertible in R, then \( R[\frac{m}{1}] \) is a splitting ring for G where \( \frac{m}{1} \) is a primitive mth root of 1. Theorem 2. If R is a splitting ring for G and all finitely generated projective indecomposable R-modules are of rank 1, then there are exactly \( s \) classes of finitely generated projective indecomposable left RG-modules over different central components each uniquely determined up to an element in the class group of R. Corollary. If the class group of R is trivial, then there are exactly \( s \)-isomorphic classes of finitely generated projective indecomposable left RG-modules. (Received November 1, 1968.)

663-481. MARK MANDELKER, University of Kansas, Lawrence, Kansas 66044. Round \( \beta X \) and round subsets of \( \beta X \).

Remote points in \( \beta R \) are characterized here as follows: A prime z-filter \( \mathcal{F} \) on R has the property that for every zero-set \( Z \in \mathcal{F} \) there exists a zero-set \( W \in \mathcal{F} \) such that \( W \subseteq \text{int } Z \) if and only if \( \mathcal{F} = \mathcal{P} \) for some remote point \( p \) in \( \beta R \). This property of z-filters is generalized to a completely regular Hausdorff space X as follows: A z-filter \( \mathcal{F} \) on X is said to be round if for every zero-set \( Z \in \mathcal{F} \) there is a zero-set \( W \in \mathcal{F} \) and a cozero-set \( S \) in X such that \( W \subseteq S \subseteq Z \). A round z-filter is characterized as the intersection of the z-filters \( \mathcal{P} \) for \( p \) in some closed subset of \( \beta X \). A related result characterizes the family of z-filters on X having a prescribed set of cluster points in \( \beta X \). The concept of remote point is generalized as follows: A subset \( A \) of \( \beta X \) is said to be \( \text{round} \) if, for any zero-set \( Z \) in \( X \), \( \text{cl } \beta_X X \) contains \( A \) then it is a neighborhood of \( A \). Among closed subsets of \( \beta X \), a round set is characterized as the set of cluster points in \( \beta X \) of a unique z-filter on X. Some examples are given. It is shown that every \( G \) in \( \beta X \) that does not meet X is round. An immediate corollary is the well-known result [Gillman-Jerison, Theorem 8.19] that for realcompact X, the intersection of all the free maximal ideals in C(X) is the family of all functions with compact support. (Received November 1, 1968.)


Let \( w = f(z) = z + a_2 z^2 + a_3 z^3 + \ldots \) be regular and univalent in the unit disk D and map D onto a region R which is starlike with respect to \( w = 0 \). Let \( d^* \) be the distance of the closest point of the boundary of R from \( w = 0 \). Let \( r_0 \) be the radius of convexity of \( w = f(z) \) and \( d_0 \) be the shortest distance from \( w = 0 \) to \( w = f(t_0 e^{i\theta}) \) for \( 0 \leq \theta < 2\pi \). It has been conjectured that \( d_0/d^* > 2/3 \). In this paper the conjecture is demonstrated for certain classes of functions, while for other functions lower estimates for \( d_0/d^* \) are found. It is shown that \( d_0/d^* > 13^{-1}(15 - 6\sqrt{3}) = .354... \) which improves
the estimate $d_0/d^* > 2 - \sqrt{3}$. Furthermore, it is demonstrated that if the mapping function $w = f(z) = z + a_{p+1}z^{p+1} + a_{2p+1}z^{2p+1} + ...$ has $p$-fold rotational symmetry, then $d_0/d^* > (p + 2)^{-1}/p$ for $p = 1, 2, ...$. In finding estimates for $d_0/d^*$, the effect of $a_{p+1}$ on certain properties of the function is discussed. In particular, the sharp lower bound for the radius of convexity is found when $a_{p+1}$ has a preassigned value. (Received November 1, 1968.)

663-483. P. L. SPERRY, University of South Carolina, Columbia, South Carolina 29208. Duals of the divisible hull of an abelian group.

It is possible to give at least three statements equivalent to the statement that $D$ is a divisible hull of the group $G$ (all groups are abelian). These statements may be formulated so that they can be dualized by the usual method of arrow reversal in the category of abelian groups. If $G$ is a group for which there is a free group such that a dual statement is true, then depending on which statement is true--$G$ is free, $G$ has a minimum system of generators in the sense of Khabbaz [Trans. Amer. Math. Soc. 98 (1961), 527-538] or $G$ satisfies the still weaker condition that there is a free group $F$ and epimorphism $f$ of $F$ onto $G$ such that if $F'$ is a free group and $f'$ is an epimorphism of $F'$ onto $G$, then there is a monomorphism $g$ of $F$ into $F'$ such that $f'g = f$. (Received November 1, 1968.)

663-484. BARTH POLLAK, University of Notre Dame, Notre Dame, Indiana 46556. Orthogonal groups over global fields of characteristic 2.

Let $F$ be a global field of characteristic 2 and $V$ a nondefective quadratic space over $F$. Let $O(V)$ denote the orthogonal group of $V$, $O'(V)$ its spinorial kernel, and $\Omega(V)$ its commutator subgroup.

**Theorem.** $O'(V) = \Omega(V)$. Since $V$ is isotropic if $\dim V > 4$, the interesting case is when $V$ is 4-dimensional anisotropic. The problem comes down to the construction of a suitable 2-dimensional nondefective space which is represented by $V$. To accomplish this, we need to use the Strong Approximation Theorem and the Reciprocity Law. This is in contrast to the case of characteristic $\neq 2$ where it is enough to construct a suitable 1-dimensional space which is represented by $V$, a task for which the Weak Approximation Theorem suffices. (Received November 1, 1968.)

663-485. E. C. BOES, New Mexico State University, Las Cruces, New Mexico 88001. The Wang sequence and some calculations in $K$-theory.

Associated to a fibration $F \to E \to S^P$, there is a Wang sequence for $K_\Lambda$ theory, $\Lambda = R$ or $C$, relating $K_\Lambda(F)$ and $K_\Lambda(E)$. The maps in this sequence have multiplicative and naturality properties similar to those of the corresponding maps in the Wang sequence for ordinary cohomology. This Wang sequence is used to calculate $K'_C(\Omega E)$ and $K'_C(SO(2n)/U(n)) \otimes Q$. Let $\widetilde{U}(n)$ be the two-fold covering of $U(n)$. Knowledge of $RU(\widetilde{U}(n))$ permits a calculation of $K'_C(SO(2n)/U(n));$ this leads to a new proof that $S^2$ and $S^6$ are the only almost-complex spheres. (Received November 1, 1968.)

663-486. J. D. DIXON, Carleton University, Ottawa 1, Ontario, Canada. Criteria for nilpotence of certain subgroups.

It is well known that a finite group is nilpotent if and only if each subgroup is subnormal. We can generalize this criterion as follows. Let $G$ be a finite group with lower central series
G = \gamma_1(G) \triangleleft \gamma_2(G) \triangleleft \ldots \text{ Theorem 1. If, for some integer } r \geq 1, \gamma_r(H) \text{ is subnormal in } G \text{ for each subgroup } H \text{ of } G, \text{ then } \gamma_r(G) \text{ is nilpotent. Theorem 2. If } G \text{ is solvable and, for some integer } r \geq 1, \text{ the } r\text{th derived group } H^{(r)} \text{ is subnormal in } G \text{ for each subgroup } H, \text{ then } G^{(r)} \text{ is nilpotent. For both theorems the converse is nearly trivial even without the assumption that } G \text{ is finite. The hypothesis of the second theorem cannot be weakened from "solvable" to "p-solvable" (p > 2) if the conclusion is to remain valid. (Received November 4, 1968.)}

663-487. WITHDRAWN.


R. P. Isaacs in his original paper on monodiffric functions (Univ. Nac. Tucuman Rev. 2 (1941), 177-201) as well as G. J. Kurowski in a latter publication (Pacific J. Math. 18 (1966), 139-147) investigated operations which are analogous to multiplication of analytic functions of a complex variable. In this paper a different approach is taken to the same problem. A new line integral is introduced with the help of which a convolution product is defined for monodiffric functions. It is shown that with respect to pointwise addition and a properly modified convolution product, the set of functions monodiffric on the Gaussian integers forms a commutative integral domain with identity. Furthermore, it is pointed out that the pseudo-powers of z satisfy the law of exponents, and the role of convolution products in obtaining monodiffric solutions for difference equations is emphasized. (Received November 1, 1968.)

663-489. R. J. GRIEGO and REUBEN HERSH, University of New Mexico, Albuquerque, New Mexico 87106. The expectation semigroup of a random evolution.

Consider n laws of evolution represented by n compatible semigroups. Suppose we switch back and forth randomly between the various laws. Then the average result defines a new law of evolution. More formally, for each i = 1, ..., n, let T_i(t) be a strongly continuous semigroup of bounded linear operators on a fixed Banach space B of real-valued functions defined on some space E. Let \( v(t) \) be a stationary Markov chain with state space \( I = \{1, \ldots, n\} \) and infinitesimal matrix Q. Define the 'random evolution' M(t) on B by \( M(t) = \sum_{i=1}^{n} T_i(0)^{(1)} T_i(1)^{(2)} T_i(2)^{(3)} \ldots T_i(N(t)) \), where \( \tau_1, \tau_2, \ldots \) are the successive jump times for \( v \) and \( N(t) \) is the number of jumps up to time t. Let \( \mathfrak{B} = B \times \ldots \times B \) (n times). For \( \vec{f} = (f_1, \ldots, f_n) \) in \( \mathfrak{B} \) let \( \|\vec{f}\| = \sum_{1 \leq i \leq n} f_i \). \( \mathfrak{B} \) is a Banach space under this norm. Define the 'expectation semigroup' \( \hat{T}(t) \) on \( \mathfrak{B} \) by \( \hat{T}(t)\vec{f}(x) = \mathbb{E}_v[M(t)\chi_{\{v\}}(x)] \) for \( x \) in E and \( i = 1, \ldots, n \). Theorem. \( \hat{T}(t) \) is a strongly continuous semigroup of bounded linear operators on \( \mathfrak{B} \) with infinitesimal generator \( \hat{A} = \text{diag}(A_1, \ldots, A_n) + Q \) where \( A_i \) is the infinitesimal generator of \( T_i(t) \). (Received November 4, 1968.)

663-490. REUBEN HERSH and R. J. GRIEGO, University of New Mexico, Albuquerque, New Mexico 87106. Probabilistic solutions of hyperbolic systems.

Let there be given linear operators \( A_i, i = 1, \ldots, n \), which generate compatible semigroups
\( T_i(t), \) \( t > 0, \) and given a matrix \( Q = (q_{ij}) \) satisfying \( q_{ij} \geq 0 \) for \( i \neq j, \sum_j q_{ij} = 0, \) \( i, j = 1, \ldots, n. \) Then Cauchy's problem for the system of equations \( u_t^i = A_i u^i + \sum_j q_{ij} u^j \) is solved by the expectation semi-group \( \tilde{T}(t) \) (see Abstract 663-489, R. J. Griego and R. Hersh, these Notices above). If the \( A_i \) are second-order parabolic differential operators, we obtain a stochastic interpretation for the most general parabolic system having a nonnegative fundamental solution. If the \( A_i \) commute with each other, \( \tilde{T}(t) \) reduces to the expected value of \( \prod_{i=1}^n T_i(\gamma_i(t)), \) where \( \gamma_i(t) \) is the occupation time in the \( i \)-th state of a Markov chain with generator \( Q. \) By taking \( A_i = c_i \frac{d}{dx}, \) we obtain a solution formula for a first-order one-dimensional hyperbolic system. By taking \( n = 2, A_1 = -A_2, q_{11} = q_{22} = -a, \) we obtain a system equivalent to the second order equation \( u_{tt} = A^2 u - 2u_{xt}. \) For \( A^2 = \sum \frac{\partial^2}{\partial x_k^2} \) we get Kac's solution of the telegraph equation as an average of randomized solutions of the wave equation. (Received November 4, 1968.)


Let \( G \) be a compact abelian group with character group \( H. \) Each closed subalgebra \( A \) of the group algebra \( L^1(G) \) induces an equivalence relation \( \sim \) on \( H : a \sim b \) if and only if \( \hat{f}(a) = \hat{f}(b) \) for every \( f \) in \( A, \) where \( \hat{f} \) is the Fourier transform of \( f. \) An equivalence relation \( \sim \) on \( H \) is said to have synthesis if it is induced by exactly one closed subalgebra, Kahane ("Idempotents and closed subalgebras of \( A(Z)\)" in Function algebras, Scott, Foresman, Chicago, 1966) has shown that any equivalence relation \( \sim \) on \( Z \) (the integers) such that \( n \sim m \) implies \( |n - m| \leq M \) (\( M \) a constant) has synthesis. A stronger result is proven, with the new condition: \( n \sim m \) and \( |n| \leq |m| \) implies \( |n - m| \leq M \sqrt{|n|} \). A similar method of proof will yield Kahane's weaker result for \( Z \times Z. \) Nontrivial necessary and sufficient conditions are given for synthesis in arbitrary discrete abelian groups. Finally, a theorem about factorization in closed subalgebras is proven. This theorem leads to the existence of a closed subalgebra \( A \) without factorization, i.e. \( A \neq A \ast A, \) yet \( A \) has the property that \( f \in A \) implies \( f \in \text{closure} (\{f \ast g : g \in A \}). \) (Received November 4, 1968.)


Let \( q : E \to B \) be the projection in a fibration with \( n \)-connected fibre \( F. \) Assume \( q \) induces an epimorphism in mod \( p \) cohomology and suppose \( [X, B] \) and \( [X, E] \) have natural group structures. Then there is defined a natural filtration \( \{F_r\} \) of the Adams type on \( [X, B] \) such that \( f \in [X, B] \) satisfies \( f \in \cap F_{r} \) if and only if \( f \) has finite order prime to \( p \) in \( [X, B]/q_*[X, E]. \) The resulting spectral sequence has \( E_2 = \text{Ext}_A(B)(H^*(B, E), H^*(X)) \) in total degrees not exceeding \( 2n. \) If \( \xi'' \) is the Whitney join of two such fibrations \( \xi \) and \( \xi', \) then the spectral sequences for \( f : E \to B \) and \( f' : E' \to B' \) are paired to that for \( f'' = f \times f'. \) The product on the \( E_2 \)-level coincides with the one obtained from the external cup-product in \( \text{Ext} \) by a change-of-rings homomorphism \( A(B'') \rightarrow A(B) \otimes A(B') \) induced by the diagonal map of \( A. \) (Received November 4, 1968.)

663-493. N. D. KAZARINOFF, University of Michigan, Ann Arbor, Michigan 48104, and E. D. ROGAK, University of Victoria, Victoria, B. C., Canada. Exterior problems for semilinear hyperbolic equations in time-dependent domains.

For \( x \in \Omega(t), \) the exterior of a bounded, smooth homeomorph of \( S^{n-1} \) for each \( t, \) and \( t > 0, \) 229
we consider finding $u$ such that

$D^2_t u - D_j (a_{ij} D_i u) + a_i D_i u + au = -f(t, x, u, D u, \ldots, D^m u),
\|u\|_{\infty}(t) = 0,
u(0, x) = U_0(x), u_t(0, x) = U_1(x),$

where $a_{ij}(x, t), a_i(x, t), f, U_0,$ and $U_1$ are given functions, $a_{ij} \xi_i \xi_j \geq c^2 \xi_i \xi_j$ uniformly in $x$ and $t,$ and $0 < i, j \leq n.$ Theorem 1. Under appropriate smoothness conditions if

(i) the supports of $f, U_0,$ and $U_1$ are compact, (ii) $f$ is independent of grad $u,$ and (iii) for each $T > 0$ there exists an $M_T > 0$ such that $f \in L^2(U_t \in [0, T] \Omega(t) \times [t])$ for all $u$ in that space, then there exists at least one weak solution to $(*)$. The main tools in the proof are the energy inequality and the Leray-Schauder fixed point theorem. Theorem 2. Under appropriate smoothness conditions if $f$ satisfies a Lipschitz condition in its last $n + 2$ arguments, there exists a unique solution to $(*)$. Of course, we assume that the lateral boundary of region is time-like. (Received November 4, 1968.)

663-494. PATRICK SUPPES, Stanford University, Stanford, California 94305. The logic of intuitive geometry.

Automaton models for the processing of geometrical perceptions and judgments about geometrical figures are considered. The problem of inferring the structure of the automaton from observable responses is considered, as well as some of the limitations of this approach to the geometry of perception. (Received November 4, 1968.)


Analytic criteria are established for the existence of periodic solutions of both general autonomous systems $\dot{x} = P(x, y),$ $\dot{y} = Q(x, y)$ and general nonautonomous systems $\dot{x} = P(x, y, t), \dot{y} = Q(x, y, t).$ In the former case, the existence of a stable limit cycle is proved whenever $x = 0, y = 0$ is the unique critical point. With regard to the latter, it is assumed that both $P$ and $Q$ are bounded periodic functions of $t$ with least period $\omega > 0$ and that for no point $(x, y)$ are $P$ and $Q$ simultaneously vanishing for every $t.$ The existence of at least one nonconstant $\omega$-periodic solution is then deduced. Principal use is made throughout of the Poincaré-Bendixson Theorem, Brouwer's Fixed-Point Theorem, and Lyapanov Function techniques. The theorems we present include as special cases the van der Pol equation $\dot{x} - x + x = 0 (\mu > 0), and the system $\dot{x} = \mu_1 (1 - y^2) x + y + e_1(t), \dot{y} = \mu_2 (1 - x^2) y - x + e_2(t) (\mu_1, \mu_2 > 0).$ More generally, they include the Liénard equation $\dot{x} - f(x) x + x = 0$ and Liénard type systems of the form $\dot{x} = f_2(y) + y + e_1(t), \dot{y} = f_1(x) y - x + e_2(t)$ whenever there exist numbers $a, b, m > 0$ such that $f(x), f_1(x) < - m$ for $|x| > a, f_2(y) < - m$ for $|y| > b,$ and $e_1(t), e_2(t)$ are bounded and $\omega$-periodic. (Received November 4, 1968.)

663-496. CYNIL NASIM, University of Calgary, Calgary 44, Alberta, Canada. Results relating the behaviour of Fourier transforms near the origin and at infinity.

It is known [Titchmarsh, P. I. Theorem 126] that under special conditions Fourier sine and cosine transforms behave asymptotically like a power of $x,$ either as $x \to 0$ or $x \to \infty$ or both. These relations suggest that other results connecting the behaviour of a function at infinity with the behaviour of its Fourier or Watson transform near the origin might exist. Theorem 1. If $g(x) \in L(0, \infty)$ and has a $\pi/2 - 1 J(2 \pi x^{1/2})$ transform $f(x), x^{1/2 - p/4} g(x)$ is of bounded variation near the origin, $\tau > (p - 1)/2$ and $p$ is a positive integer, then $\lim_{x \to \infty} (\pi/2, \Gamma(p/2)) \int_{0}^{x} x^{p/4 - 1/2} (x, (1 - x/T)^{1/2} dx = \lim_{x \to \infty} x^{1/2 - p/4} g(x).$ Note that by putting $p = 1, 2$ helps in modifying Poisson's and Hardy-Landau
summation formulae respectively. Theorem 2. If \( g(x) \in L(0,\infty) \) and has a \( \tau f(x) \) transform \( f(x) \) and \( x^{1/4} g(x) \) is of bounded variation near the origin, then for \( \nu > 1 \),
\[
\lim_{T \to -\infty} \int_{0}^{T} e^{-\nu x / 2} f(x) dx = \pi^{1/2} (\Gamma(\nu/2 + 3/4) / \Gamma(\nu/2 + 5/4)) \lim_{\lambda \to 0} x^{1/4} g(x).
\]
Note that by putting \( \nu = 1/2 \), we obtain an expression for the behaviour of Fourier sine transform near the origin which cannot be obtained directly from Fourier inversion formula. (Received November 4, 1968.)

663-497. M. B. SURYANARAYANA, University of Michigan, Ann Arbor, Michigan 48104.


Let \( G = [0,h] \times [0,k], U \subset E^m, \Gamma = \{ u : G - U, u \in [L_{\infty}(G)]^m \} \), and \([W^1(G)]^n\) the cartesian product of Sobolev spaces. Consider the boundary value problem (*) \( z_{i x y}(x,y) = f_i(x,y); z_i(x,0) = \phi_i(x); z_i(0,y) = \psi_i(y); i = 1,2,\ldots, n \), where \( \phi_i \) and \( \psi_i \) are given absolutely continuous functions and \( z = (z_1,z_2,\ldots,z_n) \). We seek the minimum of the functional \( I = \sum_{i=1}^{n} A_i z_i(h,k) \).

By writing (*) in Rashevsky-Dieudonné form (L. Cesari, Arch. Rational Mech. Anal. 29 (1968), 81-104), we can apply Cesari’s analysis of optimization problems (loc. cit., to appear) which in the present case requires \( 4n \) multipliers \( \lambda_{11}, \mu_{11}, \lambda_{21}, \mu_{21} \) satisfying (**)
\[
\lambda_{11} \mu_{11} = - \sum_{i=1}^{n} \left( \lambda_{2j} + \mu_{2j} \right) f_{j x} \alpha_{2j} z_i + \mu_{21} = - \sum_{i=1}^{n} \left( \lambda_{2j} + \mu_{2j} \right) f_{j y} \alpha_{2j} z_i; \lambda_{21} = - \sum_{i=1}^{n} \left( \lambda_{2j} + \mu_{2j} \right) f_{j x} \alpha_{2j} y_i = - \mu_{11} = \sum_{i=1}^{n} \left( \lambda_{2j} + \mu_{2j} \right) f_{j y} \alpha_{2j} y_i \lambda_{11} (h,y(x,k); \lambda_{21} (h,y(x,k); i = 1,2,\ldots, n \). Conditions are given, under which, for each \( u \) in \( T \), there is a \( z \) in \([W^1(G)]^n\) such that the generalized mixed partial \( z_{xy} \) exists and (*) is satisfied a.e. in \( G \). Also, it is proved that corresponding to these \( u \) and \( z \), there exist multipliers in \( L_{\infty}(G) \) satisfying (** a.e. in \( G \). Using these multipliers, a minimum principle is proved as in A. I. Egorov (Automat. Remote Control 25 (1964), 557-566) but under weaker smoothness conditions. Illustrative examples are given. (Received November 4, 1968.)


Fixed points of quasi-nonexpansive mappings. Preliminary report.

A self-mapping \( T \) of a subset \( C \) of a normed linear space is said to be quasi-nonexpansive on \( C \) provided \( T \) has at least one fixed point in \( C \) and if \( p \in C \) is any fixed point of \( T \) then
\[
\| Tp - p \| \leq \| x - p \| \text{ holds for all } x \in C. \text{ Theorem 1. If } C \text{ is a closed convex subset of a strictly convex normed linear space, and } T : C \to C \text{ is quasi-nonexpansive on } C, \text{ then } F(T) = \{ p : p \in C \text{ and } Tp = p \} \text{ is a closed convex set on which } T \text{ is continuous. Theorem 2. If } C \text{ is a compact convex subset of a strictly convex normed linear space, and } T : C \to C \text{ is quasi-nonexpansive on } C, \text{ and } S : C \to C \text{ is continuous on } C, \text{ then } F(T) \cap F(S) \neq \emptyset. \text{ Remark. There exist a compact convex set } C \subset l^\infty, \text{ a quasi-nonexpansive mapping } T : C \to C, \text{ and a nonexpansive mapping } S : C \to C, \text{ such that } TS = ST \text{ but } F(T) \cap F(S) = \emptyset. \text{ Theorem 3. If } C \text{ is a closed bounded convex subset of a uniformly convex Banach space, and } T : C \to C \text{ is quasi-nonexpansive on } C, \text{ and } S : C \to C \text{ is either nonexpansive or weakly continuous on } C, \text{ and } TS = ST \text{ but } F(T) \cap F(S) = \emptyset. \text{ Theorem 4. If } C \text{ is a weakly compact convex subset of a strictly convex normed linear space, and } \{ T_a \} \text{ is a commutative family of quasi-nonexpansive self-mappings of } C, \text{ then } \bigcap_a F(T_a) \neq \emptyset. \text{ (Received November 4, 1968.)}

663-499. J. S. BRADLEY, University of Tennessee, Knoxville, Tennessee 37916. Oscillation theorems for a linear delay equation of the second order.

Consider the second-order linear delay equation \( (r(x)y'(x))' + p(x)y(x - \tau(x)) = 0 \), where
r(x) > 0, p(x) > 0, 0 \leq r(x) \leq M, and r,p,\tau are continuous on some ray [a,\infty). Theorem. If 
\int_{-\infty}^{\infty} p = \infty, \int_{-\infty}^{\infty} 1/r = \infty, then all solutions have arbitrarily large zeros. One proof of this theorem uses 
techniques usually associated with oscillation theory for nonlinear equations and this proof may be 
modified to allow for various nonlinearities. A second proof, valid for r(x) = 1, uses "linear" 
techniques and is of interest since it suggests that it may be possible to relax the requirement that 
p(x) > 0. (Received November 4, 1968.)

663-500. E. L. MARSDEN, Kansas State University, Manhattan, Kansas 66502. The commutator 
in an orthomodular lattice.

For elements e and f in an orthomodular lattice, define the commutator of e and f by [e,f] = 
(e \lor f) \land (e' \lor f') \land (e' \lor f'). Then [e,f] = 0 iff e commutes with f. Theorem. Let A be a Baer 
*-algebra, and let P'(A) be the orthomodular lattice of closed projections of A. For e, f \in P'(A), 
(e - fe)' is the smallest closed projection which serves as a right identity for ef - fe. The element 
(e - fe)' = [e,f]. Theorem. Let L be an orthomodular lattice, and let J be the p-ideal (normal ideal) 
generated by the commutators in L. Then L/J is Boolean. Moreover, if I is any p-ideal for which 
L/I is Boolean, then I \supseteq J. (Received November 4, 1968.)

663-501. R. D. SINKHORN and MARK HEDRICK, University of Houston, Houston, Texas 77004.
Concerning nearly reducible matrices.

A nonnegative matrix A is said to be nearly reducible if A is irreducible but A - a_{ij}E_{ij} is 
reducible whenever a_{ij} > 0. E_{ij} is a (0,1)-matrix with a 1 only in the (i,j) position. Theorem. A 
nearly reducible doubly stochastic matrix is a full cycle permutation. Corollary. A nearly reducible 
(0,1)-matrix has at most one positive diagonal. (Received November 4, 1968.)

663-502. S. L. WARNER, Duke University, Durham, North Carolina 27706. Locally compact 
principal ideal domains.

Let A be an indiscrete, locally compact, metrizable principal ideal domain. If A is not a field, 
A possesses a unique maximal ideal \( \text{Ap} \), \( \text{Ap} \) is topologically nil, and all the powers \( \text{Ap}^n \) of \( \text{Ap} \) are open. 
If A is separable or if A possesses a compact open noetherian subring, then A is either a locally 
compact field or the (compact) valuation ring of an indiscrete, totally disconnected, locally compact 
field. If A is not a field, the residue field of A has prime characteristic. If A has prime characteristic 
and is not a field, A contains a unique discrete "coefficient field" F that is canonically isomorphic to 
its residue field; F is necessarily an absolutely algebraic field, and A is compact if F is finite; any 
absolutely algebraic field of prime characteristic may occur in this capacity. (Received November 4, 
1968.)

663-503. M. S. BERGER, University of Minneapolis, Minneapolis, Minnesota 55455. On 
periodic solutions of Hamiltonian systems near a stationary point.

Consider the periodic solutions near \( x = 0 \) of the system of \( n \) second order equations 
(*) \( \ddot{x} + Ax + f(x) = 0 \), where A is a selfadjoint matrix with positive eigenvalues \( a_1^2, a_2^2, \ldots, a_k^2 \) \( 0 < k \leq n \), 
and \( f(x) = \text{grad} F(x) \) is a locally Lipschitz \( n \)-vector function, such that \( |f(x)| = o(|x|) \). Theorem. For
each \( i (i = 1, 2, \ldots, k) \), the system (*) has a one parameter family of distinct periodic solutions \( x_i(R) \) (for \( R \) sufficiently small) with period \( T_i(R) \), such that as \( R \to 0, T_i(R) = 2\pi/a_i \) (the period of the \( i \)th normal mode of the linearized system). Furthermore if \( f(x) \) is real analytic, \( x_i(R) \) and \( T_i(R) \) depend continuously on \( R \). (This result is an extension of the Liapunov center theorem to the case of "resonance"). (Received November 4, 1968.)

663-504. T. P. WRIGHT, Florida State University, Tallahassee, Florida 32306. Extending isotopies of \( S^{n-1} \) in \( S^n \).

Given an isotopy \( f_t: S^{n-1} \to S^n \), can one find an ambient isotopy \( F_t: S^n \to S^n \) such that \( F_t|S^{n-1} = f_t \)? A necessary condition is that, for each \( t \), \( f_t \) is a locally flat embedding of \( S^{n-1} \) in \( S^n \). The sufficiency of this condition is not known. Theorem. Let \( f_t: S^{n-1} \to S^n, t \in [0,1], \) be an isotopy. Let \( C \) be a bicollar on \( S^{n-1} \) in \( S^n \). Suppose each \( \tau \in [0,1] \) has a neighborhood \( N(\tau) \) such that \( f_t \) extends to an isotopy of \( C \) in \( S^n \) for \( t \in N(\tau) \). Then there is an ambient isotopy \( F_t: S^n \to S^n, \) \( t \in [0,1], \) such that \( F_t|S^{n-1} = f_t \). (Received November 4, 1968.)

663-505. C. E. LANGENHOP, University of Kentucky, Lexington, Kentucky 40506. Laurent expansion for a nearly singular matrix.

Let \( A_0, A_1 \) be \( n \times n \) matrices of complex numbers and let \( C^n \) be the vector space of \( n \times 1 \) matrices of complex numbers. Let \( N_1 = \{ x \in C^n | A_1 x = 0 \}, N_{0,-1} = \{ 0 \} \subset C^n, \) and for \( k \geq -1 \) define \( R_{1k} = A_1 N_{0k} \) and \( N_{0k+1} = \{ x \in C^n | A_0 x \in R_{1k} \} \). In any case \( \mu = \min \{ k \geq -1 | N_{0,k+1} = N_{0k} \} \) exists and \( \mu \neq 0 \) or \( \mu = -1 \) according as \( A_0 \) is singular or not. Theorem. There exists \( \delta > 0 \) such that the matrix \( A_0 + zA_1 \) is invertible for all complex numbers \( z \) such that \( 0 < |z| < \delta \) if and only if \( N_1 \cap N_{0k} = \{ 0 \} \) for all \( k \geq 0 \). Moreover, if this holds then there exist \( n \times n \) matrices \( Q_k \) such that \( (A_0 + zA_1)^{-1} = \sum_{k=\mu}^{\infty} z^k Q_k \), the series converging for \( 0 < |z| < \delta \) for some \( \delta > 0 \), and \( Q_{-\mu-1} \neq 0 \). Standard results from linear algebra are used in analyzing the action of \( A_1 \) and \( A_0 \) on \( R_{1k} \), \( N_{0k} \) and other similar subspaces in order to define the coefficient matrices \( Q_k \). (Received November 4, 1968.)


Let \( u'(x) = f(x,u(x),lu(x)), u(a) = \mu \) and \( lu(x) = \int_a^x k(x,s,u(s)) ds \). Call this the Volterra integro-differential equation (VIDE). Techniques are discussed for the development of Runge-Kutta type methods for the numerical solution of the VIDE. The discussion is based on a general numerical analysis theory for such problems developed by the authors and reported upon at the 1968 fall SIAM meeting. We show how to develop Runge-Kutta algorithms which involve evaluations only of the functions \( f \) and \( k \). No evaluations of any derivatives of either \( f \) or \( k \) are required, yet the results obtained are as accurate (of order \( p \)) as if \( p - 1 \) derivatives of both \( f \) and \( k \) had been used. For our 4th order algorithm, the increment function requires five evaluations of \( f \). In the ordinary differential equations case, four evaluations suffice. The authors as yet have been unable to discover a suitable set of four such evaluations which will provide the necessary approximations for VIDE. They leave this as an open question. (Received November 4, 1968.)
On higher-dimensional fibered knots.

Let $k : S^n \rightarrow S^{n+2}$ be a smooth knot such that $S = S^{n+2} - k(S^n)$ fibers over $S^1$ with fiber $F$. Now $\pi_2(S)$ is the semidirect product of $\pi_1(F)$ and $Z(t)$. Since $\pi_1(S) \cong \pi_1(F)$, $t \cong 2$, $Z(t)$ may be considered to act on $\pi_1(F)$. If a presentation of $\pi_1(F)$ is given, then the action of $t$ on $\pi_1(F)$ gives rise to a matrix $A$. Theorem. If $M$ is a presentation matrix for $\pi_1(F)$ as a $\pi_1(F)$-module, then $\begin{pmatrix} M \\ A \end{pmatrix}$ is a presentation matrix for $\pi_2(S)$ as a $\pi_1(F)$-module. Theorem. If $k : S^2 \rightarrow S^4$ is the 5-twist spun trefoil knot, then $\pi_2(S)$ as a $\pi_1(F)$-module has a presentation matrix $\begin{pmatrix} G \\ t-1 \end{pmatrix}$ as a presentation matrix $G = [\pi_1(S), \pi_1(S)]$.

On the construction of the Kurosh lower radical class for associative rings.

All rings considered are to be associative. Call a subring $S$ of a ring $R$ accessible through zero extensions if there exist $S = A_0 \triangleleft A_1 \triangleleft \ldots \triangleleft A_n = R$, where $(A_i/A_{i-1})^2 = (0)$. Let $P \neq \emptyset$ be a class of rings which is homomorphically closed. Define $P_1 = \{R | R has a P-subring accessible through zero extensions\}$, and $P_2 = \{R | each nonzero homomorphic image of R has a nonzero $P_1$-ideal\}.

Let $L(P)$ denote the Kurosh lower radical class generated by $P$. The following results have been obtained. $P_2$ is a radical class which contains $L(P)$. If $L(P)$ contains all zero rings then $P_2 = L(P)$, but in general it is not true that $P_2 = L(P)$. If $P$ is hereditary then $P_2$ is hereditary. If $P$ contains no complete matrix rings over division rings and if $A$ is a ring with $D,C,C,$ on left ideals then $W(A) = P_2(A)$ where $W(A)$ is the classical Wedderburn radical of $A$ and $P_2(A)$ is the $P_2$ radical of $A$.

Factorizable semigroups.

A multiplicative semigroup $S$ is said to be factorizable if it can be written as the set product $S = AB$ of proper subsemigroups $A$ and $B$. In this case, $A$ and $B$ are called factors of the factorization $AB$. Two general problems considered are (I) Given a factorizable semigroup $S = AB$, where $A$ and $B$ are semigroups of known types, to what semigroup class does $S$ belong? (II) Can factorizations be used to characterize direct products of semigroups? Answers to (I) are given for factorizations whose factors are such semigroups as cyclic semigroups, inverse semigroups, completely simple semigroups, and groups. An affirmative answer is given (II); semigroup direct products $S \times T$ in which $S$ and $T$ each contain a certain type of idempotent are given an internal characterization which involves the existence of a particular factorization.

The existence of nontrivial universal crumpled cubes.

Notation. If $C$ and $D$ are disjoint crumpled cubes and $h$ is a homeomorphism of $Bd C$ onto $Bd D$, then $C \cup_h D$ is the space $C_1 \cup C_2$ where $x$ and $h(x)$ are identified. Definition. A crumpled cube $C$ is universal if for any crumpled cube $D$ and any homeomorphism $h : Bd C \rightarrow Bd D$, $\ldots$
Definition. A crumpled cube $C$ is semicellular at disk $E \subseteq \text{Bd } C$ if, for each open set $U \supseteq U-E$, there is an open set $V$ such that $E \subseteq V \subseteq U$ and loops in $V - E$ can be shrunk to points in $U - E$. Theorem. If $C$ is semicellular at each disk in $\text{Bd } C$, then $C$ is a universal crumpled cube.

Theorem. If the interior of $C$ is an open 3-cell and each point in $\text{Bd } C$ is a piercing point, then $C$ is semicellular at each disk in $\text{Bd } C$. Corollary. The crumpled cubes described by Gillman [Duke Math. J. 31 (1964), 247-254] and Alford [Topology of 3-manifolds and related topics, Prentice-Hall, Englewood Cliffs, N. J., 1962, pp. 29-33] are universal. Free crumpled cubes are universal. Theorem. If the interior of $C$ is an open 3-cell and $p$ is a nonpiercing point of $\text{Bd } C$, then $C \cup_h D$ is $S^3$ iff $h(p)$ is a piercing point of $\text{Bd } D$. (Received November 4, 1968.)

663-511. J. M. GANDHI, York University, Toronto 12, Ontario, Canada. Euler's numbers and the diophantine equation $x^1 + y^1 = cz^1$. III.

Theorem 1. The equation $x^1 + y^1 = cz^1$ where $x, y$ and $z$ are rational integers prime to each other and to the odd prime $l$, and $c$ is an integer with conditions $(\phi(l), t) = 1$ and $c^{l-1} \neq z^{l-1} \pmod{1^2}$, $\phi(c)$ being Euler's function is satisfied in integers; then $E(l - 3)/2 = 0 \pmod{1}$ where $E$'s are Euler's numbers. Earlier Vandaer [Amer. J. Math. (1940), 79-82] obtained the same theorem for the case of Fermat's last theorem, i.e. for $c = 1$ in the above equation. This theorem again supports Gandhi's Conjecture [Amer. Math. Monthly 71 (1964), 998-1006] that the equation $x^1 + y^1 = cz^1$ has no nontrivial integral solution for $c \neq 1$. (Received November 4, 1968.)


The large $t$ asymptotic behavior of $N(t) = \int_0^\infty e^{-tx^2} dx/x (x^2 + (\log x)^2)$ and a related family of functions is described incorrectly in the well-known Bateman Manuscript Project on higher transcendental functions. $N(t)$ is related to the special function $V(t)$ through an identity of Ramanujan, $V(t) = \int_0^\infty e^{tx} dx/T(x+1) = e^t - N(t)$. The results stated in Bateman are incorrectly attributed to G. W. Ford. They were apparently obtained by overlooking a restriction on a related theorem due to Ford. The correct asymptotic behavior of $N(t)$ is $N(t) = 1/\log t + O(1/(\log t)^2)$. Furthermore the related functions $V(t,N) - e^t$ differ radically from the description given in Bateman. Almost all of these functions diverge as $t \to \infty$ rather than approach zero as stated there. Direct numerical evaluations of $N(t)$ have been performed which verify the results of our analysis. The incorrect description of $N(t)$ mentioned above gave physically unreasonable results for a problem in neutron transport theory which stimulated our original interest in this function and Ramanujan's identity. (Received November 4, 1968.)

663-513. S. D. CHATTERJI, University of Montreal, Montreal, Canada. A general strong law.

We prove the following: Let $F$ be a norm-bounded subset of $L^p$, $0 < p < 2$. Then there is a denumerable subset $F_0$ of $F$ and a $f_0$ in $L^p$ such that for any sequence $f_n$ in $F_0$,  
\[
\lim_{n \to \infty} n^{-1/p} \|f_n - f_0\| = 0 \text{ a.e.}
\]
 Further $f_0 = 0$ is a possible choice if $0 < p < 1$. In general, $F_0$ cannot be chosen in such a way as to ensure $L^p$-convergence as well. However if there is an infinite weakly sequentially compact subset in $F_0$ (this means in the case of a bounded measure an
uniformly integrable infinite subset), then $L^p$-convergence can be ensured too. For $p = 1$, the theorem was proved recently by Komlős, Acta Math. Acad. Sci. Hungar. 18 (1967), 217-229. The direct analogue for $p = 2$ is false. A version for $p = 2$ was given earlier by Révész, ibid. 16 (1965), 310-318. We deduce this theorem of Révész as well by our methods. For $p = 1$, the theorem shows that a statement of Banach and Saks concerning $L^1$ in Studia Math. 2 (1930), 51-57, is false. Clearly the classical theorems of Kolmogorov and Marcinkiewicz in probability theory are immediate consequences of our theorem, even in slightly strengthened forms. (Received November 4, 1968.)


An element $x$ of the group $H$ is said to have trivial centralizer in $H$ if $\langle y, x \rangle$ is cyclic for all $y$ in the centralizer $c_{G}$. For $B \neq 1$ periodic and $A \neq 1$ arbitrary a complete description of all elements with trivial centralizer in the restricted wreath product $A \wr B$ is given. Definition. $AC(H) =$ subgroup generated by all elements with trivial centralizer, $PH =$ subgroup generated by all products of an even number of self-centralizing elements in $H$. Theorem. Suppose $A, B$ are nontrivial $p$-groups, and $V_1, V_2, \ldots, V_k$ are nontrivial cyclic $p$-groups. (a) $AC(A \wr B) \neq 1$ if and only if $A$ has a self-centralizing element and $B$ is cyclic. (b) If $G = A \wr B$ and $AC(G) \neq 1$ then $G / AC(G) \cong \mathbb{A} / \mathbb{A}'$. (c) If $G = \left((A \wr V_1) \wr V_2 \wr \ldots \wr V_k\right)$ and $A$ has a self-centralizing element, then $G / AC(G) \cong \mathbb{A} / \mathbb{A}'$ for $p \neq 2$ and $[G : AC(G)] = 2^{k-1} \left[A : \mathbb{A}'\right]$ for $p = 2$. The preceding generalizes earlier results by Seksenbaev (M3 34 #2671). (Received November 4, 1968.)


Hermite and Laguerre series do not converge in $L^p$ norms for $1 \leq p \leq 4/3$ or $4 \leq p \leq \infty$ no matter what weight function is used. Specifically, the following is proved. Theorem. If $p$ is a fixed number satisfying $1 \leq p \leq 4/3$ or $4 \leq p \leq \infty$, $\|x\|_p$ denotes the ordinary (unweighted) norm on $(-\infty, \infty)$, $w(x)$ is finite almost everywhere on $(-\infty, \infty)$, $f(x)$ is a function which has a Hermite series and satisfies $\|w(x)f(x)\|_p < \infty$, $s_n(x)$ is the nth partial sum of $f$'s Hermite series and $\lim_{n \to \infty} \|s_n(x) - f(x)w(x)\|_p = 0$ for every such $f(x)$, then $w(x) = 0$ almost everywhere. A similar theorem is true for Laguerre series. The method of proof is to show that norm inequalities of the form $\|a_n H_n(x)w(x)\|_p \leq C \|f(x)w(x)\|_p$ cannot be true for $p$ in the given range where $a_n H_n(x)$ is the nth term of $f$'s Hermite series. This is done by showing that if there were such a $w(x)$, then $\|H_n(x)w(x)\|_p \|H_n(x)e^{-x^2}/w(x)\|_q$ would be bounded by a constant times $z^n n^!$, and then showing that it cannot have a bound of this type. (Received November 4, 1968.)

663-516. V. V. RAO, University of Saskatchewan, Regina, Saskatchewan, Canada. Fourier coefficients of wave forms of dimension-1.

Let $g(z) (z = x + iy, y > 0)$ be a wave form of dimension-1, belonging to the group generated by the transformations $T \rightarrow T + \lambda / q$ and $T \rightarrow -1 / T$ where $\lambda$ is a positive real number and $q$ is a positive integer. Let $\{a_T\}$ denote the sequence of Fourier coefficients of $g(z)$ in the sense of Maass (S.-B. Heidelberger Akad. Wiss. Math.-Nat. Kl. 1949, no. 1). In this paper it is shown that $\sum_{T \leq T \leq x} a_T(x-t)^n$, $x$ being a positive real number, can be expressed as an absolutely convergent
infinite series of analytic functions for \( a > a_0 \) where \( a_0 \) is a real number depending on \( g(z) \).

(Received November 4, 1968.)


Theorem. Every metrizable space \( Y \) which is a continuous image of the space \( P \) of irrational numbers is also a quotient of \( P \) (under a different map, in general). Corollary. The space of rationals is a quotient of the space of irrationals. Remark. The quotient map in the theorem can be chosen to be open (or closed) if and only if \( Y \) is completely metrizable. (Received November 4, 1968.)

663-518. T. L. KRIETE, University of Miami, Coral Gables, Florida 33124. On when an almost selfadjoint operator has no selfadjoint part.

Let \( \alpha \) be a measurable, essentially bounded real function on \([0,1]\) and define \( A \) on \( L^2(0,1) \) by \((Af)(x) = \alpha(x)f(x) + \int_0^x \hat{g}(t) dt\). \( A \) is almost selfadjoint, i.e. \( \text{rank} (A - A^*) = 1 \). \( A \) is called completely nonselfadjoint if there is no reducing subspace \( N \) for \( A \) such that \( AN \) is selfadjoint. Definition. The absolutely continuous spectrum of \( \alpha \) is the set \( \Gamma_{ac}(\alpha) = \{ \gamma : \lim_{\varepsilon \to 0} m(\alpha^{-1}(\gamma - \varepsilon, \gamma + \varepsilon)) > 0 \} \), where \( m \) is Lebesgue measure on \([0,1]\). Definition. If \( \gamma \) is in the essential range of \( \alpha \), \( \gamma \) is said to be essentially invertible at \( y \) provided there is an \( x \) in \([0,1]\) so that \( \lim_{\varepsilon \to 0} m(\gamma \cap \alpha^{-1}(y - \varepsilon, y + \varepsilon))/m(\alpha^{-1}(y - \varepsilon, y + \varepsilon)) = 1 \) for every open set \( J \) containing \( x \). We prove the following. Theorem. \( A \) is completely nonselfadjoint if and only if \( \alpha \) is essentially invertible at almost every point of \( \Gamma_{ac}(\alpha) \). The theorem is proved by constructing an explicit unitary equivalence between the completely nonunitary part of \( T^* = (A^* + i/2)(A^* - i/2)^{-1} \) and its Sz.-Nagy-Foias-de Branges-Rovnyak canonical model (see R. G. Douglas, A structure theory for operators. I. J. Reine Angew. Math., to appear). A similar theorem is proved for almost unitary operators of the form \( B : f(x) \rightarrow z(x)f(x) - \int_0^x \exp(1/2(t - x))z(t)f(t) dt \), where \( z \) is measurable on \([0,1]\) and \( |z| = 1 \) a.e. (Received October 25, 1968.)


Given a compact metric space \((X,d)\), let \( \text{Lip}(X,d) \) be the Banach space of Lipschitz functions with the norm: \( ||f|| = \sup \{ ||f||_\infty, ||f||_d \} \), where \( ||f||_d = \sup \{ ||f(x) - f(y)||/d(x,y) : x \neq y \} \). For \( 0 < \alpha < 1 \), let \( \text{lip}(X,d^\alpha) \) be the Banach space of those complex functions \( f \) on \( X \) for which \( ||f(x) - f(y)|| = o(d^\alpha(x,y)) \) uniformly as \( d(x,y) \to 0 \). The norm is defined by \( ||f|| = \sup \{ ||f||_\infty, ||f||_d \} \), where \( ||f||_d = \sup \{ ||f(x) - f(y)||/d^\alpha(x,y) : x \neq y \} \). Definition 1. A metric space \((X,d)\) will be \( \beta \)-separated if there do not exist closed subsets \( A,B \) of \( X \) such that \( A \cup B = X \) and \( d(A,B) > \beta \). Definition 2. Let \( k > 0 \). A homeomorphism \( \tau \) of a metric space \((X,d)\) will be \( k \)-isometry if \( d(x,y) = d(\tau x, \tau y) \) whenever either number is smaller than \( k \). Theorem 1. Given a compact metric space \((X,d)\) which is \( \beta \)-separated, \( \beta < 1 \), every isometry \( T \) of \( \text{Lip}(X,d) \) onto \( \text{Lip}(X,d) \) has the form \( (Tf)(x) = uf(\tau x), x \in X, f \in \text{Lip}(X,d) \), where \( u \) is a constant of absolute value 1 and \( \tau \) is a \( 2 \)-isometry of \((X,d)\) onto itself. Theorem 2. The above theorem is also true if \( \text{Lip}(X,d) \) is
replaced by $\text{lip}(X,d^\alpha), 0 < \alpha < 1$, and $(X,d)$ by $(X,d^\alpha)$. We prove that in a sense these will be the best possible results on isometries. We also show that imposition of rigidity conditions on $(X,d)$ makes every 2-isometry an isometry. (Received November 4, 1968.)

663-520. L. E. MANSFIELD, University of Utah, Salt Lake City, Utah 84112. Optimal approximate integration and interpolation for functions of two variables. Preliminary report.

Approximate integration and interpolation formulas and error bounds, which are optimal in the sense of Golomb and Weinberger, are obtained for the space of functions of two variables, $B_{pq}(b,d)$, defined by Sard (Linear approximation, Chapter 4). These formulas are calculated by finding the reproducing kernel for the space and then calculating the representers for the appropriate functionals. (Received November 4, 1968.)


Consider the functional-differential equation (1) $x'' + a(t)f(t,x_t) = 0$, where $a(t)$ is $\equiv 0$, continuous; $f$ a continuous nonlinear functional bounded on $[\theta, \infty) \times U$, where $U = \{ x \in C [-h, 0] : \|x\| \equiv 1 \}$, $f(t,x_t) > 0$ if $x(t) > 0$; $f(t,x_t) < 0$ if $x(t) < 0$. Then: Theorem 1. Equation (1) has a bounded nonoscillatory solution if and only if $\int_0^\infty a(t)dt < \infty$. Theorem 2. If $f(t,x_t) = g(x_t^\gamma)$ where $g$ is a linear functional, $\gamma$ is the quotient of positive integers, then every solution of (1) is oscillatory if $\int_0^\infty a(t)dt = \infty$. (Received November 4, 1968.)


A Newton-like method which coincides with the Newton-Kantorovich method whenever the latter is well defined is applied to a class of 2 point boundary value problems which includes the Euler-Lagrange equation for simple variational problems and equations of the form $y'' = f(x,y,y')$. The Newton-Kantorovich method is not well defined for this class of problems. (Received November 4, 1968.)


Let $A$ be a partial isometry on a Hilbert space $H$ satisfying $\lim_{n \to \infty} \|A^* A^n - A^n A^*\|^{1/n} = 0$, and let $H_0$ be a subspace of $H$ defined by $H_0 = \{ x : x \in H, \lim_{n \to \infty} \|A^n x\|^{1/n} = 0 \}$. The following are proved: If $\lambda$ is a nonzero eigenvalue of $A$, then $|\lambda| = 1$, $H_0$ is closed, and $A$ splits into the direct sum of a unitary and a generalized nilpotent operator. Thus, it follows that $A$ is a spectral operator in the sense defined by N. Dunford [Pacific J. Math. 2 (1952), 559-614]. In particular, if $A^m$ is normal for a natural number $m$, then $A$ is a finite-type spectral operator (N, Dunford [Pacific J. Math. 4 (1954), 321-354], C. Apostol [Rev. Roumaine Math. Pures Appl. 12 (1967), 759-762]), and subsequently $A$ is the direct sum of a unitary and a nilpotent operator of index $\equiv m$. (Received November 4, 1968.)
On inclusion relations between parabolic Lipschitz and Lebesgue spaces.

B. F. Jones has demonstrated properties of solutions of the heat equation which are analogous to properties of solutions of Laplace's equation given by M. L. Taibleson. (See J. Math. Mech. 13 (1964), 407-480. Jones' work is to appear in J. Math. Mech. in 1968.) This investigation gives rise to the parabolic Lipschitz spaces which consist of $L_p$ functions satisfying certain mean smoothness conditions. Among his results, Jones gives inclusion relations between various of these Lipschitz spaces and between them and the parabolic Lebesgue spaces, which consist of certain fractional integrals of $L_p$ functions. In the present paper Jones' work is duplicated for periodic functions which are $p$-integrable on the torus. Then examples are given showing that the inclusion relations are best possible. An equivalent norm for defining the Lipschitz spaces is also given, showing that it suffices to consider appropriate smoothness in each variable separately. (Received November 4, 1968.)

Decomposition of the alternating group into the product of conjugacy classes.

Let $\text{Alt}(n)$ denote the alternating subgroup of $\text{Sym}(n)$, the symmetric group of $n$ symbols. $C_1$ is the conjugacy class, in $\text{Sym}(n)$, of all permutations conjugate to $(1 \ 2 \ 3 \ ... \ t)$, $1 < t \leq n$.

$|\mathfrak{m}(R)|$ is the number of symbols moved by $R \in \text{Sym}(n)$; $|c(R)|$ is the number of cycles in the standard (disjoint cycle) decomposition of $R$, and $|C^*(R)|$ is the number of these disjoint cycles with length $> 1$. $\lfloor x \rfloor$ denotes the greatest integer $\leq x$.

**Lemma.** For $P \in C_k$, $Q \in C_1$, and the composition $P \circ Q$ considered as a permutation of the symbols in $\mathfrak{m}(P) \cup \mathfrak{m}(Q)$, $|\mathfrak{m}(P) \cap \mathfrak{m}(Q)| \geq 1$ implies 

$|c(P \circ Q)| \leq |\mathfrak{m}(P) \cap \mathfrak{m}(Q)|$.

**Lemma.** For each $P \in \text{Alt}(n)$, $|\mathfrak{m}(P)| + |C^*(P)|$ is even, and $|\mathfrak{m}(P) + C^*(P)|/2 \leq \lfloor 3n/4 \rfloor$. If $n \neq 4$, a necessary condition that $\text{Alt}(n) = C_1 \circ C_1$ is $1 \leq \lfloor 3n/4 \rfloor$. This theorem generalizes the well-known result of N. Ito and O. Ore (independently) that for $n \geq 5$ every permutation in $\text{Alt}(n)$ is a commutator of permutations in $\text{Alt}(n)$, and gives a general answer to a research problem proposed by J. L. Brenner [Bull. Amer. Math. Soc. 66 (1960), 275]. A similar theorem is proved which relates $C_{4t+1} \circ C_1$ and the set of odd permutations of $\text{Sym}(n)$.

A group topology for completely distributive lattice-ordered groups.

A net $\{x_i\}_{i \in I}$ in a lattice-ordered group $G$ is said to $\alpha$-converge to $x \in G$ if $x$ is the only element of $G$ which satisfies $x = \bigvee_{i \in I_0} (x_i \wedge x) = \bigwedge_{i \in I_0} (x_i \vee x)$ for every $I_0 \subseteq I$. (See F. Papangelou, Some considerations on convergence in abelian lattice groups, Pacific J. Math. 15 (1965), 1347-1364.)

J. T. Ellis (see Group topological convergence in completely distributive lattice-ordered groups, Doctoral dissertation, Tulane University, New Orleans, Louisiana, 1968) has shown that $\alpha$-convergence in $G$ derives from a topology if and only if $G$ is completely distributive. Generalizations of some of Papangelou's results yield the following. Theorem. If $\alpha$-convergence in $G$ derives from a topology then $G$ with that topology is a Hausdorff topological group. (Received November 4, 1968.)
Let \( f(z) \) be an entire function of bounded index (see S. M. Shah, Proc. Amer. Math. Soc. 19 (1968), 1017-1022). Let as usual \( M(r) \), \( \mu(r) \), \( \nu(r) \) denote the maximum modulus, maximum term and the rank of the maximum term of \( f(z) \). \( M_1(r) \), \( \mu_1(r) \), \( \nu_1(r) \) denote the corresponding concepts for \( f'(z) \). Then, in this paper, the following theorems are proved.

**Theorem 1.** When \( f(z) \) is of bounded index with index \( N \), then for any real positive constant \( c > 0 \), \( cf(z) \), \( f(z + c) \), \( f(zc) \), \( f'(z) \) are also of bounded index, with index \( N \).

**Theorem 2.** When \( f(z) \) is of bounded index with index \( N \), then \( g(z) = \left(f(z)\right)^{\lambda} \) is also of bounded index with index \( N\lambda + \lambda - 1 \), \( \lambda \) is some positive integer.

**Theorem 3.** When \( f(z) \), \( g(z) \) are of bounded index with indexes \( N_f, N_g \), then \( (f(z) + g(z)) \) is also of bounded index with index \( N_f + N_g = \max(N_f, N_g) \).

**Theorem 4.** When \( f(z) \) is of bounded index with index \( N \), then \( \limsup_{r \to \infty} \frac{\log M_1(r)}{r/\mu(r)} \leq (N + 1) \), this bound is sharp for \( f(z) = e^z \).

**Theorem 5.** When \( f(z) \) is of bounded index with index \( N \), then 
\[
\limsup_{r \to \infty} \frac{\mu_1(r)}{\mu(r)} \leq e(N + 1), \quad \limsup_{r \to \infty} \frac{\nu_1(r)}{\nu(r)} \leq e(N + 1).
\]

**Theorem 6.** Let \( f(z) \) be an entire function, \( f_n(z) \) denote the \( n \)th partial sum of \( f(z) \), \( r_n \) denotes the maximum modulus of the zeros of \( f_n(z) \). Then if \( N \) is the index of \( f(z) \), 
\[
\limsup_{n \to \infty} \frac{\log r_n}{\log n} = (N + 1) \text{ as } n \to \infty.
\]

(Received November 4, 1968.)

Any metrizable compact convex subset of a locally convex space is a section of a compact simplex. Every infinite-dimensional Banach space X contains a compact convex set K such that there is no compact simplex in X which contains K. There is no W*-compact simplex in an infinite-dimensional dual Banach space which contains the closed unit ball. (Received November 4, 1968.)


The thermoelastic problem arising due to periodic heating of an embedded spherical cavity in an elastic half-space has been solved. The thermoelastic coupling term is retained in the heat conduction equation. The thermoelastic displacements and temperature are expressed in terms of thermoelastic displacement functions which are then found in a series form by an iteration process. Nowacki's solution for a point source in a half-space is derived as a limit case of the general solution. The problem has applications in the theory of subterranean explosions accompanied by generation of heat and seismology. (Received November 4, 1968.)

663-532. V. M. SOUNDALGEKAR, Indian Institute of Technology, Bombay, India, and PRATAP PURI, Louisiana State University, New Orleans, Louisiana. On fluctuating flow of an elastico-viscous fluid past an infinite plate with variable suction.

An exact solution is obtained for the two-dimensional flow of an elastico-viscous (Walters fluid B') incompressible fluid past an infinite porous wall under the following conditions: (i) the suction velocity normal to the plate oscillates in magnitude but not in direction about a nonzero mean, (ii) the free stream velocity oscillates in time about a constant mean. The response of the skin-friction to the fluctuating stream and suction velocity is studied for variations in the suction parameter A, the elasticity parameter k and the frequency parameter \( \omega \). It is found that the back-flow at the wall is enhanced by k, the amplitude of the skin-friction is affected by k only at large \( \omega \) when the suction velocity is constant. For very small \( \omega \), the amplitude of the skin-friction falls to a minimum and then steadily increases as \( \omega \) and k increase. Also an increase in k and \( \omega \) lead to a decrease in the phase of the skin-friction. For moderately large A and k, the phase of the skin-friction may be completely negative. (Received November 4, 1968.)

663-533. Z. W. BIRNBAUM, University of Washington, Seattle, Washington 98105. Random variables similar to Student's t.

Let \( X_{1} < \ldots < X_{m+1-r} < \ldots < X_{2m+1} < \ldots < X_{2m+1+r} < \ldots < X_{2m+1} \) be an ordered sample for the random variable X, so that \( X_{m+1} \) is the sample median and \( X_{m+1-r}, X_{m+1+r} \) two symmetrical sample quantiles, and let \( \mu \) be the median of X. For the statistic \( S = (X_{m+1} - \mu)/(X_{m+1+r} - X_{m+1-r}) \) we show the following: If X has a probability density \( f(x) \) which is symmetric about \( \mu \) and unimodal, then there exists \( \psi(m,r,\lambda) \), independent of \( f(x) \), such that \( P(|S| > \lambda) \leq \psi(m,r,\lambda) \), and \( \psi(m,r,\lambda) = O(\lambda^2) \) for \( \lambda \rightarrow +\infty \). Additional properties of \( \psi \) are obtained under further assumptions on f. (Received November 4, 1968.)
Stochastic processes with sample path functions of bounded variation.

A condition is given for a random function \( x : T \rightarrow \Omega \mathbb{F}, P \) to have its sample path functions \( x(\cdot, \omega) \) be of \( p \)-bounded variation, \( 0 < p \neq 1 \). This condition is necessary when \( x \) has independent, \( p \)-integrable increments. The sample path Stieltjes stochastic integral is defined and a few properties are given. (Received November 4, 1968.)

Representations of certain compact semigroups by \( L \)-semigroups.

An \( L \)-semigroup is defined to be a topological semigroup with the property that the Schützenberger of each \( \mathbb{W} \)-class is a Lie group. The following problem is considered: Does a compact semigroup \( S \) admit enough homomorphisms into \( L \)-semigroups to separate points of \( S \)? It is shown that this is a necessary and sufficient condition that \( S \) be isomorphic to the strict projective limit of \( L \)-semigroups. The main theorem in this regard is the Theorem. If \( S \) is an irreducible semigroup, then there are sufficiently many representations of \( S \) by \( L \)-semigroups to separate points of \( S \). If, in addition, \( S/\mathbb{W} \) is separable, then the \( L \)-semigroups may be chosen to be finite dimensional. (Received October 28, 1968.)

On uniqueness of fixed-points.

Let \( K \) be a compact convex subset of a locally convex space, and let \( \Phi \) be a semigroup of continuous affine maps on \( K \) to \( K \). Suppose that \( \Phi \) is given a topology stronger than the topology of pointwise convergence on \( K \) and with respect to which composition is a separately continuous binary operation. By the orbit of a point \( k \) in \( K \) we mean the closed convex hull of \( \Phi(k) \). M. M. Day (Illinois J. Math. 5 (1961), 585-590) proved the Theorem. If \( \Phi \) is left amenable there is a fixed-point in every orbit. Let \( \mathcal{W} \) denote the closed convex hull of \( \Phi \) in \( K \). We prove the Theorem. (1) If \( \Phi \) is left amenable, there is an element \( \psi_1 \) in \( \mathcal{W} \) such that \( \psi_1(x) \) is a fixed-point in the orbit of \( x \), each \( x \) in \( K \); (2) if \( \Phi \) is right amenable and equicontinuous on \( K \), there is an element \( \psi_2 \) of \( \mathcal{W} \) such that \( \psi_2(x) \) is the only possible fixed-point in the orbit of \( x \), each \( x \) in \( K \). Among the applications, we get a strengthening of the Kakutani fixed-point theorem with applications to the theory of almost periodic functions. (Received November 4, 1968.)


Let \( f \) and \( g \) be smooth functions and let \( a(t) = t^{-p} \) where \( 0 < p < 1 \). Let \( x(t) \) be the solution on the interval \( 0 \leq t \leq T \) of the integral equation \( x(t) = f(t) + \int_0^t a(t-s)g(x(s))ds \). Let \( N \) be a nonnegative integer, \( h = T/(N + 1) \), \( t_j = jh \), \( f_j = f(t_j) \), etc. If one replaces the integral \( \int_0^t \) by \( \int_0^{t-h} \) and then approximates \( \int_0^{t-h} \) by the trapezoidal rule, one is lead to the following algorithm for approximating
Theorem. There exists a constant $K$ (independent of $h$ and $j$) such that $|x(t_j) - X_j| \leq Kh^{1-p}$ for $0 \leq j \leq N + 1$ and $N = 1, 2, 3, \ldots$. In the case $p = 1/2$ the integral equation is of physical interest and has been studied by B. Noble (Nonlinear integral equations, Univ. of Wisconsin Press, 1964).

Noble's method was numerically unstable for certain physically interesting functions $g(x)$. The method presented here does not appear to suffer such instabilities. (Received November 5, 1968.)

663-538. J. C. DACEY, Kansas State University, Manhattan, Kansas 66502. The ortho-conditioned orthomodular product.

An orthogonality space is defined to be a set together with a symmetric irreflexive relation (orthogonality). An orthogonality space is called orthomodular if the following condition holds:

Whenever the union of the set of all points orthogonal to one point with the set of all points orthogonal to another point contains a maximal orthogonal subset then the two points are orthogonal. It is shown that every orthomodular space gives rise in a natural way to an orthocomplete orthomodular partially ordered set, and that, conversely, every orthocomplete orthomodular partially ordered set so arises.

An example of interest is the set of nonzero vectors of a Hilbert space with the inner product induced orthogonality and its corresponding (orthomodular) lattice of closed subspaces. A product of orthomodular spaces (the ortho-conditioned product) is defined which simultaneously generalizes the Cartesian product, the horizontal sum, and the Randall operational product. This product is shown to preserve orthomodularity. Applications to nonclassical probability theory are indicated. (Received November 5, 1968.)


The triangular design $(2, n - 1, 2)$ and the $L_2$-design $(2, n, 1)$ may be generalized in two directions. First they can be thought of as special cases of the series of SLB-designs $(r,k,r)$ and sets $(r, k, r - 1)$, respectively. Secondly, recalling that the treatments in a triangular ($L_2$-) design may be identified with the set of all unordered (ordered) pairs on $n$ symbols with two treatments being first associates if they have a symbol (coordinate) in common, we may consider generalizations to $r$ coordinates. Define the graph $G^n_r$ as having as vertices all ordered $r$-tuples on $n$ symbols with two vertices adjacent if they have more than one coordinate in common. Theorem. The internal stability number $\alpha(G^n_r) \leq n^2$ with equality if there exists a net $(r,n)$. The corresponding theorem for unordered $r$-tuples involves SLB-designs and is due to J. di Paola. For $r = 3$, $G^n_3$ is the cubic lattice graph, in the case of unordered triplets we obtain the tetrahedral graph. We compute the external stability number for tetrahedral graphs (for the cubic lattice graph this was done by Kalbfleisch and Stanton). Finally, progress is made toward characterizing the SLB-scheme and the lattice-scheme for $r = 3$ by means of the associated strongly regular graph. (Received November 5, 1968.)
A linear operator $T : X \to Y$ is said to be unconditionally converging (uc operator) if it sends every weakly unconditionally convergent series into an unconditionally convergent series. A. Pelczynski [Bull. Acad. Polon. Sci. 10 (1962), 641-648] introduced this class of operators and showed that every weakly compact operator is a uc operator. In certain spaces these two classes of operators coincide. These spaces are said to have property $V$. If $T : X \to Y$ is strictly singular, almost weakly compact, or completely continuous (not the same as compact), then $T$ is a uc operator; but not conversely. If a space has both property $V$ and the Dunford-Pettis property, then every uc operator is strictly singular, almost weakly compact, and completely continuous. (Received November 5, 1968.)

Let $K$ be a field of characteristic not 2 and $L = K(x)$ a simple algebraic extension. Let $r_{L/K} : W(K) \to W(L)$ be the canonical ring homomorphism, where $W$ denotes the Witt ring. Let $s : L \to K$ be a nonzero $K$-linear map. If $g : V \to L$ is a nondegenerate quadratic form over $L$, $s^*(g) = s \circ g$ is a nondegenerate quadratic form over $K$, and one obtains a group homomorphism $s^* : W(L) \to W(K)$. The composite $s^* \circ r_{L/K}$ is multiplication by $s^*(1)$. Theorem. If $[L = K]$ is even $\text{Ker}(r_{L/K})$ is contained in the annihilator of the binary form $\langle 1, -N_{L/K}(x) \rangle$. If $[L : K]$ is odd, $\text{Ker}(r_{L/K}) = 0$. Corollary (Pfister). $W(K)$ does not contain torsion elements of odd order. (Received November 5, 1968.)

Let $X$ be a compact, acyclic ANR (absolute neighborhood retract) in $\text{Int} M^3$, where $M^3$ is a 3-manifold containing no homology 3-cells which fail to be simply connected. Theorem. $X$ is an AR (absolute retract). This answers a question raised by Borsuk. The corresponding theorem for higher dimensions is well known to be false. The proof consists in showing that $X$ is contractible in each neighborhood of $X$ in $M^3$. This is done by showing that $X$ can be expressed as the intersection of a decreasing sequence of neighborhoods of $X$ in $M^3$, each neighborhood being a homotopy cube-with-handles. The proof of this fact uses results of Haken and Waldhausen. Some parts of the proof are valid with a weaker requirement than "ANR" on $X$. (Received November 5, 1968.)

An f-structure of rank $r$ with complemented frames on a manifold $M$ is a tensor $f$ of type $(1,1)$ of constant rank together with $r$ 1-forms spanning the distribution given by the tensor $f^2 + 1$. Let $N$ be a submanifold of $M$ and if $p \in N$ let $T_{N_p}$ denote the tangent space to $N$ at $p$. The following results are proved: (1) If the dimension of $T_{N_p} \cap f(T_{N_p} \cap f^2 TM_p)$ is constant on $N$ then $N$ possesses a
naturally induced f-structure. (2) If the condition in (1) is satisfied, f is integrable and transversals to N can be found that are annihilated by \( t^2 + 1 \), then the f-structure on N is integrable. (Received November 5, 1968.)

663-544. T. C. BROWN, Simon Fraser University, Burnaby 2, British Columbia, Canada.

On van der Waerden's theorem on arithmetic progressions.

For \( X = \{ x_1, \ldots, x_n \} \), define \( X\sigma = \{ x_1 + 1, \ldots, x_n + 1 \} \), \( X\sigma^2 = (X\sigma)\sigma \). We say that \( m \) arithmetic progressions \( X_1, \ldots, X_m \) are congruent if (renumbering if necessary) \( X_2 = X_1\sigma \), \( X_3 = X_1\sigma^2 \), ..., \( X_m = X_1\sigma^{m-1} \). Theorem. Let \( \kappa \) be given, and let the natural numbers be partitioned into \( \kappa \) classes. Then for every \( m, \lambda \) there are \( m \) congruent arithmetic progressions of length 1 each of which lies wholly in some one of the classes. (Different progressions may lie in different classes.) The proof is based on van der Waerden's theorem (which is the restriction of the above to \( m = 1 \)) and the following Lemma. Let \( f \) be a mapping of the set of natural numbers onto a finite set \( Y \). Then there exist an element \( y \) of \( Y \) and a number \( \kappa \) such that for arbitrary \( n \) there are \( x_1 < x_2 < \ldots < x_n \) with \( x_{j+1} - x_j \equiv \kappa \) and \( f(x_1) = f(x_2) = \ldots = f(x_n) = y \). (Received November 5, 1968.)


Let \( \mu \) be a finitely additive function from a prering \( V \) into a Banach space \( Z \) and let \( v \) be a positive volume on \( V \). The terminology used is that of Bogdanowicz, Proc. Nat. Acad. Sci. U.S.A. 53 (1965), 492-498, and Math. Ann. 164 (1966), 251-269. Assume that \( \mu \) is \( v \)-continuous and has finite variation on every set \( A \) from the prering \( V \) with respect to a bilinear operator \( u \) that is \( \mu \)-continuous and \( \mu \)-continuous on every set \( A \) from the product of Banach spaces \( Y \times Z \) into a Banach space \( W \). An integral of the form \( \int f(u,d\mu) \) has been developed whose domain, \( D_{u,\mu,v} \), is contained in the space \( M(v,Y) \) of \( v \)-measurable, \( Y \)-valued functions. This integral generalizes the Bartel integral (R. G. Bartel A general bilinear vector integral, Studia Math. 15 (1956), 337-352). Characterization of sequential convergence in the topology of the domain \( D_{u,\mu,v} \) has been obtained. It has been proved that the extension, \( \mu_v \), of \( \mu \) from the prering \( V \) into the \( \delta \)-ring of \( v \)-summable sets generates the same vectorial integral as \( \mu \). (Received November 5, 1968.)


Bochner-Raikov theorem for a generalized positive definite function. Preliminary report.

Let \( \mathcal{U} \) be a proper \( \mathbb{H}^\ast \)-algebra and let \( \tau(\mathcal{U}) = \{ xy \mid x, y \in \mathcal{U} \} \) be its trace class [see Abstract 643-17, these Notices 14 (1967), 246]. A positive definite \( \mathcal{U} \)-function is a mapping \( p \) of a topological group \( G \) into \( \tau(\mathcal{U}) \) such that \( \text{tr} \left( \sum_{i,j} a_{i,j}^\ast p(t_i t_j^{-1}) a_{i,j} \right) \equiv 0 \) for each finite subset \( \{ t_1, t_2, \ldots, t_n \} \) of \( G \) and a corresponding subset \( \{ a_1, a_2, \ldots, a_n \} \) of \( \mathcal{U} \). Let \( \mathcal{B} \) be the \( \sigma \)-algebra of subsets of \( \widehat{G} \) generated by open sets (\( \widehat{G} \) is the dual of \( G \), which is assumed to be commutative). It is shown that for each positive definite \( \mathbb{H} \)-function \( p \) on \( G \) there exists a \( \tau(\mathcal{U}) \)-valued measure \( \mu \) on \( \mathcal{B} \) such that \( p(t) = \int \mu(t,a) d\mu(a) \) for
all $t \in G$. The measure $\mu$ is positive in the sense that for each $E \in B$ there exists a $f \in \mathcal{U}$ such that $\mu E = a^* a$. (Received November 5, 1968.)


Notation is that of Nijenhuis and Richardson (Bull. Amer. Math. Soc. 72 (1966), 1-29). A positively graded ring $R = \bigoplus_{i=1}^{\infty} R_i$ is a simple graded ring (sgr) if its only homogeneous ideals are irrelevant. A homogeneous additive subgroup $U$ of $R$ is a Lie (Jordan) ideal of $R$ if $[U, R] \subseteq R$ $(U R) \subseteq R$, where $(x, y) = xy + (-1)^{xy}yx$. Theorem. If $R$ is an sgr and $U$ is a Lie ideal of $R$, then $U \subseteq Z(R)$ (the center of $R$) or $U \supseteq [R_n, R]$ for large $n$. (char $R \neq 2$.) Theorem. With $R$ as above, let $S = [R, R], [R, R]$. Either $S = \{0\}$ or $S \subseteq [R_n, R]$ for large $n$. Theorem. With $R$ as above, suppose that no irrelevant ideal of $R$ is commutative. Let $U$ be a Lie ideal of $[R, R]$ with $[R_n, [R, R]] \not\subseteq U$ for large $n$. Then $U \subseteq Z(R)$. Theorem. Let $R$ be a sgr with char $R = 2$, and let $U$ be a Lie ideal of $R$. Then one of the following is true: $U \supseteq [R_n, R]$ for large $n, [R_n, U] = \{0\}$ for large $n, U$ is contained in a commutative subring $T$ of $R$ with the property that $x \in T = x^2 \in Z(R)$. Theorem. Let $R$ be a sgr, char $R \neq 2$, and let $U$ be a Jordan ideal of $R$. Either $U$ contains an irrelevant ideal of $R$ or $(U, U) = \{0\}$. In the latter case, $x \in U = x^2 \in Z(R)$. Theorem. With $R$ and $U$ as above, if $U$ is also a subring of $R$, then either $U = \{0\}$ or $U$ contains an irrelevant ideal of $R$. (Received November 5, 1968.)


Let $X$ be a compact metric space, and let $\varphi$ be a homeomorphism of $X$ onto $X$. A finite open cover $\mathcal{U}$ of $X$ is called a generator for $(X, \varphi)$ iff for each bisequence $A_1$ of members of $\mathcal{U}$, $\cap_{n=1}^{\infty} \varphi^{-1}(A_1)$ is at most one point. Reddy (Math. Systems Theory 2 (1968), 91-92) and Keynes and Robertson (Abstract 656-13, these Notices 15 (1968), 505) have shown that $(X, \varphi)$ is expansive iff $(X, \varphi)$ has a generator. In this paper, we use generators to study the asymptotic properties of expansive homeomorphisms. Theorem. If $(X, \varphi)$ is expansive and if $x_0$ is a nonisolated fixed point, then exactly one of the following holds: (i) there is an open neighborhood $U$ of $x_0$ such that each point in $U$ is positively asymptotic to $x_0$; (ii) there is an open neighborhood $U$ of $x_0$ such that each point in $U$ is negatively asymptotic to $x_0$; (iii) there exist $p \neq x_0$ and $q \neq x_0$ such that $p$ is positively asymptotic to $x_0$ and $q$ is negatively asymptotic to $x_0$. Theorem. If $(X, \varphi)$ is expansive and if $\varphi$ has a dense positive (or negative) semi-orbit, then (iii) in the above theorem holds for each nonisolated fixed point. (Received November 5, 1968.)


Let $B^n$ be the union of $n$-disks in $E^3$ meeting precisely on a single arc $B$ on the boundary of each. $B^n$ is a n-book, the disks are the leaves of $B^n$, and $B$ is its back. The cellular hull of a subset $A$ of $E^3$ is defined to be a cellular set $C$ containing $A$ such that no proper cellular subset of $C$ contains $A$. Theorem. An arc $A$ in $E^3$ has a cellular hull that lies in a tame 2-complex if and only if there
exists a homeomorphism \( h \) mapping \( E^3 \) onto \( E^3 \) such that \( h(A) \) lies in a tame 3-book. \textbf{Theorem.} Let \( A \) be a cellular arc in the interior of an arbitrary \( n \)-book in \( E^3 \). If the set of wild points of \( A \) does not contain an arc, then \( A \) has at most one wild point that is not contained in the back of the \( n \)-book.

(Received November 5, 1968.)


A general class of polynomials \( \theta_n(x) \) is presented with generating functions, recurrence relations, integral properties and special cases. It may be noted that \( P_n^{(a,b)}(x) \) the product of Jacobi polynomials, \( (C_n^{(v)}(x))^2 \) the square of the Gegenbauer polynomial, \( L_n^{(v)}(x) \) the product of Legendre polynomial, \( L_n^{(v)}(x) \) the product of Laguerre polynomials, \( H_n^{(v)}(x) \) the Rice polynomial, \( f_n(a_1,b_1;x) \) the generalized Bessel polynomial, \( C_n^{(v)}(x) \), \( P_n^{(a,b)}(x) \) and other polynomials form particular cases of \( \theta_n(x) \). Of special interest, \( (C_n^{(v)}(x))^2 \), which occurs in quantum mechanical problems, reveals new recurrence relations, new integral properties and is seen to display the characteristics of a function in its own rights. (Received November 5, 1968.)

663-551. DANY LEVIATAN, University of Illinois, Urbana, Illinois 61801, and M. S. RAMANUJAN, University of Michigan, Ann Arbor, Michigan 48105. A generalization of the mean ergodic theorem.

The main theorem of the paper is the following: Let \( E \) be a Banach space and \( V \) be a continuous linear map of \( E \) into \( E \). Let \( A = (A_{nk}) \) be an infinite matrix of continuous linear operators \( A_{nk}: E \to E \), each of which commutes with \( V \), and \( A \) be a matrix of the Toeplitz type for the space \( E \). Let \( \{ \gamma_n \} \) be an increasing sequence of nonnegative reals. Assume that (i) \( T_n x = \sum_{k=0}^{\infty} A_{nk} V^k x \) exists for each \( x \in E \) and that for each \( x \) the sequence \( \{ T_n x \} \) is weakly relatively compact in \( E \); (ii) \( \sum_k \| A_{nk} - A_{n,k+1} \gamma_k \to 0 \) as \( n \to \infty \), and (iii) \( \| V^n \| < \gamma_n \). Then for each \( x \in E \), \( T_n x \to Px \), where \( P \) is the projection of \( E \) on to the null space \( N \) of \( 1 - V \); also, if \( \overline{E} \) denotes the closure of the range of \( 1 - V \), then \( E = N + \overline{E} \) is a topological direct sum. As an application, suitable constants \( \{ \gamma_n \} \) are determined for special matrices \( A \); in the case of the Cesaro method \( (C,1) \), with \( \gamma_n = o(n) \), the above theorem yields the mean ergodic theorem of the Yosida type. Also, for a suitably restricted class of vector valued Hausdorff methods the constants \( \{ \gamma_n \} \) with \( \gamma_n = o(\sqrt{n}) \) are shown to satisfy the requirements in the theorem. (Received November 5, 1968.)


Suppose \( L \) is a Banach lattice in which increasing sequences of bounded norm have suprema and converge in norm. Let \( F \) be the order bounded operators of finite rank on \( L \). We prove Theorem 1. If \( L \) is nonatomic then the identity operator is orthogonal to \( F \). \textbf{Theorem 2.} If \( T \) is dominated by an operator in \( F \) then \( T \) maps weakly convergent sequences into order convergent sequences. \textbf{Theorem 3.} An order bounded operator \( T \) is an order limit of finite-dimensional operators if and only if \( T \) maps bounded sequences which converge in measure into order convergent sequences. Our proofs are obtained
without using the spectral theory of Nakano (Product spaces ..., J. Fac. Sci. Hokkaido Univ. Ser. I (1953), 163-240). Applications of these results to $L^p$ spaces are given. (Received November 5, 1968.)


Extensions and applications of results announced in Abstract 656-9, these Notices 15 (1968), 504, are obtained. Suppose $S$ is a space satisfying R. L. Moore's Axioms 0, 1-4, $M$ is a Peano continuum in $S$ and $U$ is a component of $S - M$. **Theorems.** (1) (Modified Torhorst Theorem) There is a Menger regular curve in $M$ which is an irreducible continuum about $Bd(U)$. (2) (Modified Moore Theorem) If $Bd(U)$ contains no endpoint of $S$ and $V$ is a component of $S - M - U$, then the outer boundary of $U$ with respect to $V$ is a simple closed curve. (3) If $M$ contains only $k$ endpoints of $S$ and only $n$ simple closed curves, then $S - M$ has at most $k + n + 1$ components. (4) If $M$ is a Menger regular curve and $S - M$ does not have infinitely many components, then $S - M$ has two components $D$ such that $Bd(D)$ is connected and $Bd(U)$ is either a simple closed curve or a subset of an arc. (5) If $K$ is any connected, locally connected closed point set, $D$ and $E$ are components of $S - K$, $A$ and $B$ are boundary points of both $D$ and $E$ and some arc $AB$ in $K$ separates $D$ from $E$, then every arc $AB$ in $K$ separates $D$ from $E$. Related theorems and examples are obtained. (Received November 5, 1968.)


Denote by $F$ the family of functions $\lambda(x,y)$ satisfying (1) $\lambda(x,y) = \lambda(d(x,y))$, (2) $0 \leq \lambda(d) < 1$ for every $d > 0$, (3) $\lambda(d)$ is a monotonically decreasing function of $d$. Let $P$ be a subspace of a metrizable topological space $X$. Let $R$ denote the set of all contractive mappings of $P$ into itself satisfying $d(Tx, Ty) \leq \lambda(x,y)d(x,y)$, where $x, y \in P$ and $\lambda(x,y) \in F$, and $S$ denote the set of all continuous mappings of $P$ into itself such that $\cap T^n(P) = \{x\}$, a singleton. **Theorem 1.** A metrizable space $X$ is compact if $R = S$ for every nonempty closed subset $P$ of $X$. **Theorem 2.** Let $T$ be $(\epsilon, \lambda)$-uniformly locally contractive mapping of a straight set of Type II. Then $T$ is globally contractive with the same $\lambda$. **Theorem 3.** Let $T$ be $\epsilon$-contractive mapping of an $\epsilon$-chainable metric space $X$ into itself, i.e., $0 < d(x,y) < \epsilon \Rightarrow d(T(x), T(y)) < d(x,y)$, $x, y \in X$, $x \neq y$, satisfying $\exists x \in X : [T^n(x)] \subseteq [T^n(x)]$ with $\lim_{n \to \infty} T^n(x) \in X$. Then $T$ has a unique fixed point. (Received November 5, 1968.)

663-555. DANIEL REICH, Johns Hopkins University, Baltimore, Maryland 21218. A $p$-adic fixed point formula.

Let $F$ be the Banach space of functions on a polydisc $p$-adically holomorphic away from an algebraic hypersurface $[f(X) = 0]$. **Theorem 1.** If $f$ is homogeneous and splits into distinct irreducible factors mod $p$, $F$ has an orthonormal basis of the form $\{Q_i/f\}$, $Q_i$ polynomials. If $f$ has coefficients in a finite unramified extension of $Q_p$, then $[Xf(X) \neq 0 (mod p)]$ defines a variety $V^n$ over a finite field $k$. **Theorem 2.** Let $g \in F$; there is a completely continuous endomorphism $\psi_g$ of $F$ such that $(|k| - 1)^n \text{Trace} (\psi_g) = \Sigma g(x)$, where the sum runs over the fixed points of the Frobenius morphism in $V^n$. For $g = 1$, this represents the zeta function of $V^n$ (and so of any variety over $k$) as a $p$-adically
meromorphic function. A similar consequence for the L-series of varieties with group action is a generalization of the "elementary" proof of Dwork and Serre that L-series of Kummer coverings are rational to the case when the ground field may be enlarged to make the covering Kummer, provided this extension has degree prime to the degree of the covering. (Received November 5, 1968.)

663-556. H. E. STONE, Texas Christian University, Fort Worth, Texas 76129. The Hilbert basis theorem for halfrings. Preliminary report.

A semi-ideal of an hemiring H is a subset S such that S + S ⊆ S, HS ⊆ S, and SH ⊆ S. If for each s ∈ S and each h ∈ H, s + h ∈ S, then S is an ideal. A pair (u,v) of elements of H is a unital pair provided hu + h = hv and uh + h = vh for each h ∈ H and u ≠ v; H is unital if it contains a unital pair. A Noetherian hemiring is a unital hemiring with maximal condition on ideals. For x in some over-hemiring of H, denote by H[x] the subsemiring generated by H ∪ {x}. Theorem. If H is an halfring, then H[x] is Noetherian for each possible choice of x if and only if the ring of differences of H is Noetherian. An example is given of a division halfring (hence a simple halfring) whose ring of differences is not Noetherian. This shows that an analogue of the Hilbert Basis Theorem cannot hold for halfrings in general. Conversely, it is shown that the ring of differences of H is Noetherian if H is semisubtractive: For every a and b in H, one of the equations a + x = b, a = b + x has a solution x ∈ H. Therefore the Hilbert Basis Theorem holds for halfrings which are direct sums of semisubtractive halfrings. (Received November 6, 1968.)

663-557. O. A. SLOTTERBECK, University of Texas, Austin, Texas 78712. On finite factor coverings of groups. Preliminary report.

Let Ξ be a class of groups. A group G is said to be an N-Ξ group (respectively, L-Ξ group) iff G = {1} = ∪_{i=1}^{n} (K_i - N_i) where N_i ≠ K_i ⊆ G (respectively, N_i ≠ K_i ⊆ G and N_i ≠ G) and K_i/N_i ∈ Ξ. A class Ξ is N-closed (respectively, L-closed) iff every N-Ξ group (respectively, L-Ξ group) is an Ξ group. Theorem 1. If Ξ is closed under the operations of taking subgroups and homomorphic images, then every N-Ξ group has a finite normal series with Ξ factors. Corollary. A group is solvable iff it is either an N-Abelian or L-Abelian group. Theorem 2. A group is polycyclic iff it is an N-cyclic group. Theorem 3. A group is supersolvable iff it is an L-cyclic group. Various classes satisfying finiteness conditions are investigated with respect to N and L closure. Typical results, other than the obvious ones covered by Theorem 1, are the following. The class of groups satisfying either the minimum or maximum condition on subnormal subgroups is N-closed. If "normal" is substituted for "subnormal" in the preceding statement, then the class is L-closed. The class of finitely generated groups is not L-closed. (For related results see Abstract 655-34, these Notices 15 (1968), 475.) (Received November 5, 1968.)

663-558. J. L. GROSS, Princeton University, Princeton, New Jersey 08540. Manifolds in which the Poincaré conjecture is true.

A homotopy 3-cell is a compact, connected 3-manifold whose fundamental group is trivial and whose boundary is a 2-sphere. The Poincaré conjecture is true if and only if every homotopy 3-cell is a 3-cell. If M is a 3-manifold, one says that the Poincaré conjecture is true in M provided that
every imbedded homotopy 3-cell in M is a 3-cell. Equivalently, one says that M is in PC (the "Poincaré Category"). Pasting Theorem. Whenever a 3-manifold in PC is pasted to itself or to another 3-manifold in PC across a disk, a sphere, or a projective plane, the resulting 3-manifold is in PC. Moreover, if the operation of pasting across any compact surface other than a disk, a sphere, or a projective plane always preserves membership in PC, then the Poincaré conjecture is true. (Received November 5, 1968.)

663-559. R. Y. LEE, Sandia Laboratories, Division 1712, Albuquerque, New Mexico 87115. On uniform simplification of linear differential equation in a full neighborhood on a turning point.

Let $A(x, \epsilon)$ be a $2 \times 2$ matrix holomorphic in $|x| \leq x_0, 0 < |\epsilon| \leq \epsilon_0, |\arg \epsilon| \leq \theta_0$, and admits a uniform asymptotic expansion in $\epsilon$ as $\epsilon \to 0$, $|\arg \epsilon| \leq \theta_0$. Furthermore, assume that the equation

$$
(1) \quad \epsilon y' = A(x, \epsilon)y
$$

can be formally simplified by the transformation $y = (\sum_{r=0}^{\infty} P_r(x) \epsilon^r)z$ into

$$
(2) \quad \epsilon z' = B(x, \epsilon)z,
$$

where (2) is equivalent to the second order scalar equation

$$
(3) \quad \epsilon^2 v'' - (x^2/4 + \epsilon \mu(\epsilon))v = 0.
$$

(The function $\mu(\epsilon)$ is not uniquely determined, any other function having the same asymptotic expansion as $\epsilon \to 0; |\arg \epsilon| \leq \theta_0$ will serve as well.) Then results obtained so far show that corresponding to every $x$-sector (with vertex at $x = 0$) of central angle less than $\pi$, there exists a function $P(x, \epsilon)$ having the formal series

$$
\sum_{r=0}^{\infty} P_r(x) \epsilon^r
$$

as asymptotic expansion valid for $x$ in the given $x$-sector and that $y = P(x, \epsilon)z$ takes (1) into (2). In this paper it is shown that by choosing $\mu(\epsilon)$ correctly one can find a function $Q(x, \epsilon)$ having

$$
\sum_{r=0}^{\infty} P_r(x) \epsilon^r
$$

as its asymptotic expansion valid in a full neighborhood of $x = 0$, and that $y = Q(x, \epsilon)z$ takes (1) into (2). (Received November 5, 1968.)

663-560. BERRIEN MOORE, III, University of Virginia, Charlottesville, Virginia 22901. Outer factorization for vectorial Toeplitz operators. Preliminary report.

All unfamiliar definitions can be found in [E. Landesman, Pacific J. Math. 21 (1967), 113-132, and M. Rosenblum, J. Math. Anal. Appl. 23 (1968), 139-147]. Let $T$ denote a nonnegative bounded vectorial Toeplitz operator on a Hilbert space $H$ relative to a unilateral shift $S$. Denote the unique nonnegative square root by $Y$. Let $H[Y]$ and $H[Y^+]$ be the dual Hilbert spaces associated with $Y$. Our concern is the question of the existence of an outer operator $A$ such that $T = A^*A$ and thereby with considerations as to when the Szegö infimum, relative to $T$, is positive. The results of R. Douglas [J. Math. Mech. 16 (1966), 119-126] are discussed within the dual space structure and it is shown that the Szegö infimum is positive if $C \subseteq \text{Rng } Y$ where $C = \text{Ker } S^*$. Related results are identified for vectorial Laurent operators and the Kolmogorov infimum. (Received November 5, 1968.)

663-561. J. R. GOLDMAN, Harvard University, Cambridge, Massachusetts, and GIAN-CARLO ROTA, Massachusetts Institute of Technology, Cambridge, Massachusetts. The number of subspaces of a vector space.

The number $B_n$ of subspaces of an $n$-dimensional vector space over GF(q) is studied by the methods of the Umbral calculus. This calculus provides a general technique for proving theorems in particular identities, involving the $B_n$ and the Gaussian coefficient $\binom{p}{k}$. Examples include a recursion for the $B_n$, an infinite product expansion for the Eulerian generating function of the $B_n$, and a q-analog of the Pascal triangle for the $\binom{n}{k}$. (Received November 5, 1968.)

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Let $H$ be the selfadjoint operator defined for suitable $f$ in $L^2(-\infty, \infty)$ by $Hf(x) = -d^2f(x)/dx^2 + V(x)f(x)$, where $V$ is a bounded measurable function. **Theorem.** If $V$ is differentiable, $V$ and $V'$ bounded, and $-\text{sgn}(x)V'(x) \leq a|x|^{-3} + c$ for $|x| \geq b$, for some positive $a$, $b$, and $c$, and $-\text{sgn}(x)V'(x) < 0$, then $H$ is absolutely continuous. By a theorem of C. R. Putnam [Theorem 2.13.2 in Computation properties of Hilbert space operators and related topics, Springer-Verlag, Berlin, 1967] it is enough to find a selfadjoint bounded operator $A$ such that $i[H, A]$ is positive and bounded on the domain of $H$. Similar results are obtained for operators in $L^2(\mathbb{R}^3)$. (Received November 5, 1968.)


The structure of a Lie $p$-algebra $L$ over an arbitrary field $k$ of characteristic $p$ is studied, primarily in terms of the tori and Cartan subalgebras of $L$. The latter are related as in an algebraic group. Maximal tori are preserved under base extension. New "exponential operators" on sets of Cartan subalgebras are introduced and studied. If some Cartan subalgebra is a torus, this condition is preserved by these operators. If $L$ is solvable, any two Cartan subalgebras of $L$ are "conjugate" under these operators. If $B$ is a solvable $p$-ideal of $L$, $B \cap T$ is a maximal torus of $B$ for any maximal torus $T$ of $L$. (Received November 5, 1968.)

663-564. Stephen Kulik, California State College, Long Beach, California. The solution of ordinary simultaneous equations.

A solution of $n$ simultaneous equations in $n$ unknowns, (1) $f_i(x_1, x_2, \ldots, x_n) = 0$, $i = 1, 2, \ldots, n$, is obtained in the form of numerical series by introducing some parameters. Consider the equations, (2) $F(x_1, x_2, \ldots, x_n, h_iu_i) = 0$, $h_i = 1$ for $i = 1, 2, \ldots, k$, $1 \leq k \leq n$; $h_i = 0$ for $i = n - k$, which coincide with (1) for $u_i = 0$. Let $a_1, a_2, \ldots, a_n$ be approximations to a solution of (1) and at the same time, the solution of the equations not involving the parameters. Let further, the numbers $u_{0i}$ satisfy the equations $F_i(a_1, a_2, \ldots, a_n; h_iu_i) = 0$. Taylor series expansions for the solution set of (1) is now obtained in terms $u_{0i}$'s and $a_i$'s. (Received November 5, 1968.)


In a recent paper (Generators for some rings of analytic functions, Bull. Amer. Math. Soc. 73 (1967), 943-949) L. Hormander considered rings of analytic functions on a region $\Omega \subset \mathbb{C}^n$ defined as follows: **Definition 1.** Given a function $p \equiv 0$ defined on $\Omega$, $A_p$ denotes the ring of all functions $F : \Omega \to \mathbb{C}$ analytic on $\Omega$ for which there exist constants $c_1 \geq 0$ and $c_2 \geq 0$ such that $|f(z)| \leq c_1 \exp(c_2p(z))$, $z \in \Omega$. Under certain conditions on $p$ the following was proven: **Theorem 1.** Finitely many functions $f_1, \ldots, f_N \in A_p$ generate the ring $A_p$ if and only if there exist constants $\epsilon > 0$ and $c \equiv 0$ such that $|f_1(z)| + \ldots + |f_N(z)| \leq \epsilon \exp(-c p(z))$, $z \in \Omega$. Using similar methods, this may be generalized as follows: **Definition 2.** Given $f_1, \ldots, f_N \in A_p$, denote by $I$ the ideal of $A_p$...
generated by \( f_1, \ldots, f_N \), and let \( J \) be the set of all \( g \in A_p \) for which there exist constants \( c_1 \neq 0 \) and \( c_2 \neq 0 \) such that \( |g(z)| \leq c_1 \exp(c_2 p(z)) \left[ |f_1(z)| + \cdots + |f_N(z)| \right] \), \( z \in \Omega \). Then \( J \) is an ideal of \( A_p \) and \( I \subset J \). (In general, \( I \not\subset A_p \).) Theorem 2. There exists an integer \( k \equiv 1 \) such that for all \( f_1, \ldots, f_N \in A_p \) we have \( J^k \subseteq I \); in fact, we may take \( k = \text{Min} \{2n + 1, 2N - 1 \} \). There is some indication that the result is actually valid for \( k = \text{Min} \{n + 1, N \} \), and we can show that when \( n = 1 \) this is actually the case (i.e., \( J^2 \subseteq I \)) for some choices of the function \( p \). Finally, these methods also have application to various modules over the ring \( A_p \) and yield similar results. (Received November 5, 1968.)

663-566. R. C. ORR, Syracuse University, Syracuse, New York 13210. Remainder estimates for squarefree integers in arithmetic progressions.

Let \( q \) and \( q_i \) denote squarefree numbers. Let \( Q(x; k, i) = \sum_{1 \leq n \leq x, \text{n squarefree, max(R(x; k, i))}} \) and \( f(k) = \prod_{p \mid k} \frac{1}{p^2} \). Given \( k, i \) with \( (k, i) = 1 \), let \( s = k/r \) and define \( R(x; k, i) = Q(x; k, i) - xf(k)q^{s}(k)/snp(s) \). Theorem 1. For any \( A, \sum_{k \equiv y; (k, i) = \text{squarefree}} \max[R(x; k, y)] = O(x \log x)^A \), where \( y = x^{2/3} \log \log x \). Theorem 2. Let \( y < x \). Then \( \sum_{k \equiv y; (k, i) = \text{squarefree}} R(x; k, y) = O(x \log x)^2 \). The problem of finding an asymptotic for \( \sum_{p \leq x} d_3(p - 1) \), \( t \neq 0 \), is still unsolved. Theorem 1 yields the following in the squarefree case: \( \sum_{p \leq x} d_3(p - 1) = L \cdot x \log^2 x + O(x \log x \log \log x) \) where \( L \) is a nonzero constant depending on \( t \). Theorem 2 yields remainder estimates for products of squarefrees corresponding to Barban's results for products of primes [Russian Math. Surveys 21 (1966)]. A representative application of these last estimates follows: Let \( s = +1 \) and consider \( \{a_i: 1 \leq i \leq r \} \) where \( \sum_{i=1}^r a_i = 1 \) and \( 0 < a_1 \leq \cdots \leq a_r < 1 \). Suppose then \( g(n) = \sum_{d \mid n} h(d) \) where \( h(d) \) is multiplicative and \( h(d) = 1 \). Then, for any \( B \), it follows that \( \sum_{q \leq x; a_l \leq q \leq x} a_l h(q) = Kx + O(x \log x) \), where \( K \) is a constant depending on \( r \) and the function \( h \cdot K = 0 \) if and only if \( h(p) = -(p^2 - 1)/(1 - 1/p)^{r-1} \) for some prime \( p \). (Received November 6, 1968.)

663-567. L. -C. KAPPE, State University of New York, Binghamton, New York 13901. Coverings by powers and properties of groups.

We consider the following two conditions for a property \( E \) of groups: (1) If \( P(m_1, G), \ldots, P(m_k, G) \) have property \( E \) then \( P((m_1, \ldots, m_k), G) \) has property \( E \), where \( P(m, G) \) denotes the group generated by the \( m \)-th powers of elements in \( G \) and \( (m_1, \ldots, m_k) \) the greatest common divisor of the integers \( m_i \). (2) If for every \( g \in G \) there exists a power \( P(m, G) \) with property \( E \) such that \( g \in P(m, G) \) then also \( G \) has property \( E \). F. Szasz (MR 17 #494) and V. Dlab (MR 22 #8054) have shown that the property cyclic satisfies both conditions. There are wider classes of properties satisfying condition (1) as e.g., commutativity, nilpotency of fixed class or fixed derived length, and certain saturated formations (Abstract 67T-696, these Notices 14 (1967), 949). Condition (2) may be considered from two aspects. First we can show that (1) always implies (2). The converse is not true. Further we observe that for some classes of groups like finite groups and finitely generated nilpotent groups condition (2) can only be satisfied in such a way that \( P(m, G) = G \) holds for at least one of the integers \( m \). (Received November 5, 1968.)
Let K be a continuum of transfinite diameter 1. A system of points $z_1, z_2, \ldots, z_n \in K$ that maximizes $\prod_{\mu \neq \nu} |z_\mu - z_\nu|$ is called a system of Fekete points. In the case when K is a closed Jordan domain, the Fekete points will all lie on the boundary curve $\Gamma$, and can be written in the form: 

$$z_\mu = \psi(e^{i \phi_\mu})$$

where: $\phi_1 < \phi_2 < \ldots < \phi_n$, and $\psi$ is the exterior mapping function. The following lower and upper bounds are obtained for $|z_\mu - z_\nu|$ and $|\phi_\mu - \phi_\nu|$ respectively ($\mu \neq \nu$): (1) For arbitrary continua, $|z_\mu - z_\nu| \geq 2en^2$. (2) For convex sets, $|\phi_\mu - \phi_\nu| > 1/4n$. (3) For boundary curves which satisfy a weak smoothness condition, $c_1/n < \phi_{n+1} - \phi_n < c_2/n$. All these results can be extended to any point system $z_1, \ldots, z_n$ for which the fundamental polynomials of the Lagrange interpolation are uniformly bounded. (Received November 5, 1968.)

A formal system $ZF^*_c$ for set theory is presented in which the "constructive" flavor of the Zermelo-Fraenkel axioms is reflected in the underlying logic, in the sense that the law of the excluded middle is taken to hold only for formulas whose quantifiers are restricted to certain terms representing sets. These terms are obtained by introducing term-operators which correspond to the various constructions provided by the axioms. For example the power-set axiom takes the form $(x)(y)[y \in P(x) \iff y \subseteq x]$. Claim. All the main results of classical set theory are provable in $ZF^*_c$. Theorem. If $ZF^*_c$ is consistent, then so is the usual classical system $ZF_c$ (with or without the above-mentioned terms). (Received November 5, 1968.)

In a controlled, constrained, finite, time-dependent system where a set of initial and a set of terminal state variables are defined and a set of constraints is specified, it is required to find a set of system control functions which optimize a given pay-off function. In many cases, no such optimum exists. Necessary and sufficient conditions for its existence are examined. The problem requirement takes on a realistic character in a near-optimal set of values of the given pay-off function which can be found within a $(1 - \epsilon)$ confidence region. The tools for obtaining a required set of system control functions depend upon the techniques of multivariate statistical analysis and the procedures described in an earlier paper of the author, [Abstract 659-16, these Notices 15 (1968), 907]. The problem is easily modelled for computation. A numerical example illustrates the power of the concept of near-optimal pay-off in providing a wide latitude in the choice of system control functions. The theory of near-optimal pay-offs and the selection of system control functions to obtain those pay-offs parallels the theory of a classical set of boundary value problems. (Received November 5, 1968.)
Let $R(\lambda; A)$ denote the resolvent $(\lambda I - A)^{-1}$ of an operator $A$ on a Banach space. With $A$ closed and densely defined it is shown that (1) $\|A^k\| \leq M$ for all $k = 1, 2, 3, \ldots$ if and only if $\|R(\lambda; zA)^n\| \leq (2\pi k)^{1/2}N$ as $k \to \infty$. If $A$ is an operator on a Hilbert space, this can be strengthened to yield (3) $\|R(\lambda; zA)^n\| \leq N(\lambda - 1)^{-n}$ for all $\lambda > 1$, $|z| = 1$, and $n, k = 1, 2, \ldots$. (Result (4) is a consequence of the power inequality proved by C. A. Berger.) (Received November 5, 1968.)

Let $G$ be a group and let $\{G_q\}$ be the lower central series of $G$. Define a descending chain $\{G(q)\}$ of subgroups as $G(q) = G_qG_q'$, where $G_q'$ denotes the second derived group of $G$. It is clear that the factor group $G_q(q) = G(q)/G(q + 1)$ is an abelian group. If $G$ is finitely generated, so is $G_q(q)$. In particular, if $G$ is a free group $F$ of finite rank, then $G_q(q)$ is a free abelian group of finite rank.

Theorem. Let $L$ be a tame link with two components in 3-sphere $S^3$ and let $G = \pi_1(S^3 - L)$. Then $G_q(q)$ is completely determined by the Alexander polynomial $\Delta(x, y)$ of $L$. Corollary 1. If $\Delta(x, y) = 0$, then $G_q(q)$ is a free abelian group of rank $q - 1$ for $q \geq 2$. Corollary 2. $\Delta(x, y) = 0$ iff the longitudes of $L$ belong to $G(\infty) = \bigcap_q G(q)$. (Received November 5, 1958.)

Let $L$ be a Riesz space (vector lattice). A seminorm $p$ on $L$ is called a Riesz seminorm if $f, g \in L, |f| \leq |g|$, implies $p(f) \leq p(g)$. Let $\tau$ be a Hausdorff locally convex topology on $L$ generated by a family of Riesz seminorms. A sequence $\{f_n\}$ in $L$ is said to converge $\tau$-locally to $f \in L$ if there is a $\tau$-bounded set $B$ in $L$ such that for every real $r > 0$ there exists $N$ and $n \equiv N$ implies $|f - f_n| \in rB$. A $\tau$-local Cauchy sequence is defined analogously, and the space $L$ is called $\tau$-locally complete if every $\tau$-local Cauchy sequence is $\tau$-convergent. If $B$ is a $\tau$-closed, convex, solid subset of $L$, we call $\lambda$, defined by $\lambda(f) = \inf\{r > 0 \text{ and } 1/|f| \in B\}$, $\lambda(f) = \infty$ if $1/|f| \notin B$, all $r > 0$, a $\tau$-closed extended Riesz norm. In this context we prove a generalization of a theorem of I. Halpern and W. A. J. Luxemburg [Trans. Royal Soc. Canada 50 (1956), 33-39]. Theorem 1. If $L$ is $\tau$-locally complete, if $0 \leq f_n$ and $\{f_n\}$ is a $\tau$-local Cauchy sequence, then $\sup_n f_n$ exists. (3) If for some $\tau$-closed extended Riesz norm $\lambda$ and sequence $\{f_n\}$ in $L$, $\sum_{k=1}^\infty \lambda(f_n) < \infty$, then there exists $\sup_n \sum_{k=1}^n f_k$. Theorem 2. If $(L, \tau)$ is a $\tau$-locally complete, bounded, locally convex Riesz space, then $L$ is barreled and $\tau$ is the strongest locally convex topology on $L$ making it a locally convex Riesz space. (Received November 5, 1968.)
In many measuring devices the output $F(x)$ is a linear transformation (smoothing) of the input, i.e., $F(x) = \int K(x,t)f(t)dt$. To restore the original data $f(t)$, the transformation needs to be inverted. This is an improperly posed problem; in particular, small measuring errors may lead to arbitrarily large distortions of the result. The following approach circumvents these difficulties. Let $F(x)$ be measured at a set of points $\xi_i$ and let $\epsilon_i$ be the measuring error in the $i$th measurement. Consider the approximation $f(t) \approx \sum a_i(t)(F(\xi_i) + \epsilon_i)$. To determine the coefficients $a_i(t)$ minimize the norm of $I_\epsilon$, where $I_\epsilon(f,\epsilon) = f(t) - \sum a_i(t)(\epsilon_i + \int K(\xi_i,t)f(t)dt)$ is a linear operator in $f$ and $\epsilon$ giving the error in the above approximation. The choice of a norm for $I_\epsilon$ depends on a priori information about $f$ and $\epsilon$. The method can be applied to other improperly posed problems. (Received November 5, 1968.)

Let $S \supseteq R$ denote an extension of discrete rank one valuation rings. The quotient field extension $K \supset k$ is called fiercely ramified if $\overline{S} \supset \overline{R}$ is purely inseparable and $[K:k] = [\overline{S} : \overline{R}]$. The purpose of this paper is to initiate the development of a ramification theory which includes fiercely ramified extensions. Let $k$ denote the quotient field of a complete discrete rank one valuation ring $R$ of unequal characteristic and let $a$ denote the absolute ramification index of $k$; assume that $k$ contains a primitive $p$th root of unity where $p = \text{char } \overline{R}$. Let $S$ denote the integral closure of $R$ in a Galois extension $K \supset k$ of degree $p$. This study presents a method for computing $S$ in $g$ steps where $0 \leq g \leq (a/p - 1) - 1$ which entails the construction of a sequence $(S_i)$ of integral ring extensions of $R$ in $K$ where $0 \leq i \leq g$, $S_{i-1} \subseteq S_i$, and $S_g \subseteq S$. The ring $S_g$ provides a criterion for determining if $K \supset k$ is unramified, wild or fierce. Let $i$ denote the discontinuity in the sequence of ramification groups of $K \supset k$; then $i = (ap/p - 1) - p$ when $K \supset k$ is wild, and $i = (a/p - 1) - g$ when $K \supset k$ is fierce. (Received November 5, 1968.)

The elements $a, b$ of a lattice are said to form a dual-modular pair if $x \leq b = x \wedge (a \vee b) = (x \wedge a) \vee b$, and are said to form a modular pair if $x \leq b = x \vee (a \wedge b) = (x \vee a) \wedge b$. In his paper On infinite-dimensional linear spaces (Trans. Amer. Math. Soc. 57 (1945), 155-207), G. W. Mackey asked "Are there any incomplete normed linear spaces in whose lattice of closed subspaces modularity and d-modularity are equivalent?" I have this result: Theorem. If, in the lattice of all norm-closed subspaces of a real or complex inner product space $X$, every modular pair is dual-modular, then $X$ is complete. This provides the answer "no" to Mackey's question for spaces whose norm can be derived from an inner product. One would guess that the result remains valid in the general normed linear space, and in fact one is led naturally to ask the following question: If, in the lattice of all closed subspaces of a locally convex Hausdorff space $X$, every modular pair is dual-modular, is $X$ necessarily complete in its Mackey topology? The proof of the theorem for inner product spaces uses an idea due to Amemiya and Araki (A remark on Piron's paper, Publ. Res. Inst. Math. Sci., Ser. A 12 (1966-1967), 243-247). (Received November 5, 1968.)
In 1955 L. E. Fuller developed a canonical set for $m \times m$ matrices under row equivalence over a principal ideal domain $R$ modulo $p^n$, $R/Rp^n$, for $p$ a prime. If $n = 1$, $R/Rp$ is a field and the Fuller canonical matrix reduces to the Hermite canonical matrix. Employing the Fuller form, we show that the concepts of rank and row, column and null-spaces of a matrix over a field can be extended to $R/Rp^n$. Similarly, we show that such theorems as "The dimension of the column space plus the dimension of the null-space equals the number of columns of the matrix" have natural extensions to $R/Rp^n$. By an application of the Chinese Remainder Theorem the results can be stated for composite modulus. (Received November 5, 1968.)

A linearly ordered cofinal set of open covers almost implies metrizability and compactness.

A Hausdorff space $X$ satisfies property $\mathcal{U}$ iff in the collection $\mathcal{C}$ of all open covers of $X$ there is a set $\mathcal{J}$ which is linearly ordered and cofinal in $\mathcal{C}$ with respect to the partial ordering of $\mathcal{C}$ by refinement. Proposition 1. If $X$ has property $\mathcal{U}$, then either $X$ is first countable and the set $\mathcal{J}$ may be chosen as a countable set, or countable intersections of open sets are open. Proposition 2. If $X$ has property $\mathcal{U}$ and one nonisolated point in $X$ has a countable base, then the set $N$ of nonisolated points of $X$ is countably compact. Corollary. A metric space has property $\mathcal{U}$ iff it is compact except for isolated points. Proposition 3. If $X$ has property $\mathcal{U}$, is first countable, and either paracompact or completely regular, then $X$ is metrizable and $N$ is compact. (Received October 28, 1968.)

Diophantine equations of degree $n \geq 2$.

In this paper various types of homogeneous diophantine equations of degree $n \geq 2$ in $n$ variables are solved; the structure of these equations differs widely from the "Generalized Pellian Equation" which was solved by the author in a previous paper (The generalized Pellian equation, Trans. Amer. Math. Soc. 127 (1967), 76-89). As in the case of the Generalized Pellian Equation, infinite systems of solution vectors are stated for every different type of the above diophantine equations. The author discloses the connections between these solution vectors and the powers of a unit of an algebraic number field generated by the root of an algebraic equation with integral coefficients of degree $n \geq 2$ over the rational numbers. Another result of the paper states explicitly the norm of any algebraic number of that field in form of an $n$th order determinant. If the algebraic number in question is a unit of the field so that its norm equals $\pm 1$, the integral powers of this unit supply the solutions of the respective diophantine equation $n = \pm 1$. By means of complicated linear transformations, the determinant of the norm is mapped onto the determinants representing the diophantine equations mentioned above. (Received November 6, 1968.)
Theorem. A subspace of countable codimension in a locally convex space whose dual is weak* sequentially complete has weak* sequentially complete dual. Theorem. A closed subspace of countable codimension in a Mackey space whose dual is weak* sequentially complete is a Mackey space. An example is given of a nonbarreled Mackey space with weak* sequentially complete dual. Thus, this result is analogous to but does not reduce to the result announced by the authors in Abstract 660-44, these Notices 15 (1968), 1020. (Received November 6, 1968.)

663-581. W. L. REDDY, Wesleyan University, Middletown, Connecticut 06457, and ERIK HEMMINGSEN, Syracuse University, Syracuse, New York 13210. M-S coverings on n-manifolds.

Let $M$ and $N$ be compact n-manifolds and let $f: M \to N$ be a Montgomery-Samelson covering of degree $d$ with branch set $B_f$. Let $q$ be any prime dividing $d$, then, for any $m \equiv 0$, $\sum_{m}^{\infty} \dim H_{i}(B_f; \mathbb{Z}_q) = \sum_{m}^{\infty} \dim H_{i}(M; \mathbb{Z}_q)$ and $\chi(M) + (d - 1)\chi(B_f) = d\chi(N)$ where $H$ denotes Čech homology and $\chi$ denotes Euler characteristic. It follows that, if $M$ and $N$ are mod $q$ homology spheres, then $B_f$ is a homology $(n - r)$-sphere where $n$ is even. These formulas rule out large classes of degrees and Euler characteristics for various choices of $M$; for instance, if $M = \mathbb{C}P^2$, the only possibility is $d = 2$, $\chi(N) = 3$ and $\chi(B_f) = 1$ (such a map exists). (Received November 6, 1968.)


Let $A$ be nonassociative, noncommutative algebra with a nonempty set of idempotents $I$. It is assumed that with respect to any idempotent the vector space structure has a decomposition into subspaces which are identified or annihilated on the left and right by the idempotent and which satisfy certain space multiplication properties. (See J. M. Osborn, J. Algebra 2 (1965), 48.) The axioms assumed will upon definition of a radical and a primitive algebra lead to a characterization of primitive algebras as alternative or slight variations of alternative algebras (see (i)). Furthermore, A modulo its radical will be isomorphic to a direct sum of primitive algebras (Theorem).

(i) Theorem. A primitive algebra satisfying the Axioms will be isomorphic to either a primitive (Jacobsen) ring with nonzero socle, a split Cayley algebra or a modification of a four-dimensional alternative algebra. (Received November 6, 1968.)


Let $(x_{n+h}^{n+h} : n \to \infty)$ be a strictly stationary random process with $x_n \in L^\infty(G, B, \mathbb{P})$. Let $B_{0,p}$ be a Borel algebra generated by the sets $x_k^{-1}(A) \subset \Omega$, $A$ is a Borel set in $R^1$ for $-p \leq k \leq 0$. Let $B_0$ be the Borel algebra generated by $\bigcup_{p=0}^{+\infty} B_{0,p}$. Due to Wiener and Masani the nonlinear predictor $\tilde{X}_k$ for the given process $(x_{n+h}^{n+h} : n \to \infty)$ is defined as $E(x_k | B_0)$. Then we have the following Theorem. For a given strictly stationary random process $(x_{n+h}^{n+h} : n \to \infty)$ described above, there exists a martingale $\{Y_n, F_n : n \geq 1\}$ such that $\tilde{X}_k = \lim_{n \to \infty} Y_n$, $k > 0$. (Received November 6, 1968.)
Synchronizing sequences and central-definite events.

Let $\Sigma$ be a finite set of symbols, with $\Sigma^*$ the free monoid over $\Sigma$. The tape $x \in \Sigma^*$ is a subtape of $y \in \Sigma^*$ if there are tapes $z$ and $w$ such that $y = zxw$. A state automaton over $\Sigma$ is a pair $A = (S, \delta)$, where $S$ is the finite set of states of $A$ and $\delta: S \times \Sigma^* \rightarrow S$. The tape $x$ synchronizes $A$ if, for all $s$ and $t$ in $S$, $\delta(s,x) = \delta(t,x)$. A. Paz and B. Peleg (J. Assoc. Comp. Mach. 12 (1965), 399-410) define ultimate-definite and reverse ultimate-definite events (subsets of $\Sigma^*$). An event $U \subseteq \Sigma^*$ is central-definite if it has a representation of the form $\Sigma^* C \Sigma^*$, where $C \subseteq \Sigma^*$. Theorem 1. The set of tapes which synchronize an automaton $A$ is a central-definite event. Theorem 2. Each central-definite event has a unique canonical representation $\Sigma^* C \Sigma^*$ where $C \subseteq \Sigma^*$ and no tape in $C$ has a subtape in $C^t$.

Theorem 3. If $\Sigma^* C \Sigma^*$ has canonical representations $\Sigma^* C^t \Sigma^*$ and $C^t \Sigma^*$ when viewed as an ultimate-definite and reverse ultimate-definite event, respectively, then $C^t = C \cap C^t$.

(Received November 6, 1968.)


The Weak Basis Theorem is established for Fréchet spaces and for strict inductive limits of Fréchet spaces using a method based on Banach's proof that any basis for a Banach space is a Schauder basis. (Received November 6, 1968.)

663-586. S. H. DWIVEDI, 719 1/2 De Barr, Norman, Oklahoma 73069. Proximate orders and growth of meromorphic functions.

Let $f(z)$ be a meromorphic function of order $\rho$ ($0 < \lambda < \infty$) and lower order $\lambda$ ($0 \leq \lambda < \infty$). Let $T(r,f)$, $n(r,a)$, $N(r,a)\lambda(r)$, $\rho(r)$ have their usual meanings. We set $H(r) = \exp(\int r_0^r \lambda(r)/r \, dr)$, $n(r) = n(r,a) + n(r,b)$, $D(r) = \exp(\int r_0^r \rho(r)/r \, dr)$, $N(r) = N(r,a) + N(r,b)$, where $a \neq b$, $0 \leq a \leq \infty$, $0 \leq b \leq \infty$. We prove the following: Theorem 1. $\limsup \frac{T(r,f)}{H(r)} < \infty$ if and only if $\liminf \frac{T(r,f)}{D(r)} > 0$. Theorem 2. $0 < \limsup \frac{n(r)}{H(r)} < \infty$ if and only if $0 < \liminf \frac{N(r,a)}{H(r)} < \infty$. Theorem 3. $0 < \limsup \frac{n(r)}{D(r)} < \infty$ if and only if $0 < \liminf \frac{N(r,b)}{D(r)} < \infty$. Theorem 4. $n(r)/H(r) \rightarrow 0$ if and only if $N(r,a)/H(r) \rightarrow 0$. Theorem 5. $n(r)/D(r) \rightarrow 0$ if and only if $N(r,b)/D(r) \rightarrow 0$. (Received November 6, 1968.)

663-587. G. U. BRAUER, University of Minnesota, Minneapolis, Minnesota 55455. Linear transformations on a sequence space.

Let $m_0$ be the Banach space of bounded sequence with norm $\|S\| = \limsup |s_n|$. We study linear operators $T$ on $m_0$ which are representable by triangular nonnegative regular summation matrices $(t_{nk})$. If $T$ evaluates no divergent sequences then the spectrum of $T$ lies on the unit circle. If $(t_{nk})$ satisfies $\Sigma_{k=0}^{n+1} t_{n+1,k} - t_{n,k} = (t_{n+1,n+1})$, then the only real elements of the spectrum of $T$ are zero and one. The operator $T$ is never compact. (Received November 6, 1968.)
663-588. JEFF CHEEGER, University of Michigan, Ann Arbor, Michigan 48104. The differentiable pinching problem for symmetric spaces of rank one.

Calabi and Gromoll have shown independently that a sufficiently pinched Riemannian manifold is diffeomorphic to the standard sphere. In the following theorem we extend this result by allowing the model space to be an arbitrary symmetric space of rank one. Theorem. There exists a sequence of constants \( \delta_n \) such that if \( M \) is a \( \delta_n \)-pinched (resp. \( \delta_n \)-holomorphically pinched, ...) Riemannian (resp. Kaehler, ...) \( n \)-manifold, then \( M \) is diffeomorphic to \( S^n \) (resp. \( \mathbb{C}P(n) \), ...). (Received November 6, 1968.)


Let \( D \) be the unit disc \( \{|z| < 1\} \) and \( D_n \) the polydisc \( \{|z_j| < 1\}, j = 1, \ldots, n \). Given a sequence of positive numbers \( \{A_k\} \) with \( \ln A_k \) convex in \( \ln k \), let \( F_1(A_k) \) be the set of \( f(z) = \sum a_k z_k \) analytic in \( D \), such that \( f(k) \) has a continuous extension to \( D^- \) for all \( k \) and such that \( \{a_k A_k\} \) is bounded. Let \( F_2(A_k) \) be the set of \( f(z) = \sum a_m z^m \) analytic in \( D_n \) such that \( f(m) \) has a continuous extension to \( D_n^- \) for all multi-indices \( m \) and \( \{z_j | z_j = k | a_m |^2 \}^{1/2} \leq 1/A_k \). Theorem 1. If \( f \in F_1(A_k) \) and \( f(k)(1) = 0 \) for all \( k \) then a necessary and sufficient condition to conclude that \( f = 0 \) is that \( \sum k^{-3/2} \ln A_k \) diverge. Theorem 2. If \( f \in F_2(A_k) \) and \( f(m)(1,1,\ldots,1) = 0 \) for all \( m \) then the same conclusion is valid. A continuous analogue of Theorem 1 can be obtained by considering functions of the form \( f(z) = \int_0^\infty f(t) e^{it} \) dt with \( \int_0^\infty f(t) A(t) \) bounded. Half of Theorem 1 has already been proven differently by L. Carleson (Acta Math. 87 (1952), 331). Theorem 2 is proven by applying Theorem 1. (Received November 6, 1968.)

663-590. P. M. GAUTHIER, University of Montreal, Montreal, Quebec, Canada. Distribution of values of meromorphic functions which approach an asymptotic value rapidly.

Let \( f(z) \) be meromorphic in the unit disc \( D \). A sequence \( \{z_n\} \) in \( D \) is a sequence of p-points for the function \( f(z) \) if there is a sequence of non-Euclidean cercles de remplissage for \( f(z) \) whose centers are \( \{z_n\} \). Theorem. If \( f(z) \) satisfies on a boundary curve \( a \subset D \), the inequality \( |f(z)| \leq \exp[- p(1 - |z|)^2/(1 - |z|)], z \in a \), where \( p(x) \to +\infty \) as \( x \to 0 \), then each point in the end of \( a \) is the limit of a sequence of p-points. Other results of a similar nature are obtained. (Received November 6, 1968.)


Let \( A \) and \( I \) be sets, and \( \mathcal{F} \) an ultrafilter on \( I \). Functions \( f, g : I \to A \) are called \( \mathcal{F} \)-equivalent iff \( f(\mathcal{F}) = g(\mathcal{F}) \) as ultrafilters on \( A \). If \( \mathcal{V} \) is a first-order structure, we say that a retraction \( P \) of \( \mathcal{V}^I/\mathcal{F} \) is consistent iff \( P \) respects \( \mathcal{F} \)-equivalence. Theorem. \( \mathcal{V} \) is a retract of a compact topological relational structure iff \( \mathcal{V} \) is a consistent retract of every ultrapower of \( \mathcal{V} \). (This contrasts with the result of W. Węglorz that \( \mathcal{V} \) is atomic-compact iff \( \mathcal{V} \) is a retract of every ultrapower of \( \mathcal{V} \); Fund. Math. 59 (1966), 289-298.) (Received November 6, 1968.)
663-592. HEINZ BAUER and H. S. BEAR, New Mexico State University, Las Cruces, New Mexico 88001. The part metric in a convex set.

This paper extends the results announced in Abstract 658-76, these Notices 15 (1968), 740, and replaces that paper. The principal results are here stated for a convex set $C$ containing no line, rather than for a cone as before. Convex combinations are continuous on each part, with respect to the part metric $d$. If $C$ is complete in a weak linear space $L$ (i.e., every $w(L,L')$ Cauchy net in $C$ converges $w(L,L')$ to a point of $C$), then $d$ is complete. The parts in the cone of positive Radon measures on a locally compact space are characterized, and the $d$-metric is shown to be a complete metric topologically equivalent on each part to an appropriate $L^0$ metric (the $L^0$ metrics are not complete on parts). The continuous selection Theorem of the previous paper is proved for a convex set rather than a cone. This selection theorem is used to give a condition which allows the selection of representing measures $\mu_x = g_x \mu$ (on the Silov or Choquet boundary) for the points $x$ in a part $A$ of a function space or function algebra so that $x - g_x$ is continuous with respect to the $d$-metric on $A$ and the $L^0(\mu)$ metric. (Received November 6, 1968.)

663-593. D. E. BERTHOLF, Oklahoma State University, Stillwater, Oklahoma 74074.

Isomorphism invariants for quotient categories of abelian groups.

Let $\mathcal{J}$ and $\mathcal{J}$ be two Serre classes in the category $\mathcal{C}$ of abelian groups and define the category $\mathcal{C}(\mathcal{J}, \mathcal{J})$ to have the same objects as $\mathcal{C}$, and to have maps given by $\mathcal{C}(\mathcal{J}, \mathcal{J})(A,B) =$

$$\lim_{A, A' \in \mathcal{J}, B, B' \in \mathcal{J}} \text{Hom}(A', B/B').$$

If we let the class of bounded abelian $p$-groups of rank less than $\xi$ for some infinite cardinal $\xi$, be denoted by $\mathcal{E}(\xi)$, then we prove the following generalization of a result of R. J. Ensey (Pacific J. Math. 24 (1968), 71-91). Theorem. If $G$ and $H$ are totally projective $p$-groups and $\xi$ is an infinite cardinal, then $G \cong H$ in $\mathcal{C}(\mathcal{J}(\mathcal{E}(\xi))$ if and only if

(i) there exists a set $I$ of ordinal numbers and a cardinal number $\xi_1 < \xi$ such that $|I| < \xi$, $f_G(a) = f_H(a)$ for all $a \in I$, and $f_G(a) = f_H(a)$ for all $a \notin I$, and

(ii) there exists an integer $k \geq 0$ such that for all integers $r \geq 0$ and ordinal numbers $a$, $\sum_{j=0}^{\xi} f_G(a + k + j) = \sum_{j=0}^{\xi} f_H(a + j)$ and $\sum_{j=0}^{r} f_H(a + k + j) = \sum_{j=0}^{r} f_G(a + j).$ This result also corrects a result of R. J. Ensey announced in Abstract 660-18, these Notices 15 (1968), 1012. (Received November 6, 1968.)

663-594. SHIN'ICHI KINOSHITA, Florida State University, Tallahassee, Florida 32306.

A characterization of a translation in three space within a class of quasi-translations.

Let $B$ be a 3-cell and $a$ an arc in $B$ such that (i) $a \cap \text{Bdry } B$ is an endpoint of $a$ and (ii) $a$ is tame in $B$ except at a $\cap \text{Bdry } B$. Theorem 1. The Sikkema's construction of quasi-translations in three space (Abstract 650-20, these Notices 14 (1967), 921) is to associate a quasi-translation $h(a,B)$ for each pair $(a,B)$. Theorem 2. $h(a,B)$ is topologically equivalent to a standard translation if and only if $(a,B)$ is a trivial pair. (Received November 6, 1968.)
Controllability in differential games.

A definition of controllability for differential games is proposed and a necessary condition is given for linear games to be controllable in that sense. This condition generalizes the well-known controllability condition for linear control systems. (Received November 6, 1968.)

Completion of a noncomplete Lebesgue integral to a complete one.

A real-valued functional \( f \) is called a Lebesgue integral if its domain \( D(\int) \) consists of real-valued functions on a space \( X \) and the following conditions are satisfied: \( D(\int) \) is a linear lattice satisfying the Stone condition, i.e. \( D(\int) \) is a linear space and \( f \cup g, f \cap g \in D(\int) \) for all \( f, g \in D(\int) \); \( \int \) is a finite valued, positive, linear functional such that if \( f_n \in D(\int), f_n(x) \equiv 0 \) on \( X \), \( \int f_n(x) < \infty \) for \( x \in X \) and \( \sum_{n=1}^{\infty} f_n(x) < \infty \), then \( \int f = \sum_{n=1}^{\infty} f_n \). The Lebesgue integral is said to be complete if for every function \( g \) such that there exists \( f \in D(\int) \) satisfying the condition \( 0 \leq g(x) \equiv f(x) \) for all \( x \in X \) and \( \int f = 0 \), we have \( g \in D(\int) \). A condition \( C(x) \) depending on the parameter \( x \) is said to be satisfied \( \int -a.e. \) on \( X \) if \( C(x) \) holds for all \( x \notin A \) where \( A \) is a subset of \( X \) such that its characteristic function \( C_A \in D(\int) \) and \( \int C_A = 0 \). For a Lebesgue integral on \( D(\int) \) over \( X \) define the set \( D(\int_c) = \{ f: f(x) = g(x) \int -a.e. \text{ for some } g \in D(\int) \} \) and on \( D(\int_c) \) define the functional \( \int_c \) by the formula \( \int_c f = \int g, \text{ where } g \in D(\int) \) and \( f(x) = g(x) \int -a.e. \).

Theorem. Let \( \int \) be a Lebesgue integral on \( D(\int) \) over \( X \). Then \( \int_c \) is the smallest complete Lebesgue integral extending \( \int \). (Received November 6, 1968.)

\[ \text{261} \]

\[ \text{261} \]
Let $G$ be a locally compact abelian group with character group $\Gamma$, let $h$ be a continuous complex valued additive homomorphism on $\Gamma$, and let $\mu$ be a finite Borel measure on $\Gamma$ such that $h \in L^1(\mu)$. Now $\hat{\mu}$ (= the Fourier Stieltjes transform) is a function on $G$ and we define the $h$-derivative $\hat{\mu}'$ of $\hat{\mu}$ by $\hat{\mu}'(x) = \int_{\Gamma} h(\varepsilon)\overline{\hat{\mu}(x)} \, d\mu(\varepsilon)$. Then all the usual rules of differentiation apply [see J. Riss, *Eléments de calcul*, Acta Math. (1952-3), 87-89]. The purpose of this note is to expand the class of "differentiable functions" on $G$, recapture the function by integrating its derivative, and develop an "integration by parts" theorem. (Received November 6, 1968.)

Let $S$ be a regular semigroup with $E$ as its set of idempotents and $\Lambda(S)$ as its lattice of congruences. Let $\theta = \{(p \tau) \in \Lambda(S) \times \Lambda(S) : p \in \tau \in E \}$. Theorem. The relation $\theta$ is a complete congruence in the sense that (i) $\theta$ is a congruence, (ii) $\Lambda(S)/\theta$ is a complete lattice, and (iii) the natural homomorphism $\theta^\#: \Lambda(S) \rightarrow \Lambda(S)/\theta$ is a complete lattice homomorphism. (Received November 16, 1968.)

Building on work of G. W. Day (Pacific J. Math. 23 (1967), 479-489), we study countable Boolean algebras (B.A.s). Following Day, define a transfinite sequence of ideals $\Delta_\xi(A)$ of $A$ as follows: $\Delta_0(A) = 0$, $\Delta_{\xi+1}(A) = \text{pre-image in } A \text{ of the ideal generated by the atoms of } A/\Delta_\xi(A)$, $\Delta_\eta(A) = \bigcup_{\xi < \eta} \Delta_\xi(A)$ for $\eta = \text{limit ordinal}$. Let $\lambda(A) = \min \{ \xi \mid \Delta_\xi(A) = \Delta_{\xi+1}(A) \}$, and denote $\Delta(A) = \lambda(A)$. If $A$ is not superatomic, then $A/\lambda(A)$ is free on $\aleph_0$ generators. Define an increasing sequence of ideals $\Lambda_\xi(A)$ in $A/\Delta(A)$ by $\Lambda_\xi(A) = (\Delta(A) + (\Delta_\xi(A): \Delta(A)))/\Delta(A)$. Note that $\Lambda_\xi(A) = A/\Delta(A)$ for $\xi < \lambda(A)$. Call $A$ reduced if $\min \{ \xi \mid \Lambda_\xi(A) = A/\Delta(A) \} = \lambda(A)$. Proposition. If $A$ is not reduced, then $A$ is uniquely (to isomorphism) a direct product of a reduced B.A. and a superatomic B.A. Main Theorem. If $A$ and $B$ are countable, reduced B.A.s, then $A \cong B$ if and only if $\lambda(A) = \lambda(B)$ and there is an isomorphism $a$ of $A/\Delta(A)$ onto $B/\Delta(B)$ such that $a(\Lambda_\xi(A)) = \Lambda_\xi(B)$ for all $\xi$. This result is used to investigate direct product decompositions of certain countable B.A.s. (Received November 6, 1968.)

Substitution for a Lebesgue-Stieltjes type integral.

Let $f$ and $h$ be real-valued functions on the entire real axis which are of bounded variation on every closed interval. Let $g$ be a real-valued function on the entire real axis which is bounded on every closed interval. Suppose $g(x^+)$ exists for every nonnegative real number $x$, suppose $g(0^+)$ exists, and suppose $g(x^-)$ exists for every positive real number $x$ such that either $f(x^-) \neq f(x)$ or $h(x^-) \neq h(x)$. Finally, suppose the weighted refinement integral $\left[ F, (1, -1, 1) \right]^{\text{h}(x)}_{0} f(x) \, dg(x)$ exists for
every positive real number \( b \). Let \( (LS) \int_{[0,b]} f(x) \, dg(x) = \left[ F \left( x \right) \left[ g(x) - g(x-1) \right] \right]_0^b + f(0) \left[ g(b) - g(0^-) \right] \) for every positive real number \( b \), and let \( (LS) \int_{[0,b]} f(x) \, dg(x) = f(0) \left[ g(b^+) - g(0^-) \right] \). This agrees with the Lebesgue-Stieltjes integral in case \( g \) is of bounded variation on every closed interval. Let \( p(x) = (LS) \int_{[0,x]} f(u) \, dg(u) \) for every nonnegative real number \( x \), and let \( p(x) = p(0) \) for every negative real number \( x \). It is shown that if \( b \) is a positive real number such that either 
\[
(LS) \int_{[0,b]} h(x) f(x) \, dg(x) \quad \text{or} \quad (LS) \int_{[0,b]} h(x) dp(x)
\]
exists, then the other also exists, and the substitution formula 
\[
(LS) \int_{[0,b]} h(x) f(x) \, dg(x) = (LS) \int_{[0,b]} h(x) dp(x) + h(0) f(0) \left[ g(b^+) - g(0^-) \right]
\]
holds. (Received November 6, 1968.)


Let \( D_2 = \frac{1}{j(i)} \left( \partial / \partial x_1, \ldots, \partial / \partial x_n \right) \) and let \( L_0(D_2), L_1(D_2) \) be homogeneous elliptic operators with constant coefficients satisfying 
\[
L_0(\xi) + L_1(\xi) \leq C(\|\xi\|^2 + |\xi|^{\mu_1}),
\]
where \( \mu_i \) is the order of \( L_i \) and \( 0 \leq \mu_0 < \mu_1 \). Poisson kernels are constructed for the problem 
\[
\left\{ \begin{array}{l}
\epsilon^m L_1(D_2) + L_0(D_2) \right\} u_\epsilon(x) = 0, \quad x_n > 0 \\
\end{array} \right.
\]
(here \( \epsilon \) is a small parameter and \( m = \mu_1 - \mu_0 \)) with general boundary conditions on the hyperplane \( x_n = 0 \). These kernels allow one to investigate in detail the structure of the solution as \( \epsilon \to 0 \), including a possible boundary layer, and to derive corresponding Schauder estimates when the boundary operators have variable coefficients. In particular, those boundary conditions satisfied by \( u_0 = \lim_{\epsilon \to 0} u_\epsilon \) are specified. (Received November 6, 1968.)


The following free interface problem will be considered: \((*)\) 
\[
\left\{ \begin{array}{l}
u_i' = a_i(x) v_i + b_i(x) v_i + f_i(x), \\
\epsilon^m L_1(D_2) + L_0(D_2) \right\} u_\epsilon(z) = 0, \\
\end{array} \right.
\]
where \( a_i, b_i, f_i \) are continuous on \([b_1,b_2]\) and \( \epsilon \) is a small parameter. Problems like \((*)\) occur if the one-dimensional two phase Stefan problem is discretized with the method of straight lines. We then prove Theorem. Assume that \( b_i(x), c_i(x), i = 1,2 \), are positive on \([b_1,b_2]\). Assume further that \( a_1 > \beta_1 \geq \beta_2 > a_2 \). Then \((*)\) has at least one solution. The proof of this result is based on the invariant imbedding formulation presented earlier (SIAM J. Appl. Math. 16 (1968), 488-509). The technique is constructive and allows the determination of all solutions of \((*)\). (Received November 6, 1968.)


It has been shown by M. Schechter [A boundary layer phenomenon in nonlinear membrane theory, SIAM J. Appl. Math. 14 (1966), 1099-1114] that a plane infinite membrane of harmonic material with a circular hole admits of radially symmetric, deformations which are equilibrium deformations for all harmonic materials. Subclasses of the class of harmonic materials called \( \mathcal{H} \)-harmonic materials are defined and the following theorem is proved. Theorem. If \( (x,y) \rightarrow [U(x,y), V(x,y)] \) is a
mapping in the xy-plane it defines an equilibrium deformation for all \( J \)-harmonic materials iff \( U(x,y), V(x,y) \) satisfy the two differential equations \( U_x + V_y = C_1 \) and \( U_y - V_x = C_2 \) where \( C_1, C_2 \) are real constants, not both zero. This enables one to associate a certain analytic function with each such deformation and conversely. Using the above theorem two sequences of deformations, not radially symmetric, of the plane infinite membrane with a circular hole are constructed such that the hole closes in the limit and the stress at all points of any closed subset of the membrane tends to hydrostatic pressure uniformly. This result has some bearing on another theorem proved in the paper of M. Schechter referred to above. (Received November 6, 1968.)

663-605. T. W. SHOCK, Ohio State University, Columbus, Ohio 43201. Metrization of uniform spaces in ordered groups.

A (pseudo) G-metric for a set \( X \) is a distance function \( d : X \times X \to G \) (where \( G \) is an ordered abelian group) satisfying the usual axioms for a classical (pseudo) metric. Casper Goffman and L. W. Cohen [Proc. Amer. Math. Soc. 1 (1950), 750-753] gave a list of four conditions on a uniform space that are necessary and sufficient for it to be generated by some G-metric. These can be replaced by a surprisingly simple condition: Theorem 1. A uniformity \( \mathcal{U} \) is generated by some pseudo G-metric (G-metric) if and only if \( \mathcal{U} \) has a base which is linearly ordered by inclusion (and \( \cap \{ U : U \in \mathcal{U} \} = \emptyset \) ). Theorem 2. Let \( [X, \mathcal{U}] \) be a uniform space which is (pseudo) G-metrizable but not (pseudo) metrizable in the classical sense. Then \( \mathcal{U} \) can be generated by a non-archimedean (pseudo) G-metric. Theorem 3. A uniformity \( \mathcal{U} \) is generated by a non-archimedean pseudo G-metric if and only if \( \mathcal{U} \) has a base \( \mathcal{B} \), every member of which is idempotent, and which is well ordered by inclusion. (Received November 6, 1968.)

663-606. W. J. HEINZER, Purdue University, Lafayette, Indiana 47907, and JACK OHM, Louisiana State University, Baton Rouge, Louisiana 70303. Noetherian and non-noetherian commutative rings.

Some conditions are given which imply that a commutative ring \( R \) is noetherian. Two of the principal results are the following theorems. Theorem 1. \( R \) is noetherian if every \( R_p \), \( p \) prime, is noetherian and every finitely generated ideal of \( R \) has only finitely many prime divisors in the sense of Nagata [Local rings, p. 19]. Theorem 2. Let \( D, R \) be domains with the same quotient field \( K \) and suppose \( V \) is a rank 1 valuation ring of \( K \) such that \( R \not\supseteq V \) and \( D = R \cap V \). Then (i) if \( V \) is centered on a finitely generated ideal of \( D \), then \( V \) is noetherian; and (ii) if \( V \) is centered on a maximal ideal of \( D \), then \( D \) is noetherian if and only if \( R \) and \( V \) are noetherian. (Received November 6, 1968.)


Let \( M \) be a continuum (a compact connected Hausdorff space). If \( U \) is an open set in \( M \times M \) containing the diagonal, a subcontinuum \( Y \) of \( M \) is a \( U \)-subcontinuum of \( M \) provided there is a continuous function \( f : M \to Y \) such that for each \( x \in M \), \( (x, f(x)) \in U \). Theorem. A continuum \( M \) has the fixed point property if and only if, for each open set \( U \) containing the diagonal, there is a \( U \)-subcontinuum \( Y \) of \( M \) such that
Y has the fixed point property. A dendroid is an arc-wise connected, hereditarily unicoherent metric continuum. A fan is a dendroid with at most one ramification point. A dendroid M is smooth provided there is a point p ∈ M such that if x is a sequence in M converging to y, then the sequence of arcs [p,x] converges to [p,y].

Theorem. If B is a collection of smooth dendroids and fans, then π(B) has the fixed point property. (Received November 6, 1968.)


Nonexistence of a continuous right inverse for parabolic operators.

The paper considers continuous linear transformations L acting on the space $C^0(\mathbb{R}, \mathbb{C}^N)$ of indefinitely differentiable functions defined on the open subset $\Omega$ of $\mathbb{R}^n$, real n-dimensional space, and valued in $\mathbb{C}^N$, complex N-dimensional space. The paper considers the problem of determining whether or not there exists a continuous linear transformation $R: C^0(\Omega, \mathbb{C}^N) \to C^0(\Omega, \mathbb{C}^N)$ such that $LR = I$. When such an R exists, it is called a continuous right inverse of L. It is well known that hyperbolic systems with constant coefficients have a continuous right inverse on $C^0(\mathbb{R}^n, \mathbb{C}^N)$. Grothendiek has shown that if L is an elliptic system acting on $C^0(\Omega, \mathbb{C}^N)$, it has no continuous right inverse. This paper shows that no parabolic system, when the number of space variables is two or larger, and no parabolic system with constant coefficients have a continuous right inverse on $C^0(\Omega, \mathbb{C}^N)$ for any open subset $\Omega$ of $\mathbb{R}^n$. In the above considerations, the space $C^0(\Omega, \mathbb{C}^N)$ is given its usual topology, that of uniform convergence on compact subsets of $\Omega$ of the functions in $C^0(\Omega, \mathbb{C}^N)$ and their derivatives. (Received November 6, 1968.)


On the fundamental units of certain relatively biquadratic fields.

A Galois field extension $L/k$ with Klein's four-group as its Galois group will be called a four-field over $k$. Then $L = k(\sqrt{a}, \sqrt{b})$, $a, b \in k$, and $L$ contains three quadratic subextensions $K_1, K_2, K_3$ of $k$. There are just two types of number fields $k$ for which a four-field $L$ has three fundamental units (f.u.'s) (and each $K_i$ has one f.u., denoted $\epsilon_i$). The cases are (I) $k = \mathbb{Q}$ (rationals) with $L$ real, and (II) $k$ imaginary quadratic. In case (I) it is known (Kuroda, J. Fac. Sci. Tokyo 4 [1943]) that a system of f.u.'s of $L$ takes one of only seven forms (aside from permutation of indices): (i) $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, (ii) $\sqrt{\epsilon_1 \epsilon_2}$, $\sqrt{\epsilon_1 \epsilon_3}$, (iii) $\sqrt{\epsilon_2 \epsilon_3}$, (iv) $\sqrt{(\epsilon_1 \epsilon_2 \epsilon_3)}$, (v) $\sqrt{(\epsilon_1 \epsilon_2)}$, (vi) $\sqrt{(\epsilon_1 \epsilon_3)}$, (vii) $\sqrt{(\epsilon_2 \epsilon_3)}$. Certain conditions are imposed on the norm $\nu \epsilon_i$ of any $\epsilon_i$ which appears in a radical. We consider the relative case (II) of a four-field $L$ over $K = \mathbb{Q}(\sqrt{-3})$ and we show that there are essentially the same seven types of systems of f.u.'s of $L$ as in case (I). We also discuss the possibility of an algorithm to compute a system of f.u.'s for the composition of any number of quadratic fields over $k = \mathbb{Q}(\sqrt{-3})$. (Received November 6, 1968.)

663-610. E. C. ZACHMANOGLOU, Purdue University, West Lafayette, Indiana 47907. Convexity with respect to differential operators with flat characteristic cones.

In $\mathbb{R}^n$ let $P(D)$ be a differential operator of order $m$ with constant coefficients. The principal part $P_m(D)$ is the homogeneous part of $P(D)$ of order $m$. The set of zeros of $P_m(\xi)$ in $\mathbb{R}^n$ is called the characteristic cone of $P(\xi)$. An open set $\Omega$ in $\mathbb{R}^n$ is called P-convex if to every compact subset
there exists a compact subset $K_2$ of $\Omega$ such that for every distribution $u$ with compact support in $\Omega$, $\text{supp } P(-D)u \subseteq K_1$ implies $\text{supp } u \subseteq K_2$. It is well known (Malgrange) that the equation $P(D)u = f$ in $\Omega$ has a distribution (or $C^\infty$) solution $u$ for every $f$ in $C^\infty(\Omega)$ iff $\Omega$ is $P$-convex. A complete geometric characterization of $P$-convexity is known only when $n = 2$. Let $W$ be a subspace of $\mathbb{R}^n$ different from $\{0\}$. The $W$-hull of $K$ in $\Omega$, denoted by $\hat{K}(W, \Omega)$, is the union of $K$ and all components of intersections of translations of $W$ with the complement of $K$ which are relatively compact in $\Omega$.

Suppose that the characteristic cone of $P(D)$ is contained in the orthogonal complement of $W$. Then $\Omega$ is $P$-convex if for every compact subset $K$ of $\Omega$, $\hat{K}(W, \Omega)$ is compact. For first order operators this condition is also necessary for $P$-convexity. (Received November 6, 1968.)

Let $X$ be a compact metric space, and let $h: X \to X$ be a surjective homeomorphism. To generalize a theorem of Keynes and Robinson (Abstract 656-13, these Notices 15 (1968), 505-506) and Reddy (Lifting expansive homeomorphisms to symbolic flows, Math. Systems Theory 2 (1968), 91-92), we define a point generator $\mathcal{U}_{X_0}$ for $(X, h, x_0)$ to be a finite open cover for $X$ such that every bisequence $\bigcap_{n=0}^{\infty} h^{-1}([0, 1])$, $0_i \in \mathcal{U}_{X_0}$, which contains $x_0$, contains only $x_0$. Theorem. $h$ is expansive at $x_0$ iff there exists a point generator $\mathcal{U}_{X_0}$ for $(X, h, x_0)$. Theorem. $h$ is pointwise expansive iff there exists a countable set $C$ of point generators such that (i) There is some $\mathcal{U}_1 \in C$ such that every $\mathcal{U}_1 \in C$ is a refinement of $\mathcal{U}_1$. (ii) $\mathcal{U}_{n+1}$ is a refinement of $\mathcal{U}_n$. (iii) For every $x_0 \in X$, there is some $\mathcal{U}_{x_0} \in C$ such that $\mathcal{U}_{x_0}$ is a point generator for $(X, h, x_0)$. We see that if $C$ is finite for some $(X, h)$, say $C$ has $n$ elements, $\mathcal{U}_n$ is a generator for $(X, h)$, and $h$ is expansive on $X$. These characterizations allow us to prove Theorem. For compact metric $\bar{X}$, $X$, surjective homeomorphisms $f: \bar{X} \to X$, $g: X \to \bar{X}$, and covering map $\phi: \bar{X} \to X$ with $\phi f = g \phi$: (i) $f$ expansive at $\bar{x}_0 \in \bar{X}$ implies that $g$ is expansive at $x_0 = \phi(\bar{x}_0)$. (ii) $g$ expansive at $x_0 \in X$ implies that $f$ is expansive at any $\bar{x}_0 \in \phi^{-1}(x_0)$. (Received November 4, 1968.)

An $F$-manifold is a paracompact manifold modeled on the separable, infinite-dimensional Hilbert space, $H$. Theorem. Each separable $F$-manifold, $M$, is homeomorphic to an open subset $N$ of $H$. In addition, $N$ may be chosen so that (a) $N = \text{cl}(N) = \text{bd}(N)$ ("=" means "is homeomorphic to"), (b) $H - N \cong H - \text{cl}(N) \cong H$, and (c) $N \cong K \times H$, for some locally-finite, simplicial complex, $K$. Eells and Elworthy have shown very recently that if $M$ is a $C^\infty$-manifold then $M$ is $C^\infty$-diffeomorphic to $N$. Recent (unpublished) results of Burghelea, Kuiper and Mouli concerning open subsets of $H$ lead to Corollary 1. Every $F$-manifold has a unique $C^\infty$-Hilbert manifold structure. Corollary 2. Two $F$-manifolds are homeomorphic (or diffeomorphic) if and only if they have the same homotopy type. The proof of the theorem uses, among other things, a recent (unpublished) theorem due to Anderson and Shori to the effect that $M \times H \cong M$. (Received November 6, 1968.)
A ring $R$ is said to be co-Noetherian if the injective hulls of all simple $R$-modules have minimum condition on submodules. An equivalent definition is that cofinitely generated $R$-modules have cofinitely generated factors, for a suitable definition of cofinitely generated. Theorem 1. If $R$ is co-Noetherian then so is $R/I$ for any ideal $I$. Theorem 2. If $R$ is both Noetherian and co-Noetherian then $\bigcap_n j(R)^n = \{0\}$ where $j(R)$ is the Jacobson radical of $R$. Various properties of co-Noetherian rings are studied. No good internal (within $R$ itself) characterization of co-Noetherian rings is known. (Received November 6, 1968.)

The reduced wave equation is solved in an infinite channel bent at an arbitrary angle. The technique may be extended to higher dimensions and more general equations. An application has been made to the quantum mechanical three-body rearrangement scattering problem. (Received November 6, 1968.)

Let $M$ and $Q$ be combinatorial $m$- and $q$-manifolds (resp.), $M$ compact, $\dim Q \leq \dim M + 3$. Let $M_1$ and $Q_1$ be components of $\partial M$ and $\partial Q$, resp. Suppose $(M, M_1)$ is $(2m - q)$-connected and $(Q, Q_1)$ is $(2m - q + 1)$-connected. Let $f: (M, M_1, \partial M - M_1) \to (Q, Q_1, \partial Q - Q_1)$ be a continuous map on the triple such that $f|_{\partial M - M_1}$ is a PL embedding. Then $f$ is homotopic to a PL embedding. The homotopy is fixed on $\partial M - M_1$. Theorem 2. Let $h: M - \operatorname{Int} Q$ be a homeomorphism where $M$ and $Q$ are combinatorial manifolds, $M$ is compact, $\dim M \geq (2/3) \dim Q - 5/3$ and $\dim M \geq \dim Q - 4$. Given an $\varepsilon > 0$, there is a $\delta > 0$ such that if $h_0$ and $h_1$ are two PL embeddings of $M$ into $\operatorname{Int} Q$ where $d(h_0, h_1) < \delta$, $d(h_0, h_1) < \delta$ and $h_0(M) \cap h_1(M) = \emptyset$, then there is a PL embedding $H: M \times [0, 1] \to Q$ such that $H|M \times 1 = h_1, 1 = 0, 1$, and $\operatorname{diam} (M \times 1) < \varepsilon$, for all $m \in M$. Theorem 3. Let $M, M_1, Q$ and $Q_1$ be as in Theorem 1. Let $\dim Q \geq \dim M = 4$, $(M, M_1)$ is $(2m - q + 1)$-connected and $(Q, Q_1)$ is $(2m - q + 2)$-connected. Let $f$ and $g$ be two PL embeddings of $(M, M_1, \partial M - M_1)$ into $(Q, Q_1, \partial Q - Q_1)$ which are homotopic on the triple $(M, M_1, \partial M - M_1)$ rel. $(\partial M - M_1)$. Then there is an ambient PL isotopy $[h_t: Q - Q, t \in [0, 1]]$ such that $h_0 = 1, h_1 = g$ and $h_t|\partial Q - Q_1 = 1$. (Received November 6, 1968.)

A short proof of a lemma of G. R. MacLane. The level set of a function in $D = \{ |z| < 1 \}$ is defined as $\{ z: |f(z)| = \lambda, \lambda \geq 0 \}$. In G. R. MacLane's Rice Pamphlet, Asymptotic values of holomorphic functions, the following lemma (p. 10) is fundamental.

Lemma. If $f$ is holomorphic, bounded, and nonconstant in $D$, then for no $\lambda$ can $L(\lambda)$ contain a sequence
of disjoint arcs converging to a nondegenerate subarc of $C = \{ |z| = 1 \}$. The original proof of this lemma is rather involved and computational. A short geometric proof of the above lemma can be constructed as follows. Assume that for some $\lambda$ there exists a sequence of disjoint arcs $\gamma_n \subset L(\lambda)$ converging to a nondegenerate arc $\gamma \subset C$. Let $S$ denote the Riemannian image of $f$. Since $f$ is an open map and $|f| = \lambda$ on $\gamma$, we can find a sequence $\{z_n\}$ of points in $D$ which satisfy (i) $z_n \to e^{i\theta}$, where $e^{i\theta}$ is the midpoint of $\gamma$, (ii) $|f(z_n)| > \lambda$, (iii) $|f(z_n)| - a$, where $|a| = \lambda$, (iv) there are no branch points of $S$ above any of the rays $R_n = \{ w : |w| \leq |f(z_n)| \}$, arg $w = \arg f(z_n)$. The remainder of the argument depends upon lifting a maximal half open segment $[f(z_n), b_n]$ of $R_n$ into $S$ and using the Riesz uniqueness theorem. (Received November 6, 1968.)


In the derivation of a "variation of parameters" formula for the solution of the differential-difference equation $x(t) + B_0(t)x(t) + B_1(t)x(t-\tau) = f(t)$, use is made of the "adjoint equation" $z(t) + B_0^*(t)z(t) + B_1^*(t+\tau)z(t+\tau) = 0$. See for example A. Halanay (Differential equations, Academic Press, New York, 1965). In this paper the form of the Hilbert space adjoint of various linear differential-difference operators is derived, as well as the form of the adjoint boundary conditions. A theorem describing selfadjoint differential-difference operators is also proved. (Received November 6, 1968.)

663-618. J. W. BRACE, University of Maryland, College Park, Maryland 20742, and P. J. RICHETTA, Lehigh University, Bethlehem, Pennsylvania 18015. The approximation of linear operators.

Let $L(E,F)$ be the vector space of all linear maps of $E$ into $F$. Consider $G$ a subspace of $L(E,F)$ such as all continuous maps. In $G$ distinguish a subspace $H$ of maps which are to be approximated by members of a smaller subspace $N$ of $G$. Thus we always have $N \subset H \subset G \subset L(E,F)$. Then the approximation problem considered here is to find a locally convex linear Hausdorff topology on $G$ such that $H \subset \overline{N}$, $H = \overline{N}$ or the completion of $N$ is $H$. The paper develops a general approach to the problem. Specific results are given when $E$ and $F$ are Banach spaces and $N$ is all continuous linear operators with finite-dimensional range. The space $H$ is taken as the linear, continuous, weakly compact, completely continuous, or compact operators. The first three classes of operators are obtained as completions of $N$ for appropriate topologies. (Received November 6, 1968.)


Let $A_i$ denote Greendlinger eighth-groups (Proc. Amer. Math. Soc. 16 (1965), 1105). Call $h \in A_i$ basic if $h$ does not contain more than $1/8$ of any defining relation. Properties of basic elements are derived. Let $G$ denote the free product of $A_i$ with a cyclic subgroup $H$ amalgamated where $H$ is generated by a basic element in each factor. Theorem. $G$ has a solvable conjugacy problem. (Received November 6, 1968.)
An integral mean problem for bounded starlike functions.

Let $S^*$ denote the class of functions $f$ which are regular, univalent and starlike in $|z| < 1$ and which satisfy $f(0) = 0$. Let $l_2(r, l') = \int_0^{2\pi} |f'(re^{i\theta})|^2 \, d\theta$ for $0 \leq r < 1$. Theorem 1. Let $f \in S^*$ be bounded in $|z| < 1$. Then $l_2(r, l') = o(1)((1 - r)log(1/(1 - r)))^{-1}$, where $o(1) \to 0$ as $r \to 1$. Theorem 2. Given any positive function $\psi(r)$ defined in $0 \leq r < 1$ and increasing to $\infty$, however slowly, as $r \to 1$, there exists a bounded $f \in S^*$ such that $\lim \sup_{r \to 1} \psi(r)l_2(r, l')((1 - r)log(1/(1 - r))) > 0$. (Received November 6, 1968.)

Two definitions of Arens multiplication.

When $A$ is a locally convex algebra, with certain additional conditions, we may define Arens multiplication on $A''$ in essentially two ways: by letting $(x' \circ x)y = x'(xy)$, $(x'' \circ x' )x = x''(x' \circ x)$, and $(x'' \circ y'' )x' = x''(y'' \circ x')$ as usual; or by a twisted extension $(x' \circ x)y = x'(yx)$, $(x'' \circ x')x = x''(x' \circ x)$, and $(x'' \circ y'' )x' = y''(x' \circ x')$. Letting $T_x(x) = x' \circ x$ and $^tT_x(x) = (x' \circ x)^t$ we show that $(x'' \circ y'' )x' = (x'' \circ y'')$ for all $x''$, $y''$ if and only if every $T_x'$ and $^tT_x'$ maps bounded sets in $A$ into $\sigma(A', A'')$-compact subsets of $A'$. These constructions have applications to the representation of left and right multiplier algebras. (Received November 6, 1968.)

Eigenfunction expansions and similarity for nonselfadjoint operators.

Let $A = A_0 + V$ denote a closed operator with domain $D(A)$ in a Hilbert space $H$. $A_0$ is assumed to be a selfadjoint operator with spectral resolution $E_0\lambda$ such that $D(A_0) = D(A)$. This note establishes sufficient conditions for $A$ to possess an "eigenfunction expansion" and for $A_0$ and $A(A^*)$ to be similar to each other. It is assumed that $A_0 E_0 \lambda$ has an absolutely continuous spectrum and the operator $R(\lambda + i\epsilon) = (A - \lambda - i\epsilon)^{-1}$ and $R^*(\lambda + i\epsilon) = (A^* - \lambda + i\epsilon)^{-1}$ are defined for $\lambda$ in $S$ and $0 < \epsilon \leq \epsilon_0$ where $S$ is an open subset of the real line and $\epsilon_0$ is a positive number. Furthermore, it is assumed that the operators $q(\lambda + i0) = \lim_{\epsilon \to 0} R(\lambda + i\epsilon)$ and $p(\lambda + i0) = \lim_{\epsilon \to 0} R^*(\lambda + i\epsilon)$ exist as continuous invertible operators on a linear topological space $B$ dense in $H$. Now, given an "eigenfunction expansion" for $A_0 E_0 \lambda$ two sets of generalized eigenfunctions are constructed for $A(A^*)$ by means of equations analogous to the Lippmann-Schwinger equations. Using these results, a stationary and time-dependent scattering theory may be developed for the operators $A_0$ and $A(A^*)$. The results are applied to "gentle perturbations" as well as certain differential operators. (Received November 6, 1968.)

Some subspaces of a Hilbert space of analytic functions.

For a fixed positive real number $a$, let $L_2(R, a)$ be the complex-valued Lebesgue measurable functions on the line that satisfy $\int_{-\infty}^{\infty} |f(x)|^2 \exp(a|x|) \, dx < \infty$. For $\beta > 0$ let $S_\beta = \{z : |\exp(z)| < \beta\}$. If $f \in L_2(R, a)$ then the Fourier transform $\hat{f}$ is analytic on $S_\alpha/2$. The Fourier transforms form a Hilbert space $H$ with $\langle \hat{f}, \hat{g} \rangle = \int_{-\infty}^{\infty} \hat{f}(x + i/2) \overline{\hat{g}(x + i/2)} \, dx$. An inner function $F$
on $S_\beta$ is a function analytic on $S_\beta$ such that $|F(z)| \neq 1$ on $S$ and $|F(z)| = 1$ almost everywhere on the boundary. For each $t \in R$ let $\chi_t(z) = \exp(-izt)$ for all complex numbers $z$. We say that $M \subseteq H$ is invariant if $f \in H$ and $t \in R$ implies that $\{\chi_t f \} \in H$. Theorem. If $F$ is an inner function on $S_{\alpha/2}$ then $FH$ is a closed invariant subspace of $H$. (Received November 6, 1968.)


Theorem. For any positive integer $n, d$ there exists a prime $p_0$ and positive integers $t, n'$, and $d'$ such that: whenever $X$ is an $F$-variety, where $F$ is a field of characteristic $p \neq p_0$, and $X$ can be embedded in projective $n$-space as a variety of degree $\leq d$, then there exists a finite sequence of monoidal transformations $\eta_i: X_{i+1} \to X_i, 0 \leq i < t$, such that $X_0 = X$, $X_t$ is nonsingular, and for all $i$,

1. the center of $\eta_i$, say $Y_i$, is nonsingular,
2. $Y_i$ does not contain any nonsingular points of $X_i$, and
3. $X_i$ can be embedded in projective $n'$-space as a variety of degree $\leq d'$.

The proof uses Hironaka’s theorem on resolutions in characteristic zero (Ann. of Math. 79 (1964)) and an ultraproduct construction to obtain the connection between characteristic 0 and characteristic $p \neq 0$. The most general form of the theorem is proved for arbitrary varieties, with the parameter of bounded type (defined in the paper) replacing the parameters of dimension and degree. (Received November 6, 1968.)

663-625. S. P. SINGH and W. RUSSELL, Memorial University of Newfoundland, St. John’s, Newfoundland, Canada. On sequence of contraction mappings. Preliminary report.

Let $(E, d)$ be a complete $\epsilon$-chainable metric space and let $T_n (n = 1, 2, ...) be mappings of $E$ into itself, and suppose that there is a real number $k$ with $0 \leq k < 1$ such that $d(x, y) < \epsilon = d(T_n x, T_n y) \leq kd(x, y)$ for all $n$. If $u_n (n = 1, 2, ...)$ are the fixed points for $T_n$ and $\lim_{n \to \infty} T_n x = u$ respectively for every $x \in E$, then $T$ has a unique fixed point $u$ and $\lim_{n \to \infty} u_n = u$. (Received November 6, 1968.)


Let $\hat{M}$ be the injective hull of a right $R$-module $M$. If $S \subseteq M, r(S) = \{r \in R :Sr = 0\}$ is called a $M$-annihilator. A right ideal $K$ of $R$ is $M$-rationally closed if for each $K' \supseteq K$ there exists $K \subseteq B \subseteq K'$ and $a \in \text{Hom}_R (B, M)$ with $a(K) = 0$ and $a \neq 0$, [Lambek]. Let $Q$ be the complete ring of right quotients of $R$. Proposition 1. $K$ is a $\hat{M}$-annihilator if and only if $K$ is $M$-rationally closed. $K$ is $R$-rationally closed if and only if for $x \not\in K$ there exist $y, z \in R, z \neq 0$ such that $z[(yx)^{-1}K] = \{0\}$ where $(yx)^{-1}K = \{r \in R : yxr \in K\}$. Proposition 2. The lattice of $R$-rationally closed right ideals of $R$ is order isomorphic to the lattice of $Q$-rationally closed right ideals of $Q$. Proposition 3. Let $K$ be a $R$-rationally closed ideal of $R$, then (1) if $D$ is a dense right ideal of $R, D + K/K$ is dense in $R/K$. (2) If $B \supseteq K$ and $B/K$ is a $R/K$-rationally closed right ideal in $R/K$, then $B$ is $R$-rationally closed in $R$. Proposition 4. If $K$ is a $Q$-rationally closed ideal of $Q$ then $K \cap R$ is $R$-rationally closed and $O/K$ is a ring of right quotients of $R/K \cap R$. (Received November 6, 1968.)
663-627. H. W. BERKOWITZ, University of Georgia, Athens, Georgia 30601. PL approximations of embeddings of polyhedra in the metastable range. Preliminary report.

Let K be a locally finite, k-dimensional complex, and M a combinatorial m-manifold, where \( k \leq 2m/3 - 1 \). The main result is Theorem. If \( h: |K| \to M \) is a topological embedding, and \( \varepsilon: |K| \to R \) is a mapping of \( |K| \) into the positive real numbers, then there exists \( g: |K| \to M \) such that \( g \) is a piecewise linear embedding and \( d(g(x), h(x)) < \varepsilon(x) \) for each \( x \in |K| \). This theorem generalizes a theorem of Weber in which \( K \) is a finite complex. The methods used in the proof are partly based on methods used by Homma. (Received November 6, 1968.)


Let \( R_p \) be a totally ramified extension of degree \( e \) of an unramified v-ring \( R \), having residue field \( k \) of characteristic \( p \) and prime element \( \pi \), and let \( G_1 \supseteq H_1 \supseteq G_2 \supseteq H_2 \supseteq \ldots \) be the higher ramification series (HRS) of \( R_e \). When \( e = p \), Heerema has completely described the factors of the HRS \([Trans. Amer. Math. Soc. 132 (1968), 45-54]\). In this report the factors of the HRS are examined when \( e = 2p \), using generalizations of the methods of Heerema, although the results obtained are considerably more complicated. Let \( f(x) = x^{2p} + p \sum_{i=0}^{p-1} a_i x^i \) be the minimum polynomial of the extension and let \( s \) be the least positive integer such that \( a_s \) is a unit. Also, let \( R_2 \) denote a tamely ramified v-ring such that \( R_{2p} = R_2 \[11\] \). Then a partial listing of the results is as follows: \( G_1/H_1 \) is the group of order two in every case. Let \( \mathcal{B}(k) \) denote the group of derivations of \( k \), \( \mathcal{B}_0(0) \) denote the subgroup of \( \mathcal{B}(k) \) that lift to derivations on \( R_{2p} \), \( \mathcal{B}_1(k) \) the subgroup of \( \mathcal{B}(k) \) which takes \( a_{p-1} \) to 0, and let \( k^+ \) denote the additive subgroup of \( k \). If \( a_0 \not\in k^P \) and \( s \neq p, 2p \), then \( G_i/H_1 \cong k^+ \) for \( i > 1 \) and \( H_{i-1}/G_{i+1} \cong \mathcal{B}_0(k) \) for \( i = 1 \). If \( a_0 \in k^P \) and \( s = p, 2p \), then \( G_1 = H_1 \) in every case except when \( R_{2p}/R_2 \) is normal, and \( H_i/G_{i+1} \) is isomorphic to \( \mathcal{B}(k) \), \( \mathcal{B}_1(k) \) or \( \mathcal{B}_1(k) \oplus k^+ \), depending upon the value of \( i \) and \( s \). With the possible exception of the Galois automorphisms, all of the inertial automorphisms are convergent higher derivation automorphisms. (Received November 6, 1968.)


Let \( R_p = R[\pi] \) be an Eisenstein extension of degree \( p \) (\( p \neq 2 \)) of an unramified v-ring \( R \) having residue field of \( k \) of characteristic \( p \). Let \( \text{expo } R_p \) and \( \text{res } R_p \) be as in Abstract 660-25, these \( \text{Notices} \) 15 (1968), 1014. Theorem. Let \( x = \text{res } R_p \) when \( \text{expo } R_p = 0 \) and \( x = 1 \) otherwise. If \( p | \text{expo } R_p \), then a higher derivation \( \{s_i\} \) of \( k \) lifts to \( R_p \) if and only if \( x \sigma_i p (\text{res } R_p) \in k^P \) for each \( i \) and \( \sigma_i (\text{res } R_p) = 0 \) when \( p \not| i \). Otherwise, \( \{s_i\} \) lifts to \( R_p \) if and only if \( \sigma_i (\text{res } R_p) = 0 \) for each \( i \). A theorem similar to the following corollary was first proved by Wishart \([\text{Dissertation, Florida State University, 1965}]\).

Corollary. All higher derivations of \( k \) lift to \( R_p \) if and only if \( \text{res } R_p = 0 \). The proof is by constructing higher derivations of \( k \) into \( R_p \) which extend to \( R_p \). This construction makes use of a theorem of Heerema's \([Trans. Amer. Math. Soc. 132 (1968), 31-44]\) which states that higher derivations of \( k \) into \( R_p \) are completely determined by their action on a set of representatives of a \( p \)-basis of \( k \). (Received November 6, 1968.)
In what follows, all manifolds and embeddings will be smooth. Suppose that $M$ and $N$ are $n$-manifolds with $N \subset \text{Int } M$ and $H_n(M, N) = 0$. One might ask: If we have an embedding of $N$ in a manifold, $Q$, can we then embed $M$ in $Q$? The answer is no--there are counterexamples for codimension embeddings, even when $Q$ is Euclidean space. However, one has the following results.

**Theorem A.** Given $n$-manifolds $N \subset M$, $n > 4$, $H_n(M, N) = 0$; then if there is a 1-connected embedding $N \subset Q$ with $\dim Q > n$, then there is an embedding $M \times I \subset Q \times I^2$. Here 1-connected means that $f_#(\Pi_1(N)) = 0 \subset \Pi_1(Q)$.

**Theorem B.** Given $n$-manifolds $N \subset M$, $n > 4$, $H_n(M, N) = 0$; then if there is a 1-connected embedding of $N \times I \subset Q$ with $\dim Q - n > 3$, then $M \times I$ embeds in $Q$.

**Theorem C.** If $N \subset M$ are $n$-manifolds, $n > 4$, $H_n(M, N) = 0$; then if $N \times I$ embeds in $Q \times I$ and either $N$ or $\partial N$ is simply connected with $\dim Q > n$, then $M \times I$ embeds in $Q \times I$. With further restrictions involving the fundamental group, the above theorems can be somewhat sharpened. The proof of these theorems involves a detailed analysis of the handle structure of $M$ relative to $N$. (Received November 6, 1968.)

**Prolongations in semidynamical systems.**

A semidynamical system is a pair $(X, \sigma)$ where $X$ is a topological space and $\sigma$ is a continuous map of $X \times R^+$ onto $X$ satisfying the semigroup property. Prolongations as defined in dynamical systems (Andander and Seibert, Prolongations and stability in dynamical systems, Ann. Inst. Fourier, Grenoble 14 (1964), 237-268) lose a number of basic properties in semidynamical systems. In this paper, a new definition of prolongations is adopted and some results obtained therefrom. Application to stability problems is also made. (Received November 6, 1968.)

**Partial differential systems of generalized Wiener and Feynman integrals.**

We show that for appropriate complex valued functions $\sigma(\xi)$ and $\theta(t, \xi)$ the analytic Feynman, analytic Wiener, complex Wiener, limiting M Feynman, and M complex Wiener integrals of the functional $F(t, \xi, x) = \exp \left( \int_a^t \theta(t-s, x(s) + \xi) \, ds \right) \sigma[x(t) + \xi]$ satisfy certain integral, partial differential integral, and partial differential equations which are formally related to the Schroedinger equation. (Received November 6, 1968.)

**Foundation for a unified theory of sets and logic.**

Traditional set theory presumes a priori a full propositional calculus. In this paper after listing the primitives needed, all of the basic concepts of set theory are defined without resort to implicative statements. In this setting a generalization of implication is then defined. From appropriate axioms, the nature of logic then grows out of this set theory. For suggestions of some important insights produced by this approach, see Abstract 68T-108, these Notices 15 (1968), 223. (Received November 6, 1968.)
ERIC MENDELSOHN, Universite of Montreal, Montreal 3, Quebec, Canada. A characterization of the category of relational systems of given type.

The categories of relational systems of type Δ, and in particular the category of oriented graphs, are characterized by elementary axioms so that any two complete models are naturally equivalent. The arguments are analogous to those of B. Lawvere (category of sets) and D. Schomiluck (category of topological spaces). (Received November 6, 1968.)

ALESSANDRO FIGA-TALAMANCA, University of California, Berkeley, California 94720. Randomly continuous Fourier series on compact groups. Preliminary report.

Let G be a compact group, let e_i be the characters of G, we write the Fourier series of \( f \in L^1(G) \) as \( f(x) = \sum e_i * f(x) \). Let \( \xi_i \) be independent random variables with values equidistributed on G (i.e., the measure induced by \( \xi_i \) on G is the Haar measure). Theorem. \( \sum e_i * f(\xi_i) \) represents a function in \( L^\infty \) with probability one iff it represents a continuous function with probability one.

The proof is obtained using results of Kahane on random series in Banach spaces to generalize the proof of the corresponding theorem for the circle group which is due to Billard. On the other hand, in contrast with the situation in the classical case, one can find a compact group G and a series \( \sum e_i * f \) on G such that, for all choices of signs, \( \sum \pm e_i * f \) represents a bounded function and yet for no choice of signs \( \sum \pm e_i * f \) represents a continuous function. In this case G cannot be commutative. A simple example is obtained letting G = \( H_{i=1}^{\infty} H_i \) where \( H_i \) is the dihedral group in the discrete topology. (Received November 6, 1968.)

WITHDRAWN.

JOSEPH WARREN, Pennsylvania State University, University Park, Pennsylvania 16802. Holomorphic functions bounded on a spiral.

For a definition of spiral in the unit disk D and of \( k(\theta) \) and \( K(\theta) \), see Abstract 68T-144, these Notices 15 (1968), 235. Theorem. If \( f(z) \) has distinct finite asymptotic values on the spirals \( S_1, S_2, ..., S_n \), and is unbounded holomorphic in D, then \( \rho(S_i) = \lim_{\theta \to -\infty} (\log \log K(\theta)) / (\theta) \int_0^\theta \left[ 1 / k(t) \right] dt \). Theorem. If \( f(z) \) is holomorphic in D, bounded on the spirals \( S_1, S_2, ..., S_n, S_1 \cap S_j = \{ 0 \} \) if \( i \neq j \), and if \( f \) is unbounded in each of the simply connected regions bounded by \( S_1 \cup S_2, ..., S_n \), and \( |z| = 1 \), then letting \( \lambda(S_i) \) be the lower limit of the expression of the previous theorem, \( \lambda(S) \geq n \). The first theorem uses methods due to Ahlfors [Acta. Soc. Sci. Fenn., N.S., Ser. A 1 (1930)], while the second theorem uses a method due to Macintyre [J. London Math. Soc. 10 (1935)].

(Received October 30, 1968.)

HYMIE LONDON, 5208 Trans Island Avenue, Montreal 248, Quebec, Canada. Integral solution of \( y^2 + 25 = x^3 \).

Theorem. The only integral solution of the equation \( y^2 + 25 = x^3 \), y positive, is \( x = 5, y = 10 \). (Received November 6, 1968.)
Let $\mathcal{A}$ be a Banach algebra with identity $e$ over the complex field. It is possible to construct two cones, $K_1$ and $K_2$, in $\mathcal{A}$ which have the following properties: The cone $K_1$ depends only on the metric structure, the positive linear functionals being of the form $f_1 + if_2$ where the hyperplane $\{x | f_k(x) = \|f_k\|\}$ supports the unit ball at $e$. The cone $K_2$ depends only on the algebraic structure of $\mathcal{A}$ and is proper if and only if $\mathcal{A}$ is semisimple. We have $K_1 \subseteq K_2$. The point $u = (1 + i)e$ is an interior point (and hence an order unit) for both cones. The interior of $K_2$ is contained in the principal component of the set of invertible elements. The order unit $u$ (with respect to $K_2$) induces a norm $\|\cdot\|_u$ on $\mathcal{A}$ such that $\|x\|_u \leq \|x\|$ for all $x$ in $\mathcal{A}$ and for every Banach algebra norm $\|\cdot\|$ on $\mathcal{A}$. The norm $\|\cdot\|_u$ is equivalent to $\|\cdot\|$ if and only if $K_2$ is normal for the latter norm. The equivalence relation of being 'linked' (Proc. Amer. Math. Soc. 14 (1963), 438-443) can be defined in the dual cone $K'_2$ in $\mathcal{A}'$ and the equivalence classes for this relation are a generalization of the notion of Gleason part (H. S. Bear, Abstract 658-76, these Notices 15 (1968), 740). (Received November 6, 1968.)

A space is said to be a $\Sigma$-space if it has a sequence $\mathcal{F}_1, \mathcal{F}_2, \ldots$ of locally finite closed coverings with the following property: If $K_i$ is a closed subset with $K_1 \supseteq K_2 \supseteq \ldots$ and with $K_i \subset \cap \{ F \in \mathcal{F}_i | x \in F \}$ for some fixed point $x$, then $\cap K_i \neq \emptyset$. The family of this type of space has many good features in the theory of products developed by K. Morita. (Received November 6, 1968.)

Let $G$ be a Lie group with the Lie algebra $\mathfrak{g}$, $H$ a closed subgroup with the Lie algebra $\mathfrak{h}$, and $p$ a complement to $\mathfrak{h}$ in $\mathfrak{g}$. Theorem. There is a 1:1 correspondence between invariant connections on $G/H$ and bilinear functions $\beta : p \times p \to p$ satisfying $\beta(X,Y) = [\text{Ad}(h)\beta((\text{Ad}^{-1}h)X), (\text{Ad}^{-1}h)Y]_p$ for all $X$ and $Y$ in $p$, and all $h$ in $H$. Letting $\alpha(X,Y) - \beta(X,Y) + [X,Y]_p$, the curvature is given by $R(Z,X)Y = \alpha(Z,\alpha(X,Y)) - (X,\alpha(Z,Y)) - \alpha([Z,X]_p,Y) - [[Z,X]_h,Y]_p$. This generalizes results of K. Nomizu [Amer. J. Math. 76 (1954), 33-65]. (Received November 6, 1968.)

Let $\{\lambda_n\}_{n \geq 0}$ satisfy (1) $0 = \lambda_0 < \lambda_1 < \ldots < \lambda_n < \ldots | \to \infty$, (2) $\sum_{i=1}^{\infty} 1/\lambda_i = \infty$, let $E$ and $F$ be Banach spaces and let $m$ be a vector-valued measure defined on Lebesgue measurable subsets of $[0,1]$ with values in $L(E,F)$. Let $1 < q \leq \infty$ and denote by $m_{q,0,1}$ the $q$-variation of $m$ in $[0,1]$ and by $m_{q,0,1}$ the $q$-semivariation of $m$ in $[0,1]$ (see N. Dinculeanu, Vector measures). Necessary and sufficient conditions are given in order that a sequence $\{\mu_n\}$ of continuous linear operators on $E$ into $F$ should possess the representation $\mu_n = \int_0^1 e_n \lambda dm(t)$, $n = 0,1,2,\ldots$, where $m_{q,0,1} < \infty$ or where $m_{q,0,1} < \infty$. An application is given concerning the Fourier coefficients of the measure $m$. (Received November 6, 1968.)
Fibonacci numbers which are perfect cubes.

Let \( F_n \) be the nth term in the sequence of Fibonacci numbers defined by \( F_{n+2} = F_{n+1} + F_n \), \( F_1 = F_2 = 1 \), and let \( L_n \) be the nth term in the sequence of Lucas numbers defined by \( L_{n+2} = L_{n+1} + L_n \), \( L_1 = 1 \), \( L_2 = 3 \). Theorem. The only Fibonacci numbers \( F_n \), \( n \) positive, which are perfect cubes are \( F_1 = F_2 = 1 \) and \( F_6 = 8 \); the only Lucas number \( L_n \), \( n \) positive, which is a perfect cube is \( L_1 = 1 \). (Received November 6, 1968.)

WITHDRAWN.

Error formula for multidimensional Hermite interpolation.

A simple, direct proof (avoiding Peano's theorem) is given for the error in multidimensional Hermite interpolation. The formulation in two dimensions is given below. The generalization to higher dimensions is obvious. William Gordon (1968 SIAM Fall Meeting) has constructed (and thereby proved the existence of) the suitable bivariate interpolating function \( P(x,y) \). Let \( F(x,y) \) be given in a suitable domain \( D \subset E_2 \). Let \( a_i \), \( b_j > 0 \) integers. Definition. \( P(x,y) \) interpolates \( F(x,y) \) in the Hermite sense at \( (x_1,\ldots,x_M) \) relative to \( (a_1,\ldots,a_M) \) and at \( (y_1,\ldots,y_N) \) relative to \( (b_1,\ldots,b_N) \) means that the error \( E(x,y) = F(x,y) - P(x,y) \) obeys the following: \( E(x,y) \) has a zero at \( (x_i,y_j) \in D \) of multiplicity \( a_i \) for arbitrary \( y \) and \( E(x,y) \) has a zero at \( (x,y_j) \in D \) of multiplicity \( b_j \) for arbitrary \( x \) where \( i = 1(1)M, j = 1(1)N \). For smooth functions \( F(x,y) \), let \( F^{(m,n)}(x,y) \) denote \( m \) partial derivatives w.r.t. \( x \) and \( n \) partial derivatives w.r.t. \( y \). Thus \( F^{(0,0)} = F \). Let \( I = \sum a_i, J = \sum b_j, Q(x,y) = \Pi(x - x_i)^{a_i}\Pi(y - y_j)^{b_j}/(I!J!) \) where \( i = 1(1)M \) and \( j = 1(1)N \). Theorem. Let \( F^{(I,J)} \) and \( P^{(I,J)} \) exist, continuous on \( D \). Then there exists \( (\xi_x, \eta_y) \in D \) such that \( E(x,y) = [F^{(I,J)}(\xi,\eta) - P^{(I,J)}(\xi,\eta)]Q(x,y) \) for all \( (x,y) \in D \). (Received November 6, 1968.)

On the supercritical age dependent branching process.

Let \( \{Z(t); t \geq 0\} \) be an age-dependent branching process with lifetime distribution function \( G(t) \) and offspring p.g.f. \( h(s) = \sum_{0}^{\infty} p_j s^j \). Let \( m = h'(1-) \) satisfy \( 1 < m < \infty \) and \( G(0+) = 0 \). We show that if \( Y(t) = (m(t))^{-1}Z(t) \) (here \( m(t) = EZ(t) \), the mean function), then (i) \( \sum_j j \log j p_j = \infty \) implies \( Y(t) \) converges to zero in probability, (ii) \( \sum_j j \log j p_j < \infty \) implies \( Y(t) \) converges in distribution to a random variable \( W \) such that \( P(W = 0) = q \), the smallest nonnegative root of the equation \( h(x) = x \), and there exists a nonnegative continuous function \( g(x) \) defined for \( x > 0 \) with the property \( P(W \leq y) = q + \int_{0}^{y} g(x)dx \). (Received November 6, 1968.)

A general theorem on the convergence of sequences of semigroups of linear operators.

Let \( L_n \) be a sequence of Banach spaces and let \( T_n(t): L_n \rightarrow L_n \) be contraction semigroups with infinitesimal operators \( A_n \). Let \( \mathcal{Z} \) denote the Banach space of bounded sequences \( \{f_n\}, f_n \in L_n \), \( \|f_n\| = \sup_n \|f_n\| \). Then \( J(t) = \{T_n(t)f_n\} \) defines a contraction semigroup of \( \mathcal{Z} \). Let \( \mathcal{Z}_0 \subset \mathcal{Z} \) be
the subspace on which $J(t)$ is strongly continuous. Let $L$ be another Banach space and suppose \[ \lim_{n \to \infty} f_n = f \in L \] is defined for $\{f_n\} \in \mathcal{B}(P) \subset \mathcal{L}$ and $P[f_n] = \lim_{n \to \infty} f_n$ is a bounded linear operator from $\mathcal{B}(P)$ onto $L$. We say that $\{T_n(t)\}$ converges to a semigroup $T(t)$ defined on $L_0 \subset L$ if there exists $D \subset \mathcal{B}(P)$ such that $P : D \to L_0$, $J(t) : D \to D$ and $T(t)P[f_n] = P J(t)[f_n]$ defines a semigroup on $L_0$.

Theorem. Let $N = \{\{f_n\} : P[f_n] = 0\}$. Suppose $\{f_n\} \in N - \{\lambda - A_n\}^{-1} f_n \in N$ for every $\lambda > 0$. Define $B_P f_n = P[A_n f_n]$ whenever $\{f_n\}, \{A_n f_n\} \in \mathcal{B}(P)$. (B may be multivalued.) Suppose $\mathcal{C}(\lambda - B) = L$.

Then $J(t) : \mathcal{B}(P) \cap L_0 \to \mathcal{B}(P) \cap L_0$ and $T(t)P[f_n] = P J(t)[f_n]$ defines a strongly continuous contraction semigroup on $L_0 = P(\mathcal{B}(P) \cap L_0)$. In previous theorems of this type the notion of limit involved is essentially strong convergence. In the present theorem, $P$ may (in the case of Banach spaces of functions) correspond, for example, to bounded pointwise convergence or to convergence of bounded sequences uniformly on compact subsets. (Received November 6, 1968.)


Call a pair $(M, X)$ worthy if $M$ is an irreducible orientable but not closed 3-manifold and $X$ is a compact subset of int $M$ such that every map of $X$ into $S^2$ is inessential. An inverse limit sequence (ils) $\{X_i : X_{i+1} \subset X_i\}$ of compact polyhedra is free if (bounded) if there is a positive integer $j$ such that for every component $C$ of $X_i$, $\pi_1(C)$ is free [the rank of $f_i^j(H_1(C))$ is less than $j$].

Theorem. If $(M, X)$ is worthy and $X$ is the inverse limit space of a free ils and a bounded ils, then $X$ is definable by cubes-with-handles (cwh). Corollary. If $(M, X)$ is worthy and some embedding of $X$ is definable by cwh of bounded genus, then $X$ is definable by cwh. Corollary. If $(M, X)$ is worthy and $X$ is a weakly locally simply connected Peano continuum, then $X$ is definable by cwh if and only if $\pi_1(X)$ is free.

Theorem. If $(M, X)$ is worthy, $X$ is a Peano continuum, $\pi_1(X)$ is finitely generated, and $X$ is the inverse limit space of a free ils, then $X$ is definable by cwh. Theorem. If $(M, X)$ is worthy and the Čech fundamental group [A. J. Goldman, Summer institute on set theories topology, University of Wisconsin, 1955] of each component of $X$ is trivial, then $X$ is definable by cwh. (Received November 7, 1968.)

663-649. N. SANKARAN, Queen's University, Kingston, Ontario, Canada. Some remarks on Weierstrass preparation theorem.

This note gives a version of the abstract Weierstrass preparation theorem which only requires that the coefficient domain be a commutative ring with identity. Some properties of the distinguished pseudo-polynomial (known in the case of convergent power series with coefficients in a field) are also examined in this general setting. (Received November 7, 1968.)


Jacobi and Gauss-Seidel iterative methods are proposed for the numerical solution of the nonlinear network problem of Birkhoff and Kellogg. Rigorous, computable upper and lower bounds for the solution are derived. Special vectors are constructed which, when used as initial iterates in the above methods, yield sequences which converge monotonically to the solution. The monotonic iterates of the two methods are compared and it is seen that from this point of view the Gauss-Seidel method
is superior to the Jacobi method. Finally, the global convergence of both methods is established. A numerical example is included. (Received November 7, 1968.)

663-651. G. L. CAIN, JR., Georgia Institute of Technology, Atlanta, Georgia 30332. Compactification of mappings.

A compactification of a mapping (continuous) \( f : X \to f(X) = Y \) is a pair \((X^*, f^*)\) where \( X^* \) is a space containing \( X \) as a dense subspace and \( f^* \) is a compact mapping of \( X^* \) onto \( Y \) such that \( f^*|X = f \). G. T. Whyburn introduced the notion of a mapping compactification in 1953 when he showed that every mapping of one Hausdorff space onto another is a partial mapping of a compact mapping [A unified space for mappings, Trans. Amer. Math. Soc. 74 (1953), 344-350]. For \( X \) completely regular, \( Y \) regular, we introduce a class \( \mathcal{C} \) of compactifications of \( f \) and study some of its properties. Whyburn's compactification is included in \( \mathcal{C} \) in case \( X \) and \( Y \) are locally compact. (Received November 7, 1968.)


D. G. Higman [intersection matrices for finite permutation groups, J. Algebra 6 (1967), 22-42] has defined intersection numbers for a finite transitive group \( G \) of permutations on a set \( \Omega \). The author shows that these are structure constants for a certain matrix algebra. The associative law for matrix multiplication then imposes relations which the intersection numbers must satisfy. These relations are used to investigate primitive rank 4 groups which have a doubly transitive suborbital. Theorem. If \( G \) has rank 4 and has two doubly transitive suborbitals, then (i) the two doubly transitive suborbitals are paired with each other, and hence have the same length \( d \); (ii) the remaining suborbital has length \( d(d - 1)/i \), where \( i \) is an integer less than \( d - 1 \) and such that \( d + 1 - i^2 \) is a square. (Received November 7, 1968.)

663-653. L. W. BEINEKE, Purdue University, Fort Wayne, Indiana 46805. On packings of complete bipartite graphs.

A packing of a graph \( G \) with a graph \( H \) is given by a collection of edge-disjoint subgraphs of \( G \) all of which are isomorphic to \( H \). Some maximum packings of complete graphs have been studied, and maximum packings of complete bipartite graphs are considered here. Theorem. The maximum number of subgraphs in a packing of the complete bipartite graph \( K_{m,n} \) with the regular complete bipartite graph \( K_{r,r} \) is the minimum of \( [m/r \cdot [n/r]] \) and \( [[m/r] \cdot n/r] \). (Received November 7, 1968.)

663-654. R. L. CARPENTER, University of Utah, Salt Lake City, Utah 84112. A local characterization of Hol (D).

Let \( A \) be a singly-generated uniform \( F \)-algebra. There are several global conditions on \( A \) or its spectrum which guarantee that \( A \) be Hol (D) for an open subset of the plane. In this note we give a local condition. With each point \( m \) of the spectrum, we associate an algebra \( A_m \). The condition is in terms of the natural maps between these algebras and the nature of the algebras. (Received November 7, 1968.)
C. L. IRWIN, Emory University, Atlanta, Georgia 30322. Green's function for singular boundary value problems. Preliminary report.

Suppose \( I \) is a linear \( n \)th order differential expression with continuous coefficients and \( \{ y' - Qy = 0 \} \) is the vector-matrix formulation of \( Iy = 0 \). Let \( M(x,t) = \phi(x)\phi^{-1}(t) \) where \( (x,t) \in [0,\infty) \times [0,\infty) \) and \( \phi \) is a fundamental solution of \( (1) \). A function \( A \) from \([0,\infty)\) to the \( n \times n \) complex matrices so that \( \lim_{b \to \infty} [A(0) + A(b)M(b,0)] \) is a nonsingular \( n \times n \) matrix is called a boundary condition function for \( I \). The vector-matrix problem associated with \( I \) and \( A \) is \( \{ y' - Qy + H; A(0)Y(0) + \lim_{b \to \infty} A(b)Y(b) = 0 \} \). Define \( P_\lambda A = \lim_{b \to \infty} [A(0) + A(b)M(b,0)]^{-1}A(0) \) and let \( D_A \) denote the vector functions \( H \) which are continuous on \([0,\infty)\) and for which there is a solution \( Y \) of \( (2) \). Theorem 1. \( K_A \) is the Green's function and \( D_A \) is the domain of an integral inverting operator for \( (2) \).


Let \( A \) be a commutative Banach algebra with identity, \( \Delta \) the maximal ideal space of \( A \). Let \( S(\Delta) = \text{Hol}(\Delta)/Z(\Delta) \) be the sheaf of germs of locally-\( A \) functions on \( \Delta \). Theorem 2. If \( x \in \Delta \), and if the maximal ideal of the stalk of \( S(\Delta) \) is finitely generated, then there is an analytic subvariety \( V \) in some \( \mathbb{C}^n \) and a homeomorphism \( \phi \) from some neighborhood \( U \) of \( x \) onto \( V \) such that if \( a \in A \), then \( \hat{a}|_U = f \circ \phi \) where \( f \) is analytic on \( V \) and \( \hat{a} \) is the Gelfand transform of \( a \).

J. C. DERDERIAN, State University of New York at Buffalo, Amherst, New York. Pair algebras and Galois connections.

Pair algebras were introduced by J. Hartmanis and R. E. Stearns (Information and Control 7 (1964), 275-303) to study problems related to information flow in finite automata. We exhibit an intimate relationship between these and residuated mappings (or equivalently Galois connections) which leads to extensions of results of G. Birkhoff. (Received November 7, 1968.)

P. C. HAMMER, Pennsylvania State University, University Park, Pennsylvania 16802. Transitivity and measures.

Measure theory as it is now described fails to embrace some of the most important measures of mathematics. In this paper, I propose a generalization which gives more adequate coverage. Let \( E \) be a set and let \( R \) be a set with a reflexive transitive relation \( T \). Let \( \mu \) map \( E \) into \( R \). Then with \( "(\mu(p), \mu(q)) \in T" \) interpreted to mean "the measure of \( p \) is less than or equal to the measure of \( q \)",
I consider such functions \( \mu \) to provide \( R \)-valued measures on \( E \). Often there is involved also another transitive relation. Let \( X \) be a set with a reflexive transitive relation \( S \). Let \( \mu \) map a subset \( E \) of \( X \) into a set \( R \) with reflexive transitive relation \( T \). Then the measure \( \mu \) is compatible with \( S \) provided \( p, q \in E, (p, q) \in S \) implies \( (\mu(p), \mu(q)) \in T \). The set \( E \) is called the set of measurable elements of \( X \). Examples now embraced include measures of measure theory, cardinal number (of a set) and packing and covering measures. Also included are various concepts of dimension, set-valued measures and measures with classes of sets as values. (Received November 7, 1968.)

663-659. A. W. MARSHALL, Boeing Scientific Research Laboratories, Seattle, Washington, and INGRAM OLKIN, Stanford University, Stanford, California. Scaling of matrices to achieve specified row and column sums.

If \( A \) is an \( n \times n \) matrix with strictly positive elements, then there exist diagonal matrices \( D_1 \) and \( D_2 \) with strictly positive elements such that \( D_1 A D_2 \) is doubly stochastic (Sinkhorn, Ann. Math. Statist. 35 (1964), 876-879). The hypothesis can be weakened to \( A \) being fully indecomposable (Brualdi, Parter and Schneider, J. Math. Anal. Appl. 16 (1966), 31-50; Sinkhorn and Knopp, Pacific J. Math. 21 (1967), 343-348). An alternative proof is presented in which the scaling matrices are obtained as the solution of an appropriate extremal problem. This proof also yields the result that the same scaling is possible (with \( D_1 = D_2 \)) when \( A \) is strictly copositive. A similar result for \( A \) rectangular is also obtained. (Received November 7, 1968.)


Consider the initial value problem \( y' = f(t, y, \epsilon) \) where \( f : \mathbb{R} \times V \times W \rightarrow V \), \( V \) and \( W \) are Banach spaces. Solutions \( t \rightarrow \varphi(t, \epsilon) \) will satisfy the function equation \( G(\mathbf{c}, \varphi(t, \epsilon)) = 0 \), where \( G(\mathbf{c}, \varphi(t)) = \varphi(t) - \int_{0}^{t} f(s, \varphi(s), \epsilon) ds \), \( G : W \times V^R \rightarrow V^R \). Since we seek a local result, there is no loss of generality if we assume all functions are globally defined (see e.g. Hartman, Ordinary differential equations, Wiley, p. 232). If we assume that \( f \) is of class \( C^k \), then an application of the implicit function theorem yields the standard theorem on differentiability of solutions with respect to a parameter and hence with respect to initial conditions. (Received November 7, 1968.)

663-661. J. D. BRILLHART, University of Arizona, Tucson, Arizona 85721. Some modular properties of the Euler and Bernoulli polynomials.

In previous reports (Abstracts 623-2, these \( \text{Notices} \) 12 (1965), 350, and Abstract 653-156, these \( \text{Notices} \) 15 (1968), 123) various properties of the Euler polynomials \( E_n(x) \) and the Bernoulli polynomials \( B_n(x) \) were given. The present paper is concerned with mod \( p \) properties of these polynomials, where \( p \) is an odd prime. Illustrative of these results are the following: (1) If \( p \) is an odd prime not dividing the discriminant of \( E_{2m}(x) \), then \( E_{2m}(x) \) has respectively an even or an odd number of irreducible factors (mod \( p \)) as the Legendre symbol \( (-1)^{m}E_{2m}/p = +1 \) or \(-1\), where \( E_{2m} \) is the Euler number. (2) The following factorization is complete:

\[
2E_{p+1}(x) = x(x - 1) \prod_{i=1}^{p-1} [(1 - n_i| x^2 - 2x + 1] \pmod{p},
\]

where \( n_i \) are the quadratic nonresidues of \( p \). (Received November 7, 1968.)

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Dense embeddings of Hilbert spaces.

Theorem. There exist three complete inner product spaces \([S_1, Q_1], [S_2, Q_2], \) and \([S_3, Q_3]\) such that if \(N_1, N_2, \) and \(N_3\) are the norms corresponding to the inner product functions \(Q_1, Q_2, \) and \(Q_3\), respectively, then (1) if \(x\) is in \(S_1\), then \(x\) is in \(S_2\) and \(N_2(x) \leq N_1(x)\), (2) if \(y\) is in \(S_2\), then \(y\) is in \(S_3\) and \(N_3(y) \leq N_2(y)\), (3) \(S_1\) is dense in \(S_3\) with respect to \(N_3\), and (4) the orthogonal complement of \(S_1\) in \([S_2, Q_2]\) is one dimensional so that \(S_1\) is not dense in \(S_2\) with respect to \(N_2\).

Corollary. The approximate inclusion relation studied by P. H. Jessner [Abstract 594-28, these Notices 9 (1962), 466] among kernel systems \([K, E, S, Q]\), i.e. among complete inner product spaces \([S, Q]\) of functions (on a set \(E\)) with a kernel \(K\), is not transitive. Remark. Evidence from Radon-Hellinger type examples, involving absolute continuity, has heretofore led to unpublished conjectures that the aforementioned relation is transitive. (Received November 7, 1968.)

Gaussian processes with bounded paths.

The following theorem is proved: A mean-continuous Gaussian process on the unit interval with bounded paths has its points of discontinuity in a set of the first category. In particular, if the process is stationary, the paths are continuous, a result of Belayev. The method is based on expanding the process in a series with independent coefficients and applying the zero-one law to certain functionals associated with the expansion. (Received November 7, 1968.)

Algebraic varieties over real closed fields.

Let \(V\) be a variety over a real closed field \(R\) with function field \(K\), let \(V_0\) be the set of \(R\)-valued points of \(V\). Those subsets of \(V_0\) which are defined by finitely many equalities and inequalities are called elementary. An elementary set is dense in \(V\) in the Zariski topology iff there is an ordering of \(K\) for which the defining inequalities are true and no equalities occur. The image of an elementary set under a \(R\)-morphism is again elementary; this gives an extension theorem for orderings of real function fields. The ordering of \(R\) induces a topology on \(V_0\). Restricting the attention to elementary subsets one has a good concept of compactness and of connectedness, most of the usual theorems hold. Every elementary set has finitely many connected components. For projective curves \(C\) one gets a triangulation and the equivalents of the theorems of Harnack and Weichold, connecting the number of components, the genus of \(C\) and the Galois cohomology of the Jacobian of \(C\). (Received November 7, 1968.)

On the dynamo problem.

One of the possible explanations of the magnetic field of Earth is convection of charged fluid in the core of Earth. The set of resulting equations consists of 8 nonlinear evolution equations in the 3 components of velocity, 3 components of the magnetic field, the temperature deviation and the
pressure. To this set an appropriate system of initial and boundary conditions have to be added. It is shown that the full system has a nontrivial solution for which the magnetic field tends to a homogeneous one at infinity. The proof uses the invariance theorem of the degree of mapping in abstract Banach space which holds for compact operators. (Received November 7, 1968.)

663-666. LIOR TZAFRIRI, Northwestern University, Evanston, Illinois 60201. An isomorphic characterization of $L_p$ and $c_0$-spaces.

**Definition.** Let $\mathcal{S}$ be a bounded Boolean algebra of projections in a Banach space $X$. We say that $\mathcal{S}$ has a two-sided estimate if there exist a constant $K$ and a function $\psi$ defined for every sequence of complex numbers such that $K^{-1}\psi(\|P_1 x\|, \|P_2 x\|, \ldots, \|P_n x\|, \ldots) \leq \|x\| \leq K\psi(\|P_1 x\|, \|P_2 x\|, \ldots, \|P_n x\|, \ldots)$, $x \in X$, for every finite or infinite sequence of disjoint projections $P_n \in \mathcal{S}$ whose sum is the identity $I$. For a Banach space $X$ isomorphic either to $L_p$, $1 \leq p < +\infty$, on some finite measure space or to $c_0$, the images under the isomorphism of the "multiplications" by characteristic functions form a Boolean algebra of projections $\mathcal{S}$ such that (a) $\mathcal{S}$ is strongly $\sigma$-complete; (b) $X = \text{cl}(\text{im}(X_0|E \in \mathcal{S}))$ for some $x_0 \in X$; (c) $\mathcal{S}$ has a two-sided estimate (with $\psi = \sum |a_n|^p$ or $\psi = \sup |a_n|$). **Theorem.** A Banach space $X$ is isomorphic to $c_0$ or $L_p$, $1 \leq p < +\infty$, on some finite measure space $(\Omega, \Sigma, \mu)$ if and only if there exists a Boolean algebra of projections $\mathcal{S}$ in $X$ satisfying conditions $a$, $b$, $c$. The result is extended to $L_p$ on general measure spaces. (Received November 7, 1968.)

663-667. ALLAN PETERSON, University of Nebraska, Lincoln, Nebraska 68508. The distribution of zeros of extremal solutions of a fourth order differential equation for the $n$th conjugate point.

Consider the fourth order linear differential equation $L[y] = (D_3 y)'' + q_3 D_2 y + q_4 y = 0$, $x \in [a, +\infty)$, see J. Barrett, Abstract 594-36, these *Notices* 9 (1962), 469. An extremal solution of $L[y] = 0$ for $\eta_n(t)$ ($n$th conjugate point of $t$) is a nontrivial solution of $L[y] = 0$ which has $n + 3$ zeros on $[t, \eta_n(t)]$ with a zero at $t$. The author shows that if $r_{22}(a) = +\infty$ (no nontrivial solution of $L[y] = 0$ has a pair of multiple zeros on $[a, +\infty)$) then the zeros of the first principal solution $U_3(x, t)$ ($D_j U_3(x, t) = \delta_{j3}$, $j = 0, 1, 2, 3$) are strictly increasing continuous functions of $t$. The author uses this to prove that if $L[y] = 0$ is selfadjoint with $r_{22}(a) = +\infty$, then $\eta_n(t)$ is the $n$th zero of $U_3(x, t)$ in $(t, +\infty)$. Furthermore, $\eta_n(t)$ is a strictly increasing continuous function on its domain which is of the form $[t, b)$, $t \neq b \neq +\infty$. (Received November 7, 1968.)


A sequence $\sigma = (a_1, \ldots, a_p)$ of positive integers with an even sum is said to be an incidence sequence. A linear graph $G$ is said to be a realization of $\sigma$ if the vertices of $G$ can be labelled so that the incidence of the $i$th vertex equals $a_i$. An incidence sequence $\sigma$ is defined as planar if it has at least one planar realization. If $\sigma = (a_1, \ldots, a_p)$ is such that $a_1 = a_i$ ($i = 1, \ldots, p$) then $\sigma$ is said to be a regular incidence sequence. The class of planar regular incidence sequences has been completely determined. (Received November 8, 1968.)
Consider the second-order ordinary boundary value problem (1) \( y'' = f(x,y,y'), y(a) = \alpha, y(b) = \beta \) where \( f \) is a continuous real-valued function on \( I \times \mathbb{R}^2 \) with \( I \) an interval of the reals \( \mathbb{R} \), \( [a,b] \subset I \) and \( \alpha, \beta \in \mathbb{R} \). Let \( S = \{(a,b,\alpha,\beta) : a < b, [a,b] \subset I \text{ and } \alpha, \beta \in \mathbb{R}\} \subset \mathbb{R}^4 \). Suppose for each \( \sigma = (a,b,\alpha,\beta) \in S \) the boundary value problem (1) has at most one solution. If this solution exists, denote it by \( y(x;\sigma) \). Then \( y(x;\sigma) \) is a mapping on \( S \) into \( \mathbb{C}^2 \). Theorem. Suppose for each \( \sigma \in S \), (1) has at most one solution. If for some \( \sigma_0 = (a_0,b_0,\alpha_0,\beta_0) \in S \), \( y(x;\sigma_0) \) exists on \( [a_0,b_0] \) then for \( \sigma \) near \( \sigma_0 \), \( y(x;\sigma) \) exists on \( [a_0,b_0] \) and the mapping \( y(x;\sigma) \) is continuous at \( \sigma = \sigma_0 \). (Received November 7, 1968.)

Just as bivectors may be regarded as elements of a three-dimensional complex space, endowed with a pseudo-metric which allows null elements, Weyl tensors may be regarded as elements of a five-dimensional complex space endowed with a similar pseudo-metric. In analogy with the bivector case, Weyl tensors can be classified as null or nonnull. The relation of this classification to one of the invariants of the Weyl tensor is pointed out. That this classification differs significantly from the Petrov classification is seen from the fact that not only Weyl tensors of type N and type III but also some of type I are null. (Received November 7, 1968.)

In the model, fuel is consumed, heat is released and then conducted away at rates depending on the two variables, the temperature and the amount of fuel remaining. The problem is to understand how so simple a model can describe the existing sharp distinction between the fast, violent reactions called explosions and the more gentle nonexplosions. There is no discontinuity in the process nor in the rates. In suitably reduced variables the system depends on a single parameter which appears only as the small coefficient of one of the two derivatives present. As this parameter goes to zero, there appears in the field of trajectories an asymptotic void, a region \( R \) such that for any fixed, smooth (with respect to the initial values) distribution of probability over the trajectories, the probability of entering \( R \) goes to zero. Explosions are trajectories above \( R \), nonexplosions those below. To separate them analytically, one seeks an asymptotic description of trajectories near the narrow entry to \( R \) and meets with surprising difficulty. The difficulty is explained and resolved by approximating the differential equation near the entry by a Riccati equation. In the approximation the asymptotic methods previously used are WKB methods and the difficulty is the familiar one of a turning point. Formulas of Peter Debye, 1910, provide the evaluation sought. (Received November 7, 1968.)
logical space are derived from metrization theorems of Ceder and Allsbrook. In connection with these, the concept of fundamental open covering—that is, an open covering such that each point of the space lies in the interior of the intersection of all members of the covering containing the point—arises. The following property (F) is considered: To every open covering of the space X there corresponds a refining fundamental open covering. It is shown that every linearly ordered space possesses property (F). (Received November 7, 1968.)

663-673. J. W. FORD, Auburn University, Auburn, Alabama 36830, and E. S. THOMAS, JR., University of Michigan, Ann Arbor, Michigan 48104. Aligning functions.

Let K be a subset of the unit cube $I^n \subset R^n$ ($n \geq 2$). A function $f: K \rightarrow I$ can be aligned if it is topologically equivalent to the projection $\pi; I^n \rightarrow I$ given by $\pi(x_1, \ldots, x_n) = x_1$, i.e., if and only if there exist onto homeomorphisms $u: I^n \rightarrow I^n$ and $v: I \rightarrow I$ such that $f \circ u = v \circ \pi$. The function $f: K \rightarrow I$ satisfies the boundary condition provided $f^{-1}(\partial I) = K \cap \partial I^n$ (if $n = 2$ we also require that $f^{-1}(0) \cap \partial I$ and $f^{-1}(1)$ lie in disjoint arcs in $\partial I^2$). In a paper to appear in the Proc. Amer. Math. Soc., B. J. Ball and the authors show that if $K$ is a Cantor set in $I^2$ and $f: K \rightarrow I$ is continuous and satisfies the boundary condition, then $f$ can be aligned. In higher dimensions this is false, even if $K \subset \text{int } I^n$, because of the existence of wild Cantor sets. Call a Cantor set $K \subset I^n$ ($n \geq 3$) tame in $I^n$ if and only if every continuous $f: K \rightarrow I$ which satisfies the boundary condition can be aligned. Variations on this theme include corresponding results for functions from $K$ into $I^k$ with $1 < k < n$ and alternate characterizations of tameness in $I^n$. For example, a Cantor set $K \subset I^n$ ($n \geq 3$) is tame in $I^n$ if and only if it can be moved into an $(n - 1)$-hyperplane by a homeomorphism of $I^n$. (Received November 7, 1968.)


We define an object $C$ in an abelian category $\mathcal{A}$ with a torsion theory $(\mathcal{F}, \mathcal{G})$ to be cotorsion if $\text{Ext}(F, C) = 0$ for all $F \in \mathcal{F}$. Our purpose here is to give necessary and sufficient conditions for an important class of quotient objects of a cotorsion object to be cotorsion. (Received November 7, 1968.)


Let $(X, A)$ be a ringed space and $A(U)$ the functions given by the sections over $A$ for $U$ open $\subset X$. $A$ is a c.o. complete sheaf if $A(U)$ is complete in the compact open topology in $C(U)$ for all $U$ open $\subset X$.

If subsets of $A(U)$ which are uniformly bounded on compact subsets of $U$ are relatively compact in $A(U)$, $A$ is a Montel sheaf. The following is a generalization of Schwarz's lemma. Theorem. Let $A$ be a c.o. complete Montel sheaf which has a Hausdorff topology and let $D$ be an open connected subset of $X$. If $W \subset D$, $p \in W$ and $N = \{f \in W : f(p) = 0\}$, then there exists a continuous $H: W \rightarrow [0,1)$ such that $H(p) = 0$ and $\|f\|_V \leq (\|H\|_V \|f\|_W)$, for $f$ in $N$ and $V \subset W$. (Received November 7, 1968.)
A Hahn semigroup $S \cup \{0\}$ is a totally ordered semigroup with an identity $1$ and a zero element $0 < 1$ satisfying the weak cancellative laws: if $0 \neq st = st'$ in $S$ then $t = t'$, and similarly if $0 \neq ts = t's$ then $t = t'$. A Hahn valuation of a ring $A$ is a map $\sigma$ from $A$ onto a Hahn semigroup $S \cup \{0\}$ with the following properties: (i) $\sigma(a) = 0$ iff $a = 0$ in $A$; (ii) $\sigma(ab) = \sigma(a)\sigma(b)$ for all $a, b$ in $A$ and (iii) $\sigma(a - b) \leq \max(\sigma(a), \sigma(b))$ for all $a, b$ in $A$. For every $s \in S$, define $N_s = \{a \in A : \sigma(a) = s\}$, the family of subgroups $\{N_s\}_{s \in S}$ defines the Hahn topology of $A$. $A$ may not be a topological ring without further conditions. $N_1 = A^*$ is the Hahn valuation ring of $A$. A characterization of the compact topological ring $A^*$ is obtained. Theorem. Let $(A, S, \sigma)$ be a Hahn valued domain properly containing its Hahn valuation ring $A^*$. Then $A$ is a complete discrete valued field of rank $1$ with finite residue class field iff $A^*$ is compact and $A$ is a topological ring in the Hahn topology. (Received November 7, 1968.)

Let $\pi$ be a local dynamical system on a subspace $X$ of a $T_2$ space $Y$. For $y \in Y$ and $t \in \mathbb{R}$, let $J_y^t = \{z \in Y : \text{there are nets } x_i \rightarrow y, t_i \rightarrow t, \text{ and } x_i, t_i \rightarrow z\}$. Let $E(X)$ be the phase space of the global extension of $\pi$ as described by Hajek. For $X = Y$, $H^+_\pi = \bigcup \{J_x^t : \omega_x < t < + \infty\}$, and $H^-_\pi = \bigcup \{J_x^t : - \infty < t < a_\pi\}$. Theorem. $E(X)$ is $T_2$ if and only if either $H^+_\pi = \emptyset$ or $H^-_\pi = \emptyset$. Thus, whenever $X$ is an $n$-manifold, this result tells us when $E(X)$ is an $n$-manifold. For $Y$ a compactification of $X$ and $\pi$ global, the following is obtained. Theorem. $\pi$ has an extension $\rho$ on $Y$ if and only if for $y_1 \neq y_2$ in $Y$ and $t \in \mathbb{R}$, $J^t_{y_1} \cap J^t_{y_2} = \emptyset$. (Received November 7, 1968.)

For $M$ a Hodge manifold the tori $J_p(M) = H^{2p-1}(M, \mathbb{R})/H^{2p-1}(M, \mathbb{Z})$, $p = 1, \ldots, \dim M$, also have a Hodge manifold structure and there is a homomorphism, $w$, of codimension $p$ algebraic cycles algebraically equivalent to zero on $M$ into $J_p(M)$ [Weil, Amer. J. Math. 1952]. The image of $w$ is an abelian subvariety $J_{\mathfrak{a}}(M)$. The analytic moduli of $J_p(M)$ do not vary holomorphically under deformations of $M$. However, an abelian subvariety $J_{\mathfrak{a}}(M) \supset J_{\mathfrak{b}}(M)$ (which coincides with $J_{\mathfrak{a}}$ provided a special case of the Hodge conjecture is verified) satisfies Theorem. Let $M_t, t \in B$, be an analytic family of $\mathfrak{a}$ families. Then $C = \{t \in B | \dim J_{\mathfrak{b}}(M_t) \neq \emptyset\}$ is a countable union of analytic sets. If $C$ is a component of $B_d - B_{d+1}$ then $J_{\mathfrak{b}}(M_t), t \in C$, is an analytic family of Hodge manifolds. Remark. The nonempty $B_d$ may all be dense in $B$. (Received November 7, 1968.)

Let $f : M^p \rightarrow E^{n+1}$ be a $C^\infty$ immersion of a compact orientable differentiable manifold into Euclidean space of dimension $n + 1$, and let $p$ be either a point of $E^{n+1}$ not on $f(M)$. Let $v$ be the normal vector field on $M$. If one deforms the immersion of $M$ along the vector field $v$ in the positive direction, i.e. a deformation $t \rightarrow f(M) + tv$ $(0 \leq t \leq \infty)$ such that each point $f(m)$ goes into...
\( f(m) + tvf(m)' + f(M) + tv \) may intersect the point \( p \). We call such a point a cross-normal; hence, it is a triple \((p, m, t)\) where \( p = f(m) + tvf(m)' \). (In the case \( p \in M \), we assume \( m \neq p \).) In this paper we prove a number of integral equations relating the normal degree to the sum of the indices of the cross-normals. In particular, if \( n = 1 \), we show that \( (1/2\pi) \int_M^d x ds = L - 1 \), where \( L \) is the sum of the indices of the point \( p \) with manifold \( M \). If \( p \in M \), then \( L \) is the sum of the indices of the cross-normals, and \( k \) is the curvature of the curve \( M \). If \( p \in M \), then \( L \) is the linking number of the point \( p \) with the manifold \( M \) moved a small distance \( \varepsilon \) along \( v \). If \( n = 2 \), we show that the Euler characteristic of \( M \) satisfies \( \chi(M) = -2(L + 1) \), where \( L \) and 1 are as above. We have also generalized these results to higher dimension and codimension and to full normal lines. (Received November 7, 1968.)

663-680. W. J. KRIGER, Ohio State University, Columbus, Ohio 43210. On hyperfinite factors of type III.

Let \( X = \prod_{i=1}^n [0,1] \), and let \( T_i, i \in \mathbb{N} \), be the transformation of \( X \) that is given by \( (T_i x)_j = x_j \) if \( i \neq j \), \( (T_1 x)_1 = 1 - x_1 \). Denote by \( G \) the group that is generated by \( \{ T_i : i \in \mathbb{N} \} \). We put on \([0,1] \) a probability measure \( \mu \) where \( 0 < p (\{ 0 \}) = p < 1/2 \), and we put on \( X \) the corresponding product measure \( \mu^p \). The systems \( (G, \mu^p) \), \( 0 < p < 1/2 \), give rise to hyperfinite factors \( A_p \) of type III.

It was proved by Powers [Ann. of Math. 86 (1967), 138-171] that these factors are mutually non-*-isomorphic. We prove that the factor \( A_p \otimes A_{p'} \), where \( p' \) is not a rational multiple of \( p \), is a hyperfinite factor of type III that is not *-isomorphic to any of the \( A_p, 0 < p < 1/2 \). (Received November 7, 1968.)


Let the duality pairing between a Banach space \( E \) and its dual \( E^* \) be denoted by \( \langle \cdot, \cdot \rangle \). A mapping \( T \) of \( E \) into \( E^* \) is said to be monotone if \( \langle T(x) - T(y), x - y \rangle \geq 0 \) for all \( x, y \). **Theorem 1.** Let \( E \) be a reflexive Banach space which is locally-uniformly-convex and has a strictly-convex dual \( E^* \). Let \( T \) be a bounded continuous monotone mapping of \( E \) into \( E^* \). Let \( \varepsilon(r), r \geq 0 \), be a nonnegative numerical function such that \( \varepsilon(r) \rightarrow \infty \) as \( r \rightarrow \infty \) and let \( R > 0 \) be such that \( \|T(x) - t T(-x)\| \geq \varepsilon(\|x\|) \) for all \( t \) in \([0,1] \) and \( \|x\| \geq R \). Then \( T \) is surjective. Our condition in Theorem 1 is weaker than the coerciveness condition of Browder (Duke Math. J. (1963), 557-566) and Minty (Duke Math. J. (1962), 341-346). We however need that \( T \) is bounded. We then apply Theorem 1 to prove an existence and uniqueness theorem for nonlinear integral equations of Hammerstein type. **Theorem 2.** Let \( E \) be a given Banach space, \( H \) a Hilbert space. A continuous linear mapping of \( E \) into \( E^* \) and let \( K \) be a continuous linear mapping of \( E \) into \( H \) such that \( A = K^* K \), where \( K^* \) is the adjoint mapping of \( K \). (For example, \( A \) can be selfadjoint and monotone.) Let \( N \) be a continuous monotone mapping of \( E^* \) into \( E \). Then the equation \( x + AN(x) = 0 \) has a unique solution in \( E^* \). (Received November 7, 1968.)


Let \( D^N \) denote the unit polydisc in complex \( n \)-space and \( H^\infty(D^N) \) be the space of all bounded functions.
analytic complex valued functions in $D^n$. For $a = (a_1, \ldots, a_n) \in \mathbb{D}^n$, let $M_a$ be the fiber over $a$ in the maximal ideal space of $H^\infty(D^n)$. \textbf{Theorem 1.} For $a \in$ Shilov boundary of $D^n$ and $!a = (f_1, a_1, \ldots, a_n) \in H^\infty(D^n)$, we have (a) $I$ is contained in $M_a$; (b) $H^\infty(D^n)/I \cong H^\infty(D^{n-1})$; (c) $M_a$ is homeomorphic to $M_a'$ where $a' = (a_1, a_2, \ldots, a_n) \in \mathbb{D}^{n-1}$ and $M_a'$ is the fiber over $a'$ in the maximal ideal space of $H^\infty(D^{n-1})$. A similar result holds for pts which are on the topological boundary of $D^n$. \textbf{Theorem 2.} $D^n = M_{H^\infty(D^n)}$, where we consider $D^n$ as being imbedded in $M_{H^\infty(D^n)}$, the maximal ideal space of $H^\infty(D^n)$ (i.e., the Corona problem for the higher dimensional $H^\infty$-spaces).

\textbf{Theorem 3.} $H^\infty(D^n)$ is not ring isomorphic to $H^\infty(D^n)$ for $n \neq m$. (Received November 7, 1968.)

663-683. L. P. MAHER, JR, North Texas State University, Denton, Texas 76203. The bases which are possible for $\pm b$ bases infinite exponentials.

Suppose that $\exp(1/e) < b < e^e$, $B(x)$ means $b^{-x}$, $N(x)$ means $b^x$, if $F$ is any function then $F^{(k)}(x)$ means $F(F(\cdots F(x) \cdots ))$ where $F$ is iterated $k$ times, and that $F^{(0)}(x) = x$. Let $\pm b$ based infinite exponential mean real-number sequence of the form $B(n_1)(0), B(n_1)(N(n_2)(0)), B(n_1)(N(n_2)(B(n_3)(0))), \ldots$, where each $n_i$ is a nonnegative integer. Let $B^\infty$ and $N^\infty$ mean the sequences $B(1)$, $B(2)$, $B(3)$, ... and $N(1)$, $N(2)$, $N(3)$, ... \textbf{Theorem 1.} Every $\pm b$ based infinite exponential converges except the one in which each $N(n_i)$ is $N(0)$. Moreover, every nonnegative number is the sequential limit of a $\pm b$ based infinite exponential. \textbf{Theorem 2.} If a number $t$ is the limit of two $\pm b$ based infinite exponentials then there are only two such exponentials for $t$, and they are of the forms $F_1(B(N(0)(0)))$ and $F_1(N(B(0)(0)))$. Moreover, $[t]$ is countable. \textbf{Theorem 1} fails for every base $b$ such that $0 \leq b \leq \exp(1/e)$ or $b > e^e$. (Received November 7, 1968.)

663-684. J. S. COHEN, University of Maryland, College Park, Maryland 20742. Some generalizations of Hilbert Schmidt and nuclear operators. Preliminary report.

Let $E$ and $F$ be two arbitrary Banach spaces. Two classes of continuous linear operators are considered, which are closely related to the class $\Pi_p(E,F)$ of $p$-summable operators (see A. Pietsch, Absolut $p$-summierende Abbildungen in normierten Räumen, Studia Math. 27 (1967), 333-353). Let $\Pi^p(E)$ be the normed space of all weakly $p$-summable sequences with members in $E$; $\Pi^p_{\infty}(E)$ the normed space of all absolutely $p$-summable sequences; and $\Pi^p_{\infty}(E)$ the space of all sequences $(x_i)$ such that $\sup \|x_i\|_{\Pi^p_{\infty}(E)} < \infty$, where the supremum is taken over all $(x_i)$ in $\Pi^p_{\infty}(E)$ such that $\|x_i\|_{\Pi^p_{\infty}(E)} \leq 1$ and $1/p + 1/q = 1$. A continuous operator $T$ is in $\Pi^p_{\infty}(E,F)$ if the mapping $(x_i) \rightarrow (T x_i)$ induces a continuous linear operator from $\Pi^p_{\infty}(E)$ into $\Pi^p_{\infty}(F)$. Similarly, a continuous linear operator $T$ is in $\Pi^p_{\infty}(E,F)$ if the above mapping induces a continuous linear operator from $\Pi^p(E)$ into $\Pi^p(F)$. \textbf{Theorem.} An operator $T$ is in $\Pi^p_{\infty}(E,F)$ if and only if the adjoint map $T'$ is in $\Pi^p_{\infty}(E,F')$ where $1/p + 1/q = 1$ and $1 < p < \infty$. \textbf{Corollary.} For $E$ and $F$ Hilbert spaces, the class $\Pi^p_{\infty}(E,F)$ coincides with the class of Hilbert-Schmidt operators. \textbf{Theorem.} For $E$ and $F$ Hilbert spaces, the class $\Pi^p_{\infty}(E,F)$ coincides with the class of all nuclear operators. (Received November 7, 1968.)

663-685. JURGEN NEUKIRCH, Queen's University, Kingston, Ontario, Canada. A characterization of $p$-adic and finite algebraic number fields.

Let $Q$ be the field of rational numbers and $\Omega$ the field of all algebraic numbers. By $\mathbb{Q}_p^\infty$, we denote the field of all algebraic $p$-adic numbers. Under the $(p)$-characteristic of a profinite group $G$
we understand the Euler-Poincaré-characteristik of its maximal p-factorgroup \( G(p) \). With these notations we get the following grouptheoretical characterization of the field \( \mathbb{Q}_p \):

**Theorem 1.** Let \( k \) be a field in \( \Omega \) and \( G_k \) the galois group of \( k \). Then the following conditions are equivalent:

(i) \( k = \mathbb{Q}_p \),

(ii) \( G_k \) is a 2-dimensional pro-solvable group of \( (p) \)-characteristic \(-1\). Here \( G_k \) is said to be 2-dimensional if \( \text{cd}(G_k) = 2 \) for all prime numbers 1. Using classical results about prime decomposition, one deduces:

**Theorem 2.** If \( K \) and \( K' \) are finite normal algebraic number fields, then \( G_K = G_{K'} \), \( K = K' \). (Received November 7, 1968.)


Let \( A \) be a von Neumann algebra of operators on a complex Hilbert space. A closed, densely defined linear operator, \( T \), is said to be semi-Fredholm relative to \( A \) if:

(i) \( UT = TU \) for all unitary \( U \) in the commutant of \( A \),

(ii) the null projection of \( T \) or of \( T^* \) is finite relative to \( A \),

(iii) there is a projection \( E \) in \( A \) such that \( E(\mathbb{H}) \subseteq T(\mathbb{H}) \) and \( R_T \cdot E \) is finite relative to \( A \), where \( R_T \) is the projection onto the closure of the range of \( T \). The set of such operators is denoted by \( SF(A) \). The index group, \( I(A) \), and homomorphism \( I : F(A) \rightarrow I(A) \) (see Abstract 648–61, these Notices 14 (1967), 647) can canonically be extended to an index set, \( J(A) \), and a map \( J : SF(A) \rightarrow J(A) \). One generalizes the semi-Fredholm theory of Cordes-Labrousse and the Fredholm theory of Breuer (see Abstract 653–69, these Notices 15 (1968), 97) as follows. Let \( SF(A) \) be topologized with the "gap" metric and let \( \pi_0(SF(A)) \) denote the set of connected components of \( SF(A) \). The extended index map, \( J \) induces an injection of \( \pi_0(SF(A)) \) into \( J(A) \). If \( A \) is properly infinite the induced map is a bijection. (Received November 7, 1968.)


Let \( X \neq \{0\} \) and \( Y \) be normed linear spaces, \( E \) the normed linear space of bounded linear transformations from \( X \) into \( Y \), \( F_W = X \otimes Y \), and \( \lambda \) the least cross norm.

**Theorem 1.** If \( W \) is norming and \( Y \) is \( o(Y,W) \)-sequentially complete, then the unit ball \( S_E \) of \( E \) is \( \sigma(E,F_W) \)-sequentially complete. Conversely, if \( W \) is norming and \( X \) is complete, then \( E \) is \( \sigma(E,F_W) \)-sequentially complete.

Typical Corollary. For every weakly sequentially complete Banach space \( Y \), the dual Banach space is \( Y \)-sequentially complete if and only if the unit ball of \( Y \) is \( \sigma(Y,W) \)-complete if and only if the unit ball of \( E \) is \( \sigma(E,F_W) \)-complete. (Received November 7, 1968.)

663–688. JOSEPH ALTINGER, 1333 Brainard Road, Lyndhurst, Ohio 44124. Quasi split translations.

Let \( G \) be a group with subgroup \( H \). The normalizer in \( \text{Sym}(G) \) of the left translations by elements of \( H \) is the group \( QST(G;H) \) of quasi split translations of \( G \). (See C. Wells's Abstract 68T-386, these Notices 15 (1968), 553, and his Abstract 68T-437, these Notices 15 (1968), 913.) Let \( H_{ol}(H) \) denote the holomorph of \( H \) and let \( K \) denote the set of left cosets of \( H \) in \( G \). Let \( k = (G;H) \) and
let $R(H)$ be the group of right multiplications by elements of $H$. In the wreath product $Hol(H) \wr \text{Sym}(K)$, let $Q$ be the subgroup consisting of elements of the form $q = (bh_1, bh_2, \ldots, bh_k, s)$, where $b \in \text{Aut}(H)$, $h_i \in R(H)$ for $i = 1, \ldots, k$, and $s \in \text{Sym}(K)$. Theorem. $\text{QST}(G; H)$ is isomorphic to $Q$.

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663-689. C. S. QUEEN, Ohio State University, Columbus, Ohio 43201. Nonconservative function fields of genus one.

Let $F$ be a function field of one variable over a field $k$. Suppose that $k$ is the exact field of constants of $F$, the genus of $F$ is one, and $F/k$ is nonconservative, i.e., if $\bar{k}$ is an algebraic closure of $k$, then the genus of $F_{\bar{k}} = F \otimes_k \bar{k}$ is zero. Theorem 1. The characteristic of $k$ is 2 or 3 and $F/k$ has a unique singular prime divisor $P$, where the degree of $P$ is 2 or 4 if the characteristic of $k$ is 2, and is 3 if the characteristic of $k$ is 3. (i) If the characteristic of $k$ is 2, then there exist $v$ and $u$ in $F$ such that $F = k(u, v)$, where the pole divisor of $v$ is $P$. Furthermore $u$ and $v$ satisfy an equation over $k$ of one of the following three types: (i) $u^2 + u = a + dv^4$, $d$ not in $k^2$; (ii) $u^2 = a + cv^2 + dv^4$, $d$ not in $k^2$; (iii) $u^4 = a + d_1v^2 + d_2v^4$, $k[d_1^{1/2}, d_2^{1/2}]$; $k = 4$, where (i) holds if $F$ is separable of degree 2 over $k(v)$, (ii) holds if $F$ is inseparable of degree 2 over $k(v)$ and (iii) holds if $F$ is of degree 4 over $k(v)$.

(2) If the characteristic of $k$ is 3, then there exist $v$ and $u$ in $F$, $a$ and $d$ in $k$ such that $d$ is not in $k^3$, $F = k(u, v)$ and $u^3 - du = a + dv^3$, where the pole divisor of $v$ is $P$. Theorem 2. If $F_0$ is a function field of one variable over $k$ such that $F_0$ is contained in $F$, then $F = F_0$ or $F_0/k$ is of genus zero.

(Received November 7, 1968.)

663-690. D. C. KAY, University of Oklahoma, Norman, Oklahoma 73069, and M. D. GUAY, University of New Hampshire, Durham, New Hampshire. On sets which have finitely many points of local nonconvexity and a certain property $P_m$.

A set $S$ in a linear topological space is said to have point $p$ as a point of local nonconvexity (Inc) if for each neighborhood $N$ of $p$ there exist points $x$ and $y$ in $N \cap S$ such that the segment $xy$ does not belong to $S$. Results of F. A. Valentine (Local convexity and $L_n$ sets, Proc. Amer. Math. Soc. 16 (1965), 1305-1310) are used to prove Theorem 1. A closed set $S$ having a finite set $Q = \{ q_1, q_2, \ldots, q_n \}$ of Inc points, such that $S \sim Q$ is connected, is the union of $n + 1$ or fewer convex sets. (This is best possible.) We further define the following: A set $S$ has property $P_m$, or is a $P_m$ set, if for each $m$ points $p_1, p_2, \ldots, p_m$ in $S$ at least one of the segments $p_ip_j$ belongs to $S$. It may be easily established by the above results that if $S$ is a closed $P_m$ set with $|Q| = 1$ then $S$ is the union of at most $m - 1$ convex sets; since the five-pointed star is a $P_3$ set having 5 Inc points and cannot be expressed as the union of less than 3 convex sets, there is an obvious bound for the cardinality of $Q$ for which this decomposition theorem holds. Examples show that the ultimate result in this direction is Theorem 2. A closed $P_m$ set having at most 2 Inc points is the union of $m - 1$ or fewer convex sets.

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663-691. J. K. LUEDEMAN, Clemson University, Clemson, South Carolina 29631. On the embedding of topological rings.

A Hausdorff ring topology $\mathcal{J}$ on a left Ore domain $A$ is extendable if the subspace topology on $A^* (= A - \{0\})$ is uniformisable with a basis $\mathcal{B}$ satisfying (1) $(\forall a \in A^*), (\forall U \in \mathcal{B}), ((ax, ay) \in U = \cdots$
\[(x,y) \in U, (2) \left( y \in A^* \right) \cup U \cup U_2 \subseteq U \left( \left[ (x,y) \in U_1 \right] \cup \left[ (x,y) \in U_2 \right] \right) \] (xa, ya) \in U, (3) \left( y \in A^* \right) \cup U \cup U_2 \subseteq U \left( \left[ (x,y) \in V \& b'x = x'b \right] \right) = (x'a, b'(y - x) + x'a) \in U, (4) \left( y \in A^* \right) \cup U \cup U_2 \subseteq U \left( \left[ (x,y) \in V \& a'^x = a'x \right] \right) = (x'a, a'(y - x) + x'b) \in U\). Let \( Q(A) \) denote the field of left quotients of \( A \). We prove the following theorems: Theorem 1. A Hausdorff left Ore domain \( A \) is embeddable in \( Q(A) \) if and only if its topology is extendable. Theorem 2. Let \( f : (A, \mathcal{J}) \to (B, \mathcal{E}) \) be a continuous homomorphism between left Ore domains with extendable topologies. Let \( \mathcal{J}(A) \) and \( \mathcal{E}(A) \) be the uniformities on \( A^* \) and \( B^* \) associated with the extendable topologies \( \mathcal{J} \) and \( \mathcal{E} \). Then \( f \) is extendable to a continuous homomorphism \( Q(f) : Q(A) \to Q(B) \) if and only if \( f \) is a monomorphism and \( f : (A^*, \mathcal{J}(A)) \to (B^*, \mathcal{E}(A)) \) is uniformly continuous. The functorial properties of \( Q(\cdot) \) are also discussed. (Received October 30, 1968.)

663-692. LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On transient development of internal waves in stratified ocean.

An initial value investigation is made of the development of surface and internal wave motions generated by harmonic pressure distributions of fixed frequency in a fluid which is composed of two layers of different densities, the lower layer being of unlimited depth. The method for the asymptotic evaluation of the wave integrals is the same as that developed by Debnath (1967, 1968a-1968b) for the similar problem in a single homogeneous fluid. Both the steady state and the transient wave solutions have been obtained explicitly. The limiting behaviour of the solutions as time \( t \) tends to infinity is examined and the corresponding steady state solutions for the surface as well as internal waves have been derived in the following sense. It is found that there are two classes of waves---the first corresponds to usual surface waves for the homogeneous ocean with a slight modification in amplitude, and the second represents the internal waves with large amplitude for a small difference of density between the layers. Some interesting features of these wave modes have also been reported in some detail. (Received November 4, 1968.)

663-693. S. M. FAHRUDDIN, Queen's University, Kingston, Ontario, Canada. The Grothendieck groups of modules of finite rank over a valuation ring.

Let \((K, R, V, \Gamma)\) be a valuation quadruple and \( \mathcal{M}_R \) be the category of torsion free \( R \)-modules of finite rank. Denote by \( \text{Upp}(R) \) the semigroup of upper classes of \( \Gamma \), including \( \Gamma \) itself. Let \( \text{Pic}(R) \) be the semigroup \( \text{Upp}(R)/\sim \) where \( \Lambda_1, \Lambda_2 \in \text{Upp}(R) \), then \( \Lambda_1 \sim \Lambda_2 \iff \Lambda_1 + \{1\} = \Lambda_2 \) for some \( \{1\} \in \Gamma \). Then \textbf{Theorem 1.} \( \text{Gr}(\mathcal{M}_R) \cong \mathbb{Z}[\text{Pic}(R)] \), where \( \text{Gr} \) stands for Grothendieck group and \( \mathbb{Z} \) is the ring of integers. Every \( M \in \mathcal{M}_R \) has composition series in the following sense. There exists a chain \( M = M_0 \supseteq M_1 \supseteq M_2 \supseteq \ldots \supseteq M_n = (0) \) such that \( \text{Rank} (M_1) = \text{Rank} (M) - 1 \) and \( M_1 \) is pure in \( M \). Call \( M \) almost isomorphic to \( N \) if (1) they have same rank and there exist composition series for \( M \) and \( N \), say \( M = M_0 \supseteq \ldots \supseteq M_n = (0) \) and \( N = N_0 \supseteq \ldots \supseteq N_n = (0) \) such that \( M_1 \supseteq M_1 \) and \( M_n = (0) \) where \( \sigma \) is a permutation of \( n \) symbols. Notation \( M \cong N \). Then \textbf{Theorem 2.} (1) \( M \oplus M \cong N \oplus N \) if \( M \cong N \), and (2) \( M \oplus L \cong N \oplus L \cong M \oplus N \), where \( M, N, L \in \mathcal{M}_R \). (Received October 31, 1968.)

This report is concerned with the cubature error for a cross-product rule over a rectangle. Using a method similar to that of Krylov [Approximate calculation of integrals, 1962, pp. 206-212], it is shown that Peano's theorem can be used to obtain expansions for the cubature error. These error expansions provide justification for using Richardson extrapolation to estimate the cubature error for cytolitic rules. Numerical examples illustrate the estimation procedure. (Received November 1, 1968.)

663-695. T. N. SRIVASTAVA, Loyola College, Montreal, Canada. On an integral transform. II.

The symbolic notations $\psi(p) \frac{k}{\nu} f(t)$ and $\psi(p) \frac{S}{\nu} f(t)$ are used to denote the Meijer Bessel transform $\psi(p) = \frac{\nu^k}{\nu} \int_0^\infty (pt)^{1/2} \psi(p)f(t)dt$ and the Stieltjes transform $\psi(p) = \frac{\nu^k}{\nu} \int_0^\infty (pt)^{1/2} \psi(p)\cdot K(p) \cdot g(t) dt$ where $\Re(p) > 0$. Theorem 1. If $\psi(p) \frac{k}{\nu} f(t)$ and $(2x/\pi)^{1/2} \int h(p) \frac{3}{\nu} \psi(p) \cdot K(p) \cdot g(t) dt$ then $\psi(p) \psi(h(p)) \frac{S}{\nu} g(x,t)$ provided that $\psi(p) \cdot h(p)$ are continuous functions of $p$ and the integrals in the hypothesis exist. Theorem 2. If $\psi(p) \frac{k}{\nu} f(t)$ and $\psi(p) \frac{S}{\nu} f(t)$ then $\psi(p) = \frac{\nu^k}{\nu} \int_0^\infty x(t) dt$ provided that $0 < a < \nu, \Re \mu > \Re \nu > -1$ and the integrals defining $\psi(p) \cdot h(p)$ exist. The author uses these theorems to obtain an inverse Stieltjes transform of the product of Bessel and G-function of different arguments and to evaluate an infinite integral involving the product of Bessel and G-function of different arguments. (Received October 31, 1968.)

663-696. CHARLES FREIFELD, Northeastern University, Boston, Massachusetts 02115. The cohomology theory of transitive filtered modules.

Let $(L, M, \theta)$ be a transitive filtered $L$-module, where $L$ is a primitive infinite-dimensional Lie algebra. Theorem 1. The vector space $H^1(L, M)$ is finite dimensional. Let $E$ denote the $L$-module of formal germs of sections of a line bundle. Theorem 2 (Vanishing Theorem). If $E$ is not the module of sections of the trivial line bundle, then $H^1(L, E) = 0$. Also, a complete calculation of the first cohomology group will be given when $(L, E, \theta)$ is the usual representation on the set of formal power series in one variable. Theorem 3. Except for two classes of primitive infinite Lie algebras $L$, $H^1(L, L) = 0$; for those two classes, $H^1(L, L)$ is one dimensional. (Here $\theta$ is the adjoint representation.) Applications to reducibility questions are obtained. (Received November 5, 1968.)

663-697. M. A. MALIK, Sir George Williams University, Montreal, Canada. On the derivative of a polynomial.

Theorem. Let $P(z)$ be a polynomial of degree $n$ in complex domain, with $|P(z)| \equiv 1$ on $|z| = 1$ and $P(z)$ has no zero in the disk $|z| < k$, $k \equiv 1$, then $|P'(z)| \equiv n/(1 + k)$ for $|z| \equiv 1$. Corollary. If $P(z)$ is a polynomial of degree $n$ with $\max |P(z)|: |z| = 1 = 1$ and $P(z)$ has all its zeros in the disk $|z| \equiv k$, $k \equiv 1$, then $\max |P'(z)|: |z| = 1 = n/(1 + k)$. The result is best possible and extremal polynomial is $P(z) = (z + k/(1 + k))^n$. (Received November 5, 1968.)
Recently Ragab (in E-transforms, J. Res. Nat. Bur. Standards, Section B, 71B (1967), 23-37) gave a transform having MacRobert's E function as a kernel, the power of the argument being an integer. Later, Kapoor and Masood (in On a generalised L-H transform, Proc. Cambridge Philos. Soc. 64 (1968), 399) defined a transform, called the generalised L-H transform, where the kernel is Meijer's G function having the power of the argument a positive integer. They also gave inversion theorems for their respective transforms. The object of this paper is to define a more general transform where the kernel is the H function \( H_{\lambda}^{m,n} \) representing \((a_1, e_1), \ldots, (a_p, e_p)\) and \(B\) representing \((b_1, f_1), \ldots, (b_q, f_q)\). Without loss of generality, \(\lambda\) is taken to be positive. The inversion formula of the transform is obtained. The main properties of the transform are also considered. (Received November 6, 1968.)

Let \( m \neq n \) be infinite cardinals. Theorem. \( 2^m = 2^n \) if and only if there is a normal Hausdorff space of density \( m \) containing a set of power \( n \) that has no limit point. This generalizes theorems of F. B. Jones (Bull. Amer. Math. Soc. 43 (1937), 671-677) and R. W. Heath (Coll. Math. 12 (1964), 11-14). Theorem. \( m \neq n \) if and only if there is a first countable Hausdorff space of density \( m \) containing a set of power \( n \) that has no limit point. Using these equivalences and known independence results in set theory (see e.g. W. Easton, Powers of regular cardinals, Thesis, Princeton University, 1964), one immediately obtains independence theorems for topology. A space is collectionwise Hausdorff if it is collectionwise normal with respect to discrete collections of points. Theorem. If there is a first countable normal Hausdorff space which is not collectionwise Hausdorff, then there is a pointwise paracompact normal Moore space which is not collectionwise Hausdorff (and hence not metrizable). (Received November 6, 1968.)
Hausdorff space. We are interested here in the problem of explosion: If $\xi_t$ denotes the number of particles at time $t$, when do we have $p^x(\xi_t = +\infty \text{ for some } t \leq 0) > 0$? In the single-type case, assuming no dying, we have the well-known result of E. B. Dynkin: Explosion happens iff

$$\int_1^\infty \epsilon du/(h(u) - u)$$

converges for each $\epsilon > 0$, where $h$ is the generating function of new born particles. For a large class of bnp, it is shown that an analogous result holds. More generally, one has sufficient conditions for explosion or nonexplosion. (Received November 5, 1968.)

663-702. F. M. WILLIAMS, West Texas State University, Canyon, Texas 79015. Embedding in summands of infinite Abelian groups.

In (Direct sums of countable groups and related topics, J. Algebra 2 (1965), 443-450), Irwin and Richman have made the definition: The Abelian group $G$ is called a Fuchs five group if each infinite subgroup can be imbedded in a direct summand of $G$ with the same cardinal. Let $\Omega$ be the first uncountable ordinal, and let $G$ be a $p$-primary Abelian group with basic subgroup $B = \sum_\alpha < \Omega B_\alpha$, where each $B_\alpha$ is countable. Suppose that $G$ is a subgroup of $\sum_\alpha < \Omega B_\alpha$. Theorem. The following are equivalent: (1) $G$ is a direct sum of cyclic groups. (2) $G$ is the union of a chain of countable summands. (3) $G$ is a Fuchs five group. (Received November 5, 1968.)

663-703. W. G. FAIR, 425 Volker Boulevard, Kansas City, Missouri 64110. Convergence and applications of noncommutative continued fractions.

Elementary techniques and some results from functional analysis are used to prove a number of theorems concerning the convergence of continued fractions whose elements are bounded linear operators on a Banach space. Some applications are given. (Received November 8, 1968.)

663-704. G. K. WHITE, University of British Columbia, Vancouver 8, Canada. Iterated arithmetic functions.

Let $\lambda$ be any function with domain and codomain some set $X$, and suppose $\rho: Y \cup \lambda(Y) \to Z$ satisfies $\rho(\lambda(y)) = \rho(y) - 1$ for every $y \in Y$. Then $\rho$ is an index register for $\lambda$ over $Y$. It is known (see e.g. Niven [Canad. J. Math. 2 (1950), 406-408] or the author [Pacific J. Math. 12 (1962), 777-783]) that certain functions $\lambda: P \to P$ obeying some given arithmetic law will have an index register $\rho$ satisfying some related law over most or all of $P$. For example, certain multiplicative $\lambda$ have additive $\rho$, as is the case for the Euler $\phi$-function. Other $\lambda$ satisfy the following: $(C_1) \lambda(ab) = [\lambda(a), \lambda(b)]$, $(C_2) \rho(ab) = \max(\rho(a), \rho(b))$ for coprime $a, b$. The author gives a construction for finding all such related $\lambda, \rho$ when $(C_2)$ is strengthened to hold for all $a, b$ in the latter case, and determines a corresponding family of $\lambda, \rho$ in the former; he also defines some other related laws and families. (Received November 8, 1968.)


Given a Finsler space $R$, we may approximate the metric of $R$ by a Riemannian metric in the following way. At each point $p \in R$ we replace the indicatrix, which is assumed to be a convex body
in the tangent space $T_p(R)$, by its maximal inscribed ellipsoid [or its minimal circumscribed ellipsoid]. The resulting field of ellipsoids is shown to be continuous and would define a Riemannian metric if it were differentiable. This does not follow even if the given Finsler metric is differentiable. However, the field of ellipsoids can be smoothed to yield a differentiable Riemannian metric while changing the lengths of curves and the volume of the space arbitrarily little. One would not expect that such a special Riemannian metric would carry much information on the given space $R$. However, this method does enable us to extend one of the most interesting results of global surface theory (Loewner’s theorem on the torus) to two-dimensional Finsler spaces. (Received November 8, 1968.)


Let $m$ be an infinite cardinal and let $W$ be a set with cardinality $m$. Let $W^*$ be the set of all nonempty subsets of $W$ partially ordered by inclusion. An $m$-sequence in a space $X$ is a net in $X$ with indexing set $W^*$. A subset $A$ of $X$ is called $m$-open iff each $m$-sequence which converges to a point of $A$ is eventually in $A$. The space $X$ is called $m$-sequential iff its open sets and $m$-open sets coincide. Theorem. A space is $m$-sequential iff it is a quotient of an $m$-pseudometrizable space. (Received November 8, 1968.)


This paper is a continuation of work already begun dealing with the development of homological methods in the theory of locally compact abelian groups (Abstracts 658-123 and 660-11, these Notices 15 (1968), 755, 1010). The results obtained here deal with the general problem as to when a locally compact abelian group splits into the topological direct sum of a connected group and a totally disconnected one. The problem is analogous to the mixed group problem of abstract group theory. The ideas presented here parallel the development of the theory of cotorsion groups. In this case, however, we are able to make a complete determination of what might be called the "coconnected" groups. Let $\mathcal{L}$ denote the category of locally compact abelian groups, $\mathbb{R}$ the additive topological group of real numbers, and $\mathbb{Z}$ the group of integers. Some explicit results are as follows. If $G$ is in $\mathcal{L}$, then $\text{Ext}(X,G) = 0$ for all totally disconnected $X$ of $\mathcal{L}$ iff $X = \mathbb{R}^n \oplus (\mathbb{R}/\mathbb{Z})^\sigma \oplus D$ where $D$ is a discrete divisible group. A group $G$ in $\mathcal{L}$ has the property that $\text{Ext}(C,G) = 0$ for all connected $C$ in $\mathcal{L}$ iff $G = (\mathbb{R}/\mathbb{Z})^\sigma \oplus \mathbb{R}^n$. If $G$ is a group in $\mathcal{L}$, then $\text{Ext}(G,X) = 0$ for all totally disconnected $X$ of $\mathcal{L}$ iff $G = \bigoplus_\sigma \mathbb{R}^n \oplus \mathbb{R}^n$. Finally a group $G$ in $\mathcal{L}$ has the property that $\text{Ext}(G,C) = 0$ for all connected $C$ in $\mathcal{L}$ iff $G = \mathbb{R}^n \oplus M$ where $M$ contains a compact open subgroup having a cotorsion dual. (Received November 8, 1968.)

663-708. JET WIMP, 425 Volker Boulevard, Kansas City, Missouri 64110. Differential-difference properties of certain polynomials.

In this paper we investigate the differential difference properties of a large class of hypergeometric polynomials. The results include known relations for the classical orthogonal polynomials.
as special cases. Our results can also be used to generalize recurrence formulae due to Watson and Bailey. (Received November 8, 1968.)


Let \( R \) be a field complete under a discrete rank 1 valuation, \( v \), with a quasi-finite residue class field. Assume that for every finite abelian \( K/k \) (\( k/\mathbb{R} \) finite) that \( [k': N_{K/k}(K)] \leq \deg(K/k) \), and that Hilbert's Theorem 90 is true. The local norm residue map, \( f_{K/k} \), can be constructed for every abelian \( K/k \) as follows: There is \( L/K \) such that \( L = T U \) where \( U/k \) is unramified, and \( T/k \) pure ramified. Once \( f_{U/k} \) and \( f_{T/k} \) are known, it is possible to define, for \( x \in K^* \), \( f_{L/k}(x) = f_{T/k}(x)f_{U/k}(x) \) and then \( f_{K/k}(x) = f_{L/k}(x)|K \). Now \( f_{U/k}(x) = x^{y(x,k)} \), where \( F_k \) generates the group of the unramified closure of \( k \); and \( f_{T/k} \) is obtained from the following generalization by J. P. Serre (Corps locaux, p. 208, Theorem 2) of a theorem of B. Dwork (Hamb. Abh. 22 (1958)) which can in fact be proved without assuming the Local Reciprocity Law: Theorem. Let \( W/T \) be unramified of degree equal to the exponent of \( G(T/k) \). For \( x \in k^* \) there is \( A \) and \( s_{\Sigma} \in S \) in \( WT \) such that \( N_{WT/W}(A) = x \), and \( A_0^{-1}\prod_{s \in S} A_s^{-1} = 1 \), where \( \rho \) generates \( G(WT/T) \) and \( S \) is some finite subset of \( G(WT/W) \). Furthermore, \( f_{T/k}(x) = \prod_{s \in S} x^{y(A_s,WT)} \) is the local norm residue map. All of the foregoing can be generalized to include the case of a global field \( R \). (Received November 8, 1968.)

663-710. P. C. WANG, University of Iowa, Iowa City, Iowa 52240. On distinct fields over finite probability space.

Let \((\Omega, A, P)\) be a finite probability space and \( A \) be a nonempty collection of subsets of \( \Omega \) which are closed under complementation and union. By identifying to each field \( A \), there is associated an unique partition of the sample space \( \Omega \). We prove the following apparently new Theorem 1. Let \((\Omega, A, P)\) be a finite probability space where \( \Omega \) contains \( n \) elementary events. Then the total number of distinct fields defined on the sample space \( \Omega \) is

\[
\sum_{k=1}^{n} \left( \sum_{i=1}^{k} a_{n_1 \ldots n_{i-1} n_i \ldots n_k} \right)^{-1} \text{ where } n_1 \text{ appears } k_1 \text{ times, } n_2 \text{ appears } k_2 \text{ times,} \ldots, n_i \text{ appears } k_i \text{ times and the second summation is taken over all } k\text{-part partitions of } n \text{ such that } \sum_{j=1}^{k} j \cdot k_j = k, \sum_{j=1}^{k} a_{n_1 \ldots n_{j-1} n_j} = n, n_1 > n_2 > \ldots > n_k \neq 1 \text{ and } i \neq k. \text{ Consequently, if } A \text{ is a field of events defined on } \Omega \text{ then there exists } k \text{ (} k \neq 1 \text{ and } k \neq n \text{) such that } A \text{ contains precisely } 2^k \text{ distinct events.}
\]

Theorem 2. The total number of distinct fields on sample space \( \Omega \) which contains \( n \) elementary events is \( \sum_{k=1}^{n} S(n,k) \) where \( S(n,k) \) is the Stirling number of the Stirling number of the second kind. (Received November 8, 1968.)


A module \( M_R \) over a ring \( R \) with identity is a cogenerator if every right \( R \)-module can be imbedded in a direct product of copies of \( M_R \). Theorem. If \( R \) is injective and \( R_R \) is a cogenerator, then \( R \) is a cogenerator and \( R_R \) is injective. (Received November 8, 1968.)


A \textit{p-place Menger algebra} is a set \( \mathcal{D} \) of elements with a \((p+1)\)-ary operation \( \circ \) that is super-

associative, i.e. such that \( o(x_0, x_1, \ldots, x_p, y_1, \ldots, y_p) = o(x_0, o(x_1, y_1, \ldots, y_p), \ldots, o(x_p, y_1, \ldots, y_p)) \). We say \( \mathcal{A} \) is group-like if for any elements \( a_0, a_1, \ldots, a_p, b \) of \( \mathcal{A} \) there exist unique elements \( x_0, x_1, \ldots, x_p \) such that \( o(a_0, a_1, \ldots, a_i, x_i, a_{i+1}, \ldots, a_p) = b \) for \( i = 0, 1, \ldots, p \). If \( \mathcal{A} \) is a \( p \)-place group-like Menger algebra, then \( \mathcal{A} \) with the binary operation \( \cdot \) defined by \( x \cdot y = o(x, y, y, \ldots, y) \) is a group. Theorem. The cyclic group of order \( n \) is the associated group of some \( p \)-place group-like Menger algebra if and only if (1) \( n \) is odd, or (2) \( n \) is even and \( p \) is odd. (Received November 8, 1968.)

663-713. J. T. DARWIN, JR., Auburn University, Auburn, Alabama. On Fourier coefficients.

Let \( 1 < p < \infty \), and \( \{a_n\} \) be a complex number sequence. Then \( \exists \) a function \( f \in L^p[0, 2\pi] \) \( \supseteq a_n = \int_0^{2\pi} f(t) \cos nt \, dt \iff \exists \, M > 0 \exists \, (K + 1)^{-1} \sum_{n=0}^K c(K,n) \sum_{r=0}^n a_r B_K^{r,n} \rho \notin M \quad \forall \, K, s = 0, 1, \ldots \), where \( c(K,n) \) is the indicated binomial coefficient and \( B_K^{r,n} = 2^{1/2} \int_0^1 t^n (1 - t)^{K-n} \cos(2\pi rt) \, dt \). The theorem also follows for Fourier sine coefficients. (Received November 8, 1968.)

663-714. H. G. RUTHERFORD, Montana State University, Bozeman, Montana 59715. \( L \)-primes in noncommutative rings.

Let \( R \) be a ring with multiplicative identity \( 1 \). An \( L \)-preprime of \( R \) is a nonempty subset \( T \) of \( R \) which is closed under addition, multiplication, does not contain \(-1\) and for every \( a \in R \), \( \beta \in T \), \( a\beta - \beta a \in T \). \( T \) is an \( L \)-prime of \( R \) if it is maximal (i.e. not contained in any larger \( L \)-preprime). These concepts form an attempt to do for noncommutative rings what Harrison's concepts are doing for commutative ring. Two examples are Proposition 1. \( R \) has no nonzero \( L \)-primes if and only if \( R \) is a simple ring whose center is a locally finite field. Proposition 2. Let \( P \) be an \( L \)-prime of a ring \( R \); \( a \in P \) a unit in \( R \). Then \( a^{-1} \in P \) if and only if \( -a \notin P \). Let \( V \) be a finite-dimensional vector space over the rational field \( Q \), \( \dim_Q V \equiv 3 \). Let \( p \) be a prime number, \( A(p) \) the valuation ring associated with \( p \) in \( Q \). Let \( V \) be a maximal \( A(p) \)-order in \( V = \text{Hom}_Q(V, V) \). Let \( \mathcal{A}_q = \{ f \in V \mid (1/q) \cdot f \in V \} \) for any prime number. For \( q = p \), \( \mathcal{A}_p \) is the unique maximal ideal of \( V \). Theorem. The finite-\( L \)-primes of \( V \) are exactly the \( \mathcal{A}_q \), \( q \) a prime number. \( V \) has a unique infinite \( L \)-prime, namely \( \{ a \in A(p) \mid a \neq 0 \} \). (Received November 8, 1968.)

663-715. WILLIAM BARIT, Louisiana State University, Baton Rouge, Louisiana 70803. Small extensions of small homeomorphisms.

Klee [Trans. Amer. Math. Soc. 78 (1955), 30-45] proves that any homeomorphism between compact subsets of \( 1_2 \) can be extended to a space homeomorphism. A proof, somewhat similar to Klee's, using the full strength of Dugundji's extension theorem [Pacific J. Math. 1 (1949), 639-645] gives Theorem. If \( h: K_1 \to K_2 \) is an onto-homeomorphism between compact subsets of \( 1_2 \), and \( d(h, id) < \epsilon \), then there exists an onto-homeomorphism \( H: 1_2 \to 1_2 \) such that \( H|K_1 = h \) and \( d(H, id) < \epsilon \). Using methods and results of Anderson [Michigan Math. J. 14 (1967), 365-383] one also gets Theorem. If \( h: K_1 \to K_2 \) is a homeomorphism between closed subsets of the Hilbert cube with property \( Z \), and \( d(h, id) < \epsilon \), then there exists an extension \( H \) from the Hilbert cube onto itself with \( d(H, id) < \epsilon \). This result and technical modification of it are used by Anderson in problems dealing with apparent boundaries of the Hilbert cube (see R. D. Anderson, A characterization of apparent boundaries of the Hilbert cube, to appear in these Notices). (Received November 8, 1968.)

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Recent results concerning the Hilbert cube together with the largely geometric methods for obtaining them are surveyed. The following definitions are used. Let $l^\infty = \prod_{j>0} I_j^0$ and $s = \prod_{j>0} I_j^0$ where for each $j > 0$, $I_j^0 = [-1/2^j, 1/2^j]$, and $I_j = [-1/2^j, 1/2^j]$. The usual metric for $l^\infty$ is given by the following: For $x = (x_j)$, $y = (y_j) \in l^\infty$, $d(x,y) = \sum_{j>0} |x_j - y_j|^{1/2}$. Since $l^\infty$ is compact and $s$ is a dense subset of $l^\infty$, we may consider $l^\infty$ as a compactification of $s$. Let $B(l^\infty) = l^\infty \setminus s$. Clearly $B(l^\infty)$ is also a dense subset of $l^\infty$ and $B(l^\infty)$ is $\sigma$-compact, i.e. $B(l^\infty)$ is a countable union of compact sets. A closed set $K$ in $l^\infty$ has infinite deficiency if for each of infinitely many $j > 0$, $K$ projects onto a single point of $I_j^0$. The topology of $l^\infty$ has been studied largely by use of homeomorphisms "creating" infinite deficiency of "pushing" points or sets from $s$ to $B(l^\infty)$ or from $B(l^\infty)$ to $s$. Frequently such a homeomorphism is obtained as an infinite left composition $g_3 \circ g_2 \circ g_1$ of homeomorphisms of $l^\infty$ onto itself defined by relatively simple geometrically conceived homeomorphisms of Euclidean cells. The "smallness" of all but finitely many of the coordinate factors of $l^\infty$ is used to help achieve convergence. (Received November 8, 1968.)
Existence of uncomplemented subspaces of \( C(X) \) which are algebraically isomorphic to \( C(X) \). Preliminary report.

Let \( C(X) \) denote the space of continuous scalar-valued functions on a compact metric space \( X \). The \( n \)th derived set of \( X \) is denoted \( X^{(n)} \). A subset \( F \) of \( X \) is said to be pointlike if for each \( \epsilon > 0 \) and for each \( x \in F \), there exists a homeomorphism \( h \) of \( X \sim \{x\} \) onto \( X \sim F \) which is the identity function outside of an \( \epsilon \)-neighborhood of \( F \). Theorem. If at least one of the conditions (a), (b), or (c) are satisfied, then there exists a continuous mapping \( f \) of \( X \) onto itself for which the image of \( C(X) \) under the induced isometric algebra isomorphism \( f^0 \) is not complemented in \( C(X) \): (a) \( X^{(n)} \sim X^{(n+1)} \) is nonempty for each positive integer \( n \). (b) \( X \) contains a closed-open subset homeomorphic to the Cantor set. (c) \( X \) contains a subset \( Y \) such that \( Y \) is dense-in-itself and each \( y \) in \( Y \) has a closed neighborhood base \( \{F_n\}_n \) for which (i) the boundary of \( F \) contains at least two points and (ii) each \( F_n \) is pointlike in \( X \). Condition (c) is satisfied if \( X \) contains one point \( x \) with an Euclidean neighborhood. The "compact metric" requirement can be replaced with "compact Hausdorff" for conditions (a) and (b). There is a compact metric space for which no such mapping \( f \) exists. (Received November 8, 1968.)

Extensions of the hyperanalytic hierarchy.

Kleene (Trans. Amer. Math. Soc. 108 (1963), 139) defined the hyperanalytic sets as those recursive in \( \mathcal{E}^3 \) by analogy with the hyperarithmetic sets, which are those recursive in \( \mathcal{E}^2 \). A more consistent analogy is obtained by using a more powerful functional \( \mathcal{E}^3 \). We modify Kleene's schema \( S8 \) to allow computations of the form \( \mathcal{E}^3 (\lambda \omega \Phi (\alpha, \beta)) \equiv 1 \) whenever either \( \exists \alpha \omega \Phi (\alpha, \beta) \equiv 0 \) or \( \forall \alpha \omega \Phi (\alpha, \beta) \equiv 1 \). The main point of the analogy which is restored is that the class of domains of functions partial recursive in \( \mathcal{E}^3 \) is closed under function (number) quantification. It follows from the parenthetical part that the same sets are recursive in \( \mathcal{E}^2 \) as in \( \mathcal{E}^1 \). The main tool for defining a hierarchy on the sets recursive in \( \mathcal{E}^3 \) and establishing the other results which complete the analogy is an ordinal comparison theorem with a particularly simple and elegant proof. Alternatively, these follow from the fact that the sets recursive in \( \mathcal{E}^3 \) coincide with Moschovakis' hyperprojective sets. (Received November 8, 1968.)

On continuum theories of fluids suspension.

Ericksen (Arch, Rational Mech. Anal. 4 (1960)) has considered a continuum model for fluids with substructure which utilizes a single vector \( \eta \) to describe the substructure orientation at each point of the fluid. His development, however, did not include the effects of substructure interactions. Hand (J. Fluid Mech. 13 (1962)) employed Ericksen's formulation with a second order tensor assigned to each point of the fluid. This treatment can be used to analyze the motion of a broader class of substructure geometries but is subject to the same limitation mentioned above. Allen, DeSilva and Kline (Phys. Fluids 10 (1967)) introduced a triad of vectors \( \eta^\alpha_\alpha \) (\( \alpha = 1,2,3 \)) to describe the fluid substructure, and presented the field equations and boundary conditions for a fluid in which substructure deformation rates and substructure interactions are important. In this work the authors postulate new kinematical measures to be used in describing the mechanical response of fluids whose substructure deforms elastically, and write explicit constitutive equations. By employing the approxi-
mations utilized by Ericksen and Hand, in turn, the present theory is shown to reduce exactly to their formulations. (Received November 8, 1968.)


For real \( c \geq 0 \), let \( m(c) = \max_{|a|} \min_{|\beta|} |a - \beta| \) where the maximum is over all complex numbers \( a \) and \( \beta \) for which \( |a - \beta| = 2c \) and the minimum is over all Gaussian integers \( u \). We evaluate \( m \) for \( c = 7/8 \) using a result of Sawyer on minimal convex covering regions in the plane and the Lemma. If \( S \) is a closed point set in the plane, with boundary \( B \), such that there exists a point \( s \) in \( S \) with the property that \( |p - q| \leq 1 \) for all \( p, q \) in \( B \) for which \( \pi/2 \leq \theta \leq \pi \), where \( \theta \) is the smaller angle between the lines which connect \( p \) to \( s \) and \( q \) to \( s \), then \( S \) contains a Gaussian integer no matter how it is rotated or translated in the plane (i.e., \( S \) is a covering set). Evaluation of \( m(c) \) gives the smallest Cassini oval of a certain type which is a covering set. (Received November 8, 1968.)

663-724. BASIL GORDON, University of California, Los Angeles, California 90024, and LORNE HOUTEN, Washington State University, Pullman, Washington 99163. A new plane generalization of the partition function \( q(n) \).

Generating functions have been obtained for plane partitions, i.e., solutions to the equation \( n = \sum_{i,j} n_{i,j} \) with nonnegative integers, with the restriction that all the \( n_{i,j} \) are distinct.

(Received November 8, 1968.)

663-725. H. M. SCHAERF, McGill University, Montreal 2, Quebec, Canada. A factorization theorem for weights of coverings and the property of Souslin.

Let \( T \) be a covering of a set \( X \). Call a family \( U \) of subclasses of \( T \) a semi-uniformity if its union is a base of \( T \) and it contains, for each \( S \in U \), some \( R \) with the following property: For every \( x \in A \subseteq S \) there is \( B \subseteq R \) with \( x \in B \) such that all members of \( R \) intersecting \( B \) are subsets of \( A \).

Let \( \varphi(T) \) and \( W(T) \) be the least power of a semi-uniformity and a base of \( T \), respectively. Theorem 1. Max \( [\varphi(T), \text{cel } T] \leq W(T) \leq \varphi(T) \cdot \text{cel } T \). Hence \( W(T) = \varphi(T) \cdot \text{cel } T \) if \( W(T) \) is transfinite.

Corollary. The Souslin Property \( \text{cel } T = W(T) \) holds iff \( \varphi(T) = \text{cel } T \). Theorem 2. If \( T \) is the topology of a uniformizable space \( X \), then \( \varphi(T) \) is not greater than the least power of uniformities compatible with \( T \). (Received November 8, 1968.)

663-726. A. ZACHARIOU, Oklahoma State University, Stillwater, Oklahoma 74074. Steenrod operations in the cohomology of modules over Hopf algebras. Preliminary report.

Let \( A \) be a connected Hopf algebra over \( Z_2 \), with commutative diagonal. Let \( M \) be a graded \( A \)-module and comodule, which is (i) an "algebra" over \( A \) with commutative product, (ii) a "coalgebra" over \( A \) with commutative coproduct. Let \( B(M) \) be the bar resolution of \( M \) and \( \overline{B}(M) \) the reduced (normalised) bar construction. Then the following holds: Theorem. If \( F(M^*) = \text{Hom}_A(B(M),M) = \text{Hom}_K(\overline{B}(M),M) \) with differential \( \delta \), then there are maps \( \bigcup_i : F(M^*) \otimes F(M^*) \to F(M^*) \), \( i = 0,1,2,... \), such that \( \delta(f \bigcup_i g) = \delta f \bigcup_i g + f \bigcup_i \delta g + f \bigcup_{i-1} g + g \bigcup_{i-1} f \), for \( f, g \) in \( F(M^*) \). If \( H^{**}(M) = \text{Ext}_A^{**}(M,M) \) is the cohomology of \( M \), the maps \( \bigcup_i \) induce Steenrod operations \( S^i_q : H^{**}(M) \to H^{**}(M) \), having most of the properties of the Steenrod squares in the topological case. (Received November 8, 1968.)
Galois theory for a.-differential fields.

An a-differential field is defined as a partial differential field $K_a = (K, \delta_1, \delta_2, \delta_3)$ with a symmetric bilinear form $\Phi$ defined as $f_1 \otimes f_2 = \delta_1(f_1)\delta_1(f_2) + \delta_2(f_1)\delta_2(f_2) + \delta_3(f_1)\delta_3(f_2)$, $f_1, f_2 \in K$, $\alpha \in \mathbb{C}$. The notion of a-differential subfields (a.-differential extension field) is defined. Necessary and sufficient condition for the extension of the linear mapping $D_a = \alpha \delta_1 + \beta \delta_2 + \gamma \delta_3$ of an a-differential field $K_a$ to the linear mapping $L_a$ of an a-differential extension field $L_a$ of $K_a$ is presented. The notion of an a-differential mapping of an a-differential field is defined. The algebraic structure of the set of a-differential mappings and correspondence between the algebraic structure of this set of a-differential mappings and a-differential extension fields are presented. For a-differential field $\Omega_a$ with a-differential extension fields $K_{1a}$ and $K_{2a}$ of $\Omega_a$ the notion of relative a-differential mapping of $K_{1a}$ onto $K_{2a}$ over $\Omega_a$ is defined and the group $\mathcal{G}_a(K_a/\Omega_a)$ of relative a-automorphisms of $K_a$ over $\Omega_a$ set up. $\alpha$-abundant subsets of $\mathcal{G}_a(K_a/\Omega_a)$ is defined leading to the concept of a-normal-differential extension fields. Galois-type correspondence is analyzed and certain elementary extensions of a-differential fields are discussed. (Received November 8, 1968.)

Quasi-reflexivity and dual norms.

If $X$ is a Banach space with norm $p$, the dual norm $p^*$ is defined on $X^*$ by $p^*(f) = \sup \{ |f(x)| : p(x) \leq 1 \}$. A Banach space $X$ with norm $p$ is quasi-reflexive of order $\leq n$, where $n$ is a nonnegative integer, if and only if $X^*$ has a subspace $S$ of codimension $\leq n$ such that for every norm $q$ on $X^*$ equivalent to $p^*$, there exists a norm $t$ on $X$ equivalent to $p$ such that $t^*S = qS$. This result was proven for the case $n = 0$ by J. P. Williams [Proc. Amer. Math. Soc. 18 (1967), 163-165]. (Received November 8, 1968.)

Chain conditions in topological semilattices.

It is well known that Noetherian lattices are complete and that each element of such a lattice may be generated by a finite number of irreducible elements. Theorem. If $S$ is a topological semilattice in which each closed chain contains a least element, then $S$ is complete. Corollary. A topological lattice in which each closed chain contains a greatest element, then each element of $S$ that has finite breadth is the product of a finite number of irreducible elements. Corollary. In a compact, connected topological lattice with finite codimension, each element is the meet (join) of a finite number of meet (join) irreducibles. Theorem. Each element of a modular lattice that is the meet of a finite number of meet irreducibles has finite breadth. (Received November 8, 1968.)

On the cartesian product of groups.

Denote by $c(G)$ the connectivity of the graph $G$ and by $\pi(G)$ the number of vertices of $G$. Let $G \times H$ be the cartesian product of the graphs $G$ and $H$. The following properties are proved:
Theorem 1. \( c(G \times H) \equiv \min \{ c(G) \cdot c(H); c(H) \cdot c(G); c(G) \cdot c(H) + c(G) + c(H) \} \) and this is best possible. It is also shown that the connectivity of \( G \times H \) can be arbitrarily large with given connectivities for \( G \) and \( H \). Theorem 2. If \( G \) and \( H \) can both be covered by complete quadrangles such that every triangle in \( G \) (or \( H \)) belongs to exactly one of the quadrangles, then \( G \times H \) can also be covered by complete quadrangles with the same property. (Received November 8, 1968.)


A twist knot is a simple closed curve in \( \mathbb{E}^3 \) which is doubled, with twists, about an unknotted circle. This class includes the trefoil (1 full twist), the figure eight (1 full twist) and the stevedore's knot (2 full twists). Theorem 1. If \( C \) is a cube with a twist knotted hole, and \( M^3 \) is a simply connected 3-manifold obtained by sewing a solid torus onto \( C \), then \( M^3 \cong S^3 \). Theorem 2. If \( C \) is a polyhedral cube with a twist knotted hole in \( S^3 \) and \( f: C \rightarrow S^3 \) is a piecewise linear homeomorphism, then \( f \) can be extended to a piecewise linear homeomorphism of \( S^3 \) onto \( S^3 \). Definition. The statement that a knot \( K \) has Property \( P \) means that (1) \( K \) does not lead to a counterexample to the Poincaré conjecture, as in Theorem 1, and (2) any piecewise linear homeomorphism of a cube with a \( K \)-knotted hole into \( S^3 \) can be extended to a piecewise linear homeomorphism of \( S^3 \) onto \( S^3 \). Theorem 3. If \( K \) is a knot with Property \( P \) and \( K' \) is any knot, then \( K + K' \) has Property \( P \). (Received November 8, 1968.)


A notion of predictably enumerable set is defined and compared to the notion of elementary or predictably computable set [Trans. Amer. Math. Soc. 106 (1963), 139-173]. Definition. A set is predictably enumerable (p.e.) if it is the range of a one-one elementary function, or equivalently if it is the range of a recursive function with an associated elementary function \( f \) such that \( f(x) \) is an upper bound on the amount of storage used to produce \( x \) distinct elements in this range, or equivalently if it is the range of an elementary \( f \) and if there is an elementary \( g \) such that \( \{ f(0), \ldots, f(g(x)) \} \) contains at least \( x \) elements. Theorem 1. Every r.e. set which contains an infinite p.e. subset is p.e. Corollary. Every r.e. one-degree contains a p.e. set. Analogously with r.e. and recursive sets we have Theorem 2. Every infinite p.e. set has an infinite elementary subset; and if a set is p.e. in increasing order then it is elementary. In contrast with r.e. and recursive sets we have Theorem 3. There is an elementary set \( A \) such that neither \( A \) nor its complement \( \bar{A} \) is p.e.; and there is a set \( B \) which is not elementary while both \( B \) and \( \bar{B} \) are p.e. (Received November 8, 1968.)


A commutative ring \( R \) is a pseudo-valuation ring (P.V.R.) if for \( a, b \in R \), either a divides \( b \) or \( b \) divides \( a \). (All rings are commutative with unit.) Theorem 1. A ring \( R \) has the property that every finitely presented module is a summand of a direct sum of cyclic modules if and only if for each maximal ideal \( m \), \( R_m \) is a P.V.R. For a Noetherian ring, the hypothesis of Theorem 1 is

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equivalent to each of the following requirements: (1) for each maximal ideal \( m \), \( m/m^2 \) has R/m-dimension one, (2) \( R \) is a finite direct product of Dedekind domains and Artinian principal ideal rings. Rings not satisfying the hypothesis of Theorem 1 have finitely presented indecomposable modules requiring arbitrarily large numbers of generators, and also indecomposable modules which are not finitely generated. As an application, if \( G \) is a finite group and the p-Sylow subgroups of \( G \) are not cyclic, then \( G \) has infinite-dimensional, indecomposable representations over any field of characteristic \( p \). **Theorem 2.** A domain \( R \) has the property that every finitely presented module is a direct sum of cyclic modules if and only if every finitely generated ideal of \( R \) is principal.

(Received November 8, 1968.)

663-734. J. T. RENFROW, California Institute of Technology, Pasadena, California 91109. A study of rank 4 permutation groups.

See (D. G. Higman, J. Algebra 6 (1967), 22-42) for definitions. A rank 4 group will be a finite transitive permutation group \( G \) such that the subgroup, \( G_a \), fixing a point, \( a \), has four orbits. The degrees of the irreducible representations of a rank 4 group are determined from the intersection matrices. A method is given for constructing possible sets of intersection matrices and methods are given for constructing rank 4 groups corresponding to some of these matrices. **Theorem.** If \( G \) is a rank 4 group, two nontrivial orbits are paired if and only if two of the irreducible characters contained in the permutation representation are complex conjugates. **Theorem.** If \( G \) is a primitive rank 4 group and \( k \) is the length of the smallest orbit of \( G_a \), the length of the other two orbits are at most \( k(k - 1) \) and \( k(k - 1)^2 \), and the dihedral group on 7 points is the only group in which these values are attained. An algorithm is developed to determine all rank 4 representations of a finite group, using the intersection matrices. (Received November 8, 1968.)

663-735. WITHDRAWN.


Let \( S(n,k) = \{ n - k + 1, n - k + 2, \ldots, n \} \), where \( n, k \) are positive integers with \( n \neq 2k \). The theorem of Sylvester and Schur states that \( S(n,k) \) contains at least one integer which is divisible by a prime greater than \( k \). In this paper we show that \( S(n,k) \) contains at least \( t \) integers, each of which is divisible by a prime greater than \( k \), whenever \( n \) and \( k \) are sufficiently large. In particular, \( S(n,k) \) contains at least two integers divisible by primes greater than \( k \) whenever \( n \neq 2k \neq 12 \). The proof is similar to the elementary proof of the Sylvester-Schur theorem given by P. Erdös (A theorem of Sylvester and Schur, J. London Math. Soc. 9 (1934), 282-288). (Received November 8, 1968.)

663-737. WITHDRAWN.

663-738. WITHDRAWN.
Let $G$ be a finite transitive permutation group with subgroup $H$. Let $H$ have $r_i$ orbits of cardinality $c_i$ ($i = 1, \ldots, k$, where $k$ is the number of distinct cardinalities of orbits occurring in $H$). Let "A" be the assumption that the representations of $H$ on two distinct orbits of the same cardinality are isomorphic as permutation groups. Assuming "A" holds, let $H_i$ be the abstract group isomorphic to the representation on an orbit of cardinality $c_i$ ($i = 1, \ldots, k$). Theorem. If "A" holds, the centralizer of $H$ in $\text{Sym}(X)$ (where $X$ is the set permuted by $G$) is isomorphic to the direct product with $k$ factors whose $i$th factor is $H_i \wr \text{Sym}(r_i)$, that is, the complete monomial group of degree $r_i$ over $H_i$. Note. A similar but more complicated theorem can be proved when "A" does not hold. In the proof one obtains explicitly how the centralizer acts on $X$. This theorem is a generalization of a theorem of Burnside on the centralizer of an element of a symmetric group (see Sections 170-171 of his book).

Corollary. When $G$ is regarded as permuting itself by right multiplication, the centralizer of $H$ in $\text{Sym}(G)$ is isomorphic to $H \wr \text{Sym}(G:H)$. This is a rewording of a result in my Abstract 68T-386, these Notices 15 (1968), 553. See also Abstract 68T-37, these Notices 15 (1968), 913. (Received November 8, 1968.)

Twenty seven different physical situations arise. They fall in three categories: (A) Disks rotate in same sense, (B) one disk is stationary and (C) disks rotate in opposite senses. The formulation is reduced to four nonlinear differential equations. The truncating error in the numerical analysis is made arbitrarily small. Some of the results are listed here. When fluid ejection is present: (A) Transverse velocity $v'$ increases with distance from slower disk. The radial shear stress $T'$ on disks is outward. The radial velocity $u$ increases near slower disk and decreases near faster disk with Reynolds number $R$. The attraction $a$ between the disks increases with $R'$. (B) $T'$ is outward. (C) $v'$ is in opposite senses in the two halves of the region between the disks. $u$-maximum plane is in the middle. When fluid ejection and injection are absent: (A) $u'$ is outward in faster-half region. $u'$, $T'$ and $a'$ increase with $R'$. (B) Similar to (A). (C) There are two $u$-maximum planes and four planes where $u'$ is zero. When fluid injection is present: (A) $a'$ is negative. (B) Almost similar to (A). (C) When $R' = 10$ there are two $u$-maximum planes. The plane where $u'$ was maximum when fluid ejection was present turns into a $u$-minimum plane. $T$ attains greatest value. (Received November 8, 1968.)

Let $S$ be a semigroup with zero, $\mathfrak{q}$ a field, and $\phi_0[S]$ the contracted semigroup algebra of $S$ over $\mathfrak{q}$. Theorem. If every (left) ideal $I$ of $\phi_0[S]$ is equal to $\phi_0[I \cap S]$, then all (left) congruences of $S$ are Rees (left) congruences. The converse of the theorem is true for left ideals and left congruences, but not for ideals and congruences. Some related problems are discussed. (Received November 8, 1968.)
663-742. KENNETH WHYBURN, University of Washington, Seattle, Washington. Differentiable structures and cohomology on locally compact groups.

In this paper are two possibilities for differentiable functions on a locally compact group $G$ which may be written as an inverse limit of Lie groups. One arises by dualizing the inverse limit. The other comes from the Haar measure on $G$ and the Lie groups. Vector fields and differential forms are defined for each of these. The cohomology arising from the first of these gives a de Rham theorem for these groups. The cohomology arising from the other does not correspond to the Čech theory, but contains more information about the groups. Differential forms of infinite index are defined and cohomology groups indexed by the cardinals up to the dimension of the tangent space to the group are associated to $G$. Cohomology homomorphisms are shown to be induced by differentiable mappings of $G$. (Received November 8, 1968.)

663-743. NAND LAL and SAMUEL MERRILL, University of Rochester, Rochester, New York 14627. Some results on generalized $H^p$-spaces.

Let $A = A(X)$ be a logmodular algebra and $m$ a representing measure on $X$ such that the Gleason part of $m$ is nontrivial. For $1 \leq p \leq \infty$, let $H^p(dm) = \text{closure of } A$ in $L^p(dm)$ (weak* closure for $p = \infty$). Let $Z$ be the Wermer embedding function corresponding to $m$ so $H^2_m(dm) = Z \cdot H^2(dm)$. We prove that $\int \log|f|dm > -\infty$ for all $f \neq 0$ in $H^0(dm)$ if and only if the polynomials in $Z$ are dense in $H^0(dm)$. We show that if $A = H^0(d\theta)$ (the classical Hardy space) and $m$ is in a fiber such that the part of $m$ is nontrivial, then the polynomials in $Z$ are not dense in $H^0(dm)$. We characterize the extreme points of the unit ball in $H^\infty(dm)$ and those functions $f$ in $L^2(dm)$ which generate $L^2(dm)$ for the case of certain logmodular algebras on the torus $T^2$ where $m$ is Haar measure. Finally, we describe the isometries of one of these torus algebras together with those of the associated spaces $H^\infty(dm)$ and $H^1(dm)$. (Received November 8, 1968.)


These hyperspaces bear on the elimination of the distinction between countable and uncountable in topology. Suppose $(S, \tau)$ is a topological space and $A$ is a subset of $S$. A base at $A$ is defined as a collection $B_A$ of sets of $\tau$ including $A$ such that any member of $\tau$ including $A$ includes a member of $B_A$. A space $(S, \tau)$ is said to be bicomactly quasi-first-countable if and only if every point $P$ of any open set $D$ lies in a bicomact subset $\gamma$ of $D$ such that there is a countable base at $\gamma$. In the theorems to follow, if $(S, \tau)$ is such a space, $\Gamma$ denotes the collection of all bicomact sets at which $(S, \tau)$ has a countable base. There is a base for a topology $K$ on $\Gamma$: The family of all subcollections $\Gamma'$ of $\Gamma$ such that $\sum \Gamma'$ belongs to $\tau$ and every subset of $\sum \Gamma'$ belonging to $\Gamma$ is an element of $\Gamma'$. Theorem 1. A regular $T_0$ space $(S, \tau)$ is a continuous interior image of a paracompact Čech complete space if and only if it is a bicomactly quasi-first-countable space such that $(\Gamma, K)$ is a continuous interior image of a complete metric space. Theorem 2. A regular $T_0$ space $(S, \tau)$ is the range of a continuous interior transformation with uniformly paracompact Čech complete pre-images if and only if it is a bicomactly quasi-first-countable space such that $(\Gamma, K)$ is a continuous interior uniformly monotonically complete image of a metrizable space. (Received November 8, 1968.)
Let $P = \{X_t, \mathcal{F}_t\}$ be a strong Markov process with right-continuous paths in a metric state space $X$, and let $\{\varphi_t\}$ be a finite increasing additive functional of $P$. Extending the process if need be, let $\tau(\omega)$ be a random variable with $P_x(\tau > t|\mathcal{F}_t) = \exp(-\varphi_t)$. Force the process $P$ to jump at time $\tau$ to a new position with distribution $K(x_A, \Lambda)$, where $K(x, \Lambda)$ is a substochastic kernel, at which time it picks up a new $\tau$ and proceeds until it is again interrupted, and so forth. Let $A$ be the infinitesimal generator of the process $P$ and let $A\Lambda$ that of the new process $\tilde{P}$, let $K_f(x) = \int f(y)K(x, dy)$ and $\mathbb{S}_\lambda f(x) = \mathbb{E}_x(\int_0^\infty e^{-\lambda t} f(x_s) d\mathbb{F}_t)$ (if $[\mathbb{F}_t]$ is continuous), and assume $\mathbb{S}_\lambda f(x) < \infty$ for all $x$. Theorem 1. $\mathbb{A}_\lambda(A(\cdot - \mathbb{S}_\lambda(K - I)) f(x) \in \mathbb{B}(A).$ Theorem 2. Equality in Theorem 1, i.e. $f(x) \in \mathbb{B}(A)$ iff there exists nontrivial bounded measurable solutions of $f(x) - \mathbb{S}_\lambda(K - I)f(x) \in \mathbb{B}(A)$, occurs iff there exists no nontrivial bounded measurable solutions of $f(x) = \mathbb{E}_x(e^{-\lambda \tau} \int f(y)K(x, dy))$. These theorems also hold in a branching process context, for $[\mathbb{S}_\lambda f(x)] < c < 1$, where the operator $K$ is replaced by a certain nonlinear operator on $X$. If $P$ is a diffusion process and $[\mathbb{F}_t]$ a local time, then $\mathbb{S}_\lambda f(x)$ is a single-layer potential and $f(x) - \mathbb{S}_\lambda(K - I)f(x) \in \mathbb{B}(A)$ is a (nonlinear) jump condition on $f(x)$. These results are then applied to obtain classical solutions of some linear partial differential equations with nonlinear boundary conditions. (Received November 8, 1968.)

663-746. J. D. FULTON, Clemson University, Clemson, South Carolina 29631. Characterization and enumeration of linear classes of involutions over a finite field.

Two involutory matrices $A$ and $B$ over a field $F$ are said to generate a linear class if for every $\theta \in F$, $(1 - \theta)A + \theta B$ is involutory. The distinct linear classes of involutions over Galois field $GF(q)$, $q$ odd, are characterized to within a similarity transformation and are enumerated. The subgroup of the full linear $GL(n, q)$ generated by a linear class of $n \times n$ involutions over $GF(q)$ is characterized and common fixed vectors of involutions in a linear class are discussed. (Received November 8, 1968.)


The relationship between the solution space $\mathcal{J}$ of the linear third-order equation $L(y) = y''' + p_2y'' + p_1y' + p_0y = 0$ on $[a, \infty)$ and the solution space $\mathcal{J}^*$ of its formal adjoint equation $L^*(z) = ([z' - p_2z']' + p_1z' - p_0z = 0$ on $[a, \infty)$ if $p_i \in C[a, \infty)$, $i = 0, 1, 2$, and $a$ is some real number is considered. Definition. Let $\mathcal{J}_1[\mathcal{J}^*]$ denote a subspace of $\mathcal{J}$ of $\mathcal{J}^*$. The subspace $\mathcal{J}_1[\mathcal{J}^*]$ is said to be (i) nonoscillatory if no nontrivial solution in $\mathcal{J}_1[\mathcal{J}^*]$ has infinitely many zeros; (ii) weakly oscillatory if $\mathcal{J}_1[\mathcal{J}^*]$ contains both a nontrivial solution with infinitely many zeros and a solution with at most a finite number of zeros; (iii) strongly oscillatory if every solution in $\mathcal{J}_1[\mathcal{J}^*]$ has infinitely many zeros. The following results are obtained: (i) If $\mathcal{J}[\mathcal{J}^*]$ is weakly oscillatory, then $\mathcal{J}^* [\mathcal{J}]$ is oscillatory. (ii) If both $\mathcal{J}$ and $\mathcal{J}^*$ are not strongly oscillatory, then $\mathcal{J}[\mathcal{J}^*]$ has a decomposition as a direct sum of nonoscillatory or strongly oscillatory subspaces such that the zeros of linearly independent solutions in a strongly oscillatory subspace in the decomposition separate each
other. An example is given where $J$ is nonoscillatory and $\mathcal{J}$ is strongly oscillatory. (Received November 8, 1968.)

663-748. S. B. HIGGINS, Texas Christian University, Fort Worth, Texas 76129. Some generalizations of paracompactness. Preliminary report.

An indexed point set $\{p_a\}_{a \in A}$ has an accumulation point $p$ if for every neighborhood $U$ of $p$ there exist an infinite number of $a \in A$ such that $p_a \in U$. Let $\mathcal{U} = \{U_a | a \in A\}$ be a family of point sets. $\mathcal{U}$ has property $Q^*_0$ if no indexed point set $\{p_a\}_{a \in A}$, $p_a \in U_a$, has an accumulation point. $U$ has property $Q^*_1$ provided for any subcollection $\{U_{a^*}\}_{a^* \in A^*}$, with $A^* \subset A$, if there is an indexed point set $\{q_a\}_{a \in A^*}$, $q_a \in U_{a^*}$, having an accumulation point then there exists a compact set $B$ such that if $\{q_{a^*}\}_{a^* \in A^*}$, $q_{a^*} \in U_{a^*}$, is any indexed point set then $\{q_{a^*}\}_{a^* \in A^*}$ has an accumulation point $p$ and $p \in B$. A space $X$ is called a $Q^*_0(Q^*_1)$-space if every open covering of $X$ has an open $Q^*_0(Q^*_1)$-refinement. Theorem 1. Paracompactness implies $Q^*_0$; $Q^*_0$ implies $Q^*_1$; $Q^*_1$ implies metacompactness. Theorem 2. Let $f: X \rightarrow Y$ be a perfect mapping. $X$ is a $Q^*_0(Q^*_1)$-space if and only if $Y$ is a $Q^*_0(Q^*_1)$-space. Corollary. The product of a compact space and a $Q^*_0(Q^*_1)$-space is a $Q^*_0(Q^*_1)$-space. Theorem 3. In a $q$-space (see E. Michael, A note on closed maps and compact sets, Israel J. Math. 2 (1964), 173-176) the concepts of paracompactness, $Q^*_0$ and $Q^*_1$ are equivalent. Corollary. A developable space is metrizable if and only if it is a $Q^*_0(Q^*_1)$-space. (Received November 8, 1968.)

663-749. ERIK BALSLEV, State University of New York at Buffalo, Amherst, New York 14226. Discreteness of the spectrum of the Schrödinger operator.

Let $\mathcal{L}$ be the formal differential operator defined by $\mathcal{L} = -\Delta + q + v$, where $q$ and $v$ are real-valued functions on $\mathbb{R}^n$ satisfying (i) There exists $\alpha > 0$ such that $\text{ess sup}_{x \in \mathbb{R}^n} |q(t) + v(t)|^2 |t - x|^{4n-\alpha} dt < \infty$, $R > 0$ ($S_{x,R}$ denotes the set $\{t \in \mathbb{R}^n | |t - x| \leq R\}$). (ii) $q(x) \equiv 1$ for $x \in \mathbb{R}^n$. (iii) $q(x) \rightarrow +\infty$ as $|x| \rightarrow \infty$. Let $r$ be a real-valued function on $\mathbb{R}^n$ such that (iv) $0 < r(x) \equiv 1$, $x \in \mathbb{R}^n$. (v) $\text{inf}_{x \in \mathbb{R}^n} |r(x)| \equiv 1$ for $R > 0$. (vi) $r(s) \equiv s$ for $s \in S_0$ ($S_0$ is the unit sphere in $\mathbb{R}^n$). (vii) $\text{Sup}_{t \in \mathbb{R}^n} |r(t) - r(t)|/|r(t)| \rightarrow 0$ as $|x| \rightarrow \infty$. Let $-1 < \beta_i < 1$, $i = 1, 2$. Suppose that $\mathcal{V}_i$ satisfy (viii) $\text{ess sup}_{x \in \mathbb{R}^n} |x| \equiv \pi \mathcal{V}_i(1) < \infty$ for $R > 0$, $i = 1, 2$. Suppose that $\mathcal{V}_i$ satisfy (ix) $\lim_{R \rightarrow \infty} \text{ess sup}_{x \in \mathbb{R}^n} |x| \equiv \pi \mathcal{V}_i(1) < \infty$ for $R > 0$, $i = 1, 2$. Then the unique self-adjoint operator $L$ associated with $\mathcal{L}$, such that $C_0^\infty(\mathbb{R}^n) \subset D(L)$, is bounded below and has discrete sequences. (Received November 8, 1968.)


Determine the conditions for a subring $R$ of $C(X)$ so that there exists a topological space $Y$ and $R$ is the isomorphic image of $C(Y)$. (Received November 8, 1968.)


Let $G$ denote a compact topological group and let $G^\wedge$ be a complete set of mutually inequivalent,
irreducible, strongly continuous, unitary representations of G. Convolution multiplications are defined on $t^1$ and $t^2$ type spaces of operator valued functions on $G^\Lambda$. Through the use of a Parseval formula and a Bochner inversion theorem, relationships of these convolutions with the Fourier transform on $L^1(G)$ and the inverse transform on $t^1(G^\Lambda)$ are studied. (Received November 8, 1968.)

663-752. HERMAN DINGES and H. ROST, Statistical Laboratory, Catholic University of America, Washington, D. C. 20017. An order for conic distributions and the ergodic theorem.

Definition. A conic distribution $\mu$ is a monotone and linear functional on the cone $H$ of non-negative sublinear functions $h(\xi^0, \xi^1, \ldots)$ on $R^\infty$ with $\mu(\xi^1) < \infty$. Proposition 1. If $(\Omega, \mathcal{F}, m)$ is a probability space and $x^0, x^1, \ldots$ are $m$-summable then $\mu(h) = \int h(x^0, x^1, \ldots) \, dm$, $h \in H$, defines a conic distribution. Every conic distribution can be obtained that way. Proposition 2. If $T$ is a positive contraction on $L^1(\Omega, \mathcal{F}, m)$ and $x^0, x^1, x^2, \ldots$ are summable functions, then $\int h(x^0, x^1, \ldots) \, dm \preceq \int h(Tx^0, Tx^1, \ldots) \, dm$ for $h \in H$. Definition. A conic distribution $\mu$ is called more concentrated than a conic distribution $\nu$ if $\mu(h) \preceq \nu(h)$ for all $h \in H$. Proposition 3. If $\{\mu_t\}$ is a measurable family of conic distributions when $t$ varies in the probability spaces $(T, \mathcal{F}, \rho)$ and if $\int \mu_t \, d\rho(t)$ is less concentrated than the conic distribution $\nu$, then there exists a measurable family $\{\nu_t\}$ of conic distributions such that $\nu_t$ is less concentrated than $\mu_t$ and $\int \nu_t \, d\rho(t) = \nu$. Definition. A sequence of functions $x^0, x^1, \ldots \in L^1(\Omega, \mathcal{F}, m)$ is called monotone in distribution if $\int h(x^0, x^1, \ldots) \, dm \preceq \int h(x^1, x^2, \ldots) \, dm$ for $h \in H$. Proposition 4. For monotone sequences the maximal ergodic lemma holds: $\int_A x^0 \, dm \preceq 0$ if $A = \{x^0 + x^1 + \ldots + x^n > 0 \text{ for some } n\}$. The concept of monotone sequences can be generalized to pairs of sequences $\{x^i, p^i\}$ with $p^i \equiv 0$. (Received November 8, 1968.)


In commutative Moufang loops the third power of any element plays a special role; the mapping $x \to x^3$ being a centralizing endomorphism. Our objective is, for any given $n \equiv 3$, to find a class of loops with a centralizing endomorphism $x \to x^3$. We define as a $M_k$-loop (or $M$-loop of class $k$) a commutative loop with a generalized Moufang Identity: $(xy)(zx^k) = [x(yz)]x^k$. It can be shown that $M_k$ is a power-associative, inverse property loop. The Moufang theorem holds in any $M_k$. The mapping $x \to x^n, n = k + 2$ is a centralizing endomorphism. For any given $n \equiv 3$, to construct an example of a $M_k$-loop with $m$ generators, consider a Burnside group $B$ with $m$ generators and the relation $x^n = 1$. Define a new operation ($\ast$) on $G$: $x \ast y = x^{k+1}yx^{-k}, k = n - 2$. $(G, \ast)$ is a $M_k$-loop and gives rise to an ascending chain of $M_k$-loops of higher order. (Received November 8, 1968.)


A quadratic system in the plane can be written in the form $\dot{x} = b + Ax + f_2(x), x \in E^2$, where $b \in E^2, A = [a_{ij}], i, j = 1,2$, and the components of $f_2(x)$ are homogeneous quadratic forms in $(x_1, x_2)$. The following theorem characterizes all bounded quadratic systems in the plane with $f_2(x) \not= 0$.

Theorem. A quadratic system of the form (1) with $f_2(x) \not= 0$ has all of its trajectories bounded for $t \not= 0$ if and only if there exists a linear transformation which reduces it to one of the following
systems: (1) \( \dot{x} = Ax + (0, x, x) \) with \( a_{12} = 0, a_{11} < 0 \) and \( a_{22} \neq 0 \). (2) \( \dot{x} = Ax + (x_2^2, 0) \) with \( a_{21} = 0, a_{11} \neq 0, a_{22} = 0 \) and either (i) \( a_{11} < 0 \) or (ii) \( a_{11} = 0 \) and \( a_{21} = 0 \) or (iii) \( a_{11} = 0, a_{12} + a_{21} = 0 \) and \( a_{21} + a_{22} \neq 0 \).

Further, all possible phase portraits for bounded quadratic systems in the plane have been determined by studying the critical points of the above systems on the Poincaré sphere. (Received November 8, 1968.)

663-755. MAKOTO ITOH, North Carolina State University, Raleigh, North Carolina 27607.

On quadrerionic bicomplex analytic and wave-analytic (pseudo-analytic) functions.

Let \( w_1(z_1, z_2) \) and \( w_2(\overline{z}_1, \overline{z}_2) \) be two complex functions of two complex variables \( z_1 = x_1 + iy_1 \) and \( z_2 = x_2 + iy_2 \), \( w_1 \) being analytic of \( z_1 \in D_1 \) and \( z_2 \in D_2 \) while \( w_2 \) is conjugate analytic of \( z_1 \) and \( z_2 \) (i.e. \( \overline{w} \) is analytic of \( z_1 \) and \( z_2 \)). If \( w_1 \) and \( w_2 \) satisfy the simultaneous partial differential equations in \( D = D_1 \times D_2 \), then (i) \( \partial_{z_1} w_1 - \partial_{\overline{z}_2} w_2 = a_1 w_1 - \overline{a}_2 w_2 \) and (ii) \( \partial_{z_2} w_1 + \partial_{\overline{z}_1} w_2 = \beta_1 w_1 + \overline{\beta}_2 w_2 \) the quaternionic bicomplex function \( w = w_1 + jw_2 \) (\( j \) denotes Hamilton's quaternion unit) will be called bicomplex wave-analytic (or pseudo-analytic) in \( D \). In case \( a_1 = a_2 = \beta_1 = \beta_2 = 0 \), the above equations (I) become the bicomplex Cauchy-Riemann equations and such function \( w \) may be called a "bicomplex analytic" function. We shall derive some fundamental properties of such bicomplex analytic and wave-analytic functions. (Received November 8, 1968.)


Let \( V \) be a vector space over a field, and let \( S \) be an arbitrary nonempty set. Let \( \rho: S \times S \to V \) be a mapping such that the following conditions are satisfied: (1) \( \rho(x, y) = 0 \) implies \( x = y \), (2) \( \rho(x, y) + \rho(y, z) = \rho(x, z) \). The structure \([S, V, \rho] \) is called a semi-affine geometry on \( S \). The elements of \( S \) are called points of this geometry. The lines and parallelism are defined, and some results are obtained. (Received November 8, 1968.)


Meromorphic functions of elements of a commutative Banach algebra.

As an application of the extension of a commutative Banach algebra \( A \) to its Reimann sphere \( A^\infty \), we show how to define each \( h(p) \) where \( p \) is any element of \( A^\infty \) and \( h \) is a meromorphic function defined on a neighborhood of the spectrum of \( p \) (thus rational functions of \( p \) are always defined). A spectral mapping theorem is immediate. For fixed \( p \), the mapping \( h \to h(p) \) is a "pseudohomomorphism". If \( p \) is in \( A \) and \( h \) is meromorphic on the complex plane \( C \), \( h(p) \) is given by a Mittag-Leffler expansion. (Received November 8, 1968.)


Theorem. Let \( f \) be a holomorphic mapping of a bounded connected open subset \( U \) of a Banach space into itself. Suppose that the image of \( f \) stays at least \( \epsilon \) away from the boundary of \( U \) for some
\( \epsilon > 0 \). Then \( f \) has a unique fixed point. The theorem was previously known when \( U \) is finite-dimensional (M. Hervé, *Several complex variables*, local theory, Oxford University Press and Tata Institute of Research, 1963.) The theorem improves on the usual results of Leray-Schauder degree theory since (1) \( f(U) \) is not required to have compact closure, (2) \( U \) may have nontrivial topology and (3) the fixed point must be unique. For the proof, the point evaluation map gives an embedding of \( U \) as an analytic submanifold of the dual of the Banach space of bounded holomorphic functions on \( U \), and this embedding induces a Finsler metric on \( U \) in terms of which \( f \) is a contraction mapping. (Received November 8, 1968.)

663-759. ARISTIDE DELEANU, Syracuse University, Syracuse, New York 13210. *On certain maps of compact spaces inducing isomorphisms for cohomology.*

**Theorem.** Let \( X \) and \( Y \) be two compact spaces and let \( f: X \to Y \) be a map of \( X \) onto \( Y \) such that the following two conditions are satisfied: (1) For every neighborhood \( V \) of the diagonal in \( X \times X \) there exist in \( Y \) only a finite number of points \( y \) such that \( f^{-1}(y) \times f^{-1}(y) \subseteq V \). (2) There exists an integer \( k \geq -1 \) such that \( \tilde{H}^k(f^{-1}(y)) = \tilde{H}^{k+1}(f^{-1}(y)) = 0 \) for every \( y \in Y \), where Čech cohomology is used. Then the map \( f \) induces an isomorphism \( f^*: H^{k+1}(Y) \cong H^{k+1}(X) \). This represents a generalization of a result due to K. Borsuk (Bull. Acad. Polonaise Sci. 11 (1973), 499-503). (Received November 8, 1968.)


In treating elliptic and hyperbolic equations, one naturally uses local properties of the solutions. In treating equations of more general types, it is advantageous to disregard local properties and to characterize admissible boundary conditions without reference to the type of the equation in the interior of the region. Reduction of differential equation problems to problems of pseudodifferential equations implies a further nonlocalization. (Received November 27, 1968.)
During the interval from September 19 through November 1, 1968, the papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these Notices. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

**Algebra and Theory of Numbers**

69T-A1. RUDOLF WILLE, Mathematisches Institut der Universität Bonn, Bonn, West Germany.


Problem 60 in G. Birkhoff, Lattice theory, 3rd ed., Amer. Math. Soc., Providence, R. I., 1967, is solved by the following. **Theorem.** The cosets of normal subgroups of a group G form a semimodular lattice if and only if G is simple or elementary abelian. **Proof.** Suppose that the cosets of normal subgroups of a nonsimple group G form a semimodular lattice. Then the set of all minimal normal subgroups is a partition of G. By a theorem of P. Kontorowitsch (Mat. Sb. 12 (1943), 56-70), it follows that G is abelian. Therefore each element unequal to the identity generates a minimal subgroup in G, and thus G is elementary abelian. The converse is easily verified. (Received September 26, 1968.) (Author introduced by Dr. K. W. Jänich.)

69T-A2. STEVEN FEIGELSTOCK, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. On the vanishing of $G \otimes H$.

Necessary and sufficient conditions are given for the tensor product of two abelian groups to vanish. (Received September 19, 1968.)


Some known results about the system normalizers of A-groups are generalized to the $\mathfrak{Z}$-normalizers introduced by Carter and Hawkes (J. Algebra 5 (1967), 175-202). **Theorem 1.** A-groups have pronomal $\mathfrak{Z}$-normalizers, and consequently if G is an A-group and if D and $D_1$ are $\mathfrak{Z}$-normalizers of G lying in the same $\mathfrak{Z}$-covering subgroup $E$ of G, then D and $D_1$ are conjugate in $E$.

**Theorem 2.** If G is an A-group, then the $\mathfrak{Z}$-normalizers of G complement the $\mathfrak{Z}$-residual of G.

**Theorem 3.** If G is an A-group, then the $\mathfrak{Z}$-normalizers of G are characterized by their cover-avoidance property. (Received September 20, 1968.)

69T-A4. M. D. LARSEN and AHMAD MIRBAGHERI, University of Nebraska, Lincoln, Nebraska 68508. On a conjecture of J. C. Robson.

The following theorem is a partial answer to a conjecture of J. C. Robson [J. Algebra 7 (1967), 140-143]. Let $R$ be a semiprime right Noetherian ring the center of whose right quotient ring has
finite characteristic and suppose (i) \( I(a) = 0 \) implies \( r(a) = 0 \) and (ii) \( I(a) \neq 0, \, I(b) \neq 0 \) implies \( I(a) \cap I(b) \neq 0 \) for all \( a, b \in R \). [\( I(a) \) is the left annihilator of \( a \).] Then \( R \) has a unity element.

(Received September 25, 1968.)

69T-A5. SAAD MOHAMED, University of Delhi, Delhi-7, India. \( q \)-rings with chain conditions.

\( R \) is said to be a right \( q \)-ring if it has unity and each right ideal is quasi-injective (cf. Abstract 68T-A12 of Jain, Mohamed and Singh, these Noticia 15 (1968), 784). Among other results the following is proved: A right (or left) artinian (or noetherian) ring \( R \) is a right \( q \)-ring if and only if \( R \) is a left \( q \)-ring. (Received October 9, 1968.) (Author introduced by Dean R. S. Varma.)


In the following the identity \( x(xy) + (yx)x = 2(xy)x \) will be denoted by (I). A nilpotent element \( w \) of an algebra \( A \) is called a commutator nilpotent if there are elements \( u, \, v \) in \( A \) such that \( uv - vu = ml + w \) where \( m \) is in the base field. *Theorem.* If \( R \) is a ring of characteristic different from two satisfying (I), then \( R^+ \) powerassociative implies that \( R \) is powerassociative. *Theorem.* There are no nodal algebras \( A \) satisfying (I) over an algebraically closed field \( F \) of characteristic zero for which any of the following conditions hold: (1) \( A \) is Lie-admissible. (2) \( tr. \, R_n = 0 \) for all commutator nilpotents \( n \) of \( A \). (3) \( A \) is antiflexible. (4) The nilpotent elements of \( A \) are of nilindex two. (5) There is a nilpotent element of \( A \) whose nilindex is greater or equal to \( n - 1 \) where \( dim. \, A = n \). In addition, if \( A \) is a simple nodal algebra satisfying (I) over the above field \( F \) then associators of the form \( (x,y,z) \) cannot all be nilpotent. (Received October 2, 1968.) (Author introduced by Dr. E. I. Lezak.)


Let \( A \) be either a Jordan or alternative algebra with 1 of dimension \( n \) over a field \( F \), \( \text{Char.} \, F \neq 2 \).

Define \( f^1(x_1,x_2,x_3) = (1/2)(x_1 x_2 + x_2 x_1 x_3 + x_3 x_1 x_2) \), \( f^n(x_1,...,x_{2n+1}) = f^1(f^{n-1}(x_1,...,x_{2n-1}), x_{2n}, x_{2n+1}) \). A is \( f \)-nilpotent iff for some \( m \), \( f^m = 0 \) on \( A \). For \( a, b \in A \), define \( xS(a,b) = f^1(x,a,b) \). For a subalgebra \( R \) of \( A \), let \( L_A(R) \) denote the Lie algebra generated by \( S(a,b), \, a, \, b \in R \). *Theorem 1.* Let \( R \) be a \( f \)-nilpotent subalgebra of \( A \) containing 1. Then \( L_A(R) \) is nilpotent and the Fitting null component of \( A \) relative to \( L_A(R) \) is a subalgebra containing \( R \). Let \( B_a = (x \in A : xS(a,a)^n = 0) \). \( B_a \) is minimal Engel iff \( B_b \subset B_a \) implies \( B_b = B_a \). A subalgebra \( H \) of \( A \) is a Cartan subalgebra iff \( H \) contains 1, is \( f \)-nilpotent, and coincides with the Fitting null component of \( A \) relative to \( L_A(H) \). *Theorem 2.* Let \( F \) have at least \( 2^n \) elements. Then \( H \) is a Cartan subalgebra iff \( H \) is minimal Engel. *Theorem 3.* If \( F \) is algebraically closed of char. 0, then for two Cartan subalgebras \( H_1, \, H_2 \) of \( A \) there is an inner automorphism \( s \) such that \( H_1^s = H_2 \). In the Jordan case, with the exception of the "only if" part of Theorem 2, these results were obtained earlier by Jacobson (Nagoya Math. J., 1966). The results for alternative algebras are obtained using the methods of Jacobson and Barnes (Math. Z., 1967). (Received October 14, 1967.)
Let D be an integral domain with identity having quotient field K. By an averring of D we mean a domain $D_0$ such that $D \subseteq D_0 \subseteq K$. If there exists a family $\{V_\alpha\}$ of valuation rings which are overrings of D such that $D = \cap \alpha V_\alpha$ and each nonzero element of D is a nonunit in only finitely many of the $V_\alpha$'s, then D is said to be a domain of finite character. If, in addition, each $V_\alpha$ has rank one, then we say that D is a domain of finite real character. If D is a domain of finite real character in which each $V_\alpha = D_\mathfrak{p}_\alpha$ for some prime ideal $\mathfrak{p}_\alpha$ of D, then D is called a generalized Krull domain.

Theorem 1. If D is a domain of finite character, the following are equivalent: (a) Every averring of D is of finite character. (b) D is a Prufer domain. Theorem 2. If D is a domain of finite real character, the following are equivalent: (i) D is a Prufer domain. (ii) Each nontrivial valuation ring containing D has rank one. (iii) Each averring of D is of finite real character. Theorem 3. If P is a nonzero prime ideal of a generalized Krull domain, then $P$ contains a minimal prime ideal.

(Received October 17, 1968.)

69T-A9. SAMI BERAHA, Ball State University, Muncie, Indiana 47306. A rule of four reciprocities and an extension.

I. The rule is a simplified version of an earlier result. Given two matrices M and N, one the inverse of the other, such that each vector of M has $p$ elements equal to A and $q$ elements equal to B and each vector of N has $r$ elements equal to A and $s$ elements equal to B, $p + q = r + s = n$. Four reciprocities link these arguments: (1) $MN = 1$, (2) $pr = qs \equiv 1 \mod n$, (3) $(A - B)(a - b) = 1$, (4) $(pA + qB)(ra + sb) = 1$. Thus there exist solutions iff $p, q, r, \text{ and } s$ are not divisors of zero.

II. The extension is to let $p, q, r, \text{ and } s$ tend to infinity while the ratios of these four arguments remain finite. These ratios then become quadratic irrationals in consequence of a theorem by Galois on continued fractions. (Received October 16, 1968.)

69T-A10. R. M. RAPHAEL, McGill University, Montréal, Québec, Canada. A weak algebraic closure for commutative regular rings with 1.

We continue as in Abstract 68T-H29, these Notices 15 (1968), 945. Definition. $S \supseteq R$ is a weak algebraic extension of $R$ if all between rings of $R$ and $S$ are essential over $R$. Proposition 1. An algebraic extension of a regular ring is weakly algebraic. A weak algebraic extension is algebraic iff all between rings are regular. Proposition 2. Let $R$ be regular; then t.f.a.e. (1) every weak algebraic embedding with domain $R$ is onto, (2) $R$ is self-injective and algebraically closed. Definition. If $R$ is regular and it satisfies the conditions of Proposition 2, $R$ is called weakly algebraically closed. Theorem. Let $R$ be regular. Then $R$ can be embedded weakly algebraically into a weakly algebraically closed ring; this 'closure' is unique up to isomorphism, and it contains a copy over $R$, of every weak algebraic extension of $R$. Proposition 3. The weak algebraic closure of a regular ring is the complete ring of quotients of its algebraic closure. Proposition 4. "Weak algebraic"-ness is transitive. Proposition 5. The weak algebraic closure commutes with all direct products. Examples show that the result of Proposition 5 does not hold for the algebraic closure, and that the two closures can differ even in the case of a regular ring which is self-injective. (Received October 21, 1968.)
A ring is left Rickart if every left annihilator of a one-element set is generated by an idempotent. Clearly Baer rings are Rickart. The author determines conditions for a subring $R$ of the ring $L(V)$ of all linear transformations on a vector space $V$ to be left Rickart (Rickart or Baer). Principally two types of conditions are considered in this connection: conditions related to the structure of the lattice of invariant subspaces; conditions related to the form of the matrices or their graphs in terms of a suitable basis. (Received October 22, 1968.)

A set $M$ of morphisms is said to freely generate a small category $G$ if every morphism of $G$ can be written uniquely in reduced form as a composition of morphisms in $M$ and inverses of isomorphisms in $M$. Let $G$ be freely generated by $M$, and let $\equiv$ be the equivalence relation generated by a set $E$ of ordered pairs of morphisms of $G$. Let $D$ be an object of the functor category $A_11$ where $A$ is an abelian category with coproducts indexed by the morphisms of $G$. Then there is an exact sequence $0 \to S_p(D(p)) \to \bigoplus_{m \in M} S_m(D(dm)) \to \bigoplus_{p \in V} S_p(D(p)) \to D \to 0$ in $A_11$, where $V$ is the set of objects of $\equiv$, $S_p$ is the left adjoint of the $p$th evaluation functor $A_11 \to A$, and $dm$ and $rm$ denote respectively the domain and range of $m$. Theorem. If $G$ is freely generated and possesses either a morphism which is not an isomorphism or an endomorphism which is not an identity, then $\text{gl.dim. } A = 1 + \text{gl.dim. } A$ for any nontrivial abelian category $A$ with exact coproducts (or products) indexed by the morphisms of $G$. This theorem unites several previously unrelated results, for example, $\text{gl.dim. } R(G) = 1 + \text{gl.dim. } R = 1 + \text{gl.dim. } T_n(R)$ where $G$ is a free group or a free monoid and $T_n(R)$ is the ring of triangular matrices with entries in $R$ ($n > 1$). (Received October 15, 1968.)

We answer, in the affirmative, the following long-standing question of Markov: Does every infinite discrete group $G$ admit a group topology which is not discrete? (See Problem 4 on p. 269 of [Markov, On free topological groups, Amer. Math. Soc. Transl. 8 (1962), 195-272].) (Received October 25, 1968.)

In [Colloq. Math. 17 (1967), 225-234] G. Grätzer asked whether, for $k$ a variety such that every algebra in $k$ has a distinguished one element subalgebra, the property that a set $\{a_1, \ldots, a_n\}$ in an algebra $A \in k$ is weakly independent iff $\{a_1, \ldots, a_n\} = \{a_1\} \times \cdots \times \{a_n\}$ essentially characterizes $k$ as the class of modules over a ring. It is shown that above property makes $k$ equivalent to the class of modules over a semiring with unit. Further conditions are given which insures that $k$ is equivalent to the class of modules over a ring. (Received October 25, 1968.)
Let $K$ be a class of similar algebras $\mathbb{U} = \langle A, f_\beta : \beta < m \rangle$, let $L(\mathbb{U})$ (resp. $\Theta(\mathbb{U})$) be the lattice of subalgebras (resp. congruence relations) of $\mathbb{U}$, let $\Sigma(K) = \{ L(\mathbb{U}) : \mathbb{U} \in K \}$, $\Theta(K) = \{ \Theta(\mathbb{U}) : \mathbb{U} \in K \}$.

For an infinite cardinal $n$, $\mathbb{U} \in K$ is $(n, K)$-universal if $|A| = n$ and every $\mathbb{U} \in K$ with $|B| \leq n$ is isomorphic to a subalgebra of $\mathbb{U}$. $\mathbb{U}$ is $\omega$-homogeneous if every isomorphism of a finitely generated subalgebra of $\mathbb{U}$ into $\mathbb{U}$ can be extended to an automorphism of $\mathbb{U}$. For any cardinal $m$, $K_m$ is the class of algebras $\langle A, f_\beta : \beta < m \rangle$ where each $f_\beta$ is unary. (1) If $K$ is a nontrivial equational class, $K \subseteq K_1$, then there is a unique $(\mathbb{N}_0, K)$-universal, $\omega$-homogeneous algebra. Further, there are $2^{\aleph_0}$ nonisomorphic $(\mathbb{N}_0, K_1)$-universal algebras; there is no $(\mathbb{N}_0, K_m)$-universal algebra for $m \geq 2$. (2) For all $n \geq \aleph_0$, there exists a $(2^n, \Theta(K_1))$-universal lattice. (3) If $\mathbb{U} \in K_1$ and $|A| > \aleph_0$, then $\mathbb{U}$ has $2^{|A|}$ subalgebras. (4) (GCH) Let $K$ be a class of algebras such that there exists an $(n, K)$-universal algebra and each uncountable algebra $\mathbb{U} \in K$ has $2^{|A|}$ subalgebras. Then there exists a $(2^n, \Sigma(K_1))$-universal lattice. For $n \geq \aleph_0$, there exists a $(2^n, \Sigma(K_1))$-universal lattice. (Received October 31, 1968.)

**69T-A16.** A. A. MULLIN, USATACOM, Warren, Michigan 48089. **Representations as a sum of a prime number of summands.**

**Definition.** For real $x \geq 2$, put $[x]_p$ equal to the greatest prime number not exceeding $x$. E.g., by a well-known result of Bertrand-Tchebechev, $[2x]_p > [x]_p$ for every real $x \geq 2$. **Lemma.** The number $R(n)$ of representations of a natural number $n > 5$ as a sum of a prime number of natural numbers satisfies the relation: $R(n) > (1/2) \cdot \Pi([n]_p) \cdot (1 + \Pi([n+1]_p))$, where $\Pi(x)$ is the number of prime numbers not exceeding the positive real number $x$. **Related Problems.** Find usefully simple estimates (1) for the growth of the composite function $\Pi([x]_p)$ and (2) for the growth of the summatory function $\Sigma_{m \leq x} \Pi(m)$. (Received October 31, 1968.)

**69T-A17.** L. M. HERMAN, Plymouth State College, Phymouth, New Hampshire 03264. **A Loomis *-semigroup satisfies the polar decomposition axiom.**

D. J. Foulis (Canad. J. Math. 17 (1965), 40-51) defines a Loomis *-semigroup as an involution semigroup with 0, satisfying the *-cancellation axiom and axiom EP of Kaplansky, having no nonclosed projections, and in which orthogonal families of partially unitary elements are summable. In showing that the projection lattice of such a semigroup is an orthomodular geometry, Foulis proves that the left and right projections of any element are *-equivalent. This proof can be modified to show that, in fact, the polar decomposition axiom holds: Let $u$ be the partially unitary element in his proof effecting the *-equivalence of $a^*"$and $a"$. Put $b = au^*$. One may check that $a = bu$, $b$ "double commutes" with $a^*$, $b^2 = aa^*$, and $b = b^*$. Moreover, a technique of D. M. Topping (Pacific J. Math. 20 (1967), 317-325) may now be used to show that the projection lattice of a Loomis *-ring is 0-symmetric and hence also semimodular. (Received October 30, 1968.)
Analysis


The following theorem is proved: Theorem. Let the differential equation

\[(r_{n-1}(t)...(r_2(t)(r_1(t)x')')')' + P(t)G(x) = Q(t,x,x',...,x^{(n-1)}),\]

be such that

(i) \(r_i: \mathbb{R}^+ = (0, \infty), i = 1, 2, \ldots, n - 1; P: \mathbb{R}^+ \to \mathbb{R} = (-\infty, +\infty), \) continuous, and such that for every \(\lambda > 0, \int_0^\infty (\lambda P^+(t) + P^-(t))dt = +\infty\) where \(P^+ = \max\{P, 0\}, P^- = \min\{P, 0\};\)

(ii) \(G: \mathbb{R} \to \mathbb{R}, \) continuous, and \(xG(x) > 0\) for every \(x \neq 0;\)

(iii) \(Q: \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}, \) continuous, and such that there exists a continuous function \(Q_0 : \mathbb{R} \to \mathbb{R}^+, \) with \(|Q(t,i)| \leq Q_0(t)\) for every \((t,i) \in \mathbb{R} \times \mathbb{R}^n,\) and moreover, \(\int_0^\infty Q_0(t)dt < +\infty.\) Then every bounded solution of (\(*\)), which is valid for all large \(t,\) is oscillatory or such that

\[\lim_{t \to \infty} |x(t)| = 0.\]

(Received September 19, 1968.) (Author introduced by Professor Franz Schnitzer.)


Suppose \(\Omega\) is a region in the plane bounded by simple closed curves \(C_1, \ldots, C_n.\) Call a point \(\zeta \in \partial \Omega\) wild in case every arc interval about \(\zeta\) is nonrectifiable, and let \(W\) denote the set of wild points. Let \(\mu\) denote \(p\)-dimensional Hausdorff measure, and \(A(\Omega)\) all functions analytic in \(\Omega\) and continuous on \(\partial \Omega,\) with \(h(\infty) = 0\) in case \(\Omega\) is unbounded. Theorem. Suppose \(\mu\) is a complex Borel measure on \(\partial \Omega\) such that \(\int_{\partial \Omega} h(\zeta)d\mu(\zeta) = 0\) for all \(h \in A(\Omega).\) Then \(\mu < \lambda_1\), planar linear measure, an \(\partial \Omega - W,\) and the support \(\mu \supset \partial \Omega - W, \lambda_1/2(W) = 0\) is sufficient to insure \(\mu = 0\) on \(W.\) As a corollary to the above and a theorem of Bishop (A generalized Rudin-Carleson theorem, Proc. Amer. Math. Soc. 13 (1962), 140-143) we have Corollary. Let \(K\) be a closed subset of \(\partial \Omega\) of linear measure \(0,\) and let \(f(\zeta)\) be continuous on \(K.\) If \(\lambda_1/2(W) = 0,\) there exists an \(F \in A(\Omega)\) such that \(F|K = f.\) The above always holds if \(K \subset \partial \Omega - W,\) in the theorem, the density function of \(|\mu|\) with respect to \(\lambda_1\) may be related to the classes \(H^p(\Omega)\) or \(N(\Omega)\) as defined by Rudin (Analytic functions of class \(H^p,\) Trans Amer. Math. Soc. 78 (1955), 46-66) in some special cases. (Received September 26, 1968.)

69T-B3. J. W. ROBBIN, University of Wisconsin, Madison, Wisconsin 53706. The stable manifold of a semihyperbolic fixed point of a map. Preliminary report.

A uniformly \(C^k\) map is a \(C^k\) map \(f\) such that \(D^k f\) is uniformly continuous. If \(\sigma, X, Y, Y \in C^k\) Banach manifolds, a map \(\rho: \sigma \to C^k(X, Y)\) is a uniformly \(C^k\) representation iff the evaluation map \(\sigma \times X \to Y\) is uniformly \(C^k.\) A linear operator is semihyperbolic iff the unit circle separates its spectrum. A fixed point of a differentiable map is semihyperbolic iff its derivative at that point is.

Theorem. Let \(W\) be an open neighborhood of \(0\) in a Banach space \(G, \rho: \sigma \to C^k(W, G)\) a uniformly \(C^k\) representation \((r \equiv 1)\) and \(a_0 \in \sigma\) a point such that \(0 \in W\) is a semihyperbolic fixed point of \(\rho(a_0): W \to G.\) Then there is a closed splitting \(G = E \oplus F = E \times F\) of \(G,\) neighborhoods \(\sigma_0, U,\) and \(V\)
of \(a_0, 0, 0\) in \(A\) and \(E\) and \(F\) respectively, and a uniformly \(C^r\) representation \(\pi: \mathcal{A}_{\alpha} \rightarrow C^r(U, V)\) such that \(U \times V \subseteq W\) and for \(a \in \mathcal{A}_{\alpha}\) (1) \(\rho(a) (\text{graph } \pi(a)) \subseteq \text{graph } \pi(a)\); (2) \(\rho(a)\) graph \(\pi(a)\) is a contraction mapping (in a suitable metric); (3) for \(z \in U \times V, z \in \text{graph } \pi(a)\) iff \(\rho(a)^n(z) \in U \times V\) for \(n = 0, 1, 2, \ldots\).

(Received October 7, 1968.)


The notation is that of Abstract 68T-B10, these Notices 15 (1968), 791. Let \(X\) be a Banach space. Theorem 1. If \(A \in \mathcal{B}(X)\) and \(E \in \mathcal{F}(X)\), then \(i(A + E) = i(A)\), the index of \(A\). Theorem 2. Let \(E \in \mathcal{B}(X)\) be an operator such that \(A + E \in \mathcal{B}(X)\) with \(i(A + E) = 0\) for all \(A \in \mathcal{B}(X)\) having \(i(A) = 0\). Then \(E \in \mathcal{F}(X)\). Theorem 3. If \(A \in \mathcal{B}(X)\), \(E \in \mathcal{R}(X)\) and \(AE - EA\) is compact, then \(i(A + E) = i(A)\).

Definition. An operator \(A \in \mathcal{B}(X)\) is said to have property \(\beta\) if there is an \(\varepsilon > 0\) such that \(\Lambda \in \rho(A)\) for \(0 < |\lambda| < \varepsilon\). Theorem 4. An operator \(E \in \mathcal{B}(X)\) is in \(\mathcal{R}(X)\) if and only if \(A + E\) has property \(\beta\) for each \(A \in \mathcal{B}(X)\) having property \(\beta\) and commuting with \(E\). (Received September 27, 1968.)

69T-B5. S. S. TANDON, Delhi University, New Delhi-22, India. Product summability of a class of Fourier series.

If \(f(u)\) be even, \(f(u) \in L(-\pi, \pi)\) and defined by periodicity outside this range. Let \(\varphi(u) = [f(u + t) + f(u - t) - 2f(u)]\) and \(\varphi_a(u)\) be the \(a\)th mean of \(\varphi(u)\) for all \(a \geq 0\). Theorem. If \(\int_0^t|\varphi_a(u)|du = O(t/1(t/t))\) as \(t \to +\), where \(t\) is monotonic increasing and the generating sequence \(\{p_n\}\) satisfies the following conditions, \(n|p_n|/t(n + 1) < C|p_n|\), and \(\sum_{k=1}^n k|p_k - p_{k-1}|/t(k + 1) < c|p_n|\), \(\Sigma_{k=1}^n |p_k|/k 1(k + 1) < C|p_n|\), then the product summability \((\kappa_0, p_n) \cdot C_\alpha\) is Fourier effective. The particular case \(\alpha = 0\), and \(\log(n) = 1(n)\) is due to Sahney (Pacific J. Math. 13 (1963), 251-262), and the case \(p_n = 1/(n + 1)\), \(t(n) = \log n\) is a generalization of harmonic summability due to the author himself (not yet published). (Received October 8, 1968.) (Author introduced by Professor B. N. Sahney.)


If \(f(u) \in L(-\pi, \pi)\) and defined outside this range by periodicity, and be even, let \(\varphi(t) = (1/2)[f(x + t) + f(x - t) - 2s]\) and \(\Phi(t) = \int_0^t \varphi(u)|du\). Theorem. If \(\Phi(t) = O(t)\) as \(t \to +\) and \(\lim_{p \to -\infty} \int_{\pi/p}^{\pi} \Phi(t - \varphi(t + \pi/p))/t \exp[-p(1 - \cos t)]dt = 0\) and \(\eta\) is constant, then the Fourier series is summable by Borel means at the point \(x\). (Received October 8, 1968.)

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68T-B7. WITHDRAWN.


E is a locally convex, Hausdorff, topological vector space and V is a linear subspace of E'. E is V-semireflexive in case the canonical embedding of E into V' is onto. Theorem 1. If V is total, E is V-semireflexive iff the strong closure of V is minimal. Theorem 2. If V is total and \( w(V, E) \)-separable, then the following are equivalent: (i) Each bounded set in E is relatively \( w(E, V) \)-sequentially compact. (ii) Each bounded set in E is relatively \( w(E, V) \)-countably compact. (iii) E is V-semireflexive. Theorem 3. If V is total, \( w(V, E) \)-separable, strongly closed and barreled, then E is V-semireflexive iff each decreasing sequence of nonempty, \( w(E, V) \)-bounded, \( w(E, V) \)-closed and convex sets has a nonempty intersection. Theorem 4. If E[t] is a Fréchet space, then E[t] is quasireflexive of order n iff there exists a minimal subspace V of E' of codimension n such that E[t] is V-reflexive. Theorem 5. Let E be a Banach space and V total. Then E is V-reflexive iff the unit ball of E is closed in every Hausdorff locally convex topology on E comparable with \( w(E, V) \).

(Received October 16, 1968.)

69T-B9. E. G. CALYS, Washburn University, Topeka, Kansas 66621. The radius of univalence and starlikeness of certain regular functions.

Let E denote the open unit disk and let \( \mathcal{J} \) denote the class of functions \( f(z) = z + a_2z^2 + \ldots \) which are regular in E. For each real number \( a \), \( |a| < \pi/2 \), define \( \mathcal{J}_a(z) = \{ f \in \mathcal{J} | \Re\{e^{ia}f(z)/z \} > 0 \text{ for all } z \in E \} \), \( \mathcal{J}_a(\mathcal{K}) = \{ f \in \mathcal{J} | \Re\{e^{ia}f(z)/g(z) \} > 0 \text{ for some } g \in \mathcal{K} \text{ and all } z \in E \} \), \( \mathcal{J}_a(\mathcal{S}^*) = \{ f \in \mathcal{J} | \Re\{e^{ia}f(z)/g(z) \} > 0 \text{ for some } g \in \mathcal{S}^* \text{ and all } z \in E \} \). Here \( \mathcal{K} \) and \( \mathcal{S}^* \) stand for the usual classes of univalent functions. Theorem 1. If \( f \notin \bigcup_{|a| < \pi/2} \mathcal{J}_a(z) \), then \( f \) is univalent and starlike for \( |z| < \sqrt{2-1} \). Theorem 2. If \( f \notin \bigcup_{|a| < \pi/2} \mathcal{J}_a(\mathcal{K}) \), then \( f \) is univalent and starlike for \( |z| < 1/3 \). Theorem 3. If \( f \notin \bigcup_{|a| < \pi/2} \mathcal{J}_a(\mathcal{S}^*) \), then \( f \) is univalent and starlike for \( |z| < 2 - \sqrt{3} \). Remark. These radii were obtained by T. H. MacGregor (Proc. Amer. Math. Soc. 14 (1963), 514-520) for \( \mathcal{J}_0(z) \), \( \mathcal{J}_0(\mathcal{K}) \), and \( \mathcal{J}_0(\mathcal{S}^*) \) and his results are sharp. It follows that our results are also sharp. (Received October 16, 1968.)

69T-B10. J. C. WONG, University of British Columbia, Vancouver 8, British Columbia, Canada. Topologically stationary locally compact groups and amenability.

Let G be a locally compact group with a fixed left Haar measure \( \lambda \) and \( L_{\infty} = L_{\infty}(G, \lambda) \). For each compact set \( F \) in G with \( \lambda(F) > 0 \), let \( \phi_F = (1/\lambda(F)) \chi_F \) be the normalised characteristic
function of $F$. If $f \in L_\infty$, denote by $Q_L(f)$ the weak* closed convex hull of the set $\{(l/\Delta)\phi^* f : F \subset G, \text{compact and } \lambda(F) > 0\}$ (where $\Delta$ is the modular function and $g^*$ is defined by $g^*(x) = g(x^{-1}), x \in G$) and by $K_L(f)$ the set $\{a : a \in \mathcal{A}L(f)\}$ (here $\mathcal{A}$ is the constant one function on $G$). Definition. $G$ is topologically left stationary if $K_L(f) \neq \emptyset$ for all $f \in L_\infty$. Theorem. $G$ is topologically left stationary if $L_\infty$ has a topological right invariant mean. In this case $\alpha_0 \in K_L(f_0)$ iff there is a topological right invariant mean $\mu$ on $L_\infty$ such that $\mu(f_0) = \alpha_0$. This theorem is a generalisation to locally compact groups of a result in [T. Mitchell, Constant functions and left invariant means on semigroups, Trans. Amer. Math. Soc. 119 (1965), 244-261]. Some generalisations and diverse additional results in this direction are obtained. (Received October 17, 1968.)


The following generalization of the Banach-Stone theorem was proved by the author [On isomorphisms with small bound, Proc. Amer. Math. Soc. 18 (1967), 1062-1066]: If $X$ and $Y$ are locally compact Hausdorff spaces, and if there exists an isomorphism $\psi$ of $C_0(X)$ onto $C_0(Y)$ satisfying $\|\psi\| \cdot \|\psi^{-1}\| < 2$, then $X$ and $Y$ are homeomorphic. An example provided here shows that 2 is the largest number for which this generalization is valid, and that "<" cannot be replaced by "=". Example. Let $X = \{x_k : k = 0, 1, 2, \ldots\}$ be a sequence of distinct points $x_k$, where $\lim_{k \to \infty} x_k = x_0$, and $x_0$ is the only accumulation point of $X$. Let $Y = \{y_k : k = 0, 1, 2, \ldots\}$ be a sequence of distinct points $y_k$, where $\lim_{k \to \infty} y_k = y_0$, $y_0$ is the only accumulation point of $Y$, and the set $\{y_k : k \neq 1\}$ has no accumulation point in $Y$. Define $\psi$ mapping $C_0(X)$ onto $C_0(Y)$ by $(\psi(f))(y_k) = f(x_k)/2 - f(x_{-k})/2$ if $k \neq 1$, $(\psi(f))(y_0) = f(x_0)$, for $f \in C_0(X)$. Then $X$ is compact, $Y$ is not, and $\|\psi\| \cdot \|\psi^{-1}\| = 2$. (Received October 18, 1968.)

69T-B12. A. M. CHAK, West Virginia University, Morgantown, West Virginia 26505. Some generalizations of Laguerre polynomials. II.

This is the second paper of a series (see Abstract 68T-B34, these Notices 15 (1968), 798) in which we obtain another generalization of the Laguerre polynomials by considering a generalization of the operational image of the Laguerre polynomials in the sense of Laplace-Carson. The method used here is similar to that used by the author in 1954 (A generalization of Bessel-Maitland function, Ann. Soc. Sci. Bruxelles 68 (1954), 42-52). Quite a few properties have been obtained including a connection of this generalization of Laguerre polynomials with that of the Bessel-Maitland function studied in 1954 and cited above. (Received October 21, 1968.)


In the book, Introduction to theory of non-selfadjoint linear operators (Russia, 1965, p. 45) Gokhberg-Krein mention a theorem of Dolberg (1957) which allegedly gave a criterion for equality in the maximum-minimum theory of eigenvalues for a positive symmetric integral operator $K$. It is shown that Dolberg actually solved a completely different problem, namely the equation (1)

$$u = \lambda[Ku - \langle u, Kp \rangle / \langle Kp, p \rangle]Kp$$

by applying Fredholm's theory to a perturbation of finite rank (e.g.,

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of rank one defined by an arbitrary function $p$). Dolberg asserts that the corresponding variational problem is given by (2) \[ \min (Ku, u), (Ku, Ku) = 1, (Ku, p) = 0, \] which, for a restricted class of constraints $p$, is indeed equivalent to the max-min theory. However, the correct eigenvalue problem for (2) is (3) \[ u = \lambda (Ku - [(Ku, Kp)/(Kp, p)]p), \] and not (1). Dolberg’s paper again confirms the fact that the max-min theory, which was reconsidered by the method of intermediate problems [A. Weinstein, J. Math. Mech. 12 (1963), 235-245], is not reducible to perturbation theory, see also [W. Stenger, J. Math. Mech. 17 (1968), 643]. However, the problem of Dolberg shows that there are optimum choices of vectors $p$ which maximize the eigenvalues of the perturbed operator. This question will be developed in a paper of Stenger. (Received October 21, 1968.)


Analyticity and reflectivity results are established for first order elliptic semilinear systems in two independent variables. The results are obtained by adaption and modifications of techniques used by Garabedian in establishing similar results for second order systems. (Received October 17, 1968.)

69T-Bl5. J. W. NEUBERGER, Emory University, Atlanta, Georgia 30322. Product integral formulae for nonlinear expansive semigroups and nonexpansive evolution systems.

Suppose $S$ is a Banach space and $g$ is a set of functions from $[0, \infty)$ to $S$ so that (1) if $p, q$ are in $S$, $0 \leq t \leq x$, then $g_p$ is continuous, $g_p(0) = p$, $g_r(x) : r \in S \gamma S$, $\|g_p(x) - g_r(x)\| \leq \|g_p(t) - g_r(t)\|$, (2) $g'_p$ is continuous with domain $[0, \infty)$ for all $p$ in some dense subset of $S$, (3) if $x \geq 0$ and $A(x) = \{ (g_q(x), g'_q(x)) : q \in S, g'_q(x) \}$ exists, then $R(I - yA(x)) = S$ for some $y > 0$. Theorem 1. For each $x, y \geq 0$, $(I - yA(x))^{-1}$ exists and has a unique nonexpansive extension $L(y, x)$ to $S$. If $x \geq 0$, $\epsilon > 0$, there is $\delta > 0$ so that if $\{ t_{i+1} \}$ is a chain from $0$ to $x$ of mesh $< \delta$, then $\| \sum_{i=0}^{n} L(t_{i+1} - t_i, t_i) - g_p(x) \| < \epsilon$. Suppose (a) $T$ is a strongly continuous semigroup of transformations from $S$ to $S$ and, $T(0) = I$, $h_p(x) = T(x)p$, $x \geq 0$, $p \in S$, (b) $T(0) = I$, $h_p(x) : p \in S \gamma S$ if $x \geq 0$, $h'_p$ is continuous with domain $[0, \infty)$ for all $p$ in some dense subset of $S$, $h_p$ is continuous for all $p$ in $S$, (c) $c < 0$ and $\|T(x)p - T(x)q\| \leq e^{cx}\|p - q\|$ for all $p, q$ in $S$. Theorem 2. If $B$ is the infinitesimal generator of $T$, $R(I - yB) = S$ for all $y > 0$, and if $g_p(x) = e^{-cx}T(x)p$ for all $x \geq 0$, $p \in S$, then the conditions of Theorem 1 are satisfied. (Received October 25, 1968.)

69T-B16. WITHDRAWN.


The purpose of this notice is to announce a new nonoscillation theorem for the second order nonlinear differential equation (*) $y'' + q(t)y^a = 0$, where $a > 1$ is the quotient of odd integers. The function $q$ is positive and locally of bounded variation on some half line $[a, \infty)$. Theorem. Assume that $\int_{a}^{\infty} s^a q < \infty$. If there exists a positive, decreasing function $p(t)$ such that $p(t)q(t)$ is decreasing and
lim_{t \to \infty} p(t) - \delta \int_{0}^{\infty} q(s) = 0, \quad \delta = (a - 1)/2, \text{ then the equation } y'' + q(t)y^a = 0 \text{ is nonoscillatory. Since } q \text{ is not necessarily decreasing, this theorem is a distinct generalization of a well known result of Atkinson (On second order nonlinear oscillations, Pacific J. Math. 5 (1955), 643-537). A similar result is also available for the case } 0 < a < 1, \text{ thus generalizing a recent result of J. W. Heidel. The techniques used in the proof of these results can also be used to prove new nonoscillation theorems for various generalizations of (*) . These will appear elsewhere.}

(Received October 29, 1968.)

69T-B18. D. B. PRIEST, University of Mississippi, University, Mississippi 38677. A mean refinement integral of a function with respect to a function pair.

Let A, I, TBV and partition D of A be as in Abstract 67T-566, these \textit{Notices} 14 (1967), 825. Let M denote \{ f \neq 0 \} \text{ is a real-valued function on A for which there is a function pair } (f_1, f_2) \text{ such that } f(s,t) = f_1(s)f_2(t) \text{ for } (s,t) \in A \}. Suppose each of f, g, and h is a real-valued function on A and \text{ d } = [p, q; r, s] \text{ is an element of the partition D of A. Let } F(d) \text{ denote a mean evaluation of } f \text{ over the vertices of d in the sense of R. E. Lane (Proc. Amer. Math. Soc. 5 (1954), 59-66). Let } (g, h)(d) \text{ denote the number } (g(q,r) - g(p,r))(h(p,s) - h(p,r)) - (h(q,s) - h(p,s))(g(q,s) - g(q,r)). \text{ The mean integral of } f \text{ with respect to the function pair } (g, h) \text{ is defined as the refinement limit of sums of the type } \sum F(d)((g, h)(d)) \text{ where the sum is over all } d \text{ in D. Theorem 1. If } f \text{ is quasi-continuous and each of } g \text{ and } h \text{ is in TBV and either } g[1, k] \text{ or } h[1, k] \text{ is continuous for each } k \text{ in } [c, d], \text{ then } f \text{ is mean- } g, h \text{ integrable. Theorem 2. Suppose } f \text{ is a quasi-continuous function in } M, \text{ each of } g \text{ and } h \text{ is a function in } TBV \text{ and } M, \text{ and either } g \text{ or } h \text{ is continuous, then } m - \int \int A f dg dh = (m - \int \int_a (f_1(1))dg_1)(m - \int \int_c (f_2(1))dh_2) - (m - \int \int_a (f_1(1))dh_1)(m - \int \int_c (f_2(1))dg_2). \text{ (Received October 24, 1968.)}

69T-B19. M. S. ROBERTSON, University of Delaware, Newark, Delaware 19711. Coefficients of functions with bounded boundary rotation.

For } k \geq 2 \text{ let } V_k \text{ denote the class of normalized analytic functions } f(z) = z + \sum_{n=2}^{\infty} a_n z^n \text{ that, for } z \in E \{ |z| < 1 \} \text{ are regular, have } f'(0) = 1, f(z) \neq 0, \text{ and for } z = re^{i\theta}, 0 \leq r < 1, \int_0^{2\pi} |f'(z)|/|f(z)| d\theta < km. \text{ Let } S_k \text{ be the subset of } V_k \text{ whose members } f(z) \text{ are univalent in } E. \text{ For } 2 \leq k \leq 4, S_k = V_k. \text{ It is shown that if } f(z) \in V_k, \text{ then } |a_n| < k^{-1} - 1, \text{ and } f(z) \text{ maps } |z| < (k - (k^2 - 4)/2) \text{ onto a convex domain, and furthermore lim}_{k \to \infty} [k^{-1} - k/2 \max_{x \in V_k} |a_n| (f)] = 0 (n = 2, 3, ...), \text{ where } a_n(f) = f^{(n)}(0)/n!. \text{ When } f(z) \in S_k, 2 \leq k < \infty, \text{ then } |a_{n+1}| - |a_n| < 2(3/2)^3 (k^2 + k), n = 2, 3, ... \text{. (Received October 30, 1968.)}


Let } X \text{ be a commutative complex Banach algebra with identity, } m \text{ a fixed regular Borel measure on the maximal ideal space } \mathfrak{m}, \sigma(x) \text{ the spectrum of } x, \text{ and } B(\mathfrak{m}) \text{ the Banach space of complex regular Borel measures on } \mathfrak{m}. \text{ Define the complex measure } m_x \in B(\mathfrak{m}) \text{ by } m_x(E) = \int_E \sigma(x)dm \text{ where } x = \xi \text{ is the Gelfand transform. Theorem 1. The bounded linear transformation } x \mapsto m_x \text{ is 1-1 iff } m \text{ has the property } \sigma(M) = 0 \text{ a.e. (m) } = x = 0. \text{ In the algebra } C[a,b] \text{ with the usual identification of } \mathfrak{m} \text{ with } [a,b], \text{ Lebesgue measure on } [a,b] \text{ enjoys the property specified in this theorem. Theorem 2. Let } X \text{ be a complex commutative } A^*\text{-algebra with identity. If } \lambda \in B(\mathfrak{m}), \text{ then } \lambda \in \Theta(X) \text{ iff...
\[ \mathcal{Y} \in X \in \int \mathcal{X}(M) dm = \int \mathcal{X}(M) d\lambda \forall x \in X. \] Theorem 3. Let \( X \) be a commutative complex Banach algebra with identity. (i) If \( V \) is a subset of \( X \) such that \( \cup_{x \in V} \sigma(x) \) is bounded, then \( \theta(V) \) is weakly sequentially compact and weakly countably compact in \( B(\mathcal{M}) \). (ii) If \( V \) is a bounded subset of \( X \), then \( \theta(V) \) is weakly sequentially compact and weakly countably compact in \( B(\mathcal{M}) \).

Theorem 4. If \( X \) is a complex commutative \( \mathbb{A}^* \)-algebra with identity and \( B(\mathcal{M}, m) = \{ \mu \in B(\mathcal{M}) : \mu \ll m \} \), then \( \theta(X) \) is dense in \( B(\mathcal{M}, m) \).

(Received October 24, 1968.)


A topological group is called completely regular if the space of the group is completely regular. In this note the existence of right invariant Haar integrals on completely regular groups is established. (Received November 1, 1968.)


Let \( b(1), b(2), \ldots, b(n) \) be a (linear) basis of the \( n \)-dimensional vector space \( V \) over a field \( K \) of characteristic zero. Let \( \sum \) stand for summation from 1 to \( n \). Let \( W = \{ x = (x(1), x(2), \ldots, x(n) \mid x(i) \) is a nonnegative integer for each \( i \) and \( \sum x(i) = p \} \). Thus \( W \) is the set of ordered partitions of the positive integer \( p \) into \( n \) nonnegative integer parts. The set \( A = \{ a(x) = \sum x(i)b(i) \mid x \in W \} \) consists of \( q = (n + p - 1)!/(n - 1)!p! \) vectors belonging to \( V \). The set \( A \) is a homogeneous \( p \)th degree (nonlinear) basis of \( V \) in the sense made obvious by the theorems below. Theorem. A polynomial form \( u \) of degree \( p \) in \( n \) variables is uniquely determined by its values \( u(x(1), x(2), \ldots, x(n)) \) on the \( q \) points \( x \) of the set \( W \). Theorem. The \( q \) pure tensor powers \( \{ a^p \mid a \in A \} \) span the \( p \)th symmetric space, \( S^p(V) \), over \( V \). Theorem. The \( q \) pure \( p \)th Fréchet derivatives \( \{ d^p f/g \mid a \in A \} \) completely determine the homogeneous \( p \)th degree terms of the multidimensional Taylor series of a holomorphic function \( f : V \to K \) at the point \( g \in V \). Comment. The value at \( g \) of the Fréchet derivative \( df/d a \) of \( f \) with respect to \( a \in V \) is written \( df(g)/da \), and is defined to be the limit (as the scalar \( h \) approaches zero) of \( (f(g + ha) - f(g))/h \). Higher derivatives are defined inductively as usual. The foregoing theorems are all equivalent. (Received October 16, 1968.)

Applied Mathematics

69T-C1. ANDRE GLEYZAL, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland, Complex gravitation.

We propose that a pure complex analytic Riemannian geometry \( a_{\alpha \beta} = [a_{\alpha \beta}] = [a_{\alpha \beta}, 0] \), where \( ds^2 = a_{\alpha \beta}da^\alpha da^\beta \) and \( a_{\alpha \beta} = a_{\alpha \beta} \) are ten complex analytic functions \( a_{\alpha \beta}(a^\gamma) \) of the four complex coordinate variables \( a^\gamma \), represents the physical universe [cf. Abstract 64T-30, this Notices 11 (1964), 135; Naval Ordnance Lab. Tech. Rep. 66-189 and 68-112]. A relaxation of this proposal is that a test particle \( w_n, n = 0, 1, 2, \ldots \), of complex mass \( w_0 = m = m + ie/\sqrt{\gamma} \) introduced into \( a_{\alpha \beta} \) is represented by a dual geometry \( [a_{\alpha \beta}(a^\gamma), \Psi(a_{\alpha \beta}(a^\gamma), w_n)] \), where \( \Psi(a_{\alpha \beta}(a^\gamma), w_n) = \Psi(a^\gamma, w_n), \Psi \) is analytic in \( a^\gamma \) and \( w_n, \Psi \) is a functional of \( a_{\alpha \beta}(a^\gamma) \) and \( w_n \), and \( \Psi \) denotes a complex de Broglie wave for \( m \) in
Let \( a_\alpha \beta \) be the complex Schwarzschild field of \( \alpha = \gamma m_0 \sqrt{c^2 + m_0^2 \xi g / \sqrt{\gamma}} \). We consider dual geometries \( [m_0, m] = [a_\alpha \beta(a^\alpha, m_0), \mathcal{W}(a_\alpha \beta(a^\alpha, m_0), m)] \). Suppose \( a_\alpha \beta \mathcal{W}_\alpha \beta = (m_0 c^2 / \hbar^2) \mathcal{W} = 0 \) or, as in a Dirac theory, \( \gamma \mathcal{W} + (mc / \hbar) \mathcal{W} = 0 \) where \( \gamma \alpha \) is suitably defined.

A radial differential equation for \( R(r) \) may then be derived. Let the coefficients of this radial equation be suitably altered by factors \( 1 + \beta / r \) where \( |\beta| < 4 \). Then \( |\beta| r < 10^{-24} \) if \( |r| > 10^{-9} \text{cm} \). The usual eigenvalues (energy levels) of the hydrogenic dual particle \( [m_0, m] \) will result. (Received September 23, 1968.)

69T-C2. S. E. WEINSTEIN, University of Utah, Salt Lake City, Utah 84112. Solution of nonlinear equations by iterative procedures which use approximation techniques.

Traub has shown sufficient conditions for the convergence of a sequence \( \{x_i\} \) to \( x = a \), a solution to \( F(x) = 0 \), by the use of interpolatory iteration functions \(-I, I, F\). This paper extends these results to \( A, I, F\), \( -I, I, F\) iteration functions generated by approximation techniques. Three theorems giving sufficient conditions for the convergence of algorithms generated by \( A, I, F\) are presented. The essential addition to the Traub hypotheses is that the approximating polynomial \( P \) be sufficiently close to \( F \) in the sup. norm \( \|P - F\| \leq 10^{-24} \). If \( F''(a) \neq 0 \), \( F''(X) \) is continuous at \( a \), min \( \{|F_1|, |F_1 - F_1|, |F_1 - F_2|\} \) is sufficiently large and \( F \) changes sign between \( x_i \) and \( x_{i-2} \), then \( x_{i+1} = \Phi(x_i x_{i-1} x_{i-2}) \) is closer to \( a \) than the corresponding iterate obtained by false position. (Received October 21, 1968.)

69T-C3. SEYMOUR SHERMAN, Indiana University, Bloomington, Indiana 47401.

A ferromagnetic inequality.

Theorem. Let \( \phi \) be a real-valued function on the nonnegative integers. A necessary and sufficient condition that for each positive integer \( n \), each finite group \( G \), subgroups \( G_1, \ldots, G_n \) and their left cosets \( g_1 G_1, \ldots, g_n G_n \), \( \Sigma g \in G \phi([1 : g \in G_1]) \cong \Sigma g \in G \phi([1 : g \in g_i G_i]), \) is that \( \Delta^k \phi(0) \equiv 0, k = 2, 3, \ldots \), where \( \Delta \) is the forward difference operator, so that \( \Delta \phi(n) = \phi(n + 1) - \phi(n) \), and \( |A| \) is the number of elements in \( A \). The investigation of the monotonicity of correlation as a function of interactions in using ferromagnets with integral spin-1 led to the inequality on subgroups and cosets. The mathematics seems similar to that arising in error-correcting codes. (Research supported by NSF GP-7469 and Australian Research Grants Committee 66/16188.) (Received October 31, 1968.)

Logic and Foundations

69T-E1. DOV GABBAY, Hebrew University, Jerusalem, Israel. Semantics for minimal logic and applications. Preliminary report.

For terminology and notation see (1) Kripke, "Semantic analysis for intuitionistic logic"
in Formal Systems and recursive functions, Amsterdam, 1965., (2) Curry, Foundations of mathematical logic, 1963. A model for LM is a pair \((GKR)\) where \(K\) is a Kripke model and \(K_0 \subset K\). The truth value of a sentence is defined by induction exactly as in Kripke's paper except for negation. \(~A\) holds in \(H \in K\) iff for every \(H^1\) such that \(HRH^1\), and such that \(A\) holds in \(H^1\), there exists an \(H^{11} \in K_0\) such that \(H^{11}RH^1\). For this semantics, LM is complete. If we consider only LM-models \((GKR)\) such that \(K = 1\) we get a semantics for which LE is complete. If we further restrict \(K_0\) to be empty we get a semantics for which LK is complete. The models with \(K_0\) empty and \(K\) arbitrary yield Kripke's semantics for LJ. If we require \(K_0 \supseteq K - \{G\}\) we get a semantics for which LD is complete. For each semantics ultra-products are defined and the compactness theorem holds. (Received July 29, 1968.) (Author introduced by Professor Azriel Levy).

69T-E2. MIHA'LY MAKKAI, Mathematical Institute of the Hungarian Academy of Sciences, Budapest V., Rea'ltanada u. 13-15, Hungary. Structures elementarily equivalent to models of higher power relative to infinitary languages.

Let \(s, s_1, s_2\) range over the set \(S_0\) of finite sequences of natural numbers, \(\lambda(s)\) is the length of \(s\), \(s_1 \subset s_2\) means \(s_1\) is a proper initial segment of \(s_2\), \(s^n\) is \(s^n(n)\). Let \(S \subset S_0\) be such that (i) \(S \neq \emptyset\), (ii) \(s_1 \subset s_2\) and \(s_2 \in S\) then \(s_1 \in S\), and (iii) \(S\) is well founded with respect to \(s_1 \subset s_2\).

Let \(\Phi = (\phi_s : s \in S)\) be such that \(\phi_s\) is a formula of \(L(\omega_1, \omega)\) with the free variables \(v_0, \ldots, v_M(s)\) at most, \(\phi_0 = v_0 = v_0\). Let \(\psi_s\) be defined for \(s \in S\) by (iv) \(\psi_s = \exists v_0 (v_0 \phi_s)\) if \(s^n \in S\) for all \(n \in \omega\) and (v) \(\psi_s = \exists v_M(s+1) [\forall v_0 (v_0 \phi_s) \wedge (v_{M(s^n)} : s^n \in S)]\wedge (\forall v_0 \phi_s)\) otherwise. Put \(\psi(\Phi) = \psi_0\).

Theorem 1. A countable structure (not necessarily of a countable similarity type) is \(\equiv\) to an uncountable structure iff it is a model of \(\Phi\). Theorem 2. If a countable linear ordering \(A\) does not have a dense subordering, there is \(\psi \in L(\omega_1, \omega)\) such that \(B \in Mod(\psi)\) iff \(B \equiv A\).

Theorem 3. If \(\psi \in L(\kappa^+, \omega)\) has arbitrarily large models, \(\psi\) has a model of power \(\kappa\) which is \(\equiv\) to arbitrarily large models. Theorem 3 was proved independently by D. Kueker and C. C. Chang. (Received September 30, 1968.) (Author introduced by Professor R. L. Vaught.)


We follow the notation of J. H. Silver, Abstract 66T-435, these Notices 13 (1966), 721. Let \(x\) be a measurable cardinal, \(D\) a normal ultrafilter on \(x\). Theorem. If \(E\) is any \(x\)-complete nonprincipal ultrafilter on \(x\), \(L_D = L_E\). Theorem. If \(V = L_\omega\), then \(D\) is the only normal ultrafilter on \(X\), and there are exactly \(x^+\) \(x\)-complete nonprincipal ultrafilters on \(x\). Theorem. If \(V = L_\omega\) and \(S\) is any complete Boolean algebra, then the statement \([x\text{ is measurable } \rightarrow 2^x = x^+]\) is valid in \(V(S)\).

Theorem. If \(x\) is strongly compact, then for each ordinal \(\alpha\) there is a transitive model, \(M\), for ZFC, such that \(\alpha \in M\) and \(M\) satisfies \([\alpha\text{ are measurable cardinals}].\) (Received October 7, 1968.)

69T-E4. J. P. JONES, University of Calgary, Calgary, Alberta, Canada. Effectively retractable theories and degrees of undecidability.

This work is related to that of Abstract 67T-532, these Notices 14 (1967), 717. Definition. A theory \(T\) is said to be effectively retractable if each finite proper extension of \(T\) produces effectively
a strictly smaller finite proper extension of $T$. Theorem 1. Every axiomatizable effectively inseparable theory is effectively retractable. Theorem 2. Given an axiomatizable effectively retractable theory $S$ and a proper extension $T$ of $S$, there exist theories of arbitrary complete degree of unsolvability between $S$ and $T$. This result may be used to show that most of the theories proven to be essentially undecidable in [Undecidable theories, North-Holland, Amsterdam, 1953] possess subtheories of each complete degree. Theorem 2 also leads to an economical characterization of the degrees of unsolvability associated with subtheories of many familiar theories. Theorem 3. If $ZF$ is an $\omega$-consistent theory in the sense of Orey [J. Symbolic Logic 21 (1956), 246-252], then the degrees of subtheories of $ZF$ are precisely the complete degrees. Theorem 3 also holds when the theory $ZF$ is replaced by Peano arithmetic, the arithmetic of natural numbers, the arithmetic of integers, and the arithmetic of rational numbers. (Received October 25, 1968.)

69T-E5. WITHDRAWN.

69T-E6. ALBERT SADE, 364 Cours de la République, 84 Pertuis, France. Morphismes sur le groupoide ternaire des opérateurs propositionnels.

Au moyen de la représentation des fonctions propositionnelles par des polynômes sur le corps du second ordre, on construit un groupoide ternaire $\Sigma$ de ces fonctions et une table de leurs inverses qui permet de construire autant de tautologies que l'on voudra. L'étude des isotopies de $\Sigma$ et de ses restrictions fait ressortir un groupoide associatif $\Gamma$, consistant en 8 isomorphismes qui laissent invariantes les 16 restrictions de $\Sigma$ et 8 endormorphismes qui les projettent dans elles-mêmes et forment un sous groupoide de $\Gamma$. Le groupe d'automorphisme de $\Sigma$ est du second ordre et, dans la notation polonaise, est représenté par l'involution $P_7 = AK.XD.BL.EJ.OV.CM$ et son carré. Les éléments de $\Gamma$, opérant sur les seuls foncteurs atomiques, transforment deux formules logiques équivalentes quelconques en deux formules équivalentes en une tautologie en une tautologie. $P_7$ appliqué à tous les foncteurs de deux formules équivalentes, les transforme en deux formules équivalentes. $P_7$ est la seule permutation qui jouisse de cette propriété. (Received October 28, 1968.)

69T-E7. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. On a property of cofinality of ordinal numbers.

Definition. An ordinal $w$ is called cofinal to an ordinal $r$ (denoted by $w \not < r$) if and only if $w = \underset{h<r}{\bigcup} (F_h + 1)$, where $(F_h)_{h<r}$ is an increasing sequence of type $r$ of ordinals $F_h$. Theorem. For every ordinal $x$ and $y$ there exists an ordinal $v$ such that $x \not < v$ and $y \not < v$ if and only if there exists an ordinal $u$ such that $u \not < x$ and $u \not < y$. (Received October 31, 1968.)


Let $E(i,j)$ be an equivalence relation on the natural numbers $N$. Theorem. $E$ is recursively enumerable (r.e.) $\langle = \rangle$ there is a recursive class $\Omega = \{ \Pi_i \mid i \in N \}$ of finite presentations of groups such that $\Pi_i \cong \Pi_j$ if and only if $E(i,j)$. The isomorphism problem for $\Omega$ is the problem of deciding whether or not $\Pi_i \cong \Pi_j$. Corollary. Every r.e. many-one degree contains an isomorphism problem,
but the one-one degree of a simple set does not. Let $H$ be a finitely presented groups, $1 = w_1, w_2, ...$
a recursive list of the words of $H$. Take $E(i,j)$ to mean $w_i =H w_j$. Then $\Omega$ can be written down
explicitly and $\Pi_u \equiv \Pi_v$ iff $u =H v$. Now $\Omega$ admits a group structure defined by $(\Pi_u)^0 (\Pi_v) = \Pi_{uv}$.
These results answer a question of W. W. Boone. (Received October 31, 1968.)

**Statistics and Probability**

69T-F1. N. A, TSERPES, Wayne State University, Detroit, Michigan 48202. On a
colorization of idempotent probabilities on left groups. II.

A converse of the theorem announced in Abstract 68T-455, these *Notices* 15 (1968), 636, is obtained. Let $S$ be a locally compact Hausdorff semigroup and let $m$ be an idempotent probability
(Borel) measure on $S$ with support $F$. We define the following conditions: (LL) For every pair of
compact subsets $A, B$ of $S$, $A^{-1} B$ is compact. (RC) If $e=$idempotent element of $S$, then $x e = y e$ implies $x = y$. (R1) If $e =$ idempotent element and $C =$ compact, then $C e^{-1}$ is compact. Theorem 1. If $S$
satisfies (LL) and (RC), then $F$ is a compact left group. If in addition $m(x F) > 0$ for all $x \in F$, then $F$ is a compact group and $m =$ normed Haar measure on $F$. Theorem 2. If $S$
satisfies (LL) and
(R1), then $F$ is a compact kernel, i.e., $F$ is its own kernel. If in addition $m(x F) > 0$ and $m(F x) > 0$,
then $F$ is a compact group and $m =$ normed Haar measure on $F$. The proof that $F$ is a compact kernel
is new and independent of that due to Pym-Glicksberg (Pacific J. Math 12 (1962) and to Heble-Rosenblatt
(Proc. Amer. Math. Soc. 14 (1963)). Also the proof of Theorem 2 is new and independent of that used
by Mukherjea, Abstract 656-36, these *Notices* 15 (1968), 513. (Received March 26, 1968.)

69T-F2. J. C. MINNEKA, State University of New York, Lehman College, Bronx, New York
10468. On the uniqueness of positive solutions to the Wiener-Hopf equation.

Consider the equation (*) $h(x) = \int_0^\infty K(y - x) h(y)dy$ for all $x \geq 0$, where $K(x) \equiv 0$ and
$\int_0^\infty K(x)dx = 1$. For $x, y \geq 0$, define $q_1(x, y) = K(y - x)$, $q_{n+1}(x, y) = \int_0^y K(t - x) q_n(t, y)dt$. Let
g(x, y) = \sum_{n=1}^\infty q_n(x, y)$ and $G(x, y) = \int_0^y g(x, t)dt$. It is well known that if $\lim_{y \to \infty} G(0, y) = \infty$, then (*) has
a positive solution. All nontrivial nonnegative solutions are strictly positive. Theorem. If (*) has
a positive solution $h(x)$, and if for all $x > 0$ and $\Delta > 0$, $\lim_{y \to \infty} (G(0, y - x + \Delta) - G(0, y - x))$
$\cdot (G(0, y + \Delta) - G(0, y))^{-1} = r(x)$, then all positive solutions to (*) are constant multiples of
$f(x) = \int_0^x g(t, 0)r(x - t)dt + r(x)$. This theorem can be applied to obtain various sufficient conditions
for uniqueness, among them (1) $0 \leq \int_0^\infty xK(x)dx = \mu < \infty$ and (2) $\mu = 0$ and $\int_0^\infty x^2 K(x)dx < \infty$.
(Received November 20, 1967.)

**Topology**

69T-G1. PING-FUN LAM, Wesleyan University, Middletown, Connecticut 06457. Expansive
family of endomorphisms on compact groups.

Let $G$ be a topological group. A family $F$ of (topological) endomorphisms of $G$ is said to be
expansive on $G$ if there exists $V$, a neighborhood of the identity which satisfies for every
$x, y \in G, x \neq y$, there is an $f \in F$ such that $f(xy^{-1}) \not\in V$. Theorem 1. Let $G$ be a compact connected
group. Suppose $G$ admits an expansive family $F$ of endomorphisms, then $G$ is abelian. Corollary.
Let $G$ be a compact connected group. Suppose $G$ admits an expansive automorphism or endomorphism, then $G$ is abelian. An automorphism (endomorphism) $\phi$ is said to be expansive if the family $F = \{\phi^n | n = 0, 1, 2, \ldots \}$ is expansive. Theorem 1 extends the theorems of M. Eisenberg and T. S. Wu where $G$ is restricted to finite dimension and $F$ is restricted as in the Corollary. This result also gives a solution of an unsolved problem proposed by M. Eisenberg. In case $G$ is only locally compact connected, Theorem 1 is false. One, however, has Theorem 2.

Let $G$ be a connected, locally compact, maximally almost periodic group. Suppose $G$ admits an expansive family of endomorphisms, then $G$ is abelian. Theorem 2 can be considered as a generalization of Theorem 1. (Received August 19, 1968.)


Definition. A subset $\mathcal{U}$ of the power set of $(X \times X)$ is a precorrect uniformity on $X$ iff $\mathcal{U}$ satisfies the following: $(A_1)$ $U$ in $\mathcal{U}$ iff $U = U^{-1}$, $U \supseteq \Delta$, and $U$ contains a member of $\mathcal{U}$; $(A_2)$ for every $A \subseteq X$ and $U, V$ in $\mathcal{U}$ there is a $W$ in $\mathcal{U}$ such that $W[A] = U[A] \cap V[A]$; $(A_3)$ for every $A \subseteq X$ and $U$ in $\mathcal{U}$ there exist $V, W$ in $\mathcal{U}$ such that $(W \circ V)[A] = U[A]$ (cf. A. G. Mordkovič, Test for correctness of a uniform space, Soviet Math. Dokl. 7 (1966), 915-917). Let $(X, \delta)$ be a proximity space, and let $\Pi(\delta)$ be a proximity class of precorrect uniformities on $X$. Theorem. $\Pi(\delta)$ contains one and only one totally bounded symmetric uniformity. Theorem. $\Pi(\delta)$ contains a maximum and minimum element. Theorem. If $\delta$ is the usual proximity for the reals $X$, then $\Pi(\delta)$ contains at least two distinct precompact precorrect uniformities that have an open base. Theorem. If $(X, \mathcal{F})$ is a connected completely regular topological space, then there exists a precompact precorrect uniformity $\mathcal{U}$ on $X$ with an open base such that $\mathcal{F}(\mathcal{U}) = \mathcal{F}$ and every filter in $X$ is weakly Cauchy. Theorem. A completely regular topological space $(X, \mathcal{F})$ is compact iff it is complete (i.e. every weakly Cauchy filter has a cluster point) with respect to every compatible precorrect uniformity on $X$. Theorem. A precorrect space is compact iff it is complete and precompact. (Received June 11, 1968.)


This note is an outgrowth of a paper by Knaster-Kuratowski-Mazurkiewicz. We derive Brouwer's theorem from a covering theorem, the proof of which is based on a combinatorial lemma of Sperner. Our sequence of theorems are identical to the ones used by Knaster-Kuratowski-Mazurkiewicz, however, our proofs differ in that they are simpler. We conclude by noting that it may be possible to weaken the hypothesis of Sperner's lemma. Lemma 1 (Sperner). Let $\Pi$ be a simplicial subdivision of an $n$-simplex $v_0 v_1 \ldots v_n$. To each vertex $\mu$ of $\Pi$, assign an integer $\zeta(\mu)$ such that whenever $\mu \in v_0 v_1 \ldots v_k$, $\zeta(\mu) \in [i_0, i_1, \ldots, i_k]$. Then the total number of those $n$-simplexes of $\Pi$ whose vertices receive all $n$ integers $0, 1, 2, \ldots, n$ is odd. In particular, there exists at least one such simplex. Lemma 2 (Knaster-Kuratowski-Mazurkiewicz). Let $F_0, F_1, \ldots, F_n$ be $n + 1$ closed subsets of an $n$-simplex $v_0 v_1 \ldots v_n$. Moreover, let $v_{i_0} v_{i_1} \ldots v_{i_k} \subseteq F_{i_0} \cup F_{i_1} \cup \ldots \cup F_{i_k}$ for all $v_{i_0} v_{i_1} \ldots v_{i_k}$ $(0 \leq k \leq n, 0 \leq i_0 \leq i_1 < \ldots < i_k \leq n)$. Then $\cap_{i=0}^{n} F_i \neq \emptyset$. Theorem 1 (Brouwer). If $X$ is a compact convex set in $E^n$, then every continuous mapping $f : X \rightarrow X$ leaves at least one point
fixed, i.e. there exists an \( \hat{x} \in X \) such that \( \hat{x} = f(\hat{x}) \). (Received August 22, 1968.) (Author introduced by Dr. L. E. Schwartz).


A map \( f: M^n \to W^m \) is said to be in the conditionally stable range if \( k \leq n - 21 - 1 \), where \( f \) is \( 1 \)-connected. Using only techniques of surgery, it can be shown that the existence of imbeddings of \( M \) in \( W \) depends only on the existence of certain \( k \)-plane bundles over \( M \) in this range. This is in contrast to the more analytical method of [A. Haefliger, Comment. Math. Helv. 36 (1961), 47-82] where singularities of maps are studied. The results of [R. DeSapio, Ann. of Math. 82 (1965), 213-224] are also recaptured and improved, but in a more general setting. The Main Theorem. Let \( V^3 \) be a 1-connected closed manifold. Given an \( 1 \)-connected map \( f: M^n \to V^n \) of degree +1 and any \( k \)-plane bundle \( \xi \) over \( M \) which is stably equivalent to the stable normal bundle of \( f \), then \( M^n \) imbeds with normal bundle \( \xi \) in the boundary of a manifold \( W^{n+k+1} \) which is homotopy equivalent to \( V^n \), where \( 0 \leq t < [n/2] \), \( k \leq n - 21 - 1 \), and \( n + k \leq 5 \). Corollary. Let \( M^n \) be a closed \( 1 \)-connected manifold, \( 0 \leq t < [n/2] \). If \( k \leq n - 21 - 1 \), \( n + k \leq 5 \), and \( \xi \) is a \( k \)-plane bundle over \( M_0 = M \)-point which is stably equivalent to the stable normal bundle of \( M_0 \), then \( M_0 \) imbeds in \( S^{n+k} \) with normal bundle \( \xi \). This corollary is a typical consequence of the theorem. It gives imbeddings and says in addition which normal bundles are possible. The well-known fact that an \( 1 \)-connected \( M^n \) imbeds in \( S^{2n-1} \) provided \( n \geq 21 + 3 \) also follows easily. (Received September 12, 1968.)


Let \( M \) be a closed 2-manifold of genus \( k \) in \( S^3 \) and \( U \) a component of \( S^3 - M \). The limiting genus of \( U \) is defined as the least nonnegative integer \( n \) such that there exists a sequence of compact 3-manifolds-with-boundary \( H_1, H_2, \ldots \) satisfying (1) \( U = U_{H_1} \), (2) \( H_1 \subset \text{Int} H_{i+1} \), and (3) genus \( \text{Bd} H_i \leq n \) \((i = 1, 2, \ldots)\). If no such integer exists, the limiting genus is said to be infinite. Theorem. Suppose that \( M \) is locally peripherally collared from \( U \) and the limiting genus of \( U \) equals \( n > 0 \). Then there exists a set \( P \) consisting of \( n - k \) points in \( M \) such that \( M \) is locally tame from \( U \) at each point of \( M - P \). Corollary. If \( n = k > 0 \), then \( M \) is tame from \( U \). (Received October 3, 1968.)

69T-G6. WITHDRAWN.


In all the following \( G \) will denote a monotone decomposition of \( E^3 \). Definition. A compact subset \( B \) of \( E^3 \) is shrinkable to near a point \( b \) of \( B \) with respect to \( G \) if for each open set \( U \) containing \( B \setminus \{b\} \) and each \( \epsilon > 0 \) there is a homeomorphism \( h \) of \( E^3 \) onto itself such that \( h \) is the identity on \( E^3 \setminus U \), \( h(B) \subset V(b, \epsilon) \), and for each \( g \in G \) either \( h(g) < \epsilon \) or \( h(g) \subset V(g, \epsilon) \). Theorem 1. Let \( g \in G \) be the union of compact sets \( B_0, B_1 \), and \( A \) such that \( B_0 \cap B_1 = \emptyset \), \( B_1 \cap A = \{b_i\} \) \((i = 1, 2)\), and \( A \) is a tame arc from \( b_0 \) to \( b_1 \). If \( B_1 \) is shrinkable to near \( b_1 \) with respect to \( G \), then \( B_1 \cup A \) is shrinkable to
near $b_0$ with respect to $G$. Definition. A tree is a finite, 1-dimensional, connected, simplicial complex containing no simple closed curves. **Theorem 2.** If $G$ has only countably many nondegenerate elements, each a tame tree, then $E^3/G$ is topologically $E^3$. (Received October 9, 1968.)

69T-G8. L. D. LOVELAND, University of Wisconsin, Madison, Wisconsin 53706. Cross sectionally continuous 2-spheres are tame.

W. T. Eaton [Abstract 656-28, these Notices 15 (1968), 510] and Norman Hosay independently proved that a 2-sphere $S$ in $E^3$ is tame if each horizontal cross section of $S$ is either a point or a simple closed curve. Using a recent result by J. W. Cannon [Abstract 658-163, these Notices 15 (1968), 768] together with a technique given by Bing [Pushing a 2-sphere into its complement, Michigan Math. J. 11 (1964), 33-45] we adjust Eaton's proof to obtain the following Theorem. If each horizontal cross section of a 2-sphere $S$ in $E^3$ is connected and at most countably many such cross sections fail to be locally connected, then $S$ is tame. Our proof depends upon the fact that all but countably many horizontal cross sections of such a 2-sphere must be simple closed curves. (Received October 17, 1968.)

69T-G9. B. H. McCANDLESS, Kent State University, Kent, Ohio 44240. Separability in spaces having a $\sigma$-locally countable basis.

It is a well-known fact that in metric spaces, the properties second countable, separable, and Lindelöf are equivalent. The following is a generalization of that fact. **Theorem.** Let $X$ be a space with a $\sigma$-locally countable basis. Then the following conditions on $X$ are equivalent: (a) $X$ is second countable, (b) $X$ is separable, (c) $X$ is Lindelöf. It is also shown that every space with a $\sigma$-locally countable basis is first countable. (Received October 21, 1968.)


The relationship between the annulus conjecture $A_n$ and stability of homeomorphisms in $R^n$, given by Brown and Gluck [Ann. of Math. 79 (1964), 1-17], is generalized to normed linear spaces. Let $E$ be an infinite-dimensional normed linear space. Let $H(E)$ be the collection of homeomorphisms of $E$ onto itself. By a tame sphere in $E$ is meant the image of the unit sphere in $E$ under an element of $H(E)$. $A_E$ will be the conjecture that the region between two disjoint tame spheres is an annulus. $S_E$ and $I_E$ will be the conjectures that every element of $H(E)$ is stable, isotopic to the identity, respectively. (For finite dimensions, only orientation preserving homeomorphisms are considered.) **Theorem 1.** $S_E$ implies $A_E$. Corollary. $A_E$ is true in all separable, infinite-dimensional Fréchet spaces. **Theorem 2.** $S_E$ implies $I_E$. The proof of Theorem 2 is an immediate extension of the finite-dimensional proof. **Theorem 3.** For those normed linear spaces $E$ which have some hyperplane homeomorphic to $E$, $A_E$ and $I_E$ together imply $S_E$. (The finite-dimensional analog is that $A_n$ and $I_{n-1}$ together are equivalent to $S_n$.) The proofs of Theorems 1 and 3 use techniques not applicable to finite dimensions. (Received October 31, 1968.)
A dendritic continuum is a compact connected Hausdorff space in which each pair of points is separated by a third point (L. E. Ward originally called such spaces "trees"). Let \( T \) be a partially ordered set. Then \( T \) is a tree if and only if \( \{ t \leq a : t \in T \} \) is a chain for each \( a \) in \( T \). The main result characterizes dendritic continua solely in order-theoretic terms. \[ \text{Theorem 1.} \quad D \text{ is a dendritic continuum if and only if there is a partial order } \preceq \text{ on } D \text{ such that } D \text{ is a tree with an infimum, each maximal chain in } D \text{ is both order complete and order dense-in-itself, and } D \text{ has the topology generated by the subbase of all sets of the form } \{ x > a : x \in D \} \text{ or } \{ x \not\geq a : x \in D \} \text{ for } a \text{ in } D. \]

\[ \text{Theorem 2.} \quad \text{Every tree } T \text{ is order isomorphic to a subset of a semilattice tree } T^* \text{ which is cofinal in the non-end points of } T^* \text{ and such that } T^* \text{ is a dendritic continuum.} \]

If \( S \) is a topological semigroup and \( R \) is a congruence on \( S \), then \( S/R \) is not necessarily a topological semigroup. Several conditions are found which guarantee that \( S/R \) be a topological semigroup. \[ \text{Definitions.} \quad \text{Let } X \text{ and } Y \text{ be topological spaces. } \]

- \( X \) is semifirst countable if and only if, for each sequence \( \{ A_i \} \) of disjoint, closed subsets of \( X \) such that \( \bigcup_{i=1}^{\infty} A_i \) is not closed, there exists a limit point \( p \) of \( \bigcup_{i=1}^{\infty} A_i \), and there exists a subsequence \( \{ A_{i_j} \} \) and a sequence \( \{ y_{i_j} \} \) such that \( y_{i_j} \in A_{i_j} \) for each \( i_j \) and \( y_{i_j} \to p \). If \( R \) is an equivalence relation on \( X \) with projection \( p \), then \( R \) is semi-H if and only if a sequence \( x_i \to x \) in \( X \) with \( p(x_i) \to y \) in \( X/R \) implies \( p(x) = y \). Let \( f : X \to Y \) be an onto map. The map \( f \) has Property \( L \) if and only if a net \( y \to y \) in \( Y \) such that \( U f(y) \) is not closed in \( X \) implies that there exists a subnet \( y_\alpha \to y \) of \( y \) and a net \( x_\alpha \) in \( X \) such that \( f(x_\alpha) = y, \) for each \( \alpha \) and \( f(x) = y \). \[ \text{Theorem.} \quad \text{If } S \text{ is a topological semigroup and } R \text{ is a congruence on } S \text{ with projection } p, \text{ then } S/R \text{ is a topological semigroup if any one of the following is true: (1) } R \text{ induces a lower semicontinuous decomposition, (2) } S \text{ and } S/R \text{ are locally compact Hausdorff, (3) } R \text{ induces an upper semicontinuous decomposition with compact elements, (4) } S \text{ is semifirst countable, } S/R \text{ is first countable and } T_1, \text{ and } R \text{ is semi-H, (5) } p \text{ has Property } L'. \]

A regularly embedded, closed surface \( S \) in the 3-manifold \( M \) is incompressible in \( M \) iff one of the following holds: (i) \( S \) a 2-sphere, then \( S \) does not bound a 3-cell in \( M \), (ii) \( S \) not a 2-sphere, then \( \ker(\pi_1(S) \to \pi_1(M)) \) is trivial. \( F \) is a closed, orientable surface. \( g(X) \) = genus of \( X \). \[ \text{Lemma 1.} \quad F^1 \text{ is incompressible in } F \times S^1. \]

Then \( F^1 \) is orientable, and if \( g(F^1) > 1 \), then \( F^1 \) does not separate \( F \times S^1 \). \[ \text{Theorem 1.} \quad F^1 \text{ is incompressible in } F \times S^1 \text{ iff there is an isotopy } h_t \text{ of } F \times S^1 \text{ onto itself so that either (i) there is a nontrivial s.c.c. } J \subset F \text{ and } h_1(F^1) = J \times S^1, \text{ or (ii) if } p \text{ is the projection of } F \times S^1 \text{ onto } F, \text{ then } p/h_1(F^1) \text{ is a local homeomorphism onto } F. \]

\[ \text{Theorem 2.} \quad \text{If } F^1 \text{ is incompressible in } F \times S^1, \text{ then for some } k \geq 0, \text{ } g(F^1) = k(g(F) - 1) + 1. \]

\[ \text{Theorem 3.} \quad \text{If } F \neq S^2, \text{ then for any integer } k > 0, F \times S^1 \text{ can be fibered over } S^1 \text{ with fiber a surface } F' \text{ where } g(F') = k(g(F) - 1) + 1. \]

\[ \text{Corollary.} \quad \text{If } M \text{ is fibered over } S^1 \text{ with fiber } F, \text{ then for } F^1 \text{ incompressible in } M \text{ it is not true that } g(F^1) \geq g(F). \]
Corollary. If $M$ is fibered over $S^1$ with fiber $F$, then there is a nonseparating embedding $j : F \to M$ so that $(F)$ is not equivalent to $\rho^{-1}(s_0)$ for any $s_0 \in S^1$ ($\rho : M \to S^1$ is the projection). (Received October 28, 1968.)

**Miscellaneous Fields**

69T-H1. O. T. ALAS, University of Sao Paulo, Sao Paulo, Brasil. A theorem on measure theory.

Let $X$ be a Hausdorff topological space and $A(X)$ the ring generated by the topology on $X$.

Definition. A positive measure $\mu$ on $A(X)$ satisfies the condition (H) iff (1) $\mu\{x\} = 0$, $\forall x \in X$; (2) $\mu(Z) > 0$ for any nonempty open set $Z$; (3) $\mu(Y) = \inf\{\mu(K)|K \subseteq Y, K \text{ is compact}\}$, for any $Y \in A(X)$. Theorem. Let $X$ be a regular Hausdorff space. If there is a positive measure on $A(X)$, satisfying the condition (H), then every locally finite open covering of $X$ has a countable subcovering. Corollary. Let $X$ be a Hausdorff, nondiscrete, locally compact, topological group. There is a positive measure on $A(X)$, satisfying the condition (H), iff $X$ is $\sigma$-compact. (Received September 19, 1968.)

69T-H2. WITHDRAWN.


A nondeterministic on-line multitape Turing acceptor $M$ operates within time bound $f$ (where $f$ is a total recursive function) if for some $k > 0$ and every input $w$ accepted by $M$, every computation of $M$ which accepts $w$ has no more than $f(k|w|)$ steps (where $|w|$ is the length of $w$). Let $L(M)$ be the language accepted by $M$, and let $\mathcal{T}(f) = \{L(M) : M \text{ operates with time bound } f\}$. The family of quasi-realtime languages is the family $\mathcal{Q} = \mathcal{T}(i)$ where, for all $n$, $i(n) = n$. (See Abstracts 68T-H1, 68T-H6, these Notices: 15 (1968), 813, 814.) Theorem. Let $f$ be a total recursive function such that, for all $x, y > 0$, $f(x) + f(y) \leq f(x + y)$. (i) $\mathcal{T}(f)$ is an abstract family of languages closed under intersection, reversal, and linear erasing. (ii) $L \in \mathcal{T}(f)$ if and only if there is a quasi-realtime language $L_0$, a homomorphism $h$, and a constant $k > 0$ such that $h[L_0] = L$ and for every $w \in L_0$, $|w| \leq f(k|h(w)|)$. Hence, $L \in \mathcal{T}(f)$ if and only if $L = h[L_0 \cap L_2 \cap L_3 ]$ where each $L_i$ is context-free and there is a constant $k > 0$ such that, for every $w \in L_1 \cap L_2 \cap L_3$, $|w| \leq f(k|h(w)|)$. (iii) If $L \in \mathcal{T}(f)$, then there is a nondeterministic on-line Turing acceptor $M$ with one pushdown store and one nonerasing stack as storage tapes such that $M$ operates with time bound $f$ and $L(M) = L$. (Received October 24, 1968.)

69T-H4. V. I. GAVRILOV, Indian Institute of Technology, Bombay, India. The behaviour of meromorphic functions in the neighborhood of an essential singularity.

In the paper (V. I. Gavrilov, Izv. Akad. Nauk, SSSR, Ser. Math. 30 (1966), 767-788; English transl., Amer. Math. Soc. Transl. (2) 71 (1968), 181-201) the distribution of the values of meromorphic (holomorphic) functions in the neighborhood of an essential singularity was studied in terms of $M_h$-sequences ($\mu_h$-sequences). It is shown that any $\mu_h$-sequence for a holomorphic function $f(z)$ is a $M_h$-sequence and conversely any $M_h$-sequence for $f(z)$ contains a subsequence which is a $\mu_h$-sequence. The concept of a $\mu_h$-sequence is extended so as to be applicable to meromorphic as well as holomor-
phic functions and it is shown that above statement remains valid for $\phi$-sequences (in the new sense) and $\Phi$-sequences. (Received October 31, 1968.) (Author introduced by Professor P. C. Jain.)

69T-H5. ANDRE deKORVIN and R. J. EASTON, Indiana State University, Terre Haute, Indiana 47809. On the existence of expectation type maps on $B^*$-algebras.

Let $N$ and $M$ be $B^*$-algebras with identity such that $N \subseteq M$. Let $Z_N$ denote the center of $N$. Let $U(N)$ denote the collection of all unitaries in $N$ and for each $x \in N$, let $C_N(x)$ denote the norm closure of the convex hull of the collection of all $u_xu^*$, $u \in U(N)$. Theorem 1. Suppose $M$ is generated as a vector space by its unitaries and that $\{\phi_a\}$ is a complete set of states on $M$. Suppose that each $\phi_a$ diagonalizes $M$, is normal on $Z_N$, and there exists a constant $k_a$ such that $\phi_a(x^*xy^*) \leq k_a \phi_a(x^*x)\phi_a(y^*y)$, $x, y \in N$. Moreover suppose that for each $x \in N$, $C_N(x) \cap Z_N \neq \emptyset$. Then if $e_a$ is any projection in $Z_N$ with $\phi_a$ faithful on $N_{e_a}$, there exist expectation like maps $\varphi_a : M \to N_{e_a}$ such that $\phi_a(x^*au) = \phi_a(\varphi_a(au))$, $u \in U(M)$, $a \in N$, and furthermore, the $\varphi_a$'s are linear and satisfy $\varphi_a(u^*vu) = u^*\varphi_a(v)u$, $u \in U(N)$, $v \in U(M)$. Theorem 2. If $P$ has the property, whenever $\{\phi_a\}$ is an increasing net of projections in $Z_N$ with $\rho$ faithful on $N_{\phi_a}$, then $\sup_{\phi_a} \phi_a$ is again a projection in $Z_N$, then the carrier of $\rho$ exists. If $Z_N$ satisfies this condition for each $a$, then Theorem 1 holds where $e_a$ is the carrier of $\phi_a$.

(Received October 23, 1968.)

69T-H6. D. F. PHILLIPS, University of Texas, Austin, Texas 78712. Three theorems concerning real functions whose graphs are dense in the plane.

Theorem I. Each real-valued function on the set of all numbers is the sum of 2 functions whose graphs are connected point sets dense in the plane. Theorem II. Each real-valued function on the set of all numbers is the pointwise limit of a sequence of functions whose graphs are connected point sets dense in the plane. Theorem III. If the point set $M$ is the graph of a real function and $M$ is dense in the plane; then $M$ does not separate the plane. (Received October 30, 1968.) (Author introduced by Dr. H. S. Wall.)

69T-H7. P. L. MANLEY, University of Windsor, Windsor, Ontario, Canada. A characteristic property of abelian groups.

We consider the following problem: Let $G$ be a finite group, $[G:1] = p_1^{t_1} \cdots p_n^{t_n}$. If, to each $i$ ($i = 1, \ldots, n$), there is at least one index of $G$ which is a subgroup of $P_i^{t_i}$, then what type of group is $G$? P. Hall [J. London Math. Soc. 12 (1937), 198-200] showed that $G$ is a solvable group. C. V. Holmes [Amer. Math. Monthly 73 (1966), 1113-1114] proved that if the subgroup in question is a normal subgroup then $G$ is a nilpotent group, and, furthermore, that this property is uniquely possessed by nilpotent groups. We show that if the subgroup is a commutative (respectively, cyclic) normal subgroup then $G$ is an abelian (respectively, cyclic) group. Theorem 1. A finite group $G$, $[G:1] = p_1^{t_1} \cdots p_n^{t_n}$, is an abelian group (respectively, cyclic group) if and only if, to each $i$ ($i = 1, \ldots, n$), $G$ has a commutative normal subgroup (respectively, cyclic normal subgroup) with index equal to $P_i^{t_i}$. Furthermore, there is only one such subgroup. We remark that the theorem proved by Holmes is equivalent to Theorem 2. A finite group $G$ is nilpotent if and only if $[G:1] = mn$ such that $(m,n) = 1$ implies $G$ has unique normal subgroups of orders $m$ and $n$. (Received October 31, 1968.) (Author introduced by Professor Maurice Chacron.)
J. C. BRADLEY. Products of representations on discrete groups. Abstract 659-4, Page 904.

Replace \((\prod (1 - (a_i/b_i)))_{i \in I}\) by \((\prod (1 - (a_i/b_i)))_{i \in I}\) and delete the word "reducible" from the abstract.


Lines 4 and 5: Replace "power-field form" by "semi-real field form or trace".

Line 8: Insert at end of line: "in this case congruent to its semi-real trace in the complex plane, ".

R. J. ENSEY. Quotient categories and isomorphism invariants for Abelian \(p\)-groups. Abstract 660-18, Page 1012.

Line 13: "... and \(\max(f_G(a), f_H(a)) < m_1\) whenever \(f_G(a) \neq f_H(a)\)"

should read "... and \(\exists\) a cardinal number \(m_0 < m_1 \geq \max(f_G(a), f_H(a)) \geq m_0\)

whenever \(f_G(a) \neq f_H(a)\)."

The following sentence should be added: "The error was pointed out by D. E. Bertholf of Oklahoma State University who has also generalized the result of this abstract."


Line 5: After "equivalent" insert "over the space of bounded sequences", and between 

"x is a" and "sequence" insert "bounded".


Lines 1 and 4: Replace "u.s.c." by "monotone."
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