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There are partial solutions to approximately one-third of the exercises.

Graduate Studies in Mathematics, Volume 32; 2001; approximately 448 pages; Hardcover; ISBN 0-8218-0845-1; List $59; All AMS members $47; Order code GSM/32NT103

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Eva Bayer-Fluckiger, CNRS, Université de Franche-Comté, Besançon, France; David Lewis, University College, Dublin, Ireland; and Andrew Ranicki, University of Edinburgh, Scotland, Editors

This volume outlines the proceedings of the conference on “Quadratic Forms and Their Applications” held at University College Dublin. It includes survey articles and research papers ranging from applications in topology and geometry to the algebraic theory of quadratic forms and its history. Various aspects of the use of quadratic forms in algebra, analysis, topology, geometry, and number theory are addressed. Special features include the first published proof of the Conway-Schneeberger Fifteen Theorem on integer-valued quadratic forms and the first English-language biography of Ernst Witt, founder of the theory of quadratic forms.

Contemporary Mathematics, Volume 272; 2000; 311 pages; Softcover; ISBN 0-8218-2779-5; List $79; Individual member $47; Order code CONM/272NT103

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Constraint programming has become an important general approach for solving hard combinatorial problems that occur in a number of application domains, such as scheduling and configuration. This volume contains selected papers from the workshop on Constraint Programming and Large Scale Discrete Optimization held at DIMACS. It gives a sense of state-of-the-art research in this field, touching on many of the important issues that are emerging and giving an idea of the major current trends. Topics include new strategies for local search, multithreaded constraint programming, specialized constraints that enhance consistency processing, fuzzy representations, hybrid approaches involving both constraint programming and integer programming, and applications to scheduling problems in domains such as sports scheduling and satellite scheduling.

DISMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 57; 2001; 175 pages; Hardcover; ISBN 0-8218-2710-3; List $55; Individual member $33; Order code DIMACS/57NT103

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J. P. C. Greenlees, University of Sheffield, UK, Editor, with assistance from Robert R. Bruner, Wayne State University, Detroit, MI, and Nicholas Kuhn, University of Virginia, Charlottesville

This volume presents the proceedings from the AMS-IMS-SIAM Summer Research Conference on Homotopy Methods in Algebraic Topology held at the University of Colorado (Boulder). The conference coincided with the sixtieth birthday of J. Peter May. An article is included reflecting his wide-ranging and influential contributions to the subject area. Other articles in the book discuss the ordinary, elliptic and real-oriented Adams spectral sequences, mapping configuration spaces, extended powers, operads, the telescope conjecture, p-compact groups, algebraic K theory, stable and unstable splittings, the calculus of functors, the E∞ tensor product, and equivariant cohomology theories. The book offers a compendious source on modern aspects of homotopy theoretic methods in many algebraic settings.

Contemporary Mathematics, Volume 271; 2001; approximately 344 pages; Hardcover; ISBN 0-8218-2921-2; List $79; Individual member $47; Order code CONM/271NT103

**Navier--Stokes Equations**
Theory and Numerical Analysis
Roger Temam, Indiana University, Bloomington

From a review for the First Edition:

This book, in many ways remarkable, gives a detailed account of a number of results concerned with the theory and numerical analysis of the Navier-Stokes equations of viscous incompressible fluids.

—Zentralblatt für Mathematik

This book was originally published in 1977 and has since been reprinted four times (the last reprint was in 1985). The current volume is reprinted and fully retyped by the AMS. It is very close in content to the 1985 edition. The book presents a systematic treatment of results on the theory and numerical analysis of the Navier-Stokes equations for viscous incompressible fluids. Considered are the linearized stationary case, the nonlinear stationary case, and the full nonlinear time-dependent case. The relevant mathematical tools are introduced at each stage.

The new material in this book is Appendix III, reproducing a survey article written in 1998. This appendix contains a few aspects not addressed in the earlier editions, in particular a short derivation of the Navier-Stokes equations from the basic conservation principles in continuum mechanics, further historical perspectives, and indications on new developments in the area. The appendix also surveys some aspects of the related Euler equations and the compressible Navier-Stokes equations. Readers are advised to peruse this appendix before reading the core of the book.

This book presents basic results on the theory of Navier-Stokes equations and, as such, continues to serve as a comprehensive reference source on the topic.

AMS Chelsea Publishing; 2001; approximately 424 pages; Hardcover; ISBN 0-8218-2757-5; List $59; All AMS members $55; Order code CHEL/364

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NSF Mathematical Sciences Initiative

The National Science Foundation (NSF) is planning a new Mathematical Sciences Initiative (MSI). Plans call for increased funding over the next five years to support fundamental mathematical sciences research, connections of the mathematical sciences to other areas of science and engineering, and improvements in mathematical sciences education.

The MSI is needed because advances in fundamental mathematical sciences, which embrace mathematics and statistics, are closely intertwined with the discovery process in science, engineering, and technology. The mathematical sciences are accelerating progress across the spectrum of science and engineering, even in traditionally descriptive sciences.

Why is the MSI needed now? One reason is the "mathematization" of science. Science is becoming more mathematical and statistical—not only the physical and information sciences, but also the biological, geophysical, environmental, social, behavioral, and economic sciences. There is a vital need for mathematicians and statisticians to collaborate with engineers and scientists to explore the frontiers of discovery, where science and mathematics meet and interact. Another reason is that the technical work force, as well as society at large, needs more mathematical and statistical skills today than ever before. Technology-based industries fuel the growth of the U.S. economy, which, in turn, relies on large numbers of college graduates well versed in mathematics, science, and engineering. In our increasingly complex world, the need for broad mathematical and statistical literacy becomes ever more acute.

The MSI has three main components: advancing fundamental mathematics; advancing interdisciplinary collaborations among the mathematical sciences, other areas of science, and engineering; and advancing mathematical skills and mathematical literacy.

1. The first component of the initiative calls for increased support for fundamental mathematical research. The mathematical sciences have consistently renewed themselves through synthesis of preceding work and infusion of new ideas, some of which originate through the application of mathematics in other disciplines. This process of rejuvenation and evolution is indispensable for discovery at the frontiers of the mathematical sciences.

2. Many technological advances—from Doppler radar to magnetic resonance imaging to public key cryptography to the Internet—are rooted in fundamental mathematics and statistics. The concepts and structures developed by fundamental mathematics and statistics often provide just the right framework for the formulation and study of seemingly unrelated applications. Therefore, the second component of the MSI focuses on connections between the mathematical sciences and other sciences and engineering. Mathematics and statistics provide a vocabulary that grows richer as the boundaries between the disciplines become more diffuse. We must nurture and train researchers who are capable of participating in multidisciplinary collaborations. Mathematics and statistics have enabled extraordinary advances across the board, yielding new analytical, statistical, computational, and experimental tools to tackle a broad range of scientific and technological challenges previously considered intractable. With this unprecedented expansion of our capacity to discover and its potential for application to societal needs comes a demand for new mathematical and statistical techniques.

3. The third component of the initiative advances mathematical sciences skills in a broad sense. Public literacy in mathematics and statistics and appreciation of their roles in modern life are critical for societal progress. Education, research, and national work force needs are inextricably linked: we must attract the best talent from today’s youth into undergraduate, graduate, and postdoctoral programs in the mathematical sciences and train them for a broad array of careers in academia, industry, and government. It is important that U.S. students sense the excitement and see the opportunities for careers based on the mathematical sciences. The involvement of the research community in this mathematical sciences education agenda is indispensable. Providing all young citizens with the mathematical skills to enjoy productive and fruitful employment is a national priority.

The NSF has adopted a five-year investment plan for the MSI, starting in fiscal year 2002 (which begins October 1, 2001). Right now, specific plans for the initiative are taking shape. Members of the mathematical sciences community are urged to contribute their thoughts and ideas as the initiative moves forward. As plans develop information will be posted on the Web site of the NSF’s Division of Mathematical Sciences, http://www.nsf.gov/mps/dms/.

—Philippe Tondeur, Director
NSF Division of Mathematical Sciences
From Rotating Needles to Stability of Waves: Emerging Connections between Combinatorics, Analysis, and PDE

Terence Tao

Introduction

In 1917 S. Kakeya posed the Kakeya needle problem: What is the smallest area required to rotate a unit line segment (a "needle") by 180 degrees in the plane? Rotating around the midpoint requires $\pi/4$ units of area, whereas a "three-point U-turn" requires $\pi/8$. In 1927 the problem was solved by A. Besicovitch, who gave the surprising answer that one could rotate a needle using arbitrarily small area.

At first glance, Kakeya's problem and Besicovitch's resolution appear to be little more than mathematical curiosities. However, in the last three decades it has gradually been realized that this type of problem is connected to many other, seemingly unrelated, problems in number theory, geometric combinatorics, arithmetic combinatorics, oscillatory integrals, and even the analysis of dispersive and wave equations.

The purpose of this article is to discuss the interconnections between these fields, with an emphasis on the connection with oscillatory integrals and PDE. Two previous surveys ([7] and [1]) have focused on the connections between Kakeya-type problems and other problems in discrete combinatorics and number theory.

These areas are very active, but despite much recent progress our understanding of the problems and their relationships to each other is far from complete. Ideas from other fields may well be needed to make substantial new breakthroughs.

Kakeya-Type Problems

Besicovitch's solution to the Kakeya needle problem relied on two observations. The first observation, which is elementary, is that one can translate a needle to any location using arbitrarily small area; see Figure 1. The second observation is that one can construct open subsets of $\mathbb{R}^2$ of arbitrarily small area which contain a unit line segment in every direction. A typical way to construct such sets (not Besicovitch's original construction) is sketched in Figure 2; for a more detailed construction see [7].

For any $n \geq 2$ define a Besicovitch set to be a subset of $\mathbb{R}^n$ which contains a unit line segment in every direction. For any such $n$ the construction of Besicovitch shows that such sets can have arbitrarily small Lebesgue measure and can even be made to have measure zero. Intuitively this means that it is possible to compress a large number of nonparallel unit line segments into an arbitrarily small set.

In applications one wishes to obtain more quantitative understanding of this compression effect by introducing a spatial discretization. For
instance, one can replace unit line segments by $1 \times \delta$ tubes for some $0 < \delta \ll 1$ and ask for the optimal compression of these tubes. Equivalently, one can ask for bounds of the volume of the $\delta$-neighbourhood of a Besicovitch set.

Rather surprisingly, these bounds are logarithmic in two dimensions. It is known that the $\delta$-neighbourhood of a Besicovitch set in $\mathbb{R}^2$ must have area at least $C/\log(1/\delta)$; this basically follows from the geometric observation that the area of the intersection of two $1 \times \delta$ rectangles varies inversely with the angle between the long axes of the rectangles. Recently, U. Keich has shown that this bound is sharp.

This observation can be rephrased in terms of the Minkowski dimension of the Besicovitch set. Recall that a bounded set $E$ has Minkowski dimension $\alpha$ or less if and only if for every $0 < \delta \ll 1$ and $0 < \varepsilon \ll 1$ one can cover $E$ by at most $C_\delta \delta^{-\alpha+\varepsilon}$ balls of radius $\delta$. From the previous discussion we thus see that Besicovitch sets in the plane must have Minkowski dimension 2.

It is unknown if the analogous property holds in $\mathbb{R}^n$.

In one of its principal formulations, the Kakeya conjecture states that every Besicovitch set in $\mathbb{R}^n$ has Minkowski dimension $n$. (There is also a corresponding conjecture for the Hausdorff dimension, but for simplicity we shall not discuss this variant.)

Equivalently, the Kakeya conjecture asserts that the volume of the $\delta$-neighbourhood of a Besicovitch set in $\mathbb{R}^n$ is bounded below by $C_n \delta^\varepsilon$ for any $\varepsilon > 0$ and $0 < \delta \ll 1$.

The Kakeya conjecture is remarkably difficult. It remains open in three and higher dimensions, although rapid progress has been made in the last few years. The best-known lower bound for the Minkowski dimension at this time of writing is

$$\max \left( \frac{n+2}{2} + 10^{-10}, \frac{4n+3}{7} \right),$$

although I expect further improvements to follow very soon.

One can discretize the conjecture. Let $\Omega$ be a maximal $\delta$-separated subset of the sphere $S^{n-1}$ (so that $\Omega$ has cardinality approximately $\delta^{1-n}$), and for each $\omega \in \Omega$ let $T_\omega$ be a $\delta \times 1$ tube oriented in the direction $\omega$. The Kakeya conjecture then asserts logarithmic-type lower bounds on the quantity $| \bigcup_{\omega \in \Omega} T_\omega |$.

The above formulation is reminiscent of existing results in combinatorics concerning the number of incidences between lines and points.

---

1 Throughout this article, the letter $C$ denotes a constant which varies from line to line.
although a formal connection cannot be made because the nature of the intersection of two $\delta \times 1$ tubes depends on the angle between the tubes, whereas the intersection of two lines is a point regardless of what angle the lines make. However, it is plausible that one can use the ideas from combinatorial incidence geometry to obtain progress on this problem. For instance, it is fairly straightforward to show that the Minkowski dimension of Besicovitch sets is at least $(n+1)/2$ purely by using the fact that given any two points that are a distance roughly $1$ apart, there is essentially only one $\delta \times 1$ tube which can contain them both.

In the 1990s, work by J. Bourgain, T. Wolff, W. Schlag, A. Vargas, N. Katz, I. Laba, the author, and others pushed these ideas further. For instance, the lower bound of $(n+2)/2$ for the Minkowski dimension was shown in 1995 by Wolff and relies on the $\delta$-discretized version of the geometric statements that every nondegenerate triangle lies in a unique two-dimensional plane and every such plane contains only a one-parameter set of directions. However, there appears to be a limit to what can be achieved purely by applying elementary incidence geometry facts and standard combinatorial tools (such as those from extremal graph theory). More sophisticated geometric analysis seems to reveal that a counterexample to the Kakeya conjecture, if it exists, must have certain rigid structural properties (for instance, the line segments through any given point should all lie in a hyperplane). Such ideas have led to a very small recent improvement in the Minkowski bound to $(n+2)/2 + 10^{-10}$, but they are clearly insufficient to resolve the full conjecture.

The Kakeya problem is a representative member of a much larger family of problems of a similar flavour (but with more technical formulations). For instance, one can define a $\beta$-set to be a subset of the plane which contains a $\beta$-dimensional subset of a unit line segment in every direction. It is then an open problem to determine, for given $\beta$, the smallest possible dimension of a $\beta$-set. Low-dimensional examples of such sets arise in the work of H. Furstenberg, and it seems that one needs to understand these generalizations of Besicovitch sets in order to fully exploit the connection between Kakeya problems and oscillatory integrals, which we discuss below. Other variants include replacing line segments by circles or light rays, considering finite geometry analogues of these problems, or replacing the quantity $|\bigcup_{t \in B} T_t|$ by the variant $\|\sum_{t \in B} x_t \|_p$ (the relevant conjecture here is known as the Kakeya maximal function conjecture). Another interesting member of this family is the Falconer distance set conjecture, which asserts that whenever $E$ is a compact one-dimensional subset of $\mathbb{R}$, the distance set $\{|x-y| : x,y \in E\}$ is a one-dimensional subset of $\mathbb{R}$. The discrete version of this is the Erdős distance problem—what is the least number of distances determined by $n$ points—and is also unsolved. For a thorough survey of most of these questions, we refer the reader to [7]; see also [3].

The Connection with Arithmetic Combinatorics

The Kakeya problem looks very geometrical, and it is natural to apply elementary incidence geometry to bear on this problem. Although this approach has had some success, it does not seem sufficient to solve the problem.

In 1998 Bourgain introduced a new type of argument, based on arithmetic combinatorics (the
combinatorics of sums and differences), which gave improved results on this problem, especially in high dimensions. The connection between Kakeya problems and the combinatorics of addition can already be seen by considering the analogy between line segments and arithmetic progressions. (Indeed, the Kakeya conjecture can be reformulated in terms of arithmetic progressions, and this can be used to connect the Kakeya conjecture to several difficult conjectures in number theory, such as the Montgomery conjectures for generic Dirichlet series. We will not discuss this connection here, but refer the reader to [1].)

Bourgain's argument relies on the following "three-slice" idea. Let $\Omega$ and $T_o$ be as in the previous section. We may assume that the tubes $T_0$ are contained in a fixed ball. Suppose that $| \bigcup_{\omega \in \Omega} T_\omega |$ is comparable to $\delta^\alpha$ for some constant $\alpha$; our objective is to give upper bounds on $\alpha$ and eventually to show that $\alpha$ must be zero.

By choosing an appropriate set of coordinates, one can ensure that each of the three slices

$$X_t := \{ x \in \mathbb{R}^{n-1} : (x,t) \in \bigcup_{\omega \in \Omega} T_\omega \},$$

$t = 0,1/2,1$, has measure comparable to $\delta^\alpha$. Because of the $\delta$-discretized nature of the problem, one can also assume that the discrete set

$$A_t := X_t \cap \delta^\alpha \mathbb{Z}^{n-1}$$

has cardinality comparable to $\delta^{\alpha+1-n}$ for $t = 0,1/2,1$.

Morally speaking, every tube $T_\omega$ intersects each of the sets $A_0$, $A_{1/2}$, $A_1$ in exactly one point. Assuming this, we see that every tube $T_\omega$ is associated with an element of $A_0 \times A_1$. Because two points determine a line, these elements are essentially disjoint as $\omega$ varies. Let $G$ denote the set of all pairs of $A_0 \times A_1$ obtained this way. Thus $G$ has cardinality about $\delta^{1-n}$.

The sum set

$$\{ (a+b) : (a,b) \in G \}$$

of $G$ is essentially contained inside a dilate of the set $A_1/2$; this reflects the fact that the intersection of $T_\omega$ with $A_1/2$ is essentially the midpoint of the intersection of $T_\omega$ with $A_0$ and $A_1$. In particular, the sum set is quite small, having cardinality only $\delta^{\alpha+1-n}$. On the other hand, the difference set

$$\{ (a-b) : (a,b) \in G \}$$

of $G$ is quite large, because the tubes $T_\omega$ all point in different directions. Indeed, this set has the same cardinality as $G$, i.e., about $\delta^{1-n}$.

Thus, if $\alpha$ is nonzero, there is a large discrepancy in size between the sum set and difference set of $G$. In principle this should lead to a bound on $\alpha$, especially in view of standard inequalities relating the cardinalities of sum sets and difference sets, such as

$$|A-B| \leq \frac{|A+B|^3}{|A||B|}.$$  

(A summary of such inequalities can be found in [5].) However, these arguments (which are mostly graph-theoretical) do not seem to adapt well to the Kakeya application, because we are working with only a subset $G$ of $A_0 \times A_1$ rather than all of $A_0 \times A_1$.

To overcome this problem, Bourgain adapted a recent argument of W. T. Gowers which allows one to pass from arithmetic information on a subset of a Cartesian product to arithmetic information on a full Cartesian product. A typical result is:

**Theorem.** Let $A, B$ be finite subsets of a torsion-free abelian group with cardinality at most $N$, and suppose that there exists a set $G \subset A \times B$ of cardinality at least $\alpha N^2$ such that the sum set $\{ a+b : (a,b) \in G \}$ has cardinality at most $N$. Then there exist subsets $A', B'$ of $A, B$ respectively such that $A' - B'$ has cardinality at most $\alpha^{-13}N$ and $A', B'$ have cardinality at least $\delta^9 N$.

Roughly speaking, this theorem states that if most of $A+B$ is contained in a small set, then by refining $A$ and $B$ slightly, one can make all of $A-B$ be contained in a small set also. Such results are reminiscent of standard combinatorial theorems concerning the size of sum and difference sets, but the innovation in Gowers's arguments is that the control on $A'$ and $B'$ is polynomial in $\alpha$. (Previous combinatorial techniques gave bounds which were exponential or worse, which is not sufficient for Kakeya applications.)

Recently [2] Katz and the author have obtained the bound $(4n+3)/7$ by using control on both the sum set $A_0 + A_1$ and the variant $A_0 + 2A_1$, which corresponds to the slice $A_{2/3}$.

These results have remarkably elementary proofs. Apart from some randomization arguments, the proofs rely mainly on standard combinatorial tools such as the pigeonhole principle and Cauchy-Schwarz inequality, as well as on basic arithmetic facts such as

$$a+b = c+d \iff a-d = c-b,$$

$$a-b = (a-b') - (a'-b') + (a' - b),$$

and

$$a_0 + 2b_0 = a_1 + 2b_1, \quad b'_0 = b'_1$$

$$\iff a_1 - b'_1 = 2(a_0 + b_0) - 2b_1 - (a_0 + b'_0).$$

Further progress has been made by pursuing these methods, though it seems that we are still quite far from a full resolution of the Kakeya problem, and some new ideas are almost certainly needed.

One possibility may be that one would have to use combinatorial estimates on product sets in addition to sum sets and difference sets, since one has control of $\{a+tb : (a,b) \in G \}$ for all
Figure 4. Four tubes $T_t$, their shifts $\tilde{T}_t$, and the wave packets $\psi_t$. The interference between the functions $S_1 \psi_1$ will cause the $L^p$ norm to be large when $p > 2$.

t \in [0, 1]. Discrete versions of such estimates exist; for instance, G. Elekes has recently shown the bound

$$\max(|A \cdot A|, |A + A|) \geq C^{-1} |A|^{5/4}$$

for all finite sets of integers $A$. However, these bounds do not adapt well to the continuous Kakeya setting because of the difficulty in discretizing both addition and multiplication simultaneously.

A good test problem in this setting is the Erdős ring problem: Determine whether there exists a (Borel) subring of $\mathbb{R}$ with Hausdorff dimension exactly $1/2$. This problem is known to be connected with both the $\beta$-set problem and the Falconer distance set problem.

Interestingly, the Kakeya problem is also connected to another aspect of arithmetic combinatorics, namely that of locating arithmetic progressions in sparse sets. (A famous instance of this is the old conjecture of Erdős, which is still open, that the primes contain infinitely many arithmetic progressions of arbitrary length.) This difficulty arises in the Hausdorff dimension formulation of the Kakeya problem, and also in some more quantitative variants, because of the difficulty in selecting a "good" set of three slices in arithmetic progression in which to run the above argument. The combinatorial tools developed for that problem by Gowers and others may well have further applications to the Kakeya problem in the future.

Applications to the Fourier Transform

Historically, the first applications of the Kakeya problem to analysis arose in the study of Fourier summation in the 1970s.

If $f$ is a test function on $\mathbb{R}^n$, we can define the Fourier transform $\hat{f}$ by

$$\hat{f}(\xi) := \int_{\mathbb{R}^n} e^{-2\pi i x \cdot \xi} f(x) \, dx.$$}

One then has the inversion formula

$$f(x) = \int_{\mathbb{R}^n} e^{2\pi i x \cdot \xi} \hat{f}(\xi) \, d\xi.$$}

Now suppose that $f$ is a more general function, such as a function in the Lebesgue space $L^p(\mathbb{R}^n)$. The Fourier inversion formula still holds true in the sense of distributions, but one is interested in more quantitative convergence statements. Specifically, we could ask whether the partial Fourier integrals

$$S_R f(x) := \int_{|\xi| \leq R} e^{2\pi i x \cdot \xi} \hat{f}(\xi) \, d\xi$$

converge to $f$ in, say, $L^p$ norm. (The pointwise convergence question is also interesting, say for $L^2$ functions $f$, but this seems extremely difficult...
to show in two and higher dimensions. In one dimension this was proven in a famous paper by L. Carleson.) By the uniform boundedness principle, this is equivalent to asking whether the linear operators $S_R$ are bounded on $L^p(R^n)$ uniformly in $R$. By scale invariance it suffices to show this for $S_1$:

$$\|S_1f\|_{L^p(R^n)} \leq C \|f\|_{L^p(R^n)}.$$  

The operator $S_1$ is known as the ball multiplier or (when $n=2$) the disk multiplier, because of the formula

$$S_1f = \chi_B \hat{f},$$

where $B$ is the unit ball in $R^n$. In one dimension it is a classical result of Riesz that this operator is bounded on every $L^p$, $1 < p < \infty$, and so Fourier integrals converge in $L^p$ norm. (Indeed, in one dimension $S_1$ is essentially the Hilbert transform.) In higher dimensions $S_1$ is bounded in $L^2$, thanks to Plancherel's theorem; however, the behaviour in $L^p$ is more subtle. One has an explicit kernel representation which roughly looks like

$$S_1f(x) = \int \frac{e^{i|x-y|}}{(1+|x-y|)^{n+1}} f(y) \, dy;$$

to be more precise, one must use Bessel functions instead of $e^{i|x-y|}$. The kernel is in $L^p$ only when $p > \frac{2n}{n+1}$, so it might seem natural by duality arguments to conjecture that $S_1$ is bounded when $\frac{2n}{n+1} < p < \frac{2n}{n-1}$. In 1971, however, C. Fefferman proved the surprising

**Theorem.** If $n > 1$, then $S_1$ is unbounded on $L^p$ for every $p \neq 2$.

In particular, one does not have $L^p$ convergence for the Fourier inversion formula in higher dimensions unless $p = 2$.

Roughly speaking, the idea is as follows. By duality it suffices to consider the case $p > 2$. Let $R$ be a large number, and let $T$ be a cylindrical tube in $R^n$ with length $R$ and radius $\sqrt{R}$ and oriented in some direction $\omega_T$. Let $\psi_T$ be a bump function adapted to the tube $T$, and let $\hat{T}$ be a shift of $T$ in the $\omega_T$ direction. Then a computation shows that

$$|S_1(e^{2\pi i \omega_T \cdot x} \psi_T(x))| \approx 1$$

for all $x \in \hat{T}$.

To exploit this computation, one uses the Besicovitch construction to find a collection $\{T_i\}$ of tubes as above which are disjoint but whose shifts $\hat{T}_i$ have significant overlap. More precisely, we assume that

$$\bigcup_{i} \hat{T}_i \leq \frac{1}{K} \sum_{i} |T_i|$$

for some $K$ which grows in $R$ (the standard construction in Figure 2 gives $K \sim \log(R) / \log \log(R)$).

Then we consider the function

$$f(x) = \sum_{i} \varepsilon_i e^{2\pi i \omega_T \cdot x} \psi_T(x),$$

where $\varepsilon_i = \pm 1$ are randomized signs. Using Khinchin's inequality (which roughly states that one has the formula $\sum |T_i f_i|^2 \sim \sum |T_i f_i|^2$ with very high probability), one can eventually compute that

$$\|S_1f\|_p \geq C K^{1/2} \|f\|_p.$$  

Since $K$ is unbounded, we thus see that $S_1$ is unbounded.

Fefferman's theorem is an example of how a geometric construction can be used to show the unboundedness of various oscillatory integral operators. The point is that while the action of these operators on general functions is rather complicated, their action on "wave packets" such as $e^{2\pi i \omega_T \cdot x} \psi_T(x)$ is fairly easy to analyze. One can then generate a large class of functions to test the operator on by superimposing several wave packets together and possibly randomizing the coefficients to simplify the computation.

The counterexample provided by the Besicovitch construction is very weak (only growing logarithmically in the scale $R$) and can be eliminated if one mollifies the disk multiplier slightly. For instance, the counterexample does not prohibit the slightly smoother Bochner-Riesz operator $S_1^*$, defined by

$$\hat{S}_1^*(f) = \int (1-|\xi|^2) \hat{f} \hat{\psi}(\xi),$$

from being bounded for $\varepsilon > 0$, because the analogous computation gives

$$|S_1^*(e^{2\pi i \omega_T \cdot x} \psi_T(x))| \approx R^{-\varepsilon}$$

for all $x \in \hat{T}$. Indeed, the Bochner-Riesz conjecture asserts that $S_1^*$ is indeed bounded on $L^p$ for all $\varepsilon > 0$ and $2n/(n+1) < p < 2n/(n-1)$. (For other values of $p$ one needs $\varepsilon > n/2 - 1/2 - \log_2 n$.) This conjecture was proven by L. Carleson and P. Sjolin in 1972 in two dimensions, but the higher-dimensional problem is quite challenging, and only partial progress has been made so far. This conjecture would imply that the partial Fourier integrals will converge in $L^p$ if one uses a Cesàro summation method (such as the Fejer summation method, which corresponds to $\varepsilon = 1$).

The Bochner-Riesz conjecture would be disproved if one could find a collection of disjoint tubes $T$ for which the compression factor $K$ had some power dependence on $R$ as opposed to logarithmic, i.e., if $K \geq C^{-1} R^\varepsilon$ for some $\varepsilon > 0$. A more precise statement is known, namely, that the failure of the Kakeya conjecture would imply the failure of the Bochner-Riesz conjecture. (More succinctly, Bochner-Riesz implies Kakeya.)

In 1991 Bourgain introduced a method in which these types of implications could be reversed, so that progress on the Kakeya problem would (for instance) imply progress on the Bochner-Riesz
conjecture. The key observation is that every function can be decomposed into a linear combination of wave packets by applying standard cutoffs both in physical space (by pointwise multiplication) and in frequency space (using the Fourier transform). After applying the Bochner-Riesz operator to the wave packets individually, one has to reassemble the wave packets and obtain estimates for the sum. Kakeya estimates play an important role in this, since the wave packets are essentially supported on tubes; however, this is not the full story, since these packets also carry some oscillation, and one must develop tools to deal with the possible cancellation between wave packets. The known techniques to deal with this cancellation, mostly based on $L^2$ methods, are imperfect, so that even if one had a complete solution to the Kakeya conjecture, one could not then completely solve the Bochner-Riesz conjecture. Nevertheless, the best-known results on Bochner-Riesz (e.g., in three dimensions the conjecture is known \cite{6} for $p > 26/7$ and for $p < 26/19$) have been obtained by utilizing the best-known quantitative estimates of Kakeya type.

These techniques apply to a wide range of oscillatory integrals. A typical question, the (adjoint) restriction problem, concerns Fourier transforms of measures. Let $d\sigma$ be surface measure on, say, the unit sphere $S^{n-1}$. The Fourier transform $\hat{d\sigma}$ of this measure can be computed explicitly using Bessel functions and decays like $|\xi|^{-(n-1)/2}$ at infinity. In particular, it is in the class $L^p(\mathbb{R}^n)$ for all $p > 2n/(n-1)$. The restriction conjecture asserts that the same statement holds if $d\sigma$ is replaced by $\hat{f} d\sigma$ for any bounded function $f$ on the sphere. This question originally arose from studying the restriction phenomenon (that a Fourier transform of a rough function can be meaningfully restricted to a curved surface such as a sphere, but not to a flat surface like a hyperplane); it is also related to the question of obtaining $L^p$ estimates on eigenfunctions of the Laplacian on the torus (although the eigenfunction problem is far more difficult due to number theoretic issues), as well as $L^p$ estimates on solutions to dispersive PDE, as we shall see below.

The restriction conjecture is logically implied by the Bochner-Riesz conjecture and is slightly easier to deal with technically. It has essentially the same amount of progress as Bochner-Riesz; for instance, it is completely solved in two dimensions and is known to be true \cite{6} for $p > 26/7$ in three dimensions. One uses the same techniques, namely wave packet decomposition of the initial function $f$, Kakeya information, and $L^2$ estimates to handle the cancellation, in order to obtain these results.

There is an endless set of permutations on these types of oscillatory integral problems: more general phases and weights, square function and maximal estimates, more exotic function spaces, bilinear and multilinear variants, etc. There are some additional rescaling arguments available in the bilinear case, as well as some $L^2$-based estimates, but apart from this there are few effective tools known outside of Bourgain’s wave packet analysis to attack these types of problems.

One variant of the above problems comes from replacing Euclidean space by a curved manifold. There are some interesting three-dimensional examples of C. Sogge and W. Minicozzi showing that the Kakeya conjecture can fall on such manifolds, which then implies the corresponding failure of oscillatory integral conjectures such as the natural analogue of Bochner-Riesz. This may shed some light on the robustness of Kakeya estimates and their applications in variable coefficient situations. Certainly the arithmetic and geometric techniques used currently to attack Kakeya problems do not adapt well to curved space.

Applications to the Wave Equation

In the previous section we saw how Kakeya-type problems are related to oscillatory integrals. There is also a similar, and in some sense more natural, connection between Kakeya problems and linear evolution equations such as the free Schrödinger, wave, and Airy equations.

For brevity of exposition we shall restrict our attention to the solutions of the free wave equation

$$u_{tt}(t, x) = \Delta u(t, x); \quad u(0, x) = f(x), u_t(0, x) = 0$$

with initial position $f$ and initial velocity zero. However, much of our discussion has analogues for other linear evolution equations such as the Schrödinger equation.

One can solve for $u$ explicitly using the formula

$$u(t) = \cos(t \sqrt{-\Delta}) f,$$

but this does not reveal much information about the size and distribution of $u$. On the other hand, this formula does show that the wave equation is connected to the oscillatory integral problems mentioned earlier. For instance, the disk multiplier $S_1$ can be rewritten as

$$S_1 f = \chi_{[-2\pi, 2\pi]}(\sqrt{-\Delta}) f,$$

since $S_1$ preserves those Fourier modes $e^{2\pi i x \cdot \xi}$ which are (generalized) eigenfunctions of $\sqrt{-\Delta}$ with eigenvalue in $[-2\pi, 2\pi]$ and eliminates all others. In particular, we have the Fourier representation

$$S_1 f = \int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{\pi t} \cos(t \sqrt{-\Delta}) f \, dt$$

of the disk multiplier in terms of wave evolution operators.
A general class of problems is the following: given size and regularity conditions on the initial data $f$, what type of size and regularity control does one obtain on the solution?

From integration by parts (or from the above explicit formula), one has energy conservation

$$\int \frac{1}{2} |\nabla u(t, x)|^2 + \frac{1}{2} |u_t(t, x)|^2 \, dx = \int \frac{1}{2} |\nabla f(x)|^2$$

for all time $t$. This conservation law and its generalizations show that $u$ has as much regularity as $f$ when measured in $L^2$-based spaces. However, $L^2$ control by itself does not reveal whether $u$ focuses or disperses. To obtain better quantitative control on $u$, one needs other estimates, such as $L^p$ estimates.

The energy estimate is a fixed time estimate: it controls the solution at a specified time $t$. In the $L^p$ setting, fixed time estimates exist but require a lot of regularity for the initial data and therefore have limited usefulness. A typical estimate is the decay estimate

$$\|u(t)\|_{L^\infty([0,T])} \leq C(1 + |t|)^{-\frac{n-1}{2}} \sum_{0 \leq k \leq s} \|\nabla^k f\|_{L^1(\mathbb{R}^n)}$$

whenever $s > \frac{n+1}{2}$ is an integer. The necessity of this many derivatives is demonstrated by the focusing example, in which the initial data is spread out near a sphere of radius 1 and the solution $u$ focuses (with an extremely high $L^\infty$ norm) at the origin at time $t = 1$.

However, one can obtain much better estimates, requiring far fewer derivatives, if one is willing to average locally in time. The intuitive explanation for this is that it is difficult for a wave to maintain a focus point (which would generate a large $L^p$ norm for $p > 2$) for any length of time. This phenomenon is known as local smoothing. A very useful class of local smoothing estimates is known collectively as Strichartz estimates. A typical example of a Strichartz estimate is

$$\|u\|_{L^p_t([0,T])} \leq C \|\sqrt{-\Delta} f\|_{L^2(\mathbb{R}^n)}$$

in three spatial dimensions. Without the averaging in time, one would require $3/4$ of a derivative on the right-hand side rather than $1/2$; this can be seen from the Sobolev embedding theorem. These Strichartz estimates are usually proven by combining the energy and decay estimates with some orthogonality arguments.

However, even Strichartz estimates lose some regularity. One may ask if there are $L^p$ estimates other than the energy estimate which do not lose any derivatives at all. Unfortunately, even if one localizes in time and assumes $L^\infty$ control on the initial data, one still cannot do any better than $L^2$ control, as the following result of Wolff shows:

**Theorem.** If $n > 1$ and $p > 2$, then the estimate

$$\|u\|_{L^p_t([0,T])} \leq C \|f\|_{L^\infty(B(0,1))}$$

cannot hold for all bounded $f$ on the unit ball.

The argument proceeds similarly to Fefferman's disk multiplier argument. Let $(T)$ be a collection of disjoint tubes arranged using the Besicovitch set construction as in Fefferman's argument, except that we rescale the tubes to have dimensions $1 \times R^{-1/2}$ rather than $R \times \sqrt{R}$. On each of these
One can make this be the sum of all these wave trains (although logarithmic compression rate, one could get letting $e^{iRx \cdot wT}$ times a bump function adapted to $T$. Let $f$ be the sum of all these wave trains (although we may randomize the signs of these trains to simplify computations).

At time zero, the function $f$ has low $L^\infty$ norm. However, as time evolves, each wave train $T$ splits as the superposition of two pulses, one moving in the direction $\omega T$ and the other in the direction $-\omega T$. For times $1 \leq t \leq 2$ a large portion of the wave train at $T$ now lives in the shifted tube $T'$. Because of the large overlap of these tubes, the $L^p$ norm of $u$ is large for all $1 \leq t \leq 2$; as with Fefferman's argument, it is about $K^{3/2 - 1/3}$. By letting $R \to \infty$, one can make $K$ unbounded, and this gives the theorem.

Because the Besicovitch construction has a logarithmic compression rate, one could get around this obstruction by requiring an epsilon of regularity on the initial data. The local smoothing conjecture of C. Sogge asserts that no further loss of regularity occurs or, more precisely, that

$$\|u\|_{L^p([1,2] \times R^3)} \leq C_{p,\epsilon}\|1 + \sqrt{-\Delta}\|^\epsilon f\|_{L^p(R^n)}$$

for all $\epsilon > 0, n(\frac{1}{2} - \frac{1}{p}) - \frac{1}{3}$, and $2 \leq p \leq \infty$. This conjecture is easy when $p = 2$ or $p = \infty$; the most interesting case is when $p = 2n/(n - 1)$.

The local smoothing conjecture is extremely strong and would imply many of the known estimates on the wave equation. It implies the Kakeya conjecture, for a counterexample to the Kakeya conjecture could be used to strengthen Wolff's argument to disprove the local smoothing conjecture. This conjecture also implies the Bochner-Riesz conjecture; the idea is to write the Bochner-Riesz multiplier $S_f$ in terms of wave operators $\cos(t\sqrt{\Delta})$ in the manner briefly discussed earlier. However, the conjecture is far from settled; even in two dimensions the conjecture is completely proven for only $p > 74$ (due to T. Wolff), and at the critical exponent $p = 4$ the conjecture is known only for $\epsilon > 1/8 - 1/88$ [6], [8]. These estimates are all proven via Kakeya methods. Briefly, the idea is to decompose the initial data $f$ (and hence the solution $u$ by linearity) into pieces which are localized in both space and frequency. This decomposes the solution $u$ into wave packets, which are oscillatory functions which travel along light rays. One then uses Kakeya-type arguments to control how many times these light rays intersect each other, together with orthogonality arguments to control to what extent the oscillations of each wave packet reinforce each other. A recently developed, but apparently quite powerful, technique here is induction on scales, in which one assumes that the desired estimate is already proven at smaller scales and uses this hypothesis together with the Kakeya strategy just discussed to obtain the same estimate at higher scales.

There are several other wave equation estimates which are related to those discussed here. An active area of research is to obtain good bilinear or even multilinear estimates on solutions to the wave equation, as opposed to the linear estimates described here; these estimates have direct application to nonlinear wave equations, since one can often use techniques such as the method of Taylor series to write the solution of nonlinear wave equations as a series of multilinear expressions of solutions to the linear wave equation. There are some tantalizing hints that these Kakeya techniques could also be used to handle nonlinear wave equations directly (or wave equations with rough metrics, potential terms, etc.), but these ideas are still in their infancy.

Since $u$ can be written in terms of circular averages of $f$, there is also a close relationship between wave equation estimates and estimates for circular means. (Such circular means estimates can then be used, for instance, to make progress on the Falconer distance problem mentioned earlier.) There is also an extremely strong square function estimate conjectured for the wave equation which, if true, would imply the local smoothing, Bochner-Riesz, restriction, and Kakeya conjectures. It would also give estimates for other seemingly unrelated objects such as the helix convolution operator $f \mapsto f * d\sigma$, where $d\sigma$ is arclength measure on the helix $\{(\cos t, \sin t, t) : 0 \leq t \leq 2\pi\}$ in $R^3$. (The connection arises because the Fourier transform of $d\sigma$ is concentrated near the light cone.) These estimates are quite difficult, and the partial progress which has been made on them
has proceeded via Kakeya estimates. Although these deep wave equation estimates have not yet found significant applications, I am confident that they will do so in the near future.

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References

About the Cover
Around 1926 A. S. Besicovitch showed that there exist planar regions of arbitrarily small area in which one could rotate a segment of fixed length, thus solving what is now known as the Kakeya needle problem. As Terry Tao's article explains, even nowadays Besicovitch's Theorem and variations on it are a fruitful source of analysis.

One of the principal steps in Besicovitch's proof was the construction of certain complicated regions of arbitrarily small area in the plane containing needles of a fixed length and a range of directions. The original construction of such regions was quite complicated. Already in the next year's volume of the Mathematische Zeitschrift, O. Perron exhibited a simpler one, involving what are now called "Perron trees", and in the early 1960s I. J. Schoenberg simplified Perron's construction in turn, showing how these trees could be constructed recursively by what he called "sprouting". The cover illustrates Schoenberg's construction and demonstrates visually that the ratio of the areas of the regions in the third and fourth columns has limit 0.

The Mathematical Association of America received a grant from the NSF around 1960 to make a film about this topic. What was essentially a transcript of the film appeared in an often cited article by Besicovitch ("The Kakeya problem", volume 70 of the American Mathematical Monthly, 1963, pp. 697-706). The film itself may have been the first professionally produced mathematical animation. Are there any viewable copies of the film left? Is there anybody still around who took part in that project?

—Bill Casselman (covers@ams.org)
Experimental Analysis of Algorithms

Catherine C. McGeoch

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.
—Albert Einstein

In theory there is no difference between theory and practice. In practice there is.
—Yogi Berra

The algorithm and the program. The abstract and the physical. If you want to understand the fundamental and universal properties of a computational process, study the abstract algorithm and prove a theorem about it. If you want to know how the process really works, implement the algorithm as a program and measure the running time (or another quantity of interest).

This distinction between the abstract model and the physical artifact exists in the study of computational processes just as in every other area of mathematical modeling. But algorithmic problems have some unusual features. For example, we usually build models to serve as handy representations of natural phenomena that cannot be observed or manipulated directly. But programs and computers are completely accessible to the researcher and are far more manipulable than, say, weather patterns. They are also much easier to understand: hypothetically, one could obtain complete information about the behavior of a program by consulting technical documents and code. And finally, algorithms are usually invented before programs are implemented, not the other way around.

This article surveys problems and opportunities that lie in the interface between theory and practice in a relatively new research area that has been called experimental algorithmics, experimental analysis of algorithms, or algorithm engineering. Much work in this field is directed at finding and evaluating state-of-the-art implementations for given applications. Another effort focuses on using experiments to extend and improve the kinds of results obtained in traditional algorithm analysis. That is, rather than having a goal of measuring programs, we develop experiments in order to better understand algorithms, which are abstract mathematical objects. In this article we concentrate on examples from this latter type of research in experimental algorithmics.

It is natural to wonder whether such an effort is likely to bear fruit. If the ultimate goal of algorithm analysis is to produce better programs, wouldn't we be better off studying programs in their natural habitats (computers) rather than performing experiments on their abstractions? Conversely, if the goal is to advance knowledge in an area of mathematical research (algorithm analysis), are we wise to study abstract objects using imperfect, finite, and ever-changing measurement tools?

One answer to the first question is that reliable program measurement is not as easy as it sounds. Running times, for example, depend on complex interactions between a variety of products that comprise the programming environment, including the computer chip (perhaps Intel Inside), memory sizes and configurations, the operating system (such as Windows 98 or Unix), the programming language (maybe Java or C), and the brand of compiler (like CodeWarrior). These products are sophisticated, varied in design, and, especially when...
1. Array $A[1...n]$ contains $n$ distinct numbers. We want the $r^{th}$ smallest. Set $lo = 1$ and $hi = n$.

2. **Partition.** Set $x = A[lo]$. Rearrange the numbers in $A[lo...hi]$ into three groups around some index $p$ such that:
   a. For $lo < i < p$, we have $A[i] < x$.
   c. For $p < j < hi$, we have $A[j] > x$.

3. **Check Indices.** If $p = r$, stop and report the element $A[p]$. Otherwise, if $p < r$, set $lo = p + 1$ and go to Step 2. Otherwise, $p > r$, so set $hi = p - 1$ and go to Step 2.

Figure 1: Selection Algorithm $S$. The algorithm reports the $r^{th}$-smallest number from array $A[1...n]$.

There is no known general method for making accurate predictions of performance in one programming environment based on observations of running times in another. Experimental analysis of the abstraction allows us to have more control over the trade-off between generality and accuracy when making predictions about performance.

An answer to the second question is straight from the mathematician: algorithms and their analyses are beautiful and fundamental, and they deserve study by any means available, including experimentation.

Certainly algorithms existed long before the first computing machine was a gleam in Charles Babbage's eye. For example, Euclid's *Elements*, circa 300 B.C., contains an algorithm for finding the greatest common divisor of two numbers. While many algorithms have been discovered and published over the centuries, it is only with the advent of computers that the notion of analyzing algorithmic efficiency has been formalized.

Perhaps the first real surprise in algorithm analysis occurred with Strassen's discovery in 1968 of a new method for multiplying two $n \times n$ matrices. While the classic algorithm we all learned in high school requires $2n^3 - n^2$ scalar arithmetic operations, Strassen's algorithm uses fewer than $7n^{\log_2 7} - 6n^2$ operations, where $\log_2 7 < 2.808$. Therefore, this new algorithm uses fewer operations than the classic method when $n$ is greater than 654. Strassen's discovery touched off an intensive search for asymptotically better matrix multiplication algorithms. The current champion requires no more than $cn^2.376$ scalar operations for a known constant $c$. It is an open question whether better algorithms exist. (See any algorithms textbook, such as [1], for more about matrix multiplication.) These fancy algorithms are not much use in practice, however. It would be a daunting prospect even to write an error-free program for one of them, and the extra computation costs imposed by their complexity make them unlikely to outperform the classic method in typical scenarios.

Algorithm analysis is a vigorous and vital subdiscipline of computer science, providing deep new insights into the fundamental power and limitations of computation, fodder for the development of new mathematical techniques (mostly combinatorial), and, not infrequently, efficient algorithms that can have substantial impact on practice. It is clear, however, that our analytical techniques are far too weak to answer all our questions about algorithms. Computational experiments are being used to suggest new directions for research, to support or disprove conjectures, and to guide the development of new analytical tools and proof strategies.

This article presents examples from two broad research efforts in experimental algorithmics: first, to develop accurate models of computation that allow closer predictions of performance, and second, to extend abstract analyses beyond traditional questions and assumptions. We start with a short tutorial on the notations and typical results obtained in algorithm analysis.

**A Tutorial on Algorithm Analysis**

The selection problem is to report the $r^{th}$-smallest number in a collection of $n$ numbers. For example, $r = 1$ refers to the minimum of the collection, and $r = (n + 1)/2$ is the median when $n$ is odd. For convenience we assume that no duplicate numbers appear in the collection.

Figure 1 presents a well-known selection algorithm. The $n$ numbers are placed in an array called $A$ in no particular order. Each number has some position from 1 to $n$ in the array: the notation $A[i]$ refers to the number in position $i$, and $i$ is called an index. The notation $\ell = lo$ means "set $\ell$ equal to the value of $lo$".

The main operation of algorithm $S$ is to choose a partition element $x$ from a contiguous subarray of $A$ defined by indices $lo$ and $hi$ and to partition the subarray by rearranging its contents so that numbers smaller than $x$ are to its left and numbers larger are to its right. This puts $x$ at some location $p$; that is, $A[p] = x$. After partitioning, we know that $x$ is the $p^{th}$-smallest number in $A$. We are looking for the $r^{th}$-smallest number: depending on the relationship of $p$ to $r$, the algorithm either stops or repeats the process on one side or the other of $p$.

To analyze the algorithm, we derive a function that relates input size to the cost of the computational resources used by the algorithm. The precise meanings of "input size" and "cost" depend
upon the model of computation being assumed. Here we shall use the simple RAM (random access machine) model under which all scalar numbers have unit size and all basic operations on scalar values—such as arithmetic, comparison, and copying of values—have unit cost. Therefore the input size is $n$.

For our purposes it is sufficient to define cost as the number of times $x$ must be compared to elements from $A[lo \ldots hi]$ during the partitioning step. A partitioning method is known (described later in this article) that uses $hi - lo$ comparisons to partition subarray $A[lo \ldots hi]$ of size $hi - lo + 1$. The only problem remaining is to count up total costs.

Obviously, the total cost depends on which partition element $x = A[lo]$ is used each time; we may get lucky and find $p = r + 1$ after just one partitioning operation, or we may have to repeat the process several times. The worst-case cost, $C_w(n)$, is the maximal number of comparisons over all arrays of size $n$ and all $r$: a worst-case scenario holds, for example, when $r = n$ and $p$ becomes $1, 2, \ldots, n$ in successive partitioning stages. Letting $t$ denote the cost of a single comparison of $x$ to an array element, we have $C_w(n) = \sum_{i=1}^{n} t(n - i)$.

The average-case cost, $C_a(n, r)$, is found by averaging over some probability distribution on arrays of size $n$. Assume here that every number in the collection is equally likely to be in position $lo$ and selected as the partition element $x$. Then we have:

$$C_a(1, 1) = 0$$

$$C_a(n, r) = t(n - 1) + \frac{1}{n} \left( \sum_{p=1}^{r-1} C_a(n - p, r - p) + \sum_{p=r+1}^{n} C_a(n - p, r) \right).$$

Note that establishing the correctness of this recursive formula requires a proof (which exists) that the partitioning operation preserves the property that each number in the subset is equally likely to end up in position $A[lo]$ and be chosen as the partition element at a later stage.

Note also that the cost of this, and any, algorithm is described by a collection of cost functions that correspond to different scenarios. Another cost function might be obtained with other probabilistic assumptions, or a different definition of cost might be used under some other model of computation.

The first goal in algorithm analysis is the classification of cost functions according to complexity classes as defined below. Throughout, we assume that $f(n)$ and $g(n)$ map nonnegative integers to positive real numbers.

**Definition:** $f(n) \in O(g(n))$ if there exist positive constants $c$ and $n_0$ such that $f(n) \leq c \cdot g(n)$, $\forall n \geq n_0$.

**Definition:** $f(n) \in \Omega(g(n))$ iff $g(n) \in O(f(n))$.

The statements below are typical of the kinds of results obtained in classic algorithm analysis.

- Any correct selection algorithm must at least examine every element of $A$. Therefore, any algorithm that solves the selection problem must have a worst-case cost function in $\Omega(n)$. This is called a lower bound on the problem of selection.

- It is easy to see that $C_w(n) \in O(n^2)$. We have a complexity gap between the $O(n^2)$ worst-case bound and the $\Omega(n)$ lower bound on the problem. Complexity gaps generate research questions: Does an $O(n)$ algorithm exist? Should the lower bound be raised to, say, $\Omega(n^2)$, because selection is fundamentally harder than the first lower bound indicates? Or is the lower bound somewhere in between, perhaps at $\Omega(n \log n)$?

- In fact, a variation on algorithm 5 is known that has $O(n)$ worst-case cost, thus closing the complexity gap. The variation uses an elaborate strategy for choosing the partition element at each stage and is generally considered too complicated and expensive to be useful in practice.

The goal of the asymptotic analyses above is to place cost functions into complexity classes, while the goal of exact analysis is to find closed forms that retain constant factors in the leading terms. A closed form for $C_w$ is easy to obtain. Also, it can be shown that:

$$C_a(n, r) = t(n + 1)H_n - (n + 2 - r)H_{n+1-r} - (r + 1)H_r + n + 5/3 - \delta_{rn}3 - \delta_{rt}3 - 2\delta_{rn}\delta_{rt}/3,$$

where $H_n$ is the $n^{th}$ harmonic number and $\delta_{rn}$ is the Kronecker delta function. This implies that $C_a(n, r) \in O(n)$.

**Memory Sensitive Models of Computation**

In the good old days (say around 1970) it was possible to obtain quite accurate predictions of program running time by inserting appropriate time units for the constants (like $t$ above) in an exact analysis using the RAM model of computation. Nowadays, however, the RAM model is inadequate to the task.

For example, consider the seemingly innocuous instruction $x \leftarrow A[lo]$ in Step 2 of our example algorithm. The actual work to carry out this instruction is performed in a computer by the Central Processing Unit (CPU). The instruction involves about three CPU operations: (1) calculate the memory address of $A[lo]$, (2) fetch the value from that
memory address, and then (3) store the value at the memory address associated with \(x\). (Normally, array addresses need to be calculated, but scalar addresses do not.)

A fast modern CPU is capable of performing each operation in about a nanosecond. But the CPU may be forced to wait upon a slow memory, and the additional time needed for the fetch and store operations can vary by enormous factors, depending on exactly where \(A[i]\) and \(x\) reside in the memory hierarchy. Figure 2 gives a simplified diagram of a memory hierarchy ranging from a small, fast, and expensive register set, to a huge, slow, and inexpensive secondary memory.

![Figure 2: The memory hierarchy, simplified. This drawing is not to scale.](image)

The idea is to keep values that the CPU needs close to it in the fastest memory, but of course not everything will fit. Therefore, memories are designed to retain values that will be needed soon and to evict unneeded values by moving them back to slower memory. It is not always possible to predict which values will be needed next, so most memory levels operate under some kind of principle of locality, which says that values that have been recently accessed, or that are near values recently accessed, are likely to be needed soon. There may be a different version of this principle at each level of the hierarchy, and wide variations exist in computing environments with respect to number of levels, sizes of memories, and the eviction policies adopted.

Consider now the cost of fetching the value \(A[i]\). If that value has been used recently, it may be in a register, and the fetching cost is negligible. Decisions about which elements get to stay in registers are primarily made by the compiler, a translation program that converts the original program to machine-readable language. It is an NP-hard problem (about which more later) to make optimal decisions about placing variables in registers; therefore efficient optimal algorithms are not likely to be found. Modern compilers incorporate heuristic strategies for deciding which values are to be stored in registers.

If \(A[i]\) is not in a register, it may be in the Level I or Level II cache. Typical cache access times on workstations are around 5 to 10 nanoseconds. Cache sizes and eviction policies are part of the overall design of the computer. If not in a cache, \(A[i]\) is likely to be in main memory, with access times around 50 to 100 nanoseconds. But it is possible that \(A[i]\) has been placed in secondary memory, which may have access times greater than \(10^6\) nanoseconds. Main-to-secondary eviction policies are incorporated into the operating system.

Whereas the abstract RAM model assigns unit cost to one access of \(A[i]\), true memory access times can vary by factors as large as a million. Without a decent model of the memory hierarchy, we cannot predict whether a given program will take one minute or two years to run. (Of course, this worst-case scenario is not typical, but predictions about program running times that are off by three or more orders of magnitude are not unusual.)

Several threads of research in experimental algorithmics are concerned with developing new models of computation that incorporate aspects of the memory hierarchy, especially with respect to caches and to policies at the main-to-secondary memory levels.

In 1996, for example, LaMarca and Ladner introduced a RAM model with a two-layer memory (cache and main memory). They and several other researchers have since been able to reanalyze classic algorithms and data structures under the new model. These results, obtained through a combination of analytical and experimental approaches, have produced new theorems and new analysis techniques, better predictions of program performance, faster programs, and clear indications that our "common sense" understanding of what contributes to program efficiency is due for revision.

To get a feeling for the type of mathematics involved, consider reanalyzing our selection algorithm under the new two-layer model. We need to be more explicit about the partition step: Figure 3 shows one well-known partitioning method that performs \(hi – lo\) array-element comparisons for subarray \(A[lo \ldots hi]\) of size \(hi – lo + 1\).

The new model organizes main memory and the cache into contiguous blocks of scalar values. Main memory contains \(M\) blocks named \(m_0\) through \(m_{M-1}\), and the cache holds \(C\) blocks named \(c_0\) to \(c_{C-1}\), with \(C << M\). Suppose an operation accesses element \(i\) which resides in block \(m_i\). Then \(i\) and the entire block containing it are transferred to cache block \(c_j\), where \(y = x \mod C\). This transfer incurs a cost of \(t_{out}\), but any subsequent access of an element in block \(c_j\) will have smaller cost \(t_{in}\). This block will be evicted from the cache and sent back to main memory when another element is accessed from some block \(m_z\) such that \(y = z \mod C\).
Partition.

2.1. Set $x \leftarrow A[lo]$. Set $\ell = lo + 1$ and $h = hi$. Iterate the following three steps.

1. Use $\ell$ to scan up from the left, comparing each $A[\ell]$ to $x$. Stop when either $A[\ell] > x$ or $\ell = h$.
2. Use $h$ to scan down from the right, comparing each number $A[h]$ to $x$. Stop when $A[h] < x$ or $h = \ell$.
3. If $\ell < h$, swap (exchange) the numbers $A[\ell]$ and $A[h]$ and go to Partition Step 2.1. Otherwise, if $\ell = h$, go to Step 2.2 below.

2.2. The above process maintains the invariant that all numbers in $A[lo+1 \ldots \ell-1]$ are less than $x$ and numbers in $A[h+1 \ldots hi]$ are greater than $x$. At this point we also have $\ell = h$. If $A[\ell] < x$, swap $A[lo]$ and $A[\ell]$ and set $p = \ell$; otherwise, swap $A[lo]$ and $A[h-1]$ and set $p = \ell - 1$. Partitioning is now complete.

Figure 3. Partitioning $A[lo \ldots hi]$ around $x = A[lo]$.

Now suppose subarray $A[i \ldots j]$ resides in memory block $m$. The access cost for a particular array element $A[k]$, with $i \leq k \leq j$, is therefore

- $t_{out}$ if this is the first access of $A[k]$ or any of its block neighbors $A[i \ldots j]$.
- $t_{out}$ if an array element from any block $y$ such that $y = x \mod C$ has been accessed more recently than any element from block $x$ (causing an eviction).
- $t_{in}$ otherwise.

Homework Problem. Write new formulas for $C_p(n)$ and $C_a(n, r)$ under this model. Perform an exact analysis on both formulas. Would an alternative partitioning method, say one that traverses array elements from left-to-right rather than both-ends-toward-the-middle, give a lower memory access cost?

Clearly even this simple two-level model greatly complicates the analysis task. But the tighter analysis does appear to be well worth the effort in many cases. Jon Bentley (personal communication) reports that simple program modifications can reduce cache effects and improve running times by factors as large as 16. For a nice introduction to memory-sensitive models of computation, see LaMarca and Ladner's [6] analysis of four sorting algorithms under the caching model. They draw conclusions about efficiency that flatly contradict a common lore based on traditional analyses and obtain much tighter predictions of program running times than had been possible before.

Another research effort in memory-sensitive analysis concerns algorithms for data sets that must reside in secondary memory because they are too large to fit in main memory. Important applications for such algorithms include Web search engines that peruse directories containing hundreds of millions of entries, many-body simulation algorithms for research problems in the natural sciences, and algorithms for image processing and scientific visualization. The RAM model can also be extended to models that account for data transfers between main and secondary memory (which follow different rules from caches). New algorithms and analyses of old algorithms under these new models can result in huge improvements to program running times. Reductions of running times from several weeks to a few hours have been reported in the literature.

Heuristics for NP-Hard Problems

We now turn to another research thread in experimental algorithmics. The most interesting question about any solvable computational problem is whether it is tractable or intractable. A problem is tractable if it can be solved in polynomial time; that is, there exists an algorithm for the problem that has worst-case cost in $O(n^k)$ for some constant $k$. A problem is intractable if no such algorithm exists. For convenience we define intractable problems to be those having exponential lower bounds in $\Omega(c^n)$ for some constant $c > 1$ and ignore the fact that there exist functions like $n^{\log n}$ that are neither polynomial nor exponential.

Perhaps the most important technical idea to arise from the study of algorithms and problem complexity is the identification of problems that are $NP$-complete and $NP$-hard. Hundreds of practical problems that arise naturally in all spheres of industry, commerce, science, and government work have been identified as $NP$-complete and/or $NP$-hard. The technical definitions of these two problem classes are rather too involved to fit into this article, but the point is this: it is an open question whether these problems are tractable or intractable. That is, for every problem in these classes there is a huge complexity gap between an exponential worst-case bound on the best algorithm known, and a polynomial lower bound on the problem complexity. Do efficient algorithms exist for these problems, or are they fundamentally too hard to be solved in polynomial time?

For technical reasons, "NP-complete" refers to decision problems, which are always phrased as yes-or-no questions (Does input instance $I$ have property $P$?). "NP-hard" can refer to more general types of problems, such as optimization problems that involve minimizing or maximizing some quantity associated with the desired solution. The two classes are related as follows: Let $X$ and $Y$ be $NP$-complete problems, and let $Z$ be $NP$-hard but not a decision problem. Then (1) a polynomial time algorithm for $X$ exists if and only if one exists for $Y$; (2) if a polynomial time algorithm to produce optimal solutions
for $Z$ exists, then polynomial algorithms exist for $X$ and $Y$.

Thus, discovery of a polynomial-time algorithm for any NP-complete or NP-hard problem would have profound implications about the complexity of several hundred other problems of great practical interest, besides earning the discoverer a cool million dollars. (To learn more about the million-dollar Millennium Prize Problems announced by the Clay Mathematics Institute, visit www.claymath.org/prize_problems/) Proof of an exponential lower bound for any NP-complete problem would reverberate similarly. To learn more about the theory of NP-completeness, see the classic text by Garey and Johnson [3]. Three famous NP-hard problems are described below.

**Traveling Salesperson.** You are given a graph or digraph $G = (V, E)$ with positive-valued weights on its edges. A Hamiltonian tour of $G$ is a closed path over vertices and edges that visits each vertex of $G$ exactly once. The cost of the tour is the sum of the weights of edges traversed on the path. The problem is to find a minimum-cost Hamiltonian tour of any given graph $G$.

This problem is of interest to traveling salespersons who wish to tour all cities in a sales region while minimizing total travel cost. It is also of interest to any organization that must schedule regular tours of delivery trucks, like the U.S. Post Office and your favorite grocery store chain.

**Bin Packing.** Given a list of $n$ weights, all from the real number range $(0, 1)$, the problem is to organize the weights into unit-capacity bins so as to minimize the number of bins used. For example, the weight list containing $0.4, 0.3, 0.6, 0.5$ could be grouped into $(0.4, 0.3), (0.6), (0.5)$, which uses three bins, or into $(0.4, 0.6), (0.3, 0.5)$, which uses only two bins.

This problem is of interest to anyone walking into a lumberyard with a list of board lengths needed for a construction project who must group the lengths so as to minimize the number of standard 8-foot boards that must be purchased. It is of interest to any construction supply firm that must cut stock from unit-sized pieces. The problem is also of interest to anyone who wants to back up files by copying onto unit-sized floppy disks while minimizing the total number of disks needed. A two-dimensional version of the problem (where weights come in $(x, y)$ pairs) is of interest to any printer or pattern cutter who must lay out parts on unit-sized rectangular sheets so as to minimize the total number of sheets needed.

**Graph-Coloring.** Given a graph $G$, a coloring of $G$ is an assignment of colors to vertices under the constraint that no two vertices sharing an edge can have the same color. The graph-coloring problem is, for any given graph $G$, to find a coloring that minimizes the total number of distinct colors used.

Graph-coloring is a generalization of the famous map-coloring problem to nonplanar graphs.

Graph-coloring is of interest to a university registrar who assigns times (colors) to courses (nodes), under the constraint that no two courses with the same professor can meet at the same time. It is of interest to anyone who needs to create work schedules or timetables that avoid certain kinds of conflicts. And graph-coloring arises in the context of the compiler task, mentioned earlier in this article, of assigning values to registers so as to minimize memory access costs.

Recall that the CPU can work only on values that are in registers. Memory costs are incurred when a value must be transferred from another memory to a register (here we assume that this transfer cost is constant).

Consider the sequence of instructions that occurs at the beginning of our selection algorithm $S$:

1. $lo = 1$
2. $hi = n$
3. $x = A[lo]$
4. $l = lo + 1$
5. $h = hi$

Two values interfere if one must be accessed between two accesses of the other. When two values interfere, we want to place them in different registers so as to avoid the cost of evicting one to make room for the other. For example, $lo$ and $hi$ interfere because accesses of $hi$ on line (2) come between accesses of $lo$ on lines (1) and (3) and also because $lo$ is accessed between lines (2) and (5). If both $lo$ and $hi$ were assigned to register $R1$, then these values would incur the following memory transfer costs: (1) fetch $lo$; (2) evict $lo$, fetch $hi$; (3) evict $hi$, fetch $lo$; (5) evict $lo$, fetch $hi$; for a total of seven memory transfers. These two values could coexist peacefully if placed in registers $R1$ and $R2$, although this might cause interference difficulties with other values.

More generally, given a sequence of instructions in a program, we can build an interference graph $G$ where nodes represent program values and an edge is placed between two nodes if their values interfere. The problem is to assign a minimum number of registers (colors) to values (nodes) such that interfering values receive distinct register assignments.

As is the case with all NP-hard problems, no polynomial-time algorithm for finding minimal colorings is known, and it is not known whether such an algorithm exists. While the theoreticians struggle with the general problem, the compiler writers must settle for coloring algorithms that use a “small number” of colors, if not the absolute minimum. Dozens of algorithms have been proposed, which fall into some broad categories, broadly sketched in the list below. (Colors are...
assumed to be numbered 1 through \( k \), for some appropriate \( k \).

**Greedy Algorithms.** A greedy algorithm may iterate over nodes, assigning to each the least-numbered color that does not violate any edge constraint. Or an algorithm may iterate over colors, assigning each color to a large independent set (finding a maximum independent set is an NP-hard problem). Greedy algorithms are often repeated with different node or color orderings each time, and the best solution found is saved. These algorithms are fast but may produce colorings using many more colors than necessary.

**Branch-and-Bound.** Systematically generate colorings of \( G \), avoiding redundant colorings and unproductive starts, and save the best one found within a given time limit. Note that even on moderate-sized graphs only a tiny fraction of colorings can be generated from the enormous space of possibilities; various strategies for generating the best ones first may be considered.

**Heuristic Search Methods.** Start with a valid coloring. "Step" to another valid coloring by randomly modifying the current coloring in some small way (perhaps by changing the color of one node). Continue stepping until a good coloring is found. Steps toward better colorings are of course preferred, but it is important occasionally to allow steps toward worse colorings to avoid being trapped in a local minimum. A rich variety of stepping rules and strategies for controlling the stepping process have been proposed.

These general strategies can be modified to fit many varieties of NP-hard problems, including Traveling Salesperson and Bin Packing. In some cases (like these two), strategies that exploit special problem structures are known that outperform the general techniques. Note that the analysis of algorithms for NP-hard problems usually involves finding functions and complexity classes for two interesting quantities: algorithm efficiency and the quality of solution produced by the algorithm.

In the case of heuristic algorithms like those sketched above, average-case analyses are extremely difficult to obtain even under the simplest computational and probabilistic models. Analytical statements about how a coloring algorithm performs on, say, random graphs with edge probability \( p \), are rare, and statements about performance on "graph structures typical of register interference problems" are far beyond our analytical capabilities.

Computational experiments can be used to enrich our understanding of these heuristic algorithms for NP-hard problems. Experiments have been used to direct the discovery of new algorithms and new analyses, to allow prediction of algorithmic responses to variations in input properties, and to sort out which approaches work best in which kinds of scenarios.

Indeed, graph-coloring has been the focus of what might be termed a new mode of research in algorithmic studies: a coordinated, multiparticle effort to identify the state-of-the-art in algorithmic performance for some given problem. Since 1992 the DIMACS Center (an NSF- and corporate-funded center for research in discrete mathematics and theoretical computer science, located at Rutgers University) has sponsored an annual Implementation Challenge to encourage and promote experimental research on algorithms for specific problems. The focus on a particular problem allows a great deal more coordination and interaction than might otherwise be possible by independent researchers.

The second DIMACS Challenge (1993) [4] concerned algorithms for three NP-hard problems: graph-coloring, finding large cliques in graphs, and finding satisfying truth assignments for Boolean formulas. While a great deal of progress was made, one conclusion drawn at that workshop was that huge gaps remain in our understanding of the "best algorithms" for particular applications. Much more work remains to be done.

Besides the DIMACS Challenges, several other venues for experimental research on algorithms have appeared in the past decade. Two annual conferences (WAE in Europe and ALENEX in North America), the electronic *Journal on Experimental Algorithmics*, and numerous small workshops have all been developed to provide forums for researchers to report on their experimental observations.

**Questions of Methodology**

Research in experimental algorithmics produces new understanding of models, problems, algorithms, and program efficiency. Another important component of experimental algorithmics has been the development of methods and techniques for performing experiments on algorithms. As suggested in the beginning of this article, the relationship between algorithms and programs and the usual types of questions asked about algorithms are not typical of those that arise in mathematical modeling. Consequently, new problems of designing and conducting experiments and of developing appropriate statistical tools can be identified. Some research questions concerning methodologies for experimental analysis of algorithms are sketched below. See [7] for more discussion.

**How to Escape the Artifact?** We study a perfect abstract object (the algorithm) using an imperfect measuring device (the computer). Difficulties associated with numerical precision, nonrandomness in pseudo-random number generators, and inaccurate measuring tools can produce spurious and erroneous results. Tools for measuring pro-
program running times are notoriously quirky, and in some contexts it can be quite difficult to find a definition of "cost" that is useful, measurable, and generalizable. New techniques are needed for identifying and minimizing the impact of artifact on observation.

**Sampling from Large and Infinite Spaces.** It is not possible to measure the performance of a given graph-coloring algorithm over the space of all graphs of a given size $n$, nor is it always feasible to sample from the space of all colorings of a given graph. We need good methods for defining and generating input classes and input samples to support observations that can be generalized outside the realm of observation. For example, very little is known about methods for extrapolating from a finite data set to obtain the kind of asymptotic statements about complexity classes (e.g., does the data come from a function that is $O(n^2)$?) that are of interest to computer scientists.

**New Data Analysis Techniques.** Computational experiments on algorithms can produce enormous data sets. Some sets are too massive to be amenable to standard statistical techniques, so we must either make do with fewer measurements or develop faster techniques. Computational experiments can be very interactive and iterative, and it seems likely that new methods of data analysis can be developed to take advantage of these properties.

**How Important Are Algorithms?**
As the discussion in this article suggests, sufficiently precise results for all but the simplest algorithms, input models, and models of computation are quite difficult to obtain by purely analytical methods. With our current techniques, asymptotic analyses of algorithms are just barely adequate for making very rough predictions about program running times. Some researchers have wondered if we would be better off discarding our analytical models and building new empirical models based entirely on observations.

In an experiment to test the relevance of big-oh algorithm analysis to practice, Bentley [2] describes a race between two algorithms implemented under extreme conditions. He implemented an $O(n)$ algorithm (under the RAM model) using 1980's low-end technology, which required 19.5$n$ milliseconds to run, and an $O(n^3)$ algorithm in high-end 1999 technology, which required 0.58$n^3$ nanoseconds to run. At small $n$ the cubic algorithm dominates, and at large $n$ the linear algorithm dominates. Bentley found that even with this maximal difference in computing environments, the crossover point was low, near $n = 5,800$, where both implementations required around 2 minutes of running time. At $n = 10^5$ the $O(n^3)$ implementation with small coefficients required 7 days, while the $O(n)$ implementation with huge coefficients took only 32 minutes.

This and other experiences reported in the literature suggest some rough guidelines about the relative importance of algorithmic and environmental factors: In typical scenarios changes in environment (such as switching computers, compiler optimization level, or operating system; or improving programmer skills) can affect running times by factors in ranges between 2 and 100. Improvements on a larger scale can be obtained through asymptotic improvements to algorithms, by factors of $O(n)$ or more, and algorithmic improvements by factors below, say, $O(\log n)$ are not likely to have much real impact on performance.

Continuing research in the new and rapidly developing field of algorithm experimentation will allow us to refine these guidelines, to produce better understanding of environmental effects, and to generate better algorithms and tighter analyses.

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**References**


Presidential Views:
Interview with Hyman Bass

Every other year, when a new AMS president takes office, the Notices publishes interviews with the current president and with the president elect. What follows is an edited version of an interview with AMS president Hyman Bass, whose term began on February 1, 2001. The interview was conducted in October 2000 by Notices senior writer and deputy editor Allyn Jackson. An interview with past president Felix E. Browder appeared in the February 2001 issue of the Notices, pages 187-189.

Notices: First, I'd like to ask you about the AMS' role in advocating support for basic research. Where do you think the AMS is right now in this area, and how do you see its activities evolving?

Bass: I think right now we are probably in a better position than we have been in many years. A great deal of the credit goes to [past presidents] Arthur Jaffe and Felix Browder. Of course, this had a lot to do with changing conditions in the general environment for science. But I think that Arthur and Felix deserve credit for mathematics having played a significant role in improving the resource situation. The basic principle behind that, as I see it, was that mathematics moved from a stance of pursuing the narrow interests of the discipline to one of reaching out to the public with broad advocacy of support for basic science in general and doing this in partnership with the other sciences. In this mode, whatever gains mathematics enjoyed would be achieved in tandem with growth in support for science in general. The recent legislative initiative for doubling the NSF [National Science Foundation] budget over five years is confirmation of the wisdom of that strategy.

From my point of view, I don't see the need for major new initiatives by the AMS, but rather the need for sustaining the progress that's been achieved. I should add that major credit goes to the AMS Washington Office, especially the contacts that Sam Rankin has made with congressional staff there. Thanks to Sam's work, the Washington Office has become an extremely effective operation and a very productive investment on the part of the AMS.

Notices: Have you heard about the new math initiative at the NSF? What do you think of it?

Bass: Well, I don't yet know much in detail about it, but I know from earlier statements that [NSF director] Rita Colwell had, within the NSF agenda, given a high priority to redressing what were perceived as imbalances in the funding for mathematics vis-à-vis the other sciences. What was noteworthy is that this case, which had long been made by mathematicians, was now being made by a life scientist who is directing the foundation.

When things get sufficiently out of balance, even the users of mathematics are threatened. And because mathematics is an enabling science, there are lots of "users". The life sciences have always been better treated by Congress and the public, because their applications are in some sense closer to personal use and recognized social needs. When the life scientists see ample documentation that the long-term health of basic science is under threat, then I'd like to think that they are persuaded by the logic of the facts and wise policy. But it takes people with a very broad vision of science as a global enterprise to reach conclusions like that. Most people operate in a much more narrow environment, and they seek to maximize resources only on a more limited scale.

So the bottom line on this—and I would say this as an overarching statement—is that I am lucky to come into a situation in which the AMS on every front is in a healthy condition, both as an organization and as a community. The most troubled part is probably the one where I am spending much of my time, which is education. But even there I think we're making significant progress.

Notices: Let's talk a little bit about public awareness of mathematics.

Bass: For one thing, there's now a lot more writing on mathematics for the general public than there has been in recent years. This is an area where a lot of interesting progress has been made, and of course public urgings to the math community to give more attention to public outreach
and to writing good expository material helps nurture that. We have journals like the Notices which I think have been very successful at producing a lot of excellent material, not least your own writing, which I have long admired. The appointment of the new [public awareness] staff at the AMS is another step in that direction.\footnote{See "AMS Establishes Public Awareness Office" in the "Inside the AMS" section of this issue.}

**Notices:** Is there a particular emphasis you think the AMS should have in its public awareness efforts?

**Bass:** I don’t have any specific emphasis in mind, but it should be pursued on a fairly broad front, and I think that our efforts should be treated in part as a learning experience. I think we should try to assess pretty carefully what works and what doesn’t, because in these arenas we are starting to function essentially as amateurs. For example, in Washington there is a professional community that does political work, but we are not asking for the work to be done in that culture. We want the work to be done in a way that represents the culture of mathematics. In Washington we have developed a small cadre of professionally able people who have those skills. The same is true with public outreach. We should think that the product of our investment is twofold. One is to produce successful expository material and promotional material that reaches a public audience, and the other is to build a professional capacity to do this sort of work and to train others to do it. We have to think of the Washington outreach and public awareness outreach partly educationally and partly developing the professional culture for this kind of work. Once we know better how to do this, we can create internship opportunities for young people so that they can engage in this without abandoning their mathematical careers.

**Notices:** You mentioned earlier your involvement in education. What is the AMS role in K-12 education?

**Bass:** This is an area in which the AMS did not organizationally decide to move but in some sense was gradually moved into it by external developments, developments that reflect the broader growth of our professional community.

Historically, mathematicians’ involvement in K-12 education was usually seen as episodic. Certain mathematicians chose to turn their interests and reflections in those directions, just as mathematicians might become interested in philosophy or poetry or music. Interest in education was not treated as a movement in the field, but as something congenial with it. Those efforts were hospitably received in the mathematical community and were treated as a wholesome part of the general culture, but not as central to it.

The situation is quite different now, but not because of change of individual interest or concern. A lot of it has to do with the whole interlocking dynamics of expansion of the field. For one thing, the size of the field is just much, much larger. The student enrollment in our universities, the number of people who need technical training, and then in turn the number of professional mathematicians are vastly larger. Mathematics as a profession has a social fabric that is quite different from what it ever had before. No one worries that mathematics is an endangered species; the ideas have gone on for many millennia, and it’s unthinkable that mathematical ideas and culture would not continue to grow in time. But as a large professional community we inherit questions that are not about the continued growth and development of ideas, but about the sustained capacity to meet all the human needs and dependencies that are created by the large community that is trained to do this work. We have to think about the difficulties that are created by inadequate resources or dislocations caused by people having to leave the field and do something for which their mathematical training has not equipped them.

So a lot of issues that we face, even though they grew organically out of the growth of the field of mathematics, are not inherently mathematical problems. They are really problems of the professional community. By the same token, one can understand the growth and evolution of the AMS as responsive to these social dynamics in the profession. I can remember my early experiences on AMS committees, where it was hotly debated whether the AMS should become a large publishing organization. The idea of publishing was viewed as alien to its scholarly purposes and too much tinged with commercialism.

**Notices:** Nobody worries about that anymore!

**Bass:** Not only do they not worry about it, but in fact that very enterprise enables the AMS to function as a kind of enlightened patron of the very aspects of the culture that are somewhat fragile and that would not easily be supported by external organizations. But these things are always trade-offs. We are a much larger and more structured organization. The first AMS meeting I went to was at Columbia, and there were on the order of 100 or 120 people, and one attended every lecture. There was more the feeling of a philosophical inquiry rather than a major enterprise.

In the post-Sputnik era what the country needed was a cadre of highly trained technical professionals, and our system developed a very high capacity to produce that. Many people failed and many were alienated or driven away from
mathematics and science in the process, but that was considered okay, because the number of people that got through the filter was enough to meet national needs.

What we used to accomplish for a limited number of students we now must accomplish for nearly all students, without sacrificing quality levels. We need to be attentive to the ways in which the discipline has changed, to the presence of technology, to appropriate ways of presenting mathematical ideas in the classroom, and to contemporary understanding of instruction and student learning. This places great new demands on teachers. The country has undertaken to solve a problem it never has faced before—that is, to help all students attain high levels of mathematical proficiency.

One of the first things you have to do when you think about education is to decide what are the goals, what do you want people to learn? In the U.S. this is a matter for states and districts, sometimes even individual schools. Never in our nation's history have goals been articulated and shared at the national level. So the NCTM [National Council of Teachers of Mathematics] stepped into this policy vacuum. The standards NCTM created [in 1989] were based in part on a combination of educational research and the views of some disciplinary mathematicians, but largely also on the wisdom of practice and the knowledge base of professional practitioners. In my view, it was a positive event that the standards were developed by the professional organization of practicing teachers.

Creating standards is the first and the easiest step in this business. The next step is curriculum development, which is complex design work. The NSF funded many projects to develop curricula based on the NCTM standards. Starting in the mid-1990s, these curricula began entering schools. That was the first time this whole movement began to touch people's lives on a significant scale. This precipitated pockets of adverse reaction from parents, whose kids returned with homework that the parents sometimes did not know how to do or even recognize. And mathematicians are among parents. It was this concern with their children's schooling that first turned the attention of certain mathematicians toward school mathematics education.

When mathematicians first got vivid exposure to what was happening in the schools, many of them were outraged. For some it was a perceived neglect of "basic skills"; generally understood to be the teaching of standard algorithms. This was often attributed to the early introduction of technology into the classrooms. As they looked closer they were often alarmed by the seemingly fragile mathematical understanding of the teachers. It's not as if these concerns were without cause. But the question is, What do you do with what you see? We can't invent solutions that pretend that the teachers we have are not there and that some ideal community of teachers is suddenly going to appear. The teachers in the schools are not dumb or stupid and stubborn. They are actually very dedicated people who love what they do. In most cases they wouldn't be there otherwise, because there are very few incentives. Most of them are actually quite smart and able to learn things. But they have had long experience with subject matter and with kids that is very different from mathematicians' experience. Teachers are very realistic and have a real sense of survival and pragmatism, and if they feel that mathematicians are people who are going to scorn them or humiliate them, they become defensive and will not view mathematicians as a source of help. That kind of thing has happened. The mathematicians see themselves as kind of intellectual philanthropists and believe the teachers do not want to receive the wisdom they're ready to offer. So there is a lot of that kind of alienation. I think that's much of what the "math wars" are about.

I personally think the NCTM has achieved a great deal, and I think that the new PSSM document is an extraordinary achievement that has been well informed by the advice that was sought from other professional communities. The NCTM has made serious and bona fide efforts to ground its policy documents in whatever research is available and in solicited advice from other professional communities. I think that a sensible and constructive way to make improvements is to improve the way the NCTM functions. We can't invent solutions to these educational problems that ignore the professional community of teachers. The rhetoric of mathematicians who publicly protest every single fault and detail in everything the NCTM does is simply not doing the work that's going to move us forward. The NCTM has demonstrated that it can productively accommodate constructively rendered criticism.

So finally let me answer your question. The question was, What does K-12 education have to do with the AMS? What I've described so far are ways in which individual mathematicians have been drawn into this. On the national level—and this is now public policy and part of legislation—it has been recognized that this is a national problem and that, in particular, mathematicians and scientists have a special responsibility that extends their traditional roles in research and education at
the university level to concerns for K-12. This responsibility has taken concrete form in many funding programs. There is also the growing recognition of the fact that the teachers who teach in the schools and whose knowledge of mathematics we deride so much learned their mathematics primarily in mathematics departments. Therefore there is a kind of structural responsibility, even at the university level, to giving more attention to this. So for those various external reasons, the professional community of mathematicians and therefore the AMS—because it is the organization of that community—has an inherent interest in K-12 education issues.

It is appropriate for the AMS to create a legitimate and respected space for attention to that kind of concern, more or less on the order that it exists right now, with the Committee on Education, with programs at the national annual meetings, with articles in the Notices and elsewhere, and with a few other things. There’s no recipe for what order of engagement is appropriate. That’s largely up to the opinions and the practices of the community. But there are enough active mathematicians who are interested in educational issues that the level of attention given to them now seems appropriate. Mathematicians have important things to learn—about schools, about teacher education and teacher learning, about making change in schools, etc. The AMS can provide opportunities for such learning.

In addition, there is a huge number of things about math education now happening in the public domain. So at a very basic level, the AMS simply wants to remain aware of these currents. If mathematicians want to voice any concerns or opinions or contribute to educational work and policy, we need mechanisms by which connections can be made. The Committee on Education affords opportunities for this.

One motif that runs through everything we’ve talked about is outreach—outreach to the public, outreach to other disciplines, and outreach to education and to various policy arenas. The AMS, in a measured way and without undermining its central mission and its core commitments to mathematical research, is adapting flexibly to these expanding roles. As long as it performs those roles well and doesn’t let them imbalance its central mission, I think that this is a healthy mode of operation.

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Thompson and Uhlenbeck Receive National Medals of Science

On November 13, 2000, President Clinton announced the names of twelve individuals to receive the National Medal of Science, the nation's highest honor for achievements in research in science and engineering. Among those honored were two mathematicians, JOHN G. THOMPSON and KAREN K. UHLENBECK.

Thompson is the Graduate Research Professor of Mathematics at the University of Florida, Gainesville. He is considered one of the foremost group theorists of all time, and his name is associated with one of the monumental achievements of the twentieth century, the classification of all finite simple groups. He was awarded the Fields Medal in 1970.

Uhlenbeck holds the Sid W. Richardson Foundation Chair in Mathematics at the University of Texas at Austin. She made pioneering contributions to global analysis and gauge theory that resulted in advances in mathematical physics and the theory of partial differential equations. She is considered a founder of geometry based on analytical methods. She is also a leader in encouraging young women to study mathematics.

The National Medal of Science was established by Congress in 1959. It was intended to be bestowed annually by the president of the United States on a select group of individuals deserving of special recognition by reason of their outstanding contributions to knowledge in the physical, biological, mathematical, or engineering sciences. Congress expanded this definition in 1980 to recognize outstanding work in the social and behavioral sciences. In 1962 President John F. Kennedy awarded the first Medal of Science to the late Theodore Von Karmen, professor emeritus, California Institute of Technology. Including the twelve 2000 winners, 386 have been awarded the Medal of Science. Since 1995 the following mathematical scientists have received the medal: Felix E. Browder, Ronald Coifman, Leo P. Kadanoff, Richard Karp, Cathleen S. Morawetz, Louis Nirenberg, Stephen Smale, and S.-T. Yau.

A committee of twelve scientists and engineers is appointed by the president to evaluate the nominees for this award. The National Science Foundation (NSF) administers the National Medals of Science for the White House.

—Compiled from NSF press releases
Book Review

The Parrot's Theorem

Reviewed by Queena N. Lee-Chua

The Parrot's Theorem
Denis Guedj
Translated from the French by Frank Wynne
(First published as Le Théorème du Perroquet,

Scheduled U.S. Publication: September 2001
St. Martin's Press
ISBN 0-312-28055-6, Hardcover, $23.95

Literature for young adults often has elements of adventure and fantasy, as seen in the works of J. R. R. Tolkien, C. S. Lewis, and Lewis Carroll. The Parrot's Theorem is not only an adventure story but a mathematical novel as well, and it takes the reader on an odyssey through history (from Thales to Wiles). This delightful book is reminiscent of Lewis Carroll's Through the Looking Glass and Curiosa Mathematica, where mathematical tidbits are golden threads interwoven through a rich tapestry of fantasy.

The main characters are delineated in considerable depth—the deaf eleven-year-old Max Liard, whom Euclid would have classified as a solid, since Max has "length, breadth and depth" (p. 9); the reclusive bookseller Monsieur Pierre Ruche, who is bequeathed a vast library of mathematical tomes by Grosrouvre, a long-lost friend in the Amazon; and, of course, Sidney the parrot, a voluble storehouse of mathematical knowledge—theorems, proofs, and all. Not since the Cheshire Cat has such an entertaining figure been found in the pages of a mathematical fantasy book.

The highly appealing plot has all the elements of a well-crafted mystery. What exactly caused the sudden death of Grosrouvre in Brazil? What secrets lurk in Mr. Ruche's library, dubbed the Rainforest Library by Max and his family? Why are those goons after Sidney? Who kidnapped Max? And how can Archimedes, Pythagoras, Fibonacci, Cardano, and others help in ensuring that vital proofs do not fall into the wrong hands? As befits a good thriller, this book also has a satisfying twist at the end.

Yet it is the mathematics and the mathematicians that hold the novel together. In order to solve the mystery, the characters must all take a mathematical history course. And what an enlightening course it turns out to be! Here the author, a professor of the history of science at Université de Paris VIII and an award-winning film writer, is at his best. The historical accounts, many of which will be familiar to mathematically inclined readers, flow seamlessly through these pages and seem in no way contrived. How Thales used similar triangles to measure the Pyramid of Cheops is explained by Mr. Ruche through a slide show, and this concept is extended when Max measures the Obelisk in Paris. Irrational numbers and the Pythagorean theorem are the topics of a play by Max and Mr. Ruche, and the proof that $\sqrt{2}$ is irrational comes alive in a conversation between Max's siblings. To introduce a tour of The Elements, Mr. Ruche recounts the origin of one of Euclid's famous, and possibly apocryphal, dictums: When King Ptolemy asked Euclid if there were no easier way to understand mathematics, Euclid replied, "There is no royal road to geometry."

A well-crafted chapter is devoted to expounding on the three great mathematical problems of the ancient world—the squaring of the circle, the duplication of the cube, and the trisection of an angle. The author does not hesitate to use terms such as cissoid, conchoid, or trisectrix, but through the help of figures and witty conversation he manages to convey the meaning to the lay reader without sacrificing mathematical accuracy.

Queena N. Lee-Chua is associate professor of mathematics at the Ateneo de Manila University, Philippines. Her e-mail address is queena@math.admu.edu.ph.
Further explanation of these concepts is found in a couple of footnotes which, though they may detract from the flow of prose in an ordinary novel, are strategically placed here.

The author does not stick only to ancient mathematics but also includes discussion of the work of Newton and Leibniz, Cardano and Tartaglia, Galois, Gauss, Fermat, Cauchy, Pascal, Poisson, and Fibonacci. The book also includes some less familiar but no less fascinating historical figures: the Franciscan monk Luca Pacioli, whose *Summa de arithmetica*, published at the height of the Renaissance, was a masterpiece that brought Arabic algebra to the West; the physician Nicolas Chuquet, who wrote the earliest algebra textbook in French and was the first to use negative exponents; and Bernt Holmboe, whose student Niels Abel would eventually surpass him. Another chapter touches on the basics of calculus, such as limits, differentiation, and integration. There is not as much detail as is found in David Berlinski's *A Tour of the Calculus*, but probably enough to spark the curiosity of the lay reader. The mechanics of logarithms and exponentiation are gently put forth, and the subsequent segue into interest rate problems feels totally natural.

Mathematical highlights, such as Goldbach's Conjecture, Fermat's Last Theorem, and Euler's Conjecture, are also woven into the plot. Aside from stories about mathematicians' lives, the book also looks into the origins of common symbols: among them, the number 0, the equal sign, the square root symbol, the number $e$, fractional notation, and the coordinate system.

But perhaps the primary reason this book may become a classic in young adult literature is its literary beauty. The author not only has presented the mathematics in an entertaining way but has also written some lovely passages that are the linguistic equivalents of elegant mathematical proofs. He explains the structure of a quatrain ("lines one, two and four linked by rhyme, while line three is free," p. 144) and discusses what is perhaps the best exemplar: Omar al-Khayyam's *Rubaiyat*, with its haunting verses (pp. 144-5). During a discourse on friendly numbers he mentions that Pythagoras reportedly invented the word "friendship" by defining "friend" as "someone who is another me, like the numbers 220 and 284. Two numbers are friends if each is the sum of everything that measures the other" (p. 68). This becomes a portent, for shortly before dying Grosrouvre reminds Mr. Ruche, "What about us—are we friends? What is the sum of those things that define you or me? I think perhaps the time is coming when we will find out" (p. 68).

Word play abounds throughout the novel. Grosrouvre writes Mr. Ruche's first name (Pierre) as a delightful mathematical-linguistic pun, *pi*, pronounced "pee-air" (in French). Most people know that the prefix "bi-" means "two," but does it ever occur to us that Native Americans refer to a bison as "two-horn"? "Iso-" means "same," and "skelos" means "legs," hence the term "isosceles"; but are we aware that "scalene" comes from the Greek word for "limping"? Students know that algebra comes from *al-jabr*, but they may be intrigued to know that its ancient meaning is "bone-setter." Witness this exchange between Mr. Ruche and his chauffeur, Mr. Habibi (p. 157):

"*Al-jabr* means a bone-setter," said Mr. Habibi. "Where I come from, if you have a *douar*, if you break a thing, you go to *al-jabr* and he twist this way—Ow!—and that way, and set the bone back in place. Yes, *Jabr* is someone who fixes something that is broken."

Mr. Ruche jumped in, "In the story of *Don Quixote* there is an 'algebrist'—a bone-setter. I never understood why he was called an algebrist before. Cervantes must have learned the word from the Spanish Moors."

However, some typographical errors mar an otherwise engaging read. The architect I. M. Pei, is "Leo," not "Leo" (p. 199). The sine addition formula is the sum, not the difference, of two products (p. 169). In the display box, the summation sign is missing in the formula for the total surface area of all the rectangles (p. 237). A mistranslation appears in the first line of the proof of the irrationality of $\sqrt{2}$. Two numbers $a$ and $b$ are co-prime, not "both prime" (p. 95).1

Still, all's well that ends well. Max is rescued, the fate of Mr. Ruche's friend is finally revealed, the bad guys receive a well-deserved end, and Sidney? Ah, Sidney. It would be utterly improper to reveal the twist, wouldn't it? No wonder the novel became an instant bestseller in France when it first appeared in 1998. The periodical *Le Point* touts *The Parrot's Theorem* as "a beautiful book glorying in the great adventures of the human mind" (inside cover), and it is all this and more. It is evidence that there is no fundamental difference between great mathematics and great literature—both possess an elegance that resonates within the human soul.

Acknowledgment

The reviewer thanks Allyn Jackson and the anonymous referee for perceptive comments.

References


1 Editor's Note: The publisher of the U.S. edition, St. Martin's Press, told the Notices that these errors would be fixed in the U.S. edition.
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The first issue appears in January–March 2001. Subscriptions are available from AMS for the 2002 subscription year. All 2002 subscribers to Moscow Mathematical Journal will receive the inaugural 2001 issues free of charge as soon as they are published.

Here is a sample of the papers that will be appearing in this prestigious journal:

- J. Guckenheimer and Yu. Ilyashenko: The duck and the devil: canards on the staircase.
- S. Shlosman and M. Tsfasman: Random lattices and random sphere packing: typical properties.
- V. Vassiliev: On combinatorial formulas for cohomology of spaces of knots.
- S. Vladut: Isogeny classes statistics for abelian varieties over finite fields.

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Rhodes Scholarships Awarded

Six mathematics students are among the thirty-two American men and women chosen as Rhodes Scholars by the Rhodes Scholarship Trust. The Rhodes Scholars were chosen from 950 applicants who were endorsed by 327 colleges and universities in a nationwide competition. The names and brief biographical descriptions of the mathematics scholars follow.

ZACHARY J. BATTLES of State College, Pennsylvania, is a double major in computer science and mathematics at Pennsylvania State University, where he will also receive an M.S. in computer science. The winner of many scholarships and awards in science and engineering, Battles, who is blind, has been extremely active in the United States and internationally in programs for people with disabilities and has done community service in Ukraine. At Oxford he will study for the D.Phil. in numerical analysis.

THOMAS M. PALLATHY of Newark, Delaware, graduated from the University of Delaware with a B.A. in mathematics and philosophy and an M.A. in linguistics and cognitive sciences. He has published work in neuroscience and is completing a book and documentary on refugee studies. Pallathy has also assisted refugees in Kosovo and Bosnia, before and after the NATO campaign, and has advised policymakers on their plight. At Oxford he will study for the M.Phil. in development studies.

BEN M. GOODWIN of Conway, Arkansas, is a senior mathematics major at Hendrix College. A Goldwater Scholar and the winner of many awards for his research and scholarship, he has presented mathematical papers at national meetings. Goodwin is also an environmental activist and is involved with Habitat for Humanity and other community and national service organizations and projects. He will read for the B.A. in modern history and economics at Oxford.

THOMAS S. MCCaleb Jr. of Tallahassee, Florida, is in his final year at the United States Air Force Academy, where he majors in mathematics. A Cadet Wing Commander, a triathlete, and a member of the Academy parachute team, he is ranked second in the Cadet Order of Merit for his class. He plans to read for the B.A. in modern history and economics at Oxford.

SARAH S. JOHNSON of Lexington, Kentucky, is a senior at Washington University in St. Louis, where she majors in mathematics, environmental studies, and earth and planetary sciences. The recipient of both Truman and Goldwater Scholarships, she has worked on the NASA Discovery Mission Team and was a delegate to the United Nations Climate Summit. Johnson is also active as a peer advisor, as a math tutor, and with the Special Olympics. At Oxford she will read philosophy, politics, and economics.

PHILLIP N. ASSMUS of Madison, South Dakota, is a senior at Luther College in Decorah, Iowa, where he is a triple major in physics, mathematics, and computer science. A Goldwater Scholar, he has worked at the Fermi National Accelerator Laboratory and has won prizes for his work in mathematics. He founded a food drive and is active in many community and college organizations. Assmus spent a semester at the University of St. Andrews in Scotland and studied women in development in India. He intends to study for the M.Phys. in physics at Oxford.

Rhodes Scholarships provide two or three years of study at the University of Oxford in England. The value of the Rhodes Scholarship varies depending on the academic field, the degree (B.A., master’s, doctoral), and the Oxford college chosen. The Rhodes Trust pays all college and university fees and provides a stipend to cover necessary expenses while in residence in Oxford, as well as during vacations, and transportation to and from England. The total value averages approximately $27,000 per year.

—From a Rhodes Scholarship Trust announcement

Diaconis Chosen von Neumann Lecturer

Persi W. Diaconis of Stanford University was selected by the Society for Industrial and Applied Mathematics (SIAM) to deliver the 2000 John von Neumann Lecture. The lecture was titled "Geometry of Markov Chains" and was given at the SIAM 2000 Annual Meeting in Puerto Rico.

The John von Neumann Lecture prize was established in 1959 and is given in the form of an honorarium of
$2,500 plus travel expenses to deliver an invited lecture at the SIAM annual meeting. The selected lecturer surveys and evaluates a significant and useful contribution to mathematics and its applications.

—From a SIAM announcement

**2000 Maria Mitchell Women in Science Award**

The Maria Mitchell Association has announced the recipients of its Women in Science Award for 2000. They are CINDA-SUE C. DAVIS, director of the Women in Science and Engineering (WISE) Program, University of Michigan, and CATHERINE BANKS, director of the Science and Mathematics Center for Women (SMC), Texas Women's University.

Davis has been the director of the WISE program since 1984, working to provide enriched learning, research, and mentoring opportunities for girls and women from the precollege years through graduate study. She has been instrumental in developing and hosting a number of important conferences on gender-equity issues in science education. Banks is the founding director of SMC, which opened in 1986 and provides resources to enable girls and women to pursue education in science, mathematics, computer science, and engineering. The winners each receive a cash award of $5,000.

The Maria Mitchell Association makes the annual award to recognize an individual, program, or organization that encourages the advancement of girls and women in studies and careers in science and technology. Maria Mitchell (1818-1889) was the first woman astronomer and first woman astronomy professor in the United States. The award may be given in the natural and physical sciences, mathematics, engineering, computer science, or technology.

—From a Maria Mitchell Association announcement

**Deaths**

ERNST AUGUST (PETER) BEHRENS, professor emeritus, McMaster University, died on December 1, 2000. Born on May 11, 1915, he was a member of the Society for 44 years.

JOSEPH L. CARMEN, IBM, Raleigh, NC, died on October 28, 2000. Born on April 17, 1966, he was a member of the Society for 1 year.

RADIA G. LAHA, professor emeritus, Bowling Green State University, died on July 14, 1999. Born on October 1, 1930, he was a member of the Society for 36 years.

ROBERT BRUCE MCLAUGHLIN, of Herndon, VA, died on July 12, 2000. Born on August 30, 1959, he was a member of the Society for 2 years.

KENNAN T. SMITH, professor emeritus, Oregon State University, died on November 18, 2000. Born on July 17, 1926, he was a member of the Society for 51 years.

**Institute for Mathematical Sciences**

**National University of Singapore**

The National University of Singapore has recently formed the new Institute for Mathematical Sciences, whose mission is to provide an international center of excellence for mathematical research. The institute’s programs will focus on fundamental issues in and applications of the mathematical sciences and will also promote interest in those fields and in multidisciplinary research in Singapore and the region.

Each year, the institute will organize two programs, each lasting up to six months, in accordance with developing trends in the mathematical sciences and with the interests of scientists in Singapore and the region. Mathematical scientists at junior and senior levels and graduate students are expected to visit the institute for periods of varying lengths, ranging from one month to six months, and to interact with each other through workshops, seminars, and informal discussions.

From July to December 2001, the inaugural program of the institute will focus on the following areas:

**Coding Theory and Data Integrity:**

The program will be divided into three parts, each lasting six to eight weeks:

1. Mathematical foundations (computational number theory, algebraic curves, and related topics);
2. Coding and cryptography (constructions of codes and cryptosystems, and related topics);
3. Applied cryptology (implementations, commercial applications, and related topics).

Each part of the program will include a one-week tutorial and a one-week workshop.

**Organizing Committee:** Shih-Ping Chan, Robert Deng, San Ling, Harald Niederreiter (chair), Eiji Okamoto, Igor E. Shparlinski, Neil J.A. Sloane, and Chaoping Xing.

The institute invites applications for membership for participation in the above program. A limited number of fellowships, covering travel and living expenses, are available to young mathematical scientists. Applications should be received at least three (3) months before the commencement of membership.

More information and application forms are available from:

[http://www.ims.nus.edu.sg](http://www.ims.nus.edu.sg)

or by writing to:

Secretary, Institute for Mathematical Sciences
National University of Singapore
2 Science Drive 2
Singapore 117543, Republic of Singapore
Mathematics Opportunities

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a program for new or recent Ph.D.'s in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics. It addresses the full range of faculty responsibilities in teaching, research, and service, and it provides professional support for new faculty as they undertake these activities. Each year about sixty faculty members from colleges and universities throughout the country are selected to participate in a workshop preceding the MAA summer meeting, activities during MAA meetings, and an electronic discussion network. Faculty for whom the 2001-02 academic year will be the first or second year of full-time employment with significant teaching responsibilities at the college/university level are invited to apply to become Project NExT Fellows. The application deadline is April 13, 2001. For more information see the Project NExT Web page, http://archives.math.utk.edu/projnext/.

—From a Project NExT announcement

2001 Summer Program for Women in Mathematics

The George Washington University has announced the 2001 Summer Program for Women in Mathematics (SPWM 2001) to be held June 30-August 4, 2001. SPWM 2001 is an intensive five-week program for mathematically talented undergraduate women who are completing their junior years and may be contemplating graduate study in the mathematical sciences. The goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school.

Sixteen women will be selected. Each will receive a travel allowance, campus room and board, and a stipend of $1,250.

The application deadline is March 1, 2001. For further information see the university's Web site, http://www.gwu.edu/~math/spwm.html, or contact the codirectors, Murli M. Gupta (mmg@gwu.edu) or E. Arthur Robinson Jr. (robinson@gwu.edu), Department of Mathematics, George Washington University, Washington, DC 20052; telephone 202-994-4857; fax 202-994-6760.

—George Washington University announcement

Maria Mitchell Women in Science Award

The Maria Mitchell Association offers an annual award to recognize an individual, program, or organization that encourages the advancement of girls and women in studies and careers in science and technology. Maria Mitchell (1818-1889) was the first woman astronomer and first woman astronomy professor in the United States.

The award may be given in the natural and physical sciences, mathematics, engineering, computer science, or technology. The winner(s) will be chosen by a national jury of distinguished educators and scientists. The total cash award is $10,000. Funding for the award has been provided by an anonymous donor. Guidelines and nomination forms are available from the association's Web site at http://www.mmo.org/, or contact the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 2 Vestal Street, Nantucket, MA 02554; telephone 508-228-9198. Deadline for nominations is April 30, 2001.

—From a Maria Mitchell Association announcement
TIMSS Repeat Study Released

In early December 2000 the National Center for Education Statistics (NCES) released a report containing initial findings from the Third International Mathematics and Science Study—Repeat (TIMSS-R). Entitled *Pursuing Excellence: Comparisons of International Eighth-Grade Mathematics and Science Achievement from a U.S. Perspective, 1995 and 1999*, the report builds on data collected during the first round of TIMSS, which was held in 1995. The U.S. has participated in such international studies for around forty years, but TIMSS-R is the only one designed specifically to allow comparison with a prior study.

Unlike TIMSS, which collected data on mathematics and science achievement at the fourth-, eighth-, and twelfth-grade levels, TIMSS-R focused primarily on eighth-grade achievement. Forty-one countries participated in the original TIMSS, and thirty-eight participated in TIMSS-R. The latter group included twenty-six nations that had participated in TIMSS in 1995, plus twelve others that were participating for the first time. Among those countries that decided against participating in the repeat study are Austria, France, Germany, and Switzerland. In the U.S. the TIMSS-R sample consisted of about 9,000 eighth-graders in 220 schools.

The report separates the participating nations into three groups whose scores were similar. The U.S. was in the middle group, where scores were above the international average, along with the Czech Republic, Malaysia, Bulgaria, Latvia, England, and New Zealand. The top group consisted of fourteen nations, including Canada and five Asian nations. Seventeen nations were in the group scoring at or below the international average. The report also provides data about the highest-scoring students, those whose scores put them in the top 10 percent internationally. Only 9 percent of U.S. eighth-graders were among these top scorers. The five nations with the largest percentages of top-scoring students were all in Asia; these percentages ranged from 46 percent in Singapore to 33 percent in Japan.

The report discusses achievement in five mathematics content areas: fractions and number sense; measurement; geometry; algebra; and data representation, analysis, and probability. U.S. students scored below average in measurement and geometry and above average in the other areas. In none of the areas did the U.S. place among the high-scoring nations.

The TIMSS-R test included multiple-choice questions as well as "free-response" questions in which students had to present their reasoning. For example, one free-response mathematics question stated that in a club of 86 members there were 14 more girls than boys and asked students to calculate the number of boys and girls in the club. Twenty-nine percent of U.S. students got this problem right, compared to 33 percent of students internationally. The free-response sample problems in mathematics boiled down to a single numerical answer, but one of the science problems was open to multiple interpretations. The problem presented a "food web", which is a diagram showing relationships among dependencies on food sources for a small group of animals, and asked students to use the diagram to figure out what would happen to the robin population if the corn crop failed. The report states that, provided logical explanations were given, three possible answers were acceptable for full credit: the robin population decreases, the robin population increases, and the robin population stays the same. Nevertheless, only 35 percent of U.S. students got this problem right, and the international average was 26 percent.

The report includes comparative data about the performance of different U.S. student populations. For example, the report states that there was no difference between U.S. boys' and girls' performance on the mathematics part of TIMSS-R. White students outperformed black and Hispanic students by a considerable margin. Students whose parents completed college had higher scores than those whose parents did not.

Part of the report is devoted to comparing data from TIMSS and TIMSS-R. No difference was found between the 1995 achievement of eighth-graders on TIMSS and the 1999 performance of eighth-graders on TIMSS-R. Most nations that participated in both studies also found little change. The only ones registering a significant rise in average scores were Latvia, Canada, and Cyprus, and the only one with a
significant drop in average score was the Czech Republic. The study found that U.S. black eighth-graders performed significantly better in 1999 than in 1995.

A group of seventeen nations participated in the fourth-grade TIMSS testing and in the eighth-grade TIMSS-R testing. Comparing data from the two rounds of testing can provide some insight into how students fare as they move through the educational system. Thus one can compare where U.S. fourth-graders placed in this group in 1995 to where U.S. eighth-graders placed in this group in 1999. Such a comparison indicates that the performance of U.S. students deteriorates between the fourth and the eighth grade, going from around average to below average in this group. By contrast, Canadian fourth-graders performed well below average in 1995, but Canadian eighth-graders performed slightly above average in 1999. The period 1995–99 coincides with a period of reform of mathematics teaching in many U.S. school districts. However, the report cautions: "Findings from comparisons between the results of TIMSS and TIMSS-R cannot be interpreted to indicate the success or failure of mathematics and science reform efforts in the United States."

One part of TIMSS-R collected information about teachers' background and preparation. Forty-one percent of U.S. eighth-grade mathematics teachers held degrees in mathematics, compared to 71 percent internationally. Still, U.S. teachers, more frequently than their international counterparts, reported feeling very well prepared to teach mathematics. On average, 90 percent of U.S. students are being taught by teachers who reported feeling very well prepared to teach mathematics; this percentage was exceeded only in Macedonia. By contrast, in Japan, where TIMSS-R scores were quite high, just 23 percent of students are taught by teachers who reported feeling very well prepared to teach mathematics. The report also notes that, according to information collected from U.S. mathematics teachers, professional development programs tend to emphasize topics like curriculum, teaching methods, and the use of technology and tend not to emphasize strengthening teachers' knowledge of mathematics.

The report Pursuing Excellence may be downloaded from the Web site http://www.nces.ed.gov/timss/timss-r/. At the time of this writing the report was available only electronically, but a print version was forthcoming.

—Allyn Jackson

Correction: The Notices has recently undergone a change in the manner in which the publication is submitted to the printer. As a result of this transition, an erroneous font substitution occurred in the mathematical equation in the cover graphic for the February issue. The mathematics should have appeared as:

\[
V_{(2,1,0)} \otimes V_{(2,1,0)} = V_{(4,2,0)} \otimes V_{(3,2,1)} \otimes V_{(4,1,1)} \otimes V_{(2,2,2)}
\]

The Notices regrets this error.
AMS Epsilon Fund

For many years now, summer programs in mathematics have provided mathematically talented youngsters with their first serious mathematical experiences. Participation in such "Young Scholars Programs" was a major factor in the career choice of many mathematicians active in the field today. Program participants who eventually chose career paths outside of mathematics retained a deeper understanding and appreciation of the field than they would likely have gained through mathematics courses alone.

Despite their strong track record, many of these programs experienced financial difficulties during the 1990s, in part because of changes in funding from the National Science Foundation. As the mathematical community became aware of the precarious financial state of the programs, leaders in the AMS decided to take action. In 1999 the Society started the Epsilon Fund for Young Scholars to help support summer programs for mathematically talented high school students. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

At its meeting in November 2000 the AMS Board of Trustees approved the Society's engagement in a sustained effort to raise an endowment for the Epsilon Fund. In addition, a Board-designated fund of $500,000 has been created as a start for the endowment. The AMS hopes to build the endowment to around $2 million through individual donations and grants from foundations.

As a start for the program, the AMS used $75,000 from its Program Development Fund to award Epsilon grants to seven programs for activities during summer 2000. The modest size of the grants means that they cannot cover the whole cost of the programs. However, the grants can have a large effect on attracting additional funding. The programs that received Epsilon grants for summer 2000 were: All Girls/All Math (University of Nebraska, Lincoln), Hampshire College Summer Studies in Mathematics, Mathcamp, PROMYS (Boston University), Ross Young Scholars Program (Ohio State University), SWT Honors Summer Math Camp (Southwest Texas State University), and the University of Michigan Math Scholars.

The AMS will make a similar number of awards to support activities in summer 2001, for a total of around $80,000, again from the Program Development Fund. Once the Epsilon Fund endowment has reached the targeted amount, the AMS intends to award a total of $100,000 in Epsilon grants each year.

For further information about the Epsilon Fund for Young Scholars, visit the Web site http://www.ams.org/giving-to-ams/, or contact development@ams.org, telephone 800-321-4267, extension 4111, or 401-455-4111. Information about how to apply for Epsilon grants is available at http://www.ams.org/employment/epsilon.html. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at http://www.ams.org/employment/mathcamps.html.

—Allyn Jackson

AMS Establishes Public Awareness Office

In recent years AMS committees and officials have highlighted the need to raise public appreciation and understanding of mathematics. They believe that mathematicians must communicate to the general public about the essential role mathematics plays in human culture and society. Such communication could help bolster efforts to improve mathematics education and to increase funding support for mathematics research. In addition, this kind of outreach naturally feeds into efforts to increase collaborations between mathematicians and researchers in other areas of science and engineering.

To address this need, the AMS has established a Public Awareness Office at the Society's headquarters in
Life in the fast lane. It usually involves a few sacrifices. Your insurance coverage doesn't have to be one of them. Whether you're moving on or even out on your own, insurance offered through your AMS membership won't end just because you've changed jobs. It travels right in your back pocket.

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This program is administered by Seabury & Smith, a Marsh & McLennan Company. Some plans may not be available in all states.

The comprehensive healthcare insurance plan is underwritten by New York Life Insurance Company, 51 Madison Avenue, New York, NY 10010.
The member assistance and disability income protection insurance plans are underwritten by
Unum Life Insurance Company of America, 15 Corporate Place South, P.O. Box 1387, Piscataway, NJ 08855.
The catastrophe major medical and high limit accident insurance plans are underwritten by The
3600 Route 66, P.O. Box 1390, Neptune, NJ 07754-1390.
The term life insurance plan is underwritten by Connecticut General Life Insurance Company, a
Cigna Company, Hartford, CT 06132.

Inside the AMS

Providing the coverage of mathematics in the popular press is one goal, but the office will also work on a variety of other fronts. For example, efforts are under way to establish relations with other scientific societies, such as the American Physical Society and the American Chemical Society, which already have well-established public awareness efforts. The possibility of joint projects to produce radio and TV spots highlighting the importance of mathematics and science is being explored. The AMS Public Awareness Office will also provide assistance to the AMS Washington Office in developing materials for outreach efforts in the nation's capital. Other activities include organizing public events and developing Web materials about mathematics. In addition to its main purpose of promoting appreciation and understanding of mathematics, the AMS Public Awareness Office has as a secondary purpose the promotion of AMS activities and achievements.

Michael Breen and Annette Emerson have been appointed as AMS public awareness officers. A mathematician currently on leave from Tennessee Technological University, Breen has experience in radio and television broadcasting and newspaper writing. Emerson was previously the manager of the AMS Promotions Department, where among other duties she oversaw the AMS Bookstore on the Web.

For further information on the activities of the AMS Public Awareness Office, visit the Web site http://www.ams.org/public-awareness/, or contact pa-office@ams.org.

—Allyn Jackson
Reference and Book List

The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices

The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.tamu.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 979-845-6028 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines


April 13, 2001: Applications for Project NExT. See "Mathematics Opportunities" in this issue.

April 15, 2001: Applications for the second competition for NRC Research Associateships. See http://www4.nationalacademies.org/osep/rap/, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; e-mail: rap@nas.edu.


May 1, October 1, 2001: Applications for NSF/AWM Travel Grants for Women. See http://www.awm-math.org/travelgrants.html; telephone 301-405-7892; e-mail: awm@math.umd.edu.

May 1, 2001: Burroughs Wellcome Fund Career Awards at the Scientific Interface. See http://www.bwfund.org/interfaces_in_science.htm, or call Debi Linkous, program associate, telephone 919-991-5116.

August 15, 2001: Applications for the third competition for NRC Research Associateships. See http://www4.nationalacademies.org/osep/rap/, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; e-mail: rap@nas.edu.

October 1, 2001: Nominations for the Emanuel and Carol Parzen Prize.

Where To Find It

A brief index to information that appears in this and previous issues of the Notices.

AMS Bylaws—November 1999, p. 1252
AMS E-Mail Addresses—November 2000, p. 1288
AMS Ethical Guidelines—June 1995, p. 694

AMS Officers 1999 and 2000 (Council, Executive Committee, Publications Committees, Board of Trustees)—May 2000, p. 591

AMS Officers and Committee Members—October 2000, p. 1127
Conference Board of the Mathematical Sciences—September 2000, p. 913
Information for Notices Authors—January 2001, p. 39

Mathematics Research Institutes Contact Information—August 2000, p. 786
National Science Board—February 2001, p. 216
New Journals for 1999—June/July 2000, p. 688

NRC Board on Mathematical Sciences and Staff—April 2000, p. 494
NRC Mathematical Sciences Education Board and Staff—April 2000, p. 494
NSF Mathematical and Physical Sciences Advisory Committee—March 2001, p. 328

Program Officers for Federal Funding Agencies—October 2000, p. 1100
(DoD, DoE); November 2000, p. 1291 (NSF)
Submit nominations to J. H. Matis, Department of Statistics, Texas A&M University, College Station, TX 77843-3143.

MPS Advisory Committee
Following are the names and affiliations of the members of the Advisory Committee for Mathematical and Physical Sciences (MPS) of the National Science Foundation. The date of the expiration of each member’s term is given after his or her name. The Web site for the MPS directorate may be found at http://www.nsf.gov/home/mps/. The postal address is Directorate for the Mathematical and Physical Sciences, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230.

Ronald Brisbois (10/02)
Chemistry Department
Maclester College

Arturo Bronson (ex officio)
Materials Center for Synthesis and Processing
University of Texas, El Paso

Tony Chan (10/02)
Department of Mathematics
University of California, Los Angeles

Alexandre J. Chorin (10/01)
Department of Mathematics
University of California, Berkeley

Billy Joe Evans (10/02)
Department of Chemistry
University of Michigan, Ann Arbor

S. James Gates Jr. (10/03)
Physics Department
University of Maryland

Lila M. Gierasch (10/01)
Department of Biochemistry and Molecular Biology
University of Massachusetts, Amherst

Fiona Goodchild (10/03)
Materials Research Laboratory
University of California, Santa Barbara

Jacqueline N. Hewitt (10/01)
Department of Physics

Massachusetts Institute of Technology

Bernard V. Khoury (10/01)
American Association of Physics Teachers

Thomas B. W. Kirk (10/01)
Brookhaven National Laboratory

Lon Mathias (10/03)
Department of Polymer Science
University of Southern Mississippi

Julia Phillips (10/03)
Materials Science & Technologies
Sandia National Laboratories

David Siegmund (10/03)
Department of Statistics
Stanford University

J. Anthony Tyson (10/01)
Lucent Technologies

Neil deGrasse Tyson (10/03)
Hayden Planetarium
American Museum of Natural History

Book List

The Book List highlights books that have mathematical themes and hold appeal for a wide audience, including mathematicians, students, and a significant portion of the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally, the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers’ attention to older books. Suggestions for books to include on the list may be sent to the managing editor, e-mail: notices@ams.org.


*Added to the “Book List” since the title’s last appearance.
ASSOCIATE TREASURER

The American Mathematical Society is seeking applications and nominations for candidates for the position of Associate Treasurer of the Society.

The primary responsibilities of the Associate Treasurer are to know and understand the budget of the Society, to monitor the financial condition of the Society and to advise the Board of Trustees concerning the financial consequences of its decisions. The Associate Treasurer works in close cooperation with the Treasurer, and serves as Treasurer when necessary.

The Associate Treasurer is a member of the Board of Trustees, the Agenda and Budget Committee, the Investment Committee, and the Council, and serves on several other committees of the Society. As a member of the Council, the Associate Treasurer, with the Treasurer, acts as liaison between the Board of Trustees and the Council and offers advice to the Council on the financial aspects of its deliberations.

There are two other areas of major responsibility: (1) The Associate Treasurer, along with the Treasurer, serves as a liaison between the Board of Trustees and the Executive Director's office in monitoring staff appointments, promotions, and salaries. (2) The Associate Treasurer serves as the liaison trustee for Mathematical Reviews, and in that capacity, monitors the budget of the Mathematical Reviews.

While the term of office is two years, it is anticipated that the person filling this office will be reappointed biennially for a number of terms, to ensure continuity.

Applications and nominations should be sent to the chair of the search committee, Linda Keen, or to the Secretary of the Society, Robert J. Daverman.

Linda Keen  
Dept. of Math. and Comp. Science  
Lehman College, CUNY  
Bedford Park Blvd.  
Bronx, NY 10468  
keenl@g230.lehman.cuny.edu

Robert J. Daverman  
Secretary, AMS  
Department of Mathematics  
University of Tennessee  
Knoxville, TN 37996-1330  
daverman@math.utk.edu

Applications or nominations received by 15 September 2001 will be assured full consideration. The newly appointed Associate Treasurer will take office formally on 01 February 2003 but should be appointed by the Council early in 2002 to permit a smooth transition.

All necessary expenses incurred by the Associate Treasurer in the performance of duties for the Society are reimbursed, including travel and communications.
Leroy P. Steele Prizes

The selection committee for this prize requests nominations for consideration for the 2002 award. Further information about this prize can be found in the November 1999 Notices, pp. 1258-1269 (also available at http://www.ams.org/ams/prizes.html).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2002 the prize for Seminal Contribution to Research will be awarded for a paper in Geometry/Topology.

Nominations with supporting information should be submitted to the Secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, including complete biographic citations. A curriculum vitae should be included. The nominations will be forwarded by the Secretary to the prize selection committee, which will, as in the past, make final decisions on the awarding of prizes.

Deadline for nominations is March 31, 2001.
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Shmuel A. Weinberger, University of Chicago, IL, USA

Geometriae Dedicata aims to be a vehicle for excellent publications in geometry and its relationship to topology, group theory and the theory of dynamical systems.

Features of the journal include:
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2. A "Surveys in Geometry" series: this will be a series of survey articles on active areas in geometry, directed to other geometers.
3. Special issues centered on specific topics.

Forthcoming papers

"An extension theorem for Euler characteristics of groups" J. Stallings
"Sure les immeubles hyperboliques" Damien Gaboriau and Frédéric Paulin
"Actions of semisimple Lie groups on circle bundles" Dave Witte and Robert J. Zimmer
"On quasiconvex subgroups of word hyperbolic groups" G. N. Arzhantseva
"The lamplighter group as a group generated by a 2-state automation, and its spectrum" Rostislav I. Grigorchuk and Andrzej Zuk
"On Kerckhoff minima and pleating loci for quasi-Fuchsian groups" Caroline Series
"Desargues theorem, dynamics, and hyperplane arrangements" Richard Evan Schwartz
"Properly discontinuous groups of affine transformations - A survey" Herbert Abels
"Simple curves on surfaces" Igor Rivin
"On subgroup separability in hyperbolic Coxeter groups" D. D. Long, Alain W. Reid
"The proalgebraic completion of rigid groups" Haim Bass, Alexander Lubotzky, Andy R. Magid, Shahar Mozes
"Uniform growth in groups of exponential growth" Pierre de la Harpe

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Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal/.

March 2001

1–3 Fourth New Mexico Analysis Seminar, The University of New Mexico, Albuquerque, New Mexico. (Jan. 2001, p. 51)

2–3 Southeastern Analysis Meeting (SEAM XVII), University of Georgia, Athens, Georgia. (Nov. 2000, p. 1297)

8–9 DIMACS Workshop on Protein Structure and Structural Genomics: Prediction, Determination, Technology and Algorithms, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Jan. 2001, p. 51)


8–11 Workshop on Population Genetics at the Molecular Level, Centre de Recherches Mathématiques, Montréal, Quebec, Canada. (Oct. 2000, p. 1143)

9–11 Around Dynamics, A Conference to Celebrate Jack Milnor’s 70th Birthday, SUNY at Stony Brook, New York. (Feb. 2001, p. 249)

9–11 The Third International Meeting of Origami Science, Math, and Education, Asilomar, Monterey, California. (Sept. 2000, p. 977)


12–16 DIMACS Workshop on Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Jan. 2001, p. 51)

14–16 Conference on Graph Theory and Its Applications, Anna University, Chennai, India.

Topics: Interested researchers and graph theorists are invited to submit abstracts (maximum 300 words) and full-length papers neatly typed (double-spaced) on one side of size four bond paper to the organizing secretary according to the schedule. All the submitted papers will be reviewed by the program committee, and the recommended reviewed papers will be published in the proceedings of the conference. Papers are invited in the following areas and related topics: combinatorics, structural graph theory, extremal graph theory, algebraic graph theory, graph algorithm, applications of graph theory.

Information: shan@annauniv.edu.

16–18 AMS Southeastern Sectional Meeting, University of South Carolina, Columbia, South Carolina. (Jan. 1998, p. 113)

Information: W. Drady, wsd@ams.org.


25–30 Sixth International Conference on Approximation and Optimization in the Caribbean, Guatemala City, Guatemala. (Oct. 2000, p. 1145)

26–27 DIMACS Workshop on Resource Management and Scheduling in Next Generation Networks, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Jan. 2001, p. 51)

*26–29 Tenth International Conference on Approximation Theory,
Mathematics Calendar

St. Louis, Missouri.

**Topics:** This conference is a continuation of the earlier conferences on approximation theory held in Austin, College Station, and Nashville. It will cover all aspects of approximation theory and applications.

**Invited Speakers:** B. Bojanov (Univ. Sofia), M. Cheney (RPI), R. Coifman (Yale Univ.), T. Erdelyi (Texas A&M Univ.), A. Pinkus (Technion), P. Schroeder (Caltech), H.-P. Seidel (Univ. Saarbruecken), V. Temlyakov (Univ. South Carolina). In addition, there will be a lecture by the 2001 Vasil A. Popov Prize winner, to be announced at the meeting.

**Information:** The conference home page is at http://www.math.umn.edu/~rt/; further information can be requested by e-mail to s@math.umn.edu.

26-30 (NEW DATE) New Trends in Potential Theory and Applications, Bielefeld University, Bielefeld, Germany. (Nov. 2000, p. 1297)

26-30 Quadratic Forms and Related Topics, Louisiana State University, Baton Rouge, Louisiana. (Sept. 2000, p. 977)

27-29 Third Spatial Patterns in Permeable Rocks, Church College, Cambridge, UK. (Sept. 2000, p. 977)

April 2001

2-6 Conference on Levy Processes and Stable Laws, Warwick University, UK. (Jan. 2001, p. 51)


2-6 International Conference on Mathematical Modeling and Scientific Computing, Middle East Technical University, Ankara, Selcuk University, Konya, Turkey. (Feb. 2001, p. 249)


5-7 National Convention for Kappa Mu Epsilon, National Mathematics Honor Society, Washburn University, Topeka, Kansas. (Feb. 2001, p. 249)

9-11 Fourth Modelling in Industrial Maintenance and Reliability, University of Salford, UK. (Sept. 2000, p. 977)

9-13 Joint IDR-IMA Workshop: Ideal Data Representation, IMA, University of Minnesota, Minneapolis, Minnesota, (Mar. 2000, p. 938)

15-21 Spring School on Analysis, Analysis in Banach Spaces, Paseky n. Jizerou, Czech Republic.

**Topic:** The purpose of this meeting is to bring together adepts with common interest in the field. There will be opportunities for informal discussions. Graduate students and others beginning their mathematical career are encouraged to participate.

**Speakers:** Y. Benyamini (The Technion, Israel Inst. of Tech., Haifa, Israel); W. B. Johnson (Texas A&M Univ.); G. Landau (Univ. de Franche-Comte, Besançon Cedex, France); J. Lindenstrauss (The Hebrew Univ. of Jerusalem, Jerusalem, Israel); G. Schechtman (The Weizmann Inst. of Sci., Rehovot, Israel).

**Deadlines:** March 15, 2001, or January 15, 2001, for financial support.

**Information:** http://www.carlin.mff.cuni.cz/katedry/xma/ss/april01/ss.html, or e-mail: paseky@karlin.mff.cuni.cz.


18-21 IV Iberoamerican Conference on Topology and Its Applications, University of Coimbra, Coimbra, Portugal.


20-22 Riviere-Fabes Symposium on Analysis and PDE, University of Minnesota, School of Mathematics, Minneapolis, Minnesota.

Organizers: M. Safonov (chair; safonov@math.umn.edu), N. Jain, C. Kenig (Chicago), N. Krylov, W. Littman, F. Reitich.

Speakers: There will be four additional speakers giving one lecture each: P. Daskalopoulos (Univ. of Calif., Irvine), S. Hofmann (Univ. of Missouri, Columbia), M. Mitrea (Univ. of Missouri, Columbia), T. Tao (Univ. of California Los Angeles).

**Information:** See http://www.math.umn.edu/~sr/RiviereFabes.html. Support for graduate students: see Web page. Hotel and travel information: see Web page.


**Information:** For information on speakers, participants, events, and accommodations, please see http://www.cs.ubc.ca/~egethner/announcement.html, or contact E. Gethner, egethner@cs.ubc.ca.


25-26 Workshop on Mathematical Formalisms for RNA Structure, Centre de Recherches Mathématiques, Montréal, Quebec, Canada. (Oct. 2000, p. 1145)


28-29 AMS Northeastern Section Meeting, Stevens Institute of Technology, Hoboken, New Jersey. (Nov. 1998, p. 1378)

**Information:** Information will appear on the meetings pages on e-MATH.

May 2001


3-5 2001 ASL Spring Meeting (with APA), Minneapolis, Minnesota. (Feb. 2001, p. 250)

6-10 Eurocrypt 2001, Innsbruck, Austria. (Nov. 2000, p. 1297)


13-17 Fourth International Conference of SampTA, University of Central Florida, Orlando, Florida. (Dec. 2000, p. 1434)


20-24 Sixth SIAM Conference on Applications of Dynamical Systems (DS01), Snowbird Ski and Summer Resort, Snowbird, Utah. (Sept. 2000, p. 978)


21-June 8 School on High-Dimensional Manifold Topology, International Center for Theoretical Physics (ICTP), Trieste, Italy. (Jan. 2001, p. 52)


28-June 1 Harmonic Morphisms and Harmonic Maps, Centre International de Rencontres Mathématiques, Luminy, Marseille, France. (Sept. 2000, p. 978)

28-30 2001 International Conference on Computational Science, San Francisco, California. (Nov. 2000, p. 1297)
28-June 1 The Tarski Centenary Conference (including the workshop: Decidability and Complexity, May 27-29), Warszawa, Poland. (Jan. 2001, p. 52)


**June 2001**

3-8 The 6th International Conference on Mathematical Population Dynamics, Marrakech, Morocco. (Jan. 2001, p. 52)

4-8 International Conference of Computational Harmonic Analysis, City University of Hong Kong. (Dec. 2000, p. 1435)

4-9 Fractals in Graz 2001: Stochastics-Analyses-Dynamics-Geometry, Technical University of Graz, Graz, Austria. (Oct. 2000, p. 1146)

4-10 Second Goteborg Conference in Harmonic Analysis and Partial Differential Equations, Chalmers University of Technology/Goteborg University, Goteborg, Sweden. (Jan. 2001, p. 52)

6-10 The 3rd International Conference on Mathematical Biology, Guillin, China. (Sept. 2000, p. 978)

8-10 Joint Meeting of the Belgian and German Mathematical Societies 2001 BMS-DMV Meeting, University of Liege, Belgium. (Sept. 2000, p. 978)

8-11 Nonlinear Dynamics and Chaos: Where Should We Go From Here?, Burwall Conference Centre, University of Bristol, UK.

Workshop Topics: The aim of the meeting is to allow the next generation of nonlinear scientists to find an agenda for research directions of the future, focussed on three key themes: neural systems, spatially extended systems and pattern formation; applications to biology, physics, and engineering.

Invited Speakers: P. Bressloff (Utah), E. Ermentrout (Pittsburgh), J. Cowan (Chicago), J. Guckenheimer (Cornell), E. Knobloch (Leeds, UK), Y. Kuramoto (Kyoto, Japan), U. de Heiden (Witten, Germany), R. McKay (Warwick, UK), C. Jones (Brown), T. Mullin (Manchester, UK), W. Ditto (Georgia Tech), R. Roy (Maryland), A. Winfree (Arizona), M. Fink (ESPCI, France), L. Casti (Santa Fe Institute).

Information: http://www.enn.at.ac.uk/ann/colston.html.

10-13 7th International Meeting on DNA Based Computers, University of South Florida, Tampa, Florida. (Jan. 2001, p. 53)


11-22 Workshop on Fourier Analysis and Convexity, Università di Milano-Bicocca, Milano, Italy.

Organizing Committee: L. Brandolini (Bergamo), L. Colzani (Milano-Bicocca), A. Iosevich (Columbia-Missouri), G. Travaglini (Milano-Bicocca).

Topics: Many problems related to convex bodies are studied using Fourier analysis methods. The aim of this conference is to bring together leading researchers willing to learn and share problems and techniques related to the interaction between Fourier analysis and convexity.

Speakers: The following speakers will deliver minicourses: J. Beck*, Rutgers Univ.; H. Groemer*, Univ. of Arizona; N. Katz, Washington Unv. in St. Louis; A. Koldobsky*, Univ. of Missouri-Columbia; M. Kolountzas*, Univ. of Crete; A. Magyar, Univ. of Wisconsin-Madison; H. Montgomery*, Univ. of Michigan; A. Podkorytov, St. Petersbug Unv.; F. Ricci, Scuola Normale Superiore di Pisa; M. Rudelson, Univ. of Missouri-Columbia; E. Sawyer, Mc Master Unv.; T. Tao*, Univ. of California Los Angeles; A. Vargas, Univ. Autonoma de Madrid; A. Volcic, Univ. di Triest; S. Wainger*, Univ. of Wisconsin-Madison.

Information: http://www.matapp.unimib.it/convex2001/ or e-mail: convex2001@matapp.unimib.it.

12-14 M'SAB'01-The IMACS/IFAC Fourth International Symposium on Mathematical Modelling and Simulation in Agricultural and Bio-Industries, Haifa, Israel.

Scope: The symposium will include both contributed papers and special sessions in the following areas: Modeling strategies, Object-oriented modeling, Methods and tools for system identification and parameter estimation, Distributed models, Model based control, Biophysical modeling, Multi-modal models, Design of "model-biological system" experiments.

Special Activities: Special activities for the participants and their companions will include pre- and postsymposium tours to Galilee, Jerusalem, Eilat, and the Red Sea, the Dead Sea, and the Hashemite Kingdom of Jordan. A professional tour to one Israel Ministry of Agriculture Research Station is also planned. Information about the special activities is available from the Symposium Secretariat at C. Mayer, 59 Harutzmautz Road, Haifa 33033, Israel; tel: 972-4-8524254; fax: 972-4-8522491; e-mail: carel@alex.co.il.

Information: Prospective authors and participants are invited to visit the M'SAB'01 home page located at http://www.technion.ac.il/technion/agr/m2sabi01.html or contact the general chair: P. O. Gutman, Faculty of Agricultural Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel; tel: 972-4-8292811; fax: 972-4-8221529; e-mail: peo@tx.technion.ac.il.

14-21 Graphs and Patterns in Mathematics and Theoretical Physics, A Conference to Celebrate Dennis Sullivan’s 60th Birthday, SUNY at Stony Brook, New York. (Feb. 2001, p. 250)

14-23 Third International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena Resort (near Varna), Bulgaria. (Jan. 2001, p. 53)


18-23 The Fourth St. Petersburg Workshop on Simulation, St. Petersburg State Univ., St. Petersburg, Russia. (Oct. 2000, p. 1146)

18-23 Tools for Mathematical Modelling, St. Petersburg State Technical University, St. Petersburg, Russia. (Dec. 2000, p. 1435)

21-22 (NEW DATE) Integration of Diverse Biological Data, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Sept. 2000, p. 975)

23-29 International Conference on Operator Theory and Its Applications, Dedicated to the Memory of Abraham V. Strauss, Ulyanovsk, Russia. (Sept. 2000, p. 978)


25-29 International Linear Algebra Conference (9th Conference of the International Linear Algebra Society), Technion, Israel Institute of Technology, Haifa, Israel. (Dec. 2000, p. 1436)

25-29 Workshop on Groups and 3-Manifolds, Centre de Recherches Mathematiques, Universite de Montreal, Montreal, Quebec, Canada. (Feb. 2001, p. 250)

Mathematics Calendar

25-July 6 Workshop “Random Walks and Geometry”, Erwin Schrödinger Institute, Vienna, Austria. (Jan. 2001, p. 33)

27-29 MAA Summer Short Course, Ashland University, Ashland, Ohio. (Feb. 2001, p. 250)

July 2001

1-5 Warthog Delta'01 Conference on Undergraduate Teaching of Mathematics—Third Southern Hemisphere Symposium on Undergraduate Mathematics Teaching, Kruger Park, South Africa. (Sept. 2000, p. 979)

1-7 International Symposium on Computational and Applied PDEs, Zhangjiajie, Hunan, China. Description: An international symposium on computational and applied partial differential equations, this symposium is coorganized by the Hunan Institute for Computational and Applied Mathematics, Xiangtan Univ. in China; Department of Scientific Computing, Peking Univ. in China; and the Center for Computational Mathematics and Applications, The Pennsylvania State University. The goal of the meeting is to gather together researchers working in the field of theory and practical computational methods for PDEs and to discuss recent research developments in these areas, open problems, and future directions. Information: More information can be found at http://www.math.psu.edu/ccma/pde2001/.

2-6 Singapore International Symposium on Topology and Geometry (SISTAG), Singapore. (Feb. 2001, p. 250)

2-7 4th Operator Algebras International Conference: Operator Algebras and Mathematical Physics, Constanza, Romania. (Feb. 2001, p. 251)

3-5 Mathematics and Design 2001, Deakin University, Geelong, Australia. (Feb. 2001, p. 251)

3-5 Mathematics & Design 2001: Mind/Ear/Eye/Hand/Digital, School of Architecture, Australia School of Computing and Mathematics, Deakin University, Melbourne, Australia. (Nov. 2000, p. 1297)

8-13 Second ICMS Workshop on Algebraic Graph Theory, International Centre for Mathematical Sciences, Edinburgh, Scotland. (Feb. 2001, p. 251)

9-13 Workshop on Geometric Group Theory, Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada. (Feb. 2001, p. 251)

9-20 SMS-NATO ASI: Modern Methods in Scientific Computing and Applications, Université de Montréal, Montréal, Quebec, Canada. (Dec. 2000, p. 1436)

9-22 European Summer School: Asymptotic Combinatorics with Application to Mathematical Physics, Euler International Mathematical Institute, St. Petersburg, Russia. (Nov. 2000, p. 1297)

12-14 Fifth SIAM Conference on Control and Its Applications (CT01), Town & Country Hotel, San Diego, California. (Sept. 2000, p. 979)

13-27 Summer 2001 Workshop on Graphs and Combinatorial Designs, University of Hawaii at Manoa, Honolulu, Hawaii. Aim: The aim of the workshop is to gather people interested in discussing recent advances and open problems in the following areas of graph theory and combinatorial designs: cycles, colorings, and extremal problems. There will be 50-minute invited talks and 25-minute contributed talks. The invited talks will be problem-oriented with the aim of stimulating the active research of participants during the workshop, and these will be scheduled at the very beginning of the program. To keep the character of the meeting as that of an actively research-based workshop, there will be talks for three hours per day and slots for 40 contributed talks. In case of wider interest, contributions in the areas of the invited talks will be given preference. The last four days of the workshop will be entirely devoted to active research, and talks are planned on these days only in case some participants would like to report on progress achieved in solving questions tackled during the workshop or a group of people would like to have a session on a specific topic. Organizers: The main organizers of the workshop are: E. Bertram, Univ. of Hawaii (ebeath.math.hawaii.edu) and P. Horak, Kuwait Univ. (horak@kuc01.kuniv.edu.kw).


Information: More information, including social events, accommodations, preregistration form, and abstracts of the invited talks, may be found on our Web site, http://www.math.hawaii.edu/bertram/.


16-20 Probability on Geometric Structures, CRM Marseille, Luminy, France. (Nov. 2000, p. 1298)


27-29 Conference and Reunion in Honor of Arnold Ross, The Ohio State University, Columbus, Ohio.


Program: This conference is a celebration of the 45th year of the Ross Summer Mathematics Program for talented high school students. There will be lectures on a variety of mathematical topics by alumni and friends of the Ross Program.


30-August 17 School on Dynamical Systems, Trieste, Italy. (Jan. 2001, p. 54)

August 2001


6-11 2001 ASL European Summer Meeting (Logic Colloquium '01), Vienna, Austria. (Feb. 2001, p. 251)

7-10 The 4th Conference on Information Fusion, Montreal, Quebec, Canada. (Feb. 2001, p. 251)

8-12 The 9th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications, Hanoi University of Technology, Hanoi, Vietnam. (Jan. 2001, p. 54)

13-16 Fourth SIAM Conference on Linear Algebra in Signals, Systems and Control (LASSCO), Boston Park Plaza Hotel, Boston, Massachusetts. (Sept. 2000, p. 979)


19-25 9th Prague Topological Symposium (General Topology and Its Relations to Modern Algebra and Analysis), Prague, Czech Republic.

Topics: Topology (mainly set-theoretical topology, continuum, descriptive topology, categorical topology), Topological Groups and Semigroups, Topology of Banach Spaces, Topology and Computer Science, Topological Dynamics.

Information: e-mail: toposym@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/~toposym/.

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20-22 Seventh Annual International Computing and Combinatorics Conference (COCOON’01), Guilin, China. (Nov. 2000, p. 1298)

20–24 Ellipticity and Parabolicity in Analysis and Geometry (EPAG 2001), University of Potsdam, Potsdam, Germany. Description: This workshop is an activity of the Research Training Network “Geometric Analysis” of the European Commission (see http://dipart.uni.ia.it/rtn/). It particularly addresses young mathematicians with an interest in analysis and geometry for partial differential equations.

Scope: The aim of this workshop is to bring together young mathematicians interested in diverse research directions in analysis and geometry for partial differential equations. This meeting will to some extent have the character of a summer school, with reserved time for discussions. We plan to have 3 or 4 series of overview talks and a limited number of contributions describing connections between the main topics.

Topics: Spectral theory, index theory, heat equation and other parabolic equation methods, existence and regularity of solutions to elliptic/parabolic equations, geometric and spectral invariants.

Organizers: J. B. Gil (Philadelphia, USA); A. Parmeggiani (Bologna, Italy); T. Kainer (Potsdam, Germany); T. Schick (Münster, Germany); B. Menthubert (Toulouse, France); I. Witt (Potsdam, Germany).

Information: e-mail: epag2001@math.uni-potsdam.de, or http://epag2001.math.uni-potsdam.de/.

22–26 International Conference on Functional Analysis, Kyiv, Ukraine. (Jan. 2001, p. 54)


September 2001

1–5 2001 WSES International Conference on Simulation (SIM’01), Malta. (Feb. 2001, p. 251)

1–May 31 Institut Mittag-Leffler Call for Proposals, Djursholm, Sweden. (Sept. 2000, p. 979)

3–8 The Sixth International Conference on Function Spaces, Institute of Mathematics, Wroclaw University of Technology, Wroclaw, Poland.

Organizer: Institute of Mathematics, Wroclaw Univ. of Technology.

Organizing Committee: M. Burecky (secretary), R. Grzesiakiewicz (chair), H. Hudzik, J. Musielak, Cz. Ryll-Nardzewski.

Program: The topics are connected with functional analysis, e.g., operator theory, interpolation, geometry, topology, approximation in function spaces. There will be plenary lectures (about 45 min.) and short communications (not more than 20 min.).

Information: http://www.im.pw.wroc.pl/~ipe/

3–21 School on Control Theory, Trieste, Italy. (Jan. 2001, p. 54)


Local Organizers: K. Böröczky Jr. and T. Szamuely (Budapest).

Aim and Scope: The program is organized in order to encourage collaboration between specialists in higher-dimensional complex geometry and those studying arithmetic/diophantine questions. In recent years it has become apparent that the powerful geometric tools elaborated in connection with Mori's Minimal Model Program have applications over arithmetic ground fields as well. We hope that bringing together experts and graduate students specializing in higher-dimensional geometry or arithmetic will induce further cross-fertilization between the two fields and give rise to new powerful results. The main areas of research to be touched upon are: classification and minimal models of varieties, rationally connected varieties, rational and integral points, fundamental groups, and Galois groups.

Program: (1) During the first week (3–7 September) an instructional conference will take place featuring 6 speakers. Each of them will give a minicourse of 3 one-hour lectures. Roughly half of the courses will focus on geometric topics and the other half on arithmetic topics. They are intended to be accessible to graduate students and the "other half" to specialists. (2) The other two weeks (10–21 September) will be mainly devoted to research work of participants and informal discussions. There will also be a regular seminar featuring lectures on recent results by participants. The program will be elaborated in situ in order to be as up-to-date as possible. Tutorial sessions for graduate students are also planned.

Speakers: J.-L. Colliot-Thélène (Univ. de Paris-Sud), O. Debarre (IRMA, Strasbourg), B. Hassett (Rice Univ., Houston), J. Kollár (Princeton Univ.), S. Kovács (Univ. of Washington, Seattle), E. Peyre (Institut Fourier, Grenoble).

Contact: arithgeo@renyi.hu.


9–13 4th Dublin Differential Equations Conference, Dublin City University, Dublin, Ireland.

Topics: The conference covers the theory, applications, and numerical solution of Differential Equations, and the interplay between these.

Principal Speakers: C. Budd (Bath), E. Delabaere (Angers), O. Diekmann (Utrecht), P. Glendinning (Manchester), S. Howison (Oxford), A. Il'in (Ekaterinburg), M. Lavrentiev (Novosibirsk), J. Ockendon (Oxford), R. O'Malley (Washington), G. Shishkin (Ekaterinburg).

Program: In addition to lectures from the invited speakers, other participants may give shorter contributed talks.

Information: http://www.deconf.dcu.ie/ or Jurgen.Burzaff@dcu.ie.

10–14 3rd IMACS Seminar on Monte Carlo Methods MCM2001, Salzburg University, Austria. (Nov. 2000, p. 1298)

18–22 The Fifth International Workshop on Differential Geometry and Its Applications, Timisoara, Romania. (Feb. 2001, p. 251)

22–26 Application of Discrete Mathematics, Australian National University, Canberra, Australia. (Feb. 2001, p. 251)


26–28 First SIAM Conference on Imaging Science, Boston Park Plaza Hotel, Boston, Massachusetts. (Feb. 2001, p. 251)


October 2001

1–5 International Conference on Numerical Algorithms, Dedicated to Claude Brezinski on the occasion of his 60th birthday, Marrakesh, Morocco. (Oct. 2003, p. 1146)


Information: W. Drady, wd@ams.org.

24–26 DIMACS Workshop on Analysis of Gene Expression Data, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Feb. 2001, p. 252)
Mathematics Calendar

November 2001
5-8 Seventh SIAM Conference on Geometric Design (SIAG/GD) (GDO1), Holiday Inn Capitol Plaza Hotel, Sacramento, California. (Sept. 2000, p. 980)

December 2001
17-19 Eighth Cryptography and Coding, Royal Agricultural College, Cirencester, UK. (Sept. 2000, p. 980)

January 2002
6-9 Joint Mathematics Meetings, San Diego Convention Center, San Diego, California. (Nov. 1998, p. 1378)

June 2002
10-16 Aarhus Topology 2002, University of Aarhus, Aarhus, Denmark.
23-28 Fourteenth U.S. National Congress of Theoretical and Applied Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

August 2002
New Publications Offered by the AMS

Algebra and Algebraic Geometry

Abelian Groups, Rings and Modules
A. V. Kelarev, University of Tasmania, Hobart, Tasmania, Australia, R. Göbel, University of Essen, Germany, K. M. Rangaswamy, University of Colorado, Colorado Springs, P. Schultz, The University of Western Australia, Nedlands, and C. Vinsonhaler, University of Connecticut, Storrs, Editors

This volume presents the proceedings from the conference on Abelian Groups, Rings, and Modules (AGRAM) held at the University of Western Australia (Perth). Included are articles based on talks given at the conference, as well as a few specially invited papers. The proceedings are dedicated to Professor Laszlo Fuchs. The book includes a tribute and a review of his work by his long-time collaborator, Professor Luigi Salce.

Four surveys from leading experts follow Professor Salce's article. They present recent results from active research areas:
- Error correcting codes as ideals in group rings,
- Duality in module categories,
- Automorphism groups of abelian groups, and
- Generalizations of isomorphism in torsion-free abelian groups.

In addition to these surveys, the volume contains 22 research articles in diverse areas connected with the themes of the conference. The areas discussed include abelian groups and their endomorphism rings, modules over various rings, commutative and non-commutative ring theory, varieties of groups, and topological aspects of algebra. The book offers a comprehensive source for recent research in this active area of study.

Contents: Introduction: L. Salce, Laszlo Fuchs and his "moddom" work; Survey articles: A. V. Kelarev and P. Solé, Error-correcting codes as ideals in group rings; B. Olberding, Homomorphisms and duality for torsion-free modules;

K. C. O'Meara and C. Vinsonhaler, Generalizations of isomorphism in torsion-free abelian groups; P. Schultz, Automorphism groups of abelian groups; Contributed papers: D. M. Arnold, Direct sum decompositions of torsion-free abelian groups of finite rank; M. A. Avdisp and P. Schultz, The endomorphism ring of a bounded abelian p-group; E. Blagoveshchenskaya, G. Ivanov, and P. Schultz, The Baer-Kaplansky theorem for almost completely decomposable groups; A. Blass and J. Irwin, Maximal pure independent sets; D. Dikranjan and M. Tkachenko, Characterization of the tori via density of the solution set of linear equations; A. A. Fomin, Quotient divisible mixed groups; L. Fuchs and S. B. Lee, Stacked bases over h-local Prüfer domains; A. J. Giovannitti, Groups with locally defined heights and products of $\aleph_1$ groups; R. Göbel and S. Shelah, Reflexive subgroups of the Baer-Specker group and Martin's axiom; P. Hill, C. Megibben, and W. Ullery, $\Sigma$-isotype subgroups of local $k$-groups; G. Ivanov, Character modules and endomorphism rings of modules over Artinian serial rings; P. Loth, Topologically pure extensions; N. R. McConnell and T. Stokes, Rings having simple adjoint semigroup; A. Mader, L. G. Nongxa, and M. A. Ould-Beddi, Invariants of global $crq$-groups; V. H. Mikaelian, On varieties of groups generated by wreath products of abelian groups; O. Muthzbaumer, Existence of rigid indecomposable almost completely decomposable groups; W. K. Nicholson and M. F. Youssif, C2-rings and the FGF-conjecture; B. L. Osofsky, Lifting direct sum decompositions of bounded abelian $p$-groups; K. M. Rangaswamy, On modules and submodules with finite projective dimension; L. Strüngmann and S. L. Walluts, On the torsion groups in cotorsion classes; J. Trlifaj, Cotorsion theories induced by tilting and cotilting modules; J. Žemlička, Steadiness is tested by a single module.

Contemporary Mathematics, Volume 273
List $79, Institutional member $63, Order code CONM/273N

MARCH 2001
NOTICES OF THE AMS 339
Analysis

Laguerre Calculus and Its Applications on the Heisenberg Group

Carlos Berenstein, University of Maryland, College Park, Der-Chen Chang, Georgetown University, Washington, DC, and Jingzhi Tie, University of Athens, Athens

For nearly two centuries, the relation between analytic functions of one complex variable, their boundary values, harmonic functions, and the theory of Fourier series has been one of the central topics of study in mathematics. The topic stands on its own, yet also provides very useful mathematical applications.

This text provides a self-contained introduction to the corresponding questions in several complex variables: namely, analysis on the Heisenberg group and the study of the solutions of the boundary Cauchy-Riemann equations. In studying this material, readers are exposed to analysis in non-commutative compact and Lie groups, specifically the rotation group and the Heisenberg groups—both fundamental in the theory of group representations and physics.

Introduced in a concrete setting are the main ideas of the Calderón-Zygmund-Stein school of harmonic analysis. Also considered in the book are some less conventional problems of harmonic and complex analysis, in particular, the Morera and Pompeiu problems for the Heisenberg group, which relates to questions in optics, tomography, and engineering.

The book was borne of graduate courses and seminars held at the University of Maryland (College Park), the University of Toronto (ON), Georgetown University (Washington, DC), and the University of Georgia (Athens). Readers should have an advanced undergraduate understanding of Fourier analysis and complex analysis in one variable.

Contents: The Laguerre calculus; Estimates for powers of the sub-Laplacian; Estimates for the spectrum projection operators of the sub-Laplacian; The explicit solution of the \( \delta \)-Neumann problem in a non-isotropic Siegel domain; Injectivity of the Pompeiu transform in the isotropic \( H_n \); Morera-type theorems for holomorphic \( H^p \) spaces in \( H_n \) (II); Morera-type theorems for holomorphic \( H^p \) spaces in \( H_n \) (II).

AMS/IP Studies in Advanced Mathematics, Volume 22

May 2001, approximately 328 pages, Hardcover, ISBN 0-8218-2761-8, LC 00-054826, 2000 Mathematics Subject Classification: 22E30, 33C20, 42C10, 43A80, 47G30, 32W05, 32A10, 30E99, All AMS members $51, List $64, Order code AMISIP/22N

Function Theory of Several Complex Variables

Second Edition

Steven G. Krantz, Washington University, St. Louis, MO

This work departs from earlier treatments of the subject by emphasizing integral formulas, the geometric theory of pseudoconvexity, estimates, partial differential equations, approximation theory, the boundary behavior of holomorphic functions, inner functions, invariant metrics, and mapping theory. While due homage is paid to the more traditional algebraic theory (sheaves, Cousin problems, etc.), the student with a background in real and complex variable theory, harmonic analysis, and differential equations will be most comfortable with this treatment.

It is currently the only book on the subject with exercises and a large number of examples.

This item will also be of interest to those working in geometry and topology.

Contents: An introduction to the subject; Some integral formulas; Subharmonicity and its applications; Convexity; Hörmander’s solution of the \( \bar{\partial} \) equation; Solution of the Levi problem and other applications of \( \bar{\partial} \) techniques; Cousin problems, cohomology, and sheaves; The zero set of a holomorphic function; Some harmonic analysis; Constructive methods; Integral formulas for solutions to the \( \bar{\partial} \) problem and norm estimates; Holomorphic mappings and invariant metrics; Manifolds; Area measures; Exterior algebra; Vectors, covectors, and differential forms; List of notation; Bibliography; Index.

AMS Chelsea Publishing


Analysis

Second Edition

Elliott H. Lieb, Princeton University, NJ, and Michael Loss, Georgia Institute of Technology, Atlanta

Praise for the previous edition ...

I find the selection of the material covered in the book very attractive and I recommend the book to anybody who wants to learn about classical as well as modern mathematical analysis.

—European Mathematical Society Newsletter

The essentials of modern analysis ... are presented in a rigorous and pedagogical way ... readers ... are guided to a level where they can read the current literature with understanding ... treatment of the subject is as direct as possible.

—Zentralblatt für Mathematik
Lieb and Loss offer a practical presentation of real and functional analysis at the beginning graduate level... could be used as a two-semester introduction to graduate analysis... not all of the topics covered are typical. The authors introduce the subject with a thorough presentation... [an] informative exposition.

—CHOICE

Significantly revised and expanded, this new Second Edition provides readers at all levels—from beginning students to practicing analysts—with the basic concepts and standard tools necessary to solve problems of analysis, and how to apply these concepts to research in a variety of areas.

Authors Elliott Lieb and Michael Loss take you quickly from basic topics to methods that work successfully in mathematics and its applications. While omitting many usual typical textbook topics, Analysis includes all necessary definitions, proofs, explanations, examples, and exercises to bring the reader to an advanced level of understanding with a minimum of fuss, and, at the same time, doing so in a rigorous and pedagogical way.

Many topics that are useful and important, but usually left to advanced monographs, are presented in Analysis, and these give the beginner a sense that the subject is alive and growing.

This new Second Edition incorporates numerous changes since the publication of the original 1997 edition, and includes:
- a new chapter on eigenvalues that covers the min-max principle, semi-classical approximation, coherent states, Lieb-Thirring inequalities, and more
- extensive additions to chapters covering Sobolev Inequalities, including the Nash and Log Sobolev inequalities
- new material on Measure and Integration
- many new exercises
- and much more...

The Second Edition continues its no-nonsense approach to the topic that has made it one of the best selling books on the subject. It is an authoritative, straight-forward volume that readers—from the graduate student, to the professional mathematician, to the physicist or engineer using analytical methods—will find useful both as a reference and as a guide to real problem solving.

About the authors: Elliott Lieb is Professor of Mathematics and Physics at Princeton University and is a member of the US, Austrian, and Danish Academies of Science. He is also the recipient of several prizes including the 1988 AMS/SIAM Birkhoff prize. Michael Loss is Professor of Mathematics at the Georgia Institute of Technology.

Contents: Measure and integration; LP-spaces; Rearrangement inequalities; Integral inequalities; The Fourier transform; POisson's equation; Introduction to the calculus of variations; More about eigenvalues; References; List of symbols; Index.

Graduate Studies in Mathematics, Volume 14

Differential Equations

Ordinary Differential Equations
Second Edition
Stephen Salaff and Shing-Tung Yau
A publication of the International Press.

These articles arose from original lectures in differential equations given by the authors at Chung Chi College at the Chinese University of Hong Kong. This second edition makes this material available to a broader audience. Topics include: existence, uniqueness, and continuous dependence, continuation of solutions, the linear equation, power series solutions, linear systems with constant coefficients, autonomous systems, limit cycles, I, and limit cycles, II.

Distributed worldwide, except in Japan, by the American Mathematical Society.

International Press
December 2000, 72 pages, Softcover, ISBN 1-57146-065-9, 2000 Mathematics Subject Classification: 34-01, All AMS members $34, List $42, Order code INPR/35N

General and Interdisciplinary

Mikio Sato
A Great Japanese Mathematician of the Twentieth Century
Masaki Kashiwara, Kyoto University, Japan, Shing-Tung Yau, Harvard University, Cambridge, MA, and Takahiro Kawai, Kyoto University, Japan, Editors

A publication of the International Press.

This issue of the Asian Journal of Mathematics was dedicated to Professor Mikio Sato in the year of his seventieth birthday. Sato has made renowned mathematical contributions. His introduction of the sheaf of microfunctions opened a new era in analysis. His powerful collaborations with Kawai and Kashiwara led to the establishment of microlocal analysis, one of the most important discoveries in microfunctions and pseudo-differential equations. In this volume, leading researchers such as Yau, Schapira, Jacquet, Kashiwara, Melrose, and others, present their work in honor of Professor Sato.

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New Publications Offered by the AMS


International Press

December 2000, 1149 pages, Hardcover, ISBN 1-57146-081-0, 2000 Mathematics Subject Classification: 00B15, All AMS members $34, List $42, Order code INPR/39N

Sir Michael Atiyah
A Great Mathematician of the Twentieth Century
Shing-Tung Yau, Harvard University, Cambridge, MA, and Raymond H. Chan, Chinese University of Hong Kong, Hong Kong, Editors

A publication of the International Press.

This special issue of the Asian Journal of Mathematics was dedicated to Sir Michael Atiyah in the year of his seventieth birthday. Many distinguished researchers contributed articles to this volume in celebration of Atiyah's profound contributions to topology, geometry, representation theory, and partial differential equations. Distributed worldwide, except in Japan, by the American Mathematical Society.


International Press

The Third Pacific Rim Geometry Conference
Jaigyoung Choe, Seoul National University, Korea, Editor

A publication of the International Press.

This volume contains 20 papers from the proceedings of the Third Pacific Rim Geometry Conference held at the Korea University (Seoul). Topics include the existence theorem of harmonic objects via the Green function, strings of Riemannian invariants, inequalities, ideal immersions, and their applications, eigenvalue problems, Grassmann geometry and J-holomorphic curves of a 6-dimensional sphere, the structure of the image of analytic disks attached to totally real submanifolds, Mayer-Vietoris formula for determinants of elliptic operators of Laplace-Beltrami type, gauge-theoretic equations for harmonic maps into symmetric spaces, canonical flows of Einstein-Weyl manifolds, Jeffrey-Weitsman-Witten invariants of Seifert fibred manifolds, complex-analyticity of harmonic maps, and more. This item will also be of interest to those working in analysis. Distributed worldwide, except in Japan, by the American Mathematical Society.

Contents: S. Bando, The existence theorem of harmonic objects via Green function; B.-Y. Chen, Strings of Riemannian invariants, inequalities, ideal immersions and their applications; R. Chen, On some eigenvalue problems; H. Hashimoto, Grassmann geometry and J-holomorphic curves of a 6-dimensional sphere; H. Kamada, Self-duality of neutral metrics on four-dimensional manifolds; I. Kim, Marked length spectrum on the finite set of elements determines the irreducible representation in the isometry group of rank one symmetric space of noncompact type; T. Kim, Compactification and construction of asymptotically Euclidean scalar-flat Kähler surfaces; K.-T. Kim, The Wu metric and minimum ellipsoids; D. Kwon and Y.-G. Oh, Structure of the image of analytic disks attached to totally real submanifolds; Y. Lee, Mayer-Vietoris formula for determinants of elliptic operators of Laplace-Beltrami type (after Burghelea, Friedlander and Kappeler); K. Matsumoto, I. Mihai, and M. H. Shahid, Certain submanifolds of a Kenmotsu manifold; M. Mukai and Y. Ohnita, Gauge-theoretic equations for harmonic maps into symmetric spaces;
Groups of Homotopy Self-Equivalences and Related Topics

Ken-ichi Maruyama, Chiba University, Japan, and John W. Rutter, University of Liverpool, England, Editors

This volume offers the proceedings from the workshop held at the Gargnano Institute of the University of Milan (Italy) on groups of homotopy self-equivalences and related topics. The book comprises articles of current research on the group of homotopy self-equivalences, the homotopy of function spaces, rational homotopy theory, the classification of homotopy types, and equivariant homotopy theory.

Mathematicians from many areas of the globe attended the workshops to discuss their research and to share ideas. Included are two specially-written articles, by J. W. Rutter, reviewing the work done in the area of homotopy self-equivalences since 1988. Included also is a bibliography of some 122 articles published since 1988 and a list of problems. This book is suitable for both advanced graduate students and researchers.

Contents: J. W. Rutter, Homotopy self-equivalences 1988-1999; J. W. Rutter, Bibliography on $\mathcal{E}(\chi)$ 1988-1999; M. Arkowitz, G. Lupton, and A. Murillo, Subgroups of the group of self-homotopy equivalences; S. Bauer, M. Crabb, and M. Spreafico, The space of free loops on a real projective space; H.-J. Baues and Y. Drozd, Indecomposable homotopy types with at most two non-trivial homotopy groups; H.-J. Baues and N. Iwase, Square rings associated to elements in homotopy groups of spheres; P. I. Booth, Fibrations with product of Eilenberg-MacLane space fibres I; D. L. Ferrario, Self homotopy equivalences of equivariant spheres; Y. Felix, Two examples to illustrate properties of the group of self-equivalences of a finite CW complex $X$; A. Garvin, A. Murillo, P. Pavešić, and A. Viruel, Nilpotency and localization of groups of fibre homotopy equivalences; K. A. Hardie and K. H. Kamps, The homotopy groups of the homotopy fibre of an induced map of function spaces; V. Hauschild, Fibrations, self homotopy equivalences and negative derivations; K. Ishiguro, Classifying spaces and a subgroup of the exceptional Lie group $G_2$; D. Kahn and C. Schwartz, The structure of the Hurewicz homomorphism; H. J. Marcum, Joins, diagonals and Hopf invariants; K.-i. Maruyama, A subgroup of self-homotopy equivalences which is invariant on genus; K. Morisugi, Composition structure of the self maps of $SU(3)$ or $Sp(2)$; J. Mukai, Self-homotopy of a suspension of the real 4-projective space; J. Pan and M. H. Woo, Phantom elements and its applications; J. W. Rutter, Homotopy equivalences of lens spaces of one-relator groups; H. Shiga, K. Tsukiyama, and T. Yamaguchi, Principal $S^1$-bundles and forgetful maps; S. B. Smith, Rational type of classifying spaces for fibrations; M. Arkowitz, Problems on self-homotopy equivalences.

Geometry of Characteristic Classes

Shigeyuki Morita, Tokyo Institute of Technology, Japan

Characteristic classes are central to the modern study of the topology and geometry of manifolds. They were first introduced in topology, where, for instance, they could be used to define obstructions to the existence of certain fiber bundles. Characteristic classes were later defined (via the Chern-Weil theory) using connections on vector bundles, thus revealing their geometric side.

In the late 1960s new theories arose that described still finer structures. Examples of the so-called secondary characteristic classes came from Chern-Simons invariants, Gelfand-Fuks cohomology, and the characteristic classes of flat bundles. The new techniques are particularly useful for the study of fiber bundles whose structure groups are not finite dimensional.

The theory of characteristic classes of surface bundles is perhaps the most developed. Here the special geometry of surfaces allows one to connect this theory to the theory of moduli space of Riemann surfaces, i.e., Teichmüller theory. In this book Morita presents an introduction to the modern theories of characteristic classes.

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: De Rham homotopy theory; Characteristic classes of flat bundles; Characteristic classes of foliations; Characteristic classes of surface bundles; Directions and problems for future research; Bibliography; Index.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 199

New Publications Offered by the AMS

Simplicial and Operad Methods in Algebraic Topology

V. A. Smirnov, Moscow State Pedagogical Institute, Russia

In recent years, for solving problems of algebraic topology and, in particular, difficult problems of homotopy theory, algebraic structures more complicated than just a topological monoid, an algebra, a coalgebra, etc., have been used more and more often. A convenient language for describing various structures arising naturally on topological spaces and on their cohomology and homotopy groups is the language of operads and algebras over an operad. This language was proposed by J. P. May in the 1970s to describe the structures on various loop spaces.

This book presents a detailed study of the concept of an operad in the categories of topological spaces and of chain complexes. The notions of an algebra and a coalgebra over an operad are introduced, and their properties are investigated. The algebraic structure of the singular chain complex of a topological space is explained, and it is shown how the problem of homotopy classification of topological spaces can be solved using this structure. For algebras and coalgebras over operads, standard constructions are defined, particularly the bar and cobar constructions. Operad methods are applied to computing the homology of iterated loop spaces, investigating the algebraic structure of generalized cohomology theories, describing cohomology of groups and algebras, computing differential in the Adams spectral sequence for the homotopy groups of the spheres, and some other problems.

Contents: Operads in the category of topological spaces; Simplicial objects and homotopy theory; Algebraic structures on chain complexes; Aoo-structures on chain complexes; Operads and algebras over operads; Homotopy of iterated loop spaces; Homotopy theories and Eoo-structures; Operad methods in cobordism theory; Description of the cohomology of groups and algebras; Homology operations and differentials in the Adams spectral sequence; Bibliography; Index.

Translations of Mathematical Monographs, Volume 198


Mathematical Physics

Bosonic Strings: A Mathematical Treatment

Jürgen Jost, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Presented in this book is a mathematical treatment of bosonic string theory from the point of view of global geometry. As motivation, the author presents the theory of point particles and Feynman path integrals. He considers the theory of strings as a quantization of the classical Plateau problem for minimal surfaces. The conformal variance of the relevant functional, the Polyakov action or (in mathematical terminology) the Dirichlet integral, leads to an anomaly in the process of quantization. The mathematical concepts needed to resolve this anomaly via the Faddeev-Popov method are introduced, specifically the geometry of the Teichmüller and moduli spaces of Riemann surfaces and the corresponding function spaces, i.e., Hilbert spaces of Sobolev type and diffeomorphism groups. Other useful tools are the algebraic geometry of Riemann surfaces and infinite-dimensional determinants. Also discussed are the boundary regularity questions. The main result is a presentation of the string partition function as an integral over a moduli space of Riemann surfaces. Some new physical concepts, such as D-branes, are also discussed.

This volume offers a mathematically rigorous treatment of some aspects of string theory, employs a global geometry approach, systematically treats strings with boundary, and carefully explains all mathematical concepts and tools.

Titles in this series are copublished with International Press, Cambridge, MA.

Contents: Point particles; The bosonic string; Bibliography; Index.

AMS/IP Studies in Advanced Mathematics

Previously Announced Publications

Partial Differential Equations in Several Complex Variables
So-Chin Chen, National Tsing-Hua University, Hsinchu, Taiwan, and Mei-Chi Shaw, University of Notre Dame, IN
In the last few decades, significant progress was made in the study of Cauchy-Riemann and tangential Cauchy-Riemann operators; this progress greatly influenced the development of PDEs and several complex variables. After the background material in complex analysis is developed in Chapters 1 through 3, the next three chapters are devoted to the solvability and regularity of the Cauchy-Riemann equations using Hilbert space techniques. The second part of the book gives a comprehensive study of the tangential Cauchy-Riemann equations, another important class of equations in several complex variables first studied by Lewy. An up-to-date account of the $L^2$ theory for $\bar{\partial}$ operator is given.

This fairly self-contained book provides a much-needed introductory text to several complex variables and PDEs. It also provides a rich source of information to experts.

Titles in this series are copublished with International Press, Cambridge, MA.

AMS/IP Studies in Advanced Mathematics, Volume 19


Operator Theoretical Methods
A. Gheondea, R. N. Gologan, and D. Timotin, Romanian Academy, Bucharest, Romania, Editors

A publication of the Theta Foundation.

This volume contains carefully selected contributions by participants in the Seventeenth International Conference on Operator Theory held at the University of Timisoara (Romania). A large variety of topics are covered, including single operator theory, C*-algebras, spectral theory, special classes of concrete operators, and holomorphic operator functions.

Distributed worldwide, except in Romania, by the AMS.

International Book Series of Mathematical Texts

August 2000, 415 pages, Hardcover, ISBN 973-99097-2-8, 2000 Mathematics Subject Classification: 47A20, 44A60, 49J99, 47A48, 16W30, 46L05, 47A55, 35J05, 35J25, 45E10, 47A05, 47A35, 47A57, 46L51, 43A17, 47B37, 45P05, 35P05, 35A15, 47A10, 47D03, 46A32, 46B25, 46B42, 47B38, 47C05, 46L55, 22D25, 43A20, 43A80, 46A05, 46L10, 43A35, 47B25, 46E20, 93B05, 93B07, 47A10, 47A75, 47B35, 47A20, 93B36, 46L55, 42A50, 47B39, 47A35, 46L05, 20K99, 46L35, 43A07, 43A15, 43A22, 26B99, All AMS members $30, List $38, Order code THETA/1RT103

Laminations and Foliation in Dynamics, Geometry and Topology
Mikhail Lyubich, John W. Milnor, and Yair N. Minsky, SUNY at Stony Brook, NY, Editors

This volume is based on a conference held at SUNY, Stony Brook (NY). The concepts of laminations and foliations appear in a diverse number of fields, such as topology, geometry, analytic differential equations, holomorphic dynamics, and renormalization theory. Although these areas have developed deep relations, each has developed distinct research fields with little interaction among practitioners.

Of particular interest are the articles by F. Bonahon, "Geodesic Laminations on Surfaces", and D. Gabai, "Three Lectures on Laminations and Foliations on 3-manifolds", which are based on minicourses that took place during the conference.

Contemporary Mathematics, Volume 269

Algebraic Geometry 2
Sheaves and Cohomology
Kenji Ueno, Kyoto University, Japan

Modern algebraic geometry is built upon two fundamental notions: schemes and sheaves. The theory of schemes was explained in Algebraic Geometry 1: From Algebraic Varieties to Schemes (see Volume 185 in the same series, Translations of Mathematical Monographs). In the present book, Ueno turns to the theory of sheaves and their cohomology. Loosely speaking, a sheaf is a way of keeping track of local information defined on a topological space, such as the local holomorphic functions on a complex manifold or the local sections of a vector bundle. To study schemes, it is useful to study the sheaves defined on them, especially the coherent and quasicoherent sheaves. The primary tool in understanding sheaves is cohomology. For example, in studying ambleness, it is frequently useful to translate a property of sheaves into a statement about its cohomology.

The text covers the important topics of sheaf theory, including types of sheaves and the fundamental operations on them, such as coherent and quasicoherent sheaves, proper and projective morphisms, direct and inverse images, and Cech cohomology.

For the mathematician unfamiliar with the language of schemes and sheaves, algebraic geometry can seem distant. However, Ueno makes the topic seem natural through his concise style and his insightful explanations. He explains why things are done this way and supplements his explanations with illuminating examples. As a result, he is able to make algebraic geometry very accessible to a wide audience of non-specialists.

The book contains numerous problems and exercises with solutions. It would be an excellent text for the second part of a course in algebraic geometry.

Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 197
In Praise of AMS Publications

Below are AMS titles with recently received praise from well-known review publications in the mathematical community. The comments continue to attest to the quality and significance of the AMS publications made available through our publishing program. For more about these and other AMS books, visit the AMS Bookstore at www.ams.org/bookstore.

Mathematics: Frontiers and Perspectives
V. Arnold, University of Paris IX, Paris, France, and Steklov Mathematical Institute, Moscow, Russia; M. Atiyah, University of Edinburgh, Scotland; P. Lax, New York University-Courant Institute of Mathematical Sciences, NY, and B. Mazur, Harvard University, Cambridge, MA, Editors

This collection demonstrates well that mathematics is alive and vital.
—American Scientist

Individual members of mathematical societies of the IMU member countries can purchase this volume at the AMS member price when buying directly from the AMS.
2000; ISBN 0-8218-2667-2; 459 pages; Softcover; All AMS members $31, List $39, Order Code MFP.SCT103

Knotted Surfaces and Their Diagrams
J. Scott Carter, University of South Alabama, Mobile, and Masahico Saito, University of South Florida, Tampa

The authors must be congratulated on their heroic endeavors to bring known and unknown results into one book. Who should buy this book? Certainly all topologists with geometric leanings should do so, and their students too.
—Bulletin of the London Mathematical Society

This book will appeal to any reader with an interest in the history of mathematics. For instance, there are biographies of each of the major contributors at the beginnings of the chapters, and numerous smaller biographical sketches are scattered throughout the text. There are also revealing glimpses into the mathematical culture of the early twentieth century found in the numerous citations from the correspondence between these men. Curtis’ accounts of the major upheavals in the lives of German mathematicians caused by the anti-semitism of the Nazi regime are also quite engaging.
Graduate students just starting out on a research program in algebra or representation theory would benefit immensely from it. Curtis has produced a rare but necessary sort of book that fills this need for a more formal introduction to the historical mathematical background of the twentieth-century mathematics usually seen first in graduate school.
—MAA Online

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Also available in hardcover.

Domain Decomposition Methods 10
Jan Mandel, University of Colorado, Denver, and Charbel Farhat and Xiao-Chuan Cai, University of Colorado, Boulder, Editors

Experienced researchers would find these proceedings very useful. In particular, various new applications and new algorithms can be found here.
—SIAM Review

Contemporary Mathematics, Volume 218; 1998; ISBN 0-8218-0889-1; 554 pages; Softcover; Individual member $56, List $110, Institutional member $86, Order Code CONM/218CT103

Jacques Hadamard, A Universal Mathematician
Vladimir Maz’ya and Tatiana Shaposhnikova, Linköping University, Sweden

The book is filled with brief and fascinating biographies of many of these colleagues, with amusing anecdotes and inclusive quotations; and with an illuminating description of the academic milieu in which Hadamard was trained. In the second part of the book, Maz’ya and Shaposhnikova discuss the major fields of Hadamard’s mathematics. These chapters, written with clarity and authority, manage simultaneously to be accessible to the novice (with a good undergraduate training in mathematics or science) and to be enlightening to the professional.
—SIAM Review

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Riemannian Geometry
Takashi Sakai, Okayama University, Japan

A good source for teaching a somewhat advanced class in differential geometry and certainly contains enough material for a one-year course. (It is) also a good source for the working differential geometer... a fine book and worthwhile addition to any differential geometer’s library.
—Bulletin of the American Mathematical Society

Translations of Mathematical Monographs, Volume 149; 1996; ISBN 0-8218-3294-5; 358 pages; Softcover; All AMS members $47, List $59, Order Code MMON0149CT103

Riemannian Geometry
Takashi Sakai, Okayama University, Japan

A good source for teaching a somewhat advanced class in differential geometry and certainly contains enough material for a one-year course. (It is) also a good source for the working differential geometer... a fine book and worthwhile addition to any differential geometer’s library.
—Bulletin of the American Mathematical Society

Translations of Mathematical Monographs, Volume 149; 1996; ISBN 0-8218-3294-5; 358 pages; Softcover; All AMS members $47, List $59, Order Code MMON0149CT103

The Prime Numbers and Their Distribution
Gérald Tenenbaum, Université Henri Poincaré, Nancy I, France, and Michel Mendès France, Université Bordeaux I, France

It is a pleasure to read this booklet, written by experts of number theory. Due to the many results, the elegant proofs, and the informal explanation of ideas, it is highly recommended to study this small monograph thoroughly.
—Zentralblatt für Mathematik

Student Mathematical Library, Volume 6; 2000; ISBN 0-8218-1647-0; 115 pages; Softcover; All AMS members $14, List $17, Order Code STML8CT103

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MASSACHUSETTS
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Mathematical Sciences Department
Visiting Assistant Professorships

The Worcester Polytechnic Institute (WPI) Department of Mathematical Sciences invites applications for two or more anticipated visiting assistant professorships to begin in the fall of 2001. An earned Ph.D. or equivalent degree is required. Successful candidates must demonstrate strong research potential and evidence of quality teaching and will be expected to contribute to the department's research activities and to its innovative, project-based educational programs.

At least one position is anticipated in each of the following areas:

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Statistics: Applicants are especially encouraged in the areas of time series, experimental design, Bayesian methods, Monte Carlo methods, data mining methods, biostatistics, sample survey methods, and survival analysis.

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The Mathematical Sciences Department has 24 tenured/tenure-track faculty and supports B.S., M.S., and Ph.D. programs in applied and computational mathematics and applied statistics. For additional information, see http://www.wpi.edu/math/.

Qualified applicants should send a detailed curriculum vitae; a brief statement of specific teaching and research objectives; and three letters of recommendation, at least one of which addresses teaching potential, to either Math or Statistics Visitor Search Committee, Mathematical Sciences Department, WPI, 100 Institute Road, Worcester, MA 01609-2280.

Applicants will be considered on a continuing basis beginning January 1, 2001, until all positions are filled.

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Mathematical Sciences Department

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WPI is a private and highly selective technological university with an enrollment of 2,700 undergraduates and about 1,100 full- and part-time graduate students. Worcester, New England’s second largest city, offers ready access to the diverse economic, cultural, and recreational resources of the region.

The Mathematical Sciences Department has 24 tenure-track faculty and supports B.S., M.S., and Ph.D. programs in applied and computational mathematics and applied statistics. For additional information, see http://www.wpi.edu/~math/.

Nominations and applications should be sent to: Search Committee, Mathematical Sciences Department, WPI, 100 Institute Road, Worcester, MA 01609-2280. To be complete, applications must include a detailed curriculum vitae, a brief statement of specific teaching and research objectives, and the names of four references with mail/e-mail addresses and telephone/fax numbers.

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MICHIGAN
UNIVERSITY OF MICHIGAN
Department of Mathematics

The department seeks candidates for a lecturer position beginning September 2001, involving the operation and direction of its introductory program in precalculus and calculus and the training of instructors for these courses. Duties will include the direction of one large multisection precalculus or calculus course per semester; the teaching of one or two sections of the course being directed; assistance with our program to train and supervise new teachers in the introductory program; and general help with the planning, direction, and administration of the program. Applicants should have demonstrated excellence in the teaching of college mathematics, experience directing multisection courses in the first two years of college mathematics, and expertise in modern pedagogical methods. Experience in working with outreach programs is also desirable. Those who do not precisely fit this description but who are very strong in several of these areas will also be considered. A doctorate in mathematics or a closely related area is preferred, but all strong candidates will be considered. Preference will be given to candidates who are also involved in mathematical research or scholarship, including mathematics education. Rank and salary commensurate with experience.

NEVADA
UNIVERSITY OF NEVADA
Department Head

Professor and Founding Chair, Applied Mathematics and Statistics. The University of Nevada, Las Vegas, invites applications for the position of professor and founding chair of a new academic department in applied mathematics and statistics. The faculty is seeking a dynamic leader, scholar, teacher, and administrator who will direct and facilitate the growth of this new department. The new department now consists of a faculty with research interests in applied/computational mathematics and statistics. A Ph.D. in mathematics, statistics, or a related field in which the methods of applied mathematics and/or statistics are integral to the discipline is required. The successful candidate will have an established record of scholarly productivity, demonstrated excellence in teaching, and proven administrative skills. Experience in directing doctoral dissertations is desirable. Salary is commensurate with qualifications and experience. Position is contingent upon funding and pending institutional and system approval of the new department. Twelve-month-per-year position. Ph.D. in the mathematical sciences required.

Applicants should submit a detailed curriculum vitae; a statement that addresses administrative philosophy, research, and teaching interests; and the names and addresses (including e-mail) of five references to: Professor David G. Costa, Chair of the Search Committee, Department of Mathematical Sciences, University of Nevada, Las Vegas, 4505 Maryland Parkway, Box 454020, Las Vegas, NV 89154-4020; phone: 702-895-0359; e-mail: costa@unlv.edu. Review of completed applications will begin January 20, 2001, and will continue until the position is filled.

NEW JERSEY
EDUCATIONAL TESTING SERVICE
Positions Available

Educational Testing Service is the nation's leading educational assessment organization and a leader in educational research. We develop and administer achievement, occupational, and admission tests for clients in education, government, and business. We have the following positions available in Princeton, NJ:

Assessment Specialist II
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Plan, develop, and evaluate tests and testing programs in computer science and mathematics, including AP computer science. Use a high level of knowledge in all phases of test development, including item writing, review, and evaluation; test assembly; and scoring. Work independently and as part of a team; may also work with outside experts in field of specialty. Regularly provide guidance and training to less experienced assessment specialist.

May design degree in computer science or mathematics and six years of increasingly responsible professional experience, including experience in educational measurement, applied statistics, teaching or editing, are necessary. Some teaching experience is necessary; experience teaching the contents of AP computer science course is desirable. Knowledge of C++ or
Java is also desirable. An equivalent combination of education and experience may be considered.

Assessment Specialist I
Mathematics

Develop tests in the field of mathematics, including writing, reviewing, and revising test questions for the SAT program and assembling tests or pools of questions to meet specifications. Work independently and as part of a team; may also work with outside experts in field of specialty.

Master's degree in mathematics or a related field and three years of increasingly responsible professional experience, including experience in educational measurement, applied statistics, teaching, or editing. An equivalent combination of education and experience may be considered.

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EDUCATIONAL TESTING SERVICE
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NEW YORK

COLGATE UNIVERSITY
Mathematics Department

The Mathematics Department of Colgate University is accepting applications for two 1-year leave-replacement positions beginning August 2001. A Ph.D. is preferred. We invite applications representing all areas of mathematics. Colgate University is a highly selective liberal arts college with 2,700 students. Faculty members teach five courses per year and are encouraged to participate in all university programs. An application, including at least a vita, an unofficial graduate transcript, and three letters of recommendation, should be sent to:

Search Committee
Department of Mathematics
Colgate University
13 Oak Drive
Hamilton, NY 13346-1398

Screening of applications will begin January 1, 2001. Colgate is an Equal Opportunity/Affirmative Action Employer. Applications from women and minorities are encouraged.

HUNTER COLLEGE
OF THE CITY UNIVERSITY OF NEW YORK
Department of Mathematics and Statistics

Tenure-track assistant professor faculty position anticipated for September 2001. Salary range: $32,703-$57,049 depending on experience. Doctoral degree required. We are interested in mathematicians (all fields). Send CV and three letters of reference to:

Professor Ada Peluso, Chair
Department of Mathematics and Statistics
Hunter College
695 Park Avenue
New York, NY 10021

Equal Opportunity/Affirmative Action Employer (M/F/D/V).

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Pure & appl. adv. & research level, any age, usable cond. Reprints OK. One box to whole libraries sought. Contact: Collier Brown or Ryan Thomas @ Powell's Technical Bks., Portland, OR. Call 800-225-6911, fax 503-228-0505, or e-mail ryan.thomas@powells.com.
Abstract and Applied Analysis

Editor-in-Chief: A. G. Kartsatos (University of South Florida)

Aims & Scope: AAA is devoted exclusively to the publication of original research papers in the fields of abstract and applied analysis. Emphasis is placed on important developments in classical analysis, linear and nonlinear functional analysis, ordinary and partial differential equations, optimization theory, and control theory.


International Journal of Mathematics and Mathematical Sciences

Founding Managing Editor: L. Debnath (University of Central Florida)

Aims & Scope: IJMMS is devoted to the publication of original research papers, research notes, and research expository and survey articles with emphasis on unsolved problems and open questions in mathematics and mathematical sciences. All areas listed on the cover of Mathematical Reviews are included within the scope of the journal.


Journal of Applied Mathematics

Editors-in-Chief: C. Brezinski (Université des Sciences et Technologies de Lille), L. Debnath (University of Central Florida), H. Nijmeijer (Eindhoven University of Technology)

Aims & Scope: JAM is devoted to the publication of original research papers and review articles in all areas of applied, computational, and industrial mathematics.


Bundle Subscription: All three journals for $995 (print and electronic), $695 (electronic only), $695 (personal print and electronic), $195 (personal electronic only).

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Technische Universiteit Eindhoven

The Technische Universiteit Eindhoven is looking for a

Full professorship in statistics

The Department of Mathematics and Computing Science has a vacancy for a full professorship in Statistics.

The Department of Mathematics and Computing Science provides undergraduate and M.Sc. programs in Industrial and Applied Mathematics and in Computer Science. The Department has research collaborations with other Departments at the Technische Universiteit Eindhoven as well as with a large number of other universities and companies, both at home and abroad. The Department has approximately 200 employees and more than 600 students.

The chair of Statistics is one of the eight chairs in Mathematics, and one of the three chairs in the Statistics and Operations Research Group, the other ones being the chair of Combinatorial Optimization (Prof. dr. J.K. Lenstra) and on Stochastic Operations Research (Prof. dr. O.J. Boxma). The other two groups in Mathematics cover various topics in Analysis and Discrete Mathematics.

The Statistics group consists of a full professor, an associate professor and 5 assistant professors. The group provides undergraduate and graduate statistics and probability courses for mathematics students and students of other Departments. The research in the group is both fundamental and applied, with an emphasis on extreme value theory, non-parametric statistics, design of experiments and industrial statistics. The presence of the Department of Biomedical Engineering provides additional opportunities for research in biostatistics.

The Statistics group participates in two research schools: The mathematics research school Stieltjes and research school BETA in the area of production and logistics. The new scientific director of EURANDOM, Prof. dr. F. den Hollander, holds a part-time professorship in Probability in the Statistics Group.

What are your duties?
• You are expected to stimulate and coordinate the fundamental and applied research of the group, to initiate new research directions and to establish links with other research programs at our university and with EURANDOM;
• You give and coordinate courses in Statistics and Probability, and you are responsible for updating these courses; you advise MSc and PhD students;
• You fulfill key management functions in the section and the faculty.

Your skills
• A deep and broad insight into statistics, as is reflected in a strong international reputation in the field and a large number of – also recent – publications in the international scientific literature;
• Much affinity with, and knowledge of, stochastics;
• Interest in application-oriented research;
• Experience and expertise in acquisition and consultancy;
• Excellent didactical qualities;
• Leadership and management qualities.

What we have to offer
• A prominent leading position in a stimulating scientific environment, in which you will work with enthusiastic students, trainee design engineers, and PhD students, as well as postdocs from EURANDOM;
• A full-time appointment in accordance with the Collective Labour Agreement for Dutch Universities;
• A maximum salary of DFL 200,000 per annum depending on your experience;
• An extensive package of fringe benefits.

Inquiries
For more information about the content of this position, please contact the chairman of the appointment committee: Prof. dr. O.J. Boxma, e-mail: o.j.boxma@tue.nl, tel. +31 40 2472858, private tel. +31 6 2863800.
You are welcome to contact Prof. dr. O.J. Boxma if you wish to inform the committee of possible candidates for the position. For information concerning labour conditions, please contact W.C. Verhoef, personnel adviser, tel. +31 40 2472321.
For general information about Department of Mathematics and Computing Science: www.win.tue.nl.

How to respond
Please submit a written letter of application accompanied by a recent curriculum vitae within four weeks of the publication of this vacancy. Please state the vacancy code V 32854 in your letter and on the envelope.


/department of mathematics and computing science
Faculty Position
Mathematics

The Franklin W. Olin College of Engineering is a new institution that strives to provide the best and most innovative engineering education to the world's brightest and most enterprising students. The College is seeking exceptional faculty dedicated to exemplary undergraduate teaching and committed to innovation and intellectual vitality through one or more creative endeavors.

Faculty are expected to become inspirational teachers of undergraduates, work with other faculty to develop new programs, and obtain national visibility in their field. Preference will be given to experienced candidates with a record of demonstrated excellence or with conspicuous ability and motivation. We are especially interested in candidates with backgrounds in applied mathematics or statistics, however exceptional candidates in other areas such as dynamical systems, probability theory/stochastic processes are encouraged to apply. Familiarity with current issues and approaches in teaching college mathematics are important. Experience with the issues in engineering education, and with the different approaches being taken at various institutions are also attractive.

The Franklin W. Olin College of Engineering, established in 1997 by a major commitment from the Franklin W. Olin Foundation, will provide all students a full 4-year scholarship. An entirely new campus is currently under construction in Needham, MA, adjacent to Babson College. While Olin College is a completely independent institution, access to Babson’s world-class programs and other colleges near Boston’s Route 128 high-technology corridor will enrich the opportunities available to Olin faculty and students.

To apply, please send an application letter describing your teaching, research and other professional goals and accomplishments with a current resume to: Mathematics Faculty Search, c/o Dr. David V. Kerns, Jr., Provost, Franklin W. Olin College of Engineering, MS-MA, 1735 Great Plain Ave., Needham, MA 02492-1245. Email: facultysearch@olin.edu

Applications and nominations will be considered until all open positions are filled.

The Franklin W. Olin College of Engineering is an Equal Opportunity Employer.

For more information visit: www.olin.edu
Please read the “Membership Categories” section of this form to determine the membership category for which you are eligible. Then fill out this application and return it as soon as possible.

Date ____________ 20 __

Family Name: ____________ First: ____________ Middle: ____________
Place of Birth: ____________ City: ____________ State: ____________ Country: ____________
Date of Birth: ____________ Day: ____________ Month: ____________ Year: ____________

If former member of AMS, please indicate dates ____________ ____________ ____________

Check here if you are now a member of either □ MAA or □ SIAM

Degrees, with institutions and dates ____________ ____________ ____________ ____________
Present position ____________
Firm or institution ____________

City: ____________ State: ____________ Zip/Country: ____________

Primary Fields of Interest (choose five from the list at right)
00 General
01 History and biography
02 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 K-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
23 Abstract harmonic analysis
24 Integral transforms, operational calculus
29 Measures and integral transforms
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
38 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations

Secondary Fields of Interest (choose from the list at right)

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Please read the following to determine what membership category you are eligible for, and then indicate below the category for which you are applying.

**Introductory ordinary member rate applies** to the first five consecutive years of ordinary membership. Eligibility begins with the first year of membership in any category other than student and nominee. Dues are $51.

For **ordinary members** whose annual professional income is below $75,000, the dues are $102; for those whose annual professional income is $75,000 or more, the dues are $136.

For a **joint family membership**, one member pays ordinary dues, based on his or her income; the other pays ordinary dues based on his or her income, less $20. (Only the member paying full dues will receive the Notices and the Bulletin as a privilege of membership, but both members will be accorded all other privileges of membership.)

Minimum dues for **contributing members** are $204. The amount paid which exceeds the higher ordinary dues level and is purely voluntary may be treated as a charitable contribution.

For either **students** or **unemployed individuals**, dues are $34, and annual verification is required.

The annual dues for **reciprocity members** who reside outside the U.S. are $68. To be eligible for this classification, members must belong to one of those foreign societies with which the AMS has established a reciprocity agreement, and annual verification is required. Reciprocity members who reside in the U.S. must pay ordinary member dues ($102 or $136).

The annual dues for **category-S members**, those who reside in developing countries, are $16. Members can choose only one privilege journal. Please indicate your choice below.

Members can purchase a **multi-year membership** by prepaying their current dues rate for either two, three, four or five years. This option is not available to category-S, unemployed, or student members.

### 2001 Dues Schedule (January through December)

<table>
<thead>
<tr>
<th>Category</th>
<th>Rate</th>
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<tr>
<td>Ordinary member, introductory</td>
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<td>$102</td>
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<tr>
<td>Ordinary member</td>
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<td>$136</td>
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<tr>
<td>Joint family member (full rate)</td>
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<td>$136</td>
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<tr>
<td>Joint family member (reduced)</td>
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<td>Contributing member (minimum</td>
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<td>Student member (please verify)</td>
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<td>Reciprocity member (please verify)</td>
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<td>Category-S member</td>
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<tr>
<td>Multi-year membership</td>
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1 **Student Verification** (sign below)
I am a full-time student at ____________________________ currently working toward a degree.

2 **Unemployed Verification** (sign below) I am currently unemployed and actively seeking employment.

3 **Reciprocity Membership Verification** (sign below) I am currently a member of the society indicated on the right and am therefore eligible for reciprocity membership.

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### Reciprocating Societies

- Allahabad Mathematical Society
- Australian Mathematical Society
- Armenian Mathematical Society
- Balkan Society of Geometers
- Berliner Mathematische Gesellschaft
- Calcutta Mathematical Society
- Canadian Mathematical Society
- Croatian Mathematical Society
- Cyprus Mathematical Society
- Danish Mathematical Society
- Deutsche Mathematischer Vereinigung
- Edinburgh Mathematical Society
- Egyptian Mathematical Society
- European Mathematical Society
- Gesellschaft für Angewandte Mathematik und Mechanik
- Glasgow Mathematical Association
- Hellenic Mathematical Society
- Icelandic Mathematical Society
- Indian Mathematical Society
- Iranian Mathematical Society
- Irish Mathematical Society
- Israel Mathematical Union
- János Bolyai Mathematical Society
- The Korean Mathematical Society
- London Mathematical Society
- Malaysian Mathematical Society
- Mathematical Society of Japan
- Mathematical Society of Serbia
- Mathematical Society of the Philippines
- Mathematical Society of the Republic of China
- Mongolian Mathematical Society
- Nepal Mathematical Society
- New Zealand Mathematical Society
- Nigerian Mathematical Society
- Norwegian Mathematical Society
- Österreichische Mathematische Gesellschaft
- Palestinian Society for Mathematical Sciences
- Polskie Towarzystwo Matematyczne
- Punjab Mathematical Society
- Ramanujan Mathematical Society
- Real Sociedad Matemática Española
- Saudi Association for Mathematical Sciences
- Singapore Mathematical Society
- Sociedad Colombiana de Matemáticas
- Sociedad Española de Matemática Aplicada
- Sociedad Matemática de Chile

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4 [send Notices]  [send Bulletin]
Members of the Society who move or change positions are urged to notify the Providence Office as soon as possible.

Journal mailing lists must be printed four to six weeks before the issue date. Therefore, in order to avoid disruption of service, members are requested to provide the required notice well in advance.

Besides mailing addresses for members, the Society's records contain information about members' positions and their employers (for publication in the Combined Membership List). In addition, the AMS maintains records of members' honors, awards, and information on Society service.

When changing their addresses, members are urged to cooperate by supplying the requested information. The Society's records are of value only to the extent that they are current and accurate.

If your address has changed or will change within the next two or three months, please fill out this form, supply any other information appropriate for the AMS records, and mail it to:

**Customer Services**
AMS
P.O. Box 6248
Providence, RI 02940

or send the information on the form by e-mail to:
amsmem@ams.org or cust-serv@ams.org

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If mailing address is not that of your employer, please supply the following informations:

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<th>Location of employer (city, state, zip code, country)</th>
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<th>Recent honors and awards</th>
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PERSPECTIVES IN FLUID DYNAMICS
A Collective Introduction to Current Research
George Batchelor, Keith Moffatt, and Grae Worster, Editors
Provides coverage of topics including thin-film flows,
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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on e-MATH. See http://www.ams.org/meetings/. Programs and abstracts will continue to be displayed on e-MATH in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on e-MATH in an electronic issue of the Notices as noted below for each meeting.

Columbia, South Carolina
University of South Carolina
March 16-18, 2001
Meeting #963
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of Notices: January 2001
Program first available on e-MATH: February 1, 2001
Program issue of electronic Notices: April 2001
Issue of Abstracts: Volume 22, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses
David C. Brydges, University of Virginia, Analysis with functional integrals.
Albert A. Cohen, University of Paris VI, Harmonic analysis and image processing.
Herbert Edelsbrunner, Duke University and Raindrop Geomagic, Mathematics problems in the (re-)construction of shape.
Daniel J. Kleitman, Massachusetts Institute of Technology, Title to be announced.

Carl Pomerance, Bell Laboratories - Lucent Technologies, Babe Ruth, Hank Aaron, Paul Erdös, and me (Erdös Memorial Lecture).

Special Sessions
Algebraic Structures Associated with Lie Theory, Ben L. Cox, Elizabeth Jurisich, and Oleg Smirnov, College of Charleston.
Algebras, Lattices, Varieties, George F. McNulty, University of South Carolina, and Ralph S. Freese and James B. Nation, University of Hawaii.
Analytic Number Theory, Michael A. Filaseta and Ognian Trifonov, University of South Carolina, Columbia.
Approximation and Wavelets, Konstantin Osolkov, Pencho Petrushev, and Vladimir Temlyakov, University of South Carolina, Columbia.
Banach Spaces, George Androulakis, Stephen Dilworth, and Maria K. Girardi, University of South Carolina, Columbia.
Combinatorics and Graph Theory, Jerrold R. Griggs, University of South Carolina, Columbia.
Discrete and Computational Geometry and Graph Drawing, Farhad Shahrokhi, University of North Texas, and Laszlo A. Szekely, University of South Carolina, Columbia.
Geometry of Curves and Surfaces, Mohammad Ghomi and Ralph E. Howard, University of South Carolina, Columbia.
Mathematical Biology, Douglas B. Meade, Matthew Miller, and David Wethey, University of South Carolina, Columbia.
Semigroups and Evolution Equations, Anton R. Schep, University of South Carolina, Columbia.
Lawrence, Kansas
University of Kansas
March 30-31, 2001

Meeting #964
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: January 2001
Program first available on e-MATH: February 15, 2001
Program issue of electronic Notices: May 2001
Issue of Abstracts: Volume 22, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses
S. Dale Cutkosky, University of Missouri, Title to be announced.
Alexandre Eremenko, Purdue University, Title to be announced.
Ken Ono, University of Wisconsin-Madison, Title to be announced.
Yongbin Ruan, University of Wisconsin-Madison, Title to be announced.

Special Sessions
Algebraic Geometry, Christopher Peterson, Colorado State University, and B. P. Purnaprajna, University of Kansas.
C*-Algebras and Crossed Products, Steve Kaliszewski, Arizona State University, and May Nilsen, Texas A&M University.
Commutative Algebra, Craig Huneke and Daniel Katz, University of Kansas.
Complex Variables, Pietro Poggi-Corradini, Kansas State University.
Deformation Theory, Yan Soibelman and David Yetter, Kansas State University.
Harmonic Analysis and Applications, Rodolfo Torres, University of Kansas.
Number Theory, Ken Ono, University of Wisconsin-Madison, Cristian Popescu, University of Texas at Austin, and Tonghai Yang, Harvard University.
Optimal Control, Calculus of Variations, and Nonsmooth Analysis, Michael Malisoff, Texas A&M University, Corpus Christi, and Peter R. Wolenski, Louisiana State University.
PDEs and Geometry, Marianne Korten and Lev Kapitanski, Kansas State University.
Polytopes, Margaret Bayer and Carl Lee, University of Kansas.

Progress in Numerical Linear Algebra, Ralph Byers, University of Kansas.
Quantization and Operator Algebras, Albert Sheu, University of Kansas.
Set Theoretic Topology and Boolean Algebra, William Fleissner, University of Kansas.

Las Vegas, Nevada
University of Nevada
April 21-22, 2001

Meeting #965
Western Section
Associate secretary: Bernard Russo
Announcement issue of Notices: February 2001
Program first available on e-MATH: March 8, 2001
Program issue of electronic Notices: May 2001
Issue of Abstracts: Volume 22, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 27, 2001

Invited Addresses
Panagiota Daskalopoulos, University of California Irvine, Title to be announced.
Randall J. LeVeque, University of Washington, Title to be announced.
Vera Serganova, University of California Berkeley, Title to be announced.
Lynne Walling, University of Colorado, Title to be announced.

Special Sessions
Analysis and Applications of Nonlinear PDEs (Code: AMS SS G1), David G. Costa, Zhonghai Ding, and Hossein Tehrani, University of Nevada, Las Vegas.
Finite Element Analysis and Applications (Code: AMS SS B1), Jichun Li, Michael Marcozzi, George Miel, and Darrell W. Pepper, University of Nevada.
Graphs and Digraphs (Code: AMS SS C1), Michael Jacobson, University of Louisville, and K. Brooks Reid, California State University, San Marcos.
History of Mathematics (Code: AMS SS J1), Shawnee L. McMurray, California State University, San Bernardino, Adrian Rice, Randolph-Macon College, and James F. Duggan, Providence College.
Number Theory with a Geometric Flavor (Code: AMS SS K1), Arthur Baragar, University of Nevada, Las Vegas.
PDEs from Fluid Mechanics: Applied Analysis and Numerical Methods (Code: AMS SS H1), L. Steven Hou, York University and Iowa University, and Xiaoming Wang, Iowa State University.

Physical Knotting and Unknotting (Code: AMS SS D1), Jorge Alberto Calvo, North Dakota State University, Kenneth C. Millett, University of California Santa Barbara, and Eric J. Rawdon, Chatham College.

Set Theory (Code: AMS SS E1), Douglas Burke and Derrick Dubose, University of Nevada, Las Vegas.

Topology of Links (Code: AMS SS F1), Jeff Johannes, University of Nevada, Las Vegas, and Swatee Naik, University of Nevada, Reno.


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Meetings & Conferences

Computational Group Theory (Code: AMS SS C1), Robert Gilman, Stevens Institute of Technology, and Alexei Myasnikov, Vladimir Shpilrain, and Sean Cleary, City College, New York.

Deformation Quantization and Its Applications (Code: AMS SS K1), Siddhartha Sahi, Rutgers University, and Martin J. Andler, University of Versailles.

Graph Theory (Dedicated to Frank Harary on His 80th Birthday) (Code: AMS SS M1), Michael L. Gargano and Louis V. Quintas, Pace University, and Charles Suffel, Stevens Institute of Technology.

History of Mathematics (Code: AMS SS E1), Patricia R. Allaie, Queensborough Community College, CUNY, and Robert E. Bradley, Adelphi University.

Matchings in Graphs and Hypergraphs (Code: AMS SS F1), Alexander Barvinok, University of Michigan, and Alex Samorodnitsky, Institute for Advanced Study.

Quantum Error Correction and Related Aspects of Coding Theory (Code: AMS SS J1), Harriet S. Pollatsek, Mount Holyoke College, and M. Beth Ruskai, University of Massachusetts, Lowell.

Ricci Curvature and Related Topics (Code: AMS SS G1), George J. Kamberov, Stevens Institute of Technology, Christina Sormani, Lehman College, CUNY, and Megan M. Kerr, Wellesley College.

Singular and Degenerate Nonlinear Elliptic Boundary Value Problems (Code: AMS SS D1), Joe McKenna, Changfeng Gui, and Yung Sze Choi, University of Connecticut.

Stability of Nonlinear Dispersive Waves (Code: AMS SS H1), Yi Li, Stevens Institute of Technology, and Keith S. Promislow, Simon Fraser University.

Surface Geometry and Shape Perception (Code: AMS SS L1), Gary R. Jensen, Washington University, and George I. Kamberov, Stevens Institute of Technology.

Wavelets, Multiscale Analysis, and Applications (Code: AMS SS N1), Ivan Selesnick, Polytechnic University, and Gerald Schuller, Bell Laboratories.

Hoboken, New Jersey
Stevens Institute of Technology

April 28-29, 2001

Meeting #966
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: February 2001
Program first available on e-MATH: March 15, 2001
Program issue of electronic Notices: May 2001
Issue of Abstracts: Volume 22, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 6, 2001

Invited Addresses
Alexander Barvinok, University of Michigan, Ann Arbor, Title to be announced.
Robert Calderbank, AT&T Laboratories Research, Title to be announced.
Alexei Myasnikov, City College, New York, Title to be announced.
Frank Sottile, University of Massachusetts, Amherst, A Gromov-Witten invariant in the real world.

Special Sessions
Analytic Number Theory (Code: AMS SS A1), Milos A. Dostal, Stevens Institute of Technology, and Werner G. Nowak, Vienna, Austria.
Computational Algebraic Geometry and Its Applications (Code: AMS SS B1), Serkan Hosten, San Francisco State University, and Frank Sottile, University of Massachusetts, Amherst.

Morelia, Mexico

May 23-26, 2001

Meeting #967
Fifth International Joint Meeting of the AMS and the Sociedad Matemática Mexicana (SMM).
Associate secretary: John L. Bryant
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
Invited Addresses

Victor Pérez Abreu, CIMAT, Title to be announced.

Eric M. Friedlander, Northwestern University, Between algebraic and topological K-theory.

Helmut H. W. Hofer, Courant Institute, New York University, Periodic orbits, holomorphic curves, and algebraic invariants.

Ernesto A. Lacomba, UAM-I, Title to be announced.

Claude R. LeBrun, SUNY at Stony Brook, On the curvature of 4-manifolds.

Antonmaria Minzoni, IIMAS-UNAM, Title to be announced.

Special Sessions

Algebraic Geometry, Pedro Luis del Angel, CIMAT, and Javier Elizondo, IMATE-UNAM.

Algebraic Topology and K-Theory, Miguel Xicoténcatl, CINVESTAV.

Biomathematics, Jorge Velasco, UAM-I.

Combinatorics and Graph Theory, Ernesto Vallejo, IMATE-UNAM.

Complex Analysis, Enrique Ramírez de Arellano, CINVESTAV.

Differential Geometry, Adolfo Sánchez Valenzuela, CIMAT, and Raúl Quiroga, CINVESTAV.


Functional and Harmonic Analysis, Lourdes Palacios, UAM-I, and Salvador Pérez-Esteva, IMATE-UNAM.

General Topology, Alejandro Illanez, IMATE-UNAM.

Nonlinear Analysis, Gustavo Cruz, IIMAS-UNAM, and Mónica Clapp, IMATE-UNAM.

Numerical Methods in Differential Equations, Pablo Barrera, UNAM, and Francisco Solís, IIMAS-UNAM.

Quasigroups, J. D. Phyllips, and Lev V. Sabinin, University of Quintana Roo.

Representation Theory of Algebras and Related Topics, Raymundo Bautista, IMATE-UNAM.

Ring Theory, Francisco Raggi, IMATE-UNAM, and Sergio R. Lopez-Permouth, Ohio University.

Stochastic Analysis, Mogens Blatt, IIMAS-UNAM, and María E. Caballero, IAMTE-UNAM.

There will be no sessions for contributed papers.

Lyons, France

July 17-20, 2001

Meeting #968

First Joint International Meeting between the AMS and the Société Mathématique de France.
Geometry and Representation Theory of Algebraic Groups, Michel Brion, Université de Grenoble I, and Andrei Zelevinsky, Northeastern University.

History of Mathematics, Thomas W. Archibald, Acadia University, Christian Gilain, Université Pierre et Marie Curie-Paris VI, and James J. Tattersall, Providence College.

Logic and Interaction: From the Rules of Logic and the Logic of Rules, Jean-Yves Girard, Université de Marseille, and Philip Scott, University of Ottawa.

Mathematical Fluid Dynamics, Yann Brenier, Université Pierre et Marie Curie-Paris VI, Susan J. Friedlander, University of Illinois at Chicago, and Emmanuel Grenier, École Normale Supérieure de Lyon.

Mathematical Methods in Financial Modelling, Marco Avellaneda, Courant Institute, New York University, and Rama Cont, École Polytechnique.

Model Theory, Gregory L. Cherlin, Rutgers University, and Frank Wagner, Université Claude Bernard Lyon I.


Probability, Gerard Benarous, École Normale Supérieure, and George C. Papanicolaou, Stanford University.

Columbus, Ohio
Ohio State University

September 21-23, 2001

Meeting #969
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: June/July, 2001
Program first available on e-MATH: August 9, 2001
Program issue of electronic Notices: October 2001
Issue of Abstracts: Volume 22, Issue 3

Deadlines
For organizers: February 21, 2001
For consideration of contributed papers in Special Sessions: June 5, 2001
For abstracts: July 31, 2001

Invited Addresses
Alex Eskin, University of Chicago, Title to be announced.
Dennis Gaitsgory, Harvard University, Title to be announced.
Yakov B. Pesin, Pennsylvania State University, Title to be announced.
Thaleia Zariphopoulou, University of Texas at Austin, Title to be announced.

Special Sessions
$L^2$ Methods in Algebraic and Geometric Topology (Code: AMS SS C1), Dan Burghelea and Michael Davis, Ohio State University.

Algebraic Cycles, Algebraic Geometry (Code: AMS SS A1), Roy Joshua, Ohio State University.

Coding Theory (Code: AMS SS B1), Tom Dowling, Ohio State University, and Dijen Ray-Chaudhuri.

Commutative Algebra (Code: AMS SS C1), Evan Houston, University of North Carolina, Charlotte, and Alan Loper, Ohio State University.

Fractals (Code: AMS SS P1), Gerald Edgar, Ohio State University.

Group Theory (Code: AMS SS F1), Koichiro Harada, Subinder Seghal, and Ronald Solomon, Ohio State University.

Multivariate Generating Functions and Automatic Computation (Code: AMS SS H1), Robin Pemantle, Ohio State University.

Number Theory (Code: AMS SS J1), David Goss, Ohio State University.

Proof Theory and the Foundations of Mathematics (Code: AMS SS K1), Timothy Carlson, Ohio State University.

Quantum Topology (Code: AMS SS L1), Thomas Kerler, Ohio State University.

Rings and Modules (Code: AMS SS M1), S. K. Jain, Ohio University, and Tariq Rizvi, Ohio State University.

Spectral Theory of Schrödinger Operators (Code: AMS SS N1), Boris Mitryagin, Ohio State University, and Sergei Novikov, University of Maryland.

Chattanooga, Tennessee
University of Tennessee, Chattanooga

October 5-6, 2001

Meeting #970
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of Notices: August 2001
Program first available on e-MATH: August 23, 2001
Program issue of electronic Notices: November 2001
Issue of Abstracts: Volume 22, Issue 3

Deadlines
For organizers: March 5, 2001
For consideration of contributed papers in Special Sessions: June 19, 2001
For abstracts: August 14, 2001

Special Sessions
Asymptotic Behavior of Solutions of Differential and Difference Equations (Code: AMS SS B1), John R. Graef, University of Tennessee at Chattanooga, and Chuanxi Qian, Mississippi State University.
Williamstown, Massachusetts
Williams College
October 13–14, 2001

Meeting #971
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: August 2001
Program first available on e-MATH: August 30, 2001
Program issue of electronic Notices: November 2001
Issue of Abstracts: Volume 22, Issue 4

Deadlines
For organizers: March 13, 2001
For consideration of contributed papers in Special Sessions: June 26, 2001
For abstracts: August 21, 2001

Invited Addresses
Hubert Bray, Massachusetts Institute of Technology, Title to be announced.
Robin Forman, Rice University, Title to be announced.
Emma Previato, Boston University, Title to be announced.
Yisong Yang, Polytechnic University, Title to be announced.

Special Sessions
Algebraic and Topological Combinatorics (Code: AMS SS D1), Eva Maria Feichtner, ETH, Zürich, Switzerland, and Dmitry N. Kozlov, KTH, Stockholm, Sweden.
Commutative Algebra (Code: AMS SS C1), Susan R. Loepp, Williams College, and Graham J. Leuschke, University of Kansas.
Diophantine Problems (Code: AMS SS F1), Edward B. Burger, Williams College, and Jeffrey D. Vaaler, University of Texas at Austin.

Irvine, California
University of California Irvine
November 10–11, 2001

Meeting #972
Western Section
Associate secretary: Bernard Russo
Announcement issue of Notices: September 2001
Program first available on e-MATH: September 27, 2001
Program issue of electronic Notices: December 2001
Issue of Abstracts: Volume 22, Issue 4

Deadlines
For organizers: April 10, 2001
For consideration of contributed papers in Special Sessions: July 24, 2001
For abstracts: September 18, 2001

Invited Addresses
Gigliola Staffilani, Stanford University, Title to be announced.
Jonathan Weitsman, University of California Santa Cruz, Title to be announced.

San Diego, California
San Diego Convention Center
January 6–9, 2002

Joint Mathematics Meetings, including the 108th Annual Meeting of the AMS, 85th Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Meetings & Conferences

Montréal, Québec, Canada

Centre de Recherches Mathématiques, Université de Montréal

May 3–5, 2002

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: October 3, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Pisa, Italy

June 12–16, 2002

First Joint International Meeting between the AMS and the Unione Matematica Italiana.
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

Luigi Ambrosio, Scuola Normale Superiore, Title to be announced.
Luis A. Caffarelli, University of Texas at Austin, Title to be announced.
Claudio Canuto, University of Torino, Title to be announced.
L. Craig Evans, University of California Berkeley, Title to be announced.
Giovanni Gallavotti, University of Rome I, Title to be announced.
Sergiu Klainerman, Princeton University, Title to be announced.
Claudio Procesi, University of Rome, Title to be announced.

Special Sessions

The program committee for the first joint meeting of the AMS and the Unione Matematica Italiana (UMI) solicits
proposals for Special Sessions. Each Special Session must have at least one organizer from each of the sponsoring organizations (AMS and UMI). Proposals should include names of the organizers, their mailing addresses, e-mail addresses, session title, and a brief description. Proposals should be sent to Professor Lesley Sibner, Lsibner@duke.poly.edu by March 1, 2001.

**Portland, Oregon**
*Portland State University*

**June 20–22, 2002**
Western Section
Associate secretary: Bernard Russo
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

**Deadlines**
For organizers: November 20, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

**Boston, Massachusetts**
*Northeastern University*

**October 5–6, 2002**
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

**Deadlines**
For organizers: March 6, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

**Madison, Wisconsin**
*University of Wisconsin-Madison*

**October 12–13, 2002**
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

**Deadlines**
For organizers: March 12, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

**Baltimore, Maryland**
*Baltimore Convention Center*

**January 15–18, 2003**
*Joint Mathematics Meetings, including the 109th Annual Meeting of the AMS, 86th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).*
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

**Deadlines**
For organizers: April 15, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

**Phoenix, Arizona**
*Phoenix Civic Plaza*

**January 7–10, 2004**
Associate secretary: Bernard Russo
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

**Deadlines**
For organizers: April 2, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced
Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Bernard Russo, Department of Mathematics, University of California Irvine, CA 92697; e-mail: brusso@math.uci.edu; telephone: 949-824-5505.

Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-503-5041.

Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@duke.poly.edu; telephone: 718-260-3505.

Southeastern Section: John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: bryant@math.fsu.edu; telephone: 850-644-5805.

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. Up-to-date meeting and conference information is available on the World Wide Web at www.ams.org/meetings/.

Meetings:

2001

March 16-18 Columbia, South Carolina p. 360
March 30-31 Lawrence, Kansas p. 361
April 21-22 Las Vegas, Nevada p. 361
April 28-29 Hoboken, New Jersey p. 362
May 23-26 Morelia, Mexico p. 362
July 17-20 Lyon, France p. 363
September 21-23 Columbus, Ohio p. 364
October 5-6 Chattanooga, Tennessee p. 364
October 13-14 Williamsport, PA p. 365
November 10-11 Irvine, California p. 365

2002

January 6-9 San Diego, California p. 365
Annual Meeting
March 1-3 Ann Arbor, Michigan p. 366
March 8-10 Atlanta, Georgia p. 366
May 3-5 Montréal, Québec, Canada p. 366
June 12-16 Pisa, Italy p. 366
June 20-22 Portland, Oregon p. 367
October 5-6 Boston, Massachusetts p. 367

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 87 in the January 2001 issue of the Notices for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX or AMS-LaTeX may submit abstracts with such coding. To see descriptions of the forms available, visit http://www.ams.org/abstracts/instructions.html, or send mail to abs-submit@ams.org, typing help as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to abs-submit@ams.org, typing submission as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 10-August 9, 2001: Joint Summer Research Conferences in the Mathematical Sciences, Mount Holyoke College, South Hadley, MA. See pages 1327-1331, November 2000 issue, for details.

Cosponsored Conferences:


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J.L. DOOB, University of Illinois, Urbana, IL

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—METRIKA 1986, REVIEWED BY MARCEL BRELOT

2000/546 PP., SOFTCOVER/$34.95
ISBN 3-540-43599-9

R.C. LYNGGAARD and B.V. SCHUPP, University of Illinois, Urbana

COMBINATORIAL GROUP THEORY

“This book...defines the boundaries of the subject now called combinatorial group theory...it is a considerable achievement to have concentrated a survey of the subject into 579 pages...the book is a valuable and welcome addition to the literature, containing many results not previously available in a book.”
—MATHEMATICAL REVIEWS

2000/340 PP., 30 ILLUS./SOFTCOVER/$34.95
ISBN 3-540-41158-9

D. GIEBART, University of Stanford, CA and N.S. TRUDINGER, The Australian National University, Canberra, Australia

ELLiptic PARTIAL DIFFERENTIAL EQUATIONS OF SECOND ORDER

“This is a book of interest to anyone having to work with differential equations, either as a reference or as a book to learn from. The authors have taken trouble to make the treatment self-contained. It is suitable required reading for a Ph.D. student.”
—NEWSLETTER, NEW ZEALAND MATHEMATICAL SOCIETY

2001/518 PP., SOFTCOVER/$34.95
ISBN 3-540-41160-7

S. ABBOTT, Middlebury College, Middlebury, VT

UNDERSTANDING ANALYSIS

This book outlines an elementary, one-semester course which exposes students to both the process of rigor, and the rewards inherent in taking an axiomatic approach to the study of functions of a real variable. The philosophy of this book is to focus attention on questions which give analysis its inherent fascination. Does the Cantor set contain any irrational numbers? Can the set of points where a function is continuous be arbitrary? Is an infinitely differentiable function necessarily the limit of its Taylor series? Giving these topics center stage, the motivation for a rigorous approach is justified by the fact that they are inaccessible without it.

2001/264 PP., HARDCOVER/$39.95
UNDERGRADUATE TEXTS IN MATHEMATICS

L. PERKO, Northern Arizona University, Flagstaff

DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS

Reviews from the first edition—

"The text succeeds admirably...Examples abound, figures are used to advantage, and a reasonable balance is maintained between what is proved in detail and what is asserted with supporting references...Each section closes with a set of problems, many of which are quite interesting and round out the text material...this book is to be highly recommended both for use as a text, and for professionals in other fields wanting to gain insight into modern aspects of the geometric theory of continuous (i.e., not discrete) dynamical systems."
—MATHEMATICAL REVIEWS

2000/APPX. 562 PP., HARDCOVER/$40.95
TEXTS IN APPLIED MATHEMATICS, VOL. 7

J.W. STEELE, University of Pennsylvania, Philadelphia

STOCHASTIC CALCULUS AND FINANCIAL APPLICATIONS

A discussion of simple random walk and the analysis of gambling games are used to motivate the theory of martingales and continuous time stochastic process motion. The book then takes up the Ito integral and enough of the theory of the diffusion equation to be able to solve the Black-Scholes PDE and prove the uniqueness of the solution. The foundations for the martingale theory of arbitrage pricing are then prefaced by a well-motivated development of the martingale representation theorems and Girsanov theory.

2000/344 PP., HARDCOVER/$69.95
APPLICATIONS OF MATHEMATICS, VOL. 40

VLADIMIR G. BOLTYANSKI, CIMAT, Guanajuato, Mexico, and V.A. EFREMOWICH, deceased, Translated by ABE SHENZTER, York University, North York, Ontario, Canada

INTUITIVE COMBINATORIAL TOPOLOGY

This book deals with the topology of curves and surfaces as well as with the fundamental concepts of homotopy and homology, and does this in a lively and well-motivated way. The book is well suited not only as preparation for students who plan to take a course in algebraic topology but also for advanced undergraduates or beginning graduate students interested in finding out what topology is all about. The book has more than 200 problems, many examples, and over 200 illustrations.

2001/210 PP., 213 ILLUS./HARDCOVER/$49.95
UNIVERSITEXT

FRED BRAUER, University of British Columbia, Vancouver, Canada and CARLOS CASTILLO-CHAVEZ, Cornell University, Ithaca, NY

MATHEMATICAL MODELS IN POPULATION BIOLOGY AND EPIDEMIology

As the world population exceeds the six billion mark, questions of population explosion, of how many people the earth can support and under which conditions, become pressing. Some of the questions can be addressed through the use of mathematical models, but not all. The goal of this book is to search for a balance between simple and analyzable models and unsolvable models which are capable of addressing important questions such as these. This book will include both examples and exercises.

2001/528 PP., HARDCOVER/$59.95
ISBN 0-387-98902-1
TEXTS IN APPLIED MATHEMATICS, VOL. 41

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