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PAVEL ETINGOF, I.M. SINGER, and M. Frenkel, Massachusetts Institute of Technology, Cambridge, MA; VLADIMIR RETAIRK, Rutgers University, Piscataway, NJ

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SUBSCRIPTION INFORMATION: Subscription prices for Volume 53 (2006) are US$430 list; US$344 institutional member; US$258 individual member. (The subscription price for members is included in the annual dues.) A late charge of 10% of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India—US$20; in India—US$40; expedited delivery to destinations in North America—US$35; elsewhere—US$87. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 6149, Providence, RI 02940-6149, USA. All orders must be prepaid.

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Notices of the American Mathematical Society (ISSN 0002-9920) is published monthly except monthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2294 USA, GST No. 12189 2046 RTT. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to Notices of the American Mathematical Society, P.O. Box 6248, Providence, RI 02940-6248 USA. Publication here of the Society's street address and the other information in brackets above is a technical requirement of the U.S. Postal Service. Tel: 401-455-4000, email: notices@ams.org.

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Technology, Education, and the Single Salary Schedule

It is due to scientific progress and technological innovation, more than any other reason, that crop yields have gone up, that starvation has decreased, that human longevity has increased, and that the material conditions of our lives have continually improved. Our best hope for addressing resource scarcity, plagues, and other less foreseeable disasters is continued scientific and technological progress. We must produce scientists and engineers. Systemic changes are required to produce more. And the more the better. Changing the culture of mathematics and science education by increasing the percentage of mathematics and science teachers with more than a shallow knowledge of the subjects they are teaching may be the solution.

I regularly hear students tell me that they were "never good at math." Almost any student can be good at math—certainly at primary and secondary school levels. This failure is not inherent in the subject matter. I believe that my students' attitudes about math are transmitted to them by under-prepared teachers who were themselves not good at math. Richard Ingersoll at the University of Pennsylvania has found that 35 percent of high-school mathematics classes are taught by someone without even a minor in mathematics or a mathematics-related subject.\(^1\) These teachers often have to look at solution manuals to solve classroom problems.

Genuine knowledge of the subject matter will not guarantee that a teacher will be successful, much less compelling, but successful and compelling teaching certainly requires genuine subject knowledge. Mathematics and science teachers should have degrees in the subjects they teach. The "No Child Left Behind" act does nothing towards this goal. A "highly qualified" high-school mathematics teacher, for instance, must only pass a certification exam. (In some states, such as Georgia, you can score less than 50 percent and "pass".)\(^2\) The majority of mathematics-instruction certification exams, according to a study by the Education Trust, were dominated by high-school level material (mostly tenth to eleventh grade material).\(^3\) A teacher who passes a certification exam but does not have a mathematics degree is unlikely to have a confident, much less deep, knowledge of the subject matter.

According to the Center for the Study of Teaching, the best predictor of student achievement in science and mathematics is the presence of a teacher with a bachelor's degree in the subject taught and who is fully certified.\(^4\) California is attempting to put more teachers with mathematics and science degrees in the classroom; the state's university system just inaugurated an accelerated program to prepare mathematics and science majors for the classroom. In June 2005 California State University Chancellor Charles Reed said, "Math and science is tied to California's economic future. Nothing we can do could be more important than preparing math and science teachers for California students."\(^5\)

California proposes some economic incentives in the form of student loan forgiveness (up to US$19,000) in order to achieve their goal.\(^6\) The National Academies, whose recent report on educational reform emphasizes the importance of teacher content expertise, advocates programs like California's as its primary recommendation for increasing the number of mathematics and science teachers with degrees in these subjects.\(^7\) What these incentives do not address is that, according to a study by Ingersoll, 39 percent of K-12 teachers leave teaching altogether within five years (he estimates a slightly higher percentage for math/science teachers). 66 percent of math and science teachers cite "poor salary" as a reason for leaving.\(^8\)

California's plan may yield more mathematics and science teachers with degrees—but does not increase their incentive to stay in teaching after entering the profession. Higher salaries for these teachers—possibly much higher—are almost certainly required to achieve this goal. This solution, surprisingly, is not discussed in the National Academies' report.

Salary differentiation is not new in education—it is standard at universities where harder-to-attract positions (such as medical professors) are paid more than others (for instance, journalism professors).

There are two significant obstacles to this proposal; its cost and opposition from teachers’ unions. Taxpayers will have to pay these salary premiums. Taxpayers must be convinced that the costs of better mathematics and science education will be more than outweighed by the benefits. It is possible that this will not occur until some catastrophic event (such as an energy crisis or plague) inspires the recognition that continued technological innovation requires better mathematics and science education.

The second obstacle is union opposition. Teachers' unions are not opposed to paying teachers more. What they argue is that all teachers are equally valuable and all should be paid more. This position is enshrined in the "single salary schedule" used in 96 percent of public schools: under this system, teacher pay is determined by longevity and by the attainment of any advanced degrees. What is proposed here is a bifurcated salary schedule—secondary school science and mathematics teachers should be paid on a different schedule. The relative "value" of teachers of different subjects is not in question. The only issue addressed here is how to address society's (our) technological needs.

A direct benefit of this proposal would be an increase in the production of science and mathematics degrees. Some of these degree earners, originally motivated to teach, will likely be drawn to business, government, and the pursuit of advanced degrees. It is reasonable to believe that an indirect benefit will in time be a measurable change in our cultural attitudes towards mathematics and the sciences.

—C. E. Larson
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3. Ibid.
6. Ibid.
7. Rising Above the Gathering Storm, p. 5-1.
Letters to the Editor

Definitions of Fractions as a Discriminator

The timely review "Mathematicians and mathematics textbooks for prospective elementary teachers" by Raven McCrory (Notices 53, No. 1) is a start at making a critical appraisal of the recent texts written by mathematicians for a math content course given to prospective elementary school teachers. How do these texts compare to the traditional texts and to each other? As a means to address these questions, McCrory proposes to focus on the definition of fractions in each of the four mathematician-authored texts (one by S. Beckmann, one by T. H. Parker and S. J. Baldridge, one by H. H. Wu, and one by me [AMS 2003]). She concludes that the definitions are not word-for-word identical, even though it is evident that they are logically equivalent. Rather than compare the explanations for clarity, completeness, and depth, she dwells on the fact that the definitions are not literally identical. This is the wrong emphasis.

In her conclusions, McCrory writes, "The problems with definition of fractions illustrate the complexity of this endeavor, and suggest that we have a long way to go before we reach conclusive answers to the questions of what mathematics we should teach prospective elementary teachers and how it should be presented." Yet all four mathematician-authored texts include fractions. She continues in the next paragraph, "...there is no single 'correct' version of this mathematics." There certainly is. Its essential points and difficulties are written out in detail in Book VII of Euclid's Elements. That leaves us with the question of how fractions should be presented to elementary teachers. There is much more to this than the wording of definitions. Prospective elementary school teachers can learn this material, in the depth it is presented in my text, as I've observed year after year in my course. McCrory continues, "and we do not know what confusion is generated over time by the small but significant differences in what teachers are taught." If she means differences in wording, then this is nonsense. If she means the difference between how these four texts present fractions and how it is presented in one of the traditional texts that she quotes at the top of the right column of page 25, she is absolutely right.

—Gary R. Jensen
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(Received January 13, 2006)

Response to Jensen

I appreciate Gary Jensen's thoughtful response to my recent article (Notices 53, No. 1) and want to apologize for my error in citing his book. I have personally owned the book (Jensen, G. R., Arithmetic for Teachers: With Applications and Topics from Geometry, American Mathematical Society, Providence, RI, 2003) since the first week it was published and have shared it with many people. This was an oversight on my part.

The problem that Jensen points to with my article suggests that I have not made clear an essential point. It is not that the definitions of fractions in these books fail to be identical. No one would expect several different books to contain identical language in their definitions. Rather, the question is whether the definition in a given book will help future teachers make mathematical sense of other approaches or definitions he or she encounters as a student and teacher.

Jensen says that the definitions are logically equivalent, and he is no doubt right. My point is that learning a single, correct definition (especially one that is full of subtlety) may not equip a teacher to understand the logical equivalence of other definitions. These books, those by mathematicians, include nuances across definitions. While perfectly clear to the mathematically sophisticated, such subtleties are beyond the ken of most students preparing to be elementary teachers. I am not suggesting that these students could not, or do not, understand the presentation of the mathematics in a given book. At my own institution, as at Jensen's, we have very good elementary education students who work diligently to succeed and, for the most part, learn what we try to teach them. My argument goes to how they will be able to use that knowledge when confronted with a different version of fractions. These prospective teachers will see numerous treatments of fractions: their own elementary, middle and high school textbooks; the mathematics books and classes they take in college; and then the wide-ranging, sometimes inconsistent materials with which they teach; as well as district, state, and national standards for K-8 mathematics. We must pay attention to giving them the "profound understanding of fundamental mathematics" (Ma, L., Knowing and teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States, Lawrence Erlbaum, Mahway, NJ, 1998) that will enable them to see and understand the logical and practical equivalence of the many versions of fractions (and other mathematical ideas) they will encounter. Presenting correct mathematics in their undergraduate textbooks and courses is the beginning, but not the end, of this effort.

—Raven McCrory
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(Received January 27, 2006)

Molière and Mathematics

From time to time we hear of nonmathematicians being averse to mathematics. The writer Molière can help nonmathematicians appreciate math. He tells of a person who wants to learn prose. But as soon the tutor starts teaching prose to the person, the learner realizes he had been speaking and writing prose all his life. Similarly, nonmathematicians do not (wish to) realize that they have been doing certain mathematics since they started learning their nonmathematical disciplines. I consider how a taxonomy of function and sets is isomorphic to (expressing) four nonmathematical fields.
First, social science says the state may be democratic or dependent on citizen participation, isolated from citizens, or can be anarchic. Mathematics would say democracy means the state is a function of citizens, isolation means state and citizens are disjoint sets without either being a function of the other, and anarchy denotes there is only one set containing individuality alone. Second, religion speaks of asceticism where institutions may be dependent on a transforming individual spirituality, dualism means institutions isolated from spiritual individuals, and individuals escaping the world mean one set exists with the sole member as the person. Mathematics would say institutions can be a function of spirituality transforming the world, institutions and spirituality as mutually exclusive are disjoint sets, and individuals as fleeing the world mean there is one set containing spirituality alone as the member. Third, in philosophy, phenomenology says words depend on culture or values, dualism denotes that words are exclusive of values, or we have only values and existentialism. Mathematics would say that phenomenology means words are a function of values, dualism means words and values are disjoint sets, and existentialism means we have only a set containing values and no words or reasoning. Fourth, in theology, theism says God is dependent on our historical acts, deism means God and the world are mutually exclusive, while atheism says only the world exists and there is no God. In mathematics, theism would mean God is a function of history, deism denotes God and the world as disjoint sets, and atheism means only one set with one member as persons.

The above implies that religion, social science, theology, and philosophy do mathematics as soon as they articulate their own fields.

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(Received January 30, 2006)

Selected Reviews in the Bulletin

The new features in the January Bulletin are excellent, and I especially enjoyed the “Selected Mathematical Reviews”. But I suggest that these reviews be looked at critically and clarifying comments be appended where appropriate.

A case in point is the reprinted review of “The ergodic theoretical proof of Szemerédi’s theorem” (Furstenberg, Katznelson, and Ornstein, J. Analyse Math. 31, 1977). I found the reviewer’s paraphrase of the main result, Theorem 1.4, is extremely confusing. The following clarification may help readers who were as puzzled as I was. The result in question is this:

If $T$ is a measure-preserving transformation in a probability measure space and $A$ is a set of positive measure, then for any integer $k > 1$ there is an integer $n > 0$ such that the intersection of the sets $T^j(A)$, $(j = 0, \ldots, k-1)$, has positive measure.

The reviewer added the true but pointless conclusion that $A$ contains a set $B$ of positive measure (which is never mentioned again). And he considerably weakened the theorem by inserting the unnecessary hypothesis that $T$ is invertible.

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(Received February 7, 2006)

Mathematics in the Media


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(Received February 14, 2006)
Introduction

Cells—left, right, and two-sided—were introduced by D. Kazhdan and G. Lusztig in their study of the representation theory of Coxeter groups and Hecke algebras [22]. Cells are related to many disparate and deep topics in mathematics, including singularities of Schubert varieties [23], representations of $p$-adic groups [24], characters of finite groups of Lie type [25], the geometry of unipotent conjugacy classes in simple complex algebraic groups [5,6], composition factors of Verma modules for semisimple Lie algebras [21], representations of Lie algebras in characteristic $p$ [19], and primitive ideals in universal enveloping algebras [32].

In this article we hope to present a different and often overlooked aspect of the cells: as geometric objects in their own right, they possess an evocative and complex beauty. We also want to draw attention to connections between cells and some ideas from theoretical computer science.

Cells are subsets of Coxeter groups, and as such can be visualized using standard tools from the theory of the latter. How this is done, along with some background, is described in the next section. In the meantime we want to present a few examples, so that the reader can quickly see how intriguing cells are.

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We thank M. Belolipetsky, W. Casselman, J. Humphreys, and E. Sommers for helpful conversations. Some computations to generate the figures were done using software by W. Casselman, F. du Cloux, and D. Holt. In particular, the basic Postscript code to draw polygons in the Poincaré disk is due to W. Casselman, as is the photo of G. Lusztig.

The author is partially supported by the U.S. National Science Foundation.

1 D. Vogan [32] also introduced cells for Weyl groups like those of Kazhdan-Lusztig.

Let $p, q, r \in \mathbb{N} \cup \{\infty\}$ satisfy $p^{-1} + q^{-1} + r^{-1} \leq 1$, where we put $1/\infty = 0$. Let $\Delta = \Delta_{pqr}$ be a triangle with angles $(\pi/p, \pi/q, \pi/r)$. If $p^{-1} + q^{-1} + r^{-1} = 1$, then $\Delta$ is Euclidean and can be drawn in $\mathbb{R}^2$; otherwise $\Delta$ lives in the hyperbolic plane. In either case, the edges of $\Delta$ can be extended to lines, and reflections in these lines are isometries of the underlying plane. The subgroup $W = W_{par}$ of the group of isometries generated by these reflections is an example of a Coxeter group. Under the action of $W$, the images of $\Delta$ become a tessellation of the plane, with tiles in bijection with $W$ (Figure 1). Hence we can picture cells by coloring the tiles of this tessellation.

For example, the triangle $\Delta_{236}$ is Euclidean, and the associated group $W_{236}$ is also known as the

Figure 1. Generating a tessellation of the hyperbolic plane by reflections. The central white tile is repeatedly reflected in the red, green, and blue lines.
Coxeter groups are certainly the Weyl and affine Weyl groups, which play a vital role in geometry and algebra. In fact, the symmetric group $S_n$ is also known to cognoscenti as the Weyl group $A_{n-1}$, while the three Euclidean triangle groups $W_{333}$, $W_{234}$, and $W_{236}$ are examples of affine Weyl groups.

The first step towards a geometric picture of a Coxeter group is its standard geometric realization. This is a way to exhibit $W$ as a subgroup of $\text{GL}(V)$, where $V$ is a real vector space of dimension $|S|$. Suppose we have a basis $\Delta = \{\alpha_s \mid s \in S\}$ of the dual space $V^\ast$. For each $t \in S$, there is a unique point $\alpha_t \in V$ such that $\langle \alpha_s, \alpha_t^\ast \rangle = -2\cos(\pi/m_{s,t})$ for all $s \in S$, where the brackets denote the canonical pairing between $V^*$ and $V$. Each $\alpha_s$ determines a hyperplane $H_s$, namely the subspace of $V$ on which $\alpha_s$ vanishes. For each $s$, let $\sigma_s \in \text{GL}(V)$ be the linear map $\sigma_s(v) = v + \langle \alpha_s, v \rangle \alpha_s^\ast$. Note that $\sigma_s$ fixes $H_s$ and takes $\alpha_s^\ast$ to $-\alpha_s^\ast$ (Figure 5(a)). One can show that the maps $\{\sigma_s \mid s \in S\}$ satisfy $(\sigma_s, \sigma_t)_{s,t} = \text{Id}$, which implies that the map $s \mapsto \sigma_s$ extends to a representation of $W$. It is known that this representation is faithful, and thus we can identify $W$ with its image in $\text{GL}(V)$.

Next we need the Tits cone $C \subset V$. Each hyperplane $H_s$ divides $V$ into two halfspaces. We let $H_s^+$ be the closed halfspace on which $\alpha_s$ is nonnegative. The intersection $\Sigma_0 = \cap H_s^+$, where $s$ ranges over $S$, is a closed simplicial cone in $V$. The closure of the union of $W$-translates of $\Sigma_0$ is a cone $C$ in $V$; this is the Tits cone. It is known that $C = V$ exactly when $W$ is finite. Usually in fact $C$ is much less than all of $V$. Hence the Tits cone gives a better picture for the action of $W$ on $V$.

Under certain circumstances we can obtain a more succinct picture of the action of $W$ on $C$. For certain groups $W$ it is possible to take a nice “cross-section” of the simplicial cones tiling $C$ to obtain a manifold $M$ tessellated by simplices. An example can be seen in Figure 5(b) for the affine Weyl group $A_2$. This group has three generators $r, s, t$.

For more about Coxeter groups, we recommend [8, 20].

\[\text{Figure 3. } G_2 = W_{236}.\]
with the product of any two distinct generators having order three. Thus $V = \mathbb{R}^3$, and the Tits cone $\mathcal{C}$ is the upper halfspace $\{(x, y, z) \in \mathbb{R}^3 \mid z \geq 0\}$. It turns out that the action of $\tilde{A}_2$ preserves the affine hyperplane $M := \{z = 1\}$, and moreover the intersections of $M$ with translates of $\Sigma_0$ are equilateral triangles. This reveals that $\tilde{A}_2$ is none other than our triangle group $W_{333}$. A similar picture works for any affine Weyl group, except that the triangles must be replaced by higher-dimensional simplices whose dihedral angles are determined by the exponents $m_i$. For more examples we can consider the hyperbolic triangle groups $W_{pqr}$, where $p^{-1} + q^{-1} + r^{-1} < 1$. In this case the Tits cone is a certain round cone in $\mathbb{R}^3$, and the manifold $M$ is one sheet of a hyperboloid (Figure 6). Then $M$ can be identified with the hyperbolic plane; under this identification the intersections $M \cap w\Sigma_0$ become the triangles of our tessellation.

**W-graphs and Cells**

There are two main ingredients needed to define cells: descent sets and Kazhdan-Lusztig polynomials. To introduce them we require a bit more notation.

The Coxeter group $(W, S)$ comes equipped with a length function $\ell : W \to \mathbb{N} \cup \{0\}$, and a partial order $\leq$, the Chevalley-Bruhat order. Any $w \in W$ can be written as a finite product $s_1 \cdots s_N$ of the generators $s \in S$. Such an expression is called reduced if we cannot use the relations to produce a shorter expression for $w$. Then the length $\ell(w)$ is the length $N$ of a reduced expression $s_1 \cdots s_N = w$. The partial order $\leq$ can also be characterized via reduced expressions. Given an expression $s_{i_1} \cdots s_{i_N}$, a subexpression is a (possibly empty) expression of the form $s_{i_{l_1}} \cdots s_{i_{l_M}}$, where $1 \leq l_1 < \cdots < l_M \leq N$. Then $y \leq w$ if an expression for $y$ appears as a subexpression of a reduced expression for $w$. Although it is not obvious from this definition, this partial order is well-defined.

The left descent set $\mathcal{L}(w) \subset S$ of $w \in W$ is simply the set of all generators $s$ such that $\ell(sw) < \ell(w)$. There is an analogous definition for right descent set. The definition of the Kazhdan-Lusztig polynomials, on the other hand, is too lengthy to reproduce here, although it can be phrased in completely elementary terms. For each pair $y, w \in W$ satisfying $y \leq w$, there is a Kazhdan-Lusztig polynomial $P_{y,w} \in \mathbb{Z}[t]$. By definition $P_{y,y} = 1$; otherwise $P_{y,w}$ has degree at most $d(y, w) := (\ell(w) - \ell(y) - 1)/2$. These subtle polynomials are seemingly ubiquitous in representation theory; they encode deep information about various algebraic structures attached to $(W, S)$. Moreover, computing these polynomials in practice is daunting: memory is rapidly consumed in even the simplest examples. In any case, for our purposes we only need to know whether or not $P_{y,w}$ actually attains the maximum possible degree $d(y, w)$ for
a given pair $y < w$. We write $y - w$ if this is so; when $w < y$ we write $y - w$ if $w - y$ holds.

We are finally ready to define cells. The left $W$-graph $\Gamma_\ell$ of $W$ is the directed graph with vertex set $W$, and with an arrow from $y$ to $w$ if and only if $y - w$ and $L(y) \notin L(w)$. The left cells are extracted from the left $W$-graph as follows. Given any directed graph, we say two vertices are in the same strong connected component if there exist directed paths from each vertex to the other. Then the left cells of $W$ are exactly the strong connected components of the graph $\Gamma_\ell$. The right cells are defined using the analogously constructed right $W$-graph $\Gamma_r$, while $y$, $w$ are in the same two-sided cell if they are in the same left or right cell.

Figure 7 illustrates all the computations necessary to produce the cells for the symmetric group $S_3 = (s, t | s^3 = t^2 = (st)^3 = 1)$. Figure 7(a) shows $S_3$ with its partial order and with the left descent sets in boxes. For this group one can compute that $P_{y,w} = 1$ for all relevant pairs $(y, w)$. Thus all the information needed to produce $\Gamma_\ell$ is contained in the left descent sets. Figure 7(b) shows the resulting graph $\Gamma_\ell$, and Figure 7(c) shows the four left cells. Computing right descent sets shows that there are three two-sided cells, with the blue and green cells forming a single two-sided cell.

Now we can explain the coloring scheme used in Figures 3 and 4. All regions of a given color comprise a two-sided cell. Moreover, the left cells are exactly the connected components of the two-sided cells, in the following sense. Let us say two triangles are adjacent if they meet in an edge. Then by definition, a set $T$ of triangles is connected if for any two triangles $\Delta, \Delta' \in T$ it is possible within $T$ to build a sequence $\Delta_1 = \Delta, \Delta_2, \ldots \Delta_n = \Delta'$ of triangles with each $\Delta_i$ adjacent to $\Delta_{i+1}$, and such that the sequence $\Delta_i$ contains $\Delta$ and $\Delta'$. Note a significant difference between the Euclidean group $W_{36}$ and the two hyperbolic groups. For the former, each two-sided cell contains only finitely many left cells, whereas this is not necessarily the case in general. The latter phenomenon was first observed by R. Bédard [2], who also showed [3] that there are infinitely many left cells for all rank 3 crystallographic hyperbolic Coxeter groups (see the last section for the definition of crystallographic). M. Belolipetsky proved that each Coxeter group in a certain infinite family has infinitely many left cells [4].

More Examples

There are two families of Coxeter groups for which we have a good combinatorial understanding of their cells: the symmetric groups $S_n$ and the affine Weyl groups $A_n$. For the former, left cells appear naturally in the combinatorics literature in the study of the Robinson-Schensted correspondence. A lucid exposition of this connection can be found in Chapter 6 of the recently published [8].

The latter is the work of J.-Y. Shi [29]. To describe some of his results, recall that we can associate to the group $\tilde{A}_n$ a tiling of $\mathbb{R}^n$ by simplices. The simplices can be further grouped into certain convex sets called sign-type regions. Figure 8(a) shows the sixteen sign-type regions for $\tilde{A}_2$; in general for $\tilde{A}_n$ there are $n^{n+2}$ sign-type regions. One of Shi's main results is that each left cell is a union of sign-type regions. Moreover, Shi also gave an explicit algorithm that allows one to determine to which left cell a given region belongs. The algorithm requires too much notation to state here, but it is completely elementary and involves no computation of Kazhdan-Lusztig polynomials. Figure 8(b) shows the two-sided cells for $\tilde{A}_2$ [26]; one can clearly see how the regions are joined into cells.

Figures 9(a) and 9(b) depict the cells of $\tilde{A}_3$. These images were computed directly from the data in

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Figure 7. (a) top, (b) center, (c) bottom.
the most colorful way to describe them is through the permutohedron, which is a polytope $\Pi_W$ attached to a Weyl group $W$ as follows. Let $x \in V$ be a point in the standard geometric realization of $W$ such that the $W$-orbit of $x$ has size $|W|$. Then $\Pi_W$ is defined to be the closed convex hull of the points $\{w \cdot x \mid w \in W\}$. It turns out that the combinatorial type of $\Pi_W$ is independent of the choice of $x$, and moreover the structure of $\Pi_W$ is easy to understand: its faces are isomorphic to lower-rank permutohedra $\Pi_{W'}$, where $W' \subset W$ is the subgroup generated by any subset $S' \subset S$ (such subgroups are called standard parabolic subgroups). For example, the polytope underlying Figure 9(a) is the permutohedron for the symmetric group $\mathfrak{S}_4$. The eight hexagonal (respectively, six square) faces correspond to parabolic subgroups isomorphic to $\mathfrak{S}_3$ (respectively, $\mathfrak{S}_2 \times \mathfrak{S}_2$).

Now the relationship between cells of affine groups of different ranks is conjectured to be as follows. For any finite Weyl group $W$, let $\tilde{W}$ be the associated affine Weyl group. Then the intersection of the cells of $\tilde{W}$ with the face of $\Pi_W$ corresponding to the standard parabolic subgroup $P$ should produce the picture for the cells of the affine group $\tilde{P}$. This is clearly visible in Figure 9(a): the cells for $\tilde{A}_2$ (respectively, $\tilde{A}_1 \times \tilde{A}_1$) appear when one slices the cells for $\tilde{A}_3$ with hexagonal (respectively square) faces of $\Pi_{\tilde{A}_3}$. Comparing the cells for $\tilde{C}_2$ (Figure 11(b)), originally computed by R. Bédard [2], with the cells of $\tilde{C}_2$ (Figure 11(a), [26]) shows another example of this. For more along these lines see [17].

Cells and Automata

Simple examples show that $W$-graphs can be quite complicated. However, despite this complexity lurking in their construction, the cells themselves appear to be very regular. In fact, for many groups one can prove that the cells can be built using a relatively small set of rules, rules that involve no Kazhdan-Lusztig polynomial computations at all [13], [14].

Computer scientists have a formal way to work with this phenomenon, the theory of regular languages and finite state automata [1]. One starts with
a finite set $A$, called an alphabet. Words over the alphabet are sequences of elements of $A$, and any set $L$ of words over $A$ is called a language. Informally, a language is regular if its words can be recognized using a finite list of finite patterns in the alphabet, patterns that are familiar to anyone who has ever used a Unix shell (e.g., `ls *, `tex`). A finite state automaton $F$ over $A$ is a finite directed graph with edges labelled by elements of $A$. The vertices of $F$ are called states. All vertices are designated as either accepting or nonaccepting, and one vertex is set to be the initial state.

Such an automaton determines a language over $A$ as follows. One starts at the initial state and follows a directed path terminating at an accepting state. Such a path determines a word (one simply concatenates the labels of the edges along the path to produce a word). We say that this word is recognized by the automaton. The set of all words recognized by an automaton is hence a language over $A$. A basic theorem is that a language is regular exactly when it can be recognized by a finite state automaton.

For a Coxeter group $W$, the alphabet is the set of generators $S$, and the language is the set Reduced$_w$ of all reduced expressions. By a result of B. Brink and R. Howlett [10], the language Reduced$_w$ is regular. Any left cell $C$ determines a sublanguage Reduced$_w(C) := \{ w \in \text{Reduced}_w \mid w \text{ is a word in } C \}$. W. Casselman has conjectured that the language Reduced$_w(C)$ is always regular.

Figure 12 illustrates these ideas for one of the yellow left cells in $A_2$ (Figure 8(b)). This cell has the property that every element in it has a unique reduced expression; such cells were first considered by G. Lusztig [24, Proposition 3.8]. The automaton has edges labelled by elements of $\{r,s,t\}$. The initial state is the encircled light purple vertex and is nonaccepting; all other vertices are accepting. To make the connection between the automaton and the cell, start at the bottom grey triangle. Then if while following a directed path we encounter an element of $S$, we flip the indicated vertex to move to a new triangle in the cell. For another example for a cell in the hyperbolic group $W_{34}$, as well as more information about the role of automata in the context of cells, we refer to [11], [12].

For $W = A_n$, the existence of automata for Reduced$_w(C)$ follows easily from the work of P. Headley [18] and Shi. Headley proved that one can construct an automaton $F$ recognizing Reduced$_w$ in which the vertices are the sign-type regions, and in which all vertices are accepting. Hence to recognize Reduced$_w(C)$ one merely takes $F$ and makes a new automaton $F_C$ by designating only the vertices corresponding to regions in $C$ as accepting. In fact Headley's automaton makes sense for all Coxeter groups, although the examples of $C_2$ and $C_3$ already show that the above argument for Reduced$_w(C)$ breaks down. However, for affine

\textit{An exposition can be found in Chapter 4 of [8], where $F$ is called the canonical automaton.}
Weyl groups, we have conjectured that a closely related automaton works for $\text{Reduced}_n(C)$ [17].

Further Questions
The pictures in this paper certainly raise more questions than they answer. For example, in the case of affine Weyl groups, for all known examples the left cells are of "finite-type," in the sense that they can be encoded by finitely much data. Here we have, in mind descriptions of the cells using such tools as patterns among reduced expressions [2, 13, 14], sign-types [29], or similar geometric structures [2, 17].

The cells for general Coxeter groups, on the other hand, appear to be fractal in nature, and thus cannot be described in the same way. Automata provide one convenient way to treat such structures, but they are not the only way. What are other techniques, and which are natural?

The situation becomes even more intriguing when one considers relationships between cells and representation theory. For instance, Lusztig conjectured [24, 3.6] and proved [27] that an affine Weyl group $W$ contains only finitely many two-sided cells. In fact, he proved much more: he showed [28] that there is a remarkable bijection between two-sided cells and the unipotent conjugacy classes in the algebraic group dual to that of $W$. Moreover, each two-sided cell contains only finitely many left cells. Lusztig also conjectured [24, 3.6] that the number of left cells in a two-sided cell can be explicitly given in terms of the cohomology of Springer varieties [31].

For general Coxeter groups our knowledge is much more impoverished. First of all, it is not known if there are always only finitely many two-sided cells, although in all known examples it is evidently true. Perhaps the only general result is due to M. Belolipetsky, who showed that right-angled hyperbolic Coxeter groups have only 3 two-sided cells [4]. Furthermore, in joint work with M. Belolipetsky we have conjectured that the Coxeter group associated to a hyperbolic $n$-gon with $n$ distinct angles has $(n+2)$ two-sided cells.

The connection with geometry is even more tenuous. If a Coxeter group $W$ is crystallographic, which by definition means $m_i \in \{2, 3, 4, 6, \infty\}$ for all distinct generators $s$, then there is associated to $W$ an infinite-dimensional Lie group $G$ called a Kac-Moody group. In principle, $G$ provides a setting to study geometric questions about cells, since many of the standard constructions (e.g., flag varieties, Schubert varieties) make sense there. Of course, at the moment the connections with geometry are poorly understood. For instance, the fact that a two-sided cell can contain infinitely many left cells [2-4] is somewhat sobering.

If $W$ is not crystallographic, then there is no such group $G$. For such $W$ we have no candidate for an algebro-geometric picture. However, computations with many examples (cf. Figures 3 and 4) indicate that certain structures vary "continuously" in families containing both crystallographic and non-crystallographic groups and that these structures are apparently insensitive to whether or not the underlying group is crystallographic.

The situation is analogous to that of convex polytopes. In the 1980s many difficult theorems about polytopes were first proven using the geometry of certain projective complex varieties—toric varieties—built from the combinatorics of rational polytopes. Deep properties of the intersection cohomology of these varieties led to highly nontrivial theorems for rational polytopes; for some of these theorems no proofs avoiding geometry were known.

By definition rational polytopes are those whose vertices have rational coordinates. However, not every polytope is rational, and for irrational polytopes no toric variety exists. Yet irrational polytopes seem to share all the nice properties of their rational cousins.

Today we have a much better understanding of this story. Recently several researchers have developed purely combinatorial replacements for the toric variety associated to a rational polytope and using these replacements have extended various difficult results from the rational case to all polytopes; see [9] for a recent survey of these results.

For Coxeter groups, the analogy suggests developing combinatorial tools to take the role of the algebraic-geometric constructions that seem essential in the study of crystallographic groups. Recently there has been significant progress in this effort [15, 16, 30]. Nevertheless, understanding the geometry behind cells for general groups, if it exists, remains an intriguing and difficult problem.

References

\*In fact, the analogies between convex polytopes and Coxeter groups go much further than what is suggested in these paragraphs [7] and deserves a lengthy exposition of its own.


Editor's Note: This is the first part of a two-part article. In part two, which will appear in a later issue, the authors discuss the mathematical accomplishments of Serge Lang and the impact of those achievements.

In September 12, 2005, the mathematics community lost Serge Lang, who passed away in his apartment in Berkeley, California. Lang was well known as a mathematician, and also as an educator and political activist. The main force in Serge's life was his enthusiasm for mathematics. In a world of vagaries and irrational passions, he saw mathematics as a noble pursuit that represented honesty and goodness. Within mathematics alone, Serge had many facets—a researcher, an expositor, a popularizer, and a teacher. Generations of mathematicians around the world know the name Serge Lang through his numerous books and articles.

For those individuals who knew Serge, one striking feature most everyone noted was the compartmentalized manner in which he showed himself to anyone: His mathematical colleagues were told virtually nothing about his personal life, his family knew very little about his mathematical research, his political allies were only slightly informed of his mathematical interests, and even his closest friends were unaware of each other's presence in his life.

As we prepared this article discussing the many aspects of Serge's life, we chose to follow Serge's method of "file-making", where the reader is informed through the presentation of original documentation. We have sought to bring out a full picture of Serge's life by inviting contributions from a large number of individuals who knew him well. For the editors, it was fascinating to witness the diversity of these reminiscences; they represent a broad range of interests and achievements. It is clear that, with Lang's passing, we have lost someone unique and irreplaceable.

After Lang's passing, Yale University president Richard C. Levin wrote about Serge, "While having someone like this in the community is not always easy, it is salubrious." It is entirely possible Serge would have agreed with this assessment, perhaps even assigning a letter grade for President Levin's summary.

To repeat, our article is an attempt to follow Lang's insistence for an honest and complete representation, allowing readers to draw their own conclusions. With this said, we have no doubt that a common judgment will be drawn by everyone: With Lang's death, the mathematical world, and beyond, has lost someone without equal, and in time we will better understand the significance of Lang's life.

On Serge Lang's retirement from Yale University in the spring of 2005, Yale president Richard C. Levin honored him with these words:

Serge Lang, A.B., California Institute of Technology, Ph.D. Princeton University, faculty member at Yale since 1972: Your primary love has always been number theory and you have written, by one colleague's estimate, over 50 books and...
monographs, many of them concerned with this topic. Several of your monographs are the only, or nearly the only, book treatments of their important subjects. Your famous theorem in Diophantine equations earned you the distinguished Cole Prize of the American Mathematical Society. Your textbooks also have garnered accolades. Your calculus for undergraduates went through many editions in the seventies and eighties, and your algebra textbook is a standard reference in the field. So prodigious are you as a scholar that there are actual jokes in your profession about you. One joke goes: "Someone calls the Yale Mathematics Department, and asks for Serge Lang. The assistant who answers says, "He can't talk now, he is writing a book. I will put you on hold."

In your character, you are uncompromising in your insistence on what you perceive as logical consistency and rhetorical honesty, and you have questioned much received wisdom and many authorities in the external world as well as here at Yale. You are an excellent and deeply caring teacher, and in honor of this several years ago you received the Dyon Hixon Prize for teaching in Yale College. Your students keep in touch with you years after they graduate and one has created an endowed fund in your honor. Among your many monographs there is one called The Beauty of Doing Mathematics, a collection of three dialogues you gave in Paris in the '80s. Yale is grateful to you for the passion with which you understand, practice and profess the mathematical arts, and wishes you well as you continue your lifelong engagement with their illimitable splendors.

Serge Lang was born near Paris on May 19, 1927. His family lived in St. Germain en Laye. Serge's mother was a concert pianist and his father was a businessman. His sister, with whom Serge maintained an affectionate relationship all his life, currently lives in Los Angeles and is a stage and film actor. Serge's twin brother was a college basketball coach.

The family decided when Serge was a teenager to move to Los Angeles, California. Serge attended Caltech as an undergraduate and finished with a B.A. degree in physics in 1946. After spending 1.5 years in the U.S. Army, Serge entered graduate school at Princeton University in philosophy. He abandoned that study after one year and turned his attention to mathematics. That attention never deviated (except occasionally for his politics) for the rest of Serge Lang's life.

At Princeton Serge Lang fell under the spell of the great algebraic number theorist Emil Artin. Along with John Tate, a fellow student of Artin, Lang developed a passion for algebra and algebraic number theory. In later years, Lang and Tate co-edited the collected works of Artin. Lang earned his Ph.D. in 1951.

Lang's first academic position was as an instructor at Princeton. Lang also had an instructorship at the University of Chicago from 1953 to 1955. Lang's first permanent position was at Columbia University beginning in 1955. In addition to producing some terrific mathematics and directing five Ph.D. theses, Lang became passionately involved with the politics of the time (in protest against the Vietnam war). Serge ultimately resigned his position at Columbia in 1971 (without yet having arranged for another job) in protest against Columbia's treatment of anti-war protesters. It is also remarkable that, during his tenure at Columbia, Lang directed two Princeton Ph.D. students: Marvin Greenberg (1959) and Newcomb Greenleaf (1961).

After leaving Columbia University, Serge Lang landed a job at Yale University (beginning in 1972), where he spent the remainder of his career. Lang directed nine additional Ph.D. degrees while at Yale. He was awarded the AMS Frank Nelson Cole Prize (1959) for his mathematical research and the AMS Leroy P. Steele Prize (1999) for his writing. He was elected to the National Academy of Sciences in 1985.
Serge Lang was a remarkably energetic individual with eclectic and broadly ranging tastes. In addition to his passion for mathematics he loved music and the arts. He himself was an accomplished pianist and lutenist, and he enjoyed playing in public. He took a keen interest in politics, especially as it manifested man’s inability to face the truth. Lang loved to bring down individuals who obfuscated, who hid behind their rank, or who abused power. He engaged in a great many rather public battles with a wide-ranging collection of people, from social scientists at Harvard to researchers at the National Institutes of Health to education researchers at Stanford. As Lang himself put it, he “put scholarship in the service of action to stop the nonsense.”

Serge also was a prolific writer. He wrote more than 120 research articles and sixty-one books (and this does not count multiple editions and foreign translations). In fact he has 198 citations on MathSciNet. It is amazing to examine the range of mathematical topics covered by Lang’s opus: calculus, real analysis, complex analysis, differential geometry, algebra, algebraic geometry, diophantine geometry, hyperbolic geometry, math talks for undergraduates, the heat kernel, and much, much more. Perhaps Lang’s most famous and most influential book is *Algebra*, now in its third edition. In it, Lang single-handedly reorganizes and revitalizes this fundamental and central subject. The book has had an enormous impact.

Serge Lang was a man with incredible focus and self-discipline. Mathematics and politics (which he called “troublemaking”) were his primary interests, and everything else was secondary. As he grew older, he felt that he had to conserve his energy and he set other interests aside. He made hard decisions and stuck by them. As an example, when he decided to stop listening to music, he put all his recordings on the shelf, never to be picked up again.

It is astonishing how Lang’s books affected people at all levels. One high school teacher who regularly used Lang’s calculus book in his teaching said this:

As a high school teacher, I used this text with great success several times for both AP Calculus BC and AP Calculus AB courses. It is my favorite calculus text to teach from, because it is very user-friendly and the material is presented in such an eloquent way. There are no gratuitous color pictures of people parachuting out of airplanes here. Opening this book is like entering a temple: all is quiet and serene. Epsilon-delta is banished to an appendix, where (in my opinion) it belongs, but all of the proofs are there, and they’re presented in a simple (but not unsophisticated) way, with a minimum of unnecessary jargon or obtuse notation.

A somewhat recondite joke is the query “Why did Bourbaki stop writing?” The answer is that they discovered that Serge Lang is one person. Lang’s output of text connected to his many political disputes was voluminous. He also has some unpublished books of a political nature (others of his political tracts were actually published). Lang liked to say that the best way to learn a new topic is to write a book about it.

Although Lang’s first mathematical loves were algebra and number theory, his interests rapidly expanded to cover an astonishing panorama of modern mathematics. Areas that he influenced include number theory, algebraic geometry, diophantine geometry (in which he was a pioneer), diophantine approximation, differential geometry, analysis, hyperbolic geometry, Arakelov theory (in which he was a pioneer), modular forms, and many other areas as well. The scope of Lang’s books and papers is astonishing not only for its magnitude but for its breadth.

Serge Lang resigned from the AMS in 1996 in a dispute concerning an article in the AMS Notices by Denise Kirschner. He retired from Yale in the spring of 2005.

It gives a sense of Serge Lang to quote from his formal note of acceptance for the Steele Prize (which in fact had to be heavily edited because it was formulated in such strong language):

I thank the Council of the AMS and the Selection Committee for the Steele Prize, which I accept. It is of course rewarding to find one’s works appreciated by people such as those on the Selection Committee. At the same time, I am very uncomfortable with the situation, because I resigned from the AMS in early 1996, after nearly half a century’s membership. On the one hand, I am now uncomfortable with spoiling what could have been an unmitigated happy moment, and on the other hand, I do not want this moment to obscure important events which have occurred in the last two to three years, affecting my relationship with the AMS.

Torn in various directions, sadly but firmly, I do not want my accepting the Steele Prize to further obscure the history of my recent dealings with the AMS.
Perhaps Serge Lang’s greatest passion in life was learning. For Serge, learning manifested itself in many guises; but one of the most important of these was his teaching. He saw himself as a role model for his students, and he spent a great deal of time with them. He often said that the best way to learn about a university was to eat in the student cafeteria. He did so frequently. He often took his students out to eat, or invited them to his residence to listen to music. Although he did so quietly and discreetly, Serge was known to provide financial assistance to students and mathematicians who were in need. Serge is remembered fondly for spending chalk at his students.

Serge’s graduate courses frequently followed the track of the book he was currently writing. His undergraduate courses could be more freewheeling. An important point to note is the joy that Serge derived from all things mathematical. It can certainly be said that most of us mathematicians experience some sort of “high” when we learn to tackle and tame new ideas. As we get older, we become more jaded; as a result, this “high” is harder and harder to achieve. Not for Serge. He was truly engaged and fulfilled when he discovered new ideas on any level, be that an illuminating problem for one of his undergraduate texts or an insight into a new mathematical landscape. As a result, Serge Lang always remained mathematically young.

Serge Lang spent the fall of 2004 at U. C. Berkeley as a Miller Visiting Professor. He gave a number of lectures and made his presence known in many other ways. As an example, budget cutbacks had caused severe curtailment of departmental teas. People now had to pay daily for their beverages and cookies. Serge quietly contributed a substantial amount of money so that the Monday teas would be lavish: many fine cakes and pastries and lots of nice things to drink. Certainly this had a very positive effect on departmental life, and Serge asked for no particular credit for this gesture.

Serge had wide-ranging interests. He visited Berkeley every summer for the past several decades (in fact he kept an apartment there) and he would attend colloquia in departments ranging from physics to history to political science to medicine to mathematics. Of course he did not simply attend. His habit was to confront the speaker with detailed and probing questions. Frequently the sessions would become so heated and protracted that intervention was necessary.

One memorable incident—just to illustrate the eclecticism and vehemence of Serge’s interests—has Serge threatening to clobber with a bronze bust a very distinguished Princeton mathematician in the Fine Hall Professors’ Lounge because the latter would not accede to the self-evident assertion that the Beatles were greater musicians than Beethoven.

At Yale in 2001, Serge was invited to be the keynote speaker at a Pierson College Master’s Tea. He dressed in a courtier’s outfit and regaled the packed room with his theory of the similarities between Elizabethan music and classic rock of the 1960s. Lang illustrated his points by playing (classical LP records of) Ding Dong the Witch is Dead, a 1969 hit by the Fifth Estate, a 1612 piece by Michael Praetorius, and We Can Work it Out by the Beatles (1965).

Serge loved to challenge people—friend and foe alike—just for the sake of challenging them. As an instance, James Borger recalls:

“I remember one time when I was a grad student, I was standing next to him at a tea while he was explaining to a first-year student that analysis is just number theory at infinity. I said, “Come on, that’s not true.” He immediately turned up the volume, challenging me to stop bullshitting and give an example. I said, “OK, p-adic analysis,” and then walked away. But I’ve always wished I had stayed to see what his reaction would have been. We need more troublemakers like him.”

In 1998 Serge Lang published a book called Challenges. This editor (Krantz) found the volume to be particularly inspiring, for it recounted, from Lang’s personal perspective, some of his most involving and exciting political battles. The book is truly outstanding for its honesty and incisiveness. Two particular battles that stand out are The Case of Ladd and Lipsett. In the late 1970s the distinguished social scientists Everett Carll...
Ladd Jr. and Seymour Martin Lipsett set out to evaluate the American professoriate. They concocted a questionnaire to be distributed across the country, asking professors detailed questions about how they plied their trade, what values they held as members of the academic profession, and so forth. Their results were published as The 1977 Survey of the American Professoriate in 1979. Lang found the questionnaire, and the premises for the study, to be repugnant. He conducted a massive effort to discredit their work. In fact Lang published a rather massive tome, The File (Springer-Verlag, 1981), containing all his correspondence and information about the battle. In the end, Lang caused Ladd/Lipsett to lose much of their funding and a great deal of their credibility.

The Case of Samuel P. Huntington. In 1968 Samuel P. Huntington wrote a book entitled Political Order in Changing Societies. In it Huntington uses what might charitably be characterized as pseudomathematical hucksterism to "prove" that South African society in the 1960s was a "satisfied society". Serge Lang decided that nothing could be further from the truth, and in any event Huntington's methodology was suspect if not corrupt. He conducted a vigorous campaign to derail Huntington's credibility, and he twice successfully blocked Huntington's election to the National Academy of Sciences.

Serge Lang was quite proud of his efforts to instill a sense of truth and honor into our public discourse. For years after his battle with Huntington, he would give his students "Huntington tests" to ascertain their ability to think critically. Serge's battle cry was to demand whether his listeners knew "a fact from a hole in the ground". Evidently Serge did. In a particularly earthy moment, Serge liked to say that "he was inside the tent pissing in" (with allusion to Lyndon Johnson commenting about J. Edgar Hoover). For each of his battles, Lang would create what he called a "File". This was a detailed and copious collection of all his correspondence and all his data connected with any given case. Often a file would consist of several hundred pages of closely knit text. Lang would, at his own expense, send copies of his files to mathematicians and other interested parties all over the world. The Serge Lang files have been a staple of mathematical life for over forty years.

Serge Lang said of himself

I personally prefer to live in a society where people do think independently and clearly. One of my principal goals is therefore to make people think. When faced with persons who fudge the issues, or cover up, or attempt to rewrite history, the process of clarifying the issues does lead to confrontation, it

creates tension, and it may be interpreted as carrying out a "personal vendetta"...I regard such an interpretation as very unfortunate, and I reject it totally.

Serge spent hours every day on the telephone, wheeling, cajoling, instructing, and most often yelling. His collaborators relate that Serge would often phone several times a day—every day. He would learn what was the best time to phone and then phone regularly at that time. Often one would pick up the phone and hear "Serge! Let me continue to instruct you about ..." But it should be stressed that Serge was disciplined to the extreme. He did not waste time. It was amazing to watch him eat lunch in five minutes and dash back to his office to resume his writing.

In the last twelve years of his life Serge Lang developed a deep and energetic program to fight the current directions of research on the disease AIDS (Acquired Immune Deficiency Syndrome). A naive assessment of Serge's position is that HIV does not cause AIDS. But this would be an injustice to Serge. First of all, he was very careful. He very rarely made an error of fact. Secondly, he was quite a subtle thinker. His cause and his complaint, in fact, was that the search for a cure to AIDS had become politicized. At a certain point, the federal government simply commanded the National Institutes of Health to declare that HIV caused AIDS. The causal mechanism had not been identified, and the connection not logically established. To be sure, there is considerable ad hoc evidence of a link between HIV and AIDS. Certainly many of the modern treatments for AIDS are premised on that link. But Serge's assessment was that the existing data analysis does not support the conclusion that HIV causes AIDS.

The present article is a celebration of the life of Serge Lang. We present a number of vignettes, contributed by mathematicians, former students, colleagues, and friends. These are divided into pieces about Serge the man, pieces about Serge the writer, pieces about Serge the tilter at windmills, and pieces about Serge the mathematician. Our aim is to give a well-rounded picture of what a diverse and multi-faceted person we have lost. He was in many ways a thorn in our collective sides, but he was a friend to us all.
Memories of Serge Lang

Friedrich Hirzebruch, Max-Planck-Institut für Mathematik

Serge Lang was a close friend of my wife and me, of our three children, and even of some of our grandchildren. We miss his frequent telephone calls—"It's me"—the last one was on September 10, 2005. We shall miss his visit next summer and all the following summers.

My wife and I met Serge 53 years ago in Princeton when he and I were 25 years old. We became good friends. It was old Europe that all three of us liked. Serge rarely spoke about his personal past, but by asking questions we slowly learnt the basic facts. He came with his father and his sister from Paris to the United States after France had been occupied by Germany. He was a soldier in the U.S. Army from 1946 to 1947 and was stationed in Italy and Germany for part of the time. The fact that his life was disturbed by the Nazi war was not a barrier between us. How little Serge spoke of himself can also be seen from the Curriculum Vitae in his Collected Papers. The CV has thirteen brief lines, from the first one, "1927 born" to the last one "1972-present Yale".

We kept close contact with Serge, also after our return to Germany. In the summer of 1955 he visited us in the house of my parents in Hamm (Westfalen) (see top photo, right).

I was appointed to the University of Bonn in 1956 and began the series of Arbeitstagungen where the speakers are chosen by "public vote" at the beginning of the meeting. With very few exceptions Serge attended all Arbeitstagungen until 2003. During the thirty Arbeitstagungen I organized from 1957 to 1991, Serge gave thirteen lectures. The second photo from the top, right, shows Serge lecturing at one of the Arbeitstagungen. The next photo down shows him at some other Arbeitstagung activity.

During each of the twenty-five years from 1979 to 2003, Serge spent one month in Bonn, usually June; in addition, he came for three sabbatical fall terms in 1993, 1997, and 2000. He financed his June visits from 1984 to 1989 by the funds of his Humboldt Prize.

He had a stable routine: In the summer he went from Yale to Europe. For many years he visited Paris for a month until he stopped. "To everything there is a season," he used to say. In other years he went Zurich or Berlin. He never omitted Bonn until the season also ended for the Max-Planck-Institute. In 2004 and 2005 he only visited us privately for a few days. After his European visit he went to Berkeley where he enjoyed the cooler climate and we met him in a number of years.

During his visits to Bonn he gave many lectures, in seminars on his own research and for students of beginning and advanced level. In an official report to me (13 February 1997) he wrote as follows: "While at the Max-Planck, I also visit other mathematicians, both in Germany and elsewhere such as Holland. I have substantial contacts with students. I used to lecture every year in your analysis course. Last year I lectured to the high school class of Karcher's son. Thus my days at the Max-Planck, regularly for one month in June every year, and once for four months in fall of 1993, have been important periods in providing proper environment for establishing mathematical contacts at all levels, as well as learning and doing mathematics."

Serge also lectured once to my algebra course (120 students) where I used his algebra book. When I came to the lecture hall, I saw that the official student representatives were selling cheap photocopied versions of Serge's book. I told them that this was illegal. The students said "The author is far away." I replied "You are wrong. He will be here in two minutes, because he is taking over my lecture today." Serge came and did not make a great fuss about it. He even signed some of the copies before he began his lecture. Serge also lectured to the public. He wanted not only to teach mathematics, but also how to be critical and responsible. "I want to make people think." Among the public lectures I mention the "Beauty of Mathematics". He explained the hyperbolic 3-dimensional manifolds with cuspidal ends which are like the arms of an octopus. The bottom photo above shows Serge sitting at my desk in our home drawing octopuses. Behind his back there is one shelf with 40-50 of his books. When he presented me a new edition of one of his books, he threw the old edition into the wastepaper basket from where I retrieved it later.

Mathematics was the most important part of Serge's life. He worked with great self-discipline for
many hours seven days a week. I admired the way he could turn courses into books and how he continuously did fundamental research. In the early years there was time for the piano (including composing), for playing guitar and lute, for going to concerts, to the theatre and opera, and to enjoy literature. We could often do all this together with him when visiting him in his apartment in New York, where he played his piano compositions for us, and when we went out in New York to the theatre. Similar activities took place in Bonn during his visits. He enjoyed the musical life around Bonn. But then came the time of the file making. He was able to give up things he loved and to concentrate on the two parts of his later life (mathematics first and then political work). All other things had to go. "To everything there is a season." The file concerning The 1977 Survey of the American Professoriate by Ladd and Lipsett developed from 1977 to 1979. Serge put a lot of time and energy into it. We always received Serge’s mailings in installments of 20-30 pages. It was exciting reading, full of suspense. Other files followed. The mailings came regularly, the last one on the day of his death. It was certainly not easy to discuss the files with him in such a way that he did not begin to yell. "Ich bin ein unbequemer Mensch," he said. We admired his sincere way to rely only on facts, "to distinguish a fact from an opinion." He fought for honesty and precision in research and in journalism. He hated "big shots who throw their weight around". He objected to covering up because of collegiality. His heritage is his Collected Papers, his scientific and his political books. But we miss Serge as a friend.

Norbert Schappacher, University of Strasbourg

Two years ago, I gave a seminar in Zürich on the topic of intellectuals among twentieth-century mathematicians. My list included the Englishman G. H. Hardy, the Germans E. J. Gumbel and E. Kamke, the Frenchman L. Schwartz—and the French-born American Serge Lang.

The term intellectual (intellectuel) used here is a French invention of the Dreyfus affair, from the final years of the nineteenth century. Emile Zola, Anatole France, Marcel Proust, and others were the first self-declared intellectuals. The expression has a built-in partiality: it is only used for people whose opinions you sympathize with, and whose opinions and ways of expressing them are loathed by those who are on the other side.

Serge Lang was an intellectual in this European sense of the word, and he was one of the rare mathematicians of the second half of the twentieth century who can lay claim to this epithet. If colleagues sometimes felt he was overdoing things, this may actually confirm what he represented.

But Serge Lang lived and acted in the U.S. where no heritage of intellectuals exists, in spite of literary figures like Arthur Miller. So Lang had to cut one out for himself, as the Yale professor who made The New York Times by blocking Samuel Huntington’s admission to the National Academy of Sciences. In doing so he was surely helped by the ambient climate of the late 1960s and 1970s, the Free Speech Movement, etc. But his personal device, "the file", was his own creation.

Let me add something more personal: The most wonderful thing about Serge was that he was always around, and meeting him would always matter. I first saw him as a young student in Bonn during the Arbeitstagungen of 1970 and 1971; I went to his talks because I knew the name from his Algebra book. At the time I did not understand the least bit of the mathematics he was talking about; but I distinctly remember the presentation: his talk seemed to be about presenting things from the right point of view, which others working in the field had failed to see or to adopt.

I kept meeting him over the years in many places, and each time I was greeted by his charm and thrilled by his intensity. The last two summers I invited him for talks to Darmstadt and Strasbourg. Even though he would never complain about mediocre accommodation or food offered to him, Serge enjoyed being taken out to good restaurants. At least by the time we got to dessert he invariably had to raise his voice (for instance, because I still would not understand his way of putting to rest the philosophy of the Vienna circle, that he had figured out after a few months as a student of philosophy), and push people around us would start raising their eyebrows. I loved this kind of scene (with him, not in general), and I will now miss it a lot.

Barry Mazur, Harvard University

In 1958, at Princeton, I had accidentally slipped into the room in which Serge was giving his seminar in Abelian Varieties; I was transfixed by the metallic urgency, the vitality, of the voice of this chalk-wielding person; I understood absolutely nothing of the subject, but was instantaneously convinced, with that utterness of conviction that is the gift of ignorance, that abelian varieties—whatever they were—were of breathtaking importance, and furthermore, of breathtaking importance to me. That Serge (a "mathematical grown-up") would, shortly afterwards, collar me and request a series of private lectures in differential topology was astounding to me. I treasure the halting lectures I gave...
him, as a rite of passage, of immense importance. And Serge did this sort of thing, through the decades, with many of the young: he would proffer to them gracious, yet demanding, invitations to engage as a genuine colleague—not teacher to student, but mathematician to mathematician; he did all this naturally, and with extraordinary generosity and success. Serge was a gadfly with formidable tenacity. That we are personally responsible for the web of compromises that we have all come to accept, and to think are inevitable, is something he would never let us forget. That we, as editors or referees of journals, make our judgments based on some presumed social, or sociable, contract (e.g., no political articles in a math journal) does not let us off the hook when asked to examine without prejudice the underpinnings of our (usually only implicit) social contracts. Serge seemed to be, over the decades, of one age, and that age was young (with its virtues and drawbacks). He had, when he played the piano, something of a brilliant French articulation to his style, and there was a hint of this in everything he did, from his walking gait (staccato) to the way in which he pronounced certain key words in mathematics, like idea which, from Serge, would sound like EYE-dee, which has a kind of platonic zing to it. Over decades of mathematics Lang was led, more specifically, by an over-arching vision, which he pursued through the agency of various fields of mathematics. The vision, baldly put, is that geometry is an extraordinarily striking dictator of qualitative diophantine behavior. The still open Conjecture of Lang in higher dimensions continues to serve as a guiding principle to the way in which the grand subjects of geometry and number theory meet, just as Serge himself served as an inspirer of generations of mathematicians, and a spokesman for intellectual honesty.

Paul Cohen, Stanford University

I was a graduate student when Serge Lang arrived in Chicago as an assistant professor. My interests were tending towards number theory, but were not very focused. The arrival of Serge made a huge difference to me and to many other graduate students. He immediately gave courses in algebraic geometry and in algebraic number theory, also I believe, accompanied by a constant output of notes. Suddenly I had a different idea of what mathematical research was. One could just attack problems without a huge background of knowledge. His lectures were entertaining, of course, but also a little intimidating to the poor souls who might ask silly, or too elementary, questions. It was advisable to be nimble enough to dodge flying chalk coming from his direction. Another powerful memory is watching him in classes by André Weil on abelian varieties. Whereas the rest of us were totally cowed by Weil’s personality, Lang seemed to be able to follow anything and even to make corrections and amendments to Weil’s presentation. This seemed to me to be nothing short of miraculous. After Serge left Chicago to pursue his very illustrious career, I had only intermittent contact with him. When we did meet, the memory of those early days in Chicago would come flooding back, and in a way, he was a powerful force in my life. From a distance, of course, I watched his erratic battles and even was very rarely a victim of some of his outbursts. But his textbooks, his successes, gave me great pleasure. I regard him as a great mathematician, much more than an expositor, as I believe some regard him. I don’t know if we will ever see his like again, with anything approaching the enormous energy and insight which he brought to everything he touched.

When I learned of his death, a profound sadness came upon me. It was a feeling of incompleteness, that somehow I could not express my closeness to him personally, nor help him avoid some of his more acrimonious disputes. But Serge would have scoffed at such thoughts, and said that he managed perfectly well.

Stephen Smale, University of California, Berkeley

I had met Serge Lang by 1960. In fact in that year he initiated a very nice offer to me to leave Berkeley to join the faculty at Columbia University, which I accepted. During the three years I spent at Columbia I became close friends with Serge and it was his support and friendship that helped make the Columbia years such a memorable period. During my first years back at Berkeley, I became involved in the Free Speech Movement and especially the Vietnam anti-war protests. In this period I invited Serge for a visit and we shared an office for a year (1966–67, I believe). I introduced him to some local activists and Serge himself became a political activist, writing a book on the Bob Scheer campaign for political office.

During the following decades we kept in touch especially during his summer visits to Berkeley. I tried without success to get a permanent appointment in the mathematics department for Serge. We got along well and I seemed to have some immunity from his occasional outbursts of anger. I saw much less of Serge in the last decade partly because of my life in Hong Kong and Chicago.

While leaving to others assessments of Serge’s mathematical research, I want to make some brief remarks on other contributions.
Serge's Books: My opinion is that these books were a great contribution to mathematics. He gave copies of almost all to me, and I frequently used some for textbooks in courses I was teaching. They were characterized by economy and elegance and were written from a broad point of view, mathematics as a whole. I especially enjoyed his graduate analysis book. This was a book written for students in mathematics, (and mathematicians, not just analysts) and reached the heart of the subject quickly. In contrast, other texts I have encountered spend a semester or even a year on foundational material as a step in the training of an analyst. On the other hand, his book on differential geometry had the virtue of giving infinite dimensional foundations to the subject, which I found important in my own research.

Serge's Teaching and Inspiration to Students: Serge made a special and constructive effort to reach out to students of all ages. For example, he wrote a mathematics book for high school students and gave annual lectures to math club students at Berkeley. His contributions included making elementary expositions of topics in current research. On two different occasions Serge gave lectures—in German—to Don Zagier's wife's son Bernhard's school class. The talks were enjoyed by all, and greatly increased Bernhard's prestige with both his teachers and his classmates.

Serge's Files: These files and their accounts contained extensive documentation of hypocrisy of "The Establishment", in science, in and outside of mathematics. Although acknowledging their positive role in science I sometimes disagreed with him in these matters. In particular, though his criticism of the AIDS bureaucracy sometimes made sense, it was hard to go along with his attack on the HIV theory of AIDS.

In ending, let me emphasize how big an influence Serge has been in my life and how much I will miss him.

John Coates, Cambridge University

I think Serge's most remarkable quality as a colleague was his unstinting support for young mathematicians. I personally benefited from this myself when I was a young postdoc at Harvard in about 1970, and Serge came as a visitor for a semester, shortly after he had resigned from Columbia. We tend to forget when we are more established in the mathematical world how precious it is when one is trying to make one's way in research to have the support and encouragement of an older mathematician. Serge was not at all distant to young people, but went out of his way to find out what one was thinking about, and took time to discuss his own ideas and feelings about the subject with one.

We only wrote one small joint paper together (on diophantine approximation on abelian varieties), but his encouragement and friendship came at a crucial time in my own mathematical evolution, and I have always been immensely grateful for it. The second striking quality of Serge's was that he really did live for mathematics, and somehow his belief in the goodness of the endeavour to do mathematics was profoundly moving.

Dorian Goldfeld, Columbia University

Of the many people who had serious interactions with Serge, I am one of those who came away with fierce admiration and loyalty. In the mid-1960s, I was an undergraduate in the Columbia engineering school on academic probation with a C-average. In my senior year I had an idea for a theorem which combined ergodic theory and number theory in a new way, and I approached Serge and showed him what I was doing. Although I was only a C-level student in his undergraduate analysis class he took an immediate interest in my work and asked Lorch if he thought there was anything in it. When Lorch came back with a positive response, Lang immediately invited me to join the graduate program at Columbia the next year, September 1967. In the fall of 1967 I found an unfixable error in my ergodic-number theoretic theorem. Lang was not at all perturbed. He said these things happen all the time and encouraged me to move on to something else. The following year Serge refused to discuss mathematics with me or with anyone else. He said he was taking a year off from mathematics and doing politics instead, but he kept encouraging me to prove theorems and told me to talk to Gallagher, who became my official advisor.

Despite his kindness to students, everyone close to Serge has seen him explode, and this happened to me on several occasions. For example, in 1992 I organized a special year on number theory at Columbia University. I invited some of the well-known established leaders of the field such as Bombieri, Lang, Mazur, Manin, Schmidt, Szpiro, as well as many younger people. Lucien Szpiro was chief editor of Asterisque and invited me to submit a proceedings of the conference to Asterisque. Lang became utterly infuriated and blew up at me when Asterisque refused to accept his paper with Jorgenson, which I previously had invited him to submit. I ultimately told Asterisque that I would resign as editor of the proceedings and withdraw my own submitted paper unless they accepted the Jorgenson-Lang paper. Asterisque refused to budge, and I immediately followed through on my threat. The proceedings were, nonetheless, eventually published. My paper and the Jorgenson-Lang paper...
were published in 1994 by Springer-Verlag as a book.

I was very shaken for several days when I heard that Lang is with us no more. He has had a profound influence on my life and I will miss him enormously.

Jay Jorgenson, City College of New York

During the past fifteen or so years, Serge Lang and I were colleagues, co-authors, and close friends. Since Serge was known to never discuss his personal life, it is my inclination to not comment on our friendship beyond the mathematical collaboration. For instance, the reporter for the New York Times who interviewed me for Lang's obituary did not understand why I would not answer questions regarding Lang's family. Whereas I do feel compelled to respect his privacy after his passing, I have decided to accept the invitation of Steven Krantz and include comments regarding my own interactions with Serge.

As with so many others, I first learned the name Serge Lang as an undergraduate mathematics major when I purchased textbooks for my mathematics course. I met Serge for the first time in 1987, during my second year in graduate school at Stanford. I remember the level of excitement among the graduate students in anticipation of Serge's talk at Stanford. It was thrilling to see his energy during his lecture. Lang seems to be someone, I remember thinking, who has discovered what will give him the most out of life, namely his mathematics and his politics (what he himself called "trouble-making"), and he is doing it.

In 1990 I joined the faculty at Yale as a Gibbs instructor, and during that spring semester I gave a graduate course which Lang attended. As I had expected, from the audience Serge directed my lectures for himself, insisting on immediate changes in notation and topics. In one particular lecture, I presented an evaluation of spectral determinants on elliptic curves which avoided the usual approach, namely Kronecker's limit formula, and instead relied on a trick I developed in my thesis. Lang was silent during the entire presentation. When I finished, he insisted that I wait in the classroom. When he returned, he had two papers with him, one by Artin from 1923 and another by himself from 1936. He pointed out that the technique I presented in the setting of heat kernels on elliptic curves was conceptually identical to Artin's ideas in the setting of L-functions of number fields and Lang's ideas in topology, in the context of the characteristic polynomial in linear algebra. That conversation grew into our first joint paper which was published in Crelle's journal in 1994.

During the summer of 1990, Lang called me every day, several times, as he traveled through Europe and to Berkeley. We always spoke about mathematics, and he challenged me on the same point I attempted to make to him earlier: Why did I believe that one can use heat kernels and heat kernel techniques, perhaps formally in a way to be developed, in a wide range of mathematical questions? We discussed, argued, and debated, as only Serge could, until he returned to Yale that fall. He invited me to lunch one of the first days he was back. During lunch he asked if I would be willing to work with him on the mathematics we discussed. One cannot imagine my thoughts at that moment. To have a senior mathematician express interest in one's ideas is remarkable enough, but to have Lang say he wanted to work with me simply cannot be described. I found out later a more touching aspect to that conversation with Serge. Apparently, in 1988 Lang had told some faculty at Yale that his mathematical abilities were gone, and he couldn't continue; after we began working together, he then would comment to others that our work was keeping him alive.

During the first few years of our joint investigations, we spent countless hours developing a long-term program of research. Serge was a very private person, more so than I have seen with anyone else. Although I knew his private telephone numbers, I never called him at home, though he would not hesitate to call me at any time. It was very rare that he mentioned his family or his non-mathematical, or non-trouble-making, interests. We became friends, and we made a point of talking, perhaps quite briefly, each morning when he arrived in his office, each evening when he left to go home, and many times during the day.

I left Yale in December 1996. Serge was hospitalized in December and later in February, and he insisted that I visit him each day to continue our mathematical conversations. His February stay in the hospital was reported in the Yale student newspaper, and the dean of Yale College, a person Serge very much disliked, was quoted as saying that Lang was an excellent mathematician. I showed Lang the article while he was still in the intensive care unit, and, when he read the dean's comments, he screamed, "That * * * * * * * * isn't qualified to judge." Serge was stunned when his outburst resulted in my expulsion from his hospital room until the next day. As in so many other instances, Serge was right, but perhaps his message could have been delivered differently.

Later in the spring of 1997, as I was seeking employment for the upcoming year, Serge asked me to visit him at Yale so we could discuss our projects. At the time, I considered leaving academic mathematics and seeking a career change away from a university environment. We discussed the
matter in detail, and Serge pointed out to me the effect it would have on him if I were to quit mathematics. At the end of the day, I honored his request to continue our program of study and promised to not seek a nonacademic position.

We spent less time together during the next two years when I was at Oklahoma State during 1997-98 and in Greece during 1998-99. In the fall of 1999, I accepted a position at the City College of New York, in part because it allowed me to visit Serge frequently, which I did on most weekends. At that time, we began focusing our ideas, even going so far as writing a document for ourselves, establishing a "wish list" for our mathematical program. We created an outline of the articles, monographs, and books which must be completed in order to fulfill the steps of the program which we envisioned. One of the earliest words my son knew was "Serge", which he said each time the phone rang; frequently, my son was right, and indeed the call was from Serge.

It was evident to me, even in 1999, that at age 72 Serge was growing tired. Even though the learning process did keep him alive, time was catching up with him. He was always very sharp, with fascinating ideas and insight. However, he left the office earlier in the evening, and he required rest during the day. Without acknowledging the act, we altered the pace of our work, finishing what we could given the energy he had. When he was at Yale, I visited him most weekends, and when he was in Berkeley, I traveled there to continue our work.

Most everyone saw Serge as the forever young, highly energized individual. For the most part, he did not allow many people to see that time was affecting him. The fact that hardly anyone noticed that Serge was aging was, I believe, another manifestation of his level of privacy.

In August 2005, we completed another book. On the day we submitted the manuscript to Springer-Verlag, Serge was excited. To me, he momentarily regained his youth as I knew it when we delivered the manuscript to the post office. That day I saw again the dynamic person I remember lecturing at Stanford during my graduate school years. However, when we returned to the office, he needed to rest, and that evening during dinner he directed the conversation away from the specific ideas for our next project. Instead, he spoke of his wish that I always pursue our program of research, hopefully arriving at the goals we set for ourselves. During the next weeks, we spent time revising our wish list and reviewing our original mathematical plan. We continued to speak several times each day, up to and including September 12, 2005, the day Serge died.

Having spent so much time with Serge, there are many stories I could tell. Serge had many sides which affected everyone, including me, many different ways. He had friends and enemies, and perhaps I inherited some of both. As he pointed out to me quite some time ago, it is possible that my association with him was both positive and negative for me. Serge once told me that he made certain personal decisions early on in his life, and he stuck by those decisions. For me, I stand by my decision to work with Serge. He was a close and loyal friend, and I believe that I was for him. Serge was a part of my daily life for nearly fifteen years, and for me his absence is great.

Paul Vojta, University of California, Berkeley

Although I had seen Serge at Harvard once or twice in the Common Room (invariably arguing with someone), I first met him when I went down to Yale to ask him to write a letter of recommendation for me. At one point in the discussion, he asked me about my definition of integral point. I started to describe Serre’s idea of an infinite set of points with bounded denominators, but he angrily interrupted me: "I don't know what an integral point is, and neither do you!!!" However, later on that day he added a flattering paragraph to his new edition of Diophantine Geometry describing work in my thesis on integral points relative to a divisor with sufficiently many irreducible components.

Later, upon hearing about my conjectures, he called me up and invited me to come to Yale. I was a little hesitant about this, given his propensity for anger, but my advisor Barry Mazur convinced me that I should accept.

During my time at Yale, I gave two or three graduate courses. Serge always sat in the front row, paying close attention to the point of interrupting me midsentence: "The notation should be functorial with respect to the ideas!" or "This notation sucks!" But, after class he complimented me highly on the lecture.
While on sabbatical at Harvard, he sat in on a course Mazur was giving and often criticized the notation. Eventually they decided to give him a T-shirt which said, "This notation sucks" on it. So one day Barry intentionally tried to get him to say it. He introduced a complex variable \( z \), took its complex conjugate, and divided by the original \( z \). This was written as a vertical fraction, so it looked like eight horizontal lines on the blackboard. He then did a few other similar things, but Serge kept quiet—apparently he didn't criticize notation unless he knew what the underlying mathematics was about. Eventually Barry had to give up and just present him with the T-shirt.

Once, close to the end of my stay at Yale, I was in his office discussing some mathematics with him. He was yelling at me and I was yelling back. At the end of the discussion, he said that he'd miss me (when I left Yale). Now that he has left, I will miss him, too.

Gilles Lachaud, Institut de Mathématiques de Luminy

I met Serge Lang in 1972, during the AMS symposium on Harmonic Analysis on Homogeneous Spaces, held in Williamstown. He had just left Columbia for Yale, and I was with Paris 7.

At that time, French and American universities were a hotbed for the antiestablishment ideas, and Serge was involved in the Free Speech Movement. Also, he was writing his book on \( \text{SL}(2, \mathbb{R}) \): thus, in Williamstown, we discussed both alternative politics and spherical functions.

In my mind, Serge was a hunter, in mathematics as in his political and social struggles. He was chasing a precise game and nothing was able to make him deviate from this goal.

As a polemicalist, he was very proud of the File process, consisting of bringing lies on some topic to full light by sending letters to opponents, to wait for contradictions in the answers, and to send Xerox copies to all the people involved in the contest. He was sure that victory would emerge from this confrontation.

The special feature was that he wanted to work along scientific lines and to prove the statements he was defending, an uncommon and irritating position outside the mathematical community. His model was in mathematics, as opposed to social sciences, about which he used to say: "In mathematics you cannot say, 'I disagree with this statement.' You can say, 'this is false' or 'this is of poor interest,' but there is no disagreement to express whatever."

Among American mathematicians, Serge was one of those who were closest to France: the souvenirs of his life during his youth with his parents were enduring, in particular vacations on the Mediterranean seashore. Later in his life, he became a member of the Bourbaki group, stayed in France and lectured at length, in Paris and elsewhere.

In some respects, Serge was rather austere. But he had a cheerful and authentic enthusiasm for mathematics: this is reflected in his books, and this enthusiasm was contagious.

At the beginning of 2005, we exchanged our wishes by phone. As usual, he was infuriated, this time by his own illness: he was conscious of his bad health, and desperate not to be able to fulfill before his death the program on zeta functions he had in mind, after twenty years of work on analysis on groups.

We miss a great intellectual figure, and I miss a friend.

Roger Howe, Yale University

Books: Serge wrote an extraordinary array of books, from widely used texts, including calculus texts, and even a high school geometry text, to standard references, to monographs which are the only treatment of their subject in book form.

The topics tend to cluster around algebra, and especially Diophantine equations, which were his great love in mathematics, but they span a remarkable range.

For a long time, his practice was to give each year a graduate course on a new topic, and at the end, to turn the notes into a book. He had an amazing capacity to boil a subject down to its essence, which he often formulated with a few axioms or properties. He was a consummate axiomatizer. As he reached his late 70s, he gave little sign of slowing. His last several books presented joint work with Jay Jorgenson on applications of the heat equation to analysis on symmetric spaces, with a view towards automorphic forms.

Files: Besides books, a lot of Serge’s literary effort went into making files which chronicled his fights. Serge loved a good fight and he didn’t have trouble finding them. He was especially concerned with honesty, especially honesty in public rhetoric. Serge would coordinate multi-party correspondence, organizing sets of letters into packets, and circulating them with supplementary documents to a "cc list" of parties to the correspondence and other interested readers. At the end of the fight, he collected the whole into a file, and gave it a title.

Serge thought of his files as documentation of the way life works today, and especially of his First Law of Sociodynamics: the power structure does what it wants when it wants, and looks around later for justification. Serge was willing to follow
his principles and beliefs (almost) wherever they led.

Students and Teaching: A third focus of Serge's energy was teaching and students. Generations of Yale undergraduates benefited from his teaching in the broad sense. He spent hours outside of class talking with undergraduates, about mathematics, politics, music, anything. He frequently ate meals with undergraduates in the Yale dining halls. He was especially concerned with promoting clear thinking. He often found that students who came to him had been confused by poor education, but that by appropriate challenges he could help them to become independent thinkers. He referred to this process as "Recycling their brains".

Serge's attention to undergraduates was a part of his concern for the advancement of younger people generally. It was a habit with Serge to encourage younger mathematicians and be interested in their work. That was certainly so for me. I recall with gratitude his wholehearted enthusiasm for my work, and his help in promoting it, and I know of many others who similarly benefited.

Gisbert Wüstholz, Eidgenössisches Technische Hochschule, Zurich

Presumably my most expensive investment in mathematics as a student was a book with the title Algebra by an author whose name was Serge Lang. A former schoolmate had recommended it to me as a very modern new tract in algebra. He had entered university one year before me and just started with a course in algebra where the book had been recommended. Certainly the top of my list of favorite mathematics books would be Algebra by Serge Lang. The reason I like it so much is that it had a clear vision for modern and conceptual mathematics and this was put together with much mathematical taste.

It was exactly ten years later at the Arbeitstagung in 1978 in Bonn when I first met Serge personally. At the time everybody talked about "Bombieri-Lang", a paper which influenced enormously the research in transcendence. For us young students in number theory it was a big challenge to try to understand the difficult methods from geometric measure theory, the theory of plurisubharmonic functions and $L^2$-estimates. It took us away from the classical methods in transcendence theory and taught us that you need mathematics as a whole to formulate and to prove interesting new results in transcendence theory.

At that Arbeitstagung I talked quite a bit to Serge and he eventually helped me to find a postdoc position in Wuppertal. There I got into contact with people in algebraic groups and this helped me to enter into another area which had been opened by Serge: he had started in 1962 with a series of papers in transcendence theory out of which another book resulted in 1966 which turned out—at least in my eyes—to be the most influential book (Introduction to Diophantine Approximations) that he ever wrote.

Even the abc conjecture, one of the favorite subjects of Serge, has now been incorporated into transcendental context, and this indicates how important Serge's impact into diophantine geometry and transcendence has become. He had created the frame of a very active and broad area to which he substantially contributed, he had the right mathematical visions and supported enthusiastically any progress.

For many years Serge visited me at Zürich. He gave numerous talks in my seminar and to undergraduates, and he enjoyed visiting us at our house. Only one thing I never forgave him: when he together with Schinzel once had been guests at our house in Bonn he came after the main course into the kitchen where I had started to prepare a soufflé Grand Marnier. Without stopping he talked to me and distracted me so much that at the end the soufflé did not rise in the oven. I did not want to offer it to the guests but Schinzel forced us to eat it since he would not agree to throw away food. After this I essentially stopped cooking soufflés.

Jürg Kramer, Humboldt University

I had my first encounter with Serge Lang as a student in an indirect way: after having just finished my first two years as an undergraduate with the "Vordiplom" in mathematics at the University of Basel (Switzerland), Martin Eichler proposed a seminar in Lang's new book on modular forms. Since I was just a beginner in the subject, the book made me work quite hard, but anyhow, as a result I became strongly interested in the subject. Six years later, just before completing my Ph.D., Eichler asked me to accompany him to the "Arbeitstagung" taking place in June 1984 in Bonn. After arriving at the entrance hall in front of the big lecture hall at Wegelerstrasse 10, one of the first persons to meet was Serge. At the time, it was quite impressive for me to have been personally introduced to this world-known mathematician. What I didn't know at the time was that this was the start of a relationship lasting for more than twenty years.

In fact, when I returned to Bonn in 1985 to visit the Max-Planck-Institut für Mathematik (MPIM), I was caught by surprise when I met Serge again during his regular trip to Europe in June and that he immediately remembered me; as a consequence, we started to talk about mathematics, first on a quite "innocent" level. Our mathematical communication
intensified in the years 1987/88, when I was giving a course on "arithmetic surfaces" at MPIM, while at the same time Serge was preparing his book *Introduction to Arakelov Theory*.

After having completed my "Habilitation" at ETH Zürich (where Serge was also visiting on a regular basis since the late 1980s), I moved to Humboldt University (HU) in Berlin in 1994. From 1995 until 2003, Serge regularly visited HU in late May/early June for one week. During this period of almost ten years, we got to know each other more closely and our relationship deepened. In Berlin, aside from his traditional talk in our number theory seminar (demonstrating the ubiquity of the heat kernel in the last years), he was intensively arguing, discussing, and interacting with our graduate students. In addition, he always generously offered talks to gifted high school students. In particular, the high school students consider his unexpected death as an infinite loss, and it is very sad that this tradition has come to such a sudden end.

Although Serge tried to stay away from close personal relationships, it seemed to me that in his last years when coming to Berlin the ties between him, my wife Ruth, and myself got somewhat closer. We will surely miss him.

David E. Rohrlich, Boston University

Shortly after Serge's death a few people suggested to me that I write something about his mathematical contributions. The suggestion apparently stemmed from a concern that obituarists would focus on Serge's eccentricities and temper tantrums rather than on the highlights of his career. Later, when I was invited to contribute something to the present article, I decided to intertwine my personal reminiscences with some glances at Serge's mathematics, partly because I was mindful of the concern that had been expressed to me, but partly also because Serge's passion for mathematics was in my view an essential part of his persona. How a fitting memorial to Serge, for one of his noted eccentricities, his

stance here was based on principle—in the present case, a defense of an author's right to self-expression free from gratuitous editorial intervention—and what converted integrity into eccentricity was simply his stubborn insistence on continuing to do battle far beyond the point where the battle seemed worth fighting. But you had to hand it to Serge: he had the courage of his convictions. We can probably all learn something from his example.

Marvin Jay Greenberg, University of California, Santa Cruz

I was Lang's first Ph.D. student. Officially, Emil Artin is listed as my thesis advisor, but he left Princeton for Hamburg three years before I wrote my thesis. What Artin did was ask Lang to commute from Columbia to Princeton in academic year 1956–57 to continue teaching algebraic geometry, which Artin began toward the end of his graduate algebra course— an extraordinary request and an extraordinary acceptance on Lang's part. So Lang taught a course on Princeton on Abelian varieties, in the style of A. Weil, and after a few weeks, I was his only student.

Lang went away to Paris the following year, and when he returned after that, we continued meeting informally at Princeton. He told me a conjecture of his about Abelian varieties he wanted me to prove for my thesis. I had no idea how to do so, and I was extremely busy teaching four elementary courses at Rutgers that year. Then he confronted me, as he is famous for doing, and shouted at me that if I did not show some progress with his conjecture in the next two weeks, I would no longer be his student.

The following weekend, in my attic room in New Brunswick, after rereading Lang's thesis, I suddenly had a flash of insight on how to solve that problem. I needed a few more weeks to write out all the technical details, but when I told him I had the solution, he was delighted. He took me out to a fine Spanish restaurant in NYC and treated me to paella, which I'd never eaten before. He invited me to his apartment overlooking the Hudson River and played Bach's dramatic Partita #6 (which I'd never heard before) and a Brahms Rhapsody for me on his grand piano. I felt as if he had lifted me into an exalted new world of excellence.

I remained on good terms with Lang for quite a few years after I left Princeton for Berkeley. Many people were turned off by his aggressive personality, but I always enjoyed that immensely, particularly his brutal honesty and taunting sense of humor. He once told me bluntly that I would never become a great mathematician because I was afraid of making mistakes. He certainly made plenty of
Shoshichi Kobayashi, University of California, Berkeley

With no more than amateurish interest in number theory, my mathematical contact with Serge Lang is mostly through hyperbolic complex analysis. With his several conjectures and his introductory book on hyperbolic complex manifolds, he was the best promoter of the subject.

In 1985 I participated in the AMS Summer Research Institute on number theory at Humboldt State University, Arcata, in northern California. Thanks to Serge, I was invited to give an introductory talk on hyperbolic complex analysis, including Noguchi's partial answers to the function-theoretic analogue of the higher dimensional Mordell conjecture as formulated by Lang.

Since Serge was the only one driving back to Berkeley on the last day of the workshop, I accepted his offer of ride with trepidation. As I had expected, his driving was like his typing—fast. He drove German-style, flashing the headlights whenever slow-moving cars blocked us, which happened to be all cars ahead of us. We were back in Berkeley by 10:00 p.m. As a result, the only thing I remember from the workshop is this experience.

In 1987, Serge published Introduction to Complex Hyperbolic Spaces with Springer-Verlag. While writing this book, he called me, not day and night, but 9 in the morning. Quickly he found out my morning routine and the most convenient time. So usually I answered when the telephone rang around 9. When my wife answered, he invariably said "Is Kobayashi out of the shower yet?" When he stopped calling after several months, my wife said "I guess Serge finished his book."

Without Serge, the summer in Berkeley will no longer be the same. I miss him.

Hung-Hsi Wu, University of California, Berkeley

Serge Lang's life is easy to characterize: it was 90 percent mathematics and 10 percent scientific politics. I happen to be one of the few mathematicians who forged a friendship with him through politics, thereby getting a glimpse of a side of him that was perhaps denied other mathematicians.

For a mathematician to have any kind of friendship with Serge, it is a given that it could not have been completely divorced from mathematics. I was no exception.

While I had known about Serge since my undergraduate days at Columbia when he was still with that institution, my first meeting with him was in Berkeley around 1987, when he called to talk about Nevanlinna theory. At the time, he was still campaigning against Huntington's election to the National Academy of Sciences. In any case, Serge and I began political discussions with increasing frequency after that. He was at Yale, of course, and I was at Berkeley, but Serge was never shy about using the phone. While I sometimes had reservations about the tone of his writing, there was never any doubt in my mind about its substance. It may surprise some that a firebrand like Serge could use any encouragement, as it did me, but I discovered that fighting the kind of lonely battle that he did, he probably found it easier to listen to someone sympathetic to his views than to engage in a shouting match twenty-four hours a day. From time to time, he would sound me out on his strategies. One consequence of all these phone conversations was that I was privy to all his fights since about 1990, including the denouement of the Huntington case, the Gallo case, the Baltimore case, and of course the still-ongoing HIV controversy.

Because I had no strong mathematical connection with Serge, our relationship could afford to be more relaxed. Each time he called my home when my wife and I were out, he would leave a message reprimanding us for "goofing." Serge was famous for getting along with young people. He and my son were great pals, and each time my son would ask a question about mathematics, a few days later a book or two on that subject would arrive from Springer-Verlag. The author of those books was of course Serge Lang.

In the last year of his life, his big fights were still over HIV. One involved his submission of a survey to the Proceedings of the National Academy of Sciences on the state of HIV research and government actions on the so-called anti-HIV drugs. He and I knew from the beginning that it would get nowhere, but the cavalier way in which his survey was rejected was stunning. I went to his office when word had just come and, perhaps owing to the similar frustrations in my own work in mathematics education, I lost it and said in less than polite language that I had had it. In an instant, his role and mine were reversed and he soothed me with the philosophical observation that life was hard and that we just move on.

The last I heard from Serge was a phone call on September 3, 2005, after he had left Berkeley for New Haven. Four days after he passed away, his last file on HIV and the Proceedings of the National Academy of Sciences arrived. It was addressed to my son. He lived his life on his own terms to the very end.
High Expectations

Serge Lang believed that young people have a special ability to see the truth. He was a champion of youth, and his students loved him. In January 2005, Serge submitted a brief op-ed piece and a dense four page advertisement to the The Daily Californian, the U.C. Berkeley campus newspaper. The topic was a Serge standard: growing dissent against the orthodox position that AIDS is a disease and it is caused by the HIV virus. The ad contained supporting documentation for the op-ed piece, which had previously been rejected. The submission was accompanied by a personal check to pay for the ad.

Serge had been disseminating information on HIV and AIDS for roughly twelve years. The Daily Cal was a natural outlet for Serge’s challenges. After some back and forth, Serge received a rejection letter explaining why his material could not be published in The Daily Cal. Here are some excerpts that are pieced together from the Daily Cal file, which contains a copy of the rejection letter and Serge’s response.

Daily Cal: We are confident that you understand all newspapers must have the flexibility as a business to reserve the right to refuse any advertisement, letter to the editor, op-ed or press release at the discretion of their publishers.

Serge: In fact, I know that newspapers have the power to refuse advertisements, letters to the editor, op-ed or press releases, and I have known it for a long time. It’s not exactly secret information.

Daily Cal: These are the things we do to protect our readership.

Serge: I call this position Nannycism.

Daily Cal: Clearly it would have been a serious ethical error if we had elected to publish all or a portion of any op-ed letter to the editor that referenced an advertisement specifically designed to clarify or provide back-up data for the op-ed.

Serge: What you find “clear” I do not. In fact, I take an opposite view. You might have stated more accurately “clearly to us” to make your assertion more precise, instead of pretending to a universal ethical standard, applicable to others, and implying that I asked you to do something unethical.

The reply is pointed and funny throughout. Evident in every line are Serge’s high expectations of the Daily Cal staff members. As far as he was concerned, they may as well have been running The New York Times, or the world for that matter.

Serge was a champion of youth, but he had plenty of energy left over for grownups. The Daily Cal file closes with a letter to the dean of the U.C. Berkeley School of Journalism. This time, the topic was journalistic responsibility. Serge stated some of his concerns, drawing examples from the media coverage of HIV and AIDS. As always, his high expectations were in evidence. He asked, non-rhetorically, “How does one make up for defective reporting over two decades?” And he signed the missive “Informatively yours, Serge Lang.”

Allyn Jackson, Notices Deputy Editor

“It’s me”: This is what some of us in the AMS headquarters office would hear every now and again when our phones rang. No greeting, no name: Of course it was Serge Lang. Probably the very first time he called me he announced his name, but never again. When I got that first call, I mentioned it to a mathematician acquaintance, who said, “Watch out, you are in the room with the snake.” Indeed, it was with some trepidation that we AMS staffers would take these calls. Usually Lang would rail on about whatever issue he was currently campaigning about, and we would give noncommittal replies to avoid getting drawn into a debate with this tireless debater. But after a while he would soften up and crack a joke or tell a story; once he related to me the plot of a play he had seen. Sometimes he would end the call with a whimsical closing along the lines of “Toodle-oo”. Carol McConway, a former AMS employee with whom I worked on the Notices many years back, received calls from Lang on a regular basis. She told me that in one conversation he remarked that she was very smart and asked where she had gone to college. Carol, like many of the highly intelligent and capable women on the AMS staff, had never gone to college. “Aha,” Lang replied. “That’s why you are so smart. You were not ruined by the educational system.” Lang befriended another former AMS employee, Terry Drennan, who worked in the editorial department. When Terry got into a serious scrape with her boss, Lang stood up for her.

I was one of the many recipients of Lang’s innumerable and lengthy “files”. He sent me masses of documentation about his campaign about
I met Serge Lang in 1967, my sophomore year at Columbia, when I took his multivariable calculus class. This was before the days of unified calculus. All of us were math majors and many of us were spoiled by our high school experience of learning math with very little effort. So Lang would frequently throw chalk at us, or yell.

I often ate dinner at the Gold Rail with Richard (now Susan) Bassein and Eli Cohen, and if Lang was also eating there he would always join us and usually pick up the tab. Sometimes we would talk about math. Lang did not think logicians were true mathematicians, because no real mathematician would worry about whether a proof made use of the axiom of choice. Why shouldn't you use the axiom of choice? It's obviously true! Think about it! How could you not be able to construct a set by choosing one element from each set in a collection of sets? Just do it!

We also talked about politics, music, life. He shared with us his growing unease about the Vietnam war and what he viewed as Columbia's complicity in it, although he made a point of never discussing politics in class. He offered advice about unrequited love: If at first you don't succeed, try, try again, but after the third time, if you still don't succeed, give it up!

Rarely, he took someone seriously who was merely pulling his leg. Dorian Goldfeld, at that time a postdoc at Berkeley, once reported gleefully that Lang had posed him a problem which he had in fact been working on for months and had already solved. Goldfeld told him with a straight face that he would think about it, and the next day presented Lang with the solution. Astounded, Lang asked him how he had managed to find the answer so quickly, and Goldfeld explained that he had used ginseng, which greatly enhanced his mental powers. Lang excitedly promised to try it himself, but reported disappointedly a few days later that ginseng had had no discernable effect on his brain.

In the summer of 1968, I was on vacation with my parents in Berkeley. Lang was there as usual, so my folks invited him for dinner. Naturally the conversation turned to the demonstrations at Columbia and the other student protests around the country, many of them against the war. Although my mother was opposed to the war in Vietnam, she deplored the excesses of some of the demonstrators, which she attributed to their permissive upbringing. "But what about France?" Lang objected. "And Poland? Even in Poland students are protesting against the government, and you can be sure they weren't raised permissively! Young people don't want to be used by their governments. All over the world, students are fighting for their freedom!"

Joseph Gerver, Rutgers University

HIV/AIDS. As I recall, he never stated that he believed that HIV does not cause AIDS. Rather, he advocated the need to view this hypothesis with skepticism and rigor. He poked holes in research papers and other writings that uncritically assumed the hypothesis to be true. At some point I looked carefully at one of the AIDS research papers that Lang had denounced; this particular one had been written by the prominent AIDS researcher David Ho, together with some colleagues, and had been much cited in the subsequent literature. I became convinced that Lang was right in saying that the paper lacked rigor and used mathematics inappropriately. This is how it was with Lang's campaigns: He always had a valid point. Yes, he was obsessive, he could be antagonistic, he had a bulldog-like attachment to his causes and sometimes lacked a sense of proportion. And the solutions he proposed to the problems he identified were often totally impractical and naive. Still, whenever I took the time to examine his analysis of those problems, I found myself concluding that he was basically right.

I do not believe Lang pursued his causes out of a desire for fame or notoriety. Rather, he was horrified by the falsity he found all around him, and he would not let others turn their eyes from it. We have lost a person with a highly attuned sense of what is truthful and what is sham, and it is a profound loss indeed.

John Ewing, Executive Director, American Mathematical Society

I had contacts with Serge over many years, beginning when I was editor of the Mathematical Intelligencer (and he insisted on publishing a long, long article). My kids got to know him by phone—he called me at home on and off for more than a year.

Later as executive director I came in contact with him, largely because of his dispute with the Notices of the AMS (about HIV). Serge always called the office and simply said, "It's me." Everyone here knew who that was. When I explained that the ED doesn't make editorial decisions, he insisted that the "higher ups" at the AMS could do whatever they wanted. He often sent material related to various things, and always mentioned that he was sending it to the "higher ups". Phone conversations were always long, protracted affairs.

But even with long phone calls and huge quantities of written material (we have a giant file here), Serge was really a charming guy whom I instinctively liked and admired. The world is better off having had him fight for his causes, with passion and indignation.
About twelve years ago I received a package of “files” from Serge Lang. The files objected to the Washington-style cover up of the scandal surrounding the American discovery of the hypothetical AIDS virus. Impressed by the thoroughness and mathematical logic of his case, I sent my own file in response: "Dr. Lang," I wrote, "the political scandal about who discovered the hypothetical AIDS virus is from a scientific point of view no more than a distraction—a catchy story about who stole whose fake diamonds. The scientific challenge, however, is whether AIDS is a viral or a chemical alias lifestyle epidemic, caused by the long-term consumption of recreational drugs and anti-viral drugs such as the inevitably toxic DNA chain-terminator AZT."

Not much later Lang and I became allies in the AIDS debate. Lang gave seminars on AIDS, wrote for the Yale Scientific, included two AIDS chapters in his book Challenges (1998) and generated a steady flow of AIDS files, the last of which arrived here only after his death. But only now, on the sad occasion of his death, is Lang’s AIDS engagement presented as an Achilles heel of a mind that seemed otherwise irrefutable in its high standards of accuracy and precision not only by the politically correct New York Times and Yale Daily News, but even by several of his mathematical peers.

In view of this I take a last stand on behalf of our colleague, who cannot do this anymore, trying to inform his survivors with "primary evidence", rather than "condition them" with government handouts, as Lang would have said. Even if one virus could cause the twenty-six infectious and non-infectious (!) diseases that are now defined as AIDS, the following would be true:

1) AIDS would be contagious. But, there is no case report in the peer-reviewed literature of even one doctor who ever contracted AIDS from one of the 929,985 (2004) American AIDS patients in twenty-three years. Moreover, not even one of the thousands of AIDS virus researchers ever contracted AIDS from their "deadly virus", as the New York Times calls it.

2) AIDS should appear within days to weeks after infection, because the AIDS virus, like other viruses, replicates with multiplication rates of 100 to 1,000 within twenty-four hours. But AIDS is said to appear only five to ten years after infection by its hypothetical viral cause.

3) The epidemic would spread randomly like all viral epidemics. But AIDS cases in the U.S. and Europe are highly nonrandom, 80% are males, of which 1/3 are intravenous drug users and 2/3 are male homosexual users of nitrite inhalants and amphetamines and are prescribed DNA chain-terminators.

4) The epidemic would have formed a classical bell-shaped time curve, increasing exponentially and then declining exponentially owing to natural immunity within weeks to months, like a seasonal flu. Instead AIDS increased slowly over a decade and has since leveled off, without ever inducing immunity against itself.

Thus AIDS fits a lifestyle—but not a viral epidemic. I hope, therefore, that those who saw Lang’s AIDS engagement as an Achilles heel might reconsider.

I already miss Lang as an ally in the politically incorrect debate on the cause of AIDS. And I hope that you all let me join you in missing the Mensch that was hidden behind the machine. Au revoir Serge Lang!
Raoul Bott passed away on December 20, 2005. In a career spanning five decades, he has wrought profound changes on the landscape of geometry and topology. It is a daunting task to improve upon his own reminiscences [B3], [B4], [B1] and commentaries on papers [B5], punctuated as they are by insight, colorful turns of phrases, and amusing anecdotes. This article is an updated reprint of one that first appeared in the book *The Founders of Index Theory: Reminiscences of Atiyah, Bott, Hirzebruch, and Singer* (edited by S.-T. Yau, International Press, 2003). Taking a personal interest in the project, Raoul Bott introduced me to some of his friends and gave me access to his files. After the original article was completed in June 2000, he read it and verified the essential correctness of the accounts of both his life and his works. In the interest of preserving its official imprimatur as an authorized biography, the only changes I made to the 2003 version consist of this introductory paragraph, an update of the awards he received, and the suppression of a bibliography of his works not referenced in the article.

**Early Years**

Raoul Bott was born in Budapest in 1923. His lineage fully reflects the geopolitical complexity of the region at the time. His mother's family was Hungarian and Jewish, while his father's side was Austrian and Catholic. His parents divorced soon after his birth, so he grew up with his mother and stepfather. Raised as a Catholic, Raoul spent his childhood and adolescence in Slovakia, which seventy years later, after alternating between Hungary and Czechoslovakia, is today an independent country.

In the first five years of school Raoul was not a good student. This should give comfort to all parents of late bloomers. In fact, he did not earn a single A except in singing and in German. Nonetheless, he showed an early talent for breaking rules and for generating sparks—electrical sparks, that is, rigged up with wires, fuse boxes, vacuum tubes, and transformers. The schools were formal and strict, and one could get slapped or have one's ears pulled for misbehaving. For a budding original thinker, Raoul survived the schools relatively unscathed. He recalls a friar hitting him on the hand once and a teacher cuffing his ear another time, for horsing around too much.

It was by all accounts an idyllic existence, complete with a family villa, English governesses, and music lessons. This world came to an abrupt halt in 1935, when his mother died of cancer. In time his stepfather remarried.

Raoul's experimental talent found its full flowering in adolescence. He and a kindred spirit Tomy Hornak built a small box with a slit for coins. When someone dropped a coin through the slit, a display lit up saying “Thank you.” In this way they funded their early experiments.

Raoul struggled with some subjects in school and a tutor was hired to help him a few hours a week in his house. At the time Raoul and Tomy had built
a gadget to communicate by Morse code. As he was being tutored, he would hold the gadget under the table and Tomy would be sitting in the basement. Raoul received the code by getting short and long electric shocks in his hand. He then responded by pressing a button to light up a bulb in the basement. While the tutor believed that his student was listening intently to the lesson, Raoul was chatting away in Morse code under the table. In retrospect, Bott calls this his first attempt at e-mail.

Canada

In 1938, with Hitler's ascendancy and Germany's march into Czechoslovakia, Bott's stepparents flew him to the safety of England and enrolled him in an English boarding school. Since they had only transit visas for England, the following year they headed for Canada, a country that to this day has been extraordinarily welcoming to refugees and immigrants from around the world.

In the fall of 1941, after a rigorous year of preparatory studies in Ontario, Raoul Bott found himself at McGill University in Montreal. Given his electrical know-how, he chose, not surprisingly, electrical engineering as his major. His grades were respectable, but as he recalls in [B4], he was more interested in upholding the "engineering tradition of hard drinking, loud, boisterous, mischievous, and macho behavior." Mathematics was his best subject; still, it was mathematics in the engineering sense, not the kind of pure reasoning for which he became so well known years later.

With his European flair, his six-foot two-inch frame, and the conspicuous fur cap he often wore, Bott stood out from the crowd at McGill. When friends asked him where he was from, he said from Dioszeg, Czechoslovakia, and he added facetiously, where he "was a Count". After that, everyone called him the Count.

The Count sometimes spoke a very foreign tongue. In the streetcars of Montreal, Raoul and his roommate Rodolfo Guardian would occasionally engage in a deliberately loud and animated conversation. Nothing they said made sense, for they were making up the language as they went along. From the corners of their eyes, they enjoyed watching the quizzical expressions on the faces of the surrounding passengers, who were trying hard to figure out what language the two of them were speaking.

Bott loved the opera, but as a penniless student how was he to afford it? One time the famous tenor Ezio Pinza came to sing in His Majesty's Theater, the opera house of Montreal in the 1940s. For this occasion, Bott dressed up in his Sunday best and went to the theater. When the man at the entrance stopped him, Bott told him he couldn't do this because he was Ezio Pinza's nephew. Bott said it with such assurance that the man let him in. After that, Bott could go to all the shows at this theater for free.

Bott's roommate Rodolfo, equally penniless, also loved the opera. But Rodolfo did not have the nerve to sneak into the theater. When the opera Carmen was playing, Rodolfo was very eager to attend. Bott magnanimously invited him. By then, the ticket taker knew Bott very well, but he stopped Rodolfo at the entrance. Bott turned around and intoned in his authoritative voice, "It's all right. He can come in." Without any hesitation the ticket taker obeyed the order of this "nephew" of Ezio Pinza.

One New Year's Day, Raoul, Rodolfo, and some friends went to Mont Tremblant, a winter resort north of Montreal. In the most prominent and expensive hotel, a big celebration was going on. Somehow, to the envy of his friends, Raoul sneaked in. A little later, Raoul was standing on the balcony, looking down contemptuously at his friends and showing them a chicken leg he was eating. After he finished it, he threw the bone, with disdain, to his hungry friends.

(Old habits die hard. In 1960 Bott, by then a full professor at Harvard, was in India with Michael Atiyah, both giving lectures as guests of the Tata Institute of Mathematics. One day, as they walked in the streets of New Delhi, they passed by a big celebration. Bott decided to slip in uninvited, dragging Atiyah along with him. Atiyah, a professor at Oxford who was later anointed Sir Michael by the Queen and elected President of the Royal Society, was at first discomfited, but soon joined wholeheartedly in the festivities. They had a rousing time, sharing in the general merriment of complete strangers.)

Upon graduation, Bott joined the army, but the atomic bomb at Hiroshima put an end to his military career after only four months. He entered a one-year master's program in the engineering department at McGill. Gradually it dawned on him that his interest lay more in mathematics than in engineering, and he produced a very mathematical master's thesis on "impedance matching", which he said, "the department accepted with some misgivings and about whose mathematical rigor I have doubts to this very day."

At McGill Raoul met his future wife, Phyllis, an English literature major from the West Indies. Today, Phyllis remembers Raoul's first marriage proposal. At the time he was doing his short stint in the army. In full uniform, he said, "Would you marry me? Because if you do, the army will pay me more money." And then pointing through the window to his little room, he added, "And we could be living there." The proposal was not accepted. But two years later, they married. The Botts have been together ever since, and now have four children and eight grandchildren. They celebrated their golden anniversary in 1997.
Memorial Bott was asked to deliver a sermon at Harvard's Memorial Church. As he discussed the biblical passage of Eli, the wise man who counseled the young Samuel (1 Samuel 3: 3-6, 8-10), he reflected on the pivotal moment in his life that launched his mathematical career. His description of his own Eli deserves to be read in the original:

And so when I saw the two readings we just heard juxtaposed in a Scripture Service, I could not resist them. For they are appropriate to all of us, whether called to high causes or to lowly ones. And they are maybe especially appropriate to the young people of today in their search of their destiny.

For surely there never has been a time when our young people have been given such freedom and therefore such responsibility to find this destiny.

But how are we to know where we are called? And how are we to know who is calling us? These are questions beyond a mathematician’s ken. There are some who seem to have perfect pitch in these matters. There are many more who might think that they have. But with most of us, it is as it was with Samuel, and we are then truly blessed to have an advisor such as Eli. He stands for all of us Teachers as an example. For apart from communicating our call to our students, we should try and help them above all to discern theirs.

I well remember my Eli. He was the Dean of the Medical School at McGill and I approached him for help in entering the medical school there, when in 1945 the atomic bomb unexpectedly put an end to the war and to my four-month old career in the Canadian Infantry.

The Army very wisely decided to get rid of such green recruits as soon as possible, and so we all again found ourselves quite unexpectedly in charge of our own lives. I had graduated in engineering earlier that year but had already decided against that career.

The Dean greeted me very cordially and assured me that there was a great need for technically trained doctors. But, he said, seating me next to him, first tell me a little about yourself. Did you ever have any interest in botany, say, or biology? Well, not really, I had to admit. How about chemistry — Oh, I hated that course. And so it went. After a while he said, “Well, is it maybe that you want to do good for humanity?” And then, while I was coughing in embarrassment, he went on, “Because they make the worst doctors.”

I thanked him, and as I walked out of his door I knew that I would start afresh and with God’s grace try and become a mathematician.

Mathematical Career
Initially Bott wanted to stay at McGill to do a mathematics Ph.D. Because of his sketchy background, however, the McGill mathematics department recommended that he pursue a bachelor’s degree in mathematics first. It would have taken another three years. Sensing his disappointment, Professor Williams of McGill then suggested Carnegie Tech (now Carnegie-Mellon University) to Bott, where John Synge was just forming a new graduate program and would need some students.

Synge received Bott warmly at Carnegie Tech, but as they read the rules of the program together, they found that Bott would have to spend three years taking courses in the newly minted master’s program. In a flash of inspiration, Synge said, “Let’s look at the Ph.D. program.” It turned out to have hardly any requirements at all! Normally the master’s program is a prerequisite to the Ph.D. program, but perhaps recognizing a special gift in Bott, Synge put him in the Ph.D. program. In just two years Bott would walk out with his degree.

Bott found the Carnegie Tech atmosphere exceedingly supportive. The small coterie of mathematics students included Hans Weinberger, now at the University of Minnesota, and John Nash, an advanced undergraduate who after a thirty-year battle with schizophrenia received the Nobel prize in 1994. In later years Bott said of Carnegie Tech, “Being a brand new graduate program, they hadn’t learned yet how to put hurdles in front of graduate students.” Bott considers himself very fortunate to have an advisor in R. J. Duffin, for Duffin treated him as an equal from the very outset and together they published two papers on the mathematics of electrical networks.

The first of these two papers, on impedance functions [11], so impressed Hermann Weyl that he invited Bott to the Institute for Advanced Study in 1949. Thus began Bott’s initiation into the mysteries of algebraic topology. Apart from Weyl, among his main teachers were N. Steenrod, E. Specker, K. Reidemeister, and M. Morse. Of Ernst Specker, Bott said
that hat!" Now, this is the sort of order undergraduates love to obey. In no time the Dunster students had paddled to the Lowell raft. A struggle ensued, and like any good pirates, the Dunster contingent captured the admiral's hat. It was later hung, as a trophy, high in the ceiling of the Dunster House Dining Hall.

Showing true House spirit, the Dunster House Crew Team had its official team T-shirt emblazoned with "Dunster House," a pair of oars, and the exhortation: "Raoul, Raoul, Raoul your Bott."

The Harvard Houses have counterparts at Yale, where they are called Colleges. A friendly rivalry has always existed between these two august institutions, and it extends to the Houses and Colleges. Some of the Colleges at Harvard even have "sister Colleges" with which they are loosely affiliated. They would, for example, visit each other during the Harvard-Yale football games.

In the aftermath of the 1960s, many of the traditions at the Ivy League universities, such as the dress code and the parietal rules, have gone by the wayside, and for a number of years Dunster House had not had contact with Berkeley College, its sister College at Yale. One year the Berkeley College Master, a distinguished historian, decided to revive the tradition. He wrote to Bott suggesting a visit to Dunster House during the weekend of the Harvard-Yale football game. Bott readily agreed, but decided to make the occasion a memorable one. Why not fool the Yalies into thinking that Harvard has kept up, at least to a certain point, the Oxbridge tradition of High Table and academic gowns at dinner? Why not show that, perhaps, Dunster House was more "civilized" than its Yale counterpart? With enthusiasm, the Dunster House undergraduates all supported the idea.

On the appointed day, the Dunster House Dining Hall was transformed from a cafeteria into a hallowed hall, complete with linen, waiters and waitresses, and even a wine steward wearing a large medal. Unlike on a normal day, there were no T-shirts or cut-offs in sight. Every tutor was attired in a black academic gown. An orchestra sat in waiting. When the Yale Master and his tutors arrived, Bott asked, with a straight face, "Where are your gowns?" Of course, they didn't have any. "Well, no problem, you could borrow some of ours." So the Dunster tutors led them to some gowns that had just been lent from Harvard's Chapel. As Bott entered the Dining Hall with his guests, trumpets blared forth and the orchestra started playing. The undergraduates were already seated, looking prim, proper, and serious. Bott and his tutors dined with the Yale visitors at a High Table, on a stage especially set up for this occasion. The orchestra serenaded the diners with music. Everything went according to plan. But the Yale Master, ever sharp, had the last laugh. He opened his speech by saying, "I'm
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glad to see that culture has finally migrated from New Haven to Harvard."

**Bott as a Teacher**

Bott’s lectures are legendary for their seeming ease of comprehension. His style is typically the antithesis of the Definition-Theorem-Proof approach so favored among mathematical speakers. Usually he likes to discuss a simple key example that encapsulates the essence of the problem. Often, as if by magic, a concrete formula with transparent significance appears.

At a reception for new graduate students at Harvard, he once shared his view of the process of writing a Ph.D. thesis. He said it is like doing a homework problem; it’s just a harder problem. You try to understand the problem thoroughly, from every conceivable angle. Much of the thesis work is
perseverance, as opposed to inspiration. Above all, "make the problem your own."

Many of his students testify to his warmth and humanity, but he also expects the students to meet an exacting standard. He once banned the word "basically" from an advisee's vocabulary, because that word to Bott signifies that some details are about to be swept under a rug.

This insistence on thoroughness and clarity applies to his own work as well. I. M. Singer remarked that in their younger days, whenever they had a mathematical discussion, the most common phrase Bott uttered was "I don't understand," and that a few months later Bott would emerge with a beautiful paper on precisely the subject he had repeatedly not understood.

Seminar speakers at Harvard tend to address themselves to the experts in the audience. But like Steenrod, Bott often interrupts the speakers with the most basic questions, with the salutary effect of slowing down the speakers and making them more intelligible to lesser mortals.

At Michigan and Harvard, Bott directed over 36 Ph.D. theses. Some of his students have become luminaries in their own right: Stephen Smale and Daniel Quillen received the Fields Medal in 1966 and 1978 respectively, and Robert MacPherson the National Academy of Science Award in Mathematics in 1992. The accompanying sidebar presents what is, I hope, a complete list of his more note-worthy awards include: Sloan Fellowship (1956-60), the AMS Veblen Prize (1964), Guggenheim Fellowship (1976), National Medal of Science (1987), the AMS Steele Prize for Lifetime Achievement (1990), and the Wolf Prize in Mathematics (2000).

He was twice invited to address the International Congress of Mathematicians, in Edinburgh in 1958 and in Nice in 1970.

He was elected vice president of the AMS in 1974-75, Honorary Member of the London Mathematical Society (1976), Honorary Fellow of St. Catherine's College, Oxford (1983), Honorary Member of the Moscow Mathematical Society (1997), and Foreign Member of the Royal Society (2005). He has been a member of the National Academy of Sciences since 1964 and the French Academy of Sciences since 1995.

In 1987 he gave the Convocation Address at McGill University. He has also received Honorary Degrees of Doctor of Science from the University of Notre Dame (1980), McGill University (1987), Carnegie Mellon University (1989), and the University of Leicester, England (1995).

Mathematical Works
The bibliography in Raoul Bott's Collected Papers lists his publications, with some omissions, up to 1990.

When asked to single out the top three in the manner of an Olympic contest, he replied, "Can I squeeze in another one?" But after listing four as the tops, he sighed and said, "This is like being asked to single out the favorites among one's children." In the end he came up with a top-five list, in chronological order:

- [B5] Homogeneous vector bundles,
- [24] The periodicity theorem,
- [51] Topological obstruction to integrability,
- [81] Yang-Mills equations over Riemann surfaces,
- [82] The localization theorem in equivariant cohomology.

To discuss only these five would not do justice to the range of his output. On the other hand, it is evidently not possible to discuss every item in his ever-expanding opus. As a compromise, I asked him to make a longer list of all his favorite papers, without trying to rank them. What follows is a leisurely romp through the nineteen papers he chose. My goal is to explain, as simply as possible, the main achievement of his own favorite papers. For this reason, the theorems, if stated at all, are often not in their greatest generality.

Impedance
The subject of Raoul Bott's first paper [1] dates back to his engineering days. An electrical network determines an impedance function \( Z(s) \), which describes the frequency response of the network. This impedance function \( Z(s) \) is a rational function of a complex variable \( s \) and is positive-real (p.r.) in the sense that it maps the right half-plane into itself. An old question in electrical engineering asks whether conversely, given a positive-real rational function \( Z(s) \), it is possible to build a network with \( Z(s) \) as its impedance function. In some sense O. Brune had solved this problem in 1931, but Brune's solution assumes the existence of an "ideal transformer", which in practice would have to be the size of, say, the Harvard Science Center. The assumption of an ideal transformer renders Brune's algorithm not so practical, and it was Raoul's dream at McGill to remove the ideal transformer from the solution.

At his first meeting with his advisor Richard Duffin at Carnegie Tech, he blurted out the problem right away. Many days later, after a particularly fruitless and strenuous discussion, Raoul went home and realized how to do it. He called Duffin. The phone was busy. As it turned out, Duffin was calling him with exactly the same idea! They wrote up the solution to the long-standing problem in a
Morse Theory

As mentioned earlier, the paper on impedance so impressed Hermann Weyl that he invited Bott to the Institute for Advanced Study at Princeton in 1949. There Bott came into contact with Marston Morse. Morse's theory of critical points would play a decisive role throughout Bott's career, notably in his work on homogeneous spaces, the Lefschetz hyperplane theorem, the periodicity theorem, and the Yang-Mills functional on a moduli space.

In the 1920s Morse had initiated the study of the critical points of a function on a space and its relation to the topology of the space. A smooth function \( f \) on a smooth manifold \( M \) has a critical point at \( p \) if there is a coordinate system \( (x_1, \ldots, x_n) \) at \( p \) such that all the partial derivatives of \( f \) vanish at \( p \):

\[
\frac{\partial f}{\partial x_i}(p) = 0 \quad \text{for all } i = 1, \ldots, n.
\]

Such a critical point is nondegenerate if the matrix of second partials, called the Hessian of \( f \) at \( p \),

\[
H_p f = \left[ \frac{\partial^2 f}{\partial x_i \partial x_j}(p) \right]
\]

is nonsingular. The index \( \lambda(p) \) of a nondegenerate critical point \( p \) is the number of negative eigenvalues of the Hessian \( H_p f \); it is the number of independent directions along which \( f \) will decrease from \( p \).

If a smooth function has only nondegenerate critical points, we call it a Morse function. The behavior of the critical points of a Morse function can be summarized in its Morse polynomial:

\[
\mathcal{M}_i(f) := \sum t^{\lambda(p)}
\]

where the sum runs over all critical points \( p \).

A typical example of a Morse function is the height function \( f \) of a torus standing vertically on a table top (Figure 1).

![Figure 1. Critical points of the height function.](image)

The height function on this torus has four critical points of index 0, 1, 1, 2 respectively. Its Morse polynomial is

\[
\mathcal{M}_i(f) = 1 + 2t + t^2.
\]

For a Morse function \( f \) on a compact manifold \( M \), the fundamental results of Morse theory hinge on the fact that \( M \) has the homotopy type of a CW complex with one cell of dimension \( \lambda \) for each critical point of \( f \) of index \( \lambda \). This realization came about in the early 1950s, due to the work of Pitcher, Thom, and Bott.

Two consequences follow immediately:

i) The weak Morse inequalities:

\[
\# \text{ critical points of index } i \geq i^{\text{th}} \text{ Betti number.}
\]

If

\[
P_i(M) = \sum \dim H_i(M)t^i
\]

is the Poincaré polynomial of \( M \), the Morse inequalities can be restated in the form

\[
\mathcal{M}_i(f) \geq P_i(M),
\]

meaning that their difference \( \mathcal{M}_i(f) - P_i(M) \) is a polynomial with nonnegative coefficients. This inequality provides a topological constraint on analysis, for it says that the \( i^{\text{th}} \) Betti number of the manifold sets a lower bound on the number of critical points of index \( i \) that the function \( f \) must have.

ii) The lacunary principle: If no two critical points of the Morse function \( f \) have consecutive indices, then

\[
\mathcal{M}_i(f) = P_i(M).
\]

The explanation is simple: since in the CW complex of \( M \) there are no two cells of consecutive dimensions, the boundary operator is automatically zero. Therefore, the cellular chain complex is its own homology.

A Morse function \( f \) on \( M \) satisfying (1) is said to be perfect. The height function on the torus above is a perfect Morse function.

Classical Morse theory deals only with functions all of whose critical points are nondegenerate; in particular, the critical points must all be isolated points. In many situations, however, the critical points form submanifolds of \( M \). For example, if the torus now sits flat on the table, as a donut usually would, then the height function has the top and bottom circles as critical manifolds (Figure 2).

![Figure 2. Critical manifolds of the height function.](image)
One of Bott's first insights was to see how to extend Morse theory to this situation. In [9] he introduced the notion of a nondegenerate critical manifold: a critical manifold $N$ is nondegenerate if at any point $p$ in $N$ the Hessian of $f$ restricted to the normal space to $N$ is nonsingular. The index $\lambda(N)$ of the nondegenerate critical manifold $N$ is then defined to be the number of negative eigenvalues of this normal Hessian; it represents the number of independent normal directions along which $f$ is decreasing. For simplicity, assume that the normal bundles of the nondegenerate critical manifolds are all orientable. To form the Morse polynomial of $f$, each critical manifold $N$ is counted with its Poincaré polynomial; thus,

$$\mathcal{M}_f(f) := \sum P_i(M) t^{|\lambda(i)|},$$

summed over all critical manifolds.

With this definition of the Morse polynomial, Bott proved in [9] that if a smooth function $f$ on a smooth manifold $M$ has only nondegenerate critical manifolds, then the Morse inequality again holds:

$$\mathcal{M}_f(f) \geq P_1(M).$$

### Lie Groups and Homogeneous Spaces

In the 1950s Bott applied Morse theory with great success to the topology of Lie groups and homogeneous spaces. In [8] he showed how the diagram of a compact semisimple connected and simply connected group $G$ determines the integral homology of both the loop space $\Omega G$ and the flag manifold $G/T$, where $T$ is a maximal torus.

Indeed, Morse theory gives a beautiful CW cell structure on $G/T$, up to homotopy equivalence. To explain this, recall that the adjoint action of the group $G$ on its Lie algebra $g$ restricts to an action of the maximal torus $T$ on $g$. As a representation of the torus $T$, the Lie algebra $g$ decomposes into a direct sum of irreducible representations

$$g = \mathfrak{t} \bigoplus \sum E_{\alpha},$$

where $\mathfrak{t}$ is the Lie algebra of $T$ and each $E_{\alpha}$ is a 2-dimensional space on which $T$ acts as a rotation $e^{2\pi i \alpha \cdot x}$, corresponding to the root $\alpha(x)$ on $t$. The diagram of $G$ is the family of parallel hyperplanes in $t$ where some root is integral. A hyperplane that is the zero set of a root is called a root plane.

For example, for the group $G = SU(3)$ and maximal torus

$$T = \begin{bmatrix} e^{2\pi i \alpha_1} & 0 & 0 \\ 0 & e^{2\pi i \alpha_2} & 0 \\ 0 & 0 & e^{2\pi i \alpha_3} \end{bmatrix} \quad x_1 + x_2 + x_3 = 0, x_i \in \mathbb{R},$$

the roots are $\pm(x_1 - x_2), \pm(x_1 - x_3), \pm(x_2 - x_3)$, and the diagram is the collection of lines in the plane $x_1 + x_2 + x_3 = 0$ in $\mathbb{R}^3$ as in Figure 3. In this figure, the root planes are the thickened lines.

**Figure 3. The diagram of SU(3).**

For $G = SU(2)$ and

$$T = \begin{bmatrix} e^{2\pi i \alpha} & 0 \\ 0 & e^{-2\pi i \alpha} \end{bmatrix} \quad x \in \mathbb{R},$$

the Lie algebra $\mathfrak{t}$ is $\mathbb{R}$, the roots are $\pm 2x$, and the adjoint representation of $G$ on $g = \mathbb{R}^3$ corresponds to rotations. The root plane is the origin.

**Figure 4. The diagram of SU(2).**

A point $B$ in $t$ is regular if its normalizer has minimal possible dimension, or equivalently, if its normalizer is $T$. It is well known that a point $B$ in $t$ is regular if and only if it does not lie on any of the hyperplanes of the diagram. If $B$ is regular, then the stabilizer of $B$ under the adjoint action of $G$ is $T$, and so the orbit through $B$ is $G/T$.

Choose another regular point $A$ in $t$, and define the function $f$ on $\text{Orbit}(B) = G/T$ to be the distance from $A$; here the distance is measured with respect to the Killing form on $g$. Let $(B_i)$ be all the points in $t$ obtained from $B$ by reflecting about the root planes. Then Bott's theorem asserts that $f$ is a Morse function on $G/T$ whose critical points are precisely all the $B_i$'s. Moreover, the index of a critical point $B_i$ is twice the number of times that the line segment from $A$ to $B_i$ intersects the root planes. This cell decomposition of Morse theory fits in with the more group-theoretic Bruhat decomposition.

For $G = SU(3)$ and $T$ the set of diagonal matrices in $SU(3)$, the orbit $G/T$ is the complex flag manifold $F(1, 2, 3)$, consisting of all flags

$$V_1 \subset V_2 \subset \mathbb{C}^3, \quad \dim V_i = i.$$  

Bott's recipe gives 6 critical points of index $0, 2, 2, 4, 4, 6$ respectively on $G/T$ (see Figure 5). By the lacunary principle, the Morse function $f$ is perfect. Hence, the flag manifold $F(1, 2, 3)$ has the homotopy type of a CW complex with one 0-cell, two
2-cells, two 4-cells, and one 6-cell. Its Poincaré polynomial is therefore
\[ P_0(F^3(1,2,3)) = 1 + 2t^2 + 2t^4 + t^6. \]

**Figure 5. The flag manifold \( F^3(1,2,3) \).**

**Index of a Closed Geodesic**

For two points \( p \) and \( q \) on a Riemannian manifold \( M \), the space \( \Omega_{p,q}(M) \) of all paths from \( p \) to \( q \) on \( M \) is not a finite-dimensional manifold. Nonetheless, Morse theory applies to this situation also, with a Morse function on the path space \( \Omega_{p,q} \) given by the energy of a path:

\[ E(\mu) = \int_0^\mu \left( \frac{d\mu}{dt} \right)^2 dt. \]

The first result of this infinite-dimensional Morse theory asserts that the critical points of the energy function are precisely the geodesics from \( p \) to \( q \).

Two points \( p \) and \( q \) on a geodesic are **conjugate** if keeping \( p \) and \( q \) fixed, one can vary the geodesic from \( p \) to \( q \) through a family of geodesics. For example, two antipodal points on an \( n \)-sphere are conjugate points. The **multiplicity** of \( q \) as a conjugate point of \( p \) is the dimension of the family of geodesics from \( p \) to \( q \). On the \( n \)-sphere \( S^n \), the multiplicity of the south pole as a conjugate point of the north pole is therefore \( n - 1 \).

If \( p \) and \( q \) are not conjugate along the geodesic, then the geodesic is nondegenerate as a critical point of the energy function on \( \Omega_{p,q} \). Its index, according to the Morse index theorem, is the number of conjugate points from \( p \) to \( q \) counted with multiplicities.

On the \( n \)-sphere let \( p \) and \( p' \) be antipodal points and \( q \neq p' \). The geodesics from \( p \) to \( q \) are \( pp', pp'q, pp'qp, pp'qp'q \), ..., of index 0, \( n - 1 \), \( 2(n - 1) \), \( 3(n - 1) \), ..., respectively. By the Morse index theorem the energy function on the path space \( \Omega_{p,q}(S^n) \) has one critical point each of index 0, \( n - 1 \), \( 2(n - 1) \), \( 3(n - 1) \), ..., and it follows from Morse theory that \( \Omega_{p,q}(S^n) \) has the homotopy type of a CW complex with one cell in each of the dimensions \( 0, n - 1, 2(n - 1), 3(n - 1), \ldots \).

Now consider the space \( \Omega M \) of all smooth loops in \( M \), that is, smooth functions \( \mu : S^1 \to M \). The critical points of the energy function on \( \Omega M \) are again the geodesics, but these are now closed geodesics. A closed geodesic is never isolated as a critical point, since for any rotation \( r : S^1 \to S^1 \) of the circle, \( \mu = r \circ \mu : S^1 \to M \) is still a geodesic. In this way, any closed geodesic gives rise to a circle of closed geodesics. When the Riemannian metric on \( M \) is generic, the critical manifolds of the energy function on the loop space \( \Omega \) will all be circles.

Morse had shown that the index of a geodesic is the number of negative eigenvalues of a Sturm differential equation, a boundary-value problem of the form \( Ly = \lambda y \), where \( L \) is a self-adjoint second-order differential operator. For certain boundary conditions, Morse had expressed the index in terms of conjugate points, but this procedure does not apply to closed geodesics, which correspond to a Sturm problem with periodic boundary conditions.

In [14] Bott found an algorithm to compute the index of a closed geodesic. He was then able to determine the behavior of the index when the closed geodesic is iterated. Bott's method is in fact applicable to all Sturm differential equations. And so in his paper he also gave a geometric formulation and new proofs of the Sturm-Morse separation, comparison, and oscillation theorems, all based on the principle that the intersection number of two cycles of complementary dimensions is zero if one of the cycles is homologous to zero.

**Homogeneous Vector Bundles**

Let \( G \) be a connected complex semisimple Lie group, and \( P \) a parabolic subgroup. Then \( G \) is a principal \( P \)-bundle over the homogeneous manifold \( X = G/P \). Any holomorphic representation \( \phi : P \to \text{Aut}(E) \) on a complex vector space \( E \) induces a holomorphic vector bundle \( E \) over \( X \):

\[ E = G \times_G E = (G \times E)/P, \]

where \((g \phi, e) \sim (g, \phi(p)e)\). Then \( E \) is a holomorphic vector bundle over \( X = G/P \). A vector bundle over \( X \) arising in this way is called a **homogeneous vector bundle**. Let \( \mathcal{O}(E) \) be the corresponding sheaf of
holomorphic sections. The homogeneous vector bundle $E$ inherits a left $G$-action from the left multiplication in $G$:

$$h \cdot (g, e) = (hg, e) \quad \text{for} \quad h, g \in G, e \in E.$$ 

Thus, all the cohomology groups $H^q(X, O(E))$ become $G$-modules.

In [15] Bott proved that if the representation $\phi$ is irreducible, the cohomology groups $H^q(X, O(E))$ all vanish except possibly in one single dimension. Moreover, in the nonvanishing dimension $q$, $H^q(X, O(E))$ is an irreducible representation of $G$ whose highest weight is related to $\phi$.

This theorem generalizes an earlier theorem of Borel and Weil, who proved it for a positive line bundle.

In Bott’s paper one finds a precise way of determining the nonvanishing dimension in terms of the roots and weights of $G$ and $P$. Thus, on the one hand, Bott’s theorem gives a geometric realization of induced representations, and on the other hand, it provides an extremely useful vanishing criterion for the cohomology of homogeneous vector bundles.

The Peridocity Theorem

Homotopy groups are notoriously difficult to compute. For a simple space like the $n$-sphere, already, the higher homotopy groups exhibit no discernible patterns. It was therefore a complete surprise in 1957, when Raoul Bott computed the stable homotopy groups of the classical groups and found a simple periodic pattern for each of the classical groups [24].

We first explain what is meant by the stable homotopy group. Consider the unitary group $U(n+1)$. It acts transitively on the unit sphere $S^{2n+1}$ in $\mathbb{C}^{n+1}$, with stabilizer $U(n)$ at the point $(1,0,\ldots,0)$. In this way, the sphere $S^{2n+1}$ can be identified with the homogeneous space $U(n+1)/U(n)$, and there is a fibering $U(n+1) \to S^{2n+1}$ with fiber $U(n)$. By the homotopy exact sequence of a fibering, the following sequence is exact:

$$\cdots \to \pi_{k+1}(S^{2n+1}) \to \pi_k(U(n)) \to \pi_k(U(n+1)) \to \pi_k(S^{2n+1}) \to \cdots.$$ 

Since $\pi_k(S^m) = 0$ for $m > k$, it follows immediately that as $n$ goes to infinity (in fact for all $n > k/2$), the $k$th homotopy group of the unitary group stabilizes:

$$\pi_k(U(n)) = \pi_k(U(n+1)) = \pi_k(U(n+2)) = \cdots.$$ 

This common value is called the $k$th stable homotopy group of the unitary group, denoted $\pi_k(U)$.

In the original proof of the periodicity theorem [24], Bott showed that in the loop space of the special unitary group $SU(2n)$, the manifold of minimal geodesics is the complex Grassmannian $G(n,2n)$.

$$G(n,2n) = \frac{U(2n)}{U(n) \times U(n)}.$$ 

By Morse theory, the loop space $\Omega SU(2n)$ has the homotopy type of a CW complex obtained from the Grassmannian $G(n,2n)$ by attaching cells of dimension $\geq 2n+2$:

$$\Omega SU(2n) \sim G(n,2n) \cup \cup_{1}^{\infty} e_i \cup \cdots, \quad \dim e_i \geq 2n+2.$$ 

It follows that

$$\pi_k(\Omega SU(2n)) = \pi_k(G(n,2n)).$$ 

For $n \geq k$, we have

$$\pi_k(\Omega SU(2n)) = \pi_k(G(n,2n)) = \pi_{k+1}(U(2n)).$$

Using the homotopy exact sequence of the fibering

$$U(n) \to U(2n)/U(n) \to G(n,2n),$$

one gets

$$\pi_k(G(n,2n)) = \pi_k(U(n)).$$

Putting all this together, for $n$ large relative to $k$, we get

$$\pi_{k+1}(U(2n)) = \pi_k(G(n,2n)) = \pi_k(\Omega SU(2n)) = \pi_{k+1}(U(2n)).$$

Thus, the stable homotopy group of the unitary group is periodic of period 2:

$$\pi_{k+1}(U) = \pi_k(U).$$

Applying the same method to the orthogonal group and the symplectic group, Bott showed that their stable homotopy groups are periodic of period 8.

Clifford Algebras

The Clifford algebra $\mathcal{C}_k$ is the algebra over $\mathbb{R}$ with $k$ generators $e_1, \ldots, e_k$ and relations

$$e_i^2 = -1 \quad \text{for} \quad i = 1, \ldots, k, \quad e_i e_j = -e_j e_i \quad \text{for all} \quad i \neq j.$$ 

The first few Clifford algebras are easy to describe

$$\mathcal{C}_0 = \mathbb{R}, \quad \mathcal{C}_1 = \mathbb{C}, \quad \mathcal{C}_2 = \mathbb{H} = \{\text{quaternions}\}.$$ 

If $F$ is a field, denote by $F(n)$ the algebra of all $n \times n$ matrices with entries in $F$. We call $F(n)$ a full matrix algebra. It turns out that the Clifford algebras are all full matrix algebras or the direct sums of two full matrix algebras.
This table exhibits clearly a periodic pattern of period 8, except for the dimension increase after each period. The 8-fold periodicity of the Clifford algebras, long known to algebraists, is reminiscent of the 8-fold periodicity of the stable homotopy groups of the orthogonal group.

In the early 1960s Michael Atiyah, Raoul Bott, and Arnold Shapiro found an explanation for this tantalizing connection. The link is provided by a class of linear differential operators called the Dirac operators. The link between differential equations and homotopy groups first came about as a result of the realization that ellipticity of a differential operator can be defined in terms of the symbol of the differential operator.

Suppose we can find \( k \) real matrices \( e_1, \ldots, e_k \) of size \( n \times n \) satisfying

\[
e_i^2 = -1, \quad e_i e_j = -e_j e_i \quad \text{for } i \neq j.
\]

This corresponds to a real representation of the Clifford algebra \( C_k \). The associated Dirac operator \( D = D_{k,n} \) is the linear first-order differential operator

\[
D = I \frac{\partial}{\partial x_0} + e_1 \frac{\partial}{\partial x_1} + \cdots + e_k \frac{\partial}{\partial x_k},
\]

where \( I \) is the \( n \times n \) identity matrix. Such a differential operator on \( \mathbb{R}^{k+1} \) has a symbol \( \sigma_D(\xi) \) obtained by replacing \( \partial/\partial x_i \) by a variable \( \xi_i \):

\[
\sigma_D(\xi) = I \xi_0 + e_1 \xi_1 + \cdots + e_k \xi_k.
\]

The Dirac operator \( D \) is readily shown to be elliptic; this means its symbol \( \sigma_D(\xi) \) is nonsingular for all \( \xi \neq 0 \) in \( \mathbb{R}^{k+1} \). Therefore, when restricted to the unit sphere in \( \mathbb{R}^{k+1} \), the symbol of the Dirac operator gives a map

\[
\sigma_D(\xi) : S^k \to \mathrm{GL}(n, \mathbb{R}).
\]

Since \( \mathrm{GL}(n, \mathbb{R}) \) has the homotopy type of \( O(n) \), this map given by the symbol of the Dirac operator defines an element of the homotopy group \( \pi_k(\mathrm{GL}(n, \mathbb{R})) = \pi_k(O(n)) \).

The paper [33] shows that the minimal-dimensional representations of the Clifford algebras give rise to Dirac operators whose symbols generate the stable homotopy groups of the orthogonal group.

In this way, the 8-fold periodicity of the Clifford algebras reappears as the 8-fold periodicity of the stable homotopy groups of the orthogonal group.

**The Index Theorem for Homogeneous Differential Operators**

The 1960s was a time of great ferment in topology and one of its crowning glories was the Atiyah-Singer index theorem. Independently of Atiyah and Singer's work, Bott's paper [37] on homogeneous differential operators analyzes an interesting example where the analytical difficulties can be avoided by representation theory.

Suppose \( G \) is a compact connected Lie group and \( H \) a closed connected subgroup. As in our earlier discussion of homogeneous vector bundles, a representation \( \rho \) of \( G \) gives rise to a vector bundle \( G \times_H H \) over the homogeneous space \( X = G/H \). Now suppose \( E \) and \( F \) are two vector bundles over \( G/H \) arising from representations of \( H \). Since \( G \) acts on the left on both \( E \) and \( F \), it also acts on their spaces of sections, \( \Gamma(E) \) and \( \Gamma(F) \). We say that a differential operator \( D : \Gamma(E) \to \Gamma(F) \) is homogeneous if it commutes with the actions of \( G \) on \( \Gamma(E) \) and \( \Gamma(F) \). If \( D \) is elliptic, then its index

\[
\text{index}(D) = \dim \ker D - \dim \coker D
\]

is defined.

Atiyah and Singer had given a formula for the index of an elliptic operator on a manifold in terms of the topological data of the situation: the characteristic classes of \( E, F \), the tangent bundle of the base manifold, and the symbol of the operator. In [37] Raoul Bott verified the Atiyah-Singer index theorem for a homogeneous operator by introducing a refined index, which is not a number, but a character of the group \( G \). The usual index may be obtained from the refined index by evaluating at the identity. A similar theorem in the infinite-dimensional case has recently been proven in the context of physics-inspired mathematics.

**Nevanlinna Theory and the Bott-Chern Classes**

Nevanlinna theory deals with the following type of questions: Let \( f : \mathbb{C} \to \mathbb{C} P^1 \) be a holomorphic map. Given \( a \) in \( \mathbb{C} P^1 \), what is the inverse image \( f^{-1}(a) \)? Since \( \mathbb{C} \) is noncompact, there may be infinitely many points in the pre-image \( f^{-1}(a) \). Sometimes \( f^{-1}(a) \) will be empty, meaning that \( f \) misses the point \( a \) in \( \mathbb{C} P^1 \).

The exponential map \( \exp : \mathbb{C} \to \mathbb{C} P^1 \) misses exactly two points, 0 and infty, in \( \mathbb{C} P^1 \). According to a classical theorem of Picard, a nonconstant holomorphic map \( f : \mathbb{C} \to \mathbb{C} P^1 \) cannot miss more than two points.

Nevanlinna theory refines Picard's theorem in a beautiful way. For each \( a \in \mathbb{C} P^1 \), it attaches a real number \( \delta(a) \) between 0 and 1 inclusive, the deficiency index of \( a \). The deficiency index is a normalized way of counting the number of points in
the inverse image. If \( f^{-1}(a) \) is empty, then the deficiency index is 1.

In this context the first main theorem of Nevanlinna theory says that a nonconstant holomorphic map \( f: \mathbb{C} \to \mathbb{C}P^1 \) has deficiency index 0 almost everywhere. The second main theorem yields the stronger inequality:

\[
\sum_{a \in \mathbb{C}P^1} \delta(a) \leq 2.
\]

Ahlfors generalized these two theorems to holomorphic maps with values in a complex projective space \( \mathbb{C}P^n \).

In [38] Bott and Chern souped up Nevanlinna's hard analysis to give a more conceptual proof of the first main theorem.

A by-product of Bott and Chern's excursion in Nevanlinna theory is the notion of a refined Chern class, now called the Bott-Chern class, that has since been transformed into a powerful tool in Arakelov geometry and other aspects of modern number theory.

Briefly, the Bott-Chern classes arise as follows. On a complex manifold \( M \) the exterior derivative \( d \) decomposes into a sum \( d = \partial + \bar{\partial} \), and the smooth \( k \)-forms decompose into a direct sum of \((p, q)\)-forms. Let \( A^{p,q} \) be the space of smooth \((p, q)\)-forms on \( M \). Then the operator \( \partial \bar{\partial} \) makes \( A^{p,q} \) into a differential complex. Thus, the cohomology \( H^*([A^{p,q}, \partial \bar{\partial}] \) is defined.

A Hermitian structure on a holomorphic rank \( n \) vector bundle \( E \) on \( M \) determines a unique connection and hence a unique curvature tensor. If \( K \) and \( K' \) are the curvature forms determined by two Hermitian structures on \( E \) and \( \phi \) is a \( \text{GL}(n, \mathbb{C}) \)-invariant polynomial on \( \mathfrak{g}(n, \mathbb{C}) \), then it is well known that \( \phi(K) \) and \( \phi(K') \) are global closed forms on \( M \) whose difference is exact:

\[
\phi(K) - \phi(K') = d\alpha
\]

for a differential form \( \alpha \) on \( M \). This allows one to define the characteristic classes of \( E \) as cohomology classes in \( H^*(M) \).

In the holomorphic case, \( \phi(K) \) and \( \phi(K') \) are \((p, p)\)-forms closed under \( \partial \bar{\partial} \). Bott and Chern found that in fact,

\[
\phi(K) - \phi(K') = \partial \partial \bar{\partial} \beta
\]

for some \((p - 1, p - 1)\)-form \( \beta \). For a holomorphic vector bundle \( E \), the Bott-Chern class of \( E \) associated to an invariant polynomial \( \phi \) is the cohomology class of \( \phi(E) \), not in the usual cohomology, but in the cohomology of the complex \( [A^{p,p}, \partial \bar{\partial}] \).

**Characteristic Numbers and the Bott Residue**

According to the celebrated Hopf index theorem, the Euler characteristic of a smooth manifold is equal to the number of zeros of a vector field on the manifold, each counted with its index. In [41] and [43], Bott generalized the Hopf index theorem to other characteristic numbers such as the Pontryagin numbers of a real manifold and the Chern numbers of a complex manifold.

We will describe Bott's formula only for Chern numbers. Let \( M \) be a compact complex manifold of dimension \( n \), and \( c_1(M), \ldots, c_n(M) \) the Chern classes of the tangent bundle of \( M \). The Chern numbers of \( M \) are the integrals \( \int_M \phi(c_1(M), \ldots, c_n(M)) \), as \( \phi \) ranges over all weighted homogeneous polynomials of degree \( n \). Like the Hopf index theorem, Bott's formula computes a Chern number in terms of the zeros of a vector field \( X \) on \( M \), but the vector field must be holomorphic and the counting of the zeros is a little more subtle.

For any vector field \( Y \) and any \( C^\infty \) function \( f \) on \( M \), the Lie derivative \( L_X \) satisfies:

\[
L_X(fY) = (Xf)Y + fL_X(Y).
\]

It follows that at a zero \( p \) of \( X \),

\[
(L_X f)_p = f(p)(L_X Y)_p.
\]

Thus, at \( p \), the Lie derivative \( L_X \) induces an endomorphism

\[
L_p : T_p M \to T_p M
\]

of the tangent space of \( M \) at \( p \). The zero \( p \) is said to be nondegenerate if \( L_p \) is nonsingular.

For any endomorphism \( A \) of a vector space \( V \), we define the numbers \( c_i(A) \) to be the coefficients of its characteristic polynomial:

\[
\det(I + tA) = \sum c_i(A)t^i.
\]

Bott's Chern number formula is as follows. Let \( M \) be a compact complex manifold of complex dimension \( n \) and \( X \) a holomorphic vector field having only isolated nondegenerate zeros on \( M \). For any weighted homogeneous polynomial \( \phi(x_1, \ldots, x_n) \), \( \deg x_i = 2i \),

\[
\int_M \phi(c_1(M), \ldots, c_n(M)) = \sum_{p} \frac{\phi(c_1(L_p), \ldots, c_n(L_p))}{c_n(L_p)},
\]

summed over all the zeros of the vector field. Note that by the definition of a nondegenerate zero, \( c_n(L_p) \), which is \( \det L_p \), is nonzero.

In Bott's formula, if the polynomial \( \phi \) does not have degree \( 2n \), then the left-hand side of (2) is zero, and the formula gives an identity among the numbers \( c_i(L_p) \). For the polynomial \( \phi(x_1, \ldots, x_n) = x_n \), Bott's formula recovers the Hopf index theorem:

\[
\int_M c_n(M) = \sum_{p} \frac{c_n(L_p)}{c_n(L_p)} = \# \text{ zeros of } X.
\]
Bott's formula (2) is reminiscent of Cauchy's residue formula and so the right-hand side of (2) may be viewed as a residue of \( \phi \) at \( p \).

In [43] Bott generalized his Chern number formula (2), which assumes isolated zeros, to holomorphic vector fields with higher-dimensional zero sets and to bundles other than the tangent bundle (a vector field is a section of the tangent bundle).

**The Atiyah-Bott Fixed Point Theorem**

A continuous map of a finite polyhedron, \( f : P \rightarrow P \), has a Lefschetz number:

\[
L(f) = \sum (-1)^i \text{tr} f^* | H^i(P),
\]

where \( f^* \) is the induced homomorphism in cohomology and \( \text{tr} \) denotes the trace. According to the Lefschetz fixed point theorem, if the Lefschetz number of \( f \) is not zero, then \( f \) has a fixed point.

In the smooth category the Lefschetz fixed point theorem has a quantitative refinement. A smooth map \( f : M \rightarrow M \) from a compact manifold to itself is \textit{transversal} if its graph is transversal to the diagonal \( \Delta \) in \( M \times M \). Analytically, \( f \) is transversal if and only if at each fixed point \( p \),

\[
\det(1 - f_{*,p}) \neq 0,
\]

where \( f_{*,p} : T_pM \rightarrow T_pM \) is the differential of \( f \) at \( p \).

The \( C^\infty \) Lefschetz fixed point theorem states that the Lefschetz number of a transversal map \( f \) is the number of fixed points of \( f \) counted with multiplicity \( \pm 1 \) depending on the sign of the determinant \( \det(1 - f_{*,p}) \):

\[
L(f) = \sum_{p \in \text{Fix}(f)} \pm 1.
\]

In the 1960s Atiyah and Bott proved a far-reaching generalization of the Lefschetz fixed point theorem ([42], [44]). This type of result, relating a global invariant to a sum of local contributions, is a recurring theme in some of Bott's best work.

To explain it, recall that the real singular cohomology of \( M \) is computable from the de Rham complex

\[
\cdots \rightarrow T_x^* M \xrightarrow{\partial} \cdots \rightarrow T_x M \rightarrow \cdots
\]

where \( \Lambda^q = \Lambda^q T^* M \) is the \( q \)th exterior power of the cotangent bundle. The de Rham complex is an example of an \textit{elliptic complex} on a manifold.

Let \( E \) and \( F \) be vector bundles of ranks \( r_E \) and \( r_F \) respectively over \( M \). An \( \mathbb{R} \)-linear map

\[
D : \Gamma(E) \rightarrow \Gamma(F)
\]

is a \textit{differential operator} if about every point in \( M \) there is a coordinate chart \( (U, x_1, \ldots, x_n) \) and trivializations for \( E \) and \( F \) relative to which \( D \) can be written in the form

\[
D = \sum_{i=1}^{n} \partial \cdot \sum a_i(x) \partial x_i + \sum b_i(x),
\]

where \( a_i(x) \) is the \( i \)th component of the symbol of \( D \) at \( x \).

**Figure 7. A transversal map \( f \).**
\( L(T) = \sum (-1)^i \text{tr} T_i^* \).

A map \( f : M \to M \) induces a natural map
\[ \Gamma_f : \Gamma(E) \to \Gamma(f^{-1}E) \]
by composition: \( \Gamma_f(s) = s \circ f \). There is no natural way to induce a map of sections \( \Gamma(E) \to \Gamma(E) \). However, if there is a bundle map \( \phi : f^{-1}E \to E \), then the composite
\[ \Gamma(E) \to \Gamma(f^{-1}E) \xrightarrow{\phi} \Gamma(E) \]
is an endomorphism of \( \Gamma(E) \). Any bundle map \( \phi : f^{-1}E \to E \) is called a lifting of \( f \) to \( E \). At each point \( x \in M \), a lifting \( \phi \) is nothing other than a linear map \( \phi_\| : E_{f(x)} \to E_x \).

In the case of the de Rham complex, a map \( f : M \to M \) induces a linear map \( f^* : \Omega^* \to \Omega^* \) and hence a linear map
\[ \Lambda^0 f^* : \Lambda^0 \Omega^* \to \Lambda^0 \Omega^* \]
which is the lifting that finally defines the pullback of differential forms \( f^* : \Gamma(\Lambda^0 \Omega^* M) = \Gamma(\Lambda^0 \Omega^* M) \).

**Theorem 1.** (Atiyah-Bott fixed point theorem). Given an elliptic complex \( (3) \) on a compact manifold \( M \), suppose \( f : M \to M \) has a lifting \( \phi_i : \Gamma(E_i) \to \Gamma(E_i) \) for each \( i \) such that the induced maps \( T_i : \Gamma(E_i) \to \Gamma(E_i) \) give an endomorphism of the elliptic complex. Then the Lefschetz number of \( T \) is given by
\[ L(T) = \sum \frac{1}{\det(1 - f^*_x)} \text{tr} \phi_{x,x} \] for each point \( x \) in \( M \).

As evidence of its centrality, the Atiyah-Bott fixed point theorem has an astonishing range of applicability.

Here is an easily stated corollary in algebraic geometry: any holomorphic map of a rational algebraic manifold to itself has a fixed point.

Specializing the Atiyah-Bott fixed point theorem to the de Rham complex, one recovers the classical Lefschetz fixed point theorem. When they applied the theorem to other geometrically interesting elliptic complexes, Atiyah and Bott obtained new fixed point theorems, such as a holomorphic Lefschetz fixed point theorem in the complex analytic case and a signature formula in the Riemannian case. In the homogeneous case, the fixed point theorem implies the Weyl character formula.

**Obstruction to Integrability**
A subbundle \( E \) of the tangent bundle \( TM \) of a manifold \( M \) assigns to each point \( x \) of the manifold a subspace \( E_x \) of the tangent space \( T_x M \). An integral manifold of the subbundle \( E \) is a submanifold \( N \) of \( M \) whose tangent space \( T_x N \) at each point \( x \) in \( N \) is \( E_x \). The subbundle \( E \) is said to be integrable if for each point \( x \) in \( M \), there is an integrable manifold of \( E \) passing through \( x \).

By the Frobenius theorem, often proven in a first-year graduate course, a subbundle \( E \) of the tangent bundle \( TM \) is integrable if and only if its space of sections \( \Gamma(E) \) is closed under the Lie bracket.

The Pontryagin ring \( \text{Pon}(V) \) of a vector bundle \( V \) over \( M \) is defined to be the subring of the cohomology ring \( H^*(M) \) generated by the Pontryagin classes of the bundle \( V \). In [51] Bott found an obstruction to the integrability of \( E \) in terms of the Pontryagin ring of the quotient bundle \( Q := TM/E \). More precisely, if a subbundle \( E \) of the tangent bundle \( TM \) is integrable, then the Pontryagin ring \( \text{Pon}(Q) \) vanishes in dimensions greater than twice the rank of \( Q \).

What is so striking about this theorem is not only the simplicity of the statement, but also the simplicity of its proof. It spawned tremendous developments in foliation theory in the 1970s, as recounted in [C] and [H1].

**The Cohomology of Vector Fields on a Manifold**
For a finite-dimensional Lie algebra \( L \), let \( A^a(L) \) be the space of alternating \( a \)-forms on \( L \). Taking cues from the Lie algebra of left-invariant vector fields on a Lie group, one defines the differential
\[ \delta : A^a(L) \to A^{a+1}(L) \]
by
\[ \delta(\omega)(X_0, \ldots, X_a) = \sum_{i=0}^a (-1)^i \omega(X_i, X_0, \ldots, \hat{X}_i, \ldots, X_a). \]

As usual, the hat \( \hat{\cdot} \) over \( X_i \) means that \( X_i \) is to be omitted. This makes \( A^*(L) \) into a differential complex, whose cohomology is by definition the cohomology of the Lie algebra \( L \).

When \( L \) is the infinite-dimensional Lie algebra \( \Omega(L) \) of vector fields on a manifold \( M \), the formula (4) still makes sense, but the space of all alternating forms \( A^*(\Omega(L)) \) is too large for its cohomology to be computable. Gel'fan and Fuks proposed putting a topology, the continuous topology, on \( \Omega(L) \), and computing instead the cohomology of the continuous alternating forms on \( \Omega(L) \). The Gel'fan-Fuks cohomology of \( M \) is the cohomology of the complex \( (A^*(\Omega(L)), \delta) \) of continuous forms. They hoped to find in this way new invariants of a manifold. As an example, they computed the Gel'fan-Fuks cohomology of a circle.

It is not clear from the definition that the Gel'fan-Fuks cohomology is a homotopy invariant. In [71] Bott and Segal proved that the Gel'fan-Fuks cohomology of a manifold \( M \) is the singular cohomology of a space functorially constructed from \( M \). Haefliger [H] and Trauber gave a very different proof of this same result. The homotopy invariance of the Gel'fan-Fuks cohomology follows.
At the same time it also showed that the Gelfand-Fuks cohomology produces no new invariants.

**Localization in Equivariant Cohomology**

Just as singular cohomology is a functor from the category of topological spaces to the category of rings, so when a group $G$ acts on a space $M$, one seeks a functor that would incorporate both the topology of the space and the action of the group.

The naive construction of taking the cohomology of the quotient space $M/G$ is unsatisfactory because for a nonfree action, the topology of the quotient can be quite bad. A solution is to find a contractible space $EG$ on which $G$ acts freely, for then $EG \times M$ will have the same homotopy type as $M$ and the group $G$ will act freely on $EG \times M$ via the diagonal action. It is well known that such a space is the total space of the universal $G$-bundle $EG \to BG$, whose base space is the classifying space of $G$. The homotopy theorists have defined the homotopy quotient $M_G$, of $M$ by $G$ to be the quotient space $(EG \times M)/G$, and the equivariant cohomology $H^*_G(M)$ to be the ordinary cohomology of its homotopy quotient $M_G$.

The equivariant cohomology of the simplest $G$-space, a point, is already quite interesting, for it is the ordinary cohomology of the classifying space of $G$:


Since equivariant cohomology is a functor of $G$-spaces, the constant map $M \to pt$ induces a homomorphism $H^*_G(pt) \to H^*_G(M)$. Thus, the equivariant cohomology $H^*_G(M)$ has the structure of a module over $H^*(BG)$.

Characteristic classes of vector bundles over $M$ extend to equivariant characteristic classes of equivariant vector bundles.

When $M$ is a manifold, there is a push-forward map $\pi_M^* : H^*_G(M) \to H^*_G(pt)$, akin to integration along the fiber.

Suppose a torus $T$ acts on a compact manifold $M$ with fixed point set $F$, and $\phi \in H^2_G(M)$ is an equivariantly closed class. Let $P$ be the connected components of $F$ and let $\nu_P : P \to M$ be the inclusion map, $\nu_P$ the normal bundle of $P$ in $M$, and $e(\nu_P)$ the equivariant Euler class of $\nu_P$. In [82] Atiyah and Bott proved a localization theorem for the equivariant cohomology $H^*_T(M)$ with real coefficients:

$$\pi_T^* \phi = \sum_P \pi_T^* \left( \frac{1}{e(\nu_P)} \phi \right).$$

It should be noted that Berline and Vergne [BV] independently proved the same theorem at about the same time.

This localization theorem has as consequences the following results of Duistermaat and Heckman on a symplectic manifold $(M, \omega)$ of dimension of $2n$:

1) If a torus action on $M$ preserves the symplectic form and has a moment map $\mu$, then the push-forward $\mu_* (\omega^n)$ of the symplectic volume under the moment map is piecewise polynomial.

2) Under the same hypotheses, the stationary phase approximation for the integral

$$\int_M e^{-i\mu(x) / n!}$$

is exact.

In case the vector field on the manifold is generated by a circle action, the localization theorem specializes to Bott's Chern number formulas [41] of the 1960s, thus providing an alternative explanation for the Chern number formulas.

**Yang-Mills Equations over Riemann Surfaces**

In algebraic geometry it is well known that for any degree $d$ the set of isomorphism classes of holomorphic line bundles of degree $d$ over a Riemann surface $M$ of genus $g$ forms a smooth projective variety which is topologically a torus of dimension $g$. This space is called the moduli space of holomorphic line bundles of degree $d$ over $M$.

For holomorphic vector bundles of rank $k \geq 2$, the situation is far more complicated. First, in order to have an algebraic structure on the moduli space, it is necessary to discard the so-called “unstable” bundles in the sense of Mumford. It is then known that for $k$ and $d$ relatively prime, the isomorphism classes of the remaining bundles, called “semistable bundles”, form a smooth projective variety $N(k,d)$.

In [N] Newstead computed the Poincaré polynomial of $N(2,1)$. Apart from this, the topology of $N(k,d)$ remained mysterious.

In [81] Atiyah and Bott introduced the new and powerful method of equivariant Morse theory to study the topology of these moduli spaces.

Let $P = M \times U(n)$ be the trivial principal $U(n)$-bundle over the Riemann surface $M$, $\mathcal{A} = \mathcal{A}(P)$ the affine space of connections on $P$, and $\mathcal{G} = G(P)$ the gauge group, i.e., the group of automorphisms of $P$ that cover the identity. Then the gauge group $G(P)$ acts on the space $\mathcal{A}(P)$ of connections and there is a Yang-Mills functional $L$ on $\mathcal{A}(P)$ invariant under the action of the gauge group.

Equivariant Morse theory harks back to Bott’s extension of classical Morse theory to nondegenerate critical manifolds three decades earlier. The key result of Atiyah and Bott is that the Yang-Mills functional $L$ is a perfect equivariant Morse function on $\mathcal{A}(P)$. This means the equivariant Poincaré series of $\mathcal{A}(P)$ is equal to the equivariant Morse series of $L$:

$$P^*_L(\mathcal{A}(P)) = M^*_L(L).$$
Once one unravels the definition, the left-hand side of (5) is simply the Poincaré series of the classifying space of $G(P)$, which is computable from homotopy considerations. The right-hand side of (5) is the sum of contributions from all the critical sets of $L$. By the work of Narasimhan and Seshadri, the minimum of $L$ is precisely the moduli space $N(k, d)$. It contributes its Poincaré polynomial to the equivariant Morse series of $L$. By an inductive procedure, Atiyah and Bott were able to compute the contributions of all the higher critical sets. They then solved (5) for the Poincaré polynomial of $N(k, d)$.

Witten’s Rigidity Theorem

Let $E$ and $F$ be vector bundles over a compact manifold $M$. If a differential operator $D : \Gamma(E) \to \Gamma(F)$ is elliptic, then $\ker D$ and $\text{coker } D$ are finite-dimensional vector spaces and we can define the index of $D$ to be the virtual vector space

$$D = \ker D - \text{coker } D.$$

Now suppose a Lie group $G$ acts on $M$, and $E$ and $F$ are $G$-equivariant vector bundles over $M$. Then $G$ acts on $\Gamma(E)$ by

$$(g.s)(x) = g.(s(g^{-1}.x)),$$

for $g \in G$, $s \in \Gamma(E)$, $x \in M$. The $G$-action is said to preserve the differential operator $D$ if the actions of $G$ on $\Gamma(E)$ and $\Gamma(F)$ commute with $D$. In this case $\ker D$ and $\text{coker } D$ are representations of $G$, and so $D$ is a virtual representation of $G$. We say that the operator $D$ is rigid if its index is a multiple of the trivial representation of dimension 1. The rigidity of $D$ means that any nontrivial irreducible representation of $G$ in $\ker D$ occurs in $\text{coker } D$ with the same multiplicity and vice versa.

If the multiple $m$ is positive, then $m.1 = 1 + \cdots + 1$ is the trivial representation of dimension $m$. If $m$ is negative, the $m.1$ is a virtual representation and the rigidity of $D$ implies that the trivial representation 1 occurs more often in $\text{coker } D$ than in $\ker D$.

For a circle action on a compact oriented Riemannian manifold, it is well known that the Hodge operator $d + d^* : \Omega^p \to \Omega^{p+1}$ and the signature operator $d_s = d + d^* : \Omega^+ \to \Omega^-$ are both rigid.

An oriented Riemannian manifold of dimension $n$ has an atlas whose transition functions take values in $SO(n)$. The manifold is called a spin manifold if it is possible to lift the transition functions to the double cover $Spin(n)$ of $SO(n)$.

Inspired by physics, Witten discovered infinitely many rigid elliptic operators on a compact spin manifold with a circle action. They are typically of the form $d_s \otimes R$, where $d_s$ is the signature operator and $R$ is some combination of the exterior and symmetric powers of the tangent bundle. In [91] Bott and Taubes found a proof, more accessible to mathematicians, of Witten’s results, by recasting the rigidity theorem as a consequence of the Atiyah-Bott fixed point theorem.

The idea of [91] is as follows. To decompose a representation, one needs to know only its trace, since the trace determines the representation. By assumption, the action of $G$ on the elliptic complex $D : \Gamma(E) \to \Gamma(F)$ commutes with $D$. This means each element $g$ in $G$ is an endomorphism of the elliptic complex. It therefore induces an endomorphism $g^*$ in the cohomology of the complex. But $H^1 = \ker D$ and $H^1 = \text{coker } D$. The alternating sum of the trace of $g^*$ in cohomology is precisely the left-hand side of the Atiyah-Bott fixed point theorem. It then stands to reason that the fixed point theorem could be used to decompose the index of $D$ into irreducible representations.

Papers of Raoul Bott Discussed in this Article


[38] (with S. Chern) Hermitian vector bundles and the equidistribution of the zeroes of their holomorphic sections, Acta Mathematica 114 (1964), 71-112.


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References


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The next animation shows a knot.

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Percolation is a simple probabilistic model which exhibits a phase transition (as we explain below). The simplest version takes place on $\mathbb{Z}^2$, which we view as a graph with edges between neighboring vertices. All edges of $\mathbb{Z}^2$ are, independently of each other, chosen to be open with probability $p$ and closed with probability $1-p$. A basic question in this model is "What is the probability that there exists an open path, i.e., a path all of whose edges are open, from the origin to the exterior of the square $S_n := [-n,n]^2$?" This question was raised by Broadbent in 1954 at a symposium on Monte Carlo methods. It was then taken up by Broadbent and Hammersley, who regarded percolation as a model for a random medium. They interpreted the edges of $\mathbb{Z}^2$ as channels through which fluid or gas could flow if the channel was wide enough (an open edge) and not if the channel was too narrow (a closed edge). It was assumed that the fluid would move wherever it could go, so that there is no randomness in the behavior of the fluid, but all randomness in this model is associated with the medium.

We shall use 0 to denote the origin. A limit as $n \to \infty$ of the question raised above is "What is the probability that there exists an open path from 0 to infinity?" This probability is called the percolation probability and denoted by $\theta(p)$. Clearly $\theta(0) = 0$ and $\theta(1) = 1$, since there are no open edges at all when $p = 0$ and all edges are open when $p = 1$. It is also intuitively clear that the function $p \to \theta(p)$ is nondecreasing. Thus the graph of $\theta$ as a function of $p$ should have the form indicated in Figure 1, and one can define the critical probability by $p_c = \sup\{p : \theta(p) = 0\}$.

Why is this model interesting? In order to answer this we define the (open) cluster $C(v)$ of the vertex $v \in \mathbb{Z}^2$ as the collection of points connected to $v$ by an open path. The clusters $C(v)$ are the maximal connected components of the collection of open edges of $\mathbb{Z}^2$, and $\theta(p)$ is the probability that $C(0)$ is infinite. If $p < p_c$, then $\theta(p) = 0$ by definition, so that $C(0)$ is finite with probability 1. It is not hard to see that in this case all open clusters are finite. If $p > p_c$, then $\theta(p) > 0$ and there is a strictly positive probability that $C(0)$ is infinite. An application of Kolmogorov's zero-one law shows that there is then with probability 1 some infinite cluster. In fact, it turns out that there is a unique infinite cluster. Thus, the global behavior of the system is quite different for $p < p_c$ and for $p > p_c$. Such a sharp transition in global behavior of a system at some parameter value is called a phase transition or a critical phenomenon by statistical physicists, and the parameter value at which the transition takes place is called a critical value.

There is an extensive physics literature on such phenomena. Broadbent and Hammersley proved that $0 < p_c < 1$ for percolation on $\mathbb{Z}^2$, so that there is indeed a nontrivial phase transition. Much of the interest in percolation comes from the hope that one will be better able to analyze the behavior of various functions near the critical point for the simple model of percolation, with all its built-in independence properties, than for other, more complicated models for disordered media. Indeed, percolation is the simplest one in the family of the so-called random cluster or Fortuin-Kasteleyn models, which also includes the celebrated Ising model for magnetism. The studies of percolation and random cluster models have influenced each other.

Percolation can obviously be generalized to percolation on any graph $G$, even to (partially) directed graphs. One can also consider the model in which the vertices are independently open or closed, but all edges are assumed open. This version is called

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site percolation, in contrast to the version we considered so far, and which is called bond percolation. Initially research concentrated on finding the precise value of $p_c$ for various graphs. This has not been very successful; one knows $p_c$ only for a few planar lattices (e.g., $p_c = 1/2$ for bond percolation on $\mathbb{Z}^2$ and for site percolation on the triangular lattice). The value of $p_c$ depends strongly on geometric properties of $G$. Attention has therefore shifted to questions about the distribution of the number of vertices in $C(0)$ and geometric properties of the open clusters when $p$ is close to $p_c$. It is believed that a number of these properties are universal, that is, they depend only on the dimension of $G$, and not on details of its structure.

In particular, one wants to study the behavior of various functions as $p$ approaches $p_c$, or as some other parameter tends to infinity, while $p$ is kept at $p_c$. It is believed that many functions obey so-called power laws. For instance, it is believed that the expected number of vertices in $C(0)$, denoted by $\chi(p)$, behaves like $(p_c - p)^{-\gamma} p$ as $p \to p_c$, in the sense that $-\log \chi(p)/\log(p_c - p) \to \gamma$ for a suitable constant $\gamma$. Similarly one believes that $\theta(p)$ behaves like $(p - p_c)^\beta$ for some $\beta$ as $p \to p_c$, or that the probability that there is an open path from $0$ to the exterior of $S_n$ for $p = p_c$ behaves like $n^{-1/p}$ for some $p$. Even though such power laws have been proven only for site percolation on the triangular lattice or on high-dimensional lattices, it is believed that the exponents $\beta, \gamma, \rho$, etc. (usually called critical exponents), exist, and in accordance with the universality hypothesis mentioned above depend only on the dimension of $G$. For instance, bond and site percolation on $\mathbb{Z}^d$ or on the triangular lattice should all have the same exponents. Physicists invented the renormalization group to explain and/or prove such power laws and universality, but this has not been made mathematically rigorous for percolation.

$\mathbb{Z}^d$ for large $d$ behaves in many respects like a regular tree, and for percolation on a regular tree one can easily prove power laws and compute the relevant critical exponents. For bond percolation on $\mathbb{Z}^d$ with $d \geq 19$ Hara and Slade succeeded in proving power laws and in showing that the exponents agree with those for a regular tree. They have even shown that their theory applies down to $d > 6$ when one adds edges to $\mathbb{Z}^d$ between any two sites within distance $L_0$ of each other for some $L_0 = L_0(d)$.

Due to this theory we have a reasonable understanding of high-dimensional percolation. In the last few years Lawler, Schramm, Smirnov, and Werner have proven power laws for site percolation on the triangular lattice and confirmed most of the values for the critical exponents conjectured by physicists. Their proof rests on Schramm's invention of the Stochastic Loewner Evolutions or Schramm Loewner Evolutions (SLE) and on Smirnov's beautiful proof of the existence and conformal invariance properties of certain crossing probabilities. Roughly speaking this says the following: Let $D$ be a "nice" domain in $\mathbb{R}^2$ and let $A$ and $B$ be two arcs in the boundary. For $\lambda > 0$, let $P_\lambda(D, A, B)$ be the probability for $p = p_c$ that there exists an open path of site percolation on the triangular lattice in $\lambda D$ from $\lambda A$ to $\lambda B$. In fact it is nearer to take $P_\lambda(D, A, B)$ as the probability at $p_c$ of an open connection in $\lambda D$ from $A$ to $B$ on $(1/\lambda)$ times the triangular lattice. Conformal invariance says that $Q(D, A, B) : = \lim_{\lambda \to \infty} P_\lambda(D, A, B)$ exists and that $Q(D, A, B) = Q(\Phi(D), \Phi(A), \Phi(B))$ for every conformal map $\Phi$ from $D$ onto $\Phi(D)$. Further crucial ingredients are characterizations by Lawler and Werner of some SLE process on a domain by means of properties of its evolution before it hits the boundary. Conformal invariance had earlier been conjectured by physicists, and Cardy had given a formula for $Q(D, A, B)$. Smirnov's work gives a rigorous proof of Cardy's formula for percolation on the triangular lattice. Further work (see Camia and Newman (2005)) also has led to a description of the limit (in a suitable sense) as $\lambda \to \infty$ of the full pattern of the random configuration of open paths at criticality, i.e., for $p = p_c$. Since their discovery, SLE processes have led to exciting new probability theory in their own right, for instance, to power laws for the intersection probabilities of several Brownian motions (see Lawler (2005)).

So far conformal invariance results have been achieved only for site percolation on the triangular lattice. It is perhaps the principal open problem of the subject to prove conformal invariance for percolation on other two-dimensional lattices. Another related major problem is to establish power laws and universality for percolation on $d$-dimensional lattices with $2 \leq d \leq 6$. Finally, an unsolved problem of fifteen years' standing is whether there is an infinite open cluster for critical percolation on $\mathbb{Z}^d, d \geq 3$.

I thank Geoffrey Grimmett for several helpful suggestions.

**Further Reading**


Dark Hero of the Information Age: In Search of Norbert Wiener, The Father of Cybernetics

Reviewed by Michael B. Marcus

The thesis of this book is that Norbert Wiener, 1894-1964, was unknown outside the mathematical community until shortly after World War II. Then he invented cybernetics, which has the capacity to enormously transform the world for the better. The authors believe that since the promises of cybernetics have not been realized, Wiener is not the recognized genius of the Age of the Information, but its dark hero. And what, according to the authors, was the greatest of the forces that prevented the realization of the cybernetics utopia? It was a single person, Wiener's wife Margaret, née En gemann.

There are many points to be examined here. First of all, was the significance of Wiener's mathematical contributions really secondary compared to his latter work in naming and championing cybernetics? More significantly, what is cybernetics, and what does its implementation promise? Also, what did Margaret do and what good does it do us and Wiener's memory to dwell on it? And, finally, why is there such a continuing fascination with Norbert Wiener?

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I'm sure that every reader of the Notices knows that Norbert Wiener was a child prodigy. He was "home schooled" for a few years, by his overbearing father, a professor of Slavic languages at Harvard, who continued to teach and torment him until he graduated from high school at eleven. Young Norbert graduated from Tufts in 1909, at fifteen, with a B.A. in mathematics. He received his Ph.D. in philosophy from Harvard in 1913 when he was eighteen years old.

Wiener spent his first postdoctoral year in Cambridge, England, studying mathematical logic with Bertrand Russell. As P. R. Masani writes in his biography of Wiener ([4], p. 55), "Russell urged Wiener to approach mathematical philosophy from the broadest standpoint, to concentrate not just on foundations but also to look at the frontiers of mathematics and theoretical physics. This advice not only brought Wiener into contact with G. H. Hardy...but it also exposed Wiener to Bohr's atomic theory, the work of J. W. Gibbs on statistical mechanics, and the Einstein-Smoluchowski papers on Brownian motion." (Most of the biographical details of Wiener's life in this review are taken from...
Masani’s biography.) Because Russell was to be away from Cambridge in the spring of 1914, he sent Wiener off to Göttingen, where he took courses with David Hilbert, Edmund Husserl, and Edmund Landau. One must marvel at the education Wiener received and that he was able to absorb it. This must account for his enormous breadth later on and his willingness to consider questions in so many different areas of science.

In 1915 Wiener returned to Harvard as an assistant and docent lecturer in the philosophy department. But he didn’t continue with cushy appointments at prestigious universities. After this academic-year appointment he had a teaching job at a minor branch of the University of California and subsequently worked as a writer for the Encyclopaedia Americana and as a journalist. During World War I he worked as a “computer” at the U.S. Army proving grounds in Aberdeen, Maryland, and was even a private in the Army for a short time in 1918. In 1919, largely on the recommendation of W. F. Osgood of Harvard, Wiener was offered a one-year instructorship in mathematics at the Massachusetts Institute of Technology. MIT was not a prestigious research institute in 1919. The mathematics department was a service department for the engineering school.

At this point the miracles began to happen. Wiener had become increasingly interested in analysis during the years between his docent lecturership at Harvard and his appointment to MIT. After some work in functional analysis, in which he defined and studied what are now referred to as Banach spaces, he made Brownian motion mathematically rigorous by obtaining a measure, now called Wiener measure, on the space of continuous functions with the sup-norm that is supported on functions of Lip 1/2 - ε for any 0 < ε < 1/2 and that satisfies the conditions of independent increments and normality of Einstein’s model for Brownian motion. Wiener’s rigorous development of Brownian motion was done prior to Kolmogorov’s systematic description of stochastic processes.

Wiener’s interest in stochastic process and ergodic theory led him to consider stationary processes. Since these processes are not in $L^2(\mathbb{R}^1)$ and hence not amenable to Fourier analysis he invented generalized harmonic analysis to study them. Problems in generalized harmonic analysis required new, deep, Tauberian theorems, which themselves required new results in Fourier series, all of which he discovered. Correlation functions are fundamental in generalized harmonic analysis. These were to be his foremost probe in the analysis of random phenomena in biology and communication theory in the years to come. In the mid-1930s he teamed up with the electrical engineer Y. W. Lee to essentially create statistical communication theory. This is only a survey of some of Wiener’s mathematical contributions prior to World War II. In 1933 he was elected to the National Academy of Sciences.

During World War II Wiener worked with J. Bigelow on predicting the future position of aircraft, so that anti-aircraft guns would know where to aim. This led to his work in prediction theory and the closely related questions of filtering and extrapolation of stochastic processes. Moreover, beginning with his work with Lee, Wiener was also interested in constructing electrical devices to perform the operations he was analyzing. He had developed an interest in computers, stimulated by Vannevar Bush’s work on constructing a machine to solve differential equations. He and Bigelow actually built a device to carry out the prediction to be used in anti-aircraft aiming. (In fact Wiener’s theory was not practical. The amount of time an airplane could be observed was not long enough to make his brilliant theory superior to the simple deterministic model then being employed. It is significant that he, himself, pointed this out in his final report to the National Defense Research Committee.)

After the war Wiener devoted himself to applying his formidable mathematical talents to problems in biology, although not exclusively. He still produced some very good mathematics, perhaps the best being his papers with Masani on the prediction theory of multivariate stochastic processes. But he achieved fame and wide recognition outside the mathematical community by naming and popularizing cybernetics, “the science of control and communication in the animal and the machine”. Wiener didn’t leap from pure analysis to physiology. In 1933 he “became a regular participant in an interdisciplinary seminar on scientific method...the Philosophy of Science Club” ([4], p. 197), conducted by Arturo Rosenbleuth, a neurophysiologist, who was working at the Harvard Medical School. He wrote several papers with Rosenbleuth immediately following the war and dedicated his book Cybernetics [5] to him. As Masani also reports ([4], p. 218), Wiener first encountered Warren McCulloch at the neurophysiological meeting in New York in 1942 where Rosenbleuth was presenting their joint work with Bigelow on teleology. (Divulging the secret behind the relationship between Wiener and McCulloch is the height of Conway and Siegelman’s investigative reporting. We’ll get back to this later).

I called cybernetics the “science of control and communication in the animal and the machine” because this is the subtitle of Wiener’s famous book [5], published in 1948. Actually, it is not clear to me what the definition of cybernetics really is, or whether it is a science. In his 1956 book [1], W. R. Ashby states that “Cybernetics is the general study of mechanism from the standpoint of functionality and
Wiener was a great mathematical analyst. He was a great deal to say. But like Einstein's, his moral achievement were pretty much over. His years of startling mathematical pronouncements were noticed only because of his previous achievements.

People's attitudes in the United States right after World War II were very different from what they are now. Scientists and scientific achievement were held in very high regard. It was not only the atomic bomb that won the war but sonar, radar, and the brilliance to crack the enemy's codes. For a while it was widely believed that taking an impersonal scientific approach was a better way to deal with society's problems than by leaving them in the hands of self-interested, indebted politicians (as though scientists couldn't also manage to be both self-interested and indebted). Wiener's book came upon this scene with a synthesis of all activity based on the ideas of message, noise, and control. Without diminishing the significance of his vision, it is fair to say that his was the next "new thing". His ideas were immediately extolled by the influential newsweeklies.

Another aspect of the Wiener phenomenon that added to his popularity and underscored his sincerity was his morality. At the same time he extolled the enormous potential of science to do good, he also lamented its more likely uses for destruction. In the preface to Cybernetics he wrote:

Those of us who have contributed to the new science of Cybernetics thus stand in a moral position which is, to say the least, not very comfortable. We have contributed to the initiation of a new science which...embraces technical developments with great possibilities for good and evil. We can only hand it over to the world that exists about us, and this is the world of Belsen and Hiroshima. We do not even have the choice of suppressing these new technical developments. They belong to the age, and the most we can do by suppression is to put the development of the subject into the hands of the most irresponsible and most venal of our engineers. The best we can do is to see that a large public understands the trend and bearings of the present work and confine our personal efforts to those fields...most remote from war and exploitation. As we have seen, there are those who hope that the good of a better understanding of man and society which is offered by this new field of work may anticipate and outweigh the incidental contributions we are making to the concentration of power (which is always concentrated, by its very conditions of existence, in the hands of the most unscrupulous). I write...
in 1947, and I am compelled to say that it is a very slight hope.

The passages I put in italics are deleted from this quotation on page 181 of Conway and Siegelman’s book. The first one is replaced by ellipses. The second, more significant statement is not. I don’t understand why, because, from other parts of the book, one gets the impression that Conway and Siegelman were attracted to Wiener in part by his political positions.

Warren McCulloch’s educational background was much like Wiener’s. He studied philosophy and mathematics and had a degree in medicine. (Unlike Wiener he was also somewhat of a bohemian.) “He [McCulloch] became a serious student of mathematical logic, and investigated the mathematical-aspects of schizophrenia and psychopathia while serving at the Rockland Hospital for the insane” ([4], p. 218). In 1942, the year McCulloch met Wiener, he was working with Walter Pitts, trying to understand the organization of the cortex of the brain. Pitts was a self-taught “genius”, who had had a poor, troubled childhood in Detroit but who nevertheless attracted Bertrand Russell’s attention and was encouraged by Russell to study mathematical logic. In 1942 Pitts was twenty years old. Pitts went to MIT in 1943 to study with Wiener. As Masani points out ([4], p. 219), “Both McCulloch and Pitts played an absolutely positive role in the evolution of Wiener’s ideas in neurophysiology, especially on the problems of logical manipulation, Gestalt or pattern-recognition, gating, brain rhythms and sensory prosthesis.”

Wiener wrote two papers with Pitts (along with Rosenbleuth and J. Garcia Ramos) and none with McCulloch. Nonetheless, Conway and Siegelman write, “McCulloch had promoted Wiener’s theories and ideas on cybernetics with almost as much enthusiasm as Wiener himself” (p. 214). On the same page they also report that [in 1950 (or maybe 1951)], “Jerome Wiesner, who was now head of the Rad Lab [Radiation Laboratory at MIT], with Wiener’s blessings, invited McCulloch to come to Cambridge to head up a major new research effort on the brain and its cybernetic connections.” McCulloch did come to MIT, but before he did, Wiener abruptly broke off his relationship with him and Pitts and didn’t even mention them in his otherwise detailed memoir [6].

During the time I was Wiener’s graduate student assistant (1961-1963) I asked a faculty member, I don’t remember who, why there was a conflict between Wiener and McCulloch. He said it had something to do with McCulloch having had an affair with Wiener’s daughter. Beginning on page 225, Conway and Siegelman paraphrase a recollection of Jerome Lettvin, who as a medical doctor at Boston City Hospital had persuaded Pitts to study mathematics at MIT. Lettvin recalled that, during a visit with Rosenbleuth in Mexico City in 1960, Rosenbleuth told him “that Margaret told Wiener [in a letter written to him in 1951] while he was visiting Rosenbleuth in Mexico... that the boys in McCulloch’s group—Wiener’s boys—had seduced his elder daughter [Barbara] during her stay at the McCulloch home in Chicago four years earlier... Margaret alleged that not one but ‘more than one’ of the boys had seduced the chaste nineteen-year-old during her first foray away from home and the protected environment of her boarding school.” Conway and Siegelman present corroborating evidence that leaves little doubt that this story is true, although they do not claim to have seen the actual letter.

Margaret gets bashed in this book. Apparently she was enamored of Adolf Hitler long after a reasonable person of German descent should have been. Also she was very troubled by her daughters’ sexuality and made many apparently false accusations about the girls’ behavior. I first met Margaret and Wiener together in 1959 in Los Angeles. Of course I was very young then and never had more than a tangential relationship with them. Nevertheless they seemed like a loving couple to me. Wiener wrote, in 1953, in the dedication to [7], “To my wife under whose gentle tutelage I first knew freedom” ([4], p. 94).

My new wife and I had dinner with Margaret in Cambridge a couple of times in the year after Wiener’s death. She gave me his academic gown to wear when I received my Ph.D. in 1965. I was fond of her, and I think Masani was also. He filled me in on what she was doing whenever we saw each other at meetings. Masani says nothing disparaging about Margaret in his biography of Wiener.

I knew Wiener well enough to know that he was fiercely loyal and really very manly, despite his awkward appearance. He would have been devastated by the way Margaret is treated in this book and fighting mad. I could have lived without knowing all the dirt on my hero’s wife. I’m glad Wiener never had to read this book.

To be fair to the authors, their gossip is not spurious. They view the breakup of the research team of Wiener, Pitts, and McCulloch as a primary reason that cybernetics did not achieve the great success that they think was its destiny. But here I think that they are guilty of a misunderstanding of the nature of mathematical research that is prevalent among the nonmathematical public. That is, that mathematics, and perhaps scientific research in general, advances by the achievements of a very few extremely gifted individuals—people who are so deep that even their colleagues don’t understand them. This is the viewpoint of the movie Good Will Hunting and the play Proof. In this view Wiener’s separation from Pitts and McCulloch doomed their effort in using the principles of
cybernetics to explain the workings of the brain. Of course, collaboration with Wiener would have been helpful. But McCulloch and Pitts were not dummies. They were tackling a problem that is still very far from a solution. There was enormous enthusiasm in the 1950s and 1960s for the revolutionary changes that would be brought about, not only by cybernetics, but also by artificial intelligence. Progress was made, and work is continuing. But the mysteries McCulloch and Pitts were trying to answer are amongst the deepest that exist.

John von Neumann was also involved in this research. Conway and Siegelman point out that he, Wiener, McCulloch, and about twenty others, including Margaret Mead, met in several closed conferences sponsored by the Josiah Macy Foundation to explore questions "at the junction between psychology and brain science" (p. 131). The conference series ran from 1942 to 1953. Its name evolved to "Feedback Mechanisms and Circular Systems in Biology and Social Sciences Meeting", To Wiener's delight, after his book appeared, the group was happy to simply use the name "Cybernetics" to describe itself and its proceedings. But it seems clear that the goal of describing how the brain functions was too ambitious. Starting on page 243 Masani reprints a six-page letter that von Neumann wrote to Wiener in 1946 that points this out and suggests that perhaps they should first try to understand how viruses function.

Conway and Siegelman present a lot of interesting history about research funding in this period. A great deal of money was being pumped into artificial intelligence and very little into cybernetics. This, too, they blame for the absence of a cybernetics revolution. In fact, they are so convinced that great things would have occurred if only cybernetics were vigorously pursued that they deal with the absence of substantial results from the Soviet Union, where after initial hostility the government strongly supported cybernetics research, by saying that, "In the end, cybernetics did not give the Soviet Union the winning hand in the Cold War. ...the socialist system's creed of centralized planning and rigid, top-down, authoritarian rule ran counter to the most basic principles of self-governing cybernetic systems" (p. 331). Rather than finding excuses for the limited advances resulting from cybernetics, perhaps the authors might have recognized that the translation of mathematical results into concrete social advances takes a very long time and follows devious paths and that it is impossible to predict which discoveries will eventually have a significant effect on society.

Ten years before Masani's biography of Wiener appeared, Steve Heims [2] wrote a joint biography of Wiener and von Neumann, with an interesting, but I think fallacious, hypothesis (see [3]). That is, that the political positions these men took were reflected in the nature of the mathematics that they created. (In Cold War terminology, Wiener was a dove and von Neumann a hawk.) Heims' book concentrates on Wiener as a man opposed to militarism and powerful institutions, Masani's book on Wiener the mathematician and philosopher, and the book under review on Wiener's work in cybernetics. I think that a mathematician who is unfamiliar with Wiener's life and work would most enjoy Masani's book and Wiener's autobiographies [7, 6]. But both Heims' book and this book are well worth reading. Conway and Siegelman have dug up a lot of interesting material on the early days of cybernetics, and they certainly capture the enthusiasm of the early years of our information age. They also uncover many facts about Wiener's life that were not commonly known.

I think that the next biography of Wiener should be written by an historian of the mid-twentieth century who would study Norbert Wiener along with other scientific public intellectuals, like Linus Pauling, Leo Szilard, Benjamin Spock, and Phillip Morrison. I admire these figures because they spoke out against militarism. However, the other side had equally eloquent spokesmen, such as Edward Teller and John von Neumann. Why are scientists absent from public discourse today? It seems that the only people we read about, other than politicians and entertainers, are those who either make, lose, or steal a great deal of money. To compare public discourse today with that during Wiener's prime is to see how drowned our society is by materialism and superstition.

Rather than end on a discouraging note, let us return to Wiener himself. He was really a wonderful man. This is what he wrote about mathematics in 1933, when he was thirty-nine years old.

Mathematics is a subject worthy of the entire devotion of our lives. We are serving a useful place in the community by our training of engineers, and by our development of the tools of future science and engineering. Perhaps no particular discovery that we make may be used in practice; nevertheless, much of the great bulk of mathematical knowledge will be, and we are contributing to that bulk, as far as lies in us.

Moreover, a clearly framed question which we can not answer is an affront to the dignity of the human race, as a race of thinking beings. Curiosity is a good in itself. We are here but for a day; tomorrow the earth will not know us, and we shall be as though we never were. Let us then master infinity and eternity in the one way open to us;
through the power of the understanding. Knowledge is good with a good which is above usefulness, and ignorance is an evil, and we have enlisted as good soldiers in the army whose enemy is ignorance and whose watchword is Truth. Of the many varieties of truth, mathematical truth does not stand lowest. ([4], p. 341)

There was nothing "dark" about Norbert Wiener's mathematics or his morals.

Acknowledgment: I am pleased to acknowledge many discussions with David Isles which helped me shape my ideas for this review.

References

The Division of Math, Science and Technology of Nova Southeastern University invites applications for two full time, 9.5 month contract, faculty members in the area of mathematics and/or statistics at the assistant professor level, beginning August 2006, subject to budgetary approval. A Ph.D. in statistics or mathematics is required. Teaching responsibilities include 8 courses a year in statistics and/or math. Candidates should exhibit a strong commitment to undergraduate teaching and research. Experience in applied statistics and with online or computer-assisted instruction is a plus.

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NSF Fiscal Year 2007 Budget Request

This article is the 34th in a series of annual reports outlining the president's request to Congress for the budget of the National Science Foundation. Last year's report appeared in the June/July 2005 issue of the Notices, pages 637-41.

The fiscal year 2007 budget request for the National Science Foundation (NSF) is a big disappointment for mathematics. Despite a substantial increase for the NSF, as well as for the Mathematical and Physical Sciences directorate, the Division of Mathematical Sciences (DMS) is slated for an increase of only about 3%, a little bit above the expected inflation rate. What is more, the NSF's "Mathematical Sciences Priority Area", which has boosted the DMS budget over the last several years, is scheduled to end in 2007.

The Bush Administration sent its fiscal year 2007 budget request to Congress in February 2006. One component of the request is the "American Competitiveness Initiative", consisting of substantial increases for the Department of Energy's Office of Science, for the National Institute of Standards and Technology, and for the NSF. The initiative calls for a 7.9% increase for the NSF, to just over US$6 billion. This increase represents a marked shift from FY2006, when the Bush Administration sought only a 2.4% rise for the NSF. The final Congressional appropriation for fiscal 2006 brought the NSF budget up just 2.0% above the fiscal 2005 level.

A preliminary analysis by the American Association for the Advancement of Science finds that, under the terms of the FY2007 request, overall federal funding for research and development would climb by about US$2.6 billion. Even more than that amount would be absorbed as increases for weapons development and space exploration technologies, "leaving declining funding for the remainder of the [research and development] portfolio," the analysis notes. In his State of the Union address, President Bush set out the goal of doubling

Table 1: National Science Foundation (Millions of Dollars)

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</thead>
<tbody>
<tr>
<td>(1) Mathematical Sciences Research Support</td>
<td>$178.8</td>
<td>12.0%</td>
<td>$200.3</td>
<td>0.0%</td>
<td>$200.2</td>
<td>-0.4%</td>
<td>$199.3</td>
<td>3.2%</td>
<td>$205.7</td>
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<tr>
<td>(2) Other Research Support (Note a)</td>
<td>4054.7</td>
<td>5.5%</td>
<td>4277.0</td>
<td>-1.8%</td>
<td>4199.7</td>
<td>2.9%</td>
<td>4323.1</td>
<td>8.7%</td>
<td>4700.7</td>
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<tr>
<td>(3) Education and Human Resources (Note b)</td>
<td>934.9</td>
<td>1.0%</td>
<td>944.1</td>
<td>-10.7%</td>
<td>843.5</td>
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<td>796.7</td>
<td>2.4%</td>
<td>816.2</td>
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<td>(4) Salaries and Expenses (Note c)</td>
<td>201.0</td>
<td>14.7%</td>
<td>230.6</td>
<td>2.9%</td>
<td>237.3</td>
<td>10.4%</td>
<td>262.1</td>
<td>13.5%</td>
<td>297.6</td>
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<tr>
<td>(5) Totals</td>
<td>$5369.3</td>
<td>5.3%</td>
<td>$5652.0</td>
<td>-3.0%</td>
<td>$5480.8</td>
<td>1.8%</td>
<td>$5581.2</td>
<td>7.9%</td>
<td>$6020.2</td>
</tr>
</tbody>
</table>

(6) (1) as a % of the sum of (1) and (2) | 4.22% | 4.97% | 4.55% | 4.41% | 4.19% |
(7) (1) as a % of (5) | 3.33% | 3.54% | 3.65% | 3.57% | 3.42% |

*Tables prepared by Notices staff. Totals may not add up due to rounding. Note a: Support for research and related activities in areas other than the mathematical sciences. Includes scientific research facilities and instrumentation. Note b: Support for education in all fields, including the mathematical sciences. Note c: Administrative expenses of operating the NSF, including the National Science Board and the Office of the Inspector General.*
funding for "critical basic research programs in the physical sciences over the next 10 years". The emphasis on the physical sciences is reflected in the proposed budget for the National Institutes of Health, which under the terms of the request would be flat for the second year in a row.

It appears that the Bush Administration's interpretation of the term "physical sciences" does not include mathematics, as the DMS is slated for only a 3.2% increase. And outside of the NSF, there is less and less grant money available for mathematics research, due in large part to declining budgets in those agencies of the Department of Defense that have mathematics programs. Indeed, nowadays the DMS provides 77% of all federal funding for academic research in the mathematical sciences, up from about 50% a dozen years ago. By contrast, according to Samuel M. Rankin III, director of the AMS Washington Office, the NSF is currently the source of just 40% of the funding for academic research in the physical sciences.

The DMS budget was flat in fiscal 2005 and declined slightly in fiscal 2006, so the modest proposed increase for fiscal 2007 would leave the division with less, in constant dollar terms, than it had three years earlier. The requested increase for the DMS is the lowest among the six divisions within the Mathematical and Physical Sciences (MPS) directorate, which overall has a requested increase of 6.0%. "It looks to me as if the NSF or MPS is in ­tent on bringing DMS back to its historical position, as far as funding goes -t hat is, the lowest-funded division within MPS," Rankin remarked. "To me this is not recognizing current reality—the many contributions the mathematical sciences makes to toward technological innovation and competitiveness. This is surprising, given that innovation and competitiveness are on the minds of Congress and the Administration."

Table 2: Directorate for Mathematical and Physical Sciences (Millions of Dollars)

<table>
<thead>
<tr>
<th></th>
<th>2003 Actual</th>
<th>% of Total</th>
<th>2004 Actual</th>
<th>% of Total</th>
<th>2005 Actual</th>
<th>% of Total</th>
<th>Plan Request</th>
<th>% of Total</th>
<th>2007 Actual</th>
<th>% of Total</th>
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<tr>
<td>(1) Mathematical Sciences</td>
<td>$178.8</td>
<td>17.2%</td>
<td>$200.3</td>
<td>18.3%</td>
<td>$200.2</td>
<td>18.7%</td>
<td>$199.3</td>
<td>18.4%</td>
<td>$205.7</td>
<td>17.9%</td>
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<tr>
<td>(2) Astronomical Sciences</td>
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<td>18.0%</td>
<td>$195.1</td>
<td>18.2%</td>
<td>$199.6</td>
<td>18.4%</td>
<td>$215.1</td>
<td>18.7%</td>
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<tr>
<td>(3) Physics</td>
<td>$224.5</td>
<td>21.6%</td>
<td>$227.9</td>
<td>20.9%</td>
<td>$224.9</td>
<td>21.0%</td>
<td>$233.1</td>
<td>21.5%</td>
<td>$248.5</td>
<td>21.6%</td>
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<td>(4) Chemistry</td>
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<td>17.4%</td>
<td>$185.5</td>
<td>17.0%</td>
<td>$179.3</td>
<td>16.8%</td>
<td>$180.8</td>
<td>16.7%</td>
<td>$191.1</td>
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<td>$250.6</td>
<td>23.0%</td>
<td>$240.1</td>
<td>22.8%</td>
<td>$249.2</td>
<td>22.4%</td>
<td>$257.4</td>
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<td>(6) Office of Multidisciplinary Activities</td>
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<td>$31.1</td>
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<td>2.7%</td>
<td>$32.4</td>
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<tr>
<td>(7) Totals</td>
<td>$1040.7</td>
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<td>$1091.6</td>
<td>100.0%</td>
<td>$1069.4</td>
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<td>$1085.4</td>
<td>100.0%</td>
<td>$1150.3</td>
<td>100.0%</td>
</tr>
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</table>

Table 3: Compilation of NSF Budget, 2001-2007 (Millions of Dollars)

<table>
<thead>
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<tr>
<td>(1) Mathematical Sciences Research Support</td>
<td>$121.4</td>
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<td>$178.8</td>
<td>$200.3</td>
<td>$200.2</td>
<td>$199.3</td>
<td>$205.7</td>
<td>64.9%</td>
<td>69.4%</td>
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<td>Constant Dollars</td>
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<td>(2) Other Research Support (Note a)</td>
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<td>2.6%</td>
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<tr>
<td>Constant Dollars</td>
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<td>$499.8</td>
<td>$431.9</td>
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<td>-3.8%</td>
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<tr>
<td>(4) Salaries and Expenses (Note c)</td>
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<td>$201.0</td>
<td>$230.6</td>
<td>$237.3</td>
<td>$262.1</td>
<td>$297.6</td>
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<td>72.1%</td>
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<tr>
<td>Constant Dollars</td>
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<td>$98.2</td>
<td>$109.2</td>
<td>$122.1</td>
<td>$121.5</td>
<td></td>
<td>24.5%</td>
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<tr>
<td>(5) Totals</td>
<td>$4459.9</td>
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<td>$5369.3</td>
<td>$5652.0</td>
<td>$5480.8</td>
<td>$5581.2</td>
<td>$6020.2</td>
<td>22.9%</td>
<td>35.0%</td>
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<tr>
<td>Constant Dollars</td>
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<td>$2653.8</td>
<td>$2918.1</td>
<td>$2992.0</td>
<td>$2806.3</td>
<td></td>
<td>11.4%</td>
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</tbody>
</table>

Current dollars are converted to constant dollars using the Consumer Price Index (based on prices during 1982-84).

For Notes a, b, and c, see Table 1.
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almost one-third. In fact, one of the original aims of the MSPA was to double the DMS budget. Although the doubling will not have been achieved, the budget request calls for the MSPA to end in FY2007. The NSF budget request document speaks of "mainstreaming" interdisciplinary research projects that were funded through the MSPA. "Investments in formal interdisciplinary partnerships through the MSPA will be redirected to unsolicited proposals and the fundamental mathematical sciences component of the MSPA," the document states. It also says that in fiscal 2007 the DMS intends to increase by about US$5 million its support for core research in order to maintain its proposal success rate of 32% (in 2005, the DMS received 2,172 proposals and funded 687). In addition, the DMS will increase funding for Research Experiences for Undergraduates and for Enhancing the Mathematical Sciences Workforce in the 21st Century (each has a requested increase of US$500,000).

Each year in the spring, the AMS Committee on Science Policy (CSP) meets in Washington, DC. At its last meeting in 2005, the CSP decided to dispense with the customary format for its meeting, which featured a succession of presentations about federal science policy and funding. Instead, the format for the spring 2006 meeting has CSP members fanning out across Capitol Hill to meet with congressional representatives, senators, and staffers to discuss funding for research. The meetings will be arranged by the AMS Washington Office, under Rankin's direction. In its dealings with Congress, the Washington Office has generally pursued a strategy of banding together with other science organizations to speak in a unified voice for strong federal support for NSF overall, rather than seeking favored treatment for the DMS specifically. "However, we will be advocating for a better increase to the DMS budget this year," says Rankin.

—Allyn Jackson
The 2006 JPBM Communications Award was presented at the Joint Mathematics Meetings in San Antonio in January 2006.

The Joint Policy Board for Mathematics (JPBM) established its Communications Award in 1988 to reward and encourage journalists and mathematicians who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences. Presented annually, the award recognizes a significant contribution or accumulated contributions to the public understanding of mathematics, and it is meant to reward lifetime achievement. The award carries a cash prize of US$1,000. The JPBM represents the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics.


The selection committee for the 2006 award consisted of: James Arthur, Carl Cowen, Martin Golubitsky, and Elizabeth Halloran.

The 2006 JPBM Communications Award was presented to SIR ROGER PENROSE. What follows is the award citation, a brief biographical sketch, and the recipient's response to the award.

Citation
The Joint Policy Board for Mathematics presents its 2006 Communications Award to Sir Roger Penrose for the discovery of Penrose tilings, which have captured the public's imagination, and for an extraordinary series of books that brought the subject of consciousness to the public in mathematical terms.

Dr. Penrose has acquired a large public following for eight books he has written. A number of these explore ideas that relate fundamental physics, mathematical logic, and human consciousness. In *The Emperor's New Mind* (1989) and also in later volumes, he has argued that known laws of physics do not constitute a complete system and that human consciousness cannot be explained until a new physical theory of quantum gravity has been devised. These ideas have stimulated broad public debate. They have brought widespread attention to the scientific and philosophical implications of consciousness. The most recent book of Dr. Penrose, *The Road to Reality* (2005), is a bold and broadly conceived attempt to present the techniques of modern mathematics and physics before a general public audience. This year's JPBM Communication Award is a tribute to the way that Dr. Penrose has made the ideas behind high level mathematics accessible to large segments of the general public.

Sir Roger Penrose

MAY 2006

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Biographical Sketch

As a graduate student, Roger Penrose studied mathematics and physics at Cambridge University from the likes of Bondi, Dirac, Hodge, Steen, and Todd. He was awarded his Ph.D. there in algebraic geometry in 1958. After positions at various universities in both England and the United States, he was appointed the Rouse Ball Professor of Mathematics at the University of Oxford in 1973, a position he held until 1998, when he became Emeritus Rouse Ball Professor of Mathematics.

In his research career Penrose has made fundamental and remarkably diverse contributions to both mathematics and physics. Many of these concern the interplay between relativity, geometry, and topology, and are related to the attempt to unify relativity with quantum theory. In 1967, Penrose discovered twistor theory, a beautiful mathematical formalism that combines powerful techniques of algebra and geometry. In 1971, he introduced the theory of spin networks, which later became a part of the geometry of spacetime in loop quantum gravity. In 1974, he discovered what are now known as Penrose tilings, which are formed from two tiles that can only tile the plane aperiodically. Such patterns were later found, quite remarkably, to occur in the arrangement of atoms into quasicrystals.

Penrose has received many awards and honors. He was elected to fellowship in the Royal Society of London (1972) and as a foreign associate of the National Academy of Sciences (1998). He received the Wolf Foundation Prize in Physics (with Stephen Hawking, 1988) and the DeMorgan Medal of the London Mathematical Society (2004). In 1994, he was knighted for his service to science.

Response

It is a deep and unexpected honour, and a great pleasure for me, to receive the JPBM Communications Award for 2006.

I certainly believe in the importance of conveying to the general public, as far as this is possible, something of the real nature of mathematics, not only for its increasing utility across so many areas of importance to modern society, but also for its beauty and for the inner satisfaction that it brings. Perhaps these latter qualities are even more important than the more utilitarian ones; for one cannot really properly understand mathematics without having some kind of appreciation of its aesthetic qualities. Moreover it is a belief (or a faith?) of mine that there are many more out there, among those who claim no appreciation or understanding of mathematics, who actually have within themselves some genuine but unrecognized abilities in this direction.

And it is certainly not just the general public who can stand to gain from clear expositions of mathematical topics. Science in general, and mathematics in particular, have grown to enormous proportions over the years, and over the centuries. Semi-popular expositions which give clear and intuitive accounts of one area of work can be an invaluable aid to others whose expertise may lie in some area of science or mathematics which is far from that being explained. In my own experience, such accounts can have enormous value.

If, as this award seems to imply, I have contributed, in some significant way, to the spreading of scientific or mathematical knowledge and understanding, then I am indeed well pleased. Thank you very much.
At the Joint Mathematics Meetings in San Antonio in January 2006, the Mathematical Association of America (MAA) presented several prizes.

### Gung and Hu Award for Distinguished Service
The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. First given in 1990 the Gung and Hu Award is the successor to the Award for Distinguished Service to Mathematics, awarded since 1962, and has been made possible by the late Charles Y. Hu and his wife, Yueh-Gin Gung. It is worth noting that Hu was not a mathematician but a retired professor of geology. He had such strong feelings about the basic nature of mathematics and its importance in all human endeavors that he felt impelled to contribute generously to our discipline.

**Hyman Bass** of the University of Michigan received the 2006 Gung and Hu Award for “invest[ing] vast energies over several decades to strengthen the mathematical community.” An outstanding researcher and former member of Bourbaki, Bass has made many contributions to the mathematical community. He is a member of the National Academy of Sciences and has served on many NAS committees, including the Mathematical Sciences Education Board, which he chaired. A past president of the AMS, he has served on many Society committees. He has been on the boards of trustees of the Institute for Advanced Study and the Mathematical Sciences Research Institute. In recent years, Bass has turned his attention to improving school mathematics. "Hyman Bass is playing a vital role in bringing the insights of a mathematician to mathematics educators and the insights of a mathematics educator to mathematicians," the citation states. “He is conducting carefully reasoned, incremental, foundational research in order that discussion of educational issues may one day be based on more rigorous scientific findings. His service to mathematics and its teaching and learning at all levels is truly remarkable.”

### Haimo Awards for Teaching
The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions. Deborah Tepper Haimo was president of the MAA during 1991–1992.

The 2005 Haimo Awards were presented to **Jacqueline Dewar** of Loyola Marymount University, **Keith Stroyan** of the University of Iowa, and **Judy Leavitt Walker** of the University of Nebraska at Lincoln.

The award citation for Jacqueline Dewar states, "In her 32 years at Loyola Marymount University, Jackie Dewar’s enthusiasm, extraordinary energy, and clarity of thought have left a deep imprint on students, colleagues, her campus, and a much larger mathematical community.” She profoundly influenced the mathematics curriculum at Loyola Marymount, helping to shape the biomathematics program, the mathematics program for prospective secondary school teachers, and the Master of Arts in Teaching program. Her freshman-level workshop course is credited with improving the retention of mathematics majors. Outside the Loyola Marymount campus, Dewar has been active with..."
in-service programs for teachers, and with the Expanding Your Horizons conferences for middle- and high-school girls. The citation recognized Dewar for “her passionate devotion to the art of teaching.”

“Keith Stroyan’s name is synonymous with innovation in the teaching of calculus,” the award citation states. “In more than 30 years of teaching at the University of Iowa, he has constantly sought ways in which to combine past knowledge with recent discoveries and technology, and to find the mental ‘hooks’ with students’ previous experiences, current interests, and future aspirations.” One of the keys to his success is his careful training of graduate and undergraduate assistants for his courses, in which he inculcates the assistants with good teaching practices. Long before calculus reform projects started receiving grants from the National Science Foundation, Stroyan pioneered the use of computer programs to help students grasp calculus concepts. Then, with several NSF grants, he developed materials to integrate computers into calculus teaching.

“Judy Walker cares deeply about her students,” the prize citation states. “Her students testify that her courses are among the most demanding they ever had, yet consistently praise her ability to guide the direction of a class through questions. Superb at explaining mathematics and communicating the joy of discovery, she is readily available outside of class for special problem sessions, and is in demand as a doctoral thesis advisor.” One of her major innovations at the University of Nebraska was creating a freshman honors seminar for nonmajors called “The Joy of Numbers: Search for the Big Primes”, which she also adapted to serve elementary and middle school teachers. In 1997, Walker and a colleague launched ALL GIRLS/ALL MATH, a program to encourage high-school girls to pursue mathematics. She also started the Nebraska Conference for Undergraduate Women in Mathematics, which over its first seven years attracted 800 participants. The citation praised her “dynamic leadership and passionate commitment to teaching mathematics”.

Beckenbach Book Prize
The Beckenbach Book Prize, presented since 1982, is named for the late Edwin Beckenbach, a longtime leader in the MAA publications program and a professor of mathematics at the University of California, Los Angeles. The prize is awarded to an author of a distinguished, innovative book published by the MAA.

Arthur Benjamin, professor of mathematics at Harvey Mudd College and Jennifer Quinn, executive director of the Association for Women in Mathematics, received the 2006 Beckenbach Book Prize for their book Proofs that Really Count: The Art of Combinatorial Proof. “Few mathematicians are immune to the limpid charms of a clever counting argument,” the prize citation states. The book by Benjamin and Quinn “will charm you over and over again. The authors claim that counting arguments make the most compelling, natural, and memorable proofs. It is hard to disagree with them after dipping into this lovely volume...Proofs That Really Count illustrates in a magical way the pervasiveness and power of counting techniques throughout mathematics. It is one of those rare books that will appeal to the mathematical professional and seduce the neophyte.”

Chauvenet Prize
The Chauvenet Prize recognizes a member of the MAA who has written an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy.

The 2006 Chauvenet Prize was awarded to Florian Pfender and Gunter M. Ziegler, both of the Technische Universitat Berlin, for their article “Kissing Numbers, Sphere Packings, and Some Unexpected Proofs” (Notices, September 2004, pages 873–883). According to the citation, this “lucid and beautifully illustrated paper” discusses the history and progress of three classical packing problems in various dimensions: the kissing number problem, the sphere packing problem, and the lattice packing problem. The immediate backdrop for this paper is Thomas Hales’s controversial solution in 1998 to Kepler’s Conjecture, which is the general sphere packing problem for dimension three. Pfender and Ziegler’s paper clarifies the differences among the problems while also shedding light on recent developments in the kissing number problem. The authors “strip away all but the essentials so that novices may appreciate the power and beauty of these new approaches to finding answers to the kissing number problem.”

Certificates of Meritorious Service
Each year the MAA presents Certificates of Meritorious Service to honor outstanding service to sections of the MAA. Those honored in 2006 are: Kay Somers of Moravian College, Eastern Pennsylvania-Delaware Section; Calvin (Cal) Van Niewaal of Coe College, Iowa Section; Alan Tucker of Stony Brook University, Metropolitan New York Section; Ivy Knoshaug of Bemidji State University, North Central Section; Marjorie Enneking of Portland State University, Pacific Northwest Section; and William Yslas Velz of the University of Arizona, Southwestern Section.

—From MAA announcements
The Association for Women in Mathematics (AWM) presented two awards at the Joint Mathematics Meetings in San Antonio in January 2006.

**Louise Hay Award**
The Louise Hay Award for Contributions to Mathematics Education was established in 1990 to honor the memory of Louise Hay, who was widely recognized for her contributions to mathematical logic and for her devotion to students.

The 2006 Hay Award was presented to PATRICIA CLARK KENSCHAFT of Montclair State University. The award was made "in recognition of her long career of dedicated service to mathematics and mathematics education". The prize citation states that Kenschaft has "found her true calling in not only teaching university-level mathematics, but also in writing about, speaking about, and working for mathematics and mathematics education in the areas of K-12 education, the environment, affirmative action and equity, and public awareness of the importance of mathematics in society." The citation also mentions the wide range of books and articles Kenschaft has written and edited, including her latest book, *Change is Possible: Stories of Women and Minorities in Mathematics* (AMS, 2005).

She founded and directed PRIMES, the Project for Resourceful Instruction of Mathematics in the Elementary School, which was supported by fourteen Eisenhower grants and served teachers in nine urban and suburban schools. As a result of her work on this project, Kenschaft developed the book *Math Power: How to Help Your Child Love Mathematics Even If You Don't* (Addison-Wesley-Longman, 1997).

The award citation ends with this quotation from one of Kenschaft's colleagues: "She deserves to be recognized for her decades of dedication to mathematics and math education and for her innovative and unique contributions in these areas. In particular, her special attention to children and their parents, women, minorities, and the environment, all with respect to mathematics, have been and continue to be of benefit for the mathematical community and our society as a whole."

**Schafer Prize**
The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents.

ALEXANDRA OVESTKY of Princeton University was named the 2006 winner of the Schafer Prize. A senior and a Goldwater scholar, Ovetsky is the recipient of the Princeton mathematics department's Andrew H. Brown Prize for outstanding research in mathematics by a junior. The Brown Prize was given for Ovetsky's proof of a generalization of a well-known theorem of Claude Shannon from 1948. She co-authored a paper, "Surreal dimensions", which has appeared in *Advances in Applied Mathematics*. In the summer of 2004 she participated in the Research Experiences for Undergraduates program at the University of Minnesota, Duluth, where she wrote a paper on well-covered graphs. The following summer she participated in a program at the National Security Agency, and her work there is being published internally by the NSA. The prize citation quoted one of Ovetsky's mentors as saying, "She already has the research capabilities of an advanced graduate student or junior faculty member."

Two other senior mathematics majors were also recognized by AWM. ALLISON BISHOP of Princeton University was named runner-up for the Schafer Prize, and ELLEN GASPAROVIC of the College of the Holy Cross received an honorable mention.

—from AWM announcements
The Frederic Esser Nemmers Prize in Mathematics

$150,000 Award presented by Northwestern University

Previous recipients:

2004
Mikhail Gromov

2002
Yakov G. Sinai

2000
Edward Witten

1998
John H. Conway

1996
Joseph B. Keller

1994
Yuri I. Manin

The Eighth Nemmers Prize in Mathematics will be awarded in 2008 with nominations due by December 1, 2007. For further information, contact:

nemmers@northwestern.edu

or

Secretary Selection Committee for the Nemmers Prizes:
Office of the Provost
Northwestern University
633 Clark Street
Evanston, Illinois
60208-1119
U.S.A.

www.northwestern.edu/provost/awards/nemmers
Avila, Morel, and Payne Named Clay Research Fellows

The Clay Mathematics Institute (CMI) has announced the appointment of three Research Fellows: Artur Avila of the Centre Nationale de la Recherche Scientifique (CNRS), Sophie Morel of Université de Paris Sud, and Sam Payne of the University of Michigan. They were selected for their research achievements and their potential to make significant future contributions to the field.

Artur Avila, born in 1979, received his Ph.D. in 2001 at the Instituto Nacional de Matematica Pura e Aplicada (IMPA) in Rio de Janeiro, Brazil, under the direction of Welington de Melo. In his thesis Avila generalized the regular or stochastic dichotomy from the quadratic family to any nontrivial family of real analytic unimodal maps. Since then he has made numerous outstanding contributions to one-dimensional and holomorphic dynamics, spectral theory of the Schrödinger operator, and ergodic theory of interval exchange transformations and the associated Teichmüller flow.

Sophie Morel, born in 1979, is completing her Ph.D. at the Université Paris-Sud, Orsay, under the direction of Gérard Laumon. In her thesis she develops a theory of weight truncation on varieties over finite fields, with which she derives a simple description of the intersection complexes on the Baily-Borel compactifications of certain Shimura varieties over finite fields. From this she obtains a formula for the trace of the Frobenius endomorphism on the Euler characteristic of the intersection cohomology.

Sam Payne, born in 1978, is completing his Ph.D. at the University of Michigan under the direction of William Fulton. His thesis gives a surprising and simple construction of complete toric varieties on which there are no nontrivial equivariant bundles of rank two. In other work, Payne has given counterexamples to conjectures of Fujino and of Hibi (with Mircea Mustata), as well as a complete, elegant description of the equivariant Chow cohomology of toric varieties: it is the ring of integral piecewise linear polynomial functions.

Current Clay Research Fellows include Daniel Biss, Maria Chudnovsky, Ben Green, Sergei Gukov, Bo'az Klartag, Ciprian Manolescu, Maryam Mirzakhani, David Speyer, András Vasy, and Akshay Venkatesh.

Meckes Receives AIM Five-Year Fellowship

Elizabeth Meckes of Stanford University has been named the recipient of the 2006 American Institute of Mathematics (AIM) Five-Year Fellowship.

Meckes is completing her Ph.D. thesis at Stanford under the direction of Persi Diaconis. Her research interests include analysis, convex geometry, and probability theory. She has developed a new infinitesimal version of Stein's method of exchangeable pairs, which she has used in studying random matrices and eigenfunctions of the Laplacian on certain Riemannian manifolds. In addition to her thesis work, she has coauthored papers on convex geometry and on Poisson approximation. She earned a B.S. from Case Western Reserve University in 2001 and has been the recipient of a Goldwater Fellowship.

The runners-up for the AIM Fellowship are Alireza Golsefidy (Yale University), Richard Kent (University of Texas, Austin), Abhinav Kumar (Harvard University), and Benjamin Schmidt (University of Michigan, Ann Arbor).

Barenblatt Receives Timoshenko Medal

Grigory I. Barenblatt of the University of California, Berkeley, has received the Timoshenko Medal of the American Society of Mechanical Engineers (ASME). He was honored "for seminal contributions to nearly every area of solid and fluid mechanics, including fracture mechanics, turbulence, stratified flows, flames, flow in porous media, and the theory and application of intermediate asymptotics."

The Timoshenko Medal was established in 1957 and is conferred in recognition of distinguished contributions to the field of applied mechanics. Instituted by the Applied Mechanics Division of ASME, it honors Stephen P. Timoshenko, world-renowned authority in the field, and it commemorates his contributions as an author and teacher.
National Academy of Engineering Elections

The National Academy of Engineering (NAE) has announced the election of seventy-six new members and nine foreign associates, including six whose work involves the mathematical sciences. Their names, institutions, and the research for which they were elected follow.

Egon Balas, Carnegie Mellon University, for contributions to integer programming and its applications to the scheduling and planning of industrial facilities; Manuel Blum, Carnegie Mellon University, for contributions to abstract complexity theory, inductive inference, cryptographic protocols, and the theory and applications of program checkers; Leslie Greengard, Courant Institute of Mathematical Sciences, New York University, for work on the development of algorithms and software for fast multipole methods; Alvy Ray Smith of Seattle, a consultant, for the development of digital imaging, compositing, and painting that have led to fundamental changes in the graphic arts and motion picture industries; and Vladimir N. Vapnik, NEC Laboratories, for insights into the fundamental complexities of learning and for inventing practical and widely applied machine-learning algorithms. Elected as a foreign associate was Charles Anthony Richard Hoare, Microsoft Research, Cambridge, United Kingdom, for fundamental contributions to computer science in the areas of algorithms, operating systems, and programming languages.

—From an NAE announcement
Mathematics Opportunities

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period, in intervals not shorter than three consecutive months. The Research Instructorship option provides a combination of full-time and half-time support over a period of three academic years, usually one academic year full-time and two academic years half-time. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The program solicitation and deadline for the 2006 fellowships will be available in July 2006; see the DMS website, http://www.nsf.gov/div/index.jsp?div=DMS.

—From an NSF announcement

Enhancing the Mathematical Sciences Workforce in the Twenty-First Century

In an effort to increase the number of U.S. citizens, nationals, and permanent residents who are well prepared in the mathematical sciences and who pursue careers in the mathematical sciences and other scientific disciplines, the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has instituted a program titled Enhancing the Mathematical Sciences Workforce in the Twenty-First Century. This program builds on the Vertical Integration of Research and Education (VIGRE) program and includes a broadened VIGRE activity, as well as additional components for Research Training Groups in the Mathematical Sciences (RTG) and for Mentoring through Critical Transition Points (MCTP) in the Mathematical Sciences.

VIGRE grants are designed to allow departments in the mathematical sciences to carry out innovative educational programs in which research and education are integrated and in which undergraduates, graduate students, postdoctoral fellows, and faculty are mutually supportive. Integrating research and education for graduate students and postdoctoral associates, involving undergraduates in substantial learning by discovery, and developing a team approach are keys to successful VIGRE projects. VIGRE student and postdoctoral associates and their mentors may participate in international research and education collaborative activities, including activities in other countries that are integrated into and that benefit the overall VIGRE program at the institution.

The DMS expects to make nine to fifteen awards under this program in 2006. The deadline for proposals is June 6, 2006. For more information about the program and all of its components, see the website http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf05595.

—From an NSF announcement

Call for Nominations for the ICTP/IMU Ramanujan Prize

The Abdus Salam International Centre for Theoretical Physics (ICTP), in conjunction with the International Mathematical Union (IMU), is seeking nominations for the 2006 Ramanujan Prize for Young Mathematicians from Developing Countries. Researchers under forty-five years old who work in any branch of the mathematical sciences are eligible. The prize carries a cash award of US$10,000 and a travel and subsistence allowance to deliver a prize lecture at the ICTP. The deadline for receipt of nominations for the 2006 prize is July 31, 2006. For further information, see http://www.ictp.trieste.it/~sci_info/awards/Ramanujan/Ramanujan.html.

—From an ICTP announcement
Inside the AMS

AMS Archives Journals with Portico

The AMS has begun electronic archiving of eleven of its journals with Portico, a long-term archive of digital scholarly e-journals.

Portico began in 2002 as an outgrowth of JSTOR. The initial impetus was the need for a sustainable model for electronic archiving; JSTOR, by contrast, is a service that offers Internet access to journals. After extensive development of appropriate technology as well as many discussions with publishers and librarians, Portico was launched in 2005. It receives support from JSTOR, Ithaka Harbors, the Library of Congress, and the Andrew W. Mellon Foundation.

Portico will provide archival storage for the AMS journals and will also migrate the journals to upgraded systems as needed. Other participating publishers are Berkeley Electronic Press, Elsevier, John Wiley & Sons, Symposium Journals, and United Kingdom Serials Group. For more information on Portico, visit the website http://www.portico.org.

—Allyn Jackson

Department Chairs Workshop

The AMS hosted a one-day workshop for mathematical sciences department chairs at the 2006 Joint Mathematics Meetings in San Antonio, Texas. This year’s workshop focused on a number of areas of importance to department chairs, including: utilizing resources in tight budget environments, departmental assessment and long-range planning, evaluation and development of faculty and staff, and department engagement within and outside the institution. Over thirty-five department chairs and leaders came together to share ideas and experiences in a form of department chair therapy, thus creating an environment that enabled attendees to address departmental matters from new perspectives.

Workshop leaders included: Krishnaswami Alladi, chair at the University of Florida; Deanna Caveny, chair at College of Charleston; David Manderscheid, chair at the University of Iowa; and Peter March, chair at the Ohio State University.

The Department Chairs Workshop is an annual event hosted by the AMS prior to the start of the Joint Meetings. Workshop sessions have focused on a range of issues facing departments today, including personnel issues (staff and faculty), long-range planning, promotion and tenure, budget management, assessments, outreach, and junior faculty development.

Those interested in attending a future workshop should look for registration information sent out in advance of the Joint Meetings or contact the AMS Washington Office at amsdc@ams.org.

—AMS Washington Office

Correction

The article “The Incompleteness Theorem” by Martin Davis in the April 2006 Notices included an error on the last page. The sentence “For $A \subseteq V(G)$ we write $\mathcal{G}_A = \{y \mid \exists x \in A$ say that $(x, y)$ is an edge of $G\}$.” should have read “For $A \subseteq V(G)$ we write $\mathcal{G}_A = \{y \mid \exists x \in A$ such that $(x, y)$ is an edge of $G\}$.” The word “say” was erroneously substituted for the word “such”.

—Sandy Frost

Deaths of AMS Members

ADHIR K. Basu of Calcutta University, died in April 2005. Born on September 10, 1941, he was a member of the Society for 25 years.

JAAK LOHRIUS, research associate, Tartu University, Estonia, died on February 23, 2006. Born on September 28, 1937, he was a member of the Society for 8 years.

PERE RODRIGUEZ Membreu, assistant professor, University of Barcelona, died in July 2005. Born on November 19, 1955, he was a member of the Society for 15 years.

CHANDRA S. SHARMA, retired, from London, England, died in December 2005. Born on June 17, 1933, he was a member of the Society for 30 years.

JAMES G. WENDEL, retired, from Portland, OR, died on January 16, 2006. Born on April 18, 1922, he was a member of the Society for 60 years.

BENJAMIN YANDELL, a writer from Pasadena, CA, died on August 25, 2004. Born on March 16, 1951, he was a member of the Society for 5 years.
The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices
The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people’s mathematics research.

The managing editor is the person to whom to send items for “Mathematics People”, “Mathematics Opportunities”, “For Your Information”, “Reference and Book List”, and “Mathematics Calendar”. Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.ou.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines
April 7, 2006: Proposals for 2007 NSF-CBMS Regional Conferences.

See the CBMS website, http://www.cbmsweb.org/NSF/2007_call.htm, or contact: Conference Board of the Mathematical Sciences, 1529 Eighteenth Street, NW, Washington, DC 20036; telephone: 202-293-1170; fax: 202-293-3412; email: lkolbe@maa.org or rosier@georgetown.edu.


May 1, 2006: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@math.umd.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

May 31, 2006: Registration for the Thirteenth International Mathematics Competition for University Students (IMC). See the website http://www.imc-math.org or contact John E. Jayne, Department of Mathematics,

Where to Find It
A brief index to information that appears in this and previous issues of the Notices.

AMS Bylaws—November 2005, p. 1239
AMS Email Addresses—February 2006, p. 251
AMS Ethical Guidelines—June/July 2004, p. 675
AMS Officers 2005 and 2006 (Council, Executive Committee, Publications Committees, Board of Trustees)—May 2005, p. 604
AMS Officers and Committee Members—October 2005, p. 1073
Conference Board of the Mathematical Sciences—September 2005, p. 892
Information for Notices Authors—June/July 2005, p. 660
Mathematics Research Institutes Contact Information—August 2005, p. 770
National Science Board—January 2006, p. 62
NRC Board on Mathematical Sciences and Their Applications—March 2006, p. 369
NRC Mathematical Sciences Education Board—April 2006, p. 488
NSF Mathematical and Physical Sciences Advisory Committee—February 2006, p. 255
Program Officers for Federal Funding Agencies—October 2005, p. 1069 (DoD, DoE); November 2005, p. 1223 (NSF)
Stipends for Study and Travel—September 2005, p. 900
University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone +44-20-7679 7322; fax +44-20-7419 2812; email: j.jayne@imc-math.org.

June 1, 2006: Applications for fall program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See the website http://www7.nationalacademies.org/policyfellows, or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 5th Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.


October 1, 2006: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@math.umd.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years; though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers’ attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

* Added to “Book List” since the list’s last appearance.


Gödel’s Theorem: An Incomplete Guide to Its Use and Abuse, by Torkel


mathjobs.org,
the automated job application database sponsored by the AMS
visit it at: www.mathjobs.org

+ free for applicants
+ $300 per year for employers

mathjobs.org offers a paperless application process
for applicants & employers in mathematics.

registered applicants can:

> Make printable AMS coversheets
> Create their own portfolio of application documents
> Make applications online to participating employers
> Choose to make a cover sheet viewable by all registered employers

registered employers can:

> Post up to seven job ads
> Choose to receive applications electronically or register for
advertising only
> Search for and sort additional applicants in the database
> Make the application process as electronic as desired
> Manage applicant information and correspondence quickly and easily
(send email to batches of applicants; download data for use in
creating labels, merge letters, or your own database)
> Choose among many electronic functions and customizable features
(for instance, give faculty members limited or full access to the
applications)

CONTACT:
Membership and Programs Department
American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294 USA
telephone: 800.321.4267, ext. 4105
e-mail: mathjobs@ams.org
American Mathematical Society—Contributions

Dear Friends and Colleagues,

During 2005 your generous support helped the Society and our profession in many ways. I thank each of you for that support.

The Young Scholars program is in its sixth year, supporting summer workshops for talented high school students—the future of our profession. We are building an endowment, the Epsilon Fund, to support this program far into the future, and we hope to reach our goal of two million dollars over the next few years. Supporting such programs is important for mathematics.

The Centennial Fellowships play a key role for outstanding young mathematicians at the formative stages of their careers, from three to twelve years beyond the degree. These fellowships are funded by contributions from mathematicians throughout the world.

We use contributions to the General Fund to support all of our activities, including survey work, public awareness, and outreach to mathematicians in the developing world.

Your generosity allows the Society to carry out all these programs and shows that mathematicians care deeply about our profession. Thank you for that expression of caring.

John H. Ewing

Thomas S. Fiske Society

The Executive Committee and Board of Trustees have established the Thomas S. Fiske Society to honor those who have made provisions for the AMS in their estate plans. For further information contact the Development Office at 800-321-4AMS, or development@ams.org.

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The American Mathematical Society welcomes gifts made in memory or honor of members of the mathematical community or others. Unless directed toward a special fund or program, such gifts are used to support the general mission of the Society.

Gifts were made in memory of the following individuals:

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Many interesting subsets of infinite Coxeter groups, and algorithms involved in compu-
tation within those groups, are related to automata or finite state machines, as the article
by Paul Gunnells in this issue demonstrates. One automaton that arises is that which generates elements of the group one by one, the ShortLex automaton, as proven by Brigitte Brink and Bob Howlett in a classic paper many years ago. Others, more
conjectural, are those which seem to describe the Kazhdan-Lusztig cells of an arbitrary Coxeter group. The cover illustrates both of these types for the Coxeter group with Coxeter numbers 2, 3, 7. In the background the alcoves are colored in pastels according to the state of the ShortLex automaton they are associated to. In darker, primary colors are three left cells of the group, with the repetitiveness characterizing the structures determined by finite state machines singled out.

The relationship between subsets of Coxeter groups and finite state machines is just a small part of a theory not yet clearly perceived in which very complicated infinite patterns of all kinds are described by finite data structures related to the theory of languages. Here the languages are the regular languages, those recognized by finite state machines. Even in Coxeter groups, structures related to the level of complexity in language theory above regular languages, something like context-free languages, appear.

The explicit finite state machines required to draw the Kazhdan-Lusztig cells on the cover were supplied by Gunnells.

—Bill Casselman, Graphics Editor (notices-covers@ams.org)
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This prize was established in 2005 in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his Ph.D. in 1970 from MIT. He was a long-time member of the Institute for Defense Analysis Center for Communications Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is for a paper with the following characteristics: it shall report on novel research in algebra, combinatorics or discrete mathematics and shall have a significant experimental component; and it shall be on a topic which is broadly accessible and shall provide a simple statement of the problem and clear exposition of the work. The US$5,000 prize will be awarded every three years. It is expected that the first award will be made in January 2007.

Nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a complete bibliographic citation for the work that is the basis of the nomination, supplemented with brief remarks explaining what aspects of the work make it particularly suited for this prize. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations: August 15, 2006
Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal/.

May 2006

*5-6 Operator Algebra Workshop 2006, Queen's University Belfast, Belfast, Northern Ireland.
Information: A two-day workshop dedicated to all aspects of operator algebras, both selfadjoint and non-selfadjoint, will be held in the Department of Pure Mathematics of Queen's University Belfast on Friday, 5 May and Saturday, 6 May 2006.
Organizers: Martin Mathieu and Ivan Todorov.
Details: http://www.qub.ac.uk/opaw2006; email: opaw2006@qub.ac.uk.

*5-7 2006 Midwest Geometry Conference, The University of Oklahoma, Norman, Oklahoma.
Main topics include (but not limited to): Geometric flows; Complex and Riemannian geometry; Conformal geometry; Geometric Measure Theory; Minimal varieties; Symmetric Criticality and algebraic geometry; PDEs, geometric measure theory and mathematical physics.
Organizers: Chair: Shihshu Walter Wei (University of Oklahoma); Co-chair: Thomas Branson (University of Iowa); Marilyn Breen (University of Oklahoma); Andrzej Derdzinski (Ohio State University); Robert M. Hardt (Rice University); Ralph Howard (University of South Carolina); Weiping Li (Oklahoma State University); Gerard Wolseley (University of Oklahoma); and Meijun Zhu (University of Oklahoma).
Information: More information will be posted on the conference homepage as it becomes available at http://www.math.ou.edu/~weil/ngcm06.html.


Conference Description: This conference will explore new developments in applied analysis as they relate to fluid flow and wave motion, both from a theoretical and applied point of view. It will intensify scientific interactions between the two different groups of researchers, expose junior mathematicians to state-of-the-art developments in analysis and its applications and provide a stage for the dissemination of research results.
Organizers: Fernando Botelho, Thomas Hagen, Jim Jamison.
Information: http://www.msci.memphis.edu/fluidsandwaves/.

*13 Graph Theory Day 51, Montclair State University, Montclair, New Jersey.
Host: The Department of Mathematical Sciences, Montclair State University.
Information: The Mathematics Section of the New York Academy of Sciences. This one day meeting is to stimulate activity among graph theorists. Papers for contributed presentations (15 minutes) are invited.
Information: http://www.csam.montclair.edu/~lia/gtdays51/.

*14-18 3rd International Conference: Chebyshev's Mathematical Ideas and Applications to the Modern Science, Obninsk State University, Russia.
Description: On May 16, 2006 we shall celebrate the 185th birthday of the Great Russian mathematician P. L. Chebyshev. Chebyshev was a famous scientist, and on a historical level we can say that he was really the scientific "father" of the Russian St. Petersburg Mathematical School.
Information: http://www.msci.memphis.edu/fluidsandwaves/.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the Notices if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the Notices in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in the Notices. To achieve this, listings should be submitted at least six months prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the Notices. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

Applications: Please submit your application form to the Conference via the website http://www.amath.ru. Additionally you can write directly to the Organizing Committee: CHEBYSHEV-2006, Professor A. Gulin, Head of Department of Applied Mathematics, Obninsk State University, Studgorodok, I, Obninsk, 249040, Russia; email: cheb@amath.ru; phone: + (48439)37908; fax: + (48439)70822.

Languages: Russian and English.


*17-20 Combinatorial and Additive Number Theory (CANT 2006), CUNY Graduate Center, New York, New York.

Description: This is the fourth in a series of annual workshops sponsored by the New York Number Theory Seminar on problems in combinatorial and additive number theory and related parts of mathematics. A list of invited and confirmed lecturers will be posted on the conference website. Mathematicians who wish to speak at the meeting should submit a title and abstract by email to: NewYorkNumberTheory@gmail.com.

Support: It is expected that there will be some financial support, especially for graduate students and young faculty.

Organizer: Mel Nathanson.


*19-20 Groups in Galway 2006, National University of Ireland, Galway, Ireland.

Scope: Covers all areas of group theory, applications, and related fields.

Provisional List of Speakers: Cedric Bonnafé (Univ. de Franche-Comté, France), Peter Cameron (Queen Mary, Univ. of London, UK), Rod Gow (UC Dublin, Ireland), John Murray (NU, Maynooth, Ireland), Shane O'Rourke (Cork Inst. of Techn., Ireland), Gretchen Ostheimer (Hofstra University, USA), Götz Pfeiffer (NU, Galway, Ireland), Martin Quick (St. Andrews, UK), Sarah Rees (Univ. of Newcastle, UK), Chiara Tamburini (Univ. Cattolica del Sacro Cuore, Italy).

Information: Details of the talks and their scheduling will be posted at http://www.maths.nuigalway.ie/gig06/index.html closer to the event. For further information, please contact one of the conference organizers, Rachel Quinlan (Rachel.Quinlan@nuigalway.ie) or Dave Flannery (dave.flannery@nuigalway.ie).

*24-26 DIMACS Workshop on Polyhedral Combinatorics of Random Utility, DIMACS Center, CoRE Bldg, Rutgers University, Piscataway, NJ.

Organizers: Jean-Paul Doignon, Univ. Libre de Bruxelles, email: doignon@ulb.ac.be; Aleksandar Pekec, Fuqua School of Business, Duke University, email: pekec@duke.edu.

Local Arrangements: Workshop Coordinator, DIMACS Center, email: workshop@dimacs.rutgers.edu; 732-445-5928.

Description: Utility functions have a long history in economics and psychology, but have recently caught the attention of computer scientists in various applications. Random utility approaches have been extensively used in the social sciences. The fundamental idea is that utilities of agents could be hard or even impossible to precisely assess or elicit, so one should model these utilities as random variables. This modeling approach could turn out to be useful in developing and solving optimization problems and algorithms for which there is no time to or where it is impossible to assess/obtain input data precisely.

*25-31 NSM2006 "Nonstandard Methods and Applications in Mathematics", Pisa, Italy.

Description: This Congress continues the tradition of biennial meetings focused on nonstandard methods. While the fields of application of nonstandard analysis are diverse, the common methodologies and ideas make it appropriate to consider nonstandard methods as a unified mathematical field of research.


*29-June 2 International School on Partial Differential Equations, Depto. Matematicas y Mecanica, IMAS Universidad Nacional Autonoma de Mexico (UNAM), Mexico City, Mexico.

Description: This one-week school is organized around 6 short courses on various problems involving nonlinear PDEs and it is intended that young researchers and graduate students will be well-informed on the trend and current states of research in the fields. Short presentation by other participants are also encouraged.


Information: http://www.fenomec.unam.mx/.

June 2006

*2-14 Approximation Algorithms, Centre de Recherches Mathematiques, Montreal, Canada.

Description: The workshop will include lectures on the latest developments in the field of approximation algorithms, on both the approximability and the inapproximability sides.

Organizers: Joseph Cheriyan (Waterloo), Michel Goemans (MIT).

Invited Speakers: Please see website below.

Information: http://www.crm.umontreal.ca/Approximation06/.

*5-9 Workshop on Fourier Analysis, Geometric Measure Theory and Applications, Centre de Recerca Matematica, Barcelona, Spain.

Co-directors: José Maria Martell (Universidad Autónoma de Madrid), Joan Mateu (Universitat Autònoma de Barcelona), Alberto Ruiz (Universidad Autónoma de Madrid), Xavier Tolsa (Universitat Autònoma de Barcelona), Ana Vargas (Universidad Autònoma de Madrid), Joan Verdera (Universitat Autònoma de Barcelona).


*9-11 Logic and Mathematics 2006, University of Illinois at Urbana-Champaign, Urbana, Illinois.

Organizers: C. Ward Henson and Slawomir Solecki.

Meeting Topics: The focus of the meeting will be on descriptive set theory and its connections (with algebra, topology, measure theory, topological dynamics, combinatorics, etc). In part, the meeting is organized to honor Alexander S. Kechris of CalTech on the occasion of his 60th birthday.

Meeting Webpage: http://www.math uiuc.edu/Bulletin/1x2006.html. Check here for later information including titles of talks and abstracts.

*10-16 Discontinuous change in behavior issues in partial differential equations, Anogias Academiac Village, Crete, Greece.


Information: E. Kafatos and T. Pheidas; math.uoc.gr.
Mathematics Calendar

*12-15 (REVISED) Conference on 3-manifold topology in honour of Peter Shalen's 60th birthday, Centre de Recherches Mathématiques, Montreal, Canada. (Feb. 2006, p. 286)
Organizers: Steve Boyer, Dick Canary, Marc Culler, Nathan Dunfield, Benson Farb.
Speakers (Tentative): Ian Agol (Univ. of Illinois at Chicago), Mladen Bestvina (Univ. of Utah), Marc Culler (Univ. of Illinois at Chicago), Nathan Dunfield (Caltech), Cameron Gordon (Univ. of Texas), Alex Lubotzky (Hebrew Univ. of Jerusalem), Yair Minsky (Yale Univ.), Maryam Mirzakhani (Princeton Univ./Clay Institute), John Morgan (Columbia Univ.), Lenhard Ng (Stanford Univ./AIM), Peter Ozsvath (Columbia Univ.), Jake Rasmussen (Princeton Univ.), Michah Sageev (Technion).
Information: http://www.crm.umontreal.ca/Shalenfest/.

*12-16 Permutation Patterns 2006, Reykjavik University, Reykjavik, Iceland.
Conference Themes: Include (but are not limited to) enumeration questions, excluded pattern questions, study of the involvement order, algorithms for computing with permutation patterns, applications and generalisations of permutation patterns, and others.

*12-17 Boltzmann Equation and Fluidodynamic Limits. SISSA-ISAS, Trieste, Italy.
Aims and Scope: The conference wishes to provide an up-to-date overview of the recent results and open problems related to the study of the Boltzmann equation and of the connections between the macroscopic and mesoscopic description of gas dynamics.
Organizing Committee: Fabio Ancona (University of Bologna, Italy), Stefano Bianchini (SISSA-ISAS, Trieste, Italy), Camillo De Lellis (University of Zürich, Switzerland), Andrea Marson (University of Padova, Italy).
Information: http://www.sissa.it/boltzmann/.

*21-24 "Views on ODEs" Conference in Honor of Arrigo Cellina and James A. Yorke on the Occasion of their 65th Birthdays, Aveiro University, Aveiro, Portugal.
Conference: "Views on ODEs" will celebrate the 65th birthdays of Professors Arrigo Cellina and James A. Yorke and aims to bring together those enrolled in research activities related with ordinary differential equations, differential inclusions and their applications.
Main Topics: Dynamical systems; Bifurcations; Invariant measures; Chaotic attractors; Prevalence; Population dynamics; Markov operators; Semigroups; Viscosity solutions; Hamilton-Jacobi equations; Hyperbolic systems; Optimal control and differential inclusions; Variational and topological methods.
Information: http://www.dvvp-proj.org/.

*26-29 Special session on "Coding theory and cryptography", Varna, Bulgaria.
Organizers: Stefan Dodunekov, (Bulgarian Academy of Sciences), Tony Shaska, (Oakland University).
Overview: As technology becomes increasingly involved in communication, coding theory and cryptography also become increasingly important. The goal of this session is to bring together researchers in all aspects of coding theory, cryptography and related areas and explore the use of computational algebra in such areas.
Contact: T. Shaska (shaska@oakland.edu); http://www.oakland.edu/~shaska/cod_06.html.

*26-July 8 Advanced Course on Limit Cycles of Differential Equations, Centre de Recerca Matemàtica, Barcelona, Spain.
Speakers: Colin Christopher (University of Plymouth, United Kingdom): Around the Center-Focus Problem; Chengzhli Li (Peking University, China): Abelian integrals and application to weak Hilbert's 16th problem; Sergei Yakovenko (The Weizmann Institute of Science, Israel): Algebraic Solutions of Polynomial Vector Fields; Co-organizers: Armengol Gasull (Universitat Autònoma de Barcelona), Jaume Llibre (Universitat Autònoma de Barcelona).
Registration and payment: Fee: 200 euros; Deadline: May 26, 2006.
Grants: The CRM offers a limited number of grants for registration and/or accommodation addressed to young researchers. The deadline for application is April 26, 2006.
Further information: http://www.crm.es/ACDiEquations, email: ACDiEquations@crm.es.

*28-30 Workshop From Lie Algebras to Quantum Groups, Universidade de Coimbra, Portugal.
Speakers and Members of the Scientific Committee are: Helena Albuquerque, Universidade de Coimbra (Portugal), Georgia Benkart (University of Wisconsin-Madison), Alberto Elduque (Universidad de Zaragoza, Spain), George Lusztig (Massachusetts Institute of Technology), Shahn Majid (University of London (UK), Michael Semenov-Tian-Shansky, Université de Bourgogne (France).
Talks: There will be plenary talks (30 min), section talks (20-25 min) and a poster session. Proceedings will be published by the International Center of Mathematics (ICM) and will contain contributions by all the participants, after revision by the Scientific Committee.
Deadlines: April 30, 2006. For more details, including the Conference Program and information on accommodation in Coimbra, please refer to http://www.aim.esi.ilstp.pt/~jmp/ICM/Lie/index.htm. We regret to inform that we cannot offer the participants any financial support.
Information: Persons interested in participating are kindly asked to register at the conference website http://www.aim.esi.ilstp.pt/~jmp/ICM/Lie/index.htm as well as to submit an abstract for a section talk or a poster. The registration fee is 100 Euros, and there will be a reduced student fee of 50 Euros.

**July 2006**

*3-7 Iwasawa 2006 Congress, Université de Limoges, Limoges, France.
Program: Recent advances in Iwasawa Theory.
Organizing Committee: François Laubie, Abbas Movahhedi, Alain Salinanier, Stephane Vinatier.


Description: The program will be devoted (but not restricted) to subjects in contemporary Combinatorics and Graph Theory involving also relationships and applications in Algebra, Algorithms, Topology, Probability and Statistics, Mathematical Logic, Computer Science and other fields.
Organizer: DIMATIA Charles University, Prague.
10-15 Conference on Recent Developments in the Arithmetic of Shimura Varieties and Arakelov Geometry (An EMS Marie Curie Conference, supported by the European Commission), Centre de Recerca Matematica, Bellaterra, Spain.

Speakers: Ahmed Abbes (Univ. de Paris XIII, France), Pascal Boyer (Univ. de Paris VI, France), Jan H. Bruinier (Univ. Köln, Germany), Laurent Clozel (Univ. de Paris XI), Henri Darmon (McGill Univ., Canada), Jürg Kramer (Humboldt Univ. zu Berlin, Germany), Elena Mantovan (Harvard Univ., USA), Sophie Morel (Univ. de Paris XI, France), Bao Chau Ngo (Univ. de Paris XI, France), Michael Rapoport (Univ. Bonn), Damian Roesler (Univ. de Paris VII, France), Christophe Soulé (CNRS-IHES, France).

Information: http://www.crm.es/SVAG; email: SVAG@crm.es.

11-12 DIMACS Workshop on Machine Learning Techniques in Bioinformatics, DIMACS Center, CoRE Bldg, Rutgers University, Piscataway, New Jersey.

Description: Bioinformatics aims to solve biological problems by using techniques from mathematics, statistics, computer science, and machine learning. Recent years have observed the essential use of these techniques in this rapidly growing field. Examples of such applications include those to gene expression data analysis, gene-protein interactions, protein folding and structure prediction, genetic and molecular networks, sequence and structural motifs, genomics and proteomics, text mining in bioinformatics, and so on.

Organizers: Dechang Chen, Uniformed Services University of the Health Sciences, email: dchen@ums.mil; Xue-Wen Chen, University of Kansas, email: xwchen@ku.edu; Sorin Draghici, Wayne State University, email: sod@cs.wayne.edu.

Local Arrangements: Workshop Coordinator, DIMACS Center, email: workshop@dimacs.rutgers.edu; 732-445-5928.

Information: http://www.cs.rutgers.edu/Workshops/MlTechniques/.

12-16 Anomalous Transport: Experimental Results and Theoretical Challenges, Physikzentrum Bad Honnef near Bonn, Germany.

Scope: Anomalous transport phenomena such as sub- and superdiffusion, non-Gaussian probability distributions, aging and dynamical localization form a rapidly growing research area within nonequilibrium statistical physics. The seminar will provide a unique opportunity to learn about topics ranging from mathematical foundations of anomalous dynamics to the most recent experimental results in this field.

Invited Speakers: R. Artuso (Como), E. Barkai (Bar-Ilan), C. Beck (London), A. V. Chechkin (Kharkov), D. Del-Castillo-Negrete (Oak Ridge), P. Dietrich (Dresden), T. Geisel (Goettingen), R. Gorenflo (Berlin), R. Hilfer (Stuttgart), J. Kaelger (Leipzig), R. Kimmel (Ulm), J. Klafter (Tel Aviv), W. Kopf (Montpellier), A. Kusumi (Kyoto), E. Lutz (Ulm), R. Metzler (Copenhagen), M. J. Saxton (Davis), M. Shlesinger (Arlington), S. Tasaki (Tokyo), G. Vogl (Vienna), A. Vulpiani (Rome), S. Yuste (Badajoz).

Registration: Applications are welcome and should be made by using the application form on the conference web page, however, the number of attendees is limited. The seminar’s registration fee is EUR 200 and will cover accommodation and meals.

Deadline: For applications is April 30, 2006.

Information: For further information please visit the conference webpage http://anotrans.physik.hu-berlin.de, or contact one of the organizers.

13-14 Conference on Geometric Group Theory, Centre de Recherches Mathematiques, Montreal, Canada.

Description: From July 3-7: There will be five mini-courses focusing on emerging ideas in Geometric Group Theory that will be especially aimed at graduate students, but are sure to be of wider interest.

From July 10-14: There will be a workshop featuring some of the exciting new developments in the Geometric Group Theory. We expect around twenty talks during the workshop week.

Information: http://www.crm.umontreal.ca/Geometric06/index_e.html.


Organizers: Daniel Daners, Yihong Du, Chris Radford, Shusen Yan.

Deadline: Registration deadline: March 31, 2006.


16-22 Horizon of Combinatorics, Lake Balaton, Hungary.

Topics: Horizon of Combinatorics intends to gather researchers from all areas related to combinatorics. These include amongst others: Combinatorial structures (graphs, hypergraphs, matroids), designs, permutations groups), Combinatorial optimization, Combinatorial aspects of geometry and number theory, Infinite combinatorics, Algebraic combinatorics, Algorithms in combinatorics and related fields.

Information: http://www.renyi.hu/conferences/horizon.

17-21 Workshop on Singularities in PDE and the Calculus of Variations, Centre de Recherches Mathematiques, Montreal, Canada.

Focus: The development and structure of singular structures in solutions to nonlinear partial differential equations.

Participants: Amandine Aftalion (Paris VI), Giovanni Alberti (Pisa), Yanyan Li (Utah), Leopold Berlyand (Penn State), Fabrice Bethuel (Paris VI), Rustum Choksi (SFU), Manuel DelPino (U. de Chile), Carlos Garcia-Cervera (Cal-Santa Barbara), Stephen Gustafson (UofC), Robert Jerrard (Toronto), Shuichi Jimbo (Hokkaido), Bernd Kawohl (Kolm), David Kinderlehrer (Carnegie-Mellon), Robert V. Kohn (NYU), Chun Liu (Penn State), Andrea Malchiodi (SISSA, Trieste), Vincent Millot (CMU), Alberto Montero (Toronto), Yoshikazu Morita (Ryukoku, Japan), Pablo Padilla (UNAM, Mexico), Daniel Phillips (Purdue), Xi-aofeng Ren (Utah State), Maria Roginskii (Princeton), Etienne Sander (Paris-12), Sylvia Serfaty (NYU), Daniel Spirn (Minnesota), Edward Snelson (Wisconsin-Richland), Gabriella Tarantello (Rome II).

Information: http://www.crm.umontreal.ca/Singularities06/index_e.html.

August 2006

7-11 Effective Randomness, AIM Research Conference Center, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will bring together researchers who have studied effective randomness at different times, with different motivations, and drawing from different academic backgrounds, with an aim toward increasing communication and collaboration, and developing broad shared research goals and a coherent research community.

Topics: For the workshop will include effective notions of randomness such as Martin-Lof randomness; measures of relative randomness; effective dimension; Kolmogorov complexity and other concepts from algorithmic information theory; and interactions with computability theory and complexity theory.

Organizers: Joseph Miller and Denis Hirschfeldt.


14-16 Network Design: Optimization and Algorithmic Game Theory, Centre de Recherches Mathematiques, Montreal, Canada.

Description: As the network infrastructure keeps changing and new applications are emerging, the mathematical models themselves
must be adapted constantly. The workshop will explore recent developments in the field and especially the relationship between combinatorial optimization and the models used in distributed network design.

Organizers: Shek Man Mo (McGill) and Adrian Vetta (McGill).

Participants: Kamal Jain (Microsoft Research Center), Ramesh Johari (Stanford University), George Karakostas (McMaster University), Anna Karlin (University of Washington), Jochen Konemann (University of Waterloo), Kate Larson (University of Waterloo), Yishay Mansour (Tel Aviv University), Peter Marbach (University of Toronto), Sean Meyn (University of Illinois), Tim Roughgarden (Stanford University), Andreas Schulz (ETH Zentrum), Naum Shimkin (Technion), Eva Tardos (Cornell University).* (*) To be confirmed

Information: http://www.crm.umontreal.ca/Network06/.


Workshop Topics: This workshop, sponsored by AIM and the NSF, will be devoted to the study of phase transitions in several traditionally separate subjects. We propose to bring together experts in different areas to present the various intuitions, motivations, canonical examples and conceptual techniques of their areas, the hope being to come to agreement on a few key definitions and perhaps thereby to bring fresh ideas to bear on open problems.


Information: http://aimath.org/ARC/Workshops/phasetransition.html.


Topics: Riemann-Weierstrass approach to asymptotics of orthogonal polynomials, inverse problems and rational approximation, spectral theory of banded and difference operators, numerical analysis and rational approximation, integrable non linear dynamical systems, eigenvalues of random matrices, non standard orthogonal polynomials.

Main Speakers: P. Deift (Courant Institute, New York University), B. Simon (California Institute of Technology), P. A. Grunbaum (University of California, Berkeley), S. Khruschev (Attila University, Ankara, Turkey), M. E. H. Ismail (University of Central Florida), A. V. D. Kuijlaars (Katholieke Universiteit Leuven, Belgium).

Deadlines: Submission of abstracts is May 31, 2006; Registration is June 5, 2006.

Contact: Francisco Marcellán, Departamento de Matemáticas, Universidad Carlos III de Madrid, Avenida de la Universidad 30, 28911 Leganés, Spain; email: pacomar@uco3m.es.

Information: Visit the website http://www.uc3m.es/uc3m/edpco/HATK/OrthApprox/ICM06/uc3m_ICM06.html.

31-September 5 Advanced Course on Combinatorial and Computational Geometry: Trends and topics for the future. Centre de Recerca Matemàtica, Barcelona, Spain.

Speakers: János Pach (City College and Courant Institute, New York and Rényi Institute, Budapest, Hungary), Micha Sharir (Tel Aviv University, Israel).


Grants: The CRM offers a limited number of grants for registration fee and/or accommodation addressed to young researchers. The deadline for application is April 14, 2006.

Further Information: Visit: http://www.crm.es/ACoCGeometry; email: ACoCGeometry@crm.es.

September 2006

1-4 Conference on Mathematical Neuroscience, Sant Julià de Lòria (Andorra), Madrid, Spain.

Plenary Speakers: Bard Ermentrout (University of Pittsburgh), Nancy Kopell (Boston University), John Rinzel (New York University).


Information: See http://www.crm.es/CMathNeuroscience; email: CMathNeuroscience@crm.es.


Invited Speakers: X. Gual (Univ. Jaume I), L. M. Cruz-Orive (Univ. Cantabria); “Sterology”, J. M. M. Senovilla (UPV/EHU); “Geometry of submanifolds in Lorentzian geometry with applications”, O. Barjal, J. Arroyo (UPV/EHU), R. Lipowsky (Max Planck Institute); “Curvature energy minimizers. Applications to the physics of elastic and soft materials”, R. Kamin (Univ. Pennsylvania); “Materials Geometry: An Introduction Survey on Geometry and Physics; Geometry of Condensed”, Y. S. Cho (Ewha Women’s University, Korea); “Group Actions on Gauge Theory”, M. Ferr (Univ. Rologna); “Geometrical Methods in Application and Industry in Italy”, L. Verstraeten (Katholike Univ. Leuven, Belgium); “Understanding Vision Through Geometry”, A. Ferrández and J. Pastor (Univ. Murcia); “Geometry applied to DNA”, J. Martinez Aroz (Univ. Granada); “Traction of M. Cabreroz (Univ. Granada); “Physics in Action”, C. Ruiz (Univ. Granada); “Geometry in the Alhambra.”

Information: http://gigida.ugr.es/isaga06/; email: isaga06@ugr.es.

4-9 International Conference on Applied Analysis and Differential Equations, University “Al.Cuza”, Faculty of Mathematics, Iasi, Romania.

Purpose: The conference is to create a platform for international exchange of ideas and the newest results in the fields of applied analysis and differential equations.


Speakers: Please see website below.

Information: http://www.math.ueic.ro/~isaade; email: isaade@ueic.ro.

11-16 XV Fall Workshop on Geometry and Physics, Puerto de la Cruz (Tenerife, Canary Islands), Spain.

Main topics: Continuum Mechanics, Dynamical systems, Geometry Control Theory, Integrable systems, Lie algebras (groupoids) and its applications, Poisson Geometry, Classical and Quantum Field theories, Riemannian and Lorentz Geometry and Relativity, Symplectic and Contact Geometry and Topology, String Theory, Supergravity and Supersymmetry.


Deadline: July 1, 2006.


18-20 The 10th Workshop on Elliptic Curve Cryptography (ECC 2006), Fields Institute, Toronto, Canada.


18-22 Hybrid Methods and Branching Rules in Combinatorial Optimization, Centre de Recherches Mathématiques, Montreal,
Canada.

Description: Problems of combinatorial optimization (such as SAT, the problem of recognizing satisfiable boolean formulas in the conjunctive normal form) have been the subject of intensive study by two communities of researchers: Those in mathematical programming (often classified under "operations research") and those in constraint satisfaction programming (often classified under "artificial intelligence"). Recent years have seen increasing interaction between these two initially separate communities. One of the aims of the workshop is to foster this confluence.

Second Theme: Branching rules are another theme of the workshop. These rules are an important component of branch-and-bound-based exact algorithms and their choice may have an overwhelming impact on the efficiency of such algorithms.

Organizer: Vaclav Chvatal (Concordia).

Information: email: paradis@crm.umontreal.ca.

*18-22 Model Theory of Metric Structures, AIM Research Conference Center, Palo Alto, CA.

Organizers: C. Ward Henson and Itay Ben-Yaacov.

Workshop topics: This workshop, sponsored by AIM and the NSF, will focus on the use of model theoretic ideas in analysis and metric geometry, bringing together model theorists and specialists from a few key application areas for a period of intense discussions. A diverse combination of backgrounds will allow the participants to explore from new angles certain examples, applications, and theoretical problems that define the frontier of research on the model theory of metric structures.

Application deadline: June 18, 2006.

For more information: http://aimath.org/ARCC/workshops/continuumlogic.html.

*22-29 Conference on Geometry and Dynamics of Groups and Spaces in Memory of Alexander Reznikov, Max-Planck-Institut für Mathematik, Bonn, Germany.

Organizers: Mikhail Kapranov (Yale University, USA), Sergiy Kolyada (Institute of Mathematics, Ukraine), Yuri Manin (MPIM, Germany), Pieter Moree (MPIM, Germany), Leonid Polterovich (Université de Lille, France).

Contact: email: gdga06@mpim-bonn.mpg.de.

Topics: Alexander (Sasha) Reznikov (1960-2003) was a brilliant mathematician who died unfortunately very early. This conference in his remembrance focuses on topics Sasha made a contribution to. In particular: 1. Hyperbolic, Differential and Complex Geometry. 2. Geometric group theory. 3. Three dimensional topology. 4. Dynamical systems.

*29-30 16th Annual Kansas City Regional Mathematics Technology EXPO, Rockhurst University, Kansas City, Missouri.

Forum: For mathematics instructors at both the college and secondary levels to demonstrate how they use technology successfully in their teaching, to learn about new mathematics technology, and to discuss the philosophy and future of technology in the mathematics classroom.

Invited Speakers: M. Kathleen Heid (Pennsylvania State University, University Park, PA), Doug Ensley (Shippensburg University, Shippensburg, PA).


Information: Visit http://kcmathtechexpo.org; email: rgi11@bluevalleyk12.org.

October 2006

*2-6 Quantum Cryptography and Computing Workshop, Fields Institute, Toronto, Canada.

Information: http://www.fields.utoronto.ca/programs/scientific/06-07/crypto/quantum/.

*7-10 PDE Approaches to Image Processing, Mathematical Institute, University of Cologne, Cologne, Germany.

Workshop: sponsored by the ESF Programme "Global and Geometric Aspects of Nonlinear Partial Differential Equations".

Description: Recent progress in mathematical image processing shows a surprising success when one applies numerical methods to ill-posed partial differential equations. There is hardly any theory for these equations, it lags far behind their use by engineers, and the purpose of the workshop is to learn more about the underlying mathematical questions. We shall address for instance issues like anisotropic diffusion and Perona-Malik type equations.

Organizers: Bernd Kawohl (Cologne), Felix Otto (Bonn).

Information: http://www.mi.uni-koeln.de/~jhorak/workshop/

*9-13 Short-term Cardiovascular-Respiratory Control Mechanisms, AIM Research Conference Center, Palo Alto, California.

Organizers: Franz Kappel, Vera Novak, Mette Olufsen, and Hien Tran.

Workshop: This workshop, sponsored by AIM and the NSF, will be the first highly focused attempt to tackle complex problems in cardio-respiratory physiology by bringing together researchers with expertise in physiology, mathematics, and statistics. The overall objective of this workshop is to discuss methodologies to further develop mathematical models to improve understanding, diagnosis, and treatment of clinical problems related to short-term cardiovascular-respiratory regulation.

Application deadline: July 9, 2006.


*10-13 Data Mining and Mathematical Programming, Centre de Recherches Mathématiques, Montreal, Canada.

Description: Data mining is a fast-growing discipline that uses techniques from several subfields of applied mathematics, including operations research and statistics. This workshop will feature applications of exact or heuristic algorithms for solving mathematical programs (linear or nonlinear, convex or nonconvex) to the fundamental problems in data mining, in particular clustering, discrimination and search for relations.

Organizers: Pierre Hansen (HEC Montréal) and Panos Pardalos (Florida).

Information: email: paradis@crm.umontreal.ca.

*16-20 Subconvexity Bounds for L-functions, AIM Research Conference Center, Palo Alto, California.


Workshop topics: This workshop, sponsored by AIM and the NSF, will be devoted to subconvexity bounds for L-functions. In recent years, there has been substantial progress towards the subconvexity problem for GL(2) L-functions, beginning with the work of Duke, Friedlander, and Iwaniec; more recently, ideas from representation theory and dynamics have been brought to bear on the problem. Subconvexity bounds for L-functions in higher rank (and, more generally, bounds for periods) remain largely elusive. The aim of the workshop is to consolidate the existing approaches and initiate analysis of the higher rank subconvexity problem.

Application deadline: July 16, 2006.


*17-20 Polyhedral Computation, Centre de Recherches Mathématiques, Montreal, Canada.

Description: The last fifteen years have seen significant progress in the development of general purpose algorithms and software for polyhedral computation (e.g. finding lattice points, enumerating vertices, extreme rays and facets and triangulating polyhedra). This workshop will bring together researchers with both theoretical and computational expertise with polyhedral computations.

Organizers: David Avis (McGill), David Bremner (New Brunswick) and Antoine Deza (McMaster).

May 2006

NOTICES OF THE AMS 611
Workshop topics and Goals: This workshop, sponsored by AIM and the NSF, will bring together researchers with different perspectives in combinatorial representation theory: combinatorial, metric, and algebro-geometric. It has emerged from recently that Bruhat-Tits buildings play an essential, yet not well-understood role in combinatorial representation theory by providing a geometric realization to existing combinatorial models and linking them to the algebro-geometric tools of representation theory. Goals for the workshop include examining and comparing the different approaches to the saturation theorem, with an emphasis on the role of buildings.

Application Deadline: December 1, 2006.

April 2007

April 2007

May 2007

May 2007

July 2007

July 2007

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.
New Publications Offered by the AMS

Algebra and Algebraic Geometry

On Higher Frobenius-Schur Indicators
Yevgenia Kashina, DePaul University, Chicago, IL, Yorck Sommerhäuser, Universität München, Munich, Germany, and Yongchang Zhu, Hong Kong University of Science and Technology, Kowloon, Hong Kong

Contents: Introduction; The calculus of Sweedler powers; Frobenius-Schur indicators; The exponent; The order; The index; The Drinfel’d double; Examples; Bibliography; Subject index.

Memoirs of the American Mathematical Society, Volume 181, Number 855

Enumerative Geometry and String Theory
Sheldon Katz

Perhaps the most famous example of how ideas from modern physics have revolutionized mathematics is the way string theory has led to an overhaul of enumerative geometry, an area of mathematics that started in the eighteen hundreds. Century-old problems of enumerating geometric configurations have now been solved using new and deep mathematical techniques inspired by physics!

The book begins with an insightful introduction to enumerative geometry. From there, the goal becomes explaining the more advanced elements of enumerative algebraic geometry. Along the way, there are some crash courses on intermediate topics which are essential tools for the student of modern mathematics, such as cohomology and other topics in geometry.

The physics content assumes nothing beyond a first undergraduate course. The focus is on explaining the action principle in physics, the idea of string theory, and how these directly lead to questions in geometry. Once these topics are in place, the connection between physics and enumerative geometry is made with the introduction of topological quantum field theory and quantum cohomology.

Contents: Warming up to enumerative geometry; Enumerative geometry in the projective plane; Stable maps and enumerative geometry; Crash course in topology and manifolds; Crash course in $C^\infty$ manifolds and cohomology; Cellular decompositions and line bundles; Enumerative geometry of lines; Excess intersection; Rational curves on the quintic threefold; Mechanics; Introduction to supersymmetry; Introduction to string theory; Topological quantum field theory; Quantum cohomology and enumerative geometry; Bibliography; Index.

Student Mathematical Library, Volume 32

Graduate Algebra: Commutative View
Louis Halle Rowen

This book is an expanded text for a graduate course in commutative algebra, focusing on the algebraic underpinnings of algebraic geometry and of number theory. Accordingly, the theory of affine algebras is featured, treated both directly and via the theory of Noetherian and Artinian modules, and the theory of graded algebras is included to
provide the foundation for projective varieties. Major topics include the theory of modules over a principal ideal domain, and its applications to matrix theory (including the Jordan decomposition), the Galois theory of field extensions, transcendence degree, the prime spectrum of an algebra, localization, and the classical theory of Noetherian and Artinian rings. Later chapters include some algebraic theory of elliptic curves (featuring the Mordell-Weil theorem) and valuation theory, including local fields.

One feature of the book is an extension of the text through a series of appendices. This permits the inclusion of more advanced material, such as transcendental field extensions, the discriminant and resultant, the theory of Dedekind domains, and basic theorems of rings of algebraic integers. An extended appendix on derivations includes the Jacobson conjecture and Makar-Limanov's theory of locally nilpotent derivations. Gröbner bases can be found in another appendix.

Exercises provide a further extension of the text. The book can be used both as a textbook and as a reference source.

Contents: Introduction and prerequisites; Exercises—Chapter 0; Part I. Modules: Introduction to modules and their structure theory; Finitely generated modules; Simple modules and composition series; Exercises—Part I; Part II. Affine algebras and Noetherian rings: Part II. Introduction; Galois theory of fields; Algebras and affine fields; Transcendence degree and the Krull dimension of a ring; Modules and rings satisfying chain conditions; Localization and the prime spectrum; The Krull dimension theory of commutative Noetherian rings; Exercises—Part II; Part III. Applications to geometry and number theory: Part III. Introduction; The algebraic foundations of geometry; Applications to algebraic geometry over the rationals — Diophantine equations and elliptic curves; Absolute values and valuation rings; Exercises—Part III; References; Index.

Graduate Studies in Mathematics, Volume 73

Analysis

On Boundary Interpolation for Matrix Valued Schur Functions
Vladimir Bolotnikov, The College of William and Mary, Williamsburg, VA, and Harry Dym, Weizmann Institute of Science, Rehovot, Israel

Contents: Introduction; Preliminaries; Fundamental matrix inequalities; On $\mathcal{H}_2(\Omega)$ spaces; Parametrizations of all solutions; The equality case; Nontangential limits; The Nevanlinna-Pick boundary problem; A multiple analogue of the Carathéodory-Julia theorem; On the solvability of a Stein equation; Positive definite solutions of the Stein equation; A Carathéodory-Fejér boundary problem; The full matrix Carathéodory-Fejér boundary problem; The lossless inverse scattering problem; Bibliography.

Memoirs of the American Mathematical Society, Volume 181, Number 856

Stability of Spherically Symmetric Wave Maps
Joachim Krieger, Harvard University, Cambridge, MA

Contents: Introduction, controlling spherically symmetric wave maps; Technical preliminaries. Proofs of main theorems; The proof of Proposition 2.2; Proof of theorem 2.3; Bibliography.

Memoirs of the American Mathematical Society, Volume 181, Number 853

Differential Equations

Tangential Boundary Stabilization of Navier-Stokes Equations
Viorel Barbu and Irena Lasiecka, University of Virginia, Charlottesville, VA, and Roberto Triggiani

Contents: Introduction; Main results: Proof of Theorems 2.1 and 2.2 on the linearized system (2.4): $d = 3$; Boundary feedback uniform stabilization of the linearized system (3.1.4) via an optimal control problem and corresponding Riccati theory. Case $d = 3$;
Theorem 2.3(i): Well-posedness of the Navier-Stokes equations with Riccati-based boundary feedback control. Case $d = 3$;
Theorem 2.3(ii): Local uniform stability of the Navier-Stokes equations with Riccati-based boundary feedback control; A PDE-interpretation of the abstract results in Sections 5 and 6; Appendix A. Technical material complementing Section 3.1; Appendix B. Boundary feedback stabilization with arbitrarily small support of the linearized system (3.1.4a) at the level, with $\gamma_0 \in (H^1 + \epsilon(\Omega))^d \cap H$. Cases $d = 2, 3$. Theorem 2.5 for $d = 2$; Appendix C. Equivalence between unstable and stable versions of the optimal control problem of Section 4; Appendix D. Proof that $F(S) \in L(W; L^2(0, \infty; (L^2(F))^d))$. Bibliography.

Memoirs of the American Mathematical Society, Volume 181, Number 852

Discrete Mathematics and Combinatorics

The Shoelace Book
A Mathematical Guide to the Best (and Worst) Ways to Lace Your Shoes
Burkard Polster, Monash University, Clayton, Vic, Australia

Crisscross, zigzag, bowtie, devil, angel, or star: which are the longest, the shortest, the strongest, and the weakest lacing? Pondering the mathematics of shoelaces, the author paints a vivid picture of the simple, beautiful, and surprising characteristics of the most common shoelace patterns. The mathematics involved is an attractive mix of combinatorics and elementary calculus. This book will be enjoyed by mathematically minded people for as long as there are shoes to lace.

Burkard Polster is a well-known mathematical juggler, magician, origami expert, bubble-master, shoelace charmer, and "Count von Count" impersonator. His previous books include A Geometrical Picture Book, The Mathematics of Juggling, and QED: Beauty in Mathematical Proof.

This item will also be of interest to those working in general and interdisciplinary areas.

Contents: Setting the stage; One-column lacings; Counting lacings; The shortest lacings; The longest lacings; The strongest lacings; The weakest lacings; Related mathematics; Loose ends; References; Index.

Mathematical World, Volume 24
June 2006, 125 pages, Softcover, ISBN 0-8218-3933-0, LC 2006040733, 2000 Mathematics Subject Classification: 00A05, 90C27, 05A15, All AMS members US$23, List US$29, Order code MAWRLD/24

Elements of Combinatorial and Differential Topology
V. V. Prasolov, Independent University of Moscow, Russia

Modern topology uses very diverse methods. This book is devoted largely to methods of combinatorial topology, which reduce the study of topological spaces to investigations of their partitions into elementary sets, and to methods of differential topology, which deal with smooth manifolds and smooth maps. Many topological problems can be solved by using either of these two kinds of methods, combinatorial or differential. In such cases, both approaches are discussed.

One of the main goals of this book is to advance as far as possible in the study of the properties of topological spaces (especially manifolds) without employing complicated techniques. This distinguishes it from the majority of other books on topology.

The book contains many problems; almost all of them are supplied with hints or complete solutions.

Contents: Graphs; Topology in Euclidean space; Topological spaces; Two-dimensional surfaces, coverings, bundles, and homotopy groups; Manifolds; Fundamental groups; Hints and solutions; Bibliography; Index.

Graduate Studies in Mathematics, Volume 74
Logic and Foundations

The Role of True Finiteness in the Admissible Recursively Enumerable Degrees
Noam Greenberg, University of Notre Dame, IN

Contents: Introduction; Coding into the R. E. degrees; Coding effective successor models; A negative result concerning effective successor models; A nonembedding result; Embedding the 1-3-1 lattice; Appendix A. Basics; Appendix B. The jump; Appendix C. The projectum; Appendix D. The admissible collapse; Appendix E. Prompt permission; Appendix. Bibliography.

Memoirs of the American Mathematical Society, Volume 181, Number 854

Mathematical Physics

Snowbird Lectures on String Geometry
Katrin Becker, Melanie Becker, Aaron Bertram, Paul S. Green, and Benjamin McKay, Editors

The interaction and cross-fertilization of mathematics and physics is ubiquitous in the history of both disciplines. In particular, the recent developments of string theory have led to some relatively new areas of common interest among mathematicians and physicists, some of which are explored in the papers in this volume. These papers provide a reasonably comprehensive sampling of the potential for fruitful interaction between mathematicians and physicists that exists as a result of string theory.


Contemporary Mathematics, Volume 401

Number Theory

Lectures on Ergodic Theory
Paul Halmos, Santa Clara University, Santa Clara, CA

This classic book is based on lectures given by the author at the University of Chicago in 1956. The topics covered include, in particular, recurrence, the ergodic theorems, and a general discussion of ergodicity and mixing properties. There is also a general discussion of the relation between conjugacy and equivalence. With minimal prerequisites of some analysis and measure theory, this work can be used for a one-semester course in ergodic theory or for self-study.

Contents: Introduction; Examples; Recurrence; Mean convergence; Pointwise convergence; Comments on the ergodic theorem; Ergodicity; Consequences of ergodicity; Mixing; Measure algebras; Discrete spectrum; Automorphisms of compact groups; Generalized proper values; Weak topology; Weak approximation; Uniform approximation; Category; Invariant measures; Invariant measures: the solution; Invariant measures: the problem; Generalized ergodic theorems; Unsolved problems; References.

AMS Chelsea Publishing
New AMS-Distributed Publications

Algebra and Algebraic Geometry

One Semester of Elliptic Curves
Torsten Ekedahl, University of Stockholm, Sweden

These lecture notes grew out of a one semester introductory course on elliptic curves given to an audience of computer science and mathematics students, and assume only minimal background knowledge. After having covered basic analytic and algebraic aspects, putting special emphasis on explaining the interplay between algebraic and analytic formulas, they go on to some more specialized topics. These include the J-function from an algebraic and analytic perspective, a discussion of elliptic curves over finite fields, derivation of recursion formulas for the division polynomials, the algebraic structure of the torsion points of an elliptic curve, complex multiplication, and modular forms.

In an effort to motivate basic problems the book starts very slowly, but considers some aspects such as modular forms of higher level which are not usually treated. It presents more than 100 exercises and a Mathematica™ notebook that treats a number of calculations involving elliptic curves.

The book is aimed at students of mathematics with a general interest in elliptic curves but also at students of computer science interested in their cryptographic aspects.

ESA Lectures in Mathematics and Physics
Mathematics Subject Classification: 14K25, All AMS members US$30, List US$38, Order code EMSSERLEC/2

Locally Compact Groups
Markus Stroppel, University of Stuttgart, Germany

Locally compact groups play an important role in many areas of mathematics as well as in physics. The class of locally compact groups admits a strong structure theory, which allows to reduce many problems to groups constructed in various ways from the additive group of real numbers, the classical linear groups and from finite groups. The book gives a systematic and detailed introduction to the highlights of that theory.

In the beginning, a review of fundamental tools from topology and the elementary theory of topological groups and transformation groups is presented. Completions, Haar integral, applications to linear representations culminating in the Peter-Weyl Theorem are treated. Pontryagin duality for locally compact Abelian groups forms a central topic of the book. Applications are given, including results about the structure of locally compact Abelian groups, and a structure theory for locally compact rings leading to the classification of locally compact fields. Topological semigroups are discussed in a separate chapter, with special attention to their relations to groups. The last chapter reviews results related to Hilbert's Fifth Problem, with the focus on structural results for non-Abelian connected locally compact groups that can be derived using approximation by Lie groups.

The book is self-contained and is addressed to advanced undergraduate or graduate students in mathematics or physics. It can be used for one-semester courses on topological groups, on locally compact Abelian groups, or on topological algebra. Suggestions on course design are given in the preface. Each chapter is accompanied by a set of exercises that have been tested in classes.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Preliminaries; Topological groups; Topological transformation groups; The Haar integral; Categories of topological groups; Locally compact Abelian groups; Locally compact semigroups; Hilbert's fifth problem; Bibliography; Index of symbols; Subject index.

EMS Textbooks in Mathematics
Mathematics Subject Classification: 22D05, 22-01, 20E18, 22A25, 22B05, 22C05, 22D10, 22D45, 22F05, 12J10, 43A05, 54H15, 22A15, All AMS members US$46, List US$58, Order code EMSTEXT/3

Differential Equations

Dynamics on the Riemann Sphere
A Bodil Branner Festschrift
Poul Hjorth, Technical University of Denmark, Lyngby, Denmark, and Carsten Lunde Petersen, Roskilde University, Denmark, Editors

Dynamics on the Riemann Sphere presents a collection of original research articles by leading experts in the area of holomorphic dynamics. These papers arose from the Symposium Dynamics in the Complex Plane, held on the occasion of the 60th birthday of Bodil Branner. Topics covered range from Lattès maps to cubic polynomials over rational maps with Siegel disks and Gaskets as Julia sets, as well as rational and entire transcendental maps with Herman rings.

This item will also be of interest to those working in analysis.
Differential-Algebraic Equations
Analysis and Numerical Solution

Peter Kunkel, University of Leipzig, Germany, and
Volker Mehrmann, Technical University of Berlin, Germany

Differential-algebraic equations are a widely accepted tool for
the modeling and simulation of constrained dynamical
systems in numerous applications, such as mechanical
multibody systems, electrical circuit simulation, chemical
engineering, control theory, fluid dynamics and many others.

This is the first comprehensive textbook that provides a
systematic and detailed analysis of initial and boundary value
problems for differential-algebraic equations. The analysis is
developed from the theory of linear constant coefficient
systems via linear variable coefficient systems to general
nonlinear systems. Further sections on control problems,
generalized inverses of differential-algebraic operators,
generalized solutions, and differential equations on manifolds
complement the theoretical treatment of initial value
problems. Two major classes of numerical methods for
differential-algebraic equations (Runge-Kutta and BDF
methods) are discussed and analyzed with respect to
convergence and order. A chapter is devoted to index
reduction methods that allow the numerical treatment of
general differential-algebraic equations. The analysis and
numerical solution of boundary value problems for
differential-algebraic equations is presented, including
multiple shooting and collocation methods. A survey of
current software packages for differential-algebraic equations
completes the text.

The book is addressed to graduate students and researchers
in mathematics, engineering and sciences, as well as
practitioners in industry. A prerequisite is a standard course
on the numerical solution of ordinary differential equations.
Numerous examples and exercises make the book suitable as a
course textbook or for self-study.

This item will also be of interest to those working in
applications.

A publication of the European Mathematical Society (EMS). Distributed
within the Americas by the American Mathematical Society.

Contents: I. Analysis of differential-algebraic equations:
Introduction; Linear differential-algebraic equations with
constant coefficients; Linear differential-algebraic equations
with variable coefficients; Nonlinear differential-algebraic
equations; II. Numerical solution of differential-algebraic
equations: Numerical methods for strangeness-free problems;
Numerical methods for index reduction; Boundary value
problems; Software for the numerical solution of differential-
algebraic equations; Final remarks; Bibliography; Index.

EMS Textbooks in Mathematics
February 2006, 392 pages, Hardcover, ISBN 3-03719-017-5,
2000 Mathematics Subject Classification: 34A09, 65L80, All
AMS members US$54, List US$68, Order code EMSTEXT/2
Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the Notices as noted below for each meeting.

Miami, Florida

Florida International University

April 1–2, 2006
Saturday - Sunday

Meeting #1015
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: January 2006
Program first available on AMS website: February 16, 2006
Program issue of electronic Notices: April 2006
Issue of Abstracts: Volume 27, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses
Andrea R. Nahmod, University of Massachusetts, Amherst, Bilinear operators in analysis and PDEs.
Edward Odell, University of Texas at Austin, Embeddings in Banach space theory.
Karen V. H. Parshall, University of Virginia, The British development of the theory of invariants, 1841-1895.
Michael S. Vogelius, Rutgers University, Electromagnetic imaging—An applied analyst’s perspective.

Special Sessions
Approximation Theory and Orthogonal Polynomials, Doron S. Lubinsky, Georgia Institute of Technology, and Edward B. Saff, Vanderbilt University.
Commutative Algebra and Algebraic Geometry, Laura Ghezzi, Florida International University, Huy Tai Ha, Tulane University, and Aron Simis, University Federal de Pernambuco.
Composition Operators and Complex Dynamical Systems, Brian P. Kelly, University of Louisiana, Monroe, and Christopher N. B. Hammond, Connecticut College.
Financial Mathematics, Alec N. Kercheval and Craig A. Nolder, Florida State University.
Geometry of Banach Spaces and Connections with Other Areas, Edward W. Odell, University of Texas at Austin, Thomas B. Schlumprecht, Texas A&M University, and Stephen Dilworth, University of South Carolina.
Geometry of Riemannian Manifolds with Additional Structures, Tedi C. Draghici, Guo V. Grantcharov, and Philippe Rukimbira, Florida International University.
Harmonic Analysis and Partial Differential Equations, Mario Milman, Florida Atlantic University, and Marius Mitrea, University of Missouri.
History of Mathematics, Karen H. Parshall, University of Virginia.
Imaging, Homogenization, and Shape Optimization, Michael S. Vogelius, Rutgers University, and Shari Moskow, University of Florida.
Interpolation Theory and Applications, Michael Cwikel, Technion, Laura De Carli, Florida International University, and Mario Milman, Florida Atlantic University.

Invariants of Low-Dimensional Manifolds, Thomas G. Leness, Florida International University, and Nikolai N. Saveliev, University of Miami, Coral Gables.

Mathematical Models in Image and High-Dimensional Data Analysis, Hanna E. Makaruk and Robert M. Owczarek, Los Alamos National Laboratory, and Nikita Sakhanenko, University of New Mexico and Los Alamos National Laboratory.

Monomials and Resolutions, Joseph P. Brennan, North Dakota State University, and Heath M. Martin, University of Central Florida.

Nonlinear Waves, Andrea R. Nahmod, University of Massachusetts, Amherst, and Sijue Wu, University of Michigan at Ann Arbor.

Partial Differential Equations and Several Complex Variables, Shifera Berhanu, Temple University, and Hamid Meziani, Florida International University.

Qualitative Analysis of Partial Differential Equations, Congming Li, University of Colorado, and Wenxiong Chen, Yeshiva University.

Recent Developments in Fluid and Geophysical Fluid Dynamics, C. Cao and T. Tachim Medjo, Florida International University, and X. Wang, Florida State University.

Singular Integrals, Geometric Analysis, and Free Boundary Problems, Marianne Korten and Charles N. Moore, Kansas State University, and Laura DeCarli, Florida International University.

Spectral Geometry of Manifolds with Boundary and Singular Spaces, Juan B. Gil, Pennsylvania State University, Altoona, and Patrick T. McDonald, New College, University of South Florida.

Structure of Function Spaces and Applications, Jan Lang, The Ohio State University, and Osvaldo Mendez, University of Texas at El Paso.

Notre Dame, Indiana
University of Notre Dame
April 8–9, 2006
Saturday – Sunday
Meeting #1016
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: January 2006
Program first available on AMS website: February 23, 2006
Program issue of electronic Notices: April 2006
Issue of Abstracts: Volume 27, Issue 2

Deadlines
For organizers: Expired
For abstracts: Expired

Invited Addresses

Béla Bollobás, University of Memphis and Cambridge University, Inhomogeneous random graphs (Erdős Memorial Lecture).

Steve C. Hofmann, University of Missouri, Columbia, Local Tb theorems and applications in PDE.

Michael J. Larsen, Indiana University, Representation zeta functions.

Christopher M. Skinner, University of Michigan, Modular forms and special values of L-functions.

Special Sessions
Algebraic Structures of Exactly Solvable Models, Michael Gekhtman, University of Notre Dame, Mikhail Shapiro, Michigan State University, and Alexander Stolin, University of Gothenburg.


Combinatorial Algebraic Geometry, Juan C. Migliore, University of Notre Dame, and Uwe R. Nagel, University of Kentucky.

Commutative Algebra and Algebraic Geometry, Alberto Corso, University of Kentucky, Claudia Polini, University of Notre Dame, and Bernd Ulrich, Purdue University.

Developments and Applications in Differential Geometry, Jianguo Cao, Xiaobo Liu, and Brian Smyth, University of Notre Dame.

Dynamical Systems, François Ledrappier, University of Notre Dame, and Amie Wilkinson, Northwestern University.

Harmonic Analysis, PDE and Geometric Function Theory, John L. Lewis, University of Kentucky, and Steve C. Hofmann, University of Missouri, Columbia.

Holomorphic Methods and Heat Kernels in Harmonic Analysis and Quantization Theory, Brian Hall and William Kirwin, University of Notre Dame.

Mathematical Biology, Mark Alber and Bei Hu, University of Notre Dame.

Model Theory and Computability, Steven Allen Buechler and Julia Knight, University of Notre Dame, Steffen Lempp, University of Wisconsin, and Sergei Starchenko, University of Notre Dame.

New Developments in Optimization, Leonid Faybusovich, University of Notre Dame.

Nonlinear Waves, Mark S. Alber and Pavel Lushnikov, University of Notre Dame, and Ildar Gabitov and Vladimir E. Zakharov, University of Arizona.
Number Theory, Scott T. Parsell and Jonathan P. Sorenson, Butler University.
Numerical Solution of Polynomial Systems, Christopher S. Peterson, Colorado State University, and Andrew J. Sommese, University of Notre Dame.
PDEs and Geometric Analysis, Matt Gursky and Qing Han, University of Notre Dame.
Several Complex Variables, Nancy K. Stanton and Jeffrey A. Diller, University of Notre Dame.
Special Functions and Orthogonal Polynomials, Diego Dominici, State University of New York at New Paltz.
Topics in Representation Theory, Sam Evens, University of Notre Dame, and Jiu-Kang Yu, Purdue University.
Topology and Physics, Stephan A. Stolz and Bruce Williams, University of Notre Dame.
Undergraduate Mathematical Research, Francis X. Connolly, University of Notre Dame, and Zsuzsanna Szaniszlo, Valparaiso University.
Water Waves, David Nicholls, University of Illinois at Chicago.

Durham, New Hampshire
University of New Hampshire
April 22–23, 2006
Saturday – Sunday
Meeting #1017
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: February 2006
Program first available on AMS website: March 9, 2006
Program issue of electronic Notices: April 2006
Issue of Abstracts: Volume 27, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses
Ailana M. Fraser, University of British Columbia, Analytic methods in studying curvature and topology.
Dmitri Nikshych, University of New Hampshire, Algebraic theory of tensor categories.
Florian Pop, University of Pennsylvania, From topological covers to algebraic numbers.
Konstantina Trivisa, University of Maryland, College Park, On the dynamics of binary fluid mixtures.

Special Sessions
Algebraic Groups, George J. McNinch, Tufts University, and Eric Sommers, University of Massachusetts, Amherst.
Arithmetic Geometry and Modular Forms, Paul E. Gunnells and Farshid Hajir, University of Massachusetts, Amherst.
Arrangements and Configuration Spaces, Graham C. Denham, University of Western Ontario, and Alexander I. Suciu, Northeastern University.
Banach Spaces of Analytic Functions, Rita A. Hibschweiler, University of New Hampshire, and Thomas H. MacGregor, SUNY Albany and Bowdoin College.
Discrete and Convex Geometry, Daniel A. Klain, University of Massachusetts, Lowell, Barry R. Monson, University of New Brunswick, and Egon Schulte, Northeastern University.
Galois Theory in Arithmetic and Geometry, Florian Pop and David Harbater, University of Pennsylvania, and Rachel J. Pries, Colorado State University.
Global Perspectives on the Geometry of Riemann Surfaces, Eran Makover and Jeffrey K. McGowan, Central Connecticut State University.
Hopf Algebras and Galois Module Theory, Timothy Kohl, Boston University, and Robert G. Underwood, Auburn University Montgomery.
Mathematical Challenges in Physical and Engineering Sciences, Marianna A. Shubov, University of New Hampshire.
Quantum Invariants of Knots and 3-Manifolds, Charles D. Frohman, University of Iowa, and Razvan Gelca, Texas Tech University.
Symplectic and Contact Topology, Weimin Chen, Michael G. Sullivan, and Hao Wu, University of Massachusetts, Amherst.
San Francisco, California
San Francisco State University

April 29-30, 2006
Saturday - Sunday

Meeting #1018
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: February 2006
Program first available on AMS website: March 16, 2006
Program issue of electronic Notices: April 2006
Issue of Abstracts: Volume 27, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses
Lincoln Chayes, University of California Los Angeles, Mean-field approach to the problem of phase transitions in physically realistic systems.
C. Robin Graham, University of Washington, Ambient metrics, jet isomorphism and parabolic invariant theory in conformal geometry.
Vadim Kaloshin, California Institute of Technology, Non-local instabilities of the planar three body problem.
Benoit B. Mandelbrot, Yale University, The nature of roughness in mathematics, science, and art (Einstein Public Lecture in Mathematics).
Yuval Peres, University of California Berkeley, Hex, random-turn games, and the infinity Laplacian.

Special Sessions
Enumerative Aspects of Polytopes, Federico Ardila and Matthias Beck, San Francisco State University.
Fractal Geometry: Connections to Dynamics, Geometric Measure Theory, Mathematical Physics and Number Theory, Michel L. Lapidus and Erin P. Pearse, University of California Riverside, and Machiel van Frankenhuijsen, Utah Valley State College.
Geometric Dynamics and Ergodic Theory, Yitwah Cheung and Arak Goetz, San Francisco State University, and Slobodan Simic, San Jose State University.

Salt Lake City, Utah
University of Utah

October 7-8, 2006
Saturday - Sunday

Meeting #1019
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: August 2006
Program first available on AMS website: August 24, 2006
Program issue of electronic Notices: October 2006
Issue of Abstracts: Volume 27, Issue 3

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: June 20, 2006
For abstracts: August 15, 2006

Invited Addresses
William Arveson, University of California Berkeley, Title to be announced.
Alexei Borodin, California Institute of Technology, Title to be announced.
Izabella Joanna Laba, University of British Columbia, Title to be announced.
Darren Long, University of California Santa Barbara, Title to be announced.

Special Sessions

Commutative Algebra (Code: SS 3A), Paul Roberts, Anurag K. Singh, and Oana Veliche, University of Utah.

Complex Geometry, Kaehler Groups, and Related Topics (Code: SS 9A), Terrence Napier, Lehigh University, Mohan Ramachandran, State University of New York at Buffalo, and Domingo Toledo, University of Utah.

Floer Methods in Low-dimensional Topology (Code: SS 8A), Alexander Felshyn and Uwe Kaiser, Boise State University.

Harmonic Analysis: Trends and Perspectives (Code: SS 1A), Alex Iosevich, University of Missouri, and Michael T. Lacey, Georgia Institute of Technology.

Interface of Stochastic Partial Differential Equations and Gaussian Analysis (Code: SS 7A), Dava Khoshnevisan, University of Utah, and Evelia J. Nualart, University of Paris XIII.

Low Dimensional Topology and Geometry (Code: SS 4A), Mladen Bestvina and Kenneth W. Bromberg, University of Utah.

Mathematics Motivated by Physics (Code: SS 5A), Aaron J. Bertram, Yuan-Pin Lee, and Eric R. Sharpe, University of Utah.


Nonconvex Variational Problems: Recent Advances and Applications (Code: SS 10A), Marian Bocca, North Dakota State University, and Andrej Cherkaev, University of Utah.

Nonlinear Differential Equations: Methods and Applications (Code: SS 2A), David G. Costa, University of Nevada, and Zhi-Qiang Wang, Utah State University.

Random Motion in Random Media (Code: SS 11A), Firas Rassoul-Agha, University of Utah, and Tom Schmitz, Swiss Federal Institute of Technology.


Cincinnati, Ohio

University of Cincinnati

October 21–22, 2006

Saturday - Sunday

Meeting #1020

Central Section

Associate secretary: Susan J. Friedlander

Announcement issue of Notices: August 2006

Program first available on AMS website: September 7, 2006

Program issue of electronic Notices: October 2006

Issue of Abstracts: Volume 27, Issue 3

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: July 5, 2006

For abstracts: August 29, 2006

Invited Addresses

Suncica Canic, University of Houston, Title to be announced.

Bryna R. Kra, Northwestern University, Title to be announced.

Ezra N. Miller, University of Minnesota, Title to be announced.

Jon G. Wolfson, Michigan State University, Title to be announced.

Special Sessions

Algebraic Coding Theory—Honoring the Retirement of Vera Pless (Code: SS 8A), William Cary Huffman, Loyola University, and Jon-Lark Kim, University of Louisville.

Analysis and Potential Theory on Metric Spaces (Code: SS 4A), Thomas Bieske, University of South Florida, and Zair Ibragimov and Nageswari Shanmugalingam, University of Cincinnati.

Applied Algebraic Geometry and Cryptography (Code: SS 3A), Jintai Ding, Jason Eric Gower, and Timothy J. Hodges, University of Cincinnati, Lei Hu, Chinese Academy of Sciences, and Dieter S. Schmidt, University of Cincinnati.

Birational Geometry (Code: SS 2A), Mirel Constantin Caibar and Gary P. Kennedy, Ohio State University.

Boundary Value Problems for Differential Equations with Applications (Code: SS 11A), Xiaojie Hou, Philip L. Korman, and Bingyu Zhang, University of Cincinnati.

Ergodic Theory (Code: SS 1A), Nikos Frantzikinakis, Pennsylvania State University, Bryna R. Kra, Northwestern University, and Mate Wierdl, University of Memphis.

Geometric Combinatorics (Code: SS 6A), Ezra N. Miller, University of Minnesota, and Igor Pak, Massachusetts Institute of Technology.
Meetings & Conferences

Limit Theorems of Probability Theory (Code: SS 9A), Wlodek Bryc and Magda Peligrad, University of Cincinnati.

Nonlinear Functional Analysis and Applications (Code: SS 5A), S. P. Singh and Bruce Watson, Memorial University of Newfoundland.

Nonlinear Partial Differential and Its Applications (Code: SS 7A), Changyou Wang, University of Kentucky, and Guan Bo, Ohio State University.

Recent Results on Operator Algebras (Code: SS lOA), Herbert Halpern, Gary Weiss, Costel Peligrad, Shuang Zhang, and Victor G. Kaftal, University of Cincinnati.

Storrs, Connecticut
University of Connecticut
October 28-29, 2006
Saturday - Sunday
Meeting #1021
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: August 2006
Program first available on AMS website: September 14, 2006
Program issue of electronic Notices: October 2006
Issue of Abstracts: Volume 27, Issue 4

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: July 11, 2006
For abstracts: September 6, 2006

Invited Addresses
Changfeng Gui, University of Connecticut, Storrs, Title to be announced.
Niranjan Ramachandran, University of Maryland, College Park, Title to be announced.
Kannan Soundararajan, University of Michigan, Title to be announced.
Katrin Wehrheim, Institute for Advanced Study, Title to be announced.

Special Sessions
Analysis and Probability on Fractals (Code: SS 3A), Robert S. Strichartz, Cornell University, and Alexander Teplyaev, University of Connecticut, Storrs.
Combinatorial Methods in Equivariant Topology (Code: SS 1A), Tara Holm, University of Connecticut, Storrs, and Tom C. Braden, University of Massachusetts, Amherst.
Computability Theory in Honor of Manuel Lerman's Retirement (Code: SS 4A), Joseph S. Miller and David Reed Solomon, University of Connecticut, Storrs.

Nonlinear Elliptic and Parabolic Equations (Code: SS 5A), Yung-Sze Choi, Changfeng Gui, and Joseph McKenna, University of Connecticut, Storrs.
Number Theory (Code: SS 2A), Keith Conrad, University of Connecticut, Storrs, David Pollack, Wesleyan University, and Thomas A. Weston, University of Massachusetts, Amherst.
Undergraduate Mathematics Education (Code: SS 8A), Tom Roby, University of Connecticut, Storrs.

Fayetteville, Arkansas
University of Arkansas
November 3-4, 2006
Friday - Saturday
Meeting #1022
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: September 2006
Program first available on AMS website: September 21, 2006
Program issue of electronic Notices: November 2006
Issue of Abstracts: Volume 27, Issue 4

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: July 18, 2006
For abstracts: September 12, 2006

Invited Addresses
Richard P. Anstee, University of British Columbia, Title to be announced.
Arun Ram, University of Wisconsin, Title to be announced.
Donald G. Saari, University of California Irvine, Title to be announced.
Andras Vasy, Massachusetts Institute of Technology, Title to be announced.

Special Sessions
Analytic Number Theory and Modular Forms (Code: SS 2A), Matthew Boylan, University of Illinois, Urbana-Champaign, and Gang Yu, University of South Carolina.
Boundary Operators in Real and Complex Domains (Code: SS 3A), Loredana Lanzani, University of Arkansas, Fayetteville, and David E. Barrett, University of Michigan, Ann Arbor.
Dirac Operators in Analysis and Geometry (Code: SS 1A),
John Ryan, University of Arkansas, Marius Mitrea, University of Missouri, and Mircea Martin, Baker University.

Evolution Equations in Physics and Mechanics (Code: SS 4A),
John P. Albert, University of Oklahoma, Jerry L. Bona, University of Illinois at Chicago, and Jiahong Wu, Oklahoma State University.

New Orleans, Louisiana
New Orleans Marriott and Sheraton New Orleans Hotel
January 5-8, 2007
Friday - Monday

Meeting #1023
Joint Mathematics Meetings, including the 113th Annual Meeting of the AMS, 90th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: October 2006
Program first available on AMS website: November 1, 2006
Program issue of electronic Notices: January 2007
Issue of Abstracts: Volume 28, Issue 1

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: August 1, 2006
For abstracts: September 26, 2006

Davidson, North Carolina
Davidson College
March 3-4, 2007
Saturday - Sunday

Meeting #1024
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 14, 2006
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Oxford, Ohio
Miami University
March 16-17, 2007
Friday - Saturday

Meeting #1025
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions
Finite Geometry and Combinatorics (Code: SS 3A), Mark A. Miller, Marietta College.
Geometric Topology (Code: SS 2A), Jean-François Lafont, SUNY Binghamton and Ohio State University, and Ivonne J. Ortiz, Miami University.
Large Cardinals in Set Theory (Code: SS 1A), Paul B. Larson, Miami University, Justin Tatch Moore, Boise State University, and Ernest Schimmerling, Carnegie Mellon University.

Hoboken, New Jersey
Stevens Institute of Technology
April 14-15, 2007
Saturday - Sunday

Meeting #1026
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 14, 2006
Meetings & Conferences

For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Tucson, Arizona

University of Arizona

April 21–22, 2007
Saturday–Sunday

Meeting #1027
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 21, 2006
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Warsaw, Poland

University of Warsaw

July 31–August 3, 2007
Tuesday–Friday

Meeting #1028
First Joint International Meeting between the AMS and the
Polish Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Invited Addresses
Henryk Iwaniec, To be announced.
Tomasz Mrowka, To be announced.
Madhu Sudan, To be announced.

Albuquerque, New Mexico

University of New Mexico

October 13–14, 2007
Saturday–Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Murfreesboro, Tennessee

Middle Tennessee State University

November 3–4, 2007
Saturday–Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: April 4, 2007
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center

January 6–9, 2008
Sunday–Wednesday
Joint Mathematics Meetings, including the 114th Annual
Meeting of the AMS, 91st Annual Meeting of the Mathemat­
cal Association of America (MAA), annual meetings of
the Association for Women in Mathematics (AWM) and the
National Association of Mathematicians (NAM), and the
winter meeting of the Association for Symbolic Logic (ASL),
with sessions contributed by the Society for Industrial and
Applied Mathematics (SIAM).
**Bloomington, Indiana**

*Indiana University*

**April 4–6, 2008**

*Friday – Sunday*

Central Section

**Deadlines**

For organizers: September 4, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Claremont, California**

*Claremont McKenna College*

**May 3–4, 2008**

*Saturday – Sunday*

Southeastern Section

**Deadlines**

For organizers: October 4, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Rio de Janeiro, Brazil**

*Instituto Nacional de Matemática Pura e Aplicada (IMPA)*

**June 4–7, 2008**

*Wednesday – Saturday*

First Joint International Meeting between the AMS and the Sociedade Brasileira de Matemática.

**Deadlines**

For organizers: To be announced

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**New York, New York**

*Courant Institute of New York University*

**March 22–23, 2008**

*Saturday – Sunday*

Eastern Section

**Deadlines**

For organizers: August 22, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Baton Rouge, Louisiana**

*Louisiana State University, Baton Rouge*

**March 28–30, 2008**

*Friday – Sunday*

Southeastern Section

**Deadlines**

For organizers: August 28, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Central Section**

**Deadlines**

For organizers: April 1, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Southeastern Section**

**Deadlines**

For organizers: October 4, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Central Section**

**Deadlines**

For organizers: September 4, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Southeastern Section**

**Deadlines**

For organizers: August 22, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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**Southeastern Section**

**Deadlines**

For organizers: August 28, 2007

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

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Meetings & Conferences

For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Vancouver, Canada
University of British Columbia

October 4–5, 2008
Saturday - Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 9, 2008
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Huntsville, Alabama
University of Alabama, Huntsville

October 24–26, 2008
Friday - Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 24, 2008
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Shanghai, People's Republic of China
Fudan University

December 17–21, 2008
Wednesday - Sunday
First Joint International Meeting between the AMS and the Shanghai Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic Notices: Not applicable
Issue of Abstracts: Not applicable

Deadlines
For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

San Francisco, California
Moscone Center West and the San Francisco Marriott

January 6–9, 2010
Wednesday - Saturday
Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of Notices: October 2009
Program first available on AMS website: Not applicable
Program issue of electronic Notices: January 2009
Issue of Abstracts: Volume 30, Issue 1

Deadlines
For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced
Program first available on AMS website: November 1, 2009
Program issue of electronic Notices: January 2010
Issue of Abstracts: Volume 31, Issue 1

**Deadlines**
- For organizers: April 1, 2009
- For consideration of contributed papers in Special Sessions: To be announced
- For abstracts: To be announced

**New Orleans, Louisiana**

*New Orleans Marriott and Sheraton New Orleans Hotel*

**January 5-8, 2011**

*Wednesday - Saturday*

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Susan J. Friedlander
Announcement issue of Notices: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic Notices: January 2011
Issue of Abstracts: Volume 32, Issue 1

**Deadlines**
- For organizers: April 1, 2010
- For consideration of contributed papers in Special Sessions: To be announced
- For abstracts: To be announced

**San Diego, California**

*San Diego Convention Center and San Diego Marriott Hotel and Marina*

**January 9-12, 2013**

*Wednesday - Saturday*

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

**Deadlines**
- For organizers: April 1, 2012
- For consideration of contributed papers in Special Sessions: To be announced
- For abstracts: To be announced

**Boston, Massachusetts**

*John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel*

**January 4-7, 2012**

*Wednesday - Saturday*

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of Notices: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic Notices: January 2012
Issue of Abstracts: Volume 33, Issue 1

**Deadlines**
- For organizers: April 1, 2011
- For consideration of contributed papers in Special Sessions: To be announced
- For abstracts: To be announced
Whether they become scientists, engineers, or entrepreneurs, young people with mathematical talent need to be nurtured. Income from this fund supports the Young Scholars Program, which provides grants to summer programs for talented high school students.

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### Meetings and Conferences of the AMS

**Associate Secretaries of the AMS**

- **Western Section:** Michael L. Lapidus, Department of Mathematics, University of California, Riverside; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.
- **Central Section:** Susan J. Friedlander, Department of Mathematics, University of Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.
- **Eastern Section:** Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: llsibner@duke.poly.edu; telephone: 718-260-3505.
- **Southeastern Section:** Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated.**

**Up-to-date meeting and conference information can be found at** [www.ams.org/meetings/](http://www.ams.org/meetings/).

#### Meetings:

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#### Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 296 in the February 2006 issue of the Notices for general information regarding participation in AMS meetings and conferences.

**Abstracts**

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX{} is necessary to submit an electronic form, although those who use \LaTeX{} may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX{}. Visit [http://www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl). Questions about abstracts and requests for paper forms may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

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**Conferences:** (see [http://www.ams.org/meetings/](http://www.ams.org/meetings/) for the most up-to-date information on these conferences.)

June 4-June 29, 2006: Joint Summer Research Conferences, Snowbird, Utah (see November 2005 Notices, page 1296).


**Recent advances in nonlinear partial differential equations and applications:** A conference in honor of Peter D. Lax and Louis Nirenberg, June 7-10, 2006, Toledo, Spain. For more details see [http://www.mat.ucm.es/~lm06/](http://www.mat.ucm.es/~lm06/).

Free Ideal Rings and Localization in General Rings
Paul Cohn
This book presents the theory of free ideal rings (firs) in detail. There is also a full account of localization, which is treated for general rings, but the features arising in firs are given special attention.
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$140.00*: Hardback: 0-521-85337-0: 704pp

Heights in Diophantine Geometry
Enrico Bombieri and Walter Gubler
Diophantine geometry has been studied by number theorists since the time of Pythagoras and is a rich area of ideas, including Fermat’s Last Theorem and the ABC conjecture. This monograph is a bridge between the classical theory and modern approach via arithmetic geometry.
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$130.00*: Hardback: 0-521-86139-0: 668pp

Foundations of Computational Mathematics, Santander 2005
Edited by Luis Pujo, Allan Pinkus, Endre Süli, and Mike Todd
This volume is a collection of articles based on the plenary talks presented at the 2005 meeting in Santander of the Society for the Foundations of Computational Mathematics. The topics covered reflect the breadth of research within the area as well as the richness and fertility of interactions between seemingly unrelated branches of pure and applied mathematics.
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$75.00*: Paperback: 0-521-60161-8

Fundamentals of Hyperbolic Manifolds
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Classical and Quantum Orthogonal Polynomials in One Variable
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Encyclopedia of Mathematics and its Applications
$140.00*: Hardback: 0-521-78201-5: 724pp

Geometric Foundations of Computational Geometry
Michael P. Faustino
Geometric foundations of computational geometry
$75.00*: Hardback: 0-521-68161-8

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Oriented matroids play the role of matrices in discrete geometry, when metrical properties, such as angles or distances, are neither required nor available. The combination of concrete applications and computation, the profusion of illustrations, and a large number of examples and exercises make this an ideal introductory text on the subject.
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Extending Mechanics to Minds
The Mechanical Foundations of Psychology and Economics
Jon Doyle
This book deploys the mathematical axioms of modern rational mechanics to understand minds as mechanical systems that exhibit forces, inertia, and motion. Using precise mental models developed in artificial intelligence, the author analyzes motivation, attention, reasoning, learning, and communication in these mechanical terms.
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