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December 2016

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December 2016

## FEATURES



Siegel's Problem in Three Dimensions
An Introduction to the AMSNZMS Maclaurin Lecture "Siegel's Problem on Small Volume Lattices"
by Gaven J. Martin


The Graduate Student Section
Interview with Gigliola Staffilani by Alexander Diaz-Lopez
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## A Conversation with Helen Grundman, AMS Director of Education and Diversity

by Stephen Kennedy

In these last few pages of 2016, we feature Siegel's problem and the Graduate Student Section: an interview of Gigliola Staffilani; "What is Symplectic Geometry"; an interview with Helen Grundman, the new and first AMS Director of Education and Diversity; and the latest "My Professor" comic strip, responded to on our BackPage by a new "My TA" comic strip. From October through December the BackPage moves to immediately after the Graduate Student Section in order to feature the premier January Joint Mathematics Meetings at the end of the issue. -Frank Morgan, Editor-inChief

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## Game Theory A Playful Introduction

Matt DeVos, Simon Fraser University, Burnaby, BC, Canada, and Deborah A. Kent, Drake University, Des Moines, IA

This book offers a gentle introduction to the mathematics of both sides of game theory: combinatorial and classical. The combination allows for a dynamic and rich tour of the subject united by a common theme of strategic reasoning. Designed as a textbook for an undergraduate mathematics class and with ample material and limited dependencies between the chapters, the book is adaptable to a variety of situations and a range of audiences. Instructors, students, and independent readers alike will appreciate the flexibility in content choices as well as the generous sets of exercises at various levels.

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# Siegel's Problem in Three Dimensions 


#### Abstract

Editor's note: Gaven Martin kindly agreed to write for us an introduction to the story he presented in his invited lecture "Siegel's Problem on Small Volume Lattices" at the November AMS Fall Southeastern Sectional Meeting held at North Carolina State University at Raleigh.


## Gaven J. Martin

AbStract. We discuss our recent solution to Siegel's 1943 problem concerning the smallest co-volume lattices of hyperbolic 3-space.

Over the last few decades the theory of Kleinian groups-discrete groups of isometries of hyperbolic 3-space-has flourished because of its intimate connections with low-dimensional topology and geometry and has been inspired by the discoveries of W.P. Thurston. The culmination must certainly be Perelman's proof of Thurston's geometrisation conjecture, which states that compact 3-manifolds can be decomposed canonically into pieces that have

[^0]geometric structures and that the "generic" piece is hyperbolic. This is an analogue for 3-manifolds of the uniformisation theorem for surfaces and implies, for instance, the Poincaré conjecture. There have been many other recent advances. These include the proofs of the density conjecture of Agol (2004) and of Calegari and Gabai (2006); the ending lamination conjecture of Brock, Canary, and Minsky (2012); the surface subgroup conjecture of Kahn and Markovich (2012); and the virtual Haken conjecture of Agol, Groves, and Manning (2013). While we will not discuss these results here (nor even offer statements of theorems), together they give a remarkably complete picture of the structure of hyperbolic group actions and their quotient spaces-hyperbolic manifolds and orbifolds-in three dimensions.

Here we report on joint work with F. W. Gehring and T. H. Marshall [2], [3] solving an old problem of C. L. Siegel.

A lattice $\Gamma$ is the group associated with a tessellation of hyperbolic $n$-space $\mathbb{H}^{n}$ by a finite hyperbolic volume tile (fundamental domain), and an orbifold is the quotient space $\mathbb{H}^{n} / \Gamma$-a hyperbolic manifold $\Gamma$ should be torsion free. The volume of the fundamental domain is the volume


Figure 1. Siegel proved that the (2,3, 7)-tessellation of the hyperbolic plane is the unique lattice of minimal co-area.
of the orbifold $\mathbb{H}^{n} / \Gamma$, equivalently, the co-volume of the lattice $\Gamma$.

In 1943 Siegel posed the problem of identifying the minimal co-volume lattices $\Gamma$ of isometries of hyperbolic $n$-space $\mathbb{H}^{n}$ or more general rank-1 symmetric spaces. For Euclidean lattices (Bieberbach or crystallographic groups) such an infimum is obviously equal to 0 , as one can tessellate $\mathbb{R}^{n}$ by arbitrarily small hypercubes. Siegel solved the problem in two dimensions identifying the $(2,3,7)$-triangle group as the unique lattice of minimal co-area $\frac{\pi}{42}$, as illustrated in Figure 1.

Siegel in fact proved what has come to be known as the signature formula from which one may deduce the complete spectrum of co-areas of lattices of the hyperbolic plane. Každan and Margulis showed in 1968 that for each $n$ the infimum of the co-volume of lattices is positive and achieved. The set of lattice volumes is discrete in $\mathbb{R}$ for $n \geq 4$ (Wang 1972), and Thurston-Jørgensen show this set is of type $\omega^{\omega}$ when $n=3$.

At the time of Siegel's result the theory of covering spaces was not well developed, and he could only suggest a connection between minimal co-area lattices and Hurwitz's $84 g-84$ theorem of 1892 , bounding the order of the symmetry group of a Riemann surface in terms of its genus. This suggested connection was due to the many close analogies and ideas used in the proof and was confirmed by Macbeath in 1961. Selberg's lemma (1960) established the existence of torsion-free subgroups of finite-index in hyperbolic lattices, among other things. As a consequence of the Mostow rigidity theorem, the $84 g-84$ theorem in higher dimensions takes its expression in terms of bounding the order of the symmetry group of a hyperbolic manifold by its volume. Thus the largest symmetry groups arise as the finite quotients of the smallest co-volume lattices. This is interesting in light of Kojima's result (1988) that every finite group can be realized as the full isometry group of some closed hyperbolic 3-manifold.

At the same time as the excitement over the connections between 3-manifold theory and hyperbolic geometry was
running, the theory of arithmetic Kleinian groups was developing with many similar objectives. Due to formulas of A. Borel (1981) various explicit calculations can be made in the case a Kleinian group is arithmetic; for example, one can determine the maximal arithmetic Kleinian group in which it embeds. This makes the associated manifolds and orbifolds amenable to the use of techniques from algebra and number theory, as well as the well-developed topological, analytical, and geometric tools. Earlier work with Gehring, Maclachlan, and Reid (1997) motivated by this problem gave explicit criteria to determine when a two-generator Kleinian group is arithmetic. It turns out that nearly all the extremal problems one might formulate are realised by arithmetic groups: the number theory forces additional symmetries in a group, making it "smaller." Indeed many of these extremals are arithmetic and generated by two elements of finite order. With C. Maclachlan we proved that there are only finitely many such arithmetic groups, while the orbifold Dehn surgeries on two-bridge links give infinitely many nonarithmetic examples. It would seem this finite family of arithmetic groups contains all the extremal groups. The minimal co-volume lattice, the minimal co-volume nonuniform lattice, the minimal co-volume hyperbolic manifold, and the minimal volume noncompact manifold are all twogenerator and arithmetic.

The solution to Siegel's problem largely revolves around an understanding of the geometry of the two-generator subgroups of a lattice $\Gamma \subset \operatorname{Isom}^{+}\left(\mathbb{M}^{3}\right) \simeq \operatorname{PSL}(2, \mathbb{C})$. As $\operatorname{PSL}(2, \mathbb{C})$ has three complex dimensions, a two-generator subgroup $\langle f, g\rangle$ is determined uniquely up to conjugacy by the parameters
$\gamma(f, g)=\operatorname{tr}[f, g]-2, \beta(f)=\operatorname{tr}^{2}(f)-4$, and $\beta(g)=\operatorname{tr}^{2}(g)-4$ as long as $\gamma \neq 0$. This exceptional case corresponds to the elementary groups which are classified. These parameters encode important geometric information. If $\beta(f) \neq 0$, then the isometry $f$ stabilises a hyperbolic line $\ell$, the axis of $f$; translates along this axis by hyperbolic length $\tau$; and in doing so rotates about $\ell$ with a holonomy twist $\theta$; and $\beta=4 \sinh ^{2}[(\tau+i \theta) / 2]$. If the hyperbolic distance between the axes of $f$ and $g$ is $\delta$ and if the angle these axes make along the common perpendicular is $\phi$, then $\sinh ^{2}(\delta+i \phi)=4 \gamma(f, g) / \beta(f) \beta(g)$. A lattice in $\operatorname{PSL}(2, \mathbb{C})$ acting on $\mathbb{T}^{3}$ admits a finite-sided hyperbolic polyhedron as a fundamental domain, and the trace spectrum

$$
\operatorname{tr}(\Gamma)=\{ \pm \operatorname{tr}(g): g \in \Gamma\}
$$

is discrete in $\mathbb{C}$. We seek to describe the parameter space of two-generator discrete groups (in $\mathbb{C}^{3}$ ) using an intriguing semigroup of polynomial trace identities found by us earlier. For instance, iterating the trace identity $\gamma\left(f, g f g^{-1}\right)=\gamma(f, g)(\gamma(f, g)-\beta(f))$ implies that $\left\{\gamma_{i}\right\}_{i=1}^{\infty} \subset \operatorname{tr}(\Gamma)$, where $\gamma_{0}=\gamma(f, g), \gamma_{i+1}=\gamma_{i}\left(\gamma_{i}-\beta\right)$, and $\beta=\beta(f)$. The set $\left\{\gamma_{i}\right\}_{i=1}^{\infty}$ is discrete, so $\gamma_{0}$ cannot lie in a bounded component of the Fatou set of the polynomial $p_{\beta}(z)=Z(z-\beta)$ unless it is preperiodic. If $|\beta|<1$, then 0 is an attracting fixed point and the disk $D=D(0,1-|\beta|)$ lies in the basin of attraction. Thus $\gamma_{0} \notin D$ unless it is preperiodic, and groups with
$\gamma_{0}$ preperiodic are elementary. We deduce that for a nonelementary discrete group $\langle f, g\rangle$ we have $|\gamma(f, g)|+$ $|\beta(f)| \geq 1$ and by symmetry $|\gamma(f, g)|+|\beta(g)| \geq 1$. This is Jørgensen's inequality and identifies a region in $\mathbb{C}^{3}$ where parameters for discrete nonelementary groups cannot be found. Similarly, the identity $\gamma\left(f, g f g^{-1} f^{-1} g\right)=$ $\gamma(f, g)(1+\beta(f)-\gamma(f, g))^{2}$ gives the inequality $|1+\beta(f)|+$ $|\gamma(f, g)| \geq 1$ unless $\gamma(f, g)=1+\beta(f)$ and $\langle f, g\rangle$ is Nielson equivalent to a group generated by elements of orders two and three. The latter fact comes from an analysis of the preperiodic points of the polynomial $q_{\beta}(z)=z(1+\beta-z)^{2}$. We find new inequalities using other identities and considering other attracting basins. This interplay between holomorphic dynamics and twogenerator discrete groups is remarkable in itself. Indeed, generalising Jørgensen's inequality using recursion led to the very first, albeit quite primitive, picture of the Mandelbrot set by Brooks and Matelski (1978).

We describe heuristically how these inequalities provide geometric information. If $g=h f h^{-1}$ is a translate of $f$, then $\beta=\beta(f)=\beta(g)$. If $\delta$ is the distance between the axis $\ell$ of $f$ and $h(\ell)$ the axis of $g$, and supposing $\langle f, h\rangle$ is nonelementary, we deduce from the formulas above and Jørgensen's inequality that

$$
\begin{equation*}
\cosh ^{2}(\delta) \geq \frac{4(1-|\beta|)}{|\beta|^{2}} \tag{1}
\end{equation*}
$$

Using the classification of the elementary groups, there is significant information in (1). If there is an element $f \in \Gamma$ with $|\beta(f)|<2(\sqrt{2}-1)$, then $h \in \Gamma$ either commutes with $f$, is of order two, or moves the axis of $f$ a definite distance. A diophantine analysis of $\beta\left(f^{n}\right)$ turns the requirement that $|\beta|$ be small into a statement about translation lengths. If $f$ has short translation length, then every element moves the axis of $f$ a definite distance, giving a volume bound on a fundamental domain constructed around $\ell$. These are called collar-volume estimates. If there are no short translation lengths, then a ball of a certain size must be moved off itself by every element providing a definite volume. In the case at hand these estimates are very poor (Meyerhoff 1988) and do not deal with torsion elements, but they do suggest a way forward. As an example, if $f \in \Gamma$ has order 6 , then $\beta(f)=-1$, and the other inequality above gives $|\gamma(f, g)| \geq 1$ unless there is a $(2,3,6)$-triangle subgroup. This is sharp. Now $\cosh (\delta) \geq 2$ and collar-volume estimates give a lower bound of 0.1 . Known examples have smaller volume and have $(2,3,6)$ subgroups with noncompact quotient. A little more argument shows that PGL(2, $\mathcal{O}(\sqrt{-3}))$ of covolume about $0.0846 \ldots$ is the smallest lattice with an element of order $p \geq 6$ (Gehring Martin 1999).

More generally, we use polynomial trace identities to generate inequalities to provide geometric information and a priori bounds. These inequalities hold unless the parameters satisfy algebraic equations from their preperiodicity which, remarkably, arise from two-generator arithmetic subgroups if they have small volume (Gehring, Machlachlan, Martin, and Reid, 1997). An enumeration of two-generator arithmetic Kleinian groups identifies the smallest candidates. Dealing with low-order torsion
is a major obstacle, for example, groups generated by elements of orders two and three. Here $\beta(f)=-3$ and $\beta(g)=-4$ with one free parameter $\gamma=\gamma(f, g) \in \mathbb{C}$ as illustrated in Figure 2. The exterior of the lower bounded region consists of parameters for discrete free products $\mathbb{Z}_{2} * \mathbb{Z}_{3}$ and is similar to the so-called Riley slice. The interior is partially covered by disks that eliminate regions where a group cannot be nonelementary and discrete as per the inequalities found from the trace identities. Marked points are known groups, accumulating on the boundary via Dehn surgeries on two-bridge link complements as indicated in the upper left. The dust in the upper right consists of the roots of the semigroup of polynomial trace identities. The red points are roots corresponding to arithmetic groups. All these roots must lie in the bounded region, and each root provides an inequality, but need not correspond to a discrete group. By covering a large enough region, the dashed ellipse, we obtain a sufficient lower bound on volume and are left to examine the preperiodic points inside. These turn out to be maximal arithmetic lattices, so the two-generator group in question generates the entire lattice $\Gamma$. In practice such moduli spaces must be found for all groups generated by elements of order $p$ and $q$ with $2 \leq p \leq q \leq 5$.

Basically our programme of proof is predicated on the belief that the extremals for these geometric problems are two-generator arithmetic, and luckily this is the case. Our main result is the following theorem identifying the two smallest co-volumes of lattices.


Figure 2. Moduli space of Kleinian groups: $\beta(f)=-3$, $\beta(g)=-4$. The lower figure is the moduli space of Kleinian groups generated by elements of orders 2 and 3. The interior is partially covered by disks that eliminate regions where a group cannot be discrete. Marked points are known groups. The dust in the upper left consists of the roots of polynomial trace identities, with the red points corresponding to arithmetic groups.

Theorem 1. Let $\Gamma$ be a Kleinian group. Then
$\operatorname{vol}_{\mathbb{H}}\left(\mathbb{H}^{3} / \Gamma\right)=\operatorname{vol}_{\mathbb{H}}\left(\mathbb{H}^{3} / \Gamma_{0}\right)=275^{3 / 2} 2^{-7} \pi^{-6} \zeta_{k}(2) \sim 0.0390$ and $\Gamma=\Gamma_{0}$ or $\operatorname{vol}_{\mathbb{H}}\left(\mathbb{H}^{3} / \Gamma\right) \geq \operatorname{vol}_{\mathbb{H}}\left(\mathbb{H}^{3} / \Gamma_{1}\right)=283^{3 / 2} 2^{-7} \pi^{-6} \zeta_{k^{\prime}}(2) \sim 0.0408 \ldots$.

Equality here is up to conjugacy. A description of the groups $\Gamma_{0}$ and $\Gamma_{1}$ follows.

- $\Gamma_{0}$ is a two-generator arithmetic Kleinian group obtained as a $\mathbb{Z}_{2}$-extension of the index- 2 orientationpreserving subgroup of the group generated by reflection in the faces of the 3-5-3-hyperbolic Coxeter tetrahedron, and $\zeta_{k}$ is the Dedekind zeta function of the underlying number field $\mathbb{Q}\left(\gamma_{0}\right)$, with $\gamma_{0}$ a complex root of $\gamma^{4}+6 \gamma^{3}+12 \gamma^{2}+9 \gamma+1=0$.
- $\Gamma_{1}$ is a two-generator arithmetic Kleinian group, and $\zeta_{k^{\prime}}$ is the Dedekind zeta function of the underlying number field $\mathbb{Q}\left(\gamma_{1}\right)$, with $\gamma_{1}$ a complex root of $\gamma^{4}+$ $5 \gamma^{3}+7 \gamma^{2}+3 \gamma+1=0$.
This solves Siegel's problem in dimension 3:
Corollary 1. The minimal volume orientable hyperbolic orbifold is unique up to isometry and has volume

$$
\mu_{3}=275^{3 / 2} 2^{-7} \pi^{-6} \zeta_{k}(2)=0.03905 \ldots
$$

The minimal volume nonorientable hyperbolic orbifold has volume exactly half this number.

Theorem 2. Let $M$ be a finite-volume orientable hyperbolic 3-manifold and $G$ a group of orientation-preserving homeomorphisms acting faithfully on M. Then

$$
|G| \leq \frac{2}{\mu_{3}} \operatorname{vol}_{H \in H}(M)
$$

Exactly what groups establish sharpness is discussed in a 2006 paper with Conder and Torstensson [1].

## References

[1] M. D. E. Conder, G. J. MArtin and A. Torstensson, Maximal symmetry groups of hyperbolic 3-manifolds, New Zealand J. Math. 35 (2006), 37-62. MR2222175
[2] F. W. Gehring and G. J. Martin, Minimal covolume lattices I: spherical points of a Kleinian group, Annals of Math. 170 (2009), 123-161. MR2521113
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Gaven J. Martin

## ABOUT THE AUTHOR

Gaven Martin notes that this work is a three-generation project, including his advisor, Fred Gehring, and his student Timothy Marshall.


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## Searching for Small Hyperbolic Quotients

This month's cover features a variant of Figure 2 from Gaven Martin's article (See p.1244). The image was produced by Yasushi Yamashita, a collaborator of Martin's.

Hyperbolic space $\Vdash^{3}$ may be identified with the hyperboloid $x^{2}-y^{2}-z^{2}-w^{2}=1, \quad x>0$, assigned the unique metric of curvature -1 invariant under the Lorentz group $S O(1,3)$. The problem solved by Martin and his collaborators, and discussed in his article, is to find the discrete subgroup $\Gamma=\Gamma_{\min }$ of $S O(1,3)$ for which the quotient $\Gamma \backslash \mathbb{M}^{3}$ possesses minimum volume. As is well known to physicists, the group $S O(1,3)$ is isomorphic to $P S L_{2}(\mathbb{C})$. Shifting to this group makes notation much simpler, but even more crucially allows one to take advantage of the enormous literature on the relationship between complex geometry and holomorphic dynamics.

As Martin explains, a major part of finding $\Gamma$ amounts to understanding discrete groups with two generators. The cover is about the case when these have orders 2 and 3. As was already realized by Fricke and Klein, conjugacy classes of pairs of such matrices in $P S L_{2}(\mathbb{C})$ are parametrized generically by single complex parameter $\gamma=\operatorname{Tr}(h)-2$, where $h=f q f^{-1} g^{-1}$. The point with coordinate $\gamma$ corresponds to the conjugacy class of the
matrices

$$
\left[\begin{array}{rr}
0 & 1 \\
-1 & 0
\end{array}\right], \quad\left[\begin{array}{cr}
0 & -\xi \\
1 / \xi & 1
\end{array}\right] \text { with } \xi^{2}+1 / \xi^{2}-1=\gamma
$$

The cover is a map of this set, together with a little added structure. There are several interesting features of this map, and for some of them the underlying mathematics is quite complicated. The coordinates are at least fairly simple. The center of the figure is at $-3 / 2$. The endpoints of the fractal region $D$ are -4 and 1 , and the ellipse is a little bit larger than the ellipse $|z+3|+|z|=5$, which passes through endpoints of $D$.

The exterior of $D$ is where the group $\Gamma_{\gamma}$ is isomorphic to the amalgamated product $\mathbb{Z} / 2 * \mathbb{Z} / 3$. The ellipse is an artefact of the computation-it is known for simple reasons that points in its exterior parametrize such groups. Quotients parametrized by points in this region never have finite volume. Conjecturally, they satisfy the Bowditch $Q$-condition, which can be feasibly checked. The boundary of $D$ is undoubtedly fractal, although little is rigorously known about it.

The green curves are pleating rays, introduced by Linda Keen and Caroline Series around 1993, in their analysis of Schottky space.

If $K$ is a finite extension of $\mathbb{Q}$ with exactly two complex embeddings, $B$ is a quaternion algebra over $K$ that splits at the complex places but not at the real places of $K$, and $\mathfrak{o}$ is an order in $B$, then the group of elements of $\mathfrak{o}^{\times}$of norm 1 embeds into $S L_{2}(\mathbb{C})$ and has finite covolume. A group $\Gamma$ is said to be arithmetic if it is commensurable with one of these.

The red and blue points are plotted by essentially the same calculation. The polynomial identities mentioned in Martin's article give rise to certain distinguished polynomials with real coefficients, exactly two complex roots, and the other roots in $(-3,0)$. A list of polynomials with this property and whose roots lie inside the ellipse has been made by Flammang and Rhin. The calculation scans through these, and if a complex root is in $D$ it is plotted red, otherwise blue. The red ones are points for which the parameter has a chance of corresponding to a discrete group of finite covolume. The points corresponding to the minimal groups $\Gamma_{0}$ and $\Gamma_{1}$ are noted in the following figure. The horizontal symmetry occurs because $f$ has order 2. If $f, g$ generate $\Gamma$ with $f^{2}=I, g^{3}=I$, and parameter $\gamma=3 / 2+\gamma$, then the subgroup generated by $g$ and $f g f^{-1}$ has index 2 in $\Gamma$. Another group in which it has index 2 is obtained by adding $h$ such that $h g h^{-1}=f g^{-1} f^{-1}$, and this has parameter $3 / 2-\gamma$. As far as we know, the evident patterns in the pleating rays and the blue points have not been explained.


Notable points in $D$.
Finding the group $\Gamma_{\text {min }}$ has taken a long time. Martin tells us, "The project that led to a solution of Siegel's problem began in earnest more than 20 years ago at the MittagLeffler Institute, when Gehring and I were introduced to Mathematica by Hans Riesel. This enabled us to generate the trace identities I refer to in my article. One major landmark was the paper Arithmeticity, discreteness, and volume by Gehring, MacLachlan, Reid and myself, in which we formulated conditions for a Kleinian group with two generators to be contained in an arithmetic group. One very difficult problem in computation was to deal with groups whose parameters lie close to the fractal boundary of the inner region, and in that we were lucky-as we had to be because otherwise we would have faced computational intractability."

A good idea of the nature of the computations involved may be found in the article The diagonal slice of Schottky space by Caroline Series, Ser Peow Tan, and Yamashita (posted on the arXiv in 2014).

We wish to thank Yasushi Yamashita for providing the cover image as well as explaining some of the computation that went into it.

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# Gigliola Staffilani Interview 



> Gigliola Staffilani is Abby Rockefeller Mauze Professor in the Department of Mathematics at Massachusetts Institute of Technology. She is a member of the American Academy of Arts and Sciences and an AMS Fellow. She has published over fifty papers in the area of PDE and is an active member of the Association for Women in Mathematics.

Diaz-Lopez: When/how did you know you wanted to be a mathematician?

Staffilani: I didn't think of math as a profession until college. More than deciding on being a mathematician, I decided to dedicate as much time as possible to learning math and doing research. It turned out that this really meant being a mathematician.

Diaz-Lopez: Who encouraged or inspired you (mathematically or otherwise)?

Staffilani: I had a great math teacher in high school. He was assigning hard problems for homework, and I was able to solve them, which made me extremely happy. After arriving in the US, I received great support from the faculty at the University of Chicago. Because I had not taken the TOEFL exam, my paperwork was not in order, but department chair Peter May allowed me to register as a student anyway. Paul Sally, then my registration advisor, gave me a personal check for the equivalent of one monthly stipend, because my first check did not arrive as a consequence of my work permit not being issued in time.

I also received great support from Carlos Kenig, my thesis advisor, who is a master at understanding the personality of each of his students and at implementing a matching mentoring approach that is very effective. I learned a lot from him. Finally, I want to mention Jean Bourgain, my mentor at the Institute for Advanced Study, right after graduate school. His subtle humor and difficult math made for an incredibly effective learning experience.

Diaz-Lopez: How would you describe your research to a graduate student?

Staffilani: I work on PDE, more specifically dispersive PDE. These equations come from physics and cannot be explicitly solved. Thus, I invent and use analytic tools to study properties of solutions without having a formula for them. It turns out that often enough some of these equations have a very rich algebraic and geometric structure.

One of the equations I have worked on is the nonlinear Schrödinger equation $i \partial_{t} u+\Delta u= \pm|u|^{p-1} u$, where $p>1$. The interactions coming from the nonlinearity of the equation are very difficult to understand. Thus, to study the existence, stability, and long-term dynamics of solutions, we have to exploit properties of the equation itself via techniques coming from harmonic analysis, Fourier analy-

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sis, and dynamical systems, among others. In the periodic case, for example, ideas from analytic number theory have been used, starting with the work of Jean Bourgain, to estimate oscillatory sums that appear when one writes explicitly the solutions of the linear Schrödinger equation.

Diaz-Lopez: What theorem are you most proud of and what was the most important idea that led to this breakthrough?

Staffilani: I am most proud of the work I did with the "I-team," ${ }^{1}$ that is James Colliander, Markus Keel, Hideo Takaoka, and Terry Tao. We were all young, at the early stages of our careers, and that gave us the freedom to investigate certain questions from a completely different point of view. The collaboration started at a very informal lunch we all had at Stanford while Jim Colliander and I were running a small conference. We were talking about the fact that Sobolev norms, except for the $H^{1}$ norm involved in the energy, are not as suitable for dispersive equations as they are for elliptic equations. We continued the conversation after lunch, and it was there that we thought about an algorithm that would generate what we called modified energies, more in tune with the equation at hand. Formally, infinitely many energies can be defined in this way, and the challenge is to show that they are well defined. It turned out that, for equations that are integrable systems, such as Korteweg-de Vries and 1-dimensional cubic nonlinear Schrödinger equations, our algorithm recovers the infinitely many conserved integrals for these equations.

Diaz-Lopez: What advice do you have for graduate students?

[^1]Staffilani: Choosing a field and an advisor that match very well your personality and interests is extremely important. Also it is important that the advisor is active in his/her field and can recommend problems that are approachable as well as considered interesting by the experts in the area.

Diaz-Lopez: All mathematicians feel discouraged occasionally. How do you deal with discouragement?

Staffilani: I learned after many years in the profession that, eventually, things get better. I also learned that from failure sometimes comes great progress, so I focus on this. Having a life outside of mathematics-I am also a mother and wife-helps too. It balances other forces that constantly act on me: being a teacher, researcher, mentor, administrator, and so on.

Diaz-Lopez: You have won several honors and awards. Which one has been the most meaningful and why?

Staffilani: I think being inducted into the American Academy of Arts and Sciences has been an amazing honor. I felt recognized not just by the math world but by a much larger community of knowledge.

Diaz-Lopez: If you could recommend one lecture (book, paper, article, etc.) to graduate students, what would it be?

Staffilani: I loved the book Partial Differential Equations by Lawrence C. Evans. It is an amazingly clear book, and every graduate student working on PDE should read it.

Diaz-Lopez: Any final comments or advice?
Staffilani: The most important element in this profession is loving the feeling you experience when you solve a problem. Sure there are ups and downs, problems that do not get solved, but as long the research is exciting everything else will follow.

## Photo Credit

Image of Gigliola Staffilani is courtesy of Bryce Vickmark.


[^2]
# Symplectic Geometry 

Tara S. Holm<br>Communicated by Cesar E. Silva

> Symplectic
> structures are
> floppier than holomorphic functions or metrics.

In Euclidean geometry in a vector space over $\mathbb{R}$, lengths and angles are the fundamental measurements, and objects are rigid. In symplectic geometry, a two-dimensional area measurement is the key ingredient, and the complex numbers are the natural scalars. It turns out that symplectic structures are much floppier than holomorphic functions in complex geometry or metrics in Riemannian geometry.

The word "symplectic" is a calque introduced by Hermann Weyl in his textbook on the classical groups. That is, it is a root-by-root translation of the word "complex" from the Latin roots
com - plexus,
meaning "together - braided," into the Greek roots with the same meaning,

$$
\sigma v \mu-\pi \lambda \varepsilon \kappa \tau \iota \kappa o ́ \varsigma .
$$

Weyl suggested this word to describe the Lie group that preserves a nondegenerate skew-symmetric bilinear form. Prior to this, that Lie group was called the "line complex group" or the "Abelian linear group" (after Abel, who studied the group).

A differential 2 -form $\omega$ on (real) manifold $M$ is a gadget that at any point $p \in M$ eats two tangent vectors and spits out a real number in a skew-symmetric, bilinear way. That is, it gives a family of skew-symmetric bilinear functions

$$
\omega_{p}: T_{p} M \times T_{p} M \rightarrow \mathbb{R}
$$

[^3]depending smoothly on the point $p \in M$. A 2 -form $\omega \in \Omega^{2}(M)$ is symplectic if it is both closed (its exterior derivative satisfies $d \omega=0$ ) and nondegenerate (each function $\omega_{p}$ is nondegenerate). Nondegeneracy is equivalent to the statement that for each nonzero tangent vector $v \in T_{p} M$, there is a symplectic buddy: a vector $w \in T_{p} M$ so that $\omega_{p}(v, w)=1$. A symplectic manifold is a (real) manifold $M$ equipped with a symplectic form $\omega$.

Nondegeneracy has important consequences. Purely in terms of linear algebra, at any point $p \in M$ we may choose a basis of $T_{p} M$ that is compatible with $\omega_{p}$, using a skewsymmetric analogue of the Gram-Schmidt procedure. We start by choosing any nonzero vector $v_{1}$ and then finding a symplectic buddy $w_{1}$. These must be linearly independent by skew-symmetry. We then peel off the two-dimensional subspace that $v_{1}$ and $w_{1}$ span and continue recursively, eventually arriving at a basis

$$
v_{1}, w_{1}, \ldots, v_{d}, w_{d}
$$

which contains an even number of basis vectors. So symplectic manifolds are even-dimensional. This also allows us to think of each tangent space as a complex vector space where each $v_{i}$ and $w_{i}$ span a complex coordinate subspace. Moreover, the top wedge power, $\omega^{d} \in \Omega^{2 d}(M)$, is nowhere-vanishing, since at each tangent space,

$$
\omega^{d}\left(v_{1}, \ldots, w_{d}\right) \neq 0
$$

In other words, $\omega^{d}$ is a volume form, and $M$ is necessarily orientable.

Symplectic geometry enjoys connections to algebraic combinatorics, algebraic geometry, dynamics, mathematical physics, and representation theory. The key examples underlying these connections include:
(1) $M=S^{2}=\mathbb{C} P^{1}$ with $\omega_{p}(v, w)=$ signed area of the parallelogram spanned by $v$ and $w$;
(2) $M$ any Riemann surface as in Figure 1 with the area for $\omega$ as in (1);


Figure 1. The area form on a Riemannian surface defines its symplectic geometry.

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(3) $M=\mathbb{R}^{2 d}$ with $\omega_{s t d}=\sum d x_{i} \wedge d y_{i}$;
(4) $M=T^{*} X$, the cotangent bundle of any manifold $X$, thought of as a phase space, with $p$-coordinates coming from $X$ being positions, $q$-coordinates in the cotangent directions representing momentum directions, and $\omega=\sum d p_{i} \wedge d q_{i}$;
(5) $M$ any smooth complex projective variety with $\omega$ induced from the Fubini-Study form (this includes smooth normal toric varieties, for example);
(6) $M=\mathcal{O}_{\lambda}$ a coadjoint orbit of a compact connected semisimple Lie group $G$, equipped with the Kostant-Kirillov-Souriau form $\omega$. For the group $G=S U(n)$, this class of examples includes complex projective space $\mathbb{C} P^{n-1}$, Grassmannians $\mathscr{G} r_{k}\left(\mathbb{C}^{n}\right)$, the full flag variety $\mathcal{F} \ell\left(\mathbb{C}^{n}\right)$, and all other partial flag varieties.
Plenty of orientable manifolds do not admit a symplectic structure. For example, even-dimensional spheres that are at least four-dimensional are not symplectic. The reason is that on a compact manifold, Stokes' theorem guarantees that $[\omega] \neq 0 \in H^{2}(M ; \mathbb{R})$. In other words, compact symplectic manifolds must exhibit nonzero topology in degree 2 cohomology. The only sphere with this property is $S^{2}$.

Example (3) gains particular importance because of the nineteenth century work of Jean Gaston Darboux on the structure of differential forms. A consequence of his work is
Darboux's Theorem. Let $M$ be a two-dimensional symplectic manifold with symplectic form $\omega$. Then for every point $p \in M$, there exists a coordinate chart $U$ about $p$ with coordinates $x_{1}, \ldots, x_{d}, y_{1}, \ldots, y_{d}$ so that on this chart,

$$
\omega=\sum_{i=1}^{d} d x_{i} \wedge d y_{i}=\omega_{s t d}
$$

This makes precise the notion that symplectic geometry is floppy. In Riemannian geometry there are local invariants such as curvature that distinguish metrics. Darboux's theorem says that symplectic forms are all locally identical. What remains, then, are

> Tools from the 1970s and 1980s set the stage for dramatic progress in symplectic geometry and topology. global topological questions such as, What is the cohomology ring of a particular symplectic manifold? and more subtle symplectic questions such as, How large can the Darboux charts be for a particular symplectic manifold?

Two tools developed in the 1970s and 1980s set the stage for dramatic progress in symplectic geometry and topology. Marsden and Weinstein, Atiyah, and Guillemin and Sternberg established properties of the momentum map, resolving questions of the first type. Gromov introduced pseudoholomorphic curves to probe questions of
the second type. Let us briefly examine results of each kind.

If a symplectic manifold exhibits a certain flavor of symmetries in the form of a Lie group action, then it admits a momentum map. This gives rise to conserved quantities such as angular momentum, whence the name. The first example of a momentum map is the height function on a 2 -sphere (Figure 2).


Figure 2. The momentum map for $S^{1}$ acting on $S^{2}$ by rotation.

In this case, the conserved quantity is angular momentum, and the height function is the simplest example of a perfect Morse function on $S^{2}$. When the Lie group is a product of circles $T=S^{1} \times \cdots \times S^{1}$, we say that the manifold is a Hamiltonian $T$-space, and the momentum map is denoted $\Phi: M \rightarrow \mathbb{R}^{n}$. In 1982 Atiyah and independently Guillemin and Sternberg proved the Convexity Theorem (see Figure 3):
Convexity Theorem. If $M$ is a compact Hamiltonian $T$ space, then $\Phi(M)$ is a convex polytope. It is the convex hull of the images $\Phi\left(M^{T}\right)$ of the $T$-fixed points.


Figure 3. Atiyah and Guillemin-Sternberg proved that if a symplectic manifold has certain symmetries, the image of its momentum map is a convex polytope.

This provides a deep connection between symplectic and algebraic geometry on the one hand and discrete geometry and combinatorics on the other. Atiyah's proof demonstrates that the momentum map provides Morse functions on $M$ (in the sense of Bott), bringing the power of differential topology to bear on global topological questions about $M$. Momentum maps are also used to construct symplectic quotients. Lisa Jeffrey will discuss the cohomology of symplectic quotients in her 2017 Noether Lecture at the Joint Mathematics Meetings, and many more researchers will delve into these topics during the special session Jeffrey and I are organizing.

Through example (2), we see that two-dimensional symplectic geometry boils down to area-preserving geometry. Because symplectic forms induce volume forms, a natural question in higher dimensions is whether symplectic geometry is as floppy as volume-preserving geometry: can a manifold be stretched and squeezed in any which way so

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## Gromov proved that symplectic

## maps are more

 rigid than volumepreserving ones.long as volume is preserved? Using pseudoholomorphic curves, Gromov dismissed this possibility, proving that symplectic maps are more rigid than volume-preserving ones. In $\mathbb{R}^{2 d}$ we let $B^{2 d}(r)$ denote the ball of radius $r$. In 1985 Gromov proved the nonsqueezing theorem (see Figure 4):
Nonsqueezing Theorem. There is an embedding

$$
B^{2 d}(R) \hookrightarrow B^{2}(r) \times \mathbb{R}^{2 d-2}
$$

preserving $\omega_{\text {std }}$ if and only if $R \leq r$.
One direction is straightforward: if $R \leq r$, then $B^{2 d}(R) \subseteq B^{2}(r) \times \mathbb{R}^{2 d-2}$. To find an obstruction to the existence of such a map, Gromov used a pseudoholomorphic curve in $B^{2}(r) \times \mathbb{R}^{2 d-2}$ and the symplectic embedding to produce a minimal surface in $B^{2 d}(R)$, forcing $R \leq r$.

On the other hand, a volume-preserving map exists for any $r$ and $R$. Colloquially, you cannot squeeze a symplectic camel through the eye of a needle.

Gromov's work has led to many rich theories of symplectic invariants with pseudoholomorphic curves


Figure 4. The Nonsqueezing Theorem gives geometric meaning to the aphorism on the impossibility of passing a camel through the eye of a needle: A symplectic manifold cannot fit inside a space with a narrow two-dimensional obstruction, no matter how big the target is in other dimensions.
This cartoon originally appeared in the article "The Symplectic Camel" by Ian Stewart-published in the September 1987 issue of Nature-we thank him for his permission to use it. Cosgrove is a well-known cartoonist loosely affiliated with mathematicians. He did drawings for Manifold for a few years and most recently did a few sketches for the curious cookbook Simple Scoff:
https://www2.warwick.ac.uk/newsandevents/ pressreleases/50th_anniversary_cookery.
the common underlying tool. The constructions rely on subtle arguments in complex analysis and Fredholm theory. These techniques are essential to current work in symplectic topology and mirror symmetry, and they provide an important alternative perspective on invariants of four-dimensional manifolds.

Further details on momentum maps may be found in [CdS], and [McD-S] gives an account of pseudoholomorphic curves.

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## ABOUT THE AUTHOR

In addition to mathematics, Tara Holm enjoys gardening, cooking with her family, and exploring the Finger Lakes. This column was based in part on her AMS-MAA Invited Address at MathFest 2016 held in August.


Tara S. Holm

A Decade Ago in the Notices: Pseudoholomorphic Curves
"WHAT IS...Symplectic Geometry?" discusses Gromov's "nonsqueezing theorem," a key result in symplectic geometry. Gromov proved the theorem using the notion of a pseudoholomorphic curve, which he introduced in 1986.

Readers interested in these topics might also wish to read "WHAT IS...a Pseudoholomorphic Curve?" by Simon Donaldson, which appeared in the October 2005 issue of the Notices. "The notion [of pseudoholomorphic curve] has transformed the field of symplectic topology and has a bearing on many other areas such as algebraic geometry, string theory and 4-manifold theory," Donaldson writes. Starting with the basic notion of a plane curve, he gives a highly accessible explanation of what pseudoholomorphic curves are. He notes that they have been used as a tool in symplectic topology in two main ways: "First, as geometric probes to explore symplectic manifolds: for example in Gromov's result, later extended by Taubes, on the uniqueness of the symplectic structure on the complex projective plane...Second, as the source of numerical invariants: Gromov-Witten invariants."

Another related piece "WHAT IS...a Toric Variety?" by Ezra Miller appeared in the May 2008 issue of the Notices.

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This program is supported by a grant from the National Science Foundation.

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## TOPICS FOR 2017

## June 4-10, 2017

## Homotopy Type Theory

Organizers: J. Daniel Christensen (University of Western Ontario) Chris Kapulkin (University of Western Ontario) Daniel R. Licata (Wesleyan University) Emily Riehl (Johns Hopkins University) Michael Shulman (University of San Diego)

June 11-17, 2017

## Beyond Planarity: Crossing Numbers of Graphs

Organizers: Éva Czabarka (University of South Carolina)) Silvia Fernández-Merchant
(California State University, Northridge) Gelasio Salazar
(Universidad Autónoma de San Luis Potosí) Marcus Schaefer (DePaul University) László A. Székely (University of South Carolina)

June 18-24, 2017
Dynamical Systems: Smooth, Symbolic, and

## Measurable

Organizers: Jon Chaika (University of Utah)
Vaughn Climenhaga (University of Houston)
Boris Hasselblatt (Tufts University)
Bryna Kra (Northwestern University)
Daniel J. Thompson (The Ohio State University)

# Preliminary Report on the 2015-2016 

 New Doctoral RecipientsThomas H. Barr and Colleen A. Rose

This report presents a preliminary profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2015, and June 30, 2016. As of September 28, 2016, not all departments have responded to the survey instrument through which data is collected and a list of the nonresponding departments is presented below.

Data collected to this point shows that 1,631 new PhDs were awarded by the 251 mathematical sciences departments that responded in time for this report. Some attributes of the data are:

| • 1,139 Males | $\cdot 803$ US citizens | $\cdot 88 \%$ are US employed |
| :--- | :--- | :--- |
| • 492 Females | $\cdot 828$ non-US citizens | $\cdot 12 \%$ are Non-US employed |
| - 1,242 in Mathematics | $\cdot 389$ in Statistics/Biostatistics | $\cdot 52 \%$ of those employed in the US are US citizen |

Based on the data collected so far, it is likely that the final count of PhDs awarded during 2015-2016 will be slighlty lower than the number $(1,901)$ reported for 2014-2015.

Once data collection is concluded, a more detailed final report will be published in the August 2017 issue of Notices.

## Doctoral Degrees not yet Reported

As of press time for this issue of Notices, the following departments had not yet responded to the survey. Every effort will be made to collect responses for inclusion in the New Doctoral Recipients Report. In order to be included in the final report, Doctorates Granted survey forms should be sent no later than March 3, 2017. Departments yet to respond can obtain copies of the Doctorates Granted survey forms by visiting www.ams.org/annual-survey/ surveyforms, emailing ams-survey@ams.org, or calling 1-800-321-4267, ext. 4189.

## Mathematics Departments

Boston University
Bryn Mawr College
Clarkson University
College of William \& Mary
Columbia University, Applied Mathematics
Dartmouth College
Graduate Center, City University of New York
Illinois State University
Middle Tennessee State University
New Jersey Institute of Technology
Ohio State University, Columbus
Stanford University
Stevens Institute of Technology
Stony Brook University, Applied Mathematics \& Statistics
Temple University
University of California, Berkeley
University of California, Davis
University of California, Merced, School of Natural Sciences
University of California, Santa Cruz, Applied Mathematics \& Statistics
University of Colorado, Denver
University of Illinois, Urbana-Champaign
University of Puerto Rico, Rio Piedras

[^4]University of South Carolina
University of Southern Mississippi
University of Toledo
University of Vermont
University of Wisconsin, Madison
Utah State University
Western Michigan University
Yeshiva University

## Statistics \& Biostatistics Departments

Augusta University, Biostatistics \& Epidemiology
Brown University, Biostatistics
Case Western Reserve University, Epidemiology \& Biostatistics
Columbia University, Biostatistics
Cornell University, Biological Statistics \& Computational Biology
George Washington University, Statistics
Georgia Southern University, Coll of Publ Hlth, Biostatistics
Indiana University, Bloomington, Statistics
Iowa State University, Statistics
Michigan State University, Statistics \& Probability
North Carolina State University, Statistics
Northwestern University, Statistics
Oklahoma State University, Statistics
Saint Louis University, Coll of Publ Hlth \& Social Justice, Biostatistics

Statistics \& Biostatistics Departments, cont'd.
Southern Methodist University, Statistical Science
University of Alabama-Tuscaloosa, Information Systems Statistics \& Management Science
University of Albany, SUNY, Epidemiology \& Biostatistics
University of Arizona, Statistics GIDP
University of California, Berkeley, Statistics
University of California, Davis, Statistics
University of California, Los Angeles, Biostatistics, Fielding Sch of Publ Hlth
University of California, Los Angeles, Statistics
University of California, Riverside, Statistics
University of California, Santa Barbara, Statistics \& Applied Probability
University of Cincinnati, Medical College, Epidemiology \& Biostatistics Division
University of Colorado, Denver, Biostatistics \& Informatics
University of Illinois at Chicago, Epidemiology \& Biostatistics Division
University of Illinois, Urbana-Champaign, Statistics
University of Kentucky, Biostatistics
University of Kentucky, Statistics
University of Louisville, Bioinformatics \& Biostatistics
University of Minnesota-Twin Cities, School of Statistics
University of North Texas School of Public Health, Biostatistics
University of Oklahoma, Health Science Center, Biostatistics \& Epidemiology
University of South Carolina, Epidemiology \& Biostatistics University of South Florida, Epidemiology \& Biostatistics University of Texas-Sch of Publ Hlth, Biostatistics
University of Virginia, Statistics
University of Washington, Biostatistics
Vanderbilt University, School of Medicine, Biostatistics
Virginia Commonwealth University, Medical Center, Biostatistics
Virginia Polytechnic Institute and State University, Statistics
Western Michigan University, Statistics


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## THE GRADUATE STUDENT SECTION

# A Conversation with Helen Grundman, AMS Director of Education and Diversity 

Stephen Kennedy



Helen G. Grundman is the Director of Education and Diversity of the American Mathematical Society and Professor Emeritus of Mathematics at Bryn Mawr College. Her email address is hgg@ams.org.

As part of its strategic initiative on diversity and inclusion, the AMS has created the new Education and Diversity Department, appointing Helen G. Grundman as its director. The AMS's Strategic Plan describes the new department as seeking to promote diversity and inclusion at all stages of the mathematics pipeline.

Grundman is a number theorist with a Berkeley PhD, who after two years as a Moore Instructor at MIT spent twenty-five years at Bryn Mawr College. She earned campus-wide awards from Bryn Mawr for both excellent teaching and effective mentoring. In January at the Joint Mathematics Meetings she will receive the AWM's M. Gwyneth Humphreys Award for Mentoring Undergraduate Women. Notices asked Grundman to tell us about herself and to share some of her thoughts on the new department.

Notices: Please tell us about your educational background.

Grundman: When I was seven, my family moved to Detroit so that my father could be the founding headmaster of a diverse college-prep school in the inner city. At the time, black students were being turned away from the elite

[^5]college-prep schools in the area. I attended Friends School in Detroit for eight formative years, part of a learning community with students from various races, religions, ethnicities, economic backgrounds, and physical, emotional, and intellectual abilities. I know from experience that education is enhanced by diversity.

At the University of Michigan, I double majored in mathematics and psychology and completed the work for secondary teaching credentials in mathematics and general sciences. I took an extra semester to finish because I wanted to do my student teaching in the fall, allowing me to start with the students at the beginning of their school year.

After that, I took a mid-academic year job at a large parochial high school just outside of Detroit, teaching there for three and a half years. I greatly enjoyed it, particularly teaching those students who came in the door convinced that they could not do math. I missed working on advanced mathematics, but teaching is a noble profession and going to graduate school in math seemed to me to be very self-indulgent. Then a friend of mine explained that I should get a PhD in math because the field needed more female role models. This meant that I could have fun learning and doing math and do it for a good cause!

I decided to go to UC Berkeley, where I was lucky enough to take a number theory class with Emery Thomas. A topologist who had converted to a number theorist in his later years, Emery became my dissertation advisor.

Notices: Can you tell us something about your mathematical work? What areas have you worked in, what have you been doing lately?

Grundman: I consider myself an algebraic number theorist. In my thesis, "The Arithmetic Genus of Hilbert Modular Threefolds," I used algebraic and computational number theory to derive results in algebraic geometry. Since then, my research has been varied, partly because I enjoy working collaboratively, and that often means pushing one's boundaries to find common ground with a collaborator. I have a number of publications dealing with

Hilbert modular varieties and some related papers dealing with cubic number fields. I have a special love for elementary number theory, because it's usually easy to explain the problems to people who don't know much math and because it's an excellent area for involving undergraduates in research. Probably my most cited works are in inverse Galois theory, specifically on the realizability of groups as Galois groups. I've dabbled in some other areas of number theory, and in recent years have become involved in multiple projects concerning the solvability of families of Diophantine equations.

Notices: I know you've been involved with the EDGE (Enhancing Diversity in Graduate Education) program. Can you tell us about that experience, the roles you played, the effects it had on your thinking, what lessons you learned?

Grundman: EDGE is an amazing program. I taught algebra to the students the first and third years that the program existed, and had some minor roles in the following years. Several years later, Rhonda Hughes asked me to run the EDGE Mid-Atlantic Mentoring Cluster. This involved arranging gatherings of EDGE alums who were currently in the region. The most effective meetings involved a meal with a lot of socializing, followed by a more serious session in which the women shared their current situations, struggles, and victories. The feedback I received indicated that they, particularly the women in graduate school, greatly appreciated the sharing and mutual mentoring at these meetings.

The EDGE Program taught me how incredibly important it is to know that you are not alone when pursuing something like a graduate degree. These students learned ahead of time that there would be bumps in the roads to their degrees and were given tools and support that they could use to deal with difficult times. When times got rough for them, they were much less likely to get derailed, but instead became more determined. Of course, some decided on different directions, chose to change programs or final goals, but their support network helped to keep them from feeling that they had failed. These women knew that they were making choices and not letting others make their choices for them. And, of course, many of them now have PhDs and are teaching and mentoring a new generation.

Notices: Are there other relevant life or professional experiences that will inform your work in the Department of Education and Diversity?

Grundman: Probably the most relevant life experience was my time as a math graduate student. I was coming to graduate school from being a high school teacher and was not used to being treated as a student, nor to leaving issues to others to resolve. I sincerely felt that the faculty, staff, and administrators were fellow educators and colleagues and should be wanting to improve their programs. I learned the bureaucracy and helped many graduate students navigate it-particularly when they needed

> The EDGE Program taught me how incredibly important it is to know that you are not alone when pursuing something like a graduate degree.
exceptions to rules, something that women and underrepresented minority students seem less likely to know are commonly granted.

This was during a time when the Berkeley PhD program admitted a lot more students than they could support beyond the first year. Most professors didn't bother to get to know first-year students, since so many of them would not be there the next year. And then, of course, there were the exams, which caused even more students to leave. Though I am not at all opposed to graduate exams, I find it very frustrating that so many very strong students get weeded out by them. It is the students without strong egos who leave rather than asking for a second or third try.

My second or third year, a friend of mine and I were asked by a faculty member to organize something for the women graduate students. We learned that a female graduate student had been stalked by an advanced graduate student who had originally claimed to be a beginning student in a class with her. She ended up avoiding him by entirely avoiding the math building, and she eventually left the program. Somewhere in the process, she found out that this same student had also been harassing other female students. Although that stalker had graduated and was gone, there was an awareness that the women graduate students needed to be less isolated.

We applied to the graduate dean for some funds and held a picnic for women math graduate students. We worked hard to get everyone to attend. One of our goals was for all of us to learn that there really were a lot of women in the program. Another was to make sure that every woman knew at least one other. The picnic became an annual event and later morphed into the Noetherian Ring, an organization for women math graduate students that has spawned similar organizations at other schools.

At that time, the Mathematics Opportunity Committee (MOC) was able to admit capable students from underrepresented groups, who were passed over by the regular admissions committee. Those who appeared to have a weaker background were admitted to the master's program with the understanding that they could then transfer into the PhD program. What I found during my six years at Berkeley is that very few ever transferred and many felt duped. I decided that there should be a set program for these students and, during my final year, I designed and proposed such a program with the help of two friends. The PrePhD program gave students one year (with support) during which they took advanced undergraduate courses in order to fill out their backgrounds. Students who passed the courses with an appropriate grade point average were automatically transferred to the PhD program, at which point the clock would start for them to complete their exams, etc. A number of years later, I was thrilled to meet
and be thanked by the first student to complete the program and go on to receive her PhD .

All in all, my time in graduate school taught me a lot about how and why students leave mid-program. For so many of these students, their leaving did not seem to have anything to do with whether or not they would make strong mathematicians.

I should mention that prior to coming to Bryn Mawr College, I had great doubts about single-sex education. But I have learned some very good reasons for having this option for some students. I think that it's wonderful for a young woman to be on a campus where the goal is to educate women. (I think that there are plenty of campuses that give the impression that their goal is to educate men.) Equivalently, I think that it's wonderful for a black student to be on a campus with the historical goal of educating blacks. But I think that all of these campuses are enhanced by more diversity. For example, around 1950 Hampton Institute, now Hampton University, had a program in which white college students (my mother having been one of them) would attend this otherwise black college for a semester or a year, broadening the education of all of the students. I guess that my belief is that having diversity does not imply having to lose a particular institution's special focus.

Of course, all of my experiences at programs, conferences, and workshops, and my years at MIT, Bryn Mawr College, MSRI, and the Bunting Institute of Radcliffe College inform me of what does and does not help mathematics students and mathematicians at various levels to be successful. Also my work with AWM, Project NExT, and other organizations has given me a variety of additional experiences mentoring young women both before and


Helen talks to graduate student Jonah Swann at a reception in the Anna Pell Wheeler Mathematics Lounge at Bryn Mawr College.

> One clear problem is the overreliance on GRE scores.
after obtaining their PhDs, helping to inform me of the issues near this difficult transition point.

Notices: Can you tell us your thoughts on graduate education more generally?

Grundman: First of all, I want to be very clear that we, in America, have the best graduate mathematics programs in the world and we don't want to change that. Of course, we should always be working to improve our programs, if only so that they remain the best.

I think that a PhD in math should indicate a broad knowledge and understanding of mathematics along with a very deep understanding of a small area, in which one has done independent (though, perhaps, guided) research and proven new results. I would like it also to indicate an ability (though not necessarily a desire) to do more mathematics research and I would like it to indicate an ability (though perhaps one that needs some honing and, again, not necessarily a desire) to teach mathematics.

Not everyone should get a PhD in mathematics, just as not everyone should become a symphony cellist. But, in either case, in order to produce the very best of the best, we need to make sure that everyone has the opportunity to enter pathways leading there, that they are appropriately encouraged and supported, and that the filters we use to remove students from programs achieve the desired outcomes. (And we need to agree that the desired outcomes include our having a more diverse mathematics community than we have now.) We know that orchestras are becoming more diverse and of higher quality, now that blind auditions are common. We need to find analogous ways to improve the filters that we use throughout our educational process. And, as anyone who has been working in graduate admissions to improve diversity can tell you, we definitely need to improve the opportunities (and encouragement) for underrepresented minorities and women to enter, and continue on, the pathways that can lead to applying to a PhD program in mathematics. Once we have admitted well-qualified, capable students into our programs, we need to ensure that we provide them with a challenging and supportive environment that does not (perhaps inadvertently) drive them out.

Notices: What are some of the institutional or structural barriers to members of underrepresented groups in graduate education that persist? And what can we do about them?

Grundman: I'm not sure that I'd call them barriers as much as hurdles, but they are hurdles that exist for some students, and not for others. And, as I talk with more and more people, I'm learning that they still persist at every step of the way, from undergraduate advising and mentoring, through admissions, and then each stage of the graduate-school experience.

For example, there are many issues at the point of admissions; many schools are already addressing some of them. One clear problem is the over-reliance on GRE scores. On one hand, it seems like such a clearly objective measure, independent of individual biases. But, to the

> We need to understand what mechanisms are in place that cause talented and capable students to leave math graduate programs.
contrary, we have very strong evidence (a controlled experiment run by the ETS) that this sort of standardized mathematics test yields lower scores for women (on average) than they do if a simple change in the testing procedure were made. And there are similar results for students in other groups that are underrepresented in mathematics.

Another admissions problem is the desire to admit students with trajectories typical of successful students previously admitted. Though this is, in many ways, a $\log$ ical approach, it dooms us to repeat the past. Programs need more experience with students whose backgrounds are different, but they can't get that without changing
their current practices.
Admissions decisions that are not based on test scores usually depend a great deal on letters of recommendation. Obviously, letters from mathematicians known to members of the committee carry more weight than letters from unknowns. Thus, to improve diversity without sacrificing quality, members of admissions committees need to make an effort to get to know some of the professors who teach and write letters for applicants from underrepresented minorities. This would also allow the letter writers to learn more about what characteristics of the students are important to the committees.

Considering the problem more generally in graduate programs, one well-documented reason women and students from underrepresented minorities leave is because of isolation, because each is, at best, one of very few in their program. This reinforces the barrage of messages a student receives indicating that perhaps he or she doesn't belong there. These other messages are a combination of the small failures that almost all graduate students face, along with the (often accidental) micro-aggressions coming from both faculty and students, and, unfortunately, the blatantly racist and sexist actions and comments that we all would like to believe are not a part of our graduate programs.

A number of approaches have been tried to solve, or at least mitigate, some of these problems. Some institutions have drastically changed their admissions practices, with very positive results. To combat the problem of isolation, some campus-wide organizations have been formed to allow students from underrepresented minorities to see that they are not alone and to allow for co-mentoring. If that is not an option, then perhaps students can be connected with mentors on other campuses. Faculty in the students' departments can greatly reduce the effects of the negative messages, by delivering clear, honest, positive


Helen with four of her graduate students at Bryn Mawr College's graduation: Jaclyn Lang, MA '09 (PhD UCLA '16); Eva Goedhart, PhD '15; Amanda Hittson, MA '09; and Daniel Wisniewski, PhD '10.
messages to students. This is a skill that can be learned, along with the skill of avoiding micro-aggressions. Finally, departments that want to be diverse need to accept that they have a responsibility to set standards for acceptable behavior, to communicate them clearly, and to have an established procedure to be followed by people who witness racist or sexist behavior in the department or at a departmental event.

In general, I think that the biggest need is for high-quality mentoring, with at least one mentor who actually understands the student's situation. But I also think that we still have a lot to learn about success in graduate school. We need to understand what mechanisms are in place that cause talented and capable students to leave math graduate programs. If someone finds that they are not happy spending large amounts of their time working on mathematics, then it may well be that a PhD in math is not what they want. On the other hand, if they want to spend that time on math, but the real world intrudes and also demands much of their time, this should not necessitate an exit from mathematics. But few programs have easily identifiable procedures for dealing with such situations. If students enjoy math and excel at it, but circumstances make them miserable when they're physically in the math department, we should become aware of the situation and figure out what needs fixing. Again, having a good mentor in the department who can understand the problem and can help the student find ways to succeed is key.

Notices: Would you care to share some of your big-picture thinking about the mission of this office and your vision of what it can accomplish?

Grundman: The mission of the department, simply put, is to promote diversity and inclusion in the mathematical sciences and to contribute to the improvement of mathematics education. The initial focus is on diversity in graduate-level mathematics education. I'm hoping to find ways to improve the recruitment, preparation, and
success rate of students at this level, particularly members of groups that are underrepresented in the mathematical sciences, including women. Successfully increasing the number of doctoral recipients among these groups will not only diversify the population of PhD mathematicians, but will also, over time, positively affect diversity at earlier stages of the mathematics pipeline.

## Credits

Headshot of Helen Grundman is courtesy of Bryn Mawr College.
Photo of Helen at reception is courtesy of Bryn Mawr College Mathematics Department.
Photo of Helen with graduate students at graduation is courtesy of Jaclyn Lang.

## MY PROFESSOR



You look thirsty. Here's a coffee coupon. $10 \%$ off. You can thank me later.


Artwork by Sam White.
"Math without proofs is like soccer without a ball."
-Hendrik Lenstra, The Earle Raymond Hedrick Lectures, MAA MathFest 2016

## MY TA



Artwork by Sam White.

## Twitter comedy from @daemonic3, submitted by Shawn Rafalski:

"When I was little, we had to walk a mile-long spiral staircase to get to our room, uphill both ways! -M.C. Escher's kids"

[^6]

## Open hours:

Wednesday, January 4, 8:00 am - 5:30 pm Thursday, January 5, 8:00 am - 5:30 pm Friday, January 6, 8:00 am - 5:30 pm Saturday, January 7, 9:00 am - 12:00 pm

Visit www.ams.org/emp-reg for registration instructions.


# Fall2015Departmental Profile Report 

William Yslas Vélez, Thomas H. Barr, and Colleen A. Rose

This report presents a profile of mathematical sciences (MS) departments at four-year colleges and universities in the United States, as of fall 2015. The information presented includes the numbers of faculty in various categories, undergraduate and graduate course enrollments, numbers of bachelor's and master's degrees awarded during the preceding year, and the number of graduate students. Definitions of categorized terms such as "Mathematical Sciences," "Math," and "Stats" along with a description of the faculty categories are provided at the end of this report.

Data collected earlier from these departments on recruitment, hiring, and faculty salaries were presented in the Report on 2014-2015 Academic Recruitment, Hiring, and Attrition (pages 383-387 of the April 2016 issue of Notices of the AMS) and the 2015-2016 Faculty Salaries Report (pages 390-396 of the April 2016 issue of Notices of the AMS).

Detailed information, including tables, is available on the AMS website atwww.ams.org/annua1-survey

## Faculty Size

The estimated number of full-time faculty in MS for fall 2015 is 24,614. Of these, 22,373 were in Math (down slightly from 22,537 last year) and 2,241 were in Stats (down from 2,328 last year). Full-time faculty in the Doctoral Math Group increased slightly to 9,059 from 8,961 last year. In Math we estimate that the number of nondoctoral full-time faculty is 3,615 , essentally unchanged from last year, with a standard error of 99. The total part-time faculty in Math is estimated to be 7,684 (with a standard error of 222), down $4 \%$ from 8,014 last year. In Stats, the part-time faculty count is estimated to be 233, down $12 \%$ from 264 last year.

Figure F.1: Full-time Faculty by Department Grouping


Total: 24,614

Figure F.2: Full-time Doctoral Faculty by Department Grouping


Total: 20,904

Figure F. 3 Full-time Faculty Teaching Courses Outside of the Mathematical Sciences


Total: 2,183*
*(35\% teach only computer science courses

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## Doctoral Faculty

The estimated number of full-time doctoral (i.e., doctorate-holding) faculty in MS is 20,904. In Math this estimate is 18,758 (with a standard error of 99 ), down slightly from last year's number of 18,932 ; in Stats it is 2,146 , down $2 \%$ from 2,189 last year. Respectively for Math and Stats, the total doctoral tenured faculty are 11,653 and 1,011 compared to 11,909 and 1,088 for fall 2014. Sixty-six percent of all doctoral tenured faculty in Math are full professors, while $17 \%$ of all doctoral faculty are tenure-eligible. Females hold $22 \%$ of all doctoral tenured faculty and $18 \%$ of doctoral tenured full professor appointments.

Figure D.1: Full-time Tenured Doctoral Faculty by Department Grouping


Total: 12,664

Figure D.2: Full-time Tenure-eligible Doctoral Faculty by Department Grouping


Total: 3,648

Figure D.3: Full-time
Non-tenure-track Doctoral Faculty (excluding Postdocs) by Department Grouping


Total: 2,985

Features of full-time doctoral faculty data:

- $76 \%$ of all tenured doctoral faculty in the Doctoral Math Group are full professors $(3,615)$, with $71 \%$ of these appointments in Math Public departments.
- Tenure-eligible doctoral faculty increased $1 \%$ among the Doctoral Math Group, while the Bachelors and Biostatistics Groups both showed a 2\% decrease.
- Postdoctoral appointments among the Doctoral Math Group decreased to 1,231 for fall 2015. This is a $2 \%$ decrease from last year and $15 \%$ of the total full-time doctoral faculty in these departments (the same as last year). In Stats postdocs increased 9\% to 229.
- Females hold $21 \%$ of all postdoctoral appointments (the same as last year).
- $15 \%$ of the doctoral faculty in the Doctoral Math Group are in non-tenure-track positions. The majority of these faculty hold renewable (79\%) and fixed-term appointments (17\%); last year these percentages were $77 \%$ and $20 \%$, respectively.
Features of part-time doctoral faculty data:
- Total part-time doctoral faculty decreased $1 \%$ to 2,075 from 2,091 last year. Of these, $25 \%$ receive benefits, and $5 \%$ are in phased retirement.
- $27 \%$ of all part-time doctoral faculty are in Doctoral Math departments.
- Females hold $29 \%$ of all part-time doctoral faculty positions (up from 28\% last year).

Figure D.4: Full-time Tenured Doctoral Full Professor Faculty by Department Grouping


## Nondoctoral Faculty

The estimated number of nondoctoral (i.e., without a doctorate) full-time faculty in MS is 3,710, of which 3,615 are in Math and 95 are in Stats. This count is down 1\% from last year, and it represents $15 \%$ of all full-time faculty. In Math, nondoctoral tenured faculty decreased $8 \%$ from 320 to 296 this year; in Stats there were no nondoctoral tenured faculty. One hundred forty-one of the nondoctoral faculty in Math are tenure-eligible, 4\% of all tenure-eligible. Nondoctoral full-time non-tenure-track faculty increased $1 \%$ to 3,271 ; this is $88 \%$ of all nondoctoral faculty, up from $86 \%$ last year. Females composed $55 \%$ of all nondoctoral faculty.

Figure ND.1: Full-time Nondoctoral Faculty by Department Grouping


Total: 3,710

Figure ND.2: Full-time Nondoctoral Tenured Faculty by Department Grouping


Total: 296

Figure ND.3: Full-time Nondoctoral Non-tenure-track Faculty by Type of Appointment (excluding Postdocs)


Total: 3,271

Features of full-time nondoctoral faculty data:

- $30 \%$ of all tenured nondoctoral faculty in MS are full professors (88) and $88 \%$ of these appointments are in the Bachelors Group. Stats reported no faculty in this category.
- Masters and Bachelors departments combined reported the majority of the nondoctoral nontenure-track faculty holding renewable and fixed-term appointments with $70 \%$ and $79 \%$, respectively.
- Females account for $55 \%$ of full-time nondoctoral faculty in Math (down from 56\% last year), compared to females accounting for $26 \%$ of all doctoral full-time faculty and $30 \%$ of all full-time faculty in these same groups.

Features of part-time nondoctoral faculty data:

- Total part-time nondoctoral faculty decreased $6 \%$ to 5,842 from 6,187 last year. Of these faculty, $18 \%$ receive benefits and $1 \%$ are in phased retirement.
- $74 \%$ of all part-time faculty are nondoctoral; females hold $46 \%$ of these positions.
- Part-time nondoctoral faculty increased $4 \%$ to 811 in Doctoral Math departments, this is $59 \%$ of all part-time faculty in this group.

Figure ND.4: Gender of Full-time Nondoctoral Faculty Total: 3,710

Male $\square$ Female


## Female Faculty

Females account for $31 \%(7,540)$ of all full-time faculty in MS. In Math, women comprised $30 \%(6,809$ with a standard error of 158 ) of the full-time faculty $(22,373)$ in fall 2015 . For the Doctoral Math departments, women composed $16 \%$ of the combined doctorate-holding tenured and tenure-eligible faculty and $28 \%$ of the doctorate-holding non-tenure-track (including postdocs) faculty in fall 2015. In the other departments these respective percentages are: $24 \%$ and $33 \%$ in Statistics, $29 \%$ and $49 \%$ in Biostatistics, $28 \%$ and $33 \%$ in Masters, and for Bachelors faculty they are $31 \%$ and $34 \%$. Among the nondoctoral full-time faculty in Math, women compose $55 \%$. Females account for $42 \%$ of all part-time faculty in Math.

Figure FF.1: Full-time Tenured Female Doctoral Faculty
by Department Grouping

Total: 2,757
Figure FF.2: Full-time Tenure-eligible Female Doctoral Faculty by Department Grouping

Figure FF.3: Female Doctoral Full Professor Faculty by Department Grouping


Total: 1,267


Total: 1,522

Figure FF.4: Full-time Female Doctoral Non-tenure-track Faculty (excluding Postdocs) by Department Grouping


Total: 1,117

Features of full-time female faculty data:

- Females hold $14 \%$ of full-time tenured and $26 \%$ of full-time tenureeligible positions in Doctoral Math departments.
- $43 \%$ of all full-time female faculty are in the Bachelors departments.
- Biostatistics departments reported the highest percentage of fulltime female faculty (39\%), followed by the Bachelors departments (36\%), and Masters (35\%), while the Math Private Large Group reported the lowest ( $16 \%$ ).
- Females hold $21 \%$ of all postdoctoral appointments. Thirty-five percent of postdocs in Biostatistics are held by women. The majority of the Doctoral Math groups reported $22 \%$ of postdocs were held by females with only Math Public Large, Applied Math, and Statistics reporting fewer females in these positions with $20 \%, 15 \%$, and $12 \%$ respectively.
- $89 \%$ of all female nondoctoral non-tenure-track faculty appointments $(1,649)$ are renewable; $11 \%$ are fixed-term, and $1 \%$ are other types of appointments.
Features of part-time female faculty data:
- $60 \%$ of all part-time female faculty in Math are found in the Bachelors departments.
- $82 \%$ of all part-time female faculty hold nondoctoral positions. Of these faculty, $17 \%$ receive benefits and $1 \%$ are phased retirements.


## Undergraduate Course Enrollments

Total undergraduate enrollments for all groups combined increased slightly from 2,481,000 to 2,518,000 (with a standard error of 22,000 ). MS departments reported an overall increase of $2 \%$ in the number of undergraduate course enrollments per full-time faculty member.

Figure UE.1: Undergraduate Course Enrollments by Department Grouping


Total Undergraduate Enrollments (thousands): 2,518

Figure UE.2: Undergraduate Course Enrollment per Full-Time Faculty Member, Fall 2015


## Graduate Course Enrollments

Total graduate course enrollments have increased from 107,000 to 110,000 (with a standard error of 4,000). MS departments reported an overall increase of $8 \%$ in the estimated number of graduate course enrollments per full-time tenured and tenure-eligible faculty member.

Figure GE.1: Graduate Course Enrollments by Department Grouping


Total Graduate Enrollments (thousands): 110

Figure GE.2: Graduate Course Enrollment per Full-Time Tenured and Tenure-eligible Faculty Member, Fall 2015


## Bachelor's Degrees Awarded

For the period 2014-15, the estimated number of bachelor's degrees awarded in MS departments is 29,339, down slightly from the previous year's estimate of 29,673 . The standard error estimate is 348 . Of these, 11,955 were earned by females (41\%), a 3\% decrease from last year's count of 12,316. In Math, this year's estimated number of bachelor's degrees awarded is 28,043 , a count that includes 11,411 degrees earned by females, 762 Statistics-only degrees, and 1,925 Computer-Science-only degrees. This figure represents a slight drop from last year's estimate of 28,277 degrees awarded by Math departments.

Figure UD.1: Undergraduate Degrees Awarded* by Department Grouping


Total: 29,339

* Degrees awarded between July 1, 2014 and June 30, 2015.
- Math Doctoral departments awarded 8\% more bachelor's degrees this year, up 919 from last year, $42 \%$ of all degrees awarded.
- Applied Math departments showed the largest percentage increase in degrees awarded, up 27\% from last year, followed by the Math Public Large and Math Private Small Groups which both increased $10 \%$.
- Biostatistic departments showed the largest percentage decrease, down $66 \%$ from last year. Masters departments reported the largest absolute decrease of 934 degrees, netting 3,643 for 2015.
- Bachelors departments awarded $42 \%$ of all the degrees in MS, the same as last year.
- Statistics departments awarded 1,281 degrees, down $5 \%$ from 1,352 last year; females received $42 \%$ of these degrees (down from 44\% last year).
- Total Statistics-only degrees in Math departments remained essentially unchanged at 762 ; $48 \%$ of these degrees were awarded by the Bachelors Group.

Figure UD.2: Undergraduate Degrees Awarded* to Females by Department Grouping


- Among Math departments surveyed, 80\% of Computer Science degrees were awarded by Bachelors departments.
- Math Doctoral departments awarded 38\% of all degrees awarded to females, up from 34\% last year.
- Since 2010, the annual number of bachelor's degrees awarded has increased by $9 \%$, and the number of degrees awarded to females has increased by $11 \%$.

Figure UD.3: Undergraduate Degrees Awarded All Mathematics Departments Combined


## Master's Degrees Awarded

For the period 2014-2015, the estimated number of master's degrees awarded in MS departments is 7,132, an increase of $9 \%$ over the previous year's estimate of 6,546 . The standard error in this estimate is 149 . Of these, 3,034 were earned by females (43\%), the same as last year and a $7 \%$ increase over last year's 2,843. In Math, this year's estimated number of master's degrees awarded is 5,087, a count that includes 2,009 degrees earned by females, 770 Statisticsonly degrees, and 104 Computer-Science-only degrees. This figure represents a $12 \%$ increase over last year's estimate of 4,548 masters degrees awarded by Math departments.

## Figure MD.1: Master's Degrees Awarded* by Department Grouping



Total: 7,132
*Degrees awarded between July 1, 2014 and June 30, 2015.

## Overall features:

- In all groups except Biostatistics, production of master's degrees increased from last year. Most groups showed increases between $11 \%$ and $18 \%$, with the exception of Math Public Large 3\%.
- In the Statistics Group, production of master's degrees increased $4 \%$ compared with last year.
- $43 \%$ of all master's degrees were awarded to females.
- Females were awarded $47 \%$ of the master's degrees in statistics-only and 78\% of those in computer-scienceonly (up from $41 \%$ and $38 \%$, respectively).

Features of the Math Group:

- Masters departments awarded the highest percentage of degrees ( $28 \%$, up from $27 \%$ last year).
- Math Private Small awarded the smallest percentage of degrees with $3 \%$, the same as last year.
- Females received 39\% of all degrees awarded among all the Math Groups, down from $41 \%$ last year.
- $17 \%$ of degrees awarded in Math departments were in Statistics-only or Computer-Science-only. Statistics-only and Computer-Science-only degrees increased $40 \%$ and $55 \%$, respectively, over last year.


## Features of the Stats Group:

- Statistics departments awarded 1,598 degrees, an increase of 4\% over last year.
- Biostatistics departments awarded 447 degrees, down $4 \%$ from last year.

Figure MD.2: Master's Degrees Awarded* to Females by Department Grouping


Total: 3,034

* Degrees awarded between July 1, 2014 and June 30, 2015.
- Degrees awarded to females increased by $6 \%$ in the Statistics Group and decreased 7\% in the Biostatistics Group.
- $58 \%$ of all Statistics-only degrees were awarded by the Statistics Group, down from $61 \%$ last year.

From 2010 to 2015 the annual number of master's degrees from Math departments has increased by $15 \%$. The number awarded to females has increased by the same percentage over time.

Figure MD.3: Master's Degrees Awarded by Mathematics Departments


## Graduate Students

In fall 2015, the total number of full-time graduate students is estimated at 23,314, with 16,136 in Math (up from 15,939 in fall 2014) and 7,187 in Stats. The total number of full-time graduate students in Doctoral Math departments is 13,431 (up from 13,023). In Doctoral Math departments, counts of full-time and first-year graduate students who are US citizens or permanent residents have remained essentially unchanged at 7,123 and 1,827 , respectively. For the Masters Group, full-time graduate students decreased $7 \%$ to 2,705 , the number of US citizens and permanent residents is 1,930 (down from 2,022), and the number of first-year students is 1,203 (down from 1,287). Stats reported full-time first-year graduate students at 2,538, up from 2,274. Females account for $37 \%(8,597)$ of all full-time graduate students.

Figure GS.1: Graduate Students by Department Grouping


Total: 23,314
Features of full-time graduate student data:

- Full-time graduate students and full-time female graduate students increased in all groups except Math Private Small and Masters.
- Statistics departments had the largest percentage and number increase in graduate students with $11 \%$ (up from 4,597 to 5,123)
- First-year graduate students increased in all groups, except Math Public Medium, Math Private Large, and Masters; Math Public Large, Statistics, and Biostatistics Groups had the largest percentage increases with $10 \%, 10 \%$, and $17 \%$, respectively.
- US citizen and permanent resident graduate students remained essentially unchanged at 11,823 while most groups reported decreases of less than $5 \%$, the Math Public Small Group reported an $11 \%$ decrease; the Statistics Group reported an increase of $4 \%$, followed by the Math Public Medium and Math Public Large Groups which both reported increases of $3 \%$.
- Underrepresented minorities accounted for $14 \%$ of US citizen and permanent resident graduate students and 12\% of first-year graduate students. Females compose $37 \%$ of both of these categories.
- All groups reported an increase in underrepresented minorities expect Math Public Large and Stats which showed decreases of $28 \%$ and $15 \%$, respectively.
- Non-US citizen full-time graduate students and full-time female graduate student counts increased in all groups except Masters, where these respective counts decreased by $13 \%$ and $5 \%$.

Features of part-time graduate student data:

- Total part-time graduate student counts increased in all groups except in Math Private Large and Applied Math, where there were decreases of $22 \%$ and $9 \%$, respectively.
- Part-time US citizen and permanent resident graduate student counts increased $5 \%$ to 3,853, and non-US citizen counts increased $11 \%$ to 726.
- Underrepresented minorities account for $16 \%$ of part-time US citizen and permanent resident graduate students, the same as last year.

Table GS.2: Full-Time Graduate Students in All Doctoral Math Combined by Gender and Citizenship, Fall 2006-2015

|  | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total full-time graduate students | 10984 | 10936.7 | 10883 | 11286.5 | 13048 | 12514 | 12684 | 12961 | 13023 | 13431 |
| Female | 3279 | 3249 | 3193 | 3248 | 3839 | 3773 | 3771 | 3969 | 3925 | 4039 |
| \% Female | 30\% | 30\% | 29\% | 29\% | 29\% | 30\% | 30\% | 31\% | 30\% | 30\% |
| \% US Citizen \& Permanent Residents ${ }^{1}$ | 56\% | 56\% | 55\% | 56\% | 57\% | 56\% | 54\% | 53\% | 55\% | 53\% |
| \% Underrepresented minorities ${ }^{2}$ | 9\% | 9\% | 9\% | 9\% | 11\% | 8\% | 8\% | 9\% | 11\% | 15\% |
| Total first-year full-time graduate students | 2960 | 2964 | 2924 | 3040 | 3313 | 3288 | 3394 | 3623 | 3551 | 3646 |
| Female | 961 | 950 | 870 | 904 | 1019 | 1077 | 1036 | 1205 | 1193 | 1188 |
| \% Female | 32\% | 32\% | 30\% | 30\% | 31\% | 33\% | 31\% | 33\% | 34\% | 33\% |
| \% US Citizen \& Permanent Residents ${ }^{1}$ | 55\% | 56\% | 56\% | 55\% | 51\% | 50\% | 54\% | 53\% | 55\% | 53\% |
| \% Underrepresented minorities ${ }^{2}$ | 10\% | 10\% | 10\% | 9\% | 9\% | 9\% | 7\% | 10\% | 13\% | 14\% |

${ }^{1}$ Starting with 2014, departments were asked to report US citizen and permanent resident counts together; previously permanent residents were included in the non-US citizen counts. All percentages prior to 2014 have been updated to allow for comparison with previous years' data.
2 Prior to 2014 these counts only included US Citizens. Underrepresented minorities includes any person having origins within the categories American Indian or Alaskan Native, Black or African American, Hispanic or Latino, and Native Hawaiian or Other Pacific Islander.

## Faculty Categories

The faculty categories used in this report are described below. Departments were asked to report any faculty member who was considered to be full-time in the institution for the academic year and at least half-time in the department. Each faculty member was reported in exactly one of these categories.

Tenure-track faculty includes full-time faculty who hold tenured/tenure-eligible positions (i.e., only those individuals who are tenured full professors, other tenured and tenure-eligible faculty).
Postdoctoral faculty includes full-time faculty who have teaching and/or research responsibilities, but for a strictly limited term of employment (i.e., those individuals who hold a temporary position primarily intended to provide an opportunity to continue training or to further research experience).
Non-tenure-track faculty includes full-time faculty eligible for benefits and with an appointment that lasts at least one academic year. These faculty hold appointments that are renewable (potentially unlimited), fixed-term but not renewable, or temporary. Typical titles for these positions are Lecturer, Senior Lecturer, Instructor, Senior Instructor, Associate/Assistant/Full Teaching Professor, Professor of the Practice, or Clinical Professor, and similar titles for research-only faculty.
Part-time faculty includes those individuals who are hired term-by-term, paid by the course, and/or those in phased retirement.

## Department Groupings

In this report, Mathematical Sciences departments are those in four-year institutions in the US that refer to themselves with a name that incorporates (with a few exceptions) "Mathematics" or "Statistics" in some form. For instance, the term includes, but is not limited to, departments of "Mathematics," "Mathematical Sciences," "Mathematics and Statistics,""Mathematics and Computer Science," "Applied Mathematics," "Statistics," and "Biostatistics." Also, Mathematics (Math) refers to departments that (with exceptions) have "mathematics" in the name; Stats refers to departments that incorporate (again, with exceptions) "statistics" or "biostatistics" in the name but do not use "mathematics." The streamlining of language here militates against the possible objection to foreshortening the full subject names.

Starting with reports on the 2012 AMS-ASA-IMS-MAASIAM Annual Survey of the Mathematical Sciences, the Joint Data Committee implemented a new method for grouping doctorate-granting Mathematics departments. These departments are first grouped into those at public institutions and those at private institutions. These groups are further subdivided based on the size of their doctoral program as reflected in the average annual number of PhDs awarded between 2000 and 2010, based on their reports to the Annual Survey during that period.

For further details on the change in the doctoral department groupings, see the article in the October 2012 issue of Notices of the AMS at www. ams.org/ journa1s/notices/201209/rtx120901262p.pdf.

Math Public Large consists of departments with the highest annual rate of production of PhDs, ranging between 7.0 and 24.2 per year. Math Public Medium consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 6.9 per year.
Math Public Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.
Math Private Large consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 19.8 per year.
Math Private Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.
Applied Mathematics consists of doctoral-degree-granting applied mathematics departments.
Statistics consists of doctoral-degree-granting statistics departments.
Biostatistics consists of doctoral-degree-granting biostatistics departments.
Masters contains US departments granting a master's degree as the highest graduate degree.
Bachelors contains US departments granting a baccalaureate degree only.
Doctoral Math contains all US math public, math private, and applied math mathematics departments granting a PhD as the highest graduate degree.
Mathematics (Math) contains all Math Public, Math Private, and Applied Math, Masters, and Bachelors Groups above.
Stats consists of all doctoral-degree-granting statistics and biostatistics departments.
Listings of the actual departments that compose these groups are available on the AMS website at www.ams.org/ annual-survey/groups.

## Remarks on Statistical Procedures

The questionnaire on which this report is based, "Departmental Profile," is sent to all Doctoral, Masters, and Bachelors departments in the US.

Response rates vary substantially across the different department groups. For most of the data collected on the Departmental Profile form, the year-to-year changes in a given department's data are small when compared to the variations among the departments within a given group. As a result of this, the most recent prior year's response is used (imputed) if deemed suitable. After the inclusion of prior responses, standard adjustments for the remaining nonresponses are then made to arrive at the estimates reported for the entire grouping.

Standard errors were calculated for some of the key estimates for the Doctoral Math Group (Math Public, Math Private, and Applied Math), Masters Group and Bachelors, and Statistics and Biostatistics Groups. Standard errors are calculated using the variability in the data and can be used to measure how close our estimate is to the true
value for the population. As an example, the number of full-time faculty in the Masters Group is estimated at 4,343 with a standard error of 107 . This means the actual number of full-time faculty in the Masters Group is most likely between 4,343 plus or minus two standard errors, or between 4,129 and 4,557 . This is much more informative than simply giving the estimate of 4,343.

Estimates are also given for parameters that are totals from all groups, such as the total number of full-time faculty. For example, an estimate of the total number of full-time faculty in all groups except Statistics and Biostatistics combined is 22,373, with a standard error of 205.

The careful reader will note that a row or column total may differ slightly from the sum of the individual entries. All table entries are the rounded values of the individual projections associated with each entry, and the differences are the result of this rounding (as the sum of rounded numbers is not always the same as the rounded sum).

## Department Grouping Response Rates

## Survey Response Rates by Grouping

## Departmental Profile Department Response Rates

| Department Group | Number | Percent | Imputed $^{\prime}$ |
| :--- | :---: | ---: | :---: |
| Math Public Large | 26 of 26 | $100 \%$ | 8 |
| Math Public Medium | 40 of 40 | $100 \%$ | 5 |
| Math Public Small | 58 of 64 | $91 \%$ | 7 |
| Math Private Large | 24 of 24 | $100 \%$ | 5 |
| Math Private Small | 28 of 29 | $97 \%$ | 7 |
| Applied Math | 24 of $25^{2}$ | $96 \%$ | 1 |
| Statistics | 54 of 58 | $93 \%$ | 13 |
| Biostatistics | 35 of $45^{2}$ | $78 \%$ | 6 |
| Masters | 123 of 175 | $70 \%$ | 39 |
| Bachelors | 599 of 1,017 | $59 \%$ | 250 |
| Total | 1,011 of 1,503 | $67 \%$ | 341 |

1 See paragraph two under 'Remarks on Statistical Procedures.'
2 The populations for Applied Math and Biostatistics are slightly less than for the Doctorates Granted Survey because some programs do not formally "house" faculty, teach undergraduate courses, or award undergraduate degrees.

## Acknowledgments

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

## American Mathematical Society



The 2017 National Who Wants to Be a Mathematician is part of Mathemati-Con events at the meeting, open to the public on Saturday from 10 am to 4 pm , featuring James Tanton, Math Circles, Art Benjamin, Sarah Greenwald, Tim and Tanya Chartier, Math Wrangle, and the Porter Lecture by Ingrid Daubechies.


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[^7]
## BOOK REVIEW

# Mathematics of Planet Earth: Mathematicians Reflect on How to Discover, Organize, and Protect Our Planet 



A Review by Christopher K. R. T. Jones

## Mathematics of Planet Earth: Mathematicians Reflect on How to Discover, Organize, and Protect Our Planet Hans Kaper and Christiane Rousseau, Editors <br> Society for Industrial and Applied Mathematics (SIAM), 2015 <br> Paperback, US\$39.00 <br> ISBN-13: 978-1-6119-7370-9

This book is probably unlike any other reviewed in the Notices. It is certainly not a textbook, nor is it a popular mathematics book, although it is closer to the latter. It is a collection of blogs from the year on Mathematics of Planet Earth. As such it is largely lacking in editorial control. Despite this fact (or perhaps because of it) it is a real pleasure to read.
The contributions vary greatly in detail and depth. Some are merely short descriptions with a reference or two, while some are nuggets of information about our planet with complete mathematical explanations. A real gem along these lines is the piece by Robert Miller on the Gulf Stream. From popular expositions, you might
get the impression that it was merely pure luck we have the Gulf Stream and that Europe should just enjoy it while it lasts, since a warming planet may no longer sustain such a fluke. Miller shows that there are actually intrinsic physical reasons why the Gulf Stream runs and that it will not go away without something much more

[^8]> A real gem is the piece on the Gulf Stream.
drastic happening, such as the Earth no longer spinning or the sun no longer shining. There is mathematics involved in his piece, but, from the vantage point of a mathematician, it is not complicated.

This is the kind of book you might keep on a table in your living room or by your bed and occasionally pick up to read a blog or two. Each time, you will most likely be left with a warm feeling about what mathematics does in our world. You will start to build a picture of what many different people are thinking about how mathematics is applied and why it is important to get out there and see what scientific issues can and need to be addressed using varying levels of mathematics.

A key question, however, is whether the book provides a reliable guide to the problems presented by considering our planet in its current state, or even the more modest objective of listing the mathematical issues involved. I suggest that this is where the reader should be wary. This volume has not been put together with an eye to giving complete coverage of the subject, nor does it even attempt to bring a focus on the most important issues. To people who have been working in this area, it will come across as rather haphazard in both its coverage and its focus. There are many issues of central and critical importance that are either completely omitted or only summarily touched upon. To mention just a few: ice in its many forms, both on land and sea, is not well modeled nor even understood as a material, and the problems are deeply mathematical. Cloud physics is not well represented in climate models, again in part because of our lack of understanding and lack of good models. The related issue of convection raises many mathematical questions. It would be unreasonable, however, to

> It was pure genius to coin the term "Mathematics of Planet Earth."

expect this book to cover all these issues in any depth, and indeed it would have lost much of its charm had the authors attempted to make a comprehensive introduction to the area out of this variety of contributions. To their enormous credit, they have smoothed out the presentation through their many individual blogs.

The authors chose not to reprint the blogs in their original chronological sequence, but rather to gather them under theme headings. The brilliance of their thinking about the subject comes out in this organization. The section titles are compelling-"A Planet to Discover,", "A Planet Supporting Life," "A Planet Organized by Humans," and "A Planet at Risk"-and form a good guide to the various parts of the subject. Some of the séctions are much better than others, however, and some give only cursory insight into the field. Surprisingly, the last, "A Planet at Risk," is one of the weakest. Given how much work there has been on prediction issues in climate, which is essentially a mathematical issue, it is a shame that they were not able to engage this community more.

It was pure genius to coin the term "Mathematics of Planet Earth" for the special year of 2013 organized by the various mathematics societies and interest groups worldwide. My understanding is that one of the authors here (Rousseau) deserves the credit. The phrase "Planet Earth" evokes an emotional response in each of us with an almost religious fervor. It is clearly this response and the related concern we each have for the future of our planet that is our best draw to bring people into this rich area of mathematical problems. But we probably don't know yet the boundaries of the subject nor what are its core areas.

I would argue that the issues of how our climate is changing and what we need to do to sustain the planet should frame the subject. The authors clearly go beyond the boundaries such a framing would provide. They include a chapter entitled "Beyond Planet Earth." If we take climate and sustainability as defining the area, it is hard to see the relevance of celestial mechanics and finding efficient pathways for spacecraft unless we are thinking of having to abandon the planet at some point! The problem is that there is a downside to the compelling name Mathematics of Planet Earth (MPE): we live on Earth and anything that happens on it or that we do as inhabitants could be viewed as concerning Planet Earth, and if it is mathematical, then it is a subject in this area. Seen this way, the area has no limits, and if so viewed, it will fail as a banner under which to organize and encourage research.

I have to admit that I go back and forth in my own mind on this point of framing this new area. There is a significant upside to its being viewed as all-encompassing, as then anybody can join in the effort and excitement. They can feel welcome under the big tent of MPE by virtue of what they are already doing. The test of success will be
whether being part of this new area will redirect their work towards something that really is relevant to climate and sustainability. If that happens, the authors of this volume and the organizers of the MPE effort more generally will be fully vindicated in their big-tent approach. With the Society for Industrial and Applied Mathematics starting an Activity Group on MPE, they have an opportunity to forge a clear direction, and we shall see how well this approach works.

## Photo Credits

Photo of ocean wave, public domain image, National Oceanographic and Atmospheric Administration. Image of globe by Visible Earth, NASA.
Photo of Christopher Jones is courtesy of Christopher Jones.

## ABOUT THE REVIEWER

The research of Christopher K. R. T. Jones focuses on the use of dynamical systems as a tool for solving problems that originate in applications, especially the use of dynamical systems methods in the study of nonlinear wave motion in neuroscience and optics, ocean dynamics, and, more recently, cli-


Christopher K. R.T. Jones mate. He is currently director of the Mathematics and Climate Research Network, funded by the National Science Foundation. The network is a broad-based effort to engage the mathematical community in climate science and define the problems that will form an emerging area of "climate mathematics."

## COMMUNICATION <br> Celebratio Mathematica

Rob Kirby

Celebratio Mathematica is a new web publication(celebratio.org) that is an open access e-only alternative to printed collected works.


Figure 1. Three screenshots (I-r: David H. Blackwell, Michael F. Atiyah, and S.R. Srinivasa Varadhan) of current volumes in Celebratio Mathematica.

With declining budgets, math libraries are no longer buying as many printed collected works of distinguished mathematicians. A few are still printed, but a lot fewer than in the past.

A "volume" at Celebratio Mathematica normally consists of a biography (occasionally an autobiography), a CV, a complete bibliography including interesting unpublished work, a list of PhD students, perspectives on the honoree's papers and work, and photos (see Figure 1). The bibliographies are a curated list of works (including abstracts) with links to published papers and their reviews in MathSciNet ${ }^{\circledR}$ and Zentralblatt MATH (it is too

[^9]costly to track down permissions to reproduce scans of journal papers).

Photos are fun. Celebratio Mathematica volumes include photos as in Figures 2-9, ranging from traditional conference photos to the more unusual, e.g., baby pictures or mountain-climbing feats.

The publisher of Celebratio Mathematica is MSP (msp. org), and if you pass by MSP's booth in the exhibitors' hall at the Joint Mathematics Meetings, you will see a long sequence of photos we have gratefully collected from sources such as the American Mathematical Society (AMS), Mariana Cook, the Archive of American Mathematics (which has Paul Halmos' photo collection), and many individual mathematicians.

Written remarks, perspectives, and commentaries are often the most valuable part of an honoree's volume, and also one of the hardest items to obtain, for it means inducing good mathematicians to set aside time to write knowledgably about interesting mathematical history. The perspectives in the volumes for Srinivasa Varadhan or Kai Lai Chung are good examples. Others are provided by the National Academy of Sciences who allows us to include their memoirs of deceased NAS members; for their complete collection, see www.nason1ine.org/publications/ biographica1-memoirs/on1ine-col1ection.htm1.

Another remarkable website that overlaps with Celebratio Mathematica in many ways is the MacTutor History of Mathematics (www-history.mcs.st-and.ac.uk) at the University of St. Andrews in Scotland. It is run by John O’Connor and Edmund Robertson, and gives concise biographies of over 1,000 mathematicians.


Figure 2. Dusa McDuff receives an honorary doctorate from York University, 2000.
Figure 3. Bill Thurston in hawsers.
Figure 4. Joan Birman, from the Bull. L.M.S. on the occasion of her election as an honorary member of the London Mathematical Society.
Figure 5a. Hassler Whitney, age fourteen, atop a spire in the Alps.
Figure 5b. Mike Freedman, age sixteen, on Polemonium Peak, 14,080 ft., in the high Sierras.
Figure 6. Shiing-Shen Chern with Zhou En-Lai.
Figure 7. Michael Atiyah (age 3) as the "little abbot."
Figure 8. David Blackwell.
Figure 9. Mexico City, 1956 Geometry and Topology Conference (for names in the photo, see celebratio.org/ Geometry_and_Topology/groupcover/147).

# A natural question was: Who deserves a volume? 

Celebratio Mathematica began with a conversation in Evans Hall on the Berkeley campus between Jim Pitman and me, and has grown at a modest rate. The website was designed by Alex Scorpan, who also directed the software development. After a while, a formal board of editors was needed and Robert Kohn (NYU), Hendrik Lenstra (Leiden), Barry Mazur (Harvard), Ruth Williams (UCSD), and Günter Ziegler (Berlin) joined us.

A natural question was: Who deserves a volume? Of course within each volume there must be substantial mathematics. Prize winners fit this criterion, as would most (all?) Fellows of the AMS. If one's name is attached to a bit of math, that might suffice, but the basic rule of thumb decided upon was that each honoree should have left a lasting mark on mathematics.

Another question might be: Is there an age limit? These days there are a fair number of sixtieth-birthday fests, and these can come with enough surplus energy among the organizers (and perhaps money to defray costs) to produce a volume. (I sometimes say that 59 is the lower bound because that's when Atiyah's first five volumes of collected works were published.) Celebratio Mathematica is not without costs, for a volume averages $\$ 4-5,000$ to produce and then maintain as $t \rightarrow \infty$. The goal is to have a permanent repository and to establish an endowment to fund the inevitable changes in software and storage. We are grateful to Harry Lucas, a PhD student of R. L. Moore, who funded the Moore volume and the volumes of another half-dozen of Moore's mathematical descendants. PhD students and colleagues are natural candidates to edit and fund a volume for an honoree.

There is an opportunity for mathematics or statistics departments to have a "shelf" of volumes for past and present distinguished members of that particular department. The University of Chicago, for example, was the first to have a shelf of their own (it helps with obtaining funding when the president is a mathematician, Robert Zimmer in this case). Harvard University has also agreed to have a dedicated shelf, and a few more proprietary shelves are in the works. This is an excellent way for a university to showcase a department, and we encourage more to consider doing so.

We invite mathematicians to propose volumes. They should send a short description of their honoree to either me or one of Celebratio Mathematica's other editors.

There is a great deal of mathematical lore that is passed on by word of mouth, but that tends to be lost when not written down. Celebratio Mathematica is a fine place to preserve that lore.

## Credits

Parts of Figure 1 are courtesy of Professor Lotfi Zadeh, Michael Atiyah, and © Mariana Cook 2007.
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THE AUTHOR


Rob Kirby

## Movie Review : "The Accountant"

The 2016 movie "The Accountant" was leading at the box office the last time I checked though, in my opinion, it is not a great movie.

It stars Ben Affleck as an "autistic mathematics savant" working as the Accountant for criminal organizations. This theme harkens back to the trite image of the mathematician as someone unsocial but good with numbers, who can multiply 4-digit numbers in his head and find the missing money in pages of accounts. When an opponent learns that the Accountant has knocked off a number of his agents, he asks incredulously, "What did he do, hit them over the head with an adding machine?"

The movie provides as its two examples of famous mathematicians Carl Gauss and Lewis Carroll.

The high point, mathematically, is when it mentions the nonrandomness of falsified data.
-Frank Morgan.

## COMMUNICATION

## Inverse Yogiisms

Lloyd N. Trefethen

## Berra Backwards

The great New York Yankees catcher Yogi Berra died in September, 2015. Berra was famous for his quirky sayings, like these:
"It ain't over till it's over."
"When you come to a fork in the road, take it."
"It gets late early out there."
"A nickel ain't worth a dime anymore."
"I always thought that record would stand until it was broken."
"You wouldn't have won if we'd beaten you."
"Nobody goes there anymore, it's too crowded." Maybe Berra never said half the things he said, but that's not the point. We have here a brand of malapropisms that people have been enjoying for years.

It's pretty easy to spot the trick that animates these quips. Yogiisms are statements that, if taken literally, are meaningless or contradictory or nonsensical or tautological-yet nev-

## I think Yogiisms hold a special lesson for mathematicians.

 ertheless convey something true. It's a clever twist that gets us smiling and paying attention. If you like, you could argue that literature and art sometimes use the same device. A Yogiism is like a Picasso painting, you could say, messing with reality in a manner that catches our interest and still conveys a truth.But I want to stay with words and their meanings. I think Yogiisms hold a special lesson for mathematicians, because our characteristic pitfall, I propose, is the inverse Yogiism: the statement that is literally true, yet conveys something false.

At some level, we're all well aware that saying useless true things is an occupational hazard. Just think of that joke about the people lost in a hot air balloon who shout "Where are we?" to a man on the ground. "You're in a

[^10]

Yogi Berra (1925-2015) in 1953. The New York Yankees beat the Brooklyn Dodgers that year to win the World Series for the sixteenth time.
balloon!" the mathematician answers. (I have heard this joke far too often.)

So we all know in a general way about our habit of taking things literally. My proposal is that this phenomenon is more important than we may realize, and the notion of an inverse Yogiism can help us focus on it. Inverse

Yogiisms in mathematics, and in science more generally, can impede progress sometimes for generations. I will describe two examples from my own career and then mention a third topic, more open-ended, that may be a very big example indeed.

## Faber's Theorem on Polynomial Interpolation

The early 1900s was an exciting time for the constructive theory of real functions. The old idea of a function as given by a formula had broadened to include arbitrary mappings defined pointwise, and connecting the two notions was a matter of wide interest. In particular, mathematicians were concerned with the problem of approximating a continuous function $f$ defined on an interval such as [ $-1,1$ ] by a polynomial $p$. Weierstrass's theorem of 1885 had shown that arbitrarily close approximations always exist, and by 1912, alternative proofs had been published by Picard, Lerch, Volterra, Lebesgue, Mittag-Leffler, Fejér, Landau, de la Vallée Poussin, Jackson, Sierpiński, and Bernstein.

How could polynomial approximations be constructed? The simplest method would be interpolation by a degree$n$ polynomial in a set of $n+1$ distinct points in [ $-1,1$ ]. Runge showed in 1900 that interpolants in equally spaced points will not generally converge to $f$ as $n \rightarrow \infty$, even for analytic $f$. On the other hand, Chebyshev grids with their points clustered near $\pm 1$ do much better. Yet around 1912, it became clear to Bernstein, Jackson, and Faber that no system of interpolation points could work for all functions. The famous result was published by Faber in 1914, and here it is in his words and notation (translated from [4]).
Faber's Theorem. There does not exist any set $E$ of interpolation points $x_{i}^{(n)}$ in $s=(-1,1)(n=1,2 \ldots ; i=$ $1,2 \ldots n+1$ ) with the property that every continuous function $\Phi(x)$ in $s$ can be represented as the uniform limit of the degree-n polynomials taking the same values as $\Phi$ for $x=x_{i}^{(n)}$.
The proof nowadays (though not yet in 1914) makes elegant use of the uniform boundedness principle.

Faber's theorem is true, and moreover, it is beautiful. Let me now explain how its influence has been unfortunate.

The field of numerical analysis took off as soon as computers were invented, and the approximation of functions was important in every area. You might think that polynomial interpolation would have been one of the standard tools from the start, and to some extent this was the case. However, practitioners must have often run into trouble when they worked with polynomial interpolants - usually because of using equispaced points or unstable algorithms, I suspect-and Faber's theorem must have looked like some kind of explanation of what was going wrong. The hundreds of textbooks that soon began to be published fell into the habit of teaching students that interpolation is a dangerous technique, not to be trusted. Here are some illustrations.

Isaacson and Keller, Analysis of Numerical Methods(1966), p. 275:


Georg Faber (1877-1966) in 1909, a few years after his invention of Faber polynomials for series expansions on sets in the complex plane. Faber later moved to the Technische Hochschule in Munich, where he spent the years 1916-1946.

It is not generally true that higher degree interpolation polynomials yield more accurate approximations.
Kahaner, Moler, and Nash, Numerical Methods and Software (1989), p. 94:

Polynomial interpolants rarely converge to a general continuous function.
Kincaid and Cheney, Numerical Analysis (1991), p. 319: The surprising state of affairs is that for most continuous functions, the quantity $\left\|f-p_{n}\right\|_{\infty}$ will not converge to 0 .
Stoer and Bulirsch, Introduction to Numerical Analysis (1993), p. 51:

It should not be assumed that finer and finer samplings of the function $f$ will lead to better and better approximations through interpolation.
Stewart, Afternotes on Numerical Analysis (1996), p. 153:

Unfortunately, there are functions for which interpolation at the Chebyshev points fails to converge.
Gautschi, Numerical Analysis: An Introduction (1997), p. 79:

It is not possible, therefore, to conclude... that Lagrange interpolation converges uniformly on [ $a, b$ ] for any continuous function, not even for judiciously selected nodes; indeed, one knows that it does not.
Quarteroni, Sacco, and Saleri, Numerical Mathematics (2000), p. 331:

Thus, polynomial interpolation does not allow for approximating any continuous function....

What a load of inverse Yogiisms! Statements like these, which appear in so many of the textbooks, give entirely the wrong impression. In fact, polynomial interpolation in Chebyshev points is a powerful and reliable method for approximation of functions. The Chebfun software system routinely works with degrees in the thousands [2].

The flaw in the logic is that Faber's theorem says nothing if $f$ is smooth [7]. If $f$ is Lipschitz continuous, that is more than enough to guarantee convergence of interpolants in Chebyshev points, and if it has a $k$ th derivative of bounded variation, the error in the degree-n interpolant is of size $O\left(n^{-k}\right)$. If $f$ is analytic, the convergence is at a geometric rate $O\left(\rho^{-n}\right), \rho>1$. Moreover, there are methods for computing these interpolants that are fast and numerically stable, notably the so-called barycentric interpolation formula.

So the idea that polynomial interpolants can't be trusted is a myth: a myth that has drawn strength from an impeccable theorem. Make sure your functions are Lipschitz continuous or better, as is easily done in almost any application, and Faber's theorem ceases to be applicable. In fact, polynomial interpolation in Chebyshev points has the same power and robustness as discrete Fourier analysis, to which it is essentially equivalent. We must hope that the numerical analysis textbooks of future generations will begin to tell students this.

## Squire's Theorem on Hydrodynamic Instability

We now move from numerical analysis to one of the oldest problems of fluid mechanics. Consider the idealized plane Poiseuille flow of a Newtonian liquid or gas in an infinite channel between two flat plates. (The mathematics is similar for other geometries such as a circular pipe, as investigated by Reynolds in 1883.) The flow is governed by the Navier-Stokes equations, and the key parameter is the Reynolds number Re, a nondimensionalized velocity.


Herbert Brian Squire FRS (1909-1961) was a specialist in fluid mechanics, and like Faber, he worked for a time in Göttingen as a young man. His career was spent at the Royal Aircraft Establishment, the University of Manchester, the National Physical Laboratory, and Imperial College.

Will the flow be laminar, or turbulent? At low values of Re , it is the laminar solution one sees in the laboratory, a smooth parallel downstream flow with a fixed velocity profile in the shape of a parabola. At high Re, though the laminar flow remains a mathematically valid solution of the equations, what one sees in the lab are the chaotic whirls and eddies of turbulence. Now if this is so, it seems clear that for high Re, the laminar flow must be unstable in the sense that small perturbations of that flow may get amplified. In an analysis going back to Orr and Sommerfeld in 1907-08, one makes this precise by linearizing the equations about the laminar solution, obtaining a linear operator $\mathcal{L}_{\mathrm{Re}}$ that governs the evolution of infinitesimal perturbations. If $\mathcal{L}_{\text {Re }}$ has an eigenfunction corresponding to an eigenvalue in the right half of the complex plane, this represents an infinitesimal perturbation that can grow exponentially, so the flow should be unstable; and if not, it should be stable.

This brings us to the elegant result published by Herbert Brian Squire in 1933. The geometry of our planar domain is 3D, with variables $x$ (streamwise), $y$ (perpendicular to the plates), and $z$ (spanwise). Analyzing the linearized operator for this 3D flow is going to be complicated. Squire's theorem, however, tells us we can ignore the $z$
direction and just do a 2D analysis. Here is the statement from his original paper [6].

Squire's Theorem. Any instability which may be present for three-dimensional disturbances is also present for twodimensional disturbances at a lower value of Reynolds' number.

The influence of Squire's theorem can be seen all across the literature of mathematical fluid mechanics. Whenever you see an analysis involving the famous Orr-Sommerfeld equation, the authors have probably taken a 3D flow problem and reduced it to 2D. For the plane Poiseuille configuration, the theory tells us that the 2D instability sets in at a critical Reynolds number $\mathrm{Re}_{c} \approx 5772.22$, a threshold first calculated accurately by Orszag. For $\mathrm{Re}<\mathrm{Re}_{c}$ we should expect stability and laminar flow, and for $\mathrm{Re}>\mathrm{Re}_{c}$, instability and turbulence.

Here are summaries from some books.
Lin, The Theory of Hydrodynamic Stability (1967), p. 27:
We shall now show, following Squire (1933), that the problem of three-dimensional disturbances is actually equivalent to a two-dimensional problem at a lower Reynolds number.
Tritton, Physical Fluid Dynamics (1977), p. 220:
... there is a result, known as Squire's theorem, that in linear stability theory the critical Reynolds number for a two-dimensional parallel flow is lowest for two dimensional perturbations. We may thus restrict attention to these.
Drazin and Reid, Hydrodynamic Stability (1981), p. 155: Squire's theorem. To obtain the minimum critical Reynolds number it is sufficient to consider only two-dimensional disturbances.
Friedlander and Serre, eds., Handbook of Mathematical Fluid Dynamics, v. 3 (2002), p. 248:

THEOREM 1.1 (Squire's theorem, 1933). To each unstable three-dimensional disturbance there corresponds a more unstable two-dimensional one.
Sengupta, Instabilities of Flows and Transition to Turbulence (2012), p. 82:

In a two-dimensional boundary layer with real wave numbers, instability appears first for twodimensional disturbances.
These and other sources are in agreement on a very clear picture, and only one thing is amiss: the picture is wrong! In the laboratory, observed structures related to transition to turbulence are almost invariably threedimensional. Moreover, it is difficult to spot any change of flow behavior at $\operatorname{Re} \approx 5772.22$. For $\operatorname{Re}<\operatorname{Re}_{c}$, many flows are turbulent when we expect them to be laminar. For $\mathrm{Re}>\mathrm{Re}_{c}$, many flows are laminar when we expect them to be turbulent. What is going on?

The flaw in the logic is that eigenmodal analysis applies in the limit $t \rightarrow \infty$, whereas the values of $t$ achievable in the laboratory rarely exceed 100. (Thanks to the nondimensionalization, $t$ is related to the length

> Squire's theorem has told us exactly the wrong place to look for hydrodynamic instability.
of a flow apparatus relative to its width.) Consequently, high-Reynolds number flows normally do not become turbulent in an eigenmodal fashion [8]. On the one hand, the exponential growth rates of unstable eigenfunctions, known as Tollmien-Schlichting waves, are typically so low that in a laboratory setup they struggle to amplify a perturbation by even a factor of 2. This is why laminar flow is often observed with $\mathrm{Re} \gg \mathrm{Re}_{c}$. On the other hand, much faster transient amplification mechanisms are present for 3D perturbations, even for $\mathrm{Re} \ll \mathrm{Re}_{c}$. The perturbations involved are not eigenfunctions, and in principle they would die out as $t \rightarrow \infty$ if they started out truly infinitesimal: Squire's theorem is, of course, literally true. But the transient growth of 3D perturbations is so substantial that in a real flow, small finite disturbances may quickly be raised to a level where nonlinearities kick in. In assuring us that the most dangerous disturbances are two-dimensional, Squire's theorem has told us exactly the wrong place to look for hydrodynamic instability.

## $\mathbf{P}=\mathbf{N P}$ ?

The most famous problem in computer science, which is also one of the million-dollar Clay Millennium Prize Problems, is the celebrated question " $\mathrm{P}=\mathrm{NP}$ ?" This puzzle remains unresolved half a century after it was first posed by Cook and Levin in 1971.

> The precision of a mathematical formulation has encouraged us to think the truth is simpler than it is.

Some computational problems can be solved by fast algorithms, and others only by slow ones. One might expect a continuum of difficulty, but the unlocking observation was that there is a gulf between polynomial time and exponential time algorithms. Inverting an $n \times n$ matrix, say, can be done in $O\left(n^{3}\right)$ operations or less, so we deal routinely with matrices with dimensions in the thousands. Finding the shortest route for a salesman visiting $n$ cities, on the other hand, requires $C^{n}$ operations for some $C>1$ by all algorithms yet discovered. As $n \rightarrow \infty$, the difference between $n^{C}$ and $C^{n}$ looks like a clean binary distinction. And there is a great class of thousands of problems, the NP-complete problems, that have been proved to be equivalent in the sense that all of them can be solved in polynomial time or none of them can - and nobody knows which.

It's an extraordinary gap in our knowledge. If I may pick two mathematical mysteries that I hope will be resolved before I die, they are the Riemann hypothesis and " $\mathrm{P}=\mathrm{NP}$ ?" It is so crisp, and so important!

Yet Yogi Berra seems to be looking over our shoulders. Computers are millions of times more powerful than they were in 1971, increasing the tractable size of $n$ in every problem known to man, so one might expect that the gulf between P and the best known algorithms for NP, which seemed significant already in 1971, should have opened up by now to a canyon so deep we can't see its bottom. Yet in the event, nothing so straightforward has happened. Some NP-complete problems still defeat us. Others are easily solvable in many instances. For example, one of the classic NP-complete problems is "SAT," involving the satisfiability of Boolean expressions. SAT solvers have become so powerful that they are now a standard computational tool, solving problem instances of scales in the thousands and even millions [5]. Surprisingly powerful methods have been developed for other NP-complete problems too, including integer programming [1] and the traveling salesman problem itself [3].

So is there a logical flaw in " $\mathrm{P}=\mathrm{NP}$ ?", as with Faber's theorem and Squire's theorem? I would not go so far as to say this, but it is certainly the case that, once again, the precision of a mathematical formulation has encouraged us to think the truth is simpler than it is. A typical NPcomplete problem measures complexity by the worst case time required to deliver the optimal solution. Experience has shown that in practice, both ends of this formulation are negotiable. For some NP-complete problems, like SAT, the worst case indeed looks exponential but in practice it is rare for a problem instance to come close to the worst case. For others, like the "max-cut" problem, it can be proved that even in the worst case one can solve the problem in polynomial time, if one is willing to miss the optimum by a few percent (for max-cut, 13 percent is enough). A field of approximation algorithms has grown up that develops algorithms of this flavor [9]. Often these algorithms rely on tools of continuous mathematics to approximate problems formulated discretely. Indeed, the whole basis of " $\mathrm{P}=\mathrm{NP}$ ?" is a discrete view of the world, and the distracting sparkle of this great unsolved problem may have delayed the recognition that often, continuous algorithms have advantages even for discrete problems. As scientists we must always simplify the world to make sense of it; the challenge is to not get trapped by our simplifications.

## Coda

When we say something precisely and even prove that it's true, we open ourselves to the risk of inverse Yogiisms. Would it be better if mathematicians didn't try so hard to be precise? Certainly not! Rigorous theorems are the pride of mathematics, which enable this unique subject to advance from one century to the next. The point is only that we must always strive to examine a problem from different angles, to think widely about its context as well as technically about its details. Or as Yogi put it,
"Sometimes you can see a lot just by looking."

## Acknowledgments

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## ABOUT THE AUTHOR

Nick Trefethen FRS is professor of numerical analysis at the University of Oxford. His college, Balliol, is the same one where Squire was an undergraduate.


Nick Trefethen

# A man is known by the books he reads. -Emerson 

## New and Noteworthy Titles on Our BookShelf December 2016



The Joy of SET: The Many Mathematical Dimensions of a Seemingly Simple Card Game, by Liz McMahon, Gary Gordon, Hannah Gordon, and Rebecca Gordon (Princeton University Press, November 2016).

SET is a game played with a deck of eighty-one cards on which are printed some shapes. The cards have four characteristics: number of shapes (one, two, or three), type of shape (oval, diamond, or squiggle), shading (open, hatched, or solid), and color (red, green, or purple). A "set" is a collection of three cards such that, for each of the four characteristics, the cards are all the same or are all different. Facing a matrix of cards laid out on a table, players race each other to identify sets, and the player identifying the most sets wins. SET is a game even very young children can play, and watch out-they are sometimes uncannily good at it. Finding the patterns among the cards feels very much like a mathematical activity, and, as The Joy of Set shows, the game has many mathematical aspects. The book discusses connections between SET and geometry, modular arithmetic, combinatorics, probability, linear algebra, and computer simulations. It also shows how the game can be used as a springboard to mathematical exploration. The writing of the book was a family affair: Liz McMahon and Gary Gordon are mathematics professors at Lafayette College, and Hannah and Rebecca are their daughters. Hannah is currently the world champion of SET and a student of nutrition and health policy at New York University; Rebecca is a middle-school teacher. Hannah joined her parents at the MAA MathFest in Columbus in August this year, where they gave a talk titled, "Tricks with SET."

Fashion, Faith, and Fantasy in the New Physics of the Universe, by Roger Penrose (Princeton University Press, August 2016).

Roger Penrose is world famous
 for his work in mathematical physics, for which he earned the 1988 Wolf Prize (together with Stephen Hawking). Penrose has written several popular books, of which the Notices has reviewed: Shadows of the Mind: A Search for the Missing Science of Consciousness (1994), reviewed by William Faris, February 1996; and The Road to Reality: A Complete Guide to the Laws of the Universe (2004), reviewed by Brian Blank, June/July 2006. Penrose’s latest popular book, Fashion, Faith, and Fantasy in the New Physics of the Universe, was published in time for his eighty-fifth birthday. Don't fashion, faith, and fantasy have a negative effect on physics research? Not entirely, Penrose notes. Fashionable areas of physics are fashionable for sound scientific reasons, not for purely sociological ones. Faith in a time-tested theory like Newton's theory of gravitation is very much warranted, "provided that its limitations are kept appropriately under firm consideration." And fantasy? That has a role to play too: Some aspects of the universe are so odd "that if we do not indulge in what may appear to be outrageous flights of fantasy, we shall have no chance at all of coming to terms with what may well be an extraordinary fantasticalseeming underlying truth." The book devotes one chapter each to string theory (fashion), quantum mechanics (faith), and big bang cosmology (fantasy). Then, in the fourth and final chapter, Penrose describes some of his own ideas for alternative routes that could be taken in these areas, including his twistor theory and his ideas that come under the name "conformal cyclic cosmology." The book ends with an appendix that sketches some of the main mathematical ideas needed to understand the exposition.

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# Logic's Lost Genius and 

# Gentzen's Centenary 

## A Dual Review by Jeremy Avigad

Logic's Lost Genius: The Life of Gerhard Gentzen Eckart Menzler-Trott<br>American Mathematical Society<br>and London Mathematical Society, 2007, US\$97, xxii+440<br>pages<br>ISBN: 978-0-8218-3550-0<br>Gentzen's Centenary: The Quest for Consistency edited by Reinhard Kahle and Michael Rathjen Springer, Cham, $x+561$ pages<br>ISBN: 978-3-3191-0103-3

Among the mathematical disciplines, logic may have the dubious distinction of being the one that always seems to belong somewhere else. As a study of the fundamental principles of reasoning, the subject has a philosophical side and touches on psychology, cognitive science, and linguistics. Because understanding the principles of reasoning is a prerequisite to mechanizing them, logic is also fundamental to a number of branches of computer science, including artificial intelligence, automated reasoning, database theory, and formal verification. It has, in addition, given rise to subjects that are fields of mathematics in their own right, such as model theory, set theory, and computability theory. But even these are black sheep among the mathematical disciplines, with distinct subject matter and methods. The situation calls to mind the words of Georges Simenon, the prolific Belgian author and creator of the fictional detective Jules Maigret, who told Life magazine in 1958, "I am at home everywhere, and nowhere. I am never a stranger and I never quite belong." If we use his self-assessment to characterize mathematical logic instead, the description is apt.

One of the things that gives the field this peculiar character is its focus on language. Every subject has its basic objects of study, and those of logic include terms,

[^11]expressions, formulas, axioms, and proofs. Logic was not even viewed as a part of mathematics until the middle of the nineteenth century, when George Boole's landmark 1854 treatise, The Laws of Thought, took the issue of viewing propositions as mathematical objects head on. Since the time of Aristotle, mathematics was commonly described as the science of quantity, encompassing both arithmetic, as the study of discrete quantities, and geometry, as the study of continuous ones. But Boole observed that propositions obey algebraic laws similar to those obeyed by number systems, thus making room for the study of "signs" and their laws within mathematics. Put simply, we can calculate with propositions just as we calculate with numbers.

Making sequences of symbols the subject of mathematical study was the cornerstone of metamathematics, the program by which David Hilbert hoped to secure the consistency of modern mathematical methods. By


George Gentzen, as pictured on the cover of Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart Menzler-Trott.
mathematizing things like formulas and proofs, Hilbert hoped to prove, using mathematical methods-in fact, using only a secure, "finitistic" body of mathematical methods-that no contradiction would arise from the new forms of reasoning. Hilbert gave a mature presentation of his Beweistheorie, or proof theory, in the early 1920s. Ironically, the representation of formulas and proofs as mathematical objects is an important component of Gödel's incompleteness theorems as well. Published in 1931, these showed that Hilbert's program could not succeed, in the following sense: No consistent system of mathematical reasoning that is strong enough to establish basic facts of arithmetic can prove its own consistency, let alone that of any larger system that includes it.

Nonetheless, proof theory is alive and well today. Its focus has expanded from Hilbert's program, narrowly construed, to a more general study of proofs and their properties. For Hilbert, as for Gödel, a proof was a sequence of formulas, each formula of which is either an axiom or follows from previous formulas by one of the stipulated rules of inference. Perhaps these qualify as mathematical objects, but they are the kinds of mathematical objects that only a logician could love. If there is one person who should be credited with developing a mathematical theory of proof worthy of the name, it is undoubtedly Gerhard Gentzen, whose work is now fundamental to those parts of mathematics and computer science that aim to study the notion of proof in rigorous mathematical terms.

Gentzen was born in Greifswald, in the northeastern part of Germany, in 1909. He began his studies at the University of Greifswald in 1928, but after two semesters he transferred to Göttingen, where he attended Hilbert's lectures on set theory and was introduced to foundational issues. He spent a semester visiting Munich and another visiting Berlin, and then returned to Göttingen in 1931. He learned about Hilbert's program and Gödel's results from Paul Bernays, whose collaboration with Hilbert ultimately culminated in the Grundlagen der Mathematik, the two volumes of which were published in 1934 and 1939.

Hilbert and his students had studied classical firstorder arithmetic as an important example of a classical theory whose consistency one would hope to prove by finitistic methods. In 1930 Arendt Heyting presented a formal axiom-

## Gentzen's work is fundamental to the study of proof in rigorous mathematical terms.

 atization of intuitionistic first-order arithmetic in accordance with the principles of L. E. J. Brouwer. In 1932 Gentzen showed that the former could be interpreted in the latter, using an explicit translation that is now known as the double-negation translation. This showed, in particular, that the consistency of classical arithmetic can be derived from theconsistency of intuitionistic arithmetic using finitistic means. Gentzen withdrew his submission, however, when he learned that Gödel himself had obtained the same result, by essentially the same method. The translation is often referred to as the Gödel-Gentzen translation, in honor of both.

The reduction of classical arithmetic to intuitionistic arithmetic seems to have encouraged Gentzen to think about the possibility of obtaining finitistic consistency proofs of either of these theories (and hence both). The second incompleteness theorem implies that such a proof could not be carried out within the theories themselves, so the challenge was to find suitably strong principles that could be argued to conform to the vague criterion of being finitistic. In order to do


David Hilbert hoped to establish a provably consistent foundation for mathematics. so, he needed a notion of deduction that was mathematically clean and robust enough to make it possible to study formal derivations in combinatorial terms. This was the subject of Gentzen's 1934 dissertation.

In fact, Gentzen developed two fundamentally different proof systems. The first, known as natural deduction, is designed to closely model the logical structure of an informal mathematical argument. Formulas in first-order logic are built up from basic components using logical connectives such " $A$ and $B$," " $A$ or $B$," "if $A$, then $B$," and "not $A$, " as well as the quantifiers "for every $x$, $A$ " and "there exists an $x$ such that $A$. " For each of these constructs, natural deduction provides one or more introduction rules, which allow one to establish assertions of that form. For example, to prove " $A$ and $B$," you prove $A$, and you prove $B$. To prove "if $A$, then $B$," you assume $A$ and, using that assumption, prove $B$. Natural deduction also provides elimination rules, which are the rules that enable one to make use of the corresponding assertions. For example, from " $A$ and $B$," one can conclude $A$, and one can conclude $B$. The elimination rule for " $A$ or $B$ " is the familiar proof by cases: if you know that $A$ or $B$ holds, you can establish a consequence $C$ by showing that it follows from each.

Gentzen also designed a system known as the sequent calculus, with a similar symmetric pairing of rules and, for some purposes, better metamathematical properties. In both cases, Gentzen considered reductions, steps that can be used to simplify proofs and avoid unnecessary detours. The Hauptsatz (main theorem) of his dissertation, now also known as the cut elimination theorem, is a seminal and powerful tool in proof theory. It shows that appropriate reductions in the sequent calculus can be used to transform any proof into one in a suitable normal form, in which every derived formula is justified, in a sense, from the bottom up.

While Gentzen completed this work, the Nazi party was on the rise, and the political and academic environment in Germany was deteriorating. Göttingen was particularly hard hit. Hermann Weyl, who had assumed Hilbert's chair in 1930, fled to the United States in 1933 with his wife, who was Jewish, and joined the Institute for Advanced Study. Emmy Noether fled similarly to Bryn Mawr. Bernays, who had become Gentzen's advisor, was summarily dismissed from his post in 1933 because of his Jewish ancestry.

In 1935 Gentzen submitted an article to the Mathematische Annalen describing a consistency proof for arithmetic. He began by extending the system of natural deduction to intuitionistic arithmetic, adding a rule to encapsulate proof by induction. The strategy was to show that proofs in the system can be reduced to ones in normal form, since from the description of the normal forms, it was then immediate that no such proof could conclude in a contradiction. A copy of the paper was sent to Weyl, who felt that in "the immediate future it should play the rôle of the standard work on the foundations of mathematics." The paper was also discussed by Gödel and Bernays on a boat to New York in 1935, though we do not know the content of their discussions. Apparently the paper met with criticism at the Annalen, and a letter that Gentzen wrote to Bernays suggests that the problem was that it was not sufficiently clear that the reduction procedure he described would always terminate. Gentzen rewrote the argument entirely, adapting it from natural deduction to a sequent calculus for classical arithmetic. This time he assigned to each proof an ordinal less than an ordinal known as $\varepsilon_{0}$ in such a way that with each reduction the associated ordinal decreases. Since, by definition, there is no infinite descending sequence of ordinals, every reduction sequence necessarily terminates.

The revised proof was published in the Annalen in 1936, but Gentzen's original proof is independently interesting, and one can show that the associated reduction procedure does in fact terminate. This version was published in English translation in 1969 and in the original German in 1974. Jan von Plato has provided a detailed history of these results, as discussed below.

It is perhaps a sign of the marginal role that syntax plays in most branches of mathematics that Gentzen's name is generally unfamiliar outside proof theory, but he is, today, considered to be a seminal figure in computer science. Natural deduction and the sequent calculus are fundamental to automated reasoning, where normal form theorems play a key role in reducing the space that algorithms have to search to find an axiomatic derivation. The connections between deduction and computation that Gentzen foreshadowed play an important role in the study of functional programming languages, where syntactic typing judgments are used to help ensure that programs meet their specifications. His ideas have been generalized to stronger logics, including second-order and higher-order logic, as well as modal logics and other special-purpose logics designed for reasoning about computation and computational processes. Although Gentzen's structural analyses were really byproducts of his work on the consistency problem, they are fundamental to contemporary
proof theory. Sequents, rules, and normal forms are to that community of researchers what functions, operators, and derivatives are to the analyst, and now we can think of formal deduction only in those terms.

The first book under review, Logic's Lost Genius, is a biography of Gentzen that was originally published in German with the title Gentzens Problem. The work was then revised by the author, Eckart Menzler-Trott, and translated


Kurt Gödel proved Hilbert's plan hopeless. to English by Craig Smoryński and Edward Griffor. The appendices include a history of Hilbert's program by Smoryński, translations of three expository lectures delivered by Gentzen in 1935 and 1936, and a friendly and informative overview of Gentzen's work by von Plato.

Menzler-Trott does a good job of conveying the intellectual milieu of German foundational research. Readers will enjoy following the backstage exchanges between Gentzen and many important figures in early mathematical logic, including Weyl, Bernays, Gödel, Heyting, Alonzo Church, and Wilhelm Ackermann. But the greater and more moving drama is the utter collapse of the German intellectual environment as a result of the Nazi rise to power. It is painful to read firsthand reports of German science being purged of Jewish involvement by Nazi administrators and Nazi sympathizers within the academic community. And how did Gentzen respond? Disgracefully, in fact: in 1933 he voluntarily joined the paramilitary wing of the Nazi party militia, known as the Sturmabteilung, or $S A$. It is jarring to see Gentzen end letters to his old mentor in Greifswald, Martin Kneser, with a hearty "heil Hilter!" For those of us who admire Gentzen's work and contributions to logic, this raises knotty questions as to the extent to which we can admire someone's scientific contributions while isolating them from less commendable, and even deplorable, aspects of their lives.

But Menzler-Trott's portrayal of Gentzen is sympathetic. The sense we get from the stories of Gentzen's youth and from the letters he wrote to colleagues is that of someone bright, affable, creative, and polite, and above all devoted to his work. There is little to suggest that Gentzen had any interest in politics; joining the SA comes across as a pathetic attempt to keep himself in
good graces with the establishment while maximizing his chances of securing financial support to continue with his research. In Menzler-Trott's assessment, Gentzen had "a certain passive, almost phlegmatic trait in all things which did not concern mathematics."

Indeed, he seems at times to have had no real sense of what was going on around him. In April of 1934 he wrote an upbeat letter to Bernays in which he complained of the difficulty of obtaining a teaching position and casually mentioned that he had joined the SA, "as has been urgently advised from various quarters." After relating his progress on consistency proofs, he asked, cheerfully, "Are you coming again to Göttingen for the summer semester?" It is almost as though he had forgotten that Bernays had been stripped of his license to teach and was trying to convince himself that nothing had really changed.

To be sure, being generally clueless and fixated on research is not an excuse for failing to take a stand against the Nazi atrocities and the indignities that were suffered by his colleagues. But I suspect that many readers of the Notices will find Gentzen's naiveté and preoccupation to be disarmingly familiar. We see these traits in ourselves and in our colleagues, and Gentzen's story challenges us to wonder whether we would have done better and, indeed, whether there are things we could be doing better in the present day.

In any case, Gentzen's attempts to withdraw into his work did not succeed, and he did not lead a happy or easy life. In 1939 he was called to military service as a radio operator inside Germany. In 1942 he was hospitalized in a state of nervous exhaustion and then released from military service. He was then assigned to teach mathematics at the German Charles University in Prague and chose to remain there after the Third Reich fell. As part of a general backlash against Germans in Prague, he was sent to a prison camp, where he died of malnutrition in August 1945. He was thirty-five years old.

Logic's Lost Genius is not easy reading. Its primary purpose is to serve as a historical record rather than an entertaining narrative, and Menzler-Trott accumulated a litany of names, places, dates, and events throughout the course of his prodigious research effort. Whenever possible, he lets source documents speak for themselves, and as a result many facts and opinions are conveyed through letters, scholarly reviews, firsthand narratives, government reports, and academic assessments. A long chapter details the Nazi transformation of logic and foundational research from 1940 to 1945, which had the goal of establishing racial purity and a proper "German" mathematics. Telling this story cannot have been an easy task for Menzler-Trott, who often lets his personal voice rise over the dry assemblage of facts in order to express his anger, frustration, and disgust. His admiration and respect for Gentzen is apparent throughout.

The second item under review, Gentzen's Centenary, is a very different work. It comprises a collection of essays that, taken together, provide a broad appraisal
of Gentzen's mathematical legacy on the occasion of the 100year anniversary of his birth. Although a few of the articles are targeted at logicians and proof theorists, most of the articles are written with a general mathematical audience in mind.

The book is divided into four parts. The first, titled "Reflections," offers historical and philosophical views of Gentzen's work. Reinhard Kahle considers the meaning and importance of Gentzen's consistency proofs and explores modern variations on Hilbert's program. Michael Detlefsen


Paul Bernays, Gentzen's advisor, was summarily dismissed from his post in 1933 because of his Jewish ancestry. argues that Gentzen's formalist position was less farreaching than Hilbert's: whereas Hilbert felt that a finitistic consistency proof was sufficient to ground abstract mathematical methods, Gentzen held that a proper grounding of abstract mathematics would have to ascribe content to mathematical abstractions as well. Anton Setzer proposes a broader approach to Hilbert's program that combines non-mathematical and philosophical validation of basic principles with metamathematical study.

The second part, titled "Gentzen's Consistency Proofs," focuses on those. Wilfried Buchholz provides an analysis and presentation of Gentzen's original consistency proof using modern terminology and notation. Jan von Plato relies on archival work to describe the evolution of Gentzen's consistency proof from his original submission to the final result, and shows that the original version was equally prescient in introducing themes that were to become central to proof theory. Whereas Gentzen used a classical sequent calculus in the final version of the consistency proof, the one that used the notation for $\varepsilon_{0}$, Dag Prawitz presents a Gentzen-style normalization proof for a formulation of arithmetic in intuitionistic natural deduction, and Annika Siders presents a similar consistency proof for a system based on an intuitionistic sequent calculus. William Tait explains the constructive principles that can be used to ground Gentzen's original consistency proof, and Michael Rathjen gives a clean mathematical analysis of Goodstein's theorem, an interesting number theoretic result which, essentially as a result of Gentzen's analysis, can be shown to be independent of first-order arithmetic.

The third part, titled "Results," presents technical results that round out Gentzen's work. Sam Buss studies cut elimination procedures in terms of time and space complexity, Fernando Ferreira discusses a consistency
proof for second-order arithmetic due to Clifford Spector, Herman Ruge Jervell explains how properties of ordinals are established in first-order arithmetic, and Wolfram Pohlers surveys some of the methods of contemporary ordinal analysis. Only the final part, "Developments," is aimed primarily at proof theorists. It includes substantial and interesting results by leading researchers in the area. The section includes a paper by Grigori Mints, a beloved and important figure in proof theory, who passed away just as the collection was about to go to press. The volume is movingly and appropriately dedicated to his memory.

Logic's Lost Genius and Gentzen's Centenary complement each other well, offering informative overviews of Gentzen's accomplishments and the intellectual and political environment in which they emerged. Contemporary textbooks provide sufficient evidence of the importance of Gentzen's work to modern logic. But the foundational background duly reminds us that the overarching goal of research in proof theory is a better understanding of the
principles of reasoning and what it means to do mathematics. And the dark historical narrative reminds us that research doesn't take place in a vacuum; even the purest of mathematicians have to contend with the exigencies of their social, political, and institutional environments, and it is important to face them deliberately rather than accidentally.

## Acknowledgments

I am grateful to Paolo Mancosu for advice and corrections.

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Jeremy Avigad

# Mathematical Software 

## Is It Mathematics or Is It Software?

Lisa R. Goldberg

> Note: The opinions expressed here are not necessarily those of Notices. Responses on the Notices webpage are invited.

Computers can empower mathematicians to envision and to prove theorems beyond natural limitations. At the same time, computers can facilitate misdirection and error on a grand scale. Working on software development is, in effect, engaging in an interesting genre of mathematical thinking. Since we are in an age where significant progress has been achieved on both practical and intellectual levels by such thinking, the development of mathematical software is genuinely part of mathematics. That is, the conceptual breakthroughs in software development should find a home in the academic mathematical community.

## The Mandelbrot Set

In the early 1900s French mathematicians Pierre Fatou and Gaston Julia laid the groundwork for the dynamics of rational maps of one complex variable. They found that iteration of a nonlinear rational map splits the plane into a pair of invariant sets with distinct profiles. The Fatou set is a region of orderly behavior, where nearby points have similar trajectories. For example, they may tend toward an attracting periodic point. The complementary Julia set is a region of chaos, where trajectories of nearby points can differ in ways that are impossible to predict. The dynamics of rational maps was an active area of research at the time of Fatou and Julia. However, the subject faded into obscurity and lay dormant until it was reinvigorated by software half a century later.

Polynomials are the most approachable rational maps, and quadratic polynomials of the form

$$
\phi_{c}(z)=z^{2}+c
$$

[^12]exhibit a dichotomy. The Julia set of $\phi_{c}$ is either connected or totally disconnected, depending on whether the trajectory of 0 is bounded. In 1978, mathematicians Robert Brooks and Peter Matelski exploited this property to depict the set of $c$ 's for which the Julia set of $\phi_{c}$ is connected.


Figure 1. Original image of the Mandelbrot set, plotted by Robert Brooks and Peter Matelski in 1978.

The odd image in Figure 1 caught the imagination of mathematicians, who generated beautiful and suggestive portrayals of what has become known as the Mandelbrot set. The more detailed image in Figure 2 hints at its complexity.

In 1975, three years before Brooks and Matelski drew the Mandelbrot set, the physicist Mitchell Feigenbaum was studying the dynamical behavior of quadratic polynomials on the real line. For $c$ between 0 and $-3 / 4$, the map $\phi_{c}$ has a single attracting fixed point. At $c=-3 / 4$, the fixed attractor bifurcates into a period two attractor, and period doubling of the attractor continues at a decreasing set of parameter values that tend to a chaos parameter. Using an HP-65 calculator, Feigenbaum observed that successive ratios of intervals between period-doubling parameters converged to what has become known as the Feigenbaum constant, approximately 4.6692016148 . The Feigenbaum constant was discovered independently by Pierre Coullet and Charles Tresser. The parameter space for quadratic polynomials on the line embeds in the Mandelbrot set,


Figure 2. A more detailed image of the Mandelbrot set hints at its complexity.
and Feigenbaum's chaos parameter is at the tail end of a shrinking sequence of balls attached to the central cartioid, as in Figure 3.


Figure 3. Successive blowups of the neighborhood of the chaos point identified by Feigenbaum, Coullet, and Tresser.

It is impossible to decouple the evolution of the theory of rational maps from the development of the mathematical software that illuminated the theory. As Tony Phillips and I wrote in 1993 about the work of Jack Milnor on the occasion of his sixtieth birthday:

Of special note is an unconventional article entitled "Self-similarity and hairiness in the Mandelbrot set," where he presents some of his numerical experiments in holomorphic dynamics and uses them as evidence for a set of conjectures. Milnor, usually meticulous about complete mathematical arguments, here has no theorems and no proofs. Rather, he brings his mathematical
strength and imagination to bear on every detail of the design and implementation of his experiments. The resulting data are so compelling as to suggest not only what the conjectures should say, but how they can be proved.
Analogs of Feigenbaum's period-doubling descent to chaos are found throughout the Mandelbrot set. Milnor's conjectures in his 1989 paper include an assertion that the Mandelbrot set is self-similar in the neighborhood

> At each step, computers augmented human ability, allowing us to attain new advances and insights. of chaos parameters, limits of perioddoubling parameters. Scale vanishingly small neighborhoods of a chaos parameter by powers of the Feigenbaum constant and you see the same image over and over again. Conjectures from Milnor's article were proved without the direct aid of computers by Mikhail Lyubich in 1999.

Feigenbaum discovered a universal principle with a calculator. Brooks and Matelski stumbled on an odd image with a primitive computer program. Mandelbrot saw the possibilities in that image, which bears his name. Milnor designed and implemented mathematical software to formulate conjectures related to the Mandelbrot set. At each step, computers augmented human ability, allowing us to attain new advances and insights.

## Flawed Software Leads to Flawed Science

Greater demand for mathematical software has led to specialization and a growing gap between development and application. This extends our reach, but it is risky.

In 2010 economists Carmen Reinhart of the University of Maryland and Kenneth Rogoff of Harvard University released a paper entitled "Growth in a time of debt," which linked higher debt-to-GDP ratios with lower growth. This influential article gave support to policymakers who wanted to restrain borrowing at a time when governments were struggling to address the severe consequences of the 2008 financial crisis. But the analysis in the article was wrong. Among the impactful errors made by the authors was a code bug that omitted data about Australia, Austria, Belgium, Canada, and Denmark from estimates. That bug accounted for roughly 30 percent of the reported difference in growth rates in lower and higher debt-to-GDP countries.

Code bugs are not the only flaws in software. In a 2016 empirical study, biomedical scientists Anders Eklund, Thomas Nichols, and Hans Knutsson discuss the impact of an erroneous statistical assumption in software for interpreting functional magnetic resonance imaging (fMRI). The implications are profound, since fMRI is used
by brain surgeons to assess the riskiness of invasive treatment and to map damage from tumors and strokes. As Eklund, Nichols, and Knutsson emphasize in their article:

Functional MRI (fMRI) is 25 years old, yet surprisingly its most common statistical methods have not been validated using real data...In theory, we should find 5 percent false positives (for a significance threshold of 5 percent), but instead we found that the most common software packages for fMRI analysis can result in false-positive rates of up to 70 percent. These results question the validity of some 40,000 fMRI studies and may have a large impact on the interpretation of neuroimaging results.
The problem, according to the authors, is "spatial autocorrelation functions that do not follow the assumed Gaussian shape." The "Gaussian shape" is embedded in standard packages such as Excel, and it is the default for determining statistical significance throughout the social sciences, whether or not it is an appropriate description of the data.

In a 2015 article, Bob Anderson, Stephen Bianchi, and I describe a stunning false positive in a 2006 article by financial economists Andrew Ang, Robert Hodrick, Yuhang Xing, and Xiaoyan Zhang. Oblivious to a 26 -sigma outlier, the authors used the "Gaussian shape" to infer statistical significance of a financial risk factor. Their article has 2,222 citations on Google Scholar, while our article detailing the erroneous inference has 4 citations.

The damage done by erroneous inference based on an assumed "Gaussian shape" is only beginning to be understood.

## Commercial versus Open Source

The UCLA mathematics department software page lists five packages that cover a wide range of applications. The first three packages-Maple, Mathematica, and Matlabare commercial, while the last two, Octave and SageMath, are open source. These packages are essential elements of educational programs, and they facilitate quantitative research in academia and industry.

The benefits of commercial and open source software are complementary. Commercial providers have incentives to offer easy-to-use interfaces, novice-safe routines, and customer service-in other words, to serve a broad and heterogenous group of users. The motivation for open source providers is summarized by the GNU Manifesto: freedom to run a program for any purpose, freedom to study the mechanics of the program and modify it, freedom to redistribute copies, and freedom to improve and change modified versions for public use. This was written in 1983 by Richard Stallman as part of a request for support for MIT's GNU Project, the free software movement. GNU includes Octave, and GNU packages such as $R$ are part of SageMath.

Launched in 2005 by number theorist and software developer William Stein, SageMath is true to the GNU Manifesto. It has benefitted from more than 500 unpaid
contributors and serves, by a conservative estimate, more than 50,000 active users. SageMath performs standard mathematical tasks such as optimization, statistical simulation, and matrix algebra. It also performs more exotic tasks, such as the calculation of the modular degree of an elliptic curve, and its web-based computing environment, SageMathCloud, includes TEXand courseware. Despite its breadth and popularity, SageMath has not found adequate funding in the academic mathematics community, and Stein is going commercial.

## Reproducibility

Reproducibility is actually the heart of science.

## - Eric Lander

A Pythagorean triple-free split of $S \subset \mathbb{N}$ is a decomposition of $S$ into a disjoint pair of subsets, neither of which contains a complete Pythagorean triple. In the 1980s Ron Graham offered a $\$ 100$ prize for a solution to the Boolean Pythagorean Triples problem: Is there is a Pythagorean triple-free split of the natural numbers?

In May 2016 computer scientists Martin J. H. Huele, Oliver Kullman, and Victor W. Marek posted an article on arXiv arguing that there is a Pythagorean triple-free split for $S=$ $\{1,2, \ldots, N\}$ so long as $N<$ 7284, but no such split exists when $N=7285$. This is a recent addition to the library of computer-assisted results, which includes the four color map theorem

> Validation by the academic community may follow more closely the practices of science. and the proof of the Kepler conjecture. Graham awarded the $\$ 100$ prize to the authors for their publicly-available-yet-inscrutable 200terabyte proof. Validation by the academic community may follow more closely the practices of science rather than mathematics. Can independent teams of researchers with different computers and different software replicate the results?

The Reproducibility Project was an attempt by a collection of 270 scientists to replicate one hundred studies published in high-ranking psychology journals in 2008. They found that only one-third to one-half of the studies could be replicated. This is consistent with the prediction of Stanford Medical School professor John Ioannidis, who argued in a 2005 article that most published research findings are false.

Best practices for reproducible research have been developed by the Reproducibility and Open Science Working Group, which is based at New York University, UC Berkeley, and University of Washington. Their mission is to support "software tools and practices that support the sharing, preservation, provenance, and reproducibility of data, software, and scientific workflows." Reproducible science is, of course, a laudable goal, but it will require substantial discipline on the part of the academic community and perhaps a modification of its incentive structure.

## Diaspora

It is a tradition at the UC Berkeley mathematics department graduation to announce students' plans for the future as they receive their diplomas. I listened with pleasure to the graduates' hopeful next steps. Some students were enrolled in PhD programs or professional schools, others were planning to travel, still others were joining startups or well-established corporations. There were many aspiring software developers, and I thought to myself that they are not leaving mathematics, but rather, extending its reach.

## Credits

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## AWM Research Symposium 2017

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UC Santa Barbara

SPECIAL SESSIONS on a wide-range of topics in pure and applied mathematics, statistics, math education and history of mathematics. Eight of the nineteen special sessions will be organized by research networks supported by the AWM ADVANCE grant (awmadvance.org): WIN Special Session: Work from Women In Numbers; WinCompTop Special Session: Applications of Topology and Geometry; WIMB Special Session: From cells to landscapes: modeling health and disease; ACxx Special Session: Algebraic Combinatorics; WINASC Special Session: Recent Research Development on Numerical Partial Differential Equations and Scientific Computing; WINART Special Session: Representations of Algebras; WiSh Special Session: Shape Modeling and Applications; WIT Special Session: Topics in Homotopy Theory. The remaining sessions are: Women in Sage Math; Women in Government Labs; EDGE-y Mathematics: A Tribute to Dr. Sylvia Bozeman and Dr. Rhonda Hughes; SMPosium: A celebration of the Summer Mathematics Program for Women; The many facets of statistics - applied, pure and BIG; History of Mathematics; Commutative Algebra; Circadian Oscillators; Geometric Group Theory; Recent progress in Several Complex Variables; Mathematics Education.

Also featured will be POSTER SESSIONS, WIKIPEDIA EDIT-A-THON, JOBS PANEL, NETWORKING EVENT \& BANQUET,PRE-SYMPOSIUM AWM STUDENT CHAPTER EVENT HOSTED BY IPAM ON FRIDAY EVENING. For details on registration, housing, how to apply for the poster sessions, the special sessions and the rest of the program visit www.awm-math.org

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## COMMUNICATION

# Interview with Karen Saxe 

Harriet Pollatsek

Karen Saxe is the new director of the AMS Washington Office.


Karen Saxe is DeWitt Wallace Professor of Mathematics at Macalester College. She can be reached at saxe@ macalester.edu.

Pollatsek: What does the director of the Washington office do?

Saxe: It's all about communicating the beauty and importance of research in mathematics. The director helps the AMS develop a policy agenda and advocacy strategy, works with principal decision makers who impact science and education funding, and reports back to the AMS membership.

Pollatsek: Did your sabbatical year in Washington affect your interest in this job?

Saxe: When I spent the 2013-14 year in Washington, working for Senator Al Franken as the AMS Congressional Fellow, I learned first-hand how Congress operates and that I enjoy following legislation and policy discussions. I have always enjoyed working with lots of different people, and the fellowship year showed me that I could work with those in Washington. I hadn't been so sure about this given that I have always worked in academia, though I have volunteered time in Minnesota in local schools, with the League of Women Voters, and with local officials on redistricting, a mathematical interest of mine.

Pollatsek: What else attracted you to the director position?

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Saxe: I wanted to work with AMS members and staff to continually update and refine our message about how mathematics is valuable for the nation and thus why it should be funded. I want to help stakeholders in Washington feel comfortable talking about mathematics and more generally to promote public understanding of the importance and impact of mathematics.

To be a strong advocate for our discipline, I look forward to establishing relations with AMS members to become more familiar with a broad range of mathematics at the frontiers of our collective knowledge. These relations will help me draw on the good will of AMS members when I need volunteers, for everything from Congressional briefings to grassroots advocacy.

Pollatsek: What is your previous experience working with the professional societies?

Saxe: I have been an active member of AMS, and also of AWM and MAA for many years. My most significant contribution has been in my recent service as a vice president of the MAA. I currently serve on the Science Policy Committees of both AWM and MAA. I have worked over the past year with several AMS members on policy-related issues via the NSF-funded Common Vision ${ }^{1}$ project, on which I am principal investigator, and also through my involvement on the Transforming Post-Secondary Education in Mathematics (TPSE Math) Board of Directors.

The one-year grant to the Common Vision project ends September 30, 2016. The work done during the first year we view as Phase 1 of a two-part process. We now have a one-year extension, to launch collaborative work between the MAA and NCTM as we move Common Vision forward into the proposed Phase 2, now dubbed "Common Vision: Taking Action." I see my role lessening significantly on Common Vision as other groups are spurred to action and develop projects. The TPSE work is much broader, and we are beginning our implementation phase. Tara Holm and I recently wrote an article for the 2016 June/July Notices about both of these projects and their relation to each
www.maa.org/common-vision
other. ${ }^{2}$ I will continue my work on the TPSE Board.

Pollatsek: What challenges do you see as you take on this position?

Saxe: One challenge that impacts our profession is the general lack of understanding, and I would say "fear," of mathematics. This fear is evidenced by the outsized publicity given critics like Andrew Hacker [whose book The Math Myth was reviewed in the 2016 November Notices]. (Evelyn Lamb rebuts well his argument "that abstract math is scary, damaging, and should be optional in American education."3) The fact that Hacker is able to get such media attention for his cause indicates that we have not done a good job presenting our field to the public.

Criticism of our community is not limited to naïve critics such as Hacker; Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics (President's Council of Advisors on Science and Technology, 2012) also criticizes our collective enterprise. ${ }^{4}$ The challenges laid out in this report, together with the opportunities described in the more positive The Mathematical Sciences in 2025 (National Research Council, 2013) serve together as a call to action for our community to acknowledge the need for improvement as well as a platform from which to base improvement efforts. Common Vision and TPSE are just two examples of current initiatives launched in response to these two reports and aimed at encouraging and enabling positive change in the teaching of the mathematical sciences at the post-secondary level.

To present a better and more accurate image of what we do, I would like to increase the presence of mathematicians in the government sphere, at all levels. I am interested in increasing not only the number of senior members of our community in important posts within federal agencies, but also PhD mathematicians at all stages of their careers as AAAS Science and Technology Fellows (the fellowship that allowed me to serve Senator Franken), and even our undergraduate math majors as Congressional staffers and federal agency employees.

A second big challenge to our profession is a consequence of the fact that we are living in difficult fiscal times. Most AMS members are employed at institutions of higher education, many of which have been coping for several years with state disinvestment. Not only can lack of sufficient funds diminish the quality of our individual work lives, it can decrease the quality of the education offered our students and therefore negatively affect our pipeline of graduate students and post-doctoral fellows.

[^14]> We have not done a good job presenting our field to the public.

Mathematicians rely on NSF funding to support their research activities and this, too, has taken a hit in recent years. The NSF budget is determined by the House and Senate Appropriations Committees' respective subcommittees on Commerce, Justice, Science and Related Agencies. As of this interview, both the House and Senate appropriations bills have passed, and both propose disappointing 2017 budget levels for the NSF. I am looking forward to working annually through the Congressional appropriations process to secure funds to support research. Not only do we need higher levels of funding allocated each year, we need to continue to push for long-term stability of these funds to enable programmatic and infrastructure planning for continued growth. Because federal funding is an increasingly large source of support for mathematicians, AMS's engagement with the annual de-cision-making process is critical.

So far I have talked about the challenges of the job "from the outside." I know, too, that I face personal challenges as I take on this new job. Sam Rankin has held this position for over two decades; establishing relations as he has both within the AMS and also in Washington will require diligence and patience. And, I would be remiss if I didn't acknowledge a more emotional challenge-that I am moving away from teaching students and working with terrific colleagues developing programs and curricula, both of which I love and will miss.

Pollatsek: Any final comments?
Saxe: Being selected for this position to succeed Sam Rankin is an honor and a tremendous opportunity to serve the mathematics community.

For information on the federal budget process, check out these articles on the web:

## A Brief Guide to the Federal Budget and Appropriations Process <br> Appropriations Watch: FY 2017 <br> Policy Basics: Introduction to the Federal Budget Process

See these related Notices articles:
A Common Vision for Undergraduate Mathematics by Tara Holm and Karen Saxe, June/July, 2016. www.ams. org/pub7ications/journa1s/notices/201606/ rnoti-p630.pdf

Review by David Bressoud of The Math Myth by Andrew Hacker, November, 2016.

Are Mathematics Faculty Ready for Common Vision? by Marcus Jorgensen, November, 2016.

## Credit

Photo of Karen Saxe is courtesy of David Turner/Macalester College.

## Mathematics People

## Matomäki and Radziwill Awarded 2016 SASTRA Ramanujan Prize



Kaisa Matomäki

KAISA MATOMÄKI of the University of Turku, Finland, and MAKSYM RADZIWILL of McGill University and Rutgers University have been awarded the 2016 SASTRA Ramanujan Prize for their joint work on multiplicative functions in short intervals. They will share the cash award of US\$10,000.

The prize citation reads "Kaisa Matomäki and Maksym Radziwill are jointly awarded the 2016 SASTRA Ramanujan Prize for their deep and far-reaching contributions to several important problems in diverse areas of number theory and especially for their spec-


Maksym Radziwill tacular collaboration, which is revolutionizing the subject. The prize recognizes that in making significant improvements over the works of earlier stalwarts on long-standing problems, they have introduced a number of innovative techniques. The prize especially recognizes their collaboration starting with their 2015 joint paper in Geometric and Functional Analysis which led to their 2016 paper in the Annals of Mathematics in which they obtain amazing results on multiplicative functions in short intervals, and in particular a stunning result on the parity of the Liouville lambda function on almost all short intervalsa paper that is expected to change the subject of multiplicative functions in a major way. The prize notes also the very recent joint paper of Matomäki, Radziwill, and Tao announcing a significant advance in the case
$k=3$ towards a conjecture of Chowla on the values of the lambda function on sets of $k$ consecutive integers. Finally, the prize notes that Matomäki and Radziwill, through their impressive array of deep results and the powerful new techniques they have introduced, will strongly influence the development of analytic number theory in the future."

Kaisa Matomäki was born in Nakkila, Finland, in 1985. After completing her PhD at the Royal Holloway College of the University of London in 2009 under the direction of Glyn Harman, she returned to Turku, where she is an Academy Research Fellow. She is currently on maternity leave and enjoys spending most of her time with her seven-month-old daughter and three-year-old son.

Maksym Radziwill was born in Moscow, Russia, in 1988. In 1991 his family moved to Poland and in 2006 to Canada. He received his PhD from Stanford University under the direction of Kannan Soundararajan. He has been a visiting member at the Institute for Advanced Study (2013-2014) and assistant professor at Rutgers University (2014-2017) and is currently assistant professor at McGill University.

The SASTRA Ramanujan Prize is awarded annually for outstanding contributions by young mathematicians to areas influenced by the work of Srinivasa Ramanujan. The age limit for the prize has been set at thirty-two because Ramanujan achieved so much in his brief life of thirty-two years. The prize will be awarded in December 2016 at the International Conference on Number Theory at SASTRA University in Kumbakonam (Ramanujan's hometown), where the prize has been given annually.

The members of the 2016 SASTRA Ramanujan Prize Committee were:

- Krishnaswami Alladi, chair, University of Florida
- Henri Darmon, McGill University
- Winfried Kohnen, University of Heidelberg
- Hugh Montgomery, University of Michigan
- Peter Sarnak, Princeton University and Institute for Advanced Study
- Michael Schlosser, University of Vienna
- Cameron Stewart, University of Waterloo

The full list of awardees of the SASTRA Ramanujan Prize follows:

- 2005 Manjul Bhargava and Kannan Soundararajan (two full prizes)
- 2006 Terence Tao
- 2007 Ben Green
- 2008 Akshay Venkatesh
- 2009 Kathrin Bringmann
- 2010 Wei Zhang
- 2011 Roman Holowinsky
- 2012 Zhiwei Yun
- 2013 Peter Scholze
- 2014 James Maynard
- 2015 Jacob Tsimerman
- 2016 Kaisa Matomäki and Maksym Radziwill (joint prize)
-Krishnaswami Alladi, University of Florida


## Khot Awarded MacArthur Fellowship



Subhash Khot

Subhash Khot of the Courant Institute of Mathematical Sciences, New York University, has been awarded a MacArthur Fellowship for 2016. According to the prize citation, Khot "is a theoretical computer scientist whose work is providing critical insight into unresolved problems in the field of computational complexity."
The MacArthur Foundation awards unrestricted fellowships to individuals who display exceptional creativity, promise for important future advances based on a track record of significant accomplishment, and potential for the fellowship to facilitate subsequent creative work.
-From a MacArthur Foundation announcement

## CMS Doctoral Prize Awarded



Vincent X. Genest

Vincent X. Genest of the Massachusetts Institute of Technology has been awarded the 2016 Doctoral Prize of the Canadian Mathematical Society (CMS). His doctoral thesis, "Algebraic structures, superintegrable systems and orthogonal polynomials," comprises twenty-three research papers and five conference proceedings written in collaboration with other mathematicians and physicists, most of which have been published in top-tier journals. In addition to mathematics, Genest is "passionate about motorcycle
riding, weightlifting, and tennis." The Doctoral Prize is awarded annually to a doctoral student from a Canadian university who has demonstrated exceptional performance in mathematical research.

## Diamond Awarded CME-MSRI Prize

Douglas Diamond of the University of Chicago has been named the 2015 recipient of the CME-MSRI Prize in Innovative Quantitative Applications by the CME Group and the Mathematical Sciences Research Institute (MSRI) for his work in financial economics. His major interest is the study of financial intermediaries, financial crises, and liquidity. The prize recognizes individuals who contribute original concepts in mathematical, statistical, or computational methods for the study of the markets' behavior and global economics.
-From a CME-MSRI announcement

## 2016 Davidson Fellows Selected



Katherine Hudek


Meena Jagadeesan

Two high school students whose projects involved the mathematical sciences have been named 2016 Davidson Fellows. Katherine Hudek, seventeen, of Grafton, Massachusetts, was awarded a US 25,000 scholarship for her project "A New Quantum Programming Language for Specifying Quantum Computations."

Meena Jagadeesan, eighteen, of Naperville, Illinois, was awarded a US\$50,000 scholarship for her project "The Exchange Graphs of Weakly Separated Collections."

The Davidson Fellows program, a project of the Davidson Institute for Talent Development, awards scholarships to students eighteen years of age or younger who have created significant projects that have the potential to benefit society in the fields of science, technology, mathematics, literature, music, and philosophy.
-From a Davidson Fellows announcement

## NDSEG Fellowships Awarded

Ten young mathematicians have been awarded National Defense Science and Engineering Graduate (NDSEG) Fellowships by the Department of Defense (DoD) for 2016. The Fellowships are sponsored by the United States Army, Navy, and Air Force. As a means of increasing the number of US citizens trained in disciplines of military importance in science and engineering, DoD awards fellowships to individuals who have demonstrated ability and special aptitude for advanced training in science and engineering.

Following are the names of the fellows, their institutions, and the offices that awarded the fellowships:

- Kristen Altenburger, Stanford University, Office of Naval Research (ONR)
- Samuel Cogar, University of Delaware, Army Research Office (ARO)
- Daniel Fortunato, Harvard University, Air Force Research Laboratory (AFRL)
- Rina Friedberg, Stanford University, ONR
- Kristen Hunter, Harvard University, ONR
- Shuai Jiang, Brown University, ARO
- Will PaZner, Brown University, AFRL
- Mark Perlman, Stanford University, AFRL
- Evan Rosenman, Stanford University, AFRL
- Angela Zhou, Undecided, ARO


## *NSF Postdoctoral Research Fellowships Awarded

The Mathematical Sciences Postdoctoral Research Fellowship Program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) awards Fellowships each year for postdoctoral research in pure mathematics, applied mathematics and operations research, and statistics. Following are the names of the fellowship recipients for 2016, together with their PhD institutions (in parentheses) and the institutions at which they will use their fellowships.

- Alex Blumenthal (New York University), University of Maryland
- Nathaniel bottman (Massachusetts Institute of Technology), Princeton University
- Brian Collier (University of illinois Urbana-Champaign), University of Maryland
- Nicholas Cook (University of California, Los Angeles), Stanford University
- Anil Damle (Stanford University), University of California Berkeley

[^15]- NATHAN Dowlin (Princeton University), Columbia University
- Nicholas Edelen (Stanford University), Massachusetts Institute of Technology
- MARIA GILLESPIE (University of California Berkeley), University of California Davis
- Karsten Gimre (Columbia University), Harvard University
- Ryan Goh (University of Minnesota), Boston University
- Boaz HAberman (University of California Berkeley), University of Chicago
- Irina Holmes (Louisiana State University), Washington University
- Katrina Honigs (University of California Berkeley), University of Utah
- Kaitlyn Hood (University of California Los Angeles), Massachusetts Institute of Technology
- Aukosh Jagannath (New York University), University of Toronto
- CASEY JAO (University of California Los Angeles), University of California Berkeley
- Kenneth Jeffries (University of Utah), University of Michigan
- Benjamin Knudsen (Northwestern University), Harvard University
- Ben Krause (University of California Los Angeles), University of British Columbia
- Subrahmanya Krishnamoorthy (Columbia University), Freie Universitat
- Jaclyn Lang (University of California Los Angeles), Université Paris 13
- Emily Leven (University of California San Diego), University of Pennsylvania
- Kathryn MANN (University of Chicago), University of California Berkeley
- Howard Nuer (Rutgers University), Northeastern University
- James Pascoe (University of California San Diego), Washington University
- Alexander Perry (Harvard University), Columbia University
- Aaron Royer (University of Texas Austin), University of California Los Angeles
- NoAh Schweber (University of California Berkeley), University of Wisconsin, Madison
- Benjamin Schweinhart (Princeton University), Ohio State University
- Kirill Serkh (Yale University), New York University
- Kyler Siegel (Stanford University), Massachusetts Institute of Technology
- LiAm Solus (University of Kentucky), Royal Institute of Technology
- Nicholas Switala (University of Minnesota), University of Illinois at Chicago
- Michael Tait (University of California San Diego), Carnegie Mellon University
- Jesse Thorner (Emory University), Stanford University
- Nathaniel Trask (Brown University), Sandia National Laboratory
- Linh Truong (Princeton University), Massachusetts Institute of Technology
- Kurt Vinhage (Pennsylvania State University), University of Chicago
- Preston Wake (University of Chicago), University of California Los Angeles
- Kimberly Weston (Carnegie Mellon University), University of Texas at Austin
- DAN Wilson (University of California Santa Barbara), University of Pittsburgh
- JASON XU (University of Washington), University of California Los Angeles
- Letao Zhang (Rice University), Stony Brook University
-NSF announcement


## B. H. Neumann Awards Given

The Australian Mathematics Trust has honored three mathematics teachers with B. H. Neumann Awards for service to the mathematics profession. The honorees are: - Lim Chong Keang, New Era College, Selangor, Malaysia

- Greg Gamble, University of Western Australia
- Andrew Kepert, University of Newcastle.

The awards honor Bernhard H. Neumann, who supported mathematics and mathematics teaching at all levels in Australia.
-From an Australian Mathematics Trust announcement

## Seymour Papert (1928-2016)



Seymour Papert at the Learning \& Common Sense Society of Minds Symposium in 1988.

Seymour Papert died on July 31, 2016. He was known worldwide for his innovative approaches to education and for the use of technology in education. Papert helped develop Logo ${ }^{\circledR}$, a programming language for children; he was co-director of MIT's Artificial Intelligence Lab; and he cofounded MIT's Media Lab. He was born in South Africa and earned his first doctorate in 1952 from the University of Witwatersrand. Papert later moved to England and earned his second PhD from the University of Cambridge in 1959 under the direction of Frank Smithies. Papert retired in 1998 but was still very active in education, especially in his new home state of Maine, where he helped establish a program to give each seventh and eighth grader in the state a laptop. Yet he was not one to treat the computer as a panacea, writing in 1980 against "the computer being used to program the child" (Mindstorms: Children, Computers and Powerful Ideas).

## Waclaw Szymanski (19492016)



Waclaw Szymanski

WACLAW SZYMANSKi, a functional analyst who also did work in ethnomathematics, died August 7, 2016, at the age of sixtysix. He received his PhD in 1974 from the Institute of Mathematics, Polish Academy of Sciences, and taught in Poland, Mexico, Canada, and at Indiana University before joining the faculty at West Chester University in 1985, where he spent most of his career. His research in functional analysis received international recognition; Japanese mathematician Fumio Kuro coined the term "Szymanski Family" for certain mathematical objects, studied by Szymanski, that are related to von Neumann algebras.

## Photo Credits

Photo of Subhash Khot is courtesy of the John D. and Catherine T. MacArthur Foundation.
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Photo of Seymour Papert is courtesy of L. Barry Hetherington.
Photo of Waclaw Szymanski is courtesy of Lin Tan, West Chester University.
Photo of Maksym Radziwill is ${ }^{\circledR}$ The Mathematisches Forschungsinstitut Oberwolfach.

# Mathematics Opportunities 

## AMS-AAAS Mass Media Summer Fellowships

The AMS provides support each year for a graduate student in the mathematical sciences to participate in the American Association for the Advancement of Science (AAAS) Mass Media Science and Engineering Fellows Program. This summer fellowship program pairs graduate students with major media outlets nationwide, where they will research, write, and report on science news and use their skills to bring technical subjects to the general public.

The principal goal of the program is to increase the public's understanding of science and technology by strengthening the connection between scientists and journalists to improve coverage of science-related issues in the media. Past AMS-sponsored fellows have held positions at National Public Radio, Scientific American, Voice of America, The Oregonian, the Chicago Tribune, and the Milwaukee Journal Sentinel.

Fellows receive a weekly stipend of US\$500, plus travel expenses, to work for ten weeks during the summer as reporters, researchers, and production assistants in newsrooms across the country. They observe and participate in the process by which events and ideas become news, improve their ability to communicate about complex technical subjects in a manner understandable to the public, and increase their understanding of editorial decision making and of how information is effectively disseminated. Each Fellow attends an orientation and evaluation session in Washington, DC, and begins the internship in mid-June. Fellows submit interim and final reports to AAAS. A wrapup session is held at the end of the summer.

Mathematical sciences faculty are urged to make their graduate students aware of this program. The deadline to apply for fellowships for the summer of 2017 is January 15, 2017. Further information about the fellowship program and application procedures is available online at www. aaas.org/programs/education/MassMedia; or applicants may contact Dione Rossiter, Director, AAAS

Mass Media Science \& Engineering Fellows Program, 1200 New York Avenue, NW, Washington, D.C. 20005; telephone: 202-326-6645; e-mail: drossite@aaas.org. Further information is also available at www.ams.org/programs/ ams-fe11owships/media-fe110w/massmediafe11ow and through the AMS Washington Office, 1527 Eighteenth Street, NW, Washington, D.C. 20036; telephone: 202-5881100; e-mail: amsdc@ams.org.
-AMS Washington Office

## *NSF Program in Computational and DataEnabled Science and Engineering in Mathematical and Statistical Sciences

The Program in Computational and Data-Enabled Science and Engineering in Mathematical and Statistical Sciences (CDS\&E-MSS) of the NSF accepts proposals that confront and embrace the host of mathematical and statistical challenges presented to the scientific and engineering communities by the ever-expanding role of computational modeling and simulation on the one hand and the explosion in production of digital and observational data on the other. The program encourages submission of proposals that include multidisciplinary collaborations for the training of mathematicians and statisticians in CDS\&E.

[^16]The window for submission is November 25-December 9, 2016. See www.nsf.gov/funding/pgm_summ.jsp?pims_ id=504687\&org=DMS\&from=home.
-NSF announcement

## National Defense Science and Engineering Graduate Fellowships

To help increase the number of US citizens trained in disciplines of military importance in science and engineering, the Department of Defense awards National Defense Science and Engineering Graduate Fellowships to individuals who have demonstrated ability and special aptitude for advanced training in science and engineering. The fellowships are awarded for a period of three years for study and research leading to doctoral degrees in any of fifteen scientific disciplines. Application forms are available online at ndseg.asee.org/apply_online and are due December 9, 2016. For further information, see ndseg.asee.org/.
-From an NDSEG announcement

## CRM Intensive Research

## Programs

The Centre de Recerca Matemàtica (CRM) will hold two Intensive Research Programs in the spring of 2017: "Mathematics of Memory," January 16-March 6, 2017, and "Operator Algebras: Dynamics and Interactions," March 13-June 23, 2017. See www.crm.cat/en/Host/ SciEvents/IRP/Pages/CurrentIRP.aspx.
-From a CRM announcement

## STaR Fellowship Program

The Service, Teaching, and Research (STaR) Program of the Association of Mathematics Teacher Educators (AMTE) supports the development of early-career mathematics educators, including their induction into the professional community of university-based teacher educators and researchers in mathematics education. Senior and mid-career mathematics education faculty organize and facilitate STaR events, serving as mentors to Fellows. Applications are due December 1, 2016; see https:// www.amte.net/star/app7y.

## Broad Fellows Sought

The Broad Institute, a collaborative institution with Harvard University, the Massachusetts Institute of Technology, and major teaching hospitals in Boston, is seeking outstanding candidates in mathematical, computational, or physical sciences to join the Institute as Broad Fellows. The Broad Fellows program provides opportunities for exceptional young quantitative and genomic scientists with exciting and innovative scientific visions to develop independent research programs. The Institute provides full support for the Fellows to establish their own independent research program, including salary and research expenses for themselves and a small team. For further information and application instructions, see the website https://www.broadinstitute.org/partnerships/ academic-affairs/broad-fellows/broad-fellowsprogram. Applications will begin to be reviewed on November 30, 2016.
-Aviv Regev, Massachusetts Institute of Technology

## Twenty Years Ago in the Notices

## December 1996:

## A Tale of Two Sieves, by Carl Pomerance.

This masterly exposition, which received the inaugural AMS Conant prize, discusses two factorization sieves-the quadratic sieve and the number field sieve-and some of the many people who helped to develop them.
www.ams.org/notices/199612/pomerance.pdf

## For Your Information

## Departments Coordinate Job Offer Deadlines

For the past seventeen years, the American Mathematical Society has led the effort to gain broad endorsement for the following proposal:

That mathematics departments and institutes agree not to require a response prior to a certain date (usually around February 1 of a given year) to an offer of a postdoctoral position that begins in the fall of that year.

This proposal is linked to an agreement made by the National Science Foundation (NSF) that the recipients of the NSF Mathematical Sciences Postdoctoral Fellowships would be notified of their awards, at the latest, by the end of January.

This agreement ensures that our young colleagues entering the postdoctoral job market have as much information as possible about their options before making a decision. It also allows departmental hiring committees adequate time to review application files and make informed decisions. From our perspective, this agreement has worked well and has made the process more orderly. There have been very few negative comments. Last year, one hundred seventy-six (176) mathematics and applied mathematics departments and four (4) mathematics institutes endorsed the agreement.

Therefore we propose that mathematics departments again collectively enter into the same agreement for the upcoming cycle of recruiting, with the deadline set for Monday, January 30, 2017. The NSF's Division of Mathematical Sciences has already agreed that it will complete its review of applications by January 20, 2017, at the latest, and that all applicants will be notified electronically at that time.

The American Mathematical Society facilitated the process by sending an e-mail message to all doctoral-granting mathematics and applied mathematics departments and mathematics institutes. The list of departments and institutes endorsing this agreement was widely announced
on the AMS website beginning November 1, 2016, and is updated weekly until mid-January.

We ask that you view this year's formal agreement at www.ams.org/employment/postdoc-offers.htm1 along with this year's list of adhering departments.

Important: To streamline this year's process for all involved, we ask that you notify T. Christine Stevens at the AMS (aed-mps@ams.org) if and only if:
(1) your department is not listed and you would like to be listed as part of the agreement; or
(2) your department is listed and you would like to withdraw from the agreement and be removed from the list.

Please feel free to e-mail us with questions and concerns. Thank you for consideration of the proposal.

-Catherine A. Roberts<br>AMS Executive Director<br>T. Christine Stevens<br>AMS Associate Executive Director

## Inside the AMS

## Project NExT 2016 Fellows Chosen

Six mathematicians have been selected as AMS Project NExT Fellows for 2016. Their names and affiliations follow.

- Derdei Bichara, California State University, Fullerton
- Timothy Chumley, Mount Holyoke College
- Alexander Diaz-Lopez, Swarthmore College
- JaKob Kotas, University of Portland
- Benjamin Linowitz, Oberlin College
- SuZanne Marie O’Regan, North Carolina A\&T University
Project NExT (New Experiences in Teaching) is a professional development program for new and recent PhDs in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. The AMS provides funding for a number of the Fellowships.
-From an MAA announcement


## AMS Department Chairs Workshop

The annual workshop for department chairs will be held a day before the start of the Joint Mathematics Meetings in Atlanta, Georgia, on Tuesday, January 3, 2017, from 8:00 am to 6:30 pm at the Atlanta Marriott Marquis Hotel. This one-day session for mathematical sciences department chairs is organized in a workshop format so as to stimulate discussion among attendees. Sharing ideas and experiences with peers creates an environment that
enables attending chairs to address departmental challenges from new perspectives.

Workshop leaders will be Malcolm Adams, former head, Department of Mathematics, University of Georgia; Matthew Ando, chair, Department of Mathematics, University of Illinois at Urbana-Champaign; Krista Maxson, vice president of academic affairs, University of Science and Arts of Oklahoma and former chair of the Department of Mathematical Sciences, Shawnee State University; and Douglas Mupasiri, head, Department of Mathematics, University of Northern Iowa.

Past workshop sessions have focused on a range of issues facing departments, including planning and budgeting, personnel management, assessment, outreach, faculty development, communications, student learning, and departmental leadership.

The workshop registration fee of US\$150 is in addition to and separate from the Joint Meetings registration. Those interested in attending can register at bit. $1 \mathrm{y} / 2 \mathrm{cf1} \mathrm{jcr}$ by December 16, 2016. For further information, please contact the AMS Washington Office at 202-588-1100 or amsdc@ams.org.
-AMS Washington Office

## Free Grant-Writing Workshop Offered

The American Mathematical Society, in conjunction with the National Science Foundation Directorate for Education and Human Resources (NSF-EHR), is pleased to offer a free workshop titled "Writing a Competitive Grant Proposal to NSF-EHR." This grant-writing workshop will be held on Monday, January 2, 2017, from 3:00 pm to 6:00 pm at the Atlanta Marriott Marquis Hotel.

Workshop Goals:

- To familiarize participants with current direction/ priorities in EHR.
- To familiarize participants with key EHR education research and development programs.
- To consider common issues of competitive proposals.
- To prepare participants to write a competitive proposal. Topics covered will include discussion of key programs in EHR; the merit review process and merit review criteria; discussion of scenarios-short passages drawn from proposals in the EHR portfolio designed to stimulate discussion about strengths and weaknesses of a proposal; and opportunities to discuss possible proposal ideas with program officers.

This free workshop is open to all interested participants who register by December 16, 2016, at bit. $1 \mathrm{y} / 2 \mathrm{c} 7 \mathrm{baBg}$. For further information, please contact the AMS Washington Office at 202-588-1100 or amsdc@ams.org.
-AMS Washington Office

## From the AMS Public Awareness Office



Mathemati-Con 2017 On Saturday, January 7, 2017, at the Joint Mathematics Meetings in Atlanta, the AMS and MAA are sponsoring sessions that are also open to the public, which will allow students and teachers to experience mathematics in some unexpected ways. Meeting attendees will enjoy the sessions as well. Mathemati-Con begins at 10:00 am with a lecture by James Tanton and concludes at 3:00 pm with the Porter Lecture by Ingrid Daubechies. In between are Arthur Benjamin, Tim and Tanya Chartier, Sarah Greenwald, the Math Circles session, Who Wants to Be a Mathematician, the Mathematical Art Exhibition, and the Math Wrangle. As they say in Boolean algebra, "Come one, come one."
-Annette Emerson and Mike Breen AMS Public Awareness Officers paoffice@ams.org

## Deaths of AMS Members

Robert L. Borrelli, professor, Harvey Mudd College, died on September 11, 2013. Born on March 4, 1932, he was a member of the Society for 54 years.

Elliod Dent, of Washington, DC, died on December 14, 2014. Born on February 3, 1929, he was a member of the Society for 52 years.

James Do, of California, died on January 10, 2015. Born in 1952, he was a member of the Society for 2 weeks.

Laurence Domino, of Okemos, Michigan, died on December 28, 2015. Born on August 1, 1953, he was a member of the Society for 39 years.

Robert Mortimer Ellis, of Madison, Wisconsin, died on December 6, 2013. Born on September 16, 1926, he was a member of the Society for 62 years.

William T. Fishback, of Richmond, Indiana, died on March 9, 2014. Born on January 28, 1922, he was a member of the Society for 67 years.

Gerald Freilich, of Brooklyn, New York, died on December 27, 2015. Born on December 29, 1926, he was a member of the Society for 70 years.

Irene E. (MARYJOEL) Jordan, of Maitland, Florida, died on December 9, 2015. Born on February 3, 1925, she was a member of the Society for 45 years.

Ernest B. Leach, of Mayfield Heights, Ohio, died on December 29, 2015. Born on December 21, 1924, he was a member of the Society for 62 years.

William C. Mason, of Lincoln, Massachusetts, died on March 10, 2013. Born on December 1, 1926, he was a member of the Society for 55 years.

Stephen H. McCleary, of New Mexico, died on October 18, 2015. Born on February 6, 1941, he was a member of the Society for 48 years.

GARY H. Meisters, of Fort Collins, Colorado, died on December 2, 2015. Born on February 17, 1932, he was a member of the Society for 57 years.

Arnold M. Ostebee, of Minnesota, died on October 11, 2015. Born on June 24, 1950, he was a member of the Society for 36 years.

James Osterburg, of Cincinnati, Ohio, died on November 27, 2015. Born on June 1, 1944, he was a member of the Society for 41 years.

George B. Pedrick, of Oakland, California, died on December 27, 2015. Born on September 22, 1926, he was a member of the Society for 65 years.

Henry Andrew Pogorzelski, of Orono, Maine, died on December 30, 2015. Born on September 26, 1922, he was a member of the Society for 43 years.

Klaus F. Roth, of London, United Kingdom, died on November 10, 2015. Born on October 29, 1925, he was a member of the Society for 43 years.

George Seifert, of Ames, Iowa, died on December 17, 2015. Born on March 4, 1921, he was a member of the Society for 66 years.

JOSEPH STAMPFLI, of Bloomington, Indiana, died on December 28, 2015. Born on August 9, 1932, he was a member of the Society for 56 years.

Lajos F. Takacs, of Cleveland Heights, Ohio, died on December 4, 2015. Born on August 21, 1924, he was a member of the Society for 55 years.

Remi Vaillancourt, of Ottawa, Canada, died on November 29, 2015. Born on June 16, 1934, he was a member of the Society for 48 years.

William H. Warner, of Plymouth, Minnesota, died on December 27, 2015. Born on October 6, 1929, he was a member of the Society for 64 years.

George E. Weaver, of Havertown, Pennsylvania, died on December 4, 2015. Born on October 31, 1942, he was a member of the Society for 43 years.

# New Publications Offered by the AMS 

To subscribe to email notification of new AMS publications, please go to www.ams.org/bookstore-email.

## Algebra and Algebraic Geometry



On Groups of PLhomeomorphisms of the Real Line

Robert Bieri, Johann Wolfgang Goethe-Universität Frankfurt, Frankurt am Main, Germany, and Ralph Strebel, Université de Fribourg, Switzerland

This monograph studies a natural generalization of Richard Thompson's famous group F and includes Melanie Stein's generalized F-groups. The main aims of this monograph are the determination of isomorphisms among the generalized F-groups and the study of their automorphism groups. This book is aimed at graduate students (or teachers of graduate students) interested in a class of examples of torsion-free infinite groups with elements and composition that are easy to describe and work with, but have unusual properties and surprisingly small presentations in terms of generators and defining relations.
This item will also be of interest to those working in geometry and topology.
Contents: Introduction; Construction of finitary PL-homeomorphisms; Generating sets; The subgroup of bounded homeomorphisms $B$; Presentations; Isomorphisms and automorphism groups; Notes; Bibliography; Index of notation; Subject index.
Mathematical Surveys and Monographs, Volume 215
December 2016, 174 pages, Hardcover, ISBN: 978-1-4704-2901-0, LC 2016025287, 2010 Mathematics Subject Classification: 20-02, 20F05, 20E32, 20E36; 20F38, AMS members US\$88, List US\$110, Order code SURV/215


> Hyperbolically Embedded Subgroups and Rotating Families in Groups Acting on Hyperbolic Spaces

F. Dahmani, Université Grenoble Alpes, France, V. Guirardel, Université de Rennes, France, and D. Osin, Vanderbilt University, Nashville, TN

Contents: Introduction; Main results; Preliminaries; Generalizing relative hyperbolicity; Very rotating families; Examples; Dehn filling; Applications; Some open problems; References; Index.
Memoirs of the American Mathematical Society, Volume 245, Number 1156

January 2017, 154 pages, Softcover, ISBN: 978-1-4704-2194-6, Individual member US $\$ 45$, List US $\$ 75$, Institutional member US $\$ 60$, Order code MEMO/245/1156


## Algebra and Computer Science

Delaram Kahrobaei, CUNY<br>Graduate Center, City University of New York, NY, Bren Cavallo, CUNY Graduate Center, City University of New York, NY, and David Garber, Holon Institute of Technology, Israel, Editors

This volume contains the proceedings of three special sessions: Algebra and Computer Science, held during the Joint AMS-EMS-SPM meeting in Porto, Portugal, June 10-13, 2015; Groups, Algorithms, and Cryptography, held during the Joint Mathematics Meeting in San Antonio, TX, January 10-13, 2015; and Applications of Algebra to Cryptography, held during the Joint AMS-Israel Mathematical Union meeting in Tel-Aviv, Israel, June 16-19, 2014.
Papers contained in this volume address a wide range of topics, from theoretical aspects of algebra, namely group theory, universal
algebra and related areas, to applications in several different areas of computer science.

From the computational side, the book aims to reflect the rapidly emerging area of algorithmic problems in algebra, their computational complexity and applications, including information security, constraint satisfaction problems, and decision theory.
The book gives special attention to recent advances in quantum computing that highlight the need for a variety of new intractability assumptions and have resulted in a new area called group-based cryptography.
This item will also be of interest to those working in applications.
Contents: F. Bassino, C. Nicaud, and P. Weil, Generic properties of subgroups of free groups and finite presentations; C. S. Chum and X. Zhang, A new multi-server scheme for private information retrieval; C. S. Chum, B. Fine, A. I. S. Moldenhauer, G. Rosenberger, and X. Zhang, On secret sharing protocols; M. E. Habeeb, A verifiable secret sharing scheme using non-abelian groups;
A. Kalka, Non-associative public-key cryptography; A. Kalka and M. Teicher, Non-associative key establishment protocols and their implementation; D. König, M. Lohrey, and G. Zetzsche, Knapsack and subset sum problems in nilpotent, polycyclic, and co-context-free groups; A. I. S. Moldenhauer, G. Rosenberger, and K. Rosenthal, On the Tits alternative for a class of finitely presented groups with a special focus on symbolic computations; A. Sale, Geometry of the conjugacy problem in lamplighter groups; A. Weiss, A logspace solution to the word and conjugacy problem of generalized Baumslag-Solitar groups; S. Yuan, Cryptographic hash functions from sequences of lifted Paley graphs.
Contemporary Mathematics, Volume 677
December 2016, 229 pages, Softcover, ISBN: 978-1-4704-2303-2, LC 2016019097, 2010 Mathematics Subject Classification: 20-XX, 68-XX, AMS members US\$86.40, List US\$108, Order code CONM/677


## Imaginary Schur-Weyl Duality

Alexander Kleshchev, University of Oregon, Eugene, and Robert Muth, Tarleton State University, Stephenville, TN

Contents: Introduction; Preliminaries; Khovanov-Lauda-Rouquier algebras; Imaginary Schur-Weyl duality; Imaginary Howe duality; Morita equaivalence; On formal characters of imaginary modules; Imaginary tensor space for non-simply-laced types; Bibliography.
Memoirs of the American Mathematical Society, Volume 245, Number 1157

January 2017, 83 pages, Softcover, ISBN: 978-1-4704-2249-3, 2010 Mathematics Subject Classification: 20C08, 20C30, 05E10, Individual member US\$45, List US\$75, Institutional member US\$60, Order code MEMO/245/1157


> Ergodic Theory, Dynamical Systems, and the Continuing Influence of John C. Oxtoby

Joseph Auslander, University of Maryland, College Park, MD, Aimee Johnson, Swarthmore College, PA, and Cesar E. Silva, Williams College, Williamstown, MA, Editors

This volume contains the proceedings of three conferences in Ergodic Theory and Symbolic Dynamics: the Oxtoby Centennial Conference, held from October 30-31, 2010, at Bryn Mawr College; the Williams Ergodic Theory Conference, held from July 27-29, 2012, at Williams College; and the AMS Special Session on Ergodic Theory and Symbolic Dynamics, held from January 17-18, 2014, in Baltimore, MD.
This volume contains articles covering a variety of topics in measurable, symbolic and complex dynamics. It also includes a survey article on the life and work of John Oxtoby, providing a source of information about the many ways Oxtoby's work influenced mathematical thought in this and other fields.
Contents: E. Akin, Conjugacy in the Cantor set automorphism group; S. Alpern, J. Auslander, and C. E. Silva, The mathematical work of John C. Oxtoby; T. I. Andress and E. A. Robinson Jr., The Čech cohomology and the spectrum for 1-dimensional tiling systems; K. Carroll and K. Petersen, Markov diagrams for some non-Markovian systems; D. Creutz, Contractive spaces and relatively contractive maps; R. L. Devaney, Mandelpinski structures in the parameter planes of rational maps; S. Ferenczi, J. Kułaga-Przymus, M. Lemańczyk, and C. Mauduit, Substitutions and Möbius disjointness; T. French, N. Ormes, and R. Pavlov, Subshifts with slowly growing numbers of follower sets; J. Furno, Haar measures and Hausdorff dimensions of $p$-adic Julia sets; J. Hawkins, Lebesgue measure theoretic dynamics of rational maps; A. Hill, The inverse problem for canonically bounded rank-one transformations; S. Kass and K. Madden, Higher dimensional shift spaces with shear; J. Kułaga-Przymus, M. Lemańczyk, and B. Weiss, Hereditary subshifts whose simplex of invariant measures is Poulsen; D. Ralston, Substitutions, symbolic codings, and discrepancy; B. D. Springer, Nearly continuous even Kakutani equivalence of nearly continuously rank-one transformations; K. Yancey, Dynamics of self-similar interval exchange transformations on three intervals.
Contemporary Mathematics, Volume 678
December 2016, 328 pages, Softcover, ISBN: 978-1-4704-2299-8, 2010
Mathematics Subject Classification: 37A05, 37B05, 37A40, 37P40, 37B50, 37B10, 37A30, 37A20, 01A70, AMS members US\$86.40, List US\$108, Order code CONM/678

> $L^{p}$-Square Function Estimates on Spaces of Homogeneous Type and on Uniformly Rectifiable Sets

Steve Hofmann, University of Missouri, Columbia, Dorina Mitrea, University of Missouri, Columbia, Marius Mitrea, University of Missouri, Columbia, and Andrew J. Morris, University of Missouri, Columbia

Contents: Introduction; Analysis and geometry on quasi-metric spaces; $T(1)$ and local $T(b)$ theorems for square functions; An inductive scheme for square function estimates; Square function estimates on uniformly rectifiable sets; $L^{p}$ square function estimates; Conclusion; References.
Memoirs of the American Mathematical Society, Volume 245, Number 1159

January 2017, 108 pages, Softcover, ISBN: 978-1-4704-2260-8, 2010 Mathematics Subject Classification: 28A75, 42B20; 28A78, 42B25, 42B30, Individual member US\$45, List US\$75, Institutional member US\$60, Order code MEMO/245/1159


> Classification of Actions of Discrete Kac Algebras on Injective Factors

Toshihiko Masuda, Kyushu University, Fukuoka, Japan, and Reiji Tomatsu, Hokkaido University, Sapporo, Japan

Contents: Introduction; Preliminary; Canonical extension of irreducible endomorphisms; Kac algebras; Classification of modular kernels; Classification of actions with non-trivial modular parts; Classification of centrally free actions; Related problems; Appendix; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 245, Number 1160

January 2017, 118 pages, Softcover, ISBN: 978-1-4704-2055-0, 2010 Mathematics Subject Classification: 46L65; 46L55, Individual member US\$45, List US\$75, Institutional member US\$60, Order code MEMO/245/1160

## MEMOIRS <br> American Mathematical Society

Rectifiable Measures, Square Functions Involving Densities and the Cauchy Transform


> Rectifiable Measures, Square Functions Involving Densities, and the Cauchy Transform

Xavier Tolsa, ICREA, Barcelona, Spain, and Universitat Autònoma de Barcelona, Spain

Contents: Introduction; Preliminaries; A compactness argument; The dyadic lattice of cells with small boundaries; The Main Lemma; The stopping cells for the proof of Main Lemma 5.1; The measure $\tilde{\mu}$ and some estimates about its flatness; The measure of the cells from BCF, LD, BSD and BCG; The new families of cells $\mathrm{BS} \beta$, NTerm, NGood, NQgood and NReg; The approximating curves $\Gamma^{k}$; The small measure $\tilde{\mu}$ of the cells from $\mathrm{BS} \beta$; The approximating measure $v^{k}$ on $\Gamma_{e x}^{k}$; Square function estimates for $\nu^{k}$; The good measure $\sigma^{k}$ on $\Gamma^{k}$; The $L^{2}\left(\sigma^{k}\right)$ norm of the density of $v^{k}$ with respect to $\sigma^{k}$; The end of the proof of the Main Lemma 5.1; Proof of Theorem 1.3: Boundedness of $T_{\mu}$ implies boundedness of the Cauchy transform; Some Calderón-Zygmund theory for $T_{\mu}$; Proof of Theorem 1.3: Boundedness of the Cauchy transform implies boundedness of $T_{\mu} ;$ Bibliography.
Memoirs of the American Mathematical Society, Volume 245, Number 1158

January 2017, 130 pages, Softcover, ISBN: 978-1-4704-2252-3, 2010 Mathematics Subject Classification: 28A75, 42B20; 28A78, Individual member US\$45, List US\$75, Institutional member US\$60, Order code MEMO/245/1158

## Differential Equations



The Role of Advection in a Two-Species Competition Model: A Bifurcation Approach
Isabel Averill, Bryn Mawr College, PA, King-Yeung Lam, Ohio State University, Columbus, and Yuan Lou, Renmin University, China

Contents: Introduction: The role of advection; Summary of main results; Preliminaries; Coexistence and classification of $\mu-\nu$ plane; Results in $\mathcal{R}_{1}$ : Proof of Theorem 2.10; Results in $\mathcal{R}_{2}$ : Proof of Theorem 2.11; Results in $\mathcal{R}_{3}$ : Proof of Theorem 2.12; Summary of asymptotic behaviors of $\eta_{*}$ and $\eta^{*}$; Structure of positive steady states via Lyapunov-Schmidt procedure; Non-convex domains; Global bifurcation results; Discussion and future directions; Appendix A. Asymptotic behavior of $\tilde{u}$ and $\lambda_{u}$; Appendix B. Limit eigenvalue problems as $\mu, \nu \rightarrow 0$; Appendix C. Limiting eigenvalue problem as $\mu \rightarrow \infty$; Bibliography.
Memoirs of the American Mathematical Society, Volume 245, Number 1161

January 2017, 106 pages, Softcover, ISBN: 978-1-4704-2202-8, 2010 Mathematics Subject Classification: 35J57, 35B32, 92D25, Individual member US\$45, List US\$75, Institutional member US\$60, Order code MEMO/245/1161


Differential Galois Theory through Riemann-Hilbert Correspondence

## An Elementary Introduction

Jacques Sauloy, Institut de Mathématiques de Toulouse, France

Differential Galois theory is an important, fast developing area which appears more and more in graduate courses since it mixes fundamental objects from many different areas of mathematics in a stimulating context. This book presents a transcendental approach to differential Galois theory, made possible through Riemann-Hilbert correspondence. This allows for a down-to-earth introduction to some of the most abstract parts of both theories, at an elementary level. A large variety of examples, exercise, and theoretical constructions, often via explicit computations, offers first-year graduate students an accessible entry into this exciting area.
This item will also be of interest to those working in number theory.
Contents: Part 1. A quick introduction to complex analytic functions: The complex exponential function; Power series; Analytic functions; The complex logarithm; From the local to the global; Part 2. Complex linear differential equations and their monodromy: Two basic equations and their monodromy; Linear complex analytic differential equations; A functorial point of view on analytic continuation: Local systems; Part 3. The Riemann-Hilbert correspondence: Regular singular points and the local Riemann-Hilbert correspondence; Local Riemann-Hilbert correspondence as an equivalence of categories; Hypergeometric series and equations; The global Riemann-Hilbert correspondence; Part 4. Differential Galois theory:Local differential Galois theory; The local Schlesinger density theorem; The universal (Fuchsian local) Galois group; The universal group as proalgebraic hull of the fundamental group; Beyond local Fuchsian differential Galois theory; Appendix A. Another proof of the surjectivity of $\exp : \mathrm{Mat}_{n}(\mathbf{C}) \rightarrow \mathrm{GL}_{n}(\mathbf{C})$; Appendix B. Another construction of the logarithm of a matrix; Appendix C. Jordan decomposition in a linear algebraic group; Appendix D. Tannaka duality without schemes; Appendix E. Duality for diagonalizable algebraic groups; Appendix F. Revision problems; Bibliography; Index.
Graduate Studies in Mathematics, Volume 177
January 2017, approximately 279 pages, Hardcover, ISBN: 978-1-4704-3095-5, LC 2016033241, 2010 Mathematics Subject Classification: 12H05, 30F99, 34-01, 34A30, 34Mxx, AMS members US\$71.20, List US\$89, Order code GSM/177

# Discrete Mathematics and Combinatorics 



Game Theory

A Playful Introduction
Matt DeVos, Simon Fraser University, Burnaby, BC, Canada, and Deborah A. Kent, Drake University, Des Moines, IA

This book offers a gentle introduction to the mathematics of both sides of game theory: combinatorial and classical. The combination allows for a dynamic and rich tour of the subject united by a common theme of strategic reasoning. Designed as a textbook for an undergraduate mathematics class and with ample material and limited dependencies between the chapters, the book is adaptable to a variety of situations and a range of audiences. Instructors, students, and independent readers alike will appreciate the flexibility in content choices as well as the generous sets of exercises at various levels.
This item will also be of interest to those working in applications.
Contents: Combinatorial games; Normal-play games; Impartial games; Hackenbush and partizan games; Zero-sum matrix games; Von Neumann's Minimax Theorem; General games; Nash equilibrium and applications; Nash's Equilibrium Theorem; Cooperation; n-player games; Preferences and society; On games and numbers; Linear programming; Nash equilibrium in high dimensions; Game boards; Bibliography; Index of games; Index.
Student Mathematical Library, Volume 80
January 2017, 360 pages, Softcover, ISBN: 978-1-4704-2210-3, LC 2016035452, 2010 Mathematics Subject Classification: 91-01, 91A46, 91A06, 91B06, All Individuals US\$39.20, List US\$49, Institutional member US\$39.20, Order code STML/80

## Geometry and Topology



## Cartan for Beginners

Differential Geometry via Moving Frames and Exterior Differential Systems, Second Edition

Thomas A. Ivey, College of Charleston, SC, and Joseph M. Landsberg, Texas A\&M University, College Station, TX

The second edition of this introduction to Cartan's approach to differential geometry presents thorough and modern treatments of the theory of exterior differential systems (EDS) and the method of moving frames. Including new chapters and updates from the first edition, key concepts are developed incrementally, with motivating examples leading to definitions, theorems, and proofs. This text has
numerous exercises and examples throughout and is suitable for a one-year graduate course in differential geometry. The second edition features three new chapters: on Riemannian geometry, emphasizing the use of representation theory; on the latest developments in the study of Darboux-integrable systems; and on conformal geometry, written in a manner to introduce readers to the related parabolic geometry perspective.
Contents: Moving frames and exterior differential systems; Euclidean geometry; Riemannian geometry; Projective geometry I: Basic definitions and examples; Cartan-Kähler I: Linear algebra and constant-coefficient homogeneous systems; Cartan-Kähler II: The Cartan algorithm for linear Pfaffian systems; Applications to PDE; Cartan-Kähler III: The general case; Geometric structures and connections; Superposition for Darboux-integrable systems; Conformal differential geometry; Projective geometry II: Moving frames and subvarieties of projective space; Linear algebra and representation theory; Differential forms; Complex structures and complex manifolds; Initial value problems and the Cauchy-Kowalevski theorem; Hints and answers to selected exercises; Bibliography; Index.
Graduate Studies in Mathematics, Volume 175
January 2017, 455 pages, Hardcover, ISBN: 978-1-4704-0986-9, LC 2016025601, 2010 Mathematics Subject Classification: 35N10, 37K35, 53A05, 53A20, 53A30, 53A55, 53C10, 53C15, 58A15, 58A17, AMS members US\$71.20, List US\$89, Order code GSM/175

## Math Education



# Algebraic Inequalities: New Vistas 

> Titu Andreescu, The University of Texas at Dallas, Richardson, TX, and Mark Saul, Executive Director, Julia Robinson Math Festivals

Beginning with simple arithmetic inequalities and building to sophisticated inequality results, this book leverages topics in the traditional high school curriculum to make accessible to the reader more sophisticated results. The theme of each chapter prepares the student for learning other mathematical topics, concepts, or habits of mind, while helping to build intuitions and formal knowledge along the way.
In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.
Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).
Contents: Some introductory problems; Squares are never negative; The arithmetic-geometric mean inequality, part I; The arithmetic-geometric mean inequality, part II; The harmonic mean; Symmetry in algebra, part I; Symmetry in algebra, part II; Symmetry in algebra, part III; The rearrangement inequality; The Cauchy-Schwarz inequality.

December 2016, 126 pages, Softcover, ISBN: 978-1-4704-3464-9, LC 2016038784, 2010 Mathematics Subject Classification: 97H30, AMS members US\$32.80, List US\$41, Order code MCL/19

## Probability and Statistics



## Large Deviations

S. R. S. Varadhan, Courant Institute of Mathematical Sciences, New York, NY

Based on a graduate course on large deviations at the Courant Institute, this book focuses on three concrete sets of examples: (i) diffusions with small noise and the exit problem, (ii) large time behavior of Markov processes and their connection to the Feynman-Kac formula and the related large deviation behavior of the number of distinct sites visited by a random walk, and (iii) interacting particle systems, their scaling limits, and large deviations from their expected limits. The examples are worked out in detail, and in the process the subject of large deviations is developed, giving the reader a flavor of how large deviation theory can help in problems that are not posed directly in terms of large deviations.
Titles in this series are co-published with the Courant Institute of Mathematical Sciences at New York University.
Contents: Introduction; Basic formulation; Small noise; Large time; Hydrodynamic scaling; Self-diffusion; Nongradient systems; Some comments about TASEP; Bibliography.
Courant Lecture Notes, Volume 27
December 2016, 104 pages, Softcover, ISBN: 978-0-8218-4086-3, LC 2016036484, 2010 Mathematics Subject Classification: 60-02, 60F10, AMS members US\$28.80, List US\$36, Order code CLN/27

## New AMS-Distributed Publications

## Algebra and Algebraic Geometry



## Metric Geometry of Locally Compact Groups

Yves Cornulier, Université de Paris-Sud, Orsay, France, and Pierre de la Harpe, Université de Gèneve, Switzerland

The main aim of this book is the study of locally compact groups from a geometric perspective, with an emphasis on appropriate metrics that can be defined on them. The approach has been successful for finitely generated groups and can be favorably extended to locally compact groups.

Basic results in the subject are exposed with complete proofs; others are stated with appropriate references. Most importantly, the development of the theory is illustrated by numerous examples, including matrix groups with entries in the the field of real or complex numbers, or other locally compact fields such as p -adic fields, isometry groups of various metric spaces, and last but not least, discrete groups themselves.
The book is aimed at graduate students, advanced undergraduate students, and mathematicians seeking some introduction to coarse geometry and locally compact groups.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Tracts in Mathematics, Volume 25

September 2016, 241 pages, Hardcover, ISBN: 978-3-03719-166-8,
2010 Mathematics Subject Classification: 20F65; 20F05, 22D05,
51F99, 54E35, 57M07, 57T20, AMS members US\$59.20, List US\$74,
Order code EMSTM/25

## Analysis



## Compactness

Properties of Perturbed Sub-Stochastic $C_{0}{ }^{-}$ Semigroups on $L^{1}(\mu)$ with Applications to Discreteness and Spectral Gaps
Mustapha Mokhtar-Kharroubi, Université de Franche-Comté, Besançon, France

The author gives a systematic approach to compactness properties or essential compactness properties (spectral gaps) for substochastic semigroups in $L^{1}$ induced by singular potentials. This construction is based in particular on new a priori estimates peculiar to $L^{1}$ spaces and on local weak compactness arguments. Various applications to convolution semigroups, weighted Laplacians, and Witten Laplacians on 1-forms are given.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30\% discount from list.
Mémoires de la Société Mathématique de France, Number 148
September 2016, 87 pages, Softcover, ISBN: 978-285629-839-8, 2010 Mathematics Subject Classification: 47D06, 47B07, 47B34, 47B65, 35P15, AMS members US\$41.60, List US\$52, Order code SMFMEM/148


## Elliptic PDEs, Measures and Capacities

Elliptic PDEs, Measures<br>and Capacities<br>From the Poisson Equation to Nonlinear Thomas-Fermi Problems<br>From the Poisson Equation to Nonlinear Thomas-Fermi Problems

Augusto C. Ponce, Université Catholique de Louvain, Louvain-la-Neuve, Belgium

Partial differential equations (PDEs) and geometric measure theory (GMT) are branches of analysis whose connections are usually not emphasized in introductory graduate courses. Yet one cannot dissociate the notions of mass or electric charge, naturally described in terms of measures, from the physical potential they generate. Having such a principle in mind, this book illustrates the beautiful interplay between tools from PDEs and GMT in a simple and elegant way by investigating properties such as existence and regularity of solutions of linear and nonlinear elliptic PDEs.
Inspired by a variety of sources, from the pioneer balayage scheme of Poincaré to more recent results related to the Thomas-Fermi and Chern-Simons models, the problems covered in this book follow an original presentation, intended to emphasize the main ideas in the proofs. Classical techniques such as regularity theory, maximum principles and the method of sub- and supersolutions are adapted to the setting where merely integrability or density assumptions on the data are available. The distinguished role played by capacities and precise representatives is also explained.

Other special features are: the remarkable equivalence between Sobolev capacities and Hausdorff contents in terms of trace inequalities; the strong approximation of measures in terms of capacities or densities, normally absent from GMT books; and the rescue of the strong maximum principle for the Schrödinger operator involving singular potentials.
This book invites the reader on a trip through modern techniques in the frontier of elliptic PDEs and GMT and is addressed to graduate students and researchers with a deep interest in analysis. Most of the chapters can be read independently, and only a basic knowledge of measure theory, functional analysis, and Sobolev spaces is required.
This item will also be of interest to those working in differential equations. F A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.
EMS Tracts in Mathematics, Volume 23
October 2016, 463 pages, Hardcover, ISBN: 978-3-03719-140-8, 2010 Mathematics Subject Classification: 28-02, 31-01, 35-02, 35R06; 26B20, 26B35, 26D10, 28A12, 28A25, 28A33, AMS members US\$55.20, List US\$69, Order code EMSTM/23

## Geometry and Topology

Mém@ires

ROLLING OF
MANTFLDS AND
CONTROLLABLITY IN
DIMENSION THREE
dimension three
Numéro 147
Nouvelle serie
$\begin{array}{llllll}2 & 0 & 1 & 6 & \begin{array}{l}\text { Yacine CHITOUR } \\ \text { Petri KOKKONEN }\end{array}\end{array}$
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## Rolling of Manifolds and Controllability in Dimension Three

Yacine Chitour, Université<br>Paris-Sud XI, France, and Petri Kokkonen, Université Paris-Sud XI, France, and University of Eastern Finland, Kuopio, Finland

The authors present the rolling (or development) of a smooth connected complete Riemannian manifold over another manifold with the same dimension. The rolling is assumed to be without spinning or slipping. Relying on geometric control theory, the authors provide an intrinsic description of the two constraints "without spinning" and "without slipping" in terms of the Levi-Civita connections, by defining corresponding vector fields distributions in the appropriate state space.
When the two manifolds are three dimensional, the authors give a complete local characterization of the reachable sets and, in particular, they identify necessary and sufficient conditions for the existence of a non-open orbit. In addition to the trivial case where the manifolds are (locally) isometric the authors show that (local) noncontrollability occurs if and only if the manifolds are either warped products or contact manifolds with additional restrictions that the authors precisely describe.
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30\% discount from list.
Mémoires de la Société Mathématique de France, Number 147
July 2016, 162 pages, Softcover, ISBN: 978-285629-838-1, 2010 Mathematics Subject Classification: 53C05, 53C29, 70G45, AMS members US\$41.60, List US\$52, Order code SMFMEM/147

# Classified Advertisements 

## Positions available, items for sale, services available, and more

## CALIFORNIA <br> UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Tenure-Track Position

The Department of Statistics and Applied Probability invites applications for a tenure-track Assistant Professor position in Statistics, starting July 1, 2017. Qualifications: research and teaching excellence and PhD in Statistics, biostatistics and related fields. Preference will be given for research on methodology and applications relating to biometrics or environmental science. To apply, submit resume, statement of research, teaching objectives and have four letters of reference sent (at least one of which is directed towards teaching). Materials should be submitted electronically via UC Recruit: https://recruit.ap.ucsb.edu/app7y/ JPF00835. Review of applications begins November 21, 2016 and continues until the position is filled. Additional information: www.pstat.ucsb.edu.

The Department is especially interested in candidates who can contribute to the diversity and excellence of the academic community through research, teaching and service.

The University of California is an Equal Opportunity/Affirmative Action Employer and all qualified applicants will receive consideration for employment without regard to race, color, religion, sex, sexual orientation, gender identity, national origin, disability status, protected veteran status, or any other characteristic protected by law.

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## CONNECTICUT

## FAIRFIELD UNIVERSITY Mathematics Department

The Mathematics Department, in the College of Arts and Sciences at Fairfield University, invites applications for one tenure-track position in Mathematics at the rank of Assistant Professor, to begin in September 2017. We seek a highly qualified candidate with demonstrated excellence in teaching-a commitment to teaching, to innovation in teaching and to using technology in the classroom, as well as strong evidence of research potential. A doctorate in Mathematics or Mathematics Education is required. Teaching duties will be evenly split between the Department of Mathematics and the Department of Educational Studies and Teacher Preparation (ESTP) in Fairfield's Graduate School of Education and Allied Professions. In the Mathematics Department, where we offer a BS and an MS in Mathematics, the successful candidate will teach 3 courses/9 credit hours per year. The rest of the workload will be in ESTP where the candidate will be the program advisor for secondary math education, teach secondary math methods and the student teaching seminar, and contribute to accreditation work.
Fairfield University is a Catholic, Jesuit comprehensive university with about 3,600 undergraduate students, 1,000 graduate students and a strong emphasis on liberal arts education. The Department of Mathematics has an active faculty of 14 full-time tenured or tenure track members.
Fairfield offers competitive salaries and compensation benefits. The picturesque
campus is located on Long Island Sound in southwestern Connecticut, about 50 miles from New York City. Fairfield is an Affirmative Action/Equal Opportunity Employer. For more information see webpages at https://www.fairfie7d. edu/undergraduate/academics/ schools-and-colleqes/colleqe-of-arts-and-sciences/programs/ mathematics and https://www. fairfield.edu/graduate/academics/ graduate-school-of-education-and-allied-professions/programs-and-certificates/graduate-programs/ secondary-education. Applicants should submit a letter of application, a curriculum vitae, teaching statement, research statement, and three letters of recommendation commenting on the applicant's experience and promise as a teacher and scholar on MathJobs.Org or in hard copy to Dr. Irene Mulvey, Chair of the Department of Mathematics, Fairfield University, 1073 North Benson Rd., Fairfield CT 06824-5195. Full consideration will be given to complete applications received by December 15, 2016. We will be interviewing at the Joint Mathematics Meetings in Atlanta, January 4-7, 2017. Please let us know if you will be attending.

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## UNIVERSITY OF CONNECTICUT Department of Mathematics Assistant, Associate, or Full Professor

The Department of Mathematics at the University of Connecticut, Storrs, invites applications for full-time, 9-month tenuretrack faculty positions at the rank of Assistant, Associate, or Full Professor in Mathematics beginning in Fall 2017. We

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.
The 2016 rate is $\$ 3.50$ per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $\$ 10$ charge, announcements can be placed anonymously. Correspondence will be forwarded.
Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.
There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.
Upcoming deadlines for classified advertising are as follows: February 2017-December 2, 2016; March 2017-January 2, 2017; April 2017February 3, 2017; May 2017—March 9, 2017; June/July 2017—May 9, 2017; August 2017—June 6, 2017.
US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).
Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.
Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to classads@ams . org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.
are seeking exceptionally well-qualified individuals with outstanding research programs. The successful candidate will be expected to teach mathematics courses at all levels and to have a vigorous externally funded research program.
UConn has grown rapidly in the past decade to become one of the nation's Top 20 public universities, with an ambitious goal, at this transformational time in its history, to join the ranks of the greatest universities in the world. As one of the University's emphasized STEM programs, supported by the \$1.7B Next Generation Connecticut (nextgenct.uconn.edu) investment, the math department enjoys an active and dynamic academic environment. The department currently has 43 research faculty members with diverse research interests (including financial mathematics and actuarial science, algebra and number theory, analysis, applied math, geometry and topology, mathematical logic, math education, numerical analysis, partial differential equations, and probability) and a strong record of external funding. Faculty members in the department participate in a range of interdisciplinary projects with Physics, Philosophy, Life Sciences, Statistics, and with the Neag School of Education. The department has moved into a new building in fall 2016.
The successful candidates for these positions will be expected to contribute to research and scholarship through extramural funding (in disciplines where applicable), high quality publications, impact as measured through citations, performances and exhibits (in disciplines where applicable), and national recognition as through honorific awards. In the area of teaching, the candidate will share a deep commitment to effective instruction at the undergraduate and graduate levels, development of innovative courses and mentoring of students in research, outreach and professional development. Successful candidates will also be expected to broaden participation among members of under-represented groups; demonstrate through their research, teaching, and/or public engagement the richness of diversity in the learning experience; integrate multicultural experiences into instructional methods and research tools; and provide leadership in developing pedagogical techniques designed to meet the needs of diverse learning styles and intellectual interests.

Minimum Qualifications: A PhD or an equivalent foreign degree in mathematics or a closely related area by August 22, 2017, demonstrated evidence of excellent teaching and outstanding research; and a deep commitment to promoting diversity through academic and research programs. Equivalent foreign degrees are acceptable.

Preferred Qualifications: An outstanding research program in an area that complements the research activity in the department; a record of attracting external funding; a commitment to effective teaching at the graduate and
undergraduate levels including integrating technology into instruction, such as online instruction; and the ability to contribute through research, teaching, and/ or public engagement to the diversity and excellence of the learning experience.
Appointment Terms: These are fulltime, 9-month, tenure-track positions with an anticipated start date of August 23,2017 . The successful candidate's academic appointment will be at the Storrs campus. Faculty may also be asked to teach at one of UConn's regional campuses as part of their ordinary workload. Rank and salary will be commensurate with qualifications and experience.
To Apply: Submit a cover letter, curriculum vitae, teaching statement (including teaching philosophy, teaching experience, commitment to effective learning, concepts for new course development, etc.), research and scholarship statement (innovative concepts that will form the basis of academic career, experience in proposal development, mentorship of graduate students, etc.), commitment to diversity statement (including broadening participation, integrating multicultural experiences in instruction and research and pedagogical techniques to meet the needs of diverse learning styles, etc.); and five letters of reference, one of which addresses the applicant's teaching online at www.mathjobs.org.jobs.

Evaluation of applications will begin on November 21, 2016. Employment of the successful candidate will be contingent upon the successful completion of a pre-employment criminal background check. Questions or requests for further information should be sent to the Hiring Committee at mathhiring@uconn.edu.
All employees are subject to adherence to the State Code of Ethics which may be found at www.ct.gov/ethics/site/ default.asp.

The University of Connecticut is committed to building and supporting a multicultural and diverse community of students, faculty and staff. The diversity of students, faculty and staff continues to increase, as does the number of honors students, valedictorians and salutatorians who consistently make UConn their top choice. More than 100 research centers and institutes serve the University's teaching, research, diversity, and outreach missions, leading to UConn's ranking as one of the nation's top research universities. UConn's faculty and staff are the critical link to fostering and expanding our vibrant, multicultural and diverse University community. As an Affirmative Action/Equal Employment Opportunity employer, UConn encourages applications from women, veterans, people with disabilities and members of traditionally underrepresented populations.

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## UNIVERSITY OF CONNECTICUT Department of Mathematics Assistant Professor in Residence

The Department of Mathematics at the University of Connecticut invites applications for positions at the rank of Assistant Professor in Residence beginning in Fall 2017. The number of positions is subject to budgetary constraints.
The University of Connecticut (UConn) is in the midst of a transformational period of growth supported by the \$1.7B Next Generation Connecticut (nextgenct.uconn.edu) and the \$1B Bioscience Connecticut (biosciencect. uchc.edu) investments and a bold new Academic Plan: Path to Excellence (i ssuu. com/uconnprovost/docs/academic-plan-single-hi-optimized 1). We are pleased to continue these investments by inviting applications for faculty positions in the Department of Mathematics at the rank of Assistant Professor in Residence.

The Mathematics Department has active programs in various areas of mathematical research including mathematics education. The education group works on issues across the $\mathrm{K}-20$ spectrum, with particular emphasis on mathematics content, and on applications of mathematics education research to instruction in the department. This group works closely with the Center for Research in Mathematics Education, based in the Neag School of Education. Successful candidates who are interested in mathematics education will have an opportunity to work on writing and implementing grants within these units.

These positions are open only to candidates who have received a PhD in mathematics, and are suitable for candidates interested in a career at UConn with an emphasis on teaching excellence. In particular, the committee will be looking within the applications for specific evidence of outstanding teaching and experience with educational technology and interactive technological teaching tools. Applicants with an interest in mathematics education are strongly encouraged to apply.
Minimum qualifications include: A PhD in mathematics, a record of outstanding teaching, and an interest in mathematics education. Equivalent foreign degrees are acceptable.

Preferred qualifications include: Experience with educational technologies, such as online homework systems, message boards, videos, smart boards, or classroom personal response systems ("clickers"); experience teaching large lectures; commitment to effective teaching, the ability to contribute through research, teaching, and/or public engagement to the diversity and excellence of the learning experience.
Appointment Terms: These are fulltime, 9-month, non-tenure track positions which may lead to long-term multi-year contracts. The teaching load for these positions are approximately six courses
per year. These positions have an anticipated start date of August 23, 2017. The successful candidate's academic appointment will be at the Storrs campus. Faculty may also be asked to teach at one of UConn's regional campuses as part of their workload. Rank and salary will be commensurate with qualifications and experience.

To Apply: Submit a cover letter, curriculum vitae, teaching statement (including teaching philosophy, teaching experience, commitment to effective learning, concepts for new course development, etc.), a commitment to diversity statement (including broadening participation, integrating multicultural experiences in instruction and research and pedagogical techniques to meet the needs of diverse learning styles, etc.); and three letters of reference, at least two of which address the applicant's teaching online at www. mathjobs.org.jobs.

The review of applications will begin on November 21, 2016. Employment of the successful candidate will be contingent upon the successful completion of a preemployment criminal background check.

Questions or requests for further information should be sent to the Hiring Committee at vaphiring@math.uconn.edu.

All employees are subject to adherence to the State Code of Ethics which may be found at www.ct.gov/ethics/site/ default.asp.

The University of Connecticut is committed to building and supporting a multicultural and diverse community of students, faculty and staff. The diversity of students, faculty and staff continues to increase, as does the number of honors students, valedictorians and salutatorians who consistently make UConn their top choice. More than 100 research centers and institutes serve the University's teaching, research, diversity, and outreach missions, leading to UConn's ranking as one of the nation's top research universities. UConn's faculty and staff are the critical link to fostering and expanding our vibrant, multicultural and diverse University community. As an Affirmative Action/Equal Employment Opportunity employer, UConn encourages applications from women, veterans, people with disabilities and members of traditionally underrepresented populations.

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## DISTRICT OF COLUMBIA

## THE GEORGE WASHINGTON UNIVERSITY Department of Mathematics

The Department of Mathematics of The George Washington University invites applications for a tenure-track position at the rank of assistant professor. Our priorities are analysis, combinatorics and logic. The successful candidate will be expected to teach at all undergraduate and graduate levels, excel in research,
interact with researchers in mathematics and other disciplines, and become actively involved in the life of the department and the university.
Basic Qualifications: Applicants must possess a PhD in Mathematics, have an excellent research record as evidenced by publications, work in progress and research funding, and excellent teaching credentials as demonstrated by letters from peers or supervisors.

Application Procedure: Applications should be submitted on-line at www. mathjobs.org and include cover letter, CV, research statement, teaching statement, and four letters of recommendation including one addressing teaching ability. Only complete applications will be considered. Review of applications will begin December 1, 2016 and will continue until the position is filled. Employment offers are contingent on the satisfactory outcome of a standard background screening. The George Washington University is an Equal Opportunity/Affirmative Action employer that does not unlawfully discriminate in any of its programs or activities on the basis of race, color, religion, sex, national origin, age, disability, veteran status, sexual orientation, gender identity expression, or on any other basis prohibited by applicable law.

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## KANSAS

## UNIVERSITY OF KANSAS The Department of Mathematics

The Department of Mathematics at the University of Kansas invites applications for a tenure-track faculty position in Probability and/or Statistics expected to begin as early as August 18, 2017. Candidates must demonstrate an outstanding record of research and must be strongly committed to excellence in teaching. Requirements for the positions include a PhD in mathematics or a closely related field and a research interest in probability and/or statistics.
For a complete announcement and to apply online, go to https://employment. ku.edu and click "Search Faculty Jobs," and search key word: "PROB STAT." A complete online application includes: C.V., cover letter, research and teaching statements, and the names and contact information for four references. In addition, at least four recommendation letters (teaching ability must be addressed in at least one letter) should be submitted electronically to https://www. mathjobs.org/jobs/jobs/, position \#9137. Initial review of applications will begin November 1, 2016 and continue as long as needed to identify a qualified pool. For additional information about the position, please contact: Gloria Prothe/864-3651/ gprothe@ku.edu. KU is
an EO/AAE, full policy policy.ku.edu/ IOA/nondiscrimination.

## UNIVERSITY OF KANSAS Visiting Assistant Professor

Applications are invited for three Visiting Assistant Professor positions, one in algebra and/or combinatorics, one in computational mathematics, and one in probability and/or statistics expected to begin on August 18, 2017. These are nontenure track, limited term appointments for three consecutive academic years (272 days). For a complete announcement and to apply online go to https:// employment.ku.edu and click "Search Openings" and search by "VAP". In addition at least four recommendation letters (teaching ability must be addressed in at least one letter) should be submitted to MathJobs.orglat/https://www.mathjobs. org/jobs/jobs/9269/(for COMP MATH; 9270 for PROB STATS; 9271 for ALG COMB). Review of applications will begin December 1, 2016 and will continue as long as needed to identify a qualified pool. KU is an EO/AAE, full policy policy. ku.edu/IOA/nondiscrimination.

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## MARYLAND

## JOHNS HOPKINS UNIVERSITY Non-Tenure-Track J.J. Sylvester Assistant Professor

Subject to availability of resources and administrative approval, the Department of Mathematics solicits applications for non-tenure-track Assistant Professor positions beginning Fall 2017.

The J.J. Sylvester Assistant Professorship is a three-year position offered to recent PhDs with outstanding research potential. Candidates in all areas of pure mathematics, including analysis, mathematical physics, geometric analysis, complex and algebraic geometry, number theory, and topology are encouraged to apply. The teaching load is three courses per academic year.

To submit your applications go to www. mathjobs.org/jobs/jhu. Applicants are strongly advised to submit their other materials electronically at this site.

If you do not have computer access, you may mail your application to: Appointments Committee, Department of Mathematics, Johns Hopkins University, 404 Krieger Hall, Baltimore, MD 21218. Application should include a vita, at least four letters of recommendation of which one specifically comments on teaching, and a description of current and planned research. Write to cpoole@jhu.edu for questions concerning these positions.
Applications received by November 15, 2016 will be given priority. Johns

Hopkins University is committed to active recruitment of a diverse faculty and student body. The University is an Affirmative Action/Equal Opportunity Employer of women, minorities, protected veterans and individuals with disabilities and encourages applications from these and other protected group members. Consistent with the University's goals of achieving excellence in all areas, we will assess the comprehensive qualifications of each applicant.

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## MASSACHUSETTS

## WILLIAMS COLLEGE <br> Tenure-Track Assistant Professor of Mathematics

The Williams College Department of Mathematics and Statistics invites applications for two tenure-track positions in mathematics, beginning fall 2017, at the rank of assistant professor (in an exceptional case, a more advanced appointment may be considered). We are seeking highly qualified candidates who have demonstrated excellence in teaching and research and who are committed to working with an increasingly diverse student body. The teaching load is four 12 -week semester courses per year and a pass-fail Winter Study class every other January. Preference will be given to candidates who will have a PhD in mathematics by September 2017. We welcome applications from members of groups traditionally underrepresented in the field.

Applicants can apply electronically at MathJobs.Org. Evaluations of applications will begin on or after November 15 and will continue until the position is filled. All offers of employment are contingent upon completion of a background check dean-faculty.wil1iams.edu/ prospective-faculty/background-check-policy.

For more information on the Department of Mathematics and Statistics, visit math.williams.edu/.

Williams College is a coeducational liberal arts institution located in the Berkshire Hills of western Massachusetts. The college has built its reputation on outstanding teaching and scholarship and on the academic excellence of its approximately 2,000 students. Please visit the Williams College website (www. williams.edu).

Beyond meeting fully its legal obligations for nondiscrimination, Williams College is committed to building a diverse and inclusive community where members from all backgrounds can live, learn, and thrive.

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## MICHIGAN

## MICHIGAN STATE UNIVERSITY Department of Mathematics

Michigan State University has a tenuresystem faculty position to begin in the fall 2017 for the Actuarial Science program. The search seeks candidates at the rank of Assistant Professor, but outstanding candidates at higher ranks may be considered. Candidates must have a PhD in mathematics or statistics with published research directly related to actuarial science, risk analysis, or financial mathematics. Associate status with the Society of Actuaries, the Casualty Actuarial Society, or similarly recognized actuarial professional association is desirable. The successful candidate will be appointed either in the Department of Mathematics, or in the Department of Statistics and Probability, or jointly between the two departments.

Applications should include curriculum vitae, research statement, teaching statement, and should arrange for at least 4 letters of recommendation to be sent by the letter writers through www. mathjobs. org, one of which must specifically address the applicant's teaching ability.

000055

## NORTH CAROLINA

## NORTH CAROLINA STATE UNIVERSITY Mathematics Department

The Mathematics Department at North Carolina State University invites applicants for an Endowed, Tenured Professorship in Mathematics. We are seeking exceptionally well-qualified, world-renowned individuals with research interests compatible with those in the department. All areas of pure and applied mathematics will be considered. Candidates must have a PhD in the mathematical sciences, and qualifications for appointment as a tenured associate or full professor. The successful applicant must have an internationally recognized research program, demonstrated ability to attract external funding, a strong record of mentoring PhD students and post-docs, and a commitment to effective teaching at the undergraduate and graduate levels. The Department of Mathematics has strong research programs in both pure and applied mathematics. Many members of the department participate in interdisciplinary programs and research groups on campus and in the broader Research Triangle community. More information about the department can be found at

## www.math.ncsu.edu,

To submit your application materials, go to www.mathjobs.org/jobs/ncsu. For consideration, applicants should submit a vita and a list of four references with complete contact information. You will then be given instructions to go to
https://jobs.ncsu.edu/postings/ 74279 and complete a Faculty Profile for the position.

Write to math-jobs@math.ncsu.edu for questions concerning this position.

NC State University is an equal opportunity and affirmative action employer. All qualified applicants will receive consideration for employment without regard to race, color, national origin, religion, sex, age, veteran status, or disability. In addition, NC State University welcomes all persons without regard to sexual orientation or genetic information. Persons with disabilities requiring accommodations in the application and interview process please call (919) 515-3148. We welcome the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners.

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## UTAH

## UNIVERSITY OF UTAH Department of Mathematics

The Department of Mathematics at the University of Utah invites applications for the following positions:

- Full-time tenure-track or tenured appointments at the level of Assistant, Associate, or Full Professor in all areas of mathematics.
- Full-time tenure-track or tenured appointments at the level of Assistant, Associate, or Full Professor in all areas of statistics. These positions are part of a University-wide cluster hiring effort in statistics, with particular emphasis in mathematics, computer science, and bioengineering. Successful candidates will have strong interdisciplinary interests.
- Three-year Burgess, Tucker, and Wylie Assistant Professor Lecturer positions.
- Research Training Group (RTG) postdoctoral positions in Algebraic Geometry and Topology.
Please see our website at www.math. utah.edu/positions for information regarding available positions and application requirements. Applications must be completed through www.mathjobs.org/ jobs/Utah. Review of complete applications for tenure-track positions will begin on November 1, 2016, and will continue until the positions are filled. Completed applications for postdoctoral positions received before January 1, 2017, will receive full consideration. The University of Utah is an Equal Opportunity/Affirmative Action employer and educator. Minorities, women, veterans, and those with disabilities are strongly encouraged to apply. Veterans' preference is extended to qualified veterans. Reasonable disability accommodations will be provided with adequate notice. For additional information about the University's commitment to equal opportunity and access see: Www. utah.edu/nondiscrimination/.

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## VIRGINIA

## UNIVERSITY OF VIRGINIA Department of Mathematics Assistant Professor in Mathematics

The Department of Mathematics at the University of Virginia, Charlottesville, VA, invites applications for a tenure-track Assistant Professor full time position. Applicants must present evidence of outstanding accomplishments and promise in both research and teaching. We seek candidates dedicated to our mission and passionate about teaching in a world class institution.

In addition to developing external funding to support research endeavors, candidates will be expected to teach at the graduate and undergraduate levels and provide service to the University, Department and professional organizations. The appointment is anticipated to begin with the fall term of 2017, with an anticipated start date of July 25, 2017. Applicants must be on track to receive a PhD in the relevant field by May 2017 and must hold a PhD at the time of appointment. Preference will be given to applicants whose research program is in Algebraic Geometry or Analysis, but all candidates whose research interests complement the strengths of the department's current faculty will be considered.

To apply, candidates must submit a Candidate Profile through Jobs@UVa (https://jobs.virginia.edu), search on posting number 0619419 and electronically attach the following: a cover letter of interest describing research agenda and teaching experience, a curriculum vitae, and contact information for four references.

Review of applications will begin November 1, 2016; however, the positions will remain open until filled.

In addition, please submit the following required documents electronically through www.mathjobs.org: A cover letter, an AMS Standard Cover Sheet, a curriculum vitae, a publication list, a description of research, and a statement about teaching interests and experience. The applicant must also have at least four letters of recommendation submitted, of which one must support the applicant's effectiveness as a teacher.

Questions regarding the application process in JOBS@UVa should be directed to: Zvezdana Kish, zk4g@virginia.edu (434) 924-9437.

For additional information about the position contact: Math-employment@ Virginia.EDU.

The University will perform background checks on all new faculty hires prior to making a final offer of employment.

The University of Virginia is an equal opportunity/affirmative action employer. Women, minorities, veterans, and persons with disabilities are encouraged to apply.

## UNIVERSITY OF VIRGINIA Department of Mathematics Postdoctoral Fellow and Lecturer in Mathematics

The Department of Mathematics at the University of Virginia invites applications for several postdoctoral positions, including Whyburn Instructorships, to begin the fall semester of 2017. These positions carry a three-year appointment. Preference will be given to candidates who have received their PhD within the last three years. Applicants must be on track to receive a PhD in the relevant field by May 2017 and must hold a PhD at the time of appointment. Applicants must present evidence of outstanding accomplishments and promise in both research and teaching. All candidates whose research interests complement the strengths of the department's current faculty will be considered. Information about the department may be found at www.math. virginia.edu.

Review of applications will begin on November 15, 2016; however, the positions will remain open until filled.

To apply candidates must submit a Candidate Profile through Jobs@UVa (https://jobs.virginia.edu), search on posting number 0619358 and electronically attach the following: a cover letter of interest describing research agenda and teaching experience, a curriculum vitae, and contact information for four references.

In addition, please submit the following required documents electronically through www.mathjobs.org: A cover letter, an AMS Standard Cover Sheet, a curriculum vitae, a publication list, a description of research, and a statement about teaching interests and experience. The applicant must also have at least four letters of recommendation submitted, of which one must support the applicant's effectiveness as a teacher.

Questions regarding the application process in Jobs@UVa should be directed to: Zvezdana Kish, zk4g@virginia.edu (434) 924-9437

For additional information about the position contact: math-employment@ virginia.edu.

The University will perform background checks on all new faculty hires prior to making a final offer of employment.

The University of Virginia is an equal opportunity and affirmative action employer. Women, minorities, veterans, and persons with disabilities are encouraged to apply.

## CHINA

## CENTER FOR APPLIED MATHEMATICS <br> Tianjin University, China <br> Tenured/Tenure-Track/ Postdoctoral Positions

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required. Light or no teaching load, adequate facilities, spacious office environment, and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars. The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board, and Professor William Y.C. Chen serves as the Director. TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines. For more information, please visit www. cam. tju.edu.cn/ or contact Ms. Debbie Renyuan Zhang at zhangry@tju.edu.cn, telephone: 86-22-2740-5389.

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## CONSULTANT WANTED

Mathematician with courage and insight wanted-namely, the courage to give a fair, open-minded reading to a paper written by an outsider (my degree is in computer science, and most of my career has been spent as a researcher in the computer industry), and the insight to see that it contains an abundance of publishable content even if the problem solution it sets forth contains major errors. I will offer shared authorship if the mathematician's help leads to publication. Contact: Peter at peteschorer@gmail.com; the paper is "A Solution to the $3 x+1$ Problem" on occampress.com.

This section contains new announcements of worldwide meetings and conferences of interest to the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. New announcements only are published in the print Mathematics Calendar featured in each Notices issue.
An announcement will be published in the Notices if it contains a call for papers and specifies the place, date, subject (when applicable). A second announcement will be published only if there are changes or necessary additional information. Asterisks (*) mark those announcements containing revised information.
In general, print announcements of meetings and conferences carry only the date, title and location of the event.
The complete listing of the Mathematics Calendar is available at:www.ams.org/meetings/calendar/mathca7
All submissions to the Mathematics Calendar should be done online via: www.ams.org/cgi-bin/mathca1/mathca1submit.p7
Any questions or difficulties may be directed to mathca1@ams. org.
Disclaimer The publisher reserves the right to reject any posting not in keeping with the Society's standards. Acceptance of a posting shall not be construed as approval of the accuracy or the legality of any information therein.

## November 2016

21-25 International Conference on Nonlinear Partial Differential Equations: A Celebration of Professor Norman Dancer's Seventieth Birthday
Location: University of New England, Armidale, Australia.
URL: sydney.edu.au/science/maths/u/PDESeminar/ n7pde2016

## December 2016

7-11 New Trends in Optimization and Variational Analysis for Applications. An International Conference in Honor of Professor Michel Théra for his Seventieth Birthday
Location: University of Quy Nhon, Vietnam.
URL: https://sites.google.com/a/qnu.edu.vn/ newtovaa/home

## 9-11 Tech Topology Conference

Location: Georgia Institute of Technology, Atlanta, GA.
URL: ttc.gatech.edu

January 2017
12-14 11th International Young Researcher Workshop on Geometry, Mechanics, and Control
Location: University of La Laguna, Tenerife, Canary Islands, Spain.
URL: gmcnet.webs.ul1.es/?q=activity-detai11/1762

## 16-19 Model Theory and Applications

Location: University of Mons, Centre Vésale Avenue du Champ de Mars, B-7000 Mons Belgique.
URL: www.mathconf.org/mta2017
16-20 5 IberoAmerican Meeting on Geometry, Mechanics, and Control
Location: University of La Laguna, Tenerife, Canary Islands, Spain.
URL: 5ibamgmc.webs.ull.es
25-26 22th Seminar on Mathematical Analysis and Its Applications Location: Department of Mathematics, Basic Science Faculty, University of Bonab, Bonab,5551761167, Iran.
URL: smaa22.bonabu.ac.ir
March 2017
6-10 Random Structures in Statistical Mathematical Physics
Location: CIRM Luminy, Marseille.
URL: khanin-sh1osman.weeb7y.com/research-school.
htm 1
8 - 11 51st Spring Topology and Dynamical Systems Conference (STDC 2017)
Location: NJCU (School of Business), Jersey City, NJ, USA.
URL: https://sites.google.com/site/
2017springconftopanddynamics/home
13-17 Resonances: Geometric Scattering and Dynamics Location: CIRM, Luminy, France.
URL: programme-scientifique.weeb7y.com/1604.htm1
15-17 Critical Issues in Mathematics Education 2017: Observing for Access, Power, and Participation in Mathematics Classrooms as a Strategy to Improve Mathematics Teaching and Learning
Location: Mathematical Sciences Research Institute, Berkeley, CA.
URL: www.msri.org/workshops/836
21-24 Quiver Grassmannians and Their Applications
Location: University of Wuppertal, Germany.
URL: www2.math.uni-wuppertal.de/~weist/quivgras. htm 7

24-24 The 2nd Biennial Conference on Financial Mathematics Location: Farmingdale State College, Farmingdale, NY.
URL: www.farmingdale.edu/arts-sciences/ mathematics/index.shtm1

30 - April 1 Vojtech Jarnik International Mathematical Competition Location: University of Ostrava, Ostrava, Czech Republic.
URL: vjimc.osu.cz

## April 2017

8-9 Graduate Student Combinatorics Conference 2017
Location: University of Kansas, Lawrence, Kansas, United States.
URL: www.math.ku.edu/conferences/2017/GSCC/index. htm 1

17-21 Applied Probability at The Rock
Location: Ayers Rock Resort, NT, Australia.
URL: www.maths.adelaide.edu.au/APatR
24-27 Qualitative Methods in KPZ Universality
Location: CIRM Luminy, Marseille.
URL: khanin-sh1osman.weebly.com/conference.htm1

May 2017
15-19 Spring School ContrOpt 2017: Control and Optimization Location: Hotel Rosa Beach, Skanes, Monastir, Tunisia. URL: www. 1ama.univ-savoie.fr/ContrOpt2017

## June 2017

1-2 International Conference on Mathematics and Mechanics
Location: Nice Riviera Hotel \& Spa, France, Nice.
URL: mathematics. conference-site.com
5-7 ATFA17-Aspects of Time-Frequency Analysis Location: Politecnico di Torino, Italy.
URL: nuhag.eu/atfa17
12 - 16 LLAVEFEST: A Broad Perspective on Finite and Infinite Dimensional Dynamical Systems (FIDDS-17)
Location: Universitat de Barcelona (Barcelona, Spain).
URL: www.crm.cat/en/Activities/Curs_2016-2017/ Pages/C_FIDDS.aspx

25 - 30 BIOMATH 2017: International Conference on Mathematical Methods and Models in Biosciences
Location: Skukuza Camp, Kruger Park, South Africa. Hosted by University of Pretoria.
URL: www.biomath.bg/2017
26 - 29 11th Annual International Conference on Mathematics: Teaching, Theory \& Applications
Location: Athens, Greece.
URL: www.atiner.gr/mathematics
26-29 11th Annual International Conference on Statistics: Teaching, Theory \& Applications
Location: Athens, Greece.
URL: www.atiner.gr/statistics
July 2017
17-20 An Annual International Symposium on the Future of STEAM (Sciences, Technology, Engineering, Arts, and Mathematics) Education
Location: Athens, Greece.
URL: www.atiner.gr/stema

17 - 21 28th IFIP TC7 Conference on System Modeling and Optimization Second Announcement
Location: Ankara, Turkey.
URL: ifip17.iam.metu.edu.tr
29 - August 5 Fourth Mile High Conference on Nonassociative Mathematics
Location: University of Denver, Denver, Colorado.
URL: www.math.du.edu/milehigh

August 2017
13-20 8th International Conference on Differential and Functional Differential Equations
Location: Faculty of Social and Humanitarian Sciences, Peoples' Friendship University of Russia (RUDN University), 10/2 Miklukho-Maklaya str., 117198 Moscow, Russia.
URL: dfde2017.mi.ras.ru
27 - September 1 23rd Czech and Slovak International Conference on Number Theory
Location: Ostravice, Czech Republic.
URL: ntc.osu.cz
28 - September 1 EUROCOMB 2017-European Conference on Combinatorics, Graph Theory, and Applications
Location: TU Wien, Vienna, Austria.
URL: www.dmg.tuwien.ac.at/eurocomb2017/
31 - September 1 Connections for Women Workshop: Geometric and Topological Combinatorics
Location: Mathematical Sciences Research Institute, Berkeley, CA.
URL: www.msri.org/workshops/812
July 2019
8-12 Equadiff 2019
Location: Leiden, The Netherlands.
URL: https://equadiff2015.sciencesconf.org/

# MEETINGS \& CONFERENCES OF THE AMS 

## DECEMBER TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: www.ams.org/meetings/

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 88 in the January 2016 issue of the Notices for general information regarding participation in AMS meetings and conferences.
Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LATEX is
necessary to submit an electronic form, although those who use IETEX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LATEX. Visit www.ams.org/ cgi-bin/abstracts/abstract.pl//. Questions about abstracts may be sent to absinfo@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

## MEETINGS IN THIS ISSUE

| 2017 |  |  |
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| January 4-7 | JMM 2017- Atlanta | p. 1324 |
| March 10-12 | Charleston, South Carolina | p. 1327 |
| April 1-2 | Bloomington, Indiana | p. 1328 |
| April 22-23 | Pullman, Washington | p. 1329 |
| May 6-7 | New York, New York | p. 1330 |
| July 24-28 | Montréal, Quebec, Canada | p. 1331 |
| September 9-10 | Denton, Texas | p. 1331 |
| September 16-17 | Buffalo, New York | p. 1331 |
| September 23-24 | Orlando, Florida | p. 1331 |
| November 4-5 | Riverside, California | p. 1332 |
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| January 10-13 | San Diego, California | p. 1332 |
| March 24-25 | Columbus, Ohio | p. 1332 |
| April 14-15 | Portland, Oregon | p. 1333 |
| April 14-15 | Nashville, Tennessee | p. 1333 |
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| June 11-14 | People's Republic of China | p. 1333 |


| September 29-30 | Newark, Delaware | p. 1333 |
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| January 16-19 | Baltimore, Maryland | p. 1334 |
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| January 6-9 | Washington, DC | p. 1334 |
| Conferences in Cooperation with the AMS |  |  |
| Indian Mathematics Consortium |  |  |
| December 14-17, 2016 |  |  |
| Banaras Hindu University |  |  |
|  | Varanasi, India |  |

See www.ams.org/meetings/ for the most up-to-date information on these conferences.

## ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of WisconsinMadison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 925210135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

# Meetings \& Conferences of the AMS 

> IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www. ams.org/meetings/.
> Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL.

## Atlanta, Georgia

## Hyatt Regency Atlanta and <br> Marriott Atlanta Marquis

January 4-7, 2017
Wednesday - Saturday

## Meeting \#1 125

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of Notices: October 2016
Program first available on AMS website: November 2016 Issue of Abstracts: Volume 38, Issue 1

## Deadlines

For organizers: Expired
For abstracts: Expired
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ national.htm1.

## Joint Invited Addresses

Ingrid Daubechies, Duke University, Mathematics for art investigation (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

Lisa Jeffrey, University of Toronto, Real loci in symplectic manifolds (AWM-AMS Noether Lecture).

Donald Richards, Penn State University, Distance correlation: a new tool for detecting association and measuring correlation between data sets (AMS-MAA Invited Address).

Alice Silverberg, University of California, Irvine, Through the cryptographer's looking-glass, and what Alice found there (AMS-MAA Invited Address).

## AMS Invited Addresses

Tobias Holck Colding, Massachusetts Institute of Technology, Arrival time.

Carlos E. Kenig, University of Chicago, Overview: The focusing energy critical wave equation (AMS Colloquium Lectures: Lecture I).

Carlos E. Kenig, University of Chicago, The focusing energy critical wave equation: the non-radial case (AMS Colloquium Lectures: Lecture III).

Carlos E. Kenig, University of Chicago, The focusing energy critical wave equation: the radial case in 3 space dimensions (AMS Colloquium Lectures: Lecture II).

John Preskill, California Institute of Technology, Quantum computing and the entanglement frontier (AMS Josiah Willard Gibbs Lecture).

Barry Simon, Caltech, Spectral Theory Sum Rules, Meromorphic Herglotz Functions and Large Deviations.

Gigliola Staffilani, Massachusetts Institute of Technology, The many faces of dispersive and wave equations.

Richard Taylor, Institute for Advanced Study, Galois groups and locally symmetric spaces.

Anna Wienhard, Ruprecht-Karls-Universität Heidelberg, A tale of rigidity and flexibility - discrete subgroups of higher rank Lie groups.

## AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.jointmathematicsmeetings.org/meetings/abstracts/abstract.p7?type=jmm.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

Advanced Mathematical Programming and Applications, Ram N. Mohapatra, University of Central Florida, Ram U. Verma, University of North Texas, and Gayatri Pany, Indian Institute of Technology.

Advances in Mathematics of Ecology, Epidemiology and Immunology of Infectious Diseases, Abba Gumel, Arizona State University.

Advances in Numerical Analysis for Partial Differential Equations, Thomas Lewis, University of North Carolina at Greensboro, and Amanda Diegel, Louisiana State University.

Advances in Operator Algebras, Michael Hartglass, University of California, Riverside, David Penneys, University of California, Los Angeles, and Elizabeth Gillaspy, University of Colorado, Boulder.

Algebraic Statistics (a Mathematics Research Communities Session), Mateja Raic, University of Illinois at Chicago, Nathanial Bushnek, University of Alaska, Anchorage, and Daniel Irving Bernstein, North Carolina State University.

An Amicable Combination of Algebra and Number Theory (Dedicated to Dr. Helen G. Grundman), Eva Goedhart, Lebanon Valley College, Pamela E. Harris, Williams College, Daniel P. Wisniewski, DeSales University, and Alejandra Alvarado, Eastern Illinois University.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems and their Applications, Aghalaya S. Vatsala, University of Louisiana, Gangaram S. Ladde, University of South Florida, and John R. Graef, University of Tennessee at Chattanooga.

Analytic Number Theory and Arithmetic, Robert Lemke Oliver, Tufts University, Paul Pollack, University of Georgia, and Frank Thorne, University of South Carolina.

Analytical and Computational Studies in Mathematical Biology, Yanyu Xiao, University of Cincinnati, and XiangSheng Wang, University of Louisiana at Lafayette.

ApREUF: Applied Research Experience for Undergraduate Faculty, Shenglan Yuan, LaGuardia Community College, CUNY, Jason Callahan, St. Edwards University, Eva Strawbridge, James Madison University, and Ami Radunskaya, Pomona College.

Applications of Partially Ordered Sets in Algebraic, Topological, and Enumerative Combinatorics, Rafael S. González D'León, University of Kentucky, and Joshua Hallam, Wake Forest University.

Arithmetic Properties of Sequences from Number Theory and Combinatorics, Eric Rowland, Hofstra University, and Armin Straub, University of South Alabama.

Automorphic Forms and Arithmetic, Frank Calegari, University of Chicago, Ana Caraiani, Princeton University, and Richard Taylor, Institute for Advanced Study.

Bases in Function Spaces: Sampling, Interpolation, Expansions and Approximations, Shahaf Nitzan and Christopher Heil, Georgia Institute of Technology, and Alexander V. Powell, Vanderbilt University.

Character Varieties (a Mathematics Research Communities Session), Nathan Druivenga, University of Kentucky, Brett Frankel, Northwestern University, and Ian Le, Perimeter Institute for Theoretical Physics.

Coding Theory for Modern Applications, Christine A. Kelley, University of Nebraska-Lincoln, Iwan M. Duursma, University of Illinois Urbana-Champaign, and Gretchen L. Matthews, Clemson University.

Combinatorial and Cohomological Invariants of Flag Manifolds and Related Varieties, Martha Precup, Northwestern University, and Rebecca Goldin, George Mason University.

Commutative Algebra: Research for Undergraduate and Early Graduate Students, Nicholas Baeth, University of Central Missouri, and Courtney Gibbons, Hamilton College.

Complex Analysis and Special Functions, Brock Williams, Texas Tech University, Kendall Richards, Southwestern University, and Alex Solynin, Texas Tech University.

Continued Fractions, James McLaughlin, West Chester University, Geremías Polanco, Hampshire College, and Nancy J. Wyshinski, Trinity College.

Control and Long Time Behavior of Evolutionary PDEs, Louis Tebou, Florida International University, and Luz de Teresa, Instituto de Matemáticas, UNAM.

Discrete Geometry and Convexity (Dedicated to András Bezdek on the occasion of his 60th birthday), Krystyna Kuperberg, Auburn University, Gergely Ambrus, Renyi Institute of Mathematics, Braxton Carrigan, Southern Connecticut State University, and Ferenc Fodor, University of Szeged.

Discrete Structures in Number Theory, Anna Haensch, Duquesne University, and Adriana Salerno, Bates College.

Dynamical Systems, Jim Wiseman, Agnes Scott College, and Aimee Johnson, Swarthmore College.

Dynamics of Fluids and Nonlinear Waves, Zhiwu Lin, Jiayin Jin, and Chongchun Zeng, Georgia Institute of Technology.

Ergodic Theory and Dynamical Systems, Mrinal Kanti Roychowdhury, University of Texas Rio Grande Valley, and Tamara Kucherenko, City College of New York.

Fusion Categories and Quantum Symmetries, Julia Plavnik, Texas A\&M University, Paul Bruillard, Pacific Northwest National Laboratory, and Eric Rowell, Texas A\&M University.

Gaussian Graphical Models and Combinatorial Algebraic Geometry, Rainer Sinn, Georgia Institute of Technology, Seth Sullivant, North Carolina State University, and Josephine Yu, Georgia Institute of Technology.

Graphs and Matrices, Sudipta Mallik, Northern Arizona University, Keivan Hassani Monfared, University of Calgary, and Bryan Shader, University of Wyoming.

Group Actions and Geometric Structures, Anna Wienhard, Universität Heidelberg, and Jeffrey Danciger, University of Texas at Austin.

Group Representations and Cohomology, Hung Nguyen, The University of Akron, Nham Ngo, The University of Arizona, Andrei Pavelescu, University of South Alabama, and Paul Sobaje, University of Georgia.

Harmonic Analysis (In Honor of Gestur Olafsson's 65th Birthday), Jens Christensen, Colgate University, and Susanna Dann, Technische Universität Wien-Vienna, Austria.

History of Mathematics, Adrian Rice, Randolph-Macon College, Sloan Despeaux, Western Carolina University, and Daniel Otero, Xavier University (AMS-MAA-ICHM).

Hopf Algebras and their Actions, Henry Tucker, University of California, San Diego, Susan Montgomery, University of Southern California - Los Angeles, and Siu-Hung Ng, Louisiana State University.

Inverse Problems and Applications, Vu Kim Tuan and Amin Boumenir, University of West Georgia.

Inverse Problems and Multivariate Signal Analysis, M. Zuhair Nashed, University of Central Florida, Willi Freeden, University of Kaiserslautern, and Otmar Scherzer, University of Vienna.

Lie Group Representations, Discretization, and Gelfand Pairs (a Mathematics Research Communities Session), Matthew Dawson, CIMAT, Holley Friedlander, Dickenson College, John Hutchens, Winston-Salem State University, and Wayne Johnson, Truman State University.

Mapping Class Groups and their Subgroups, James W. Anderson, University of Southampton, UK, and Aaron Wootton, University of Portland.

Mathematics and Music, Mariana Montiel, Georgia State University, and Robert Peck, Louisiana State University.

Mathematics in Physiology and Medicine (a Mathematics Research Communities Session), Kamila Larripa, Humboldt State University, Charles Puelz, Rice University, Laura Strube, University of Utah, and Longhua Zhao, Case Western Reserve University.

Mathematics of Cryptography, Nathan Kaplan and Alice Silverberg, University of California, Irvine (AMS-MAA).

Mathematics of Signal Processing and Information, Rayan Saab, University of California, San Diego, and Mark Iwen, Michigan State University.

Measure and Measurable Dynamics (In Memory of Dorothy Maharam, 1917-2014), Cesar Silva, Williams College.

Minimal Integral Models of Algebraic Curves, Tony Shaska, Oakland University.

NSFD Discretizations: Recent Advances, Applications, and Unresolved Issues, Talitha M. Washington, Howard University, and Ronald E. Mickens, Clark Atlanta University.

New Developments in Noncommutative Algebra \& Representation Theory, Ellen Kirkman, Wake Forest University, and Chelsea Walton, Temple University.

Nonlinear Systems and Applications, Wenrui Hao, Ohio State University.

Open \& Accessible Problems for Undergraduate Research, Allison Henrich, Seattle University, Michael Dorff, Brigham Young University, and Nicholas Scoville, Ursinus College.

Operator Theory, Function Theory, and Models, William Ross, University of Richmond, and Alberto Condori, Florida Golf Coast University.

Orthogonal Polynomials, Doron Lubinsky and Jeff Geronimo, Georgia Institute of Technology.

PDE Analysis on Fluid Flows, Xiang Xu, Old Dominion University, and Geng Chen and Ronghua Pan, Georgia Institute of Technology.

PDEs for Fluid flow: Analysis and Computation, Thinh Kieu, University of North Georgia, Emine Celik, University of Nevada, Reno, and Hashim Saber, University of North Georgia.

Partition Theory and Related Topics, Amita Malik, University of Illinois at Urbana-Champaign, Dennis Eichhorn, University of California, Irvine, and Tim Huber, University of Texas-Rio Grande Valley.

Problems in Partial Differential Equations, Alex Himonas, University of Notre Dame, and Dionyssios Mantzavinos, State University of New York at Buffalo.

Public School Districts and Higher Education Mathematics Partnerships, Virgil U. Pierce and Aaron Wilson, University of Texas Rio Grande Valley.

Pure and Applied Talks by Women Math Warriors Presented by EDGE (Enhancing Diversity in Graduate Education), Candice Price, University of San Diego, and Amy Buchman, Tulane University.

Quantum Groups, Shuzhou Wang and Angshuman Bhattacharya, University of Georgia.

Quaternions, Johannes Familton, Borough of Manhattan Community College, Terrence Blackman, Medgar Evers College, and Chris McCarthy, Borough of Manhattan Community College.

RE(UF)search on Graphs and Matrices, Cheryl Grood, Swarthmore College, Daniela Ferrero, Texas State University, and Mary Flagg, University of St. Thomas.

Random Matrices, Random Percolation and Random Sequence Alignments, Ruoting Gong, Illinois Institute of Technology, and Michael Damron, Georgia Institute of Technology.

Real Discrete Dynamical Systems with Applications, M. R. S. Kulenovic, University of Rhode Island, and AbdulAziz Yakubu, Howard University.

Recent Advances in Mathematical Biology, Zhisheng Shuai, University of Central Florida, Guihong Fan, Columbus State University, Andrew Nevai, University of Central Florida, and Eric Numfor, Augusta University.

Recent Progress on Nonlinear Dispersive and Wave Equations, Dana Mendelson, Carlos Kenig, and Hao Jia, University of Chicago, Andrew Lawrie, University of California, Berkeley, Gigliola Staffilani, Massachusetts Institute of Technology, and Magdalena Czubak, University of Colorado Boulder.

Representations and Related Geometry in Lie Theory, Laura Rider, Massachusetts Institute of Technology, and Amber Russell, Butler University.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs, Darren A. Narayan, Rochester Institute of Technology, Tamas Forgacs, California State University, Fresno, and Ugur Abdulla, Florida Institute of Technology (AMS-MAA-SIAM).

Sheaves in Topological Data Analysis, Mikael VejdemoJohansson, CUNY College of Staten Island, Elizabeth Munch, University at Albany, SUNY, and Martina Scolamiero, École polytechnique fédérale de Lausanne.

Spectral Calculus and Quasilinear Partial Differential Equations, Shijun Zheng, Georgia Southern University, Marius Beceanu, State University of New York - Albany, and Tuoc Van Phan, University of Tennessee, Knoxville.

Spin Glasses and Disordered Media, Antonio Auffinger, Northwestern University, Aukosh Jagannath, New York University, and Dmitry Panchenko, University of Toronto.

Statistical Methods in Computational Topology and Applications, Yu-Min Chung and Sarah Day, College of William \& Mary.

Stochastic Matrices and Their Applications, Selcuk Koyuncu, University of North Georgia, and Lei Cao, Georgian Court University.

Stochastic Processes and Modelling, Erkan Nane, Auburn University, and Jebessa B. Mijena, Georgia College and State University.

Symmetries, Integrability, and Beyond, Maria Clara Nucci, Università di Perugia, ITALY, and Sarah Post, University of Hawaí at Manoa.

Symplectic Geometry, Moment Maps and Morse Theory, Lisa Jeffrey, University of Toronto, and Tara Holm, Cornell University (AMS-AWM).

Teaching Assistant Development Programs: Why and How? Solomon Friedberg, Boston College, Jessica Deshler, West Virginia University, Jeffrey Remmel, University of

California, San Diego, and Lisa Townsley, University Of Georgia.

The Mathematics of the Atlanta University Center, Talitha M. Washington, Howard University, Monica Jackson, American University, and Colm Mulcahy, Spelman College (AMS-NAM).

The Modeling First Approach to Teaching Differential Equations, Chris McCarthy, City University of New York, and Brian Winkel, US Military Academy, West Point.

Theory and Applications of Numerical Algebraic Geometry, Daniel Brake, University of Notre Dame, Robert Krone, Queen's University, and Jose Israel Rodriguez, University of Chicago.

Topics in Graph Theory, Songling Shan, Vanderbilt University, and Xiaofeng Gu, University of West Georgia.

Topology, Representation Theory, and Operator Algebras (A Tribute to Paul Baum), Efton Park and Jose Carrion, Texas Christian University.

Women in Analysis (In Honor of Cora Sadosky), Alexander Reznikov, Vanderbilt University, Oleksandra Beznosova and Hyun-Kyoung Kwon, University of Alabama, and Katharine Ott, Bates College.

Women in Topology, Jocelyn Bell, Hobart and William Smith Colleges, Eleanor Ollhoff, University of Tennessee, Candice Price, University of San Diego, and Arunima Ray, Brandeis University.

## Charleston, South Carolina

## College of Charleston

## March 10-12, 2017

Friday - Sunday

## Meeting \#1 126

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced Issue of Abstracts: Volume 38, Issue 2

## Deadlines

For organizers: Expired
For abstracts: January 17, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Pramod N. Achar, Louisiana State University, Title to be announced.

Hubert Bray, Duke University, Title to be announced.
Alina Chertock, North Carolina State University, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the ab-
stract submission form found at www. ams.org/cgi-bin/ abstracts/abstract.p1.

Active Learning in Undergraduate Mathematics (Code: SS 21A), Draga Vidakovic, Georgia State University, Harrison Stalvey, University of Colorado, Boulder, and Darryl Chamberlain,Jr., Aubrey Kemp, and Leslie Meadows, Georgia State University.

Advances in Long-term Behavior of Nonlinear Dispersive Equations (Code: SS 27A), Brian Pigott, Wofford College, and Sarah Raynor, Wake Forest University.

Advances in Nonlinear Waves: Theory and Applications (Code: SS 23A), Constance M. Schober, University of Central Florida, and Andrei Ludu, Embry Riddle University.

Algebras, Lattices, Varieties (Code: SS 19A), George F. McNulty, University of South Carolina, and Kate S. Owens, College of Charleston.

Analysis and Control of Fluid-Structure Interactions and Fluid-Solid Mixtures (Code: SS 6A), Justin T. Webster, College of Charleston, and Daniel Toundykov, University of Nebraska-Lincoln.

Analysis, Control and Stabilization of PDE's (Code: SS 13A), George Avalos, University of Nebraska-Lincoln, and Scott Hansen, Iowa State University.

Bicycle Track Mathematics (Code: SS 25A), Ron Perline, Drexel University.

Coding Theory, Cryptography, and Number Theory (Code: SS 17A), Jim Brown, Shuhong Gao, Kevin James, Felice Manganiello, and Gretchen Matthews, Clemson University.

Commutative Algebra (Code: SS 1A), Bethany Kubik, University of Minnesota Duluth, Saeed Nasseh, Georgia Southern University, and Sean Sather-Wagstaff, Clemson University.

Computability in Algebra and Number Theory (Code: SS 8A), Valentina Harizanov, The George Washington University, Russell Miller, Queens College and Graduate Center - City University of New York, and Alexandra Shlapentokh, East Carolina University.

Data Analytics and Applications (Code: SS 2A), Scott C. Batson, Lucas A. Overbuy, and Bryan Williams, Space and Naval Warfare Systems Center Atlantic.

Factorization and Multiplicative Ideal Theory (Code: SS 16A), Jim Coykendall, Clemson University, and Evan Houston and Thomas G. Lucas, University of North Carolina, Charlotte.

Fluid-Boundary Interactions (Code: SS 26A), M. Nick Moore, Florida State University.

Frame Theory (Code: SS 22A), Dustin Mixon, Air Force Institute of Technology, John Jasper, University of Cincinnati, and James Solazzo, Coastal Carolina University.

Free-boundary Fluid Models and Related Problems (Code: SS 7A), Marcelo Disconzi, Vanderbilt University, and Lorena Bociu, North Carolina State University.

Geometric Analysis and General Relativity (Code: SS 20A), Hubert L. Bray, Duke University, Otis Chodosh, Princeton University, and Greg Galloway and Pengzi Miao, University of Miami.

Geometric Methods in Representation Theory (Code: SS 15A), Pramod N. Achar, Louisiana State University, and Amber Russell, Butler University.

Geometry and Symmetry in Integrable Systems (Code: SS 10A), Annalisa Calini, Alex Kasman, and Thomas Ivey, College of Charleston.

Graph Theory (Code: SS 5A), Colton Magnant, Georgia Southern University, and Zixia Song, University of Central Florida.

Knot Theory and its Applications (Code: SS 3A), Elizabeth Denne, Washington \& Lee University, and Jason Parsley, Wake Forest University.

Nonlinear Waves: Analysis and Numerics (Code: SS 18A), Anna Ghazaryan, Miami University, St é phane Lafortune, College of Charleston, and Vahagn Manukian, Miami University.

Numerical Methods for Coupled Problems in Computational Fluid Dynamics (Code: SS 11A), Vincent J. Ervin and Hyesuk Lee, Clemson University.

Oscillator Chain and Lattice Models in Optics, the Power Grid, Biology, and Polymer Science (Code: SS 14A), Alejandro Aceves, Southern Methodist University, and Brenton LeMesurier, College of Charleston.

Recent Trends in Finite Element Methods (Code: SS 9A), Michael Neilan, University of Pittsburgh, and Leo Rebholz, Clemson University.

Representation Theory and Algebraic Mathematical Physics (Code: SS 12A), Iana I. Anguelova, Ben Cox, and Elizabeth Jurisich, College of Charleston.

Riemann-Hilbert Problem Approach to Asymptotic Problems in Integrable Systems, Orthogonal Polynomials and Other Areas (Code: SS 24A), Alexander Tovbis, University of Central Florida, and Robert Jenkins, University of Arizona.

Rigidity Theory and Inversive Distance Circle Packings (Code: SS 4A), John C. Bowers, James Madison University, and Philip L. Bowers, The Florida State University.

## Bloomington, Indiana <br> Indiana University

April 1-2, 2017
Saturday - Sunday
Meeting \#1 127
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: February 2017
Program first available on AMS website: February 23, 2017
Issue of Abstracts: Volume 38, Issue 2

## Deadlines

For organizers: Expired
For abstracts: February 7, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Ciprian Demeter, Indiana University, Title to be announced.

Sarah Koch, University of Michigan, Title to be announced.

Richard Evan Schwartz, Brown University, Modern scratch paper: Graphical explanations in geometry and
dynamics (Einstein Public Lecture in Mathematics).

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www. ams.org/cgi-bin/ abstracts/abstract.p1.

Algebraic and Enumerative Combinatorics with Applications (Code: SS 6A), Saúl A. Blanco, Indiana University, and Kyle Peterson, DePaul University.

Analysis and Numerical Computations of PDEs in Fluid Mechanics (Code: SS 20A), Gung-Min Gie, University of Louisville, and Makram Hamouda and Roger Temam, Indiana University.

Analysis of Variational Problems and Nonlinear Partial Di fferential Equations (Code: SS 11A), Nam Q. Le and Peter Sternberg, Indiana University.

Automorphic Forms and Algebraic Number Theory (Code: SS 2A), Patrick B. Allen, University of Illinois at Urbana-Champaign, and Matthias Strauch, Indiana University Bloomington.

Commutative Algebra (Code: SS 19A), Ela Celikbas and Olgur Celikbas, West Virginia University.

Computability and Inductive Definability over Structures (Code: SS 3A), Siddharth Bhaskar, Lawrence Valby, and Alex Kruckman, Indiana University.

Dependence in Probability and Statistics (Code: SS 7A), Richard C. Bradley and Lanh T. Tran, Indiana University.

Differential Equations and Their Applications to Biology (Code: SS 27A), Changbing Hu, Bingtuan Li, and Jiaxu Li, University of Louisville.

Discrete Structures in Conformal Dynamics and Geometry (Code: SS 5A), Sarah Koch, University of Michigan, and Kevin Pilgrim and Dylan Thurston, Indiana University.

Extremal Problems in Graphs, Hypergraphs and Other Combinatorial Structures (Code: SS 25A), Amin Bahmanian, Illinois State University, and Theodore Molla, University of Illinois Urbana-Champaign.

Financial Mathematics and Statistics (Code: SS 24A), Ryan Gill, University of Louisville, Rasitha Jayasekera, Butler University, and Kiseop Lee, Purdue University.

Fusion Categories and Applications (Code: SS 26A), Paul Bruillard, Pacific Northwest National Laboratory, and Julia Plavnik and Eric Rowell, Texas A\&M University.

Harmonic Analysis and Partial Differential Equations (Code: SS 9A), Lucas Chaffee, Western Washington University, William Green, Rose-Hulman Institute of Technology, and Jarod Hart, University of Kansas.

Homotopy Theory (Code: SS 12A), David Gepner, Purdue University, Ayelet Lindenstrauss and Michael Mandell, Indiana University, and Daniel Ramras, Indiana University-Purdue University Indianapolis.

Model Theory (Code: SS 14A), Gabriel Conant, University of Notre Dame, and Philipp Hieronymi, University of Illinois Urbana Champaign.

Multivariate Operator Theory and Function Theory (Code: SS 21A), Hari Bercovici, Indiana University, Kelly Bickel, Bucknell University, Constanze Liaw, Baylor University, and Alan Sola, Stockholm University.

Nonlinear Elliptic and Parabolic Partial Differential Equations and Their Various Applications (Code: SS 13A),

Changyou Wang, Purdue University, and Yifeng Yu, University of California, Irvine.

Probabilistic Methods in Combinatorics (Code: SS 22A), Patrick Bennett and Andrzej Dudek, Western Michigan University.

Probability and Applications (Code: SS 16A), Russell Lyons and Nick Travers, Indiana University.

Randomness in Complex Geometry (Code: SS 1A), Turgay Bayraktar, Syracuse University, and Norman Levenberg, Indiana University.

Representation Stability and its Applications (Code: SS 23A), Patricia Hersh, North Carolina State University, Jeremy Miller, Purdue University, and Andrew Putman, University of Notre Dame.

Representation Theory and Integrable Systems (Code: SS 18A), Eugene Mukhin, Indiana University,Purdue University Indianapolis, and Vitaly Tarasov, Indiana University, Purdue University Indianapolis.

Self-similarity and Long-range Dependence in Stochastic Processes (Code: SS 10A), Takashi Owada, Purdue University, Yi Shen, University of Waterloo, and Yizao Wang, University of Cincinnati.

Spectrum of the Laplacian on Domains and Manifolds (Code: SS 4A), Chris Judge and Sugata Mondal, Indiana University.

Topics in Extremal, Probabilistic and Structural Graph Theory (Code: SS 15A), John Engbers, Marquette University, and David Galvin, University of Notre Dame.

Topological Mathematical Physics (Code: SS 17A), E. Birgit Kaufmann and Ralph M. Kaufmann, Purdue University, and Emil Prodan, Yeshiva University.

## Pullman, Washington <br> Washington State University

April 22-23, 2017
Saturday - Sunday
Meeting \#1 128
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: February 2017
Program first available on AMS website: March 9, 2017
Issue of Abstracts: Volume 38, Issue 2

## Deadlines

For organizers: Expired
For abstracts: February 28, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

Andrew S. Raich, University of Arkansas, Title to be announced.

## Invited Addresses

Michael Hitrik, University of California, Los Angeles, Spectra for non-self adjoint operations and integrable dynamics.

Daniel Rogalski, University of California, San Diego, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the
abstract submission form found at www.ams.org/cgibin/abstracts/abstract.p1.

Analysis on the Navier-Stokes equations and related PDEs (Code: SS 9A), Kazuo Yamazaki, University of Rochester, and Litzheng Tao, University of California, Riverside.

Clustering of Graphs: Theory and Practice (Code: SS 18A), Stephen J. Young and Jennifer Webster, Pacific Northwest National Laboratory.

Combinatorial and Algebraic Structures in Knot Theory (Code: SS 5A), Sam Nelson, McKenna College, and Allison Henrich, Seattle University.

Combinatorial and Computational Commutative Algebra and Algebraic Geometry (Code: SS 21A), Hirotachi Abo, Stefan Tohaneanu, and Alexander Woo, University of Idaho.

Commutative Algebra (Code: SS 3A), Jason Lutz and Katharine Shultis, Gonzaga University.

Fixed Point Methods in Differential and Integral Equations (Code: SS 1A), Theodore A. Burton, Southern Illinois University in Carbondale.

Geometric Measure Theory and it's Applications (Code: SS 20A), .

Geometry and Optimization in Computer Vision (Code: SS 15A), Bala Krishnamoorthy, Washington State University, and Sudipta Sinha, Microsoft Research, Redmond, WA.

Inverse Problems (Code: SS 2A), Hanna Makaruk, Los Alamos National Laboratory (LANL), and Robert Owczarek, University of New Mexico, Albuquerque \& Los Alamos.

Mathematical \& Computational Neuroscience (Code: SS 12A), Alexander Dimitrov, Washington State University Vancouver, Andrew Oster, Eastern Washington University, and Predrag Tosic, Washington State University.

Mathematical Modeling of Forest and Landscape Change (Code: SS 11A), Nikolay Strigul and Jean Lienard, Washington State University Vancouver.

Microlocal Analysis and Spectral Theory (Code: SS 17A), Michael Hitrik, University of California, Los Angeles, and Semyon Dyatlov, Masssachusetts Institute of Technology.

Noncommutative algebraic geometry and related topics (Code: SS 16A), Daniel Rogalski, University of California, San Diego, and James Zhang, University of Washington.

Partial Differential Equations and Applications (Code: SS 8A), V. S. Manoranjan, C. Moore, Lynn Schreyer, and Hong-Ming Yin, Washington State University.

Recent Advances in Applied Algebraic Topology (Code: SS 14A), Henry Adams, Colorado State University, and Bala Krishnamoorthy, Washington State University.

Recent Advances in Optimization and Statistical Learning (Code: SS 19A), Hongbo Dong, Bala Krishnamoorthy, Haijun Li, and Robert Mifflin, Washington State University.

Recent Advances on Mathematical Biology and Their Applications (Code: SS 7A), Robert Dillon and Xueying Wang, Washington State University.

Several Complex Variables and PDEs (Code: SS 10A), Andrew Raich and Phillip Harrington, University of Arkansas.

Special Session on Analytic Number Theory and Automorphic Forms (Code: SS 6A), Steven J. Miller, Williams College, and Sheng-Chi Liu, Washington State University.

Theory and Applications of Linear Algebra (Code: SS 4A), Judi McDonald and Michael Tsatsomeros, Washington State University.

Undergraduate Research Experiences in the Classroom (Code: SS 13A), Heather Moon, Lewis-Clark State College.

# New York, New York Hunter College, City University of New York 

May 6-7, 2017
Saturday - Sunday
Meeting \#1 129
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: March 2017
Program first available on AMS website: March 22, 2017
Issue of Abstracts: Volume 38, Issue 2

## Deadlines

For organizers: Expired
For abstracts: March 14, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Jeremy Kahn, City University of New York, Title to be announced.

Fernando Coda Marques, Princeton University, Title to be announced.

James Maynard, Magdalen College, University of Oxford, Title to be announced (Erdős Memorial Lecture).

Kavita Ramanan, Brown University, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgibin/abstracts/abstract.p1.

Analysis and numerics on liquid crystals and soft matter (Code: SS 16A), Xiang Xu, Old Dominion University, and Wujun Zhang, Rutgers University.

Applications of network analysis, in honor of Charlie Suffel's 75th birthday (Code: SS 18A), Michael Yatauro, Pennsylvania State University-Brandywine.

Banach Space Theory and Metric Embeddings (Code: SS 10A), Mikhail Ostrovskii, St John's University, and Beata Randrianantoanina, Miami University of Ohio.

Cluster Algebras in Representation Theory and Combinatorics (Code: SS 6A), Alexander Garver, Université du Quebéc à Montréal and Sherbrooke, and Khrystyna Serhiyenko, University of California at Berkeley.

Cohomologies and Combinatorics (Code: SS 15A), Rebecca Patrias, Université du Québec à Montréal, and Oliver Pechenik, Rutgers University.

Common Threads to Nonlinear Elliptic Equations and Systems (Code: SS 14A), Florin Catrina, St. John’s University, and Wenxiong Chen, Yeshiva University.

Commutative Algebra (Code: SS 1A), Laura Ghezzi, New York City College of Technology-CUNY, and Jooyoun Hong, Southern Connecticut State University.

Computability Theory: Pushing the Boundaries (Code: SS 9A), Johanna Franklin, Hofstra University, and Russell Miller, Queens College and Graduate Center, City University of New York.

Computational and Algorithmic Group Theory (Code: SS 7A), Denis Serbin and Alexander Ushakov, Stevens Institute of Technology.

Cryptography (Code: SS 3A), Xiaowen Zhang, College of Staten Island and Graduate Center-CUNY.

Current Trends in Function Spaces and Nonlinear Analysis (Code: SS 2A), David Cruz-Uribe, University of Alabama, Jan Lang, The Ohio State University, and Osvaldo Mendez, University of Texas at El Paso.

Differential and Difference Algebra: Recent Developments, Applications, and Interactions (Code: SS 12A), Omár León-Sanchez, McMaster University, and Alexander Levin, The Catholic University of America.

Geometric Function Theory and Related Topics (Code: SS 19A), Sudeb Mitra, Queens College and Graduate CenterCUNY, and Zhe Wang, Bronx Community College-CUNY.

Geometry and Topology of Ball Quotients and Related Topics (Code: SS 5A), Luca F. Di Cerbo, Max Planck Institute, Bonn, and Matthew Stover, Temple University.

Hydrodynamic and Wave Turbulence (Code: SS 11A), Tristan Buckmaster, Courant Institute of Mathematical Sciences, New York University, and Vlad Vicol, Princeton University.

Infinite Permutation Groups, Totally Disconnected Locally Compact Groups, and Geometric Group Theory (Code: SS 4A), Delaram Kahrobaei, New York City College of Technology and Graduate Center-CUNY, and Simon Smith, New York City College of Technology-CUNY.

Nonlinear and Stochastic Partial Differential Equations: Theory and Applications in Turbulence and Geophysical Flows (Code: SS 8A), Nathan Glatt-Holtz, Tulane University, Geordie Richards, Utah State University, and Xiaoming Wang, Florida State University.

Operator algebras and ergodic theory (Code: SS 17A), Genady Grabarnik and Alexander Katz, St John's University.

Qualitative and Quantitative Properties of Solutions to Partial Differential Equations (Code: SS 20A), Blair Davey, The City College of New York-CUNY, and Nguyen Cong Phuc and Jiuyi Zhu, Louisiana State University.

Recent Developments in Automorphic Forms and Representation Theory (Code: SS 21A), Moshe Adrian, Queens College-CUNY, and Shuichiro Takeda, University of Missouri.

Representation Spaces and Toric Topology (Code: SS 13A), Anthony Bahri, Rider University, and Daniel Ramras and Mentor Stafa, Indiana University-Purdue University Indianapolis.

## Montréal, Quebec Canada

McGill University

July 24-28, 2017
Monday - Friday
Meeting \#1 130
The second Mathematical Congress of the Americas (MCA 2017) is being hosted by the Canadian Mathematical Society (CMS) in collaboration with the Pacific Institute for the Mathematical Sciences (PIMS), the Fields Institute (FIELDS), Le Centre de Recherches Mathématiques (CRM), and the Atlantic Association for Research in the Mathematical Sciences (AARMS).
Associate secretary: Brian D. Boe
Announcement issue of Notices: January, 2017
Program first available on AMS website: To be announced Issue of Abstracts: N/A

## Deadlines

For organizers: Expired
For abstracts: March 31, 2017

## Denton, Texas

## University of North Texas

September 9-10, 2017
Saturday - Sunday

## Meeting \#1131

Central Section
Associate secretary: Georgia Benkart Announcement issue of Notices: June 2017
Program first available on AMS website: July 27, 2017
Issue of Abstracts: Volume 38, Issue 3

## Deadlines

For organizers: February 2, 2017
For abstracts: July 18, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Mirela Çiperiani, University of Texas at Austin, Title to be announced.

Adrianna Gillman, Rice University, Title to be announced.

Kevin Pilgrim, Indiana University, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgibin/abstracts/abstract.p1.

Dynamics, Geometry and Number Theory (Code: SS 1A), Lior Fishman and Mariusz Urbanski, University of North Texas.

Fractal Geometry and Ergodic Theory (Code: SS 5A),

Mrinal Kanti Roychowdhury, University of Texas Rio Grande Valley.

Lie algebras, Superalgebras, and Applications (Code: SS 3A), Charles H. Conley, University of North Texas, and Dimitar Grantcharov, University of Texas at Arlington.

Noncommutative and Homological Algebra (Code: SS 4A), Anne Shepler, University of North Texas, and Sarah Witherspoon, Texas A\&M University.

Numbers, Functions, Transcendence, and Geometry (Code: SS 6A), William Cherry, University of North Texas, Mirela Ciperiani, University of Texas Austin, Matt Papanikolas, Texas A\&M University, and Min Ru, University of Houston.

Real-Analytic Automorphic Forms (Code: SS 2A), Olav K. Richter, University of North Texas, and Martin WesterholtRaum, Chalmers University of Technology.

## Buffalo, New York

State University of New York at Buffalo
September 16-17,2017
Saturday - Sunday
Meeting \#1 132
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: June 2017
Program first available on AMS website: August 3, 2017
Issue of Abstracts: Volume 38, Issue 3

## Deadlines

For organizers: February 16, 2017
For abstracts: July 25, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm7.

## Invited Addresses

Inwon Kim, University of California at Los Angeles, Title to be announced.

Govind Menon, Brown University, Title to be announced.
Bruce Sagan, Michigan State University, Title to be announced.

## Orlando, Florida

## University of Central Florida, Orlando

## September 23-24, 2017

Saturday - Sunday
Meeting \#1 133
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of Notices: June 2017
Program first available on AMS website: August 10, 2017
Issue of Abstracts: Volume 38, Issue 4

## Deadlines

For organizers: February 23, 2017
For abstracts: August 1, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Christine Heitsch, Georgia Institute of Technology, Title to be announced.

Jonathan Kujawa, University of Oklahoma, Title to be announced.

Christopher D. Sogge, Johns Hopkins University, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www. ams.org/cgi-bin/ abstracts/abstract.p1.

Algebraic Curves and their Applications (Code: SS 3A), Lubjana Beshaj, The University of Texas at Austin.

Commutative Algebra: Interactions with Algebraic Geometry and Algebraic Topology (Code: SS 1A), Joseph Brennan, University of Central Florida, and Alina Iacob and Saeed Nasseh, Georgia Southern University.

Fractal Geometry, Dynamical Systems, and Their Applications (Code: SS 4A), Mrinal Kanti Roychowdhury, University of Texas Rio Grande Valley.

Graph Connectivity and Edge Coloring (Code: SS 5A), Colton Magnant, Georgia Southern University.

Structural Graph Theory (Code: SS 2A), Zixia Song, University of Central Florida.

## Riverside, California

University of California, Riverside
November 4-5, 2017
Saturday - Sunday
Meeting \#1 134
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: September 2017
Program first available on AMS website: September 21, 2017
Issue of Abstracts: Volume 38, Issue 4

## Deadlines

For organizers: April 14, 2017
For abstracts: September 12, 2017
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Paul Balmer, University of California, Los Angeles, Title to be announced.

Pavel Etingof, Massachusetts Institute of Technology, Title to be announced.

Monica Vazirani, University of California, Davis, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www. ams.org/cgi-bin/ abstracts/abstract.p7.

Combinatorial aspects of the polynomial ring (Code: SS 1A), Sami Assaf and Dominic Searles, University of Southern California.

## San Diego, California

## San Diego Convention Center and San Diego Marriott Hotel and Marina

## January 10-13, 2018

Wednesday - Saturday
Meeting \#1 135
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of Notices: October 2017
Program first available on AMS website: To be announced Issue of Abstracts: Volume 39, Issue 1

## Deadlines

For organizers: April 1, 2017
For abstracts: To be announced

## Columbus, Ohio

## Ohio State University

March 17-18,2018
Saturday - Sunday
Meeting \#11 36
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: To be announced
For abstracts: To be announced
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectiona1.htm7.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the
abstract submission form found at www.ams.org/cgibin/abstracts/abstract.pl.

Recent Advances in Approximation Theory and Operator Theory (Code: SS 1A), Jan Lang and Paul Nevai, The Ohio State University.

## Portland, Oregon

## Portland State University

April 14-15, 2018
Saturday - Sunday
Meeting \#1 137
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: To be announced
For abstracts: To be announced
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Sándor Kovács, University of Washington, Seattle, Title to be announced.

Elena Mantovan, California Institute of Technology, Title to be announced.

Dimitri Shlyakhtenko, University of California, Los Angeles, Title to be announced.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgibin/abstracts/abstract.p1.

Inverse Problems (Code: SS 2A), Hanna Makaruk, Los Alamos National Laboratory (LANL), and Robert Owczarek, University of New Mexico, Albuquerque \& Los Alamos.

Pattern Formation in Crowds, Flocks, and Traffic (Code: SS 1A), J. J. P. Veerman, Portland State University, Alethea Barbaro, Case Western Reserve University, and Bassam Bamieh, UC Santa Barbara.

## Nashville, Tennessee

## Vanderbilt University

April 14-15, 2018
Saturday - Sunday
Meeting \#1138
Southeastern Section
Associate secretary: Brian D. Boe

Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: To be announced For abstracts: To be announced

## Boston, Massachusetts

## Northeastern University

April 21-22, 2018
Saturday - Sunday
Meeting \#1 139
Eastern Section
Associate secretary: Steven H. Weintraub Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: September 21, 2017
For abstracts: March 6, 2018

## People’s Republic of China

## Fudan University

June 11-14, 2018
Monday - Thursday
Meeting \#1 140
Associate secretary: Steven H. Weintraub Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: To be announced
For abstracts: To be announced

## Newark, Delaware

## University of Delaware

September 29-30, 2018
Saturday - Sunday
Eastern Section
Associate secretary: Steven H. Weintraub Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: February 28, 2018
For abstracts: To be announced

## San Francisco, California

## San Francisco State University

October 27-28, 2018

## Sunday

Western Section
Associate secretary: Michel L. Lapidus Announcement issue of Notices: August 2018
Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: April 30, 2018
For abstracts: August 28, 2018

## Baltimore, Maryland <br> Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16-19, 2019
Wednesday - Saturday
Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: October 2018
Program first available on AMS website: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

## Honolulu, Hawaii

## University of Hawaii at Manoa

March 29-31, 2019
Friday - Sunday
Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of Notices: To be announced Program first available on AMS website: To be announced

Issue of Abstracts: To be announced Deadlines
For organizers: To be announced For abstracts: To be announced

## Denver, Colorado <br> Colorado Convention Center

## January 15-18, 2020

Wednesday - Saturday
Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced Program first available on AMS website: November 1, 2019 Issue of Abstracts: To be announced

## Deadlines

For organizers: April 1, 2019
For abstracts: To be announced

# Washington, District of Columbia 

## Walter E. Washington Convention Center

January 6-9, 2021
Wednesday - Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of Notices: October 2020
Program first available on AMS website: November 1, 2020 Issue of Abstracts: To be announced

## Deadlines

For organizers: April 1, 2020
For abstracts: To be announced


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Part I: Issue at-a-Glance, is organized by specific Volume and Issue. This Part provides readers with a quick snapshot of the main articles featured in each specific Notices issue.
Part II: Societal Record, is organized alphabetically under each category heading. This Part provides readers with a searchable listing of all content of record for the Society, including: elections, awards, meetings, news, opportunities, annual AMS reports, surveys, grants, fellowships, etc.

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Hampshire College Summer Studies in Mathematics
Joseph Baldwin Academy for Eminent Young Scholars
MathlLy
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New York Math Circle High School Summer Program
PROMYS (Program in Mathematics for Young Scientists)
PROTaSM (Puerto Rico Opportunities for Talented Students inMathematics)

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Name (please write name as you would like it to appear on your badge)

Mailing Address

Telephone
Fax:
In case of emergency (for you) at the meeting, call: Day \# $\qquad$ Evening \#:

Email Address $\qquad$ Additional email address for receipt $\qquad$
Acknowledgment of this registration and any hotel reservations will be sent to the email address(es) given here. Check this box to receive a copy in U.S. Mail:
Affiliation for badge $\qquad$ Nonmathematician guest badge name:
(Note fee of US\$20
$\square$ I DO NOT want my program and badge to be mailed to me on 12/9/16. (Materials will be mailed to the address listed above unless you check this box.)

## Registration Fees

Membership please $\checkmark$ all that apply. First row is eligible to register as a member.
For undergraduate students, membership in PME and KME also applies.

| $\square$ AMS $\quad \square$ MAA | $\square$ ASL | $\square$ CMS | $\square$ SIAM |
| :--- | :---: | :--- | :--- |
| Undergraduate Students Only: | $\square$ PME | $\square$ KME |  |
| Other Societies: | $\square$ AWM | $\square$ NAM | $\square$ YMN |

$\square$ AMATYC

| Joint Meetings by |  | by Dec 20 | at mtg | Subtotal |
| :---: | :---: | :---: | :---: | :---: |
| 口 | Member AMS, MAA, ASL, CMS, or SIAM | M US\$ 316 | US\$ 416 |  |
| $\square$ | Nonmember | US\$ 502 | US\$ 640 |  |
| - | Graduate Student Member (AMS, MAA |  |  |  |
|  | ASL, CMS, or SIAM) | US\$ 71 | US\$ 83 |  |
| $\square$ | Graduate Student (Nonmember) | US\$113 | US\$ 125 |  |
| $\square$ | Undergraduate Student (Member AMS, ASL, CMS, MAA, PME, KME, or SIAM) | US\$ 71 | US\$ 83 |  |
| $\square$ | Undergraduate Student (Nonmember) | US\$113 | US\$ 125 |  |
| $\square$ | High School Student | US\$ 7 | US\$ 13 |  |
| $\square$ | Unemployed | US\$ 71 | US\$ 83 |  |
| $\square$ | Temporarily Employed | US\$ 258 | US\$ 295 |  |
| $\square$ | Developing Countries Special Rate | US\$ 71 | US\$ 83 |  |
| $\square$ | Emeritus Member of AMS or MAA | US\$ 71 | US\$ 83 |  |
| $\square$ | High School Teacher | US\$ 71 | US\$ 83 |  |
| $\square$ | Librarian | US\$ 71 | US\$ 83 |  |
| $\square$ | Press | US\$ 0 | US\$ 0 |  |
| $\square$ | Exhibitor (Commercial) | US\$ 0 | US\$ 0 |  |
| $\square$ | Artist Exhibitor (work in JMM Art Exhibit) | US\$ 0 | US\$ 0 |  |
| $\square$ | Nonmathematician Guest of registered mathematician | US\$ 20 | US\$ 20 |  |
|  |  |  | \$ |  |
| AMS Short Course: Random Growth Models (1/2-1/3) |  |  |  |  |
| $\square$ | Member of AMS | US\$ 112 | US\$ 146 |  |
| $\square$ | Nonmember | US\$ 170 | US\$200 |  |
| $\square$ | Student, Unemployed, Emeritus | US\$ 60 | US\$ 81 |  |
|  |  |  | \$ |  |

MAA Minicourses (see listing in text)
I would like to attend: $\square$ One Minicourse $\square$ Two Minicourses
Please enroll me in MAA Minicourse(s) \# $\qquad$ and \# $\qquad$
Price: US\$ 100 for each minicourse.
(For more than 2 minicourses, call or email the MMSB.) \$ $\qquad$

## Graduate School Fair

ㅁ Graduate Program Table US\$80 US\$ 80 (includes table, posterboard \& electricity) \$
\$ $\qquad$

## Receptions \& Banquets

$\square$ Graduate Student/First-Time Attendee Reception (1/4) (no charge) - NAM Banquet (1/6)
\#___Chicken \#___Braised Short Ribs \#___ Veg US\$ 65
\#___Kosher (Additional fees apply for Kosher Meals.) US\$ 104
Total for NAM Banquet
\$ $\qquad$
$\square \quad$ AMS Dinner (1/7)
Regular Price $\qquad$ _US\$ 69
Student Price \#___US\$ 30
Total for AMS Dinner

Total for Registrations and Events
\$
\$ $\qquad$
Registration for the Joint Meetings is not required for the short course but it is required for the minicourses and the Employment Center. To register for the Employment Center, go to http://www.ams.org/profession/employment-services/ employment-center For questions, email: emp-info@ams.org.

## Payment

Registration \& Event Total (total from column on left)
Hotel Deposit (only if paying by check)
If you send a hotel deposit check, the
deadline for this form is December 1.

## Total Amount To Be Paid

\$

## Method of Payment

$\square$ Check. Make checks payable to the AMS. For all check payments, please keep a copy of this form for your records.

Credit Card. All major credit cards accepted. For your security, we do not accept credit card numbers by postal mail, email or fax. If the MMSB receives your registration form by fax or postal mail, it will contact you at the phone number provided on this form. For questions, contact the MMSB at mmsb@ams.org.

Signature:
$\square$ Purchase Order \# $\qquad$ (please enclose copy)

## Other Information

Mathematical Reviews field of interest \#
$\square \quad$ I am willing to serve as a judge for the MAA Undergraduate Student Poster Session
$\square$ For planning purposes for the MAA Two-year College Reception, please check if you are a faculty member at a two-year college.
$\square \quad$ I am a mathematics department chair.
$\square$ Please do not include my name and postal address on any promotional mailing lists. (The JMM does not share email addresses.)
$\square$ Please do not include my name on any list of JMM participants other than the scientific program if I am, in fact, making a presentation that is part of the meeting.
$\square \quad$ Please $\checkmark$ this box if you have a disability requiring special services.

## Deadlines

| To receive badges/programs in the mail: | Nov. 22, 2016 |
| :--- | :--- |
| Hotel reservations with check deposit: | Dec. 1, 2016 |
| Hotel reservations, changes/cancellations |  |
| through the JMM website: | Dec. 6, 2016 |
| Advance registration for the Joint Meetings, short |  |
| course, minicourses, and tickets: | Dec. 20, 2016 |
| 50\% refund on advance registration, banquets, <br> minicourses, and short course, cancel by | Dec. 29, 2016* |
| *no refunds issued after this date. |  |

## Mailing Address/Contact:

Mathematics Meetings Service Bureau (MMSB)
P. O. Box 6887

Providence, RI 02940-6887 Fax: 401-455-4004; Email: mmsb@ams.org
Telephone: 401-455-4144 or 1-800-321-4267 $\times 4144$ or $\times 4137$

## 2017 Joint Mathematics Meetings Hotel Reservations - Atlanta, GA





 Deposit enclosed (see front of form)
 number provided on the reverse of this form.
Date and Time of Arrival
Name of Other Adult Room Occupant (s)
Housing Requests:.(example: rollaway cot, crib, nonsmoking room, low floor)
ㅁ I have disabilities as defined by the ADA that require a sleeping room that is accessible to the physically challenged. My needs are:
$\square$ I am a member of a hotel frequent-travel club and would like to receive appropriate credit. The hotel chain and card number are:
who is making the reservation.

| Order of choice | Hotel | Single | Double <br> 1 bed-2 people | Double <br> 2 beds- 2 people | Triple <br> 3 adults- 2 beds | Quad 4 adults-2 beds | Rollaway Cot Fee <br> (add to special requests if reserving online) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hyatt Regency Atlanta (co-hqtrs) | US\$ 175 | US\$ 175 | US\$ 175 | US\$ 195 | US\$ 215 | Rollaways available (at no charge) only in king-bedded rooms. |
|  | Student Rate | US\$ 140 | US\$ 140 | US\$ 140 | US\$ 160 | UD\$ 180 |  |
|  | Marriott Marquis Atlanta (co-hqtrs) | US\$ 175 | US\$ 175 | US\$ 175 | US\$ 175 | US\$ 175 | Rollaways available (at no charge) only in king-bedded rooms. |
|  | Student Rate | US\$ 140 | US\$ 140 | US\$ 140 | US\$ 140 | US\$ 140 |  |
|  | Hilton Atlanta - First Tier <br> (Rooms will be available at this price until they run out. When they run out, rooms will be priced at second tier | US\$ 139 | US\$ 139 | US\$ 139 | US\$ 159 | US\$ 179 | Rollaways available (at no charge) only in king-bedded rooms. |
|  | Hilton Atlanta - Second Tier | US\$ 149 | US\$ 149 | US\$ 149 | US\$ 169 | US\$ 189 |  |

People interested in suites should contact the MMSB directly by email at mmsb@ams.org or by calling 800-321-4267, ext. 4137 or 4144; (401-455-4137 or 401-455-4144).

## Attend the

## 2017 AMS DINNER

## at the Joint Mathematics Meetings in Atlanta, GA!

Join your colleagues on this special occasion of celebration in the mathematical community. The AMS will recognize long-term members as well as honor the recipients of Programs That Make a Difference Awards, Award for Impact on the Teaching and Learning of Mathematics, and the Exemplary Programs Award. Enjoy delicious meals from gourmet food stations, special entertainment, and enter to win fun prizes at the raffle table!

# This evening of celebration will be held on Saturday, January 7th 

with a reception at 6:30 pm and doors opening at 7:30pm.

## Tickets are $\$ 69$ including tax and gratuity.

## The student discount price is $\$ 30^{\prime \prime}$.

Purchase your tickets when registering for the Joint Mathematics Meetings. *Quantity is limited.


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## American Mathematical Society



## Riot at the Calc Exam and Other Mathematically Bent Stories

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4817-3; Order code MBK/62

## Famous Puzzles of Great Mathematicians <br> Miodrag S. Petkovi

...The book would be the ideal graduation present for a mathematics major, an ideal prize for the winner of an integration contest, an ideal book to have lying around a mathematics department (if properly chained down, that is).
-MAA Reviews
2009; 325 pages; Softcover; ISBN: 978-0-82 I8-48I4-2; Order code MBK/63
bookstore.ams.org

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A publication of Delta Stream Media, an imprint of Natural Math. Distributed in North America by the American Mathematical Society

Natural Math Series,Volume 4; 2016;84 pages; Softcover; ISBN: 978-0-9776939-0-0; Order code NMATH/4

40\% off for AMS members and $25 \%$ off for non-AMS members from Nov. 16-Dec. 15, 2016.

## Gallery of the Infinite <br> Richard Evan Schwartz

Written in a playful yet informative style, this book is a mathematician's unique view of the infinitely many sizes of infinity.
2016; I87 pages; Softcover; ISBN: 978-I-4704-
2557-9; Order code MBK/97

## The Case of Academician Nikolai Nikolaevich Luzin

Sergei S. Demidov and Boris V. Lëvshin, Editors
Translated by Roger Cooke
This book chronicles the 1936 attack on mathematician Nikolai Nikolaevich Luzin during the USSR campaign to "Sovietize" all sciences.

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[^14]:    ${ }^{2}$ Also see the opinion piece by Jorgensen in the 2016 November Notices.
    3www.s1ate.com/artic1es/hea1th_and_science/ education/2016/03/andrew_hacker_s_the_math_myth_ is_a_great_examp1e_of_mathematics_i11iteracy.htm1.
    ${ }^{4}$ See the October 2012 Notices for two articles about this report.

[^15]:    ${ }^{*}$ The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at: www.nsf.gov/dms and for the Directorate of Education and Human Resources at www.nsf.gov/dir/index.jsp?org=ehr. To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions atwww.nsf.gov/mps/dms/about.jsp.

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