# Notices of the American Mathematical Society

Edited by Everett Pitcher and Gordon L. Walker

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MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

<table>
<thead>
<tr>
<th>Meeting No.</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
</tr>
</thead>
<tbody>
<tr>
<td>668</td>
<td>October 25, 1969</td>
<td>Cambridge, Massachusetts</td>
<td>Sept. 9, 1969</td>
</tr>
<tr>
<td>670</td>
<td>November 22, 1969</td>
<td>Claremont, California</td>
<td>Oct. 8, 1969</td>
</tr>
<tr>
<td>672</td>
<td>January 22-26, 1970</td>
<td>Miami, Florida</td>
<td>Nov. 6, 1969</td>
</tr>
<tr>
<td></td>
<td>August 24-28, 1970</td>
<td>Laramie, Wyoming</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(75th Summer Meeting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>January 21-25, 1971</td>
<td>Atlantic City, New Jersey</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(77th Annual Meeting)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be September 2, and October 1, 1969.*

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Second-class postage paid at Providence, Rhode Island, and additional mailing offices.

Copyright ©, 1969 by the American Mathematical Society
Printed in the United States of America
The seventy-fourth summer meeting of the American Mathematical Society will be held at the University of Oregon, Eugene, Oregon, from Tuesday, August 26, through Friday, August 29, 1969. All sessions of the meeting will be held on the campus of the university. The times listed below for the events of the meeting are PACIFIC DAYLIGHT SAVING TIME in all cases.

There will be two sets of Colloquium Lectures. Professor Raoul Bott of Harvard University will present four lectures entitled "On the periodicity theorem of the classical groups and its applications." These addresses will be given on Tuesday, August 26, at 1:30 p.m. and on Wednesday, Thursday, and Friday at 8:30 a.m. The other Colloquium Lecturer will be Professor Harish-Chandra of the Institute for Advanced Study. His topic will be "Harmonic analysis on semisimple Lie groups." Professor Harish-Chandra's four lectures will be given on Tuesday, August 26, at 2:45 p.m. and on Wednesday, Thursday, and Friday at 9:40 a.m. The first two addresses of each series will be presented in the Ballroom of the Erb Memorial Union; the remaining Colloquium Lectures will be given in Room 150 of the Science Building.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be three one-hour addresses. Professor Robion C. Kirby of the University of California, Los Angeles, will speak on Thursday, August 28, at 1:30 p.m. The title of his lecture is "On the existence and uniqueness of triangulations of manifolds." Professor Murray Gerstenhaber of the University of Pennsylvania will present a lecture entitled "Algebraic deformation theory" at 2:45 p.m. on Thursday, August 28. The last invited address at the meeting will be given by Professor Paul F. Baum of Brown University at 1:30 p.m. on Friday, August 29. The title of his talk is "Vector fields and Gauss-Bonnet." All three lectures will be given in the Ballroom of the Erb Memorial Union.

There will be several sessions for contributed ten-minute papers. These sessions are scheduled at 10:50 a.m. on Wednesday, Thursday, and Friday, and at 2:45 p.m. on Friday.

This meeting will be held in conjunction with meetings of the Mathematical Association of America, the Society for Industrial and Applied Mathematics, Pi Mu Epsilon, and Mu Alpha Theta. The Mathematical Association of America will meet from Monday through Wednesday. The Association will present Professor Errett Bishop of the University of California, San Diego, as the Earle Raymond Hedrick Lecturer. The topic of Professor Bishop's lectures is "The constructive point of view." The Hedrick Lectures will be given on Monday, August 25, at 9:15 a.m. and 1:30 p.m.; and on Tuesday at 9:00 a.m. The Society for Industrial and Applied Mathematics will meet on Wednesday. The SIAM program will include the von Neumann Lecture, which will be given at 8:00 p.m. on Wednesday by Professor George Carrier of Harvard University. The title of his address will be "Singular perturbation theory in geophysics." Pi Mu Epsilon and Mu Alpha Theta will meet concurrently with the Society and the Association.

On Thursday, August 28, at 8:00 p.m. in the Ballroom, there will be a special panel on Scientific and Technical Communication reviewing the recently published report of the National Academy of Sciences -- National Academy of Engineering Survey of Scientific and Technical Communication (SATCOM) under that title. The discussion will be lead by a panel under
the chairmanship of Dr. F. J. Weyl, acting president of Hunter College.

COUNCIL AND BUSINESS MEETINGS

The Council of the Society will meet on Tuesday, August 26, at 5:00 p.m. in Room 101 of the Erb Memorial Union. The Business Meeting of the Society will be held on Thursday, August 28, at 4:00 p.m. in the Ballroom of the Erb Memorial Union.

At the Business Meeting, the Council will recommend changes in the By-Laws of the Society, consisting of the repeal of Article III, Section 3, and Article XI, Section 3, together with related editorial changes effective December 31, 1971. The Sections require a Committee on Printing and Publishing and define its function. The amendment would eliminate the committee and would not shorten the term of any Council member.

REGISTRATION

The Registration Desk will be in the Taylor Lounge of the Erb Memorial Union. It will be open on Sunday from 2:00 p.m. to 8:00 p.m.; on Monday from 8:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m. and on Friday from 9:00 a.m. to 1:00 p.m. The telephone number will be 342-1411, Extension 2811.

The registration fees will be as follows:

<table>
<thead>
<tr>
<th>Category</th>
<th>Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member</td>
<td>$3.00</td>
</tr>
<tr>
<td>Member’s family</td>
<td>0.50</td>
</tr>
<tr>
<td>Students</td>
<td>No charge</td>
</tr>
<tr>
<td>Others</td>
<td>7.50</td>
</tr>
</tbody>
</table>

EMPLOYMENT REGISTER

At the January 1969 annual meeting, the Joint Committee on Employment Opportunities voted not to have an Employment Register at the Eugene, Oregon, Meeting.

EXHIBITS

Book exhibits and exhibits of educational media will be displayed in Rooms 108-113 on the first floor of the Erb Memorial Union on Tuesday, Wednesday, and Thursday.
cribs are available for a small charge with advanced notice. All rooms have private baths. Linen, blankets, towels, and soap are furnished; and daily maid service is provided. Rooms in the College Inn will be available from noon Sunday, August 24, through noon Saturday, August 30. Limited off-street parking is available for residents of the College Inn.

Upon arrival on campus, all guests who have reservations for or wish accommodations in the University Residence Halls should go to the main desk in Carson Hall for room assignments. Those with reservations at the College Inn should check in at the main desk of the Inn.

**FOOD SERVICE**

For those staying in the University Residence Halls, meals will be served cafeteria style beginning with lunch on Sunday, August 24, and continuing through lunch on Friday, August 29. Hours for food service are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Breakfast</th>
<th>Lunch</th>
<th>Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7:00 a.m.</strong></td>
<td>7:00 a.m.</td>
<td>12:00 p.m.</td>
<td>5:30 p.m.</td>
</tr>
<tr>
<td><strong>8:15 a.m.</strong></td>
<td></td>
<td>1:00 p.m.</td>
<td>6:30 p.m.</td>
</tr>
</tbody>
</table>

On Wednesday, August 27, there will be a picnic on the Football Practice Field at 5:30 p.m. in lieu of the regular dinner.

For residents of the College Inn, meals will be available cafeteria style beginning with breakfast on Monday, August 25, and continuing through breakfast on Saturday, August 30. Hours and prices for service are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Breakfast</th>
<th>Lunch</th>
<th>Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7:00 a.m.</strong></td>
<td>($1.00)</td>
<td>8:00 a.m.</td>
<td>4:30 p.m.</td>
</tr>
<tr>
<td><strong>8:00 a.m.</strong></td>
<td></td>
<td>9:00 a.m.</td>
<td>6:00 p.m.</td>
</tr>
</tbody>
</table>

The Erb Memorial Union will be open for coffee and snacks during the following hours:

<table>
<thead>
<tr>
<th></th>
<th>Monday through Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7:00 a.m.</strong></td>
<td>7:00 a.m.-10:00 p.m.</td>
<td>7:00 a.m.-11:00 p.m.</td>
<td>9:00 a.m.-11:00 p.m.</td>
<td>9:00 a.m.-10:00 p.m.</td>
</tr>
<tr>
<td><strong>10:00 a.m.</strong></td>
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<td></td>
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<tr>
<td><strong>11:15 a.m.</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>12:00 p.m.</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>1:00 p.m.</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>2:00 p.m.</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>3:00 p.m.</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4:00 p.m.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5:00 p.m.</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>6:00 p.m.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On Monday through Friday, the Erb Memorial Union cafeteria will serve meals at the following times:

<table>
<thead>
<tr>
<th></th>
<th>Breakfast</th>
<th>Lunch</th>
<th>Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7:00 a.m.</strong></td>
<td>7:00 a.m.-10:00 a.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10:00 a.m.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>11:00 a.m.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1:15 p.m.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5:00 p.m.</strong></td>
<td></td>
<td></td>
<td>5:00 p.m.- 6:15 p.m.</td>
</tr>
</tbody>
</table>

On Saturday, August 23 and August 30, breakfast will be served from 9:00 a.m. to 11:00 a.m. and lunch from 11:00 a.m. to 1:15 p.m.

A list of nearby restaurants will be available in the registration area at the Erb Memorial Union.

**HOTELS AND MOTELS**

There are a number of motels and hotels in the area, some of which are listed below with coded information which is explained at the end of the list. Those motels preceded by an asterisk (*) are within walking distance of the University of Oregon. Participants should make their own reservations with hotels and motels.

*CITY CENTER LODGE (503) 344-5233
476 East Broadway - 48 rooms

<table>
<thead>
<tr>
<th>Room Type</th>
<th>Single</th>
<th>Double</th>
<th>Twin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$7.50</td>
<td>9.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.50- 9.00</td>
<td>11.00 - 13.00</td>
<td>12.00 - $16.00</td>
</tr>
<tr>
<td>Extra person</td>
<td>1.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Rollaway beds</td>
<td>1.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hide-a-beds</td>
<td>2.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Code: RT-FP-SP-AC
8 blocks from campus

*CONTINENTAL MOTEL (503) 343-3376
390 East Broadway - 63 rooms

<table>
<thead>
<tr>
<th>Room Type</th>
<th>Single</th>
<th>Double</th>
<th>Twin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$9.00</td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.00 - $16.00</td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td>Extra person</td>
<td>2.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children</td>
<td>2.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Code: RT-FP-SP-TV-AC
5 blocks from campus

*EUGENE HOTEL (503) 344-1461
222 East Broadway - 171 rooms

<table>
<thead>
<tr>
<th>Room Type</th>
<th>Single</th>
<th>Double</th>
<th>Twin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$8.00</td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Code: RT-CL-FP-TV-AC
10 blocks from campus

*MANOR HOTEL (503) 345-2331
599 East Broadway - 26 rooms

<table>
<thead>
<tr>
<th>Room Type</th>
<th>Single</th>
<th>Double</th>
<th>Twin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$7.00</td>
<td>8.00</td>
<td>9.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.50</td>
<td></td>
</tr>
</tbody>
</table>

Code: FP-SP-TV-AC
6 blocks from campus
**Motel Flagstone (503) 343-7725**

1601 Franklin Boulevard - 32 rooms

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>$7.00</td>
<td>$8.00</td>
</tr>
<tr>
<td>Double</td>
<td>10.00</td>
<td>11.00</td>
</tr>
<tr>
<td>Twin</td>
<td>12.00</td>
<td></td>
</tr>
<tr>
<td>Extra person</td>
<td>1.00 - 2.00</td>
<td></td>
</tr>
</tbody>
</table>

Code: FP-SP-TV-AC

3 blocks from campus

**New Oregon Motel (503) 345-8731**

1655 Franklin Boulevard - 72 rooms

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>$11.00</td>
<td></td>
</tr>
<tr>
<td>Double</td>
<td>13.00</td>
<td></td>
</tr>
<tr>
<td>Twin</td>
<td>15.00</td>
<td></td>
</tr>
<tr>
<td>Extra person</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

Code: RT-FP-SP-TV-AC

3 blocks from campus

**Thunderbird Motel (503) 342-5201**

205 Coburg Road - 130 rooms

<p>| | | |</p>
<table>
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</thead>
<tbody>
<tr>
<td>Single</td>
<td>$10.00</td>
<td></td>
</tr>
<tr>
<td>Double</td>
<td>13.00</td>
<td></td>
</tr>
<tr>
<td>Twin</td>
<td>16.00</td>
<td></td>
</tr>
<tr>
<td>Extra person</td>
<td>3.00</td>
<td></td>
</tr>
</tbody>
</table>

Code: RT-SP-CL-TV-FP-AC

2 miles from campus

**The Timbers (503) 343-3345**

1015 Peal Street - 60 rooms

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>$9.00</td>
<td></td>
</tr>
<tr>
<td>Double</td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td>Twin</td>
<td>13.00</td>
<td></td>
</tr>
<tr>
<td>Extra person (Single)</td>
<td>$2, (Twin)</td>
<td>$1</td>
</tr>
</tbody>
</table>

Code: FP-TV-AC

8 blocks from campus

**Travel Inn Motel**

2121 Franklin Boulevard - 115 rooms

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>$8.50</td>
<td></td>
</tr>
<tr>
<td>Double</td>
<td>10.00</td>
<td></td>
</tr>
<tr>
<td>Twin</td>
<td>12.00 - $14.00</td>
<td>$16.00</td>
</tr>
</tbody>
</table>

Suites to fit any need and priced according to number of people occupying same

Code: RT-CL-FP-SP-TV-AC

5 blocks from campus

RT-Restaurant SP-Swimming Pool
CL-Cocktail Lounge TV-Television
FP-Free Parking AC-Air Conditioned

**Entertainment**

There will be a reception on Tuesday, August 26, from 4:00 p.m. to 6:00 p.m. for adults only in the University Art Museum.

On Thursday, August 28, there will be a bus excursion to Crater Lake National Park. The tour, from 8:30 a.m. to 5:00 p.m., will follow different routes to and from the park. There will be a box lunch picnic on the rim of the lake. The charge for the tour will be $8.00, and tickets will be on sale in the registration area. Tickets will be slightly less for residents of the dormitories. This tour is not recommended for small children.

The traditional SIAM Beer Party will be held on Wednesday, August 27, at 9:00 p.m. at the College Inn following the von Neumann Lecture.

A picnic will be held on Wednesday, August 27, at 5:30 p.m. on the Football Practice Field. In case of rain, it will be held at Hayward Field. Tickets are provided for residents of the University Residence Halls. For others, tickets will be on sale in the Registration area, $3.00 for adults and $1.50 for children 11 years of age and under.

Several tours will be available, including tours of the university collection of Oriental art, the Chase Gardens, and the Weyerhaeuser lumber and paper mill. Also open for visitors will be the Museum, the Miniature Wagon Museum, and the Museum of Natural History. Recreational facilities on campus include bowling, swimming, tennis, and canoeing. There are public golf courses nearby.

**Travel**

During the summer, Western Oregon is on Pacific Daylight Saving Time.

Eugene is serviced by United Airlines and Air West, both with connecting flights from Portland and San Francisco. Greyhound and Trailway bus lines have frequent service to Portland with several express buses. Greyhound has good service to San Francisco. Southern Pacific maintains one passenger train a day from the north and one from the south. Those driving from the east have several interesting possibilities. For example, State Highway 242 over the McKenzie Pass from Bend to Eugene goes through spectacular lava beds, mountain, forest, and river scenery. For further information about other routes and about the many vacation opportunities in Oregon, write to the Travel
CAMPING

There are several excellent campgrounds and trailer parks within driving distance of Eugene. For a complete list of these facilities, write to the Convention Bureau, Eugene Area Chamber of Commerce, 230 East Broadway, Eugene, Oregon 97401.

Western Oregon is richly endowed by nature with outdoor recreational opportunities for the camper, fisherman, hiker and climber, as well as the sightseer. Advance information about these can be obtained from either of the above organizations. Further information will be available in the registration area.

PARKING

At this time of year, there should be ample parking on campus within a few minutes' walk of the meeting rooms and the dormitories. Some of the larger lots have been indicated on the accompanying map. Parking permits will not be required.

WEATHER

Typical temperatures in Eugene for the last week of August range from highs of about 80° to lows of about 50°. Since it is usually rather cool in the evening, long sleeves or sweaters will be necessary. Rain is unusual in August, but it has occurred. At this time of year, Eugene can experience considerable smoke from agricultural burning.

BOOKSTORE

The University Co-operative Store, 895 E. 13th Avenue, is open from 8:15 a.m. to 5:00 p.m., Monday through Friday.

LIBRARIES

The Science Library, located in the basement of the Science Building, will be open from Monday through Thursday 8:00 a.m. to 10:00 p.m.; Friday, 8:00 a.m. to 9:00 p.m.; Saturday, 8:00 a.m. to 5:00 p.m.; Sunday, 2:00 p.m. to 10:00 p.m. The Main Library will be open from Monday through Friday, 7:30 a.m. to 9:30 p.m.; Saturday, 7:30 a.m. to 5:00 p.m.; Sunday, 2:00 p.m. to 9:30 p.m.

MEDICAL SERVICE

Emergency medical service will be available at Sacred Heart General Hospital, 12th and Alder Street, 24 hours a day.

ADDRESS FOR MAIL AND TELEGRAMS

Individuals may be addressed at Mathematical Meetings, Erb Memorial Union, University of Oregon, Eugene, Oregon 97403. The telephone number of the Message Center will be 503-342-1411, Extension 2812.

COMMITTEE

### TIMETABLE
(Pacific Daylight Saving Time)

<table>
<thead>
<tr>
<th>SUNDAY</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 24</td>
<td>9:00 a.m. - 4:00 p.m.</td>
<td>Board of Governors</td>
</tr>
<tr>
<td></td>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>Erb Memorial Union 101</td>
</tr>
<tr>
<td></td>
<td>7:00 p.m.</td>
<td>REGISTRATION - Erb Memorial Union, Taylor Lounge</td>
</tr>
<tr>
<td></td>
<td>7:00 p.m. - 7:10 p.m.</td>
<td>Ballroom</td>
</tr>
<tr>
<td></td>
<td>7:11 p.m. - 7:19 p.m.</td>
<td>Films</td>
</tr>
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<td>7:20 p.m. - 7:30 p.m.</td>
<td>ALLENDOERFER FILMS ON ARITHMETIC AND SET THEORY (animated and in color)</td>
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<td>7:45 p.m. - 8:29 p.m.</td>
<td>ASSOCIATIVE PROPERTY</td>
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<td>8:40 p.m. - 9:07 p.m.</td>
<td>DISTRIBUTIVE PROPERTY</td>
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<td>FILMS OF THE UNIVERSITY OF ILLINOIS ARITHMETIC PROJECT</td>
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<td></td>
<td>SOME ARTIFICIAL OPERATIONS, a fourth grade taught by Miss Phyllis Klein</td>
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<td>GRAPHING WITH SQUARE BRACKETS, a fifth grade with Professor David A. Page</td>
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<thead>
<tr>
<th>MONDAY</th>
<th>AMS</th>
<th>MAA</th>
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<tbody>
<tr>
<td>August 25</td>
<td>8:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Erb Memorial Union, Taylor Lounge</td>
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<tr>
<td></td>
<td>9:00 a.m. - 9:15 a.m.</td>
<td>Ballroom</td>
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<td></td>
<td>9:15 a.m. - 10:15 a.m.</td>
<td>Welcome on Behalf of the University</td>
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<td>10:30 a.m. - 11:30 a.m.</td>
<td>The Earle Raymond Hedrick Lectures: The Constructive Point of View, Lecture I E. A. Bishop</td>
</tr>
<tr>
<td></td>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Finite-Dimensional Hilbert Spaces P. R. Halmos</td>
</tr>
<tr>
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<td>2:40 p.m. - 3:40 p.m.</td>
<td>The Earle Raymond Hedrick Lectures: Lecture II E. A. Bishop</td>
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<tr>
<td></td>
<td>3:50 p.m.</td>
<td>Pitfalls in Automatic Computation, or Why a Math Book Isn’t Enough G. E. Forsythe</td>
</tr>
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<td>3:50 p.m. - 4:30 p.m.</td>
<td>Panel Discussion on Assistance to Developing Colleges F. M. Stewart, Moderator</td>
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<td>4:30 p.m. - 4:50 p.m.</td>
<td>Presentations by Members of the Panel: L. L. Clarkson C. F. Smith W. E. Marsh</td>
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<td>7:30 p.m.</td>
<td>General Discussion by the Panel and the Audience</td>
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<td>SIAM Council Meeting - Eugene Hotel, Showcase Room</td>
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<tr>
<td></td>
<td>7:00 p.m.</td>
<td>Ballroom</td>
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<td>7:00 p.m. - 7:43 p.m.</td>
<td>Films</td>
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<td>FILMS OF THE MAA MATHEMATICS TODAY SERIES</td>
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<td>PREDICTING AT RANDOM, A Lecture by David Blackwell (in color)</td>
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<tr>
<td>7:55 p.m. - 9:00 p.m.</td>
<td>THE CLASSICAL GROUPS AS A SOURCE OF ALGEBRAIC PROBLEMS, A Lecture by Charles Curtis (b &amp; w)</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Erb Memorial Union, Taylor Lounge</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Erb Memorial Union 108-113</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>The Earle Raymond Hedrick Lectures: Lecture III E. A. Bishop</td>
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<tr>
<td>10:10 a.m. - 11:00 a.m.</td>
<td>Business Meeting, Presentation of Lester R. Ford Awards</td>
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<tr>
<td>11:10 a.m. - 12:00 p.m.</td>
<td>What is Convexity? Victor Klee</td>
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<tr>
<td>12:15 p.m.</td>
<td>Colloquium Lectures: On the periodicity theorem of the classical groups and its applications, Lecture I Raoul Bott Ballroom</td>
<td></td>
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<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>Colloquium Lectures: Harmonic analysis on semisimple Lie groups, Lecture I Harish-Chandra Ballroom</td>
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<tr>
<td>3:15 p.m.</td>
<td>PI MU EPSILON Contributed Papers - Science Building 150</td>
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<tr>
<td>4:00 p.m. - 6:00 p.m.</td>
<td>RECEPTION - University Art Museum</td>
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<tr>
<td>5:00 p.m.</td>
<td>Council Meeting, Erb Memorial Union 101</td>
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<tr>
<td>6:30 p.m.</td>
<td>PI MU EPSILON Banquet - Erb Memorial Union 213-214</td>
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<tr>
<td>7:00 p.m.</td>
<td>Films</td>
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<tr>
<td>7:00 p.m. - 7:12 p.m.</td>
<td>SETS, CROWS AND INFINITY</td>
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<tr>
<td>7:13 p.m. - 7:23 p.m.</td>
<td>BIG NUMBERS. . . LITTLE NUMBERS</td>
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<td>7:24 p.m. - 7:28 p.m.</td>
<td>THE SEVEN BRIDGES OF KÖNIGSBERG</td>
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<td>7:29 p.m. - 7:41 p.m.</td>
<td>POSSIBLY SO PYTHOGORAS</td>
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<tr>
<td>7:42 p.m. - 7:50 p.m.</td>
<td>NEWTON'S EQUAL AREAS</td>
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<td>7:51 p.m. - 7:58 p.m.</td>
<td>TRIO FOR THREE ANGLES</td>
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<tr>
<td>8:15 p.m. - 8:25 p.m.</td>
<td>LIMIT by R. C. Fisher</td>
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<td>8:26 p.m. - 8:36 p.m.</td>
<td>NEWTON'S METHOD by H. S. Wilf</td>
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<tr>
<td>8:37 p.m. - 8:47 p.m.</td>
<td>THE DEFINITE INTEGRAL AS A LIMIT by R. C. Fisher</td>
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<tr>
<td>8:48 p.m. - 8:58 p.m.</td>
<td>THE THEOREM OF THE MEAN by F. P. Welch</td>
<td></td>
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<tr>
<td>8:59 p.m. - 9:07 p.m.</td>
<td>VOLUME BY SHELLS by G. F. Leger</td>
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<tr>
<td>9:08 p.m. - 9:18 p.m.</td>
<td>A FUNCTION IS A MAPPING by A. G. Fadell</td>
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<thead>
<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>8:00 a.m.</td>
<td>PI MU EPSILON Breakfast - Erb Memorial Union 115-116</td>
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<tr>
<td>8:30 a.m. - 9:30 a.m.</td>
<td>Colloquium Lectures: Lecture II&lt;br&gt;Raoul Bott&lt;br&gt;Ballroom</td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Erb Memorial Union, Taylor Lounge&lt;br&gt;EXHIBITS - Erb Memorial Union 108-113</td>
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<tr>
<td>9:00 a.m.</td>
<td>MU ALPHA THETA Governing Council - Erb Memorial Union 213</td>
</tr>
<tr>
<td>9:40 a.m. - 10:40 a.m.</td>
<td>Colloquium Lectures: Lecture II&lt;br&gt;Harish-Chandra&lt;br&gt;Ballroom</td>
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<tr>
<td>10:40 a.m.</td>
<td>PI MU EPSILON Contributed Papers - Science Building 150</td>
</tr>
<tr>
<td>10:40 a.m. - 12:15 p.m.</td>
<td>Session on Analysis I&lt;br&gt;Deady Hall 102</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Analysis II&lt;br&gt;Deady Hall 106</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Analysis III&lt;br&gt;Deady Hall 208</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Applied Mathematics&lt;br&gt;Allen Hall 214</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Topology I&lt;br&gt;Science Building 16</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Algebra I&lt;br&gt;Allen Hall 215</td>
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<tr>
<td>10:50 a.m. - 12:00 p.m.</td>
<td>Session on Number Theory&lt;br&gt;Allen Hall 216</td>
</tr>
<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>The Application of Mathematics to Mathematics&lt;br&gt;R. P. Feynman</td>
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<tr>
<td>2:40 p.m.</td>
<td>Panel Discussion on Mathematics in Two-Year Colleges&lt;br&gt;J. F. Ellis, Moderator</td>
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<tr>
<td>2:40 p.m. - 3:25 p.m.</td>
<td>Presentations by Members of the Panel: A Non-Mathematician Looks at Junior College Mathematics&lt;br&gt;J. F. Ellis</td>
</tr>
<tr>
<td>3:25 p.m. - 4:10 p.m.</td>
<td>A Comprehensive Community College Mathematics Program&lt;br&gt;J. C. Knutson</td>
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<tr>
<td>5:30 p.m.</td>
<td>General Discussion by the Panel and the Audience</td>
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<tr>
<td>8:00 p.m.</td>
<td>PICNIC - Football Practice Field</td>
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<tr>
<td>8:00 p.m.</td>
<td>SIAM - VON NEUMANN LECTURE - Singular perturbation theory and geophysics - George Carrier - Ballroom</td>
</tr>
<tr>
<td>9:00 p.m.</td>
<td>SIAM BEER PARTY - College Inn</td>
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<td>8:30 a.m. - 5:00 p.m.</td>
<td>CRATER LAKE EXCURSION</td>
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<td>8:30 a.m. - 9:30 a.m.</td>
<td>Colloquium Lectures: Lectures III</td>
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<td>Raoul Bott</td>
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<td>9:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Erb Memorial Union, Taylor Lounge</td>
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<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Erb Memorial Union 108-113</td>
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<tr>
<td>9:00 a.m. - 12:30 p.m.</td>
<td>CBMS Council Meeting, Eugene Hotel, Alcove Room</td>
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<tr>
<td>9:40 a.m. - 10:40 a.m.</td>
<td>Colloquium Lectures: Lecture III</td>
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<td>Science Building 150</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Analysis IV</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Analysis V</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Topology and Geometry</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Topology II</td>
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<tr>
<td>10:50 a.m. - 12:15 p.m.</td>
<td>Session on Logic and Foundations</td>
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<tr>
<td>10:50 a.m. - 11:45 a.m.</td>
<td>Session on Algebra II</td>
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<tr>
<td>10:50 a.m. - 11:45 a.m.</td>
<td>Session on Algebra III</td>
</tr>
<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>Invited address: On the existence and uniqueness of triangulations of manifolds</td>
</tr>
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<td>Invited address: Algebraic deformation theory</td>
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<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>Invited address: Algebraic deformation theory</td>
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<td>Invited address: Algebraic deformation theory</td>
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<tr>
<td>4:00 p.m.</td>
<td>Business Meeting</td>
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<tr>
<td>8:00 p.m.</td>
<td>SATCOM Panel - &quot;The use of hard-won information and insight&quot;</td>
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<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Mathematical Association of America</th>
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</thead>
</table>
| 1:30 p.m. - 2:30 p.m. | Invited address: Vector fields and Gauss-Bonnet  
Paul F. Baum  
Ballroom |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Analysis VIII  
Deady Hall 106 |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Analysis IX  
Deady Hall 208 |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Analysis X  
Deady Hall 102 |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Statistics, Probability, and Numerical Analysis  
Allen Hall 214 |                                                      |
| 2:45 p.m. - 4:55 p.m. | Session on Topology V  
Science Building 16 |                                                      |
| 2:45 p.m. - 4:55 p.m. | Session on Geometry  
Science Building 30 |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Algebra VI  
Allen Hall 215 |                                                      |
| 2:45 p.m. - 4:40 p.m. | Session on Algebra VII  
Allen Hall 216 |                                                      |
| 8:30 a.m. - 9:30 a.m. | Colloquium Lectures: Lecture IV  
Raoul Bott  
Science Building 150 |                                                      |
| 9:00 a.m. - 1:00 p.m. | REGISTRATION - Erb Memorial Union, Taylor Lounge |                                                      |
| 9:40 a.m. - 10:40 a.m. | Colloquium Lectures: Lecture IV  
Harish-Chandra  
Science Building 150 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Analysis VI  
Deady Hall 106 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Analysis VII  
Deady Hall 208 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Topology III  
Deady Hall 102 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Topology IV  
Allen Hall 214 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Algebra IV  
Allen Hall 215 |                                                      |
| 10:50 a.m. - 12:15 p.m. | Session on Algebra V  
Allen Hall 216 |                                                      |
PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain the schedule, the time limit will be strictly enforced.

TUESDAY, 1:30 P.M.

Colloquium Lecture I, Ballroom
On the periodicity theorems of the classical groups and its applications
Professor Raoul Bott, Harvard University

TUESDAY, 2:45 P.M.

Colloquium Lecture I, Ballroom
Harmonic analysis on semisimple Lie groups
Professor Harish-Chandra, Institute for Advanced Study

WEDNESDAY, 8:30 A.M.

Colloquium Lecture II, Ballroom
On the periodicity theorem of the classical groups and its applications
Professor Raoul Bott, Harvard University

WEDNESDAY, 9:40 A.M.

Colloquium Lecture II, Ballroom
Harmonic analysis on semisimple Lie groups
Professor Harish-Chandra, Institute for Advanced Study

WEDNESDAY, 10:50 A.M.

Session on Analysis I, Room 102, Deady Hall
10:50-11:00
(1) Kolmogorov type consistency theorems for products of locally compact, B-compact spaces
Professor Ronald B. Kirk, Southern Illinois University (667-123)
11:05-11:15
(2) Existence and substitution for weighted g-summability
Professor Fred M. Wright, Iowa State University (667-143)
11:20-11:30
(3) Dual spaces of weighted spaces
Professor W. H. Summers, University of Arkansas (667-34)
11:35-11:45
(4) Radon-Nikodym densities and Jacobians
Professor Mitsuru Nakai, University of California, Los Angeles, and Nagoya University, Japan (667-142)
11:50-12:00
(5) On a characterization of absolute continuity in locally compact Hausdorff space
Professor Jamil A. Siddiqi, University of Sherbrooke (667-128)
12:05-12:15
(6) A characterization of amenability
Professor Melven R. Krom, University of California, Davis, and Professor Myren Krom*, Sacramento State College (667-153)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
WEDNESDAY, 10:50 A.M.

Session on Analysis II, Room 106, Deady Hall
10:50-11:00
(7) Canonical example of a general complemented algebra. Preliminary report
Professor George R. Giellis, U. S. Naval Academy, and Professor Parfeny
P. Saworotnow*, Catholic University of America (667-147)
11:05-11:15
(8) A characterization of C*-algebras. Preliminary report
Professor Ellen Torrance, Mount Holyoke College (667-77)
11:20-11:30
(9) On the zeros of functions with derivatives in $H_1$ and $H_{\infty}$
Professor James H. Wells, University of Kentucky (667-91)
11:35-11:45
(10) Uniform algebras with nonextendible homomorphisms
Professor Raymond W. Honerlah, Marquette University (667-178)
11:50-12:00
(11) A converse of the Stone-Weierstrass theorem
Professor Robert E. Mullins, Marquette University (667-49)
12:05-12:15
(12) The Stone-Weierstrass theorem and completeness of orthogonal systems
Mr. Bruce D. Craven, University of Melbourne, Australia, and University
of Washington (667-165)

WEDNESDAY, 10:50 A.M.

Session on Analysis III, Room 208, Deady Hall
10:50-11:00
(13) Differentiable functions from $C^\alpha(M)$ to $C^\alpha(M)$
Professor Robert R. Welland, Northwestern University (667-156)
11:05-11:15
(14) Absolute Abel-type summability methods
Professor David Borwein and Mr. Jawaid H. Rizvi*, University of West­
ern Ontario (667-99)
11:20-11:30
(15) On a fundamental result of Zeller in summability
Professor John J. Sember, Simon Fraser University (667-96)
11:35-11:45
(16) Expansions of confluent functions
Professor Evelyn Frank, University of Illinois at Chicago Circle (667-97)
12:05-12:15
(18) On simultaneous Chebyshev approximations with polynomials
Professor David A. Sprecher, University of California, Santa Barbara
(667-50)

WEDNESDAY, 10:50 A.M.

Session on Applied Mathematics, Room 214, Allen Hall
10:50-11:00
(19) Griffiths' theorems for the ferromagnetic Heisenberg model
Professor C. A. Hurst, University of Adelaide, South Australia, and
Professor Seymour Sherman*, Indiana University (667-82)
11:05-11:15
(20) The steady-state thermoelastic mixed boundary-value problem for the elastic layer
Professor Ranjit S. Dhaliwal, University of Calgary (667-179)

11:20-11:30
(21) Simple waves in multidimensional gas flow
Dr. G. M. Schindler, McDonnell Douglas Corporation, Douglas Aircraft Division, Long Beach, California (667-120)

11:35-11:45
(22) Hydrodynamic flow due to an axially oscillating infinite cylinder
Dr. K. L. Arora* and Dr. P. R. Gupta, Punjab Engineering College, Chandigarh, India (667-4)
(Introduced by Professor R. P. Bambah)

11:50-12:00
(23) A vibration problem in parabolic coordinates
Professor Willie R. Callahan, St. John's University (667-31)

12:05-12:15
(24) Axisymmetric Boussinesque's problem with couple stresses
Professor Pratap Puri, Louisiana State University in New Orleans (667-180)
(Introduced by Professor P. K. Kulshrestha)

Wednesdays, 10:50 A.M.

Session on Topology I, Room 16, Science Building
10:50-11:00
(25) Estimates for the number of real-valued continuous functions
Professor W. Wistar Comfort* and Professor Anthony W. Hager, Wesleyan University (667-136)

11:05-11:15
(26) Structure spaces and Wallman-type compactifications
Mr. Charles M. Biles, Humboldt State College (667-121)

11:20-11:30
(27) On extending spaces using inverse limits
Professor Phillip Zenor, Auburn University (667-78)

11:35-11:45
(28) Nest generated intersection rings in Tychonoff spaces
Professor Anne K. S. Steiner* and Professor Eugene F. Steiner, Iowa State University (667-167)

11:50-12:00
(29) On the lattices of compactifications. II. Preliminary report
Mr. T. Thrivikraman, Madurai University, India (667-16)
(Presented by Professor M. Rajagopalan)

12:05-12:15
(30) On spaces of countable and pointwise-countable type. Preliminary report
Professor Jerry E. Vaughan, University of North Carolina at Chapel Hill (667-150)

Wednesdays, 10:50 A.M.

Session on Algebra I, Room 215, Allen Hall
10:50-11:00
(31) On a result of Schenkman on products of abelian groups
Professor William R. Scott, University of Utah (667-137)

11:05-11:15
(32) Centralizing endomorphisms of Moufang loops. Preliminary report
Professor Hala Pflugfelder* and Professor Orin Chein, Temple University (667-36)
11:20-11:30
(33) One-parameter inverse semigroups
Professor Carl A. Eberhart* and Professor John Selden, University of Kentucky (667-29)

11:35-11:45
(34) Commutative power joined cancellative semigroups without idempotent
Professor Takayuki Tamura*, University of California, Davis, and Professor Mirio Sasaki, Iwate University, Japan (667-93)

11:50-12:00
(35) On injective semilattices
Professor Naoki Kimura, University of Arkansas (667-54)

WEDNESDAY, 10:50 A.M.

Session on Number Theory, Room 216, Allen Hall
10:50-11:00
(36) An interesting conjecture regarding Genocchi numbers
Professor J. M. Gandhi, York University (667-176)

11:05-11:15
(37) A simplified proof of Chebyshev's theorem
Mrs. Nina Spears, Kansas State University (667-163)
(Introduced by Professor William T. Spears)

11:20-11:30
(38) A function in the theory of Diophantine approximation
Dr. Thomas W. Cusick, State University of New York at Buffalo (667-33)

11:35-11:45
(39) Infinite nonsolvable classes of the Delaunay-Nagell Diophantine equation
x^3 + my^3 = 1
Professor Leon Bernstein, Illinois Institute of Technology (667-170)

11:50-12:00
(40) On bases for the set of integers
Professor C. T. Long, Washington State University, and Professor Norman Woo*, Fresno State College (667-13)

THURSDAY, 8:30 A.M.

Colloquium Lecture III, Room 150, Science Building
On the periodicity theorem of the classical groups and its applications
Professor Raoul Bott, Harvard University

THURSDAY, 9:40 A.M.

Colloquium Lecture III, Room 150, Science Building
Harmonic analysis on semisimple Lie groups
Professor Harish-Chandra, Institute for Advanced Study

THURSDAY, 10:50 A.M.

Session on Analysis IV, Room 106, Deady Hall
10:50-11:00
(41) Positive length but zero capacity
Professor John B. Garnett, University of California, Los Angeles (667-161)

11:05-11:15
(42) Martin boundary induced by a transition operator
Dr. S. P. Lloyd, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (667-30)
11:20-11:30
(43) Extremal elements of the convex cone $A_n$ of functions
Mr. Roy M. Rakestraw, Oklahoma State University (667-114)

11:35-11:45
(44) Geometrical properties of equipotential surfaces
Professor Browne Shaffer, Fairfield University (667-134)

11:50-12:00
(45) Potential theory on Banach spaces of functions
Mr. Peter A Fowler, California State College, Hayward (667-168)

12:05-12:15
(46) Extreme points of the unit cell in Lebesgue-Bochner functions spaces. Preliminary report
Professor K. Sundaresan, Carnegie-Mellon University (667-9)

THURSDAY, 10:50 A.M.

Session on Analysis V, Room 208, Deady Hall
10:50-11:00
(47) A new operator in the principal function problem
Professor Leo Sario*, University of California, Los Angeles, and Professor Mitsuru Nakai, University of California, Los Angeles, and Nagoya University, Japan (667-141)

11:05-11:15
(48) A uniqueness theorem for second order quasilinear hyperbolic equations
Professor Albert E. Hurd, University of California, Los Angeles (667-151)

11:20-11:30
(49) On bounded in the mean solutions of the equations $\Delta u = Pu$
Mr. Kwang-Nan Chow and Professor Moses Glasner*, California Institute of Technology (667-59)

11:35-11:45
(50) The energy equality for weak solutions of the Navier-Stokes equations
Professor Marvin Shinbrot, Northwestern University (667-76)

11:50-12:00
(51) On a solution to the two-dimensional supersonic initial value problem of a nonviscous compressible fluid by Bergman's integral operator method
Dr. Paul Rosenthal, Stanford University (667-112)

12:05-12:15
(52) A remark on the Bergman metric
Mr. Jacob Burbea, Stanford University (667-129)

THURSDAY, 10:50 A.M.

Session on Topology and Geometry, Room 102, Deady Hall
10:50-11:00
(53) Mutual aposyndesis in products of continua. Preliminary report
Mr. Leland E. Rogers, University of California, Riverside (667-46)

11:05-11:15
(54) Continua not separated by any nonaposyndetic subcontinuum
Professor Eldon J. Vought, California State Polytechnic College, Pomona (667-32)

11:20-11:30
(55) Some order theoretic characterizations of the 3-cell
Professor Edward D. Tymchatyn, University of Saskatchewan (667-43)

11:35-11:45
(56) Commutative rims in clans with zero
Professor Edward N. Ferguson, University of Florida (667-47)

(57) WITHDRAWN.
11:50-12:00
(58) A refinement of the four-vertex theorem
Professor Stanley B. Jackson, University of Maryland (667-58)

THURSDAY, 10:50 A.M.

Session on Topology II, Room 214, Allen Hall
10:50-11:00
(59) Atiyah-Jänich theory in von Neumann algebras
Professor Manfred Breuer, University of Kansas (667-41)

11:05-11:15
(60) Constructing G-spaces. Preliminary report
Mr. Louis A. Feldman, Stanislaus State College (667-182)

11:20-11:30
(61) Rational homotopy type and cohomology operations
Professor David P. Kraines, Haverford College (667-25)

11:35-11:45
(62) On extending homeomorphisms to Fréchet manifolds
Professor Richard D. Anderson and Mr. John D. McCharen*, Louisiana State University (667-124)

11:50-12:00
(63) The product theorem for nonseparable infinite-dimensional manifolds
Professor Richard M. Schori, Louisiana State University (667-158)

12:05-12:15
(64) Subspaces of Banach spaces whose duals are L₁ spaces
Mr. Mordecai Zippin, University of California, Berkeley (667-85)

THURSDAY, 10:50 A.M.

Session on Logic and Foundations, Room 16, Science Building
10:50-11:00
(65) A canonical form for the additive group of nonstandard models of arithmetic
Professor Robert G. Phillips, University of South Carolina (667-133)

11:05-11:15
(66) Z is not translatable into the ramified "Principia"
Mr. James R. Royse, San Francisco State College (667-183)

11:20-11:30
(67) Russell's paradox in set spaces (Axiomatic set theory is not set theory)
Professor Hidegoro Nakano, Wayne State University (667-27)

11:35-11:45
(68) Uncountable does not imply nonpairable
Mr. K. Demys, Santa Barbara, California (667-63)

11:50-12:00
(69) Toward perfect proofs
Professor Robert L. Stanley, Portland State University (667-20)

12:05-12:15
(70) A generalized unified field theory. Preliminary report
Professor Alvin C. Sugar, California State Polytechnic College (667-21)

THURSDAY, 10:50 A.M.

Session on Algebra II, Room 215, Allen Hall
10:50-11:00
(71) Valuations, primes and irreducibility in rings of polynomials and rational functions
Dr. Ronald P. Brown, University of Oregon (667-122)

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11:05-11:15
(72) α-Liouvillian extensions and \( \omega \)-simple-\( \alpha \)-differential extensions of \( \alpha \)-differential fields
Professor Abdul M. Sayied, Boston College (667-98)

11:20-11:30
(73) Derivations of Lie algebras. IV
Professor George F. Leger*, Tufts University, and Professor Eugene M. Luks, Bucknell University (667-130)

11:35-11:45
(74) A unification of the theories of Jordan and alternative algebras
Professor Richard E. Block, University of California, Riverside (667-160)

THURSDAY, 10:50 A.M.

Session on Algebra III, Room 216, Allen Hall
10:50-11:00
(75) On similarity of ST and TS
Professor Charles S. Ballantine, Oregon State University (667-24)

11:05-11:15
(76) Theorems of the alternative for complex linear inequalities
Professor Adi Ben-Israel, Northwestern University (667-5)

11:20-11:30
(77) A note on cones of positive operators
Professor Emilie V. Haynsworth, Auburn University (667-135)

11:35-11:45
(78) Some criteria for completely positive quadratic forms. Preliminary report
Professor Thomas L. Markham, University of South Carolina (667-83)

THURSDAY, 1:30 P.M.

Invited Address, Ballroom

On the existence and uniqueness of triangulations of manifolds
Professor Robion C. Kirby, University of California, Los Angeles

THURSDAY, 2:45 P.M.

Invited Address, Ballroom

Algebraic deformation theory
Professor Murray Gerstenhaber, University of Pennsylvania

THURSDAY, 4:00 P.M.

Business Meeting of the Society, Ballroom

FRIDAY, 8:30 A.M.

Colloquium Lecture IV, Room 150, Science Building

On the periodicity theorem of the classical groups and its applications
Professor Raoul Bott, Harvard University

FRIDAY, 9:40 A.M.

Colloquium Lecture IV, Room 150, Science Building

Harmonic analysis on semisimple Lie groups
Professor Harish-Chandra, Institute for Advanced Study

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Session on Analysis VI, Room 106, Deady Hall
10:50-11:00

(79) Some self-dual locally compact groups
Dr. Lawrence J. Corwin, Massachusetts Institute of Technology (667-173)

11:05-11:15

(80) Multipliers and sets of uniqueness for $L^p$
Dr. Alessandro Figà-Talamanca, University of California, Berkeley, and
Dr. G. I. Gaudry*, Yale University (667-11)

11:20-11:30

(81) A Plancherel theorem for positive definite measures on locally compact
abelian groups
Professor Loren N. Argabright* and Mr. Jesús Gil de Lamadrid, University
of Minnesota (667-145)

11:35-11:45

(82) The integral representation of bi-invariant positive definite functions

11:50-12:00

(83) Ergodic properties of automorphisms of a locally compact group. II
Professor M. Rajagopalan, University of Illinois and Madurai University,
India (667-95)

12:05-12:15

(84) Groups of a generalized Fourier transform
Dr. R. W. Preisendorfer, University of California, San Diego (667-104)

Session on Analysis VII, Room 208, Deady Hall
10:50-11:00

(85) A Tauberian theorem characterizing the regularity of growth of a class of
entire functions
Professor Simon Hellerstein, Professor Daniel F. Shea, University of
Wisconsin, and Professor J. Williamson*, University of Hawaii (667-117)

11:05-11:15

(86) Successive derivatives of analytic functions
Professor James D. Buckholtz, University of Kentucky (667-89)

11:20-11:30

(87) The relation between bounded index and exponential type for entire functions
Professor Benjamin Lepson, U.S. Naval Research Laboratory, Washing-
ton, D. C., and Catholic University of America (667-154)

11:35-11:45

(88) Entire functions mapping arbitrary countable dense sets (and their comple-
ments) onto each other
Professor Karl E. Barth and Professor Walter J. Schneider*, Syracuse
University (667-70)

11:50-12:00

(89) On orders and types of an entire function over $c^k$
Professor J. G. Krishna*, University of Illinois, and Mr. I. H. N. Rao,
Andhra University, India (667-159)

12:05-12:15

(90) A Weierstrass point of a class of modular groups. Preliminary report.
Mr. John Kasdan, University of California, Los Angeles (667-162)
(Introduced by Professor John B. Garnett)
FRIDAY, 10:50 A.M.

Session on Topology III, Room 102, Deady Hall

10:50-11:00
(91) On commuting functions without common fixed points
Mr. John Philip Huneke, University of Minnesota and Ohio State University
(667-116)

11:05-11:15
(92) Sequence of mappings and fixed points
Professor S. P. Singh, Memorial University of Newfoundland (667-8)

11:20-11:30
(93) On the existence of proper maps between Euclidean spaces
Dr. John N. Johnson, Boeing Company, Seattle, Washington (667-169)

11:35-11:45
(94) Function spaces of CW-complex
Professor Carlos R. Borges, University of California, Davis (667-74)

11:50-12:00
(95) Metric spaces in which a strengthened form of Blumberg's theorem holds
Professor Jack B. Brown, Auburn University (667-92)

12:05-12:15
(96) Uniformly continuous functions and uniformly equivalent metrics. Preliminary report
Mr. Gladwin E. Bartel, Washington State University (667-111)

FRIDAY, 10:50 A.M.

Session on Topology IV, Room 214, Allen Hall

10:50-11:00
(97) Retracting diffeomorphisms of n-spheres
Professor Jack M. Robertson, Washington State University (667-40)

11:05-11:15
(98) On differentiable extensions
Professor Robert H. Bowman, Vanderbilt University (667-73)

11:20-11:30
(99) A global existence and uniqueness theorem for generalized parallelism
Professor Alan B. Poritz, University of Pennsylvania (667-140)

11:35-11:45
(100) Lagrangian foliations of symplectic manifolds. Preliminary report
Professor Alan Weinstein, University of California, Berkeley (667-55)

11:50-12:00
(101) On the immersion of an n-dimensional manifold in (n + 1)-dimensional Euclidean space
Professor Benjamin R. Halpern, University of California, Berkeley (667-60)

12:05-12:15
(102) (O, SO)-bordism
Professor George E. Mitchell, West Virginia University (667-166)

FRIDAY, 10:50 A.M.

Session on Algebra IV, Room 215, Allen Hall

10:50-11:00
(103) 2-transitive symmetric designs with k = 2p
Professor Noboru Ito and Professor William M. Kantor*, University of Illinois (667-61)

11:05-11:15
(104) Characterizations of the groups $D_4^2(q^3)$ and $PSU_5(q^2)$, $q = 2^n$
Mr. L. Gomer Thomas, University of Washington (667-181)
11:20-11:30
(105) On the step-by-step conjugation of p-subgroups of a group
Professor William M. Kantor and Professor Gary M. Seitz*, University of Illinois at Chicago Circle (667-44)

11:35-11:45
(106) Dense subgroups of finite groups
Dr. Terence M. Gagen, University of Illinois, and Professor Paul M. Weichsel*, University of Illinois and Sydney University, Australia (667-23)

11:50-12:00
(107) Decomposition numbers of p-solvable groups
Mr. Forrest A. Richen, University of Michigan (667-109)

12:05-12:15
(108) Idempotents in group algebras and exceptional characters
Professor Ray O. Hamel, Northwestern University (667-68)

FRIDAY, 10:50 A.M.

Session on Algebra V, Room 216, Allen Hall
10:50-11:00
(109) New proofs for the lattice property of systems ordered by a semi-associative law
Mr. Samuel S. Huang* and Professor Dov Tamari, State University of New York at Buffalo (667-175)

11:05-11:15
(110) On the coordinatization of orthomodular posets
Mr. Richard H. Schelp, Kansas State University (667-69)
(Introduced by Professor R. J. Greechie)

11:20-11:30
(111) On the existence of an atomic nonatomistic orthomodular poset
Professor Richard J. Greechie, Kansas State University (667-66)

11:35-11:45
(112) Ortho-implication algebra
Mr. Robert J. Kimble, Jr., U. S. Naval Academy, Annapolis, Maryland (667-56)
(Introduced by Professor James C. Abbott)

11:50-12:00
(113) Numerical invariants on orthomodular lattices
Miss Mary Katherine Bennett, Dartmouth College (667-115)

12:05-12:15
(114) σ-completions of Boolean algebras. Preliminary report
Mr. Joseph Diestel, West Georgia College (667-100)

FRIDAY, 1:30 P.M.

Invited Address, Ballroom
Vector fields and Gauss-Bonnet
Professor Paul F. Baum, Brown University

FRIDAY, 2:45 P.M.

Session on Analysis VIII, Room 106, Deady Hall
2:45-2:55
(115) On a theorem of H. Goldstine
Dr. Carl L. DeVito*, University of Arizona, and Dr. Robert R. Welland, Northwestern University (667-67)
3:00-3:10
(116) Conditional expectations and vector measures
Professor M. M. Rao, University of Vienna and Carnegie-Mellon University (667-10)

3:15-3:25
(117) A representation theorem for infinitely-linear bounded continuous operators
Professor Vernon E. Zander, West Georgia College (667-94)

3:30-3:40
(118) Product integral representation of time dependent nonlinear evolution equations in Banach spaces
Professor G. F. Webb, Vanderbilt University (667-26)

3:45-3:55
(119) Polynomials of infinitesimal generators of contraction semigroups
Dr. Karl E. Gustafson, University of Colorado (667-35)

4:00-4:10
(120) A collection of sequence spaces
Professor James R. Calder and Professor Joe B. Hill*, Auburn University (667-110)

4:15-4:25
(121) $G_2^n$ spaces. Preliminary report
Professor Donald O. Koehler, Miami University (667-125)

4:30-4:40
(122) Generalized inverses
Professor Gustave Rabson, Clarkson College of Technology (667-79)

FRIDAY, 2:45 P.M.

Session on Analysis IX, Room 208, Deady Hall

2:45-2:55
(123) Two subspaces
Professor Paul R. Halmos, University of Hawaii (667-19)

3:00-3:10
(124) Pairs of unitary operators whose powers are close together
Professor Lawrence J. Wallen, University of Hawaii (667-106)

3:15-3:25
(125) An extension of Hölder maps
Professor Thomas L. Hayden* and Professor James H. Wells, University of Kentucky (667-107)

3:30-3:40
(126) Symmetric operators with piecewise $C^2$ spectral functions
Professor Richard C. Gilbert, California State College, Fullerton (667-148)

3:45-3:55
(127) Perturbation nonlinearly dependent on the eigenvalue
Professor Frank H. Brownell, University of Washington (667-3)

4:00-4:10
(128) Lower bounds for eigenvalues with displacement essential spectra
Mr. David W. Fox, The Johns Hopkins University (667-149)

4:15-4:25
(129) Expansion of powers of a class of linear differential operators
Dr. Murray S. Klamkin*, Ford Scientific Laboratory, Dearborn, Michigan, and Professor Donald J. Newman, Yeshiva University (667-155)

4:30-4:40
(130) Higher-order iterative solution of quadratic operator equations
Dr. Roy B. Leipnik, Naval Weapons Center, China Lake, California (667-71)
Session on Analysis X, Room 102, Deady Hall
FRIDAY, 2:45 P.M.

Session on Analysis X, Room 102, Deady Hall
FRIDAY, 2:45 P.M.

(131) An integral equation involving the Whittaker function
Professor Hari M. Srivastava, West Virginia University (667-186)

2:45-2:55

(132) Minimum of a norm subject to a system of D. E.
Professor Morteza Anvari, University of British Columbia and Aria-Mehr
University (667-53)

3:00-3:10

(133) On the continuity of two-point boundary value functions with respect to
coefficients
Dr. Allan Peterson, University of Nebraska (667-185)

3:15-3:25

(134) A Picone identity for strongly elliptic systems
Professor Kurt Kreith, University of California, Davis (667-14)

3:30-3:40

(135) An integrodifferential system which occurs in reactor dynamics
Professor Thomas A. Bronikowski, Marquette University (667-72)

3:45-3:55

(136) The asymptotic behaviour of linear ordinary differential equations
Dr. Jack W. Macki and Dr. James S. Muldowney*, University of Alberta
(667-138)

4:00-4:10

(137) A two point boundary problem for matrix differential equations
Professor Garret J. Etgen, University of Houston (667-103)

4:15-4:25

(138) On the limit cycles of \( y'' + \mu F(y') + y = 0 \)
Professor Craig Comstock, University of Michigan (667-52)
(Introduced by Professor M. S. Ramanujan)

4:30-4:40

(139) Some properties of LP-stability and K-stability
Professor M. Kursheed Ali, Fresno State College (667-113)

4:45-4:55

(140) A geometric problem involving a differential equation with deviating argument
Professor D. R. Smith, University of California, San Diego (667-2)

FRIDAY, 2:45 P.M.

Session on Statistics, Probability, and Numerical Analysis, Room 214, Allen Hall
FRIDAY, 2:45 P.M.

2:45-2:55

(141) Generating random normal numbers
Mr. Samuel Gorenstein, IBM, New York, New York (667-45)

3:00-3:10

(142) Existence of optimal stochastic control laws. Preliminary report
Dr. Vaclav E. Beneš, Bell Telephone Laboratories, Murray Hill, New
Jersey (667-101)

3:15-3:25

(143) A method of proof and some probabilistic applications
Professor D. Truax, University of Oregon, and Professor R. Fischler*,
University of Toronto (667-12)
(Introduced by Professor William A. O'N. Waugh)
3:30-3:40
(144) Some applications for Kramer's generalized sampling theorem
Professor Abdul Jabbar Jerri, Clarkson College of Technology (667-62)

3:45-3:55
(145) Optimizing fifth-order explicit Runge-Kutta formulas
Mr. A. R. Crawley* and Professor H. A. Luther, Texas A & M University (667-152)

4:00-4:10
(146) Seventh order Runge-Kutta formulas
Professor Diran Sarafyan, Louisiana State University in New Orleans (667-81)

4:15-4:25
(147) Convergence of semidiscrete approximations of linear transport equations

4:30-4:40
(148) Smooth error term for numerical integration, etc. Preliminary report
Professor Lincoln E. Bragg, University of Kentucky (667-118)
(Introduced by Professor Robert B. Hughes)

FRIDAY, 2:45 P.M.

Session on Topology V, Room 16, Science Building
2:45-2:55
(149) Weakly equivalent topologies
Professor Murray R. Kirch, State University of New York at Buffalo (667-108)

3:00-3:10
(150) Cofinal completeness and paracompactness
Professor Norman R. Howes, University of Dallas and Texas Instruments (667-39)

3:15-3:25
(151) Decisive convergence structures
Professor Darrell C. Kent, Washington State University (667-127)

3:30-3:40
(152) Stratifiable spaces are σ-spaces
Professor Robert W. Heath, University of Washington and Arizona State University (667-18)

3:45-3:55
(153) On a subclass of M-spaces. II. Preliminary report
Mr. Thomas W. Rishel, University of Pittsburgh (667-37)

4:00-4:10
(154) Topological spaces with a σ-point finite base
Professor Charles E. Aull, Virginia Polytechnic Institute (667-15)

4:15-4:25
(155) Compact subsets in function spaces
Professor S. K. Kaul, University of Saskatchewan, Regina Campus (667-88)

4:30-4:40
(156) Multifunctions and bitopological spaces
Professor Raymond E. Smithson, University of Wyoming (667-28)

FRIDAY, 2:45 P.M.

Session on Geometry, Room 30, Science Building
2:45-2:55
(157) Some semicontinuity theorems concerning the facial structure of convex sets
Professor Victor L. Klee and Mr. Michael S. Martin*, University of Washington (667-157)
3:00-3:10
(158) A characterization of the generalized convex kernel
Mr. Arthur G. Sparks, Clemson University (667-1)

3:15-3:25
(159) Hilbert space arcs with orthogonal chords
Professor George W. Batten, Jr., University of Houston (667-172)

3:30-3:40
(160) Point sets which are norm intervals
Mr. David Moe, Jacksonville State University (667-64)

3:45-3:55
(161) A lower-bound conjecture for convex polytopes
Dr. Peter McMullen, Western Washington State College (667-86)

4:00-4:10
(162) Facial cones and local similarity of polytopes
Mr. Joe D. Flowers, Oklahoma State University of Agriculture and
Applied Science (667-119)

4:15-4:25
(163) Inscribed polyhedra of maximum volume
Mr. Joel D. Berman and Mr. Kit Hanes*, University of Washington (667-90)

4:30-4:40
(164) Graphs without 5-ways
Professor John L. Leonard, University of Arizona (667-144)

4:45-4:55
(165) A simple condition of the traveling salesman's problem
Professor Hwa S. Hahn, West Georgia College (667-139)

FRIDAY, 2:45 P.M.

Session on Algebra VI, Room 215, Allen Hall
2:45-2:55
(166) Nilpotency index of a certain ring. Preliminary report
Professor Seymour Bachmuth and Professor Horace Y. Mochizuki*, University of California, Santa Barbara (667-48)

3:00-3:10
(167) Nil ideals of rings satisfying maximum condition on right annihilators
Professor Robert C. Shock, Southern Illinois University (667-171)

3:15-3:25
(168) \(\pi\)-reducible rings
Dr. Vlastimal Dlab, Carleton University (667-87)

3:30-3:40
(169) Rings in which minimal left ideals are projective
Professor Robert Gordon, University of Utah (667-102)

3:45-3:55
(170) Quasi-projective and quasi-injective modules
Mrs. Anne B. Koehler, Miami University (667-126)

4:00-4:10
(171) Ultrapowers of Zariski rings
Professor Loren C. Larson, Saint Olaf College (667-51)

4:15-4:25
(172) Families of valuations and the ideal transform
Professor Elbert M. Pirtle, University of Missouri-Kansas City (667-174)

4:30-4:40
(173) R-automorphisms of the ring of restricted power series over R. Preliminary report
Professor N. Sankaran, Queen's University (667-146)
FRIDAY, 2:45 P.M.

Session on Algebra VII, Room 216, Allen Hall

2:45-2:55
(174) On a problem in partial difference equations
Professor Calvin T. Long, Washington State University (667-7)

3:00-3:10
(175) Measures of clustering of partitions
Mr. T. A. Darden, Professor Abraham P. Hillman*, and Mr. W. C. Huffman, University of New Mexico (667-131)

3:15-3:25
(176) Enumerating finite rings
Dr. Robert L. Kruse*, Sandia Laboratories, Albuquerque, New Mexico, and Mr. D. T. Price, Wheaton College (667-84)

3:30-3:40
(177) Some extensions of the congruence concept for incomplete nondeterministic automata
Professor Raymond T. Yeh, Pennsylvania State University (667-22)

3:45-3:55
(178) Congruences on ternary relations and symmetrisations
Mr. T. R. Sundararaman* and Professor Dov Tamari, State University of New York at Buffalo (667-177)

4:00-4:10
(179) On the number of polynomials of an idempotent algebra, I
Professor G. A. Grätzer* and Dr. J. Plonka, University of Manitoba (667-38

4:15-4:25
(180) Global dimension in categories of diagrams. Preliminary report
Professor William T. Spears, Kansas State University (667-164)

4:30-4:40
(181) $S$-objects in an abelian category
Professor George B. Williams, Hamline University (667-57)

Seattle, Washington

R, S. Pierce
Associate Secretary

NEWS ITEMS

SPECIAL YEAR ON FUNCTIONS OF SEVERAL COMPLEX VARIABLES

The Mathematics Department of the University of Maryland is making plans to hold a special year in Functions of Several Complex Variables during the 1969-1970 academic year. Several visitors proficient in the field, including Professors Martin Eichler, Michel Herve, Hans Maass, Reinhold Remmert, Edoardo Vesentini, will spend all or part of the academic year in College Park. Also, an International Conference in Functions of Several Complex Variables will be held from April 5 to April 17, 1970. Further information on this conference will be available at a later date.

SALEM PRIZE

The Salem prize for 1969 was awarded to Dr. Richard Hunt of Princeton University for his work on the convergence of Fourier series. The prize, established in 1968, is given every year to a young mathematician who is judged to have done outstanding work on Fourier Series and related topics. The recipient in 1968 was Dr. Nicholas Varpoulos. The jury in both cases consisted of Professors A. Zygmund, C. Pisot, and J-P Kahane.
PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred and Sixty-Eighth Meeting
Massachusetts Institute of Technology
Cambridge, Massachusetts
October 25, 1969

The six hundred and sixty-eighth meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology on Saturday, October 25, 1969.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses. Professor Hyman Bass of Columbia University will give an address entitled "$K_2$ of global fields." Professor John T. Tate of Harvard University will announce the title of his address at a later date.

There will be sessions for contributed papers both morning and afternoon. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of September 9, 1969.

Leonard Gillman
Associate Secretary
Rochester, New York

The Six Hundred Sixty-Ninth Meeting
Louisiana State University
Baton Rouge, Louisiana
November 21-22, 1969

The six hundred and sixty-ninth meeting of the American Mathematical Society will be held at Louisiana State University at Baton Rouge, Louisiana, November 21-22, 1969.

By invitation of the Committee to Select Hour Speaker for Southeastern Sectional Meetings, Professor Marvin Rosenblum of the University of Virginia will speak on "Shifts and Hilbert space factorization problems"; Professor Swarupchand M. Shah of the University of Kentucky will speak on "Univalent functions with univalent derivatives"; Professor Nickolas Heerema of Florida State University will speak on "Higher derivations and automorphisms of complete local rings."

Abstracts of contributed papers should be sent to the American Mathematical Society, Providence, Rhode Island, so as to arrive prior to the deadline date of October 8, 1969.

The registration desk will be located in the basement of the Mathematics Building, Lockett Hall, where all sessions will be held. Registration hours will be 9 a.m. to 5 p.m. on Friday, November 21, and 9 a.m. to 12 noon on Saturday, November 22.

Baton Rouge, which is approximately 75 miles northwest of New Orleans, is located on U.S. 61 and U.S. 190. It is served by Delta, Southern and Trans-Texas Airlines, and by Greyhound and Trailways Bus Companies. Passenger train service is extremely limited. From the New Orleans Airport to the Baton Rouge campus is a drive of less than an hour and a half. Several persons might wish to rent jointly a car at the airport and drive to Baton Rouge.

Meals and snacks will be available at campus cafeterias and off-campus establishments. Coffee and doughnuts will be served each morning in the basement of Lockett Hall. A beer party at the Capitol House Hotel is planned for Friday evening. Tickets for this may be purchased at the time of registration.

Pleasant Hall is a university-owned hotel located on campus within a five-
A minute walk of Lockett Hall. The accommodations are adequate but not luxurious. There are about 95 rooms with private baths and 45 rooms with hall baths available. Rates are as follows:

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<thead>
<tr>
<th></th>
<th>With private bath</th>
<th>With hall bath</th>
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<tbody>
<tr>
<td>1 person</td>
<td>$7.00</td>
<td>$4.00</td>
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<tr>
<td>2 persons</td>
<td>$10.00</td>
<td>$7.00</td>
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<tr>
<td>3 persons</td>
<td>$12.00</td>
<td>$9.00</td>
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Make reservations at Pleasant Hall Reservation Desk, Louisiana State University, Baton Rouge, Louisiana 70803.

Additional accommodations, convenient to the University, with approximate driving times to the university are given below:

**JACK TAR CAPITOL HOUSE**
Lafayette at Convention, Baton Rouge, La. 70821

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<td>$9.00 to $15.00</td>
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<td>14.00 to 19.00</td>
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**PRINCE MURAT INN** (5 minutes)
1480 Nicholson Drive, Baton Rouge, La. 70821

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<td>$10.00 up</td>
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The following motels might not have accommodations available for Saturday night because of a home football game, but they do expect to have facilities available for Friday night.

**BATON ROUGE TRAVELodge MOTEL**
427 Lafayette Street, Baton Rouge, La. 70821

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<tr>
<td>$11.50</td>
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<tr>
<td>15.50 (two beds)</td>
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**HOLIDAY INN-South** (15 minutes) (Intersection of I-12 and U. S. 61)
9940 Airline Highway, Baton Rouge, La. 70821

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<th>Single</th>
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<tr>
<td>$11.50</td>
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<tr>
<th></th>
<th>2.00</th>
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<td>each additional person.</td>
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</table>

Reservations should be made directly with Pleasant Hall, the hotel, or one of the motels. It is suggested that reservations be made as early as is practical.

O. G. Harrold
Associate Secretary

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**The Six Hundred Seventieth Meeting**
Claremont Graduate School
Claremont, California
November 22, 1969

The six hundred seventieth meeting of the American Mathematical Society will be held at the Claremont Graduate School, Claremont, California, on November 22, 1969.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, hour addresses will be presented by Professor Solomon Feferman of Stanford University and Professor R. D. Richtmyer of the University of Colorado. There will be several sessions for contributed papers at the meeting. The deadline for submission of abstracts for this meeting is October 8, 1969.

Additional information concerning the meeting will be given in the October issue of these Notices.

R. S. Pierce
Associate Secretary

Seattle, Washington
Six Hundred Seventy-First Meeting  
University of Michigan  
Ann Arbor, Michigan  
November 29, 1969

The six hundred seventy-first meeting of the American Mathematical Society will be held at the University of Michigan, Ann Arbor, Michigan, on Saturday, November 29, 1969. All sessions of the meeting will be held in the Michigan Union, which will also have accommodations for two hundred guests.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Dr. Alan Baker of Cambridge University and the Universities of Michigan and Colorado will address the Society at 11:00 a.m. His subject will be announced later. Professor Avner Friedman of Northwestern University will speak at 1:45 p.m. on the topic "Free boundary problems for parabolic equations."

There will be sessions for the presentation of contributed ten-minute papers both morning and afternoon. Those having time preferences for the presentation of their papers should so indicate on their abstracts. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of October 8, 1969. There will be a session for late papers if one is needed.

There will be two special sessions of selected twenty-minute papers, one on Partial Differential Equations under the chairmanship of Professor James B. Serrin of the University of Minnesota, and one on Number Theory under the chairmanship of Professor Donald J. Lewis of the University of Michigan. In fact, both of these will probably be double sessions, meeting both morning and afternoon. Most of the papers to be presented at these sessions will be by invitation. Anyone contributing an abstract for the meeting, who feels that his paper would be particularly appropriate for one of these special sessions, should indicate this clearly on his abstract and submit it three weeks earlier than the above deadline, namely by September 17, 1969, to allow time for additional handling.

Information about travel and accommodations will appear in the October issue of the Notices.

The University of Michigan hopes to sponsor a symposium on function algebras and rational approximation on Friday, November 28, 1969, the day before the meeting itself. If this materializes, further details will be given in the October Notices.

Paul T. Bateman  
Associate Secretary  
Urbana, Illinois
COMMENTS ON THE REFERENDUM

The referendum on Resolution B and Resolutions 1 through 5 raised a variety of comment in Letters to the Editor or in letters to the Secretary. These were of three kinds, referring to the conduct of the referendum, to the social merit or the content of Resolutions 1 through 5, or to matters seemingly irrelevant to the referendum. The Secretary is attempting in this article to cover only the breadth of opinion offered on the conduct of the referendum, not the comments on the substance of Resolutions 1 through 5. He notes in passing that his previous attempt to extract thoughts from letters (pp. 485-487 of the current volume of these Notices) produced at least one sharp contrary letter.

In my opinion, Resolutions 1 through 5 do not conflict with Resolution B, since scholarship cannot exist in a vacuum. However, the position of the Council implies that such a conflict does exist.

Your resolution B contradicts only resolution #2.

It is unclear that resolutions 3 and 5 conflict with resolution B, as implied by your recommendations.

...in which way should a recommended vote for Resolution B imply as a consequence a recommendation against Resolutions 1 and 2?

How being for Resolution B should lead one to be against Resolution 1 or Resolution 3 is quite beyond me.

This note is to express my great dismay that the Council of the Society should have thought that the aims of Resolutions 1, 2, 4, and 5 are in any way inconsistent with the goals of the Society as expressed in Resolution B.

...seems to imply that the AMS not only should not speak with one voice, it should not speak at all.

Either explain why there is a conflict between these Resolutions or say nothing at all.

I find suspect the Council's purpose in presenting its resolutions as an only alternative to the others, some of which are equally undeserving of support. Does this action represent the members or merely the convenience of the Council?

I feel there is too much arm-twisting in the hand column of the ballot.

While I feel that it is important that the membership be informed as to the Council's position on such matters, as was done by the letter on the reverse side of the ballot, I feel that it is extremely inappropriate to include statements such as "The Council recommends a vote FOR Resolution B" on the face of the ballot.

I feel that AMS members are quite capable of making up their own minds on these important issues without the need for advice from the Council.

[The Secretary notes that the Council debated at length the form of the referendum and instructed him firmly, if informally, on the form of the ballot, including instruction to place FOR and AGAINST boxes one above the other rather than on the same line and to include the phrase "The Council recommends a vote against Resolution n
as a consequence of its position on Resolution B.

I would much rather resign than vote on these resolutions. No doubt the AMS will be an interesting political forum (and quickly too!), but I do not care to participate.

It always has been my belief that the function of a learned society is to be a learned society. Its purpose is not to proclaim the political views of its members, no matter how eminent they may be professionally. Furthermore, I have been dismayed to see how men of great distinction in their own field can be so wrong, yet be so self-righteous, when they come to consider social, political, and moral problems.

It is my sincere hope that the AMS will remain calm, adult, and intelligent in the midst of a world now indulging in childish tantrums.

As long as the AMS is a Mathematical Society, then I am proud to be a member. When it becomes a political society, count me out!

To [vote] for Proposition B could serve only one purpose: to indicate that we felt that the recent decision to move an AMS meeting from Chicago was in violation of this policy. I have discussed this with mathematicians on both sides of the question, and they have agreed that this is the intent of Proposition B. I believe that those who support Proposition B take an unreasonably narrow view of the above policy.

As I am not a U.S. citizen, I feel somewhat out of place in voting on [Resolution 1].

I write you in your capacity as Secretary of the AMS to remind you that the AMS... is supposed to be a professional organization and that only! Neither is supposed to be a POLITICAL, MILITARY, or RELIGIOUS entity. ...There is still, as we Mathematicians say, one and only one proper step you and your cabal can take. Having thus prostituted your offices and your trusts, RESIGN!

Everett Pitcher
Secretary

NEWS ITEM

MATHEMATICS RESEARCH CENTER

Dr. Alan Berman, Director of Research at the Naval Research Laboratory, has announced the establishment of a Mathematics Research Center within the laboratory's Mathematics and Information Sciences Division. The purpose of the new center will be to conduct research in areas of mathematics that are vital to a complete understanding of physical and scientific phenomena governing the Naval environment. Among the initial areas of investigation will be functional analysis, ordinary and partial differential equations, special functions, approximation theory, functions of a complex variable, diophantine approximations, stochastic processes, control theory, and numerical methods. When fully staffed, the center is expected to consist of about 10 permanent members and an equal number of term appointees, primarily holders of sabbatical appointments from universities and other government laboratories. A limited number of term appointments will also be available to outstanding young Ph.D.'s.
Editor, The Notices

In the June 1969 issue of the Notices is a letter from Dr. William G. Spohn, Jr. calling for the publication of the work of the late Professor H. Rademacher, including lecture notes and problem collections.

In this respect I can give the following information:

1. The collected works of Professor Rademacher are in the process of being published by the M.I.T. Press under the general direction of Professor Gian-Carlo Rota. It includes Professor Rademacher's published papers, as well as some technical reports, but not his books, lecture notes or problem collections.

2. An almost finished manuscript of a book on Analytic Number Theory has been edited by three former students of Professor Rademacher, namely Professor J. Lehner, Dr. M. Newman, and myself, and will be published by Springer, presumably in the Grundlehren der Mathematischen Wissenschafren.

3. The notes of Professor Rademacher on Dedekind sums, that served as the basis for the Hedrick lectures of 1963 are being edited for publication as a Carus Monograph by the Mathematical Association of America.

4. The question of lecture notes is far more difficult. I do not think that one should publish notes that the author did not want to have published. The bulk of his lecture notes refer to topics in analytic number theory and have been incorporated (by Professor Rademacher himself) in the manuscript of his book to be published by Springer. His Lecture Notes from the Tata Institute are also available (unfortunately, as far as I know, only in mimeographed form) and the text of his Philips Lectures at Haverford College has been published under the title "Lectures on Elementary Number Theory" (Blaisdell Publishing Company). Finally, as already mentioned, his notes on Dedekind Sums will appear as a Carus Monograph. As for his other Lecture notes, such as his beautifully prepared lectures on Equations with Partial Derivatives, or on the Calculus of Variations or on his own proof of the Jordan Curve Theorem, I believe that these should not be published. If Professor Rademacher had wanted to make them generally available, he would have done so himself, or at least would have indicated his intention, as he did e.g. in the case of the Dedekind Sums notes. As a matter of fact, I am not even certain that he kept the corresponding "Notes" on Partial Differential Equations, etc. in his file.

5. As one who has had the privilege to attend Professor Rademacher's unforgettable Problem Proseminar, I know only too well what may have prompted the suggestion to include the problem collections in any publication of collected works. This, however, is extremely difficult. First of all, Professor Rademacher rarely ran the Proseminar all by himself, and even when that was the case, he used to solicit problems also from his colleagues. Next, many of the problems discussed at the Proseminar were taken from other collections, such as Polya and Szegő's Lehrsatze Und Aufgaben, etc. Under these conditions it would be difficult to select the problems that originated with Professor Rademacher--and it does not seem appropriate to me to publish under his name problems due to other mathematicians.

6. It seems certain to me that the mathematical notes of Professor Rademacher contain a great wealth of important mathematical ideas. The sheer bulk of those notes could not possibly be published with any hope of success, and I believe it would be better to let them lie where they are, rather than to attempt to publish them.

Emil Grosswald

Editor's Note: The editors have been led to believe that at some future time the mathematical notes of Professor Rademacher may be made available to qualified mathematicians who want to work on them.
The American Mathematical Society has enlisted the cooperation of other mathematical organizations to study the whole complex of problems of communication in an increasingly productive mathematical community. In particular, one committee is working on the specific question of financial support of publication. As chairman of this committee, I would like to solicit comments from the readers of the Notices on present or future modes of spreading equitably the growing costs of publication.

The committee is not in the first instance concerned with proposals to eliminate any mathematics from publication, nor with more radical schemes relegating some papers to more limited and less expensive distribution. Rather, assuming that more mathematicians will publish more mathematics in more journals and books (following the history of other disciplines like chemistry or medicine) we are asking how this can be paid for. Present devices include journal subscriptions (the reader's share), page charges (the author's or his sponsor's share as a part of the cost of the research), subsidies by societies (the mathematical dues payer is taxed) or by other organizations—universities or government agencies. The major sources of journal income have been the first two, but the second, or at least its present form (page charges assessed against the author but never in fact paid directly by him), has been the target of criticism and has recently proved somewhat unreliable in the face of government expenditure limitations.

Once again, I request comments, experiences, philosophy, and especially concrete suggestions, to be sent to me at Northwestern University.

Daniel Zelinsky

H. W. Gould's letter (on page 624 of the June Notices) reminded me of the fact that the practice of printing Aleph upside down goes back at least as far as 1926. Around 1930, Professor J. E. Littlewood gave a course of lectures for which the textbook was his Theory of Real Functions (Heffer, Cambridge, 1926). My fellow student Patrick DuVal pointed out that the letter Aleph was printed upside down throughout (including the sheet of errata) with one exception: \$_1$ in the middle of page 38.

H. S. M. Coxeter

Gould's letter (Notices, June 1969) and the Editor's comments were very satisfying regarding the lack of uniformity in printing Hebrew Alephs. I suggest the font for aleph be modified so as to eliminate the problem entirely and improve its esthetic appeal. Thus, let us have alephs that look like this $\aleph$. Clearly only antisymmetricites will object.

Frank B. Cannonito

ACTIVITIES OF OTHER ASSOCIATIONS

STUDENT MEMBERSHIP
ASSOCIATION FOR SYMBOLIC LOGIC

The Association for Symbolic Logic wishes to remind students that they are eligible to become members of the ASL at the student rate of $3 annually. Membership includes a one-year subscription to the Journal of Symbolic Logic. Membership applications, along with student status certification blanks, appear in the Bulletin of Information of the ASL. The Bulletin of Information may be obtained by writing to the Association for Symbolic Logic, P.O. Box 6248, Providence, Rhode Island 02904. Dues should be sent to the same address.
PERSONAL ITEMS

Dr. J.C. AGRAWAL of Purdue University has been appointed to a professorship at California State College, California, Pennsylvania.

Dean A. ADRIAN ALBERT of the University of Chicago has been elected to membership on the Board of Trustees of the Institute for Advanced Study.

Mr. SURESH CHANDRA ARYA of Tribhuvan University, Nepal, has been appointed Principal of the Government Arts and Science College, Daman, India.

Dr. RAYMOND BALBES of the University of Missouri, St. Louis, has been appointed to a visiting assistant professorship at the University of Illinois at Chicago Circle.

Dr. JAN LIST BOAL of the University of South Carolina has been appointed to a professorship at Georgia State College.

Dr. DAVID R. BRILLINGER of the London School of Economics, England, has been appointed to a professorship at the University of California, Berkeley.

Professor ALFRED CARASSO of Michigan State University has been appointed to an assistant professorship at the University of New Mexico.

Professor Emeritus RICHARD COUR-ANT of New York University has been awarded the degree of Doctor Technices, Honoris Causa by the Technical University of Denmark, Lyngby, Denmark.

Dr. THEODORUS J. DEKKER of Bell Telephone Laboratories, Murray Hill, New Jersey, has been appointed staff member at the Mathematical Centre, Amsterdam, Netherlands.

H. PETER DEMBOWSKI of the Johann Wolfgang Goethe University, Frankfurt, has been appointed to a professorship at the University of Tubingen, Germany.

Dr. KLAUS DIETZ of the University of Sheffield has been appointed a mathematician at WHO, Epidemiology-Communications Research Division, Geneva, Switzerland.

Professor JAMES EELLS of Cornell University has been appointed to the Chair of Mathematical Analysis at the University of Warwick, Coventry, England.

Dr. A. N. FELDZAMEN of the State University of New York has been appointed Director of Production with the Encyclopaedia Britannica Educational Corporation, Chicago, Illinois.

Professor PAUL C. FIFE of the University of Minnesota has been appointed to a professorship at the University of Arizona.

Professor CHARLES E. FORD of the University of Toronto has been appointed to an assistant professorship at Washington University, St. Louis.

Mr. ROBERT FRED GORDON of IBM Corporation, White Plains, New York, has been appointed Operations Research Specialist with Hoffmann-La Roche, Nutley, New Jersey.

Professor HELMUT HASSE of the University of Hawaii has been appointed professor emeritus at the University of Hamburg, Germany.

Professor SUE-CHIN LIN of the University of Miami has been appointed a member of the Institute for Advanced Study for the academic year 1969-1970.

Dr. DAVID LOVELOCK of the University of Bristol, England, has been appointed to an associate professorship at the University of Waterloo, Ontario, Canada.

Professor ANASTASIOS MALLIOS of the Royal Hellenic Research Foundation has been appointed to a professorship at the University of Athens, Athens, Greece.

Professor JOHN H. MATHEWS of Michigan State University has been appointed to an assistant professorship at California State College, Fullerton.

Mr. RAMON MIRELES of TRW Systems, Houston, Texas, has been appointed Senior Operations Research Analyst at The Fluor Corporation, Ltd., Los Angeles, California.

Professor RICHARD G. MONTGOMERY of Clark University has been appointed to an assistant professorship at Humboldt State College.

Professor S. A. NAIMPALLY of the University of Alberta has been appointed to a professorship at the Indian Institute of Technology, Kankur, India.
Professor JAMES C. T. POOL of the University of Massachusetts has been appointed Assistant Division Director at the Argonne National Laboratory, Argonne, Illinois.

Professor LOWELL SCHOENFELD of Pennsylvania State University has been appointed to a professorship at the State University of New York at Buffalo.

Dr. C. J. SCRIBA of the University of Hamburg, Germany, has been appointed to a professorship at the Technical University of Berlin.

Professor DOROTHY B. SHAFFER on leave from Fairfield University has been appointed an NSF Faculty Fellow at New York University, Courant Institute of Mathematical Sciences.

Dr. JASMER SINGH of the Ministry of Railways, Government of India, has been appointed Under Secretary at the Ministry of Defense, Government of India, New Delhi.

Professor JOSEPH SPEAR of Northeastern University received the honorary Doctor of Science degree from the University at the 1969 commencement exercises, having completed 50 years of service at Northeastern. He is now with the Division of Programmed Instruction in Mathematics.

Professor EDWARD L. SPITZNAGEL of Northwestern University has been appointed to an associate professorship at Washington University, St. Louis.

Dr. FRANK STENGERT of the University of Michigan has been appointed to an associate professorship at the University of Utah.

Dr. ROBERT S. STEPLEMAN of the University of Maryland has been appointed to an assistant professorship at the University of Virginia.

Professor ALLEN R. STRAND of Muskingum College has been appointed to an assistant professorship at Colgate University.

Dr. SIEGFRIED THOMEIER of the Mathematical Institute, Aarhus, Denmark, has been appointed to a professorship at the Memorial University of New Foundland.

Mr. D. M. WEISS of Leesona-Mods Laboratories has been appointed a systems engineer with Gibbs and Hill, Consulting Engineers, New York City.

Professor KENNETH W. WESTON of the University of Notre Dame has been appointed to an associate professorship at Marquette University.

Mr. DAVID WESTREICH of Bronx Community College of the City University of New York has been appointed a lecturer at Brooklyn College.

Mr. RICHARD M. WOODWARD of Wayne State University has been appointed a research engineer at Ford Motor Company, Dearborn, Michigan.

Professor JEONG SHENG YANG of the University of Miami has been appointed to an assistant professorship at the University of South Carolina.

PROMOTIONS

To Dean of the Graduate School, St. Louis University: EDWIN G. EIGEL, Jr.

To Professor, Mississippi State College: JOHN L. TILLEY; Northwestern University: ADI BEN-ISRAEL; Temple University: THEODORE MITCHELL.

To Associate Professor, San Diego State College: HUNG-TA HO; Stanford University: MARY V. SUNSERI.

To Senior Lecturer, University of Stellenbosch, South Africa: THOMAS P. DREYER.

DEATHS

Professor Emeritus W. RANDOLPH CHURCH of the United States Naval Postgraduate School died on February 8, 1969, in Monterey, California, at the age of 65. He was a member of the Society for 39 years.

Dr. JON H. FOLKMAN of The Rand Corporation, Santa Monica, California, died on January 23, 1969, at the age of 30. He was a member of the Society for 7 years.

Professor FREDERICK W. JOHN of New York University died on January 29, 1969, at the age of 75. He was a member of the Society for 57 years.

Professor FRED L. KIOKEMEISTER of Mt. Holyoke College died on May 26, 1969, at the age of 56. He was a member of the Society for 33 years.

Dr. WILLIAM C. KRATHWOHL of Illinois Institute of Technology died on April 16, 1969, at the age of 86. He was a
member of the Society for 56 years.

Professor ROBERT M. LEWIS of New York University, Courant Institute of Mathematical Sciences died on November 7, 1968, at the age of 43. He was a member of the Society for 9 years.

Professor DAVID S. MORSE of Union College died on January 9, 1969, at the age of 70. He was a member of the Society for 47 years.

Professor CHARLES K. PAYNE of New York University died on May 5, 1969, at the age of 89. He was a member of the Society for 40 years.

Dr. HAROLD TINNAPPEL of Bowling Green State University died on March 9, 1968, at the age of 51. He was a member of the Society for 23 years.

Professor THOMAS O. WALTON of Kalamazoo College died on November 29, 1968, at the age of 78. He was a member of the Society for 48 years.

NEWS ITEMS

A. ADRIAN ALBERT

A. Adrian Albert, Dean of the Division of the Physical Sciences and Eliakim Hastings Moore Distinguished Service Professor at the University of Chicago, has been named a member of the Board of Trustees of the Institute for Advanced Study, Princeton, New Jersey. He is one of the few academicians ever invited to serve on the board.

Dean Albert, who is known for his research on the theory of linear algebras, is a past president of the American Mathematical Society. He served as chairman of the Division of Mathematics of the National Council from 1952 to 1955 and also has been chairman of the Section of Mathematics for the National Academy of Sciences. Dean Albert received his Ph.D. degree in mathematics from the University of Chicago in 1928, and he holds honorary degrees from the University of Notre Dame and Yeshiva University. He joined the faculty of the University in 1931, served as chairman of the Department of Mathematics from 1958 to 1962, and was appointed Dean of the Division of the Physical Sciences in 1962.

SEVENTH INTERNATIONAL SYMPOSIUM ON FUNCTIONAL EQUATIONS

The Faculty of Mathematics of the University of Waterloo is sponsoring a symposium on functional equations in Waterloo, Ontario, and the Lake of Bays on September 1-13, 1969. There will be 30 to 50 half-hour talks. In addition, ample time will be left for discussion and for the posing and/or solving of open problems in theory and applications of functional equations. Seminars on specialized topics may also be organized. This symposium is the seventh in a series of meetings held since 1962 in Europe and North America. Approximately 50 participants are being invited. Qualified mathematicians who are working in functional equations and their applications and who wish to participate are invited to write to the following members of the symposium committee: J. Aczel, University of Waterloo; M. A. McKiernan, University of Waterloo; and A. M. Ostrowski, University of Basle, Switzerland.
MEMORANDA TO MEMBERS
Backlog of Mathematical Research Journals

Information on this important matter is being published twice a year, in the February and August issues of the Notices, with the kind cooperation of the respective editorial boards.

It is important that the reader should interpret the data with full allowance for the wide and sometimes meaningless fluctuations which are characteristic of them. Waiting times in particular are affected by many transient effects, which arise in part from the refereeing system. Extreme waiting times as observed from the published dates of receipt of manuscripts may be very misleading, and for that reason, no data on extremes are presented in the table at the bottom of this page.

Some of the columns in the table are not quite self-explanatory, and here are some further details on how the figures were computed.

Column 2. These numbers are rounded off to the nearest 50.

Column 3. For each journal, this is the estimate, as of the indicated dates, of the total number of printed pages which will have been accepted by the next time that manuscripts are to be sent to the printer, but which nevertheless will not be sent to the printer at that time. (Pages received but not yet accepted are being ignored.)

Column 4. Estimated by the editors (or the Editorial Department of the American Mathematical Society in the case of the Society’s journals) and based on these factors: manuscripts accepted, manuscripts received and under consideration, manuscripts in galley, and rate of publication. There is no fixed formula.

Column 5. The first quartile (Q₁) and the third quartile (Q₃) are presented to give a measure of the dispersion which will not be too much distorted by meaningless extreme values. The median (Med.) is used as the measure of location. The observations were made from the latest issue received in the Headquarters Offices before the deadline date for this issue of the Notices. The waiting times were measured by counting the months from receipt of manuscript in final revised form to the month in which the issue was received at the Headquarters Offices, It should be noted that when a paper is revised, the waiting time between receipt by editors of the final revision and its publication may be much shorter than is the case for a paper which is not revised, so these figures are to that extent distorted on the low side.

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<th>Approx. No. pages per year</th>
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</table>

* NR means that no response was received to a request for information.
** Dates of receipt of manuscripts not indicated in this journal.
*** The latest issue of this journal consisted of only two articles.
# First issue to be published January 1970.
COMMITTEE ON THE WELFARE OF MATHEMATICIANS AND STUDENTS IN INVOLUNTARY SERVICE

The Council of the American Mathematical Society has appointed the under-named as a special committee to concern itself with developing a means to help advanced mathematics students maintain their mathematical interests, identity, and competence during a period of service in the armed forces or other enforced absence from the mathematical community, such as a prison term for draft resistance.

Letters have been sent to all colleges with an M.A. program, asking for information about such students, so that the AMS can build a roster. When this is ready, we expect to query these students about their interest in the program, and then to help place them in contact with college or university mathematicians who are willing to correspond with them and to serve as a mathematical mentor during the period of service. In many cases, we assume that these mathematicians will be faculty members who have already had some prior contact with the student and have a continuing interest in him. In cases where a student does not name a specific individual, the committee will attempt to select one.

In order that we have a supply of possible mentors on hand, we ask that any persons interested in becoming a part of this program send his name and any relevant information to Dr. Gordon L. Walker, Attention: Mentor Program, American Mathematical Society, P.O.Box 6248, Providence, Rhode Island 02904.

J. L. Kelley

R. C. Buck Chandler Davis

REFERENDUM

The Council ordered a referendum by mail on six Resolutions, labeled B and 1 through 5 for reference. Some background information and the text of the resolutions can be found on page 627 of the June issue of these Notices. The ballots were mailed on May 15, 1969. Since the stated closing date for counting the ballots was June 15, a Sunday, the effective date was Monday, June 16. Tellers for the election were Albert Wilansky and Murray Schechter. They have reported the following results to the Secretary:

<table>
<thead>
<tr>
<th>Resolution</th>
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<th>Against</th>
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<tr>
<td>B</td>
<td>6731</td>
<td>538</td>
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<tr>
<td>1</td>
<td>1860</td>
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<td>2</td>
<td>1903</td>
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<td>3</td>
<td>1167</td>
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<td>5171</td>
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<tr>
<td>5</td>
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Three or four members questioned whether the ballot was secret when the instructions required the name of the voter on the envelope. The authorization for conducting the referendum by mail is in the By-laws, Article IV, Section 8 but the manner of conducting it is not described. Accordingly, the procedures and practices for elections were used. Ballots in good order are separated from their envelopes by administrative and clerical staff in the Providence office. The Tellers receive only the ballots, except that in a handful of cases of possible irregularity they see both the ballot and the envelope to judge whether the ballot is valid.

The results of the election will be reported formally to the Council in August and the Council will report formally to the membership in the Bulletin as usual.

Everett Pitcher
Secretary
NEWS ITEMS AND ANNOUNCEMENTS

THE SATCOM REPORT

The Committee on Scientific and Technical Communication of the National Academy of Sciences in cooperation with the National Academy of Engineering has recently published a full 306 page report and a 30 page synopsis, entitled Scientific and Technical Communication, A Pressing National Problem and Recommendations for Its Solution. It is the result of an intensive three-year study of the problems involved in coping with the processing and dissemination of the ever-increasing, almost staggering amount of available scientific and technical information which increases in geometrical progression and which is the result of a yearly 27 billion dollars worth of U. S. research and development activities. The Committee on Scientific and Technical Information (SATCOM) was formed in 1966 at the suggestion of the National Science Foundation to survey the present status of information transfer and to recommend specific courses of action concerning future coordination of public and private communication activities. This action was felt necessary not only because of an expanding body of scientific knowledge and the appearance of new fields, but also because of a growing need for the specialist to be aware of the effect that formerly unassociated, but now related, research has on his particular area of interest.

At present, the Committee discovered, the groups in most dire need of more useful information services are the professionals, particularly practitioners in engineering, medicine, and agriculture. The process of consolidating such information needs to be more rapid and efficient to meet this need and the even greater demands of the future. Simplifying, abstracting, reviewing, indexing are becoming more and more essential. "For the near future, we regard the expansion of consolidation and reprocessing activities as the most vital thrust in fostering the prompt and effective application of scientific and technical information."

The responsibility for meeting these greater demands lies with the scientific and technical societies, the "for profit" information organizations, and the federal government. The Committee suggest that federal and private organizations cooperate in working toward common goals rather than duplicating services, and that sponsors of research and development make certain that the results of their work are made readily available.

The basic principles of the Committee's philosophy include (1) responsiveness of the management of information services to the needs of the scientific community they serve; (2) organization and coordination of information programs with a logical division of functions helping to maintain flexibility; and (3) reprocessing existing knowledge to make it adaptable to its users. These services can be accomplished with the necessarily diversified, structurally stable systems already in existence, and should be able to rely more and more on computer systems as the latter develop in sophistication.

SATCOM has suggested 55 specific recommendations falling under five general headings which include: (1) Planning, coordination, and leadership at the national level; (2) Consolidation and reprocessing -- services for the user; (3) Classical services (abstracting and indexing, libraries, formal and semiformal publication, and meetings); (4) Personal informal communication; and (5) Studies, research, and experiments. Their recommendations include several basic objectives.

The first step to successful coordination, the Committee feels, is the establishment of a widely representative, prestigious, nongovernmental body called the Joint Commission on Scientific and Technical Communication, which would be responsible to the Councils of the National Academy of Science and the National Academy of Engineering. One of their many responsibilities would be to see that appropriate groups within the organizations help
to manage government-sponsored projects. Another would be to see that information transfer be effected on an international as well as a national level, since the United States' contribution to the entire body of specialized information is only a fraction of the world knowledge in this realm.

The Committee also strongly urges that scientific and technical societies take more responsibility for identifying needs for the consolidation and reprocessing of new material and that they help to create an awareness among potential users of the resulting critical reviews and data compilations. Also, these societies should stimulate education in the use of this consolidated material, assisted by increased attention from practitioners in the fields to which they apply. Abstracting agencies should, as well, engage more actively in identifying difficulties, finding solutions, and testing arrangements to make reprocessing more financially feasible.

Other suggestions include government support of literature-access services and grants from appropriate agencies to assist libraries, as well as bibliographic control of semiformal publications, and the desirability of informal personal communication assisted by opportunities such as organized conferences and institutional exchanges of personnel during sabbatical leaves.

MOS HELPS IN RECRUITING

The Mathematical Offprint Service (MOS) offers an unusual opportunity to institutions seeking new recruiting methods to attract faculty members to their mathematics departments. MOS caters to the individual mathematician by providing him on a monthly basis with the titles of current articles of interest to him, and with reprints of articles of particular interest. By subscribing to this service, a mathematician has available to him an extensive coverage of current material in the world's primary mathematical journals, including a broad selection of publications in applied mathematics, which few university or industrial libraries are able to supply through journal subscriptions.

Several institutions have announced that in order to offer more complete coverage of mathematical literature to their faculty members, they will provide a subscription to MOS for each mathematician on their staff. It is our suggestion that other institutions follow suit in making this attractive offer as a recruiting device.

MOS is supported in part by a grant from the National Science Foundation. The NSF procurement regulations permit grantees and contractors to charge information services related to their research projects directly against grants and contracts. NSF grantees should refer to NSF 63-27, "Grants for Scientific Research," June 1963, page 13 (as amended December 1963).

ROCKY MOUNTAIN JOURNAL OF MATHEMATICS

The Rocky Mountain Mathematics Consortium has announced the establishment of the Rocky Mountain Journal of Mathematics, a quarterly periodical which will publish both primary research articles and survey articles in all fields of mathematics. The journal, which will be managed and edited within the Rocky Mountain region, will hold to national standards of excellence. It will not seek to publish primarily authors from the region, but will solicit articles nationally and internationally.

The first issue of the Rocky Mountain Journal of Mathematics will appear in March 1970. It will contain a series of lectures presented under the auspices of the Rocky Mountain Mathematics Consortium at a summer seminar on the Mathematical Theory of Scattering, which was held from July 20 through August 16, 1969, at Northern Arizona University in Flagstaff. The seminar was supported by a grant from the National Science Foundation and was directed by Professor Calvin Wilcox of the University of Arizona. The program of the seminar consisted of four series of lectures: "The abstract theory of scattering" by Tosio Kato and S. T. Kuroda; "Representation theory and scattering theory" by P. D. Lax and R. S. Phillips; "Eigenfunction expansions and scattering
theory: by N. Shenk and D. Thoe; and "The theory of the coulomb interaction" by J. D. Dollard.

Professor Robert McKelvey, executive director of the Rocky Mountain Mathematics Consortium, will serve as business manager for the journal; Professor William Scott will serve as managing editor; the Brigham Young University Press will handle composition and printing; and the American Mathematical Society will handle all subscription servicing. Subscriptions to the Rocky Mountain Journal of Mathematics will be available from the American Mathematical Society at the following prices: list, $20; agents' subscriptions for foreign addresses, $17; individuals, $10.

NATIONAL SCIENCE FOUNDATION INSTITUTIONAL SCIENCE DEVELOPMENT GRANTS

The National Science Foundation has announced the award of five institutional science development grants. These grants are designed to support institutions judged to have substantial potential for elevating the quality of their scientific activities and for maintaining this new high level of excellence. Among the universities receiving support under this program are the University of Arizona and the University of Virginia. The award to the University of Arizona will enable the departments of astronomy, chemistry, mathematics, and physics to complete their planned program for science development and will provide funds to augment the staff in mathematics. The grant to the University of Virginia will provide equipment for the departments of astronomy, biology, chemistry, material sciences, mathematics, and physics; and funds will also be used for the continued development of the university's Center for Advanced Studies.

CONFERENCE ON ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

A conference on Ordinary and Partial Differential Equations will be held at the University of Dundee, Scotland, from March 23 to March 27, 1970. The conference is to be supported with the aid of funds from the Scientific Affairs Division of the North Atlantic Treaty Organization. The following mathematicians have accepted invitations to give lectures: Professor F. V. Atkinson, University of Toronto, "Multi-parameter spectral theory of ordinary differential equations"; Professor G. Fichera, University of Rome, "Some topics in the theory of partial differential equations"; Professor K. Jorgens, University of Munich, "Spectral theory of partial differential equations." Full details on the conference may be obtained by writing to Dr. B. D. Sleeman, Department of Mathematics, The University, Dundee, Scotland.

MAA INFORMATION BOOKLET

The Mathematical Association of America has released its new Information Booklet for 1969. It describes the services and materials available through the MAA for the improvement in mathematics and the teaching of mathematics in colleges and universities. The booklet lists information on publications, films, employment, meetings of the Association, lecturers and consultants, and guidance of students. An address for further information on each category appears in the booklet. For a copy, write to A. B. Willcox, Mathematical Association of America, 1225 Connecticut Avenue, N. W., Washington, D. C. 20036.

CONFERENCE ON DECISION PROBLEMS IN GROUP THEORY

A conference on Decision Problems in Group Theory concerned with both positive and negative results and featuring talks by world authorities on most recent results on the Burnside conjecture, applications to 3-manifolds, and degrees of unsolvability will be held August 31 through September 6, 1969, at the University of California, Irvine. For further information, write to Professor Frank B. Cannonito, Department of Mathematics, University of California, Irvine 92664.
**ABSTRACTS OF CONTRIBUTED PAPERS**

*The Seventy-Fourth Summer Meeting*  
*University of Oregon*  
*August 26-29, 1969*

667-1. ARTHUR G. SPARKS, Clemson University, Clemson, South Carolina 29631. *A characteriza-
tion of the generalized convex kernel.*

Let \( S \) be a set in \( E_2 \). For \( x \) and \( y \) in \( S \), let \( \rho(x,y) \) denote the minimum number of segments that a polygonal line joining \( x \) to \( y \) and lying in \( S \) can have. **Definition.** For \( x \) in \( S \), let \( K^n_x \) be defined by \( K^n_x = \{ y \in S | \rho(x,y) \leq n \} \). The set \( K^n = \{ x \in S | K^n_x = S \} \) is called the \( n \)th order kernel of \( S \). **Definition.** \( S \) is said to be an \( L_n \) set if for every \( x \) in \( S \), it is true that \( K^n_x = S \), i.e., \( K^n = S \). In the following theorems, let \( S \) be a compact simply-connected set in \( E_2 \) and let \( \mathcal{L}_n \) be the set of all maximal \( L_n \) subsets of \( S \). **Theorem 1.** \( \cap \mathcal{L}_n = K^0 \). **Theorem 2.** Let \( \mathcal{L}_n' \subseteq \mathcal{L}_n \) be such that \( \cup \mathcal{L}_n' = S \), then \( \cap \mathcal{L}_n' = K^0 \). (Received December 20, 1968.)

667-2. D. R. SMITH, University of California, San Diego, La Jolla, California 92037. *A geometric problem involving a differential equation with deviating argument.*

Consider the equation (*) \( \phi'(x) = \phi(h(x)) \cdot h(x)^{-1} \) for real \( x \) in a fixed interval \( I \) containing the origin, where \( h = h(x) \) is a given continuous strictly monotonic increasing function mapping \( I \) into \( I \) satisfying \( x < h(x) \) for \( x \neq 0 \) and \( h(0) = 0 \). The problem is to determine \( \phi : I \rightarrow \mathbb{R} \) satisfying (*) and such that \( \phi(0) = 0 \). This is an inverse problem for the mean value theorem of differential calculus. **Theorem.** Let \( h \) be analytic at the origin, say \( h(x) = \gamma x(1 + \sum_{\nu=0}^{\infty} h_\nu x^\nu) \) with \( 1 < \gamma < e \). Then the above problem has, up to an arbitrary multiplicative constant, at most one solution \( \phi \) of the form (**) \( \phi(x) = x^a \cdot \sum_{\nu=0}^{\infty} \phi_\nu x^\nu \) for constants \( a, \phi_0, \phi_1, \ldots \). More precisely, the exponent \( a \) and the coefficients \( \phi_\nu \) in (**) are explicitly uniquely determined. The resulting series (**) defines a solution if and only if (**) converges; convergence of (**) is shown to hold in certain special cases. (It is conjectured that convergence always holds.) (Received March 6, 1969.)


Consider the problem \( H_0 u - G_\lambda u = \lambda u \) for unknown real scalars \( \lambda \) and unknown vectors \( u \) in the domains of both \( H_0 \) and \( G_\lambda \), where both are operators in the Hilbert space \( X \), \( H_0 \) being selfadjoint with known spectral measure \( E_0 \) and \( G_\lambda \) being symmetric and dependent on \( \lambda \). Under a variety of conditions making \( G_\lambda \) small with respect to \( H_0 \), for a given finite \( m \)-dimensional manifold of solutions of the unperturbed problem (\( G_\lambda = 0 \)) we show how the above perturbed problem with so associated \( u \) becomes equivalent to an \( m \)-dimensional determinant equation, transcendental in \( \lambda \) with \( u \) eliminated. Such is clearly computationally useful, reduces to standard perturbation formulas if \( G_\lambda \) is independent of \( \lambda \), and has the added advantage of showing no added complications if the eigenvalues split under perturbation. (Received March 4, 1969.)

The flow field set up by an infinitely long cylinder oscillating harmonically along its axis in an incompressible viscous fluid is studied in two parts. Part I deals with the solution when the oscillating cylinder is immersed in an infinite expanse of fluid, while in Part II the fluid is confined between two co-axial cylinders with the inner cylinder oscillating and the outer one at rest. The results express clearly the dissipative effect of finite viscosity. Other aspects like skin friction and energy dissipation are also studied. (Received February 13, 1969.)

667-5. ADI BEN-ISRAEL, Department of Engineering Sciences, The Technological Institute, Northwestern University, Evanston, Illinois 60201. Theorems of the alternative for complex linear inequalities.

For any set $S \subseteq \mathbb{C}^n$, $S^* = \{ y \in \mathbb{C}^n : \text{Re}(y,S) \geq 0 \}$. Theorem 1. Let $A_i \in \mathbb{C}^{m \times n_i}$ ($i = 1, \ldots, 4$), $A_1 \neq 0$, $A_2 
eq 0$, $T$ a polyhedral cone in $\mathbb{C}^m$, $S_1$, polyhedral cones in $\mathbb{C}^{n_i}$ ($i = 1,2,3$), $S_2^*$ pointed. Then exactly one of the following systems is consistent: (I) $\sum_{i=1}^{4} A_i x_i \in T$, $0 \neq x_1 \in S_1$, $x_2 \in S_2$; or $x_1 \in S_1$, $x_2 \in \text{int } S_2$, $x_3 \in S_3$. (II) $y \in -T^*$, $A_1^H y \in \text{int } S_1^*$, $0 \neq A_2^H y \in S_2^*$, $A_3^H y \in S_3^*$, $A_4^H y = 0$. Theorem 2. Let $T$, $A_i$, $S_i$ ($i = 1,3,4$) be as in Theorem 1. Then exactly one of the following systems is consistent: (I) $A_1 x_1 + A_3 x_3 + A_4 x_4 \in T$, $0 \neq x_1 \in S_1$, $x_3 \in S_3$. (II) $y \in -T^*$, $A_1^H y \in \text{int } S_1^*$, $A_3^H y \in S_3^*$, $A_4^H y = 0$. Theorem 3. Let $T$, $A_i$, $S_i$ ($i = 2,3,4$) be as in Theorem 1. Then exactly one of the following systems is consistent: (I) $A_2 x_2 + A_3 x_3 + A_4 x_4 \in T$, $x_2 \in \text{int } S_2$, $x_3 \in S_3$. (II) $y \in -T^*$, $0 \neq A_2^H y \in S_2^*$, $A_3^H y \in S_3^*$, $A_4^H y = 0$. Corollaries of these theorems include the transposition theorems and theorems of the alternative of Gordan, Stiemke, Motzkin, Tucker, Slater and others. (Received February 5, 1969.)

667-6. WITHDRAWN.

Given that the number \( n \) has the canonical representation \( n = \Pi_{i=1}^{q} a_i \), the problem of finding the number \( F(n) = F(a_1, \ldots, a_q) \) of ordered nontrivial factorizations of \( n \) is equivalent to that of finding the number of ordered partitions of the vector \( (a_1, \ldots, a_q) \) into nonzero vectors with nonnegative integral components. The author has shown (Addition theorems for sets of integers, Pacific J. Math. 23 (1967), 107-112) that \( F(n) = 2 \sum_{d|n} \mu(d) F(n/d) \) for \( n \geq 1 \) and this leads to a partial difference equation of order \( r \). For example, for \( r = 3 \), we have

\[
\begin{align*}
&2f(a, b, c) - 2f(a - 1, b, c) - 2f(a, b - 1, c) - 2f(a, b, c - 1) \\
&+ 2f(a - 1, b, c - 1, d) + 2f(a - 1, b - 1, c, d) + 2f(a, b - 1, c - 1, d) - 2f(a - 1, b - 1, c - 1, d - 1) = 0
\end{align*}
\]

with \( f(0, 0, 0) = 1 \), \( f(0, 0, 0) = 0 \), \( f(0, 0, 0) = 0 \), and this is shown that the solution can be obtained by fully expanding the polynomial \( 2^{a_1-1}[(x + (x + 1)]^b(x + (x + 1)(y + 1)]^c \) and then replacing \( x_k \) and \( y_k \) for all values of \( k \) by the binomial coefficients \( \binom{a}{b+k} \) and \( \binom{b}{y-k} \) respectively.

It is conjectured that this method holds for all \( r \) and this suggests the possible existence of a powerful transform method for the solution of a large class of partial difference equations. (Received February 17, 1969.)

S. P. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Sequence of mappings and fixed points.

Theorem 1. Suppose (1) \( T: X \to X \) is a mapping such that \( d(Tx, Ty) \leq K d(x, y) \) for all \( x, y \in X \); where \( K \) (\( K \) is a constant as given by Rakotch) is a constant \( 0 \leq K < 1 \) and with fixed point \( u \), (2) \( T_n: X \to X \) has a fixed point \( u_n \) for each \( n = 1, 2, \ldots \), (3) the sequence \( \{T_n\} \) converges uniformly on the subset \( \{u_n \mid n = 1, 2, \ldots \} \). Then the sequence \( \{u_n\} \) converges to \( u \). The theorems of S. B. Nadler, Jr., and of Kai-Wang Ng are included as easy corollaries to this theorem. Theorem 2. Let \( (X, d) \) be a locally compact metric space, let \( T_n: X \to X \) be a contractive mapping with fixed point \( u_n \) for each \( n = 1, 2, \ldots \) and let \( T: X \to X \) be a contraction mapping with fixed point \( u \). If the sequence \( \{T_n\} \) converges pointwise to \( T \), then the sequence \( \{u_n\} \) converges to \( u \). (\( T: X \to X \) is contractive if \( d(Tx, Ty) \leq d(x, y) \) for all \( x, y \in X \).) (Received March 12, 1969.)


Let \( X \) be a locally compact Hausdorff space, \( \Sigma \) the \( \sigma \)-ring of Borel sets in \( X \) and \( \mu \) a regular positive Borel measure. Let \( E \) be a Banach space. Let \( \mathcal{B}_p \) (\( 1 \leq p \leq \infty \)) be the Lebesgue-Bochner function space \( L^p(x) \mu (\Sigma) \). Let \( U \) and \( U_p \) be the unit cells in \( E \) and \( \mathcal{B}_p \) respectively. Theorem 1. If \( f \in U \), \( \|f\| = 1 \) then \( f \) is an extreme point of \( U \) and only if there exists an atom \( A \) in \( \Sigma \) and an extreme point \( e \) in \( V \) such that \( f = X_A e / \mu (A) \). Theorem 2. If \( E \) is a separable conjugate Banach space and \( 1 < p \leq \infty \) then a function \( f \in \mathcal{B}_p \), \( \|f\| = 1 \) is an extreme point of \( U_p \) and only if for \( x \) a.e. in the support of \( f \) \( f(x)/\|f(x)\| \) is an extreme point of \( U \). Similar results are obtained for various classes of Banach spaces \( E \) and the results are extended to Orlicz-Bochner function spaces. Applications of the results to the theory of operators and approximation are indicated. (Received March 20, 1969.)
Let \( \Phi \) be a Young's function, \( \Phi(x) = 0 \) iff \( x = 0 \), and \( L_p^\Phi(\Sigma) \) an Orlicz space on \((\Omega, \Sigma, \mu)\), \( \mu \) any measure. **Theorem 1.** For any \( \sigma \)-field \( \mathcal{B} \subset \Sigma \), \( \mathcal{A} \) a unique map \( E^\mathcal{A} : L_p^\Phi(\Sigma) \to L_p^\Phi(\mathcal{B}) \).

For \( \mathcal{B} = \Sigma \), \( E^\mathcal{B} \) is a vector measure (v.m.) of weakly \( \Psi \)-bounded variation rel. to \( \mu \) (cf. e.g. Dinculeanu's *Vector measures*). It is (not) reflexive. A similar representation holds, more generally, for the \( L_P^0 \)-spaces of \( F \). (Received March 17, 1969.)

**667-11.** ALESSANDRO FIGÀ-TALAMANCA, University of California, Berkeley, California 94720, and G. I. GAUDRY, Yale University, New Haven, California 06520. **Multipliers and sets of uniqueness for \( L_P^0 \).**

Let \( G \) be an LCA group and denote by \( L_p^0(G) \) the space of multipliers of \( L_p(G) \). **Theorem 1.** The inclusions \( L^1_p \subset L^2_p \subset L^2_p \) are all strict if \( G \) is infinite, and \( 1 < p_1 < p_2 < 2 \). **Corollary 1.** If \( 1 < p < \infty \), then \( L^1_p \neq L^2_p(G) = C^0_0(G) \) if and only if \( G \) is finite. **Theorem 2.** Let \( G \) be a nondiscrete LCA group and \( E \) a set of positive measure in \( G \). Then there exists a set \( E_0 \subset E \) whose measure is arbitrarily close to that of \( E \) and which is a set of uniqueness for all \( L_p(G) \) \( (1 \leq p < 2) \): i.e. if \( \mu \) is a measure supported by \( E_0 \) and \( \hat{\mu} \neq L^1_p(G) \) for some \( p \) in the range \([1, 2]\), then \( \mu = 0 \). (This extends a result of Katnelson for \( G = \) the circle group.) **Corollary 2.** If \( G \) is any noncompact LCA group and \( p > 2 \), there exist functions \( f \in L^1_p(G) \) with \( \hat{f} \) not a measure. **Theorem 3.** If \( G \) is noncompact, the multipliers of \( \hat{f} \in L^1_1 : \hat{f} \in L^1_1(X) \to L^1_1(G) \) are precisely the Fourier-Stieltjes transforms on \( X \). (Received March 24, 1969.)

**667-12.** DONALD R. TRUAX, University of Oregon, Eugene, Oregon 97405, and R. FISCHLER, University of Toronto, Toronto, Canada. **A method of proof and some probabilistic applications.**

We use a theorem of Kolmós (Acta Math. Hungar. 18 (1968), 217-219) to the effect that any sequence of sets \( \{A_n\} \) contains a subsequence \( \{A'_n\} \) such that for all subsequences of \( \{A'_n\} \) the Cesàro averages of the indicator functions converge with probability one to prove among other things: **Theorem.** A measure preserving transformation is strongly mixing iff for each set \( B \) and each subsequence of integers there is a further subsequence of integers for which the indicator functions of the subsequence of iterates of \( B \) converge in the Cesàro sense. **Theorem** (Converse to the Demoivre-Laplace Theorem). If a sequence \( \{A_n\} \cdot \mathcal{P}(A_n) = p \) is such that each subsequence
contains a further subsequence for which the statement of the Demyivre-Laplace theorem holds, then the $A_n$ are asymptotically independent in the sense that for all sets $A$, \( \lim P(A_n \cap A) = P(A) \).

(Received March 24, 1969.)


In two papers, (On bases for the set of integers, "Publ. Math. (Debrecen) 1 (1950), 232-242, and Some direct decompositions of the set of integers, "Math. of Comp. 18 (1964), 537-546), de Bruijn defines and considers some of the properties of bases for the set of integers. In this paper we generalize his notion of an A-base to that of an $\mathfrak{A}$-base where $\mathfrak{A} = \{A_i\}_{i \geq 1}$ where $A_i$ contains $m_i \geq 2$ elements including zero for each $i$. We give two sets of sufficient conditions for forming $\mathfrak{A}$-bases and a rather general method of constructing (infinitely many, but not all) nonsimple $\mathfrak{A}$-bases. The latter construction depends on a result of Long (Addition theorems for sets of integers, "Pacific J. Math. 23 (1967), 107-112). (Received March 24, 1969.)

667-14. KURT KREITH, University of California, Davis, California 95615. A Picone identity for strongly elliptic systems.

If $u(x)$ and $v(x)$ are solutions of Sturm-Liouville equations $(au')' + cu = 0$ and $(gv')' + hv = 0$ and if $v(x) \neq 0$, then the Sturm-Picone Theorem follows easily from the identity \[ \frac{u(au'v - guv')}{v}' = (h - c)u^2 + (a - g)u'^2 + g(u' - uv'/v)^2. \] This identity admits a direct generalization to the case where the Sturm-Liouville equations are replaced by selfadjoint strongly elliptic systems. Using this identity one can generalize a Sturmian theorem for strongly elliptic systems due to Kuks, establish a maximum principle, and derive bounds for eigenvalues associated with such systems. (Received March 27, 1969.)

667-15. CHARLES E. AULL, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Topological spaces with a $\sigma$-point finite base.

A hereditary collectionwise normal space with a $\sigma$-point finite base has a $\sigma$-disjoint base. Arhangelski$'$s theorem that a $T_3$ space with a $\sigma$-point finite base is metrizable iff it is perfectly normal and collectionwise normal follows. A $T_3$ space with a $\sigma$-point finite base if quasi-developable in the sense of Bennett, [H. R. Bennett, "Quasi-developable spaces," Topology Conference, Arizona State University, 1967, pp. 314-317.] A hereditary metacompact (screenable) quasi-developable space has a $\sigma$-point finite base ($\sigma$-disjoint base). A hereditary countably paracompact space with a $\sigma$-point finite base has a $\sigma$-disjoint base iff it is either hereditary collectionwise normal, hereditary screenable or hereditary paracompact. See also the author's abstract (Abstract 69T-G60, these Notices 16 (1969), June issue). (Received April 3, 1969.)
All spaces considered are locally compact and Hausdorff. Let $\mathcal{A}$ be the category of all lattices which are lattices of compactifications of spaces, a morphism from $K(X)$ to $K(Y)$ in $\mathcal{A}$ being a lattice isomorphism from $K(X)$ into $K(Y)$ preserving the zero element. Let $\mathcal{B}$ be the category of all the spaces each of which is $\beta X \setminus X$ for some space $X$, where $\beta X$ is the Stone-Cech compactification of $X$, morphisms being continuous surjections. Let $\mathcal{J}$ associate to each $K(X) \in \mathcal{A}$, the corresponding $\beta X \setminus X \in \mathcal{B}$. Theorem 1. $\mathcal{J}$ extends to a surjective contravariant functor from $\mathcal{A}$ to $\mathcal{B}$. The lattice with only a single element which is the one-point-$\beta$-compactification of some space is a universal element for $\mathcal{J}$. Theorem 2. $X$ and $Y$ are spaces. Then there exists a lattice isomorphism $\Gamma$ from the dual ideal generated by some $aX \in K(X)$ into $K(Y)$ such that $aX$ goes to the zero-element of $K(Y)$ if and only if there is a continuous function $h$ from $\beta Y \setminus Y$ into $\beta X \setminus X$; further $\Gamma$ uniquely specifies and is specified by $h$. Theorem 3. If $\Gamma$ is a nontrivial meet-complete homomorphism from $K(X)$ into $K(Y)$, then $\Gamma$ restricted to the dual atoms is a bijection; further $\Gamma$ induces uniquely in a natural manner a continuous map $h: \beta X \setminus X \rightarrow Y \setminus Y$. (Received March 27, 1969.)

**WITHDRAWN**

A $\sigma$-space is a space with a $\sigma$-discrete network (or net). A $T_1$-space is stratifiable if there is a function $k$ from $X \times N$ into the open subsets of $X$ such that (i) for each open set $Q$, $Q = \bigcup_{n=1}^{\infty} k(Q,n) = \bigcup_{n=1}^{\infty} k(Q,n)$ and (ii) if $Q \subseteq R$, then $k(Q,n) \subseteq k(R,n)$ for all $n$ (see Borges, Pacific J. Math. (1966), 1-16). Theorem. Every stratifiable space is a $\sigma$-space. Note that the converse is not true (Heath, Proc. Amer. Math. Soc. (1966), 868-870). But note that $\sigma$-spaces are semi-stratifiable (see Heath, pp. 153-167, and Creede, pp. 318-323, Topology Conference, Arizona State University, 1967, Tempe, Ariz.), whereas a paracompact semi-stratifiable (even semi-metric) space need not be a $\sigma$-space (Ernest S. Berney, presented at the 1969 Auburn Topology Conference (to appear)). (Received April 3, 1969.)

One way to get an example of a pair of subspaces in "generic position" (Dixmier's "position p") in a Hilbert space is to form a direct sum, let one subspace be one of the "axes", and let the other subspace be the graph of a suitable closed linear transformation. Theorem. That's the only way. Alternatively: Each such generic pair of subspaces can be obtained from a bounded operator $T$ as the graph of $T$ and the graph of $-T$. The proofs are operatorial transcriptions of trivial facts from plane geometry: Each pair of lines in a plane can be represented as an axis and a linear graph, or, alternatively, by rotation through half the angle between them, as two linear graphs, each the reflection of the other in one of the coordinate axes. Dixmier's result on the unitary equivalence of pairs of subspaces in generic position is an easy corollary. (Received April 4, 1969.)
Mathematical proof-methods have evolved with increasing utility. Classical nonformal proofs enjoyed (1) brevity, (2) creativity and (3) strength. Frege's, Russell-Whitehead's and Zermelo's systems added (4) unambiguity and (5) rigor but lost (1) (still unregained). Gentzen's C(onstructional) N(atural) D(education) added (6) spontaneity, but lost (2) and (3), being rigid in proof-pattern and limited to quantification theory. Takeuti's CND and Stanley's Linear R(educational) ND restored (3), and RND added (7) (practical) discoverability, via largely automatic proof-composing. All major methods translate syntactically into LRND (also LCND) form. Three context theorems (duals exist for LCND) emerge: (I) If \( \Gamma \vdash F \equiv G \), then \( \exists \vDash H[\Gamma] \equiv \vDash H[G] \). (II) If \( \vdash F \supseteq G \), then \( \exists \vDash H\left[\Lbrack\Box F\supseteq \vDash H[G]. (III) If \( \vdash (H[x = x])y/x' \) is \( \vdash H[y = y] \) and \( \vdash F \equiv (\exists x)G \), then \( \exists \vDash H\left[\Lbrack\Box IP \vdash \vDash H[G] \right) \). \( \Box ' \), 'Q', 'E', 'P' mean closure, linear proof-step, and quantificational, existential, positive contexts). Novel, strong use of (I)-(III) in LRND breaches main enduring barriers to (1) by restoring major creative options (2), preserves (3)-(7) in unweakened, practical measure, and is argued to be the first acceptable evidence of foreseeably "perfect" proofs in this 7-fold sense. (Received April 9, 1969.)

A generalized unified field theory. Preliminary report.

We solve, as a corollary, the famous unsolved problem, outstanding for fifty-three years, of establishing a unified field theory. Einstein failed to solve it. The existing kinetic theory reduces thermodynamics to an undefined and empirically unconfirmed impact dynamics. A statistical version yet! Using an appropriate version of my gravitational law \( F = c_1 u^2 + c_2 u^4 - c_3 u^4.1 \), which applies to both molecular and macroscopic bodies, we define axiomatically and confirm by the advance of the perihelion of mercury and the universal gas constant of thermodynamics, an impact dynamics, which in turn is used to properly reduce my axiomatic version of thermodynamics to it. Finally, we include in this unification a new theory of electromagnetism based on invariant mass and variant charge and using my modification of the inverse square law. Furthermore, all of this is formulated within the framework of the essential and important axiomatic methodology (using sets of quantified and unquantified open sentences--open since they contain variables, which can be replaced by the primitives of an interpretation). I have shown in an unpublished paper that epsilonics can be eliminated by the inf and sup concepts. It follows, therefore, that we also have reduced much of theoretical physics to abstract algebra. (Received March 7, 1969.)

Some extensions of the congruence concept for incomplete nondeterministic automata.

An automaton \( M \) is a triple \( (S, A, \delta) \), where \( S \) and \( A \) are nonempty finite sets of states and input symbols respectively, and \( \delta \subseteq (S \times A) \times S \). For each \( a \in A \), we define \( \delta_a = \{(s,t) \in S^2 | ((s,a), t) \in \delta \} \). \( \theta \subseteq S^2 \) is called weakly reflexive iff \( \theta \subseteq (I_S \cap \theta \circ \theta) \). \( \theta \subseteq S^2 \) is called a generalized congruence relation of degree \( n \) (GCR \( n \)) on \( M = (S, A, \delta) \) iff \( \theta \) is weakly reflexive, symmetric and...
Let $R_1(n)$ be the set of $n \times n$ complex matrices $S$ for which $S^*X = 0$ iff $S^*X = 0$. Here $S^*$ denotes the conjugate transpose of $S$. Let $R_2(n)$ be the set of $S$ such that $C^*SC \in R_1(n)$ for every $n \times n$ complex matrix $C$. Let $R_3(n)$ be the set of $S \in R_1(n)$ for which $X^*SX = 0$ implies $SX = 0$. Then $R_1(n) \supseteq R_2(n) \supseteq R_3(n)$. Theorem 1 (presumably known). If $S$ and $T$ are in $R_1(n)$, then rank $ST = \text{rank } TS$. Corollary. If $S$ and $T$ are in $R_1(n)$ and $n \geq 3$, then $ST$ is similar to $TS$. Theorem 2. If $S$ and $T$ are in $R_1(n)$ and one of $S$ or $T$ is in $R_2(n)$, then $ST$ is similar to $TS$. Theorem 3. If $S$ and $T$ are in $R_2(n)$, then $ST$ is similar to $TS$. Theorem 3 generalizes Carlson’s result for the case where $S$ and $T$ are hermitian (Pacific J. Math. 15 (1965), 1124, Remark 1, in which an obvious misprint occurs on the first line). An example is given of an $S$ and $T$ both in $R_1(4)$ (in fact, both are normal) with $S \in R_2(4)$ (in fact, $S$ is hermitian), for which $(ST)^2$ has rank different from that of $(TS)^2$ and hence $ST$ is not similar to $TS$. A more explicit characterization is given for $R_2(n)$ and $R_3(n)$ for the present (complex) case, and generalizations are given to more abstract algebraic settings. (Received May 1, 1969.)
Let $P_k$ be a $k$ stage Postnikov tower built up from $K(Q, n_j)$s. Theorem. The loop space $\Omega P_k$ has the homotopy type of a product of $K(Q, n_j - 1)s$. From this result we draw several consequences.

1. Let $X$ be a simply connected space such that $\pi_*(X)$ is a $Q$ module of finite type. Then $\Omega X$ has the homotopy type of a product of $K(Q, n_j)$s.

2. Let $\Phi$ be a higher order cohomology operation defined on certain classes of dimension greater than 1. Then $\Phi$ is essentially a matrix Massey product.

3. Let $a \in \pi_*(X) \otimes Q$ be a nonzero element. Then $a$ is detected by a functional matrix Massey product.

(Received May 2, 1969.)


A space $S$ is called a set space if $S$ is associated with two operations $\emptyset$ and $\cap$ such that for every $x, y \in S$ we have either $x \subseteq y$ or $y \subseteq x$ but not both. Every set space $S$ Russell's paradox is a Theorem. For the set $U = \{ u : u \subseteq u \} \subset S$ there is no  $x \in S$ such that $U = \{ u : u \subseteq x \}$. Every axiomatic set theory is a set space. Therefore, there is no axiomatic set theory equivalent to set theory because $U$ is a set for set theory but not for axiomatic set theory by this theorem. (Received May 5, 1969.)

667-28. RAYMOND E. SMITHSON, University of Wyoming, P. O. Box 3036, University Station, Laramie, Wyoming 82070. Multifunctions and bitopological spaces.

It has long been known that a multivalued function distinguishes open and closed sets. By using this fact and a generalization of the product topology we are naturally led to two topologies for a set. Thus let $\{ F_a : a \in \Gamma \}$ be a collection of multifunctions on a set $X$ into topological spaces $X_a$ (i.e., $F_a : X \to X_a$). Let $\tau(\Gamma)$ be the smallest topology on $X$ for which each $F_a$ is upper (lower)

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semicontinuous. We then obtain conditions under which the bitopological space satisfies separation axioms and under which it is quasi-metrizable. Theorem. Suppose for each \( x, y \in X \), there is an \( F_x \) and an \( F_y \) such that \( F_x(y) \cap F_y(x) \). If each \( X \) is regular and if each \( F(x) \) is closed, then for \( x \neq y \) there is \( A \subseteq U \) and \( V \subseteq Y \) such that \( x \in U, y \in V \) and \( U \cap V = \emptyset \). This condition is called pairwise Hausdorff. We also obtained similar theorems for regularity and second countability. Finally, if \( T^2 \) is countable, \( X^a \) regular and second countable for each \( a \) and if \( F(x) \) is compact for all \( a \) and all \( x \), then \((X^a,\delta,\gamma)\) is quasi-pseudo-metrizable. If in addition the first condition in the above theorem holds, then \((X^a,\delta,\gamma)\) is quasi-metrizable. (Received May 5, 1969.)

667-29. CARL A. EBERHART and JOHN SELDEN, University of Kentucky, Lexington, Kentucky 40506. One-parameter inverse semigroups.

A pair \((H,f)\) is a one-parameter inverse semigroup provided \( H \) is an inverse semigroup and \( f \) is a homomorphism from \( \mathbb{R} \), the multiplicative real numbers \( \{x \in \mathbb{R} \mid x \geq 1\} \), into \( H \) such that \( f(P) \) generates \( H \).

There exists \( (F,g) \) together with the operation defined by \((x,y,z) \rightarrow \begin{pmatrix} xzr/yA \rightarrow zr, xrt/yA zr \end{pmatrix} \) where \( A b = \min\{a,b\} \). Let \( g : \mathbb{R} \to F \) be given by \( g(x) = (x,x,x) \). Then \((F,g)\) is a one-parameter inverse semigroup. Furthermore, if \((H,f)\) is a one-parameter inverse semigroup, then there is a unique homomorphism \( h : F \to H \) such that \( f = hg \).

For each subgroup \( M \) of \( \mathbb{R} \), the positive multiplicative real numbers, define a relation \( \sigma_M \) on \( F \) by \((x,y,z) \sigma_M (r,s,t) \iff xzs/rt \in M \). Theorem. The correspondence \( M \to \sigma_M \) is a lattice isomorphism from the subgroups of \( \mathbb{R} \) onto the group congruences on \( F \). Define two relations \( \alpha \) and \( \beta \) on \( F \) by \((x,y,z) \alpha (r,s,t) \iff xs = yr \) and \( z = t \). Theorem. \( \alpha \) and \( \beta \) are congruences on \( F \). Further \( \alpha \wedge \beta = 0 \), the smallest congruence on \( F \), and \( \alpha \vee \beta = \sigma_M \}, \) the smallest group congruence on \( F \). Let \( T = \{\alpha, \beta\} \cup \{\sigma_M \} \) be a subgroup of \( R^1 \). Theorem. Every non-zero congruence \( \sigma \) on \( F \) can be written uniquely in the form \( \sigma = (\alpha \cup \beta) \) where \( \delta \in T \) and \( 1 \) is an ideal of \( F \). (Received May 7, 1969.)


It is shown that a measure-valued Green's kernel is associated with the Martin boundary as defined by Ackoglu and Sharpe [Trans. Amer. Math. Soc. 132 (1968), 447-460]. The kernel has the properties (1) each bounded excessive function is the derivative of an integral involving the function, the kernel, and a reference measure, (2) the boundary part of the integral gives the regular part of the function in the Riesz decomposition, (3) the off-boundary part of the integral gives the pure potential part of the function. Under separability assumptions, the entities drop isomorphically to a certain quotient space which is metrizable, and in this space there is sample path convergence to the boundary w. p. 1. (Received May 9, 1969.)

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The partial differential equations governing the motion of plates when transverse shear and rotatory inertia are considered are first transformed into parabolic coordinates, and then solutions are obtained which satisfy all of the eight different types of boundary conditions entering into the theory. Algorithms are given showing how to calculate the frequency for the higher modes of vibration, and it is proved that this theory is a decided improvement over the classical theory, primarily because coupling between flexural and shear motions are considered. (Received May 12, 1969.)

667-32. ELDON J. VOUGHT, California State Polytechnic College, Pomona, California. 91766. Continua not separated by any nonaposyndetic subcontinuum.

Certain theorems that apply to compact, metric continua that are separated by none of their subcontinua can be generalized and strengthened in those continua that are separated by none of their nonaposyndetic subcontinua. For those of the former type, Bing (Amer. J. Math. 70 (1948)) has proved that if the continuum is aposyndetic at a point, it is locally connected at the point. The same conclusion is possible if the continuum is not separated by any nonaposyndetic subcontinuum. Bing (Amer. J. Math. 70 (1948)) has also proved that if a continuum is separated by no subcontinuum and cut by no point, it is a simple closed curve. A second result of this paper is to prove that if no nonaposyndetic subcontinuum separates and no point cuts the continuum, then it is a cyclically connected continuous curve; in fact this yields a characterization of hereditarily locally connected, cyclically connected continua. A third theorem characterizes an hereditarily locally connected continuum as an aposyndetic continuum that is separated by no nonaposyndetic subcontinuum. This is a somewhat stronger result than the known equivalence of hereditary local connectedness and hereditary aposyndesis. (Received May 14, 1969.)


Let \( \theta \) denote an irrational number and let \( a_i \) (\( i = 1, 2, \ldots \)) denote the partial quotients of the continued fraction for \( \theta \). Let the numerators \( p_n \) and denominators \( q_n \) of the convergents \( p_n/q_n \) (\( n = 1, 2, \ldots \)) to \( \theta \) be defined as follows: 

\[
p_1 = a_1, \quad p_2 = a_1 a_2 + 1, \quad p_n = a_n p_{n-1} + p_{n-2} (n \geq 3);
\]

\[
q_1 = 1, \quad q_2 = a_2, \quad q_n = a_n q_{n-1} + q_{n-2} (n \geq 3).
\]

Define \( \lambda(\theta) = \liminf_{n \to \infty} n (\sum_{j=n+1}^{\infty} |q_j \theta - p_j|) \). It is shown that the function \( \lambda(\theta) \) has the following properties: (l) if for some positive integer \( j \) the continued fraction for \( \theta' \) has partial quotients \( a'_i \) (\( i = 1, 2, \ldots \)) which satisfy \( a'_i = a_m \) (\( m = 1, 2, \ldots \)), then \( \lambda(\theta') = \lambda(\theta) \); (2) \( \lambda(\theta) > 0 \) if and only if the partial quotients \( a_i \) are bounded; (3) for all \( \theta, \ z^{-1}(1 + 5^{-1/2}) \leq \lambda(\theta) \leq 0, \) and the upper bound is attained. The interest of the function \( \lambda(\theta) \) in the theory of Diophantine approximation is discussed. (Received May 16, 1969.)
The dual spaces of a large class of weighted spaces of continuous functions are characterized as those spaces of Radon measures which can be factored into a product of a weight function and a bounded Radon measure. This result allows us to obtain useful representations for a base for the equicontinuous subsets of these duals and for the extremal points of the members of this base. Finally, these ideas are applied to obtain, among other results, a generalization of our representation theorem for the biequicontinuous completed tensor product of weighted spaces (see Abstract 663-2, these Notices 16 (1969), 87). (Received May 19, 1969.)

Let (unbounded, in general) $A$ be a generator (here, to mean the infinitesimal generator of a strongly continuous contraction semigroup of linear operators on a complex Banach space); due to the wide range of applicability there has been much interest in additive and multiplicative perturbation criteria preserving the generator property. Let $p$ be a polynomial, real or complex coefficients, and let $\rho(A)$ denote the resolvent set for $A$. Theorem. Let $A$ be a generator, and suppose $p(A)$ is dissipative. If there exists $\mu$, $\Re \mu > 0$, in the image of $\rho(A)$ under transformation by $p$, such that the zeros of $\mu - p$ all lie in $\rho(A)$, then $p(A)$ is a generator. Remark. Actually, it is sufficient that $\mu$ and the zeros of $\mu - p$ exist in certain Fredholm resolvents of $A$. Remark. A little experimentation reveals, due to the freedom in choosing $\mu$, that the above theorem has wide applicability (roughly, except in the case that the spectrum of $A$ is the whole left half-plane). (Received May 19, 1969.)

A loop $G$ is called an $M_k$ loop if the law $(xy)(zx)^k = (x(yz))^k$ holds for all $x,y,z$ in $G$. In a report given at the January 1969 Meeting in New Orleans (Abstract 663-753, these Notices 16 (1969), 306), the first author showed that every $M_k$ loop is Moufang. Theorem. In an $M_k$ loop, the $(k - 1)$st power of every element is in the nucleus. Theorem 2. A necessary condition for a Moufang loop to have a centralizing endomorphism $x \mapsto x^{k-1}$ is that $G$ be an $M_k$ loop. Theorem 3. The map $x \mapsto x^2$ is not a centralizing endomorphism of a Moufang loop $G$ unless $G$ is a group. Any $M_k$ law implies a set of $M_s$ laws, where $s$ is a module. Examples of noncommutative $M_k$ loops are found for arbitrary $k \equiv 1 \pmod 3$ and for $k \equiv 1 \pmod 2$. In some of these examples, $x \mapsto x^{k-1}$ is a centralizing endomorphism, and in others it is not. (Received May 22, 1969.)

Definition. A topological space $X$ is weakly-$k$ iff $F \subseteq X$ has the property that $F \cap C$ is finite
for all compact $C$ in $X$ implies that $F$ is closed. Then, given the definitions of c-space [Abstract 671-473, these Notices 14 (1967), 698] and class $c$ of spaces [Ishii, Tsuda, Kunugi, "On the product of M-spaces," Proc. Japan Acad. 44 (1968), 897], the following hold: Theorem 1. Every weakly-k and c-space is a k-space; every k-space is weakly-k. Theorem 2. A space belongs to class $c$ if it is both weakly-k and M. (Received May 21, 1969.)

667-38. G. A. GRATZER and J. P. DONKA, University of Manitoba, Winnipeg 19, Canada. On the number of polynomials of an idempotent algebra. 1.

All algebras $\mathcal{A}$ are assumed to be idempotent. For $n \geq 2$, let $p_n(\mathcal{A})$ denote the number of essentially $n$-ary polynomials of $\mathcal{A}$. Theorem 1. Assume that $\mathcal{A}$ has a commutative binary polynomial. Let $n$ be the smallest integer with $p_n(\mathcal{A}) \neq 1$. Then the sequence $p_n(\mathcal{A}), p_{n+1}(\mathcal{A}), \ldots,$ is strictly increasing, in fact, $p_{m+1}(\mathcal{A}) \geq p_m(\mathcal{A}) + (m - 1)$. Theorem 2. Let $\mathcal{A}$ have a commutative and associative binary polynomial. Then $p_{m+1}(\mathcal{A}) \geq p_m(\mathcal{A}) + 1 + \max\{m + 1, p_m(\mathcal{A})\}$, if $p_m(\mathcal{A}) \neq 1$. Theorem 3. In addition to the conditions of Theorem 2 assume that $p_2(\mathcal{A}) = 3$. Then $p_m(\mathcal{A}) \geq 2^m - 1$ for all $m \geq 2$. Examples with $p_m(\mathcal{A}) = 2^m - 1$ are given. (Received May 26, 1969.)

667-39. NORMAN R. HOWES, 4529 Mockingbird Lane, Dallas, Texas 75205. Cofinal completeness and paracompactness.

For definitions see Abstract 663-136, these Notices 16 (1969), 125. A net $\mathfrak{N}$ on X in a uniform space $(X,U)$ is cofinally Cauchy if for each $U \in \mathcal{U}$ there is a cofinal $C \subseteq D$ with $\psi(C) \times \psi(C) \subseteq U$. $(X,\mathcal{U})$ is cofinally complete if each cofinally Cauchy net quasi-converges. Theorem. A $T_1$ space is paracompact iff it is cofinally complete with respect to some uniformity for $X$. Theorem. A paracompact space is Lindelöf iff it is pre-Lindelöf with respect to the uniformity consisting of all neighborhoods of the diagonal. (Received May 23, 1969.)


Let $H_n$ represent the $C^\infty$-orientation preserving diffeomorphisms of the n-sphere onto itself using the $C^k$ topology. Connectedness properties of this space have been studied although much remains to be done. (See J. W. Milnor, "On manifolds homeomorphic to the 7-sphere," Ann. of Math. 64(1956), 399-405, and S. Smale, "Diffeomorphisms on the 2-sphere," Proc. Amer. Math. Soc. 10(1959), 621-626.) This result shows the problem becomes much easier in the coarser topology obtained by taking the pointwise counterpart of the $C^k$ topology. In particular; Theorem. The rotation group on $S^n$ is a strong deformation retract of $H_n$ under the pointwise $C^k$ topology for all $n$. Mappings are constructed using a variation of the one given by Alexander. (See J. W. Alexander, "On the deformation of an n-cell," Proc. Nat. Acad. Sci. U.S.A. 9 (1923), 406-407.) (Received May 22, 1969.)

667-41. MANFRED BREUER, University of Kansas, Lawrence, Kansas 66044. Atiyah-Janich theory in von Neumann algebras.

Let $A$ be a semifinite properly infinite von Neumann algebra. Let $\mathcal{F}(A)$ be the space of Fredholm elements of $A$ as defined in Abstract 648-61, these Notices 14 (1967), 647. Let $X$ be a compact
space. A vector bundle $\xi$ over $X$ is called an $A$-bundle if its transition functions assume their values in the group of regular elements of some finite reduced subalgebra of $A$. Let $M_A(X)$ be the monoid of equivalence classes of $A$-vector bundles. Let $K(X)$ be the corresponding universal group. Theorem. If $A$ is countably decomposable, then $\mathcal{N}(A)$ is a classifying space for $K_A$. I.e., let $[X, \mathcal{N}(A)]$ denote the group of homotopy classes of maps of $X$ into $\mathcal{N}(A)$. Then there is a natural isomorphism $[X, \mathcal{N}(A)] \to K_A(X)$ which is defined similarly as in the case of the full operator algebra. (See M. Atiyah, "K-theory," Benjamin, New York, 1967) - the proof depends on the generalized Kuiper theorem (Abstract 663-316, these Notices 16 (1969), 177). (Received May 28, 1969.)


Let $G$ be a locally compact topological group with a closed subgroup $K$, and let $\varphi$ be a continuous positive definite function defined on $G$. If the equation $\varphi(kxk') = \varphi(x)$ is satisfied for all $k, k' \in K$ and $x \in G$, then $\varphi$ is said to be bi-invariant with respect to $K$. Let $B$ be the set of all elementary normalized positive definite functions on $G$, bi-invariant with respect to $K$, endowed with the weak-* topology induced by $L^1(G)$. Gelfand has shown that if $G$ is unimodular, $K$ is compact, and a certain subalgebra of $L^1(G)$ is commutative, then to each bi-invariant $\varphi$ there corresponds a Radon measure $\mu$ on $B$ such that $\varphi(x) = \int_B \varphi(x)d\mu$ for all $x \in G$. This same result can be established, and with greater ease, by the use of Choquet's theorem. Moreover Gelfand's three conditions listed above can be replaced with the single one that $G$ have a countable neighborhood basis. We may also take a slightly more general definition of bi-invariance. (Received May 19, 1969.)

667-43. EDUARD TYMCHATYN, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. Some order theoretic characterizations of the 3-cell.

Let $X$ be a space with a closed partial order $\preceq$ (i.e. $\preceq$ is a partial order and a closed subset of $X \times X$). For $x \in X$ let $L(x) = \{y \in X | y \preceq x\}$. Let $\operatorname{Min}(X) = \{y \in X | L(y) = \{y\}\}$. Define $\operatorname{Max}(X)$ dually. A totally ordered continuum in $X$ is called an order arc. Theorem 1. Let $X$ be a compact 3-manifold with boundary $S^2$ and let $\theta \in X - S^2$. If $X$ admits a closed partial order such that $\operatorname{Max}(X) = S^2$, $\operatorname{Min}(X) = \{\theta\}$, and for each $x \in X$, $L(x)$ is an order arc, then $X$ is a 3-cell. Theorem 2. Let $X$ be as in Theorem 1. Suppose $X$ admits a metric $p$ such that for each $x \in S^2$ there is a unique line segment $T_x$ with endpoints $\theta$ and $x$. If for each $x \in S^2$, $T_x \cap S^2 = \{x\}$ and $X \subset \cup T_x | x \in S^2\}$, then $X$ is a 3-cell. By placing extra conditions on the partial order one gets a similar theorem without assuming that $X$ is a manifold. (Received June 2, 1969.)


Let $P$ and $Q$ be conjugate $p$-subgroups of the finite group $G$. In the case $P$ is $p$-Sylow in $G$, Alperin (J. Algebra 6 (1967), 222-241) has shown how to conjugate from $P$ to $Q$ in a sequence of steps. Under suitable conditions this result is extended to the case where $P$ is not necessarily Sylow in $G$. Examples are provided by certain $p$-subgroups of multiply transitive groups and of groups having split $(B,N)$-pairs. As a corollary it is shown that if $P$ is abelian and the extension of Alperin's theorem holds for $P$, then $N_G(P)$ controls fusion in $P$. (Received June 2, 1969.)
Generating random normal numbers.

A new method of generating random normal numbers is proposed, based on the binomial distribution. Using a generator of uniformly distributed random numbers whose binary digits have been shown, by statistical tests reported in the literature, to be random and not periodic, the standardized sum of these digits is used as an approximation to the normal distribution. Since the digits are themselves random, that is they represent samples from independent Bernoulli distributions, the De Moivre-Laplace limit theorem is applicable to the sum of the digits. Computations show that the sum of 64 Bernoulli distributed random variables leads to a better approximation to the normal distribution than does the sum of 12 uniformly distributed random variables. This latter method is in fairly common use as an approximation to the normal distribution because of its simplicity and its minimal computer memory storage requirements, characteristics which are shared by the summing of the binary digits. (Received June 2, 1969.)

Mutual aposyndesis in products of continua. Preliminary report

C. L. Hagopian [Doctoral Dissertation, Arizona State University, 1968] introduced the notion of mutual aposyndesis (M is mutually aposyndetic at $x, y$ iff there are 2 disjoint continua $H$ and $K$ with $x \in H^0$ and $y \in K^0$). Extending this from 2 points to $n$ points yields the concept of $n$-mutual aposyndesis (with 2-mutually aposyndetic equivalent to mutually aposyndetic), which lies between aposyndesis and locally connectedness. Theorem 1. The product of any three regular Hausdorff continua (not necessarily compact) is $n$-mutually aposyndetic for each $n$. The proof of Theorem 1 also contains the following strengthening of the result of R. W. Fitzgerald ["The cartesian product of non-degenerate compact continua is $n$-point aposyndetic," Topology Conference, Arizona State University, 1967, pp. 324-326]. Theorem 2. The product of any two regular Hausdorff continua (not necessarily compact) is countable-set aposyndetic. (Received June 2, 1969.)

Commutative rims in clans with zero.

Let $S$ be a topological semigroup. The compact subset $B$ is a rim of $S$ if for each closed subset $Y$ of $S$ there is some integer $n$ such that the map $H^n(S) \to H^n(Y)$ induced by inclusion is not onto. (Alexander-Spanier Cohomology) Lemma. Let $S$ be a continuum semigroup containing 0 and 1 and let $B$ be a rim of $S$. If $T$ is a continuum subsemigroup containing 0 and 1, then $S = BT$. Theorem. Let $S$ and $B$ be as above and assume $B$ is connected and the elements of $B$ commute. Then $S$ is commutative. Counterexamples to possible extensions will be given. (Received June 4, 1969.)

Nilpotency index of a certain ring. Preliminary report.

Let $R$ be an associative ring generated by $x_1, x_2, \ldots$ where one can divide by 2, 3 and 5, and let $L$ be the Lie ring of $R$ generated by $x_1, x_2, \ldots$ under ring commutation. Theorem 1. If $x^3 = 0$ for
all x in L, then the nilpotency index of R is 12. Theorem 1 improves results by Heineken and Higgins [see R. H. Bruck's Notes, Canberra, 1963]. (Received June 6, 1969.)

667-49. ROBERT E. MULLINS, Marquette University, Milwaukee, Wisconsin 53233.
A converse of the Stone-Weierstrass theorem.

For X any set, B(X) with sup norm will denote the Banach algebra of all bounded real-valued functions defined on X. A subalgebra A of B(X) will be called an F-algebra if A is closed relative to uniform convergence, contains constants and separates the points of X. An algebra A will be called a minimal algebra if A is an F-algebra which does not properly contain another F-algebra. If X is given a compact Hausdorff topology and A is C(X), the algebra of all continuous real-valued functions on X, the Stone-Weierstrass theorem states that A is minimal in B(X). The following theorem is a converse to the Stone-Weierstrass theorem: Theorem 1. If A is a minimal algebra in B(X), then A = C(X) for some compact Hausdorff topology on X. A necessary and sufficient condition for an F-algebra to contain a minimal algebra is given in the next theorem. Theorem 2. Let A be an F-algebra in B(X) and let Z be the Stone-Čech compactification of X where X is given the Tychonoff topology induced by A. Then A contains a minimal algebra if and only if the closure of Z \ X in Z contains at most one point of X. (Received June 6, 1969.)

667-50. DAVID A. SPRECHER, University of California, Santa Barbara, California 93106.
On simultaneous Chebyshev approximations with polynomials.

Consider the space C of continuous functions on [0,1]; for f \in C let E_j(f) stand for the degree of best approximation by algebraic polynomials of degree j. Theorem 1. Given two polynomials p_m and p_n, 0 \leq m < n, of respective degrees m and n, then there is a function f \in C such that ||f - p_j|| = E_j(f) for j = m, n if and only if if p_n - p_m changes sign at least m + 1 times in [0,1].

Theorem 2. Let p_0 = a_0, p_1 = b_0 + b_1 x, p_2 = c_0 + c_1 x + c_2 x^2 be given. Then there is a function f \in C such that ||f - p_j|| = E_j(f) for j = 0, 1, 2 if and only if if there are points 0 < x_1 < x_2 < 1 such that p_1(x_j) = p_2(x_j) and x_j/2 < (a_0 - b_0)/b_1 < (x_1 + x_2)/2 (b_1 > 0, c_2 < 0 or b_1 < 0, c_2 > 0), or else (x_1 + x_2)/2 < (a_0 - b_0)/b_1 < (x_2 + 1)/2 (b_1 < 0, c_2 < 0 or b_1 > 0, c_2 > 0). (Received June 6, 1969.)

667-51. LOREN C. LARSON, Saint Olaf College, Northfield, Minnesota 55057. Ultrapowers of Zariski rings.

Let D be a nonprincipal ultrafilter over a countably infinite set and let *X denote the ultrapower of a set X with respect to D. If X \subseteq Y, we consider *X \subseteq *Y. Let R be a commutative ring with unity and A an ideal of R such that \bigcap_{n=1}^{\infty} A^n = 0. If B is an ideal of R, then *B is an ideal of *R. Set \mu(A) = \bigcap_{n=1}^{\infty} *\left(A^n\right) and B_\mu(A) = (*B + \mu(A))/\mu(A). Theorem 1. If R is a Zariski ring with respect to A and R* is the A-adic completion of R, then (R, R_\mu(A)) and (R*, R_\mu(A)) are flat couples.

Theorem 2. If (R; P_1, ..., P_n) is a semilocal (Noetherian) ring and A = Rad R, then (R_\mu(A); P_1, ..., P_n, \mu(A)) is a semilocal ring. Theorem 3. If R is a Zariski ring and A = Rad R, then the following are equivalent: (1) R_\mu(A) is Noetherian; (2) R_\mu(A) is semilocal; (3) R_\mu(A) is a Zariski ring with respect to A_\mu(A); (4) R is semilocal. Theorem 4. If (R; P_1, ..., P_n) is a semilocal ring and A = Rad R, then R_\mu(A) \cong R_\mu(P_1) \oplus ... \oplus (R_\mu(P_n) \mu(P_n R_n) \cap R_\mu(P_1))^\perp where R_i = R_{P_i} for i = 1, ..., n. (Received May 29, 1969.)
667-52. CRAIG COMSTOCK, University of Michigan, Ann Arbor, Michigan 48104. On the limit cycles of \( y'' + \mu F(y') + y = 0 \).

In studying the oscillations of the Liénard-type equation (1) \( y'' + \mu F(y') + y = 0 \) or its derivative \( x'' + \mu f(x)x' + x = 0 \) the classical theorems due to Levinson and Smith, and others, have required as a hypothesis that sufficiently far from the origin \( F \) (or \( f \)) tends to infinity. Recently several authors have shown that (2) \( y'' + \mu \sin y' + y = 0 \) has an infinite number of periodic solutions. Following D'heudene's proof of this we show that (1) has an infinite number of periodic solutions for a wide class of oscillatory functions \( F(y') \). (Received May 29, 1969.)

667-53. MORTEZA ANVARI, Aria-Mehr University, Tehran, Iran. Minimum of a norm subject to a system of D. E.

Let (1) \( \dot{y}(t) = P(t)y(t) + Q(t)x(t), y = y(0), y_0 = y(T) \), where \( x(t) = (x_1(t), \ldots, x_n(t)) \), \( y(t) = (y_1(t), \ldots, y_n(t)) \), and \( P(t) \) & \( Q(t) \) are matrices of suitable order, be an initial value problem and

\[
\|x\| = \left( \sum_{k=1}^{n} \int_{0}^{T} |x_k(t)|^{2} dt \right)^{1/2}. 
\]

The minimum of \( \|x\| \) subject to the initial value problem (1) is determined in terms of \( P(t), Q(t), y \) and \( y_0 \). (Received June 9, 1969.)

667-54. NAOHI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. On injective semilattices.

It will be shown that a semilattice is injective if and only if it is complete as a semilattice (so that both the sup and the inf are defined in it) and the complete distributivity of the sup over the multiplication does hold. Also a few relevant results will be given. (Received June 9, 1969.)


Let \((M^{2n}, \Omega)\) be a symplectic manifold. A lagrangian foliation of \( M \) is an integrable distribution of \( n \)-planes on \( M \) for which the restriction of \( \Omega \) to each integral manifold is identically zero.

**Theorem.** (1) All lagrangian-foliated symplectic manifolds of the same dimension are locally isomorphic. (2) The integral manifolds carry a natural flat torsionless affine connection. (3) Every flat torsionless affine manifold is an integral manifold of a lagrangian foliation of its cotangent bundle. (4) The generating functions of infinitesimal automorphisms of a lagrangian-foliated symplectic manifold are precisely those functions which are affine on each integral manifold. (Received June 11, 1969.)

667-56. ROBERT J. KIMBLE, Jr., U. S. Naval Academy, Annapolis, Maryland 21402. Ortho-implication algebra.

A semiorthomodular lattice is defined as an upper semilattice \((K, \leq, \lor, 1)\) with greatest element \( 1 \) in which every principal filter \([b]\) is orthocomplemented under the partial operation \( x_{b}^{\perp} \setminus x \in [b] \). In such a semilattice we define ortho-implication by \( a \rightarrow b = (a \lor b)_{b}^{\perp} \). This operation is shown to satisfy \( K1: (ab)_{a} = a; K2: (ab)_{b} = (ba)_{a}; K3: (ab)(bc) = bc \) and \( K4: b[ab]_{c} = bc \). Conversely an abstract algebra \((K, \cdot)\) satisfying \( K1-K4 \) determines an associated semiorthomodular lattice.
under $aa = 1$, $a \equiv b \Leftrightarrow ab = 1$ and $a \sqcup b = (ab)b$. The theory of orthomodular lattices is obtained as a special case by requiring the existence of a least element $0$ satisfying $K5: 0a = 1 \forall a$. Lattice meet and ortho-complementation are then given by $a \cap b = (a0 \sqcup b0)0$ and $a^\perp = a0$. The treatment is similar to that of semi-Boolean and Boolean algebras using $K1$, $K2$ and $K3: a(bc) = b(ac)$. (Cf. Abbott, "Sets, lattices and Boolean algebras," Allyn and Bacon, Boston, 1969, Chapter 7.) (Received June 11, 1969.)

667-57. GEORGE B. WILLIAMS, Hamline University, Saint Paul, Minnesota 55101. S-objects in an abelian category.

An abelian group $G$ is an S-group if whenever $K$ is a direct summand of $G$, then $G \cong G \oplus K$ (cf. R. A. Beaumont, "Abelian groups $G$ which satisfy $G \cong G \oplus K$ for every direct summand $K$ of $G$" (to appear)). $G$ is an ID-group if $G$ has an isomorphic proper direct summand (cf. R. A. Beaumont and R. S. Pierce, "Isomorphic direct summands of abelian groups," Math. Ann 153 (1964), 21-37). In addition to extending most of the results of R. A. Beaumont on S-groups to a general abelian category the following are shown. (1) If $G$ is a $C_3$-category (i.e., satisfies the Grothendieck axiom A. B. 5), then an ID-object $A$ in $G$ contains a nonzero S-object. (2) If, in particular, $G$ is an abelian group whose torsion subgroup is an ID-group, then $G$ has a nonzero direct summand which is an S-group. (3) There are conditions such that an S-object $A$ in a complete $C_3$-category is isomorphic to an interdirect sum of countably many copies of $A$. (4) If $T$ is a torsion abelian group, $T$ an S-group, and $T = K \oplus L$ where $K$ contains no nonzero S-groups, then $T \not\cong L$. (Received June 11, 1969.)

667-58. STANLEY B. JACKSON, University of Maryland, College Park, Maryland 20742. A refinement of the four-vertex theorem.

A simple closed curve $J$ in the plane is called strongly conformally differentiable if it has, at each point, a unique pencil of general tangent circles and a unique general osculating circle. If such a curve $J$ is not a circle, a point (or circular arc) of $J$ is called a global vertex if the general osculating circle here never meets $J$ again, i.e., supports $J$ globally. The global vertices are called positive or negative according as $J$ lies to the right or left of the osculating circle. It is established that $J$ must have at least two positive and at least two negative global vertices. The concept of global vertex is more restrictive than the usual definitions of vertex since it is a property of the curve in the large. If ordinary vertices are defined as extrema of the curvature, the maxima and minima must alternate on the curve. In contrast, it is shown by an example that the positive and negative global vertices need not alternate on $J$ and that the total number of global vertices need not be even. (Received June 12, 1969.)

667-59. KWANG-NAN CHOW and MOSES GLASNER, California Institute of Technology, Pasadena, California 91109. On bounded in the mean solutions of the equation $\Delta u = Pu$.

Let $R$ be a noncompact Riemannian manifold and $P$ a nonnegative smooth $n$-form on $R$ not identically zero. Consider a positive increasing convex function and set $d = \lim_{t \to \infty} f(t)/t$. Denote by $P\Phi$ the solutions of $\Delta u = Pu$ on $R$ such that $\Phi([u])$ is majorized by a solution, by $PN$ the nonnegative solutions on $R$ and by $PB$ the bounded solutions on $R$. **Theorem.** If $d = \infty$, then

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dim $P \Phi > i$ if and only if $\text{dim } PB > i$; if $d < \infty$, then $\text{dim } P \Phi > i$ if and only if $\text{dim } PN > i$. Here $i = 0$ or 1. It is known that $P \neq 0$ implies that $\text{dim } PN > 0$. These are analogous of Parreau's (Ann. Inst. Fourier Grenoble 3 (1951), 103-197) results for the case $P \equiv 0$. (Received June 12, 1969.)

667-60. BENJAMIN HALPERN, University of California, Berkeley, California 94720.

On the immersion of an $n$-dimensional manifold in $(n + 1)$-dimensional Euclidean space.

Let $M$ be a compact, closed, connected, $n$-dimensional, $(n \geq 2)$, smooth (infinitely differentiable) manifold and $I: M \to \mathbb{R}^{n+1}$ a smooth immersion of $M$ into $(n + 1)$-dimensional Euclidean space. For each $p \in M$ consider the hyperplane $T_p$ in $\mathbb{R}^{n+1}$ drawn through $I(p)$ and tangent to $I(M)$. Theorem. If $\bigcup_{p \in M} T_p \neq \mathbb{R}^{n+1}$, then $M$ is diffeomorphic to the $n$-sphere, $I$ is actually an embedding, there exists a unique open starshaped set $V \subset \mathbb{R}^{n+1}$ such that $\partial V = I(M)$, and $\mathbb{R}^{n+1} - \bigcup_{p \in M} T_p = \text{int}(\text{kernel } V)$, where kernel $V = \{p \in V | p + (1 - t)q \notin V \text{ for all } q \in V \text{ and } 0 \leq t \leq 1\}$. Conversely, if $I(M) = \partial V$ for some open starshaped set $V \subset \mathbb{R}^{n+1}$ with $\text{int}(\text{kernel } V) \neq \emptyset$, then $\bigcup_{p \in M} T_p \neq \mathbb{R}^{n+1}$. (Received June 12, 1969.)

667-61. NOBORU ITO and WILLIAM M. KANTOR, University of Illinois, Chicago, Illinois 60680.

2-transitive symmetric designs with $k = 2p$.

Let $G$ be a finite 2-transitive group of degree $v$ having an intransitive subgroup $H$ of index $v$ with an orbit of length $k \neq 1$, $v - 1$. There is a symmetric design associated with the pair $G,H$. Using this design and a result of Wielandt (Math. Z. 63 (1956), 478-485), the possible groups $G$ are enumerated when $k = 2p$, $p$ prime, and $G$ is not similar to a 2-transitive collineation group of a finite projective space. Under these assumptions, $G$ must be similar to $\text{PSL}(2,11)$ in its representation of degree 11, or to a group of degree 16 of the form $E_6$, with $E$ a normal elementary abelian subgroup of order 16 and $\text{PSL}(2,5) \not\subseteq S \subseteq S_6$. (Received June 18, 1969.)

667-62. ABDUL JABBAR JERRI, Clarkson College of Technology, Potsdam, New York 13676.

Some applications for Kramer's generalized sampling theorem.

Kramer's generalization of Shannon's sampling theorem takes us from a signal represented by a finite Fourier transform to a signal represented by another and more general finite integral transform. In this paper, we will attempt to show that the already obtained results for Kramer's theorem are of use in the field of finite integral transforms. Also by introducing such transforms one can treat some communications problems. An example is the case of representing a signal which is the output of a time variant filter. (Received June 18, 1969.)

667-63. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93103. Uncountable does not imply nonpairable.

Pairability is defined to mean that 1-1 correspondence can be achieved between the members of two sets. Countability of two equal sets implies not only pairability but also a demonstrable definition of nextness, i.e., an ordinating or "threading" rule, and hence an explicitly writable sequence. Proof of uncountability does not ensure or imply proof of nonpairability. If two sets are
pairable they are equal in Cantor's sense. And it may be that an infinite uncountable set is pairable with an infinite countable set, and hence equal to it. **Example.** Between any two rational real numbers is an irrational real number. But the following theorem is also true: Let R be the set of rational reals and I, the set of irrational reals. Between any two irrationals (i,I) is a rational (r), the axis of reals being hence given by \( \ldots \text{irr irrational} \ldots \). Hence \( (R = I) = (N = c) \). The failure to recognize that nonpairable and uncountable were not synonymous vitiated Cantor's theorem of 1863, which proved only the latter property of I, which as is now evident, does not imply the former. Indeed in this case, R and I are pairable, even though I is uncountable. (Received June 18, 1969.)

667-64. DAVID MOE, Jacksonville State University, Jacksonville, Alabama 36265. **Point sets** which are norm intervals.

In Proc. Amer. Math. Soc. 17 (1966), 202-206, J. R. Calder introduced the notion of a norm interval in a normed linear space. If M is a norm interval, it is shown that M is a closed sphere relative to some norm on the linear span of M. Then a characterization is obtained of those point sets in a given linear space which are norm intervals relative to some norm on the space. (Received June 18, 1969.)

667-65. WITHDRAWN.

667-66. RICHARD J. GREECHIE, Kansas State University, Manhattan, Kansas 66502. **On the existence of an atomic nonatomistic orthomodular poset.**

**Theorem.** There exists an orthomodular poset P such that (1) P admits a full set of states, (2) if \( x \in P \) with \( x \neq 0 \) then x is finite (every maximal chain from 0 to x is finite) or \( x' \) is finite, (3) if \( x \in P \) with \( x \neq 0 \) then x dominates an atom of P, (4) every Boolean subalgebra of P is countable, (5) if \( \{ x,y,z \} \) is a pairwise compatible subset of P then x is compatible with y v z, and (6) there exists a coatom \( x \in P \) such that sup \( \{ a: a \text{ is an atom and } a \neq x \} \) does not exist. Hence there exists a "nice" orthomodular poset which is atomic (3) but not atomistic (6). (Such pathology could not occur if P were an orthomodular lattice.) This result answers a conundrum which was implicitly posed in a paper concerned with the algebraic structure of quantum mechanics (Comm. Math. Phys. 6 (1967), 267). (Received June 13, 1969.)
Let $E$ be a real locally convex, topological vector space. Let $\tau$ be the Mackey topology of $E$, assume that $\tau$ is nonmetrizable and let $\mathcal{U}$ be a fixed fundamental system of circled, closed, convex $\tau$-neighborhoods of zero in $E$. A net \[ x_a \] in a directed set $A$ of points of $E$ will be called a Mackey net provided: (i) The cardinality of $A$ is equal to that of $\mathcal{U}$. (ii) No countable subset of $A$ is cofinal; i.e., no subnet of \[ x_a \] is a sequence. \[ \text{Theorem.} \] Let $E$ be a complete locally convex space and suppose that $E$ contains a strongly dense subset whose cardinality is equal to that of $U$. Then $E$ is semi-reflexive iff every bounded, weakly Cauchy, Mackey net in $E$ is weakly convergent to a point of $E$.

In order to prove this theorem we first prove a number of facts about Mackey nets and we give a characterization of weak compactness using these nets. All of our theorems remain valid in the event that $\tau$ is metrizable provided that Mackey nets are replaced by sequences. (Received June 13, 1969.)

Let $G$ be a finite group with a family of exceptional characters $\chi_1, \ldots, \chi_w$, $w > 2$ associated with irreducible characters $\psi_1, \ldots, \psi_w$ of the normalizer of an Abelian CC-subgroup $A$ of $G$. We may denote the special classes (those nonidentity conjugacy classes of $G$ meeting $A$) by $K_1, \ldots, K_w$. Let \[ c_{ijk} \] be the structure constants associated with these classes and for $g_i \in K_i \cap A$ let $a_i = (\psi_1 - \psi_2)(g_i)$ and $b_i = (\psi_1 - \psi_2)(g_i)$. Then $d = \sum_{i,j,k=1}^{w} a_i b_j c_{ijk}$ is an integer divisible by $|A|$, and $|G|/|A|/|d|$ is the common degree of the exceptional characters. Generalizations of this result, together with other results, show how exceptional characters are determined by the behavior of the special classes. These results are found by evaluating characters on certain idempotents in the group algebra and may be proved directly using orthogonality relations. (Received June 13, 1969.)

The results of D. J. Foulis, "Baer *-semigroups," Proc. Amer. Math. Soc. 11 (1960), 648-654, concerning the coordinatization of orthomodular lattices by Baer *-semigroups are extended to the coordinatization of orthomodular posets by OM-partial Baer *-semigroups. Let $S$ be a set, $R \subseteq S \times S$ a relation, and $S = (S, R, \cdot, *)$ a partial involution semigroup with 0. An element $f \in S$ is called a projection if $(f,f) \in R$ and $f = f \cdot f = f^*$. A partial involution semigroup $S$ with 0 is called an OM-partial Baer *-semigroup if (1) for every $x \in S$ there exists a projection $x' \in S$, such that $(x',y) \in R$ and $y = x' \cdot y$ if and only if $(x,y) \in R$ and $x \cdot y = 0$ and (2) if $P(S) = \{ e \mid e \in S, e$ is a projection, $e = e'^* \}$ and $x, y \in P(S)$ with $x \neq y$ (i.e., $x = xy$), then $(x', y) \in R$. \[ \text{Theorem 1.} \] If $S$ is an OM-partial Baer *-semigroup then $P(S)$ is an orthomodular poset. Conversely if $P$ is an orthomodular poset then there exists an OM-partial Baer *-semigroup $S_0(P)$ such that $P(S_0(P))$ is order isomorphic to $P$ preserving orthocomplementation, i.e., $P(S_0(P))$ coordinatizes $P$. \[ \text{Theorem 2.} \] If $P$ is an orthomodular poset and $S$ is any OM-partial Baer *-semigroup coordinatizing $P$, then $S_0(P)$ (as in Theorem 1) can be "embedded" in $S$. (Received June 13, 1969.)
667-70. KARL E. BARTH and WALTER J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. Entire functions mapping arbitrary countable dense sets (and their complements) onto each other.

In Hayman's book "Research problems in function theory," p. 17, the following question (attributed to Erdős) is posed: If \( A = \{ a_1, a_2, \ldots \} \) and \( B = \{ b_1, b_2, \ldots \} \) are countable dense subsets of the complex plane, does there exist an entire function \( f(z) \) such that \( f(z) \notin B \) if and only if \( z \notin A \)? The authors answer this question in the affirmative by constructing \( f \) as the sum of functions \( h_1, h_2, \ldots \) where the \( h_n \)'s are polynomials which are, respectively, very small, in absolute value, on the disks \( |z| < n \). Also the \( h_n \)'s are so chosen that for the \( n \)th partial sum \( P_n \) one has

(i) \( \{ P_n (\bigcup_{k=1}^{n} \{ a_k \}) \} \subseteq B \),
(ii) \( \{ P_{n-1}^{-1} (\bigcup_{k=1}^{n-1} \{ b_k \}) \cap \{ |z| < n \} \} \subseteq A \); in addition, in \( |z| < n - 1 \),
\( \{ P_{n-1}^{-1} (\bigcup_{k=1}^{n-1} \{ b_k \}) \} \) exactly coincides with \( \{ P_{n-1}^{-1} (\bigcup_{k=1}^{n-2} \{ b_k \}) \} \); in \( |z| < n - 2 \), \( \{ P_{n-1}^{-1} (\bigcup_{k=1}^{n-2} \{ b_k \}) \} \) exactly coincides with \( \{ P_{n-2}^{-1} (\bigcup_{k=1}^{n-2} \{ b_k \}) \} \); etc. This follows by using Rouché's theorem and methods not too different from the techniques in an earlier abstract of the authors' (Abstract 658-145, these Notices 15 (1968), 762) --the arguments, while very straightforward, involve a fair bit of bookkeeping. The fact that \( f^{-1}(B) = A \) then follows by Hurwitz's theorem. (Received June 16, 1969.)


For any integer \( v \geq 2 \), an iterative polynomial algorithm \( Q_{n+1} = P_v(Q_n) \) is found which converges \( v \)-ically to a solution of the operator equation \( QHQ^+ = I \), where \( H \) is a Hermitian operator on a Hilbert space. Extensions to related equations and sets of equations are considered. The results depend on a property of truncated binomial expansions. An application is the rapid iterative orthogonalization of a matrix. (Received June 16, 1969.)

667-72. THOMAS A. BRONIKOWSKI, Marquette University, Milwaukee, Wisconsin 53233. An integrodifferential system which occurs in reactor dynamics.

Consider the real system \( u'(t) = - \int_0^c a(x)T(x,t)dx, \ T_T(x,t) = (b(x)T_x(x,t))_x - q(x)T(x,t) + \eta(x)u(t) \) for \( 0 < x < c, \ 0 < t < \infty \). The initial and boundary conditions are \( u(0) = u_0, T(x,0) = f(x) \) \( 0 < x < c \), \( a_1 T(0,t) + a_2 T(c,t) = 0, b_1 T(c,t) + b_2 T_x(c,t) = 0 \) \( 0 < t < \infty \), where \( |a_1| + |a_2| > 0, |b_1| + |b_2| > 0 \). After establishing existence and uniqueness criteria, the asymptotic behavior as \( t \to \infty \) of the solution \( u(t), T(x,t) \) is studied. Under several additional conditions the associated Sturm-Liouville problem \( (b(x)y')' + (\lambda - q(x))y = 0, a_1 y(0) + a_2 y'(0) = 0, b_1 y(c) + b_2 y'(c) = 0 \) has only positive eigenvalues. In this case \( u(t) \) and \( T(x,t) \) both tend to zero exponentially as \( t \to \infty \). If, however, the Sturm-Liouville problem has zero for an eigenvalue, then \( u(t), T(x,t) \) need not tend to zero. The limiting behavior is obtained explicitly. Finally, the continuity of the solutions on \( a, \eta, f \) (considered as points in \( L_2(0,c) \)) is established. (Received June 19, 1969.)

Suppose that M is a $C^\infty$-manifold. A sequence $0^m M \rightarrow 1^m M \rightarrow \ldots$ of $C^\infty$ manifolds with associated $C^\infty$ maps $0^m \rightarrow 1^m \rightarrow \ldots$ is called an extended sequence of length $m$ of $M$ provided that $0^1 M = M$ and

(i) for each integer $1 \leq k \leq m$ there exists an imbedding $k^{-1} \rightarrow k^1 M$ with $0^1$ onto such that $k^{-1} \pi_1 T^k k^1 M = k^{-1} \pi_1 T^k (k^1 + 1) M$.

(ii) for each integer $1 \leq k < m$, $k^{-1} \pi_1 (k^1 M) = k^{-1} \pi_1 (k^1 M)$.

Where $T^k M$ is the tangent bundle of $k^1 M$, and $T^k k^1 \pi_1 : T^k M \rightarrow k^1 M$ the natural projection. If $J^k (k^1 M)$ denotes the tensor algebra of $k^1 M$ we show that each element of $J^k (k^1 M)$ may be extended (or lifted) to a unique element of $J^k (k^1 M)$ for each positive integer $k$. In the case $k = 1$ this extension coincides with the complete lift defined by K. Yano and S. Kobayashi (J. Math. Soc. Japan 18 (1966), 194-210).

(Received June 19, 1969.)


We prove, among other results, that if $X$ is compact Hausdorff and $K$ is a CW-complex then the function space $K^X$ (compact-open topology) is stratifiable (therefore, paracompact and perfectly normal). Subsequently, we improve several well-known results concerning the equality of the cartesian product topology and its associated $k$-topology. (Received June 19, 1969.)

667-75. WITHDRAWN.

667-76. MARVIN SHINBROT, Northwestern University, Evanston, Illinois 60201. The energy equality for weak solutions of the Navier-Stokes equations.

It is known that every weak (Hopf) solution of the Navier-Stokes equations satisfies an energy inequality. It is shown here that this inequality is actually an equality. The question has relevance to the important question of uniqueness of weak solutions of the Navier-Stokes equations. (Received June 19, 1969.)

667-77. ELLEN TORRANCE, Mount Holyoke College, South Hadley, Massachusetts 01075. A characterization of $C^*$-algebras. Preliminary report.

Let $(A, \| \cdot \|)$ be a complex Banach algebra with identity $e$, $\| e \| = 1$, and let $H$ be the set of hermitian elements of $A$: $h \in H$ if and only if $\| e + i\alpha h \| = 1 + o(\alpha)$ as $\alpha \rightarrow 0$, $\alpha \in \mathbb{R}$. It is shown by a semi-inner-product calculation that $H + iH$ is a closed linear subspace of $A$. Let $(g + i h)^* = g - i h$ for all $g, h \in H$. Theorem. $(H + i H, \| \cdot \|, *)$ is a $C^*$-algebra if and only if $h^2 \in H + i H$ whenever $h \in H$. (Received June 18, 1969.)

667-78. PHILLIP ZENOR, Auburn University, Auburn, Alabama 36830. On extending spaces using inverse limits.

Theorem 1. If $X$ is completely regular, then there is an inverse system $(X_\alpha, t^\alpha_{\beta}, A)$ of totally bounded metric spaces with uniformly continuous bonding maps over a directed set such that: 1. The Hewett realcompactification of $X$ is homeomorphic to $\lim \downarrow (X_\alpha, t^\alpha_{\beta}, A)$. 2. The Stone-$\check{C}$ech compactifica-
tion of $X$ is homeomorphic to $\lim\left(\overset{\ast}{X}_{a}, \overset{\ast}{f}_{b}, A\right)$, where $\overset{\ast}{X}_{a}$ denotes the completion of $X_{a}$. Theorem 2. The completely regular space $X$ is realcompact if and only if $X$ is the inverse limit of separable metric spaces over a directed set. (Received June 26, 1969.)


Let $S$ and $T$ be nonempty sets and $L : S \rightarrow T$. A subset $B$ of $S$ is called a branch of $L$ if $B \subseteq L^{-1}(t)$ consists of exactly one element of $S$ for each $t$ in $L(S)$. A mapping $\phi : T \rightarrow T$ is called a projection on $L(S)$ if $\phi(T) \subseteq L(S)$ and $\phi(t) = t$ for all $t$ in $L(S)$. Call this bijection $\phi_{L}$. Let $\pi^{*} : B \rightarrow S$ be the insertion mapping. Define $L^{+} = \pi^{*} \phi_{L}^{-1} \phi$. Clearly we have (1) $L = LL^{+}$ and (2) $L^{+} = L^{+}LL^{+}$. If $S$ and $T$ are Hilbert spaces and $(\ker L)$ and $L(S)$ are closed then we may choose $B = (\ker L)^{\perp}$ and $\phi$ the projection on $L(S)$. In this case, we have $\pi^{*}$ the adjoint of the projection on $B$ and (3) $LL^{+} = \pi$. The usual properties of generalized inverses of matrices follow easily. We also prove (A) $L^{+} = L^{*}$ if and only if $\|Ls\| = \|s\|$ for every $s$ in $L^{*}(T)$. (B) If $L : X \rightarrow Y$, $M : Y \rightarrow Z$ are linear transformations such that $\phi_{ML}(x) = \phi_{M} \phi_{L}(x)$ for every $x$ in $(ML)^{*}(Z)$ then $(ML)^{+} = L^{+}M^{+}QMM^{+}$ where $Q$ is the projection of $M(Y)$ on $ML(X)$. If $L$ is a differential operator the set of solutions of $Lf = g$ satisfying a given set of (suitable) boundary conditions is a branch so that the Green's function is the kernel of the generalized inverse with respect to this branch. (Received June 20, 1969.)

667-80. ANAND M. CHAK and A. K. AGARWAL, West Virginia University, Morgantown, West Virginia 26506. An extension of a class of polynomials, II.

In this paper we continue the study (paper in press; cf. A. M. Chak, "An extension of a class of polynomials, I," these Notices 13 (1966), 610) of our Appell Set of polynomials to the base $(u)$, that is, the class of polynomials $\{H_{n}(x)\}$ in $x$ which satisfy the functional equation $D_{u}H_{n}(x) = H_{n-1}(x)$, for $n = 1, 2, \ldots$, where $D_{u}$ is a very general operator, linear and distributive, which converts a polynomial of degree $n$ in $x$ into one of degree $n - 1$. In particular, $D_{u}x^{n} = u_{n}x^{n-1}$ where $(u)$ is a given sequence of real or complex numbers subject to the only restriction that $u_{0} = 0$, $u_{1} = 1$ and $u_{n} \neq 0$ for $n \geq 1$; (cf. M. Ward, "A calculus of sequences," Amer. J. Math. 58 (1936), 255-266). In the first part of the paper the algebraic structure of our class of polynomials has been studied while in the second part some of its subsets have been examined, that is which have properties analogous to the regular and cyclic sets of N. Nielsen ("Traité élémentaire des nombres de Bernoulli," 1923) and M. Ward ("A certain class of polynomials," Ann. of Math. 31 (1930), 43-51). (Received June 20, 1969.)

667-81. DIRAN SARAFYAN, Louisiana State University, New Orleans, Louisiana 70122. Seventh order Runge-Kutta formulas.

An infinite class of seventh order ten-stage Runge-Kutta formulas is established. From this class a few formulas are selected as more effective. It will be shown that where the directional function is exempt of the dependent variable, these formulas behave as of the eighth order. The
selection of an appropriate step-size will be treated and the estimation of errors discussed. The
formulas will be available to participants and interested parties. (Received June 23, 1969.)

667-82. C. A. HURST, University of Adelaide, South Australia 5001, and SEYMOUR SHERMAN,
Indiana University, Bloomington, Indiana 47401. Griffiths' theorems for the ferromagnetic Heisenberg
model.

R. B. Griffiths [J. Math. Phys. 8 (1967), 478] has shown that for an Ising ferromagnet with
possibly long range interactions the spin-spin correlation is nonnegative and a monotone increasing
function of the interactions. In this paper it is shown that for the corresponding Heisenberg ferro-
magnet the correlation is nonnegative but not necessarily a monotone increasing function of inter-
action. This answers a question of Freeman Dyson ["Existence of a phase transition in a one-
Grants Comm. Contract 66/15220. (Received June 23, 1969.)

667-83. THOMAS L. MARKHAM, University of South Carolina, Columbia, South Carolina 29208.
Some criteria for completely positive quadratic forms. Preliminary report.

Let \( B \) denote the closed convex cone of completely positive quadratic forms, and assume
\[ Q = \sum_{i,j=1}^{n} a_{ij} x_i x_j \]
with associated symmetric matrix \( A = (a_{ij}) \). Theorem. \( Q \) is in \( B \) if and only if
there exists a \( t \times n \) nonnegative matrix \( C \) such that \( A = C^T C \). Those matrices (corresponding to
forms in \( B \)) which admit a lower-triangular nonnegative factorization are characterized in terms of
certain minors of the matrices, and the factorization is shown to be unique. A useful by-product of
this characterization is the well-known lower-triangular factorization of positive-definite matrices.
(Received June 23, 1969.)

667-84. ROBERT L. KRUSE, Sandia Laboratories, Box 5800, Albuquerque, New Mexico 87115,
and D. T. PRICE, Wheaton College, Norton, Massachusetts 02766. Enumerating finite rings.

The purpose of this paper is to obtain order-of-magnitude bounds on the number of finite rings
of a given order. Since the primary decomposition of the additive group of a ring induces a ring
direct sum, in studying finite rings it is always sufficient to consider only rings whose order is a
power of a prime \( p \). The principal result of the paper is that if the number of pairwise nonisomorphic
rings of order \( p^n \) is written in the form \( f(n,p) n^3 \), then \( \lim_{n \to \infty} f(n,p) = 4/27 \) independent of the prime \( p \).
(Received June 23, 1969.)

667-85. MORDECAY ZIPPIN, University of California, Berkeley, California 94720. Subspaces
of Banach spaces whose duals are \( L_1 \) spaces.

Let \( X \) be a separable Banach space whose dual is an \( L_1 \) space. Results of Michael, Pełczyński,
Lazar, and Lindenstrauss show that \( X \) has a structure of the form \( X = \bigcup_{n=1}^{\infty} E_n \) where \( \{E_n\} \) is a
sequence of subspaces of \( X \), directed by inclusion and each \( E_n \) is isometric to \( l_1^n \) (= the space of
\( n \)-tuples \( \lambda = (\lambda_1, \ldots, \lambda_n) \) of numbers with \( \| \lambda \| = \max |\lambda_i| \)). An infinite-dimensional structure for

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such spaces $X$ is given by the following Theorem. Let $X$ be a separable Banach space whose dual is an $L_1$ space. Then $X$ contains a subspace $E$ isometric to $c_0$ (= the space of sequences $\lambda_n$ such that $\lim_n \lambda_n = 0$ and $\|\lambda\| = \sup |\lambda_n|$) such that there is a projection of norm 1 from $X$ onto $E$. (To appear in Proc. Amer. Math. Soc.) (Received June 23, 1969.)


For $1 < j < d < v$, let $\phi_j(v,d) = [\mu_j(v,d)]$ be the minimum [maximum] possible number of $j$-faces of a $d$-polytope with $v$ vertices, and let $f_j(v,d)$ be the number of $j$-faces of the cyclic polytope $C(v,d)$. The well-known upper-bound conjecture states that $\mu_j(v,d) = f_j(v,d)$. We propose the following lower-bound conjecture for facets: The relation $\phi_{d-1}(v,d) = \min [r | f_{d-1}(r,d) \leq v]$ holds provided $d \leq 4$ or $d \leq 5$ and $v \leq \lceil(d^2 + 2d + 5)/4 \rceil$, except that if $d$ is odd and $v = f_{d-1}(r,d) - 2s + 1$, for some $r \geq d + 3$ and $s$ with $1 \leq s < (d+1)/2$ then $\phi_{d-1}(v,d) = r + 1$. If $d \leq 5$ and $v < \lceil(d^2 + 2d + 5)/4 \rceil$ then $\phi_{d-1}(v,d) = d + 2$ if $v = r + (s + 1)(t + 1)$ for some $r \geq 0$, $s \geq 1$, $t \geq 1$ with $r + s + t = d$, and $\phi_{d-1}(v,d) = d + 3$ otherwise. Certain cases of the conjecture can be proved. Indications are that the lower-bound for faces of intermediate dimension $1 \leq j \leq d - 2$ will be even more complicated. (Received June 23, 1969.)

667-87. VLASTIMAL DLAB, Carleton University, Ottawa 1, Canada. $\pi$-reducible rings.

Earlier, perfect rings have been characterized in terms of hereditary torsions [Abstract 69T-A144, these Notices 16 (1969), this issue]. The following two theorems concerning a particular class of perfect rings can be easily proved. Theorem. The properties (i) - (iii) of a ring are equivalent: (i) There are only two torsions in Mod $R$. (ii) $R$ possesses a (transfinite left) composition sequence with $R$-isomorphic factors. (iii) $R$ is isomorphic to the ring of all $m \times m$ matrices over a local (i.e. with a unique maximal left ideal) ring possessing a (transfinite left) composition sequence. Theorem [cf. R. Courter, Canad. J. Math. 21 (1969), 430-446]. A $\pi$-reducible ring can be characterized in any of the following equivalent ways: (i) $R$ is a finite direct sum of rings described in previous theorem. (ii) $R$ is a direct sum of its primary (i.e. corresponding to minimal torsions) parts. (iii) $R$ is a right perfect ring whose idempotent two-sided ideals form a sublattice of the lattice of all left ideals of $R$. (iv) Every $R$-module is rationally complete [see G. D. Findlay and J. Lambek, Canad. Math. Bull. 1 (1958), 77-85]. (Received June 23, 1969.)

667-88. S. K. KAUL, University of Saskatchewan, Regina, Saskatchewan, Canada. Compact subsets in function spaces.

Let $X$, $Y$ be topological spaces, and let $(X,Y)$ denote the set of all continuous functions from $X$ to $Y$. We say that $F \subset (X,Y)$ is a uniformly regular set if for any open covering $V$ of $Y$ there exists an open covering $U$ of $X$ such that $U$ refines $f^{-1}(V)$ for each $f \in F$. We say $F \subset (X,Y)$ is regular at $x$ in $X$ if for any open set $V$ in $Y$ and $G \subset F$ such that the closure of $G(x) = \{g(x) : g \in G\}$ lies in $V$, there exists an open set $U$ containing $x$ such that $g(u) \subset V$ for each $g$ in $G$.

Theorem 1. If $Y$ is regular or Hausdorff, $F \subset (X,Y)$ and $F(x)$ is compact for each $x$ in $X$, then the following are equivalent: (a) $F$ is uniformly regular, (b) $F$ is regular, and (c) $F$ is evenly continuous.
Theorem 2. Suppose $Y$ is regular and Hausdorff and $X$ is separable. If $F$, a regular set in $(X,Y)$, is a closed subset of $(X,Y)$ with the compact open topology, and $F(x)$ is compact for each $x$ in $X$, then $F$ is sequentially compact. (Received June 23, 1969.)

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Successive derivatives of analytic functions.

The Whittaker constant $W$ is the largest positive number $C$ with the following property: If $F$ is a transcendental entire function of exponential type less than 1, then infinitely many derivatives of $F$ have no zero in the disc $|z| \leq C$. The Goncharov constant $G$ is the largest positive number $C$ with the following property: If $f$ is analytic in $|z| \leq 1$ and is not a polynomial, then for infinitely many $n$, $f^{(n)}$ has no zero in the disc $|z| \leq C/n$. It is known that $0.7259 < W < G < 0.7378$, and the conjecture that $G = W$ is over twenty years old. Theorem 1. There exists an entire $F$ such that $0.4 < |F^{(n)}(0)| \leq 1$, $n = 0, 1, 2, \ldots$, and such that each of $F, F', F'', \ldots$ has a zero on the circle $|z| = W$. Theorem 2. $G = W$. (Received June 23, 1969.)

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Inscribed polyhedra of maximum volume.

A condition necessary to maximize the volume of the convex hull of $n$ points on the unit sphere in $E^3$ is that each point $p$ be proportional to the sum of all products $p_1 \times p_2$, where $p_1, p_2, p$ are vertices of a face of the convex hull. This condition is used to obtain inscribed polyhedra of $n$ vertices with maximum volume when $n = 4, 5, 6, 7, 8$. Also, for each of several polyhedral types there is only one polyhedron satisfying the condition and hence only one polyhedron whose vertices give a relative maximum for the volume function. (Received June 23, 1969.)

JAMES H. WELLS, University of Kentucky, Lexington, Kentucky 40506. On the zeros of functions with derivatives in $H_1$ and $H_{\infty}$.

Let $z_k = r_k e^{i\theta_k} (k = 1, 2, \ldots)$ be a complex number sequence in $0 < |z| < 1$ satisfying $\sum (1 - |z_k|) < \infty$ and having as its set of limit points on the unit circle $|z| = 1$ a Carleson set $E$.

Theorem 1. If $\sum_{k=1}^{\infty} \text{dist}(e^{i\theta_k}, E)^\alpha < \infty$ for some $\alpha > 1$, there exists a function $f$ analytic in $|z| < 1$ such that (*) $f(0) = 1$, $f(z_k) = 0$ $(k = 1, 2, \ldots)$ and $f'$ is in the Hardy space $H_1$. Theorem 2. If there exist constants $M$ and $\alpha \geq 1$ such that $\text{dist}(z_k, E)^\alpha < M(1 - |z_k|)$ $(k = 1, 2, \ldots)$, then there exists a function $f$ analytic in $|z| < 1$ such that (*) holds and whose derivatives of all orders are bounded in $|z| < 1$. The proofs depend upon estimating the growth of a Blaschke product near its singularities on $|z| = 1$. (Received June 23, 1969.)

JACK B. BROWN, Auburn University, Auburn, Alabama 36830. Metric spaces in which a strengthened form of Blumberg's theorem holds.

A subset $W$ of a metric space $X$ is uncountably dense (c-dense) in $X$ if and only if every open subset of $X$ contains uncountably many (c-many) elements of $W$. $c$ is the cardinality of the continuum.
The following propositions are considered: Proposition B (Proposition C). If \( f \) is a real-valued function with domain \( X \), then there exists an uncountably dense (a \( c \)-dense) subset \( W \) of \( X \) such that \( f|W \) is continuous at each element of a dense subset \( D \) of \( W \). **Theorem 1.** If \( X \) is a metric space such that no open subset of \( X \) is the union of a first category set and a set which has no uncountable nowhere dense subset, then Proposition B holds in \( X \). **Theorem 2.** If \( X \) is a separable metric space, then Proposition C holds in \( X \) if and only if no open subset of \( X \) is the union of a first category set and a set which has no nowhere dense subset of cardinality \( c \). **Corollary.** Proposition C holds for real functions with a connected domain. (Received June 23, 1969.)

667-93. TAKAYUKI TAMURA, University of California, Davis, California 95616, and MORIO SASAKI, Iwate University, 3-18-UEDA, Morioka, Iwate, Japan. Commutative power joined cancellative semigroups without idempotent.

A commutative semigroup \( S \) is called power joined if for \( a, b \in S \) there exist positive integers \( m \) and \( n \) such that \( a^m = b^n \). A commutative power joined cancellative semigroup \( S \) without idempotent is called a power joined \( N \)-semigroup. The homomorphic image of \( S \) into the semigroup \( \mathbb{R}_+ \) of all positive rational numbers with addition is unique up to isomorphism. **Theorem.** A semigroup \( S \) is a power joined \( N \)-semigroup if and only if \( S \) is isomorphic to a subdirect product of a periodic abelian group \( G \) and a positive rational semigroup \( R \) with addition. Further the author discusses how to construct all subdirect products of \( G \) and \( R \) when \( G \) and \( R \) are given. This paper generalizes J. Higgin's result for finitely generated \( N \)-semigroups. [Thesis, University of California, Davis, 1966.] (Received June 23, 1969.)

667-94. VERNON E. ZANDER, West Georgia College, Carrollton, Georgia 30117. A representation theorem for infinitely-linear bounded continuous operators.

Let \( Z, W, Y_t (t \in T) \) be Banach spaces. Let \( y^t = (y_{1}^t) \), where \( y_{1}^t \in Y_t \) for all \( t \in T \), be a fixed \( T \)-tuple such that \( \sum_{T} \| y_{1}^t \| - 1 < \infty \). Denote by \( P_T(Y_t, y^t) \) the family of all tuples \( y = (y_t) \) such that \( y_t \in Y_t \) for all \( t \in T \) and \( \sum_{T} \| y_t - y^t \| < \infty \). Let \( (X_t, V_t, v_t) (t \in T) \) be probability volume spaces and let \( (X, V, v) \) be the product probability volume space (Abstract 658-109, these Notices 15 (1968), 750). Let \( L(v_t, Y_t) (t \in T) \) be spaces of Bochner summable functions and let \( K(v, Z) \) be the space of vector-valued volumes (Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 492-498). Define \( f_t(x_t) = y_{1}^t \) for all \( x_t \in X_t \) for all \( t \in T \). See Abstract 69T-B163, these Notices 16 (1969), this issue, for the definition of the infinitely-linear vectorial integral. **Theorem.** To every infinitely-linear bounded continuous operator \( h \) from the multiplicative product space \( P_T(L(v_t, Y_t), f^t) \) into the Banach space \( W \) corresponds a unique vector volume \( \mu \in K(v, W_0) \) where \( W_0 \) is the space of all infinitely-linear bounded continuous operators from the multiplicative product space \( P_T(Y_t, y^t) \) into the space \( W \), such that \( h(f) = \int f(t, d\mu) \) for all \( f \in P_T(L(v_t, Y_t), f^t) \), where \( u(y, w) = w(y) \) for all \( y \in P_T(Y_t, y^t), w \in W_0 \). This correspondence establishes an isometry and isomorphism of the space \( P_T(L(v_t, Y_t), f^t) \) and the space \( K(v, W_0) \). (Received June 23, 1969.)

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Let G be a locally compact group with a left Haar measure. It is not known whether G admits a continuous automorphism T onto itself such that T is ergodic but G being noncompact. We prove here the following: (i) If G is nilpotent then the automorphism T of G is ergodic only when G is compact. (ii) If such a noncompact G exists, then there is also a totally disconnected locally compact group with similar property. So it is enough to consider the problem only for totally disconnected G. (iii) If every G is totally disconnected and every compact set S generates a compact group H, then G should be compact. (iv) If T is given to be an inner automorphism $x \rightarrow a \rightarrow xa$ where $a \in G$ is of compact order and G is totally disconnected, then G must be compact. (v) If $G = \bigcup_{n=1}^{\infty} H_n$ where each $H_n$ is a compact group, then G must be compact. (Received June 23, 1969.)

On a fundamental result of Zeller in summability.

In 1952 K. Zeller (Math. Z. 56, 134-151) showed that if an infinite matrix $A = (a_{nk})$ sums all sequences of bounded variation and satisfies the condition

$$\chi = \lim_{n} \sum_{k=1}^{\infty} a_{nk} = \sum_{k=1}^{\infty} \lim_{n} a_{nk} = 0,$$

then $A$ sums all "sufficiently slowly" oscillating sequences. Hence it sums bounded divergent sequences. Theorem. The condition $\chi = 0$ is equivalent to the condition that $A$ be a weakly compact operator. As a corollary, $\chi = 0$ implies that $A$ has no continuous inverse on any infinite-dimensional subspace of the space of sequences of bounded variation, i.e., $A$ is strictly singular. Compactness is strictly stronger than weak compactness. (This announcement is intended to replace Abstract 69T-B89, these Notices 16 (1969), 666, which contained several incorrect statements.) (Received June 5, 1969.)

Expansions of confluent hypergeometric functions are discussed, as well as some of their properties. (Received June 24, 1969.)

The Galois type theorem of $\alpha$-differential fields (See Abstract 663-727, these Notices 16 (1969), 299) now seems well established. It is of interest to analyze certain elementary and simple $\alpha$-differential extensions of $\alpha$-differential fields. The notions of $\alpha$-Liouvillian extensions and $\omega$-simple-$\alpha$-differential extensions are clearly set up. These notions finally enable us to establish that any elementary solution $\phi_m$ of the family of partial differential equations $\Delta \phi = 0$ is classified by an element $\omega$ of an $\alpha$-differential field $\Omega_0$ and a quadratic expression $\omega^2 + \gamma^2 + \tau^2$ for $\omega$ such that $\alpha \Delta p^2 + \alpha \Delta q^2 + \alpha \Delta r^2 = 0$. The extension field $\Omega_0(p_m, q_m, r_m) \equiv \Omega_m$ obtained by such a quadratic decomposition of $\omega$ is then extended by an $\alpha$-Liouvillian extension of $\Omega_m$. (Received June 25, 1969.)
Let $s_n = u_0 + u_1 + \ldots + u_n (n = 0, 1, \ldots)$. The absolute Abel-type summability methods $|A_{\lambda}|$ and $|A'_{\lambda}|$ are defined as follows: $s_n - s |A_{\lambda}|$ if $\phi(y) = (1 + y)^{-\lambda} - \sum_{n=0}^{\infty} (n + \lambda)^{\alpha} (y/(1 + y))^n$ is of bounded variation in $[0, \infty)$ and tends to $s$ as $y \to \infty$. $s_n - s |A'_{\lambda}|$ if $\phi(y) = \lambda \int_0^y (1 + t)^{-\lambda} - \sum_{n=0}^{\infty} (n + \lambda)^{\alpha} (t/(1 + t))^n$ is of bounded variation in $[0, \infty)$ and tends to $s$ as $y \to \infty$.

Theorem 1. For $\lambda > 0$, $s_n - s |A_{\lambda}|$ if and only if $s_n - s |A'_{\lambda}|$ and $\lim_n u_n = 0$.

Theorem 2. For $\lambda > 0$, $s_n - s |A'_{\lambda}|$ if and only if $s_n - s |A_{\lambda}|$.

Theorem 3. If $\lambda > -1$, $H$ is a regular Hausdorff summability method and $s_n - s |A_{\lambda}|$, then $s_n - s |A_{\lambda}H|$. The corresponding results for ordinary summability methods have been proved by Borwein in Proc. Cambridge Philos. Soc. 53 (1957), 318-322, and J. London Math. Soc. 35 (1960), 71-77. (Received June 25, 1969.)

Let $B$ be a Boolean algebra. By a $\sigma$-completion of $B$ is meant a pair $(B, \sigma)$ where $B$ is a Boolean $\sigma$-algebra and $\sigma$ is a monomorphism $h: B \to B_\sigma$ satisfying (i) the $\sigma$-subalgebra of $B_\sigma$ generated by $h(B)$ is $B_\sigma$; and (ii) whenever a countable set $\{p_n: n \in \mathbb{N}\}$ of elements of $B$ has a supremum -- say $p$ -- in $B$ then $\sup_n h(p_n) = h(p)$. Theorem. Every Boolean algebra has a $\sigma$-completion. Questions of minimality, and completeness of this $\sigma$-completion are under study as well as those of assumption that a $\sigma$-completion be in fact a measure algebra. (Received June 25, 1969.)
667-102. ROBERT GORDON, University of Utah, Salt Lake City, Utah 84112. Rings in which minimal left ideals are projective.

It is shown that the left socle of a ring $R$ is a direct summand of $R_R$ iff it is a projective left $R$-module and contains no infinite sets of orthogonal idempotents. A consequence is that a ring with finitely generated left socle and no nilpotent minimal left ideals is a ring direct sum of a semisimple artinian ring and a ring with zero left socle. It is also shown that a ring with projective, essential, finitely generated left socle has maximal and minimal condition on an annihilator left and right ideals. It follows that such a ring is semiprimary if it is left or right perfect. However, an example is given of a left noetherian ring with projective, essential left socle which is artinian modulo its radical, but which is not even semiperfect. In another example, a left perfect ring $R$ with projective, nonessential left socle is constructed in which every indecomposable direct summand of $R_R$ has a unique simple submodule. (Received June 25, 1969.)


Consider the system of two first order matrix equations (1): $Y' = K(x,\lambda)Z$, $Z' = -G(x,\lambda)Y$, where each of $K$ and $G$ is an $n \times n$ symmetric matrix of continuous real-valued functions on $X$: $a \leq x \leq b$, $L: L_1 \leq \lambda \leq L_2$, with $K$ positive definite on $X \times L$. A solution pair $\{Y(x,\lambda), Z(x,\lambda)\}$ is nontrivial and conjoined provided for each $\lambda$ on $L$, det $Y(x,\lambda)$ has at most a finite number of zeros on $X$ and $Y^*Z = Z^*Y$ (* denotes transpose). The values of $\lambda$ for which there exists a nontrivial, conjoined solution pair $\{Y(x,\lambda), Z(x,\lambda)\}$ of (1) satisfying the two point boundary conditions (2): $A(\lambda)Y(a,\lambda) - B(\lambda)Z(a,\lambda) = 0$ on $L$, det $[\Omega(\lambda)Y(b,\lambda) - \Delta(\lambda)Z(b,\lambda)] = 0$, where $A$, $B$, $\Omega$ and $\Delta$ are $n \times n$ continuous matrices on $L$ having the properties: $A^*B = B^*A$, $\Omega^*\Delta = \Delta^*\Omega$, $A^*A + B^*B$ is positive definite and $\Omega^*\Gamma + \Delta^*\Delta$ is positive definite, are called the eigenvalues of the system (1), (2). Motivated by the work of W. M. Whyburn [Pacific J. Math. 5 (1955), 147-160] and using the methods suggested by F. V. Atkinson ["Discrete and continuous boundary problems," New York, 1964, Chapter 10] together with the matrix generalization of the polar coordinate transformation, sufficient conditions for the existence of eigenvalues are obtained. (Received June 24, 1969.)


Let $M(x,y; \theta) = f(\theta)\exp[i g(x,y; \theta)]$, where $g(x,y; \theta) = (1/2)(x^2 + y^2) \cot \theta - xy \csc \theta + \theta^2/2$, $f(\theta) = 1/\sqrt{2\pi} |\sin \theta|^{1/2}$, $\gamma = \exp[i\pi/4]$. Define an operator $F_\theta$ on a function $f$ such that $(f F_\theta)(y) = \gamma^{1/2} f(x) M(xy; \theta) dx$; $-\infty < \theta < \infty$, $-\infty < y < \infty$. Then (i) $F_\theta F_\phi = F_{\theta+\phi}$, for all real $\theta, \phi$. (ii) $F_0 = I$, the identity operator. (iii) $F_{1/2} = I^{1/2} F$, where $F$ is the classical integral Fourier operator. (iv) $F_\rho = iR$, where $R$ is the reverse operator. (v) $F_\theta = F_{-\phi}$ if $\theta = \phi$ (mod $4\pi$). (vi) $CF_\theta = F_{-\theta} C$, for all real $\theta$; and $C$ is the complex conjugate operator. This generalizes $CF = F^{-1}C$. (vii) If $\theta = 4\pi/n$, then $F_\theta$ generates a cyclic group of order $n$. This generalizes the fourth order cyclic group property of $F$. (viii) If $\theta = 4\pi/n$, then $F_\theta$ and $C$ generate a dihedral group of order $2n$; which generalizes the eighth order dihedral group property of $F$. (Received June 26, 1969.)
667-106. LAWRENCE J. WALLEN, University of Hawaii, Honolulu, Hawaii 96822. Pairs of unitary operators whose powers are close together.

Let $U$ and $V$ be unitary operators on a Hilbert space. Suppose $U$ and $V$ both are diagonalizable (i.e., have a spanning set of eigenvectors) and suppose further that $\sup_n \| U^n - V^n \| < 1$. Then $U$ and $V$ are unitarily equivalent. The proof is an adaptation to a noncommutative situation of our proof that in a real normed algebra $\sup_n \| x^n - 1 \| < 1$ implies $x = 1$ (Proc. Amer. Math. Soc. 18 (1967), 956).

(Received June 25, 1969.)


Let $X$ and $Y$ be Banach Spaces and $S$ a subset of $X$. Suppose $T$ is a map from $S$ into $Y$ such that $\| T(u) - T(v) \| \leq \| u - v \| ^{\alpha}$ for all pairs $u, v$ in $S$. If $T$ can always be extended to a map from $X$ into $Y$ satisfying the same inequality for all pairs $u, v$ in $X$ we say "$e(X, Y, \alpha)$ holds." Theorem. If $Y$ is a Hilbert space $H$ and $0 < \alpha$, then $e(X, H, \alpha)$ holds for $2\alpha < 1$. If $X$ is an $L^p$ space and $1 \leq p \leq 2$, then $e(L^p, H, \alpha)$ holds for $2\alpha \leq p$. Furthermore in each case the range of $\alpha$ cannot be increased. The proof uses a version of the Riesz Thorin Theorem for weighted direct sums. (Received June 25, 1969.)


Let $T$ and $T'$ denote topologies for a set $X$. M. K. Fort, Jr., has defined $T$ and $T'$ to be weakly equivalent if they determine the same collection of subsets of $X$ which have nonempty interior. Denote by $L(T)$ the collection of all semiopen sets in the space $(X, T)$ (a set is semiopen if it is contained in the closure of its interior). Theorem 1. Let $T$ and $T'$ be topologies for $X$ with $T'$ stronger than $T$. Then $T$ and $T'$ are weakly equivalent iff $T' \subset L(T)$. The proof makes use of the following Lemma. The simple extension of a topology $T$ by a set $A$ is weakly equivalent with $T$ iff $A \in L(T)$.

Corollary 1. Let $T$ and $T'$ be topologies for $X$ such that $T \cup T' \subset L(T \cap T')$. Then $T$ and $T'$ are weakly equivalent. Corollary 2. Let $T$ and $T'$ be topologies for $X$ such that $T \subset L(T')$ and $T' \subset L(T)$. Then $T$ and $T'$ are weakly equivalent. Theorem 1 above and Theorem 1 of Abstract 653-102, these Notices 15 (1968), 107, yield Theorem 2. Every topology is weakly equivalent with an extremally disconnected topology. (Received June 25, 1969.)

667-109. FORREST A. RICHEN, University of Michigan, Ann Arbor, Michigan 48104. Decomposition numbers of p-solvable groups.

In the character theory of finite groups one decomposes each ordinary irreducible character $\chi_i$ of a group into an integral linear combination of $p$-modular irreducible characters $\epsilon_{ij} \chi_j = \sum d_{ij} \epsilon_{ij}$. The nonnegative integers $d_{ij}$ are called the $p$-decomposition numbers. Let $G$ be a $p$-solvable group whose $p$-Sylow subgroups are abelian. If $G/O_{p^2}(G)$ is cyclic the $p$-decomposition numbers
are $\leq 1$. This condition is far from necessary as any group $G$ with an abelian, normal $p$-Sylow subgroup $P$ and with $G/P$ abelian has $p$-decomposition numbers $\leq 1$. The difficulty in sharpening these results is equivalent to the difficulty of deciding when a character of a normal subgroup extends to a character of its inertia group. A result of Brauer and Nesbitt together with the first result yields the following: A group $G$ has a normal $p$-complement and abelian $p$-Sylow subgroups if and only if each irreducible character of $G$ is irreducible as a $p$-modular character. (Received June 25, 1969.)


Suppose $a = (a_i)$ is a real number sequence and $\mathcal{G}$ is the collection of reversible transformations from the positive integers into the positive integers. $d_a$ denotes the set to which the number sequence $x = (x_i)$ belongs if and only if there exists a $k > 0$ such that $h_a(x) = \sup_{p} \in \mathcal{G} \left( \sum_{i=1}^{\infty} |x_p(i)| |a_i| \right) < k$. $h_a$ is a norm on $d_a$. **Theorem 1.** $d_a = \ell_1$ if and only if $a$ is in $m - c_0$. **Theorem 2.** $d_a = m$ if and only if $a$ is in $\ell_1$. **Theorem 3.** If $d_a = \ell_1$ then $h_a$ is equivalent to the ordinary norm on $\ell_1$ and if $d_a = m$ then $h_a$ is equivalent to the ordinary norm on $m$. J. R. Retherford [Proc. Amer. Math. Soc., 17 (1968), 766] has shown that the space "d", which is $d_a$ when $a = (1/i)$, has a semishrinking (Schauder) basis which is not shrinking. **Theorem 4.** If $a$ is in $c_0 - \ell_1$ then $(d_a, h_a)$ has a semishrinking (Schauder) basis which is not shrinking. **Theorem 5.** $(d_a, h_a)$ is congruent to the first conjugate space of a linear space but is isomorphic to the second conjugate space of a linear space if and only if $a$ is in $\ell_1$. Additional properties of these spaces are established. (Received June 26, 1969.)


Let $\rho$ be a metric on a metric space $X$ and $f: X \to X$ a one-to-one function. Then $\sigma_f(x, y) = \rho(f(x), f(y))$ is also a metric on $X$. If $f, f^{-1}, g$ and $g^{-1}$ are uniformly continuous functions from $X$ to $X$, then the metrics $\sigma_f$ and $\sigma_g$ are uniformly equivalent. If $f, f^{-1}$ and $g$ are uniformly continuous, $g^{-1}$ is continuous, and $\sigma_f$ and $\sigma_g$ are uniformly equivalent, then $g^{-1}$ is uniformly continuous. As an application, let $X$ be the reals, $\rho(x, y) = |x - y|$, $f(x) = x$ and $g(x) = x/(1 + |x|)$. Then $\sigma_g(x, y) = |x/(1 + |x|) - y/(1 + |y|)|$ is equivalent but not uniformly equivalent to the usual metric on the reals. (Received June 26, 1969.)

667-112. PAUL ROSENTHAL, Stanford University, Stanford, California 94305. On a solution to the two-dimensional supersonic initial value problem of a nonviscous compressible fluid by Bergman's integral operator method.

S. Bergman in a paper to appear has indicated how his integral operator method can be used to solve the initial value problem in the supersonic region. In a series of papers, Bergman, Bergman and Bojanic, and Stark have investigated and derived certain formulas relating to the supersonic case. Utilizing some of these results and notation, we study the initial value problem in the supersonic case as to weak solutions as well as approximations to the regular solution. (Received June 27, 1969.)

Aaron Strauss (Trans Amer. Math. Soc. 119 (1965), 37-50) introduced a new type of integral stability for ordinary differential equations (\( L^p \)-stability) which is in a sense an approach parallel to asymptotic stability. Wolfgang Hahn (J. Differential Equations 3 (1967), 440-448) generalized this concept and proposed a new type of integral stability (K-stability). He also defined K - D behavior for a particular behavior of the solution. In his paper he studied this behavior using Liapunov functions. However, the conditions given are not enough to guarantee K-stability. In this paper we prove a theorem about uniform K-stability and its converse for linear differential equations. Asymptotic stability implies K-stability but the converse is not true. We give a simple criterion under which this is possible. We formulate two propositions about \( L^p \)-instability, one using the well-known Wazewski's lemma and the other using the Liapunov functions. (Received June 24, 1969.)

Extremal elements of the convex cone \( A_n \) of functions.

Let \( A_n \), \( n \geq 1 \), denote the convex cone of nonnegative real functions defined on \([0,1]\) of the real line which are alternating of order \( n \) (cf. G. Choquet, "Theory of capacities," Ann. Inst. Fourier 5 (1953 and 1954), 169). The extremal elements of \( A_1 \) are the functions which assume exactly one positive value in \([0,1]\). The positive constant functions and the functions which are a positive constant on \((0,1]\) and zero at 0 are extremal elements of \( A_n \), \( n \geq 1 \). The functions \( e(m, a, n - 1; x) = m[a^{n-1} - (a - x)^{n-1}], x \in [0, a] \) and \( ma^{n-1} \) for \( x \in [a,1] \), where \( m > 0 \) and \( 0 < a \leq 1 \), are extremal elements of \( A_n \), \( n \geq 2 \). The only other extremal elements of \( A_n \), \( n \geq 3 \), are those functions \( e(m, 1, k); 1 \leq k \leq n - 2 \). It is shown that the extremal elements of \( A_n \) form a compact set in a compact convex set which meets every ray of \( A_n \) exactly once but does not contain the origin. It then follows from Choquet's theorem (cf. R. R. Phelps, "Lectures on Choquet's theorem," 1966, p. 5) that for each function in \( A_n \) an integral representation in terms of extremal elements of \( A_n \) is possible. (Received June 26, 1969.)

Numerical invariants on orthomodular lattices.

To each finite orthomodular lattice \( L \) there is associated a set of nonnegative integers \([b_i], i \in L\), called the orthogonality numbers of \( L \). Boolean lattices can be characterized in terms of these numbers. Formulas for the orthogonality numbers of direct products and horizontal products are given, as well as a necessary condition for the modularity of \( L \). (Received June 26, 1969.)

On commuting functions without common fixed points.

Let "function of \( X\)" mean continuous function mapping the topological space \( X \) onto \( X \). A topological space \( X \) has the fixed point property (F. P. P.) provided every function of \( X \) has a fixed point. Let a topological space, \( X \), have C.F.P. provided any two functions of \( X \) which commute under composition have a common fixed point. In "Recent developments in fixed-point theory" (invited
address, No. 665 meeting of the AMS), E. R. Fadel felt that simply connected manifolds which have F.P.P. might have C.F.P. This is in general not true as two commuting functions of complex projective
2-space, \( \mathbb{CP}^2 \), without common fixed points are constructed. The procedure Consider an isometry, \( h \), of \( \mathbb{CP}^2 \) which has a finite number of fixed points, and consider a closed \( h \)-invariant neighborhood of each of the fixed points. \( \mathbb{CP}^2 \) is locally \( \mathbb{R}^4 \) so each neighborhood looks like a closed 4-ball. For each of these neighborhoods \( \mathbb{B}^4 \), commuting functions of \( \mathbb{B}^4 \) can be constructed (using known functions of an interval) without common fixed point which coincide with \( h \) on the boundary of \( \mathbb{B}^4 \). So \( h \) can be redefined in disjoint neighborhoods of each of its fixed points to yield two commuting functions of \( \mathbb{CP}^2 \) without common fixed points. (Received June 26, 1969.)

667-117. SIMON HELLERSTEIN and DANIEL F. SHEA, University of Wisconsin, Madison, Wisconsin 53706, and J. WILLIAMSON, University of Hawaii, Honolulu, Hawaii 96822. A Tauberian theorem characterizing the regularity of growth of a class of entire functions.

Let \( N(r, \cdot) \) and \( T(r, \cdot) \) denote the standard functionals of Nevanlinna's theory. Let \( E^\lambda \) denote the family of entire functions of order \( \lambda \), \( 0 \leq \lambda \leq \infty \), having only negative zeros and let 
\[ k(q, \rho) = \frac{\sin \pi \rho}{(q + \sin \pi \rho)} \text{ if } q \leq \rho \leq q + 1/2 \] 
and 
\[ k(q, \rho) = \frac{\sin \pi \rho}{(q + 1)} \text{ if } q + 1/2 < \rho \leq q + 1. \]
An earlier result of two of the authors is the following Theorem. If \( f \in E^\lambda \) of genus \( q \) and lower order \( \mu \) then for any \( \rho \) satisfying \( q \leq \rho \leq \lambda \), 
\[ \lim_{r \to \infty} \frac{N(r,0)}{T(r,f)} = k(q, \rho) \text{ and } \lim_{r \to \infty} \sup \frac{N(r,0)}{T(r,f)} \text{ exists.} \]
(Received June 26, 1969.)

667-118. LINCOLN E. BRAGG, University of Kentucky, Lexington, Kentucky 40506. Smooth error term for numerical integration, etc. Preliminary report.

A universal polynomial approximation formula is described. The coefficients are expressed as averaging integrals over simplices (intervals, in one dimension) spanned by the data points. The error term coefficient is expressed as an integral over the simplex spanned by all of the data points plus the evaluation point \( x \). Taking all data points distinct yields the Lagrange polynomial approximation; coincident in pairs, Hermite approximation; and all at one point, Taylor series approximation. There being no division by differences involved, it is clear from the formulas for the coefficients that they are smooth functions of the data points and of \( x \). In particular, one can integrate and differentiate the error term coefficient explicitly (under the integral signs). Thus, if one wants to substitute the approximation formula into an integration, differentiation, or other such algorithm, one need not rely on an intuitive feeling that the result is correct and could be justified with better analysis. The derivation is based on the fundamental theorem of calculus, \( f(b) = f(a) + (b - a) \int_a^b f'(t) \, dt \), rather than the commonly used mean value theorem. (Received June 26, 1969.)
Let P be a convex polytope in n-dimensional Euclidean space E_n, and let \( f_P \) denote the support functional of P. For each face F of P, define \( C(P,F) = \{ x : f_P(x) = x \cdot v \text{ for every } v \in \text{ext}(F) \} \). Then C(P,F) is a convex cone with vertex at the origin and is called the facial cone of P with respect to F. The facial cone has the property \( \dim(F) + \dim(C(P,F)) = n \). The polytope P is locally similar to the polytope Q if and only if there exists a one-to-one correspondence between \( \text{ext}(P) \) and \( \text{ext}(Q) \), say \( v \rightarrow w \), such that \( C(P,v) = C(Q,w) \). If F and G are faces of the polytopes P and Q, respectively, then \( F + G \) is a face of \( P + Q \) if and only if the relative interiors of \( C(P,F) \) and \( C(Q,G) \) have a nonempty intersection. (Received June 26, 1969.)

Solved is the problem of simple waves in unsteady and steady flow of a nonviscous polytropic gas in n-dimensional space where all flow variables depend on a single generating function. The complete solution is presented in terms of n arbitrary expressions of this generating function. A boundary value problem is formulated which specifies these arbitrary expressions. Described is a special method which permits a convenient discussion of the flow field. Application of the general results is made to solve a few selected problems. (Received June 26, 1969.)

Let X be a completely regular Hausdorff space. \( \mathcal{U} \) is a Wallman base on X means \( \mathcal{U} \) is a normal, disjunctive base for the closed subsets of X. The space \( w\mathcal{U} \) of \( \mathcal{U} \)-ultrafilters is a Hausdorff compactification of X iff \( \mathcal{U} \) is a Wallman base on X [Brooks, Fund. Math. 60 (1967)]. Let \( \mathcal{F} \) be a subring (with unity) of C(X). Denote the structure space of \( \mathcal{F} \) by \( H[\mathcal{F}] \). In Gillman and Jerison's "Rings of continuous functions" we find \( wZ(X) = H[C(X)] (= \beta X) \). We investigate a similar situation for \( \mathcal{F} \). An ideal I of \( \mathcal{F} \) is a filter ideal means \( Z[I] (= \{ f \in I \}) \) is a \( Z[\mathcal{F}] \)-filter. Let \( F[\mathcal{F}] \) denote the structure space of maximal filter ideals of \( \mathcal{F} \). We obtain Theorem 1. Every maximal filter ideal is prime. Theorem 2. F[\mathcal{F}] is a Hausdorff compactification of X iff \( Z[\mathcal{F}] \) is a Wallman base on X. Moreover, when \( F[\mathcal{F}] \) is a Hausdorff compactification of X, then \( F[\mathcal{F}] = wZ[\mathcal{F}] \). Hence, there is a direct relationship between ultrafilters of zero sets of functions in \( \mathcal{F} \) and certain ideals of \( \mathcal{F} \). Theorem 3. Suppose \( \mathcal{F} \) is a subring of C(X) where \( Z[\mathcal{F}] \) is a Wallman base for X. Let \( T = wZ[\mathcal{F}] \). Then the maximal filter ideals of \( \mathcal{F} \) are given by \( M^T = \{ T \in \mathcal{F} : f \in \text{cl}_{Z(f)} \} \) (\( T \in T \)). (Received June 26, 1969.)

The set of extensions of a valuation v on a field which is maximal with respect to v (i.e. linearly compact) to the polynomial ring of the field is shown in this study to depend only on the value group.
and residue class field of \( v \). The method is to associate with each such extension a determining invariant called its "signature", which is, roughly, a pair of sequences, one in the algebraic closure of the residue class field of \( v \) and the other in the divisible closure of the value group of \( v \). Also, signatures are associated with various mathematical objects by means of extensions of the above sort which arise naturally from them. For example, the sets of all monic irreducible polynomials over a maximal field, of all finite Harrison primes of the polynomial ring of a global field, and of a valuation on the field of rational functions over a global field are each shown to be naturally bijective with certain easily described sets of signatures. Finally, these objects are studied by means of their associated signatures, e.g. Eisenstein's criterion is extended to give necessary and sufficient conditions for irreducibility over a maximal field. (Received June 30, 1969.)

667-123. RONALD B. KIRK, Southern Illinois University, Carbondale, Illinois 62901. Kolmogorov type consistency theorems for products of locally compact, B-compact spaces.

Let \( \{X_a : a \in A\} \) be a family of locally-compact, B-compact spaces. (For the definition of a B-compact space, see Abstract 663-329, these Notices 16 (1969), 181.) For \( F \subseteq A \), let \( X_F = \prod \{X_a : a \in F\} \) and for \( G \subseteq F \subseteq A \), denote by \( \prod_{F \subseteq G} \) the projection mapping from \( X_F \) onto \( X_G \). Denote by \( \mathcal{F} \) the set of all finite subsets of \( A \) and let \( M = \{m_F : F \in \mathcal{F}\} \) be a family of Baire (Borel) measures such that \( m_F \) is defined on \( X_F \). The family \( M \) is said to be Baire (Borel) consistent if whenever \( G, F \in \mathcal{F} \) with \( G \subseteq F \), it follows that \( m_G(A) = m_F(\prod_{F \subseteq G}^{-1}[A]) \) for every Baire (Borel) set \( A \) in \( X_G \).

The following theorems are proved. Theorem 1. If \( \{m_F : F \in \mathcal{F}\} \) is a Baire consistent family of Baire measures, then there is a unique measure \( m \) on the Baire cylinder algebra in \( X_A \) such that the projection of \( m \) on the space \( X_F \) is \( m_F \) for each \( F \in \mathcal{F} \). Theorem 2. If \( \{m_F : F \in \mathcal{F}\} \) is a Borel consistent family of regular Borel measures, then there is a unique measure \( m \) on the Borel cylinder algebra in \( X_A \) such that the projection of \( m \) on \( X_F \) is \( m_F \) for each \( F \in \mathcal{F} \). (Received June 24, 1969.)

667-124. RICHARD D. ANDERSON and JOHN D. McCHAREN, Louisiana State University, Baton Rouge, Louisiana 70803. On extending homeomorphisms to Fréchet manifolds.

A Fréchet manifold (F-manifold) is a separable metric space admitting an open cover by sets homeomorphic to a countably infinite product of lines. A closed set \( K \) in an F-manifold \( M \) has Property \( Z \) provided that for each nonnull set \( U \) of \( M \) with trivial homotopy groups, \( U \setminus K \) is also nonnull with trivial homotopy groups. Using recent results of James E. West ("Approximating homotopies by isotopies in Fréchet manifolds," to appear, Bull. Amer. Math. Soc.) and T. A. Chapman ("Infinite deficiency in Fréchet manifolds," submitted to Proc. Amer. Math. Soc.), necessary and sufficient conditions are obtained in order that a homeomorphism between closed subsets of an F-manifold be extended isotopically to a homeomorphism of the manifold. Theorem. If \( f \) and \( g \) are closed embeddings of a complete separable metric space \( X \) into an F-manifold \( M \) such that \( f(X) \) and \( g(X) \) have Property \( Z \), then there exists an isotopy \( \{H_t\}_t \) of \( M \) onto \( M \) such that \( H_0 = id \) and \( H_1 \cdot f = g \) if and only if \( f \) and \( g \) are homotopic. Theorem. If \( f \) is a continuous map of \( X \) into \( M \) such that \( f|A \) is a homeomorphism of \( A \) onto a closed subset of \( M \) with Property \( Z \), where \( A \) is a closed subset of \( X \), then there exists a homeomorphism \( h \) of \( X \) into \( M \) such that \( h|A = f|A \) and \( h(X) \) has Property \( Z \) in \( M \). (Received June 27, 1969.)

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G₂ⁿ spaces. Preliminary report.

A complex vector space X will be called a G₂ⁿ space if and only if there is a mapping \( \langle \cdot, \ldots, \cdot \rangle \) from \( X^{2n} \) into \( \mathbb{C} \) such that \( x_k \cdot (x_1, \ldots, x_{2n}) = \langle x_1, \ldots, x_{2n} \rangle \) for all \( k \), and \( x, \ldots, x \rangle^{1/2n} \) defines a norm. The basic models for these spaces are the \( L^{2n} \) spaces. One also has that every inner product space is a G₂ⁿ space for each \( n \), and hence G₂ⁿ spaces (for a fixed \( n \)) are not all isometrically isomorphic. One can show that a normed linear space is a G₂ⁿ space if and only if it satisfies a generalized parallelogram law. Of particular interest are the CBS G₂ⁿ spaces in which \( \| x_1, \ldots, x_{2n} \| = \| x_1 \| \ldots \| x_{2n} \| \). In these spaces, one defines \( A \) to be symmetric if and only if \( \langle x_1, \ldots, A(x_j), \ldots, x_{2n} \rangle = \langle x_1, \ldots, A(x_j), \ldots, x_{2n} \rangle \) for all \( j \) and \( k \). It is easy to show that such operators are scalar, and that these operators include the multiplication operators on \( L^{2n} \). Finally, from the proof of the characterization of G₂ⁿ spaces it follows that \( U \) is an isometry if and only if \( \langle U(x_1), \ldots, U(x_{2n}) \rangle = \langle x_1, \ldots, x_{2n} \rangle \) for all \( x_1, \ldots, x_{2n} \). This then provides one with a way to construct all of the isometries of a finite dimensional G₂ⁿ space. (Received June 27, 1969.)

Quasi-projective and quasi-injective modules.

Let \( R \) be a ring with \( 1 \), and let all modules be left unital \( R \)-modules. A module is quasi-projective iff for every epimorphism \( q: M \rightarrow B \), \( \text{Hom}_R(M,B) = \text{Hom}_R(M,M) \circ q \) (L. E. T. Wu and J. P. Jans, "On quasi-projectives," Illinois J. Math. 11 (1967), 439-448). A characterization is given for quasi-projective modules when \( R \) is left perfect. This result for quasi-projective modules is used to improve and extend a proposition which appeared in a paper by M. Harada ("Note on quasi-injective modules," Osaka J. Math. 2 (1965), 351-356). The theorem which is obtained by this extension is a characterization for quasi-injective modules when \( R \) is an Artinian ring which has a finitely generated, injective cogenerator. (Received June 27, 1969.)

Decisive convergence structures.

Let \( (S,q) \) be a convergence space, let \( \pi(q) \) denote the finest pretopology (i.e., closure space) coarser than \( q \), and let \( \mathcal{V}_q(x) \) denote the \( q \)-neighborhood filter at \( x \). For each element \( x \) in \( S \), let \( \mathcal{J}_q(x) \) be the filter obtained by intersecting all nonprincipal ultrafilters which fail to \( q \)-converge to \( x \); \( \mathcal{J}_q(x) \) is called the \( q \)-anti-neighborhood filter at \( x \). If \( \mathcal{J} \) \( q \)-converges to \( x \) whenever \( \mathcal{J} \) is a nonprincipal ultrafilter and \( \mathcal{J} \geq \mathcal{V}_q(x) \), then \( (S,q) \) is said to be almost pretopological. If \( \mathcal{J} \) fails to \( q \)-converge to \( x \) whenever \( \mathcal{J} \) is a nonprincipal ultrafilter and \( \mathcal{J} \geq \mathcal{J}_q(x) \), then \( (S,q) \) is said to be decisive. The following statements about a convergence space \( (S,q) \) are equivalent:

(a) For each convergence space \( (T,p) \), \( (S \times T, \pi(q) \times \pi(p)) = (S, \pi(q)) \times (T, \pi(p)) \).
(b) \( (S,q) \) is both decisive and almost pretopological. (Received June 27, 1969.)
667-128. JAMIL A. SIDDIQI, University of Sherbrooke, Sherbrooke, Quebec, Canada. On a characterization of absolute continuity in locally compact Hausdorff space.

Recently Lawrence Zalcman has given a characterization of the absolute continuity of a complex Baire measure with respect to a positive Baire measure both defined on a compact metric space. It is shown that the same characterization is valid for Baire measures defined on any locally compact Hausdorff space. (June 27, 1969.)

667-129. JACOB BURBEA, Stanford University, Stanford, California 94305. A remark on the Bergman metric.

Using a conveniently chosen complete system \( \{ \varphi_p \} \) of orthonormal functions, we obtain a new expression for the Bergman metric. This new form yields bounds for various distortion problems. Further, by considering the principal minors of this metric, we give a new interpretation for the Bergman metric. It should be noted that this method uses the minimum integral principle only and can also be adjusted to the kernel function obtained from a system of analytic functions which are orthonormal when integrating over the distinguished boundary of the domain. Finally this method yields new proofs for the fact that the metric is positive definite. (Received June 27, 1969.)

667-130. GEORGE F. LEGER, Tufts University, Medford, Massachusetts 02155, and EUGENE LUKS, Bucknell University, Lewisburg, Pennsylvania 17837. Derivations of Lie algebras. IV.

Let \( L \) be a nilpotent Lie algebra and let \( t(L) \) be the dimension of a maximal toroidal subalgebra of the derivation algebra, \( D(L) \), of \( L \). In D. L. A. III, Duke J. Math. 30, (1963), it was conjectured that \( t(L) \not\geq 1 \) implies \( D(L) \) solvable. This is shown to be true if \( L \) has a nonsingular derivation but false in general. A counterexample is given to the conjecture \( t(L) \not\geq t(J) + t(L/J) \) for every ideal \( J \). Lie algebras with \( L^3 = 0 \) and \( t(L) = 1 \) are constructed. Examples of nonisomorphic nilpotent Lie algebras are given which have isomorphic derivation algebras and holomorphs. (Received June 27, 1969.)


A partition of \( n \) into \( d \) nonnegative parts is a \( d \)-tuple \( B = (b_1, \ldots, b_d) \) of nonnegative integers \( b_i \) with \( b_1 \leq b_2 \leq \ldots \leq b_d \) and \( b_1 + b_2 + \ldots + b_d = n \). In \( B \), let \( c_j \) be the number of values of \( i \) for which \( b_i = j \). Let \( S(B) \) be the set of \( V = \{ v_1, v_2, \ldots, v_h \} \) such that each \( v_k \) is either a \( c_j \) or a sum \( c_j + c_{j+1} \) and every nonzero \( c_j \) is thus involved in exactly one \( v_k \). Let \( g \) be a fixed positive integer, \( a_k = \max(0, v_k - g) \), and \( a(B) \) be the maximum of \( a_1 + a_2 + \ldots + a_h \) for all the \( V \) in \( S(B) \). Let \( M(a,n,d) \) be the number of partitions \( B \) of \( n \) into \( d \) nonnegative parts with \( a(B) = a \). Then \( M(a,n,d) = M(a,n-a,d) + M(a-1,n,d-1) + M(a-1,n-1,d-1) - M(a-1,n-a+1,d-1) - M(a-1,n-a,d-1) - M(a-2,n-1,d-2) + M(a-2,n-a+1,d-2) + M(a-1,n-2d+2g+2,d-g-1) \). (Received June 27, 1969.)
Convergence of semidiscrete approximations of linear transport equations.

A technique will be presented for establishing the convergence of semidiscrete methods of approximating linear transport equations. The technique is based on the "positivity" property of the system of hyperbolic equations which approximate the transport equation. A priori bounds are deduced by using (1) the properties of Riemann functions that are associated with the hyperbolic systems, (2) the "positivity" of the hyperbolic systems. The a priori bounds are then used to deduce convergence in the classical way. (Received June 27, 1969.)

A canonical form for the additive group of nonstandard models of arithmetic.

Definition. Let F be a countably infinite direct sum of the rational numbers and let $F_0$ denote the class of functions $g: F \times F \rightarrow Z$ (Z is the standard model of arithmetic) such that (i) $g(a,0) = 0$, (ii) $g(a,b) = g(b,a)$, (iii) $g(a,b) + g(a+b,c) = g(a,b+c) + g(b,c)$, (iv) $g(a,-a) = 0$. If $g \in F_0$, we denote by $F_g Z$ the set $F \times Z$ with the addition $(a,x) + (b,y) = (a+b, x+y+g(a,b))$. Then $F_g Z$ is an abelian group and we have Theorem. Let $\ast Z$ be a countable nonstandard model of arithmetic. Then there are functions $g \in F_0$ such that $\ast Z$, regarded as an additive group, is isomorphic to $F_g Z$, the isomorphism having the property that $n$ in $Z$ is mapped onto $(0,n)$ in $F_g Z$. (Received June 30, 1969.)

Geometrical properties of equipotential surfaces.

Let $E_1$ and $E_2$ be compact convex sets in $\mathbb{R}^n$, $E_1 \cap E_2 = \emptyset$, $d_{\mu_1}$ and $d_{\mu_2}$ be positive bounded measures on $E_1$ and $E_2$, and let $\Phi(r) \in C^2$ be a decreasing function of $r$. Let the potentials $V(x_0) = \int \Phi(r_1)d\mu_1$ and $U(x_0) = \int \Phi(r_2)d\mu_2$ be defined where $r_i$ denotes the Euclidean distance from $x_0 \in E_i$, $x \in E_1$, $i = 1,2$. Consider the equipotential surfaces $V_{\lambda} : V(x) = \lambda$ and $U : U(x_\lambda) = \lambda$. Several theorems are obtained, these represent extensions of the results of J. P. Kahane [Proc. Amer. Math. Soc. 13 (1962), 617-618]. Theorem 1. Let $M_1$ and $M_2$ be two points $\in V_{\lambda}$, $M = [M_1 + M_2]/2$, then the hyperplane $\pi$ through $M \cap M_1 \cap M_2$ must intersect $E_1$. Theorem 2. Let $M_1, M_2 \in U_{\lambda}$, then $\pi$ cannot separate $E_1$ and $E_2$. Corollary. Any hyperplane containing the normal to $U_{\lambda}$ cannot separate $E_1$ and $E_2$. If in addition it is assumed that $0 < \Phi''(r) \leq -\Phi'(r)/r$, (i.e. $-\log r, r^{-1}$), theorems are obtained dealing with bounds for the curvature of any curves on $U_{\lambda}$ and $V_{\lambda}$ in terms of the minimum distance from $x$ to $E_1$ or $E_2$. (Received June 30, 1969.)

A note on cones of positive operators.

One consequence of the Krein-Rutman Theorem on positive operators is the following: If a real $n \times n$ matrix $A$ is a positive operator on a closed, full, pointed cone $C$ in $\mathbb{R}^n$, then the spectral radius $\rho(A)$ is an eigenvalue. The set of positive operators on a cone $C$ in $\mathbb{R}^n$ forms a cone $\mathfrak{P}(C)$ in $\mathbb{R}^n^2$. Theorem. Suppose $A = (A_{ij})$ is a $tn \times tn$ matrix with square blocks $A_{ij}$ all belonging to $\mathfrak{P}(C)$ for some cone $C$ in $\mathbb{R}^n$. Then $A$ is a positive operator on the "direct sum cone" $K = \sum_{i=1}^t C_i = \sum_{i=1}^t C_i$.
In connection with cones of positive operators we generalize the notion of an 
\(M\)-matrix, that is a matrix \(A = kI - B\), where \(B\) is nonnegative and \(k > \rho(B)\). Suppose \(\pi(C)\) is a cone 
of \(n \times n\) matrices which are positive operators on a given cone \(C\). We define an \(M_C\)-matrix \(A = kI - B\), 
where \(B \in \pi(C)\) and \(k > \rho(B)\), and show that some of the properties of regular \(M\)-matrices carry 
over to this larger class. In particular, the inverse of an \(M_C\)-matrix is in \(\pi(C)\) and the root of 
minimal absolute value of any \(M_C\)-matrix is positive. (Received June 30, 1969.)

667-136. W. WISTAR COMFORT and ANTHONY W. HAGER, Wesleyan University, Middletown, 
Connecticut 06457. \Estimates for the number of real-valued continuous functions.\n
Let \(C(X)\) be the set of real-valued continuous functions on the infinite, completely regular 
Hausdorff space \(X\). We improve the familiar, trivial estimate \(|C(X)| \leq 2 \delta X\) (where \(\delta X\) is the smallest 
of the numbers \(|D|\), with \(D\) dense in \(X\)) with this \textbf{Theorem}. \(|C(X)| \neq (wX)^{wcX} \leq 2 \delta X\). (Here \(wX\) is 
the weight of \(X\) and \(wcX\) is the smallest cardinal \(m\) for which each open cover of \(X\) admits a sub­
family, with \(m\) or fewer members, whose union is dense in \(X\).) \textbf{Corollary}. \((wX)^{wcX} = (w \delta X)^{wcX}\). 
Assuming that no cardinal pairs \((k,n)\) exist with \(\infty \leq k < n < \kappa \in \aleph_0\), a condition which follows from 
the generalized continuum hypothesis, we show that spaces \(X\) for which \(|C(X)| < (wX)^{wcX}\) exist in 
profusion. In all, 36 numbers of the form \(a^b\) are considered, where \(a\) and \(b\) belong to a 6-element 
set of cardinal numbers naturally associated with \(X\). In each case we prove that \(a^b\) is, or by an 
example that it is not, an upper bound for \(|C(X)|\). (Received June 30, 1969.)

667-137. WILLIAM R. SCOTT, University of Utah, Salt Lake City, Utah 84112. \On a result of 
Schenkman on products of abelian groups.\n
Amer. Math. Soc.} 21 (1969), 202-204) proved that if \(G = AB\) with \(A\) and \(B\) finitely generated abelian 
subgroups, then \((i)\) \(G\) contains a nontrivial normal subgroup contained in \(A\) or \(B\), and \((ii)\) \(G\) contains a 
proper normal subgroup containing \(A\) or \(B\). However, the proof contains an error. In this paper, 
part of Schenkman's proof plus further argument is used to prove that \((i)\) holds in case \(A\) and \(B\) are 
abelian and one of them satisfies the minimum condition, while \((ii)\) is true if \(A\) and \(B\) are abelian 
with one satisfying the minimum condition and the other the maximum condition. The status of 
Schenkman's theorems remains unresolved. (Received June 30, 1969.)

667-138. JACK W. MACKI and JAMES S. MULDOWNEY, University of Alberta, Edmonton 7, 
Alberta, Canada. \The asymptotic behaviour of linear ordinary differential equations.\n
\textbf{Theorem}. Let \(H(t)\) be an \(n \times n\) nonsingular positive definite Hermitian matrix of absolutely 
continuous functions on \([0, \omega]\). If \(H' + A^*H + HA \neq 0\) a.e. on \([0, \omega]\) (\(A^*\) is the conjugate transpose of 
\(A\)), then \(\lim_{t \to \omega} t^{-1/2} \log \det H(t) + Re \int_0^t Tr A = -\infty\) is necessary and sufficient for the existence 
of a solution \(x_0(t)\) to \(x' = A(t)x\) such that \(\lim_{t \to \omega} x_0(t) = 0\). Applications to second order vector 
equations of the form \(\mathbf{0} = H(t)x'^2 + O(t)x = 0\) are given that improve on theorems of Hartman and 
Coppel. (Received June 30, 1969.)
A simple condition of the traveling salesman's problem.

If one assumes that a traveling path from one city to another is through a straight line edge, like the airline route, a simple necessary condition is given by Theorem 1. Unless all given cities are collinear, the shortest traveling salesman's route must be a simple closed path. Corollary 1. For any given finite number of noncollinear points on the plane, there exists a simple closed path connecting all of these points. Corollary 2. If a route forms a convex polygon then it is the only shortest route. Theorem 2. For given n points on the plane, the number of simple closed paths connecting all these points cannot exceed \( k!(n - k)^k \), where \( k \) is the number of interior points of the convex hull. (Received June 30, 1969.)
Existence and substitution for weighted \( g \)-summability.

Let \( (w_1, w_2, w_3) \) be an ordered triple of real numbers such that \( w_1 + w_2 + w_3 = 1 \). Let \( g \) be a real-valued function on the entire real axis which is of bounded variation on every closed interval. For a bounded real-valued function \( f \) on a closed interval \([a, b]\), the concept of \( f \) being \( (w_1, w_2, w_3) \) \( g \)-summable over \([a, b]\) is introduced, and the integral \( \int_{a}^{b} f(x)dg(x) \) is defined when \( f \) has this property. This concept of \( (w_1, w_2, w_3) \) \( g \)-summability is a generalization of the concept of mean \( g \)-summability defined in Abstract 658-142, these Notices 15 (1968), 761. An existence theorem for the integral defined here is given which generalizes the existence theorem presented in Abstract 662-13, these Notices 15 (1968), 1033. A convergence theorem for the integral here is then proved which is analogous to the Lebesgue Dominated Convergence Theorem for the Lebesgue-Stieltjes integral. Finally, a substitution theorem for the integral here is proved which extends the substitution theorem for the weighted refinement integral given in Abstract 663-416, these Notices 16 (1969), 206-207. (Received June 30, 1969.)

Graphs without 5-ways.

The number \( k_r(n) \) is defined as the smallest integer having the property that any graph (with neither loops nor multiple edges) having \( n \) vertices and \( k_r(n) \) or more edges must have an \( r \)-way (i.e., contains at least two vertices joined by \( r \) independent paths). The values of \( k_2(n) \), \( k_3(n) \), and \( k_4(n) \) are known. We establish that \( k_5(2n + 1) = 5n + 1 \) and \( k_5(2n) = 5n - 2 \). Unlike the case \( r = 4 \), the graphs with \( n \) vertices, \( k_r(n) - 1 \) edges, and no \( r \)-ways cannot be simply characterized when \( r = 5 \). (Received June 30, 1969.)

A Plancherel theorem for positive definite measures of locally compact abelian groups.

Let \( G \) be a locally compact abelian group and \( \Gamma \) its dual. We use \( K(G) \) to denote the set of all continuous complex-valued functions on \( G \) which have compact support; we regard \( K(G) \) as a subalgebra of \( L^1(G) \). A positive definite measure on \( G \) is a locally bounded linear functional on \( K(G) \) with the property that \( \mu(f^* f) > 0 \) for all \( f \in K(G) \). Theorem 1. If \( \mu \) is a positive definite measure on \( G \), then there is a uniquely determined positive measure \( \hat{\mu} \) on \( \Gamma \) such that \( \mu(f^* f) = \int_{\Gamma} \hat{f} \hat{g} \hat{\mu} \) for all \( f, g \in K(G) \). This gives a way of associating with each positive definite measure \( \mu \) on \( G \) a positive measure \( \hat{\mu} \) on \( \Gamma \), and it can be shown that the transform \( \mu = \hat{\mu} \) is, in a sense, an extension of the Fourier-Stieltjes transform for bounded measures. If \( \mu \) is Haar measure on a closed subgroup \( H \) of \( G \), then \( \hat{\mu} \) is a Haar measure on the annihilator of \( H \); in particular, if \( \mu \) is the point evaluation at \( e \), then \( \hat{\mu} \) is a Haar measure on \( \Gamma \) and Theorem 1 reduces to the standard Plancherel theorem. (Received June 30, 1969.)
667-146. N. SANKARAN, Queen's University, Kingston, Ontario, Canada. **R-automorphisms of the ring of restricted power series over R.** Preliminary report.

Let R be a commutative ring with identity and Ω be an ideal-adic topology on R. We shall denote by R[x] the ring of restricted power series over R, and assume Ω to be Hausdorff. We show that if f(x) ∈ R[x] with f(0) ∈ Ω where Ω is the ideal generating Ω, then there exists a unique R-endomorphism of R[x] associated with f(x) which is a substitution map if R is complete under Ω.

The converse is valid if we assume that Ω is the principal ideal generated by f(0). We also give conditions for an R-endomorphism of R[x] to be 1-1 and/or onto. The results presented here are analogues of the results of M. J. O'Malley (Ph.D. Thesis, Florida State University, Tallahassee).

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667-147. GEORGE R. GISSLIS, U.S. Naval Academy, Annapolis, Maryland 21402, and PARFENY P. SAWOROTNOW, Catholic University of America, Washington, D. C. 20017. **Canonical example of a general complemented algebra.** Preliminary report.

In his previous papers the second author (Saworotnow) gave different examples of simple complemented algebras [Proc. Amer. Math. Soc. 8 (1959), 49-55; also pp. 56-62]. Here is an example of a complemented algebra, which need not be simple. Let \( \mathfrak{G} \) be a proper \( \ast \)-algebra and let \( \beta \) be a one-to-one bounded linear operator on \( \mathfrak{G} \) such that \( (x)y = \beta(xy) \) for all \( x, y \in \mathfrak{G} \). Let \( \mathfrak{U} \) be the range of \( \beta \), \( \mathfrak{U} = \{ \beta(x) | x \in \mathfrak{G} \} \). Then \( \mathfrak{U} \) is a complemented algebra with respect to the norm \( ||\beta(x)|| = ||x|| \). \( \mathfrak{U} \) is a proper two-sided \( \ast \)-algebra if and only if the inverse of \( \beta \) is bounded. Conversely, each semisimple complemented algebra is of this form. It follows that each proper complemented algebra is a Hilbert module [Duke Math. J. 35, 191-198].

(Received June 30, 1969.)

667-148. RICHARD C. GILBERT, California State College, Fullerton, California 92631. **Symmetric operators with piecewise \( C^2 \) spectral functions.**

Let A be a closed symmetric operator with deficiency index \((1,1)\) in a Hilbert space \( H \). Suppose that \( \sigma_0(t) = (E_0(t)g_0, g_0) \) is \( C^2 \) everywhere, where \( E_0(t) \) is the spectral function of a selfadjoint extension \( A_0 \) of A in \( H \), and \( g_0 \) is an element of norm 1 in a deficiency subspace of A. A generalized resolvent \( R(\lambda) \) of A corresponding to a minimal selfadjoint extension or dilation \( A^\dagger \) of A is given by the formula \( R(\lambda) = R_0(\lambda) - \left[ \Theta(\lambda) + Q(\lambda) \right]^{-1}(\cdot, g(\lambda))g(\lambda) \), where \( R_0(\lambda) \) is the resolvent of \( A_0 \), and \( \Theta(\lambda) \) and \( Q(\lambda) \) are two functions which are analytic in the upper half-plane with nonnegative imaginary parts. Let \( \Phi_{11}(\lambda) = - \left[ \Theta(\lambda) + Q(\lambda) \right]^{-1}, \Phi_{12}(\lambda) = \Phi_{21}(\lambda) = i\Phi_{11}(\lambda)Im \left[ Q(\lambda) \right], \Phi_{22}(\lambda) = - \Phi_{11}(\lambda)[Im \left[ Q(\lambda) \right]]^2 + iIm \left[ Q(\lambda) \right] \). Then for \( j = 1,2 \), \( \rho_{jj}(\xi) = \lim_{\eta \to 0+}(1/\pi)^{\xi} \int_{0}^{\xi}Im \left[ \Phi_{jj}(\sigma + i\eta) d\sigma \right] \) exists for all \( \xi \). There exists a sequence \( \{ \eta_n \} \) such that \( \rho_{11}(\xi) = \rho_{21}(\xi) = \lim_{n \to \infty}(1/\pi)^{\xi} \int_{0}^{\xi}Im \left[ \Phi_{22}(\sigma + i\eta_n) d\sigma \right] \) exists for all \( \xi \). If \( \rho(\lambda) = (c_{RS}(\lambda)), A^\dagger \) is unitarily equivalent to the multiplication operator in \( L^2_0(-\infty, \infty) \). If \( \rho_0(t) \) is piecewise \( C^2 \), one may use the above result together with preceding results of the author to obtain a spectral representation of \( A^\dagger \).

(Received June 30, 1969.)
A new construction of comparison operators for lower bounds to eigenvalues is given for a class of selfadjoint operators that includes Schrödinger operators of atomic systems. The construction uses the notions of (abstract) separation of variables and pairwise couplings in tensor products of Hilbert spaces. The required displacement of essential spectra is done with noncompact operators that are tensor products of operators of finite rank with identities. The resolution of the spectral problems that arise for several specializations of the comparison operators can be accomplished by separation of variables and diagonalization of finite Hermitian matrices. (Received June 30, 1969.)


A subset A of a topological space X is a generalized $F_\sigma$ in X if for every set U open in X containing A there exists an $F_\sigma$-set F in X such that $A \subseteq F \subseteq U$. A subset is called a generalized $G_\delta$ if its complement is a generalized $F_\sigma$. A space is of countable (pointwise-countable) type provided every compact set (point) is contained in a compact set which has a countable fundamental system of neighborhoods. A. V. Arkhangel' skii ["Bicom pact sets and the topology of spaces," Trans. Moscow Math. Soc. (1965), 1-62] proved that a Tychonoff space X is of pointwise-countable type if it is a union of $G_\delta$-sets in its Stone-Čech compactification $\beta X$. Theorem. A Tychonoff space is of countable type iff it is a generalized $G_\delta$ in $\beta X$. M. Henriksen and J. R. Isbell ["Some properties of compactifications," Duke Math. J. 25 (1958), 83-105] proved that a Tychonoff space is of countable type iff $\beta X - X$ is Lindelöf. Theorem. If X is a Tychonoff space of pointwise-countable type, then $\beta X - X$ is realcompact, but the converse is false (under the assumption of the continuum hypothesis). Some other results are obtained. (Received June 30, 1969.)

A uniqueness theorem for second order quasilinear hyperbolic equations.

A uniqueness theorem is proved for weak solutions of quasilinear second-order hyperbolic equations of the form $u_{tt} - \sum_{i=1}^{n} (a_i(x,t,u,u_x))u_{x_i}$ = $b(x,t,u)$. The weak solutions are assumed to satisfy a timewise upper Lipschitz bound $u(x,t_1) - u(x,t_2) \leq (t_1 - t_2)K(t)$ for all $0 < t_1 < t_2$ where $K(t)$ is an $L^1$ function. Together with the obvious assumptions, the equation is supposed to satisfy a symmetry condition $a_i^T = a_j$ and the convexity conditions $\sum_{i=1}^{n} (a_i^T u_x) u_{x_i} \leq 0$ and $\sum_{i=1}^{n} (a_i^T u_x) u_{x_i} \leq 0$, $k = 1, 2, ..., n$. As a corollary, a uniqueness theorem proved for systems by Olešnik is generalized. (Received June 30, 1969.)

Optimizing fifth-order explicit Runge-Kutta formulas.

A previous study of fifth-order Runge-Kutta formulas, of usual explicit type, is continued. The goal is the best from the point of view of truncation error. It can now be stated that amongst all those related to Newton-Cotes, Legendre-Gauss, Radau and Lobatto quadrature, the best is found in the Lobatto group. It can be exhibited. (Received June 30, 1969.)
Let $S$ be a semigroup and \( \varnothing(S) \) the power set of $S$. For any finite set $\mathfrak{m} \subset S \times \varnothing(S)$ and any $x \in S$ let $C_{\mathfrak{m}}(x)$ be \( \{ \text{let } T_{\mathfrak{m}}(x) \} \) the number of ordered pairs $(a,P) \in \mathfrak{m}$ such that $x \in P$ [such that $ax \in P$]. Theorem 1. The semigroup $S$ is left amenable (see F. P. Greenleaf, "Invariant means on topological groups," Van Nostrand, New York, (1969), 4) iff for every pair $\mathfrak{m}, \mathfrak{n}$ of finite subsets of $S \times \varnothing(S)$, there is an element $x \in S$ such that $C_{\mathfrak{m}}(x) \leq T_{\mathfrak{n}}(x) + C_{\mathfrak{n}}(x)$. Theorem 2. If $S$ is a group, then $S$ is left amenable iff for every finite subset $\mathfrak{m}$ of $S \times \varnothing(S)$ there is an element $x \in S$ such that $C_{\mathfrak{m}}(x) = T_{\mathfrak{m}}(x)$. Theorem 3. If $S$ is a group containing a free subgroup having two generators, then there is a two element set $\mathfrak{m} \subset S \times \varnothing(S)$ such that $T_{\mathfrak{m}}(x) > C_{\mathfrak{m}}(x)$ for all $x \in S$.

The relation between bounded index and exponential type for entire functions.

The class of entire functions of bounded index [Proc. Sympos. Pure Math., vol. 11, Amer. Math. Soc., 1968, pp. 298-307] was defined by the author and various properties of this class were studied by him in a series of papers starting in 1965. In particular, the author proved, in 1966, the fundamental result that an entire function of bounded index is necessarily of exponential type. The proof made use of some basic results of Shah and others on the rates of growth of entire functions and of their derivatives. An alternative proof of this theorem, based upon the Wimon-Valiron theory, was communicated to the author in 1967 by Clunie. Shah, by an extension of the author's method, sharpened the result as follows: Theorem. Let $f(z)$ be an entire function of bounded index $N$. Then $f(z)$ is of exponential type not exceeding $N + 1$. The purpose of this note is to give an elementary proof of the above, based upon a portion of the author's original method as well as some inequalities of a potential-theoretic character derived recently by Bose for entire functions of bounded index [Proc. Amer. Math. Soc. 21 (1969), 257-262]. (Received June 30, 1969.)

Expansion of powers of a class of linear differential operators.

In two previous papers, the authors had shown that $x^2D^2n = [x^2D^2 - (n - 1)D - n]$ plus generalizations. In a recent paper, Berković and Kvalwasser, Izv. Vysš. Ucebn. Zaved. Matematika, no. 5 (72)(1968), 3-16, generalized the above two results by finding the explicit polynomial expansions of $[x^2D^2 + aD - n]$ and $[x^2D^2 + ax^2 - n]$. Here we obtain a still further generalization by finding, in a more direct manner, the polynomial expansions of $[(xD + a + 1 - n)(xD + a + 1 - 2n) \ldots (xD + a + 1 - rn)]^n$, $[(x^2D + a + 1 - n)(x^2D + a + 1 - 2n) \ldots (x^2D + a + 1 - rn)]^n$ and other related nth order operators. These expansions are then used to solve certain nth order linear differential equations. (Received June 30, 1969.)
Differentiable functions from $C^0(M)$ to $C^0(M)$.

Let $M$ be a compact manifold without boundary. Let $E$ be the locally convex space of $C^0$ real-valued functions on $M$, and let $L(E,E)$ be the space of continuous linear functions from $E$ to $E$. Let $C^1(E,E)$ be the space of $C^1$ functions (in the sense of Fréchet) from $E$ to $E$. We subdivide $E$ into equivalence classes of equally continuous functions, analyse the behaviour of the Fréchet derivative $Df_\varphi$ of elements $f$ in $C^1(E,E)$ as $\varphi$ varies over $E$, show why the classical form of the implicit function theorem fails and give a weak version of this result for functions $f$ in $C^1(E,E)$. (Received June 30, 1969.)

Some semicontinuity theorems concerning the facial structure of convex sets.

Suppose $K$ is a $d$-dimensional compact convex subset of $\mathbb{R}^d$ with boundary $B$. Let $B_1$ and $B_u$ denote respectively the sets of points of lower and upper semicontinuity of the face-function which assigns to each point $p$ of $B$ the union of all segments in $B$ that cross $p$. Theorem 1. Both $B_1$ and $B_u$ are dense in $B$. $B_1$ is almost all of $B$ in the sense of category (that is, $B_1$ contains a dense $G_δ$ subset of $B$) but not necessarily (for $d \geq 3$) in the sense of measure. Now let $C(K)$ be the set of all continuous real-valued functions on $K$. For each $f \in C(K)$ the lower convex envelope of $f$ is the restriction to $K$ of the supremum of all affine functions on $\mathbb{R}^d$ which are majorized on $K$ by $f$, and $B_c$ is the subset of $B$ at which every lower convex envelope is continuous. Theorem 2. $B_c \subseteq B_1$ and $B_c$ is almost all of $B$ in the sense of category. Theorem 3. $B_c = B_1$ when $d \leq 3$ and $B_c = B$ when $d = 2$. Theorems 2 and 3 extend results of H. S. Witsenhausen [IEEE Trans. Automatic Control AC-13 (1968), 5-21] concerning the convergence of an algorithm for a class of minimax control problems. The second part of Theorem 3 proves a conjecture of J. B. Kruskal. (Received June 30, 1969.)

The product theorem for nonseparable infinite-dimensional manifolds.

In this paper the main theorem is that if $M$ is any metric manifold modeled on any metric topological vector space $F$ where $F$ is homeomorphic to $F^\infty$ ($F \cong F^\infty$), then $F \times M \cong M$. Bessaga and Kadec ["On topological classification of nonseparable Banach spaces" (to appear)] have proved that if $H$ is any infinite-dimensional Hilbert space or reflexive Banach space, then $H \cong H^\infty$. Anderson and Schori [Bull. Amer. Math. Soc. 75 (1969), 53-56] proved that if $M$ is any separable metric manifold modeled on an infinite-dimensional separable Fréchet space $s$, then $s \times M = M$. Most of the techniques in the separable case do not apply in the theorem of this paper. A theorem by Michael [Duke Math. J. 21 (1954), 163-171] bridges one of the main gaps. It gives some rather general conditions when local properties of a topological space imply the corresponding global properties. David W. Henderson [Bull. Amer. Math. Soc. (to appear); Topology (to appear)] used the product theorem in the separable case as the final step in proving that separable Fréchet manifolds can be embedded as open subsets of Hilbert space. (Received June 30, 1969.)
Let $f$ be an entire function over the complex $k$-dimensional space $\mathbb{C}^k$ and let $M(r) = M(r_1, \ldots, r_k) = \max\{ |f(z)| : z = (z_1, \ldots, z_k) \in \mathbb{C}^k, |z_j| \leq r_j \text{ for } 1 \leq j \leq k \}$. The concepts of order and type of $f$, for the special case $k = 1$, admit a number of "natural" extentions. Ronkin and Fuks, "Introduction--several complex var.," Amer. Math. Soc.) discussed such extentions based on a comparison of $\ln^+ M(r)$ with $\sum A_j r_j^{a_j} (A_j, a_j \text{ being real and } > 0, \text{ for } 1 \leq j \leq k)$ as $\|r\| = \sum r_j - + \infty$. Here one might consider as $r_j - + \infty$ for $1 \leq j \leq k$, instead of as $\|r\| - + \infty$. But this is shown to give no new concepts. A systematic study of the extended concepts, based on a comparison of $\ln^+ M(r)$ with $A_1 r_1^{a_1} r_2^{a_2} \cdots r_k^{a_k}$ as $r_j - + \infty$ for $1 \leq j \leq k$, is pursued. Some properties of these concepts turn out to be more elegant, although less easy to prove, than their known analogues; in particular, the order-points of $f$ constitute the boundary of an "infinite rectangle". The relations among the different growth-concepts, including the ones due to Gol'dberg based on special exhaustions of $\mathbb{C}^k$, are studied. In particular, the property that $f$ is of finite order, limited to the concepts under consideration, is independent of the notion of order. (Received June 20, 1969.)

A unification of the theories of Jordan and alternative algebras.

The main results in the structure and representation theory of alternative and of Jordan algebras are proved for a larger class of algebras defined by simple identities which hold in alternative and in Jordan algebras. Write $(x,y,z) = (xy)z - x(yz)$ and $[x,y] = xy - yx$. Let $A$ be a finite-dimensional algebra of characteristic $\neq 2$, satisfying (1) $(x,y,x) = 0$, (2) $(x^2, y,x) = 0$ (i.e. $A$ is a noncommutative Jordan algebra), (3) $([x,y], z,x) = 0$, and (4) $(x, [y,z], w) + (x,w,[y,z]) = 0$ ($x,y,z,w \in A$) (i.e. $[A,A]$ is completely alternative). Theorem 1. A minimal ideal of $A$ is either simple or trivial. Theorem 2. If $A$ is nil, then $A$ is nilpotent. Theorem 3. If $A$ is simple, then $A$ is alternative or Jordan (a semisimple (nilradical 0) $A$ is a direct sum of these). Theorem 4. The Wedderburn Principal Theorem holds for $A$. Let $I$ be the set of identities given by (1), (2), (3), (4). Theorem 5. If $A$ is separable, then any 1-birepresentation of $A$ is completely reducible, and if $A$ is central simple, then any irreducible 1-birepresentation is alternative or Jordan. These theorems generalize work of Schafer (Proc. Nat. Acad. Sci. 60 (1968), 73-74). They also hold with (3), (4) and "alternative" replaced by "associative". Theorems 1, 2, and 3 hold without (4), but Theorems 4 and 5 do not. (Received June 30, 1969.)

Positive length but zero capacity.

In 1959 A. G. Vitushkin gave an example of a compact plane set of positive length but of zero analytic capacity. We give another simpler example. The set if $K \times K$ where $K$ is the Cantor set obtained from $[0,1]$ by removing middle halves. (Received June 30, 1969.)
A Weierstrass point of a class of modular groups. Preliminary report.

For prime \( p \), the normalizer of \( \Gamma_0^0(p) \) in \( SL(2, \mathbb{C}) \) is shown to be \( \Gamma_0^0(p) \cup \Gamma_0^0(p)^{0 \cdot 1} \). The genera of \( \Gamma_0^0(p) \) and of the normalizer are determined, and a theorem of B. Schoenberg is applied to prove the Theorem. \( i \) is a Weierstrass point of \( \Gamma_0^0(p) \) for all prime \( p \neq 11 \). (Received June 30, 1969.)

A simplified proof of Chebyshev's theorem.

For a positive variable \( x \) let \( \pi(x) \) denote the number of prime numbers less than or equal to \( x \). Chebyshev's theorem asserts that \( \pi(x) \) has the order of magnitude of \( x / \log x \). Equivalently, \( \psi(x) \) has the order of magnitude of \( x \), where \( \psi(x) = \sum \mu(n) \), \( \mu \) is the Möbius function, and \( \ast \) is arithmetic convolution. The work of H. N. Shapiro (Proc. Amer. Math. Soc. 1 (1950), 346-348) has shown that to prove Chebyshev's theorem it is sufficient to prove that \( \psi(x) = O(x) \) as \( x \to \infty \). Using a method of Tatuzawa and Iseki (Proc. Japan Acad. 27 (1951), 340-342) and a strong type of Möbius transformation, one may give a simplified convolution proof that \( \psi(x) = O(x) \) for \( x \neq 1 \). (Received June 30, 1969.)

Global dimension in categories of diagrams. Preliminary report.

Let \( I \) be a finite poset; \( \sigma \) be an abelian category with enough projectives; and \( \sigma^I \) be the set of covariant functions from \( I \) to \( \sigma \). B. Mitchell asks in Chapter IX of his book, "Theory of categories," if \( \text{gl. dim} \, \sigma^I - \text{gl. dim} \, \sigma \) is independent of \( \sigma \), and proved that it was for certain classes of posets. In this paper, an example is given of a poset \( I \) with 15 elements such that \( \text{gl. dim} \, \sigma^I - \text{gl. dim} \, \sigma \) takes on the values 4, 4, and 3 where \( \sigma \) is the category of left \( R \)-modules and \( R \) is, respectively, a field of characteristic 2, the integers, and a field of characteristic 0. (Received June 30, 1969.)

The Stone-Weierstrass theorem and completeness of orthogonal systems.

Using the Stone-Weierstrass theorem, a completeness theorem is proved, which includes as special cases Plancherel's theorem, the corresponding theorem for Hankel transforms, the completeness of various polynomial systems, and various expansions in Jacobian elliptic functions. Let \( X \) and \( Y \) be \( \sigma \)-compact Hausdorff spaces, \( \mu \) resp. \( \nu \) measures on \( X \) resp. \( Y \). For a suitable map \( \sigma : X \times X \to \mathcal{C} \), define maps \( T \) and \( Q \) on functions of compact support in \( L^2_\mu(X) \) resp. \( L^2_\nu(Y) \) by \( (T\sigma)(\ast) = \int_X \sigma(\ast, y) d\nu(y) \in L^2_\mu(X) \) and \( (Qf)(\ast) = \int_Y f(x) \sigma(y, \ast) d\mu(y) \in L^2_\nu(Y) \). Denote by \( S \) the vector space of functions \( T \sigma \), where \( \sigma \) has compact support; denote by \( A \) the algebra generated by \( S \); denote by \( S^* \) the closure of \( S \) in the norm \( \| \cdot \| = L^2(X) \) norm + uniform norm. If \( \sigma \) satisfies suitable conditions, principally an orthogonality condition resulting in \( \| T \sigma \| = \| \sigma \| (L^2 - \text{norm}) \), if certain subspaces of functions of compact support in \( L^2_\mu(X) \) separate points, and if \( A \subset S^* \), then \( T \) and \( Q \) extend to \( L^2_\mu(Y), L^2_\mu(X) \), and \( f = TQf \) (\( f \in L^2_\mu(X) \)). (Received June 30, 1969.)
In this paper a bordism theory is developed for compact manifolds which have an oriented boundary. Let $\Omega_{n}^{O,SO}$ denote this bordism group in dimension $n$. It is shown that there is an isomorphism $\Omega_{n}^{O,SO} \cong \Omega_{n-1}^{SO} \oplus \Omega_{n-2}^{O}$, where $\Omega_{n-1}^{SO}$ and $\Omega_{n-2}^{O}$ denote the oriented and unoriented bordism groups, respectively. The bordism classes in $\Omega_{n}^{O,SO}$ are characterized via characteristic numbers. As usual, a generalized homology theory arises, which is denoted by $\Omega_{*}^{O,SO}(X,A)$. This generalized homology theory is shown to be isomorphic to the generalized homology theory of the quotient spectrum $MO/MSO$. To relate this bordism theory to the usual oriented bordism theory, an isomorphism $\Omega_{n}^{O,SO}(X,A) \cong \Omega_{n-1}(X,A) \times \mathbb{RP}(\infty)$ is established. (Received June 30, 1969.)

Nest generated intersection rings in Tychonoff spaces.

Tychonoff spaces are characterized by possessing separating nest generated intersection rings which exhibit the hereditary nature of complete regularity [Duke Math. J. 33 (1966), 743-746]. The Wallman compactifications and realcompactifications associated with such rings are considered. Negative answers are given to two questions of Alo and Shapiro [J. Austral. Math. Soc. 9 (1969), 489-495], who studied normal bases which are intersection rings (not necessarily nest generated). As a result, nest generation cannot be relaxed if one is to obtain realcompactifications in Wallman compactifications in the way the Hewitt realcompactification is obtained in the Stone-Čech compactification. The zero-sets of algebras (certain inverse-closed subalgebras of $C(X)$) are separating nest generated intersection rings. In fact, there is a one-to-one correspondence between algebras and such rings. The family of all compactifications provided by these rings on a given locally compact space is a complete lattice. (Received June 30, 1969.)

Potential theory on Banach spaces of functions.

Let $X$ be a locally compact Hausdorff space, $C$ the space of continuous scalar-valued compact support functions on $X$. Let $D = D(X, \xi)$ be a $B$-space of (equivalence classes of) scalar-valued functions on $X$, locally integrable with respect to a dense Radon measure $\xi \geq 0$, and verifying the Dirichlet axioms (Beurling and Deny, Proc. Nat. Acad. Sci, 45 (1959)). Pure potentials are elements of $D^{+}$, and $f \in D^{+}$ is a potential if there exists an associated $\mu$ such that $\phi \in D \cap C$ implies $(\phi, f) = \int \phi \, d\mu$. Let $S: D \rightarrow D^{+}$ verify $(u, S(u)) = \|u\|^{2}$ and $\|S(u)\| = \|u\|$. Theorem 1. (i) A pure potential is a potential. (ii) If $D$ is uniformly convex and $S(u)$ is a pure potential, then $u \in D^{+}$. Theorem 2. Let $D$ be uniformly convex and smooth, $\omega_{j}, \omega_{0} \subset X$ open, $\overline{\omega}_{j}$ compact, $\overline{\omega}_{j} \cap \overline{\omega}_{0} = \emptyset$. Then there exists $u \in D$ verifying (i) $0 \leq u \leq 1$, a.e. $\xi$, (ii) $u = 0$ a.e. $\xi$ on $\omega_{0}$, $u = 1$ a.e. $\xi$ on $\omega_{j}$, (iii) $S(u)$ is a potential with associated measure $\mu = \mu^{+} - \mu^{-}$ with supports $\mu^{+} \subset \overline{\omega}_{j}$, $\mu^{-} \subset \overline{\omega}_{0}$. (Received June 30, 1969.)
On the existence of proper maps between Euclidean spaces.

A continuous map from X to Y (Euclidean spaces) is said to be proper if the inverse image of compact sets in Y is compact in X. We give examples of continuous proper maps from $\mathbb{R}^m$ onto $\mathbb{R}^k$ for any $m, k \neq 1$, except when $m > k = 1$. **Theorem.** There do not exist any continuous proper maps from $\mathbb{R}^m$ onto $\mathbb{R}$ if $m > 1$. Let $S^m$ denote the unit sphere in $\mathbb{R}^{m+1}$. **Corollary.** Let $f$ be a continuous map from $S^m$ onto $S^1$ with $m > 1$, then for each $q$ in $S^m$ $f(S^m - \{q\}) = S^1$. (Received June 30, 1969.)

Infinite nonsolvable classes of the Delaunay-Nagell diophantine equations $x^3 + my^3 = 1$.

The well-known Delaunay-Nagell theorem about the diophantine equation $x^3 + my^3 = 1$ states that this equation has at most one solution $(x_1, y_1)$ with nonzero components for a fixed natural $m$, and that in this case $x_1 + y_1 m^{1/3}$ is either the fundamental unit of the algebraic number field generated by $m^{1/3}$ or its square. The question of the existence of such a single solution has not been investigated, except for the almost trivial case $m = D_3^2$, $D$ a natural. In this paper a large family of infinite classes of values of $m$ are enumerated for which $x^3 + my^3 = 1$ has no solution. These results are mainly based on previous papers of the author and Helmut Hasse concerning explicit units of algebraic number fields generated by $m^{1/n}$ ($n > 2$, $m$ a nonperfect nth power natural). (Received June 30, 1969.)

Nil ideals of rings satisfying maximum condition on right annihilators.

Let $R$ denote a ring with unity satisfying the maximum condition on right annihilators, $N$ the prime radical of $R$. It is known that if $N$ is nil, it need not be nilpotent. However, the following are equivalent: (1) $N$ is nilpotent, (2) $N$ is right $T$-nilpotent, (3) $x \in N$ implies that there exists a positive integer $n(x)$ such that $N^n(x) = 0$. $N$ contains all nil right ideals and all nil left ideals and $N$ is nilpotent if $N^R = \text{finite dimensional}$ or if $N^R$ is finite dimensional and the left singular ideal is nilpotent. Even in the case when $N$ is not nilpotent, it is "close to being nilpotent" in that there exists a nilpotent ideal $K$ of $R$ such that $K^R$ is large in $N^R$. (Received July 1, 1969.)

667-172. GEORGE W. BATTEN, JR., University of Houston, Houston, Texas 77004. Hilbert space arcs with orthogonal chords.

Let $H$ be an infinite-dimensional Hilbert space. It is shown that an arc $\Gamma$ in $H$ for which every pair of nonoverlapping chords is orthogonal can be constructed as follows. Let $\{x_1, x_2, \ldots\}$ be an infinite orthonormal sequence in $H$. Let $\Gamma_1$ be the line segment from $x_0 \in H$ to $x_0 + ax_1$. Suppose, for the sake of induction, that $\Gamma_n$ has been defined as a polygonal arc with vertices $v_0, v_1, \ldots, v_n$ in that order and that $\Gamma_n$ has the property that any pair of nonoverlapping chords with endpoints coinciding with vertices of $\Gamma$ is orthogonal. Let $i$ be an integer such that $\|v_{i+1} - v_i\|$ is maximal. Define $\Gamma_{n+1}$ to be the polygonal arc with vertices $u_0, u_1, \ldots, u_{n+1}$ in that order, where $u_j = v_j$ for $j \neq i$, $u_{i+1} = (1/2)(v_{i+1} + v_i) + (1/2)\|v_{i+1} + v_i\|v_{i+1}$, $u_j = u_{j-1}$ for $i + 1 \neq j$. Then the sequence $\{\Gamma_n\}$
converges to an arc $\mathbf{T}$ having the required properties. Furthermore, every such arc can be con­structed by this method. It follows, in particular, that every such arc must lie on a sphere in $H$. This construction extends easily to higher dimensional structures. (Received July 1, 1969.)

667-173. LAWRENCE J. CORWIN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Some self-dual locally compact groups.

Let $G$ be a locally compact Abelian group, and let $f: G \rightarrow G^*$ be a topological isomorphism of $G$ onto its dual with the following two properties: $f$ and $f^*: G \rightarrow G^*$ are identical, and $G$ has a compact open subgroup $H$ such that $f(H) = H^\perp$. (All known self-dual groups are direct sums of such groups $G$ with $\mathbb{R}^n$ and finite groups.) $G$ can be determined from $H$ and certain maps taking $H^*$ into the unit circle. This analysis of $G$ enables one to construct new self-dual groups. (Received July 1, 1969.)

667-174. ELBERT M. PIRTLE, University of Missouri, Kansas City, Missouri 64110. Families of valuations and the ideal transform.

Let $R$ be an integral domain with quotient field $L$, and let $A$ be an ideal of $R$. $T(A) = \bigcup \{ R: A^n \mid n = 1, 2, 3, \ldots \}$ is called the transform of $A$. Brewer proved that if $A$ is a finitely generated ideal of $R$, then $T(A) = \bigcap R_{p_a}$, where $\{ p_a \}$ is the collection of prime ideals which do not contain $A$. Using families of valuations, in certain cases we obtain a characterization of $T(B)$ in terms of valuation overrings of $R$, where $B$ is an arbitrary ideal of $R$. We also obtain a new characterization of $K$-domains in terms of the ideal transform. As a corollary, this gives another characterization of Dedekind domains in the class of 1-dimensional Prüfer domains by using a result of Gilmer. (Received July 1, 1969.)

667-175. SAMUEL S. HUANG and DOV TAMARI, State University of New York at Buffalo, Amherst, New York 14226. New proofs for the lattice property of systems ordered by a semi-associative law.

The theorem "$(T_n, \preceq)$ is a lattice" is proved by H. Friedman and D. Tamari [J. Combinatorial Theory 2 (1967), 215-242]. Here new methods of proof are presented with the aim to replace some of the "hard" combinatorial analysis by conceptual considerations. The outline of a proof based on induction follows: Observe that (a) $T_2$ is a lattice; (b) for $n \geq 2$, the subsystems $S_n$, $S'_n \in T_n$, $S_n = \{ E \in T_n \mid E = (Ax_n) \}$, $S'_n = \{ E \in T_n \mid E = (x_0A) \}$ are order isomorphic to $T_{n-1}$, therefore, by induction hypothesis, lattices, and sublattices in $T_n$. The two semi-associative laws induce two, mutually inverse, 1-1 maps $\mathfrak{N}: T_n \rightarrow T_n - S_n - U_n = T_n - S'_n$ [resp. $\mathfrak{N}: U'_n \rightarrow U_n$] by

$$(A(BC)) = ((AB)C) \text{ resp. } ((AB)C) = (A(BC)).$$

Denote, e.g., by $\mathfrak{N}^k$ the kth iteration of $\mathfrak{N}$, by $\mathfrak{N}^0$ the identity map of $T_n$. One can prove the following two propositions: (1) $\forall E \in T_n \forall k \in N \forall G \in S_n (G \mathfrak{N}^k = E)$, i.e. each element of $T_n$ is image of a (unique) element of $S_n$ under iterated $\mathfrak{N}$-mapping; (2) $E \lor F = G (E$ and $F$ have the l.u.b. $G) = E \mathfrak{N}^k \lor F = G \mathfrak{N}^k$, i.e. in particular, the existence of a unique l.u.b. is invariant under $\mathfrak{N}$ transformation. Combining (1) and (2), one obtains the theorem. Remark. The proof of proposition (2) still requires considerable "hard" combinatorial analysis depending on the theory of indices and their commutation rules (l.c.). (Received July 1, 1969.)

We conjecture that $G_{2N} = (-1)^N \frac{1}{2} \sum_{n=0}^{N/2} \frac{1}{n! n^N}$. The first few Genocchi numbers are $G_2 = -1$, $G_4 = -3$, $G_6 = +17$, $G_8 = -155$, $G_{10} = +2073$, etc. The $\Sigma$ notation has the following meaning: $\Sigma k^2 \Sigma (k+1)^2 \Sigma (k+2)^2 \cdots \Sigma (k+N)^2 = \frac{2}{k} G_{2N}$. Since $G_{2N} = 2(2^{2N} - 1) B_{2N}$ we get an explicit formula for Bernoulli's numbers $B_{2N}$ and a detailed study of which may probably give some information about the numerators of $B_{2N}$ which is needed in the proof of Fermat's Last Theorem. (Received July 1, 1969.)


An equivalence $\mathcal{E}$ on $S$ is a $T$-congruence (T is a ternary relation) if $(s_1, s_1', s_1'') \in T$, $s_1 \mathcal{E} s_1'$ and $s_1' \mathcal{E} s_1''$. The canonical projection $\pi: S \to S/\mathcal{E} = S'$ induces a map $T \to T' = \pi^* T$, a (partial) binary operation (b.o.) on $S'$. Such congruences exist (e.g. the universal one), the intersection of a family of $T$-congruences is a $T$-congruence, and the $(\omega)$ (unique) smallest $T$-congruence $\mathcal{E}$ determines the b.o. $T'$ induced by $T$. $\mathcal{E} = \cup \mathcal{E}_k$ where $\mathcal{E}_0 \subset \mathcal{E}_1 \subset \cdots$ is an ascending chain of equivalences on $S$ starting with the identity $1 = \mathcal{E}_0$ and generated by $(s_1, s_1', s_1'') \in T$, $s_1 \mathcal{E}_1 s_1'$ and $s_1' \mathcal{E}_1 s_1''$. Application to the construction of the symmetrisation $\Sigma_M$ of a b.o. $M = T_M$. Put $S = M \cup \{1\} \cup \overline{M}$ (an anti-isomorphic copy of $M$; $1 \notin M$) and define $T_S$ over $S$ by $T_S = T_M \cup T_{\overline{M}} \cup \{(s, 1, s), (1, s, s) | s \in S\} \cup \{(c, b, a), (a, c, b), (b, c, a), (c, a, b)\}$. The canonical map $M \to \Sigma_M$ is the universal object in the category of morphisms $\theta: M \to \Sigma_M$ for any symmetric b.o. $M$ n.a.s.c. for the symmetrisability of $M$ (embeddability in an $M_\rho$) are proven. [For definition of symmetric b.o. etc., see D. Tamari [1] "Embedding p. multipl. systems," these Notices 7 (1960), 760; or [2] Portugal. Math. 21 (1962), 157]. (Received July 1, 1969.)

667-178. RAYMOND W. HONERLAH, 582 Ewald Street, Salem, Oregon 97302. Uniform algebras with nonextendible homomorphisms.

Let $X$ be a compact Hausdorff space and consider a pair $(A, \phi)$ where $A$ is a proper closed subalgebra of complex $C(X)$, $1 \notin A$, and $\phi$ is a complex homomorphism which does not extend to any larger subalgebra. Some conclusions about such a pair were presented in Abstract 663-415, these Notices 16 (1969), 206. The author has previously shown that such pairs do not exist when $X$ fails to contain perfect subsets, whereas examples of such pairs are immediate when $X = T$, the unit circle. Suppose $X$ is perfect, then a strong Urysohn's lemma is proved which gives a map of $X$ into $[0, 1]$, hence onto $T$. This yields a proof of the existence of pairs $(A, \phi)$ on $X$. Existence of such pairs on arbitrary spaces containing perfect subsets then follows by taking full extensions. Finally, the connection between extension of homomorphisms and relative maximality of $\hat{A}$ is established. (Received July 1, 1969.)
The steady-state thermoelastic mixed boundary-value problem for the elastic layer.

Steady-state thermoelastic problems for the elastic layer have been discussed by Sneddon and Lockett [Quart. Appl. Math. 18, 145-153] and Martin and Payton [J. Math. Mech. 13, 1-30]. The present paper is concerned with determination of the steady-state thermal stresses in an elastic layer with one face stress free and the other face resting on a rigid frictionless foundation. The free surface of the layer is subjected to a known (arbitrary function of the radial coordinate) heat flux on a circular area of radius unity and on the rest of the face temperature is given to be zero. The surface in contact with the foundation is assumed to be insulated. By using Hankel transforms, the determination of temperature in the layer is reduced to the solution of dual integral equations. The dual integral equations are further reduced to a single Fredholm integral equation for the determination of an unknown function \( \Phi(t) \). The temperature, stresses and displacements have been obtained in terms of finite integrals involving the function \( \Phi(t) \). For large (compared to unity) thickness of the layer, \( \Phi(t) \) has been obtained as a polynomial in \( t \). Using numerical integration, numerical values of the various physical quantities have been tabulated. (Received July 1, 1969.)

Axisymmetric Boussinesque's problem with couple stresses.

The problem of a concentrated normal load acting on the bounding plane of a semi-infinite elastic medium in the presence of couple stresses is investigated. Papkovitch functions developed by Mindlin and Tiersten (Arch. Rat. Mech. and Anal. 11 (1962)) have been used to solve the problem. Hankel transform technique has been applied. The displacements are then expressed in the form of definite integrals. The solution approaches the classical solution as the material constant \( \ell \) of couple stress \( \rightarrow 0 \). The results have been generalized to any distribution of normal loads on the bounding plane. The particular case of constant normal load on a circular area on the bounding plane has been discussed. It is found that the couple stresses introduce a discontinuity in the radial displacement in the bounding plane at the boundary of the stressed circular area, and the couple stress does not effect the radial displacement in the bounding plane outside the stressed area and has an effect of \( O(\ell^2) \) on the normal displacement in the bounding plane. To discuss the nature of the singularity at the point of application of the load, the results in the transform domain are expanded for large \( \ell \), the transform variable. It is found that the order of the singularity is the same as in the classical case, but with a different structure. (Received June 27, 1969.)

Characterizations of the groups \( D^2_4(q^3) \) and \( PSU_5(q^2), q = 2^n \).

The Steinberg groups \( D^2_4(q^3), q = 2^n \), and the unitary groups \( PSU_5(q^2), q = 2^n \), are characterized in terms of the centralizer in the group of an element of order 2 in the center of a Sylow 2-subgroup of the group. The connections between these results and similar characterizations of other finite simple groups are discussed. (Received June 26, 1969.)

Let $G$ be a compact connected Lie group with closed subgroups $H^0, H^1, ..., H^r$ satisfying the following inclusion relationships with their normalizers: $H^{i-1} \subseteq N(H^i) \cap N(H^l), 1 \leq i \leq r$. Let $\Sigma$ be the set of $G$-conjugacy classes of these subgroups and let $\tilde{X}$ be a $\Sigma$-space [R. S. Palais, "The classification of $G$-spaces," Mem. Amer. Math. Soc. No. 36, 1960]. Let $X^1$ be an $(H^1)/H^1$-principal bundle over $\tilde{X}(H^1)$. Then there is a $G$-space $X$ such that for each $i$, $X^i$ is the coordinate bundle with fibre $G/H^i$ associated with the $(H^i)/H^1$-principal bundle $X^i$. This result is a partial converse to a strengthening of a result announced last summer (cf. Abstract 658-52, these Notices 15 (1968), 733).

(Received June 23, 1969.)

JAMES R. ROYSE, Department of Philosophy, San Francisco State College, San Francisco, California 94132. Z is not translatable into the ramified "Principia".

$S_0$ is a certain fragment of elementary number theory $Z$. For each $i$, $S_{i+1}$ is the predicative extension of $S_i$. $S$ is the union of all the $S_n$'s. $R_n$ is ramified set theory of order $n$, with the axiom of infinity. $R$ is the union of all the $R_n$'s. ($R$ is essentially "Principia Mathematica" without the axiom of reducibility.) Cons (T) is an arithmetic sentence expressing the consistency of $T$. Theorem 1. For each $n$, $Z \vdash \text{Cons}(S_n)$. Theorem 2. For each $n$, $R_n$ is translatable into $S_n$. Thus: Theorem 3. For each $n$, $Z \vdash \text{Cons}(R_n)$. If $Z$ were translatable into $R$, it would be translatable into some $R_n$. But, then, by 3, $Z$ could prove its own consistency, which is impossible. Hence: Theorem 4. $Z$ is not translatable into $R$. (Received June 30, 1969.)

WITHDRAWN.

ALLAN PETERSON, University of Nebraska, Lincoln, Nebraska 68508. On the continuity of two-point boundary value functions with respect to coefficients.

Consider the $n$th order differential equation $t[y] = y^{(n)} + p_1(x)y^{(n-1)} + ... + p_n(x)y = 0$. Recently T. Sherman [Bull. Amer. Math. Soc. 74 (1968), 923-925] announced that the first conjugate point $\eta_1(a)$ is a continuous function of the coefficients $p_i(x), i = 1, ..., n$. The author is concerned with the question of when are the two-point boundary value functions $r_{n-k,k}(a), k = 1, ..., n - 1$, continuous functions of the coefficients $p_i(x), i = 1, ..., n$. Theorem. If for $t[y] = 0, r_{n-k,k+1}(a) = r_{n-k+1,k-1}(a) = \infty$ $(r_{n-1,k}(a) = \infty; \ r_{2,n-1}(a) = \infty)$, then $r_{n-k,k}(a)$ is a continuous function of the coefficients $p_i(x), i = 1, ..., n, k = 2, 3, ..., n - 2$. The author also has a theorem which gives conditions under which the $m$th conjugate point $\eta_m(a)$ for the fourth order quasi differential equation $L_4[y] = (D_4^2y)^3 + q_3D_2y + q_2y = 0$ of J. Barrett is a continuous function of the coefficients of $L_4[y] = 0$. (Received June 20, 1969.)
An integral equation involving the Whittaker function.

Recently, the author [H. M. Srivastava, "Certain properties of a generalized Whittaker transform," Mathematica 10 (33) (1968), 385-390] introduced the integral equation \( (*) \int_{0}^{\infty} (pt)^{\sigma - 1/2} e^{-p/2} W_{k,m}(x)^{\frac{1}{2}} f(t) dt = \delta^{(n, \sigma)}_{q,k,m} [f; \rho] \), which evidently provides an elegant unification of the various generalizations of the classical Laplace transform and where, if \( f(t) = \Omega(t e^{t}) \) for large \( t > 0 \), and \( f(t) = \Omega(t^{c}) \) for small \( t > 0 \), then, for existence of \( (*) \), \( \text{Re} [q + \rho - 2 \epsilon] > 0 \) and \( \text{Re} (\sigma + c + 1) > |\text{Re}(m)| \).

ABSTRACTS PRESENTED TO THE SOCIETY

During the interval from April 25 through June 24, 1969, the papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these Notices. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

Algebra & Theory of Numbers

69T-A96. WITHDRAWN.


An algebra P is (semi) primal if every partial [subalgebra preserving] finitary operation over P with finite domain can be extended to a polynomial over P. Theorem. (SPA) [cf. Abstract 69T-A74, these Notices 16 (1969), 655-656, Report (1)]. If P is semiprimal, then P is finitarily semicategorical wk. Thus if \( \mathcal{O}(P) \) is finite, then P is semicategorical. Moreover if P - \( \mathcal{G} \) and \( \mathcal{G} \) is finitely generated, then \( \mathcal{G} \) is uniquely isomorphic to a direct product of finitely many subalgebras of P. This generalizes results of Foster, "Functional completeness in the small," Math. Ann. 148 (1962), Section 14; Foster, "Families of algebras with unique (sub) direct factorization," Ibid, 166 (1966) Theorem 15.1; and Foster-Pixley, "Semicategorical algebras," Math. Z. 83 (1964), Theorems 4.1 and 7.1. Various characterizations and properties of finite semiprimal algebras [Foster-Pixley, Ibid] are shown to extend to the infinite case; some new characterizations are provided. Proposition. Let \( \mathcal{F} \) be a field with "subtraction" and "inversion" taken as primitive operations. Then \( \mathcal{F} \) is semiprimal iff (i) \( \mathcal{F} \) is a purely algebraic extension of its prime field P. (ii) No element of \( \mathcal{F} \) has any proper conjugates in \( \mathcal{F} \) over P. Thus the field Q of rationals, \( Q(\sqrt{2}) \), \( Q(\sqrt{3}) \), and so on, are semiprimal. (Received March 7, 1969.) (Author introduced by Professor Ralph McKenzie.)

69T-A98. ADI BEN-ISRAEL, Engineering Sciences Department, The Technological Institute, Northwestern University, Evanston, Illinois 60201. On cone monotonicity of complex matrices.

Theorem. Let A \( \in \mathbb{C}^{m \times n} \), B \( \in \mathbb{C}^{k \times n} \) and let \( S \subset \mathbb{C}^m \), T \( \in \mathbb{C}^k \) be closed convex cones. Then the following are equivalent: (i) Ax \( \in S \iff Bx \in T \), (ii) \( B^HT^* \subset clA^HS^* \). If S is polyhedral then (ii) may be replaced by (ii') \( B^HT^* \subset A^HS^* \). Corollary. Let A \( \in \mathbb{R}^{m \times n} \), S a polyhedral cone in \( \mathbb{R}^m \) then the following are equivalent: (i) Ax \( \in S \iff x \geq 0 \), (ii) \( R_A^T \subset A^TS \), (iii) A has a left inverse whose rows lie in \( S^* \). For \( S = \mathbb{R}_+^m \) this is a theorem of O. L. Mangasarian (SIAM Rev. 10 (1968), 939-941) and for \( m = n \) a theorem of Collatz. (Received February 5, 1969.)
radical in a class of naring.

Recently Zavlakav (Solvability and nilpotence in Jordan rings, Akad. Nauk. SSSR Algebra i Logica Seminar 5 (1966), 37-58) has proved that if \( J \) is a Jordan ring, then every finitely generated subring \( A \) has the property that for each positive integer \( m \) there is a positive integer \( f(m) \) such that \( A^f(m) \subseteq A_m \). Call a naring \( N \) an \( s \)-naring if for each ideal \( I \) in \( N \), \( I^s \) is also an ideal in \( N \) (\( s \equiv 2 \)). Sufficient conditions are given which insure that an \( s \)-naring has the Zavlakav property. This enables one to define the Lavitzki radical for such a class of narrings. Here \( A_1 = A^s \), and \( A_{m+1} = A_m^s \). It is shown that Jordan rings, alternative rings, Lie rings, and standard rings satisfy these conditions and hence have the Zavlakav property. (Received April 14, 1969.)

On integral domains of finite character. II.

Let \( D \) be an integral domain with quotient field \( K \). By an overring of \( D \) we mean any ring between \( D \) and \( K \). We say that \( D \) is a domain of finite character provided there exists a family \( \{V_\alpha\} \) of valuation rings which are overrings of \( D \) having the following two properties: (i) \( D = \bigcap_\alpha V_\alpha \) and (ii) each nonzero element \( x \in D \) is a nonunit in only finitely many \( V_\alpha \)'s. If, in addition to (i) and (ii), each \( V_\alpha \) is a quotient ring of \( D \), then we say that \( D \) is a domain of Krull type. A domain of finite character with the property that each \( V_\alpha \) has rank one is called a domain of finite real character. In each case, the family \( \{V_\alpha\} \) is called a defining family for \( D \). Theorem 1. A domain of finite character is completely integrally closed if and only if it is a domain of finite real character. Following Gilmer and Heinzer in \( \text{[Math. Z. 103 (1968), 306-317]} \) we say that a representation \( D = \bigcap_{\lambda \in \Lambda} W_\lambda \) of an integral domain \( D \) as an intersection of valuation overrings of \( D \) is an \( S \)-representation for \( D \) provided

\[
\bigcap_{\lambda \in \Lambda} W_\lambda \supseteq D
\]

for each proper subset \( \Lambda \) of \( \Lambda \). Theorem 2. A domain \( D \) of Krull type has one and only one defining family \( \{V_\alpha\} \) such that \( D = \bigcap_\alpha V_\alpha \) is an \( S \)-representation for \( D \). Furthermore, if \( \{W_\lambda\} \) is any defining family for \( D \), then \( \{V_\alpha\} \subseteq \{W_\lambda\} \). The analogue of Theorem 2 is false for domains of finite character, true for domains of finite real character. (Received April 21, 1969.)

Permutations of finite abelian groups.

Let \( A \) be a finite Abelian group of order \( de = m \) written multiplicatively. \( A \) is the direct product of \( k \) cyclic groups \( C_{m_i} \) of order \( m_i \) (\( i = 1, \ldots, k \)). Let \( e \) be the exponent of \( A \); then \( a^e = 1 \) for each \( a \in A \). Let \( \theta = \prod_{i=1}^k (d, m_i) \). Then \( A \) contains exactly \( a = \theta^{-1} \) \( d \)-th powers \( u_1, \ldots, u_a \). Let \( a_1, \ldots, a_\epsilon \in A \) and let \( (a_r)^d = 1, r = 1, \ldots, a_\epsilon \), and define a mapping \( \phi : x \mapsto a_\epsilon x^c \) \( (x^d = u_r, r = 1, \ldots, a_\epsilon, x \in A) \), where \( (c, m) = 1 \). The set \( H(d) \) of all such mappings is a group of permutations of \( A \). The order of \( H(d) \) is \( \varphi(d', e') \) where \( \varphi \) is the Euler \( \varphi \)-function and \( d' = (\epsilon, d), d'e' = \epsilon \). Moreover, \( K(d) \), the group of permutations of \( A \) of the form \( x \mapsto a_\epsilon x^c \) \( (x^d = u_r, a_r^d = 1, r = 1, \ldots, a_\epsilon, x \in A) \) is a normal subgroup of \( H(d) \) and the factor group \( H(d)/K(d) \) is isomorphic to the reduced residue system \( \{d', e'\} \). (This generalizes theorems due to Wells, in \"Groups of permutation polynomials,\" Monatsh. Math. 71 (1967), 248-262, and in \"A generalization of the regular representation of finite Abelian groups,\" Monatsh. Math. 72 (1968), 152-156.) (Received April 28, 1969.) (Author introduced by Professor Charles F. Wells.)
69T-A102. EUGENE W. JOHNSON, University of Iowa, Iowa City, Iowa 52240, JOHNNY A. JOHNSON, University of Houston, Houston, Texas 77004, and JOHN P. LEDIAEV, University of Iowa, Iowa City, Iowa 52240. A structural approach to Noether lattices.

Let $\mathcal{L}$ be a Noether lattice and let $A \in \mathcal{L}$. A sequence $\{B_n\}$ of elements of $\mathcal{L}$ is a completely regular $A$-sequence if $B_{n+1} \lor A^n = B_n$ for all $n \geq 1$. $\mathcal{L}$ is $A$-complete if, given any completely regular $A$-sequence $\{B_n\}$, it follows that $B_n = (\bigcap B_n) \lor A$ for all $n \geq 1$. Theorem 1. A Noether lattice $\mathcal{L}$ is a union of a collection of semilocal Noether lattices which determine $\mathcal{L}$ to within isomorphism.

Let $J(\mathcal{L})$ denote the greatest lower bound of the collection of maximal elements of $\mathcal{L}$. Theorem 2. A semilocal Noether lattice $\mathcal{L}$ which is $J(\mathcal{L})$-complete is determined (up to isomorphism) by the structure of the submultiplicative lattice consisting of those elements all of whose associated primes are maximal. Theorem 3. A semilocal Noether lattice $\mathcal{L}$ which is $J(\mathcal{L})$-complete is a direct product of finitely many local Noether lattices. Theorem 4. Any semilocal Noether lattice $\mathcal{L}$ can be embedded in the direct product of finitely many local Noether lattices. Thus the general embedding problems for $\mathcal{L}$ are largely reduced to embedding problems for local Noether lattices. (Received April 28, 1969.)

69T-A103. JOHN McKAY, Atlas Laboratory, Chilton, Didcot, Berkshire, U. K. Generators for the Mathieu group $M_{24}$.

Let $M_{24} = \langle x, y \rangle$ where $x^2 = y^3 = (xy)^{23} = 1$; then in the natural permutation representation either (1) $x = (0 \ 1 \ 2 \ 3 \ 21)(5 \ 20)(6 \ 18)(7 \ 10)(11 \ 17)(12 \ 15)(2 \ 4 \ 8 \ 9 \ 13 \ 14 \ 16 \ 19)$ and $y = (0 \ oo)(1 \ l2)(2 \ 3)(4 \ 5 \ 21)(6 \ 19 \ 20)(7 \ 11 \ 18)(8 \ 9 \ 10)(12 \ 16 \ 17)(13 \ 14 \ 15)$, or (2) $x = (0 \ oo)(1 \ 22)(2 \ 3)(4 \ 21)(5 \ 10)(6 \ 7)(8 \ 9)(11 \ 20)(12 \ 13)(14 \ 19)(15 \ 16)(17 \ 18)$ and $y = (0 \ oo \ l2)(4 \ 22)(5 \ 11 \ 21)(6 \ 8 \ 10)(12 \ 14 \ 20)(15 \ 17 \ 19)$ 3 7 9 13 16 18 to within conjugacy and inversion. (Received April 24, 1969.)


An "O" error term for Ingham's theorem (A Tauberian theorem for partitions, Ann. of Math. 42 (1941), 1075-1090) is obtained in place of the "o(1)" type of error, and applications are made to various partition functions. In particular, for the unrestricted partition function $p(n)$, the error term $O(n^{-1/3 + \varepsilon})$ is obtained in place of $o(1)$, with the constant of the exponent determined by the accumulated errors in the proof of the Tauberian theorem itself rather than by properties of the generating function of $p(n)$. (Received May 2, 1969.)

69T-A105. H. LAKSER, University of Manitoba, Winnipeg 19, Canada. Injective hulls of Stone algebras.

Our notations and methods are those of C. C. Chen and G. Grätzer, "Construction of Stone lattices, I-III," these Notices 14 (1967), 527, 715 and 650. In these Notices 16 (1969), 407, R. Balbes and G. Grätzer characterized injective Stone algebras and remarked that, if $B$ is a Boolean algebra, $B[\mathcal{L}] = \{x, y \in B^2 | x \leq y\}$ is a Stone algebra. An extension $L_1$ of a Stone algebra $L$ is said to be
essential if, given any Stone algebra $M$ and any homomorphism $f : L_1 \rightarrow M$ whose restriction to $L$ is $1-1$, then $f$ is $1-1$. An injective hull of $L$ is an essential injective extension of $L$. Let $(C(L), D(L), \phi_L)$ be the triple associated with $L$, and let the ideal $R(L)$ of $C(L)$ be $\text{Ker } \phi_L$ and the ideal $K(L) = \{ x \in C(L) | x \cap R(L) = (0) \}$. **Theorem 1.** The injective hull of the Stone algebra $L$ is $B_0 \times B_1^2$ where $B_0$ is the completion of $C(L)/K(L)$ and $B_1$ is the completion of the Boolean algebra generated by $D(L)$.

**Theorem 2.** The extension $L_1$ of the Stone algebra $L$ is essential iff $D(L_1)$ is an essential extension of $D(L)$ and $C(L_1)/K(L_1)$ is an essential extension of $C(L)/(K(L_1) \cap C(L))$. It also follows that if $L_1$ is an essential extension of $L$ then $K(L) = K(L_1) \cap C(L)$. (Received May 5, 1969.) (Author introduced by Professor George A. Grätzer.)

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**69T-A106.** JIRI SICHLER, University of Manitoba, Winnipeg 19, Manitoba, Canada. **Rich small categories.**

A small category $K$ is called rich if any full category of algebras is isomorphic to a full subcategory of $\text{Jet}^K$, the category of all functors $F : K \rightarrow \text{Jet}$. The couple $(\text{card } (\text{Obj}(K)), \text{card } (\text{Mor}(K)))$ is called the size of $K$. For $a \geq 2$, $(a, \beta)$ is the size of a rich category if and only if $\beta \geq 4$. $(1, \beta)$ is the size of a rich category if and only if $\beta \geq 5$. Given a category $K$ of size $(a, \beta)$, there is a rich category $L$ of size $(a + 2, \beta + 4)$ containing $K$ as full subcategory. Any category $M$ of size $(1, \beta)$ (i.e. a monoid with $\beta$ elements) is contained in a rich category $R$ of size $(1, \beta + 4)$. Any such $M$ is also contained in a nonrich monoid $P$ of cardinality $\beta + 1$. No small discrete category is rich.

Examples of nonrich monoids are the following: any group (Z. Hedrlín and J. Lambek), commutative monoid, monoid with right zero element, monoid with left-zero multiplication. (Received May 5, 1969.) (Author introduced by Professor George A. Grätzer.)

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**69T-A107.** ALFRED HORN, University of California, Los Angeles, California 90024. **Injective and projective semilattices.**

Let $K$ be the category of join semilattices $(A, +)$. **Theorem.** An algebra $A$ is $K$ injective if and only if $A$ is complete and satisfies the distributive law $a + \Pi b_i = \Pi (a + b_i)$. Equivalently, if and only if $A$ is the dual of a complete Heyting algebra. If $A \in K$, let $A_0$ be the result of adding a smallest element to $A$ if $A$ does not have a smallest element. **Theorem.** An algebra $A$ is $K$ projective if and only if $A_0$ is isomorphic with a ring of finite sets. In particular, if $A$ is finite, $A$ is $K$ projective if and only if $A_0$ is a distributive lattice. (Received May 8, 1969.)

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**69T-A108.** FRANK S. BRENNEMAN, Lehigh University, Bethlehem, Pennsylvania 18015. A note on the Eilenberg-MacLane cohomology of groups.

Let $(\mathfrak{G}, \mathfrak{P})$ be the category of ordered pairs $(A, \rho)$ where $A, \mathfrak{P}$ are groups and $\rho : A \rightarrow \mathfrak{P}$ is a group homomorphism. Define $T : (\mathfrak{G}, \mathfrak{P}) \rightarrow \text{Ab}$ by $T(A, \rho) = \text{Der}(A, \mathfrak{M}) = \{ f : A \rightarrow \text{Hom}(M, \mathfrak{P}) | f(xy) = \rho(x)f(y) + f(x) \}$ where $M$ is a $\mathfrak{P}$-module. Let $J : (\mathfrak{G}, \mathfrak{P}) \rightarrow \text{Z}(\mathfrak{P})$ be defined by $J((A, \rho)) = \text{cok}(d)$ where $d : B_2(A) \rightarrow B_1(A)$ is defined on basis elements of $B_2(A); d_2([x_1 x_2]) = \rho(x_1) [x_2] + [x_1] - [x_1 x_2]$. **Proposition.** $\text{Der}((\mathfrak{G}, \mathfrak{P})) \cong \text{Hom}(\text{Z}(\mathfrak{P}), \mathfrak{P}) \ast J$. If $G$ is the free group cotriple on $(\mathfrak{G}, \mathfrak{P})$ and if the cotriple cohomology is denoted by $H_n^G(A, \mathfrak{M})$, we have **Theorem.** $H_n^G(A, \mathfrak{M}) = \text{Der}(A, \mathfrak{M})$ if $n = 0$, $= H^{n+1}(A, \mathfrak{M})$ if $n > 0$.
where $H_k(A,M)$ denotes the Eilenberg-MacLane cohomology of groups. (References. N. Shimada, H. Uehara, and F. Brenneman, "Cotriple cohomology in relative homological algebra," unpublished manuscript. M. Barr and J. Beck, "Acyclic models and triples," Proc. conference on categorical algebra, La Jolla.) (Received May 7, 1969.)


A polyadic group is an algebra with one associative, $m$-ary operation in which certain equations are solvable. (See E. L. Post, "Polyadic groups," Trans. Amer. Math. Soc. 49 (1940), 208-350.) The existence of free polyadic groups is proven by construction. This construction is then used to give an independent proof of the Post Coset Theorem. Finally, it is shown that every subpolyadic group of a free polyadic group is free. This theorem generalizes the corresponding, well-known result from group theory and utilizes that result in its proof. (Received April 2, 1969.) (Author introduced by Professor E. J. Peake.)

69T A110. HYMIE LONDON, 5208 Trans-Island Avenue, Montreal 248, Quebec, Canada. Integer solution of $y^2 + 18 = x^3$.

Theorem. The only integer solution of the Diophantine equation $y^2 + 18 = x^3$ with $y$ positive is $x = 3$, $y = 3$. (Received May 7, 1969.)


The theory of polygenic functions and that of pseudo-conformal geometry, originally due to Kasner, are extended to a complex vector space $V$, and its dual $V^*$, either finite or infinite dimensional. Polygenic and pseudo-conformal linear transformations on $V$ are introduced. Every polygenic linear transformation $T$ can be expressed as a sum $T_1 + T_2$, of a unique direct linear pseudo-conformal transformation $T_1$ and a unique reverse linear pseudo-conformal transformation $T_2$. A linear polygenic functional $\lambda \otimes \mu$ is uniquely decomposable into a sum $(\lambda, \mu)_1 + (\lambda, \mu)_2$, where $(\lambda, \mu)_1$ and $(\lambda, \mu)_2$ are direct pseudo-conformal linear functionals, $\lambda$ is any contravariant vector, and $\mu$ is a covariant vector. The theory of such pseudo-conformal functionals is developed. Isocline planes in the contravariant space $V$ and in the covariant space $V^*$ are defined, the pseudo-angle $\theta$ between vectors in the same isocline plane is defined, and the theory of pseudo-angles $\theta$ between contravariant vectors and covariant vectors is studied. The transformation theory or polygenic linear functionals is developed. The pseudo-conformal group $G^*$ and the direct pseudo-conformal group $G$ are characterized by the pseudo-angle $\theta$. (Received May 5, 1969.)

69T A112. CHANG MO BANG, Vanderbilt University, Nashville, Tennessee 37203. Countably generated modules over complete discrete valuation rings.

Let $R$ be a complete discrete valuation ring and $(c, R, k)$ the class of all countably generated reduced $R$-modules of torsion-free rank $k$. The classification of $(c, R, k)$ up to isomorphism is done.
by Ulm in case $k = 0$ ["Zur Theorie der abzählbar-unendlichen Abelschen Gruppen," Math. Ann. 107 (1933), 774-803], by Kaplansky and Mackey in case $k = 1$ "A generalization of Ulm's theorem," Summa Brasil. Math. 2 (1951), 195-202, and by Rotman and Yen in case $k$ is finite ["Modules over a complete discrete valuation ring," Trans. Amer. Math. Soc. 89 (1961), 242-254]. The classification of $(c,R,k)$ is completed for all values of $k$ by the following theorem. **Theorem.** Let $M, M' \in (c,R,k)$. Then $M$ and $M'$ are isomorphic if and only if they have the same Ulm invariants and the same height equivalences. Here, the height equivalences are the natural generalization of height equivalences as defined in the Rotman and Yen paper referred to above. (Received May 19, 1969.)

**69T-A113. STEPHEN U. CHASE, Cornell University, Ithaca, New York 14850.** **Inseparable Galois theory and a theorem of Jacobson.**

Let $K/k$ be a finite purely inseparable modular field extension. We construct a cocommutative coconnected Hopf $k$-algebra $H(K/k)$ which measures $K$ to itself in the sense of M. E. Sweedler, "The Hopf algebra of an algebra as applied to field theory," J. Algebra 8 (1968), 262-276. $H(K/k)$ is generated by sequences of divided powers, and has degree $[K:k][K:k]$ over $k$. Moreover, $K$ acts, via Hopf algebra endomorphisms, on $gr(H(K/k))$, the associated graded Hopf algebra of $H(K/k)$.

**Theorem.** There exists a one-to-one lattice-inverting correspondence between (1) Fields $F$ such that $k \subset F \subset K$ and $K$ is a tensor product, over $F$, of primitive extensions of equal exponent, and (2) Hopf subalgebras $H$ of $H(K/k)$, generated by sequences of divided powers of equal length, such that $gr(H)$ is stable under the action of $K$ on $gr(H(K/k))$. The theorem generalizes, to arbitrary exponent, Jacobson's Galois theory of purely inseparable field extensions of exponent one. (Received May 14, 1969.)

**69T-A114. MARK E. WATKINS, Syracuse University, Syracuse, New York 13210.** **When non-abelian groups act regularly on graphs.**

A finite group $G$ acts regularly on a finite simple graph $X$ with automorphism group $A(X)$ if $G \cong A(X)$, and for each $u,v \in V(X)$, there exists a unique $\phi \in A(X)$ such that $\phi(u) = v$. We say that $G$ is in **Class R** if $G$ acts regularly on $X$ for some $X$. The abelian groups in **Class R** have been characterized by C-Y Chao (Proc. Amer. Math. Soc. 15 (1964), 291-292) and M. H. McAndrew (Abstract 625-103, these Notices) 12 (1965), 575). For non-abelian groups we announce the following: The dihedral group $D_m$ is in **Class R** if and only if $m \equiv 6$. The groups of order $p^3$ for odd prime $p$ are in **Class R** except for the group of order 27 or 3 generators, which remains unresolved. The direct products of groups ($\not\cong C_2$) in **Class R** are also in **Class R**. The dicyclic groups (including the quaternion groups) are never in **Class R**. (Received May 15, 1969.)

**69T-A115. THOMAS P. WHALEY, University of Florida, Gainesville, Florida 32601.** **On endomorphisms of partial algebras.**

Grätzer (Universal algebra, Van Nostrand, Princeton, N.J., 1968) introduces the notions of endomorphisms, full endomorphisms, and strong endomorphisms for a partial universal algebra $\mathcal{U}$. It is clear the $\text{End}_s(\mathcal{U}) \subseteq \text{End}_f(\mathcal{U}) \subseteq \text{End}(\mathcal{U})$. It is stated that these are subsemigroup inclusions and the problem is posed (Problem 16) to characterize these as a triple of semigroups. We give an example
in which $\text{End}_S(\mathbf{U})$ is not closed under composition. **Theorem.** Let $S$ be a subgroup with 1 and let $T$ be a semigroup containing 1. Then there is a partial algebra $\mathbf{U}$ with an isomorphism $\varphi$ of $S$ onto $\text{End}_S(\mathbf{U})$ if and only if $s_1, s_2 \in S$ and $s_1 s_2 \in T$ imply that $s_2 \in T$. **Theorem.** Let $A$ be a set, $S$ a semigroup of mappings of $A$ into $A$ and $T$ a subsemigroup of $S$ containing the identity map. Then there is a partial algebra $\mathbf{U} = \langle A; F \rangle$ with an isomorphism $\varphi$ of $S$ onto $\text{End}_S(A; F)$ if and only if the following hold:

(i) If $\varphi : A \to A$ and if for each finite sequence $x$ of elements of $A$, $\varphi(x) \in x^*$ (the closure of $x$ under $S$ and $T^{-1}$), then $\varphi \in S$.

(ii) If $\varphi \in S$ and if for each finite sequence $x$ of elements of $A$, $x \in \varphi(x)^*$, then $\varphi \in T$. (Compare B. Jonsson, J. Pionka, Colloq. Math. 19 (1968), 1-8; K. Driessel and M. Stone, Abstract 697-A53, these Notices 16 (1969), 564.) (Received May 16, 1969.)

69T-A116. KEITH DENNIS, Rice University, Houston, Texas 77001. **Subdirectly irreducible group rings.**

A ring is subdirectly irreducible if the intersection of all its nonzero ideals is a nonzero ideal. McCoy, Duke Math. J. 12 (1945), 381-387, separates commutative subdirectly irreducible rings into two types: (1) There exists at least one element which is not a zero divisor. (2) Every element is a zero divisor. **Theorem 1.** Let $R$ be a commutative ring and $G$ an abelian group. Then the group ring $R(G)$ is a subdirectly irreducible ring of type 1 if and only if (1) $G$ is a finite $p$-group for some prime $p$, (2) $R$ is a subdirectly irreducible ring of type 1, (3) $p|0$, where $(j)$ is the minimal ideal of $R$. **Corollary.** Let $R$ be a commutative ring with unit and $G$ an abelian group. Then $R(G)$ is a noetherian (artinian) subdirectly irreducible ring if and only if (1) $G$ is a finite $p$-group for some prime $p$, (2) $R$ is subdirectly irreducible, (3) the characteristic of $R$ is $p^k$, (4) $R$ is noetherian (artinian). **Theorem 2.** If $R$ is a commutative subdirectly irreducible ring of type 2, then $R(G)$ is never subdirectly irreducible for any nontrivial group $G$. (Received May 16, 1969.)

69T-A117. KENNETH W. NEWMAN, Cornell University, Ithaca, New York 14850. **A structure theorem for free coconnected Hopf algebras.**

Let $k$ be a perfect field of char. $p > 0$. Let $P_n = k [Y_0, Y_1, \ldots, Y_n]$ (commutative or noncommutative). Give $P_n$ a Hopf algebra structure by letting $Y_0$ be a primitive and letting $Y_1$ ($1 \leq 1, \ldots, n$) be the $p^k$th term of a sequence of divided powers over $Y_0$. (This construction is possible by Lemma 7 in M. E. Sweedler, Trans. Amer. Math. Soc. 127 (1967), 515-526.) **Theorem.** Let $A$ be a commutative, finite-dimensional, graded, local, augmented algebra over a perfect field of char. $p > 0$. Then the free coconnected, commutative (noncommutative) Hopf algebra on $A^*$ can be written as a product of commutative (noncommutative) $P_n$'s. (Received May 19, 1969.)

69T-A118. DANIEL A. ROBINSON, Georgia Institute of Technology, Atlanta, Georgia 30332. **Holomorphy theory of extra loops.**

A loop $(G, \cdot)$ is said to be an extra loop if and only if $(xy \cdot z)x = x(y \cdot zx)$ for all $x, y, z \in G$. Recently F. Fenyves (Extra loops, I, Publ. Math. Debrecen 15 (1968), 235-238) showed that extra loops are Moufang loops which are isomorphic to each of their loop isotopes. If $A$ is a group of automorphisms of a loop $(G, \cdot)$, the $A$-holomorph of $(G, \cdot)$ is defined in the usual manner. In this paper the following results are established. **Theorem 1.** The $A$-holomorph of a loop $(G, \cdot)$ is an extra loop if
and only if \((G, \cdot)\) is an extra loop and each \(a \in A\) is nuclear in that \(x^{-1} \cdot xa\) is in the nucleus of \((G, \cdot)\) for all \(x \in G\). Theorem 2. If \((G, \cdot)\) is an extra loop, then each of the inner mappings \(R(x,y) = R(x)R(y)R(xy)^{-1}\) is a nuclear automorphism of \((G, \cdot)\). (Received May 19, 1969.)


It is not true that a group which obeys the maximal condition for normal subgroups may always be cancelled in direct products. However, we show the following: Theorem. Let \(C\) be a group which obeys the maximal condition for normal subgroup. Suppose further that if \(C_a\) is an arbitrary homomorphic image of \(C\), then \(C_a\) is not isomorphic to a proper normal subgroup of itself. Then \(C\) may be cancelled in direct products. Some generalizations of this result are indicated. (Received May 21, 1969.)

69T-A120. R.J. WARNE, West Virginia University, Morgantown, West Virginia 26505. A class of regular bisimple semigroups.

Let \(I^0\) denote the nonnegative integers. A band \(X\) will be called naturally \(R\)-ordered if \(X\) is a disjoint union of equipotent right zero semigroups \(\{R_n : n \in I^0\}\) such that if \(a \in R_n\) and \(f \in R_m\), then \(e < f\) if and only if \(n > m\). Such bands are componentwise ordered in the sense of the author [R.J. Warne, "Regular D-classes whose idempotents obey certain conditions," Duke Math. J. 33 (1966), 187-196]. If \(Y\) is a semigroup, \(E_Y\) will denote its set of idempotents. Let \(T\) be a semigroup which is a disjoint union of isomorphic right groups \(\{T_k : k \in I^0\}\) such that \(T_k T_r \subseteq T_{\max(k,r)}\) and such that \(E_T\) is a naturally \(R\)-ordered band. Let \(C\) be the bicyclic semigroup. Let \((n, r) \rightarrow a_{(n,r)}\) be an antihomomorphism of \(C\) into the semigroup of endomorphisms of \(T\) such that for \(n, k \in I^0\), \(a_{(k,k)}\) is an inner right translation of \(T\) determined by some \(e \in E_{T_k}\) and \(\alpha_{(n,k)} \subseteq T_n\). Let \(S = \{(n, k), g_k: g \in T_k, (n, k) \in C\}\) under the multiplication \(\{(n, k), g_k\}(r, s), h_s) = ((n, k)(r, s), g_k a_{(r,s)} h_s)\) where juxtaposition denotes multiplication in \(C\) and \(T\). Then, \(S\) is a bisimple semigroup such that \(E_S\) is a naturally \(R\)-ordered band. Conversely, every such semigroup is obtained by the above construction. We note that the semigroups considered here are not inverse semigroups in general. (Received May 21, 1969.)


The structures of powers of binary relations and their graphs defined on an arbitrary class of elements \(\Sigma\) are examined in relation to general issues of convergence and oscillation. Two sets of definitions are introduced: "asymptotic dyadic periodicity" and "asymptotic relational periodicity" (convergence may be treated as periodicity with period unity); the two approaches are shown to be formally equivalent. The methods of the paper for obtaining the asymptotic forms for powers of relations are essentially algebraic in nature. The characterizations found for binary relations defined on classes \(\Sigma\) of arbitrary cardinality are examined in relation to results known to govern the

69T-Al22. WITHDRAWN.


We shall say that an ECA (equational class of algebras) has kernels if for any algebra $A$ in the class with admissible equivalence relations $K, J, L$, then $B \neq K \equiv J$ and $K \equiv L = JL \equiv L$. An ECA has kernels if for some $n$ there exist derived operators $h, f_1, k_1 (i = 1, \ldots, n)$ such that $f_i (x, x, y) = k_1 (y)$ and $h(x, f_1 (x, y, z), \ldots, f_n (x, y, z)) = y$. The following three conditions are equivalent for an ECA.

(i) Every admissible ternary relation $R(x, y, z)$ satisfies $R(x, y, z) = (\exists x)R(x, y, z) \cap (\exists y)R(x, y, z) \cap (\exists z)R(x, y, z)$. (ii) Every four admissible binary relations $x_1, x_2, x_3, y$ satisfy $(x_1 \cap x_2 \cap x_3) y = (Mx_1 \cap x_2 \cap x_3 y) \cap (x_1 \cap Mx_2 \cap x_3 y) \cap (x_1 \cap x_2 \cap Mx_3 y)$. Here $M$ is the maximal relation on the algebra.

(iii) There exists an operator $f$ such that $f(x, x, y) = f(x, y, x) = f(y, x, x) = x$. (Received June 2, 1969.)


L. Carlitz established (Duke Math. J. 19 (1952), 65-70), by combinatorial arguments, that the arithmetic functions $|\mu(n)|$, $1, n^n, \ldots, n^r$ are algebraically independent over the complex field. Subsequently, J. Popken showed (G. Szego, et al, editors, "Studies in mathematical analysis and related topics," 1962, pp. 285-293) that more purely algebraic arguments can be used to generalize such results for multiplicative functions. The present study extends Carlitz' and Popken's results by proving analogous propositions for generalized multiplicative functions (i.e., arithmetic functions $f$ such that $f(a \cdot b) = f(a) \cdot f(b)$, if the mosaics of $a$ and $b$ have no prime in common). Lemma. Let $\mu^*$ be the modified Möbius function (Amer. Math. Monthly 74 (1967), 1100-1102). Then the finite set of generalized arithmetic functions $|\mu^*(n)|$, $1, n^n, \ldots, n^r$ is algebraically independent over the complex field. (Received June 2, 1969.)

69T-Al25. CLIFTON T. WHYBURN, East Carolina University, Greenville, North Carolina 27834, and 4 Mount Bolus Road, Chapel Hill, North Carolina 27514. The distribution of $r$th powers in a finite field.

Let $p$ be an odd prime, $k$ an integer and suppose $r$ is an integer such that $1 < r - 1$. $GF(p^k)$ is represented as $Z_p[x]/p$ where $P$ is of degree $k$ and irreducible in $Z_p[x]$. By elementary means, the following results are obtained: Theorem 1. Let $\epsilon = 1$ or 2 and $k = \epsilon (mod 2)$. If $\nu^*$ denotes the number of elements of $C_{r, 1}$ (a coset of the group of $r$th powers in $GF(p^k)$), having degree not exceeding $[k/2]$, then $\nu^* < p^{-k} + p^{-\epsilon} - p^{-[\epsilon + k]} [2r]$. Theorem 2. If $\nu^*$ denotes the number of elements of $C_{r, 1}$ having coefficients $< p^{-1/2}$ in absolute value, then $\nu^* < (2p^{-1/2})^k \cdot [2r]$. A refinement of Theorem 2 is achieved in case $2r \cdot p^{-1}$. (Received June 2, 1969.)
On a class of algebras satisfying \( x(xy) + (yx)x = 2(xy)x \). Preliminary report.

In the following the identity \( x(xy) + (yx)x = 2(xy)x \) will be denoted by (I). **Theorem.** A simple strictly power-associative algebra \( A \) over a field of characteristic \( \neq 2, 3 \) and degree greater than two which satisfies (I) can not be a quasi-associative algebra and hence is a commutative Jordan algebra (cf. Kosier, Trans. Amer. Math. Soc. 102 (1962), 299-318). The next theorem is an extension to characteristic \( \neq 2, 3 \) of some of the results of M. Rich (cf. Abstract 69T-A6, these Notices 16 (1969), 310). **Theorem.** There are no nodal algebras \( A \) satisfying (I) over an algebraically closed field \( F \) for which any of the following conditions hold: (1) The associators \( (x, x, y) \) and \( (y, x, x) \) are equal for all \( x \) and \( y \) in \( A \). (2) The nilpotent elements of \( A \) are all of nilindex two. (3) There is a nilpotent element of \( A \) whose nilindex is greater or equal to \( n - 1 \) where \( \dim A = n \). (4) All associators of the form \( (x, y, x) \) are nilpotent. (Received May 20, 1969.) (Author introduced by Professor R. D. Boswell, Jr.)

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logical properties of the ring of differential polynomials.

The ring \( R \) of differential polynomials over a universal differential field (E. R. Kolchin, Amer. J. Math. 75, 753-824) provides an example of a principal right and left ideal domain, not a field, that is a right \( V \)-ring (i.e., each simple right \( R \)-module is injective), and can be shown to have a unique simple right \( R \)-module \( M \). Such a ring was conjectured to exist by Carl Faith. The fact that \( M \) is a semisimple injective cogenerator for \( \text{Mod-} R \) readily implies that every right \( R \)-module has a maximal submodule. Bass proved that if a ring \( A \) satisfies the d.c.c. on principal right ideals, then \( A \) has a bounded number or orthogonal idempotents and every right \( A \)-module has a maximal submodule. The above example shows that the converse is false, thus answering a question raised by Bass (Trans. Amer. Math. Soc. 95 (1960), 470). More generally, the following result is established: **Theorem.** Let \( k \) be a differential field with derivation \( D \), and let \( k[y, D] \) denote the ring of differential polynomials. Then the following are equivalent: (1) \( R = k[y, D] \) is a right \( V \)-ring having, up to isomorphism, a unique simple right \( R \)-module \( M \). (2) Every linear (homogeneous) differential equation in \( D \) over \( k \) has a solution (nontrivial solution) in \( k \). (Received June 4, 1969.)

On the Jacobson radical of a group algebra.

Let \( G \) be a discrete group and \( H \) a normal subgroup of finite index \( n \). Let \( K \) be any field and let \( K G \) denote the Jacobson radical of the group algebra \( KG \). It was shown that \( (JKG)^n \) is contained in \( (KG)^n \) (JKH). (This result was independently obtained by D. S. Passman and is to appear in his forthcoming paper "Radicals of twisted group algebras"). Utilizing this result the following theorem was proved. **Theorem.** Let \( G \) be a solvable group with derived series \( G = G^0 \supset G^1 \supset \ldots \supset G^{(i)} \supset \ldots \supset G^{(k)} = 1 \). Let \( K \) be an algebraically closed field of characteristic \( p \). If \( JKG \neq 0 \) then there exists an element \( g \), of order \( p \), in \( G^{(i)} \) for some \( i \), \( 0 \leq i \leq k - 1 \), such that the distinct conjugates of \( g \) under the action of \( G^{(i+1)} \) are finite in number. This theorem gives a sufficient condition for the group
algebra of solvable group to be semisimple. Further it was shown that if \( H \) is a normal subgroup of \( G \) and \( G/H \) is locally finite, then \( JKH \subset JKG \). With the aid of this result it was established that the necessary condition for \( JKG \neq 0 \), stated in the theorem, becomes also sufficient with extra assumption that \( G^{(i)}/G^{(i+1)} \) are periodic for \( 0 \leq i \leq k - 2 \). (Received May 28, 1969.)

69T-A129. JONATHAN S. GOLAN, The Hebrew University, Jerusalem, Israel. A characteriza-

Let \( R \) be a ring with \( 1 \). A left \( R \)-module \( Q \) is quasi-projective iff, for all epimorphisms
\[
\varphi: Q \to M, \text{Hom}(Q,M) = \text{Hom}(Q,Q) \varphi.
\]
\( Q \) is regular quasi-projective iff \( Q \cong P/T \) where \( P \) is projective and \( T \) is a fully-invariant submodule. An epimorphism \( \varphi: Q \to M \) is called a quasi-projective cover iff \( \ker(\varphi) \) is small in \( Q \) and \( Q/T \) is not quasi-projective for all \( 0 \neq T \subset \ker(\varphi) \). [For elementary facts see Wu and Jans "On quasi-projectives," Illinois J. Math. 11 (1967), 439-448.] The following are equivalent: (1) \( R \) is left perfect. (2) Every flat left \( R \)-module is quasi-projective. (3) Every left \( R \)-module has a regular quasi-projective cover. (4) \( R \) is semiperfect and every quasi-projective left \( R \)-module has a projective cover. Corollary. If \( R \) is commutative then \( R \) is left perfect iff every quasi-projective left \( R \)-module has a projective cover. (Received June 5, 1969.)

69T-A120. GEORGE IVANOV, School of General Studies, Australian National University, Canberra, 2600 Australia. Rings with zero singular ideal.

We answer a question posed by Goldie ("Torsion-free modules and rings," J. Algebra 1 (1964), 268-287). Theorem. Let \( R \) be an indecomposable Artinian ring with zero left singular ideal. Then there are integers \( m,k \) with \( m \not\equiv k \) and integers \( n_1,\ldots,n_k \) such that \( R \) is isomorphic to a blocked matrix ring \( H \) such that the \((i,j)\)th block of an arbitrary element of \( H \) is an arbitrary \( n_i \times n_j \) matrix with entries from an additive abelian group \( H_{ij} \). The groups \( H_{ij} \) have the following properties:

(i) There is a partial multiplicative structure on \( \bigcup_{i,j} H_{ij} \) such that \( H_{ij} H_{jt} \subseteq H_{it} \), for all \( i,j,t \).
(ii) For every \( i \), \( H_{ii} \) is a completely primary ring. (iii) For all \( i,j \), \( H_{ij} \) is an Artinian left \( H_{ii} \)-module and a (unital) right \( H_{jj} \)-module. (iv) For every \( i \) such that \( m \not\equiv i \equiv k \), \( H_{ii} \) is a sfield.
(v) If \( i \not\equiv j \) and \( m \equiv j \equiv k \), then \( H_{ij} = 0 \). (vi) For every pair \( s,t \) there is a sequence \( s = r_1, r_2,\ldots,r_n = t \) such that for each \( j \) there is an \( i \), \( m \equiv i \equiv k \), such that \( H_{ir_j} \neq 0 \) and \( H_{ir_{j+1}} \neq 0 \). (vii) For all \( s,t \), if \( \varphi \in H_{st} \) and \( H_{j\varphi} = 0 \) for every \( j \) such that \( m \not\equiv j \equiv k \), then \( \varphi = 0 \). Conversely, any matrix ring \( H \) satisfying the above conditions is indecomposable, (left) Artinian, and has zero left singular ideal. We prove a more general and slightly stronger result. (Received June 6, 1969.) (Author introduced by Professor Hanna Neumann.)

69T-A131. AHMAD SHAFAAT, Carleton University, Ottawa, Ontario, Canada. On the structure of certain idempotent semigroups.

A normal band is an idempotent semigroup satisfying the identity \( xyzx = xzyx \). In this paper results like the following are proved: Every normal band is a subcartesian product of some bands of order 2 and 3. (Received June 5, 1969.)

69T-A132. JERZY PLONKA, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the number of polynomials of a universal algebra. III.

Theorem. Let \( \mathfrak{G} \) be a universal algebra without constants. If for some integer \( n > 1 \), \( \mathfrak{G} \) has an \( n \)-ary polynomial \( p(x_0,\ldots,x_{n-1}) \) satisfying \( p(x_0,\ldots,x_{n-1}) = p(x_1,\ldots,x_{n-1}, x_0) \), then \( \mathfrak{G} \) has an essen-
tially \((2m + 1)\) -ary polynomial for each \(m \geq 1\). Let \(p_n(\mathbb{U})\) denote the number of essentially \(n\) -ary polynomials. **Corollary 1.** If \(p_1(\mathbb{U}) = 0\) and for some \(n \geq 1\), \(p_{2n+1}(\mathbb{U}) = 0\), then \(m\) divides \(p_m(\mathbb{U})\) for all prime \(m\). **Corollary 2.** The sequence \(\langle 0, p_1, \ldots, p_n, \ldots \rangle\) satisfying \(p_1 > 0\), and \(p_n > 0\) only if \(n\) is a prime, is representable if \(n\) divides \(p_n\) for all \(n\). **Corollary 3 (G. Grätzer).** If \(p_0(\mathbb{U}) = 0\), and for some \(n \geq 1\), \(p_{2n+1}(\mathbb{U}) = 0\), then for every \(m > 1\) either \(p_m(\mathbb{U}) = 0\) or \(p_m(\mathbb{U}) \equiv m\). (Received June 2, 1969.) (Author introduced by Professor George A. Grätzer.)

69T-Al33. JOHN N. MORDESON, Creighton University, Omaha, Nebraska 68131, and BERNARD VINOGRAD, Iowa State University, Ames, Iowa 50010. Relative \(p\)-bases.

We say that a purely inseparable field extension \(L/K\) has property \(R\) if and only if every relative \(p\)-base of \(L;K\) generates \(L/K\). We have investigated the following question: Let \(C\) be a set of intermediate fields such that every element of \(C\) has property \(R\). When does this imply that \(L/K\) is of bounded exponent? Here we report the following result: Let \(G \in C\) if and only if \(L/G\) preserves relative \(p\)-independence. Then \(L/K\) is of bounded exponent. Hence, in particular, if every intermediate field of \(L/K\) has property \(R\), then \(L/K\) has bounded exponent. (Received June 2, 1969.)

69T-Al34. GEORGE GRÄTZER and R. PADMANABHAN, University of Manitoba, Winnipeg 19, Manitoba, Canada. On commutative, idempotent, and nonassociative groupoids.

Let \(\mathbb{U} = \langle A; \cdot \rangle\) be a commutative, idempotent, nonassociative groupoid. For \(n \geq 2\), let \(p_n(\mathbb{U})\) denote the number of essentially \(n\)-ary polynomials of \(\mathbb{U}\). **Theorem 1.** \(p_n(\mathbb{U}) \equiv (2^n - (-1)^n)/3\) for all \(n \equiv 2\). **Theorem 2.** Let \(\langle G; + \rangle\) be an abelian group satisfying \(3x = 0\). Define \(x \cdot y = 2x + 2y\). Then \(p_n(\langle G; \cdot \rangle) = (2^n - (-1)^n)/3\), for all \(n \equiv 2\). And conversely, every groupoid \(\mathbb{U}\) satisfying \(p_n(\mathbb{U}) = (2^n - (-1)^n)/3\) for \(n = 2, 3, 4\) can be so constructed from an abelian group. **Theorem 3.** Let \(\mathbb{U}\) be an idempotent algebra satisfying \(p_2(\mathbb{U}) = 1\), \(p_3(\mathbb{U}) = 3\). Then \(p_n(\mathbb{U}) \equiv (2^n - (-1)^n)/3\) for all \(n \equiv 2\). The proof of Theorem 3 uses Theorem 2 of Abstract 69T-Al32, "On the number of polynomials of an idempotent algebra, I," these Notices, this issue. (Received June 11, 1969.)

69T-Al35. KENNETH A. BYRD, North Carolina State University, Raleigh, North Carolina 27607. Some characterizations of uniserial rings.

Let \(R\) be an associative ring with identity. \(R\) is called uniserial if it is left and right artinian and if every one-sided ideal is principal. The following statements are equivalent: (1) \(R\) is uniserial, (2) every quasi-injective right (left), unital \(R\)-module is quasi-projective, (3) every quasi-projective right (left), unital \(R\)-module is quasi-injective, (4) \(R\) is quasi-Fröbenius and every finitely generated, quasi-projective right (left) unital \(R\)-module is quasi-injective, (5) \(R\) is quasi-Fröbenius and every finitely generated, quasi-injective, right (left), unital \(R\)-module is quasi-projective. (Received June 12, 1969.)

69T-Al36. JAMES A. GERHARD and AHMAD SHAFAAT, University of Manitoba, Winnipeg 19, Manitoba, Canada. Semivarieties of idempotent semigroups.

A semivariety [see Shafat, "Characterizations of some universal classes of algebras," J. London Math. Soc. (in press)] is a class of algebras which satisfy a set of implications of the form
In this paper we show that not all semivarieties of idempotent semigroups are varieties. In fact we completely describe the lattice of subsemivarieties of normal semigroups, that is idempotent semigroups which satisfy the identity $(xyzx = xzyx)$. In addition to the eight subvarieties there are five subsemivarieties which are not varieties. These semivarieties are defined by the implications $xz = yz \rightarrow xy = yx$, $zx = zy \rightarrow xy = yx$, $zx = zy \rightarrow uxy = uyx$, $xz = yz \rightarrow xyu = yxu$. We also show that the atomic quasiprimitive classes of idempotent semigroups are exactly the atomic varieties of idempotent semigroups, and prove several results describing relations between implications in general, and those identities which hold in normal semigroups. These last results are proved in the context of idempotent semigroups, and not just for normal semigroups. 

(Received June 6, 1969.)


Various identities and theorems concerning the outer and inner products of several elements of a quadratic ring $R$ are deduced. If $v_1$, $v_2$, $v_3$ are any three pure elements of a quadratic ring $R$ with a direct involutorial automorphism $T$, then $(v_1 \times v_2, v_3) = 0$. If $z_1 = x_1 + v_1$, $z_2 = x_2 + v_2$, $z_3 = x_3 + v_3$ are three elements of a commutative quadratic ring $R$, then $(z_1 \times z_2, z_3) = -(x_1v_2)v_3 + v_3(x_2v_1)$. If $v_1'$, $v_2'$, $v_3'$ are three pure elements of a quadratic ring $R$ with a direct involutorial automorphism $T$, then $(v_1' \times v_2') \times v_3' = (v_1 \times v_2)\times v_3$. If $T$ is a reverse involutorial automorphism, then $(v_1 \times v_2) \times v_3 = (v_3(v_2v_1') - (v_2v_1)v_3)/2$. For three general elements $z_1$, $z_2$, $z_3$ of a commutative quadratic ring $R$ with a reverse involutorial automorphism $T$ it is found that $(z_1 \times z_2) \times z_3 = (x_2v_1)x_3 - x_3(x_1v_2)$. The scalar quadruple product and the outer quadruple product are studied. If $z_1, z_2, z_3, z_4$ are four elements of a commutative quadratic ring $R$ with an involutorial automorphism $T$, then $(z_1 \times z_2) \times (z_3 \times z_4) = 0$. (Received June 16, 1969.)


By a definition due to K. Morita a quasi-normal matrix $A$ is a matrix with complex elements such that $AA^CT = A^TAC$ where $A^T$ denotes the transpose of $A$ and $A^C$ the complex-conjugate of $A$. Among the results obtained, relating to normal as well as quasi-normal matrices, are the following: If $A$ is normal, $AB$ and $BA$ are normal if and only if $A^CTAB = BAA^CT$ and $B^CTBA = ABB^CT$; if $A = LW = WL$ is the polar form of $A$ (with $W$ unitary and $L$ Hermitian), $AB$ and $BA$ are normal if and only if $B = NW^CT$ where $N$ is normal and $LN = NL$; if $A$ and $B$ are quasi-normal and $AB$ is normal, then $BA$ is normal; if $A$ and $B$ are quasi-normal, $AB$ is normal if and only if $A^CTAB = BAA^CT$ and $AB^CT = B^CTBA$; if $A$ is normal and $B$ quasi-normal, then $AB$ is quasi-normal if and only if $AB^CT = BB^CTA$ and $B^CA^CT = A^TACBC$; and others. (Received June 17, 1969.)
A new unitary perfect number. Preliminary report.

A divisor d of n is a unitary divisor if (d, n/d) = 1. Let \( \sigma^*(n) \) be the sum of the unitary divisors of n. Subbarao and Warren, Canad. Math. Bull. 9 (1966), 147-153, called an integer n unitary perfect if \( \sigma^*(n) = 2n \), and gave four examples: 6, 60, 90, and 87,360. Because \( \sigma^* \) is multiplicative, it is easy to verify that the integer \( 2^{18} \times 3 \times 5^4 \times 7 \times 11 \times 13 \times 19 \times 37 \times 79 \times 109 \times 157 \times 313 \) is unitary perfect. (Received June 18, 1969.)

The lattice of equational classes of distributive pseudo-complemented lattices.

One can construct a subdirectly irreducible distributive pseudo-complemented lattice \( \bar{B} \) from a Boolean lattice B by adjoining a new unit element 1. Let \( B_n \) be the Boolean lattice with length n \( (n \geq 1) \) and \( \mathcal{E}_n \) the equational class of distributive pseudo-complemented lattices generated by \( B_n \).

**Theorem 1.** The following are equivalent for a distributive pseudo-complemented lattice \( L \) (n \( \geq 1 \)): (i) (E) \( (\wedge_{i=1}^n x_i)^{*} \vee \bigwedge_{i=1}^n (x_1 \wedge \ldots \wedge x_i^*) = 1 \) holds in L. (ii) Every proper prime filter of L is contained in at most n distinct maximal filters. (iii) Every proper prime ideal of L contains at most n distinct minimal prime ideals. (iv) L = \( \bigvee_{i=1}^{n+1} Q_i \) for any \( n + 1 \) distinct minimal prime ideals \( Q_i \). (v) \( L \in \mathcal{E}_n \). **Theorem 2.** The following are equivalent for any distributive lattice L in which every closed interval is pseudo-complemented (n \( \geq 1 \)): (i) Every closed interval of L satisfies (E). (ii) L = \( \bigvee_{i=1}^{n+1} Q_i \) for any \( n + 1 \) pairwise incomparable prime ideals \( Q_i \). (iii) \( \bar{B}_{n+1} \) is not a (lattice) homomorphic image of L. **Theorem 3.** The lattice of equational classes of distributive pseudo-complemented lattices is the chain \( \mathcal{E}_0 \subset \mathcal{E}_1 \subset \ldots \subset \mathcal{E}_n \subset \ldots \subset 1 \). (Received June 19, 1969.)

**69T-A141.** WALLACE S. MARTINDALE, University of Massachusetts, Amherst, Massachusetts 01002. Primitive rings with involution whose symmetric elements satisfy a generalized polynomial identity.

Using techniques due to Amitsur [Trans. Amer. Math. Soc. 114 (1965), 210-226], we prove the following result. **Theorem.** Let R be a dense ring of linear transformations of a vector space V over a division ring D, with CR \( \subset R \), where C is the center of D. Furthermore assume that R has an involution and that the set S of symmetric elements of R satisfies a generalized polynomial identity over C (in the sense of the above mentioned paper of Amitsur). Then D is finite dimensional over C and R contains nonzero transformations of finite rank. (Received June 19, 1969.)

**69T-A142.** RALPH GELLAR, University of New Mexico, Albuquerque, New Mexico. Spectrum of X satisfying \( 0 \leq X^P \leq X \).

Let X lie in a Dedekind \( \sigma \)-complete partially ordered real linear algebra with unit. **Theorem.** If for some integer p \( \geq 2 \), \( 0 \leq X^P \leq X \), then the complex spectrum of X is contained in the "spoked wheel" \{ \( \lambda : |\lambda| \leq a \) and \( |\lambda - X^P| \leq \beta \} \cup \{ \lambda : a \leq |\lambda| \leq 1 \) and \( \lambda^{P-1} \) positive real} where \( a > 0 \), \( a = (1/p)(1/p-1) \) and \( \beta = a - a^p \). This estimate is sharp. The theorem holds for positive square matrices,
and it is conjectured that this estimate remains sharp when consideration is limited to the eigenvalues of such matrices. (Received June 19, 1969.)


The author gives an efficient method for finding infinitely many solutions of the inequality $|x + \theta y + \theta^2 z| \max(y^2, z^2) < 1$, where the numbers 1, $\theta$, $\theta^2$ form an integral basis of a totally real cubic field. The method is applicable to a wider class of ternary linear forms whose coefficients form an integral basis of a totally real cubic field, but the details are more complicated. (Received June 23, 1969.)

69T-A144. VLASTIMIL DLAB, Carleton University, Ottawa 1, Ontario, Canada. A characterization of perfect rings.

The (hereditary) torsions in the category $\text{Mod}_R$ of all left unital $R$-modules form an atomistic (i.e. each element contains an atom) complete lattice $A_R$; all the fundamental torsions (i.e. joins of atoms in $A_R$) form a lattice ideal of $A_R$. Theorem. A ring $R$ is right perfect in the sense of H. Bass [Trans. Amer. Math. Soc. 95 (1960), 466-488] if and only if every torsion in $\text{Mod}_R$ is fundamental and closed under taking direct products; in fact, then the number of all torsions in $\text{Mod}_R$ is finite and equal to $2^n$ for a natural $n$. The result cannot be improved in the sense that there exist (i) a ring with 4 fundamental torsions which is not perfect and (ii) a ring with 3 torsions closed under taking direct products which is not perfect. (Received June 23, 1969.)

69T-A145. WITHDRAWN.

69T-A146. MORRIS JACK DeLEON, Florida Atlantic University, Boca Raton, Florida 33432. The diophantine equation $XYZ - WY^2 + X^2 = Y/D$.

Let $f(y)$ be an arbitrary function which depends only on $y$. If $(x,y,z,w)$ is a solution of the diophantine equation $XYZ - WY^2 + X^2 = f(y)$, then $(y - x, y, y - z - 2, y - x - z + w - 1)$, $(x + y, y, z - 2, w + z - 1)$, and $(x, y, z + y, w + x)$ are also solutions. Let $D$ be any nonzero integer. Using elementary methods, the solutions to $XYZ - WY^2 + X^2 = Y/D$ are characterized. (Received June 23, 1969.)

69T-A147. RAPHAEL FINKELSTEIN, Bowling Green State University, Bowling Green, Ohio 43402, and HYMIE LONDON, McGill University, Montreal, Quebec, Canada. On the diophantine equation $y^2 + 100 = x^3$.

The only integer solutions of the diophantine equation $y^2 + 100 = x^3$ are $(x,y) = (5, 5), (10, 30)$ and $(34, 198)$. (Received June 24, 1969.)
Let $P$ be an order in a finite-dimensional associative algebra with 1, $E$, over the quotient field $K$ of an integral domain $R$. If $E$ is central simple, $R$ is a DVR, and $P$ is maximal, then $P$ is known to be a hereditary ring. Besides the maximal orders there are other hereditary orders in $E$ and much is known about these. In this paper examples are given of orders of global dimensions $n - 1$ in $n \times n$ matrices over the quotient field of a DVR, for any $n$. Also, the following theorems are proved:

**Theorem.** If $R$ is a Dedekind ring and $E$ is a quaternion algebra, then $\lfloor \text{GDP} \rfloor = 1$ for any order $P$.

**Theorem.** If $R$ is a Dedekind ring, $E$ is separable, and $P$ is an order of finite global dimension, then $\text{GD } P = \text{id}_P P$. Here $\text{id}_P P$ means the injective dimension of $P$ as a left (or right) module over itself and $\lfloor \text{GDP} \rfloor$ is the supremum of the projective dimensions of finitely generated left $P$ modules of finite projective dimension. (Received May 23, 1969.)

**Analysis**

69T-B133. V. C. NAIR, Regional College of Education, Ajmer, India, and Ohio State University, Columbus, Ohio 43210. On the $H$ function. III.

Recently, Bajpai ["Some integrals involving Gauss's hypergeometric function and Meijer's $G$ function," Proc. Cambridge Philos. Soc. 63 (1967), 1049] evaluated certain integrals in terms of infinite series of $G$ functions. This paper generalizes Bajpai's results replacing the $G$ function by the more general $H$ function. Incidentally, a mistake [Formula (2.6)] in Bajpai's paper is corrected. The new results are used in summing certain finite and in infinite series of $H$ functions and from them some recurrence relations are deduced for the $H$ function. (Received February 3, 1969.)

69T-B134. RICHARD A. ALÔ, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, and ANDRE de KORVIN, Indiana State University, Terre Haute, Indiana 47809. Algebras of functions generated by some Abelian semigroups.

Abelian semigroups which have "many" multiplicative homomorphisms are investigated and results similar to the Stone Representation Theorem and its generalization due to Kaplansky are obtained. It is shown that the selfadjoint algebra generated by the monomorphic image of an abelian semigroup $\mathcal{A}$ with identity is uniformly dense in the space of complex-valued continuous functions defined on the set $M$ of multiplicative maps of the semigroup into the complex plane with norm less than or equal to 1. Applying this theorem it is possible to show that the unit interval taken as an abelian semigroup under supremum multiplication is isomorphic to a semigroup of continuous functions on a compact space that generates as an algebra all the continuous functions on that space. Using a result due to Kaplansky, this can be generalized to the following. If each $h$ in $M$ has its range contained in a compact subset $K_h$ of a fixed $C^*$ algebra $B$ with unit, then the smallest self-adjoint algebra generated by the monomorphic image of $\mathcal{A}$ and the identity function is uniformly dense in the space of all continuous $B$-valued functions on $M$. (Received February 19, 1969.)
Zero-one set functions.

U, F, R, R' A' RB' R ~ and the notion of integral are as in previous abstracts of the author, such as Abstract 656-14, these Notices 15 (1968), 506. Theorem 1. Suppose T is a function from R^+ into R^+, K is a number, T(0) = 0, and K \int f - g \int |T(f) - T(g)| is in R^+ for all f and g in R^+. Then, if h is in R, h - h^2 = 0, g is in R^+ and \int_0^h g exists, then \int hT(G) = T(\int g). Corollary 1. If each of f and g is in R^+, then c(\max{f, g}) = \max{c(f), c(g)} and c(\min{f, g}) = \min{c(f), c(g)}: Theorem 2. If each of f and g is in R^+, then the following two statements are equivalent: (1) If h is in R, h - h^2 = 0 and \int_0^h f = 0, then \int_0^h g exists; (2) if t is in RB and \int_0^t f exists, then \int_0^t g exists. (Received March 24, 1969.)

Some results concerning semi-Fredholm operators on subprojective Banach spaces.

Let B(X) denote the Banach algebra of all continuous transformations from a Banach space X into itself. Let K(X) denote the ideal of compact operators of B(X). Let N^+, N^1 and N denote the sets of right, left and two-sided null divisors in the Banach algebra B(X)/K(X), respectively. Let \Phi^+(X) denote the set of all T \in B(X) such that the dimension of the null space of T is finite and the range of T is closed. Let \Phi^-(X) denote the set of all T \in B(X) such that the codimension of the range of T is finite. \Phi^+(X) and \Phi^-(X) are known as the semi-Fredholm operators of B(X). Let X denote the conjugate space of X and for any set M \subset B(X), let [M]^C denote the complement of M in B(X). \Pi denotes the natural homomorphism of B(X) onto B(X)/K(X). Subprojective spaces were introduced in [R. J. Whitley, "Strictly singular operators and their conjugates," Trans. Amer. Math. Soc. 113 (1964), 252-261]. Theorem 1. If X is a subprojective Banach space, then \Pi^(-1)(N^+) = [\Phi^-(X)]^C. Theorem 2. If X is a reflexive Banach space and X^* is subprojective, then \Pi^(-1)(N^+) = [\Phi^-(X)]^C. Corollary. If X = l_p^p, 1 < p < \infty, or any Hilbert space, then (a) \Pi^(-1)(N^+) = [\Phi^-(X)]^C, (b) \Pi^(-1)(N^T) = [\Phi^-(X)]^C, and (c) \Pi^(-1)(N) = [\Phi^+(X) \cup \Phi^-(X)]^C. (Received March 26, 1969.)

Reflection principle for solutions of elliptic equation with analytic coefficients.

Reflection principles are given for solutions of two different types of elliptic equations with analytic coefficients in the complex plane. The first type is \Re a w = b w + \Re \overline{w}. Let w be a solution defined in a simply connected domain. If the boundary value of Re w(\nu) is analytic on a portion of the x-axis and satisfies some further conditions of analyticity, then w can be continued analytically across the x-axis, onto the entire mirror image. The second type is \sum_{k=1}^n L_k u = 0 where L_k = \sum_{p+q} A_k p q(x, y) \Re \overline{u} = 0. It is shown that a solution u, defined in a simply connected domain, having analytic Dirichlet data on a portion of the x-axis and satisfying some further conditions, can again be continued across the x-axis, onto the entire mirror image. (Received March 28, 1969.) (Author introduced by Professor Avron Douglis.)
Let $H$ be a semigroup and let $\hat{H}$ denote all the continuous, complex-valued, nonzero, semicharacters with absolute value between zero and one if $H$ is topological, and let $\hat{H}$ denote all nonzero semicharacters if $H$ is nontopological. Let $\hat{H}^*$ in either case denote those elements of $\hat{H}$ which take on values zero or one in absolute value. Definition. We say that $H$ has the $*$-property if $\hat{H}^* = \hat{H}$. Two such examples are (1) any idempotent semigroup, and (2) any compact, abelian, semigroup with identity such that $\hat{H}^*$ separates points in $H$. If $H$ is any semigroup with identity satisfying the $*$-property and $|\hat{H}|$ denotes all $|f|$ such that $f \in H$, then we obtain Lemma 1. The simple functions on $\mathcal{A}$ are the same as the linear span of $|\hat{H}|$. Lemma 2. $|\hat{H}|$ is a linearly independent set. We use these to obtain Theorem. For $H$ is above, if $\mathcal{A}$ is generated by the kernels of elements of $\hat{H}$, then there is an isomorphism between $M(\mathcal{A})$ under convolution product and the algebra of all functions on $|\hat{H}|$ under pointwise multiplication. Corollary. If $H$ is discrete, abelian, and also a union of groups, then there exists an isomorphism between $M(\mathcal{A})$ under convolution product and the algebra of all functions on $|\hat{H}|$ under pointwise multiplication, where $\mathcal{A}$ is the algebra generated by the annihilators of $\hat{H}$ of points in $H$. (Received March 31, 1969.)

Orthogonality in non-Archimedean spaces. III and IV.

Notation is as in Abstract 69T-B119, these Notices 16 (1969), 675. A unitary operator need not be diagonalizable, even if $F$ is algebraically closed. Eigenvectors corresponding to distinct eigenvalues of a unitary operator need not be o. The residue space of $X$ is $\sum_{F \in \mathbb{F} \cdot [0 \cdot 1] \cdot \|F\| = r} / \{\|F\| < r\}$. $X$ is maximally complete if $X$ is not a proper subspace of a normed space with the same residue space. Theorem. $X$ is maximally complete iff $X$ is spherically complete. An example is given of an orthogonalizable space in which not every maximal o. set is an o base. Let $W$ be a 0-dimensional compact Hausdorff space. Theorem. If $W$ is metrizable, $F(W)$, the space of continuous functions (with "sup" norm) from $W$ to $F$, is orthogonalizable and of denumerable type. Corollaries. (1) $F(W)$ is separable iff $F$ is separable and $W$ is metrizable; (2) $F(W)$ has countable residue space iff $F$ has countable residue field and $W$ is metrizable. (Received April 11, 1969.)

Idempotent measures and a compactness condition. II.

A locally compact semigroup $S$ satisfies condition (L) if $AB^{-1} = \{x \in S; \text{there is } b \in B \text{ such that } xb \in A\}$ is compact for every pair of compact $A, B \subseteq S$. Theorem 1. Joint weak* continuity of the convolution $m \cdot n$ $(m, n, \text{regular probability measures on } S)$ implies (L) and its "dual" condition (R) (defined analogously using $A^{-1}B$). $M \cdot N$ is continuous (weak* topology) in $m$ if (L) holds. On the other hand, if $m \cdot n$ is continuous in $m$, then $Ax^{-1}$ is compact for every compact $A \subseteq S$ and every $x \in S$.

For definition of $m \cdot n$ see Williamson, J. London Math. Soc. 42 (1967), 19. Theorem 2. If $S$ is abelian, then for $m$ to be idempotent on $S$ is equivalent to being the Haar measure on a compact subgroup of $S$. (Glicksberg has proved this when $S$ is compact, Pacific J. Math. 11 (1961)). Theorem 3.
Let \( m^{n} \) be continuous in \( m \) (it suffices \( S \) to satisfy \( L \)); then \( N^{-1} \sum_{j=1}^{N} m^{j} \) either converges to 0 in the weak* topology or weakly to an idempotent measure \( \mathbb{P} \) such that \( m \mathbb{P} = \mathbb{P} m = \mathbb{P} = \mathbb{P}^2 \). (Theorems 2 and 3 still hold even when \( S \) is locally compact semitopological semigroup, i.e., multiplication being separately continuous.) [For Part I, see Abstract 68T-B73, these Notices 15 (1968), 1041.]

(Received April 28, 1969.)

69T-B141. GODFREY LEONARD ISAACS, Herbert H. Lehman College of the City University of New York, Bronx, New York 10468. A limitation theorem for absolute summability.

Suppose that the Laplace-Stieltjes integral \( \int_{0}^{\infty} e^{-uszA(u)} dA(u) \) is summable \( |C,k| \) \((k \neq 0, \Re(s) < 0)\). Then it is known that the inverted fractional integral \( \int_{w}^{\infty} (u-w)^{k-1} \frac{dA(u)}{u} \) (which is certainly summable \( |C,k| \) and thus \( (C,k) \)) satisfies \( e^{-ws} - kR_k(w) = o(1) \) \(|C,0|\) as \( w \to \infty \), if \( k \) is not an integer \( (L. S. Bosenquet, Proc. London Math. Soc. (3) 3 (1953), 267-304; G. L. Isaacs, J. London Math. Soc. 33 (1958), 406-418). The present result states that if \( k \) is not an integer then \( e^{-ws} - kR_k(w) = o(1) \) \(|C,0|\) for each positive \( \delta \), and that the result is best possible in the sense that \( \delta \) may not, in general, be replaced by 0 (in fact, the left side of the conclusion may then be unbounded, even with \( s \) replaced by \( X \), where \( X \) is as large as we please). (Received April 28, 1969.)


Let \( S \) denote the unit circle, \( I \) the unit interval. Let \( f \) be a continuous real-valued function on \( S \times I \) with \( a = \min f < \max f = b \). Assume that for all but an at most countable set of \( z \in [a,b] \), all but an at most countable set of \( (x_0,y_0) \in f^{-1}(z) \) possess neighborhoods \( U(x_0,y_0) \) such that

1. \( U(x_0,y_0) \cap f^{-1}(z) \) defines its first coordinates as a continuous function of its second coordinates and vice versa;
2. the projections of \( U(x_0,y_0) \cap f^{-1}(z) \) into both \( S \) and \( I \) contain nondegenerate intervals.

Then there is a set \( E \subset S \times I \) with projections into \( S \) and \( I \) of measure zero, such that \( f(E) \) is nonmeasurable. One therefore has, e.g., Corollary. There are sets \( H \) and \( Z \) of measure zero contained in the real line, whose vector sum is nonmeasurable. (Received May 5, 1969.)


Let \( A_n \), \( A \) be maximal dissipative (nonlinear) operators on a Banach space \( X \) having a uniformly convex dual space. Let \( D \subset X \) be such that \( \lim_{n \to \infty} \|A_n x - Ax\| = 0 \) for all \( x \in D \) and \( (\lambda I - A)^{-1}(D) \) is dense in \( X \) for some sequence of real numbers \( \lambda \) which converges to \(+\infty\). Then \( \lim_{n \to \infty} \|\exp(tA_n)x - \exp(tA)x\| = 0 \) for all \( x \in X \), the limit being uniform for \( t \) in compact subintervals of \([0,\infty)\). This theorem is applied to prove various cases of the Trotter product formula for nonexpansive semigroups \( \lim_{n \to \infty} \|\exp((t/n)A)\exp((t/n)B)x\| = \exp(t(A + B))x \). (Received May 9, 1969.)
Inverse relations for hypergeometric polynomials.

The author has defined the generalized hypergeometric polynomials \( P_n(x) \) and exhibited the inverse relation \( x^n/n! = \frac{((b)_n/((a)_n)}{((a+\beta)_n/((a+\beta+n))}(a+\beta+n)\psi_k(x) \) which contains a variety of special cases involving the classical polynomials. The prototype is a new inverse relation for the generalized Laguerre polynomials \( P_n(x) \) occurring when \( A = B = 0 \). The derivation is based on a pair of inverse relations, \( t_n = \sum_{k=0}^{n} (a+\beta k)_n/s_k \) and \( s_n = \sum_{k=0}^{n} (a+\beta k)_n/t_k \), which is equivalent to a pair originally obtained by Gould (Duke Math. J. 28 (1961), 193-202) in connection with his Vandermonde convolution transform. (Received May 15, 1969.)

The conjugate of a product of closed operators.

Let \( X, Y, Z \) be Banach spaces, let \( A \) be a densely-defined closed linear operator from \( X \) to \( Y \) and let \( B \) be a densely-defined closed linear operator from \( Y \) to \( Z \). Let \( E' \) denote the conjugate of an operator \( E \). Theorem. If the range of \( A \) is closed in \( Y \) and has finite codimension, the \( (BA)' = A'B' \). (Received May 12, 1969.)

Spiral functions with finite sets of asymptotic values.

The term spiral function is used here to denote a function holomorphic in the unit disk and having \( \infty \) as a spiral asymptotic value. Barth and Schneider (On a question of Seidel concerning holomorphic functions bounded on a spiral, to appear) have constructed a spiral function which has no finite asymptotic values but is bounded on a spiral. A technique of exponentiation is described which uses the function mentioned above to construct a spiral function with precisely one finite spiral asymptotic value. This technique then provides an iterative process where a spiral function with precisely \( n \) finite asymptotic values can be constructed where \( n \) is any nonnegative integer. (Received May 12, 1969.) (Author introduced by Professor Robert L. Hall.)

On the absolute Ces\'aro summability of Fourier series. I.

In this note the following theorem has been proved. Theorem. Let \( f(x) \in L (-\pi, \pi) \) and be periodic with period \( 2\pi \). If at every point \( y \) on the closed interval \( [-\pi, \pi] \) there are a function \( g_y(x) \) and a \( \delta > 0 \) such that (i) \( g_y(x) = f(x) \) for \( |x - y| < \delta \) and (ii) the Fourier series of \( g_y(x) \) is summable \( |C, 1|_k, k \equiv 1 \), then the Fourier series of \( f(x) \) is summable \( |C, 1|_k \). This generalizes a result of Randel (Duke Math. J. 7(1940), 204-207) who has obtained an extension of a well-known theorem of N. Wiener on the absolute convergence of Fourier series. (Received September 24, 1968.)
Approximation of the conformal map onto a near-circle.

Let \( w = F(z) \) be the function which conformally maps the open unit disc onto the interior of the near-circle \( L \), which is defined by the polar equation \( r = 1 + \epsilon p(\theta) \) such that \( F(0) = 0 \) and real directions at the origin correspond. \( F(z) \) is approximated by the function \( F^*(z) = z + \left( \frac{\epsilon z}{2\pi} \right) \oint_0^{2\pi} \frac{(e^{i\theta} + z)/(e^{i\theta} - z) p(\theta) d\theta}{1} \) which is obtained from \( F(z) \) by truncating all terms of \( F(z) \) which have degree greater than 1 with respect to \( \epsilon \). It is assumed that \( p(\theta) \) is the real part of the image of the unit circle of a function \( Q(y) \) which is analytic in the closed unit disc and \( \epsilon < 1/4M \) where \( |Q(z)| \leq M \) for \( |z| \leq 1 \). Under these hypotheses the following theorems are proved: Theorem 1. \( F^*(z) = z + \epsilon zQ(z) \), and \( F^*(z) \) is conformal and starlike for \( |z| \leq 1 \). Theorem 2. Let the image of the unit circle under \( F^* \) be denoted by \( L^* \). Then the maximum distance between \( L \) and \( L^* \) measured along rays drawn from the origin is less than \( 7.5M^2 \epsilon^2 \). These results are then extended to mappings from arbitrary bounded domains into near-circles. (Received May 19, 1969.)

69T-B149. CHARLES M. SCHNEEBERGER, University of Michigan, Ann Arbor, Michigan 48104. Commutators on a separable \( L_p \)-space. Preliminary report.

Let \( \mathcal{L}(\mathcal{B}) \) denote the algebra of all bounded operators on a Banach space \( \mathcal{B} \). A commutator in \( \mathcal{L}(\mathcal{B}) \) is an operator \( C \) for which there exist operators \( A \) and \( B \) in \( \mathcal{L}(\mathcal{B}) \) satisfying \( C = AB - BA \).

A point in the spectrum of a bounded operator is called a limit point if either the point is an eigenvalue whose eigenspace is infinite dimensional or it is a cluster point of the spectrum. Suppose \( 1 < p < \infty \), and let \( (X, \mathcal{J}, \mu) \) be a measure space such that \( L_p(X, \mathcal{J}, \mu) \) is a separable infinite dimensional \( L_p \)-space. Theorem. Each compact operator on \( L_p(X) \) is a commutator. Theorem. Let \( C \) be a bounded multiplication operator on either \( L_p \) or \( L[0, 1] \); then \( C \) is a commutator if and only if it has two or more limit points in its spectrum or it has zero as its only limit point. A similar but more restrictive condition can be given for a multiplication operator on \( L_p(X) \) to be a commutator; however, some multiplication operators have not been classified as either commutators or noncommutators. (Received May 21, 1969.)

69T-B150. JOHN L. LEWIS, University of Illinois, Urbana, Illinois 61801. Some theorems on the \( \cos \pi \lambda \) inequality. Preliminary report.

Let \( u \) be subharmonic in \( \Delta(0; R) = \{z; |z| < R \} \), \( 0 < R < +\infty \). Let \( m(r) = \inf_{|z|=r} u(z) \), \( M(r) = \max_{|z|=r} u(z) \), \( 0 \leq r < R \). Let \( E \) be the union of a finite number of closed intervals contained in \( [-R, 0] \), and let \( E^+= \{r; -r \in E \} \). Suppose there exists \( M(R) \), \( 0 < M(R) < +\infty \), and \( \lambda \in (0, 1) \) such that (1) \( u(z) \equiv M(R) \), \( z \in \Delta(0; R) \); (2) \( m(r) \leq (\cos \pi \lambda)M(r), r \in E^+ \). Theorem 1. There exists a bounded subharmonic function \( h \) in \( \Delta(0; R) \) which is harmonic in \( \Delta(0; R) - E \), and which satisfies (3) \( h(-r) = (\cos \pi \lambda)h(r), r \in E^+ \), (4) \( h(Re^{i\theta}) = 1, |\theta| < \pi \), (5) \( M(r) \equiv M(R)h(r) \equiv M(R)C(\lambda) \exp\left\{} \frac{\lambda}{2\pi n(r,R)} \right\}^r \), where \( C(\lambda) \) is an absolute constant depending only on \( \lambda \). Theorem 2. Let \( V \) be a nonconstant subharmonic function in \( \mathcal{C} \) of order \( \rho \) and lower order \( \kappa \). Let \( F = \{r; m(r) \equiv (\cos \pi \lambda)M(r)\} \). (6) If \( 0 \leq \rho \leq \lambda < 1 \), then \( \log \text{dens } F \equiv 1 - \rho / \lambda \). (7) If \( 0 \leq k < \lambda < 1 \), then \( \log \text{dens } F \equiv 1 - k / \lambda \). (8) If \( \lim_\lambda \frac{M(r)}{\lambda} = A \), \( 0 < A \equiv +\infty \). Theorem 3. Given \( \epsilon > 0 \)
and \( \rho, \lambda \) such that \( 0 = \rho < \lambda < 1 \), there exists an entire function \( f \) of order \( \rho \), and of very regular growth such that \( V = \log |f| \) satisfies \( \log \text{ dens } F \leq 1 - \rho A + \epsilon \). (Received May 12, 1969.)

69T-B151. ERWIN O. KREYSZIG, MANFRED KRACHT, Mathematisches Institut, University of Düsseldorf, Düsseldorf, Germany. On a class of Bergman operators.

\( L \) defined by \( Lu = u_{zz} + b(z, z^*)u_{z} + c(z, z^*)u = 0 \) is said to be of class \( P \) if the solutions of \( Lu = 0 \) can be generated by a Bergman operator \( B \) defined by \( u(z, z^*) = (Bf)(z) = \int g(z, z^*, t) f(z(1-t)/2)(1-t)^{-1/2} \, dt \) where \( g \) is a polynomial in \( t \). Necessary and sufficient conditions for \( L \in P \) are known (E. Kreyszig, Math. Nachr. 37 (1968), 197-202). It is shown that if \( L \in P \), then \( B \) can be represented in a form free of integrals, and that the class \( P \) includes certain equations which are related to the Laplace and wave equations and have been investigated in detail by K. W. Bauer and E. Peschl [J. Reine Angew. Math. 221 (1966), 48-84, 176-196; Arch. Math. 18 (1967), 285-289]. (Received May 23, 1969.)

69T-B152. MORRIS MARDEN, University of Wisconsin, Milwaukke, Wisconsin 53201. Logarithmic derivative of an entire function.

Let \( f \) denote an entire function of finite order \( \rho \). In this note it is shown that \( f'(z)/f(z) \) is \( O(|z|^{\rho-1}) \) away from the zeros of \( f \) and for \( |z| \) sufficiently large. This result is then used to derive in a new and simpler way the representation given in a previous paper [M. Marden, Proc. Amer. Math. Soc. 19 (1968), 1045-1051] for \( f'(z)/f(z) \). In addition, the previous paper's extension of Lucas' theorem from polynomials to entire functions is in this note shown by example to be a best possible result. (Received June 4, 1969.)

69T-B153. WITHDRAWN


Let \( A \) be any selfadjoint strongly elliptic partial differential operator with real, reasonably smooth coefficients on a bounded open set \( \Omega \) in \( \mathbb{R}^n \). Let \( h \) be a real-valued square-integrable function defined on \( \Omega \). Let \( g \) be any continuous real-valued function defined for all real numbers and such that \( g(\infty) = \lim_{x \to -\infty} g(x) \) and \( g(-\infty) = \lim_{x \to -\infty} g(x) \neq g(\infty) \) for all real \( x \). Let \( w_1, \ldots, w_k \) be real-valued functions spanning the generalized null space of \( A \). Then a sufficient condition that \( Au + g(u) = h \) have a weak solution \( u \) with generalized Dirichlet data zero is that

\[
\int_{\Omega} \left( b_{1,w_1}^2 + \cdots + b_{k,w_k}^2 \right) dx \geq \int_{\Omega} \left( b_{1,w_1}^2 + \cdots + b_{k,w_k}^2 \right) dx > \int_{\Omega} h(b_{1,w_1} + \cdots + b_{k,w_k}) dx \text{ for all real } b_{1}, \ldots, b_{k} \text{ with } b_{1}^2 + \cdots + b_{k}^2 = 1, \text{ where } \Omega = \{ x \in \Omega; b_{1,w_1}(x) + \cdots + b_{k,w_k}(x) > 0 \} \text{ and } \Omega \supset \{ x \in \Omega; b_{1,w_1}(x) + \cdots + b_{k,w_k}(x) < 0 \}.
\]

This sufficient condition differs from a necessary condition only by the replacement of \( > \) in the above by \( \geq \), and is sharp in that sense. (Received June 6, 1969.)

69T-B155. WITHDRAWN.
The index-zero theorem for Brelot sheaves. Preliminary report.

Let \( \mathbf{V} \) be a complete presheaf of vector spaces of continuous real-valued functions on a locally compact Hausdorff space \( W \), such that \( (W, \mathbf{V}) \) satisfies the Brelot axioms for "harmonic" functions and \( W \) has a countable basis. Let \( 0 \rightarrow \mathbf{V} \rightarrow \mathcal{P} \rightarrow 2 \rightarrow 0 \) be the fine resolution of \( \mathbf{V} \) constructed in the author's paper "Flux in axiomatic potential theory. I: Cohomology" [Invent. Math., to appear].

**Theorem.** If \( W \) is compact, then \( \Gamma(W, \mathbf{P}) \) has finite codimension in \( \Gamma(W, 2) \), and moreover \( \text{dim} \ H^0(W, \mathbf{V}) = \text{dim} \ H^1(W, \mathbf{P}) \). In the classical setting, this is the theorem that the index of a second-order elliptic equation (i.e., operator in a trivial bundle) is zero. The proof is valid under somewhat weaker assumptions on \( \mathbf{V} \) than those of Brelot. (Received June 11, 1969.)

**69T-B157.** U. KRENGEL, Ohio State University, Columbus, Ohio 43210. On the existence of finite strong generators for nonsingular transformations.

Let \((\Omega, \mathcal{F}, \mu)\) be a probability space for which \( \mathcal{F} \) is countably generated mod \( \mu \). **Theorem.** If \( T \) is a nonsingular invertible transformation in \((\Omega, \mathcal{F}, \mu)\) without \( T \)-invariant probability measure \( \mu_0 \ll \mu \), then \( T \) possesses a strong generator of size 2, i.e., there exists a set \( E \in \mathcal{F} \) such that the sets \( E, E^c, T^{-1}E, T^{-1}E^c, T^{-2}E, \ldots \) generate \( \mathcal{F} \) mod \( \mu \). (Parry, Helmberg, and Simons proved the existence of a countable strong generator.) (Received June 11, 1969.) (Author introduced by Professor Wolfgang Krieger.)

Suppose that \( G \) is a complete normed Abelian group, \( K \) is a near ring of transformations from \( G \) to \( G \) (see Abstract 642-136, these Notices 14 (1967), 103), and \( S \) is the set of real numbers. Let \( OAI \) be the subset of \( OA \) to which \( V \) belongs only in case if \( y \) is in \( S \) then each of \( 1 + V(y, y^+) \), \( 1 + V(y^-, y) \), \( 1 + V(y^+, y) \), and \( 1 + V(y, y^-) \) has an inverse in \( K \). **Theorem.** There is a reversible function \( J \) from \( OAI \) onto \( OAI \) which has the following properties: If \( U \) is in \( OAI \), then (i) \( J \{ U \} = U \), (ii) \( \sum_{a \in A} n^{[b]}(1 + J[U]) b \cdot n^{[b]}(1 + U)^{-1} P = P \) for each \( P \) in \( G \), and (iii) \( J[U](a,b) = \sum_{a \in A} U(a + U)^{-1} \). **Corollary.** If \( U \) is in \( OAI \), then \( W(x,y)P = (R) \chi^R U(W(x,y)P) + P \) for each \( \{x,y,P\} \), \( P \neq Q \), and \( \{x,y\} \) is in \( S \times S \), then \( W(x,y)P \neq W(x,y)Q \). (Compare Abstract 650-9, these Notices 14 (1967), 918.) (Received June 16, 1969.)

**69T-B158.** JAMES V. HEROD, Georgia Institute of Technology, Atlanta, Georgia 30332. Coalescence of solutions for nonlinear Volterra equations.

**69T-B159.** C. ROBERT WARNER, University of Maryland, College Park, Maryland 20742. A generalization of the Šilov-Wiener Tauberian theorem.

Let \( G \) be a locally compact abelian group with character group \( \Gamma \). **Theorem 1.** If \( E \) is a closed subset of \( \Gamma \) which contains no nonempty perfect subset (i.e., \( E \) is scattered), then \( E \) is a Calderón set (= Ditkin set). [Since, in general, there are Calderón sets which are not scattered, this shows that the following result is a generalization of the Šilov-Wiener Tauberian theorem.] **Theorem 2.** Suppose \( f \) belongs to \( L^1(G) \), \( I \) is a closed ideal in \( L^1(G) \), and \( Z(I) \subset Z(f) \). If the intersection of the boundaries of \( Z(I) \) and \( Z(f) \) is a Calderón set, then \( f \) belongs to \( I \). **Theorem 3.** Let \( E \) be a closed subset of \( \Gamma \), and
suppose that for each \( f \) in \( L^1(G) \) such that \( E \subset Z(f) \), the intersection of the boundaries of \( Z(f) \) and \( E \) is a Calderón set. Then \( E \) is a Calderón set. [This theorem shows that the only sets known to be spectral sets by application of the Šilov-Wiener Tauberian theorem--or the generalization of it--are actually Calderón sets.] (Submitted to The Journal of Functional Analysis, April, 1968.) (Received June 17, 1969.)


Let \( K \) be a bounded closed convex subset of a Banach space \( B \), \( f: K \rightarrow K \) be nonexpansive and for \( x \in K \), \( x \) denotes the smallest closed convex subset of \( K \) which contains \( x \) and is invariant under \( f \). Then \( f \) has a fixed point under any one of the following conditions: (1) \( K \) is weakly compact and for any sequence \( \{x_n\} \) in \( K \), there exists \( r \) such that for any \( x = \sum \frac{n+p}{i} t_i x_i \) (\( t_i \geq 0 \), \( \sum t_i = 1 \)), \( \|f(x) - x\| \leq r \sup_{i \geq n} \|f(x_i) - x_i\| \). (2) \( K \) is weakly compact, \( f \) is strictly contractive and for each \( x \in K \), either \( \delta(x) = 0 \) or \( \sup_{n \geq 1} \|f^n(z) - f^n(z)\| < \delta(x) \) for some \( y, z \in x \) and integer \( k \). \( f \) need not be strictly contractive if \( k = 1 \) or if \( f \) is asymptotically regular. It extends two results of L. P. Belluce and W. A. Kirk (Proc. Amer. Math. Soc. 20 (1969), 141-146). (3) \( f \) is uniformly continuous on \((K,d)\) into \( K \), where \( d \) is the metric for \( K \) determined by \( d(x,y) = \sup \{\|f(x) - x\|, \|f(y) - y\|\} \). \( x \neq y \). This is the case if there exists \( 0 \neq r < 1 \) such that \( \|f(x) - f(y)\| \leq r\|x - y\| \) for all \( x, y \in K \). (Received June 18, 1969.)

69T-B161. HARI M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506. On a generalized integral transform. II. Preliminary report.

In an earlier paper [H. M. Srivastava, "On a generalized integral transform," Math. Z. 108 (1969), 197-201] the author has introduced a double integral transform which not only provides a generalization of a class of integral transforms whose kernels are expressible as a Meijer's G-function, or the product of two G-functions, but also offers the possibility of their appropriate extensions in two variables. Relationships with the various known transforms as well as the inversion theorem have already been given by the author [loc. cit. pp. 198-200]. The present note incorporates a number of interesting properties of the generalized integral transform. Since most of the functions of frequent occurrence in problems of analysis, both pure and applied, are special cases of the kernel used, a detailed and systematic study of this transform is expected to yield deeper, general and useful results. (Received June 23, 1969.)

69T-B162. K. V. RAJESWARA RAO, Purdue University, Lafayette, Indiana 47907. Reproducing formulas for automorphic forms of dimension 2 and applications.

Let \( \Gamma \) be a Fuchsian group of convergence type acting freely on the unit disc \( U \) of the complex plane with a fundamental domain \( \Omega \). Let \( A(\Gamma) \) be the Hilbert space of holomorphic automorphic forms \( G \) of dimension 2 belonging to \( \Gamma \) which satisfy \( \int_M |G|^2 \ dx dy < \infty \). For any function \( f \) on \( U \) denote by \( \Theta(f)(z) \) the sum of the series \( \sum_{\tau \in \Gamma} f(\tau z)T(\tau) \) whenever (every arrangement of) it converges uniformly on compact subsets of \( U \). Set \( f_k(z) = z^k \). \textbf{Theorem 1.} \( \Theta(f)(z) \in A(\Gamma) \) and, if \( G(z) = \sum_{k=0}^\infty A_k z^k \) is in \( A(\Gamma) \), then \( \pi \cdot A_k = (k + 1) \cdot \int_M G(z) \ dz_k \), \( k = 0, 1, 2, \ldots \). \textbf{Theorem 2.} For any \( s \) in \( \Gamma \) let \( \eta = S \), \( f_s(z) = \)
7/ (1 - z) and, for G in A(T) let \( C(S, G) = \int_{\Omega} G \cdot \overline{G(z)} dz \) be its period along S. Then \( \pi C(S, G) = \int_{\Omega} G \cdot \overline{G(z)} dz \).

Combining these with the author's earlier results (J. Math. Mech. 18 (1969), 629-644) the following can be proved. **Theorem 3.** The map \( f \to \Theta(f) \) is injective on the space of polynomials in z.

**Theorem 4.** \( f \to \Theta(f) \) is a bounded linear map from the Hardy space \( H^2(U) \) into \( A(T) \). **Theorem 5.** There exists no Banach space B of functions holomorphic on U such that, for every \( \Gamma \) under consideration, (i) \( \Theta(f) \) exists whenever \( f \) is in B and (ii) \( f \to \Theta(f) \) is a bounded linear map of B onto \( A(T) \). **Theorem 6.** If \( S \) is parabolic, then \( \Theta(fS)(z) = 0 \) for all \( z \) in \( U \). (Received June 23, 1969.)


Let \( Z, W, Y, (t \in T) \) be a family of Banach spaces. Let \( y' = (y'_t) \), where \( y'_t \in Y_t \) for all \( t \in T \), be a fixed T-tuple such that \( \sum_T \|y'_t\| < \infty \). Denote by \( P_T(y'_t, Y'_t) \) the family of all tuples \( y = (y_t) \) such that \( y_t \in Y_t \) for all \( t \in T \) and \( \sum_T \|y_t - y'_t\| < \infty \). Let \( (X', V', v') (t \in T) \) be a family of probability volume spaces and let \( (X, V, v) \) be the product probability volume space (Abstract 658-109, these Notices) 15 (1968), 750). Let \( L(v_t, Y_t) (t \in T) \) be spaces of Bochner summable functions and let \( K(v, Z) \) be the space of vector-valued volumes (Bogdanowicz, Proc. Nat. Acad. Sci. USA 53 (1965), 492-498). Define \( f'_t(x'_t) = y'_t \) for all \( x'_t \in X_t \) for all \( t \in T \). Let \( u: P_T(Y_t, Y'_t) \times Z \to W \) be an infinitely-linear bounded continuous operator (Abstract 658-108, these Notices) 15 (1968), 750). Let \( Y_0 \) be the space of bounded linear operators from \( Z \) into \( W \). For \( f \in P_T(L(v_t, Y_t), f'_t) \) let \( S_f \) be the set of those \( t \in T \) such that \( \|f_t - f'_t\| > 0 \) and define an operator \( U \) by \( U(f)(x) = u((f_t(x'_t))_{S_f}, x'_t, y'_t) 
\) when meaningful and \( 0 \in Y_0 \) otherwise. **Theorem.** For every infinitely-linear bounded continuous operator \( u: P_T(Y_t, Y'_t) \times Z \to W \) the corresponding operator \( U \) maps the multiplicative product space \( P_T(L(v_t, Y_t), f'_t) \) into the space \( L(v, Y_0) \) of Bochner summable functions. Thus an integral can be defined for \( f \in P_T(L(v_t, Y_t), f'_t) \), \( \mu \in K(v, Z) \) by \( \int u(f, d\mu) = \int_{Y_0} U(0, d\mu) \), where \( u(y, z) = y(z) \) for all \( y \in Y_0, z \in Z \). (Received June 23, 1969.)

69T-B164. DON BRATTON, 2000 W. Hemlock Street, Oxnard, California 93030. Powergrowth algebras.

A designates a topological algebra (see Abstract 69T-B69, these Notices) 16 (1969), 571). A is called a powergrowth algebra when for each \( x \in A \) there exists a scalar \( \lambda \neq 0 \) such that \( \lambda^{-1} x \) is powerbounded (see Abstract 69T-B105, these Notices) 16 (1969), 671). Each quasi-complete powergrowth algebra is Gelfand. If A is Hausdorff and powergrowth and a field, it is identical to \( \mathbb{C} \cdot 1 \), where \( \mathbb{C} \) designates the complex plane. If A is a powergrowth algebra, Hausdorff and quasi-complete, then for each \( x \in A \), the spectrum \( \sigma(x) \) is compact and the function \( \lambda - (x - \lambda)^{-1} \) is a holomorphic function \( C(x) = A. (x - \lambda)^{-1} = -\lambda^{-1} \sum x^n \lambda^{-n} \) and the critical value for convergence is \( \lambda > r(x) \). Let A be a commutative powergrowth algebra and assume that \( P \) (Abstract 69T-B105 cited) is a neighborhood of 0. For each closed ideal I of A such that \( I \neq A \) there exists a maximal ideal M of A such that M is closed and \( M \supseteq I \). In addition, there exists a compact space \( K \) and a representation \( \varphi \) of A into \( \mathbb{C}(K) \) continuous for the topology of uniform convergence in K such that \( \sigma(\varphi(x)) = \sigma(x) \) for all \( x \in A \). (Received June 24, 1969.)

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Let $X$ be a completely regular Hausdorff space and let $C(X)$ denote the ring of all real-valued continuous functions on $X$. Theorem 1. If $I$ is a maximal ideal of $C(X)$ or an ideal $O_p'$, then the maximal ideals of $I$ are exactly all ideals of the form $I \cap M$ where $M$ is a maximal ideal of $C(X)$ not containing $I$. Call an ideal $I$ which is contained in a unique maximal ideal nearly real if the corresponding maximal ideal is real. Theorem 2. Let $X$ be a realcompact space and let $I$ and $J$ be ideals of $C(X)$ each contained in a unique maximal ideal. If both $I$ and $J$ are nearly real or neither is nearly real, then any ring isomorphism from $I$ to $J$ can be extended to an automorphism of $C(X)$. As an application we get Theorem 3. A realcompact space $X$ is homogeneous iff every pair of real maximal ideals of $C(X)$ are isomorphic. Proofs and refinements of these theorems are to appear elsewhere. (Received May 2, 1969.)

69T-B166. MAX SHIFFMAN, California State College, Hayward, California 94542. A property of arbitrary point sets, including nonmeasurable sets: 'Average' measure.

Given any point set $S$ (on an interval of a line, or in Euclidean space, etc.). Let $m_i(S)$ and $m_e(S)$ be its interior and exterior Lebesgue measures. If $S_1$ and $S_2$ are two disjoint sets, place $d_i = m_i(S_1 \cup S_2) - m_i(S_1) - m_i(S_2)$ and $d_e = m_e(S_1) + m_e(S_2) - m_e(S_1 \cup S_2)$. These 'deficiencies' have the known property of being $\geq 0$. It is proved here that $d_i \leq d_e$. Another formulation is to place $m_a(S) = (m_i(S) + m_e(S))/2$ so that $m_a(S)$ is the 'average' measure of $S$; then $m_a(S_1 \cup S_2) \leq m_a(S_1) + m_a(S_2)$. This subadditivity inequality holds all the more if $S_1$ and $S_2$ are not disjoint, and indeed $m_a(\bigcup S_n) \leq \sum m_a(S_n)$ for any finite or countable number of sets $S_n$. (After completing the present article, the author found that this had been obtained in a posthumous book by Carathéodory.) More generally, $\alpha m_i(S) + \beta m_e(S)$ is subadditive and monotone increasing if and only if $\alpha \geq 0, \beta \geq 0$. The condition that it agree with Lebesgue measure for all Lebesgue measurable sets is that $\alpha + \beta = 1$, and that it be complementary is that $\alpha = 1, \beta = 0$. (Received June 24, 1969.)

Applied Mathematics


Magnetohydrodynamic flow due to axial oscillations of an infinite cylinder (insulator) is studied in two parts. In the first part inviscid conducting fluids are considered, whereas in the second part the conductivity is taken to be infinite. A radial magnetic field $k/r$, where $k$ is a constant and $r$ the radial distance, is impressed across the fluid. The results are expressed in terms of interaction parameter in the first part and a dimensionless quantity, the ratio of interaction parameter and magnetic Prandtl number, in the second part. In both the cases the induced magnetic field is found to be discontinuous at the surface of the oscillating cylinder. Other aspects such as skin friction, energy dissipation, induced current density and electric field distribution are studied. (Received March 27, 1969.) (Author introduced by Professor J. W. Bebernes.)
A new method for solving boundary value problems of the first kind for (*)
\[ \frac{\partial^2 u}{\partial x_1^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2} + B(r^2)u = 0 \]
is provided by the following Theorem. Let D be star like with respect to the origin and B(r^2) an entire function, such that B(r^2) < 0 in D. Furthermore, let \( \partial D \) be a Lyapunov boundary and \( f(\xi) \) a continuous function on \( \partial D \). Then there exists a unique solution to (*) which may be represented in the form \( u(\xi) = h(\xi) - \int_{\partial D} R(\xi, \eta; r, r) h(\eta) d\eta \), where \( \xi = (x_1, \ldots, x_n) \), \( r = \left\| \xi \right\| \) and \( R(\xi, \eta; z, z*) \) is the Riemann function associated with (*) for \( n = 2 \) (R. P. Gilbert, "Function theoretic methods in partial differential equations", Academic Press, New York, 1969).

Furthermore, \( h(y) \) may be represented as a double layer potential \[ h(y) = \left( \frac{\pi^2}{2} \right)^{1/2} \int_{\partial D} \mu(\xi) \left( \frac{1}{\left\| \xi - y \right\|^2} \right) d\xi, \quad y \in \partial D. \]

(Received April 21, 1969.)

Cosets and ferromagnetic correlation inequalities.

Let \( N = \{ 1, \ldots, n \} \) and \( \mathfrak{J} \) be the group of subsets of \( N \) under the operation of symmetric difference, which is denoted by juxtaposition. Let \( J = \bigoplus_{A \subset N} J_A \) be an element of the real group algebra of \( \mathfrak{J} \) such that \( (\forall A \subset N) J_A \neq 0 \) (ferromagnetic requirement). Let \( \pi = \bigoplus_{A \subset N} \pi_A \) = \( \exp(J) \). For any subgroup \( \mathfrak{J}_0 \) of \( \mathfrak{J} \) and subsets \( R, S \subset N \) it is shown that \( \sum_{B \in \mathfrak{J}_0} \pi_B \pi_B = \pi_{RS} \neq 0 \). The special case \( \mathfrak{J}_0 \), the one element subgroup of \( \mathfrak{J} \), implies the result of Griffiths [Phys. Rev. 65 (1964), 117] that, for an Ising ferromagnet, the correlations are monotone increasing functions of the interactions.

To appear in "Communications in Mathematical Physics". Supported by NSF GP-7946. (Received May 21, 1969.)

Consider the linear quadratic cost control problem \[ \dot{x} = A(t)x + B(t)u, \quad x(0) = x_0, \] with a cost functional \[ J[u] = \left( \frac{1}{2} \right) \int_0^T (x, Q(t)x) + (u, R(t)u) dt. \] Let \( S \) be the space of cubic splines on a mesh of norm \( h \) on the interval \([0, T]\). Then it is shown that the Rayleigh-Ritz method of minimizing \( J[u] \) over \( S \) leads to an approximation to \( J(\cdot) \) of order \( O(h^6) \). Further, a computable error bound is exhibited. In addition, if the computed spline control \( \bar{u} \) is used to control the dynamical system, then one can prove that the cost \( J[\bar{u}] \) differs from the optimal cost by \( O(h^6) \). Finally, the algebraic system resulting from the Rayleigh-Ritz Spline approach is shown to be numerically well conditioned. Numerical results and comparisons will follow. (Received April 30, 1969.)
Unsteady flow of Reiner-Rivlin fluids between two porous planes in a magnetic field.

We consider the unsteady flows of non-Newtonian conducting Reiner-Rivlin fluids in the space between two porous parallel planes in the presence of a constant transverse magnetic field. The pressure gradient in the direction of the main flow considered here is any function of time. It has been found that the solution of this problem reduces to the system of Volterra integral equations of the second kind with a regular kernel. The effects of cross viscosity over the velocity, magnetic field and the pressure field are examined and the solution of the stationary flow has also been obtained. (Received June 16, 1969.)

Geometry

An arc A in the conformal plane is said to have the nesting property if, whenever point q2 separates q1 and q3 on A, then circle C2 separates circles C1 and C3, where C1, C2, C3 are general osculating circles at q1, q2, q3 respectively. It is shown that an arc has local cyclic order three if and only if (a) the arc crosses every general osculating circle at each interior point and (b) the general osculating circles at the endpoints do not cover the entire plane. This leads to the theorem that an arc is of local cyclic order three if and only if it has the nesting property. It is shown by a counterexample that the first theorem is false if condition (b) on the endpoints is omitted. It is also shown by a counterexample that the second theorem is false if the nesting property is required only for the osculating circles rather than for the larger class of general osculating circles. (Received June 12, 1969.)

Intrinsic measures on complex manifolds and holomorphic mappings. I, II.

Part I. Let \( \lambda_n \) be the hyperbolic distance on \( B^n \), the unit ball in \( C^n \). For \( r \neq 0 \) and \( A \subset B^n \), let \( B_{\lambda_n}^r(A;r) \) be the set of points in \( B^n \) which have \( \lambda_n \)-distance less than \( r \) from \( A \). Then \( \lambda_n \) satisfies and are invariant under the automorphism group of \( B^n \). Let \( \lambda_k^k \) be the \( k \)-dimensional Hausdorff measure on \( B^n \) defined by \( \lambda_k^k \). \( \lambda_k^k \) is unique among invariant measures (on \( k \)-dimensional real submanifolds of \( B^n \)) which satisfy a certain additional condition. Thus \( \lambda_k^k \) is identical with the \( k \)-dimensional measure defined on \( B^n \) by the Poincaré-Bergman metric. Part II. Let \( M \) be a complex manifold, \( S \subset M \). Define \( \gamma_k^k \) to be \( \inf \{ \sum_i \lambda_n^k (E_i) : E_i \subset B^n, f_i : B^n \to M \text{ holomorphic}, S \subset \bigcup_i f_i(E_i) \} \), and \( \gamma_k^k = \sup \{ \sum_i \lambda_n^k (f_i(E_i \cap S)) : E_i \text{ disjoint Borel in } M, f_i : M \to B^n \text{ holomorphic} \} \). These are outer measures on \( M \) and Borel sets are measurable. If \( f : M \to N \) is holomorphic and \( S \subset M \), then \( \gamma_k^k(f(S)) \geq \gamma_k^k(S) \), and the same for \( \gamma_k^k \). On \( B^n \) \( \gamma_k^k = \gamma_k^k = \lambda_k^k \), \( \gamma_k^k \) is the largest and \( \gamma_k^k \) the smallest.
measure on $M$ with these properties. On a hyperbolic (Kobayashi) manifold $x^n$ is a Borel measure on $k$-dimensional real submanifolds of $M$. Both tight and taut manifolds (Wu) are hyperbolic. Other measures are defined and their relationships discussed. (Received June 13, 1969.) (Author introduced by Professor Shing-Shen Chern.)

**Logic and Foundations**


Let $x > \lambda$ be infinite cardinals, $x$ uncountable. We say that a subset $x$ of $x$ is somewhat homogeneous for a partition of the $n$-element subsets of $x$ into $\lambda$ pieces if the $n$-element subsets of $x$ meet fewer (in the sense of cardinality) than $\lambda$ members of the partition. With this concept in mind, we define, for $\delta$ a cardinal less than $\lambda$, the partition relation $x \prec (x)_{\lambda, \delta}^n$ by "for each partition of the $n$-element subsets of $x$ into $\lambda$ pieces there exists a size $x$ subset of $x$ whose $n$-element subsets meet at most $\delta$ of these pieces." Theorem. $x \prec (x)_{\lambda, \delta}^n$ iff the cofinality of $x$ is either at most $\delta$ or greater than $\lambda$. Theorem. Suppose that $x \prec (x)_{\lambda, \delta}^2$. Then $2^\lambda \not\subseteq x$ implies $2^\delta \not\subseteq x$ and $\lambda^+ \not\subseteq x$ implies $\delta^+ \not\subseteq x$.

Corollary. If $\forall \lambda \exists \delta < \lambda (x \prec (x)_{\lambda, \delta}^2)$ then $x$ is strongly inaccessible. (Received April 23, 1969.)

69T-E50. J. H. SCHMERL, University of California, Berkeley, California 94720, and SAHARON SHELAH, Hebrew University, Jerusalem, Israel. On models with orderings.

Consider a countable first-order language which includes a symbol denoting a linear ordering. A structure $M$ is $\lambda$-like iff $M = x$ and each proper initial segment has fewer than $x$ elements. For the definition of $\rho_a$-cardinals see, for example, Gaifman [Israel J. Math. 5 (1967)]. Theorem 1. If for each $n < \omega$ $T$ has a $\lambda$-like model for some $\rho_a$-cardinal $x$, then $T$ has a $\lambda$-like model for each uncountable $\lambda$. Theorem 1 extends results of Helling [these Notices 12 (1965), 723], partially confirming a conjecture of Fuhrken. Theorem 2. If for each $a < \omega^1$, $T$ has a $\lambda$-like $\omega$-model for some $\rho_a$-cardinal $x$, then $T$ has a $\lambda$-like $\omega$-model for each uncountable $\lambda$. Theorems 1 and 2 generalize to uncountable languages and $\lambda$-logics and have applications to compactness, completeness and transfer properties in languages with generalized quantifiers. These results were found by the first author several months after, though independently of, the second author. (Received April 25, 1969.)

69T-E51. SAHARON SHELAH, The Hebrew University, Jerusalem, Israel. On classes with only homogeneous models. II.

This notice solves problem D in Keisler's "Some model theoretic results for $\omega$-logic," Israel J. Math. 4, p. 249, and partially solves problem C. Let $T$ be a complete first-order theory, D the set of complete types in the variables $x_1,...,x_n$ for $n < \omega$, and $\text{Sp}$ the class of cardinals in which $T$ has only homogeneous models. Theorem 1. If $|T| < \lambda \not\in \text{Sp}$, then there exists $\mu_0 < \mu_{|T|}$, $\mu_0 > |D|$, such that $\mu \not\subseteq \mu_0$ implies $\mu \not\in \text{Sp}$; and $\mu < \mu_0$, $\mu > |T|$, implies $\mu \not\in \text{Sp}$. Corollary. If a denumerable complete first-order theory has only homogeneous models of power $\aleph_1$, then it is categorical in $\aleph_1$. Let $\{p_i : i \in I\}$ be a set of types, and $\text{SQ}$ the class of infinite cardinals $\lambda$ such that every model of $T$ which omits $\{p_i : i \in I\}$ is homogeneous if it has power $\lambda$. Theorem 2. If $2^{|T|} \not\subseteq \lambda < \mu \not\in \text{SQ}$,
\[ \lambda \in \text{SQ}, \text{then either } z^\lambda \geq \mu \text{ and } |T| < x < \lambda \text{ implies } x \notin \text{SQ}, \text{or } \lambda = \sum_{\chi < \lambda} \chi^y \text{ and } |T| < x < \lambda \text{ implies } z^x = \lambda, x = \sum_{\chi < x} \chi^x. \] (Received February 11, 1969.) (Author introduced by Professor Michael O. Rabin.)

69T-E52. BARUCH GERSHUNI, Frans van Mierisstraat 37, Amsterdam, Nederland.

Systematics of the totalities.

There arises the necessity of letting the totality "word" also participate in the author's general theory of totalities. This can be done in the following way: A totality is called (innerly) separated, if, in a true representation of it, its elements are separated from one another by spaces, which may of course be occupied by commas or semicolons or the like. A totality is called connected, if these spaces are occupied by commas or the like, or if it is bracketted. One defines now the word abc... as a totality \( G_1 \), which is neither separated nor connected. One defines now the totalities \( G_2 = \{a,b,c,...\}; G_3 = (a,b,c,...); G_5 = \{a,b,c,...\} \), which are called variety (or manifoldness); plural class; singular class; and set respectively by letting \( G_{i+1} \) have one property, which \( G_i \) has not.

One defines so the variety by its being separated; the plural class by its being moreover innerly connected by commas or the like; the singular class - by its being moreover collectively connected towards its exterior by usual parentheses; and finally the set - by its being moreover collectively connected towards its inner by braces. The sequence \( G_1,G_2,G_3,G_4,G_5 \) is a continuous sequence of totalities, i.e. it is not possible to insert between these totalities other totalities. They are called basic totalities. (Received March 24, 1969.)

69T-E53. ALBERT SADE, 364 Cours de la République, Pertuis, Vaucluse, France.

Sur le système d'axiomes de Gotlib.

Le système \( G_1: x \vee x = x; G_2: x = x \vee y; G_3: x = x; G_4: (x = y) = (x \vee x = y \vee y) \) a pour solutions, sur \( GF(2): (i) x \vee y = 1; x \vee y arbitraire; (ii) x = y = xy + x + 1; x \vee y = xy + x + y. \) Il admet la solution unique (ii), si on lui adjoint l'axiome Gc de cancellation. \( x = y = xy + x + 1, x \vee y = xy \) satisfait \( G_1, G_2, G_3, G_4 \), non \( G_3 \). \( x = y = xy + x + 1, x \vee y = xy \) satisfait \( G_1, G_3, G_4, G_2, G_4 \), non \( G_1 \). \( G_3 \). Sur GF(2), le système \( G_2,G_3,G_4,G_5: x = y = Nx \vee y, a la solution unique x \vee y = xy + x + y, Nx = x + 1. \) Toute solution de G2 satisfait G1 et G4. x \vee y = xy + x + 1, Nx = x + 1 satisfait tous les axiomes, sauf G3. x = y = x + y, Nx = x + 1 satisfait tous les axiomes, sauf Gc. G1 et G4 sont inutiles. (Received April 8, 1969.)

69T-E54. CARL F. ECKBERG, Purdue University, Lafayette, Indiana 47907. Some results in recursive analysis.

Let \( R, [0,1] \) and property (A) be as in Abstract 69T-E47, these \( \text{Notice} \) 16 (1969), June issue.

Let \( \theta \) be a rec. fn. s.t. for any infinite r.e. set \( W_x \), \( \phi(x) : N \rightarrow W_x \) is a recursive bijection. Theorem 1. For all \( x, n \in N \), let \( a_n(x) = \sum_{i=0}^{n} 3/10^k(i) \) where \( k(i) = \phi(i)+1 \). Then \( a_n(x) \) satisfies (A) whenever:

1. \( W_x \) is creative or 
2. \( W_x \) is simple but not hypersimple; 
3. \( a_n(x) \) does not satisfy (A) if \( W_x \) is hypersimple. In Zaslavskii's "Some properties of constructive real numbers and constructive func-

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tions" [Amer. Math. Soc. Transl. (2) 57 (1966), 1-84] in Theorems 5.1, 5.2, 5.3, 5.5 the functions constructed behave pathologically on a perfect set in R - [0,1]. Theorem 2. Any fn., establishing Theorem 5.5 attains its sup on a perfect set. However by using Theorem 1(1) and a construction due to O. Averbh, we may obtain simpler proofs of Theorems 5.1, 5.2, 5.3, and the constructed functions will be pathological on just one point of R - [0,1]. From Theorem 1(3), we also obtain Theorem 3. If \( W_x \) is hypersimple and \( a = \lim_{n \to \infty} a_n(x) \), then there is no recursive operator \( F \) total on [0,1] s.t. \( F \) is right continuous at \( a \) but not continuous at \( a \). (Received April 11, 1969.)

69T-E55. RONALD BJÖRN JENSEN, Rockefeller University, New York, New York 10021. Souslin's hypothesis = weak compactness in L.

Let \( \mathfrak{x} \) be a regular cardinal and let \( V \) be the constructible closure of a bounded subset of \( \mathfrak{x} \).

Then the following conditions are equivalent: (i) \( \mathfrak{x} \) is weakly compact. (ii) The \( \mathfrak{x} \) Souslin hypothesis holds. (iii) If \( A \subseteq \mathfrak{x} \) is Mahlo in \( \mathfrak{x} \), then for some \( \beta < \mathfrak{x} \), \( A \cap \beta \) is Mahlo in \( \beta \). (We call A Mahlo in \( \mathfrak{x} \) iff A has a nonempty intersection with every closed, unbounded subset of \( \mathfrak{x} \). The \( \mathfrak{x} \) Souslin hypothesis says that every linear ordering whose intervals satisfy the \( \mathfrak{x} \) antichain condition contains a dense subset of cardinality \( < \mathfrak{x} \).) The proof actually shows: If \( V = L[B] \) for a \( B \subseteq \mathfrak{x} \) s.t. for some \( \Pi_1 \) statement \( \phi(B) \), \( \phi \) is true of \( (L[B],B) \) but is not true of \( (L[\beta\cap\beta],\beta) \) in \( L[\beta\cap\beta] \) for any \( \beta < \mathfrak{x} \), then (ii), (iii) fail. Moreover, for strongly inaccessible \( \mathfrak{x} \), if \( B \subseteq \mathfrak{x} \), \( V = L[B] \), and (iii) fails, then (ii) fails. It is not known whether (iii) is a large cardinal property in ZF (e.g. can (iii) hold for \( \mathfrak{x} = \mathfrak{w}_{\omega + 1} \) ?). (Author introduced by Professor Hao Wang.) (Received May 8, 1969.)

69T-E56. MIRO BENDA, University of Wisconsin, Madison, Wisconsin 53706. On saturated reduced direct products.

A filter \( D \) is said to be \( \sigma \)-incomplete if there is \( \{ X_n | n < \omega \} \subseteq D \) such that \( \bigcap_{n < \omega} X_n = 0 \). A filter \( D \) over \( I \) is said to be \( \mathfrak{x} \)-compact if for any \( X \subseteq S(I) - J \), \( J = \{ x \subseteq I | x \in D \} \), \( |X| < \mathfrak{x} \), and such that for any \( s \in S(\omega \times \omega) \cap s \subseteq S(I) - J \) there is a \( y \in S(I) - J \) for which \( y - x \in J \) for each \( x \subseteq \omega \). A filter \( D \) over \( I \) is said to be \( \mathfrak{x} \)-separable if for any \( \lambda < \mathfrak{x} \) and any \( \{ x_n | n < \lambda \} \subseteq S(I) - J \) such that \( x_n \cap \lambda \in J \) if \( \beta < \lambda \) and \( \{ y_n | n < \lambda \} \subseteq S(I) - J \) such that \( y_n \subseteq x_n \) and \( y_n \cap \beta = 0 \) for each \( \beta < \mathfrak{x} \). Filters which are \( \sigma \)-incomplete, \( \mathfrak{x} \)-good, \( \mathfrak{x} \)-compact and \( \mathfrak{x} \)-separable are all \( \mathfrak{x} \)-excellent. Let \( \mathfrak{W} \) be a structure and let \( L(\mathfrak{W}) \) be the language of \( \mathfrak{W} \) enlarged by constants \( c \) for \( \mathfrak{W} \subseteq |\mathfrak{W}| < |\mathfrak{W}| \). Let \( \phi \) be a set of formulas of \( L(\mathfrak{W}) \); if for each \( \Sigma_0(\mathfrak{W}) \subseteq \phi \) such that \( |\Sigma_0(\mathfrak{W})| < \mathfrak{x} \) and \( \Sigma_0(\mathfrak{W}) \) is finitely satisfiable in \( \mathfrak{W}, \Sigma_0(\mathfrak{W}) \) is satisfiable in \( \mathfrak{W} \), then \( \mathfrak{W} \) is called to be \( (\phi, \mathfrak{x}) \)-saturated. \( L^0(\mathfrak{W}) \) is the set of all quantifier-free formulas of \( L(\mathfrak{W}) \). Theorem. \( D \) is \( \mathfrak{x} \)-excellent over \( I \) if and only if \( \Pi_1^1 / D \) is \( L^0(\Pi_1^1 / D, \mathfrak{x}) \)-saturated for each \( \mathfrak{W} \subseteq |\mathfrak{W}| \). Corollary. If \( S(I)/D \) is atomless and \( D \) is \( \mathfrak{x} \)-excellent, then \( \Pi_1 \) / \( D \) is \( \mathfrak{x} \)-saturated for each \( \mathfrak{W} \subseteq |\mathfrak{W}| \). The corollary is also true when \( S(I)/D \) has finitely many atoms. For \( \mathfrak{x} = \omega_1 \), the corollary is a result of L. Pacholinski and C. Ryll-Nardzewski. (Received May 8, 1969.)

69T-E57. JAMES E. BAUMGARTNER, University of California, Berkeley, California 94720. Undefinability of \( n \)-ary relations from unary functions. Preliminary report.

An \( n \)-ary relation \( R \) on \( X \) is called an \( n \)-tournament on \( X \) if for any \( n \)-element subset \( y \) of \( X \) there exist \( a_1, \ldots, a_n, b_1, \ldots, b_n \in X \) such that \( \langle a_1, \ldots, a_n \rangle \in R \), \( \langle b_1, \ldots, b_n \rangle \not\in R \) and \( y = \{ a_1, \ldots, a_n \} = \{ b_1, \ldots, b_n \} \). Suppose \( A \) and \( X \) are infinite sets, \( f : A \to A \) and \( X \subseteq A \). Theorem 1. In \( \langle A, f, S \rangle \), \( S \subseteq A \) no \( n \)-tournament on \( X \) is (first-order) definable. Corollary 2. In \( \langle A, f, S \rangle \), \( S \subseteq A \) no linear ordering of \( X \)
is definable. Corollary 3. If $f: \omega \rightarrow \omega$, then in $(\omega, f, S)_{Sc \omega}$ the ordinary ordering on the natural numbers is not definable. This improves a theorem of McNaughton (Trans. Amer. Math. Soc. 117 (1965), 329-337) which says that in $(\omega, f, S)_{Sc \omega}$ addition is not definable. (Received May 12, 1969.)


Using the formulation of algebraic logic given by A. Copeland, Sr., Michigan J. Math. 3(1955), which requires only five primitive operations, $\exists, Q, P, T, T^{-1}$, in addition to the Boolean operations, $\wedge, \vee, \neg$, and following the theory of cylindric algebras of Tarski, the following classes are defined in the natural way: set Copeland algebras (SCP), representable Copeland algebras (RCP), the equational closure of SCP (SCP'), locally finite-dimensional Copeland algebras (LCP), Copeland algebras of formulas of some first-order language (FCP), and the class of Copeland algebras (CP).

Theorem. (i) SCP $\subset$ RCP $\subset$ SCP' $\subset$ CP; (ii) FCP $\subset$ RCP; (iii) FCP $\subset$ LCP; (iv) LCP $\cap$ SCP $\not\subset$ RCP. For $U \in$ LCP one can define the substitution operator, $S_{\tau}$, for every $\tau \in Z^Z$, where $Z$ denotes the set of integers. Let $s \in Z$ denote the successor function on $Z$. Theorem. $U \in$ FCP iff $U \in$ LCP and $T(a) = S_{\tau}(a)$ for every $a$ in $A$. Theorem. (i) RCP is not elementary in the wider sense; (ii) SCP is not finitely axiomatizable; (iii) CP is not finitely axiomatizable.

(Received May 14, 1969.) (Author introduced by Professor J. Donald Monk.)


For definitions see Schmerl and Shelah [Abstract 69T-E50, these Notices 16(1969), this issue].

Theorem. For each $n < \omega$ there is a sentence $\sigma_n$ of first-order logic such that for any cardinal $\kappa$, $\sigma_n$ has a $\kappa$-like model iff $\kappa$ is not a $\rho_\alpha$-cardinal. This theorem refutes a conjecture of Fuhrken. In conjunction with Theorem 1 of the cited abstract it implies the following Corollary. Assume there exists an uncountable $\rho_0$-cardinal; then $\kappa$ is a $\rho_\alpha$-cardinal iff each theory $T$ which has a $\kappa$-like model also has a $\alpha$-like model for each uncountable $\alpha$. (Received May 23, 1969.)

69T-E60. DOV GABBAY, Hebrew University, Jerusalem, Israel. Montague type semantics for modal predicate $T$ without equality.

Our structures are of the form $\Delta = (A_1, R, 0)$, $i \in D$, $0 \in D$, $R \in D \times \times D$, where $A_1$ are ordinary predicate structures all having the same domain. Let $[\phi]$ be defined as follows: $[\phi] = \text{value given by } A_1$ for atomic $\phi$: $[\phi \land \psi]_1 = T$ ($[\neg \phi]_1 = T$), $[\exists x \phi(x)]_1 = T$ iff $[\phi]_1 = T$ and $[\neg \psi]_1 = T$ ($[\phi]_1 = F$, $[\phi(x)]_1 = T$ for some $a \in A_1$, resp.) and $[\Box \phi]_1 = T$ iff $[\phi]_1 = T$ ($[\phi]_1 = F$), $[\neg \phi]_1 = T$) $\in R$. $\phi$ holds at $A_1$ iff $[\phi]_0 = T$. $R$ has the properties (1) $F_1 = [C C D] (i, C) \in R$ is a filter, (2) (i, $C) \in R - i : i \in C$.

Theorem. (a) Modal predicate $T$ without Barcan formula is complete for the above semantics. (b) Adding (3): each $F_1$ is closed under arbitrary intersections, provides a semantics for which $T$ with Barcan is Barcan is complete. (See Cresswell, Notre Dame J. (1967), 186.) Remark. The semantics without (1), (2), (3) is essentially Montague's ("Pragmatics and intensional logics," to appear in Dialectica) semantics without equality. It can be axiomatized by adding to the predicate calculus the rule $\forall \phi \leftrightarrow \psi = \Box \phi \leftrightarrow \Box \psi$. D. Kaplan first announced axiomatization of Montague semantics. (Received May 27, 1969.) (Author introduced by Professor Azriel Levy.)

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On Robinson's and Specker's theorems.


**Theorem 1 (Robinson's Theorem with amalgamation).** Let $\mathfrak{U}_0$ and $\mathfrak{U}_1$ be relational systems with index sets $I_{\mathfrak{U}_0}$ and $I_{\mathfrak{U}_1}$ respectively, and let $X \subseteq \mathfrak{U}_0$, $f \in \mathfrak{U}_1^X$. If $(\mathfrak{U}_0 \cap \mathfrak{U}_1, \mathfrak{U}_0 \cap \mathfrak{U}_1, f(x))_{x \in X}$ then there exists a relational system $\mathfrak{U}$ with index set $I_{\mathfrak{U}}$ such that $I_{\mathfrak{U}} = I_{\mathfrak{U}_0} \cup I_{\mathfrak{U}_1}$, and there exist elementary embeddings $g_0: \mathfrak{U}_0 \rightarrow \mathfrak{U}$ and $g_1: \mathfrak{U}_1 \rightarrow \mathfrak{U}$ such that $g_0(x) = g_1(f(x))$ for all $x \in X$.

**Theorem 2 (Specker's Theorem with dual amalgamation).** Let $\mathfrak{U} = \langle A, U, V, R \rangle_{i \in I}$, $i \in I$, $U, V \subseteq A$, and let $\mathfrak{U} = (U, R, f(x))_{x \in X}$ such that $\langle A, U, V, R \rangle_{i \in I}$, where $U, V \subseteq A$, and let $\mathfrak{U} = (U, V, f(x))_{x \in X}$ such that $\langle A, U, V, R \rangle_{i \in I}$, where $A, U, V \subseteq A$, and let $\mathfrak{U} = (U, V, f(x))_{x \in X}$. Then there exists $\mathfrak{V} = \langle B, V, V', S \rangle_{i \in I}$, where $V, V' \subseteq B$, such that $\mathfrak{U} \cong \mathfrak{V}$ and an isomorphism $g: \langle V, S, f(x) \rangle_{i \in I} \cong \langle V', S, f(x) \rangle_{i \in I}$ such that $g[X] = f$.
set of \((K,\epsilon)\) and that the axiom of Choice (C) is valid in the Synergistic model \((S,\tau)\). Therefore, C is consistent with Z-F. (Received June 24, 1969.)

69T-E65. C. M. da BARROS, NEPEC, Rau Almirante Alexandrino, 537 Rio de Janeiro, GB, Brazil, and NEWTON C. A. da COSTA, Caixa Postal 1170, Campinas, Sao Paulo, Brazil. The predicate calculus of order \(\omega\) as the underlying logic of set theory.

In this work the authors formalize the Zermelo-Fraenkel system of set theory using the predicate calculus of order \(\omega\) as the underlying logic (Hilbert and Ackermann, "Principles of mathematical logic," Chapter IV, 15, 1950). A new treatment of Bourbaki's concept of species of structures and other related notions are presented. For instance, the so-called procedures of deduction (Bourbaki, "Set theory," Chapter IV, 1968) correspond to formulas of second order whose free propositional variables "represent" species of structures. The notions of category, functor and natural transformation may be interpreted as abstracts (Quine, "Mathematical logic," 1950), introduced by contextual definitions, corresponding to propositions or predicates associated to formulas of order \(\leq 2\), via the principle of abstraction:

\[(E \forall \xi(G_1) \ldots (G_n)(E[G_1, \ldots, G_n] \sim \tau(G_1, \ldots, G_n))\) (Hilbert and Ackermann, loc. cit., p. 157). Species of local structures (Ehresmann, C. R. Acad. Sci. Paris 234 (1952), 587) may be interpreted as abstracts of order 3, whose free propositional variables denote species of structures in Bourbaki's sense and procedures of induction. (Received June 24, 1969.)

Statistics and Probability

69T-F11. THEODORE E. HARRIS, University of Southern California, Los Angeles, California 90007. Changing random point processes. II.

For notation see Abstract 69T-F6, these Notices 16 (1969), 580. Particles in \(R_1\) initially at \(x_1, x_0, x_1, \ldots\) move respectively to \(y_1, y_0, y_1, \ldots\); i.e., \(a = [x_i, y_i] \in M_2\). Let \(S(a, A)\), \(a \in M_1\), \(A \in M_2\), be a transition pr. from \(M_1\) to \(M_2\) with \(S(a + x, A + x) = S(a, A)\), all \(x \in R_1\), and \(S(a, [\xi_1 = a]) = 1\). Given \(a\), appropriate "iteration" of \(S\) generates a Markov process \(\xi_1, \xi_2, \ldots\), state space \(M_2\), where each \(\xi_n = a\), and \(\xi_n\) is the set of positions at time \(n\). Choose \(a\) randomly either as a regular stationary point process or with the corresponding Palm pr. measure. In the latter case let \(\theta_n\) be the Markov process \(\xi_n - w_n\) where \(w_n\) is the position at time \(n\) of the particle initially at 0. For each \(n\) the successive spacings of \(\theta_n\) form a stationary sequence; let \(R^n(\cdot)\) be its covariance function. In certain cases \(R^1\) can be chosen so that \(R^1 = R^2 = \ldots\), all finite, thereby giving compactness properties insuring that the processes \(\theta_n\) and \(\xi_n\) have stationary pr. distributions. (Received April 30, 1969.)

69T-F12. MAREK KANTER, University of California, Berkeley, California 94720. Linear functionals on a probability space.

Let \(X\) be a real-valued stochastic process with independent increments on a parameter set \(T\) which can either be \(R\), the real line, \(I\), the unit interval, or \(Z\), the set of integers. Let \(S\) be the real linear space of sample paths of the process. Let \(F\) be the smallest sigma field of subsets of \(S\) with respect to which all increments of \(X\) when treated as functionals on \(S\) are measurable. Let \(P\) be the
probability measure induced on \((S,F)\) via the Kolmogorov extension theorem. Let \(S^*\) be the set of all linear functionals on \(S\) that are measurable with respect to the completion of \(F\) under \(P\). If \(T\) is \(Z\) and \(X\) is symmetrically distributed under \(P\) then every \(f\) in \(S^*\) is equal to a stochastic integral \(a.s., i.e., an expression of the form \(\int_T h(t)dX(t)\) where \(h\) is a Borel measurable function. If \(T\) is \(I\) and \(X\) is continuous in probability, lattice valued, and such that its sample paths have bounded variation \(a.s., then the same result holds; if the further condition that \(X\) is symmetrically distributed is assured then the result also holds if \(T\) is \(R\). (Received February 17, 1969.) (Author introduced by Professor Jacob Feldman.)


Theorem. If \(\xi_k\) and \(a_k\) are sequences of random variables and real numbers, respectively, such that \(\sum P(|\xi_k| > a_k) < \infty\) and \(1/n(a_1 + \ldots + a_n) \to 0\) as \(n \to \infty\), then with probability one,

\[
\frac{1}{n}(\xi_1 + \ldots + \xi_n) \to 0.
\]

Two sequences \(\xi_k\) and \(\xi_k^{*}\) are equivalent (\(\xi_k \sim \xi_k^{*}\)) if for some sequence \(a_k\) of real numbers, the hypothesis of the theorem holds for the sequences \(\xi_k - \xi_k^{*}\) and \(a_k\). Corollary. If \(\xi_k\) and \(\xi_k^{*}\) are equivalent in the sense of Khinchin, then \(\xi_k \sim \xi_k^{*}\), but not conversely. As in the Khinchin case, if \(\xi_k\) obeys the strong law of large numbers and \(\xi_k^{*} \sim \xi_k^{*}\), then \(\xi_k^{*}\) obeys the law also. (Received June 19, 1969.)

69T-G96. GIOVANNI A. VIGLINO, Wesleyan University, Middletown, Connecticut 06457. \(\widetilde{T}_n\) spaces.

Let \(n\) denote a positive integer. Definition. A space \((X,\tau)\) is \(\widetilde{T}_n\) if for any two distinct points \(p,q\) of \(X\) there exists \(O_i \in \tau, 1 \leq i \leq n\), such that \(p \in O_1, q \notin \bigcap_{i=1}^{n} O_i\). Note that \(\widetilde{T}_1\) is simply the Hausdorff separation property while \(\widetilde{T}_2\) is the Urysohn separation property. It is clear that a regular space is \(\widetilde{T}_n\) for every \(n\). One can easily construct an example of a space that is \(\widetilde{T}_n\) for every \(n\) but which is not regular. Theorem. There exist spaces which are \(\widetilde{T}_n\) but not \(\widetilde{T}_{n+1}\) for any \(n\). (Received March 14, 1969.)

69T-G97. C. J. MOZZOCHI, 18 Tuxis Road, Madison, Connecticut 06443. On symmetric generalized uniform spaces. II.

Let \(R\) denote the reals. Let \(I\) denote the positive integers. Let \(\Delta = \{(x,y) \mid x \in R, y \in R, x = y\}\). Let \(\Delta^* = \{(x,y) \mid x \in R, y \in R, x = -y\}\). Let \(I_n = [-1/n, 1/n]\) for each \(n \in I\). Let \(B_n = ((I_n \times I_n) - \Delta^*) \cup \Delta\) for each \(n \in I\). Let \(S = \{B_n \mid n \in I\}\). Theorem. \(S\) is a base for a symmetric generalized uniformity \(\nu\) on \(R\) that has the following properties: (1) For every \(U,V\) in \(\mathcal{U}\), \((U \cap V) \in \mathcal{U}\). (2) \((0,0) \notin B_0\) for every \(n \in I\); so that \(\mathcal{U}\) does not have an open base, and \(\mathcal{U}\) is not \(p\)-correct. (3) \((B_n \star B_m) \cap ((R \times R) - B_m) \neq \emptyset\) for every \(m,n\) in \(I\). (4) The neighborhood system of \(0\) is a convergent filter in \((R, \mathcal{J}(\mathcal{U}))\) that is not Cauchy with respect to \(\mathcal{U}\). (5) \((R, \mathcal{U})\) is complete. (6) \(\mathcal{J}(\mathcal{U})\) is not compact; so that \(\mathcal{U}\) is not totally bounded. Remark. This example provides answers to two questions raised by the author (cf. Abstract 658-54, these Notices 15 (1968), 733). By a suitable
modification of the above construction it is possible to prove the following Theorem. There exists a symmetric generalized uniformity \( \nu \) on \( R \) without an open base that generates the usual topology for \( R \) such that for every \( U, V \) in \( \nu \), \((U \cap V) \in \nu \). Theorem. There exists a totally bounded symmetric generalized uniform space that is not \( p \)-correct or \( p \)-correct of degree \( n \) for every \( n \in \mathbb{N} \) (cf. Abstracts 68T-2, 68T-161, these Notices 15 (1968), 188, 347). (Received March 21, 1969.)

69T-G98. CHARLES L. HAGOPIAN, California Institute of Technology, Pasadena, California 91109. A cutpoint theorem for plane continua.

Let \( M \) be a plane continuum and let \( x \) be a point of \( M \). F. B. Jones ["Concerning nonaposyndetic continua," Amer. J. Math. (1948), 404] defines \( K_x \) to be the set consisting of \( x \) and all points \( y \) of \( M - \{x\} \) such that \( M \) is not aposyndetic at \( x \) with respect to \( y \). The set \( K_x \) is closed. However \( K_x \) need not be connected. Theorem. For each point \( y \) in the set \( K_x - (x \text{-component of } K_x) \) there exists a point \( z \) in \( M - \{x, y\} \) such that \( y \) cuts \( M \) weakly between \( x \) and \( z \). (Received April 9, 1969.) (Author introduced by Dr. F. B. Jones.)


Given a topological property \( P \) and a set \( X \), let \( P(X) \) denote the set of topologies on \( X \) with property \( P \). A topological space \((X, \mathcal{T})\) is minimum \( P \) (maximum \( P \)) if \( \mathcal{T} \) is a minimum (maximum) element in \( P(X) \). Definition. A topological space, \((X, \mathcal{T})\), is completely homogeneous if every one-to-one mapping of \( X \) onto itself is a homeomorphism. Theorem 1. Given a topological space, \((X, \mathcal{T})\), the following three conditions are equivalent: (1) \((X, \mathcal{T})\) is completely homogeneous. (2) \((X, \mathcal{T})\) is minimum \( P \) for some topological property \( P \). (3) \((X, \mathcal{T})\) is maximum \( P \) for some topological property \( P \). Theorem 2. The only completely homogeneous topologies on \( X \) are the following: (1) The indiscrete topology. (2) The discrete topology. (3) Topologies of the form \( \mathcal{T} = \{G : |X - G| \notin m \text{ where } \mathbb{N}_0 \notin m \notin |X|\} \). A well-known example of a minimum \( P \) space is the space of finite complements, which is minimum \( T_1 \). Theorem 2 points out that there are relatively few minimum or maximum topologies on a given set \( X \). In particular, assuming the continuum hypothesis, there are only four minimum or maximum topologies on the reals. These are the indiscrete topology, the topology of finite complements, the topology of countable complements, and the discrete topology. (Received April 9, 1969.) (Author introduced by Professor Wolfgang J. Thron.)

69T-G100. ROBERT F. BROWN, University of California, Los Angeles, California 90024. H-manifolds have no nontrivial idempotents.

An H-space is a triple \((X, m, e)\) where \( m : X \times X \to X \) is a map, \( e \in X \), and \( m(x, e) = m(e, x) = x \) for all \( x \in X \). Let \( H_e(X) \) be the space of all maps \( m : X \times X \to X \) such that \((X, m, e)\) is an H-space. Elements of \( H_e(M) \) are considered to be equivalent if they are in the same path component. A point \( x \in X \) is a nontrivial idempotent of \((X, m, e)\) if \( m(x, x) = x \) and \( x \neq e \). Theorem. Let \((M, m, e)\) be an H-space where \( M \) is a compact connected triangulable n-dimensional manifold with or without boundary, \( n \geq 3 \). There exists \( m' \notin H_e(M) \) equivalent to \( m \) such that \((M, m', e)\) has no nontrivial idempotents. The proof depends on a theorem of F. Wecken (Math. Ann. 118 (1942), 544-577). (Received April 28, 1969.)
Topological manifolds must now be studied in their own right. Our classification theorem for PL structures on a topological (= TOP) manifold, Bull. Amer. Math. Soc., July 1969, which contains a TOP·PL 'Caïns-Hirsch' theorem, yields readily the following: (I) Every TOP manifold triad (W; V, V') (perhaps noncompact) of dimension \( \geq 6 \) admits a (locally finite) TOP handle decomposition on V. TOP handlebody theory works formally like the PL and DIFF theories in dim \( \geq 6 \). But in dim 4 or 5 (one or both) there exists a closed manifold M with no TOP handle decomposition.

- M \( \times S^1 \times \cdots \times S^1 \) is never homotopy equivalent to a closed PL manifold. (II) A restricted relative TOP microbundle transversality theorem holds. It is like Williamson's PL theorem, Ann. of Math. 83, p. 9, but the source S must be a TOP open manifold, and the expected dimension of zero-section pre-image must be \( \geq 5 \). (III) Surgery can be done in TOP on manifolds of dim \( \geq 6 \) (\( \geq 5 \) if rel boundary). (IV) G/TOP \( \cong \Omega^n G/\Omega^n G/\text{PL} \). In dim \( \geq 5 \) many TOP classifications can now be pushed as far as for PL. (Received April 1, 1969.)

69T-G102. MURRAY EISENBERG, University of Massachusetts, Amherst, Massachusetts 01002. Expansiveness of T-algebras. Preliminary report.

Let T be a discrete group. Ellis [Trans. Amer. Math. Soc. 101 (1961), 384-395] has shown that each T-invariant subalgebra A of \( C(\mathbb{R}) \) gives rise to a point-transitive transformation group \((IAI, T)\) on a compact space \(IAI\). It is known that when \(IAI\) is zero-dimensional, \((IAI, T)\) is expansive if and only if A is finitely generated. We prove Theorem 1. If \((IAI, T)\) is expansive, then A is finitely generated. Theorem 2. There exists a T-invariant subalgebra A of \( C(\mathbb{R}) \) generated by a single element such that \((IAI, T)\) is not expansive. (Received April 30, 1969.)

69T-G103. RUSSELL G. WOODS, McGill University, Montreal, Quebec, Canada. The dimension of \( pX - X \) for \( \sigma \)-compact X.

**Theorem.** Let X be a locally compact \( \sigma \)-compact Hausdorff space, and let n be a nonnegative integer. Then the (Lebesgue) dimension of \( pX - X \) is n if and only if n is the largest integer satisfying the following condition: there exists a finite cover \( \mathcal{U} \) of X such that every refinement of \( \mathcal{U} \) contains \( n + 1 \) members whose intersection is not contained in any compact subset of X. **Corollary.** For each positive integer n the dimension of \( p\mathbb{R}^n - \mathbb{R} \) is n. (Received May 2, 1969.)

69T-G104. R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306. A mapping theorem and criteria for taming 3-spheres in \( S^4 \).

**Theorem 1.** Let X and Y be metric spaces, f a mapping of X onto Y. Let Z be the mapping cylinder of f. If Y is locally collared in Z, then \( X \times \mathbb{R} \cong Y \times \mathbb{R} \). **Corollary.** If f is locally shrinkable by a pseudo isotopy, then \( X \times \mathbb{R} \cong Y \times \mathbb{R} \). **Theorem 2.** Let S be a 3-sphere in \( S^4 \), W a complementary domain of S. Assume that the following hold: (a) W is 1-ULC. (b) Each point of S has a mapping cylinder neighborhood in \( \overline{W} \). (i.e., for each point p \( \in S \) there exists a neighborhood V of p in S, an open 3-manifold U, a proper mapping \( \phi \) of U onto V, and an embedding of Z(\( \phi \)) into \( \overline{W} \) which is the identity on V.) Then \( \overline{W} \) is a 4-cell. The proof of Theorem 1 uses Brown's collaring theorem, an
elementary engulfing lemma, and the monotone union technique. Theorem 2 needs McMillan’s cellularity criterion and Armentrout’s results on shrinking cellular decompositions of 3-manifolds. (Received May 26, 1969.)

69T-G105. MILTON ULMER, Wesleyan University, Middletown, Connecticut 06457. C-embedded \( \mathfrak{M} \)-Spaces. Preliminary report.

For \( p \in \prod_{a \in A} X_a = X \), we set \( \Sigma(p) = \{ x \in X : \left| \{ a : x \notin X_a \} \right| \leq \aleph_0 \} \). Extending results recorded and improved by Engelking (Fund. Math. 59 (1966), 221-231) of Corson (Amer. J. Math. 81 (1959), 785-796) and others, we show that each continuous real-valued function on \( \Sigma(p) \) extends to \( X \) provided that (a) \( X \) is pseudocompact; or (b) each \( X_a \) is P-space; or (c) each \( X_a \) is first-countable. (The sufficiency of a fourth condition, (d), has been established in unpublished work by Comfort-Negrepontis: (d) each product of finitely many of the spaces \( X_a \), with repetitions allowed, is weakly Lindelöf in the sense that each open cover admits a countable subfamily with dense union.) Hypotheses such as (a), (b), (c), or (d) cannot be eliminated entirely, however. Specifically, there is a space \( X = \prod_{a \in A} X_a \), and a continuous function on one of its spaces \( \Sigma(p) \) to \([0,1]\), not extendable continuously to some point in \( X \). In the simplest of several instances, one can choose all but one of the spaces \( X_a \) to be the one point-compactification of a discrete space with \( \aleph_1 \) points; and one can choose the exceptional space to be discrete at all but one of its points. (Received May 26, 1969.) (Author introduced by Professor W., W., Comfort.)

69T-G106. RICHARD J. TONDRA, Iowa State University, Ames, Iowa 50010. Connected 2-manifolds with finite domain rank.

Results presented in Abstract 68T-G5, these Notices 15 (1968), 810, have been extended as follows: Theorem 1. Let \( M \) be a compact, connected 2-manifold of genus \( n \) with \( p \equiv 0 \) boundary components and let \( r(M) = 1 \) if \( M \) is orientable and 0 otherwise. Then \( DR(M) = ((p + 2)(p + 1)/2)((n - 1)/2)(1 - r(M)) + (n + 1) \). Theorem 2. Let \( M \) be a connected 2-manifold. Then \( DR(M) \) is finite iff \( M \) is homeomorphic to a domain of a compact, connected 2-manifold. (Received May 19, 1969.)

69T-G107. JAMES M. McPHERSON, University of New South Wales, Australia, and University of Virginia, Charlottesville, Virginia 22903. An invariant of local type for arcs in a 3-manifold. Preliminary report.

\( k \) is a knot or an arc in \( M^3 \), with one wild point \( w \). \( P_1 \) is the penetration index of \( k \) with respect to solid tori, i.e. \( P_1 \) is the smallest integer \( n \) such that there exists a sequence of solid torus neighborhoods \( \{ V_i \} \) of \( w \), each \( V_i \) meeting \( k \) on its boundary in \( n \) points. \( P_0 \) is the usual (2-sphere) penetration index. Choose a 3-cell \( E_0 \supseteq w \), such that every 2-sphere in \( \text{Int } E_0 \) meets \( k \) in \( P_1 \) points; define a \( k \)-sequence to be a sequence of solid tori \( \{ V_i \} \) in \( E_0 \) such that (i) for all \( i \), \( N(k \cap \text{Bd } V_i) = P_1 \), \( V_{i+1} \subseteq \text{Int } E_i \) for some 3-cell \( E_0 \), and \( V_i = w \); and (ii) if \( V_i \) is a solid torus with \( N(k \cap \text{Bd } V_i) = P_1 \), and \( V_{i+1} \subseteq V \subseteq V_i \), then either \( V_{i+1} \) has nonzero order in \( V \), or \( \text{Cl}(V_i - V) \cong S^1 \times S^1 \times I \). Theorem. Let \( k \) have \( P_1 \leq 2 \) and \( P_0 > 3P_1 \). Then \( k \) has a \( k \)-sequence \( \{ V_i \} \) in some 3-cell in \( M \). Moreover, if \( k_1 \) is of the same local type as \( k \) [S. J. Lomonaco, Trans. Amer. Math. Soc. 129 (1967), 323] and \( [U_j] \) is a \( k_1 \)-sequence, then the sequences \( [k(V_i)] \) and \( [k(U_j)] \) are cofinal. (\( k(T) \) denotes the knot type of a longitude of \( \text{Bd } T \).) Corollary. There exist uncountably many arcs which are wild at one endpoint, for each \( P_0 \geq 5 \). In fact, there exist uncountably many noninvertible such arcs. (Received May 26, 1969.)

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Let $X$, $Y$ be locally compact nonhomeomorphic topological Hausdorff spaces such that $C_0(X)$ and $C_0(Y)$ are isomorphic Banach spaces. Let $d = \inf \{ \| T \| \; | T \text{ is an isomorphism of } C_0(X) \text{ onto } C_0(Y) \}$. M. Cambern in ["On isomorphisms with small bound," Proc. Amer. Math. Soc. 18 (1967), 1062-1066] proved that $d \geq 2$. Denote by $|S|$ the cardinal number of a set $S$. Theorem 1. If $X,Y$ satisfy the first axiom of countability, then $|X| = |Y|$. Denote by $x'$ the set of accumulation points of the space $X$. Theorem 2. If (i) $X' = \emptyset$ or if (ii) $X'$ is finite and $Y'$ is infinite. An example with $X'$ and $Y'$ finite and $d = 2$ is given in [M. Cambern, Abstract 69T-B11, these Notices 16 (1969), 317].

Theorem 3. If $X'$ and $Y'$ are finite and $d < 3$, then $|Y'|/|X'| \leq \frac{2d}{3 - d}$.

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Author introduced by Dr. Micha A. Perles.

69T-Gl09. NORMAN HOWES and RAY CHANDLER, University of Dallas, Dallas Station, Texas 75061, and Texas Instruments, Incorporated, Dallas, Texas 75222. Accumulation complete and sequential spaces.

Arhangel'skii (Soviet Math. Dokl. 4 (1963), 1726) and Franklin (Fund. Math. 61 (1967), 51) characterized Fréchet and sequential spaces as continuous pseudo-open images and quotients respectively of $I^0$ spaces. Franklin investigated: When is a sequential space Fréchet? Accumulation complete (A.C.) spaces are such that each sequence that accumulates to a point $p$ has a subsequence that converges to $p$. Theorem. A sequential space is Fréchet if and only if it is A.C. For additional definitions see Abstract 654-33, these Notices 15 (1968), 346. Theorem. A function $f$ from $X$ onto $Y$ is pseudo-open if and only if for each WOS $\mathcal{C} \subseteq Y$ quasi-converging to $p$ there is a WOS $\mathcal{E} \subseteq f^{-1}(\mathcal{C})$ quasi-converging to a point of $f^{-1}(p)$. An onto function $f$: $X \rightarrow Y$ is $N_0$ pseudo-open if and only if for each sequence $(x_n) \subseteq Y$ that quasi-converges to $p$ there is a WOS $\mathcal{E} \subseteq f^{-1}(\{x_n\})$ that quasi-converges to a point of $f^{-1}(p)$. Theorem. A space $X$ is continuous if and only if each sequentially continuous function on $X$ is continuous.

Theorem. In a CLB space sequential $\Rightarrow$ Fréchet $\Rightarrow 1$. (Received May 15, 1969.)

69T-GII0. DOUGLAS E. CAMERON, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Maximal Lindelöf spaces. II. Preliminary report.

(For definitions and other results, see Abstract 68T-H24, these Notices 15 (1968), 944.) Lemma. Finite products of nontrivial quasi-$P$ spaces are quasi-$P$; infinite products never are. Theorem 5. If a finite product of spaces is maximal Lindelöf, then each component space is maximal Lindelöf. Theorem 6. Finite products of maximal Lindelöf $T_2$ spaces are maximal Lindelöf $T_2$ spaces.

Corollary 1. Finite products of Lindelöf spaces for which there exist stronger maximal Lindelöf $T_2$ spaces are Lindelöf. Theorem 7. A maximal Lindelöf space $(X,T)$ is separable, first countable, or second countable if and only if cardinality $X \leq N_0$. Theorem 8. Let $(X,T)$ be a topological space such that $X = \bigcup_{n=1}^{N} B_1$ and each $B_1$ is maximal Lindelöf in its relative topology. Then $(X,T)$ is maximal Lindelöf if and only if each $B_1$ is closed in $T$. Example. Not all maximal Lindelöf spaces are $T_2$.

(Received June 2, 1969.)

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Let $M^n$ and $Q^q$ be topological manifolds. Then a map $f: M \to Q$ is a merision if and only if for all $x \in M$, there exist local charts $h: R^m \to M$, $g: R^q \to Q$ ($x \in h(R^m)$) such that $g^{-1} fh: R^m \to R^q$ is the natural inclusion if $m \ge q$ and projection on the first $q$ coordinates if $m \le q$. In the former case $f$ is an immersion and in the latter a submersion. Let $TM$ denote the tangent microbundle $M \overset{\Delta}{\to} M \times M \overset{p_1}{\to} M$.

A bundle map $\varphi: TM \to TQ$ is a representation if and only if its restriction to some neighborhood of $\Delta(M)$ in $M \times M$ is a merision on each fiber. If $f: M \to Q$ is a merision, then its differential $df: TM \to TQ$ is a representation. Let $N$ be a closed subset of $M$, $M$ a locally flat submanifold of $M'$, where $M'$ is of dimension $n = \max(m, q)$. Let $g: U \to Q$ be a merision of a neighborhood $U$ of $N$ in $M'$. Let $\mathcal{J}_N(M, Q)$ be the s.s. complex of germs (in $M'$) of merisions of $M$ in $Q$ which agree with the germ of $g$, and $\mathcal{E}_N(M, Q)$ the s.s. complex of germs of representations agreeing with the germ of $dg$. Then the differential gives a s.s. map $d: \mathcal{J}_N(M, Q) \to \mathcal{E}_N(M, Q)$. Theorem. If $m < q$ or if every component of $M - N - N$ meets $M' - M$, then $d: \mathcal{J}_N(M, Q) \to \mathcal{E}_N(M, Q)$ is a homotopy equivalence. (Received June 2, 1969.)

69T-G112. ANDREW C. CONNOR, University of Georgia, Athens, Georgia 30601. An algebraic characterization of the 3-sphere.

Theorem. If $M$ is a closed 3-manifold such that the fundamental group of the complement of each polyhedral simple closed curve in $M$ is isomorphic to a knot group (that is, the fundamental group of the complement of a polyhedral simple closed curve in $S^3$), then $M = S^3$. (Received June 5, 1969.)


The following are proved: A highly divergent sequence such that every subsequence has a side point has an uncountable number of side points. For definitions, see author's, 1965, abstracts in these Notices or "Sequences in topological spaces," Prace Mat. 11 (1968), 329-336. In a $T_4$ space no highly divergent sequence in $X$ converges in $\beta X$. For the rest of the theorems $X$ is $T_{3\frac{1}{2}}$. If the set of values of a highly divergent sequence in $X$ is a zero set then the sequence does not converge in $\beta X$. Sequences in $X$ with side points do not converge in $\beta X$. If sequences in $X$ either converge in $\beta X$ or have all their side points in $X$, then $X$ is pseudo-compact. (Received June 11, 1969.)


For integers $n \geq 3$, let $(X_n, \rho)$ be a metric space such that (1) $X_n \subset (K_n, \rho)$, a compact $n$-dimensional metric space; (2) $X_n = K_n - \cup_{i=1}^{[n/2]} A_i$, where the $A_i$ are mutually disjoint and closed in $K_n$; and (3) $\mu \dim (X_n, \rho) = [n/2]$ and $\dim X_n = n - 1$. (Here $\mu \dim$ denotes metric dimension and $\dim$ denotes covering dimension.) K. Sitnikov [Dokl. Akad. Nauk SSSR 88 (1953), 21-24] has constructed such spaces. The present result is that there exists a complete metric $\sigma$ on $X_n$ equivalent to $\rho$ such that $\mu \dim (X_n, \sigma) \leq [n/2] + 1$. It is known that $d_2(X, d) \leq \mu \dim (X, d)$ for all metric spaces $(X, d)$, where $d_2$ is the metric-dependent dimension function introduced by K. Nagami and J. H. Roberts [Proc. Amer. Math. Soc. 16 (1965), 601-604]. Thus the present result gives a negative
answer to the following question posed by Richard E. Hodel [Topology Conference, Arizona State University, 1967, Tempe, 1968, pp. 162-167]. Is $d_2(X,d) = \dim X$ for all complete metric spaces $(X,d)$?

Problem. For integers $n \geq 3$, do there exist complete metric spaces $(X_n,d)$ with $\mu \dim (X_n,d) = \lceil n/2 \rceil$ and $\dim X_n = n - 1$? (Received June 11, 1969.)

69T-G115. WILLIAM LAURENCE YOUNG, University of Illinois, Urbana, Illinois 61801. A product space with the fixed point property.

R. H. Bing, Amer. Math. Monthly, 76 (1969), 119-132, utilizes an example of a 1-dimensional arcwise connected continuum $X$ in $E^3$ with the fixed point property. His question (5) asks if $X \times I$ has the fixed point property. The answer is yes, and the proof given uses standard techniques of point-set topology. (Received June 11, 1969.)

69T-G116. HOWARD W. LAMBERT, University of Iowa, Iowa City, Iowa 52240. Replacing certain maps of 3-manifolds by homeomorphisms.

Let $f$ be a map of a 3-manifold $M_1$ onto a 3-manifold $M_2$. Call $f$ boundary preserving if $f|\text{Bd } M_1$ is a homeomorphism onto $\text{Bd } M_2$ and $f|\text{Int } M_1 = \text{Int } M_2$. Let $S_f = \{x \in M_1$ and $f^{-1}(x)$ is nondegenerate$\}$. Following the definition given by D. R. McMillan in Abstract 69T-G41, these

Notices 16 (1969), 582-583, we say $X \subset M$ is strongly $n$-acyclic (over $Z$) if for each open set $U \subset M$ containing $X$ there is an open set $V$ so that $X \subset V \subset U$ and injection of $H_n(V,Z)$ into $H_n(U,Z)$ is trivial.

Theorem 1. Suppose $M_1$ is a compact, connected, irreducible 3-manifold with nonempty boundary and $M_2$ is an orientable irreducible 3-manifold. Then if $f$ is a boundary preserving map of $M_1$ onto $M_2$ and there is a closed set $S$ so that $f(S) \subset X \subset \text{Int } M_2$ and each component of $X$ is strongly 1-acyclic, then $M_1$ is homeomorphic to $M_2$. D. R. McMillan in the given reference has already obtained a version of the following: Corollary 1. Suppose $M_1$ is an irreducible 3-manifold with nonempty boundary. Then if $f:M_1 \rightarrow M_2$ is boundary preserving and $f(\text{Cl } S_f)$ is 0-dimensional, then $M_1$ is homeomorphic to $M_2$. Theorem 2. Suppose $M_1$, $M_2$ are cubes with one hole and the commutator subgroup of $\Pi_1(M_1)$ is finitely generated. Then if $f$ is a boundary preserving map of $M_1$ onto $M_2$ and $f(\text{Cl } S_f)$ shrinks to a point in $M_2$, then $M_1$ is homeomorphic to $M_2$. (Received June 16, 1969.)

69T-G117. ERNEST S. BERNEY, Arizona State University, Tempe, Arizona 85281. A completely regular semimetric space with no $\sigma$-discrete network. Preliminary report.

A completely regular, compact semimetric space $X$ is constructed which has no $\sigma$-discrete network thus answering a question of A. V. Arkhangel'ski! if $\omega_1 = 2^\omega$, then $X$ is also Lindelöf and hence answers a question of R. W. Heath: Is every regular Lindelöf semimetric space cosmic? Also, if $\omega_1 = 2^\omega$, it is shown that $X \times X$ is not normal, hence not Lindelöf. (Received June 18, 1969.)


Reasonableness of a transitive homeomorphism group $G$ of a topological space $M$ (i.e., $G$ is a topological group and the function $f:G/G_m \rightarrow M$ defined by $f(gG_m) = g(m)$ is a homeomorphism where $G_m = \{g \in G|g(m) = m\}$ for $m \in M$) has been investigated by a number of people. (See, for example, Gerald S. Ungar, "Local homogeneity," Duke Math. J. 34 (1967), 693-700.) Reasonableness is generalized to semigroups of mappings ($s$-reasonableness). Two topologies similar to the compact-
open topology are introduced and their s-reasonableness and reasonableness are investigated. Ungar's concept of uniformly locally homogeneous is extended to quasi-uniform spaces and generalized by replacing homeomorphisms with mappings. Connections are drawn between the properties of \( G, G_m \), and \( M \). If \( M/R \) is a decomposition space of \( M \), then the relationship between the reasonableness of \( G \) and the reasonableness of a group of homeomorphisms of \( M/R \) is considered. (Received June 19, 1969.)


K is a negligible subset of the complete metric space \( X \) if \( X \) is homeomorphic to \( X - K \). K is extractable from \( X \) if such a homeomorphism exists which is isotopic, through homeomorphisms of \( X \) onto itself, to the identity on \( X \), and the isotopy satisfies given smallness conditions. A Fréchet manifold is a manifold modelled on an infinite-dimensional Fréchet space. Recent results about negligible subsets of separable Fréchet manifolds are generalized to include nonseparable cases. For the separable case, see R. D. Anderson's papers in Bull. Amer. Math. Soc. 74 (1968), 771-792, and 75 (1969), 64-67. The following results are corollaries to a more general theorem proved in the paper: Theorem. Countable unions of locally compact sets are extractable from Fréchet manifolds. Theorem. For \( i = 1, 2, \ldots \), suppose \( M_i \) is a Fréchet manifold which is locally collared in \( X \), and \( K_i \) is a locally closed subset of \( M_i \). Then \( \bigcup K_i \) is extractable from \( X \). (Received June 19, 1969.)


Let \( \beta N \) be the Stone-Čech compactification of the integers \( N \). Theorem 1. If \( p \) is a P-point of \( \beta N/N \), then \( \beta N/N/\{p\} \) is not normal. This question is attributed to L. Gillman in a paper by W. W. Comfort and S. Negrepontis, "Homeomorphs of three subspaces of \( \beta N/N \)," Math. Z. 107 (1968), 55. Let \( D \) be an uncountable discrete set, and let \( E_0 \) be the set of points in \( \beta D/D \) in the closures of countable subsets of \( D \). Theorem 2. There is a two-valued bounded continuous function on \( E_0 \) which cannot be extended continuously to \( \beta D \). This question is raised by N. J. Fine and L. Gillman in "Extension of continuous functions in \( \beta N \)," Bull. Amer. Math. Soc. 66 (1960), 381. (Received June 12, 1969.)


Throughout, suppose \( M^3 \) and \( N^3 \) are compact piecewise-linear 3-manifolds either both with or both without boundary, and suppose \( f \) is a monotone continuous mapping from \( M^3 \) onto \( N^3 \) where, if \( S_f \) is the set of \( x \in N^3 \) such that \( f^{-1}(x) \) is nondegenerate, \( S_f \subset \text{Int} N^3 \). A compact connected subset \( X \) of \( \text{Int} M^3 \) is strongly acyclic if for each open set \( U \subset M^3 \) such that \( X \subset U \), there is an open set \( V \) such that \( X \subset V \subset U \) and such that for \( i > 0 \), the image of \( H_i(V; Z) \) in \( H_i(U; Z) \) is zero under the inclusion induced homomorphism. Let \( A^1_f \) be the set of \( x \in N^3 \) such that \( f^{-1}(x) \) is not strongly acyclic. Let \( C^2_f \) be the set of \( x \in N^3 \) such that \( f^{-1}(x) \) is not cellular. Theorem. If \( A^2_f \) is 0-dimensional, then \( C^2_f \) is a finite set. Corollary 1. If \( S^2_f \) is 0-dimensional, then \( C^2_f \) is a finite set. Corollary 2. If \( A^1_f \) is 0-dimensional, then \( N^3 \) can be obtained from \( M^3 \) by cutting out of \( M^3 \) a finite number of disjoint 3-manifolds each with boundary a 2-sphere, and replacing each by a 3-cell. Corollary 3. If \( A^1_f \) is
Negligible sets for real connectivity functions.

Definition. If \( g \) is a function from the interval \( I = 0,1 \) into \( I \) and \( M \) is a subset of \( I \), then the statement that \( M \) is \( g \)-negligible means that every function from \( I \) into \( I \) which agrees with \( g \) on \( I - M \) is a connectivity function. 

Theorem 1. If \( M \) is a subset of \( I \), then the following are equivalent:

(i) there exists a function \( g \) from \( I \) into \( I \) such that \( M \) is \( g \)-negligible, and

(ii) every subinterval of \( I \) contains \( c \)-many points of \( I - M \) (\( c \) is the cardinality of the continuum).

Theorem 2. If \( g \) is a connectivity function from \( I \) into \( I \), then the following are equivalent:

(i) \( g \) is dense in \( I \),

(ii) every nowhere dense subset of \( I \) is \( g \)-negligible, and

(iii) there is a dense subset of \( I \) which is \( g \)-negligible.

"Nowhere dense" cannot be replaced by "countable" in (ii), but the following is true. Theorem 3.

There exists a function \( g \) from \( I \) into \( I \) such that every subset of \( I \) which has cardinality less than \( c \) is \( g \)-negligible. Theorem 1 implies that there is a function \( g \) from \( I \) into \( I \) which has a negligible set of length one, but since well-ordering techniques are used in the proof of Theorem 1, it might be difficult to imagine what such a function might be like. An effective procedure can be described by which such a function can be constructed. (Received June 23, 1969.)

An \( H \)-manifold is a triple \((M, m, e)\) where \( M \) is a compact connected manifold without boundary, \( e \in M \), and \( m : M \times M \to M \) is a map such that \( m(x, e) = m(e, x) = x \) for all \( x \in M \). Define \( m_k : M \to M \) for \( k \geq 2 \) by \( m_k(x) = x \) and, for \( k \geq 2 \), let \( m_{k-1}(x) = m(x, m_{k-1}(x)) \).

A \([\text{primitive}]\) \( k \)-th root of unity in an \( H \)-manifold \((M, m, e)\) is a point \( x \in M \) such that \( m_k(x) = e \) and \( m_{j}(x) \neq e \) for \( j < k \). Call \( k \)-th roots \( x \) and \( x' \) equivalent if there is a path \( C \) in \( M \) from \( x \) to \( x' \) such that \( [m_k C] = 0 \in \pi_k(M, e) \).

An equivalence class of \( k \)-th roots of unity is called totally primitive if every element in it is primitive. Let \((M, m, e)\) be an \( H \)-manifold and set \( \pi = \pi_1(M, e) \). For each integer \( k \), let \( \pi_k \) denote \( \pi / k \pi \). Theorem. For any integer \( k \geq 2 \), the number of totally primitive classes of \( k \)-th roots of unity in \((M, m, e)\) is precisely \( \left| \pi_k \right| \pi_1 \) \( (1 - p^{-\left(p_1\right)}) \) where, for each prime \( p \), \( (p_1, p) \) denotes the number of infinite cyclic groups plus the number of cyclic groups of order a power of \( p \) in the canonical decomposition of \( \pi \).

(Received June 9, 1969.)

The monotone mapping problem asks if each monotone map \( f (f^{-1}(y)) \) is a compact connected set of Euclidean \( n \)-space \( E^n \) onto itself is compact (inverses of compact sets are compact). G. T. Whyburn gave an affirmative solution for \( n = 2 \) and L. C. Glaser a negative one for \( n \geq 4 \). The solution is completed by exhibiting a monotone noncompact map \( f_{2} \) of \( E^3 \) onto itself where \( f_{1} \) is the noncompact one-to-one map of Euclidean half space \( E^3_{+} \) onto \( E^3_{+} \) described by Glaser and \( f_{2} \) is a monotone map of \( E^3 \) onto \( E^3_{+} \) whose point inverses are finite graphs, annuli, and cartesian products of arcs with \( \theta \)-curves. This last map \( f_{2} \) is a by-product of a monotone map of a 3-cell \( I^3 \) onto itself all of whose point inverses intersect the base of \( I^3 \). (Received June 23, 1969.)
Miscellaneous Fields


The graphs $G_n$ are defined as follows: Let $G_1 = C_{2m_1}$, the cycle on $2m_1$ points, and recursively define the cartesian product $G_n = G_{n-1} \times C_{2m_2}$ for $n \geq 2$. Then the genus of $G_n$ is $1 + 2^{n+2}(\prod_{i=1}^{n} m_i)$, $(n-2)$ provided $n \geq 2$ and $m_i \neq 2$, $i = 1, \ldots, n$. Now define the graphs $H_n$ by $H_1 = P_{m_1}$, the path on $m_1$ points, and $H_{n+1} = H_n \times P_{m_n}$, for $n \geq 2$. Let $m_1$, $m_2$, and $m_3$ be even; then the genus of $H_n$ is $1 + (1/4)((n-2) \prod_{i=1}^{n} m_i \sum_{i=1}^{n} ((1/m_i) \prod_{j=1}^{n} m_j))$, for $n \geq 3$; and 0 otherwise. In both cases, the method of proof is to establish the existence of an imbedding of the bipartite graph in question in a compact orientable surface in such a manner that every face is a quadrilateral. For bipartite graphs such imbeddings are minimal, and the genus may be computed using the Euler formula. (Received April 7, 1969.)


N. J. Hicks (Rend. Circ. Mat. Palermo 12 (1963), 137-149) considered $\mathbb{R}^n$ a Riemannian manifold with connexion $\Gamma$, and $M^k$ a submanifold with induced metric and connexion. Every point in $M$ has a neighborhood $U \subset M$ with a frame $N_1, \ldots, N_{n-k}$ of orthonormal fields normal to $U$. For vectors $X$ tangent to $U$ at a point, $L_i(X) = \tan X N_i$ and $S_i(X) = \nor X N_i$ $(1 \leq i \leq n - k)$. This paper further develops methods to generalize results known when $n - k = 1$. A point $m$ in $M$ is said to be totally principal if $L_1, \ldots, L_{n-k}$ share the same principal directions at $m$, and $m$ is said to be totally umbilic if every $L_i$ is a multiple of the identity on $M_m$ $(1 \leq i \leq n - k)$. Lemma. If one of these defined conditions occurs at $m$ for one normal frame, it occurs at $m$ for all normal frames. Theorem 1. If $M$ has constant curvature and $M$ consists entirely of totally principal points, then every point in $M$ has a neighborhood $U \subset M$ with normal frame $N_1, \ldots, N_{n-k}$ such that $S_1 = S_2 = \cdots = S_{n-k} = 0$ throughout $U$. Theorem 2. If $M$ has constant curvature $K$, $k \geq 2$, and $M$ is connected and consists entirely of totally umbilic points, then $M$ has constant curvature $K \geq K$. (Received April 28, 1969.)


This paper generalizes the notion of independence in a graph $G$ and extends theorems of Shannon, Rosenfeld, and the author concerning the (direct) product of graphs (cf. Abstract 658-11, these Notices 15 (1968), 722). A set $S$ of vertices of $G$ is $d$-independent, $d = 0, 1, 2, \ldots$, iff $v_1, v_2 \in S$ implies $d(v_1, v_2) > d$, where $d(\cdot, \cdot)$ is the "shortest path" length in $G$. The $d$-independence number $\beta_d$ of $d$-clique, $d$-preserving map, $d$-universal, and $d$-capacity are defined analogously. $G$ is totally universal if $G$ is $d$-universal for all $d$. Theorem 1. If $G$ is reducible to a $d$-independent set by a $d$-preserving map (is $d$-reducible), $G$ is $d$-universal. Shannon showed this for $d = 1$. Theorem 2 extends Rosenfeld's n.a.s. conditions for $d$-universality from $d = 1$ to arbitrary $d$. Theorem 3. If $G$ has no odd cycles, $G$ is 1-reducible. Let $C_n$ be the $n$-gon graph. Theorem 4. $C_n$ is $d$-reducible iff $C_n$ is $d$-universal iff $d + 1 > n/2$ or $d + 1 \mid n$. Otherwise, given integers $j, k \geq 2$; $r, s$ such that
\[ 0 < r, s < d + 1; \text{and } m = (d + 1)r + s, n = (d + 1)k + s, \beta_d(C_m \times C_n) = \beta_d(C_m) \cdot \beta_d(C_n) + \min([js/(d + 1)], [kr/(d + 1)]). \]

Corollary. \( C_3, C_4, C_6 \) are the only totally universal (hence only totally reducible) cycles. Other results concerning \( d \)-capacity are obtained. (Received May 8, 1969.)

69T-H42. ROBERT E. BECK, Villanova University, Villanova, Pennsylvania 19085.

Connections on semisimple Lie groups.

The results announced in Abstract 663-34, these Notices 16 (1969), 96, are extended to real semisimple Lie groups. Theorem 1. The set of left invariant connections on a real semisimple Lie group \( G \) which have (a) zero curvature and (b) maximal geodesics through the identity as one-parameter subgroups is in one-to-one correspondence with the set of decompositions of \( g \), Lie algebra of \( G \), into the vector space direct sum of an ideal and a subspace. Theorem 2. The only bi-invariant connections on a real semisimple Lie group with properties (a) and (b) are the plus and minus connections of Cartan. (Received May 12, 1969.)


Let \( G = K \times H \) be a semidirect product of semisimple Lie groups over \( \mathbb{R} \) and let \( V(G) = G \times H \rightarrow V \) be the vector bundle over \( G/H \) associated with the irreducible finite-dimensional \( H \) module \( V \). We confine our remarks here to first order invariant operators having local value \( \sum_{a} a_{a}(0)D^{a} \) at 0 in canonical coordinates \( (a_{a}(0) \in L(V, W)) \). The \( a_{a}(0) = a_{a}(0) \) for \( a = (0, \ldots, 1_{m}, ..., 0) \), \( 1 \leq m \leq r = \text{dim } k^{r} \), are shown to be characterized by \( Q_{1}(d^{m} \otimes v) = a_{m}(0)v \) where \( Q_{1} \) is an \( h^{r} \)-module homomorphism \( J_{1}(g) \otimes h^{r} \rightarrow W \) and this leads to a multiplication table \( [x, a_{m}(0)] + \sum_{i=1}^{r} \beta_{i}(X)a_{i}(0) = 0, X \in h^{r} \), dual to the representation \( \Gamma; h^{r} \rightarrow \text{End } k^{r} \) determined by \( \gamma \). Reduction of this dual representation, suitably complexified, involves solving canonical irreducible multiplication tables in \( L(V^C, W^C) \) relative to root space decompositions of simple complex Lie algebras and canonical bases for \( V^C \) and \( W^C \). The canonical coefficients (which will be linear combinations of the \( a_{i}(0) \)) can then be readily calculated; this has been carried out in detail for various low rank situations. Various compatibility requirements arise from the necessity of satisfying natural "cutoff" conditions. As an application one obtains canonical variations (and generalizations) of some of the results of Gelfand, NaUMark, et al., on this subject. (Received May 19, 1969.)

69T-H44. ROBERT L. CONSTABLE, Department of Computer Science, Cornell University, Ithaca, New York 14850. The operator gap.

Let \( \{ \phi_{i}( ) \} \) be an acceptable indexing of the partial recursive functions and \( \Phi = \{ \phi_{i}( ) \} \) a measure of computational complexity (cf. Blum, M. "A machine-independent theory of computational complexity," J. Assoc. Comput. Mach. 14 (1967), 322-336). Let \( \overline{R}_{\Phi} = \{ t( ) : \phi_{i}(x) \leq t(x) \text{ a.e. } x \} \) where lower case English letters denote recursive functions, Theorem (Operator Gap). For all \( \Phi \) and all general recursive operators \( F[ ] \) there are arbitrarily large recursive \( t( ) \) such that \( t(n) < \phi_{i}(n) < F[t( )](n) \) i.o. implies \( F[t( )](m) < \phi_{i}(m) \) i.o. Corollary 1. For all \( \Phi \) there is no general recursive operator \( F[ ] \) such that for all \( t( ) \) \( R_{\Phi} \subset F[t( )] \). Corollary 2. For all \( \Phi \) and for all increasing general recursive operators \( F[ ] \) there are arbitrarily large \( t( ) \) such that \( F[t( )] = F[t( )] \). These results generalize to operators a weak form of Borodin's Gap Theorem (1969 ACM Symposium on the Theory of Computing). (Received May 22, 1969.)
Nested iterated substitution and full AFL's.

Definition. An iterated substitution \( f \) is nested if \( a \in f(a) \) for all \( a \). A superAFL is a family of languages closed under nested iterated substitution (n.i.s.), union with unitary sets and intersection with regular, containing at least one nonunitary set. \( S(L) \) is the substitution closure of \( L \); \( S^\infty(L) \) is the least superAFL containing \( L \). Theorem 1. Every superAFL is a full AFL closed under substitution and containing the context-free languages which form the smallest superAFL. Theorem 2. Let \( L \) be a full AFL. (A) Each member of \( S^0(L) \) can be obtained from \( L \) by intersection with regular and one application of n.i.s. (B) If \( L \) is full principal, so is \( S^0(L) \). (C) If \( L \) has a decidable emptiness problem, so does \( S^0(L) \). Corollary. The index languages are a super AFL but not the least superAFL containing the one-way stack languages. Definition. If \( D \) is an AFA, let the corresponding well-nested AFA be \( D^w \), the AFA using as data structure a pushdown store of data structures of \( D \). Theorem 3. \( L(D^w) = S(L(D)) \).

An extension of the Farkas-Minkowski theorem.

Let \( f(x) \) and \( g_i(x) \), \( i \in I \), be mappings of the n-space \( R^n \) to the real numbers. For any finite set \( I \) of positive integer values of the index \( i \) we denote by \( \Gamma_i \) the set \( \{x \mid g_i(x) \leq 0, \ i \in I \} \) and by \( g(x) \) the vector in \( k \)-space \( (g_i(x)), (i \in I) \) where \( k = |I| \) is the cardinality of \( I \). A polyhedral set will be a finite intersection of closed half-spaces; the positive orthant of \( R^n \) is the set \( \{x \mid x \in R^n, x \geq 0 \} \) and the scalar product of two vectors \( x \) and \( y \) will be written \( xy \). Then the Farkas Minkowski theorem (C. Berge and A. Ghouila Houri, "Programming, games and transportation networks," Wiley, 1965, Chapter 61) is extended in the following way: Theorem. Suppose \( f(x) \) and \( g_i(x) \), \( (i \in I) \) to be concave and \( X \subseteq R^n \) to be convex. Let \( I = L \cup N \) be a partition of \( I \) defined by \( i \in L \) if and only if \( g_i(X_0) \) is linear affine, and finally assume the following constraint qualification: There exists \( x_0 \in \Gamma_i \cap X \) such that (i) if \( i \in N \) then \( g_i(x_0) > 0 \) and (ii) if \( X \) is not polyhedral, then \( x_0 \) belongs to the relative interior of \( X \). Under these conditions if \( X \cap \Gamma_i \cap \{x \mid f(x) > 0 \} = \emptyset \), then there exists a vector \( \gamma \in R^{|I|} \) such that \( f(x) + \gamma g(x) \leq 0 \) for all \( x \in X \). If in addition \( \{x \mid f(x) > 0 \} \cap X = \emptyset \), then \( \gamma \neq 0 \). (Received May 27, 1969.)
The main result of this paper is, roughly, the following: **Theorem.** If $G$ is a planar cubic graph with interchange graph $G'$, and $M$ is the $2n \times 2n$ square matrix formed by "stacking" two copies of the vertex-(directed) edge matrix of $G'$, then the number of edge 3-colorings of $G = (1/2^n) |\text{Permanent}(M)|$. A similar but much weaker result is derived for arbitrary graphs. (Received May 19, 1969.) (Author introduced by Professor Sanford L. Segal.)

**69T-H50. WITHDRAWN.**

**69T-H51. BEN WEGBREIT, Aiken Computation Laboratory, Harvard University, Cambridge, Massachusetts 02138. A generator of context-sensitive languages.**

The existence of a context-sensitive grammar, $G_u$, which acts as a "generator" of all context-sensitive languages is established. Specifically, $G_u$ has the property that for each context-sensitive language, $L$, there exists a regular set, $R_L$, and an e-limited gsm, $g_L$, such that $L = g_L(L(G_u) \cap R_L)$. It follows that the family of context-sensitive languages is a principal AFL. An analogous result is proved for deterministic context-sensitive languages. (Received June 18, 1969.)
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