Interview with Ian Agol

Photo courtesy of Ian Agol.

Editor's Note: *Since this interview, Agol has won the three million dollar Breakthrough Prize in Mathematics.*

Ian Agol, Professor of Mathematics at the University of California, Berkeley, proved the virtual Haken Conjecture in 2012. This interview was conducted by email by Alexander Diaz-Lopez.

Diaz-Lopez. *When did you know you wanted to be a mathematician?*

Agol. I was interested in physics and math entering Caltech, and I had mapped out the courses I would need to take in order to take string theory as a senior. However, I did poorly in an advanced physics class as a sophomore. Hence I decided to become a mathematics major. I also preferred the rigor of mathematics over the approximations and heuristics of physics.

Diaz-Lopez. *Who encouraged or inspired you?*

Agol. I had excellent mathematics teachers all throughout my education, but my geometry teacher Spreck Rosecrans my first year in high school inspired me by giving me extra problems to work on.

Diaz-Lopez. *How would you describe your research to a graduate student?*

Agol. I study 3-dimensional manifolds, with an emphasis on manifolds admitting hyperbolic metrics. I was interested in knot theory as a first-year graduate student, but drifted towards 3-manifold theory when I learned that Thurston's geometrization theorem allowed a complete and practical classification of knots. As my career has progressed, I have delved into deeper subjects in mathematics, including Riemannian geometry and geometric group theory, in order to solve questions in 3-manifold topology. Most of the revolutions in the subject have come from other fields (dynamics, partial differential equations, mathematical physics, algebraic geometry, symplectic geometry, number theory), which is exciting because it requires one to continuously learn new mathematics, giving one new perspectives on the topic.

Diaz-Lopez. *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

Agol. A few years ago, work I did resolved the virtual Haken conjecture. This work relied on expertise of my collaborators Daniel Groves and Jason Manning, as well as on the much deeper work of Dani Wise and his collaborators (especially Haglund, Hsu, and Sageev) and the work of Kahn and Markovic. One remarkable thing about this topic is the way that it married quite disparate techniques and relied on results from dynamics and deep ideas in geometric group theory initiated by Gromov. Anyway, even though the question was regarding a problem in 3-manifold topology, the method of resolution (whose strategy was formulated by Wise) went into a broader category of word-hyperbolic groups and high-dimensional cube complexes. Nevertheless, some of my key contributions to the problem were to extend the intuition I had developed regarding hyperbolic 3-manifolds to the broader categories required to solve the problem, such as hyperbolic Dehn filling (in the joint work with Groves and Manning), geometrically finite Kleinian groups, and hierarchies for Haken 3-manifolds. Moreover, it seems that the broader category was necessary to solve the problem, since one could carry out certain inductive arguments, which would not have been possible considering only the category of hyperbolic 3-manifolds. So I would say that the most important idea in the proof was to port 3-manifold techniques and the intuition developed regarding them over to geometric group theory, although I should make it clear that I wasn't the first to develop these generalizations, but was building on the work of many others.

"I suggest spending part of your time thinking about very hard problems"

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Diaz-Lopez. *What advice do you have for graduate students?*

Agol. I suggest spending part of your time thinking about very hard problems (sure, even the Riemann hypothesis). It might not seem like it, but in graduate school you have much more time than you will as a postdoc or professor (if that's your career goal), and you can afford to waste some time on difficult problems (if you don't mind the life of a graduate student). You might even solve it! As a graduate student, I spent some time thinking about the Poincaré conjecture and questions related to the virtual Haken conjecture, as well as the linearity of braid groups. I also thought about a problem about energy of links posed by my advisor, which I didn't solve, as well as many other problems that seemed more accessible at the time. Eventually I found a do-able problem and wrote a thesis. Even though it was frustrating to get nowhere on the harder problems, the time I spent turned out to be extremely valuable in my career, because when progress was made by others, I could immediately appreciate the difficulty of what was done. Moreover, the problems I listed above have now all been solved by me or others. The problem on energy of links that I originally thought was simpler than the other problems turned out to be solved historically last (using the remarkable insights of Marques and Neves).

Diaz-Lopez. *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

Agol. Yes, one thing about math research is that, unless you are not challenging yourself enough, you spend most of your time not solving problems, which can be frustrating. Having collaborators can be quite helpful, partly to vent your emotions. But also verbal discussion of mathematics puts you in a different frame of mind. Together with the useful comments of a mathematician who may have a different knowledge base, this can help outline the difficulties and sometimes lead to new avenues to pursue.

Diaz-Lopez. *You have won several honors and awards. Which one has been the most meaningful and why?*

Agol. The Senior Berwick Prize. Although I couldn't make it to the UK for the award presentation, it is meaningful because it was a prize for a paper in the *Journal of Topology* regarding Thurston's virtual fibering conjecture. The *Journal of Topology* came into existence after the editorial board of *Topology* resigned because they didn't approve of the publishing practices of Elsevier, and they established the new *Journal of Topology* to replace it, published by the London Mathematical Society. This was risky, because it takes time for a new journal to build a reputation. I admired the board's action, and thus to support the journal, I decided to submit the paper, rather than submit it to a more-established journal. Thus, I was pleased to receive some recognition for the paper, which certainly justified publishing there. Now I am on the editorial board!

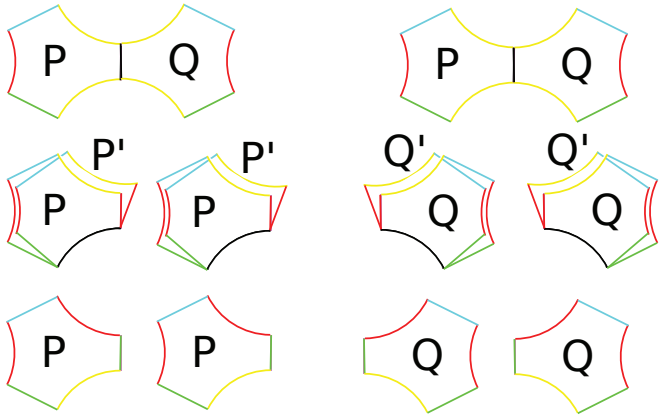

Image courtesy of Ian Agol.

An illustration of one step in the proof of the virtual Haken conjecture, showing how to "reverse-engineer" a hierarchy of manifolds. See "Virtual properties of 3-manifolds", by Ian Agol, Proceedings of the International Congress of Mathematicians, 2014, `www.icm2014.org/en/vod/proceedings`. See also "Getting Into Shapes: From Hyperbolic Geometry to Cube Complexes and Back", by Erica Klarreich, Quanta, October 2, 2012.

Diaz-Lopez. *If you were not a mathematician, what would you be?*

Agol. I have an identical twin brother Eric who is an astrophysicist, so I could certainly see myself as a physicist, but only of the theoretical sort - I did terribly in lab classes.

Diaz-Lopez. *If you could ask a question to one famous mathematician, who would it be and what would you ask?*

Agol. Let's imagine we could resurrect Poincaré and explain to him the resolution of his conjecture (however long that might take). Then I would ask him what he thought of the solution, and whether he was surprised that it took so long to discover.

Diaz-Lopez. *If you could recommend one lecture (book, paper, article, etc.) to graduate students, what would it be?*

Agol. I'm fond of the paper "Shapes of polyhedra and triangulations of the sphere" by William Thurston. It's not his most difficult result, but it's insightful, and it makes it unnecessary to read the Deligne-Mostow papers, which I find quite difficult.

Alexander Diaz-Lopez is a PhD student at the University of Notre Dame. Diaz-Lopez is the first graduate student member of the *Notices* Editorial Board.