Experimental Mathematics: Examples, Methods and Implications
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The Importance of MathML to Mathematics Communication
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Extreme 3D visualization (see page 557)
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Emmanuel Lesigne, Université François Rabelais, Tours, France

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The Unity of Mathematics
In Honor of the Ninetieth Birthday of I.M. Gelfand

PAVEL ETNYBERG, Massachusetts Institute of Technology, Cambridge, MA; VLADIMIR S KAPRARI, Rutgers University, Piscataway, NJ; and I.M. SINGER, Massachusetts Institute of Technology, Cambridge, MA (Eds.)

A tribute to the vision and legacy of Israel Gelfand, the invited papers in this volume reflect the unity of mathematics as a whole, with particular emphasis on the many connections among the fields of geometry, physics, and representation theory. Written by leading mathematicians, the text is broadly divided into two sections: the first is devoted to developments at the intersection of geometry and physics, and the second to representation theory and algebraic geometry.


Complex Numbers from A to ...Z

TIJU ANDRESCU, University of Ruan, Bates, USA; DORIN ANDRICA, "Babes-Bolyai" University, Cluj-Napoca, Romania

Complex Numbers from A to ...Z is an introduction to this fascinating subject, with a particular emphasis on key concepts and elementary results concerning these numbers. The reader learns how complex numbers can be used to solve algebraic equations and to understand the geometric interpretation of complex numbers and the operations involving them.

The theoretical part of the book is augmented with rich exercises and problems at various levels of difficulty. A special feature of the book is the last chapter, a selection of outstanding Olympiad and other important mathematical contest problems solved by employing methods presented in the text.

The target audience includes undergraduates, high school students, instructors, mathematical contestants (such as those training for Olympiads and the W. L. Putnam Mathematical Competition) and their coaches. The work may serve as an engaging supplemental undergraduate text for an introductory course on complex numbers or number theory.


Time-Frequency and Time-Scale Methods

Adaptive Decompositions, Uncertainty Principles, and Sampling

Jeffrey A. Hogan, University of Arkansas, Fayetteville, AR; Joseph D. Lakey, New Mexico State University, Las Cruces, NM

Developed in this book are several deep connections between time-frequency (Fourier/Gabor) analysis and time-scale (wavelet) analysis, emphasizing the powerful adaptive methods that emerge when separate techniques from each area are properly assembled in a larger context. While researchers at the forefront of developments in time-frequency and time-scale analysis are well aware of the benefits of such a unified approach, there remains a knowledge gap in the larger community of practitioners about the precise strengths and limitations of Fourier/Gabor analysis versus wavelets. This book fills that gap by presenting the interface of time-frequency and time-scale methods as a rich area of work.


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Promotion YB589
502 Experimental Mathematics: Examples, Methods and Implications

David H. Bailey and Jonathan M. Borwein

Computing software and hardware advances have made numerical experimentation in mathematics an increasingly important research tool. The authors present examples of such experimentation and consider some of the consequences for mathematicians and mathematics of the availability of such tools.

532 The Importance of MathML to Mathematics Communication

Robert Miner

MathML is the markup language used for displaying mathematics on webpages and other HTML documents. The author describes the current structure and features of MathML and how its usage is affecting and can impact mathematical communication.

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Gender and Mathematics—Again

The president of Harvard stands up in a room full of women undergraduates and about one third of all U.S. mathematicians graduating this year are women. The president of Harvard says that maybe women can’t do first-rate science and math for genetic reasons. If this sounds like the start of a bad joke, it is: a bad joke that really happened.

So many women are doing mathematics and science at such high levels that we cannot imagine any meaningful interpretation of his comment. With about half of all U.S. undergraduates mathematics and about one third of all U.S. Ph.D.’s going to women, you might think there’s no problem anymore. You would be wrong.

Recently, one of us got an email which read in its entirety: “Women and mathematics remind me of Dr. Johnson’s sally about women and preachers.” (At Boswell’s report of hearing a woman preachers, he said, “Sir, a woman’s preaching is like a dog’s walking on his hinder legs. It is not done well; but you are surprised to find it done at all.”

Earlier, one of us heard an accomplished postdoc at another institution described offhandedly by a colleague as “the little girl.” You do not have to be at Harvard to notice that women still are not entirely welcome or completely accepted. And Harvard is not the only place in the mathematical community where women faculty are absent or scarce.

Samuel Johnson’s eighteenth-century comment is not only smug but unfortunately still relevant. The notion that women are an oddity is destructive to women and to mathematics. And this notion still exists. Talent needs nurturing—not coddling, but nurturing—and it is difficult to nurture people you regard as freaks.

What about the “little girl” comment? Remarks like this may seem innocuous, but they aren’t, and potentially hurt feelings are only the tip of the iceberg. To explain this, we need to define the notion of gender schema. A schema is, roughly, a cluster of expectations. For example, one part of the “driver” schema is “stops for red lights.” That is why we feel safe going when the light turns green.

As anyone who has studied calculus knows, the cumulative effect of small differences can be very large indeed. In her book *Why So Slow?* (MIT Press) Virginia Valian gives persuasive evidence of how the cumulative effect of small differences in treatment, differences resulting from gender schemas, has a major impact on women in supposedly male domains.

Consider the following scenario, which has recurred over the thirty-plus years each of us has been in mathematics. We meet a mathematically talented, enthusiastic young woman and think to ourselves, “YES! This one can’t miss!” Her teachers encourage her and follow her career with interest. But a few years down the road something goes awry—maybe a lot, more often just a little, then a little more. Her work is fine, but somehow the men have passed her by. Questions and self-doubt appear. Maybe she loses energy. Why continue to bang her head against this particular wall?

Have you ever heard a grown man described in a professional context as a little boy? We thought not. Are little girls invited to give hour addresses? Again, we thought not. “Juvenile” is part of the common gender schema for women but not for men. So is “not worth listening to.” (Read Valian’s book for the evidence; it’s extensive.) Many aspects of the schema “woman” essentially clash with schemas such as “professional” or “mathematician.” Aspects of the schema “man” do not.

Women can lose momentum and heart as a result of the accumulation of reactions to them based on these schemas. Gross comments are bad enough, but accumulated microscopic inequities devastate. To persevere despite them takes a tough skin; to avoid being distracted by them—well, neither of us has figured that one out yet. Not many people have.

It is especially insidious that both women and men hold these schemas. A basic study, replicated often, sends the same vita or the same academic paper to some people under a male name, to other people under a female name. The “woman” is ranked lower when men do the ranking. And also when women do the ranking.

What can be done? We can become more conscious of our own gender schemas and the way they influence our judgments. Like Dracula, the more pernicious of these schemas tend to be destroyed by light. We can consciously increase the number of women—the talent pool is deep—in our departments, our conferences, the major committees of our professional organizations. The more women there are, the more normal their presence becomes, the more they become just people, and the easier it is for them to do their work.

We know this is true because there are institutions, both academic and nonacademic, even subfields of mathematics, in which women participate in sufficient numbers that their presence is no longer remarkable, hence no longer remarked on. There are existence proofs. Next step: a universal quantifier.

Why bother?

Because it is right. Because it is just. Because it is dumb to discourage talented people simply because their genitals go in instead of out. Because a “mathematician” schema that does not include women turns off talented students. And because the profession is ill served by the ensuing waste of talent if we do not change.

As Barbara Grosz of Harvard said, “It’s time to put solutions in place.”

In this op-ed we have addressed the situation only for women. The situation for underrepresented minorities is even worse. The schemas they encounter are far more insidious. We hope the mathematics community will realize that Grosz’s words apply to this situation as well and will move swiftly to action.

—Judith Roitman, University of Kansas
Carol Wood, Wesleyan University
Former presidents of the Association for Women in Mathematics
Experimental Mathematics: Examples, Methods and Implications

David H. Bailey and Jonathan M. Borwein

The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.

—Jacques Hadamard

If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.

—Kurt Gödel

Introduction

Recent years have seen the flowering of "experimental" mathematics, namely the utilization of modern computer technology as an active tool in mathematical research. This development is not limited to a handful of researchers nor to a handful of universities, nor is it limited to one particular field of mathematics. Instead, it involves hundreds of individuals, at many different institutions, who have turned to the remarkable new computational tools now available to assist in their research, whether it be in number theory, algebra, analysis, geometry, or even topology. These tools are being used to work out specific examples, generate plots, perform various algebraic and calculus manipulations, test conjectures, and explore routes to formal proof. Using computer tools to test conjectures is by itself a major timesaver for mathematicians, as it permits them to quickly rule out false notions.

Clearly one of the major factors here is the development of robust symbolic mathematics software. Leading the way are the Maple and Mathematica products, which in the latest editions are far more expansive, robust, and user-friendly than when they first appeared twenty to twenty-five years ago. But numerous other tools, some of which emerged only in the past few years, are also playing key roles. These include: (1) the Magma computational algebra package, developed at the University of Sydney in Australia; (2) Neil Sloane's online integer sequence recognition tool, available at http://www.research.att.com/njas/sequences; (3) the inverse symbolic calculator (an online numeric constant recognition facility), available at http://www.cecm.sfu.ca/projects/ISC; (4) the electronic geometry site at http://www.eg-models.de; and numerous others. See
Nearly forty years later, we observe a record of sustained exponential progress that has no peer in the history of technology. Hardware progress alone has transformed mathematical computations that were once impossible into simple operations that can be done on any laptop.

Many papers have now been published in the experimental mathematics arena, and a full-fledged journal, appropriately titled *Experimental Mathematics*, has been in operation for twelve years. Even older is the AMS journal *Mathematics of Computation*, which has been publishing articles in the general area of computational mathematics since 1960 (since 1943 if you count its predecessor). Just as significant are the hundreds of other recent articles that mention computations but which otherwise are considered entirely mainstream work. All of this represents a major shift from when the present authors began their research careers, when the view that "real mathematicians don't compute" was widely held in the field.

In this article, we will summarize some of the discoveries and research results of recent years, by ourselves and by others, together with a brief description of some of the key methods employed. We will then attempt to ascertain at a more fundamental level what these developments mean for the larger world of mathematical research.

**Integer Relation Detection**

One of the key techniques used in experimental mathematics is integer relation detection, which in effect searches for linear relationships satisfied by a set of numerical values. To be precise, given a real or complex vector \((x_1, x_2, \ldots, x_n)\), an integer relation algorithm is a computational scheme that either finds the \(n\) integers \((a_i)\), not all zero, such that \(a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0\) (to within available numerical accuracy) or else establishes that there is no such integer vector within a ball of radius \(A\) about the origin, where the metric is the Euclidean norm: \(A = (a_1^2 + a_2^2 + \cdots + a_n^2)^{1/2}\).

Integer relation computations require very high precision in the input vector \(x\) to obtain numerically meaningful results—at least \(dn\)-digit precision, where \(d = \log_{10} A\). This is the principal reason for the interest in very high-precision arithmetic in experimental mathematics. In one recent integer relation detection computation, 50,000-digit arithmetic was required to obtain the result [9].

At the present time, the best-known integer relation algorithm is the PSLQ algorithm [26] of mathematician-sculptor Helaman Ferguson, who, together with his wife, Claire, received the 2002 Communications Award of the Joint Policy Board for Mathematics (AMS-MAA-SIAM). Simple formulations of the PSLQ algorithm and several variants are given in [10]. The PSLQ algorithm, together with related lattice reduction schemes such as LLL, was recently named one of ten "algorithms of the century" by the publication *Computing in Science and Engineering* [4]. PSLQ or a variant is implemented in current releases of most computer algebra systems.

**Arbitrary Digit Calculation Formulas**

The best-known application of PSLQ in experimental mathematics is the 1995 discovery, by means of a PSLQ computation, of the "BBP" formula for \(\pi\):

\[
\pi = \sum_{k=0}^{m} \frac{1}{16^k} \left( \frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).
\]

This formula permits one to directly calculate binary or hexadecimal digits beginning at the \(n\)-th digit, without needing to calculate any of the first \(n-1\) digits [8], using a simple scheme that requires very little memory and no multiple-precision arithmetic software.

It is easiest to see how this individual digit-calculating scheme works by illustrating it for a similar formula, known at least since Euler, for \(\log_2\):

\[
\log_2 = \sum_{n=1}^{\infty} \frac{1}{n^n}.
\]

Note that the binary expansion of \(\log_2\) beginning after the first \(d\) binary digits is simply \(2^d \log_2\), where by \(\lfloor \cdot \rfloor\) we mean fractional part. We can write

\[
\lfloor 2^d \log_2 \rfloor = \left\lfloor \sum_{n=1}^{\lfloor 2^d \rfloor} \frac{1}{n^n} \right\rfloor + \left\lfloor \sum_{n=\lfloor 2^d \rfloor}^{\infty} \frac{1}{n^n} \right\rfloor + \left\lfloor \sum_{n=\lfloor 2^d \rfloor}^{\infty} \frac{2^n}{n} \right\rfloor + \left\lfloor \sum_{n=\lfloor 2^d \rfloor}^{\infty} \frac{2^n}{n} \right\rfloor,
\]

where we insert "mod \(n\)" in the numerator of the first term of (2), since we are interested only in the fractional part after division by \(n\). Now the expression \(2^d \mod n\) may be evaluated very rapidly by means of the binary algorithm for exponentiation, where each multiplication is reduced.
modular n. The entire scheme indicated by formula (2) can be implemented on a computer using ordinary 64-bit or 128-bit arithmetic; high-precision arithmetic software is not required. The resulting floating-point value, when expressed in binary format, gives the first few digits of the binary expansion of \( \log_2 3 \) beginning at position \( d+1 \). Similar calculations applied to each of the four terms in formula (1) yield a similar result for \( \pi \). The largest computation of this type to date is binary digits of \( \pi \) beginning at the quadrillionth \( (10^{15}) \)-th binary digit, performed by an international network of computers organized by Colin Percival.

The BBP formula for \( \pi \) has even found a practical application: it is now employed in the g95 Fortran compiler as part of transcendental function evaluation software.

Since 1995 numerous other formulas of this type have been found and proven using a similar experimental approach. Several examples include:

\[
\pi \sqrt{3} = \frac{9}{32} \sum_{k=0}^{\infty} \frac{1}{64^k} \left( \frac{16}{6k+1} - \frac{8}{6k+2} - \frac{2}{6k+4} - \frac{1}{6k+5} \right),
\]

(3)

\[
\pi^2 = \frac{1}{8} \sum_{k=0}^{\infty} \frac{1}{64^k} \left[ \frac{144}{(6k+1)^3} - \frac{216}{(6k+2)^3} - \frac{72}{(6k+3)^3} - \frac{54}{(6k+4)^3} + \frac{9}{(6k+5)^3} \right].
\]

(4)

\[
\pi^2 = \frac{2}{27} \sum_{k=0}^{\infty} \frac{1}{128^k} \left[ \frac{243}{(12k+1)^5} - \frac{405}{(12k+2)^5} - \frac{81}{(12k+3)^5} - \frac{27}{(12k+4)^5} \right.
\]

\[
\left. - \frac{72}{(12k+5)^5} + \frac{9}{(12k+6)^5} - \frac{9}{(12k+7)^5} - \frac{5}{(12k+8)^5} + \frac{1}{(12k+9)^5} \right].
\]

(5)

\[
\sqrt{3} \arctan \left( \frac{\sqrt{3}}{7} \right) = \sum_{k=0}^{\infty} \frac{1}{27^k} \left( \frac{3k+1}{3k+2} \right).
\]

(6)

\[
\frac{25}{2} \log \left[ \frac{781}{256} \left( \frac{57 - \sqrt{55}}{57 + \sqrt{55}} \right)^2 \right] = \sum_{k=0}^{\infty} \frac{1}{5^k} \left( \frac{5k+2}{5k+3} \right).
\]

(7)

Formulas (3) and (4) permit arbitrary-position binary digits to be calculated for \( \pi \sqrt{3} \) and \( \pi^2 \). Formulas (5) and (6) permit the same for ternary (base-3) expansions of \( \pi^2 \) and \( \sqrt{3} \arctan \left( \sqrt{3}/7 \right) \). Formula (7) permits the same for the base-5 expansion of the curious constant shown. A compendium of known BBP-type formulas, with references, is available at [5].

One interesting twist here is that the hyperbolic volume of one of Ferguson’s sculptures (the

"Figure Eight Knot Complement", see Figure 1), which is given by

\[
V = 2\sqrt{3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(6n+1)^3} - \frac{24}{(6n+3)^3} + \frac{6}{(6n+4)^3} - \frac{2}{(6n+5)^3}.
\]

has been identified in terms of a BBP-type formula by application of Ferguson’s own PSLQ algorithm. In particular, British physicist David Broadhurst found in 1998, using a PSLQ program, that

\[
V = \frac{\sqrt{3}}{9} \sum_{n=0}^{\infty} \frac{(-1)^n}{27^n} \times \left[ \frac{18}{(6n+1)^3} - \frac{18}{(6n+2)^3} - \frac{24}{(6n+3)^3} + \frac{6}{(6n+4)^3} + \frac{2}{(6n+5)^3} \right].
\]

This result is proven in [15, Chap. 2, Prob. 34].

Does \( \pi \) Have a Nonbinary BBP Formula?

Since the discovery of the BBP formula for \( \pi \) in 1995, numerous researches have investigated, by means of computational searches, whether there is a similar formula for calculating arbitrary digits of \( \pi \) in other number bases (such as base 10). As a result of these searches have not been fruitful.

Recently, one of the present authors (JMB), together with David Borwein (Jon’s father) and William Galway, established that there is no degree-1 BBP-type formula for \( \pi \) for bases other than powers of two (although this does not rule out some other scheme for calculating individual digits). We will sketch this result here. Full details and some related results can be found in [20].

In the following, \( \mathbb{R}(z) \) and \( \mathbb{S}(z) \) denote the real and imaginary parts of \( z \), respectively. The integer \( b > 1 \) is not a proper power if it cannot be written as \( c^m \) for any integers \( c \) and \( m > 1 \). We will use the notation \( \text{ord}_b(z) \) to denote the \( p \)-adic order of the rational \( z \) in \( \mathbb{Q} \). In particular, \( \text{ord}_p(p) = 1 \) for prime \( p \), while \( \text{ord}_p(q) = 0 \) for primes \( q \neq p \), and \( \text{ord}_p(w z) = \text{ord}_p(w) + \text{ord}_p(z) \). The notation \( \nu_b(p) \) will mean the order of the integer \( b \) in the multiplicative group of the integers modulo \( b \). We will say that \( p \) is a primitive prime factor of \( b^m - 1 \) if \( m \) is the least integer such that \( p | (b^m - 1) \). Thus \( p \) is a primitive prime factor of \( b^m - 1 \) provided \( \nu_b(p) = m \). Given the Gaussian integer \( z \in \mathbb{Q}[i] \) and the rational prime \( p = 1 \pmod{4} \), let \( \theta_p(z) \) denote \( \text{ord}_p(z) - \text{ord}_p(z) \), where \( p \) and \( \bar{p} \) are the two conjugate Gaussian primes dividing \( p \) and where we require \( 0 < \mathbb{S}(p) < \mathbb{R}(p) \) to make the definition of \( \theta_p \) unambiguous. Note that

\[
\theta_p(w z) = \theta_p(w) + \theta_p(z).
\]

Given \( \kappa \in \mathbb{R} \), with \( 0 < b \in \mathbb{Z} \) and \( b \) not a proper power, we say that \( \kappa \) has a \( Z \)-linear or \( Q \)-linear

\[3\text{Reproduced by permission of the sculptor.}\]
Machin-type BBP arctangent formula to the base $b$ if and only if $\kappa$ can be written as a $2$-linear or $Q$-linear combination (respectively) of generators of the form

$$\arctan \left( \frac{1}{b^m} \right) = 3 \log \left( 1 + \frac{i}{b^m} \right) = b^m \sum_{k=0}^{\infty} \frac{(-1)^k}{b^{2mk}(2k+1)}.$$  

(9)

We shall also use the following result, first proved by Bang in 1886:

**Theorem 1.** The only cases where $b^m - 1$ has no primitive prime factor(s) are when $b = 2$, $m = 6$, $b^m - 1 = 3^2 \cdot 7$ or when $b = 2^N - 1, N \in Z, m = 2$, $b^m - 1 = 2^{N+1}(2^{N-1} - 1)$.

We can now state the main result:

**Theorem 2.** Given $b > 2$ and not a proper power, there is no $Q$-linear Machin-type BBP arctangent formula for $\pi$.

**Proof:** It follows immediately from the definition of a $Q$-linear Machin-type BBP arctangent formula that any such formula has the form

$$\pi = \frac{1}{n} \sum_{m=1}^{M} r_m n m \log(b^m - i),$$

(10)

where $n > 0 \in Z, r_m \in Z$, and $M \geq 1, n_M \neq 0$. This implies that

$$\prod_{m=1}^{M} (b^m - i)^{r_m} \in e^{nm} Q^x = Q^x.$$  

(11)

For any $b > 2$ and not a proper power, it follows from Bang's Theorem that $b^M - 1$ has a primitive prime factor, say $p$. Furthermore, $p$ must be odd, since $p = 2$ can only be a primitive prime factor of $b^M - 1$ when $b$ is odd and $m = 1$. Since $p$ is a primitive prime factor, it does not divide $b^{2M} - 1$, and so $p$ must divide $b^{2M+1} = (b^M + i)(b^M - i)$. We cannot have both $p | b^{2M} + i$ and $p | b^M - i$, since this would give the contradiction that $p(b^M + i) - (b^M - i) = 2i$. It follows that $p \equiv 1 \mod{4}$ and that $p$ factors as $p = pp$ over $Z[i]$, with exactly one of $p, \overline{p}$ dividing $b^M - i$. Referring to the definition of $\theta$, we see that we must have $\theta_x(p(b^M - i) = 0$. Furthermore, for any $m < M$, neither $p$ nor $\overline{p}$ can divide $b^m - i$, since this would imply $p | b^{4m} - 1, 4m < 4M$, contradicting the fact that $p$ is a primitive prime factor of $b^{2M} - 1$. So for $m < M$, we have $\theta_x(p(b^M - i) = 0$. Referring to equation (10) and using equation (8) and the fact that $n_M \neq 0$, we get the contradiction

$$0 = n_M \theta_x(p(b^M - i) = \sum_{m=1}^{M} n_m \theta_x(p(b^M - i) = \theta_x(p(Q^x)) = 0.$$  

(12)

Thus our assumption that there was a $b$-ary Machin-type BBP arctangent formula for $\pi$ must be false.

**Normality Implications of the BBP Formulas**

One interesting (and unanticipated) discovery is that the existence of these computer-discovered BBP-formulas has implications for the age-old question of normality for several basic mathematical constants, including $\pi$ and $\log 2$. What's more, this line of research has recently led to a full-fledged proof of normality for an uncountably infinite class of explicit real numbers.

Given a positive integer $b$, we will define a real number $\alpha$ to be $b$-normal if every $m$-long string of base-$b$ digits appears in the base-$b$ expansion of $\alpha$ with limiting frequency $b^{-m}$. In spite of the apparently stringent nature of this requirement, it is well known from measure theory that almost all real numbers are $b$-normal, for all bases $b$. Nonetheless, there are very few explicit examples of $b$-normal numbers, other than the likes of Champernowne's constant $0.123456789101112131415\ldots$. In particular, although computations suggest that virtually all of the well-known irrational constants of mathematics (such as $\pi$, $\sqrt{2}$, $\log 2$, etc.) are normal to various number bases, there is not a single proof—not for any of these constants, not for any number base.

Recently one of the present authors (DHB) and Richard Crandall established the following result.

Let $p(x)$ and $q(x)$ be integer-coefficient polynomials, with $\deg p < \deg q$, and $q(x)$ having no zeroes for positive integer arguments. By an equidistributed sequence in the unit interval we mean a sequence $(x_n)$ such that for every subinterval $(a, b)$, the fraction $\# \{x_n \in (a, b) \}/n$ tends to $b - a$ in the limit. The result is as follows:

**Theorem 3.** A constant $\alpha$ satisfying the BBP-type formula

$$\alpha = \sum_{n=1}^{\infty} \frac{p(n)}{b^n q(n)}$$

is $b$-normal if and only if the associated sequence defined by $x_0 = 0$ and, for $n \geq 1$, $x_n = \{bx_{n-1} + p(n)/q(n)\} / (b^n q(n))$ (where $\{ \cdot \}$ denotes fractional part as before), is equidistributed in the unit interval.

For example, $\log 2$ is $2$-normal if and only if the simple sequence defined by $x_0 = 0$ and $x_n = 2x_{n-1} + 1/n$ is equidistributed in the unit interval. For $\pi$, the associated sequence is $x_0 = 0$ and

$$x_n = \left\{ \frac{120n^2 - 89n + 16}{512n^4 - 1024n^3 + 712n^2 - 206n + 21} \right\}.$$  

Full details of this result are given in [11] [15, Section 3.8].

It is difficult to know at the present time whether this result will lead to a full-fledged proof of normality for, say, $\pi$ or $\log 2$. However, this approach
has yielded a solid normality proof for another class of reals: Given \( r \in [0, 1) \), let \( r_n \) be the \( n \)-th binary digit of \( r \). Then for each \( r \) in the unit interval, the constant

\[
\alpha_r = \sum_{n=1}^{\infty} \frac{1}{3^n 2^{3n+r_n}}
\]

is 2-normal and transcendental [12]. What's more, it can be shown that whenever \( r \neq s \), then \( \alpha_r \neq \alpha_s \). Thus (13) defines an uncountably infinite class of distinct 2-normal, transcendental real numbers. A similar conclusion applies when 2 and 3 in (13) are replaced by any pair of relatively prime integers greater than 1.

Here we will sketch a proof of normality for one particular instance of these constants, namely \( \alpha_0 = \sum_{n=1}^{\infty} 1/(3^n 2^n) \). Its associated sequence can be seen to be \( x_0 = 0 \) and \( x_n = \{2x_{n-1} + c_n\} \), where \( c_n = 1/n \) if \( n \) is a power of 3, and 0 otherwise. This associated sequence is a very good approximation to the sequence \((2^n \alpha_0)\) of shifted binary fractions of \( \alpha_0 \). In fact, \(|(2^n \alpha_0) - x_n| < 1/(2n)\). The first few terms of the associated sequence are

\[
0, 0.1, 0.2, 0.121, 0.232, 0.31321, 0.42412321, ...
\]

and so forth. The clear pattern is that of triply repeated segments, each of length \( 2 \cdot 3^m \), where the numerators range over all integers relatively prime to and less than \( 3^{m+1} \).

Note the very even manner in which this sequence fills the unit interval. Given any subinterval \((c, d)\) of the unit interval, it can be seen that this sequence visits this subinterval no more than \( 3n(d-c)+3 \) times, among the first \( n \) elements, provided that \( n > 1/(d-c) \). It can then be shown that the sequence \((2^n \alpha_0)\) visits \((c, d)\) no more than \( 8n(d-c) \) times, among the first \( n \) elements of this sequence, so long as \( n \) is at least \( 1/(d-c) \). The 2-normality of \( \alpha_0 \) then follows from a result given in [28, p. 77]. Further details on these results are given in [15, Sec. 4.3], [6], [12].

Euler's Multi-Zeta Sums

In April 1993, Enrico Au-Yeung, an undergraduate at the University of Waterloo, brought to the attention of one of us (JMB) the curious result

\[
\sum_{k=1}^{\infty} \left( 1 + \frac{1}{2} + \cdots + \frac{1}{k} \right)^2 k^{-2} = 4.59987\ldots \approx \frac{17}{4} \zeta(4) = \frac{17\pi^4}{360}
\]

where \( \zeta(s) = \sum_{n=1}^{\infty} n^{-s} \) is the Riemann zeta function. Au-Yeung had computed the sum in (14) to 500,000 terms, giving an accuracy of five or six decimal digits. Suspecting that his discovery was merely a modest numerical coincidence, Borwein sought to compute the sum to a higher level of precision. Using Fourier analysis and Parseval's equation, he wrote

\[
\frac{1}{2\pi} \int_0^{\pi} (\pi - t)^2 \log^2(2 \sin \frac{t}{2}) \, dt = \sum_{n=1}^{\infty} \left( \frac{\zeta(2n+1)}{(2n+1)^2} \right)^2.
\]

The series on the right of (15) permits one to evaluate (14), while the integral on the left can be computed using the numerical quadrature facility of Mathematica or Maple. When he did this, Borwein was surprised to find that the conjectured identity (14) holds to more than 30 digits. We should add here that by good fortune, 17/360 = 0.047222... has period one and thus can plausibly be recognized from its first six digits, so that Au-Yeung's numerical discovery was not entirely far-fetched.

Borwein was not aware at the time that (14) follows directly from a 1991 result due to De Doelder and had even arisen in 1952 as a problem in the American Mathematical Monthly. What's more, it turns out that Euler considered some related summations. Perhaps it was just as well that Borwein was not aware of these earlier results—and indeed of a large, quite deep and varied literature [21]—because pursuit of this and similar questions had led to a line of research that continues to the present day.

First define the multi-zeta constant

\[
\zeta(s_1, s_2, \ldots, s_k) := \sum_{n_1 > n_2 > \cdots > n_k > 0} \prod_{j=1}^{k} n_j^{-s_j} \sigma_j^{-n_j},
\]

where the \( s_1, s_2, \ldots, s_k \) are nonzero integers and the \( \sigma_j := \text{signum}(s_j) \). Such constants can be considered as generalizations of the Riemann zeta function at integer arguments in higher dimensions.

The analytic evaluation of such sums has relied on fast methods for computing their numerical values. One scheme, based on Holder Convolution, is discussed in [22] and implemented in EZFace+, an online tool available at http://www.cecm.sfu.ca/projects/ezface+. We will illustrate its application to one specific case, namely the analytic identification of the sum

\[
S_{2,3} = \sum_{k=1}^{\infty} \left( 1 - \frac{1}{2} + \cdots + (-1)^{k+1} \frac{1}{k} \right)^2 (k+1)^{-3}.
\]

Expanding the squared term in (16), we have
Mathematica have some rather effective integration facilities, not only for obtaining analytic results directly, but also for obtaining high-precision numeric values. However, these products do have limitations, and their numeric integration facilities are typically limited to 100 digits or so, beyond which they tend to require an unreasonable amount of run time.

Fortunately, some new methods for numerical integration have been developed that appear to be effective for a broad range of one-dimensional integrals, typically producing up to 1000 digit accuracy in just a few seconds' (or at most a few minutes') run time on a 2004-era personal computer, and that are also well suited for parallel processing [13, 14, 16, p. 312]. These schemes are based on the Euler-Maclaurin summation formula [3, p. 180], which can be stated as follows: Let \( m \geq 0 \) and \( n \geq 1 \) be integers, and define \( h = (b - a)/n \) and \( x_i = a + jh \) for \( 0 \leq j \leq n \). Further assume that the function \( f(x) \) is at least \((2m + 2)\)-times continuously differentiable on \([a, b]\). Then

\[
\int_a^b f(x) \, dx = h \sum_{j=0}^n f(x_j) - \frac{h}{2} (f(a) + f(b)) + \sum_{k=1}^m \frac{B_{2k}}{(2k)!} \left( f^{(2k-1)}(b) - f^{(2k-1)}(a) \right) - E(h),
\]

for some \( \xi \in (a, b) \). In the circumstance where the function \( f(x) \) and all of its derivatives are zero at the endpoints \( a \) and \( b \) (as in a smooth, bell-shaped function), the second and third terms of the Euler-Maclaurin formula (20) are zero, and we conclude that the error \( E(h) \) goes to zero more rapidly than any power of \( h \).

This principle is utilized by transforming the integral of some \( C^\infty \) function \( f(x) \) on the interval \([-1, 1]\) to an integral on \((-\infty, \infty)\) using the change of variable \( x = g(t) \). Here \( g(x) \) is some monotonic, infinitely differentiable function with the property that \( g(x) - 1 \) as \( x \to -\infty \) and \( g(x) + 1 \) as \( x \to -\infty \), and also with the property that \( g'(x) \) and all higher derivatives rapidly approach zero for large positive and negative arguments. In this case we can write, for \( h > 0 \),

\[
\int_{-1}^1 f(x) \, dx = \int_{-\infty}^\infty f(g(t))g'(t) \, dt = h \sum_{j=-\infty}^\infty w_j f(x_j) + E(h),
\]

for some \( \xi \in (a, b) \). The only proof known is the following, true for all \( n > 0 \):

\[
\zeta(\tfrac{1}{2}) = \frac{1}{30} \log^2(2) - \frac{17}{32} \zeta(2) - \frac{11}{720} \pi^4 \log(2) + \frac{7}{4} \zeta(3) \log^2(2) + \frac{1}{8} \pi^2 \log^2(2) - \frac{1}{8} \pi^2 \zeta(3).
\]
where $x_i = g(h)$ and $w_i = g'(h)$ are abscissas and weights that can be precomputed. If $g'(t)$ and its derivatives tend to zero sufficiently rapidly for large $t$, positive and negative, then even in cases where $f(x)$ has a vertical derivative or an integrable singularity at one or both endpoints, the resulting integrand $f(g(t))g'(t)$ is, in many cases, a smooth bell-shaped function for which the Euler-Maclaurin formula applies. In these cases, the error $E(h)$ in this approximation decreases faster than any power of $h$.

Three suitable $g$ functions are $g_1(t) = \tan t$, $g_2(t) = \text{erf} t$, and $g_3(t) = \tan(\pi/2 \cdot \sinh t)$. Among these three, $g_3(t)$ appears to be the most effective for typical experimental math applications. For many integrals, "tanh-sinh" quadrature, as the resulting scheme is known, achieves quadrature convergence: reducing the interval $h$ in half roughly doubles the number of correct digits in the quadrature result. This is another case where we have more heuristic than proven knowledge.

As one example, recently the present authors, together with Greg Fee of Simon Fraser University in Canada, were inspired by a recent problem in the American Mathematical Monthly [2]. They found by using a tanh-sinh quadrature program, together with a PSLQ integer relation detection program, that if \( C(a) \) is defined by

$$C(a) = \int_0^1 \arctan(\sqrt{x^2 + a^2}) \, dx$$

then

$$C(0) = \pi \log 2/8 + G/2,$$

$$C(1) = \pi/4 - \pi \sqrt{2}/2 + 3 \arctan(\sqrt{2})/\sqrt{2},$$

$$C(\sqrt{2}) = 5\pi^2/96.$$ 

Here \( G = \sum_{k=0}^{\infty} (-1)^k/(2k+1)^2 \) is Catalan's constant—the simplest number whose irrationality is not established but for which abundant numerical evidence exists. These experimental results then led to the following general result, rigorously established, among others:

$$\begin{align*}
\int_0^\infty \arctan(\sqrt{x^2 + a^2}) \, dx &= \frac{\pi}{2\sqrt{a^2 - 1}} \left[ 2\arctan(\sqrt{a^2 - 1}) - \arctan(\sqrt{a^2}) \right].
\end{align*}$$

As a second example, recently the present authors empirically determined that

$$\begin{align*}
\int_0^1 \log x \arctan(\sqrt{x^2 + (x - 2)^2} \, dx &= \frac{1}{81648} - \frac{1}{229635}L_4(6) \\
&+ 29852550L_3(7) \log 3 - 1632960L_4(6)m^2 + 27760320L_5(5)c(3) \\
&- 275184L_3(4)m^4 + 3628800L_4(3)c(5) - 30008L_5(2)m^6 \\
&- 5703012L_4(1)c(7)).
\end{align*}$$

where \( L_n(s) = \sum_{n=1}^{\infty} [1/(3n - 2)^s - 1/(3n - 1)^s] \). Based on these experimental results, general results of this type have been conjectured but not yet rigorously established.

A third example is the following:

$$\frac{24}{\sqrt{7}} \int_{\pi/3}^{\pi/2} \log \left| \frac{\tan t + \sqrt{7}}{\tan t - \sqrt{7}} \right| \, dt = \mathcal{L}_7(2)$$

where

$$\mathcal{L}_7(s) = \sum_{n=0}^{\infty} \left[ \frac{1}{(7n + 1)^s} + \frac{1}{(7n + 2)^s} - \frac{1}{(7n + 3)^s} + \frac{1}{(7n + 4)^s} - \frac{1}{(7n + 5)^s} + \frac{1}{(7n + 6)^s} \right].$$

The "identity" (21) has been verified to over 5000 decimal digit accuracy, but a proof is not yet known. It arises from the volume of an ideal tetrahedron in hyperbolic space, [15, pp. 90-1]. For algebraic topology reasons, it is known that the ratio of the left-hand to the right-hand side of (21) is rational.

A related experimental result, verified to 1000 digit accuracy, is

$$0 \approx -2J_2 - 2J_3 + J_4 + 2J_6 + 2J_{10} + 3J_{12} + 3J_{13} + J_{14} - J_{15} - J_{16} - J_{17} + J_{18} + J_{20} - J_{22} - J_{23} + 2J_{25},$$

where \( J_n \) is the integral in (21), with limits \( n\pi/60 \) and \( (n+1)\pi/60 \).

The above examples are ordinary one-dimensional integrals. Two-dimensional integrals are also of interest. Along this line we present a more recreational example discovered experimentally by James Klein—and confirmed by Monte Carlo simulation. It is that the expected distance between two random points on different sides of a unit square is

$$\frac{2}{3} \int_0^1 \int_0^1 \sqrt{x^2 + y^2} \, dx \, dy + \frac{1}{3} \int_0^1 \int_0^1 \sqrt{1 + (u - v)^2} \, du \, dv = \frac{1}{9} \sqrt{2} + \frac{5}{9} \log(\sqrt{2} + 1) + \frac{2}{9},$$

and the expected distance between two random points on different sides of a unit cube is

$$\frac{4}{3} \int_0^1 \int_0^1 \int_0^1 \sqrt{x^2 + y^2 + (z - w)^2} \, dx \, dy \, dz + \frac{1}{5} \int_0^1 \int_0^1 \int_0^1 \sqrt{1 + (y - u)^2 + (z - w)^2} \, du \, dv \, dz = \frac{4}{75} + \frac{17}{75} \sqrt{2} - \frac{25}{75} \sqrt{3} - \frac{7}{75} \pi + \frac{7}{25} \log(1 + \sqrt{2}) + \frac{7}{25} \log(7 + 4\sqrt{3}).$$

See [7] for details and some additional examples. It is not known whether similar closed forms exist for higher-dimensional cubes.
Ramanujan's AGM Continued Fraction

Given \( a, b, \eta > 0 \), define

\[
R_\eta(a, b) = \frac{a}{\eta + \frac{b^2}{\eta + \frac{4a^2}{\eta + \ldots}}}
\]

This continued fraction arises in Ramanujan's Notebooks. He discovered the beautiful fact that

\[
\frac{R_\eta(a, b) + R_\eta(b, a)}{2} = R_\eta\left(\frac{a + b}{2}, \sqrt{ab}\right).
\]

The authors wished to record this in [15] and wished to computationally check the identity. A first attempt to numerically compute \( R_1(1, 1) \) directly failed miserably, and with some effort only three reliable digits were obtained: 0.693… With hindsight, the slowest convergence of the fraction occurs in the mathematically simplest case, namely when \( a = b \). Indeed \( R_1(1, 1) = \log 2 \), as the first primitive numerics had tantalizingly suggested.

Attempting a direct computation of \( R_1(2, 2) \) using a depth of 20000 gives us two digits. Thus we must seek more sophisticated methods. From formula (1.11.70) of [16] we see that for \( 0 < b < a \),

\[
R_1(a, b)
\]

(22) \[= \frac{\pi}{2} \sum_{k \in \mathbb{Z}} \frac{aK(k)}{K^2(k) + a^2 \pi^2 \operatorname{sech}\left(n\pi \frac{K(k')}{K(k)}\right)},\]

where \( k = b/a = \theta_2^2/\theta_3^2, k' = \sqrt{1-k^2} \). Here \( \theta_2, \theta_3 \) are Jacobian theta functions and \( K \) is a complete elliptic integral of the first kind.

Writing the previous equation as a Riemann sum, we have

\[
\mathcal{R}(a) := \mathcal{R}_1(a, a) = \int_0^\infty \frac{\operatorname{sech}(\pi x/(2a))}{1 + x^2} \, dx
\]

(23)

\[= 2a \sum_{k=1}^\infty \frac{(-1)^{k+1}}{1 + (2k - 1)a^2},\]

where the final equality follows from the Cauchy-Lindelof Theorem. This sum may also be written as \( \mathcal{R}(a) = \frac{2a}{i-a} F\left(\frac{1}{2a} + \frac{1}{2}, \frac{1}{2}; \frac{1}{2a} + \frac{3}{2}; 1\right) \). The latter form can be used in Maple or Mathematica to determine

\[
\mathcal{R}(2) = 0.9749909879872096719900334529\ldots.
\]

This constant, as written, is a bit difficult to recognize, but if one first divides by \( \sqrt{2} \), one can obtain, using the Inverse Symbolic Calculator, an online tool available at the URL http://www.cecm.sfu.ca/projects/ISC/ISCmain.html, that the quotient is \( \pi/2 - \log(1 + \sqrt{2}) \). Thus we conclude, experimentally, that

\[
\mathcal{R}(2) = \sqrt{2} \left[ \frac{\pi}{2} - \log(1 + \sqrt{2}) \right].
\]

Indeed, it follows (see [19]) that

\[
\mathcal{R}(a) = 2 \int_0^1 \frac{t^{1/a}}{1 + t^2} \, dt.
\]

Note that \( \mathcal{R}(1) = \log 2 \). No nontrivial closed form is known for \( \mathcal{R}(a, b) \) with \( a \neq b \), although

\[
\mathcal{R}_1\left(\frac{1}{4\pi^2} \beta\left(\frac{1}{4}, \frac{1}{4}\right) \cdot \frac{\sqrt{2}}{8\pi^2} \beta\left(\frac{1}{4}, \frac{1}{4}\right) - \frac{1}{2} \sum_{n \in \mathbb{Z}} \frac{\operatorname{sech}(n\pi)}{1 + n^2}\right)
\]

is close to closed. Here \( \beta \) denotes the classical Beta function. It would be pleasant to find a direct proof of (23). Further details are to be found in [19], [17], [16].

Study of these Ramanujan continued fractions has been facilitated by examining the closely related dynamical system \( t_0 = 1, t_1 = 1 \), and
There are some exceptional cases. Jacobsen-Masson theory [17], [18] shows that the even/odd fractions for $R_1(i, i)$ behave “chaotically”; neither converge. Indeed, when $a = b = i$, $(t_n(i, i))$ exhibit a fourfold quasi-oscillation, as $n$ runs through values mod 4. Plotted versus $n$, the (real) sequence $t_n(i)$ exhibits the serpentine oscillation of four separate “necklaces”. The detailed asymptotic is

$$ t_n(i, i) = \sqrt{2 \pi} \cosh \frac{\pi}{2} \sqrt{n} \left( 1 + O \left( \frac{1}{n} \right) \right) $$

$$ \times \begin{cases} (-1)^{n/2} \cos(\theta - \log(2n)/2) & n \text{ is even} \\ (-1)^{(n+1)/2} \sin(\theta - \log(2n)/2) & n \text{ odd} \end{cases} $$

where $\theta := \arg(1 + i)/2$.

Analysis is easy given the following striking hypergeometric parametrization of (24) when $a = b \neq 0$ (see [18]), which was both experimentally discovered and is computer provable:

$$ t_n(a, a) = \frac{1}{2} F_n(a) + \frac{1}{2} F_n(-a), $$

where

$$ F_n(a) := -\frac{a^{n+1-w}}{\omega \beta(n+\omega,-\omega)} \, \, _2F_1 \left( \omega, \omega; n+1+\omega; \frac{1}{2} \right). $$

Here

$$ \beta(n+1+\omega,-\omega) := \frac{\Gamma(n+1)}{\Gamma(n+1+\omega)\Gamma(-\omega)}, \quad and $$

$$ \omega := 1-1/a $$

Indeed, once (25) was discovered by a combination of insight and methodical computer experiment, its proof became highly representative of the changing paradigm; both sides satisfy the same recursion and the same initial conditions. This can be checked in Maple, and if one looks inside the computation, one learns which confluent hypergeometric identities are needed for an explicit human proof.

As noted, study of $R$ devolved to hard but compelling conjectures on complex dynamics, with many interesting proven and unproven generalizations. In [23] consideration is made of continued fractions like

$$ S_1(a) = \frac{1^2 a_1^2}{1 + \frac{2^2 a_2^2}{1 + \frac{3^2 a_3^2}{1 + \ldots}}}, $$

for any sequence $a = (a_n)_{n=1}^\infty$ and convergence properties obtained for deterministic and random sequences $(a_n)$. For the deterministic case the best results obtained are for periodic sequences, satisfying $a_j = a_{j+c}$ for all $j$ and some finite $c$. The dynamics are considerably more varied, as illustrated in Figure 4.
Coincidence and Fraud

Coincidences do occur, and such examples drive home the need for reasonable caution in this enterprise. For example, the approximations

\[ \pi \approx \frac{3}{\sqrt{163}} \log(640320), \quad \pi \approx \frac{2}{9801} \frac{4412}{4412} \]

occur for deep number theoretic reasons: the first good to fifteen places, the second to eight. By contrast

\[ e^\pi - \pi = 19.999099979189475768 \ldots, \]

most probably for no good reason. This seemed more bizarre on an eight-digit calculator. Likewise, as spotted by Pierre Lanchon recently,

\[ e = 10.1011011111000101000101100\ldots \]

while

\[ \pi = 11.00100100011111011010100100\ldots \]

have 19 bits agreeing in base two—with one reading right to left. More extended coincidences are almost always contrived, as illustrated by the following:

\[ \sum_{n=1}^{\infty} \frac{[n \tanh(n \pi/2)]}{10^n} \approx \frac{1}{81}, \quad \sum_{n=1}^{\infty} \frac{[n \tanh(n \pi)]}{10^n} \approx \frac{1}{81}. \]

The first holds to 12 decimal places, while the second holds to 268 places. This phenomenon can be understood by examining the continued fraction expansion of the constants \( \tanh(n \pi/2) \) and \( \tanh(n \pi) \): the integer 11 appears as the third entry of the first, while 267 appears as the third entry of the second.

Bill Gosper, commenting on the extraordinary effectiveness of continued-fraction expansions to “see” what is happening in such problems, declared, “It looks like you are cheating God somehow.”

A fine illustration is the unremarkable decimal \( \alpha = 1.4331274267223117583 \ldots \) whose continued fraction begins [1, 2, 3, 4, 5, 6, 7, 8, 9, ...] and so most probably is a ratio of Bessel functions. Indeed, \( I_0(2)/I_1(2) \) was what generated the decimal. Similarly, \( \pi \) and \( e \) are quite different as continued fractions, less so as decimals.

A more sobering example of high-precision “fraud” is the integral

\[ \tau_2 := \int_0^\infty \cos(2x) \prod_{n=1}^{\infty} \cos \left( \frac{x}{n} \right) \, dx. \]

The computation of a high-precision numerical value for this integral is rather challenging, due in part to the oscillatory behavior of \( \prod_{n=1}^{\infty} \cos(x/n) \) (see Figure 2), but mostly due to the difficulty of computing high-precision evaluations of the integrand function. Note that evaluating thousands of terms of the infinite product would produce only a few correct digits. Thus it is necessary to rewrite the integrand function in a form more suitable for computation. This can be done by writing

\[ f(x) = \cos(2x) \prod_{k=1}^{m} \cos \left( \frac{x}{k} \right) \exp(f_m(x)), \]

where we choose \( m > x \), and where

\[ f_m(x) = \sum_{k=m+1}^{\infty} \log \cos \left( \frac{x}{k} \right). \]

The log cos evaluation can be expanded in a Taylor series [1, p. 75], as follows:

\[ \log \cos \left( \frac{x}{k} \right) = \sum_{j=1}^{\infty} \frac{(-1)^{j-1}(2j-1)B_{2j}}{(2j)!} \frac{x^{2j}}{k^{2j}}, \]

where \( B_{2j} \) are Bernoulli numbers. Note that since \( k > m > x \) in (28), this series converges. We can now write

\[ f_m(x) = \sum_{k=m+1}^{\infty} \sum_{j=1}^{\infty} \frac{(-1)^{j-1}(2j-1)B_{2j}}{j(2j)!} \frac{x^{2j}}{k^{2j}}. \]

This can now be written in a compact form for computation as

\[ f_m(x) = -\sum_{j=1}^{\infty} a_j b_{j,m} x^{2j}, \]

where

\[ a_j = \frac{(2j-1)\zeta(2j)}{j\pi^{2j}}, \]

\[ b_{j,m} = \zeta(2j) - \sum_{k=1}^{m} \frac{1}{k^{2j}}. \]
Computation of these \(b\) coefficients must be done to a much higher precision than that desired for the quadrature result, since two very nearly equal quantities are subtracted here.

The integral can now be computed using, for example, the tanh-sinh quadrature scheme. The first 60 digits of the result are the following:

\[
0.3926990816987241548078304229099378605246454341872315926812\ldots
\]

At first glance, this appears to be \(\pi/8\). But a careful comparison with a high-precision value of \(\pi/8\), namely

\[
0.392699081698724154807830422909937860524646174921888227621868\ldots
\]

reveals that they are not equal: the two values differ by approximately \(7.407 \times 10^{-43}\).

Indeed, these two values are provably distinct. The reason is governed by the fact that

\[
\sum_{n=1}^{\infty} \frac{1}{(2n+1)} > \sum_{n=1}^{\infty} \frac{1}{(2n+1)}
\]

See [16, Chap. 2] for additional details.

A related example is the following. Recall the sinc function

\[
sinc(x) := \frac{\sin x}{x}.
\]

Consider the seven highly oscillatory integrals below:

\[
I_1 := \int_0^\infty \sin(x) \, dx = \frac{\pi}{2},
\]

\[
I_2 := \int_0^\infty \sin(x) \sin\left(\frac{x}{3}\right) \, dx = \frac{\pi}{2},
\]

\[
I_3 := \int_0^\infty \sin(x) \sin\left(\frac{x}{3}\right) \sin\left(\frac{x}{5}\right) \, dx = \frac{\pi}{2},
\]

\[
I_4 := \int_0^\infty \sin(x) \sin\left(\frac{x}{3}\right) \sin\left(\frac{x}{11}\right) \, dx = \frac{\pi}{2},
\]

\[
I_5 := \int_0^\infty \sin(x) \sin\left(\frac{x}{3}\right) \sin\left(\frac{x}{13}\right) \, dx = \frac{\pi}{2}.
\]

However,

\[
I_6 := \int_0^\infty \sin(x) \sin\left(\frac{x}{3}\right) \cdots \sin\left(\frac{x}{15}\right) \, dx = 467807924713440738696537864469 = 9356158494460907310521750000 \pi \approx 0.499999999992646\pi.
\]

When this was first found by a researcher using a well-known computer algebra package, both he and the software vendor concluded there was a "bug" in the software. Not so! It is easy to see that the limit of these integrals is \(2\pi/1\), where

\[
\pi_1 := \int_0^\infty \cos(x) \prod_{n=1}^{\infty} \cos\left(\frac{x}{n}\right) \, dx.
\]

This can be seen via Parseval's theorem, which links the integral

\[
I_N := \int_0^\infty \sin(a_1x) \sin(a_2x) \cdots \sin(a_Nx) \, dx
\]

with the volume of the polyhedron \(P_N\) given by

\[
P_N := \{x : \sum_{k=2}^{N} a_k x_k \leq a_1, |x_k| \leq 1, 2 \leq k \leq N\},
\]

where \(x = (x_2, x_3, \cdots, x_N)\). If we let

\[
C_N := \{(x_2, x_3, \cdots, x_N) : -1 \leq x_k \leq 1, 2 \leq k \leq N\},
\]

then

\[
I_N = \frac{\pi}{2a_1} \text{Vol}(P_N).
\]

Thus, the value drops precisely when the constraint \(\sum_{k=2}^{N} a_k x_k \leq a_1\) becomes active and bites the hypercube \(C_N\). That occurs when \(\sum_{k=2}^{N} a_k > a_1\).

In the above, \(\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{15} < 1\), but on addition of the term \(\frac{1}{17}\), the sum exceeds 1, the volume drops, and \(I_N = \pi/2\) no longer holds. A similar analysis applies to \(\pi_2\). Moreover, it is fortunate that we began with \(\pi_1\) or the falsehood of the identity analogous to that displayed above would have been much harder to see.

**Further Directions and Implications**

In spite of the examples of the previous section, it must be acknowledged that computations can in many cases provide very compelling evidence for mathematical assertions. As a single example, recently Yasumasa Kanada of Japan calculated \(\pi\) to over one trillion decimal digits (and also to over one trillion hexadecimal digits). Given that such computations—which take many hours on large, state-of-the-art supercomputers—are prone to many types of error, including hardware failures, system software problems, and especially programming bugs, how can one be confident in such results?

In Kanada's case, he first used two different arctangent-based formulas to evaluate \(\pi\) to over one trillion hexadecimal digits. Both calculations
agreed that the hex expansion beginning at position 1,000,000,000,001 is B446EFBD215388C4E014. He then applied a variant of the BBP formula for \( \pi \), mentioned in Section 3, to calculate these hex digits directly. The result agreed exactly. Needless to say, it is exceedingly unlikely that three different computations, each using a completely distinct computational approach, would all perfectly agree on these digits unless all three are correct.

Another, much more common, example is the usage of probabilistic primality testing schemes. Damgård, Landrock, and Pomerance showed in 1993 that if an integer \( n \) has \( k \) bits, then the probability that it is prime, provided it passes the most commonly used probabilistic test, is greater than \( 1 - \frac{k}{4}^{2^{-k}} \), and for certain \( k \) is even higher [23]. For instance, if \( n \) has 500 bits, then this probability is greater than \( 1 - \frac{1}{4}^{2^{500}} \). Thus a 500-bit integer that passes this test even once is prime with prohibitively safe odds: the chance of a false declaration of primality is less than one part in Avogadro's number \((6 \times 10^{23})\). If it passes the test for four pseudorandomly chosen integers \( a \), then the chance of false declaration of primality is less than one part in a googol \((10^{100})\). Each of these four tests can occasionally be wrong, but not for any substantial chunk of the input space. Such tests thus draw into question the distinction between a probabilistic test and a "provably" test.

Another interesting question is whether these experimental methods may be capable of discovering facts that are fundamentally beyond the reach of formal proof methods, which, due to Gödel's result, we know must exist; see also [24].

One interesting example, which has arisen in our work, is the following. We mentioned in Section 3 the fact that the question of the 2-normality of \( \pi \) reduces to the question of whether the chaotic iteration \( x_0 = 0 \) and

\[
x_n = \left\{ 16x_{n-1} + \frac{120n^2 - 89n + 16}{512n^4 - 1024n^3 + 712n^2 - 206n + 21} \right\}
\]

where \( \{ \cdot \} \) denotes fractional part, are equidistributed in the unit interval.

It turns out that if one defines the sequence \( y_n = \lfloor 16x_n \rfloor \) (in other words, one records which of the 16 subintervals of \((0, 1)\), numbered 0 through 15, \( x_n \) lies in), that the sequence \( (y_n) \) when interpreted as a hexadecimal string, appears to precisely generate the hexadecimal digit expansion of \( \pi \). We have checked this to 1,000,000 hex digits and have found no discrepancies. It is known that \( (y_n) \) is a very good approximation to the hex digits of \( \pi \), in the sense that the expected value of the number of errors is finite [15, Section 4.3] [11]. Thus one can argue, by the second Borel-Cantelli lemma, that in a heuristic sense the probability that there is any error among the remaining digits after the first million is less than \( 1.465 \times 10^{-8} \) [15, Section 4.3]. Additional computations could be used to lower this probability even more.

Although few would bet against such odds, these computations do not constitute a rigorous proof that the sequence \( (y_n) \) is identical to the hexadecimal expansion of \( \pi \). Perhaps someday someone will be able to prove this observation rigorously. On the other hand, maybe not—maybe this observation is in some sense an “accident” of mathematics, for which no proof will ever be found. Perhaps numerical validation is all we can ever achieve here.

**Conclusion**

We are only now beginning to digest some very old ideas:

Leibniz's idea is very simple and very profound. It's in section VI of the *Discours de métaphysique*. It's the observation that the concept of law becomes vacuous if arbitrarily high mathematical complexity is permitted, for then there is always a law. Conversely, if the law has to be extremely complicated, then the data is irregular, lawless, random, unstructured, patternless, and also incompressible and irreducible. A theory has to be simpler than the data that it explains, otherwise it doesn't explain anything. —Gregory Chaitin [24]

Chaitin argues convincingly that there are many mathematical truths which are logically and computationally irreducible—they have no good reason in the traditional rationalist sense. This in turn adds force to the desire for evidence even when proof may not be possible. Computer experiments...
can provide precisely the sort of evidence that is required.

Although computer technology had its roots in mathematics, the field is a relative latecomer to the application of computer technology, compared, say, with physics and chemistry. But now this is changing, as an army of young mathematicians, many of whom have been trained in the usage of sophisticated computer math tools from their high school years, begin their research careers. Further advances in software, including compelling new mathematical visualization environments (see Figures 6 and 7), will have their impact. And the remarkable trend towards greater miniaturization (and corresponding higher power and lower cost) in computer technology, as tracked by Moore’s Law, is pretty well assured to continue for at least another ten years, according to Gordon Moore himself and other industry analysts. As Richard Feynman noted back in 1959, “There’s plenty of room at the bottom” [27]. It will be interesting to see what the future will bring.

References
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WHAT IS...

a Horseshoe?

Michael Shub

The Smale horseshoe is the hallmark of chaos. With striking geometric and analytic clarity it robustly describes the homoclinic dynamics encountered by Poincaré and studied by Birkhoff, Cartwright-Littlewood, and Levinson. We give the example first and the definitions later.

Consider the embedding $f$ of the disc $\Delta$ into itself exhibited in the figure. It contracts the semidiscs $A$, $E$ to the semidiscs $f(A)$, $f(E)$ in $A$ and it sends the rectangles $B$, $D$ linearly to the rectangles $f(B)$, $f(D)$, stretching them vertically and shrinking them horizontally. In the case of $D$, it also rotates by 180 degrees. We don't really care what the image $f(C)$ of $C$ is as long as it does not intersect the rectangle $B \cup C \cup D$. In the figure it is placed so that the total image resembles a horseshoe, hence the name.

It is easy to see that $f$ extends to a diffeomorphism of the 2-sphere to itself. We also refer to the extension as $f$ and work out its dynamics in $\Delta$, i.e., its iterates $f^n$ for $n \in \mathbb{Z}$.

Necessarily there are three fixed points $p$, $q$, $s$. The point $q$ is a sink in the sense that all points $z \in A \cup E \cup C$ converge to $q$ under forward iteration, $f^n(z) \to q$ as $n \to \infty$.

The points $p$, $s$ are saddle points. If $x$ lies on the horizontal through $p$, then $f^n$ squeezes it to $p$ as $n \to \infty$; while if $y$ lies on the vertical through $p$, then the inverse iterates of $f$ squeeze it to $p$. With respect to linear coordinates centered at $p$, $f(x, y) = (kx, my)$ where $(x, y) \in B$ and $0 < k < 1 < m$. Similarly, $f(x, y) = (-kx, -my)$ with respect to linear coordinates on $D$ at $s$.

The sets

$$W^s = \{ z : f^n(z) \to p \text{ as } n \to \infty \},$$
$$W^u = \{ z : f^n(z) \to p \text{ as } n \to -\infty \}$$

are the stable and unstable manifolds of $p$. They intersect at $r$, which is what Poincaré called a homoclinic point. The figure shows these invariant manifolds only locally. Iteration extends them globally.

The key part of the dynamics of $f$ happens on the horseshoe

$$\Lambda = \{ z : f^n(z) \in B \cup D \text{ for all } n \in \mathbb{Z} \}.$$  

Everything there is explained as the “full shift on the space of two symbols”. Take two symbols, 0 and 1, and look at the set $\Sigma$ of all bi-infinite sequences $a = (a_n)$ where $n \in \mathbb{Z}$ and for each $n$, $a_n$ is 0 or $a_n$ is 1. Thus $\Sigma = \{0, 1\}^\mathbb{Z}$ is homeomorphic to the Cantor set. The map $\sigma : \Sigma \to \Sigma$ that sends $a = (a_n)$ to $\sigma(a) = (a_{n+1})$ is a homeomorphism called the shift map. It shifts the decimal point one slot rightward. Every dynamical property of the shift map is possessed equally by $f|_{\Lambda}$, because there is a homeomorphism $h : \Sigma \to \Lambda$ such that the diagram
between the stable and unstable manifolds of a true of

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vertical and contract vectors parallel to the

The mere existence of a transverse intersection

For instance, \((\cdots 11.111 \cdots)\) codes the point \(p\),

The utility of Smale's analysis is this: every
dynamical system having a transverse homoclinic
point, such as \(r\), also has a horseshoe containing
\(r\) and thus has the shift chaos. Nowadays, this
fact is not hard to see, even in higher dimensions.
The mere existence of a transverse intersection
between the stable and unstable manifolds of a
periodic orbit implies a horseshoe. In the case of
flows, the corresponding assertion holds for the
Poincaré map. To recapitulate,

\[
\begin{array}{c}
\Sigma \\
h \downarrow \\
\Lambda \\
\end{array}
\begin{array}{c}
\sigma \\
h^{-1} \\
f \\
\end{array}
\begin{array}{c}
\Sigma \\
\Lambda \\
\end{array}
\]

commutes. The conjugacy \(h\) is easy to describe.
Given any \(a \in \Sigma\), there is a unique \(z \in \Lambda\) such that
\(f^n(z) \in B\) whenever \(a_n = 1\), while \(f^n(z) \in D\) whenever
\(a_n = 0\). Thus \(\sigma\) codes the horseshoe dynamics.
For instance, \((\cdots 11.111 \cdots)\) codes the point \(p\),
\((\cdots 0.000 \cdots)\) codes \(s\), while \((\cdots 111.0111 \cdots)\)
codes \(r\).

\(\sigma\) has \(2^n\) periodic orbits of period \(n\), and so
must \(f|\Lambda\). The set of periodic orbits of \(\sigma\) is dense
in \(\Sigma\) and so must be the set of periodic orbits of
\(f|\Lambda\). Small changes of initial conditions in \(\Sigma\) can
produce large changes of a \(\sigma\)-orbit, so the same must
be true of \(f|\Lambda\). In short, due to conjugacy, the chaos
of \(\sigma\) is reproduced exactly in the horseshoe.

The utility of Smale's analysis is this: every
dynamical system having a transverse homoclinic
point, such as \(r\), also has a horseshoe containing
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The mere existence of a transverse intersection
between the stable and unstable manifolds of a
periodic orbit implies a horseshoe. In the case of
flows, the corresponding assertion holds for the
Poincaré map. To recapitulate,

\[\text{transverse homoclinicity} \Rightarrow \text{horseshoe} \Rightarrow \text{chaos}.\]

Since transversality persists under perturbation, it
follows that so does the horseshoe and so does its
chaos.

The analytical feature of the horseshoe is
hyperbolicity, the squeeze/stretch phenomenon
expressed via the derivative. The derivative of \(f\)
stretches tangent vectors that are parallel to the
vertical and contracts vectors parallel to the
horizontal, not only at the saddle points, but
uniformly throughout \(\Lambda\). In general, hyperbolicity
of a compact invariant set such as \(\Lambda\) in any
dimension is expressed in terms of expansion and
contraction of the derivative on subbundles of the
tangent bundle. Smale unified such examples as
the horseshoe and the geodesic flow on manifolds
of negative curvature, defining what is now called
uniformly hyperbolic dynamical systems. The study
of these systems has led to many fruitful discoveries
in modern dynamical systems theory.

David Ruelle has called Smale's 1967 article [3]
"a masterpiece of mathematical literature". It is
still worth reading today. Hyperbolic dynamics
flourished in the 1960s and 1970s. Anosov
proved the stability and ergodicity of the
hyperbolic systems that now bear his name. Sinai
initiated the more general investigation of the
ergodic theory of hyperbolic dynamical systems,
and in particular showed that the Markov partitions
of Adler and Weiss could be constructed for all
hyperbolic invariant sets, thus giving a coding
similar to the two-shift coding for the horseshoe.
This work was carried forward by Ruelle and Bowen.
The invariant measures they found, now called
Sinai-Ruelle-Bowen (SRB) measures, describe the
asymptotic dynamics of most Lebesgue points in
the manifold, even for dissipative systems. Uniformly
hyperbolic dynamical systems are remarkable. They
exhibit chaotic behaviour. By the work of Anosov,
Smale, Palis, and Robbin, they are structurally
stable; that is, the dynamics of a perturbation of a
uniformly hyperbolic system is topologically
conjugate to the original. By the work of Sinai,
Ruelle, and Bowen, they are described statistically.

In the early days of the 1960s it was hoped that
uniformly hyperbolic dynamical systems might be
in some sense typical. While they form a large open
set on all manifolds, they are not dense. The search
for typical dynamical systems continues to be a
great problem. For progress see the survey [1].
Hyperbolic periodic points, their global stable and
unstable manifolds, and homoclinic points remain
some of the principal features of, and tools for,
understanding the dynamics of chaotic systems.

Indeed, transversal homoclinic points are proven
to exist in many of the dynamical systems
encountered in science and engineering from celestial
mechanics, where Poincaré first observed them,
to ecology and beyond.

References

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Smale's horseshoe map takes a disk into itself. Points in A and E are mapped into A, points in C into E.

Upon iteration, any point that finds itself in A, C, or E eventually winds up in A, converging to an attracting fixed point in that region. The interesting dynamics happens to points that never arrive in A, C, E. This is the intersection of all the sets

\[ \Omega_{0,n} = \{ x \mid f^k(x) \in B \cup D \text{ for } 0 \leq k \leq n \}, \]

defined recursively by

\[ \Omega_{0,0} = B \cup D \]
\[ \Omega_{0,n+1} = \Omega_{0,n} \cap f^{-(n+1)}(B \cup D). \]

These converge to a collection \( \Omega_{0,\infty} \) of horizontal bands distributed vertically like Cantor dust.

Points in \( \Omega_{0,\infty} \), as iterations of \( f \) are applied to them, approach a set on which \( f \) is invertible. This...
is the set $\Omega_{-\infty, \infty}$ of points $x$ for which all $f^n(x)$ lie in $B \cup D$. The condition that $f^{-n}(x)$ lie in $B \cup D$ is equivalent to the condition that $x$ lie in $f^n(B \cup D)$. This set is approximated by sets $\Omega_{-m,n}$ with $m$ and $n$ large, and amounts to a two-dimensional Cantor dust.

Because the map $f$ is an affine transformation on $B$ and $D$, these sets are easy to calculate, and it is easy to see that the connected components converge to isolated points. Each one of the connected sets $\Omega_{-m,n}$ can be labeled by a finite string $s_{-m} \ldots s_{-1} s_0 s_1 \ldots s_n$ of characters $B$ and $D$ where $s_k = B$ if $f^k(\Omega) \subseteq B$, otherwise $D$. Thus $B,D$ is the set of points $x$ with $x$ in $D, f^{-1}(x)$ in $B$. The points to which the sets converge are indexed by infinite strings $(s_k)$ for $k$ in $\mathbb{Z}$. The limit set is stable under $f$, which acts as a left shift. The image of $BB,DB$ under $f$ is $BBD,B$.

The coordinate succession is semi-inverted:

There are exactly two points fixed by $f$, namely $B = \ldots BB.BB\ldots$ and $D = \ldots DD.DD\ldots$, which are both hyperbolic—attracting horizontally and repelling vertically. The points with an infinite string $BBBB\ldots$ at the right end are those attracted to the first, those with an infinite terminal string $DDDD\ldots$ to the second. Points $BBBB\ldots$ with an infinite string of $B$ at both right and left other than $B$ itself are homoclinic points, attracted to $B$ upon iteration by both $f$ and $f^{-1}$.

Hyperbolic fixed points and a homoclinic point

The simplest homoclinic point is $BBB.BB\ldots$, which lies on both the horizontal line through the hyperbolic point $P = \ldots BBB.BB\ldots$ and the image under iterations of $f$ applied to a small vertical segment through $P$.

More homoclinic points

But the homoclinic points are in fact dense in $\Omega_{-\infty, \infty}$, as the picture above barely suggests. In fact, as it is I hope it also suggests, the horseshoe map and the network of homoclinic points in very general dynamical systems are intimately related. This was Smale’s original insight on the beach at Rio, mentioned in Shub’s article in this issue.

—Bill Casselman
The Works of Archimedes: Translation and Commentary
Volume 1: The Two Books On the Sphere and the Cylinder

Reviewed by Alexander Jones

Ancient Greek mathematics is associated in most people's minds with two names: Euclid and Archimedes. The lasting fame of these men does not rest on the same basis. We remember Euclid as the author of a famous book, the *Elements*, which for more than two millennia served as the fundamental introduction to ruler-and-compass geometry and number theory. About Euclid the man we know practically nothing, except that he lived before about 200 B.C. and may have worked in Alexandria. He wrote works on more advanced mathematics than the *Elements*, but none of these have survived, though we have several fairly basic books on mathematical sciences (optics, astronomy, harmonic theory) under his name. All his writings dive straight into the mathematics with no introductions. There are hardly even any unreliable anecdotes about Euclid.

Archimedes, by contrast, is not just an author to us but a personality. He was famous in his time, not only among mathematicians and intellectuals.

He was the subject of a biography—now lost alas!—and stories about him are told by ancient historians and other writers who generally took little interest in scientific matters. The stories of Archimedes' inventions; his solution of the "crown problem"; the machines by which, as an old man, he defended his native city, Syracuse, from the besieging Roman fleet in 212 B.C.; and his death—still doing geometry—at the hands of a Roman soldier when Syracuse at last fell have never lost their appeal. Archimedes became paradoxically emblematic of two stereotypes of the mathematician: a man who could harness reasoning to the seemingly superhuman performance of practical tasks, yet whose preoccupation with abstract problems could make him fatally oblivious to his surroundings.

Alongside the public Archimedes of the stories, we also have the private Archimedes of the writings. In the manner of his time, Archimedes wrote his mathematics in the form of substantial books built up of theorems that cumulatively lead to the
proof of a series of major results.Copies of these books were sent, in the form of papyrus rolls, to other mathematicians to be read, copied, and appreciated. The third century B.C., when Archimedes lived, was the heyday of the Greek mathematical sciences. Mathematicians were scattered about the Greek-speaking world, but there was a particular concentration of them as of other intellectuals, in Alexandria. Archimedes sent his books from Syracuse to three Alexandrian mathematicians: Eratosthenes, remembered for his measurement of the Earth and his scientifically based world map; Conon, whose identification of a new constellation in honor of Berenice, the queen of Egypt, was immortalized by the poets Callimachus and Catullus; and one Dositheus. There was a perverse, teasing streak in Archimedes' relations with his Alexandrian correspondents. At a time when, reflecting the cosmopolitan nature of Greek culture after the conquests of Alexander the Great, writers had embraced a standard form of Greek in place of the many local dialects, Archimedes persisted in writing his mathematics in the provincial Doric dialect of his native city. In the letters that he prefaced to his books, he speaks in less-than-flattering terms of the mathematicians to whom he is sending them, and on one occasion he sent them a list of theorems without proofs, including two false ones that were laid as a trap to catch anyone claiming priority of discovery. In spite of his best efforts to get their backs up, Archimedes' contemporaries evidently thought well of his work, and later people not only preserved them but wrote commentaries on some of them, in some instances also stripping the texts of their dialect for easier reading. We can still read the commentaries of a very late Alexandrian philosopher-mathematician, Eutocius, who lived at the time of the emperor Justinian, in the sixth century of our era.

It was about Eutocius' time that Constantinople began supplanting Alexandria as the focus of Greek learning—the Islamic conquest of Egypt was just a century away—and the selection of ancient Greek literature that was to survive through the Middle Ages was to a large extent determined by which books were brought to Constantinople. There was a very high risk of loss, especially in the seventh and eighth centuries, when classical learning, especially in the sciences, was at a low ebb, but somehow a dozen or so papyrus rolls of works of Archimedes were still around and in copyable condition by the time intellectual conditions were improving, about A.D. 800. (A few works of Archimedes, and apparently some others falsely attributed to him, were translated into Arabic about this time.) During the next two hundred years or so, a large number of manuscripts of ancient philosophical, scientific, and mathematical works were produced. These were codices, manuscripts of parchment sheets bound like modern books, and they could hold the equivalent of many of the old rolls. The parchment codices were exceedingly costly, both because of the materials (good parchment was always expensive) and because of the calligraphy, to say nothing of special skills such as copying geometrical diagrams. Nevertheless, at least three codex collections of Archimedes' works were made, each containing a different selection. Yet it seems as if no one in the Byzantine Empire read them. Archimedes' name continues to crop up in Byzantine literature, but he is the Archimedes of the old anecdotes, not the mathematical writer. No further copies of the Archimedes codices were made. In fact, by about A.D. 1300 all three codices were in situations where Byzantine scholars could no longer read them. Two of them had somehow made their way to western Europe, where Greek learning was still rather scarce. Perhaps they were part of a royal gift, like the manuscript of Ptolemy's *Almagest* that the Byzantine emperor Manuel Comnenus gave to the Norman king William I of Sicily about 1160. Whatever the story, by 1300 these manuscripts had become part of the small collection of Greek manuscripts in the papal library (one of them was in pretty bad condition). After the papacy was moved to Avignon in 1309, the papal manuscripts seem to have been dispersed, and only one of the Archimedes manuscripts eventually resurfaced in the fifteenth century. Greek humanism was now in full flower in Italy, and good copies were made of this survivor before it again vanished, this time for good. Apparently the other manuscript had never been copied, though a painstaking Latin translation of the contents of both manuscripts had been made by the Dominican scholar William of Moerbeke in 1269, and this Latin version has survived. The third codex met a different fate: around 1300 it was dismantled, its parchment leaves were cut in two, to some extent cleaned, and rewritten with a Greek prayer book. In this new guise, as a palimpsest (recycled manuscript) it passed through one or more monastic libraries. By the end of the nineteenth century it was in the library of a monastery in Constantinople, and a scholar writing a catalogue of Greek manuscripts in that city noticed and copied a bit of the partially erased text. The Danish classicist Johan Ludvig Heiberg, who had recently published an edition of Archimedes' works based on the manuscripts then known, recognized that the lines copied from the palimpsest were part of Archimedes' *On the Sphere and the Cylinder*, and he made haste to get access to the manuscript. Heiberg succeeded in transcribing a substantial part of the faded Archimedean texts written crossways underneath the prayers; they turned out to include two works that were otherwise entirely unknown and a third
Uncovering New Views on Archimedes

In 1996 when as a postdoc Reviel Netz launched his project of translating the works of Archimedes, his historian colleagues were not encouraging. "They said that obviously it would be a misguided project," he recalled. "In scholarly terms it would be incomplete because I would not have access to the palimpsest." That Netz forged ahead anyway proved to be fortuitous. When the long-lost Archimedes palimpsest resurfaced in 1998, his work on translating the first volume, On the Sphere and Cylinder, was ideal preparation for working on the palimpsest. "I was incredibly lucky," he says.

The program of preserving, imaging, and transcribing the Archimedes palimpsest is under way at the Walters Art Museum in Baltimore. Netz, now at Stanford University, and classics scholar Nigel Wilson of Oxford University are leading an international team analyzing the content of the palimpsest. The museum's work on Sphere and Cylinder has been completed. The imaging of The Method is also finished, and the Stanford Linear Accelerator is now helping to uncover additional information. Transcription of The Method is about one-third finished.

The palimpsest is the only extant source for The Method, and Netz lost no time in setting about studying it. He has formed some intriguing new insights. For example, in The Method Archimedes constructs in the course of a proof a one-to-one correspondence between two infinite (in fact, uncountable) sets. "This was completely unknown," Netz states. "The assumption was that Greek mathematicians never made this kind of claim" about two infinite sets being the same size.

Another work unique to the palimpsest is The Stomachion. This mere half-page fragment has puzzled scholars. "There wasn't an interpretation of it," Netz says. "No one really ventured to say what it was." He has now conjectured that the purpose of The Stomachion was to treat a combinatorics problem, a surprising idea, for it was not thought that combinatorics existed in ancient mathematics.

In fact Netz is changing many of the standard views on Archimedes and his work. "The original picture historians had of Archimedes is as a practical engineer," Netz remarks. "I don't find any evidence for this... He was strictly a mathematician." Even in Archimedes' works on physics Netz sees mathematics as the ultimate goal. For example, Netz believes that Archimedes invented statics as a way of deriving results in geometry. The strategy was to use imaginary situations involving bodies in equilibriums to derive proportions that lead to measurements of geometric figures.

Netz sees in Archimedes' work a personality that is "very playful, cunningly and even maliciously playful." Hellenistic mathematicians produced works that juxtaposed different things in surprising ways and that set challenges and puzzles for readers. Netz says this style of presentation has parallels in the larger cultural tendencies of Alexandrian and Hellenistic society. His next planned book, Ludic Proof: Greek Mathematics and the Alexandrian Aesthetic, will examine the playful strands running through Hellenistic mathematics.

"Archimedes, among the truly great, is relatively neglected," Netz comments. "There is a Newton industry and an Einstein industry, but there isn't an Archimedes industry, and there ought to be one." He believes one barrier to the study of Archimedes has been the lack of a complete and faithful translation of his works into English. Netz's translation is so faithful, he says, that one could use it for serious historical studies. It is not an easy read, but then it is not an easy thing to plunge into the mind of a writer from a completely different culture and time. The translation is "very Greek—it's a Greek book," Netz says. He has not transcribed the mathematics into modern notation, preferring instead to let the reader puzzle through the mathematics just as Archimedes' contemporaries did. Says Netz, "If you are really interested in Archimedes, invest the extra effort to see what he did."

—Allyn Jackson
that had hitherto only been available in Moerbeke's Latin version. After a rather obscure history in the twentieth century, the Archimedes palimpsest is now in private hands and the subject of an ambitious project of conservation and research intended to recover text that Heiberg was unable to read. In the meantime, Heiberg's second edition, which is very good, remains after a century the basis for studying what Archimedes wrote.

Astonishingly, given Archimedes' fame and the importance of the works, Archimedes' books have never really been translated into English. Thomas Heath, who made an excellent translation of Euclid's *Elements*, published *The Works of Archimedes* in 1897 (before Heiberg's rediscovery of the palimpsest), but except for the prefatory letters this was not a translation so much as a mathematical paraphrase that not only uses modern notation but liberally reorganizes parts of the proofs. E. J. Dijksterhuis's *Archimedes*, first published in 1956, contains mathematical paraphrases of the proofs that are closer to the reasoning of the original than Heath's, and his book, which has been reprinted by Princeton University Press, is still the best general introduction to Archimedes' thought. But neither Heath nor Dijksterhuis gives a complete restatement of all the proofs, and the "feel" of Archimedes' writing is entirely lost.

Now Reviel Netz offers us the first installment of a translation of all the works of Archimedes that survive in Greek. The first volume contains a single large work, *On the Sphere and the Cylinder*, together with the commentary on that work by Eutocius. *On the Sphere and the Cylinder* is the book in which Archimedes proved several relations between the volumes and surface areas of spheres, segments of spheres, and cylinders, including of course the proof that the volume of a sphere is two-thirds the volume of the smallest containing cylinder; a diagram illustrating this relation is reputed to have been inscribed on Archimedes' tomb. The central proofs employ what are traditionally called "exhaustion methods", which are rigorous limit arguments founded on inscribed or circumscribed solids (such as the solids of rotation of polygons). Because exhaustion arguments are not easy to transfer from one geometrical figure to another, the proofs have a certain virtuosic character, very different from the more readily generalizable approaches of European mathematicians to such problems leading up to the calculus. In another book, *The Method* (to appear in a subsequent volume), Archimedes presented a more heuristic and less rigorous strategy for demonstrating volume and area relations by comparison of infinitesimal slices of figures. This work, however, was unknown to modern mathematicians until Heiberg discovered it in the palimpsest.

As well as the intrinsic interest of Archimedes' mathematics in *On the Sphere and the Cylinder*, there are two "loose ends" that provoked Eutocius to collect for us some remarkable specimens of Greek geometry that we would not otherwise know about. These are the solutions by several geometers of the problem of finding two mean proportions between given magnitudes (i.e., given $A$ and $D$, to find $B$ and $C$ such that $A:B=B:C=C:D$) and another problem mathematically equivalent to solving a cubic equation. Both problems fall into the class that Greek mathematicians sensed from experience, though they could not prove, to be insoluble using only the postulates of Euclid's *Elements*, and the solutions that Eutocius preserves illustrate how they extended their "toolbox" by allowing certain mechanical constructions, intersections of conic sections, or special curves.

In due course Netz will follow this volume with two more. Of the books awaiting translation, some are, like *On the Sphere and the Cylinder*, works of pure mathematics, while others give a mathematical treatment of problems in statics such as centers of gravity of figures and conditions of stability of floating solids. Netz does not intend to include in his scope the medieval Arabic tradition of Archimedes' works. In fact, it is likely that none of the works that pass under Archimedes' name in Arabic, aside from those that we also have in Greek, are authentic, although several contain interesting mathematics. An edition and translation collecting these would be a worthwhile project.

Netz's English Archimedes could hardly be more different from Heath's. To begin with, it is ruthlessly literal. Archimedes, like all Greek geometers, wrote his mathematics in continuous prose, using words to represent concepts and relations, and letters of the Greek alphabet to name them. The vocabulary and sentence structures of Greek geometrical writing were highly standardized and formulaic by the third century B.C., a quality that gives the arguments something of the same clarity and freedom from ambiguity that notation provides in modern mathematics. But a mathematical argument written out as prose (sometimes referred to as "rhetorical" mathematics) has two characteristics in contrast to notation: the relation to the spoken word is much more immediate, so that one can read the argument aloud correctly even if one does not understand the mathematics at all, but it takes much more space on the page to write—and more time for the eye to take in—each step of a proof. Mathematicians are often more comfortable with translations of early mathematics that employ notations to compress the argument, whereas nonmathematicians interested in the history of science may find the rhetorical style more approachable, though in the case of Archimedes they soon find that the
easy flow of words expresses some very difficult mathematics.

Netz does not hold that it is the translator's business to cater to the comfort of the reader; he writes (p. 3) that "the purpose of a scholarly translation as I understand it is to remove all barriers having to do with the foreign language itself, leaving all other barriers intact." And whatever one may say of Netz's Archimedes, it cannot in any way be charged with looseness. He maintains a close and consistent correspondence between the Greek terminology and his English equivalents. The formulaic character of Greek geometrical prose makes it often possible to omit certain common words without ambiguity, and the geometers took full advantage of this means of shortening their sentences: for example, the conventional Greek way of saying "the angle contained by AB and BC" would translate word-for-word as "the contained by ABC." Netz carefully marks all the implied words by enclosing them in angle brackets, as in this case "the <angle> contained by ABC." He also adheres more than any other translator I know to the Greek word order, even when this goes against natural habits of English. This is particularly apparent when ratios are being expressed and manipulated: for example, a sentence that might be represented by the notation

\[ Z : H = AC : CB = AD^2 : DB^2 \]

is rendered: "it is: as CD to CB, that is Z to H, the <square> on AD to the <square> on DB." Reading long stretches of this is hard going, though one soon gets used to the oddness, and the reader will probably find that writing out each step symbolically as one works through the proofs makes them easier to follow and verify. It is a pity that Netz has not provided such a step-by-step synopsis to accompany the translation.

For Archimedes' text Netz has relied on Heiberg's edition. (In subsequent volumes he intends to take account of new readings arising from study of the palimpsest.) The figures are a different matter. Heiberg, like most scholars editing works of Greek mathematics, preferred to redraw the figures that accompany each theorem according to the sense of the text rather than reproduce with necessary corrections the drawings that appear in the manuscripts. In many cases the resulting reconstructed figures look quite different from the transmitted ones, though they are usually mathematically equivalent. Netz has made a close study of the role of figures in Greek mathematics, and in his earlier book, The Shaping of Deduction in Greek Mathematics (1999), he showed how the figures were intended as an integral and indispensable part of the proofs, not mere adjuncts to aid visualization. So here Netz has gone back to the manuscripts and produced the first "edition" of the figures, reporting all significant variations in the manuscript versions. Like his close translation, this brings the reader a step closer to seeing Archimedes' mathematics as an ancient reader would have seen it.

Netz's intention, which I have quoted, of "leaving all other barriers intact" sounds forbidding, but it applies only to the bare translation: taking the book as a whole, he provides the reader with a great deal of help in getting over those barriers. This help takes several forms. There are frequent footnotes clarifying and justifying unobvious steps in Archimedes' arguments and cross-referencing them with the relevant parts of Eutocius' commentary, which appears later in the volume. After each theorem Netz gives two sections of commentary: one discussing textual matters (this will be of
particular interest to people comparing the translation to Heiberg’s edition of the Greek), the other offering more general remarks. The general comments are not, for the most part, mathematical in scope, but discuss linguistic and stylistic aspects of the text. For mathematical commentary, Netz refers us to Eutocius and Dijksterhuis, who are certainly worthy and sensible guides, though Dijksterhuis is difficult to use in close comparison with Netz’s translation, because the lettering of the diagrams is different, and Eutocius is writing for a reader more conversant in the idioms of ancient mathematics than most modern readers are likely to be. A possible consequence of Netz’s decision not to provide his own mathematical commentary is that there may be places where he has not detected that the text as transmitted by the manuscripts and edited by Heiberg is incorrect. I have noticed one place where this seems to be the case: in the fourth theorem of Book 2 (p. 203, step 24) a statement about ratios in Heiberg’s text is both false and different from what Eutocius seems to have read in his copy of the same passage.

In general the translation has been made consistently and with care, and I have noticed very few misprints, except in the bibliography, where the typesetters seem to have been uncharacteristically creative (and, oddly, Lewis Carroll appears disguised as Carol wherever he turns up). The typography, layout, and draftsmanship of the diagrams are of a standard that one can unfortunately no longer take for granted in scholarly books. A regrettable corollary is the high price; individuals planning to own all three volumes will need deep pockets.

The photograph at left is taken in ultraviolet light, showing some of the underlying text of Archimedes’ Sphere and Cylinder.

The photograph below is a processed image, making the Archimedes undertext and drawing appear in red.
Mathematics in the Near East: Some Personal Observations

David Mumford

In October 2003 I visited Turkey, and in June 2004 I made a circuit through Lebanon, Israel, and the Palestinian West Bank. There is a lot of mathematics going on in the Near East, some in places fairly well known to the West and others in places that are less well known. I would like to share with my colleagues here in the U.S. something about the conditions in the places I visited as well as some of the things the mathematicians in these universities would like from their colleagues in the West. I had been in Israel twice before, in September 1967 and in June 1995, but had never before visited any of the largely Muslim countries there. As the newspapers have been filled with articles on the wars there (not merely now, but in fact for most of my life), I felt a strong urge to get a first-hand impression of what the Near East is like and how people are living. As a mathematician, I wanted to meet and talk with mathematicians and see what conditions were like for them. In particular, during these visits I promised to help some of those I met by publicizing a few of their needs and initiatives. We in the West can help and thereby further the development of mathematics in this part of the world.

As these were my first visits to largely Muslim countries, I was curious to see if there was a different "feel" to Islam. In fact, what I found was that the campuses were extremely lively and full of energy and not all that different from large state universities in the U.S. For instance, on all the campuses I visited half or so of the women did not cover their heads at all, and I saw no one in a burka. In the country the women were more conservative, and almost everywhere—in cities and small towns—you hear the call to prayer at dawn, something I loved, as I like to wake up early. But only on Friday at noon did there seem to be large crowds going to the mosques. I loved visiting the mosques: their austere beauty is very peaceful and moving (though sadly I have been denied entry to the al-Aqsa mosque on each visit to Jerusalem by one or the other party).

Turkey, the Middle East Technical University (METU), and Boğaziçi University

Though I had heard of Mustafa Kemal Ataturk, it was not until my trip to Turkey that I appreciated

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1 Following what I believe is the most widespread usage, the term "Near East" is used here to refer to the land in the convex hull of Turkey, Iran, and Egypt. The "Middle East" is somewhat more restricted.
the amazing transformation he brought about in his country, transforming the disintegrating Ottoman Empire into a modern country now on the verge of joining the EU. I had not realized that he was a champion of women’s rights and universal education and embraced an integration of Western ideas with Islam. The universities reflect his philosophy.

METU is in Ankara and was founded in 1956 to train Turks and, more widely, people from all over the Middle East in the sciences and engineering. The language of instruction is English, and today it has over 20,000 students, of whom about 1,000 come from outside Turkey. It is a huge and bustling campus, with much green still-undeveloped space. A strong mathematics program was started at the outset through the guidance of the distinguished Turkish mathematician Cahit Arf and through bringing Masatoshi Ikeda to METU. They created an environment in which research was held to an international standard. They have a large and active Ph.D. program—sixty-six students are currently working for Ph.D.’s—and have forty faculty members covering all areas of mathematics. Many other students from there go to the West to complete their Ph.D.’s; we have had a stream of strong METU students coming to Brown. Their quandary is that they want to maintain their successful local Ph.D. program, yet if their students go to the West, they are exposed to more ideas, because the mathematical community is so much larger.

I had a very warm welcome from Safak Alpay, a functional analyst and the chair; Ayse Berkman, a group theorist and the vice chair; and Ersan Akyildiz, an algebraic geometer. My lectures were about vision and applied topics, and I met some excellent people from the Computer Engineering Department as well, in particular Sibel Tari, who has started up a strong modern program in vision (hard because the area often falls between departmental lines). Ayse is married to a colleague and model theorist, David Pierce, who proved to me that it is possible to learn Turkish, though it seemed impossible at first to even remember names. As in almost all places outside the U.S.-European loop, one of their greatest needs is to have more visitors who can help to keep them in touch with the developments in the West. Their website is http://www.math.metu.tr. I was astonished to learn that Bob Langlands is fluent in Turkish and has visited several times.

Going there for a sabbatical or as a visitor for a year is very possible, and I believe many mathematicians would find it fascinating (though salaries
The "Wall" with (left to right) Iyad Suwan, Suwan's father, and the author. in Turkey are a problem, as I discuss below. Ankara has one of the best archeological museums in the world, a reflection of the fact that so many civilizations have flourished there in the last ten millennia, and it is close to Cappadocia, amazing for its physical beauty and for its underground churches and whole subterranean villages. All of Turkey, in fact, is covered with interesting places to visit, although Ankara itself is not very beautiful. What Ankara lacks in charm, Istanbul makes up for in spades. Every era has left incredible structures, from the early Roman cistern through the Topkapi palace and the immense "covered market" of the Ottoman era.

The primary center of mathematics in Istanbul is Boğaziçi University (pronounced 'bowzichi'), which means 'Bosphorus' in Turkish). Boğaziçi was formerly Robert College, founded in 1869 by two Americans, Christopher Robert and Cyrus Hamlin, with a decree from the Ottoman sultan. It sits on a spectacular site overlooking the Bosphorus and flourished for a long time with American philanthropy and trustees, teaching an American-style education in English to Turks. In 1971 the college was taken over by the Turkish government and renamed Boğaziçi University, though the medium of instruction remains English. It has about 10,000 students and is a wonderful and beautiful campus just to the north of the city. Like METU, they have vigorous master's and Ph.D. programs and offer a full range of advanced mathematics courses. However, also like METU, they face a dilemma about whether to send their students abroad for their Ph.D. or not. Their website is http://www.math.boun.edu.tr.

I was welcomed by the chair, Betul Tanbay, a very dynamic woman working in operator algebras. She has created a new international mathematical institute, called the Boğaziçi Center for Mathematical Sciences (BMM in Turkish), which is expected to open in 2005 and already has a new building on the campus of Boğaziçi University, where workshops and semester-long programs are planned in the style of the other major mathematics institutes. Last summer a workshop on M-theory organized by Rahmi Guven was held at Boğaziçi. Because of its location exactly on the boundary between Europe and Asia (traditionally drawn along the Bosphorus) and, more importantly, because of the political neutrality of Turkey, to which everyone can get a visa, it seems an ideal location for bringing together the Western and Near Eastern mathematical communities.

I should mention one problem which seems endemic to academic institutions in many countries but is especially bad today in Turkey: salaries. The situation in Turkey, which was brought up by colleagues both in Ankara and Istanbul, is that the public universities, such as METU and Boğaziçi, have traditionally been the best in the country, but they pay very low salaries. Today many private universities have been started, especially in science and engineering, that are tempting professors away by offering more reasonable salaries. Thus professors are split between those maintaining the high standards of the older public universities and those who feel the future is with the new private universities. I had a good talk about this with a Brown Ph.D. in computer vision, Aytul Ercil, who has shifted to Sabanci University, where she can offer a strong program in vision. There is very little a foreigner can do to help this situation, except to be aware and sympathetic, unless you are on an external advisory committee or the like.

Lebanon and the American University of Beirut (AUB)

Lebanon is arguably the most complex country in the Middle East from a sociopolitical point of view. It has deep divisions going back over a millennium between Christians, Muslims, and, until recently, Jews. But it is much more complex: there are Maronite Christians, other Christians, Druze, Muslims, Palestinian refugees, and many other subgroups of the population that have alternately fought and made alliances with each other and with Israelis. A Ph.D. in Lebanese history is required to read the newspaper intelligently! The economy that sustains the country is even murkier. While my son, Jeremy, and I were there, there was an incident in which some people in a mob were shot by the police. We heard a dozen theories about which group had instigated the mob, the confrontation, the police response, and why. On the positive side, we were taken on a fascinating tour through some of the remaining old districts of Beirut by AUB professor Jihad Touma. Here many of these groups were living in close proximity, and Jihad strongly recommended returning at 4:00 a.m. to sample the bread fresh from the ovens of an ancient bakery (something he assured us would be quite safe to do).

In the midst of all this confusion, the American University in Beirut plays a remarkable role: like...
Boğaziçi University, it was founded by an American, Daniel Bliss, in 1866 and has been a neutral force for higher education and medical training. To this day it is governed by an independent board of trustees, now fully international, which gives it insulation from the byzantine politics of Lebanon. During the ghastly twenty-year civil war, it remained an island of peace, receiving only one hit from the bombardment while the center of Beirut was leveled, as it was considered an indispensable part of the city by all communities.

AUB has about 7,000 students and offers many master's programs but no Ph.D.'s. The student body is very mixed: Christian and Muslim, Lebanese and foreign (from sixty-seven countries!). It sits on the corniche of Beirut overlooking the Mediterranean, about half an hour's walk west of Beirut's center. For the visiting mathematician, the most exciting part of AUB is their new Center for Advanced Mathematical Sciences (CAMS), which was started with the help of Sir Michael Atiyah and Nicola Khuri (a physicist at Rockefeller University). Though still small, they are hosting a large number of workshops and summer schools under the leadership of the director, Wafic Sabra. This is a great place to visit, and I strongly recommend it. Their website is http://www-1b.cams.aub.edu.lb.

There is only one hitch! You cannot enter Lebanon if there are any Israeli stamps on your passport or if your ticket shows you have been or are going to Israel. Lebanon and Israel are still technically at war, though Israel has withdrawn from all but contested parts of Lebanon and there are only the usual skirmishes. To make our trip, Jeremy and I bought a round-trip ticket to Amman, stopping in Beirut, and made separate arrangements to get from Amman to Israel and back. This makes it impossible for Israelis to come to Beirut and is a major obstacle to CAMS's desire to be open to everyone. A peace treaty between Lebanon and Israel is conceivable sometime in the future, but right now no one is holding their breath.

The West Bank and Birzeit University

My son and I were very fortunate in having a guide to help us travel in the West Bank, without whom this leg of our trip would certainly not have been possible. Iyad Suwan is a graduate student working with Achi Brandt at the Weizmann Institute. He is Palestinian but is fortunate in that his family has lived in East Jerusalem for three generations, so his Israeli identity card, identifying him as a Palestinian from Jerusalem, allows him to enter Israel and work at the Weizmann.

We traveled to the West Bank by flying to Amman, taking a taxi to the Allenby (or King Hussein) Bridge, crossing the border there, and taking a shared minivan to East Jerusalem, where Iyad met us. It is 44 miles from Amman to Jerusalem, but the trip took us six hours, mainly because it took three hours for the Israeli soldiers at the border to allow us to enter. We met Iyad in East Jerusalem and went to dinner that night at his house, which is fifty feet from "the Wall" (see photo). It has to be seen to be believed. Iyad's grandfather built the house in the 1940s, and the extended family has lived there ever since, paying taxes to whoever runs the city of Jerusalem. But now Iyad is moving because his children are 45 minutes and a checkpoint away from the hospital, which is less than a mile away across the Wall.

Going to Birzeit the next day, the Wall was massively visible from many points. Nor did there seem to be any spots in the West Bank where there were not Israeli settlements as well as Palestinian towns in view. My impression was of a fractal-like interpenetration which one cannot imagine undoing. There were stark contrasts: the huge settlement...
in the middle of the rocky, uncultivable desert that constitutes much of the West Bank. We took a "service taxi", the ubiquitous minibus, to Birzeit to avoid bringing a car through the various checkpoints. Besides the delays of waiting in line, the problem of living with the checkpoints seems to be that the rules are never clear, rather like Kafka’s story The Castle. And even if the rules are clear, some of them are enforced only rarely but are always a threat (for instance, the rule prohibiting West Bank Palestinians from working in Israel, which, when enforced, financially ruins a worker). This was quite a surprise to us when we encountered it at the Allenby Bridge, and Iyad counseled us later to be patient.

In the minibus I sat next to a young Arab woman in head scarf studying reinforced concrete. She was deep in her math but told me they make their own concrete in the West Bank, though they must import some key components for construction. Birzeit amazed me: the students looked as well dressed and happy as their counterparts in the U.S. Though there was a meeting at noon to discuss Palestinian students jailed by Israel, the campus did not feel as angry as I had expected. But clearly the university operates under great stress. In the last few years a new checkpoint has effectively shut them down intermittently. I have described the above not to make a political point—I am aware of the Israeli reasons for both the Wall and the checkpoints—but to explain the conditions under which the Palestinians work and do math.

Birzeit has been operating as a university since 1974 and today has about 5,000 students, all from the West Bank and Gaza, mixed Muslim and Christians. They offer a master’s program in some areas but not in mathematics. Their greatest need is to have more contacts with universities in the West to which they can send their best students. Walking with Alaeddin Elyyan, the chair of the mathematics department, we happened to meet two students working on Cauchy’s theorem; their best students seemed well prepared to make the jump to a regular graduate program. Birzeit has had support and collaboration with the EU and especially with France, where they have an interdisciplinary mathematics and economics program jointly with the University of Paris and hope to start a master’s program jointly with the University of Tours. Their website is http://home.birzeit.edu/math/.

Israel, the Weizmann Institute, Haifa University, and Bar Ilan University

After our visit to Birzeit, Iyad drove us to the Weizmann Institute, which is a truly unique and extraordinary place. Our main hosts at the Weizmann were Ronen Basri, from computer science, and Eitan Sharon, who was born there, went on to get his Ph.D. there with Achi Brandt, and is now a postdoc with me at Brown. Achi is also the advisor of Iyad, and at dinner in his house we learned a lot about the history of Israel. The Weizmann Institute is a combination of a research laboratory and a small and select graduate school (700 students and a comparable number of researchers) covering most of the sciences. The closest analog in the U.S. might be Rockefeller University, if it covered more fields, or Caltech. It was started in 1934 as a chemistry research institute under Chaim Weizmann. It has a gorgeous campus where the faculty live in a communal atmosphere, almost like a kibbutz. Scientifically, it is far and away the most exciting place I visited. I am good friends with and admire tremendously the work of people in several departments there.

Most of us in the U.S. know quite a few people in the seven Israeli universities and do not need my fairly random comments. The stimulus to my trip was an invitation to the University of Haifa, where I was hosted by Larry Manevitz. Overlooking Haifa from a hilltop, the university is moving ahead strongly by virtue of a major grant from the Rothschild Foundation, which gives it funds for a diverse array of new programs. I then visited my old friend Mina Teicher, now vice president for research at Bar-Ilan University. Bar Ilan is a religious institution, but, like the historically Christian AUB and like Birzeit, the scientists share a nonsectarian culture that keeps religious politics at arm’s length. They have just opened the interdisciplinary Gonda Brain Research Center, bringing in Moshe Abeles, the best brain researcher in Israel (in my view), as director. Having been part of a group that sought to create something very much like this at Brown but with only partial success due to lack of funds, I can appreciate how much Mina and Moshe have accomplished at Bar Ilan. Economically, the Israeli universities and their science programs seem as well endowed, if not better, as comparable universities and programs in the U.S.

For me, three things stand out after these trips. One is the desirability of U.S. mathematicians engaging productively with all parts of this fascinating and troubled region through the universal language of mathematics. The second is the hope that the mathematical community in the Near East, on both sides of the religious divide, can bypass the political stalemate and find ways to enrich their mathematical collaboration. The third is the great potential for fruitful interchange between the isolated but scientifically motivated Turkish and Arab universities and American mathematicians.
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The Importance of MathML to Mathematics Communication

Robert Miner

Compared to plain text, mathematical notation is hard to represent, edit, display, and process with computers. On the surface the formalism and rigor of mathematics seem as if they should lend themselves to computing. However, advances in user interfaces and electronic communication have led us to expect that information should flow seamlessly from application to application, intelligently doing the right thing for the given context. Double-click on a picture and the expectation is that it will open in a photo editor or viewer. Drag that same image into your word processor and it should be imported automatically into the document. For mathematics notation, these expectations are particularly demanding, since math is used in a wide variety of applications with very different requirements. Not only should double-clicking on an equation open it in an equation editor, but dragging it onto a computer algebra system or graphing program should be supported as well. Similarly, math should "work" with screen readers for the visually impaired, page composition engines in publishing workflows, search engines, online testing systems, and more. Devising a world-wide software infrastructure that facilitates the use of mathematics in all these contexts is a challenging problem with a long history. Recent years, however, have witnessed a significant step forward in the form of a standardized encoding for mathematical notation called Mathematical Markup Language, or MathML.

MathML is an XML-based encoding for mathematics. XML (short for eXtensible Markup Language) has emerged as the dominant data format underlying the information infrastructure of the World Wide Web. XML defines a method of representing structured data types, essentially the now familiar pointy-bracket tagging of HTML. XML itself merely defines the common syntax and leaves it to specific areas of application to define appropriate data types. MathML is one such data type. XML is significant because it is becoming deeply embedded in the software systems and workflows that will shape the information landscape for years to come. Because of MathML, mathematics is a full-fledged part of that information landscape, and this bodes well for the scientific community.

In this article we will look closer at MathML and examine some of the implications it has for scientific communication. We will describe some of the ways it is currently being put to use in publishing, e-learning, searching, knowledge management, and accessibility for the visually impaired and learning disabled. We will also describe areas of active research and suggest some of the possibilities that may lie ahead.

The Mathematical Software Landscape

The challenges of dealing with math notation have long had a polarizing effect on mathematical software development. On one hand, because supporting math is difficult and expensive, generic applications such as word processors, databases, and web browsers have not usually supported math directly on economic grounds. Instead, math functionality is delegated to third-party math add-ons. On the other hand, within particular communities with a strong need for math, software packages...
such as \TeX, Maple, and Mathematica have arisen, with integrated, specialized support for math.

The appeal of specialized software is that it is less constrained by the requirements of a diverse user group and can therefore focus on the math processing, adding value through ease of authoring, quality of output, computational power, and so on. But the downside of specialized software is that to the extent its functionality overlaps that of generic applications, it tends to get left behind, struggling to keep up with desirable new features as they evolve in mainstream generic applications. More importantly, specialized systems tend to be difficult to interface with other software, a key consideration in networked information environments. By contrast, adding math functionality to generic applications via third-party add-ons allows math components to benefit from the functionality of the highly engineered host application, and component architectures lend themselves well to reuse in networked environments. However, the add-on component approach has pitfalls of its own, since extension mechanisms in generic applications have tended to be inadequate for math and lack of standards has hindered interoperability.

As a result, prior to the rise of the Web, the math software landscape was balkanized, consisting of a collection of specialized applications and math add-on components that couldn’t easily share data or interoperate to any great degree. A number of these software packages achieved success within particular communities, such as commercial publishing, mathematical computation, and research mathematics. An obvious example is \TeX, which came closer to providing a standard electronic format for math than anything before it. To the limited extent that math applications of this era talked to each other, they probably exchanged \TeX code. However, in nonnetworked desktop computing environments before the Internet and the Web became ubiquitous, there was more emphasis on self-contained end-to-end solutions for particular tasks and less demand for interoperability.

The success of the Web, however, dramatically changes the value of information exchange. The process of taking information and moving it around via email, publishing it in print and in electronic format, and making it almost instantly accessible using search engines and web browsers has tremendous value and offers thought-provoking opportunities. By enabling this kind of information exchange, the Web has had a profound impact on society in general and the way scholarship is conducted in particular. As a measure of its success, we now take it for granted that by typing "soliton" and "Cameroon" into Google, within seconds one can discover that there is a physicist at the Université Yaoundé I who is interested in the topic and whose latest paper can be read online for a fee.

The technological key to information exchange in the context of the Web has been standards. Standard protocols and formats for exchanging structured data, such as HTTP, HTML, and XML are fundamental to the software architecture of the Web. In the case of math, MathML has become established as the standard format for math in XML, and this stands to benefit mathematical software development in a number of ways. Just by existing, MathML draws attention to the requirements of mathematics in XML and Web contexts, which is significant. Having a standard format also provides a clear direction for developing conversion and import and export capabilities in specialized math applications. A standard also encourages better, more uniform extension architectures in generic applications, which benefit add-on math components as well. Finally, and not least significantly, a standard decreases the risk of investment and increases the potential for return, which stimulates software development. As a measure of the effectiveness of MathML in this regard, the World Wide Web Consortium (W3C) maintains a list of over fifty MathML-aware software packages, about half of which have appeared in the last year.

About MathML

To better understand how MathML is being used, it is useful to know a bit about MathML itself. MathML is an encoding of the visual presentation and semantic content of mathematical expressions. It was developed under the auspices of the W3C, the body that has responsibility for most Web-related standards such as HTML, XML, and many others. The initial impetus behind the Math Activity at W3C was to provide a better way of displaying equations in Web pages than as graphics. However, MathML rapidly evolved into a general means of representing and communicating mathematical expressions in XML.

MathML was first published as a W3C Recommendation in 1998, which means it is comparatively old and mature for a Web technology. MathML appeared within a few weeks of the initial release of the XML specification itself and was the first major application of XML. Because of this it has had a high profile within the XML community and has been an important test case for many subsequent Web technologies. In particular, math was a motivating example when the namespace mechanism for mixing XML markup languages in a single document was devised.

In order to support the diverse demands placed on math in different contexts, MathML is an information-rich encoding. This reflects a fundamental design decision to make low-level information explicitly available in the markup, information that is usually inferred from context by human readers. While this is usually advantageous and
sometimes essential for machine processing, it means MathML is not suitable for writing by hand. It is a text-based format, however, so it can be read by persistent humans. In this regard, it is somewhat analogous to other low-level structured formats such as PostScript, for example.

One particularly notable feature of MathML that distinguishes it from most other math encodings is that it contains two separate vocabularies, which can either be used alone or in conjunction. One vocabulary, termed presentation markup, describes the visual appearance of an expression. By contrast, content markup attempts to capture the meaning, or mathematical semantics, of an expression. An example is useful for illustrating the differences between the two. Consider the expression $(x + 2)^3$.

The presentation markup for this expression is:

```xml
<math>
  <msup>
    <mrow>
      <mi>x</mi><mo>+</mo><mn>2</mn>
    </mrow>
    <mn>3</mn>
  </msup>
</math>
```

The content markup for the same expression is:

```xml
<math>
  <apply>
    <power/>
    <apply>
      <plus/>
      <ci>x</ci><cn>2</cn>
    </apply>
    <cn>3</cn>
  </apply>
</math>
```

There are several interesting points to be made. The first is that MathML requires that all tokens, or indivisible units of text, be explicitly tagged to indicate their roles. In presentation markup, the markup elements `<mi>`, `<mn>`, and `<mo>` indicate identifiers, numbers, and operators respectively. Identifying the structural roles of tokens in markup is important for use with XSLT, a powerful stylesheet-driven transformation language for XML data types. XSLT has little facility for parsing text data, so XSLT-based applications would have difficulty applying standard math typesetting conventions to tokens if it weren't for the low-level token tagging in MathML. Operator tokens are quite general in Presentation MathML. They include "grouping operators" such as parentheses as well as more traditional operators like "+". Operators can be "stretchy" in the case of notations such as arrows, bars, and parentheses.

A second notable feature of presentation markup is that it goes to some lengths to make the hierarchical markup structure reflect the underlying mathematical structure. Thus, for example, the `<msup>` element denoting the superscript construct has two child elements representing the base and the script. This is in contrast to encodings such as TeX, where the script would typically just embellish the final parenthesis. Similarly, the MathML markup introduces invisible `<mrow>` elements to group the operands together with the "+" operator. Stretchy operators such as parentheses also tend to encourage proper grouping of arguments, since they automatically stretch to the height of the enclosing element. This usually has the effect of requiring that an expression semantically grouped by parentheses is also structurally enclosed by a corresponding `<mrow>` element.

The content markup for $(x + 2)^3$ is quite different. Here we have a functional, LISP-like representation, where operators are applied to arguments. Note that the parentheses do not appear directly in this representation. They are artifacts of a particular visual presentation of the expression. This abstraction gives one the ability to generate multiple presentations of the same expression. A common approach for associating a particular visual rendering with a Content MathML expression is to use XSLT stylesheets to transform it to Presentation MathML. As noted above, XSLT is a rule-based transformation language for XML, and it is supported by most contemporary web browsers. The Connexions Project at Rice University [1] uses this technique to allow users to choose notational preferences. Other groups have used it to localize math expressions: for example, using "tan" for the tangent function in the U.S. and "tg" in France.

An obvious limitation of Content MathML is its scope. It covers only a modest collection of mathematical concepts, covering up to roughly the first year of calculus in the U.S. curriculum. However, Content MathML defines a mechanism for extending its usage by referencing external repositories of semantic definitions. In particular, there is a close relationship between Content MathML and OpenMath, an organization which maintains such "content dictionaries" in a standard format. Content MathML and OpenMath are being used in a number of formal systems and theorem-proving research projects. The EU-funded MowGLI project [2] and Ontario Research Center for Computer Algebra are particularly noteworthy in this regard.
Science, Technical, and Medical (STM) Publishing

As many commentators have noted, mathematics research literature has a long lifespan. An article can remain relevant for many decades. Consequently, making more effective use of the breadth and depth of the literature is an enticing possibility. By connecting ideas and thinkers through time and space, information technology has enormous potential for mathematics. Organizations such as MathSciNet, JSTOR, the arXiv, and many others have begun to suggest the outlines of what is possible. A large amount of material has already been made available in electronic format, and re-digitization projects continue to push the frontier of electronically accessible documents back into the past. Schemes for durable references to electronic publications have been established by STM publishers, and articles and bibliographic databases have been cross-linked. To make this substantial achievement happen, STM publishers have developed cross-media publishing workflows, which involve creating and managing content in print, over the Web, in databases, and so on.

Across the publishing industry as a whole, XML-centric workflows have become the strategy of choice to meet the challenges of cross-media publishing. In the XML-centric model, articles are stored in XML format in a central repository and formatted for various media by stylesheet-driven composition engines. Fueled by public and private investment throughout the dot-com boom, there is widespread support for XML in current publishing software systems. But in order for STM publishers to take advantage of these XML-centric publishing systems, they must be able to deal with math, and that’s where MathML comes in.

In the last several years there has been a proliferation of MathML software targeting STM publishing, reflecting a significant shift toward XML workflows amongst major content providers. STM composition software, XML editors, and conversion software have added MathML support. This has been greatly facilitated by the fact that MathML is a completely integrated XML data type that can be accessed and manipulated through standard APIs in XML software systems. This enables math add-ons to do a better job more easily and in an interoperable way.

At the same time, MathML has been incorporated into a number of XML document types used in STM publishing such as DocBook and the Journal Archiving and Interchange format used by the National Library of Medicine and its PubMed system. As a result, a number of publishers are now running pilot projects using MathML, and a few have embarked on plans to shift major workflows to an XML-centric model using MathML. John Wiley & Sons is conducting pilot projects using MathML. The American Physical Society and American Institute of Physics have already begun using MathML directly for production purposes. The oldest and probably largest volume MathML-based workflow is that of the U.S. Patent Office, operated under contract by Reed-Elsevier. That workflow handles thousands of equations per week. Other projects are currently gearing up, and the next year or two promise a substantial increase in MathML use in production workflows.

Gaps in software support remain. Ironically, MathML support in web browsers continues to present challenges. The newer Netscape and Mozilla-based browsers such as Firefox now have built-in MathML support, and the free MathPlayer extension from Design Science adds native-quality MathML support to Internet Explorer for Windows. But the Safari browser for Macintosh does not support MathML yet. Support for MathML in page layout programs has yet to be developed, and TeX conversion is another area requiring further work. The Hermes &AAPX-to-MathML translator, being developed as part of the MowGLI project, seems promising in this regard.

Math-Aware Searching

One of the most interesting possibilities of MathML is the potential for enhancing searching of technical literature and educational material. By integrating mathematics with the surrounding document in a highly structured way, MathML opens the door to mathematical keyword searching: type an equation into a search engine and get back a list of papers in which it occurs. MathML could also play a role in automated or semiautomated creation of metadata, where the content of a document is analyzed by a software agent to suggest keywords from subject taxonomies or other metadata ontologies. In this way, MathML may have a role in enhancing existing search systems geared toward bibliographic metadata.

In most cases, current searching of online STM content is limited to keyword searches on text. As a result, a researcher typically needs to know appropriate keywords in advance to search for the desired mathematical subject matter. This is limiting in three ways. First, searching is frequently restricted to abstracts and bibliographic metadata, and appropriate keywords may not appear there, even though the full document may contain the desired information. This is especially likely for material in secondary topics, background information, and introductory material. This is the classic metadata problem of insuring appropriate keywords are accessible and appear in close proximity to the resources that they describe.

Secondly, keywords describing mathematical objects are typically overgeneral. One can search for "quadratic polynomial", but there is no effec-
tive way to narrow the search to a particular polynomial or class of polynomials. This is particularly limiting for educational resources, where the same generic label applies to many different treatments of the same material. Searching for “rate of work” with Google produces some 20,000 references. Finding out which of these documents might shed light on the particular rate-of-work problem in your child’s homework assignment is a laborious, and likely fruitless, task.

Finally, it is commonplace in technical subjects to be confronted with mathematical problems of a type beyond one’s experience for which one does not know appropriate keywords at all. A variant of this problem arises when different fields of study have different terminology for identical mathematical objects. In such instances, a problem may in fact be well understood, but the researcher has no way to discover what keywords will find the answer. While the problem of differing nomenclature also affects mathematical notation, at least in some cases the problem is more tractable.

At the Future of Math Communication II workshop [3], held at MSRI in 1999, Rob Corless of the Ontario Research Centre for Computer Algebra related an incident that makes this point well. In the course of working on a nonlinear initial value problem that arose in conjunction with a dynamical system he was studying, he needed to understand the behavior of a certain power series. He knew that Neil Sloane of AT&T Research had recently set up a website where one could search the Handbook of Integer Sequences and Series [4], so he entered the coefficients of his power series. It turned out that the series was known, and the initial value problem had been solved in generality by Gilbert Labelle in a paper in the European Journal of Combinatorics. Not being a combinatorialist, Corless thought it unlikely he would have found this information in any other way. In particular, Corless would not have found it using text-based keyword searches, since he did not know in advance that the solution had anything to do with combinatorics.

Of course, a key point in this anecdote is that integer sequences and series have an obvious, unique, easy-to-type canonical form that make mathematical keyword searches particularly easy in this very narrow area. However, it is important to note that many other kinds of mathematical objects also have easily computed canonical forms. Similarly, other techniques involving normalizing expressions and pattern matching can be quite effective for suitably restricted kinds of mathematical content, especially when augmented by metadata. Consequently, it is reasonable to suppose that mathematical search technology can be extended without undue effort to the point where search success stories of the kind Corless describes could at least be replicated for many categories of content, if not in full generality.

However, to scale up mathematical searching and integrate it with text searching to any appreciable degree, the ability to automatically identify and normalize large classes of mathematical expressions is essential. Here MathML plays an important role in two ways. First, since it takes pains to insure markup structure generally reflects mathematical structure, it significantly simplifies the computational complexity of recognizing and manipulating expressions. This facilitates the creation of specialized, math-aware search engines. Second, by integrating math with text in a common XML-based format, MathML makes math accessible to generic XML search technologies. The XQuery standard currently under development, for example, is expected to make a substantial impact in this area. Because of their obvious commercial potential, generic XML search technologies are attracting widespread investment and support, so the potential benefit to STM searching is highly leveraged.

The potential benefit of math-aware searching has been recognized in many quarters. The National Science Foundation awarded Design Science a grant through the National Science Digital Library (NSDL) program to investigate ways of enhancing math searching. The MowGLI project is also investigating applications of MathML to math searching. A number of other projects, both commercial and academic, are under way. For example, a workshop on the topic, funded by Design Science, was held at the Institute for Math and its Applications at the University of Minnesota in April 2004 [5]. Searching and related topics also featured prominently at a conference on Mathematical Knowledge Management, held in conjunction with the Joint Mathematics Meetings in January 2004.

E-learning

E-learning is another area where MathML has natural applications. In many ways, MathML has its roots in online learning, as much of the original motivation for MathML was to provide a better means of incorporating mathematics into Web pages for educational purposes. However, in practice, MathML has probably had a larger impact to date as a backend technology used to add math support in course management systems, online assessment systems, and the like.

From the outset, the Web has had a strong appeal as a medium where educational content can be dynamic and interactive and where concepts can be presented in multiple educational modalities: text, images, sound, animation, and even manipulatives and simulations. Much effort has been invested in the creation of such content, in many cases with highly engaging and effective results. Different projects have used a wide variety
of Web technologies, including some that make use of MathML. One of the larger projects making extensive use of MathML to create dynamic educational content is the Homework Help feature of Microsoft's MSN Premium service.

However, highly dynamic educational Web content is problematic in several ways. First, it is quite difficult to create. It requires not only skill in educational design but also substantial technical skill. Dynamic Web content also places great demands on bandwidth and browser technology. But education is precisely where one finds the broadest diversity of browsers and platforms and where the requirement of universal access is strongest. As a consequence, efforts to use the Web as a means of integrating rich media into math curricula have generally had mixed success.

At the same time, use of the Web to provide a learning environment where students and teachers can interact via various modes of electronic communication has grown to the point where it is now commonplace. Such uses range from email and simple course homepages, where syllabi, assignments, and office hours are posted, to sophisticated learning management systems (LMS) such as WebCT, Blackboard, eCollege, and others. Because electronic communication has an entirely different dynamic than face-to-face interactions, it can lower barriers to participation for students who otherwise might sit silent and unnoticed in the back row. Some students may be more comfortable interacting with teachers and their peers via electronic means, where there is a slight element of anonymity and the ability to consider questions and responses without the real-time pressure of face-to-face interaction.

MathML is now widely used behind the scenes for adding math support to online collaboration tools. To a large extent, this has been a consequence of the fact that MathML fostered the development of a number of interoperable math-aware components that LMS vendors have been able to utilize, WebEQ, webMathematica, techexplorer, and MathWYG are among the most common. In particular, a number of LMS vendors now provide math support in their whiteboard and message board systems using these components. Typically such systems use a MathML equation editor component that can be embedded in a Web page for authoring, together with server-side components for processing MathML for display.

Another area where MathML is being used is online assessment. For lower-level mathematics, where MathML is most successful at capturing mathematical semantics, several systems utilize computer algebra systems and other techniques to perform automatic scoring. Some systems, such as MapleTA, are even able to analyze student errors to provide adaptive hints and customize tutorials to target student skill deficits. In a similar vein, Integre Technical Publishing is investigating ways of using standard XML tools to take advantage of the structured nature of MathML to analyze student work through an NSF-funded NSDL grant.

Accessibility
One last area where MathML is making a noteworthy contribution is accessibility for the visually impaired and learning disabled. As the work force has aged, disabilities and impairments of all sorts have grown to affect nearly two-thirds of adults. Most are mild, but, according to a survey by Forster Research commissioned by Microsoft, 17 percent of computer users have a mild visual difficulty or impairment and 9 percent have a severe visual difficulty or impairment.

The needs of users vary substantially depending on the nature and degree of the impairment. Individuals with severe visual impairments often rely on tactile feedback through braille displays and embossers, as well as audio renderings of the math. Individuals with low vision typically require audio renderings, in conjunction with conventional type-set representations using large font sizes or high-contrast colors. Those with learning disabilities such as dyslexia benefit most from synchronized highlighting of visual display along with an audio rendering.

In the United States, the American Disabilities Act (ADA), the Individuals with Disabilities Education Act (IDEA), and other federal legislation require schools and publishers to provide accessible versions of texts in many circumstances. A majority of states have similar laws mandating accessibility of content for organizations that receive state funding, and a majority of states require that textbook publishers provide versions of their textbooks for the blind. Similar laws also apply in Europe and elsewhere. Currently, in the case of mathematics, these requirements are typically met by providing a text equivalent for equations. For example, in an HTML page, this typically means equations are displayed using images, with a textual ALT description of an image.

Unfortunately, text descriptions of mathematical expressions only meet the letter of the law and do not really address user needs. At a practical level, the preparation of text descriptions is labor intensive and error prone. At a deeper level, for audio rendering of mathematics, the ability to "navigate" around a long expression is critical to comprehension. Moreover, static text cannot take advantage of locale or user preference information to choose the language or customize the vocabulary.

MathML facilitates solutions to all of these problems. Most assistive technology utilizes standard software APIs developed for HTML and XML. By making the math notation available through these
standard methods, MathML enables screen readers and other assistive technologies to properly handle the math with minimal additional effort. And since MathML tries to insure that markup structure reflects the underlying semantics, navigation is also greatly facilitated. Finally, since the audio rendering of a MathML expression is generated on the client machine, it can take full advantage of locale and user preference information.

Design Science has received an Small Business Innovation Research grant from the National Science Foundation to add accessibility functionality to its MathPlayer extension for Internet Explorer. MathPlayer 2.0 has a demo “speak expression” feature that works with leading screen readers, and support for expression navigation and synchronized highlighting is underway. A number of other groups are also exploring ways of using MathML for accessibility, and it is anticipated MathML will soon be incorporated into standard XML formats for accessibility currently under development.

**Conclusion**

From a certain point of view, MathML is merely another data format for math notation—not the first, and assuredly not the last. However, as has so often been the case in the history of technology, the larger significance of MathML depends only tangentially on its particular strengths and weaknesses as a technology. Instead, MathML is significant because of the opportunity it represents for math and science to participate more fully in the information revolution that is one of the great intellectual movements of our time. MathML reserves a place at the XML technology table for the interests of math and science.

Many groups have already seized upon the opportunity presented by MathML and are using it in innovative ways in STM publishing, searching and knowledge management, e-learning, and accessibility. Moreover, momentum is building, as development efforts cross-pollinate and reinforce one another. While many of these projects are still in their early stages, the future generally looks bright for electronic communication in the sciences. In a world where math phobia is pervasive and the cold calculus of the marketplace rarely favors academia, that is no small achievement.

**References**

On January 18, 2005, the Wolf Foundation announced that the 2005 Wolf Prize in Mathematics will be awarded to Gregory A. Margulis of Yale University "for his monumental contributions to algebra, in particular to the theory of lattices in semi-simple Lie groups, and striking applications of this to ergodic theory, representation theory, number theory, combinatorics, and measure theory" and to Sergei P. Novikov of the University of Maryland, College Park, and the L. D. Landau Institute for Theoretical Physics "for his fundamental and pioneering contributions to algebraic and differential topology on one hand, and to mathematical physics on the other hand." Margulis and Novikov will share the $100,000 prize, which will be presented by the president of the State of Israel, Moshe Katsav, at a special ceremony at the Knesset (parliament) in Jerusalem on May 22, 2005.

Gregory A. Margulis
At the center of the work of Gregory Margulis lies his proof of the Selberg-Piatetski-Shapiro Conjecture, affirming that lattices in higher rank Lie groups are arithmetic, a question whose origins date back to Poincaré. This was achieved by a remarkable tour de force, in which probabilistic ideas revolving around a noncommutative version of the ergodic theorem were combined with p-adic analysis and with algebraic geometric ideas showing that "rigidity" phenomena, earlier established by Margulis and others, could be formulated in such a way ("super-rigidity") as to imply arithmeticity. This work displays stunning technical virtuosity and originality, with both algebraic and analytic methods. The work has subsequently reshaped the ergodic theory of general group actions on manifolds.

In a second tour de force, Margulis solved the 1929 Oppenheim Conjecture, stating that the set of values at integer points of an indefinite irrational nondegenerate quadratic form in more than three variables is dense in $R^n$. This had been reduced (by Raghunathan) to a conjecture about unipotent flows on homogeneous spaces, proved by Margulis. This method transformed to this ergodic setting a family of questions till then investigated only in analytic number theory.

A third dramatic breakthrough came when Margulis showed that Kazhdan's "Property T" (known to hold for rigid lattices) could be used in a single arithmetic lattice construction to solve two apparently unrelated problems. One was the solution to a problem posed by Rusiewicz, about...
Novikov made a fundamental and striking contribution to two separate fields in mathematics, while he is one of those rare mathematicians who brings deep, key mathematical ideas to bear on difficult pivotal problems of physics, in ways that are stunning and compelling for both mathematicians and physicists.

Born in Russia in 1936, Sergei P. Novikov graduated from Moscow State University in 1960. In 1965, he received his Ph.D. in physics and mathematics from the Steklov Institute of Mathematics in Moscow. Since 1971, Novikov has been head of the Mathematical Division at the L. D. Landau Institute for Theoretical Physics in Moscow. Since 1992, he has been a professor in the Department of Mathematics and at the Institute for Physical Science and Technology at the University of Maryland, College Park. Novikov received the Lenin Prize of the USSR in 1967 and the Fields Medal in 1970. In 1981, he was elected as a full member of the Academy of Sciences of the USSR. He is a foreign associate of the U.S. National Academy of Sciences.

**Sergei P. Novikov**

Sergei P. Novikov is awarded the Wolf Prize for his fundamental and pioneering contributions to topology and to mathematical physics. His early work in algebraic and differential topology includes such milestones as the calculation of cobordism rings and stable homotopy groups, proof of the topological invariance of rational Pontrjagin classes, formulation of the "Novikov Conjecture" on higher signature invariants, and proof of the existence of closed leaves in two-dimensional foliations of the 3-sphere.

In the early 1970s Novikov turned his attention to mathematical physics, initially contributing to general relativity and conductivity of metals. He constructed a global version of Morse theory on manifolds and loop spaces that had novel applications to quantum field theory (multivalued action functionals). His most significant achievements in mathematical physics flow from his introduction of algebraic-geometric methods to the study of completely integrable systems. These include a systematic study of finite-gap solutions of two-dimensional integrable systems, formulation of the equivalence of the classification of algebraic-geometric solutions of the KP equation with the conformal classification of Riemann surfaces, and work (with Krichever) on "almost commuting" operators that appear in string theory and matrix models ("Krichever-Novikov algebras", now widely used in physics).

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Novikov's work is characterized by extraordinary depth, technical power, creative synthesis of ideas and methods from different areas of mathematics, and a grand architectural unity of its final form. Though his work addresses deep unsolved problems, his solutions are housed in new conceptual and methodological frameworks of broad and enduring application. He is one of the mathematical giants of the last half century.

Born in 1946 in Russia, Margulis received his Ph.D. in 1970 from Moscow State University. Starting in 1970, he was associated with the Institute for Problems in Information Transmission at that university, first as junior scientific worker, later as senior staff member, and from 1986 until he left in 1991, as leading scientist. Since 1991, Margulis has been a professor of mathematics at Yale University. He received the Fields Medal in 1978. He is a foreign honorary member of the American Academy of Arts and Sciences and a member of the U.S. National Academy of Sciences.

**About the Wolf Prize**

The Israel-based Wolf Foundation was established by the late German-born inventor, diplomat, and philanthropist Ricardo Wolf. A resident of Cuba for many years, Wolf became Fidel Castro's ambassador to Israel, where he lived until his death in 1981. Five annual Wolf Prizes have been awarded since 1978 to outstanding scientists and artists "for achievements in the interest of mankind and friendly relations among peoples, irrespective of nationality, race, color, religion, sex, or political view." The prizes of $100,000 in each area are given every year in four out of five scientific fields in rotation: agriculture, chemistry, mathematics, medicine, and physics. In the arts, the prize rotates among architecture, music, painting, and sculpture. To date, a total of 224 scientists and artists from twenty-one countries have been honored.  

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For a list of previous Wolf Prize winners in mathematics, see the Web page [http://www.aquanet.co.il/wolf/wolf5.html](http://www.aquanet.co.il/wolf/wolf5.html).
The 2005 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the 111th Annual Meeting of the AMS in Atlanta in January 2005.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring accurate mathematical information to non-mathematical audiences. The award carries a cash prize of $1,000.


The 2005 JPBM Communications Award was presented to BARRY CIPRA. The text that follows presents the award citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Citation
The Joint Policy Board for Mathematics presents its 2004 Communications Award to Dr. Barry Cipra who, for nearly twenty years, has written about mathematics of every kind—from the most abstract to the most applied. His lucid explanations of complicated ideas at the frontiers of research have appeared in dozens of articles in newspapers, magazines, and books.

While some of his audience undoubtedly consists of mathematicians themselves, he writes for scientists and scholars who are mathematically literate. In this way, he has reached many thousands of scientists. Dr. Cipra's work has educated mathematicians and nonmathematicians alike by exposing them to current and deep mathematical ideas about the beauty and power of mathematics. Barry Cipra has given his readers a greater understanding of the ideas of mathematics, but most importantly he has changed their perception of the nature of mathematics.

Biographical Sketch
Barry Cipra received his doctoral degree in mathematics from the University of Maryland in 1980. After a brief career as an academic, he turned to freelance writing, and he has continued with that work for the past 15 years. He has written many articles for Science magazine, one of the premier journals of scientific exposition. Examples of the intriguing titles of his articles are "Simple recipe creates acid test for primes" and "How to play platoonic billiards". He is a regular contributor to SIAM News, writing many dozens of articles that are accessible and illuminating. He has authored five volumes of What's Happening in the Mathematical Sciences for the AMS, each including a compilation
“Let me cut to the chase: every mathematics library requires a copy of this book. . . . Every supervisor of higher degree students requires a copy on their shelf. Welcome to the rich world of computer-supported mathematics!”

—Mathematical Reviews

Mathematics by Experiment
Plausible Reasoning in the 21st Century
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—American Scientist

of expository articles on recent mathematical developments aimed at the mathematically literate public. Those volumes have been widely distributed (and admired) in the scientific community and in Washington.

Cipra received the 1991 Merten M. Hasse Prize from the Mathematical Association of America for an expository article on the Ising model, published in the December 1987 issue of the American Mathematical Monthly. He is the author of Mistakes...and how to find them before the teacher does... (a calculus supplement), published by A K Peters, Ltd.

Cipra completed his Ph.D. degree under the direction of Michael Razar, with much help from Steve Kudla. He was a Moore Instructor at the Massachusetts Institute of Technology, a research instructor at the Ohio State University, and an assistant professor at St. Olaf College in Northfield, Minnesota, before turning to freelance writing.

Response
It is a great honor to receive the JPBM Communications Award. To be able to write about mathematics for a living—to meet so many first-rate mathematicians and learn about their exciting work—is a pleasure beyond description. This is an amazing age in which to be reporting on mathematics and its applications. I never would have guessed, in 1987, that I would wind up reporting on the proofs of Fermat’s Last Theorem and the Kepler Conjecture (and, very possibly, the Poincaré Conjecture). I have witnessed an incredible growth in the applications of mathematics, especially in biology, which fifteen years ago was barely a whisper at math meetings and now is a prominent theme at many. Perhaps most surprisingly, I’ve seen mathematics go from a virtual nonentity in popular culture to become the basis (or McGuffin) of award-winning plays and movies.

I’ve been helped by many people over the years. Chief among them are Klaus Peters, Lynn Steen, Ed Block, Paul Sally, and Sam Rankin. I would like to thank my editors, especially Gail Corbett, Tim Appenzeller, and Paul Zorn, who have made the final, published versions of my articles so much better than their first drafts. Indeed, the key to writing, I’ve found, is expressible in a familiar mathematical term: iteration. The hard part, as mathematicians well know, is making sure the iterative process converges to the desired result.
At the Joint Mathematics Meetings in Atlanta in January 2005, the Mathematical Association of America presented the following honors.

**Gung and Hu Award for Distinguished Service**
The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the Association. The 2005 Gung and Hu Award was presented to Gerald L. Alexanderson of Santa Clara University. The award citation states, "Jerry has a long record of able service to mathematics as a practitioner, teacher, administrator, professional organization leader, publicist, advocate, and enthusiast whose love for mathematics and its people comes through clearly in his public talks and widely-read books." Alexanderson is known for his work on the Mathematical People volumes of interviews with mathematicians. Since 1975 he has been the associate director of the William Lowell Putnam competition. He served as chair of his department for thirty-five years and received a President's Special Recognition Award in 1996 for this service to his institution. Since 1994 he has been chair of the Board of Trustees of the American Institute of Mathematics. During his fifty years of MAA membership he has served in many different capacities, including serving as MAA president and secretary.

**Haimo Awards for Distinguished College or University Teaching**
The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2005 Haimo awards were presented to Gerald L. Alexanderson of Santa Clara University, Aparna Higgins of the University of Dayton, and Deborah Hughes-Hallett of the University of Arizona and Harvard University.

"Jerry Alexanderson is a master teacher, an inspiration to both students and colleagues," the citation states. "In his 47 years of teaching at Santa Clara University (35 years of which he was department chair), he has consistently had the reputation for being not only the best, but also one of the most demanding teachers. His classes are amusing, entertaining, and highly informative, an impressive mix of challenging mathematics and historical anecdotes, delivered clearly and concisely. Many mathematicians (and former students in other careers) discovered the excitement of mathematics in the first course they took with Jerry, and his personal advice and encouragement continues to guide many of those careers today."

"Aparna Higgins is one of the dynamos of the U.S. mathematical community," the award citation states. "Her ease with and genuine connection to students is remarkable; her dedication to teaching and mentoring is recognized by colleagues near and far...[Her] love of all things mathematical and the desire to encourage others fuels her charisma, energy, and enthusiasm. Her joy is contagious in the classroom, at MAA student chapter meetings, in her REU [Research Experiences for Undergraduates] summer programs, and with Project NExT Fellows." Higgins has received two other teaching awards, from her own university and from the Ohio section of the MAA.

"Deborah Hughes-Hallett is known for her superb skills in the classroom," the prize citation
Princeton = Math

COMPLEXITIES
Women in Mathematics
Bette Anne Case and Anne M. Leggett, Editors

This eye-opening book presents the stories of dozens of women who have pursued careers in mathematics. The contributors offer their own narratives, recount the experiences of women who came before them, and offer guidance for those who will follow in their career paths. The one thing they share in common is a genuine passion for mathematics.

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Certificates of Meritorious Service
The Certificate of Meritorious Service is presented for service at the national level or for service to an MAA section. Those honored in 2005 were Charles Carle of Allegheny College, Allegheny Mountain Section; Jon Scott of Montgomery College, Maryland-District of Columbia-Virginia Section; Barbara Ososky of Rutgers University, New Jersey Section; Roy Deal, Jr., of Oklahoma State University, Oklahoma-Arkansas Section; and Ernie Solheid of California State University at Fullerton, Southern California-Nevada Section.
At the Joint Mathematics Meetings in Atlanta in January 2005, the Association for Women in Mathematics (AWM) presented the following honors.

Hay Award
The Louise Hay Award for Contributions to Mathematics Education recognizes outstanding achievements in any area of mathematics education, to be interpreted in the broadest possible sense.

The 2005 Hay Award was presented to SUSANNA S. EPP of DePaul University. "For the past twenty-five years, she has committed herself to helping students come to understand the unspoken logic and language that underlie mathematical thought," the award citation states. She has written a very popular and well-regarded textbook, *Discrete Mathematics with Applications*, and has coauthored a volume in the University of Chicago School Mathematics Project (UCSMP) Secondary Series. Epp has written many articles on the teaching and learning of mathematics and has been active in national efforts to improve mathematics education. At DePaul University, she developed more than a dozen successful courses. The prize citation says, "For her selfless contributions to mathematics education, her role as a mentor, her scholarship, her administrative skills, her human qualities of kindness, absolute honesty and trustworthiness, and her willingness to listen, the Association for Women in Mathematics is pleased to designate Susanna S. Epp as the Fifteenth Annual Louise Hay Awardee."

Schafer Prize
The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman honors outstanding achievement in mathematics by a female undergraduate.

The 2005 Schafer Prize was awarded to MELODY CHAN, a senior at Yale University. She has excelled in a wide variety of mathematics courses and was awarded the prestigious Hart Lyman Prize. She has made presentations at the Yale Math Club, earned an honorable mention in the Putnam Competition, and is vice president of the Yale chapter of Phi Beta Kappa. She participated in the Budapest Semesters in Mathematics in Hungary and in Research Experiences for Undergraduates (REU) programs. At the University of Minnesota at Duluth REU, she wrote three professional-level papers on the concept of the distinguishing number.

Runners-up for the Schafer Prize are MARGARET L. DOIG, University of Notre Dame, and ELENA FUCHS, University of California, Berkeley. ANNALIES VUONG, University of California, Santa Barbara, received an honorable mention.
Happy 100th, Baley Price!

G. Baley Price, Distinguished Professor Emeritus of the University of Kansas, turned 100 years old on March 14, 2005. Known for his dedication to teaching and to public service, Price had a hand in many key developments in the American mathematical community during the twentieth century. He has been a member of the AMS since January 1, 1929.

Griffith Baley Price was born in 1905 and received his bachelor's degree from Mississippi College in Clinton, Mississippi, in 1925. He received his doctorate in 1932 from Harvard University, where, like his fellow students C. B. Morrey and Hassler Whitney, he was a student of G. D. Birkhoff. The title of Price's dissertation was "Double pendulum and similar dynamical systems". After a short stint teaching at Brown University, in 1937 he joined the faculty of the University of Kansas, where he remained for the rest of his career. In 1970 he was named the first E. B. Stouffer Distinguished Professor of Mathematics. He retired in 1975.

Price devoted a great deal of time and effort to service on behalf of the mathematics profession and the general public. While at Brown University he had become acquainted with R. G. D. Richardson, who was then AMS secretary and who was instrumental in launching Mathematical Reviews, which began publication in 1940. Price served on the publicity committee that helped drum up subscriptions for the fledgling journal. Around this time he served on the AMS Council and was also named an associate secretary of the Society, but his World War II service prevented his serving in that capacity. In 1970 he was named the first E. B. Stouffer Distinguished Professor of Mathematics. He retired in 1975.

During the 1950s Price worked to get the mathematical community involved in undergraduate and school education. While he was president of the Mathematical Association of America (MAA) in 1957–58, Price collaborated with AMS president Richard Brauer and National Council of Teachers of Mathematics president Harold Fawcett to appoint a committee to establish the School Mathematics Study Group (SMSG). This group launched the "new math" program that transformed mathematics teaching in schools across the country. While sometimes dismissed as a failure, SMSG was often successful when teachers were properly trained. Price participated in writing sessions for SMSG and also taught in a summer institute designed to train teachers to use the "new math" curriculum.

Price served as an AMS associate secretary from 1946 until 1949 and was editor of the Bulletin from 1950 until 1957. He was also very active in the MAA, serving not only as president but also as a vice president and as a longtime member of the MAA Board of Governors. During the 1950s he was on the MAA Committee on the Undergraduate Program in Mathematics. He received the MAA's Distinguished Service Award in 1970. He was the first chairman of the Conference Board of the Mathematical Sciences (CBMS) (1959–60) and also served as CBMS executive secretary (1960–62).

A University of Kansas alumnus who became a publishing sales representative was so impressed with Price when they first met in the 1960s that in 2004 he committed nearly a million dollars toward the establishment of an endowed chair in Price's honor. Not long thereafter, Price himself made a gift of $500,000 to the university for an endowed professorship in honor of his wife, Cora Lee Beers Price, who held a Ph.D. in English and was on the faculty of the University of Kansas. She passed away in late 2004.

Still active in his retirement, Price wrote a history of the mathematics department at the University of Kansas as well as historical pieces for the AMS centennial celebration in 1988 and the MAA's seventy-fifth anniversary in 1990. His long career of service to mathematics has made him a beloved member of the community. Happy Birthday, Professor Price!

—Allyn Jackson
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In the Monte Carlo simulations that follow, three bandwidth choices are parameter combination. The LSCV bandwidth, the “Stanton” bandwidth, and the least squares cross validation problem (ref. LSCV func). The IID bandwidth is the sample size, and it is defined as \( h_{IID} = \hat{\sigma}T^{-1/5} \), where \( \hat{\sigma} \) is the sample standard error.

The Stanton bandwidth is the one actually used in Stanton (1997) and is based on equations (ref. above). In particular, “inverting” these equations yields:

\[
\begin{align*}
\hat{\mu}(x_t) &= \frac{1}{\Delta}E[y_{t+\Delta} - y_t | x_t] + o(\Delta) \\
\hat{\sigma}(x_t) &= \sqrt{E[(y_{t+\Delta} - y_t)^2 | x_t] \frac{1}{\Delta} + o(\Delta)}
\end{align*}
\]

The Stanton procedure is to apply the Nadaraya-Watson (N) regression estimator to construct nonparametric estimates of the conditional equations (ref. drifft) and (ref. diff):

\[
\begin{align*}
\hat{\mu}(x_t) &= \frac{1}{\Delta} \sum_{t=1}^{T-h} \left( x_{t+h} - x_t \right) K \left( \frac{x_{t+h} - x_t}{h} \right) \\
\hat{\sigma}(x_t) &= \sqrt{\sum_{t=1}^{T-h} \left( x_{t+h} - x_t \right)^2 K \left( \frac{x_{t+h} - x_t}{h} \right) \frac{1}{\Delta}}
\end{align*}
\]


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Fedkiw Receives NAS Award for Initiatives in Research

RONALD FEDKIW of Stanford University has been awarded the National Academy of Sciences (NAS) Award for Initiatives in Research. He has been honored for "his many innovations in the modeling and numerical simulation of flows and his pioneering contributions to physically based computer graphics."

The award carries a cash prize of $15,000 and is awarded annually to recognize innovative scientists under thirty-five years old and to encourage research likely to lead toward new capabilities for human benefit. The prize alternates among the fields of physical sciences, engineering, and mathematics.

—From an NAS announcement

Hughes Receives AAAS Lifetime Mentor Award

RHONDA J. HUGHES of Bryn Mawr College has been named the recipient of the 2004 AAAS Lifetime Mentor Award of the American Association for the Advancement of Science (AAAS). She has helped fifty-seven women and minority students earn graduate degrees in mathematics, including seventeen at the doctoral level.

Hughes and Sylvia Bozeman of Spelman College developed the Spelman-Bryn Mawr Summer Mathematics Program and the Enhancing Diversity in Graduate Education (EDGE) Program, both designed to help young women through college and graduate school. Hughes is a former president of the Association for Women in Mathematics.

The AAAS Mentor Award for Lifetime Achievement honors AAAS members who have mentored and guided significant numbers of underrepresented students toward a Ph.D. degree in the sciences. The recipient receives $5,000 and a commemorative plaque.

—From an AAAS announcement

Lander Receives Public Understanding Award

The American Association for the Advancement of Science (AAAS) has named ERIC S. LANDER to receive its Public Understanding of Science and Technology Award for 2004.

Lander is the founding director of the newly created Broad Institute of the Massachusetts Institute of Technology and of Harvard University. He was cited by AAAS "for his excellence in communicating complex scientific ideas, and their implications for society, to the general public and policy-makers, while actively engaged in a demanding and aggressive research program."

A former Rhodes scholar, Lander earned his undergraduate degree in mathematics with highest honors from Princeton University in 1978 and then received his Ph.D. in mathematics from Oxford University in 1981. His honors and awards include a MacArthur Fellowship in 1987. He was elected a member of the U.S. National Academy of Sciences in 1997 and the U.S. Institute of Medicine in 1999.

The AAAS Award for Public Understanding of Science and Technology, established in 1987, recognizes scientists and engineers who make outstanding contributions to the popularization of science. It carries a monetary prize of $5,000.

—From an AAAS news release

Boyd Awarded CRM-Fields Prize

DAVID BOYD of the University of British Columbia has been awarded the 2005 CRM-Fields Prize. The prize, awarded annually by the Centre de Recherches Mathématiques (CRM) in Montreal and the Fields Institute in Toronto, recognizes exceptional contributions by a mathematician working in Canada. The prize carries a cash award of 5,000 Canadian dollars (approximately US$3,850), and the recipient is expected to present a lecture at the CRM and at the Fields Institute.
Boyd was recognized for seminal contributions to analytic number theory, particularly his explorations of the deep connections between the Mahler measure of polynomials and special values of their associated $L$-functions.

Boyd received his B.Sc. from Carleton University in 1963 and his M.A. in 1964 and Ph.D. in 1966 from the University of Toronto. He has taught at the University of Alberta and the California Institute of Technology. He has been teaching at the University of British Columbia since 1971 and is currently a full professor. Boyd is a fellow of the Royal Society of Canada. His awards include the E. W. R. Steacie Prize and both the Coxeter-James and Jeffery-Williams prize lectureships of the Canadian Mathematical Society (CMS). His service to the Canadian mathematical community includes terms as vice president of the CMS, as chair of the NSERC mathematics grant selection committee, and as acting director of the Pacific Institute for the Mathematical Sciences.


—From a Fields Institute announcement

AIM Five-Year Fellow Announced

JÖEL KAMNITZER of the University of California, Berkeley, has been named the recipient of the 2005 American Institute of Mathematics (AIM) Five-Year Fellowship.

Kamnitzer will receive his Ph.D. in 2005 from Berkeley. His research interests include algebraic geometry, representation theory, and combinatorics. His thesis, titled “Mirkovic-Vilonen cycles and polytopes”, gives a combinatorial description of a family of varieties arising in geometric representation theory. The fellowship will provide sixty months of full-time research, as well as funds for travel and equipment.

The runners-up for the AIM Fellowship are Spyros Alexakis (Princeton University), Beth Samuels (Yale University), Lior Silberman (Princeton University), and Lauren Williams (Massachusetts Institute of Technology).

—From an AIM announcement

Packard Fellowships Awarded

The David and Lucile Packard Foundation has awarded sixteen Fellowships for Science and Engineering for the year 2004. Two mathematical scientists were among the awardees.

MANJUL BHARGAVA of Princeton University and ALEXANDRU D. IONESCU of the University of Wisconsin, Madison, will each receive an unrestricted research grant of $625,000 over five years.

The fellowships are awarded to researchers in mathematics, natural sciences, computer science, and engineering who are in the first three years of a faculty appointment.

—From a Packard Foundation announcement

Maz'ya Awarded Celsius Gold Medal

VLADIMIR MAZ'YA of the Ohio State and Liverpool Universities received the Celsius Gold Medal of the Royal Society of Sciences of Uppsala on August 31, 2004. The citation states that Maz'ya received the medal "for his outstanding research in partial differential equations and hydrodynamics".

Maz'ya authored more than four hundred papers and fifteen books in various branches of pure, applied, and numerical analysis. In particular, as early as 1960 he discovered the equivalence of Sobolev embeddings and isoperimetric-isocapacitary inequalities, which had great impact on further development on the theory of function spaces, spectral theory, and differential geometry. In 1968 he constructed counterexamples related to the nineteenth and twentieth Hilbert problems. He solved F. John's problem on time-harmonic waves above a submerged body in 1978. Recently he found regularity criteria of Wiener's type for higher order elliptic equations.

The Royal Swedish Society was founded in 1710 and is the oldest scholarly society in Sweden. The Gold Medal is the Society's highest award and was initiated in 1960. It is awarded every two years in different areas of science, going to a mathematician every sixth year. Among previous recipients in the mathematical sciences are L. Carleson, L. Hörmander and J. Peetre.

—From a Royal Society of Sciences of Uppsala announcement
Mathematics Opportunities

Enhancing the Mathematical Sciences Workforce in the Twenty-First Century

In an effort to increase the number of U.S. citizens, nationals, and permanent residents who are well prepared in the mathematical sciences and who pursue careers in the mathematical sciences and other scientific disciplines, the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has instituted a program called Enhancing the Mathematical Sciences Workforce in the Twenty-First Century. This program builds on the Vertical Integration of Research and Education (VIGRE) program and includes a broadened VIGRE activity as well as additional components for Research Training Groups in the Mathematical Sciences (RTG) and for Mentoring through Critical Transition Points (MCTP) in the Mathematical Sciences.

VIGRE grants are designed to allow departments in the mathematical sciences to carry out innovative educational programs in which research and education are integrated and in which undergraduates, graduate students, postdoctoral fellows, and faculty are mutually supportive. Integrating research and education for graduate students and postdoctoral associates, involving undergraduates in substantial learning by discovery, and developing a team approach are keys to successful VIGRE projects. VIGRE student and postdoctoral associates and their mentors may participate in international research and education collaborative activities, including activities in other countries that are integrated into and benefit the overall VIGRE program at the institution.

The DMS expects to make eighteen or nineteen awards under this program in 2005. The deadline for proposals is September 16, 2005. For more information about the program and all of its components, see the website http://www.nsf.gov/pubs/2003/nsf03575/nsf03575.htm.

—From an NSF announcement

NSF Graduate Teaching Fellowships in K–12 Education

The National Science Foundation (NSF) has instituted the Graduate Teaching Fellowships in K–12 Education (GK–12) to support fellowships and associated training that enable graduate students in NSF-supported science, technology, engineering, and mathematics (STEM) disciplines to acquire additional skills that will broadly prepare them for professional and scientific careers in the twenty-first century. Through interactions with teachers in kindergartens through high schools, graduate students can improve communication and teaching skills while enriching STEM instruction in these schools. Expected outcomes include improved communication, teaching, and team-building skills for the fellows; professional development opportunities for K–12 teachers; enriched learning for K–12 students; and strengthened partnerships between institutions of higher education and local school districts.

The deadline for letters of intent is May 4, 2005, and full proposals are due June 2, 2005. For more details, see http://www.nsf.gov/pubs/ods/getpub.cfm?nsf05553.

—From an NSF announcement

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period, in intervals not shorter than three consecutive months. The Research Instructorship option provides a combination of full-time and half-time support over a period of three academic years, usually one academic year full time and two academic years half time. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The deadline for applications is October 19, 2005. For more information and application instructions, see the NSF website at http://www.nsf.gov/pubsys/ods/getpub.cfm?nsf05510.

—From an NSF announcement

550 NOTICES OF THE AMS VOLUME 52, NUMBER 5
SASTRA Ramanujan Prize

The Shanmuga Arts, Science, Technology, Research Academy (SASTRA), based in the state of Tamil Nadu in South India, has instituted a SASTRA Ramanujan Prize of $10,000 to be awarded annually to a mathematician not exceeding the age of thirty-two for outstanding contributions in an area of mathematics influenced by the late Indian mathematical genius Srinivasa Ramanujan. Young mathematicians all over the world are eligible for this award. The age limit has been set at thirty-two because Ramanujan achieved so much in his brief life of thirty-two years, and also to encourage doctoral and post-doctoral research.

SASTRA, based in Tanjore in South India, started in 1984 as a college of engineering. It grew considerably in size in areas outside of engineering, attaining the status of a university in 2001. In 2003 SASTRA opened the Srinivasa Ramanujan Center and a branch campus in Kumbakonam, the hometown of Ramanujan, and also purchased Ramanujan's home to maintain it as a museum. In this connection SASTRA conducted an international conference in December 2003 in Kumbakonam. The President of India, Abdul Kalam, inaugurated the conference and declared Ramanujan's home a museum and national treasure. SASTRA will conduct an international conference each year in Kumbakonam during December 20-22 to coincide with Ramanujan's birthday which is on December 22.

Starting in December 2005, the SASTRA Ramanujan Prize will be awarded at each of these annual conferences. The winner will be invited to give a talk at the conference. Nominations for the first SASTRA Ramanujan prize must be made by July 31, 2005. A panel of experts will select the winner from the nominations. The nomination must include the vita of the nominee, some selected papers, three letters supporting the nomination, and be sent to: SASTRA Ramanujan Prize, Department of Mathematics, University of Florida, Gainesville, FL 32611, USA; or to sastraprize@math.ufl.edu. Information is available on the webpage http://www.math.ufl.edu/sastrapri ze/.

—Krishnaswami Alladi, University of Florida

News from The Fields Institute

The Fields Institute for Research in the Mathematical Sciences has announced its thematic program for the 2005-2006 academic year, on renormalization and universality in mathematics and physics and on holomorphic dynamics, laminations, and hyperbolic geometry. The fall program, organized by Pavel Bleher, Mikhail Lyubich, and Michael Yampolsky, will aim to give a broad perspective of applications of renormalization ideas. Activities and dates for the fall program on renormalization and universality follow.


—From a Fields Institute announcement

News from Oberwolfach

The Mathematisches Forschungsinstitut Oberwolfach (MFO) has announced its scientific program for 2006. The new program is available on the website http://www.mfo.de or http://www.oberwolfach.org. There one can also find information on the scheduled weeks for miniworkshops and guidelines for proposals. The deadline for proposals for workshops in 2007 is the end of July 2005.

—MFO announcement
Comments on Female Math Ability Spark Reaction

In remarks made during an economics conference in Cambridge, Massachusetts, in January 2005, Harvard University President Lawrence Summers suggested that innate differences might account for the disparity in men’s and women’s achievement in mathematics and science. After his remarks elicited a torrent of protest, Summers posted an apology on the Harvard website.

Among those registering protest were leaders of several mathematics organizations, including the AMS. Below are excerpts from some of the statements by these leaders.

David Eisenbud, AMS president, and James Arthur, AMS president elect:

The speculations made by Lawrence Summers, President of Harvard University, at a conference on January 14, 2005, about the causes of the current shortage of women in science were inappropriate. His high position at Harvard places on him a high burden of responsibility. His remarks may be damaging and counterproductive to a cause he and all educators should support. We who strive to make our subject areas attractive and accessible to all express our dismay at such remarks.

January 21, 2005

Martin Golubitsky, president, Society for Industrial and Applied Mathematics (SIAM):

The widely reported speculations made by the president of Harvard University, Lawrence Summers, about possible causes of the current shortage of women in science have fueled controversy and had some potentially unfortunate effects. One of the most serious is the possible discouragement of talented women with strong potential for excellence in science, engineering, and mathematics. SIAM’s position is that it is essential to encourage women (as well as men) to pursue studies in science and mathematics. SIAM wishes to emphasize strongly the many outstanding accomplishments of women in mathematics and its applications.

February 2, 2005

Carolyn Gordon, president, Association for Women in Mathematics:

Regarding Lawrence H. Summers's remarks on the underrepresentation of women in mathematics and science, the real news is that despite cultural barriers, women are entering these fields in greater and greater numbers. About a third of all United States citizens who have received Ph.D.’s in mathematics recently are women. About half of all undergraduate mathematics degrees in the United States go to women. Yes, there is still a shortage of women on the mathematics and sciences faculties of many American universities, including Harvard. So universities should hire more of these excellent women and then treat them as if they value them. We call on Lawrence Summers, as well as the leaders of all educational institutions, to take positive action to encourage the influx of women and minorities into mathematics, science, and engineering. [This statement was endorsed by the AWM Executive Committee and appeared as a letter to the editor in the New York Times on January 28, 2005.]

International Council for Industrial and Applied Mathematics (ICIAM):

Recent remarks of the President of Harvard University have led to media speculation that innate differences in the mathematical abilities of men and women make it less likely that women will succeed in science and mathematics. ICIAM does not accept this notion. ICIAM members are well aware that there are many barriers (whether financial, cultural, or practical) that face women who want to pursue mathematical or scientific careers at the highest levels. The unbroken career paths that are typical of successful male careers in mathematics take no account of the specific responsibilities of women related to childbearing and family. As an international organization representing the world’s applied mathematicians, ICIAM is committed to removing the educational inequalities in mathematics that exist in many parts of the world and to improving the access to careers in the mathematical sciences for all men and women.

February 1, 2005

—Allyn Jackson
Login Now Required for Online Notices

Following what has become a standard procedure for many online publications, the AMS has begun requiring users to log into an AMS user account to access the online version of the Notices. The Notices remains freely available to all, and setting up an online account does not require membership in the AMS. This change was made primarily as a way to emphasize to online Notices readers that the worldwide availability of the online Notices is supported by dues paid by the AMS membership.

—Ally Jackson

AMS Holds Workshop for Department Chairs

The AMS hosted a one-day workshop for mathematical sciences department chairs at the 2005 Joint Mathematics Meetings in Atlanta, Georgia. This year’s workshop focused on a number of areas of importance to department chairs including faculty evaluations, strategic planning, effective use of resources, and accountability. Over thirty department chairs and leaders came together to share ideas and experiences in a form of “department chair therapy”, thus creating an environment that enabled attendees to address departmental matters from new perspectives.

Workshop leaders included Krishnaswami Alladi, department chair of mathematics, University of Florida; Deanna Caveny, department chair of mathematics, College of Charleston; Peter March, department chair of mathematics, Ohio State University; and Robert Olin, dean of arts and sciences, University of Alabama-Tuscaloosa.

The Department Chairs Workshop is an annual event hosted by the AMS prior to the start of the Joint Meetings. Past workshop sessions have focused on a range of issues facing departments today, including personnel issues (staff and faculty), long range planning, hiring, promotion and tenure, budget management, assessments, outreach, stewardship, junior faculty development, communication, and departmental leadership.

If you are interested in attending a future workshop, please look for registration information sent out in advance of the Joint Meetings or contact the AMS Washington office at amsdc@ams.org.

—AMS Washington office

Deaths of AMS Members

Hayk Badalyan, professor, Yerevan State University, Armenia, died on October 27, 2004. Born on June 22, 1915, he was a recent member of the Society.

Malcolm K. Brachman, president, Northwest Oil, Dallas, died on January 11, 2005. Born on December 9, 1926, he was a member of the Society for 47 years.

Vincent McBrien, retired, Harvard University, died on February 2, 2005. Born on April 21, 1916, he was a member of the Society for 60 years.

Marilyn Molloy, professor, Our Lady of the Lake University, San Antonio, died on March 16, 2003. Born on April 24, 1931, she was a member of the Society for 36 years.

Donald R. Morrison, professor emeritus, University of New Mexico, Albuquerque, died on September 5, 2004. Born on May 3, 1922, he was a member of the Society for 57 years.

George W. Peglar, professor emeritus, from San Francisco, died on January 7, 2005. Born on September 2, 1922, he was a member of the Society for 51 years.

Werner Schwamburg, of Bochum, Germany, died on January 3, 2005. Born on June 9, 1926, he was a member of the Society for 7 years.

Jozef Joachim Telera, professor, Polish Academy of Sciences, Warsaw, died on January 28, 2005. Born on March 24, 1943, he was a member of the Society for 13 years.
Reference and Book List

The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices
The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.ou.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines
April 15, 2005: Nominations for Maria Mitchell Women in Science Award. See http://209.68.19.123/museums/wnmInsc.php, or contact the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 4 Vestal Street, Nantucket, MA 02554; telephone 508-228-9198.


May 31, 2005: Registration for International Mathematics Competition for University Students. See http://www.imc-math.org or contact John E. Jayne, Department of Mathematics, University College London, Gower Street, London WCIE 6BT, United

Where to Find It
A brief index to information that appears in this and previous issues of the Notices.

AMS Bylaws—November 2003, p. 1283
AMS E-mail Addresses—December 2004, p. 1365
AMS Ethical Guidelines—June/July 2004, p. 675
AMS Officers 2004 and 2005 (Council, Executive Committee, Publications Committees, Board of Trustees)—May 2005, p. 564
AMS Officers and Committee Members—October 2004, p. 1082
Conference Board of the Mathematical Sciences—September 2004, p. 921
Information for Notices Authors—June/July 2004, p. 670
Mathematics Research Institutes Contact Information—August 2004, p. 810
National Science Board—January 2005, p. 76
New Journals for 2003—June/July 2004, p. 672
NRC Board on Mathematical Sciences and Their Applications—March 2005, p. 361
NRC Mathematical Sciences Education Board—April 2005, p. 465
NSF Mathematical and Physical Sciences Advisory Committee—February 2005, p. 261
Program Officers for Federal Funding Agencies—October 2004, p. 1078 (DoD, DoE); December 2004, p. 1368 (NSF)
Kingdom; telephone +44-20-7679 7322; fax +44-20-7419 2812; email: j.jayne@imc-math.org.

June 1, 2005: Applications for fall program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See http://www7.nationalacademies.org/policyfellow/or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 5th Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.


July 31, 2005: Nominations and applications for the Monroe H. Martin Prize. Contact R. Roy, Director, Institute for Physical Science and Technology, University of Maryland, College Park, MD 20742-2431.


October 1, 2005: Nominations for Lucien Godeaux Prize, Contact J. Aghion, c/o Secretariat of the Royal Society of Sciences of Liege, Institute of Mathematics of the University of Liege, 12 Grande Traverse, Sart Tilman Bat. B37, B-4000 Liege 1, Belgium; email: jaghion@ulg.ac.be.


Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers’ attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to “Book List” since the list’s last appearance.


Reference and Book List


* The Road to Reality: A Complete Guide to the Laws of the Universe, by
About the Cover
Extreme 3D visualization
The background image of this month's cover is a photograph included by Jonathan Borwein and David Bailey, perhaps somewhat whimsically, in their article on experimental mathematics. The photograph was taken for a publicity brochure for the now defunct New Media Innovation Centre in downtown Vancouver, British Columbia, an organization partially sponsored by Simon Fraser University, to which Borwein is affiliated. The two young men, who are graduate students in the department of Electrical and Computer Engineering at the University of British Columbia, are in a kind of box with what might be called surround-projection. The approximate spheres are displayed in duplicate at rapidly alternating times in synchronization with the goggles they are wearing, so that what they see is a simulated 3D image, not just the flat projections on the walls on their box. The projections are interactive, controlled by input through a key pad held by Timothy Chen, the student on the right. The project the students are involved in is part of Mr. Chen’s student work at U. B. C. What is being projected is a flow field of spheres in a cylinder with various obstacles interactively superimposed into the flow. The inset photographs are screen displays produced by Mr. Chen from the same project.

It’s hard to imagine exactly what role such high end visualization technology will play in mathematical research, but not impossible. One likely application for similar, but not quite so sophisticated, display systems might very well be in public presentations. The effects can be spectacular.

Brian Corrie of Simon Fraser University provided us with the digital version of the background photograph.

—Bill Casselman, Graphics Editor
(notices-cover@ams.org)

Reference and Book List


Dear Friends and Colleagues,

During 2004 your generous support helped the Society and our profession in many ways. I thank each of you for that support.

The Young Scholars program is in its fifth year, supporting summer workshops for talented high school students—the future of our profession. We are building an endowment, the Epsilon Fund, to support this program indefinitely, and we hope to reach our goal of two million dollars over the next few years. Young scholars programs work and supporting them is important for mathematics.

The Centennial Fellowships play a key role for outstanding young mathematicians at the formative stages of their careers, from three to twelve years beyond the degree. The fellowships are funded directly by contributions from mathematicians throughout the world.

We use contributions to the General Fund to support all of our activities, including survey work, public awareness, and outreach to mathematicians in the developing world.

Your generosity allows the Society to carry out all these programs and shows that mathematicians care deeply about our profession. Thank you for that expression of caring.

John H. Ewing

Thomas S. Fiske Society

The Executive Committee and Board of Trustees have established the Thomas S. Fiske Society to honor those who have made provisions for the AMS in their estate plans. For further information contact the Development Office at 800-321-4AMS, or development@ams.org.

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Henry M. Schaefer
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Steven H. Weintraub

Bequests Received

Barbara J. Beechler
Gifts in Memory and Gifts in Honor

The American Mathematical Society welcomes gifts made in memory or honor of members of the mathematical community or others. Unless directed toward a special fund or program, such gifts are used to support the general mission of the Society.

Gifts were made in memory of the following individuals:
Maurice Auslander by Bernice L. Auslander
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Gifts were made in honor of the following individuals:
Mike Breen by Rhode Island College
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£ Donors who have given to the AMS Epsilon Fund, the endowment for the support of young scholars programs.

The names of donors who have given $1,000 or more in a single year are affixed to a plaque that is prominently displayed in the Society's headquarters.

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Except for the members at large of the Council, the month and year of the first term and the end of the present term are given. For members at large of the Council, the last year of the present term is listed.

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Jean E. Taylor 2/03-1/08
Carol S. Wood 2/02-1/07
Mathematics Calendar

May 2005

*2-4 2nd Symposium on Networked Systems Design and Implementation (NSDI'05), Boston Park Plaza Hotel, Boston, Massachusetts.

Description: NSDI '05 is a symposium focused on the design principles of large-scale distributed and networked systems. We believe systems as diverse as scalable Web services, peer-to-peer file sharing, sensor nets, and distributed network measurement share a set of common challenges. Progress in any of these areas requires a deep understanding of how researchers are addressing the challenges of large-scale systems in other contexts. Our goal is to bring together researchers from across the systems community—including operating systems, distributed systems, and computer networking—to foster a cross-disciplinary approach to addressing our common research challenges. NSDI will provide a high-quality, single-track forum for presenting new results and discussing ideas that overlap these disciplines.

Information: http://www.usenix.org/nsdi05@email:conference@usenix.org; tel: 510-528-8649.

*7-9 Nebraska Commutative Algebra Conference: WiegandFest, The University of Nebraska-Lincoln, Lincoln, Nebraska.

Description: The conference is open to anyone working in commutative algebra. The purpose is for recent Ph.D.s and graduate students in the field to meet experienced researchers, to learn about recent developments in commutative algebra, to exchange ideas with one another, and to develop collaborations for future research. This gives us in the commutative algebra community a wonderful opportunity to celebrate the many important contributions of Roger and Sylvia Wiegand. The primary topics will be aspects of representation theory, homological algebra, computational algebra, and ideal theory that interact with commutative algebra.

Support: We have some support for travel and local expenses from the UNL Department of Mathematics; additional support from the National Science Foundation is anticipated. In the allocation of NSF funds, particular consideration will be given to advanced graduate students, recent Ph.D.s, and members of underrepresented groups. In order to be considered for support you need to register.

Deadline: Deadline of registration is extended to April 15.

Information: For more information, contact the organizers: Luchezar L. Avramov; email: avramov@math.unl.edu. David A. Jorgensen: email: djorgena@uta.edu; tel: 817-272-5507. Aihua Li; email: 11a@mail.montclair.edu; tel: 973-655-7271. Conference web site: http://dreadnought.uta.edu/~dave/wiegandfest.html.

*12-14 Groups in Galway 2005, National University of Ireland, Galway, Ireland.

Information: The annual conference 'Groups in Galway' will be held at National University of Ireland, Galway, this year on 12-14 May. The current conference is in honour of Professor Martin Newell. The scope of the conference covers all areas of group theory, applications, and related fields.

Speakers: Rex Dark (NUl, Galway), Warwick de Launey (Center for Communications Research, USA), Colin Campbell (St Andrews), Ailla Detinko (NUl, Galway/Belarus), Franciscos de Giovanni (Università di Napoli, Hermann Heinzeke (Universität Wuerzburg), Luise-Charlotte Kappe (SUNY), Tom Laffey (UC Dublin), John Mc Dermott (NUl, Galway), Des MacHale (UC Cork), Mike Newman (Australian National University), Rachel Quinlan (UC Dublin/Galway), Stewart Stonehewer (Warwick).

Registration and Contact Information: All who are interested are invited to attend. There is no formal registration (and in respect to participation in the meeting, this fact should be noted). All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the Notices in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the Notices prior to the meeting in question. To achieve this, listings should be received in Providence eight months prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the Notices. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.
particular no registration fee), although it would be appreciated if intending attendees send an email to one of the organizers (addresses below). Details of the talks and their scheduling will be posted at http://www.maths.nuigalway.ie/gig05.html closer to the event. For further information, please contact one of the conference organizers, Jim Cruickshank (james.cruickshank@nuigalway.ie) or Dare Flannery (dane.flannery@nuigalway.ie).

* 13-15 1st KAIST International Symposium on Enhancing University Mathematics Teaching, Korea Advanced Institute of Science and Technology, Daejeon, Korea.

Program: The Department of Mathematics of the Korea Advanced Institute of Science and Technology announces its initial international symposium on the enhancement of the teaching of mathematics in universities and colleges. This symposium provides an opportunity for university and college faculty to give presentations of their successful approaches to teaching undergraduate mathematics. Presentations may include teaching styles, uses of technology, successful classroom demonstrations, and much more. Hands-on workshops will highlight specific methods. There are specially designed sessions for graduate students, with presentations both by and for teaching assistants.

Coordinators: Young Han Choe, Deane Arganbright.

Abstract Deadline: April 1, 2005.

Information: The following symposium Web page provides details for submissions and registration, together with the program outline, fees, featured speakers, contacts, and other useful information: http://math.kaist.ac.kr/2005.

* 16-20 Mini-invasive Procedures in Medicine and Surgery: Mathematical and Computational Challenges, Centre de recherches mathematique, Montreal, Quebec, Canada.

Information: Workshop and Spring School. The objective of this tandem School-Workshop is to explore several facets of mini-invasive procedures in medicine and surgery, identity issues, problems, trends, and mathematical and computational challenges in their modeling, simulation, and integration in the medical and surgical processes. It will be structured around the following themes: medical imaging and geometrical modeling, fluid-structure interactions in health problems, static/dynamical processes in medicine and surgery, identify issues, problems, trends, and mathematical and computational challenges in their modeling, simulation, and integration in the medical and surgical processes. It will be structured around the following themes: medical imaging and geometrical modeling, fluid-structure interactions in health problems, static/dynamical design and control of (implantable) medical devices, finite element based computer aided design/manufacturing.

Information: http://www.crm.umontreal.ca/Med05/.

* 23-June 3 11th European Intensive Course on Complex Analysis and its Generalizations (with applications to partial differential equations), University of Coimbra, Coimbra, Portugal.

Program: Week 23-27: Matrix Differential Riccati Equations - Gerhard Junk (Aachen University, Germany); Orthogonal Polynomials on the unit circle. From Gabor Szego to Barry Simon - Francisco Marcellan Espanol, (Universidad Carlos Ill de Madrid, Spain); Week 30-3: Monogenic, hypermonogenic and holomorphic Clifford functions - Eric Lehman (Universite de Caen, France); Transform Analysis: The Hilbert Transform - Fred Brackx (University of Ghent, Belgium); Transform Analysis: The Continuous Wavelet Transform - Nele De Schepper (University of Ghent, Belgium).

Deadlines: There is no deadline for the registration. Living expenses can be partially covered for some Ph.D. students if they do not have support from their own institution and if there is enough money available.

Organizer: H. Maloney, A. Branquinho, J. Carvalho e Silva, P. Cerejeiras.


* 29-June 4 (REVISED) Spring School in Analysis: Function Spaces and Applications, Paseky nad Jizerou, Czech Republic.

Topics: Hardy Operators, Function Spaces, Embeddings, Whitney Extension Problem, Helly’s intersection theorem.

Main speakers: W. Desmond Evans (University of Wales, Cardiff, U.K.); Pavel Shvartsman (Technion, Haifa).

Organizers: Jaroslav Lukes, Lukas Pick (Charles University, Prague, Czech Republic).


June 2005

* 1-3 Workshop on PDE and Harmonic Analysis, Norwegian University of Science and Technology, Trondheim, Norway.

Aim: The aim is to bring together, in an informal, workshop setting, a small group of researchers working on problems related to PDE and Harmonic Analysis. Ph.D. students are also very welcome.

Organizing committee: S. Selberg, E. Malinnikova, P. Hasto.


* 1-4 Classics in PDE: A meeting in Honor of Nina Nikolaevna Uraltseva 70th Birthday, KTH, Stockholm, Sweden.

Organizing Committee: Ari Laptev, Gregory Seregin, Henrik Shahgholian.

Tentative list of speakers: D. Apushkinskaya, Saarbruecken; A. Arkhipova, St. Petersburg; H. Brezis, Paris; L. A. Caffarelli, Austin, Texas; A. Chang, Princeton; M. J. Esteban, Paris; A. Friedman, Ohio; L. Gamba, Austin, Texas; M. Giaquinta, Pisa; A. Grigor'yan, London; C. Kenig, Chicago; L. Kondrat'ev, Moscow; A. Nazarov, St. Petersburg; L. Nirenberg, NY; A. Petrosyan, West Lafayette; G. S. Weiss, Tokyo; Y. Zhikov, Russia.

Information: Henrik Shahgholian, chair, email: henrik@math.kth.se; Marie Lundin, Administrative Secretary, email: malund@math.kth.se.

* 16-27 Mini-invasive Procedures in Medicine and Surgery: Mathematical and Computational Challenges, Centre de recherches mathematique, Montreal, Quebec, Canada.

Objective: Of this tandem School-Workshop is to explore several facets of mini-invasive procedures in medicine and surgery, identify issues, problems, trends, and mathematical and computational
Tentative Program: A preliminary version of the detailed Program will be posted in the web page until May 23.

**General Information:** Indications such as "Technical support! "How to arrive to..." and others will be posted on the web page until May 16.


**Organizers:** H. Malonek, A. Branquinho, J. Carvalho e Silva, P. Cerejeiras.


**Organizer:** Institute of Mathematics, Vietnamese Academy of Science and Technology, and Quy Nhon University.

**Sponsors:** Institute of Mathematics, Vietnamese Academy of Science and Technology, National Basic Research Program in Natural Science, Quy Nhon University.

**Topics:** ODE and Dynamical Systems, Wavelet Analysis, p-adic Analysis, PDE and Microlocal Analysis, Numerical Analysis, Complex Analysis, Harmonic Analysis, Fractional Analysis, Stochastic Analysis, Nonlinear Analysis and Differential Geometry, Ill-posed and Inverse Problems.

**Information:** Please send the Registration Form to Dr. Nguyen Huu Dien, Institute of Mathematics, VAST 18 Hoang Quoc Viet Road, CauGlay District, 10307, Hanoi Vietnam; email: nahdien@math.ac.vn; [http://www.math.ac.vn/conference/icaaa2005/](http://www.math.ac.vn/conference/icaaa2005/).

*5-9 Representation Theory, Geometry and Automorphic Form. International Conference in honor of J. Bernstein's 60th birthday, Tel-Aviv University, Tel-Aviv, Israel.*

**Organizers:** Alexander Berllinson, Alexander Braverman, Dennis Gaitsgory, Alexander Goncharov, David Kazhdan, Wilfried Schmid, Marie-France Vigneras.


**Information:** For more information as well as for support applications please contact Alexander Braverman (braval@math.huji.ac.il) or Dennis Gaitsgory (gaitsgde@math.uchicago.edu).

*5-July 21 Joint Summer Research Conference, Snowbird Resort, Snowbird, Utah.*

**Information:** [http://www.ams.org/meetings/arc.html](http://www.ams.org/meetings/arc.html).

*10-11 CAM 2005, University of Central Oklahoma, Edmond, Oklahoma.*

**Workshop Topic:** A cryptology workshop designed for the non-specialist.

**Organizers:** Dr. Jesse Byrne, 100 N University Drive, Edmond, OK 73034; email: jbyrne@ucok.edu; tel: 405-974-5575 and Dr. Charlotte Simmons, 100 N University Drive, Edmond, OK 73034; email: csimmons@ucok.edu; tel: 405-974-5316.

**Speakers:** Confirmed speakers: D.J. Bernstein (University of Illinois at Chicago), Robert Lewand (Goucher College), and Alice Silverberg (University of California at Irvine).

**Travel Funds:** This workshop is supported by the NSA. Travel funds are available to defray the expenses of participants. Graduate students, junior faculty, women, minorities, and persons with disabilities are especially encouraged to participate and to apply for support. Early application for support is encouraged.

**Information:** [http://www.math.ucok.edu or contact an organizer.](http://www.math.ucok.edu)

*10-15 Summer School on Harmonic, Wavelet, P-adic analysis, Quy Nhon, Vietnam.*

**Description:** Continuation of the Second International Conference on Abstract and Applied Analysis ICAAA 2005; see June 4-9, 2005.


**Workshop Topics:** Recent development in inverse problems, multiscale analysis, and homogenization reveals that these fields share several fundamental concepts in common. The main purpose of this workshop is to bring together researchers coming from these fields to share their new ideas and to earn benefit from others different viewpoints.

**Invited Speakers:** Elena Beretta, Eric Bonnetier, Yves Capdeboscq, Andrej Cherkaev, Soo Young Chung, Mathias Fink, Leslie Greenberg, Tomas Hou, Masaru Ikehata, Jun Yub Lee, Mikiyoung Lim, Graeme Milton, Gen Nakamura, Jin Keun Seo, Gunther Uhlmann, Michael Vogelius.

**Information:** [http://www.math.snu.ac.kr/bk21/workshop/](http://www.math.snu.ac.kr/bk21/workshop/).


**Dedication:** To the 60th Anniversary of Academician Andrei Izmailovich Subbotin (February 16, 1945-October 14, 1997).

**Description:** It is planned to discuss the present state of control theory and theory of generalized solutions of Hamilton-Jacobi equations.

**Topics:** Scope of the seminar include: Generalized solutions of Hamilton-Jacobi equations. Control of dynamic systems under conditions of conflict and uncertainty. Problems of estimation and identification in dynamic systems. Inverse problems and distributed control systems. Numerical algorithms for constructing solutions of optimal control problems and boundary value problems for Hamilton-Jacobi equations.

**Organizers:** The Ural Branch of the Russian Academy of Sciences, the Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, and the Ural State University.


**Languages:** Russian and English.


*27-30 14th International Scientific Congress CNIC 2005: 40 Years at the Service of Science and Technology, Havana, Cuba.*


**July 2005**

*4-8 MODELLING 2005- Third IMACS Conference on Mathematical Modelling in Applied Sciences and Engineering, University of West Bohemia, Pilsen, Czech Republic.*

**Information:** The event is organized by Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia in Pilsen, Czech Republic, in cooperation with other Czech universities and research institutions in the week preceding the 17th IMACS World Congress IMACS 2005 in Paris.

**Purpose:** The purpose of the Conference is to stimulate research and, in an informal setting, to foster the interaction of researchers in different fields of science.

**Topics:** Computational modelling in general, Computer models in fluid dynamics and biomechanics, Computer models in mechanics engineering, Optimal design problems and structural optimization, Multiscale modelling, Qualitative properties of nonlinear boundary value problems, Advanced numerical methods related to the above fields.

**Invited Speakers:** A. Cepek, Czech Technical University, Czech Republic; Z. Dostal, Technical University Ostrava, Czech Republic; C. Farhat, Stanford University, USA; R. Glowinski, University of Houston, USA; J. Jarusek, Mathematical Institute, Academy of Sciences of the Czech Republic; S. Korotov, University of Jyvaskyla, USA.
Mathematics Calendar

Finland, Y. Kuznetsova, University of Houston, USA, J. Lopez-Gomez, Universidad Complutense de Madrid, Spain, J. Malík, Institute of Geonics, Academy of Sciences of the Czech Republic, J. Mandel, University of Colorado at Denver, USA, B. Miara, ESIIE, France, U. Sautangazin, Space Research Institute, Republic of Kazakhstan, M. Tabata, Kyushu University, Japan, M. Vrty, Mathematical Institute AS CR, Czech Republic, R. Van Keer, University of Gent, Belgium.

Website: The MODELLING website has now been opened up at http://www.modelling.zcu.cz/ and more details may be found there.

* 6-23 35th Saint-Flour Probability Summer School, Saint-Flour, France.
Information: http://math.univ-bpclermont.fr/stflour/.

Aim: The aim of this workshop is to bring together the many disparate groups of researchers who work on coagulation-fragmentation (CF) equations and related processes. A key feature will be the exchange of ideas between theoreticians and practitioners involved with CF equations. This should promote a greater awareness of existing results and outstanding problems in this area and provide mathematicians with new mathematical challenges and experimental scientists, engineers and industrialists with new analytical, modelling, and numerical techniques.
Organizing Committee: Fernando da Costa (CAMGSD, Institute Superior Técnico, Lisboa). Michael Grinfeld (Mathematics, University of Strathclyde), Wilson Lamb (Mathematics, University of Strathclyde), Jonathan Wattis (Mathematical Sciences, University of Nottingham). The meeting is organized in collaboration with the Centro de Análise Matemática, Geometria e Sistemas Dinâmicos (CAMGSD) of Institute Superior Técnico, Lisboa, Portugal.
Deadlines: Those wishing to contribute a talk and/or apply for financial support should complete their application by May 2, 2005. Applications will not be accepted after June 6th which is also the date by which payment of the registration fee must be made in order to secure a place. Online registration is available at the meetings' website.
Information: http://www.icms.org.uk/meetings/2005/coagfrac/

* 7-9 OTFUSA2005: Conference on Operator Theory, Function Spaces and Applications—Dedicated to the 60th birthday of Frank-Olme Speck. Department of Mathematics, University of Aveiro, Aveiro, Portugal.
Description: To celebrate the 60th birthday of Professor Frank Speck and to bring together those enrolled in the research activities related with operator theory, function spaces and related applications.
Topics: Convolution type operators and related classes of singular operators, Bessel potential and pseudo-differential operators, factorisation theory, operator relations and normalisation problems; The theory of function spaces and distributions around Lebesgue, Lorentz, Sobolev, Besov and Triebel-Lizorkin spaces, embeddings, interpolation, traces and extensions, representation formulas, oscillation; Applications to mathematical physics, wave diffraction and scattering theory, elliptic boundary value problems, mixed problems in canonical domains, localisation, interface problems, boundary integral methods, boundary-domain methods, explicit solutions, regularity, singularities, fractal analysis and asymptotic behaviour.

* 10-17 Dimitriana Summer School on Stochastic Differential Geometry and Applications in Finance, Thermo Aitolokarnianias, Greece.
Program: Designed for graduate students of mathematics with a background in stochastic analysis. Previous knowledge of differential geometry (manifold theory) is desirable but not necessary. Topics covered include Brownian motions and stochastic equations on manifolds, interest rate curves and stochastic movements on manifolds, Riemannian Brownian motion? connection? stochastic flows, martingale fields.
Sponsors: Under the auspices of the University of Patras, Greece, and exclusive sponsoring of the National Bank of Greece.
Lecturers: Freddy Delbaen, David Ellworthy, Michel Emery, Damir Filipovic, Yue-Mei Li, Marc Yor.
Funding: Available for all participants.
Information: A. Arvanitoyeorgos (arvanit@math.upatras.gr), V. Papakonstantinou (papakon@math.upatras.gr), http://www.math.upatras.gr/dimitrana.

* 18-24 Methods of Logic in Mathematics 2005, Euler International Mathematical Institute, St. Petersburg, Russia.
Topics: Its main themes include computability theory, model theory, set theory and their connections with other areas of mathematics, most notably algebra, arithmetic, computation and geometry. The meeting will consist of invited talks, contributed paper sessions and tutorials. Two of the tutorials will be given by Simon Thomas (Rutgers) and Zili Sela (Hebrew University).
Invited Speakers: Include M. Arslanov (Kazan), Su Gao (North Texas), E. Gordon (Eastern Illinois), E. Griffor (Michigan), W. Hodges (QMC London), T. Hyttinen (Helsinki), J. Iovino (San Antonio), L. Kalimullin (Kazan), A. Katchatchev (Omsk), K. Selma (Hebrew University), R. Soare (Chicago), S. Thomas (Rutgers), M. Viljanen (Helsinki), A. Weiermann (Munster, Utretch), B. Zilber (Oxford), Yi Zhang (Guangzhou).
For more information contact Edward Griffor by email at egriffor@uiuc.edu or Elena Novikova at novikova@pdmi.ras.ru.

* 25-August 12 Summer Research Institute on Algebraic Geometry, University of Washington, Seattle, Washington.
Information: email: rba@ams.org.

August 2005

* 2005 Mathematical Modeling of Infectious Diseases: Dynamics and Control, Institute for Mathematical Sciences, National University of Singapore, Singapore 118402.
Information: KP Chua, Administrative Officer, Institute for Mathematical Sciences, National University of Singapore, 3 Prince George's Park, Singapore 118402; tel: (+65) 6874 1893; Facsimile: (+65) 6873 8292; http://www.imas.nus.edu.sg.

* 5-11 4th USENIX Security Symposium, Baltimore, Maryland.
Description: The USENIX Security Symposium brings together researchers, practitioners, system administrators, system programmers, and others interested in the latest advances in security of computer systems.
Information: Contact: Conference Department, email: conference@usenix.org; tel: 510-528-8649; http://www.usenix.org/sec05/.

* 1-9 XVI Coloquio Latinoamericano de Algebra, Colonia, Uruguay.
Description: This biannual event is the premier conference linking algebraists and algebraic geometers from all of Latin America.
Topics: Besides the plenary talks and general courses this meeting will have seven thematic parallel sessions on the following topics: Commutative Algebra and Algebraic Geometry, Non-associative Algebras and Ring Theory, Group Theory, Hopf Algebras and Algebraic Combinatorics, Homological Methods and Representation Theory, Number Theory, Operator Algebras. A special session on Applications of Algebra will also be held.
Speakers: A list of the confirmed speakers includes the following:

Organizing and Scientific Committee: Walter Ferrer Santos (Co-ord.), Gerardo Gonzalez-Sprinberg, Alfredo Jones, Alvaro Rittatore, Andrea Solotar.

Deadline: May 1st, 2005.
Information: http://www.cmat.edu.uy/cmat/eventos/16clia/en; Walter Ferrer: email: wrferrer@cmat.edu.uy

Organizers: Wee Teck Gan, Steven Kudla, and Yuri Tschinkel.
Workshop Topics: This workshop, sponsored by AIM and the NSF, will consider some recent applications of Eisenstein series to problems in arithmetic geometry and number theory. A central goal of the workshop will be to try to understand the common structural properties of the Eisenstein series occurring in applications.

List of Speakers: K. H. Karlsen (CMA/Uuniversity of Oslo), P. Laurence (Univ. of Rome), M. Avellaneda (New York Univ.), R. Oskesental (Univ. of Oslo), J-P. Fouque (North Carolina State Univ.), K. Solna (Univ. of California, Irvine), N. Touzi (ENSEA), C. Schwab (Swiss Fed. inst. of Tech., Zurich), M. Jonsson (Royal Inst. of Tech.), T. Zariphopoulou (Univ. of Texas at Austin), A. Ilhan (Princeton Univ.), D. Talay (INRIA), J. Tysk (Uppsala Univ.), W. Schachermayer (Vienna Univ. of Tech.), L. Karatzas (Columbia Univ.), H. M. Soner (Koch Univ.), H. Pham (Jussieu), J. Teichmann (Vienna Univ. of Tech.), S. Howison (Univ. of Oxford).
Information: Contact: Teitur Arnarson, Dept. of Math, KTH, 100 44 Stockholm, Sweden; email: teitur@math.kth.se.

*28-September 2 5th Conference on Differential Geometry, Mangalia, Romania.
Invited Participants: Geometers; Members of BSG-AMS, EMIS.
Institutional Organizers: Balkan Society of Geometers, University Politehnica of Bucharest-Department of Mathematics, University of Bucharest-Faculty of Mathematics and Informatics, Society of Mathematical Sciences from Romania, Callatis High School of Mangalia.
Deadlines: Submission of applications: June 20, 2005. Selected scientific papers will be published in BSG journals.
Coordinators: Constantin Udriste, Gabriel Pinopol.
Information: email: ybalan@mathem.pub.ro, udriste@mathem.pub.ro (scientific); callatis@callatisdl.net (educational).

*29-31 Algebraic Methods and Applications in Dynamical Systems-Special session in the 5th IASTED International Conference on Modeling, Simulation, and Optimization (MSO 2005), Oranjested, Aruba.
Description: This special session will focus on the algebraic methods and applications in continuous or discrete dynamical systems. In recent years, algebraic techniques are increasingly applied to many real world problems producing dynamic systems. Through the presentations we propose to identify common themes and recent developments regarding algebraic approach to solving dynamical system problems. Furthermore, this session will provide participants opportunities to interact and establish research connections with each other. The deadline to submit your paper to this special session is April 15, 2005. Please send your paper directly to Aihua Li, the organizer. Email submission is preferred. The address is: liahua@montclair.edu. Please see the conference web site for instruction.
Organizer: Aihua Li, Department of Mathematical Sciences, Montclair State University, 1 Normal Avenue, Montclair, NJ 07043, USA. Office phone: 973-655-7271.
Information: http://www.iasted.org/conferences/2005/aruba/c471.htm

*29-September 2 Numerical Methods for Optimal Control in High Dimensions, AIM Research Conference Center, Palo Alto, CA.
Organizers: Doron Levy, Ian Mitchell, and Adam Oberman.
Workshop Topics: This workshop, sponsored by AIM and the NSF, will be devoted to problems of optimal control, broadly interpreted to include stochastic control problems and differential games. It is a standard practice to formulate these problems in terms of a multi-dimensional Hamilton-Jacobi-Bellman (HJB) equation. The workshop will focus on computational methods for tackling high dimensional HJB and related equations.
Application Deadline: May 29, 2005.

September 2005

*5-9 10th International Workshop on Functional Analysis, Liege, Belgium.
Description: This Functional Analysis Conference is a joint venture of the University of TRIER (Germany) and of the University of Liege (Belgium); it is organized on the occasion of the 65th birthday of Professor Jean Schmets (University of Liege).
Organizers: Francoise Bastin (University of Liege, Belgium), Su­samne Dierolf (University of TRIER, Germany), Jochen Wengenroth (University of TRIER, Germany).

*12-17 International Conference on Mathematical Analysis of Random Phenomena, Hotel Abou Nawas Hammamet, Hammamet, Tunisia.
Inscription deadline: June 15, 2005.

Mathematics Calendar

Hydrodynamics, mechanics of gas and plasma. Information technologies. Program of Conference consists of plenary (40 minutes）and sectional (20 minutes）talks. Working languages of Conference are Kazakh, Russian and English.


Organizing Committee: Postal address: Institute of Mathematics, Pushkin str., 125, Almaty, 050010, Kazakhstan; tel: +7(3272)913764; fax: +7(3272)913764; email: im40@math.kz; http://www.math.kz/confer.htm.


Information: Contact Information: Secretary ICCMSE 2005 (Mrs. Eleni Ralli-Simou); email: iccmse@uos.gr, Postal Address: 26 Meneou Street, Amithrea Paleon Faliron, GR-175 64, Athens, Greece, Fax: +30210 94 20 091 or + 30 2710 237397; http://www.uos.gr/~iccmse/.

October 2005

7—8 Twenty-Fifth Annual Southeastern-Atlantic Regional Conference on Differential Equations, University of Dayton, Dayton, Ohio.

Organizer: Muhammad N. Islam.

Principal Speakers: T. A. Burton (Northwest Research Institute, Washington, Retired from Southern Illinois University), Functional Differential Equations; Srdjan Stojanovic (University of Cincinnati, Ohio), Nonlinear Partial Differential Equations—methods in Financial Mathematics; Avner Friedman (Ohio State University, Ohio), Partial Differential Equations (tentative); Konstantina Trivisa (University of Maryland), Nonlinear Partial Differential Equations and Applied Mathematics. (tentative)

In addition to the principal speakers, there will also be sessions of twenty minute contributed talks. Funding pending from the National Science Foundation, travel support funds will be available for advanced graduate students and recent Ph.D. recipients. Women and minority participants are especially encouraged to participate in this conference and to apply for support.

Information: http://academic.udayton.edu/search-cde-25/. To get instructions on registration, lodging, submission of abstracts, and application for support. Please pass this announcement along to all who might be interested in participating in the conference. If you have questions about the conference, please send e-mail to searchde25!lnotes.udayton.edu or call Muhammad Islam (937-229-2109) or Mark Oxley (937-255-3636 ex 4515).

20—22 3rd Symposium on Stochastic Algorithms: Foundations and Applications (SAGA’05), Moscow State University, Moscow, Russia.

Scope: The 3rd Symposium on Stochastic Algorithms, Foundations and Applications (SAGA’05) will be held in Moscow (Russia) from 20th to 22nd October 2005. The symposium offers the opportunity to present research in the analysis, implementation, experimental evaluation and real-world application of stochastic algorithms. In particular, the focus of SAGA’05 is on new algorithmic ideas involving stochastic decisions and the design and evaluation of stochastic algorithms within realistic scenarios. Thus, the symposium wants to foster the co-operation between practitioners and theoreticians from this research area.

Information: Further information can be found at http://math.nsc.ru/departmen/cma/SAGA2005 or http://www.ams.kcl.ac.uk/events/saga05/.


Information: The Call for Presentations deadlines for G005 are fast approaching.

Deadline Dates: Minisymposium proposals: April 1, 2005; Abstracts for all contributed and minisymposium presentations: May 2, 2005.

Conference Webpage: http://www.siam.org/meetings/g005/.

For additional information, contact SIAM Conference Department at meetings@siam.org.

November 2005

3—4 (NEW DATE) DIMACS Workshop on The Epidemiology and Evolution of Influenza, DIMACS Center, CoRE Bldg, Rutgers University, Piscataway, New Jersey. (Oct. 2004, p. 1969)

December 2005

5—9 30th Australasian Conference in Combinatorial Mathematics and Combinatorial Computing (30ACCMCC), The University of Queensland, Brisbane, Australia.

Invited Speakers: Simon Blackburn, Royal Holloway, University of London, U.K.; Matthew Brown, The University of Adelaide, Australia; Mike Gannell, The Open University, U.K.; Lily Khadjavi, Loyola Marymount University, U.S.A.; Curt Lindner, Auburn University, U.S.A.; Brendan McKay, The Australian National University, Canberra; Wal Wallis, Southern Illinois University, Carbondale, U.S.A.

Contributed talks are welcome in all areas of combinatorics, graph theory, combinatorial computing and applications.

Deadline: A closing date for abstracts and registration will be announced later; this will be around late October 2005.


Email the Director at sbjmaths.uq.edu.au for further information, or check the conference web page.

January 2006

12—15 Joint Mathematics Meetings, San Antonio, Texas.


April 2006

1—2 AMS Southeastern Section Meeting, Florida International University, Miami, Florida.


8—9 AMS Central Section Meeting, University of Notre Dame, Notre Dame, Indiana.


22—23 AMS Eastern Section Meeting, University of New Hampshire, Durham, New Hampshire.


29—30 AMS Western Section Meeting, San Francisco State University, San Francisco, California.


The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2006

12—15 2006 International Conference on Applied Mathematics and Interdisciplinary Research-Nankai, Nankai University, Tianjin, P. R. China.

Information: The website of the meeting is at http://www.isan.nankai.edu.cn. Please check out the website for further information.


Contact and Information: ICNPAA-2006, S. Sivasundaram, 104 Snow Goose Ct, Daytona Beach, Fl. 32119; email: info@icnpaa.com, meenithi@aol.com; http://www.icnpaa.com.
New Publications Offered by the AMS

Algebra and Algebraic Geometry

Geometric Methods in Group Theory
José Burillo, Universitat Politécnica de Catalunya, Barcelona, Spain, Sean Cleary, The City College of New York (CUNY), Murray Elder, University of St. Andrews, Fife, Scotland, Jennifer Taback, Bowdoin College, Brunswick, ME, and Enric Ventura, Universitat Politécnica de Catalunya, Barcelona, Spain, Editors

This volume presents articles by speakers and participants in two AMS special sessions, Geometric Group Theory and Geometric Methods in Group Theory, held respectively at Northeastern University (Boston, MA) and at Universidad de Sevilla (Spain). The expository and survey articles in the book cover a wide range of topics, making it suitable for researchers and graduate students interested in group theory.

Contents: M. Cardenas and F. F. Lasheras, Properly 3-realizable groups: a survey; A. Martino and S. O Rourke, Free actions on \(2^n\)-trees: a survey; G. Levitt, Characterizing rigid simplicial actions on trees; J. González-Menéndez, Improving an algorithm to solve multiple simultaneous conjugacy problems in braid groups; E. Godêlê and L. Paris, On singular Artin monoids; O. Bogopolskii, A surface groups analogue of a theorem of Magnus; V. Addepalli and E. C. Turner, Shift automorphisms of finite order; V. Shpilrain, Counting primitive elements of a free group; R. Weidmann, A rank formula for amalgamated products with finite amalgam; D. Kahrobaei, A simple proof of a theorem of Karrass and Solitar; S. W. Margolis, J. Meakin, and Z. Sunik, Distortion functions and the membership problem for submonoids of groups and monoids; J. Belk and K.-U. Bux, Thompson's group \(F\) is maximally nonconvex; S. Cleary and J. Taback, Seesaw words in Thompson's group \(F\); X. Martin, Piecewise-projective representation of Thompson's group \(F\); T. Dymarz, Bijective quasi-isometries of amenable groups; I. Bumagin, On definitions of relatively hyperbolic groups; G. Baumslag, Embedding wreath-like products in finitely presented groups; L. G. Păun, Metric properties of the lamplighter group as an automaton group; F. Dahmani, An example of noncontracting weakly branch automaton group; A. Akhmedov, Travelling salesman problem in groups.

Contemporary Mathematics, Volume 372

Analysis

Analyzable Functions and Applications
O. Costin and M. D. Kruskal, Rutgers University, Piscataway, NJ, and A. Macintyre, University of London, UK, Editors

The theory of analyzable functions is a technique used to study a wide class of asymptotic expansion methods and their applications in analysis, difference and differential equations, partial differential equations and other areas of mathematics. Key ideas in the theory of analyzable functions were laid out by Euler, Cauchy, Stokes, Hardy, E. Borel, and others. Then in the early 1980s, this theory took a great leap forward with the work of J. Écalle. Similar techniques and concepts in analysis, logic, applied mathematics and surreal number theory emerged at essentially the same time and developed rapidly through the 1990s. The links among various approaches soon became apparent and this body of ideas is now recognized as a field of its own with numerous applications.

This volume stemmed from the International Workshop on Analyzable Functions and Applications held in Edinburgh (Scotland). The contributed articles, written by many leading experts, are suitable for graduate students and researchers interested in asymptotic methods.

Contents: S. Alt-Mokhtar, A singularly perturbed Riccati equation; T. Aoki, T. Kawai, T. Koike, and Y. Takei, On global aspects of exact WKB analysis of operators admitting infinitely many phases; M. Aschenbrenner and L. van den Dries,

**Differential Equations**

Nonlinear Partial Differential Equations and Related Analysis

Gui-Qiang Chen, George Gasper, and Joseph Jerome, Northwestern University, Evanston, IL, Editors

The Emphasis Year on Nonlinear Partial Differential Equations and Related Analysis at Northwestern University produced this fine collection of original research and survey articles. Many well-known mathematicians attended the events and submitted their contributions for this volume.

Eighteen papers comprise this work, representing the most significant advances and current trends in nonlinear PDEs and their applications. Topics covered include elliptic and parabolic equations, Navier-Stokes equations, and hyperbolic conservation laws. Important applications are presented from incompressible and compressible fluid mechanics, combustion, and electromagnetism. Also included are articles on recent advances in statistical reliability in modeling, simulation, level set methods for image processing, shock waves, boundary layers, errors in numerical solutions, stability, instability, and singular limits.

The volume is suitable for researchers and graduate students interested in partial differential equations.

**Contents:** M. Bendahmane and K. H. Karlsen, Uniqueness of entropy solutions for doubly nonlinear anisotropic degenerate parabolic equations; A. Bertozzi, J. Greer, S. Osher, and K. Vixie, Nonlinear regularizations of TV based PDEs for image processing; J. L. Bona and V. V. Varlamov, Wave generation by a moving boundary; G.-Q. Chen and K. Trivisa, Analysis on models for exothermically reacting, compressible, and heat conducting fluids; P. Constantin, Eulerian-Lagrangian hydrodynamic equations; P. Degond, F. Méhats, and C. Ringhofer, Quantum hydrodynamic models derived from the entropy principle; E. Feireisl, Mathematics of viscous, compressible, and heat conducting fluids; A. Friedman, Symmetry-breaking bifurcations for free boundary problems; J. Glimm, J. W. Grove, Y. Kang, T. Lee, X. Li, D. H. Sharp, K. Q. Ye, Y. Yu, and M. Zhao, Errors in numerical solutions of spherically symmetric shock physics problems; P. E. Jabin and C. Klingenberg, Existence to solutions of a kinetic aerosol model; J. W. Jerome, Functional analytic methods for evolution systems; Y. Li, Stability of Riemann solutions with large oscillation for the Euler equations; Z. Lin, Some recent results on instability of ideal plane flows; A. Rousseau, R. Temam, and J. Tribbia, Boundary conditions for an ocean related system with a small parameter; D. Serre, A remark on Y. Brenier's approach to...
Long-time behavior of solutions to the Boussinesq system at Born-Infeld electro-magnetic fields; R. Shvydkoy and S. Friedlander, On recent developments in the spectral problem for the linearized Euler equation; K. Song, Transonic flow arising from 2-D Riemann problems; X. Wang, A note on long-time behavior of solutions to the Boussinesq system at large Prandtl number.

Contemporary Mathematics, Volume 371

Discrete Mathematics and Combinatorics

Integer Points in Polyhedra — Geometry, Number Theory, Algebra, Optimization

Alexander Barvinok, University of Michigan, Ann Arbor, Matthias Beck, San Francisco State University, Christian Haase, Duke

University, Durham, NC, Bruce Reznick, University of Illinois at Urbana-Champaign, and Volkmar Welker, Philipps-Universität Marburg, Germany, Editors

The AMS-IMS-SIAM Summer Research Conference on Integer Points in Polyhedra took place in Snowbird (UT). This proceedings volume contains original research and survey articles stemming from that event. Topics covered include commutative algebra, optimization, discrete geometry, statistics, representation theory, and symplectic geometry. The book is suitable for researchers and graduate students interested in combinatorial aspects of the above fields.

Contents: J. Agapito, A weighted version of quantization commutes with reduction for a toric manifold; M. Beck, J. A. De Loera, M. Develin, J. Pfeifle, and R. P. Stanley, Coefficients and roots of Ehrhart polynomials; B. Chen, Ehrhart polynomials of lattice polyhedral functions; Y. Chen, I. Dinwoodie, A. Dobra, and M. Huber, Lattice points, contingency tables, and sampling; C. Cochet, Kostka numbers and Littlewood-Richardson coefficients; C. Haase, Polar decomposition and Brion's theorem; P. Hersh and V. Welker, Gröbner basis degree bounds on Tor$^1_{[A]}(k,k)$, and discrete Morse theory for posets; J. B. Lasserre, Integer programming duality and superadditive functions; F. Santos, The Cayley trick and triangulations of products of simplices; M. Beck, B. Chen, I. Fukshansky, C. Haase, A. Knutson, B. Reznick, S. Robins, and A. Schürmann, Problems from the Cottonwood Room.

Contemporary Mathematics, Volume 374

Probability

Heads or Tails: An Introduction to Limit Theorems in Probability

Emmanuel Lesigne, Université François Rabelais, Tours, France

Everyone knows some of the basics of probability, perhaps enough to play cards. Beyond the introductory ideas, there are many wonderful results that are unfamiliar to the layman, but which are well within our grasp to understand and appreciate. Some of the most remarkable results in probability are those that are related to limit theorems—statements about what happens when the trial is repeated many times. The most famous of these is the Law of Large Numbers, which mathematicians, engineers, economists, and many others use every day.

In this book, Lesigne has made these limit theorems accessible by stating everything in terms of a game of tossing of a coin: heads or tails. In this way, the analysis becomes much clearer, helping establish the reader's intuition about probability. Moreover, very little generality is lost, as many situations can be modelled from combinations of coin tosses.

This book is suitable for anyone who would like to learn more about mathematical probability and has had a one-year undergraduate course in analysis.

Contents: Prerequisites and overview; Modeling a probabilistic experiment; Random variables; Independence; The binomial distribution; The weak law of large numbers; The large deviations estimate; The central limit theorem; The moderate deviations estimate; The local limit theorem; The arcsine law; The strong law of large numbers; The law of the iterated logarithm; Recurrence of random walks; Epilogue; Biographies; Bibliography; Index.

Student Mathematical Library, Volume 28


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Analysis

Introduction à l'étude des espaces de Banach
Analyse et probabilités

Daniel Li, and Hervé Queffélec, Université de Lille 1, France

Devoted to the study of Banach spaces, this book emphasizes connections with classical analysis, harmonic analysis, and probability theory. It is suitable for beginning graduate students. The study is taken from the beginning and then worked out thoroughly, presenting several fundamental results which were obtained during the period 1950–2000: Grothendieck's theorem, Dvoretzky's theorem, Rosenthal's dichotomy theorem, and Gowers's dichotomy theorem, etc., with applications.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Préliminaire. Topologies faible et préfaible. Filtres, ultrafiltres. Ordinaux; Notions fondamentales de probabilités; Bases dans les espaces de Banach; Convergence inconditionnelle; Variables aléatoires banachiques; Type et cotype des espaces de Banach. Factorisation par un espace de Hilbert; Opérateurs p-somnateurs. Applications; Quelques propriétés des espaces Lp; L'espace L1; Sections euclidiennes; Espaces de Banach séparables sans la propriété d'approximation; Processus gaussiens; Sous-espaces réflexifs de L1; Quelques exemples d'utilisation de la méthode des sélecteurs; Espace de Pisier des fonctions presque sûrement continues. Applications; Annexe. Algèbres de Banach. Groupes abéliens compacts; Bibliographie; Index des notations; Index des noms cités; Index terminologique.

Cours Spécialisés—Collection SMF, Number 12

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Une introduction aux motifs
Motifs purs, motifs mixtes, périodes

Yves André, École Normale Superieure, Paris, France

"Motives" were introduced 40 years ago by A. Grothendieck as "a systematic theory of arithmetic properties of algebraic varieties as embodied in their groups of classes of cycles". This text provides an exposition of the geometric foundations of the theory (pure and mixed) and a panorama of major developments that have occurred in the last 15 years. The last part is devoted to a study of periods of motives, with emphasis on examples (polyzeta numbers, notably). It is suitable for graduate students and research mathematicians interested in number theory.

This item will also be of interest to those working in number theory.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Partie I. Motifs purs: Sources: géométrie énumérative, cohomologie, théorie de Galois; 0-Catégories rigides, catégories tannakiennes; Cycles algébriques et cohomologies (cas des variétés projectives lisses); Motifs purs de Grothendieck; Les conjectures standard; Groupes de Galois motiviques; Les conjectures de pléniété et de semi-simplicité des réalisations enrichies; Effectivité; Comment contourner les conjectures standard; Applications de la théorie des cycles motivés; Filtrations sur les anneaux de Chow et nilpotence; Structure de la catégorie des motifs purs pour une équivalence adéquate quelconque; MOTIFS PURS VIRTUELS ATTACHES AUX CONGRUENCES (transition vers la mixité); Partie II. Motifs mixtes: Pourquoi des motifs mixtes? Le formalisme élémentaire des variétés multivariées; Motifs mixtes de Voevodsky; Twists et cohomologie motivique; Propriétés fondamentales de DM_{gm}(k); Complexes de faisceaux motiviques; Exemples: 1-motifs et motifs de Tate mixtes; Vers le cœur de DM_{gm}(k); Réalisations mixtes et régulateurs; Partie III. Périodes: Relations de périodes; MOTIFS ET VALEURS SPECIALES DE LA FONCTION Z; MOTIFS ET NOMBRES POLYZETA; Bibliographie; Index terminologique.

Panoramas et Synthèses, Number 17
Geometry and Topology

Cobordisme complexe des espaces pro-finis et foncteur T de Lannes
François-Xavier Dehon,
Université de Nice, France

The author shows that the continuous MU-cohomology of the mapping spaces from the classifying space of a commutative compact Lie group to the pro-p-completion of a space whose p-adic cohomology is torsion free is the image of the p-completed MU-cohomology of the target space by a functor T analogous to the functor T associated to the classifying space of the cyclic group of order p.

The book is suitable for graduate students and research mathematicians interested in geometry and topology.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Cobordisme complexe des espaces pro-finis; Cohomologie des espaces fonctionnels de source le classifiant d’un groupe de Lie compact commutatif; Appendice; Bibliographie; Index.

Mémoires de la Société Mathématique de France. Number 98

Number Theory

Cohomologies p-adiques et applications arithmétiques (III)
Pierre Berthelot, Université de Rennes I, France, Jean-Marc Fontaine and Luc Illusie,
Université de Paris-Sud, Orsay, France, Kazuya Kato, Kyoto University, Japan, and Michael Rapoport, University of Köln, Germany, Editors

This volume contains survey papers on p-adic methods in arithmetic geometry. Topics covered include Galois representations, p-adic L-functions of modular forms, and Iwasawa theory of modular forms. The book is suitable for graduate students and research mathematicians interested in number theory, algebra, and algebraic geometry.

This item will also be of interest to those working in algebra and algebraic geometry.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: J.-M. Fontaine, Arithmétique des représentations galoisennes p-adiques; K. Kato, p-adic Hodge theory and values of zeta functions of modular forms; P. Schneider and J. Teitelbaum, Correction to "p-adic boundary values".

Astérisque, Number 295
Journals at de Gruyter

■ Advances in Geometry
Managing Editors: T. Grundhöfer, K. Strambach
ISSN 1615-715X
Approx. 530 pages.
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Managing Editor: R. Weissauer
Editors: J. Cuntz, G. Huisken, Y. I. Manin, E. Vielhag, P. Voja
ISSN 0075-4102
Approx. 2900 pages.
Annual subscription rate:
Print only or online only: € 2298.00
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■ Journal of Group Theory
Managing Editor: J. S. Wilson
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Search Committee: Numerical Analysis
Department of Mathematics
Oregon State University
Corvallis, OR 97331-4605

Additionally three letters of recommendation, one of which addresses teaching, are required. They should be sent directly to the above address. For full consideration, complete application materials must arrive by April 15, 2005. Further information is available at http://www.math.oregonstate.edu/hiring.

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Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classifieds@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.
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2004; 307 pages; Softcover; ISBN 0-8218-3678-1; List $29; All AMS members $23; Order code KNOTN049

Also of Interest ...

Supersymmetry for Mathematicians: An Introduction
V. S. Varadarajan, University of California, Los Angeles

In this book, V. S. Varadarajan presents a cogent and self-contained exposition of the foundations of supersymmetry for the mathematically-minded reader.

Titles in this series are copublished with the Courant Institute of Mathematical Sciences at New York University.

Courant Lecture Notes, Volume 11: 2004; 300 pages; Softcover; ISBN 0-8218-3574-2; List $39; All AMS members $31; Order code CLN/11N049

The Stationary Tower
Notes on a Course by W. Hugh Woodin
Paul B. Larson, Miami University, Oxford, OH

Written by a leading figure in modern set theory, this book is the first detailed treatment of his method of the stationary tower that is accessible to graduate students.

University Lecture Series, Volume 32: 2004; 132 pages; Softcover; ISBN 0-8218-3604-8; List $29; All AMS members $23; Order code ULECT/32N049

Transformation Groups for Beginners
S.V. Duzhin, Steklov Institute of Mathematics, St. Petersburg, Russia, and B. D. Chebotarevsky, Minsk, Belarus

The modern mathematical way of treating symmetry is through transformation groups. This book offers an easy introduction to these ideas for the relative novice.

Student Mathematical Library, Volume 25: 2004; approximately 256 pages; Softcover; ISBN 0-8218-3643-9; List $39; All AMS members $31; Order code STML/25N049
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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Programs and abstracts will continue to be displayed on the AMS website in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on the AMS website in an electronic issue of the Notices as noted below for each meeting.

Santa Barbara, California
University of California Santa Barbara
April 16-17, 2005
Saturday - Sunday
Meeting #1007
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: February 2005
Program first available on AMS website: March 3, 2005
Program issue of electronic Notices: April 2005
Issue of Abstracts: Volume 26, Issue 3

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional1.html.

Invited Addresses
Mei-Chu Chang, University of California Riverside, Set addition and set multiplication.

Michael Kapovich, University of California Davis, Generalized triangle inequalities and their applications.
Mihai Putinar, University of California Santa Barbara, Positive polynomials, a hilbertian perspective.
James Sethian, University of California Berkeley, Advances in advancing interfaces: New techniques for propagating fronts in wave propagation and materials sciences.

Special Sessions
Algebraic Geometry and Combinatorics, Alexander Yong and Allen Knutson, University of California Berkeley.
Arithmetic Geometry, Adebisi Agboola, University of California Santa Barbara, and Cristian Dumitru Popescu, University of California San Diego.
Automorphisms of Surfaces, Anthony Weaver, Bronx Community College of the City University of New York, and Peter Turbek, Purdue University Calumet.
Complexity of Computation and Algorithms, Mark Burgin, University of California Los Angeles.
Curvature in Group Theory and Combinatorics, Laura M. Anderson, State University of New York at Binghamton.
Noel Patrick Brady, University of Oklahoma, Robin Forman, Rice University, and Jonathan P. McCammond, University of California Santa Barbara.
Function Theory, Mihai Putinar and Stephan R. Garcia, University of California Santa Barbara.
Geometric Methods in Three Dimensions, Daryl Cooper, David Darren Long, and Martin G. Scharlemann, University of California Santa Barbara.

Geometry and Physics, Xianzhe Dai, University of California Santa Barbara, and Zhiqin Lu, University of California Irvine.

History of Mathematics, Shawnee L. McMurran, California State University San Bernardino, and James J. Tattersall, Providence College.

Noncommutative Geometry and Algebra, Kenneth R. Goodearl, University of California Santa Barbara, J. T. Stafford, University of Michigan, and J. J. Zhang, University of Washington.

Recent Advances in Combinatorial Number Theory, Mei-Chu Chang, University of California Riverside, and Van Ha Vu, University of California San Diego.

Representation Theory of Algebras (in Honor of Claus Michael Ringel), Alex Martenskovsky, Northeastern University, Dan Zacharia, Syracuse University, Birge K. Huisinga-Zimmermann, University of California Santa Barbara, and Edward L. Green, Virginia Polytechnic Institute & State University.

Ricci Flow/Riemannian Geometry, Guofang Wei and Rugang Ye, University of California Santa Barbara.

Mainz, Germany

June 16-19, 2005
Thursday - Sunday

Meeting #1008
Joint International Meeting with the Deutsche Mathematiker-Vereinigung (DMV) and the Oesterreichische Mathematische Gesellschaft (OMG)
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: February 2005
Program first available on AMS website: Not applicable
Program Issue of electronic Notices: Not applicable
Issue of Abstracts: Not applicable

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses
Hélène Esnault, University of Essen, Deligne's integrality theorem in unequal characteristic and rational points over finite fields.
Richard Hamilton, Columbia University, The Ricci flow.

Michael J. Hopkins, Massachusetts Institute of Technology, Title to be announced.
Christian Krattenthaler, University of Lyon-I, Exact and asymptotic enumeration of vicious walkers with a wall interaction.
Frank Natterer, University of Muenster, Imaging and inverse problems for partial differential equations.
Horng-Tzer Yau, New York University and Stanford University, Dynamics of Bose-Einstein condensate.

Special Sessions
Affine Algebraic Geometry, Shreeram Abhyankar, Purdue University, Hubert Flenner, Ruhr University Bochum, and Makar Limanov, Wayne State University.
Algebraic Combinatorics, Patricia Hersh, Indiana University-Bloomington, Christian Krattenthaler, University of Lyon-I, and Volklmar Welker, Philips University Marburg.
Algebraic Cryptography, Dorian Goldfeld, Columbia University, Martin Kreuzer and Gerhard Rosenberger, Universitat Dortmund, and Vladimir Shpilrain, The City College of New York.
Algebraic Cycles, Eric Friedlander and Marc Levine, Northwestern University, and Fabien Morel, Université Paris.
Algebraic Geometry, Yuri Tschinkel, Georg-August-Universität Göttingen, and Brendan E. Hassett, Rice University.
Dirac Operators, Clifford Analysis and Applications, Klaus Gürlebeck, University of Weimar, Mircea Martin, Baker University, John Ryan, University of Arkansas, and Michael Shapiro, IPN Mexico.
Function Spaces and Their Operators, Ernst Albrecht, Universität des Saarlandes, Raymond Mortini, Université de Metz, and William Ross, University of Richmond.
Functional Analytic and Complex Analytic Methods in Linear Partial Differential Equations, R. Meise, University of Düsseldorf, B. A. Taylor, University of Michigan, and Dietmar Vogt, University of Wuppertal.
Geometric Analysis, Victor Nistor, Pennsylvania State University, and Elam Schrohe, Universität Hannover.
Geometric Topology and Group Theory, Cameron MCA Gordon, The University of Texas at Austin, Cynthia Hog-Angeloni, Johann Wolfgang Goethe-Universität, and Wolfgang Metzler, University of Frankfurt.
Group Theory, Luise-Charlotte Kappe, SUNY at Binghamton, Robert Fitzgerald Morse, University of Evansville, and Gerhard Rosenberger, Universität Dortmund.
Hilbert Functions and Syzygies, Uwe Nagel, University of Kentucky, Irena Peeva, Cornell University, and Tim Römer, Universität Osnabrück.
History of Mathematics (including a special workshop on Mathematics and War), Thomas W. Archibald, Acadia
Annandale-on-Hudson, New York
Bard College

October 8-9, 2005
Saturday – Sunday

Meeting #1009
Eastern Section

Associate secretary: Lesley M. Sibner
Announcement issue of Notices: August 2005
Program first available on AMS website: August 25, 2005
Program issue of electronic Notices: October 2005
Issue of Abstracts: Volume 26, Issue 4

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: June 21, 2005
For abstracts: August 16, 2005

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/specialsessions.html.

Invited Addresses
Persi Diaconis, Stanford University, Title to be announced (Erdős Memorial Lecture).
Harold Rosenberg, University of Paris VII, Title to be announced.
Alice Silverberg, University of California Irvine, Title to be announced.
Christopher Sogge, Johns Hopkins University, Title to be announced.
Benjamin Sudakov, Princeton University, Title to be announced.

Special Sessions
Algebraic and Geometric Combinatorics (Code: SS 12A), Cristian P. Lenart, State University of New York at Albany, and Lauren L. Rose and Sheila Sundaram, Bard College.
Extremal and Probabilistic Combinatorics (Code: SS 11A), Benjamin Sudakov, Princeton University.
Meetings & Conferences

Geometric Group Theory (Code: SS 1A), Sean Cleary, The City College of New York, and Melanie I. Stein, Trinity College.

Geometric Transversal Theory (Code: SS 3A), Richard Pollack, Courant Institute, New York University, and Jacob Eli Goodman, The City College of New York.

Global Theory of Minimal Surfaces (Code: SS 6A), David A. Hoffman, Mathematical Sciences Research Institute, and Harold Rosenberg, University of Paris VII.

History of Mathematics (Code: SS 2A), Patricia R. Allaire, Queensborough Community College, CUNY, Robert E. Bradley, Adelphi University, and Jeff Suzuki, Bard College.

Homological Aspects of Commutative Algebra (Code: SS 4A), Alexandre Tchernev, University of Albany, SUNY, and Janet Vassilev, University of Arkansas.

Infinite Groups (Code: SS 10A), Anthony M. Gaglione, United States Naval Academy, Benjamin Fine, Fairfield University, and Dennis Spellman, Philadelphia University.


Special Functions and Orthogonal Polynomials: Theory and Applications (Code: SS 7A), Diego Dominici, State University of New York at New Paltz.

Theory of Infinite-Dimensional Lie Algebras, Vertex Operator Algebras, and Related Topics (Code: SS 5A), Antun Milas, SUNY at Albany, Alex J. Feingold, Binghamton University, and Yi-Zhi Huang, Rutgers University.

Invited Addresses

Alberto Bressan, Pennsylvania State University, Title to be announced.
Assaf Naor, Microsoft Research, Title to be announced.
Prasad V. Tetali, Georgia Institute of Technology, Title to be announced.
Rekha R. Thomas, University of Washington, Title to be announced.

Special Sessions

Commutative Ring Theory (Code: SS 1A), David F. Anderson and David E. Dobbs, University of Tennessee at Knoxville.

Mathematical Aspects of Wave Propagation Phenomena (Code: SS 2A), Boris P. Belinskiy, University of Tennessee at Chattanooga, and Anjan Biswas, Tennessee State University.

Lincoln, Nebraska

University of Nebraska in Lincoln

October 21-23, 2005
Friday - Sunday

Meeting #1011
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: August 2005
Program first available on AMS website: September 1, 2005
Program issue of electronic Notices: October 2005
Issue of Abstracts: Volume 26, Issue 4

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: July 5, 2005
For abstracts: August 30, 2005

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Howard A. Masur, University of Illinois at Chicago, Billiards in Polygons: Connections of Geometry and Complex Analysis to Dynamical Systems.
Alejandro Uribe, University of Michigan, Title to be announced.
Judy Walker, University of Nebraska, Title to be announced.
Jack Xin, University of Texas, Title to be announced.

Special Sessions

Algebraic Geometry (Code: SS 1A), Brian Harbourne, University of Nebraska-Lincoln, and Bangere P. Purnaprajna, University of Kansas.
Combinatorial Matrix Theory (Code: SS 10A), Leslie Hogben, Iowa State University, and Bryan L. Shader, University of Wyoming.

Dynamic Equations on Time Scales (Code: SS 5A), Lynn H. Erbe and Allan C. Peterson, University of Nebraska-Lincoln.

Geometric Methods in Group Theory and Semigroup Theory (Code: SS 6A), Susan M. Hermiller and John C. Meakin, University of Nebraska-Lincoln, and Zoran Sunik, Texas A&M University.

Geometry of Differential Equations (Code: SS 11A), Jeannine Nielsen Clelland, University of Colorado, Irina A. Kogan, North Carolina State University, and Zhijun Qiao, University of Texas-Pan American.

Graph Theory (Code: SS 8A), Andrew J. Radcliffe, University of Nebraska-Lincoln, Zsuzsanna Szaniszlo, Valparaiso University, and Jonathan Cutler, University of Nebraska-Lincoln.

Large Cardinals in Set Theory (Code: SS 4A), Paul B. Larson, Miami University, Justin Tatch Moore, Boise State University, and Ernest Schimmerling, Carnegie Mellon University.

Mathematical Ecology (Code: SS 9A), David Logan, University of Nebraska-Lincoln, and William Robert Wolesensky, College of St. Mary.

Mathematical and Engineering Aspects of Coding Theory (Code: SS 3A), Lance Perez and Judy Walker, University of Nebraska-Lincoln.

Randomness in Computation (Code: SS 7A), John M. Hitchcock, University of Wyoming, Aduri Pavan, Iowa State University, and Vinodchandran Variyam, University of Nebraska-Lincoln.

Recent Progress in Operator Algebras (Code: SS 2A), Allan P. Donsig and David R. Pitts, University of Nebraska-Lincoln.

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Matthew Foreman, University of California Irvine, Title to be announced.
Mark Haiman, University of California Berkeley, Title to be announced.
Wilhelm Schlag, California Institute of Technology, Title to be announced.
Hart H. Smith, University of Washington, Title to be announced.

Special Sessions
Algebraic Combinatorics and Geometry (Code: SS 7A), Sara C. Billey, University of Washington, and Mark Haiman, University of California Berkeley.
K-Theory in M-Theory (Code: SS 6A), Gregory D. Landweber, University of Oregon, and Charles F. Doran, University of Washington.

Noncommutative Algebra and Noncommutative Birational Geometry (Code: SS 3A), Arkady Dmitrievich Berenstein, University of Oregon, and Vladimir Retakh, Rutgers University.

Partial Differential Equations with Applications (Code: SS 4A), Alexander Panchenko, Washington State University, R. E. Showalter, Oregon State University, and Hong-Ming Yin, Washington State University.

Regular Algebras and Noncommutative Projective Geometry (Code: SS 2A), Brad Shelton, University of Oregon, Michaela Vancliff, University of Texas at Arlington, and James J. Zhang, University of Washington.


Resolutions (Code: SS 1A), Christopher Alan Francisco, University of Missouri, and Irena Peeva, Cornell University.

Eugene, Oregon
University of Oregon

November 12-13, 2005
Saturday - Sunday

Meeting #1012
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: September 2005
Program first available on AMS website: September 29, 2005
Program issue of electronic Notices: November 2005
Issue of Abstracts: Volume 26, Issue 4

Deadlines
For organizers: April 12, 2005
For consideration of contributed papers in Special Sessions: July 26, 2005
For abstracts: September 20, 2005

Taiwan

December 14-18, 2005
Wednesday - Sunday

Meeting #1013
First Joint International Meeting between the AMS and the Taiwanese Mathematical Society.
Associate secretary: John L. Bryant
Announcement issue of Notices: June 2005
Program first available on AMS website: Not applicable
Program issue of electronic Notices: Not applicable
Issue of Abstracts: Not applicable

Deadlines
For organizers: To be announced
Meetings & Conferences

For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

San Antonio, Texas
*Henry B. Gonzalez Convention Center*

January 12-15, 2006
*Thursday - Sunday*

**Meeting #1014**
*Joint Mathematics Meetings, including the 112th Annual Meeting of the AMS, 89th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).*

Associate secretary: Matthew Miller
Announcement issue of Notices: October 2005
Program first available on AMS website: November 1, 2005
Program issue of electronic Notices: January 2006
Issue of Abstracts: Volume 27, Issue 1

**Deadlines**
For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

**AMS Invited Addresses**

Charles L. Fefferman, Princeton University, *Title to be announced.*

Dusa McDuff, SUNY at Stony Brook, *Title to be announced.*

Miami, Florida
*Florida International University*

April 1-2, 2006
*Saturday - Sunday*

**Meeting #1015**
*Southeastern Section*

Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

**Deadlines**
For organizers: September 1, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Notre Dame, Indiana
*University of Notre Dame*

April 8-9, 2006
*Saturday - Sunday*

**Meeting #1016**
*Central Section*

Associate secretary: Susan J. Friedlander
Announcement issue of Notices: February 2006
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

**Deadlines**
For organizers: September 9, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Durham, New Hampshire
*University of New Hampshire*

April 22-23, 2006
*Saturday - Sunday*

**Meeting #1017**
*Eastern Section*

Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

**Deadlines**
For organizers: September 22, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California
*San Francisco State University*

April 29-30, 2006
*Saturday - Sunday*

**Meeting #1018**
*Western Section*

Associate secretary: Michel L. Lapidus
Meetings & Conferences

Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 30, 2005
For consideration of contributed papers in Special Sessions:
   To be announced
For abstracts: To be announced

Special Sessions
History of Mathematics (Code: SS1A), Shawnee L. McMurran,
California State University, San Bernardino, and James J.
Tattersall, Providence College.

Salt Lake City, Utah
University of Utah
October 7-8, 2006
Saturday - Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 7, 2006
For consideration of contributed papers in Special Sessions:
   To be announced
For abstracts: To be announced

Cincinnati, Ohio
University of Cincinnati
October 21-22, 2006
Saturday - Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 21, 2006
For consideration of contributed papers in Special Sessions:
   To be announced
For abstracts: To be announced

New Orleans, Louisiana
New Orleans Marriott and Sheraton New Orleans Hotel
January 4-7, 2007
Thursday - Sunday
Joint Mathematics Meetings, including the 113th Annual
Meeting of the AMS, 90th Annual Meeting of the Mathe-
matical Association of America (MAA), annual meetings of
the Association for Women in Mathematics (AWM) and the
National Association of Mathematicians (NAM), and the
winter meeting of the Association for Symbolic Logic (ASL),
Meetings & Conferences

with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: October 2006
Program first available on AMS website: November 1, 2006
Program issue of electronic Notices: January 2007
Issue of Abstracts: Volume 28, Issue 1

Deadlines
For organizers: April 1, 2006
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Oxford, Ohio
Miami University
March 16-17, 2007
Friday - Saturday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

San Diego, California
San Diego Convention Center
January 6-9, 2008
Sunday - Wednesday
Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: October 2007
Program first available on AMS website: November 1, 2007
Program issue of electronic Notices: January 2008
Issue of Abstracts: Volume 29, Issue 1

Deadlines
For organizers: April 1, 2007
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Washington, District of Columbia
Marriott Wardman Park Hotel
and Omni Shoreham Hotel
January 7-10, 2009
Wednesday - Saturday
Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic Notices: January 2009
Issue of Abstracts: Volume 30, Issue 1

Deadlines
For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

San Francisco, California
Moscone Center West and the San Francisco Marriott
January 6-9, 2010
Wednesday - Saturday
Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: John L. Bryant
Announcement issue of Notices: October 2009
Program first available on AMS website: November 1, 2009
Program issue of electronic Notices: January 2010
Issue of Abstracts: Volume 31, Issue 1

Deadlines
For organizers: April 1, 2009
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced
Meetings & Conferences

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5-8, 2011

Wednesday – Saturday

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Susan J. Friedlander

Announcement issue of Notices: October 2010

Program first available on AMS website: November 1, 2010

Program issue of electronic Notices: January 2011

Issue of Abstracts: Volume 32, Issue 1

Deadlines

For organizers: April 2, 2011

For consideration of contributed papers in Special Sessions:

To be announced

For abstracts: To be announced
Meetings and Conferences of the AMS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. Up-to-date meeting and conference information can be found at www.ams.org/meetings/.

Meetings:

2005
April 16-17 Santa Barbara, California p. 583
June 16-19 Mainz, Germany p. 584
October 8-9 Annandale-on-Hudson, New York p. 585
October 15-16 Johnson City, Tennessee p. 586
October 21-23 Lincoln, Nebraska p. 587
November 12-13 Eugene, Oregon p. 587
December 14-18 Taiwan p. 587

2006
January 12-15 San Antonio, Texas Annual Meeting p. 588
April 1-2 Miami, Florida p. 588
April 8-9 Notre Dame, Indiana p. 588
April 22-23 Durham, New Hampshire p. 588
April 29-30 San Francisco, California p. 588
October 7-8 Salt Lake City, Utah p. 589
October 21-22 Cincinnati, Ohio p. 589
October 28-29 Storrs, Connecticut p. 589
November 3-4 Fayetteville, Arkansas p. 589

2007
January 4-7 New Orleans, Louisiana Annual Meeting p. 589

Important Information regarding AMS Meetings
Potential organizers, speakers, and hosts should refer to page 100 in the January 2005 issue of the Notices for general information regarding participation in AMS meetings and conferences.

Abstracts
Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX{} is necessary to submit an electronic form, although those who use \LaTeX{} may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX{}. Visit http://www.ams.org/cgi-bin/abstracts/abstracts.pl.

Questions about abstracts and requests for paper forms may be sent to abs-info@ams.org.

Paper abstract forms must be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that abstract deadlines are strictly enforced.

Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)
Springer for Mathematics

Linearity, Symmetry and Prediction in THE HYDROGEN ATOM
Stephanie F. Singer, Haverford College, Pennsylvania
This undergraduate textbook concentrates on how to make predictions about dimensions of the basic states of a quantum system from only two ingredients: the symmetry and the linear model of quantum mechanics. This method, known as representation theory or group theory, combines linear algebra, analysis and abstract algebra and finds wide applications in crystallography, classification of manifolds with symmetry, atomic structure, and so on.
2005, Approx. 370 p. 50 illus., (Undergraduate Texts in Mathematics) Hardcover 0-387-24637-1 ▶ Approx. $79.95

Probability: A Graduate Course
Allan Gut, University of Uppsala, Sweden
This graduate textbook follows the ideology that rather than being a purely mathematical discipline, probability theory is an intimate companion of statistics. The book starts with the basic tools, and goes on to chapters on inequalities, characteristic functions, convergence, followed by the three main subjects, the law of large numbers, the central limit theorem, and the law of the iterated logarithm. After a discussion of generalizations and extensions, the book concludes with an extensive chapter on martingales.

NOW IN SOFTCOVER
Stochastic Calculus for Finance I
The Binomial Asset Pricing Model
Steven E. Shreve, Carnegie Mellon University, Pennsylvania
2005, 185 p., (Springer Finance) Softcover 0-387-24968-0 ▶ $34.95

NOW IN SOFTCOVER
Combinatorial Commutative Algebra
Ezra Miller, University of Minnesota and Bernd Sturmfels, UC Berkeley
This book introduces combinatorial commutative algebra, with an emphasis on combinatorial techniques for multigraded polynomial rings, semigroup algebras, and determinantal rings.
2005, 420 p., (Graduate Texts in Mathematics, Vol. 227) Softcover 0-387-23707-0 ▶ $49.95

Research Problems in Discrete Geometry
Peter Brass, City College, New York; William O.J. Moser, McGill University, Canada; and Janos Pach, New York University
Based on William O.J. Moser's problem collection, the authors state a much extended variety of problems, providing historical background and comprehensive references.
2005, Approx. 500 p. 116 illus., Hardcover 0-387-23815-8 ▶ Approx. $69.95

Mathematics Is Not a Spectator Sport
George Phillips, University of St. Andrews, Fife, UK
This book is intended for students at the start of their mathematical journey. Topics include early algebraic ideas such as the Euclidean algorithm, geometrical constructions created by the Greeks, and ancient Babylonian and Chinese proofs of the Pythagorean theorem.
2005, Approx. 255 p. 100 illus., Hardcover 0-387-25528-1 ▶ Approx. $49.95

Ramanujan's Lost Notebook
Part I
George E. Andrews, Pennsylvania State University and Bruce Berndt, University of Illinois, Urbana-Champaign
This is the first of approximately four volumes devoted to providing statements, proofs, and discussions of all the claims made by Srinivasa Ramanujan in his lost notebook and all his other manuscripts and letters published with the lost notebook.
2005, Approx. 450 p., Hardcover 0-387-25529-X ▶ Approx. $89.00

Nonholonomic Mechanics and Control
A.M. Bloch, University of Michigan
"A well-written and comprehensive reference that can be used as a graduate-level textbook, complete with exercises..." -IEEE Transactions on Automatic Control
2003, 483 p. 49 illus., (Interdisciplinary Applied Mathematics, Vol. 24) Hardcover 0-387-95535-6 ▶ $69.95

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