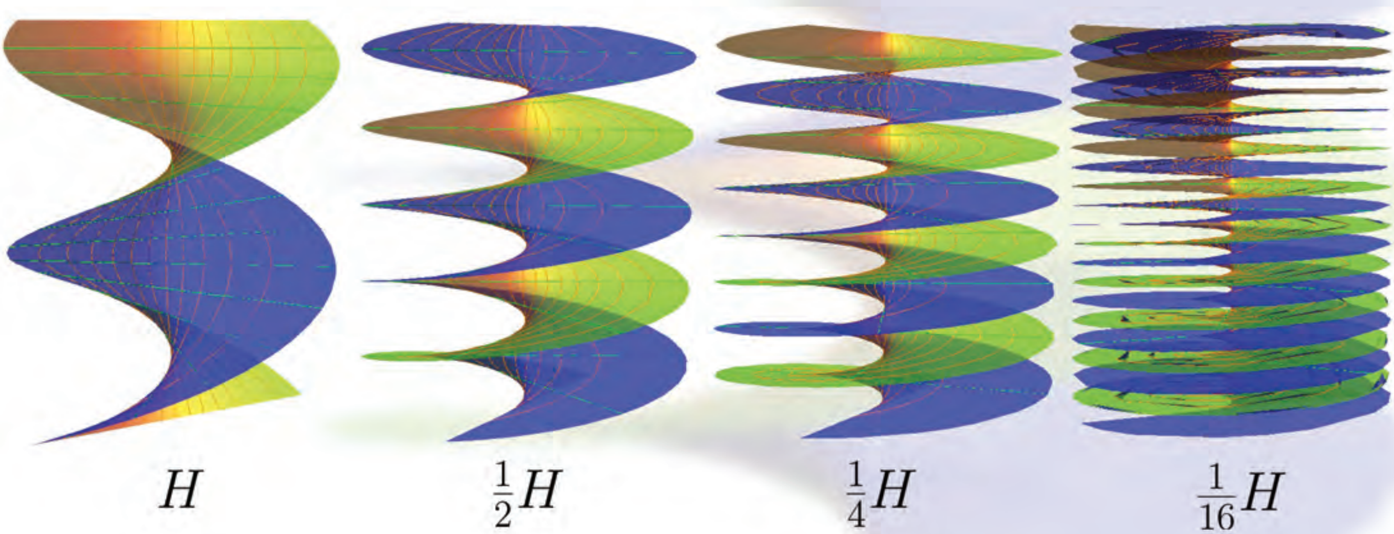
ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices
of the American Mathematical Society

April 2017 Volume 64, Number 4



AMS
AMERICAN MATHEMATICAL SOCIETY

About the Cover: How Minimal Surfaces Converge to a Foliation
(see page 307)

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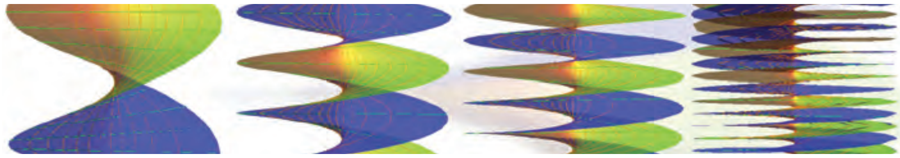

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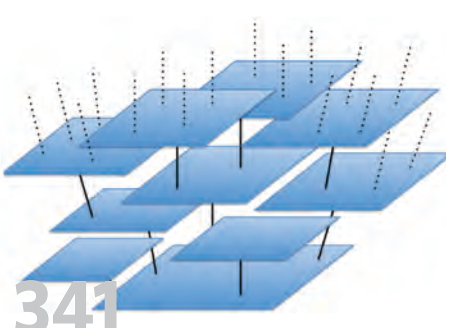

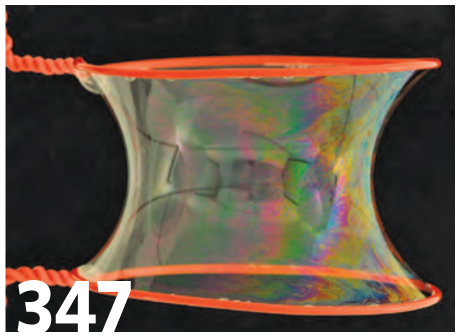

Most fitting celebrations of April as Mathematics and Statistics Awareness Month are the AMS prize announcements, the spring sectional meetings (sampled herein for all), and the Association for Women in Mathematics Research Symposium (also sampled herein). After an article tracing the history of Mathematics Awareness Month back to World War II, we have a report from IMAGINARY on how to share mathematics with the world, and an opinion piece on the National Math Festival. The BackPage announces the January caption contest winner and the new April contest. Celebrate with us, and add your own contributions to the commentary on the Notices *webpage.* ***—Frank Morgan, Editor-in-Chief***

Notices

of the American Mathematical Society

FROM THE SECRETARY, CONT'D.

Announcements of winners for the :

IN EVERY ISSUE

ABOUT THE COVER

The cover illustration shows how embedded minimal disks in R^3 or in balls of increasing radius can converge to a smooth foliation while the curvature blows up along a vertical axis. Inside a ball of fixed radius, curvature blow up can occur along infinitely many arcs. On the other hand, if the curvature is bounded, the convergence is smooth. Joaquín Pérez's survey (Section 4, page 347) describes such results from the work of Colding and Minicozzi on limits of embedded minimal disks and more recent results with Bill Meeks and Antonio Ros.

Notices
of the American Mathematical Society

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[Notices of the American Mathematical Society (ISSN 0002-9920) is published monthly except bimonthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2294 USA, GST No. 12189 2046 RT****. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to Notices of the American Mathematical Society, P.O. Box 6248, Providence, RI 02904-6248 USA.] Publication here of the Society's street address and the other bracketed information is a technical requirement of the U.S. Postal Service.

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2017 Leroy P. Steele Prizes


Dusa McDuff


Dietmar Salamon


Leon Simon


James Arthur

The 2017 Leroy P. Steele Prizes were presented at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017. The Steele Prizes were awarded to DUSA MCDUFF and DIETMAR SALAMON for Mathematical Exposition, to LEON SIMON for Seminal Contribution to Research, and to JAMES ARTHUR for Lifetime Achievement.

Citation for Mathematical Exposition: Dusa McDuff and Dietmar Salamon

Dusa McDuff and Dietmar Salamon are awarded the Steele Prize for Mathematical Exposition for their book *J-Holomorphic Curves and Symplectic Topology*, American Mathematical Society Colloquium Publications, 52, 2004.

The field of symplectic topology went through a rapid phase of development following Gromov's 1985 paper that introduced *J*-holomorphic curves into symplectic topology and intertwined this field with algebraic geometry and string theory. Techniques revolving around *J*-holomorphic curves have been a basic ingredient in the solution of many classical and crucial questions in symplectic topology, as well as in the discovery of new structures. More than thirty years after its publication, the influence of Gromov's paper in the rapidly developing field of symplectic topology is as strong as in the beginning, and many of the most exciting research topics in the field (like, for example, mirror symmetry) involve in one way or another the notion of a *J*-holomorphic map.

The book *J -Holomorphic Curves and Symplectic Topology* is a comprehensive introduction to Gromov's theory of *J*-holomorphic curves, explaining from the beginning and in detail the essential notions and results, as well as many of its spectacular applications in symplectic topology. While being among the main contributors to this development, McDuff and Salamon spent nearly a decade assembling the foundations of this subject into a mammoth 700-page book. It has since served as the most standard and undisputed reference in the field and as the main textbook for graduate students and others entering the field. The use of the abbreviation M-S in the context of *J*-holomorphic curves and symplectic topology has now become routine and causes no confusion.

This book begins with a sixteen-page overview of the subject of symplectic topology, the theory of *J*-holomorphic curves, and its applications to symplectic topology, algebraic geometry, and mirror symmetry. This overview is informative to those outside the field who are just curious and serves as a guide to the book. Each chapter begins with its own very informative introduction. The chapters and sections are structured so that the main statements are formulated as early as possible while their proofs are delayed.

In some ways, the McDuff and Salamon book on *J*-holomorphic curves is the symplectic analogue of Lazarsfeld's *Positivity in Algebraic Geometry* and Griffiths and Harris's *Principles of Algebraic Geometry*. This book, together with McDuff and Salamon's *Introduction to Symplectic Topology* and their many other contributions, has been a great help to both junior and senior symplectic geometers.

Biographical Sketch: Dusa McDuff

Dusa McDuff was born in London in 1945, grew up in Edinburgh, and in 1971 received her PhD from Cambridge University under the direction of George Reid. She spent six months in Moscow in 1969–1970 as a student of I. M. Gelfand, who had a profound influence on her mathematics. After working on topics in topology and foliation theory (often in collaboration with Graeme Segal), she was moving into the area of symplectic geometry just as Gromov published his pioneering paper and has remained there ever since. After holding positions at York, Warwick, and Stony Brook Universities, she is currently Helen Lyttle Kimmel '42 Professor of Mathematics at Barnard College, Columbia University. She received the Ruth Lyttle Satter Prize (1991), gave a plenary address to the ICM (1998), and was AWM Noether Lecturer (1998) and AMS Colloquium Lecturer (2014). She is a fellow of the Royal Society of London (1994), a member of the US Academy of Sciences (1999) and of the American Philosophical Society (2013), and has honorary degrees from the universities of York, Edinburgh, St. Andrews, Strasbourg, and Pierre and Marie Curie, Paris.

Biographical Sketch: Dietmar Salamon

Dietmar Salamon was born in Bremen in 1953 and completed his PhD at the University of Bremen in 1982 under the direction of Diederich Hinrichsen. After postdoctoral positions at the University of Madison–Wisconsin and at ETH Zurich, he took up a lectureship at the University of Warwick in 1986, where he became professor in 1994. In 1998 he moved to ETH Zurich to take up a professorship. His field of research is symplectic topology and related subjects.

He was an invited speaker at the ECM 1992 in Paris, at the ICM 1994 in Zurich, and at the ECM 2000 in Barcelona. He delivered the Andrejewski Lectures in Goettingen (1998) and at the Humboldt University Berlin (2005) and the Xth Lisbon Summer Lectures in Geometry (2009). He is a member of the Academia Europaea and a fellow of the AMS.

He is the author of several books, including the joint monographs *Introduction to Symplectic Topology* and *J-Holomorphic Curves and Symplectic Topology* with Dusa McDuff, and of over seventy research papers. He has supervised twenty PhD students.

Response from Dusa McDuff and Dietmar Salamon

It is a great honor to receive the Leroy P. Steele Prize for Mathematical Exposition for our book *J-Holomorphic Curves and Symplectic Topology*.

Our collaboration started in 1990 at a conference at the University of Warwick. During the preceding year we had both given lecture courses on symplectic topology and decided to put our acts together to write a monograph about that newly emerging subject, not imagining how much effort would go into this in the course of the following quarter of a century. In December 1993 at a conference in Tel Aviv—coincidentally in honor of Misha Gromov's fiftieth birthday—we decided that the theory of J-holomorphic curves, together with the vast amount of introductory material, would be too much for a single volume.

So one book turned into two, and the one on J-holomorphic curves "bubbled off." It was initially conceived as a fairly brief introduction, together with a proof of the gluing theorem, and appeared less than a year later, even before our *Introduction to Symplectic Topology*, in the AMS University Lecture Series under the title *J-Holomorphic Curves and Quantum Cohomology*. About a decade later we decided to correct errors and include more details and applications, after which the manuscript tripled in size to almost 700 pages and was published in 2004 under the title *J-Holomorphic Curves and Symplectic Topology* in the AMS Colloquium Publications series. An updated and corrected second edition appeared in 2012.

Our work on both manuscripts required a certain amount of compromise, as well as extensive arguing, as often we were approaching the subject from rather different points of view, which one might characterize as more geometric (Dusa) versus more analytic (Dietmar). However, we both found this process stimulating, and in the end it led to a much better result than either of us could have achieved on our own.

We learnt a great deal from each other, as well as from many other researchers in the field, to whom we wish to express our deep gratitude. We are very happy that our books helped many others to study this beautiful subject and also are deeply honored that our efforts have been recognized with the award of this prize.

Citation for Seminal Contribution to Research: Leon Simon

The Steele Prize for Seminal Contribution to Research is awarded to Leon Simon for his fundamental contributions to geometric analysis and in particular for his 1983 paper "Asymptotics for a class of nonlinear evolution equations, with applications to geometric problems," published in the *Annals of Mathematics*.

In this groundbreaking paper, Simon addressed the basic question of what a minimal variety must look like near a singularity. This is a question of fundamental importance, since singularities in minimal varieties (as well as solutions to many other nonlinear problems) are generally unavoidable. Once one knows that singularities occur, one naturally wonders what they are like. The first answer, already known to Federer and Fleming in 1959, is that they weakly resemble cones. Unfortunately, the simple proof leaves open the possibility that a minimal variety looked at under a microscope will resemble one blowup, but under higher magnification, it might (as far as anyone knows) resemble a completely different blowup. Whether this ever happens is one of the most fundamental questions about singularities.

Simon recasts this as a question of long-time behavior of solutions to gradient flows. He then uses this formulation to prove the uniqueness of the tangent cone at a singularity where there is a tangent cone with an isolated singularity at the vertex. Simon also obtains, in the same paper, a similar uniqueness result for other variational problems. A couple of years earlier, in a landmark paper, Allard and Almgren had proven uniqueness under an additional integrability assumption on the cross-section by entirely different methods.

In the 1960s Łojasiewicz showed that, for an analytic function on Euclidean space, any limit point of its gradient flow is in fact a limit. Simon realized in a brilliant way that many fundamental problems in analysis and geometry can be recast as infinite-dimensional Łojasiewicz theorems and, by ingenious analytic arguments, deduced from the finite-dimensional Łojasiewicz theorem.

The significance of Simon's pioneering paper extends well beyond these results. In fact, Simon obtained these results as an application of a strikingly original and general method that he developed in the paper, based on the Łojasiewicz inequality in real algebraic geometry, known now as the Łojasiewicz–Simon inequality. The basic analytic ingredient that Simon developed to carry out this method has proven to be an extraordinarily powerful tool of far-reaching impact. It has since been applied or adapted in uniqueness and related questions in a very large number of contexts, ranging from differential geometry to fluid dynamics and superconductivity.

Simon himself subsequently used this inequality, together with a host of further new ideas, to show regularity of the singular set of a minimal variety.

Leon Simon's paper has had extraordinary impact on analysis, geometry, and applied mathematics. Hundreds of papers have been written either directly applying the Łojasiewicz–Simon inequality or based upon the insights contained in this paper. Without a doubt Simon's ideas will continue to be applied and further developed in future work.

Biographical Sketch: Leon Simon

Leon Simon is emeritus professor of mathematics at Stanford University. Born July 6, 1945, in Adelaide, South Australia, he received his bachelor's degree at the University of Adelaide in 1967 and his PhD, written under the direction of James Michael, from the same institution in 1971. After briefly holding a lectureship at Flinders University in Adelaide, he took a postdoctoral assistant professorship at Stanford University from 1973 to 1976. After holding professorships at Minneapolis, Melbourne University, and the Australian National University in Canberra, he returned to Stanford as professor of mathematics in 1986. He was chair of mathematics at Stanford for the period 1998–2001.

Simon's main research interests are in geometric measure theory and partial differential equations, in particular the theory of minimal surfaces and related problems in the geometric calculus of variations.

He was elected Fellow of the Australian Academy of Sciences in 1983, the American Academy of Arts and Sciences in 1994, and the Royal Society in 2003. He was awarded a Sloan Fellowship in 1975, an Australian Mathematical Society Medal in 1983, the Bôcher Prize of the American Mathematical Society in 1994, and a Humboldt Award in 2005. He gave an invited talk at the ICM in 1983 and is an AMS Fellow. In the course of his career, he has supervised the thesis work of eighteen graduate students.

Response from Leon Simon

I am very honored to be chosen for this award. The cited work was carried out during my time at the Australian National University in Canberra, and I owe a great debt to a number of people, including Robert Bartnik, John Hutchinson, Peter Price, and Neil Trudinger, who were responsible for the congenial and very active research environment during that time. I am of course also indebted to those who provided me with inspiration and support in the period prior to that, including James Michael (1920–2001), who was an inspiring undergraduate teacher and who supervised my PhD work, and David Gilbarg (1918–2001), Rick Schoen, and S.-T. Yau during my postdoctoral period at Stanford. I'm also greatly indebted to Robert Hardt, who acquainted me with many of the finer points of geometric measure theory during our collaborations at the University of Minnesota and the University of Melbourne in 1977–1979.

Citation for Lifetime Achievement: James Arthur

The 2017 Steele Prize for Lifetime Achievement is awarded to James Arthur for his fundamental contributions to number theory and harmonic analysis, and in particular for his proof of the Arthur–Selberg trace formula.

Introduction of L-functions into the theory of automorphic forms began with a conjecture of Ramanujan, its proof by Mordell, and the exploitation of Mordell's ideas by Hecke, who had already had experience with Euler products in the context of Dedekind ζ-functions and related L-functions. Later Selberg introduced methods from the spectral theory of second-order differential equations on a half-line, as well as a form of the Frobenius reciprocity theorem, familiar from the representation theory of finite groups. In the context of discrete subgroups of Lie groups it became known as the Selberg trace formula. For groups with compact quotient, it is hardly more difficult than the Frobenius theorem itself. For groups with quotients of finite volume but not compact, not only its formulation but also its proof required ingenuity and a good deal of skill in the use of the spectral theory.

The first trace formula for general groups was established by Arthur in the 1970s in a series of three papers. Starting from the particular case of SL(2) which had been established by Selberg in 1956, Arthur has built a whole mathematical framework and introduced many major tools in noncommutative harmonic analysis in order to prove the trace formula for a general reductive group. The final result is now called the Arthur–Selberg trace formula. The proof in itself takes sixteen long and difficult papers that Arthur published between 1974 and 1988. This is considered to be a major achievement in mathematics.

As Langlands suggested at the end of the 1960s, the trace formula is a powerful tool for proving the Langlands principle of functoriality, especially in the so-called endoscopic case. For this purpose, one first needs to stabilize the Arthur–Selberg trace formula. Arthur published eight papers between 1997 and 2003 on the stabilization process. Using the stable trace formula and the Fundamental Lemma proved in 2008 by Ngô Bảo Châu, Arthur has recently been able to establish the Langlands functoriality for the standard representations of the classical groups (symplectic, orthogonal, and unitary).

As a consequence, he has obtained explicit formulas for the multiplicities in the automorphic discrete spectrum for those classical groups. The Arthur–Selberg trace formula is a central tool in Lafforgue's proof of the Langlands correspondence for function fields.

Arthur's contribution to mathematics is fundamental. His work already has had, is having, and will have an enormous impact on several branches of mathematics. But his service to the mathematical community is also very impressive. Arthur played an important role in shaping the work of several important national and international committees and organizations. All this culminated when he served as President of the American Mathematical Society.

In 1992 Arthur was elected a Fellow of the Royal Society. He was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 2003 and a foreign member of the National Academy of Sciences in 2014. In 2015 he was awarded the Wolf Prize in Mathematics.

Biographical Sketch: James Arthur

James Arthur is a university professor and holds the Ted Mossman Chair in Mathematics at the University of Toronto. He was born in Hamilton, Ontario, in 1944 and received a BSc from the University of Toronto in 1966, an MSc from the University of Toronto in 1967, and a PhD from Yale University in 1970. He then held positions in mathematics at Princeton University, Yale University, and Duke University before returning to the University of Toronto in 1979.

Arthur is a fellow of the Royal Society of Canada, a fellow of the Royal Society of London, a Foreign Honorary Member of the American Academy of Arts and Sciences and a Foreign Associate of the National Academy of Sciences. His various honors and awards include an honorary doctorate at the University of Ottawa in 2002, the Canada Gold Medal in Science and Engineering in 1999, and the Wolf Prize in Mathematics in 2015. He has given several addresses at International Congresses of Mathematicians, including a Plenary Lecture at the congress in Seoul, Korea, in 2014, and he gave a Plenary Lecture at the first Mathematical Congress of the Americas in Guanajuato, Mexico, in 2013. He is presently working on Beyond Endoscopy, a proposal by Robert Langlands for using the trace formula to study the general principle of functoriality.

Arthur has served mathematics in several senior administrative roles. He was a member of the Executive Committee of the International Mathematics Union from 1991 to 1998 and the Academic Trustee for Mathematics on the Board of Trustees of the Institute for Advanced Study from 1997 to 2007. He also served as president of the AMS from 2005 to 2007. He lives in Toronto with his wife, Penny. They have two sons: James, a poet in the creative writing program at Johns Hopkins University; and David, a computer engineer at Google, in Mountain View, California.

Response from James Arthur

I am thrilled and honored to receive the Steele Prize for Lifetime Achievement. It is a cliché, but true nonetheless, for me to say that I feel humbled to look down the list of past winners. I would like to thank the Steele Prize Committee for selecting me. I would also like to thank the AMS and the many mathematical colleagues in particular who donate their time to serve on prize committees and to participate in the many other activities that do so much to help our subject thrive.

I was not a prodigy in mathematics as a child. As a matter of fact, I am quite happy that my record for the Putnam exams was not available to the Prize Committee. But I do remember being fascinated even as a child by what was said to be the magic and power of mathematics. These feelings have remained with me throughout my professional life, and they have motivated me more than any specific theorem or result.

I am very grateful to Robert Langlands for his encouragement, both during my time as a graduate student and since then. I am also grateful to him personally and as a member of the larger community for what he has given to mathematics. His mathematical discoveries truly are magical and powerful. They are becoming more widely known among mathematicians today, and I have no doubt that they will bring pleasure and inspiration to many generations of mathematicians to come.

Much of my mathematical life has been connected in one way or another with what has become known as the Arthur–Selberg trace formula. It is now a very general identity that, like other things in mathematics, links geometric objects (such as closed geodesics) with spectral objects (such as eigenvalues of a Laplacian). The trace formula has many different terms, but as we are beginning to understand them now, each of these sometimes arcane quantities (either geometric or spectral) seems to have its own particular role in the larger scheme of things. I have been fortunate that the trace formula has assumed a more central role than might have been imagined earlier. I am excited to think that there is now a well-defined (if also rather imposing) strategy for using the trace formula to attack what is known as the principle of functoriality, the central tenet of the Langlands program.

About the Prizes

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. The Steele Prizes for Mathematical Exposition and Seminal Contribution to Research each carry a cash award of US$5,000; the Prize for Lifetime Achievement, a cash award of US$10,000.

FROM THE AMS SECRETARY

The list of previous recipients of the Steele Prizes may be found on the AMS website at `www.ams.org/profession/prizes-awards/prizes`.

The members of the Committee to Select the Winner of the Steele Prize for 2017 were:

- Paul F. Baum
- Tobias H. Colding
- Simon Donaldson
- Phillip Griffiths
- Carlos E. Kenig
- Nancy J. Kopell
- Vladimir Markovic (Chair)
- Yuval Peres
- Karen Uhlenbeck

The Prize for Seminal Contribution to Research is awarded on a six-year cycle of subject areas. The 2017 prize was given in geometry/topology, the 2018 prize will be given in discrete mathematics/logic, the 2019 prize is open, the 2020 prize in analysis/probability, and the 2021 prize in algebra/number theory.

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Photo of James Arthur is courtesy of James Arthur.

2017 Ruth Lyttle Satter Prize

Laura DeMarco

Laura DeMarco was awarded the 2017 Ruth Lyttle Satter Prize in Mathematics at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

The 2017 Ruth Lyttle Satter Prize in Mathematics is awarded to Laura DeMarco of Northwestern University for her fundamental contributions to complex dynamics, potential theory, and the emerging field of arithmetic dynamics.

In her early work, DeMarco introduced the bifurcation current to study the stable locus in moduli spaces of rational maps, and she constructed a dynamically natural compactification of the moduli spaces with tools from algebraic geometry, potential theory, and geometric topology. Both ideas were groundbreaking, opening new directions of research in complex dynamics. In recent joint work with M. Baker, she formulated a far-reaching conjecture about arithmetically special points in these moduli spaces, analogous to (and containing overlap with) the André–Oort and related conjectures in arithmetic geometry. They proved cases of the conjecture with methods involving a remarkable confluence of ideas from complex dynamics and disparate fields such as logic, number theory, and analysis on Berkovich spaces. With K. Pilgrim, she has constructed new invariants of polynomial maps in terms of metric trees and additional planar topological information. This led to two striking results, one on the algorithmic enumeration of cusps for certain curves in the space of cubic polynomials, addressing a problem first formulated and studied by J. Milnor, and the other a generalization of the well-known theorem that the Mandelbrot set is connected. Finally, in her most recent work, she has established direct connections between the theory of bifurcations in complex dynamics and the study of rational points on elliptic curves.

Biographical Sketch

Laura DeMarco is a professor at Northwestern University. She earned her PhD in 2002 from Harvard, where she studied with Curtis McMullen. Her undergraduate degree is in mathematics and physics from the University of Virginia, and she obtained an MA at the University of California Berkeley. DeMarco held an NSF Postdoctoral Fellowship and Dickson Instructorship at the University of Chicago. She became an assistant professor at the University of Chicago before moving to (and subsequently being tenured and promoted to professor at) the University of Illinois at Chicago. While there, DeMarco received the NSF CAREER Award and a Sloan Fellowship. She also became a Fellow of the American Mathematical Society. During the academic year 2013–2014, DeMarco was the Kreeger–Wolf Distinguished Visiting Professor in the mathematics department at Northwestern University. She moved to Northwestern in 2014. Laura DeMarco was awarded a Simons Fellowship in 2015.

About the Prize

The Satter Prize is awarded every two years to recognize an outstanding contribution to mathematics research by a woman in the previous six years. Established in 1990 with funds donated by Joan S. Birman, the prize honors the memory of Birman's sister, Ruth Lyttle Satter. Satter earned a bachelor's degree in mathematics and then joined the research staff at AT&T Bell Laboratories during World War II. After raising a family, she received a PhD in botany at the age of forty-three from the University of Connecticut at Storrs, where she later became a faculty member. Her research on the biological clocks in plants earned her recognition in the United States and abroad. Birman requested that the prize be established to honor her sister's commitment to research and to encourage women in science. The prize carries a cash award of US$5,000.

The Satter Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize, the following individuals served as members of the selection committee:

- Estelle L. Basor
- Georgia Benkart
- Benson Farb (Chair)

The complete list of recipients of the Satter Prize follows:

1991 Dusa McDuff
1993 Lai-Sang Young
1995 Sun-Yung Alice Chang
1997 Ingrid Daubechies
2001 Bernadette Perrin-Riou, Karen E. Smith, Sijue Wu
2003 Abigail Thompson
2005 Svetlana Jitomirskaya
2007 Claire Voisin
2009 Laure Saint-Raymond
2011 Amie Wilkinson
2013 Maryam Mirzakhani
2015 Hee Oh
2017 Laura DeMarco

—AMS Satter Prize Committee

Photo Credit

Photo of Laura DeMarco courtesy of Marc Harris.

2017 Bôcher Memorial Prize

András Vasy

András Vasy was awarded the 2017 Bôcher Memorial Prize at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

The 2017 Bôcher Memorial Prize is awarded to András Vasy for his fundamental paper "Microlocal analysis of asymptotically hyperbolic and Kerr-de Sitter spaces," *Inventiones Mathematicae* **194** (2013), 381–513. This paper resolved a thirty-five-year-old conundrum in geometric scattering theory regarding an effective meromorphic continuation of Green functions in these settings. In so doing, it developed a systematic microlocal framework for the Fredholm analysis of nonelliptic problems. This paper was seminal for numerous subsequent works, including two by Vasy in collaboration with P. Hintz: "Semilinear wave equations on asymptotically de Sitter, Kerr-de Sitter, and Minkowski spacetimes," *Analysis & PDE* **8** (2015), 1807–1890, and the recently posted paper, "The global nonlinear stability of the Kerr-de Sitter family of black holes." The committee also recognizes Vasy's outstanding contributions to multibody scattering and to propagation of singularities for solutions to wave equations on regions with singular boundaries.

Biographical Sketch

András Vasy was born and grew up in Budapest, Hungary. He attended the Apáczai Csere János Gimnázium (high school) in Budapest and the United World College of the Atlantic in Llantwit Major, Wales, before undergraduate studies at Stanford University in mathematics and physics. He received his PhD in mathematics from the Massachusetts Institute of Technology under the supervision of Richard Melrose in 1997. Subsequently, he held positions at the University of California Berkeley, the Massachusetts Institute of Technology, and Northwestern University before joining Stanford University in 2005, where he is currently professor of mathematics. He received a Sloan Research Fellowship and a Clay Research Fellowship and was a speaker in the partial differential equations section of the 2014 ICM in Seoul.

Response from András Vasy

It is a great honor to receive the 2017 Bôcher Memorial Prize. I am very grateful that the prize recognizes the development of microlocal analysis, along with the role I played in it. Microlocal analysis is a powerful unified approach dealing with many problems in analysis, from partial differential equations to integral geometry and inverse problems. It is this unified aspect that appeals to me particularly, and I very much hope that future generations of mathematicians will derive as much joy from working on and with it as I do.

The work leading to this prize could not have happened without the support of many people. My parents, Margit and Géza; my siblings, Benedek and Júlia; as well as my wife, Sara, and my daughter, Marguerite, supported me in this endeavor in a multitude of ways, including forgiving me for spending so much time thinking about mathematics and for creating such a happy environment for my life.

I am also grateful to my teachers throughout the years who led me to the delights of mathematics and physics: my Budapest Apáczai Gimnázium (high school) math and physics teachers, Péter Pósfai and Ferenc Zsigri, and my Stanford undergraduate and MIT graduate instructors, especially Steven Chu, Leon Simon, and Victor Guillemin; it is thanks to Leon's inspiring lectures that I ended up doing mathematics. I also learned a lot from my collaborators; I am very grateful for all the discussions that undoubtedly played a role in how I approach the area. But most of all I am extremely grateful to my PhD advisor, Richard Melrose. My view of the subject was fundamentally shaped by what I learned from him as a student, a collaborator, and a colleague; I believe that the insights I acquired through interactions with him form the key part of the work that is now being recognized by the AMS.

About the Prize

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–1910. Bôcher was also one of the

FROM THE AMS SECRETARY

founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the AMS or the paper should have been published in a recognized North American journal. The prize is given every three years and carries a cash award of US$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the 2017 prize selection committee are the following individuals:

- Fang Hua Lin (Chair)
- Michael E. Taylor
- Ruth J. Williams

The complete list of recipients of the Bôcher Prize follows:

1923 G. D. Birkhoff
1924 E. T. Bell, Solomon Lefschetz
1928 J. W. Alexander
1933 Marston Morse, Norbert Wiener
1938 John von Neumann
1943 Jesse Douglas
1948 A. C. Schaeffer, D. C. Spencer
1953 Norman Levinson
1959 Louis Nirenberg
1964 Paul J. Cohen
1969 I. M. Singer
1974 Donald S. Ornstein
1979 Alberto P. Calderón
1984 Luis A. Caffarelli, Richard B. Melrose
1989 Richard M. Schoen
1994 Leon Simon
1999 Demetrios Christodoulou, Sergiu Klainerman, Thomas Wolff
2002 Daniel Tataru, Terence Tao, Fanghua Lin
2005 Frank Merle
2008 Charles Fefferman, Carlos Kenig, Alberto Bressan
2011 Assaf Naor, Gunther Uhlmann
2014 Simon Brendle
2017 András Vasy

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2017 Frank Nelson Cole Prize in Number Theory


Henri Darmon

Henri Darmon was awarded the 2017 Cole Prize in Number Theory at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

Henri Darmon of McGill University is awarded the Cole Prize in Number Theory for his contributions to the arithmetic of elliptic curves and modular forms. The prize recognizes, in particular, the papers "Generalized Heegner cycles and p-adic Rankin L-series" (with Massimo Bertolini and Kartik Prasanna and with an appendix by Brian Conrad), published in 2013 in the *Duke Mathematical Journal*, and "Diagonal cycles and Euler systems, II: The Birch and Swinnerton-Dyer Conjecture for Hasse-Weil-Artin L-functions" (with Victor Rotger), published in 2016 in the *Journal of the American Mathematical Society*. These works, which are themselves only high points of a long sequence of influential papers, prove p-adic analogues of the Gross-Zagier formula, thus relating the value of a p-adic L-function to a cohomology class constructed from the geometry of modular curves. In certain situations, these cohomology classes can be used to control the Mordell-Weil group of an elliptic curve, thus establishing new cases of the conjecture of Birch and Swinnerton-Dyer.

Biographical Sketch

Henri Darmon was born in Paris, France, in 1965, before moving to Canada in 1968, first to Quebec City and then to Montreal at the age of eleven. He earned a bachelor of science degree with joint honours in mathematics and computer science at McGill University in 1987 and a PhD in mathematics from Harvard University in 1991, under the supervision of Benedict Gross. After a postdoctoral instructorship at Princeton University under the mentorship of Andrew Wiles, he returned to his undergraduate alma mater in 1994, where he is currently a James McGill Professor in the Department of Mathematics and Statistics. He has delivered an invited lecture in the number theory section of the 2006 ICM in Madrid, the Earle Raymond Hedrick lectures of the MAA in 2003, and two plenary AMS lectures at the annual joint meetings in Orlando (1996) and San Antonio (2015).

Response from Henri Darmon

I am tremendously honored to receive the Frank Nelson Cole Prize of the American Mathematical Society, as well as humbled by the thought of the many close friends, towering influences, and fortuitous events that were instrumental for the articles mentioned in the prize citation (referred to henceforth as [BDP] and [DR]).

Above all, I thank my collaborators, Massimo Bertolini, Kartik Prasanna, and Victor Rotger, who deserve this recognition as much as I do. Massimo and I have known each other since our graduate student days almost thirty years ago, and since then have shared many mathematical dreams and written twenty-five papers together. Without Massimo's friendship my career would have been very different: less successful for sure and also far less enjoyable. We started working with Kartik in 2006, at a time when our earlier ideas had largely played themselves out and we were eager for fresh perspectives. We learned a great deal from Kartik, who made us venture outside our comfort zone and expand our horizons. Our collaboration lasted roughly four years and culminated in the p-adic Gross–Zagier formula described in [BDP]. Over a memorable summer in Barcelona in 2010, Victor encouraged me to extend this formula to the setting, originally explored by Gross, Kudla, and Schoen in the early 1990s, of diagonal cycles in the triple product of modular curves. I was a bit reluctant at first to embark on this project, fearing it would interfere with my two main preoccupations at the time: questions surrounding elliptic curves and the Birch and Swinnerton-Dyer conjecture, and the search for a counterpart of the theory of complex multiplication for real quadratic fields. It is fortunate that Victor prevailed, because a few months later we discovered close connections with both topics, thanks to the extra "miracle ingredient" of p-adic variation of modular forms and associated cohomology classes. One of the contributions of [DR] is a proof of the weak Birch and Swinnerton-Dyer conjecture in analytic rank zero for elliptic curves over Q twisted by ring class characters of real quadratic fields.

Although this is hardly a mainstream result, proving it had become something of a personal obsession since the late 1990s, when I realised it would follow from a conjectural extension of Heegner points to the setting of real quadratic fields that I proposed back then and on which I was—and continue to be—quite stuck.

I can hardly do justice in a short paragraph to all the mathematical giants on whose shoulders Massimo, Kartik, Victor, and I have stood, but let me at least try. Much of my work with Massimo over the years has been guided by Barry Mazur's grand vision of the Iwasawa theory of elliptic curves. Some p-adic variants of the influential results of Benedict Gross, Don Zagier, and Jean-Loup Waldspurger, in the spirit of Leopoldt's p-adic analogue of Dirichlet's analytic class number formula, are explored in [BDP]. The seminal ideas of John Coates and Andrew Wiles originally used to study the arithmetic of elliptic curves with complex multiplication, along with the spectacular refinements and variations that arose in the work of Victor Kolyvagin, Francisco Thaine, Karl Rubin, and Kazuya Kato, are a cornerstone of [DR]. Both [BDP] and [DR] exploit the notion of p-adic families of modular forms pioneered by Haruzo Hida and his school and rest on an approach towards p-adic L-functions that grew out of the work of Coates–Wiles, as systematized and vastly extended by Robert Coleman, Kazuyo Kato, and Bernadette Perrin-Riou. Out of such excellent ingredients, even a mediocre cook can make a good stew!

Lady Luck has played an inordinate role in my career and deserves a paragraph of her own. I was fortunate to be the PhD student of Dick Gross from 1987 to 1991, in the heady days when the Gross–Zagier formula was still fresh but starting to assert its profound and lasting influence on number theory through its role in such breakthroughs as the 1989 work of Kolyvagin on the Birch and Swinnerton-Dyer conjecture. As a postdoc in Princeton from 1991 to 1994, I had the privilege of witnessing first hand Andrew Wiles's momentous announcement of his proof of Fermat's Last Theorem and the Shimura–Taniyama conjecture. If there is one merit I can claim with some confidence, it is the knack for being at the right place at the right time, which served me well in my formative years. I also thank my family, most of all my parents, my wife, and my daughter, for their love and support, and my colleagues at McGill University, Concordia University, and the Centre de Recherches Mathématiques in Montréal for providing the most pleasant, stimulating, and supportive environment, bar none, that a research mathematician could ask for.

About the Prize

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years. The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years.

These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole and has received contributions from Society members and from Cole's son, Charles A. Cole. The Cole Prize carries a cash award of US$5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize the members of the selection committee were the following individuals:

- Guy Henniart
- Michael J. Larsen (Chair)
- Akshay Venkatesh

The complete list of recipients of the Cole Prize in Number Theory follows:

1931 H. S. Vandiver
1941 Claude Chevalley
1946 H. B. Mann
1951 Paul Erdős
1956 John T. Tate
1962 Kenkichi Iwasawa, Bernard M. Dwork
1967 James B. Ax, Simon B. Kochen
1972 Wolfgang M. Schmidt
1977 Goro Shimura
1982 Robert P. Langlands, Barry Mazur
1987 Dorian M. Goldfeld, Benedict H. Gross, Don B. Zagier
1992 Karl Rubin, Paul Vojta
1997 Andrew J. Wiles
2002 Henryk Iwaniec, Richard Taylor
2005 Peter Sarnak
2008 Manjul Bhargava
2011 Chandrashekhar Khare and Jean-Pierre Wintenberger
2014 Yitang Zhang, Daniel Goldston, János Pintz, Cem Y. Yildirim
2017 Henri Darmon

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Photo of Henri Darmon is courtesy of Henri Darmon.

2017 Leonard Eisenbud Prize for Mathematics and Physics


László Erdős


Horng-Tzer Yau

LÁSZLÓ ERDŐS and HORNG-TZER YAU were awarded the 2017 Leonard Eisenbud Prize for Mathematics and Physics at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

The 2017 Leonard Eisenbud Prize is awarded to László Erdős and Horng-Tzer Yau for proving the universality of eigenvalue statistics of Wigner random matrices. In the 1950s, Eugene Wigner, motivated by the study of the complex spectra of highly excited nuclei, initiated an investigation of random matrices of a seemingly simple form: $N \times N$ symmetric, Hermitian, or quaternion self-dual random matrices with independent, identically distributed entries. Lacking a truly microscopic theory, Wigner proposed randomly selecting a quantum Hamiltonian. Since then, modeling through random matrices has been surprisingly useful: in determining the distribution of the zeros of the Riemann ζ-function; in characterizing the spectra of quantum Hamiltonians whose classical limit generates a chaotic dynamics; in developing lattice gauge theories; and in carrying out statistical analysis on large data sets. By browsing the *Oxford Handbook of Random Matrix Theory*, one gains an appreciation of how much the subject has flourished.

A quantity of prime interest is the gap probability, i.e., the distribution of the distance between neighboring eigenvalues. For matrices whose entries have a Gaussian distribution, F. Dyson computed the gap probability in the limit of large N. In fact, he determined the entire local spectrum of eigenvalues and found a structure now known as Pfaffian, or determinantal, point processes with a specific defining kernel. At the time, pioneering numerical simulations suggested that the eigenvalue statistics are universal, in the sense that for large N there is no longer a dependence on the particular distribution of the matrix entries. In applications, this kind of robustness is a crucial assumption. The true Hamiltonian will not resemble a particular Wigner matrix. But for the purpose of predicting universal features, such a model may suffice.

A proof of the universality conjecture remained as an unsolved, challenging problem for many years. K. Johansson established the desired result for complex Hermitian matrices under the assumption that the distribution of the entries has a Gaussian component. László Erdős and Horng-Tzer Yau, jointly with collaborators, introduced and perfected the technique of using Dyson's Brownian motion as an interpolating scheme. The dynamics start with the eigenvalues of A and reach the eigenvalues of a Gaussian random matrix at time $t \to \infty$.

To summarize their amazing result: Let $\{A_{ij}\}_{1\leq i,\, j\leq N}$ be a collection of either real or complex random variables, independent up to symmetry, such that $\mathbb{E}(A_{ij}) = 0$, $\mathbb{E}(|A_{ij}|^2) = N^{-1}$, and $\mathbb{E}(|\sqrt{N}A_{ij}|^{4+\epsilon}) < C$ for some constants C and $\epsilon > 0$. Then the typical distance between eigenvalues is of the order N–1. We focus at a point $E \in \mathbb{R}$ at which the average density of eigenvalues is strictly positive and consider the eigenvalues lying in the interval $[E-\ell N^{-1}, E+\ell N^{-1}]$ with arbitrary $\ell > 0$. Shift these eigenvalues by E and rescale by N. Then we arrive at the collection of eigenvalues $\{\lambda_j^{(N)}\}_{j=1,\dots,n}$, with random n, located in the interval $[-\ell, \ell]$. The assertion of the theorem is that in the limit $N \to \infty$, the point process $\{\lambda_j^{(N)}\}_{j=1,\dots,n}$ converges to the limiting point process for the eigenvalues of the Gaussian Wigner random matrix $\{\lambda_j^{(G)}\}_{j=1,\dots,n}$.

A complete formulation can be found in Erdős and Yau's review "Universality of local spectral statistics of random matrices," published in the *Bulletin of the AMS* in 2012, where the notion used for the convergence of point processes is specified.

The initial breakthrough was published in 2010 in "Bulk universality for Wigner matrices," *Communications on Pure and Applied Mathematics* (with S. Péché, J. Ramírez, and B. Schlein), with important subsequent improvements in "Universality of random matrices and local relaxation flow," *Inventiones Mathematicae* (2011, with B. Schlein), "Bulk universality for generalized Wigner matrices," *Probability Theory and Related Fields* (2012, with J. Yin), "Gap universality of generalized Wigner and β-ensembles," *Journal of European Mathematical Society* (2015), and "Fixed energy universality for generalized Wigner matrices," *Communications on Pure and Applied Mathematics* (2016, with P. Bourgade and J. Yin).

Biographical Sketch: László Erdős

László Erdős was born in Budapest in 1966 and completed his university education in mathematics at the Lorand Eötvös University in 1990. He received his PhD at Princeton University in 1994 under the supervision of Elliott H. Lieb. After postdoc positions in Zürich and New York he first became a faculty member at Georgia Tech in Atlanta, then obtained a chair professorship at the Ludwig-Maximilian University in Munich, Germany. Since 2013 he has been professor at the Institute of Science and Technology Austria, near Vienna. He was an invited speaker at ICM 2014. He is a corresponding member of the Austrian Academy of Sciences, an external member of the Hungarian Academy of Sciences, and member of the Academia Europaea. Erdős's research focuses on mathematical physics, in particular many-body quantum mechanics, disordered quantum systems, and random matrices.

Response from László Erdős

It is a great pleasure and honor to be selected as a co-recipient of the 2017 Leonard Eisenbud Prize. I am grateful to the committee for this recognition of our work.

I am very fortunate to have learned the importance of combining physical motivations with sharp analysis from the very beginning of my career, starting in the Budapest dynamical system school led by Doma Szász and continuing at Princeton under the guidance of Elliott Lieb, whose infallible scientific taste and mathematical mastery have shaped my research ever since. Finally, I owe most of my research aptitude to my former postdoctoral advisor and long-term collaborator, Horng-Tzer Yau, with whom sharing this prize is a great distinction.

A very special acknowledgment goes to our younger collaborators with whom we shared many parts of this long journey toward the solution of the Wigner-Dyson-Mehta conjecture. The results would not have been possible without the multitude of ideas and indefatigable engagement by Paul Bourgade, Antti Knowles, Benjamin Schlein, and Jun Yin, together with shorter but essential collaborations with Jose Ramírez and Sandrine Péché. I thank all of them.

Biographical Sketch: Horng-Tzer Yau

Horng-Tzer Yau received his BSc from National Taiwan University in 1981 and his PhD degree from Princeton University in 1987, under the supervision of Elliott Lieb. He has held faculty positions at the Courant Institute and Stanford University, and since 2005 he has been a professor of mathematics at Harvard University. Yau received the Henri Poincaré Prize in 2000 and was the Distinguished Visiting Professor at the Institute for Advanced Study from 2013 to 2014. He has received fellowships from the Sloan Foundation, Packard Foundation, and MacArthur Foundation and has been a member of the American Academy of Arts and Sciences since 2001 and the National Academy of Sciences since 2013. Currently, Yau is a Simons Investigator and the Editor-in-Chief of *Communications in Mathematical Physics*. His work focuses on quantum many-body systems, interacting particle systems, and random matrix theory.

Response from Horng-Tzer Yau

It is a great pleasure and honor to receive this prize. As a student, I saw E. Wigner many times in the colloquium at Princeton. During those years, it never occurred to me that one day I would work on a problem in his area of interest. My coworker, László Erdős, and I came to the universality problem accidentally after many years of working on random Schrödinger equations, which were believed to exhibit random matrix statistics. At the time, study of the universality of random matrices was under the reign of integrable methods. It was fortunate for us that the Green's function method and probabilistic tools were mature enough by then to be applied to this problem. These tools allowed us to understand the universality problem through analytic methods and to make the connection with Dyson's work.

I would like to take this opportunity to thank my thesis adviser, E. Lieb, who taught me to believe in the simplicity of mathematics and physics. I also would like to thank Raghu Varadhan, from whom I learned probability theory during my postdoctoral time at the Courant Institute. In addition to Erdős, I am also indebted to my other coworkers. Among them, Paul Bourgade, Benjamin Schlein, and Jun Yin collaborated with Erdős and me on several papers and generated many key ideas in this project. I also would like to thank the committee for selecting this work for the Leonard Eisenbud Prize. Finally, I would like to thank my wife, Chuan-Chuan, for her patience and care through my career.

About the Prize

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English." He was one of the founders of the Physics Department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the AMS during 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US$5,000 prize will be awarded every three years for a work published in the preceding six years.

FROM THE AMS SECRETARY

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize, the members of the selection committee were

- Ezra Getzler
- Alice Guionnet
- Herbert Spohn (Chair)

The complete list of recipients of the Leonard Eisenbud Prize for Mathematics and Physics follows:

- Hirosi Ooguri, Andrew Strominger, Cumrun Vafa (2008)
- Herbert Spohn (2011)
- Gregory W. Moore (2014)
- László Erdős, Horng-Tzer Yau (2017)

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Photo of Horng-Tzer Yau is courtesy of Horng-Tzer Yau.

2017 Levi L. Conant Prize


David H. Bailey


Jonathan Borwein


Andrew Mattingly


Glenn Wightwick

David H. Bailey, Jonathan Borwein, Andrew Mattingly, and Glenn Wightwick were awarded the 2017 Levi L. Conant Prize at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017 for their article "The computation of previously inaccessible digits of π^2 and Catalan's constant," published in the *Notices of the American Mathematical Society* in 2013.

Citation

This fascinating article will delight any mathematician who has ever been intrigued by the mysteries of the digits of π. The reader is first taken on a historical journey from Archimedes to the computer age with many interesting anecdotes along the way. For example, "Isaac Newton devised an arcsine-like scheme to compute digits of π and… sheepishly acknowledged 'I am ashamed to tell you to how many figures I carried these computations, having no other business at the time.' Newton wrote these words during the plague year 1666, when, ensconced in a country estate, he devised the fundamentals of calculus and the laws of motion and gravitation."

The remarkable "BBP" formula, discovered by the first author along with Peter Borwein and Simon Plouffe, allows one to calculate binary or hexadecimal digits of π beginning with the nth digit without first calculating any of the preceding $n-1$ digits. We are led through an elementary proof of the BBP formula but also learn about the nonconventional search that originally led to this formula, along with similar formulas for Catalan's constant and π^2.

Most intriguing are the insights into the age-old question of whether the digits of π are truly randomly distributed. A real number α is said to be b-normal, where b is a positive integer, if every string of base b digits appears in the base b expansion of α with the expected limiting frequency. The first two authors and Richard Crandall observed that the normality of real numbers such as π that admit BBP formulas can be reduced to proving the equidistribution in (0,1) of a related recursively defined sequence. In particular, we are shown an explicit sequence x_n in (0,1) such that $16x_n$ appears to produce exactly the hexadecimal expansion of π, with an explicit minuscule bound on any possible errors. If this sequence can be proven to be randomly distributed, it will follow that π is 16-normal.

Computations of digits of π have practical applications: running paired computations of π provides a strenuous integrity test of computer hardware and software. Well beyond such applications, however, few mathematical objects have piqued the public interest as powerfully as π. Next π Day, we can amaze our friends by reciting the sequence of ten digits of π starting from position 17,387,594,880, namely 0123456789!

We are saddened that the second author, a frequent contributor to the *Notices*, did not live to receive this prize. Borwein's creative work and beautiful expositions will be sorely missed.

Biographical Sketch: David H. Bailey

David H. Bailey received his PhD in mathematics from Stanford University in 1976 and in his subsequent career worked at the NASA Ames Research Center and then at the Lawrence Berkeley National Laboratory. He recently retired from the Berkeley Lab but continues as a research associate with the University of California Davis, Department of

Computer Science. His published work includes over two hundred papers in experimental mathematics, computational number theory, parallel computing, high-precision computing, fast Fourier transforms, and mathematical finance. Perhaps his best-known paper, coauthored with Peter Borwein and Simon Plouffe, describes a new formula for π, discovered by a computer program, that permits one to directly calculate binary digits of π, beginning at an arbitrary starting position, without needing to calculate any of the preceding digits. Bailey operates several blogs and writes articles on mathematics, computing, and science for the *Huffington Post* and the *Conversation*. He has previously received the Chauvenet and Merten Hesse Prizes from the Mathematical Association of America, the Sidney Fernbach Award from the IEEE Computer Society, and the Gordon Bell Prize from the Association of Computing Machinery.

Response from David H. Bailey

I am truly honored to be a co-recipient of this year's Levi L. Conant Prize. It is remarkable how the number π, after more than two millennia, continues to amaze, delight, and inspire the general public and professional mathematicians alike. We have learned so much, and yet there is so much more that we still do not know, such as the age-old question of whether and why the digits of π are normal—whether given a positive integer b, every m-long string of base b digits appears in the base b expansion of π with the limiting frequency $1/b^m$. We do not know the answer to this question even for $b = 2$ and $m = 1$, let alone for all m or all b. Computationally exploring questions such as this is a delight and opens an avenue for mathematicians to work hand-in-hand with computer scientists, such as our coauthors Andrew Mattingly and Glenn Wightwick, to make significant contributions. With new theoretical results, combined with ever-more-powerful computer tools, we can look forward to uncovering additional interesting facts about π in the years to come.

This article was the brainchild of our coauthor Jonathan M. Borwein, who sadly passed away on August 2, 2016, in what can only be described as a monumental loss to the world mathematical community. My own career was deeply intertwined with Jon's, dating back to 1985 when I read a paper by Jon and his brother Peter on their new nth-order convergent algorithms for π. Since then Jon and I have collaborated on five books and more than eighty papers, encompassing a large fraction of my career, and so I owe him a deep debt of gratitude for his inspiration and support. Jon's fascination with π, as well as his delight in bringing the excitement of new findings on π to the general public, was matched only by his indefatigable energy in pursuing a wide range of mathematical research, ranging from optimization and experimental mathematics to biomedical imaging and mathematical finance, using state-of-the-art computer tools to discover and understand new results. For decades to come we will be mining his enormous published corpus (over five hundred papers and twenty-eight books) for insights and inspiration.

Biographical Sketch: Andrew Mattingly

Andrew Mattingly holds a bachelor of science degree with honours in applied mathematics and meteorology from Monash University (Melbourne, Australia) and a master of science in astronomy from Swinburne University (Melbourne, Australia). He is employed as a software architect with IBM Australia. While he specializes in IBM's mainframe systems, he has experience with distributed and supercomputing environments, in particular, IBM's Blue Gene supercomputer. Andrew also operates a remote optical observatory in outback Australia for the benefit of astronomy students at Wheaton College in Massachusetts.

Response from Andrew Mattingly

I am very honoured to receive the Levi L. Conant Prize in the company of my esteemed coauthors. I am grateful to Glenn Wightwick for inviting me to participate in the Pi Day project that led to our winning paper. This collaboration with Glenn, David H. Bailey, and the late Jon Borwein led to many subsequent collaborations in experimental mathematics, awakening my enthusiasm for mathematics that, apart from brief encounters in the course of my astronomical pursuits, had lain dormant for decades while I pursued a career in computer software. I very much appreciate the guidance and patience offered by Jon and David during the preparation of this paper, as we wrangled the IBM Blue Gene into producing the desired numerical results.

Biographical Sketch: Glenn Wightwick

Glenn Wightwick is the deputy vice chancellor and vice president (research) at the University of Technology Sydney (UTS), where he is responsible for research activity and research policy development, postgraduate education, industry liaison, intellectual property, and commercialization. Prior to joining UTS, he worked for IBM for over twenty-seven years in a number of roles related to high-performance and scientific computing. He led the establishment of IBM research and development laboratories in Australia, as director of IBM Research–Australia and director of IBM Australia Development Laboratory and also held the position of chief technologist for IBM Australia. He was appointed an IBM distinguished engineer in 2003 and elected to the IBM Academy of Technology in 2000.

Glenn Wightwick is recognized as a leader in developing Australia's information technology industrial research and development base and as a significant contributor to innovation across the nation. He has a distinguished industrial research and development track record. A fellow of the Australian Academy of Technological Sciences and Engineering, Wightwick has also served on the Australian Research Council (ARC) College of Experts and the Board of National ICT Australia and has led national bodies and committees such as the NSW Digital Economy Industry Taskforce. He has a bachelor of science from Monash University.

FROM THE AMS SECRETARY

Response from Glenn Wightwick

I am absolutely delighted and deeply honoured to receive the 2017 Levi L. Conant Prize along with my collaborators David H. Bailey, Jonathan Borwein, and Andrew Mattingly for our paper in the *Notices of the AMS*. The computations associated with this research work were undertaken on an IBM Blue Gene supercomputer and were partly motivated by a public event at the University of Technology Sydney (where I now work) to celebrate international Pi Day in 2011. Even though I am not a practicing mathematician, the opportunity to contribute to a large computation involving π connects me back to some of my first interactions with computers at school in 1976. I was fortunate then to have access to DEC PDP-11/750 and an Apple and used them to compute π using various algorithms, including a Monte Carlo method which revealed (rather painfully!) fundamental limitations in the underlying pseudorandom number generator. This began a lifelong love of computation, and I have been very fortunate to work on numerical weather models, seismic processing algorithms, computational chemistry problems, and bioinformatics. I would very much like to acknowledge my coauthors on this paper and the many colleagues over the years whom I have interacted with. In particular, I would like to acknowledge Lance Leslie, who taught me everything I know about numeric weather prediction. Finally, I was deeply saddened by the passing of Jonathan Borwein in August 2016. He was one of the world's experts in π, and he will be sadly missed by many inside and outside the mathematical community.

About the Prize

The Levi L. Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic Institute. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize, the selection committee consisted of the following individuals:

- John C. Baez
- Carolyn Gordon (Chair)
- Serge L. Tabachnikov

The complete list of recipients of the Conant Prize follows.

2001 Carl Pomerance
2002 Elliott Lieb, Jakob Yngvason
2003 Nicholas Katz, Peter Sarnak
2004 Noam D. Elkies
2005 Allen Knutson, Terence Tao
2006 Ronald M. Solomon
2007 Jeffrey Weeks
2008 J. Brian Conrey, Shlomo Hoory, Nathan Linial, Avi Wigderson
2009 John W. Morgan
2010 Bryna Kra
2011 David Vogan
2012 Persi Diaconis
2013 John Baez, John Huerta
2014 Alex Kontorovich
2015 Jeffrey C. Lagarias, Chuanming Zong
2016 Daniel Rothman
2017 David H. Bailey, Jonathan Borwein, Andrew Mattingly, Glenn Wightwick

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2017 Joseph L. Doob Prize

John Friedlander and Henryk Iwaniec

John Friedlander and Henryk Iwaniec were awarded the Joseph L. Doob Prize at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017 for their book *Opera de Cribro*, published in 2010 as volume 57 of the American Mathematical Society Colloquium Publications.

Citation

This monograph by two top masters of the subject is dedicated to the study of sieves in number theory and to their applications. Its Latin title could be translated literally as "A Laborious Work Around the Sieve," but the Latin has a conciseness easily missed in any translation.

The Eratosthenes sieve, going back to the 3rd century BCE, was a simple but efficient method to produce a table of prime numbers, and for a long time it was the only way to study the mysterious sequence of primes, at least experimentally. It was only in 1919 that the Norwegian mathematician Viggo Brun obtained the first quantitative results of the correct order of magnitude for the density of sifted sequences by combining the sieve with ideas from combinatorics. From another direction, the introduction of complex variable methods by Hardy, Ramanujan, and Littlewood and of techniques of harmonic analysis by Vinogradov helped to obtain the correct conjectures about the distribution of prime numbers of special type and of their fine distribution, such as the study of the sequence of gaps between prime numbers.

For a long time Brun's method and its refinements by Buchstab and many others were the only tools at the mathematician's disposal for obtaining unconditional results on the arithmetical structure of sequences of integers until, in 1950, Selberg put forward a new, simple, elegant method to study such questions. Selberg's method and Brun's combinatorial method were independent of each other and gave rise to new deep results on the arithmetic structure of special sequences. In the 1950s and early 1960s the new ideas of Linnik and Rényi gave origin to the so-called Large Sieve, particularly apt to the study of the distribution of sequences of integers in arithmetic progressions.

In the next thirty years many very deep results on classical questions previously considered to be inaccessible were obtained. Suffice it here to mention the asymptotic formula for the number of primes representable as the sum of a square and of a fourth power, obtained by Friedlander and Iwaniec in 1998, and a similar result by Heath-Brown in 2001 for the number of primes which are the sum of a cube and of twice a cube. So it was time for a new book dealing not only with the sieves per se but, in fact, with the very deep new techniques needed for the applications. The first nine chapters of this monograph deal with the sieves, followed by three chapters dedicated to the optimization of parameters. The next ten chapters are dedicated to specific problems, including several milestone results. The last three chapters, which are a most original contribution to this monograph, deal with the future by raising new questions, giving partial answers, and indicating new ways of approaching the problems.

Two long appendices deal with technical results of general application. The bibliography, with 161 entries, is a major complement to this work. Everything is well written, the motivations of the arguments are well explained, and the numerous examples help the student to understand the subject in depth. These features distinguish this unique monograph from anything that had been written before on the subject and lift it to the level of a true masterpiece.

The selection committee thanks Professor E. Bombieri for writing the citation.

Biographical Sketch: John Friedlander

John Friedlander was born in Toronto, less than a mile from his current office. Following a BSc in Toronto and an MA in Waterloo, he received his PhD at Penn State in 1972 working under the supervision of S. Chowla. His first position was that of assistant to A. Selberg at the Institute for Advanced Study. After further positions at IAS, MIT, Scuola Normale Superiore (Pisa), and the University of Illinois (Urbana), he returned to the University of Toronto in 1980, where he was mathematics department chair from 1987 to 1991 and, since 2002, has been University Professor of Mathematics. He was awarded the Jeffery-Williams Prize of the Canadian Mathematical Society (1999) and the CRM-Fields (currently CRM-Fields-PIMS) Prize, given by the Canadian Mathematical Institutes (2002). He has given an invited lecture at the ICM in Zurich in 1994, been a Research Professor at MSRI Berkeley in 2001–2002, and was a Killam Research Fellow during the period 2003–2005. He is a Fellow of the Royal Society of Canada, a Founding Fellow of the Fields Institute, and a Fellow of the American Mathematical Society. His best friend, Cherry, has been sharing her life with him and, amongst many other things, has been largely responsible for creating the space-time during which he has found a chance to think about mathematics.

Biographical Sketch: Henryk Iwaniec

Henryk Iwaniec was born on October 9, 1947, in Elblag, Poland. He graduated from Warsaw University in 1971, and he received his PhD in 1972. In 1976 he defended his habilitation thesis at the Institute of Mathematics of the Polish Academy of Sciences, where he held various positions from 1971 until 1983. In 1983 he was promoted to extraordinary professor (which is one step below the ordinary professor) and was elected to member correspondent of the Polish Academy of Sciences. Henryk Iwaniec spent the year 1976–1977 at the Scuola Normale Superiore di Pisa and the year 1979–1980 at the University of Bordeaux I. He left Poland in 1983 to take visiting positions in the United States: at the Institute for Advanced Study in Princeton (1983–1984), at the University of Michigan in Ann Arbor (summer 1984), as Ulam Distinguished Visiting Professor at Boulder University (fall 1984), and again at IAS in Princeton (January 1985–December 1986). Iwaniec was appointed as New Jersey State Professor of Mathematics at Rutgers University, where he has held this position from January 1987 until the present. Iwaniec was elected to the American Academy of Arts and Sciences in 1995, to the National Academy in 2006, and to the Polska Akademia Umiejetnosci in 2006 (foreign member). He received the Docteur Honoris Causa of Bordeaux University in 2006. Iwaniec twice received first prizes in the Marcinkiewicz contest for student works in the academic years 1968–1969 and 1969–1970. Among several other prizes he received are the Alfred Jurzykowski Award (New York, 1991); the Wacław Sierpiński Medal (Warsaw, 1996); the Ostrowski Prize (Basel, 2001, shared with Richard Taylor and Peter Sarnak); the Frank Nelson Cole Prize in Number Theory (AMS, 2002, shared with Richard Taylor); the Leroy P. Steele Prize for Mathematical Exposition (AMS, 2011); the Stefan Banach Medal (Polish Academy of Sciences, 2015); and the Shaw Prize in Mathematical Sciences (Hong Kong, 2015, shared with Gerd Faltings). Henryk Iwaniec was an invited speaker at the International Congresses of Mathematicians in Helsinki (1978), Berkeley (1986), and Madrid (2006).

Response from John Friedlander and Henryk Iwaniec

We are grateful to the American Mathematical Society and to the Joseph L. Doob Prize Selection Committee for having chosen our book *Opera de Cribro* for this award.

We are, in particular, gratified by the recognition that this prize brings to the (beloved by us) subject of our book. The study of sieve methods in number theory began its modern history with the works of Viggo Brun just about one hundred years ago. Brun's works were of an elementary (though not at all easy) combinatorial nature, yet led to theorems about prime numbers that still today have found no other source of proof. The first few following decades saw further development of the sieve mechanisms themselves given by many people, most notably Atle Selberg. Beginning in the 1970s, the subject entered into a new period during which it has become possible to incorporate into the sieve structure deep results coming from several of the main sources which power modern analytic number theory more generally. These include, most frequently, harmonic analysis, both classical and automorphic; algebraic tools of various types; and arithmetic geometry. But anything is fair game. Basically, the modern sieve takes from mathematics anything it can use, and the more surprising the source, the more intensely the beauty shines through.

We also greatly appreciate the timing of the Joseph L. Doob Prize. Although we spent five years working intensively on our *Opera*, it of course actually incorporates works of the authors dating back over a considerably longer period of time. This prize represents to us a milestone of our collaboration almost precisely forty years after it began in Pisa reading the preprint of *The Asymptotic Sieve*, written by Enrico Bombieri.

About the Prize

The Doob Prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob (1910–2004). Paul Halmos (1916–2006) was Doob's first PhD student. Doob received his PhD from Harvard in 1932 and three years later joined the faculty at the University of Illinois, where he remained until his retirement in 1978. He worked in probability theory and measure theory, served as AMS president in 1963–1964, and received the AMS Steele Prize in 1984 "for his fundamental work in establishing probability as a branch of mathematics and for his continuing profound influence on its development." The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize of US$5,000 is given every three years.

The Doob Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize, the members of the selection committee were the following:

- Lawrence Craig Evans (Chair)
- William Fulton
- Mark Goresky
- Fan Chung Graham
- Philip J. Holmes

The complete list of recipients of the Doob Prize follows.

2005 William P. Thurston
2008 Enrico Bombieri and Walter Gubler
2011 Peter Kronheimer and Tomasz Mrowka
2014 Cédric Villani
2017 John Friedlander and Henryk Iwaniec

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2017 Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student


David H. Yang

DAVID H. YANG was awarded the 2017 Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

The recipient of the 2017 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student is David H. Yang for his outstanding research in algebraic geometry. Yang is an author of five papers, with two more in preparation. Three of his papers have appeared or will appear in the *Memoirs of the American Mathematical Society, Journal für die reine und angewandte Mathematik,* and *Research in the Mathematical Sciences.* His letters of support describe his work as truly exceptional. His joint paper with senior collaborators Ein and Lazarsfeld builds on David's earlier single-author work. Starting in his freshman year at MIT, Yang's research was guided by Professors Joe Harris at Harvard and Roman Bezrukavnikov at MIT. Yang has also excelled in contest math. He was a Putnam Competition Fellow for the last three consecutive years. He also won two gold medals in the International Mathematics Olympiad.

Biographical Note

David Yang was born in California, where he spent most of his early childhood. He moved to New Hampshire to attend Phillips Exeter Academy, where he was first exposed to algebraic geometry. After graduating, David matriculated at the Massachusetts Institute of Technology, where he is currently a senior. It was at MIT that David started pursuing research in algebraic geometry. He plans to continue his research after graduating from MIT.

Response from David H. Yang

It is a great honor for me to receive the 2017 Frank and Brennie Morgan Prize. First, I would like to thank my research advisors, Roman Bezrukavnikov and Joe Harris, who have deeply influenced how I view mathematics. I have been fortunate to spend much of my undergraduate career in mathematically stimulating environments, including the vibrant departments of MIT and Harvard and the REU at Emory University, and I am very grateful for the atmosphere that they provided. I would also like to thank many mathematicians who took the time to impart their wisdom to me, including Clark Barwick, Lawrence Ein, Pavel Etingof, Dennis Gaitsgory, Rob Lazarsfeld, Ivan Losev, Davesh Maulik, Bjorn Poonen, and Jason Starr. Finally, I would like to thank my friends and family, whose support has always been vital to my work.

Citation for Honorable Mention: Aaron Landesman

AARON LANDESMAN is recognized with an Honorable Mention for the 2017 Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student. He has authored eight papers in algebraic geometry, number theory, and combinatorics. Three of his papers were accepted in *Annales de l'Institut Fourier, Research in Number Theory*, and *Order*. His work on interpolation of varieties is an important contribution to the area, proving fundamental theorems while introducing new specialization techniques which are expected to be of further use for algebraic geometers. He has already received the Mumford Prize, the Hoopes Prize, and the Friends Prize from Harvard for his undergraduate research. Landesman conducted further research at the Emory and Minnesota Twin Cities REUs. He is currently a PhD student at Stanford, where he enjoys an NSF Graduate Fellowship.

Biographical Note

Aaron Landesman, raised in New York City, is a graduate of Hunter College High School and Harvard University. Aaron is currently pursuing a PhD at Stanford University, focusing on algebraic and arithmetic geometry. His interest in math was piqued at a young age by his father, and together they wrote a book of three-dimensional mazes.

In high school Aaron's passion for mathematics grew through attending MathCamp, math team, and courses at Columbia. In college he concentrated in mathematics, with a secondary degree in computer science. During his summers Aaron conducted math research at the Minnesota Twin Cities REU and the Emory REU. He also enjoys playing chess, solving puzzle hunts, and public speaking.

Response from Aaron Landesman

It is an incredible honor to receive the Honorable Mention for the 2017 AMS-MAA-SIAM Morgan Prize. I would like to thank Mrs. Morgan and the AMS, MAA, and SIAM for awarding this prize and supporting undergraduate research. Many past Morgan Prize winners have had a profound impact on my development, so it is a privilege to be counted among them. I would like to thank my mentors, including Joe Harris and Anand Patel, who advised my senior thesis; David Zureick-Brown and Ken Ono, who advised my research at the Emory REU; and Vic Reiner and Gregg Musiker, who advised me at the Minnesota Twin Cities REU. I extend thanks to all my professors and teachers, particularly Dennis Gaitsgory, Curtis McMullen, Wilfried Schmid, Michael Thaddeus, and Eliza Kuberska. Finally, I would like to thank my friends and family for their support and inspiration.

About the Prize

The Morgan Prize is awarded annually for outstanding research in mathematics by an undergraduate student (or students having submitted joint work). Students in Canada, Mexico, or the United States or its possessions are eligible for consideration for the prize. Established in 1995, the prize was endowed by Mrs. Frank (Brennie) Morgan of Allentown, Pennsylvania, and carries the name of her late husband. The prize is given jointly by the AMS, the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) and carries a cash award of US$1,200.

Recipients of the Morgan Prize are chosen by a joint AMS-MAA-SIAM selection committee. For the 2017 prize, the members of the selection committee were:

- Jacob Fox (Chair)
- Anant P. Godbole
- Steven J. Leon
- Sarah Dianne Olson
- Ken Ono
- Melanie Matchett Wood

The complete list of recipients of the Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student follows.

1995 Kannan Soundararajan
1996 Manjul Bhargava
1998 Jade Vinson (1997)
1999 Daniel Biss
2000 Sean McLaughlin
2001 Jacob Lurie
2002 Ciprian Manolescu
2003 Joshua Greene
2004 Melanie Wood
2005 Reid Barton
2006 Jacob Fox
2007 Daniel Kane
2008 Nathan Kaplan
2009 Aaron Pixton
2010 Scott Duke Kominers
2011 Maria Monks
2012 John Pardon
2013 Fan Wei
2014 Eric Larson
2015 Levent Alpoge
2016 Amol Aggarwal
2017 David Yang

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AMS SPRING SECTIONAL SAMPLER

Photo courtesy of College of Charleston.

Spring Southeastern Sectional Meeting
March 10–12, 2017 (Friday–Sunday)
College of Charleston, Charleston, SC

Image courtesy of Indiana University Communications.

Spring Central Sectional Meeting
April 1–2, 2017 (Saturday–Sunday)
Indiana University, Bloomington, IN

Image courtesy of WSU Photo Services.

Spring Western Sectional Meeting
April 22–23, 2017 (Saturday–Sunday)
Washington State University, Pullman, WA

Image courtesy of Hunter College, CUNY.

***Spring Eastern Sectional Meeting**
May 6–7, 2017 (Saturday–Sunday)
Hunter College, City University of New York, New York, NY
* A sampler from this meeting will appear in the May issue of *Notices*.

Make the time to visit any of the AMS Spring Sectional Meetings listed above.

In this sampler, the speakers below have kindly provided introductions to their Invited Addresses for the upcoming AMS Spring Central Sectional (Indiana University, April 1–2) and the AMS Spring Western Sectional (Washington State University, April 22–23) Meetings.

Spring Central Sectional Meeting

Spring Western Sectional Meeting

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DOI: http://dx.doi.org/10.1090/noti1530

Michael Hitrik

Spectra for Non-self-adjoint Operators and Integrable Dynamics

This creates a challenge, but also provides an opportunity.

Non-self-adjoint operators arise in many settings, from linearizations of equations of mathematical physics to operators defining poles of Eisenstein series. Roughly speaking, any physical system where the energy is not conserved, typically due to some form of damping or a possibility of escape to infinity, will be governed by a non-self-adjoint operator. From the point of view of mathematics, an essential feature making the study of non-self-adjoint operators notoriously difficult is the potential instability of their spectra. When A is a self-adjoint matrix, we know from the spectral theorem that the operator norm of the *resolvent* $(A - zI)^{-1}$ is equal to the reciprocal of the distance from z to the spectrum of A. In contrast, let us consider a (non-self-adjoint) Jordan block matrix

$$J_n = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

of size n, with n large. The spectrum of J_n is equal to $\{0\}$, while the operator norm of the resolvent $(J_n - zI)^{-1}$ grows exponentially with n, roughly like $1/|z|^n$, for z in the open unit disc in the complex plane. As one can imagine, things are even more intricate for, say, differential operators. This creates a challenge, but also provides an opportunity, accounting for some of the complex and fascinating traits in the spectral behavior of non-self-adjoint operators.

For most operators, we cannot compute the spectra exactly, and, as is natural in many physical situations, one studies *asymptotic* properties for problems depending on a parameter. To make things uniform, we consider a small parameter h and refer to the study of the limit $h \to 0$ as the semiclassical approximation. It is important to remember, however, that h may mean different things in different settings: as the name suggests, it could be the Planck constant (quantum mechanics), but also the temperature (Fokker–Planck equation), or the reciprocal of the square root of an eigenvalue (spectral geometry), of frequency (scattering theory), of Reynolds number (fluid mechanics), or of the power of a line bundle (complex geometry). Roughly speaking, h measures the slow variation of the medium compared to the length of the waves, hence the alternative name short wave approximation. In PDE theory, this is the basis of *microlocal analysis*, where $h \sim 1/|\partial_x|$. Adopting the philosophy of the quantum-classical correspondence principle of Niels Bohr, whereby the behavior of quantum mechanical systems should reduce to classical physics in the semiclassical limit $h \to 0$, we are then led to the following basic problem: can we describe the distribution of complex eigenvalues of a non-self-adjoint operator, in the semiclassical régime, in terms of the underlying classical dynamics? The latter is given by a Hamilton flow on the classical phase space, and to implement the quantum-classical correspondence in a non-self-adjoint environment, one frequently needs to consider complex such flows on the complexification of the real phase space.

In joint work with Johannes Sjöstrand, we have carried out a detailed spectral analysis for non-self-adjoint perturbations of self-adjoint operators in dimension two under suitable assumptions of analytic and dynamical nature. Specifically, let us consider a non-self-adjoint operator of the form

$$P_\varepsilon = P_0 + i\varepsilon Q,$$

acting on a compact two-dimensional real-analytic Riemannian manifold M. Here, to fix the ideas, we may take $P_0 = -h^2\Delta$ to be the semiclassical Laplacian, and Q is a multiplication by an analytic real-valued function q on M, which we can think of as a damping coefficient or a potential. The small parameter $\varepsilon > 0$ measures the strength of the non-self-adjoint perturbation, which should not be too weak. The eigenvalues of P_ε (see Figure 1) are confined to a thin band near the real domain, and thanks to a classical result originating in work of Torsten Carleman, we know that the distribution of the real parts of the eigenvalues of P_ε near the energy level $E = 1$, say, is governed by the same Weyl law as that for the unperturbed self-adjoint operator P_0. The distribution of the imaginary parts of the eigenvalues of P_ε is much more subtle, however, and is expected to depend on the dynamics of the geodesic flow and its relation to the non-self-adjoint perturbation.

In order to obtain a rigorous justification of the intuition above, let us assume that the geodesic flow is *completely integrable*. The phase space T^*M is then foliated by Lagrangian tori, invariant under the geodesic flow, and the flow on each such torus becomes linear when expressed in suitable canonical coordinates. Following the ideas of the method of averaging (Birkhoff normal forms), we are led to consider the long-time averages of q along the geodesic flow. The following two radically different cases occur, depending on the type of the dynamics: periodic

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I am most grateful to Johannes Sjöstrand, Joe Viola, and Maciej Zworski for their very helpful comments on this note.

DOI: http://dx.doi.org/10.1090/noti1495

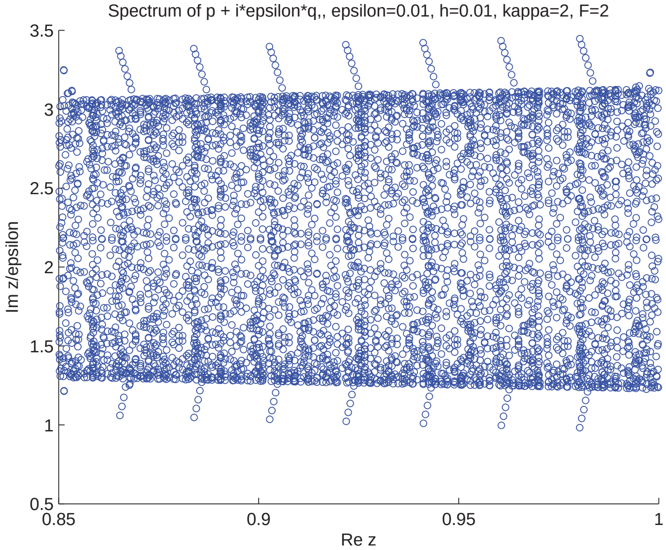

Figure 1. Numerical computation of the real and normalized imaginary parts of the eigenvalues of the non-self-adjoint operator $P_\varepsilon = P_0 + i\varepsilon Q$ on the two-dimensional torus, when Q is multiplication by a trigonometric polynomial q of degree 2. The legs of the spectral centipede represent the influence of rational tori for the classical flow.

(rational tori) or dense (irrational or, better, Diophantine tori). In joint work with Sjöstrand and San Vũ Ngọc, we have obtained a complete semiclassical description of the spectrum of P_ε near a complex energy level of the form $1 + i\varepsilon F$, assuming that F is given by the average of q along a Diophantine torus and that no other torus has this property, roughly speaking. It turns out that the spectrum in this region has the structure of a distorted two-dimensional lattice, given by a quantization condition of Bohr–Sommerfeld type.

What about spectral contributions of rational tori? These have remained rather mysterious for quite some time, until recent work with Sjöstrand demonstrated that rational tori can produce eigenvalues of P_ε close to the edges of the spectral band, isolated away from the bulk of the spectrum, and that a complete semiclassical description of these extremal, "rational" eigenvalues can be achieved. Somewhat surprisingly perhaps, the extremal eigenvalues turn out to have the structure of the legs in a spectral "centipede" as in Figure 1, with the body of the centipede agreeing with the range of torus averages of q.

The following vague philosophical remark may perhaps clarify the reason for a detailed study of dimension two: as one knows from basic quantum mechanics texts, for problems in dimension one, the Bohr–Sommerfeld quantization rules work very well to determine spectra of self-adjoint observables, whose energy surfaces are real curves. For complex Hamiltonians in dimension two, energy surfaces are *complex curves*, and this can sometimes be used to obtain spectral results that are more precise than those for real Hamiltonians.

The spectral structures described above are expected to occur also for other genuinely two-dimensional non-self-adjoint analytic spectral problems and notably for the distribution of scattering resonances for convex obstacles in $\mathbf{R}^3$, assuming that the boundary geodesic flow is completely integrable or is a small perturbation of it. The study of these fascinating issues, which is very much at the heart of the current research, will also be touched upon in the talk.

Credits

All images are courtesy of Michael Hitirik.

THE AUTHOR

Michael Hitrik

Andrew Raich

Closed Range of $\bar\partial$ in L^2 on Domains in $\mathbb{C}^n$

At the AMS Western Sectional Meeting, I will talk about my recent work on the L^2 theory of the Cauchy–Riemann operator $\bar\partial$ in several complex variables and the solvability/regularity of solutions u to the Cauchy–Riemann equation $\bar\partial u = f$ in a domain Ω in $\mathbb{C}^n$.

In one complex variable, we have the Cauchy kernel as an incredibly powerful tool to solve $\bar\partial u = f$. of the Cauchy kernel rests largely on the facts that it is holomorphic and can be used to build a variety of operators. Integrating against the Cauchy kernel on Ω produces a solution to the Cauchy–Riemann operator, while integrating against the Cauchy kernel on the boundary $b\Omega$ builds an operator that (re)produces holomorphic functions. Also, for dimensional reasons, the boundary of any domain $\Omega \subset \mathbb{C}$ lacks any complex structure

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DOI: http://dx.doi.org/10.1090/noti1494

The Riemann Mapping Theorem fails in the most spectacular way possible.

(a one real dimensional object cannot carry any complex structure), which, philosophically, allows for a result like the Riemann Mapping Theorem to hold.

In several variables, the Cauchy kernel has no higher dimensional analog. All of the replacements have serious deficiencies; for example, they are noncanonical or not holomorphic. Moreover, domains in $\mathbb{C}^n$ have boundaries that are $(2n-1)$-real dimensional manifolds, so there is an $(n-1)$-complex dimensional structure as part of the boundary. Not coincidentally, the Riemann Mapping Theorem fails in the most spectacular way possible—the unit ball and unit polydisk (product of real 2D disks) are not biholomorphic. We are left in a landscape where the solvability and regularity of the Cauchy–Riemann equations $\bar{\partial}u = f$ depend on subtle geometric and potential-theoretic quantities associated to the boundary bΩ. This makes the analysis much more difficult than either the one complex dimensional case or the analogous real problem $du = f$ on Ω.

To discuss the $\bar{\partial}$-problem in any depth, we must frame the problem more carefully. Given a $(0,q)$-form f on a domain Ω, the problem is to find a $(0,q-1)$-form u so that $\bar{\partial}u = f$. As with d, $\bar{\partial}^2 = 0$, so solving $\bar{\partial}u = f$ has the *necessary* condition $\bar{\partial}f = 0$. The question then becomes to solve $\bar{\partial}u = f$ when $\bar{\partial}f = 0$. Before the 1960s, the inhomogeneous Cauchy–Riemann equation was studied in the C^∞-topology. In the 1960s, Hörmander and Kohn pioneered the use of L^2 techniques to solve the equation. From a functional analytic viewpoint, the switch from C^∞ to L^2 is a major improvement, because the space $C^\infty(\Omega)$ is nonmetrizable, while $L^2(\Omega)$ is a Hilbert space, which allows for the use of tremendously powerful tools. Hörmander showed that $\bar{\partial}u = f$ can be solved in $L^2_{0,q}(\Omega)$ at every form level ($1 \le q \le n$) if and only if Ω satisfies a curvature condition called pseudoconvexity. Moreover, he also showed that there is no L^2 cohomology for $\bar{\partial}$ on pseudoconvex domains. Pseudoconvexity can be understood as the complex analysis version of convexity. For example, pseudoconvexity is invariant under biholomorphisms while convexity is not.

A common approach to solving the inhomogeneous Cauchy–Riemann equations is to establish a Hodge theory for $\bar{\partial}$. To do this, we need to introduce the L^2 adjoint of $\bar{\partial}$, $\bar{\partial}^*$. Computationally, $\bar{\partial}^*$ is obtained via integration by parts, and as every calculus student knows, integration by parts introduces a boundary term. For a form to be in the domain of $\bar{\partial}^*$ this boundary term must vanish. This vanishing condition is the source of trouble for proving regularity results. Although $\bar{\partial} \oplus \bar{\partial}^*$ provides an elliptic system, the inverse to $\bar{\partial}$ (when it exists) *never* has an elliptic gain. The $\bar{\partial}$-Neumann Laplacian $\Box = \bar{\partial}\bar{\partial}^* + \bar{\partial}^*\bar{\partial}$ acts diagonally on forms, and componentwise it is nothing more than the ordinary Laplacian. However, inverting $\Box$ never gains two derivatives. Sometimes the solving operator gains one derivative, sometimes it gains a fractional derivative, and sometimes the inverse is just continuous on L^2. It is in the domain of $\bar{\partial}^*$ that the geometric information of the boundary is encoded, and it is the geometry of the boundary that determines how nice solutions of $\bar{\partial}u = f$ can be.

In the decades since the 1960s the exploration into the inhomogeneous Cauchy–Riemann equation $\bar{\partial}u = f$ has expanded to various function spaces, complex manifolds, the tangential Cauchy–Riemann equations on CR manifolds, and, more recently, nonpseudoconvex domains and unbounded domains. This is where my colleague Phil Harrington and I enter the picture. To understand the more subtle geometry, we work on answering the question, What is the replacement condition for pseudoconvexity for optimal solvability of $\bar{\partial}u = f$ in $L^2_{0,q}(\Omega)$ for a fixed q, $1 \le q \le n$? In my talk, I will outline the progress that we have made on this problem and put our results in context. Time permitting, I will also discuss our progress on solving the inhomogeneous Cauchy–Riemann equation on unbounded domains and the additional complications that this entails.

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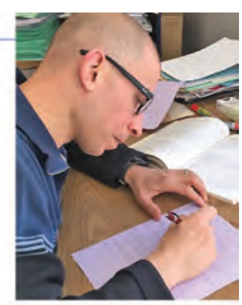
THE AUTHOR



Andrew Raich

Daniel Rogalski

Noncommutative Projective Surfaces

In this note we introduce the reader to the subject of noncommutative projective geometry, which will be the focus of our talk at the AMS regional meeting in

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DOI: http://dx.doi.org/10.1090/noti1502

Pullman, WA, in April 2017. In the talk we will begin with an overview of noncommutative geometry and then give a survey of some results about noncommutative surfaces which have been the subject of our own research. In this short space, however, we primarily describe the main idea of the subject and then at the end briefly mention one of the interesting examples that will be explored in more detail in the talk. Given our informality here, we do not provide references. The reader can find more information in the author's more extensive course notes on the subject [BRSSW].

We assume the reader is familiar with the definition of a ring. We always assume that rings have an identity element 1, and in fact all of our examples will be algebras over the complex numbers $\mathbb{C}$, which means that the ring contains a copy of the field $\mathbb{C}$ and can be thought of as a $\mathbb{C}$-vector space. In a first course in abstract algebra, the discussion of rings invariably focuses almost exclusively on commutative rings, where $ab = ba$ for all elements a and b. Perhaps the matrix ring $M_n(\mathbb{C})$ might be mentioned as an example of a noncommutative ring. Of course, it makes sense to get comfortable first with commutative rings, which are essential to many applications of ring theory. Plenty of interesting rings arising in nature are noncommutative, however, such as the various quantum algebras that arise in physics to describe the interaction of operators (such as position and momentum) that do not necessarily commute. One simple but already nontrivial example of a noncommutative algebra is the *quantum plane*, $A_q = \mathbb{C}\langle x, y\rangle/(yx - qxy)$, for a nonzero scalar $q \in \mathbb{C}$. Here, $\mathbb{C}\langle x, y\rangle$ is the free associative algebra in two variables, that is, the $\mathbb{C}$-vector space with basis all words in the (noncommuting) variables x and y, with product given by concatenation. To obtain A_q we mod out by the ideal generated by one relation, which says that the variables commute "up to scalar." It is not hard to see that the elements of A_q can be identified with the usual commutative polynomials $\sum_{i,j\geq 0} a_{ij}x^i y^j$ in the variables, but the multiplication is changed so that every time we move a y to the right of an x, a factor of q appears. We describe some properties of this algebra below.

Commutative ring theorists have long enjoyed the advantage of the tight connection of their subject with algebraic geometry. This means both that ring-theoretic results may have an application to geometry and that geometric intuition can help to prove and interpret results about rings. In a course on algebraic geometry over the complex numbers, a student learns first about the most fundamental varieties: affine n-space $\mathbb{A}^n = \{(a_1, a_2, \dots, a_n) | a_i \in \mathbb{C}\}$, and projective n-space $\mathbb{P}^n$, which can be identified with the set of lines through the origin in $\mathbb{A}^{n+1}$, with the line through the point $(a_0, a_1, \dots, a_n)$ written as $(a_0 : a_1 : \cdots : a_n)$. Both affine and projective spaces have a natural topology called the Zariski topology, and affine varieties are defined to be closed subsets of affine spaces, while projective varieties are defined to be closed subsets of projective spaces. In the more modern approach to the subject via the theory of schemes, the connection between varieties and rings becomes more important, and one learns how to define a projective variety all at once using a graded ring. In this note, a *graded* $\mathbb{C}$*-algebra* will be a $\mathbb{C}$-algebra R with a $\mathbb{C}$-vector space decomposition $R = R_0 \oplus R_1 \oplus \dots$, where $R_n R_m \subseteq R_{n+m}$ for all n, m, and where for simplicity we always assume that $R_0 = \mathbb{C}$ and that R is generated by R_1 as an algebra over $\mathbb{C}$. For a commutative graded $\mathbb{C}$-algebra, one defines $\operatorname{Proj} R$ to be the set of homogeneous prime ideals in R, that is, those prime ideals P such that $P = \bigoplus_{n\geq 0}(P \cap R_n)$, excluding the *irrelevant ideal* $R_{\geq 1}$ spanned by all elements of positive degree. The maximal elements of this space $\operatorname{Proj} R$ correspond to the points of a projective variety, while the other points are called "generic" and are important to the scheme-theoretic point of view. For example, the polynomial ring $R = \mathbb{C}[x_0, x_1, \dots, x_n]$ in $n + 1$ variables over $\mathbb{C}$ is graded, where R_n is simply the span of monomials of degree n, and in this case $\operatorname{Proj} R = \mathbb{P}^n$. As a more explicit example, the homogeneous primes of $\mathbb{C}[x, y]$ are (0), $(ax + by)$ for $a, b \in \mathbb{C}$ not both 0, and (x, y). Removing the irrelevant ideal (x, y), the maximal elements are those ideals of the form $(ax + by)$ which correspond to points $(a : b)$ in $\mathbb{P}^1$.

Noncommutative projective geometry is an attempt to understand noncommutative graded rings in a geometric way.

Noncommutative projective geometry is an attempt to understand noncommutative graded rings in a geometric way or to associate some kind of interesting geometric space to a noncommutative ring, similar to the way that the Proj construction associates a projective variety to a commutative graded ring. The development of the subject was surely delayed by the fact that the most obvious analog of the Proj construction for a noncommutative ring does not lead to an interesting theory, even for such fundamental examples as the quantum plane. There is a natural definition of prime ideal for a noncommutative ring: P is prime if whenever $IJ \subseteq P$ for ideals I and J, $I \subseteq P$ or $J \subseteq P$ (this definition reduces to the usual one for commutative rings). However, in contrast to the commutative case, the quantum plane A_q usually has very few prime ideals. In particular, if q is not a root of unity, the only homogeneous prime ideals of A_q are (0), (x), (y), and (x, y). Removing the irrelevant ideal (x, y), one is not left with a very interesting space. On the other hand, A_q behaves a lot like a commutative polynomial ring in two variables in many other ways, so one might hope it would have a geometry more similar to $\operatorname{Proj} \mathbb{C}[x, y] = \mathbb{P}^1$.

As a general principle, often one has to find a particular way of formulating a concept in the commutative case, not always the most elementary or obvious way, to get something that generalizes to a useful notion for noncommutative rings. In fact, there are two more subtle ways of thinking about how $\mathbb{P}^1$ is connected with the ring $\mathbb{C}[x, y]$ which *do* generalize well to the quantum plane. First, given any graded $\mathbb{C}$-algebra A, a *point module* over A is a graded left module $M = \bigoplus_{n \geq 0} M_n$ (that is, $A_i M_j \subseteq M_{i+j}$ for all i, j) such that $M = AM_0$ and $\dim_{\mathbb{C}} M_n = 1$ for all $n \geq 0$. The point modules for $\mathbb{C}[x, y]$ are the cyclic modules $M_{(a:b)} = \mathbb{C}[x, y]/(ax + by)$, as $(a : b)$ varies over points in $\mathbb{P}^1$. One can even say that the point modules are *parametrized* by $\mathbb{P}^1$ in a way that is made more precise in algebraic geometry. It turns out that the point modules for A_q are also parametrized by $\mathbb{P}^1$, being of the form $N_{(a:b)} = A_q/A_q(ax + by)$ where $A_q(ax + by)$ is the *left* ideal generated by $ax + by$. Thus, the space of point modules associated to A_q is always the projective line $\mathbb{P}^1$.

An even more subtle approach involves generalizing the idea of sheaves. A sheaf on a projective variety can be defined by taking an open cover by rings and taking a module over each ring such that the modules "glue together" appropriately. This is the most natural definition geometrically, but it turns out that there is a purely algebraic way of getting at this notion. Given a commutative graded $\mathbb{C}$-algebra A, we define the *category* of finitely generated $\mathbb{Z}$-graded left A-modules A-gr and define A-qgr to be the quotient category of A-gr by the subcategory of modules M which have $M_n = 0$ for $n \gg 0$. The category A-qgr turns out to be the same as the category of *coherent sheaves* on Proj A. The category of coherent sheaves on a variety is extremely fundamental to the study of the variety, similar to the way that the representations of a finite group are fundamental to the study of the group. Fortunately, the category A-qgr as defined above makes perfect sense for any noncommutative graded algebra A, and thus it intuitively represents some kind of "category of coherent sheaves" *even though there is no noncommutative space that these objects are sheaves on*! Thus the category becomes the fundamental object of study in the noncommutative case, and in fact A-qgr is called the *noncommutative projective scheme* associated to the algebra A. One may prove that the noncommutative projective scheme of the quantum plane, A_q-qgr, is in fact equivalent to $\mathbb{C}[x, y]$-qgr, the usual category of coherent sheaves on $\mathbb{P}^1$. We say that A_q is a *noncommutative coordinate ring* of $\mathbb{P}^1$.

Most of the attention in noncommutative geometry has focused on analogs of surfaces and higher-dimensional varieties, where things become more complicated. Artin and Schelter defined a notion of *regular algebra* which captures those noncommutative graded algebras which behave *homologically* like commutative polynomial rings; these are the rings that correspond geometrically to noncommutative projective spaces. The noncommutative $\mathbb{P}^2$s were classified by Artin, Schelter, Tate, and Van den Bergh. One of the most important such examples is the *Sklyanin algebra*

$$S = \mathbb{C}\langle x, y, z\rangle/(ax^2 + byz + czy, \\ ay^2 + bzx + cxz, az^2 + bxy + cyx)$$

for general scalars $a, b, c \in \mathbb{C}$. It turns out that noncommutative $\mathbb{P}^2$s can have fewer points than one might at first expect. In particular, the point modules for S turn out to be parametrized by an *elliptic curve* E, and the beautiful theory of elliptic curves in algebraic geometry plays a big role in the analysis of the properties of the algebra S. The noncommutative projective scheme S-qgr is similar in some ways to the category of coherent sheaves on $\mathbb{P}^2$, but is not the same—it is a genuinely noncommutative $\mathbb{P}^2$, with only an elliptic curve's worth of points!

In the talk we plan to describe our long-term program to develop a minimal model program for noncommutative surfaces, which has driven a lot of our recent research. To close this note, we wish to mention a weird example that is particularly close to our heart. One of the joys of noncommutative algebra is the wide abundance of examples and counterexamples which constantly point out the myriad ways in which noncommutative rings are more complicated (or we could say, with obvious bias, more interesting!) than commutative rings. Because of this zoo of examples, there are still many foundational questions in noncommutative geometry about what assumptions on graded rings are necessary to get a good theory. It is natural to restrict to the study of graded rings which are *Noetherian* (after Emmy Noether), which means that they do not have infinite ascending chains of left or right ideals. This is a basic structural assumption, just as in the commutative case, and one which is satisfied by most reasonable examples. However, it is not enough to prevent certain kinds of pathologies in noncommutative geometry. In particular, there turn out to be many Noetherian graded algebras R whose space of point modules is so wild that it is not parametrized by a projective variety. One simple example of such an R can be obtained as follows. First define $B = \mathbb{C}\langle x, y, z\rangle/(zx - pxz, xy - ryx, yz - qzy)$, where $p, q, r \in \mathbb{C}$ are general scalars satisfying $pqr = 1$. This is a very nice noncommutative $\mathbb{P}^2$; in fact, B has point modules parametrized by $\mathbb{P}^2$, and B-qgr is equivalent to coherent sheaves on $\mathbb{P}^2$. The subalgebra $R = \mathbb{C}\langle x - y, y - z\rangle \subseteq B$ generated by $x - y$ and $y - z$ inside B is Noetherian, yet has point modules corresponding to an *infinite blowup* of $\mathbb{P}^2$, where each of infinitely many points in the orbit of a certain automorphism of $\mathbb{P}^2$ gets replaced by a whole projective line. The noncommutative projective scheme R-gqr also has unusual properties. Ring theoretically, the strangeness of R shows up in the failure of R to be *strongly Noetherian*; that is, the base extension $R \otimes_{\mathbb{C}} C$ is not Noetherian for some Noetherian commutative $\mathbb{C}$-algebra C. There is now a whole theory of rings like this, which are called *naïve blowups*. They form one of the main classes of examples in the theory of what are known as *birationally commutative surfaces*, as have been classified by the author and Stafford, and in a more general case by Sierra.

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Photo Credit

Photo of Daniel Rogalski is courtesy of Daniel Rogalski.

THE AUTHOR

Daniel Rogalski

Sarah C. Koch

Postcritical Sets in Complex Dynamics

I'll begin by introducing some key objects and ideas from the field of complex dynamics, specifically looking at the dynamics of quadratic polynomials. I will then discuss the work I plan to present at the AMS meeting in April 2017.

Iterating Quadratic Polynomials

For each complex number c, consider the map

$$p_c : \mathbb{C} \to \mathbb{C} \quad \text{given by} \quad p_c : z \mapsto z^2 + c.$$

Although p_c is just a quadratic polynomial, a wealth of complicated and deep behavior can emerge when it is iterated. Given $z_0 \in \mathbb{C}$, the *orbit of* z_0 is the sequence $z_n := p_c(z_{n-1})$. There is a nonempty compact subset $K_c \subseteq \mathbb{C}$ associated to iterating p_c called the *filled Julia set of* p_c; see Figure 1. By definition K_c consists of all $z_0 \in \mathbb{C}$ such that the orbit of z_0 is bounded.

In complex dynamics, the orbits of the critical points (those points where the derivative of the map vanishes) play an important role. For example, the filled Julia set

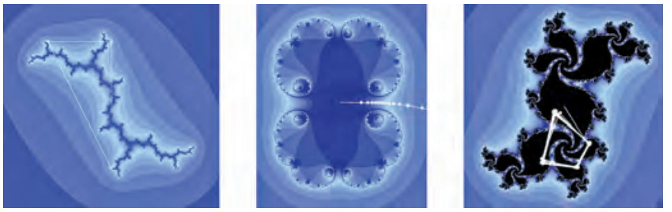

Figure 1. Filled Julia sets K_c of three quadratic polynomials p_c are drawn in black. The critical point $z_0 = 0$ is in the center of each of these pictures, and its orbit is drawn in white. Left: **The set K_c is connected and has no interior. The critical point eventually maps to a repelling cycle of period 2.** Center: **The set K_c is a Cantor set. The critical point escapes to infinity under iteration.** Right: **The set K_c is connected, and every interior point (including the critical point) is attracted to an attracting cycle of period 4.**

K_c is connected if and only if the orbit of the critical point $z_0 = 0$ is bounded, that is, if and only if $0 \in K_c$. As another example, if p_c possesses an attracting periodic cycle, then the critical point is necessarily attracted to it under iteration.

The Mandelbrot Set

Moving from the z-plane to the c-parameter plane, one considers the *Mandelbrot set*

$$M := \{c \in \mathbb{C} \mid K_c \text{ is connected}\} = \{c \in \mathbb{C} \mid 0 \in K_c\}.$$

The Mandelbrot set (see Figure 2) is a fundamental object in the field of complex dynamics. It has been thoroughly studied over the past few decades with much success, and

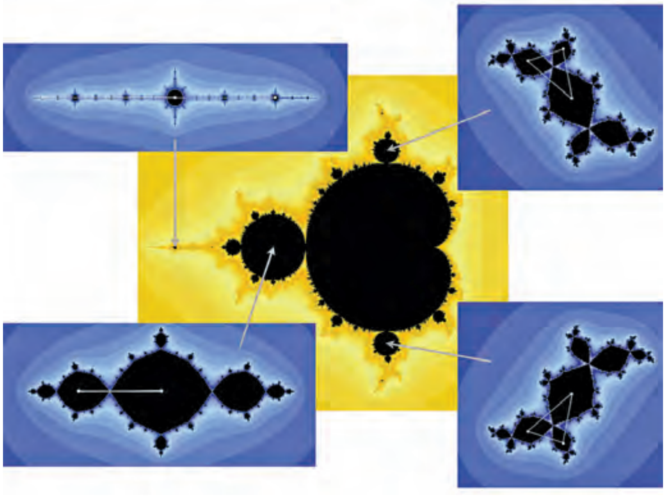

Figure 2. The picture in the center is the c-parameter plane for the family $p_c(z) = z^2 + c$; the Mandelbrot set is drawn in black. Each of the surrounding pictures contains a filled Julia set K_c, and the arrows indicate the corresponding value of c.

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DOI: http://dx.doi.org/10.1090/noti1493

it continues to be a central topic of research. It is compact and connected. Among the connected components of its interior are the *hyperbolic components*, which consist of those parameters c such that the polynomial p_c possesses an attracting periodic cycle.

Postcritically Finite Quadratic Polynomials

Of particular interest are the polynomials p_c for which the orbit of the critical point $z_0 = 0$ is finite. Such a polynomial is said to be *postcritically finite*. If p_c is postcritically finite, then the critical point is either periodic or preperiodic; that is, it eventually maps to a periodic cycle under p_c. Therefore, if p_c is postcritically finite, then $0 \in K_c$, so K_c is connected. It follows that the parameters c for which p_c is postcritically finite are all contained in M. In fact, they can be found explicitly, as demonstrated in the following example.

Example. Suppose we are interested in finding all parameters $c \in M$ for which the critical point $z_0 = 0$ is periodic of period 3. The equation defined by the *critical orbit relation* $(p_c)^3(0) = 0$ is

$$c(c^3 + 2c^2 + c + 1) = 0.$$

We are interested in the three roots of $c^3 + 2c^2 + c + 1$ (there is a degenerate solution at $c = 0$ which we ignore). One root is real, and the other two are complex conjugates. Thus, there are three quadratic polynomials p_c for which 0 is periodic of period 3. The filled Julia sets of these polynomials are drawn in Figure 2: the two at the top, and the one at the lower right.

More generally, each $c \in M$ for which 0 is periodic under p_c is an algebraic integer, as it is determined by the critical orbit relation $(p_c)^n(0) = 0$. Each of these parameters is contained in a hyperbolic component of M, and every hyperbolic component contains one and only one such parameter, called its *center*. This has been incredibly fruitful for understanding the structure of M; indeed, using the dynamics of p_c as c runs over all centers, one can essentially catalog and organize all hyperbolic components. The arrows in Figure 2 point to four centers of hyperbolic components in M.

The parameters $c \in M$ for which 0 is not periodic itself but strictly preperiodic are also algebraic integers. Indeed, each of these parameters is determined by a critical orbit relation $(p_c)^m(0) = (p_c)^n(0)$. The strictly preperiodic parameters are dense in the boundary of M.

If c is algebraic, then the critical orbit

$$0 \longmapsto c \longmapsto c^2 + c \longmapsto (c^2+c)^2 + c \longmapsto \cdots$$

is also algebraic. Therefore, if p_c is postcritically finite, then the critical orbit is a finite algebraic subset of $\mathbb{C}$. One question to explore is, Which finite algebraic subsets of $\mathbb{C}$ arise as critical orbits of p_c as $c \in M$ runs over all postcritically finite parameters?

There is no reason to restrict our attention to just postcritically finite quadratic polynomials; there is a completely analogous discussion for rational maps.

Postcritically Finite Rational Maps

Let $f : \mathbb{P}^1 \to \mathbb{P}^1$ be a rational map on the Riemann sphere of degree $d \geq 2$. By the Riemann–Hurwitz formula, f has $2d - 2$ critical points, counted with multiplicity; let C_f be the set of critical points of f. The map f is *postcritically finite* if the *postcritical set*

$$P_f := \bigcup_{n>0} f^n(C_f)$$

is finite.

In My Talk

Based on joint work with L. DeMarco and C. McMullen, we study the subsets $X \subseteq \mathbb{P}^1$ that arise as P_f for some postcritically finite rational map f and also investigate the extent to which the combinatorics of $f : P_f \to P_f$ can be specified. We employ a variety of results to explore this problem, ranging from Belyi's celebrated theorem [B] to analytic techniques used in the proof of Thurston's topological characterization of rational maps [DH], one of the most central theorems in complex dynamics.

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Credits

Figures 1 and 2 are produced with FractalStream, which is written by M. Noonan and freely available online.

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ABOUT THE AUTHOR

Sarah Koch works in complex dynamics and Teichmüller theory. She is honored to have received an NSF-CAREER award in 2015 and the University of Michigan Class of 1923 Memorial Teaching Award in 2016.

Sarah Koch

2017 AWM LECTURE SAMPLER

Photo from the banquet at the 2015 AWM Research Symposium. Back row (l-r): 2015 AWM Presidential Award winner Sylvia Bozeman; Past AWM President Kristin Lauter; former AWM President Jill Pipher. Front row (l-r): AWM Executive Committee member Talitha Washington; 2015 Keynote Speaker Shirley Malcom; and former AWM President Ruth Charney.

The AWM Research Symposium 2017 (April 8–9, 2017 at UCLA) will showcase the research of women in the mathematical professions. Some of the plenary speakers and members of the committee have offered to share a sneak-peek of their presentations with *Notices*.

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DOI: http://dx.doi.org/10.1090/noti1529

Kristin Lauter and Ami Radunskaya

AWM Research Symposium April 2017: Introduction

As the recent book and movie *Hidden Figures* bring to light, women's contributions to mathematics have often been overlooked and minimized. The 2017 AWM Research Symposium celebrates the mathematical contributions of many women mathematicians at all stages in their careers.

On the fortieth anniversary of AWM in 2011, AWM launched a new tradition of biennial AWM Research Symposia. The first one was held at Brown University in 2011, then Santa Clara University in 2013, University of Maryland in 2015, and now UCLA in 2017. These are large weekend research meetings on the model of AMS sectional conferences, bringing together more than three hundred female research mathematicians from around the world and featuring more than one hundred fifty research talks by female mathematicians.

The conference at UCLA on April 8–9 will feature eighteen special sessions, including eight sessions for Research Networks for Women supported by the AWM ADVANCE grant, four plenary talks, a poster session for graduate students, a reception for student chapters, a jobs panel, exhibit booths for sponsors, an awards banquet, and an AWM Presidential Award!

The AWM Research Symposia were originally inspired by the annual meetings of the Korean Women in Mathematical Sciences society. The plenary speakers at AWM Symposia have included a Fields Medalist, a president of the IMU, and an NSF math institute director. This year's plenary speakers are no less eminent: AWM Past President Ruth Charney, the 2016 Blackwell–Tapia Prize winner Mariel Vazquez, AWM Sadosky Prize winner Svitlana Mayboroda, and the first AWM Sonya Kovalevsky Lecturer Linda Petzold.

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Photo of Ami Radunskaya is courtesy of Pomona College.

Kristin Lauter, AWM past president, is principal researcher and research manager at Microsoft Research in Redmond and affiliate professor of mathematics at the University of Washington. Her e-mail address is klauter@microsoft.com.

Ami Radunskaya, AWM president, is professor of mathematics at Pomona College and co-director of the EDGE (Enhancing Diversity in Graduate Education) program. Her e-mail address is aer04747@pomona.edu.

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DOI: http://dx.doi.org/10.1090/noti1503

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Kristin Lauter

In 1999 **Kristin Lauter** received an AWM Mentoring Grant to study with Jean-Pierre Serre in Paris at the College de France (sketch by Enrico Bombieri) and wrote two papers with appendices by Serre. She enjoys soccer, sailing, bike trips with her husband, and doing art with her twin daughters.


Ami Radunskaya

Ami Radunskaya, known as "Dr. Rad" by her students, enjoys transdisciplinary research, using mathematics to answer questions in medicine, power systems, and economics. The second photo shows her working with her father, Roy Radner, on a recently published PNAS article, "Dynamic pricing of network goods." She enjoys hiking, making music, cooking, and eating with family and friends.

Ruth Charney

Searching for Hyperbolicity

Geometry has come a long way since Euclid. In the nineteenth century, hyperbolic geometry came into its own with the work of Felix Klein and to this day has continued to play a central role in differential geometry and low-dimensional topology. In the late 1980s, Mikhail Gromov introduced a notion of hyperbolicity that applies to a very general class of metric spaces (not necessarily manifolds) and in the process launched the field of geometric group theory.

In geometric group theory, one studies groups acting by isometries on a metric space and explores connections between algebraic properties of the group and geometric properties of the space. Gromov's hyperbolicity condition is a particularly powerful property that has been deeply explored and applied to understand groups that act by isometries on such spaces. In recent years, there has been much interest in extending these techniques to broader classes of groups and spaces. Many interesting spaces do not fully satisfy Gromov's hyperbolicity condition and yet display some aspects of hyperbolic behavior. How can we identify such behavior and put it to good use?

How can we identify such behavior and put it to good use?

Gromov's definition of hyperbolicity is based on a thin triangle condition. Let X be a geodesic metric space, that is, a space in which the distance between two points is equal to the minimal length of a path connecting them. We say that X is hyperbolic if there exists a constant δ such that for any geodesic triangle Δ in X, each side of Δ is contained in the δ-neighborhood of the other two sides. A classic example of this is when X is a tree and all triangles are tripods. In this case, we can take $\delta=0$; that is, each side of a triangle is contained in the union of the other two sides. At the other extreme, it is easy to see that the Euclidean plane fails the thin triangle condition for every choice of δ.

Now let's combine the behavior of the tree and the plane. Take Y to be the universal cover of $T^2 \vee S^1$, the wedge of a torus and a circle. Envision Y as a collection of horizontal planes with vertical edges emanating from every lattice point, creating a tree of flats as in Figure 1. If we stay in a horizontal plane, our surroundings do not look at all hyperbolic and triangles can get arbitrarily wide, but if we travel mostly in an upward direction, not spending too much time in any horizontal plane, then our behavior begins to look hyperbolic!

In joint work with H. Sultan, along with subsequent work of M. Cordes and D. Murray, we consider geodesic rays in a metric space X that have hyperbolic-like behavior and encode this information in a new type of boundary for X, called the Morse boundary. We show that the Morse boundary has properties analogous to boundaries of Gromov hyperbolic spaces and (providing the Morse boundary is nonempty) it provides an effective tool to study groups acting isometrically on X.

Photo Credit

Photo of Ruth Charney courtesy of Michael Lovett.

ABOUT THE AUTHOR

Ruth Charney

Ruth Charney has always been intrigued by the interaction between algebra and topology. She was drawn into the field of geometric group theory at its inception in the late 1980s and has been happily ensconced there ever since. In addition to mathematics, she loves traveling, hiking, dancing, and exploring restaurants

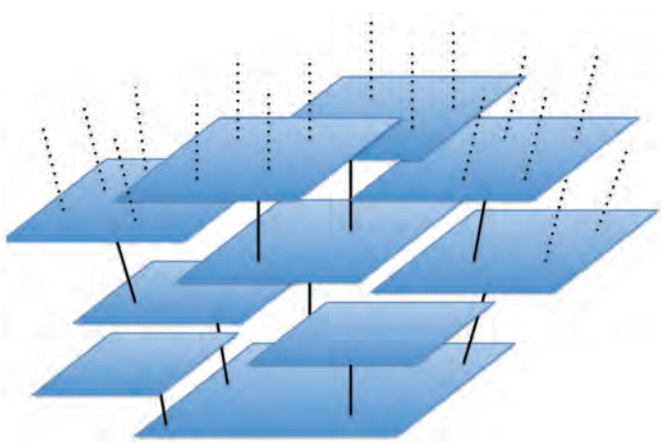

Figure 1. A tree of flats looks hyperbolic vertically, though not horizontally.

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DOI: http://dx.doi.org/10.1090/noti1497

Svitlana Mayboroda

Localization of Eigenfunctions

At the forefront of modern technology, matter structured at the nanometer and atomic levels increasingly reveals its wavelike nature. This provides new remarkable opportunities and challenges in analysis and partial differential equations. In recent years, pushing the limits of fundamental physics, cutting-edge experiments with ultracold atoms have created new states of matter and offer the possibility of controlling quantum entanglement, with major potential applications to cryptography and quantum computing. In engineering, electronic waves confined in quantum wells have yielded high efficiency LEDs, which are about to revolutionize the energetics of lighting, as recognized by the 2014 Nobel Prize. At these scales, even the slightest disorder or irregularity can trigger one of the most puzzling and ill-understood phenomena—wave localization.

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DOI: http://dx.doi.org/10.1090/noti1498

What is wave localization? It is an astonishing ability of physical systems to maintain vibrations in small portions of their original domains of activity while preventing extended propagation. One should not, in this context, think solely in terms of mechanical vibrations. Light is a particular example of an electromagnetic wave, wifi is delivered by waves, sound is a pressure wave, and, from the vantage point of quantum physics, even matter can be perceived as a type of wave. In mathematical terms, localized eigenfunctions φ of a self-adjoint elliptic operator L=-div $A\nabla+V$ in a domain Ω satisfy $L\varphi=\lambda\varphi$, where φ is extremely close to zero outside some small subset of Ω. This phenomenon can be triggered by irregularities of the coefficients of A, disorder in the potential V, special features of the shape of Ω, or an intricate mixture of all of the above. Over the past century this has been a source of persistent interest in condensed matter physics, engineering, and mathematics. However, it has remained a mystery whether it is possible to directly translate knowledge of A, V and Ω into the specific information on the location and frequencies of localized eigenfunctions or, better yet, to design systems with desired localization patterns.

In 2012, together with M. Filoche, we introduced a new concept of the "localization landscape." This has turned out to have some remarkable and powerful features. Indeed, in a joint work with D. Arnold, G. David, M. Filoche, and D. Jerison, we show that the landscape function, which is defined as a solution to Lu=1 on Ω, reveals a clear disjoint partition of Ω into independent regions, and this partition predicts localization domains with exponential accuracy, with the rate of decay governed by the so-called Agmon metric associated to $1/u$ [A+2016]. That is, given any system, one needs only to solve a simple equation Lu=1, and determine the corresponding "valley lines" to obtain a map of subregions which host the localized eigenfunctions (see Figure 1). Furthermore, modulo this exponentially small error, the part of the spectrum of L on Ω not exceeding the maximum of $1/u$ can be fully diagonalized, that is, bijectively mapped on the combined spectrum of the subregions determined by the landscape. In particular, an eigenfunction can be massive simultaneously in two landscape subregions if and only if the eigenvalues of these subregions are exponentially close (a fairly unlikely event). In this sense, one could view $1/u$ as an *effective* quantum potential which exhibits clear structure even when V is highly disordered or when is absent and the localization is caused by the geometry and/or by coefficients of A. In other words, $1/u$ suitably quantifies the *uncertainty principle*.

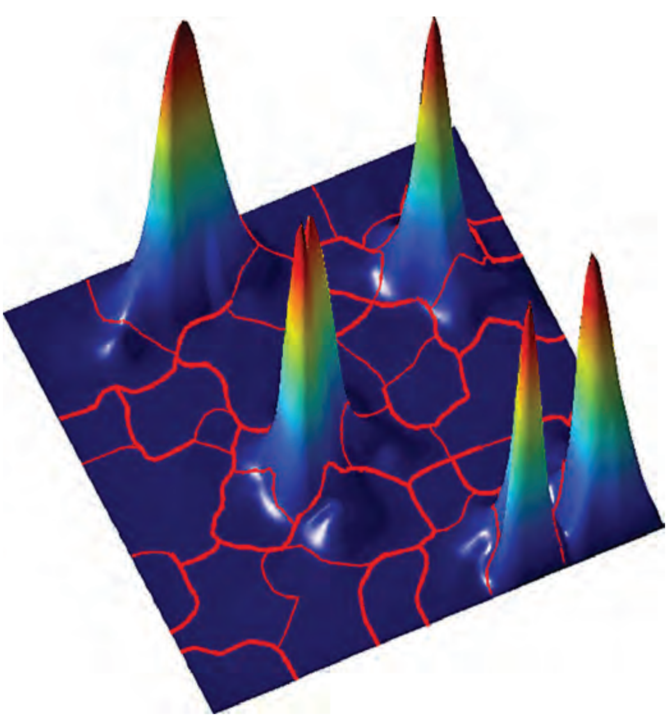

Figure 1. A network of valleys of a localization landscape (in red) and the first five localized eigenfunctions for the Schrödinger operator with disordered potential.

Going further, the localization landscape furnishes a new version of the Weyl law. It appears to give the first universal estimate on the counting function and on the density of states for small eigenvalues in the range where the classical Weyl law notoriously fails. In fact, numerical experiments show that already minima of $1/u$ give a very good approximation for the bottom of the spectrum, but these observations for now remain mathematically inaccessible.

Finally, in the intervening years, these results have found immediate applications in theoretical and experimental physics and in energy engineering to predict the vibration of plates, the spectrum of the bilaplacian with Dirichlet data, the efficiency and quantum droop of GaN LEDs governed by the Poisson–Schrödinger self-consistent system, and the spectral properties of the Schrödinger operator with Anderson or Anderson–Bernoulli potentials in bounded domains.

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Credits

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ABOUT THE AUTHOR

Svitlana Mayboroda works at the interface of harmonic analysis, geometric measure theory, and partial differential equations, with applications to condensed matter physics and engineering.

Svitlana Mayboroda

Linda Petzold

Inference of the Functional Network Controlling Circadian Rhythm

Petzold's lecture focuses on the use of computing and mathematics to better understand circadian rhythm, the process by which living organisms manage to follow a 24-hour cycle. Difficulties in following this cycle are experienced, for example, in jet lag and shift work. Circadian rhythm disorders are known risk factors for heart disease, obesity, and diabetes, as well as numerous psychiatric and neurodegenerative diseases. In mammals, the Suprachiasmatic Nucleus (SCN), a brain region of about 20,000 neurons, serves as the master circadian clock, coordinating timing throughout the body and entraining the body to daily light cycles. The extent to which cells in the SCN can synchronize and entrain to external signals depends both on the properties of the individual oscillators (neurons) and on the communication network between individual cell oscillators. Petzold's lecture explores both the development of mathematical models for the oscillators and inference of the structure of the network that connects the neurons.

Linda Petzold

ABOUT THE AUTHOR

Linda Petzold is a computational and applied mathematician working on modeling, analysis, simulation, and software applied to networked systems in biology and medicine.

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DOI: http://dx.doi.org/10.1090/noti1496

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A New Golden Age of Minimal Surfaces

Joaquín Pérez

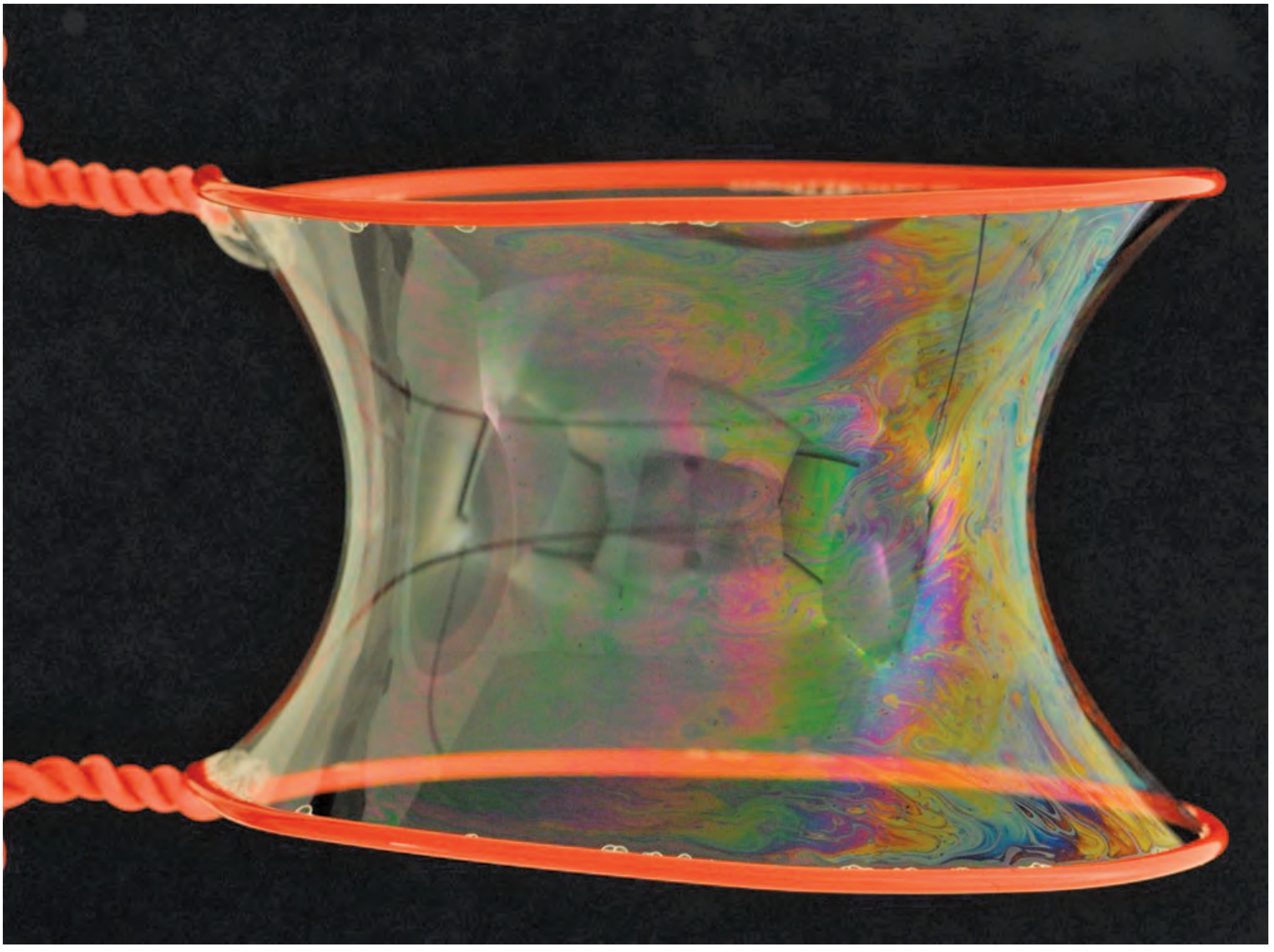

Figure 1. A minimal surface, like this soap film, is characterized by the property that small pieces minimize area for given boundary.

Abstract. We give a brief tour of some of the recent developments in classical minimal surface theory, especially those where the work of Colding and Minicozzi on compactness properties of embedded minimal disks in Euclidean three-space has been instrumental. Along the way, we will discuss some of the main open problems. This article is a translated and revised version of an expository paper to appear in the *Gaceta de la Real Sociedad Matemática Española*.

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Research partially supported by the Spanish MEC/FEDER grant No. MTM2014-52368-P.

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DOI: http://dx.doi.org/10.1090/noti1500

Introduction

A *minimal surface,* like the soap film of Figure 1, has the property that small pieces minimize area for a given boundary, even though the whole surface may be unstable. At first, there were few explicit examples (see Figure 2): the plane, the catenoid of Euler (1741), and the helicoid of Meusnier (1776).

Many of the greatest mathematicians in history have been challenged by minimal surfaces; some of them made spectacular advances in a relatively definite period, producing golden ages of this theory: The first one occurred approximately in the period 1830–1890, when renowned mathematicians such as Enneper, Scherk, Schwarz, Riemann, and Weierstrass made major advances on minimal surfaces through the application of the newly created field of complex analysis by providing analytic formulas for a general minimal surface. Also in this period, fundamental research by Plateau on surface tension gave a physical interpretation to the problem of minimizing area with a given contour, which allowed the spread of this minimization problem beyond mathematics, to the point that since then it is customary to refer to it as the Plateau problem. A second golden age of minimal surfaces took place from about 1914 to 1950, with the incipient theory of partial differential equations: here, we highlight the contributions of Bernstein, Courant, Douglas (who in 1936 won the first Fields Medal[1] for his solution of the Plateau problem), Morrey, Morse, Radó, and Shiffman. A third golden age started in the 1960s, when giants of the stature of Almgren, Alt, Calabi, do Carmo, Chern, Federer, Finn, Fleming, Gackstatter, Gulliver, Hardt, Hildebrandt, Jenkins, Lawson, Nitsche, Osserman, Serrin, Simon, and Simons opened new routes through the use of multiple techniques, from Riemann surfaces to geometric measure theory, passing through integrable systems, conformal geometry, and functional analysis. The appearance of computers was crucial for the discovery in the eighties of new examples of complete minimal surfaces without self-intersections. The abundance of these newly discovered examples led to new problems and conjectures about the classification and structure of families of surfaces with prescribed topology. More recent major contributors are too numerous to list here.

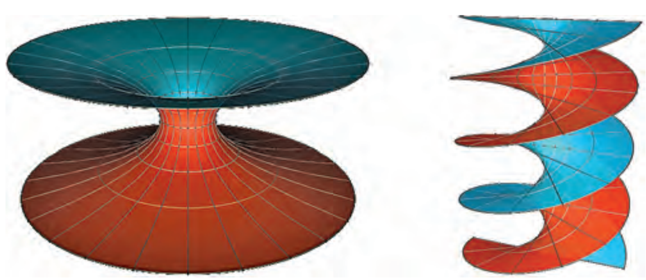

Figure 2. The first explicit examples of minimal surfaces were the plane, the catenoid (Euler, 1741), and the helicoid (Meusnier, 1776).

[1]*Shared with Ahlfors for his work on Riemann surfaces.*

In this article we hope to convince the reader that as with the previous milestones, we are currently witnessing a new golden age of minimal surfaces, mostly favored by a new tool discovered in 2004: the so-called Colding–Minicozzi theory. This work, published in an impressive series of four articles in the same issue of *Annals of Mathematics* [2], analyzes the convergence of sequences of embedded minimal disks without imposing a priori uniform bounds on area or curvature. We will sketch how this theory has helped to solve open problems that were considered inaccessible until recently, and we will venture, with all the reservations that predictions deserve, to expose some of the most interesting open problems in this field.

The theory of minimal surfaces is a confluence of many branches of mathematics.

In order to develop these objectives in a limited number of pages, we must pay the price of not going into detail. There are many articles, books, and chapters of books where interested readers can satisfy their curiosity, such as the volume by Colding and Minicozzi [1] or the survey by Meeks and me [3].

Basic Results

The theory of minimal surfaces is a confluence of many branches of mathematics. We can define minimality in at least eight different but equivalent ways, based on the theory that we are most passionate about.

Let $X\colon M \to \mathbb{R}^3$ be an isometric immersion of a Riemannian surface in three-dimensional Euclidean space, and let $N\colon M \to \mathbb{S}^2(1) \subset \mathbb{R}^3$ be its unit normal or Gauss map (here $\mathbb{S}^2(1)$ denotes the sphere of radius 1 and center the origin of $\mathbb{R}^3$). If we perturb X in a relatively compact domain $\Omega \subset M$ by a compactly supported differentiable function $f \in C_0^\infty(\Omega)$, then $X + tfN$ is again an immersion for $|t| < \varepsilon$ and $\varepsilon > 0$ small enough. The mean curvature $H \in C^\infty(M)$ of X (arithmetic mean of the principal curvatures) is related to the area functional $A(t) = \text{Area}((X+tfN)(\Omega))$ by means of the *first variation of area formula*:

$$A'(0) = -2\int_\Omega fH\,dA, \tag{1}$$

where dA is the area element of M. Now we can state the first two equivalent definitions of minimality.

Definition 1. A surface $M \subset \mathbb{R}^3$ is *minimal* if it is a critical point of the area functional for all variations with compact support.

Definition 2. A surface $M \subset \mathbb{R}^3$ is *minimal* when its mean curvature vanishes identically.

Locally and after a rotation, every surface $M \subset \mathbb{R}^3$ can be written as the graph of a differentiable function $u = u(x,y)$. In 1762, Lagrange wrote the foundations of the calculus of variations by finding the PDE associated to a critical point of the area functional when the surface is a graph:

Definition 3. A surface $M \subset \mathbb{R}^3$ is *minimal* if around any point it can be written as the graph of a function $u = u(x, y)$ that satisfies the second-order, quasi-linear elliptic partial differential equation

$$(1 + u_x^2)u_{yy} - 2u_x u_y u_{xy} + (1 + u_y^2)u_{xx} = 0. \tag{2}$$

The above PDE can also be written in divergence form:

$$\operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right) = 0. \tag{3}$$

Neglecting the gradient in the denominator of (3) leads to the celebrated Laplace equation. This means that on a small scale (where u is close to a constant), minimal surfaces inherit properties of harmonic functions, such as the maximum principle, Harnack's inequality, and others. On a large scale, dramatic changes appear in the way that global solutions to the Laplace and minimal surface equations behave; perhaps the paradigmatic example of this dichotomy is Bernstein's theorem: the only solutions of (3) defined in the whole of $\mathbb{R}^2$ are the affine functions, while of course there are many global harmonic functions.

A consequence of the second variation of area formula (i.e., the expression for $A''(0)$) shows that every minimal surface minimizes area locally. This property justifies the word *minimal* for these surfaces (not to be confused with being a global area minimizer, which is a much more restrictive property: the unique complete surfaces in $\mathbb{R}^3$ that minimize area globally are the affine planes).

Definition 4. A surface $M \subset \mathbb{R}^3$ is *minimal* if every point $p \in M$ admits a neighborhood that minimizes area among all surfaces with the same boundary.

Definitions 1 and 4 place minimal surfaces as 2-dimensional analogues of geodesics in Riemannian geometry and connect them with the calculus of variations. Another functional of great importance is the *energy,*

$$E = \int_\Omega |\nabla X|^2 dA,$$

where again $X\colon M \to \mathbb{R}^3$ is an isometric immersion and $\Omega \subset M$ is a relatively compact domain. Area and energy are related by the inequality $E \geq 2A$, with equality occurring exactly when X is conformal. The fact that every Riemannian surface admits local conformal (isothermal) coordinates allows us to give two other equivalent definitions of minimality.

Definition 5. A conformal immersion $X\colon M \to \mathbb{R}^3$ is *minimal* if it is a critical point of the energy functional for every compactly supported variation or, equivalently, when every point on the surface admits a neighborhood that minimizes energy among all surfaces with the same boundary.

The classical formula $\Delta X = 2HN$ that links the Laplacian of an isometric immersion $X\colon M \to \mathbb{R}^3$ with its mean curvature function H and Gauss map N leads us to the next definition.

Definition 6. An isometric immersion $X = (x_1, x_2, x_3)\colon M \to \mathbb{R}^3$ of a Riemannian surface in three-dimensional Euclidean space is said to be *minimal* if its coordinate functions are harmonic: $\Delta x_i = 0$, $i = 1, 2, 3$.

From a physical point of view, the so-called Young's equation shows that the mean curvature of a surface separating two media expresses the difference of pressures between the media. When both media are under the same pressure, the surface that separates them is minimal. This happens after dipping a wire frame (mathematically, a nonnecessarily planar Jordan curve) in soapy water. However, soap bubbles that we all have blown have nonzero constant mean curvature, because they enclose a volume of air whose pressure is greater than the atmospheric pressure.

Definition 7. A surface $M \subset \mathbb{R}^3$ is *minimal* if each point $p \in M$ has a neighborhood that matches the soap film spanned by the boundary of this neighborhood.

To give the last definition of minimality, remember that the differential dN_p at each point $p \in M$ of the Gauss map N is a self-adjoint endomorphism of the tangent plane T_pM. Therefore, there exists an orthonormal basis of T_pM where dN_p diagonalizes (principal directions at p), being the opposite of the eigenvalues of dN_p, the so-called *principal curvatures* of M at p. As the mean curvature H is the arithmetic mean of the principal curvatures, the minimality of M is equivalent to the vanishing of the trace of dN_p or, equivalently, to the property that the matrix of dN_p in any orthonormal basis of T_pM is of the form

$$dN_p = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}.$$

After identifying N with its stereographic projection onto the extended complex plane, the Cauchy–Riemann equations allow us to enunciate the eighth equivalent version of minimality.

Definition 8. A surface $M \subset \mathbb{R}^3$ is *minimal* when its stereographically projected Gauss map $g\colon M \to \mathbb{C} \cup \{\infty\}$ is a meromorphic function.

Minimal surfaces appear frequently in nature.

In fact, for a minimal surface $M \subset \mathbb{R}^3$, not only is the Gauss map meromorphic but also the whole immersion can be expressed by means of holomorphic data: as the third coordinate function x_3 of M is a harmonic function, then it admits locally a conjugate harmonic function x_3^*. Thus, the *height differential* $dh := dx_3 + i dx_3^*$ is a well-defined holomorphic 1-form on M, and the surface can be conformally parameterized by the explicit formula

$$X\colon M \to \mathbb{R}^3, X(p) = \Re \int_{p_0}^{p} \left(\frac{1}{2}\left(\frac{1}{g} - g\right), \frac{i}{2}\left(\frac{1}{g} + g\right), 1\right) dh, \tag{4}$$

where $\Re$ stands for real part and p_0 is the point of M that we choose to be sent by X to the origin in $\mathbb{R}^3$ (i.e.,

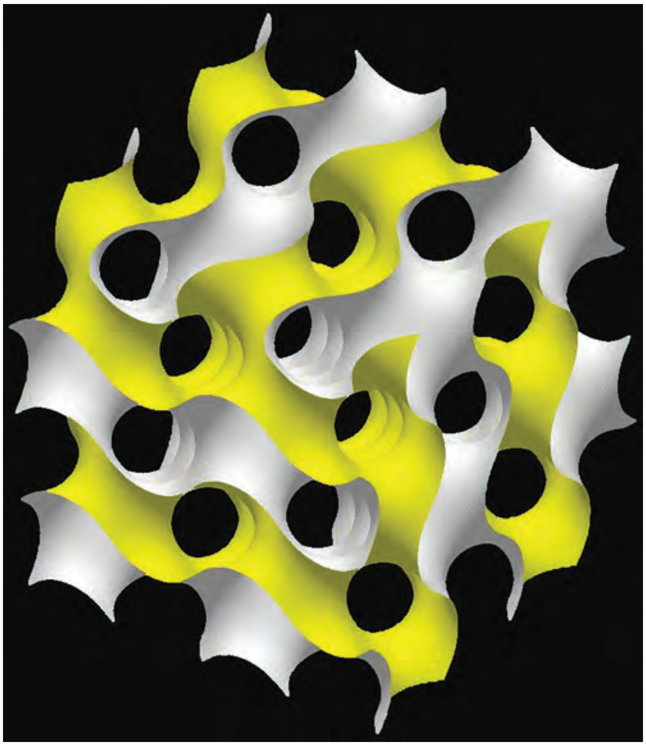

Figure 3. (left) The gyroid (A. Schoen, 1970) has been observed in diblock copolymer systems; (right) nature often seeks optimal forms in terms of perimeter and area such as minimal surfaces.

Figure 4. Frei Otto modeled the Olympic stadium in Munich on minimal surfaces.

formula (4) defines X up to an ambient translation). The pair (g, dh) is usually called the *Weierstrass data* of M.

Minimal surfaces appear frequently in nature, not only in soap films or, more generally, interfaces separating immiscible fluids at the same pressure but also for example in diblock copolymers, smectic liquid crystals (materials that have uniformly spaced layers with fluidlike order within each layer), crystallography, semiconductor technology,…even in the cuticular structure in the wing scales of certain insects! See Figure 3.

Minimization properties for this class of surfaces have motivated renowned architects such as Frei Otto to use them to design optimal structures such as the cover of the Olympic stadium in Munich (Figure 4). The beauty of their balanced forms has awakened the interest of sculptors such as Robert Engman and Robert Longhurst. From a purely mathematical viewpoint, minimal surfaces have been studied in other ambient spaces besides Euclidean space, giving rise to applications in such diverse problems as the positive mass and the Penrose conjectures in mathematical physics, the Smith conjecture on diffeomorphisms of finite order of the three-dimensional sphere, and Thurston's geometrization conjecture in 3-manifold theory.

Classical Minimal Surface Theory

By classical theory we will mean the study of connected, orientable, complete, embedded minimal surfaces in $\mathbb{R}^3$. Let $\mathcal{M}_C$ be the class of complete embedded minimal surfaces $M \subset \mathbb{R}^3$ with finite genus. In order to understand this last word, recall that the maximum principle for harmonic functions implies that there are no compact minimal surfaces without boundary in $\mathbb{R}^3$; therefore, complete minimal surfaces must have topological ends (roughly speaking, ways to go to infinity intrinsically on the surface). After compactifying topologically a minimal surface M by adding a point to each end, we define the genus of M as the genus of its compactification. If $g \in \mathbb{N} \cup \{0\} \cup \{\infty\}$ and $k \in \mathbb{N} \cup \{\infty\}$, we let $\mathcal{M}_C(g, k)$ be the subset of $\mathcal{M}_C$ that consists of those surfaces with genus g and k topological ends. When both g and k are finite, we will say that the surface has *finite topology.*

A surface $M \subset \mathbb{R}^3$ is called *proper* if every intrinsically divergent sequence of points of M also diverges in $\mathbb{R}^3$. Roughly speaking, a complete surface is proper when its topological ends are placed at infinity in $\mathbb{R}^3$ (an infinite roll of paper that wraps infinitely often and limits to an infinite cylinder from its inside or outside is an example of a complete surface that is not proper). We will denote by $\mathcal{M}_P$ the subset of $\mathcal{M}_C$ formed by the proper minimal surfaces, and we let $\mathcal{M}_P(g,k) = \mathcal{M}_P \cap \mathcal{M}_C(g,k)$.

Our goal in this section is to describe the main examples of minimal surfaces in these families, attending to their topology, conformal structure, asymptotic behavior, and the main results of classification. As we go through this description, we will discuss some of the most interesting open problems.

Complete Minimal Surfaces with Finite Topology

The trivial example in this class is the plane. The first nontrivial examples of minimal surfaces (Figure 2) belong to this class: the catenoid discovered by Euler in 1741 (genus zero and two ends) and the helicoid found by Meusnier in 1776 (genus zero, one end). Both surfaces support multiple characterizations; among the most classic ones we will mention that the catenoid is the unique minimal surface of revolution together with the plane (Euler) and that the helicoid and the plane are the unique ruled minimal surfaces (Catalan). A special mention among examples in this family is deserved by the Costa torus, the first complete minimal surface of finite topology discovered after the aforementioned ones (after 206 years!), which has genus 1 and 3 ends, and its generalization to any finite genus $g \geq 1$, discovered by Hoffman and Meeks, also with three ends; see Figure 5.

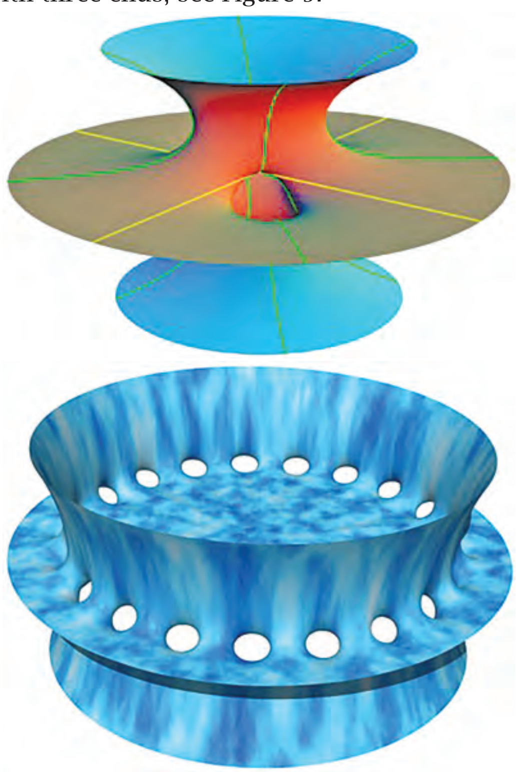

Figure 5. (top) The Costa torus; (bottom) a Hoffman–Meeks minimal surface.

Regarding the relationship between $\mathcal{M}_C$ and $\mathcal{M}_P$ in the case of finite topology, we should highlight a deep result of Colding and Minicozzi which asserts that every complete embedded minimal surface with finite topology is proper. Its proof is an application of the famous theory by Colding and Minicozzi, a topic which we will talk about a little later.

For our discussion of the case of finite topology, we will distinguish two subcases, depending on whether the number of ends of the minimal surface is one or more than one.

Surfaces with finite genus and one end. In 2005, Meeks and Rosenberg applied Colding–Minicozzi theory to show that the plane and the helicoid are the only possible examples in $\mathcal{M}_P(0,1)$ (i.e., they gave the full classification of the simply connected properly embedded minimal surfaces). By the above properness result of Colding and Minicozzi, the same uniqueness result holds in $\mathcal{M}_C(0,1)$. As for the asymptotic behavior of surfaces in $\mathcal{M}_P(g,1) = \mathcal{M}_C(g,1)$ with $1 \leq g < \infty$, Bernstein and Breiner proved in 2011 that every surface in $\mathcal{M}_P(g,1)$ is asymptotic to a helicoid and conformally parabolic.[2] For this reason, surfaces in $\mathcal{M}_P(g,1)$ are usually called *helicoids of genus g*. On existence results in this line, it is worth mentioning that Hoffman, Weber, and Wolf discovered in 2009 a helicoid of genus one with the conformal structure of a rhombic torus minus one point and that Hoffman, Traizet, and White have recently proven the existence of examples in $\mathcal{M}_P(g,1)$ for each finite $g \geq 1$ (arXiv 2015). An important open problem about $\mathcal{M}_P(g,1)$ is the possible uniqueness of examples with a given genus: this uniqueness is known in the case $g = 0$, and it is conjectured that there exists a unique helicoid of genus g for each $g \geq 1$, but even the local version of this result is not known.

Surfaces with finite genus and k ends, $2 \leq k < \infty$. The main structural result in this case is due to Collin, who proved in 1997 that if $M \in \mathcal{M}_P(g,k)$ has g, k finite and $k \geq 2$, then M lies in a particularly well-studied family: surfaces with *finite total curvature*, i.e., those where the Gaussian curvature K is integrable:

$$\int_M K\,dA = -\int_M |K|\,dA > -\infty. \tag{5}$$

(Note that since the mean curvature, the sum of the principal curvatures, is zero, the Gauss curvature, the product of the principal curvatures, is nonpositive.) By previous work of Huber and Osserman, condition (5) implies that M is conformally equivalent to a compact Riemann surface $\mathbb{M}$ of genus g to which we have removed k points (in particular, M is conformally parabolic), and both the Gauss map $g\colon M \to \mathbb{C} \cup \{\infty\}$ and the height differential dh of M extend to holomorphic objects defined on $\mathbb{M}$. This allows the application of powerful tools of complex analysis and algebraic geometry of compact Riemann surfaces; in some way, and given the lack of compactness of a complete minimal surface in $\mathbb{R}^3$, those with finite total curvature are the closest ones to being compact.

[2] *M is conformally parabolic if it does not admit any nonconstant, nonpositive subharmonic function.*

The asymptotic behavior of these minimal surfaces is also well known: every end is asymptotic to a plane or half-catenoid. On uniqueness results, we highlight the following ones:

1. Schoen proved in 1983 that if $M \in \mathcal{M}_C(g,2)$ has finite total curvature, then M is a catenoid. This is an application of the famous reflection method of moving planes of Alexandrov, which is based on the maximum principle for the equation (2).
2. In 1991, López and Ros characterized the catenoid as the only surface in $\mathcal{M}_C(0,k)$ with finite total curvature together with the plane. Again the idea is based on the maximum principle, but now applied to what has since been dubbed the *López–Ros deformation,* a 1-parameter family of minimal surfaces defined in terms of the Weierstrass data (g, dh) of a given minimal surface M. The López–Ros deformation only exists under a certain hypothesis on the flux map of the original surface.[3]
3. In 1984, Costa classified the surfaces in $\mathcal{M}_C(1,3)$ with finite total curvature. These surfaces reduce to the Costa torus and a 1-parametric family of thrice-punctured tori discovered by Hoffman and Meeks by deforming the Costa torus (and studied later by Hoffman and Karcher).

The previous result by Costa was the first complete description of a *moduli space* $\mathcal{M}_C(g,k)$ that does not reduce to a single surface: $\mathcal{M}_C(1,3)$ has the structure of a noncompact 1-dimensional manifold, identifiable with an open interval. Generalizing this result, in 1996 Pérez and Ros endowed the moduli spaces $\mathcal{M}_C(g,k)$ ($0 \le g < \infty$, $2 \le k < \infty$) with a differentiable structure of dimension $k-2$ around each minimal surface $M \in \mathcal{M}_C(g,k)$ with an additional nondegeneracy assumption that affects the linear space of Jacobi functions on M, which are the solutions $u\colon M \to \mathbb{R}$ to the second-order, linear elliptic PDE

$$\Delta u - 2Ku = 0 \quad \text{on } M,$$

where K is the Gaussian curvature of M. Until now, all known examples in $\mathcal{M}_C(g,k)$ satisfy this nondegeneracy hypothesis. We highlight that the dimension of the space of nondegenerate surfaces in $\mathcal{M}_C(g,k)$ does not depend on the genus g, but only on the number of ends k.

A major open problem is the *Hoffman–Meeks conjecture:* If $M \in \mathcal{M}_C(g,k)$, then $k \le g+2$. The best known result to date in this regard is due to Meeks, Pérez, and Ros (arXiv 2016), who proved the existence of an upper bound for k depending only on g, again by application of the Colding–Minicozzi theory.

Another important open problem consists of deciding if there exist surfaces in some moduli space $\mathcal{M}_C(g,k)$ that do not satisfy the nondegeneracy condition mentioned above, and if they do exist, provide any "reasonable" structure to the space $\mathcal{M}_C(g,k)$ around such a singular surface (as an *orbifold* ?).

Minimal Surfaces with Infinite Topology

Next we enter the world of classical minimal surfaces with infinite topology, i.e., those that have either infinitely many ends or infinite genus. The most basic examples in this family were discovered by Riemann in the nineteenth century (and posthumously published by his disciple Hattendorf) and consist of a 1-parametric family of properly embedded minimal surfaces, invariant by a translation, with genus zero and infinitely many ends asymptotic to equally spaced parallel planes. The Riemann minimal examples admit the following fascinating characterization: together with the plane, the helicoid and the catenoid, they are the unique properly embedded minimal surfaces in $\mathbb{R}^3$ that can be foliated by circles and lines in parallel planes (indeed, Riemann discovered these examples by imposing this property); see Figure 6.

The Riemann minimal examples show how the periodicity of a surface can be regarded as a method to produce examples of infinite topology: if the quotient surface by the group of isometries is not simply connected, then the lifted surface in $\mathbb{R}^3$ has infinite topology. The same thing happens with other examples of minimal surfaces discovered in the nineteenth century, such as those shown in Figure 6:

1. The singly periodic Scherk minimal surface (second from the left in Figure 6) is invariant by a cyclic group of translations. The quotient surface by the cyclic group has genus zero and four ends (asymptotic to half-planes; these ends are called *Scherk type ends*); viewed in $\mathbb{R}^3$, this surface has infinite genus and one end. We can see this example as a desingularization of two orthogonal planes by introducing infinitely many alternating holes forming a 45° angle with the planes along their line of intersection. As in the case of the Riemann minimal examples, the singly periodic Scherk minimal surface can be deformed by a 1-parametric family of singly periodic, properly embedded minimal surfaces, obtained by desingularization of planes that intersect with an angle $\theta \in (0, \pi)$.
2. The doubly periodic Scherk minimal surface (third from the left in Figure 6) is invariant by an infinite group generated by two translations of linearly independent vectors. Again, the quotient surface has genus zero and four ends (of Scherk type); viewed in $\mathbb{R}^3$, this surface has infinite genus and one end. It can be considered as the desingularization of two infinite families of equally spaced vertical half-planes, one family inside $\{(x,y,z) \mid z > 0\}$ and the other one in $\{(x,y,z) \mid z < 0\}$, in such a way that half-planes in different families cut at right angles. This surface also lies in a 1-parameter family of properly embedded, doubly periodic minimal surfaces, each of which desingularizes two infinite families of vertical half-planes in the open upper and lower half-spaces of $\mathbb{R}^3$, where the parameter is the angle

[3] *The flux of a minimal surface $M \subset \mathbb{R}^3$ is the linear map $F\colon H_1(M) \to \mathbb{R}^3$ that associates to each 1-dimensional homology class $[c] \in H_1(M)$ the integral along a representative $c \in [c]$ of the unit vector field along c that is tangent to M and orthogonal to c. The condition for the López–Ros deformation to be well defined on M is that the range of F is at most 1.*

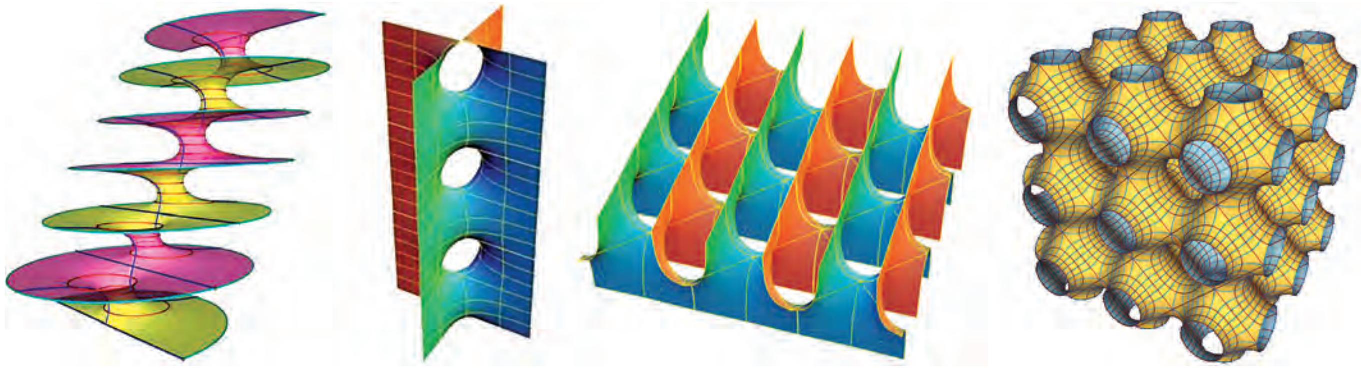

Figure 6. From left to right: a Riemann minimal example, singly and doubly periodic Scherk surfaces, and the triply periodic Schwarz P-surface.

$\theta \in (0, \pi)$ that the half-planes in the two families form. There is a direct relationship between the Scherk singly and doubly minimal surfaces, which reflects the fact that every harmonic function admits (locally) a conjugate harmonic function.

3. The triply periodic Schwarz P-surface (Figure 6, right) is invariant by the group generated by three translations of linearly independent vectors. The quotient surface by this lattice of translations is compact with genus three and lives in a three-dimensional cubic torus. Viewed in $\mathbb{R}^3$, this surface has infinite genus and one end. The Schwarz P-surface is one of the most famous *triply periodic minimal surfaces,* a class of surfaces with multiple applications to crystallography and material science: the isometry group of each triply periodic minimal surface $M \subset \mathbb{R}^3$ is a crystallographic group, and the quotient surface M over the lattice Γ of translations of rank 3 that leaves M invariant divides the three-dimensional torus $\mathbb{R}^3/\Gamma$ into two regions of equal volume called *labyrinths*. The gyroid (Figure 3, left) is another famous triply periodic minimal surface with compact quotient of genus three. The classification of triply periodic embedded minimal surfaces with quotient of genus three (the lowest possible nontrivial value) is another major open problem.

In view of the above examples, we could ask ourselves if the only method of producing minimal surfaces with infinite topology is by imposing periodicity. The answer is no, as shown in 2007 by Hauswirth and Pacard, who used *gluing techniques*[4] to merge a Hoffman–Meeks minimal surface (we mentioned these surfaces when describing examples in $\mathcal{M}_C(g, 3)$ in the subsection "Complete Minimal Surfaces with Finite Topology") with two halves of a Riemann minimal surface $\mathcal{R}$. In Figure 7 (left) we can see a schematic representation of one of the examples by Hauswirth and Pacard, when the central surface to be merged is the Costa torus (i.e., $g = 1$). Also by gluing techniques, but using Riemann surfaces with nodes, Traizet was able to prove in 2012 the existence of a complete, nonperiodic minimal surface with infinite genus and infinitely many ends asymptotic to half-catenoids (Figure 7 right). In summary, there are lots of examples in the case of infinite topology.

As for uniqueness results for minimal surfaces of infinite topology, it is clear in light of the previous paragraph that we must distinguish in some way the families that we have found: a reasonable starting point could be imposing some kind of periodicity. Here, it is worth mentioning the following classification results for moduli spaces of periodic minimal surfaces with prescribed topology:

1. The Riemann minimal examples are the unique properly embedded minimal tori with finitely many planar ends in a quotient of $\mathbb{R}^3$ by a translation (Meeks, Pérez, and Ros, 1998). The number of ends must be even, and when we fix this number the corresponding moduli space is a noncompact manifold of dimension 1.
2. The Scherk doubly periodic minimal surfaces are the unique properly embedded minimal surfaces with genus zero and finitely many ends in a quotient of $\mathbb{R}^3$ by two independent translations (Lazard–Holly and Meeks, 2001). Again the number of ends is necessarily even, and for a fixed number of ends, the moduli space is diffeomorphic to an open interval.
3. The moduli spaces of properly embedded minimal tori with any fixed finite number of parallel planar ends in a quotient of $\mathbb{R}^3$ by two linearly independent translations were described in 2005 by Pérez, Rodríguez, and Traizet. Each of these moduli spaces (for any fixed even number of ends) is a noncompact manifold of dimension 3 whose surfaces are called *KMR examples* (in honor of Karcher, Meeks, and Rosenberg, who previously found 1-parameter families of these surfaces in this moduli space).
4. The moduli spaces of properly embedded minimal surfaces with genus zero and finitely many ends of Scherk type in a quotient of $\mathbb{R}^3$ by a translation were

[4] *This technique consists of a sophisticated application of the implicit function theorem to the mean curvature operator defined between certain Banach spaces.*

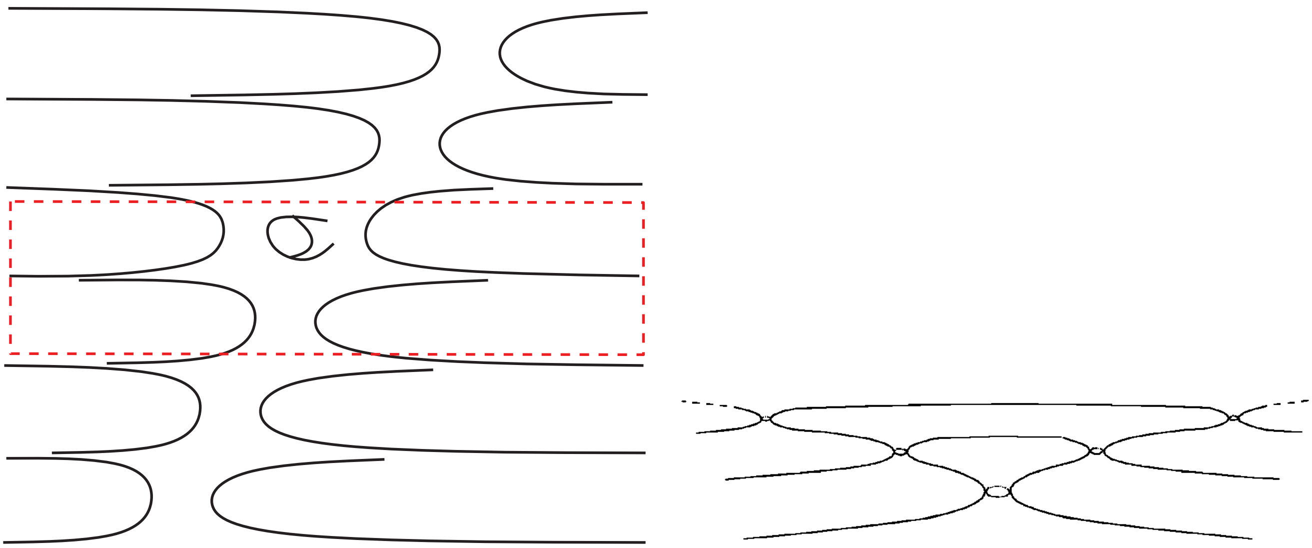

Figure 7. From left to right: schematic representations of a Hauswirth–Pacard minimal surface with genus 1 and of the example of infinite genus by Traizet: two nonperiodic, complete embedded minimal surfaces with infinite topology.

classified in 2007 by Pérez and Traizet. In this case, these moduli spaces are noncompact manifolds of dimension $2k - 3$ (here $2k$ is the number of ends), whose surfaces were discovered by Karcher in 1988 as a generalization of the Scherk simply periodic minimal surfaces.

The four uniqueness results listed above have a common flavor. First, periodicity is used in a strong way, since it allows working in the quotient of $\mathbb{R}^3$ by the corresponding group of isometries, and the quotient minimal surfaces always have finite total curvature in the sense of equation (5); in this setting, one can control the asymptotic geometry and the conformal representation of the minimal surfaces under study. Second, the desired uniqueness follows from a continuity argument: one starts by proving that any surface M in each of these moduli spaces can be deformed within the moduli space (openness part) until arriving at a point M_∞ in the boundary of the moduli space, which turns out to be a properly embedded minimal surface with simpler topology or periodicity than those in the original moduli space (compactness part); this compactness requires a rather complete understanding of the possible limits of sequences of minimal surfaces in the original moduli space. Once we have arrived at M_∞, the desired global uniqueness follows from an inverse function theorem argument (local uniqueness around M_∞) that needs the previous classification of the moduli space of minimal surfaces to which M_∞ belongs. This last aspect reveals a stratified structure in the moduli spaces of embedded minimal surfaces with prescribed topology and periodicity: the boundary of a given moduli space is the union of other moduli spaces of minimal surfaces with simpler topology or periodicity. For instance, the description of the moduli space in item 3 above requires solving the classification problem in item 4.

The strategy sketched in the preceding paragraph fails badly if we seek classification results for minimal surfaces with infinite topology without imposing periodicity, but in this case the Colding–Minicozzi theory is of great help, as we will explain next.

Colding–Minicozzi Theory

Consider the following question:

Problem 9. *What are the properly embedded minimal surfaces in $\mathbb{R}^3$ with genus zero?*

Suppose that $M \subset \mathbb{R}^3$ is a surface that meets the conditions of Problem 9. As explained above, in the case that M has only one end we know that M is a plane or a helicoid (Meeks and Rosenberg). If M has k ends with $2 \leq k < \infty$, then M is a catenoid by the theorems of Collin and López–Ros. It remains to study the case that M has infinitely many ends. If we knew that such an M were invariant by a translation T, then it would not be difficult to check that the quotient surface of M by the cyclic group generated by T is a torus with finitely many ends, and hence M is a Riemann minimal example by the 1998 result of Meeks, Pérez, and Ros. Therefore, a way of solving Problem 9 is to prove that if the number of ends of M is infinite, then M is periodic.

Often we face the problem of understanding the possible limits of a sequence of embedded minimal surfaces. As a trivial example, let us think of a surface $M \subset \mathbb{R}^3$ that is invariant by a translation of vector $v \in \mathbb{R}^3 - \{0\}$. The constant sequence $\{M_n := M - nv = M\}_{n\in\mathbb{N}}$ has as trivial limit M itself. This naive example suggests a possible way to solve Problem 9: suppose that a properly embedded minimal surface $M \subset \mathbb{R}^3$ with infinitely many ends is a solution to this problem. As the number of ends of M is infinite, one can deduce that M has infinite total curvature, whence we can find a divergent sequence of points $p_n \in M$

where the unit normal to M takes the same value. It is reasonable to try to conclude that $\{M_n = M - p_n\}_n$ has (at least) a convergent subsequence as a step to demonstrate the desired periodicity of M. We have thus transformed Problem 9 into another one, perhaps more ambitious:

Problem 10. *Under what conditions can we extract a convergent subsequence from a given sequence of embedded minimal surfaces?*

Suppose that $\{M_n\}_n$ is a sequence of embedded minimal surfaces in an open set $A \subset \mathbb{R}^3$. Also assume that $\{M_n\}_n$ has at least one accumulation point, since we do not want the M_n to completely escape and have nothing to analyze in the limit. Each surface M_n can be locally written as the graph of a function u_n defined in an open subset of the tangent plane of M_n at a given point, and the size of the domain of u_n can be uniformly controlled if we have uniform local bounds for the Gaussian curvatures (equivalently, for the second fundamental forms) of the surfaces. If in addition we have uniform local area bounds for the M_n, then we will control the number of graphs that lie in a given region of A. Therefore, working in a very small (but uniform) scale, we will deduce that every surface M_n gives rise to a single graphing function u_n. Thus we have transformed Problem 10 about convergence of surfaces into another problem, the convergence of graphs. In this setting, the uniform local curvature bounds for the M_n produce equicontinuity of the u_n, and the fact that we are working locally produces uniform boundedness for the u_n. Therefore, the Arzelá–Ascoli Theorem insures that a subsequence of the u_n converges uniformly to a limit function u_∞ that can be proven to satisfy the same PDE (2) as the u_n. A prolongation argument now implies that a subsequence of $\{M_n\}_n$ converges to an embedded minimal surface in A, and thus our Problem 10 is solved in this case.

If we do not have local area bounds for the M_n but still assume local uniform curvature bounds, reasoning similar to the above leads to the conclusion that a subsequence of $\{M_n\}_n$ converges to a natural generalization of the notion of minimal surface: a *lamination* whose leaves are minimal surfaces. Without going into detail, a lamination $\mathcal{L}$ of A is a closed union (in the induced topology on A) of surfaces embedded in A, called *leaves* of $\mathcal{L}$, with a certain local product structure. This means that we can take local coordinates in A that transform the leaves into the product of a two-dimensional disk with a closed subset of $\mathbb{R}$, which we can think of as the heights of disjoint copies of that disk placed horizontally (see Figure 8). This local product structure endows the leaves of $\mathcal{L}$ with the structure of smooth, pairwise disjoint surfaces. A lamination is said to be minimal if its leaves are all minimal surfaces. For example, if Z is a nonempty closed subset of $\mathbb{R}$, then the collection of horizontal planes $\mathcal{L}_Z = \{P_z = \mathbb{R}^2 \times \{z\} \mid z \in Z\}$ is a minimal lamination of $A = \mathbb{R}^3$ whose leaves are the planes P_z. In the case that a lamination $\mathcal{L}$ of A does not leave any empty spaces in A, we call it a *foliation* of A ($\mathcal{L}_Z$ is a foliation of $\mathbb{R}^3$ when $Z = \mathbb{R}$). The theory of minimal laminations is a natural extension of the one of minimal surfaces. However, we still have not

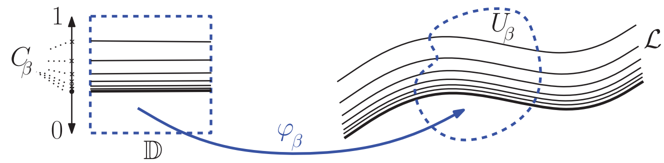

Figure 8. The open set A is covered by images U_β of local charts φ_β, each one transforming a collection of disks at heights lying in a closed subset C_β of $[0,1]$ into portions of the leaves of $\mathcal{L}$.

provided any examples of a nontrivial minimal lamination of $\mathbb{R}^3$ that does not consist of a single embedded minimal surface other than a collection of planes $\mathcal{L}_Z$ as above.

Coming back to our Problem 10, what can we say about the limit of the M_n if these embedded minimal surfaces do not have uniform local bounds for their second fundamental forms? Here is where the theory of Colding–Minicozzi comes to our aid. Following the previous notation, the lack of uniform local curvature bounds implies that the Gaussian curvature of the M_n blows up at some point of A; i.e., the following set is nonempty:

$$\widehat{S} = \left\{ x \in A \mid \sup |K_{M_n \cap \overline{\mathbb{B}}(x,r)}| \to \infty, \ \forall r > 0 \right\}, \tag{6}$$

where K_Σ denotes the Gaussian curvature of a surface Σ and $\overline{\mathbb{B}}(x,r)$ is the closed ball centered at $x \in \mathbb{R}^3$ with radius $r > 0$. The Colding–Minicozzi theory describes the limit of (a subsequence of) the M_n in the above scenario under an additional hypothesis: each M_n must be topologically a compact disk that is contained in a ball of radius $R_n > 0$, say centered at the origin, with boundary ∂M_n contained in the boundary sphere of that ball. The description of this limit is very different depending upon whether the sequence of radii R_n diverges or stays bounded.

Theorem 11 (Colding–Minicozzi)**.** *Given $n \in \mathbb{N}$, let M_n be an embedded minimal disk in a closed ball $\overline{\mathbb{B}}(R_n) = \overline{\mathbb{B}}(\vec{0}, R_n)$ with $\partial M_n \subset \partial\mathbb{B}(R_n)$. If $R_n \to \infty$ and $\widehat{S} \cap \mathbb{B}(1) \neq \emptyset$, then a subsequence of the M_n converges to a foliation of $\mathbb{R}^3$ by parallel planes, away from a straight line*[5] *(called the* singular set of convergence*), along which the curvature of M_n blows up when $n \to \infty$.*

To better understand the last result, we will use the following example. Consider the standard vertical helicoid $H = \{(x,y,z) \mid x \sin z = y \cos z\}$. Take a sequence of positive numbers λ_n tending to zero, and consider for each $n \in \mathbb{N}$ the homothetic copy $M_n = \lambda_n H$ of H by ratio λ_n that is again minimal and simply connected. As n increases, M_n can be thought of as a new view of H from a viewpoint that becomes further and further away, as in Figure 9. The farther away we look at the helicoid H, the more it looks like a collection of horizontal planes separated by smaller and smaller distances, and so in the

[5]*In fact, Colding and Minicozzi proved that the singular set of convergence is a Lipschitz arc transverse to the limit foliation. Using the uniqueness of the helicoid as the only nonflat surface in $\mathcal{M}_P(0,1)$, Meeks deduced in 2004 that this Lipschitz arc is indeed a line.*

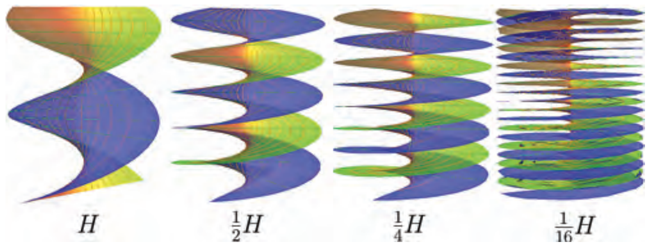

Figure 9. Homothetic images of the same vertical helicoid H, with ratio $\lambda_n = 1/2^n$.

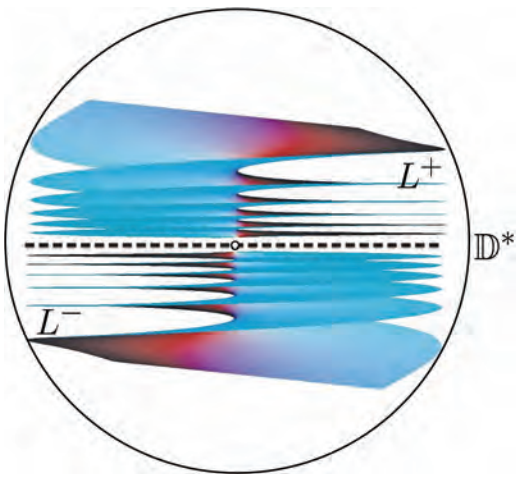

Figure 10. A minimal lamination of the punctured unit ball, with three leaves and a singularity at the origin.

limit we obtain the foliation of $\mathbb{R}^3$ by horizontal planes. Observe that each leaf of this limit foliation is flat (its Gaussian curvature is identically zero) and the Gaussian curvatures of the M_n converge to zero away from the z-axis. However, since the Gaussian curvature of H along the z-axis is constant -1, the Gaussian curvature of M_n along the same axis is $-1/\lambda_n^2$, which tends to infinity. In other words, the singular set $\widehat{S}$ defined in (6) is the z-axis in this example. Also note that the limit foliation is perfectly regular along $\widehat{S}$; it is only the convergence of the M_n to the limit that fails along $\widehat{S}$. This limit object is known as a *limiting parking garage structure* with one column: away from the z-axis, the structure becomes arbitrarily flat and horizontal (this is where cars park), and to travel from one parking floor to another one, cars have to go up the ramp (around the column at the z-axis). Well, Theorem 11 tells us that the general behavior of the limit of the embedded minimal disks M_n when the radii R_n tend to infinity is essentially the same as this example.

The description when the radii R_n remain bounded can also be visualized with an example. In 2003, Colding and Minicozzi produced a sequence of minimal disks M_n embedded in the closed unit ball $\overline{\mathbb{B}}(1)$ and of helicoidal appearance, such that the number of turns that the boundary curve ∂M_n makes around the z-axis tends to infinity as $n \to \infty$, and the limit of the M_n is a minimal lamination $\mathcal{L}$ of $\mathbb{B}(1) - \{\vec{0}\}$ that consists of three leaves: one is the horizontal punctured unit disk $\mathbb{D}^* = \{(x, y, 0) \mid 0 < x^2 + y^2 < 1\}$, and the other two are nonproper minimal surfaces L^+, L^- that rotate infinitely many times from above and below $\mathbb{D}^*$, accumulating on $\mathbb{D}^*$ as in Figure 10. In this case, the Gaussian curvature of the M_n blows up at the origin, but this time $\vec{0}$ is a genuine singularity of the limit lamination $\mathcal{L}$, which does not admit a smooth extension across $\vec{0}$.

The theoretical description by Colding and Minicozzi for the limit of a sequence of compact, embedded minimal disks $M_n \subset \overline{\mathbb{B}}(R_n)$ with $\partial M_n \subset \partial\mathbb{B}(R_n)$ and bounded radii R_n is very technical, and we will omit it here. Instead, we will simply mention that after extracting a subsequence, the M_n converge to a *minimal lamination with singularities.* The singularities of such a limit singular minimal lamination form a closed set, and each singularity is of one of the following two types:

(a) Isolated singularities, in which case Figure 10 shows essentially the behavior of the limit object: there is a leaf D^* of the lamination that limits to the singularity p (in fact, D^* extends smoothly across p to an embedded minimal disk D) and one or two nonproper leaves, which rotate infinitely many times and accumulate at D^*. Furthermore, portions of the M_n outside a solid cone of axis $p + (T_pD)^\perp$ can be written as multivalued graphs over annular regions of T_pD, and as $n \to \infty$, these annular regions converge to a punctured disk, at the same time that the number of turns of the multivalued graphs inside the M_n become arbitrarily large and the multivalued graphs collapse into D^* as in Figure 11 (left).

(b) Nonisolated singularities, each of which is the limit of at least one sequence of isolated singularities, as in Figure 11 (right).

It should also be noted that in the above description, the set $\widehat{S}$ where the Gaussian curvatures of the disks M_n blow up consists of not only the singularities of the limit lamination $\mathcal{L}$ but also possibly embedded arcs of class $C^{1,1}$ around which $\mathcal{L}$ is a local foliation, as in Figure 11 (left). In particular, these arcs are not singularities of $\mathcal{L}$ except for their end points, as in the convergence of the M_n to $\mathcal{L}$ in Theorem 11.

The previous description leads us directly to the study of the singularities of a minimal lamination of an open subset of $\mathbb{R}^3$. Does this set have any reasonable structure? This question is another open central problem in minimal surface theory. Along this line, it is worth mentioning two recent results of Meeks, Pérez, and Ros (2016):

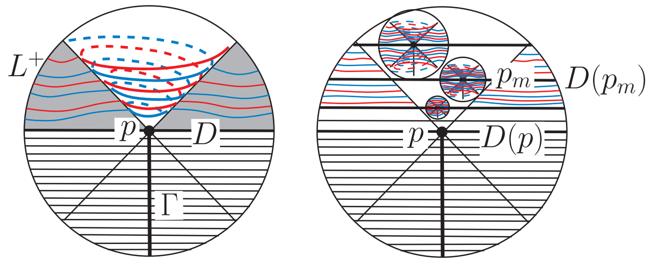

Figure 11. Left: schematic representation of an isolated singularity p of a singular minimal lamination $\mathcal{L}$ obtained as a limit of embedded minimal disks M_n, with a nonproper leaf L^+ at one side of the disk leaf D that passes through p. Locally around p and outside a solid cone of vertex p and axis $p + (T_pD)^\perp$, L^+ is a multivalued graph. The other side of D is foliated by leaves of $\mathcal{L}$. The convergence is singular along a $C^{1,1}$ arc Γ. Right: at a nonisolated singularity p, the disk leaf $D = D(p)$ passing through p is also the limit of the corresponding disk leaves $D(p_m)$ associated to isolated singularities p_m that converge to p.

Theorem 12 (Local Removable Singularity Theorem). *Let $\mathcal{L} \subset \mathbb{B}(1) - \{\vec{0}\}$ be a minimal lamination. Then $\mathcal{L}$ extends to a minimal lamination of $\mathbb{B}(1)$ (i.e., the singularity at $\vec{0}$ is removable) if and only if the Gaussian curvature function $K_{\mathcal{L}}$ of the lamination does not blow up at the origin faster than the square of the extrinsic distance to $\vec{0}$; i.e., $|K_{\mathcal{L}}|(x) \cdot \|x\|^2$ is bounded in $\mathcal{L}$.*

It follows from Theorem 12 that if the function $|K_{\mathcal{L}}|(x) \cdot \|x\|^2$ is bounded in a minimal lamination $\mathcal{L} \subset \mathbb{B}(1) - \{\vec{0}\}$, then $|K_{\mathcal{L}}|(x) \cdot \|x\|^2$ extends across the origin with value zero. Another consequence of this theorem is that in the example of Figure 10, the Gaussian curvature of the disks M_n blows up faster than the square of the distance to the origin as $n \to \infty$.

Another result about singularities of minimal laminations is a global version of Theorem 12 that classifies the minimal laminations of $\mathbb{R}^3 - \{\vec{0}\}$ with quadratic decay of curvature:

Theorem 13. *Let $\mathcal{L} \subset \mathbb{R}^3 - \{\vec{0}\}$ be a nonflat minimal lamination such that $|K|(x) \cdot \|x\|^2$ is bounded. Then $\mathcal{L}$ extends across the origin to a minimal lamination of $\mathbb{R}^3$ that consists of a single leaf M, which is a properly embedded minimal surface with finite total curvature. In particular, $|K|$ decays much faster than quadratically with the distance to the origin: $|K|(x) \cdot \|x\|^4$ is bounded in M.*

We have said that in order to apply the theory of Colding–Minicozzi to a sequence M_n of embedded minimal surfaces, we need to assume that the M_n are compact disks with boundaries in ambient spheres. This condition is not really a restriction, as it can be naturally obtained by a rescaling argument so that the injectivity radius function of the rescaled minimal surfaces is uniformly bounded away from zero (Meeks, Pérez, and Ros [4]).

Classification of the Properly Embedded Minimal Surfaces in $\mathbb{R}^3$ with Genus Zero

To finish our brief tour through the current state of the classical minimal surface theory, we return to Problem 9 on the properly embedded minimal surfaces M in $\mathbb{R}^3$ with genus zero. In the first paragraph of the section "Colding–Minicozzi Theory" we explained that the problem reduces to proving that if M has infinitely many ends, then M is periodic. This strategy, which uses Colding–Minicozzi theory as we have mentioned above, was the one used by Meeks, Pérez, and Ros [5] to prove the following result:

Theorem 14. *Every properly embedded minimal surface $M \subset \mathbb{R}^3$ with genus zero is either a plane, a helicoid, a catenoid, or one of the Riemann minimal examples. In particular, M is foliated by circles or straight lines in parallel planes.*

A final remark about the proof of Theorem 14 is in order. The theory of Colding and Minicozzi yields only the *quasi-periodicity* of M (this means that if $\{p_n\}_n$ is a divergent sequence of points in M, then a subsequence of $\{M - p_n\}_n$ converges to a properly embedded minimal surface in $\mathbb{R}^3$ with genus zero and infinitely many ends). The key to proving the desired periodicity of M once we know it is quasi-periodic is a fascinating application of the theory of integrable systems and more precisely of the holomorphic Korteweg-de Vries equation, a third-order PDE that models mathematically the behavior of waves on shallow water surfaces.

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Joaquín Pérez

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Joaquín Pérez received a PhD in mathematics from the University of Granada under Antonio Ros in 1996. He is a member of the International Relations Committee of the Royal Mathematical Society of Spain and director of the Research Mathematical Institute IEMath-GR, one of the four venues of the Spanish Institute of Mathematics.

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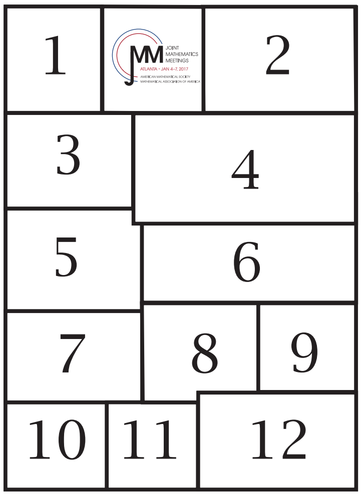

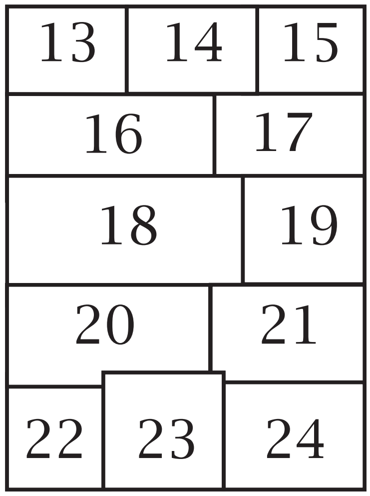

1) Attendee Kevin Grace, LSU
2) AMS Booth
3) AMS Director of Education and Diversity Helen G. Grundman (right) receives the M. Gweneth Humphreys Award from AWM Past President Kristin Lauter (left).
4) JMM Opening Day Ribbon Cutting; pictured left to right: Gerard Venema, MAA Associate Secretary; Kenneth A. Ribet, AMS President; Robert L. Bryant, AMS Immediate Past President; Francis Su, MAA President; Michael Pearson, MAA Executive Director; Catherine A. Roberts, AMS Executive Director; and Barbara Faires, MAA Secretary.
5) AMS Associate Executive Director, Publishing Robert Harington at *Mathemati-con.*
6) The AMS celebration dinner.
7) Attendees
8) Former MAA Vice President Lloyd E. Douglas with Jim Maxwell, AMS Coordinator of Special Projects.
9) David Bailey receiving the Conant Prize from AMS Immediate Past President Robert L. Bryant.
10) Mathematical Art Exhibition winner by Doug Dunham and John Shier
11) James G. Arthur (right) receives the 2017 AMS Leroy P. Steele Prize from AMS Immediate Past President Robert L. Bryant (left).
12) Attendees enjoying the art exhibits.
13) MAA Press Booth
14) WWTBAM contestants
15) Janet Heine Barnett receives a 2017 MAA Deborah and Franklin Tepper Haimo Award for teaching effectiveness and influence from MAA President Francis Su.
16) *Mathemati-con* crowd
17) Mathematical Mime Tanya Chartier
18) The lively WWTBAM crowd with Glenn Stevens, Boston University; Ken Ono, Emory University; Robert L. Bryant, AMS Immediate Past President; Kenneth A. Ribet, AMS President; and Catherine A. Roberts, AMS Executive Director judging from the front row.
19) Dietmar Salamon and Dusa McDuff receive the 2017 AMS Leroy P. Steele Prize for Exposition from AMS Immediate Past President Robert L. Bryant.
20) AWM's Women in Mathematics Panel Discussion: Mentoring Women in Mathematics. Panelists pictured left to right: Helen G. Grundman, Bryn Mawr College; Suzanne Weekes, Worcester Polytechnic Institute; Emina Soljanin, Rutgers University; Kristin Lauter, Microsoft Research; and Deanna Haunsperger, Carleton College.
21) Attendees Joseph Gallian and Aparna Higgins enjoy a laugh.
22) Donald St. P. Richards' AMS-MAA Invited Address on Distance Correlation
23) Lillian Pierce's MAA Invited Address "From Gauss to Today"
24) The YP17 HCSSiM Reunion Breakfast

Photographs by Kate Awetry, JMM 2017 Photographer; Annette Emerson, Samantha Faria, AMS.

JOINT MATHEMATICS MEETINGS
ATLANTA • JAN 4–7, 2017
AMERICAN MATHEMATICAL SOCIETY
MATHEMATICAL ASSOCIATION OF AMERICA

See photo key on page 359.

Starting this April, "Mathematics Awareness Month" has become "Mathematics and Statistics Awareness Month."

Further, the Joint Policy Board for Mathematics—consisting of the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics—has decided that to simplify coordination efforts there will no longer be an annual theme.

In this issue of *Notices* we feature four pieces on Mathematics and Statistics Awareness Month: Michael Barany traces the origins of mathematics awareness to World War II; Andreas Daniel Matt describes the IMAGINARY international math exhibitions; David Eisenbud and Ronald Wasserstein provide short opinion pieces on the celebration of mathematics and statistics.

The World War II Origins of Mathematics Awareness

Michael J. Barany

Comments are invited at the *Notices* website: www.ams.org/journals/notices/.

Since ancient times, advocates for mathematics have argued that their subject is foundational for many areas of human endeavor, though the areas and arguments have changed over the years. Much newer, however, is the idea that mathematicians should systematically try to promote the usefulness or importance of mathematics to the public. This effort, which I shall generically call "mathematics awareness," was largely an American invention. One outward manifestation was the 1986 inauguration, by President Ronald Reagan, of the first Mathematics Awareness Week. Every year since then, mathematicians and mathematics educators in the United States have dedicated a week—or, beginning in 1999, the month of April—to raising public awareness of "the importance of this basic branch of science to our daily lives," as Reagan put it.

To understand the origins of mathematics awareness, one must follow the money.

While today's mathematics awareness is focused on schools and on peaceful applications of mathematics, a direct line connects it to its origins in a very different kind of activity: mathematicians promoting their expertise to leaders of the American war effort during World War II. Recent mathematics awareness has focused on encouraging more people to take up the discipline. However, wartime and early postwar mathematics awareness centered on securing resources for those already in the profession. To understand the origins of mathematics awareness, one must follow the money.

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DOI: http://dx.doi.org/10.1090/noti1506

A War of Mathematics

World War II was not the first war that mathematicians attempted to characterize as "a war of mathematics," but it was the first one where the characterization appeared to stick. In addition to lobbying elite policymakers, mathematicians wrote articles for the popular press and offered radio broadcasts that attempted to explain why mathematics mattered in terms the masses could understand. For example, Bennington P. Gill, who served as AMS treasurer from 1938 to 1948, gave an interview in 1942 with WNYC for their series on *The Role of Science in War*; see www.wnyc.org/story/bennington-p-gill/.

A Discipline in Need

For most of the discipline's history, mathematicians have supported their research either through independent wealth or through patronage from the wealthy. Universities and a select few other academic institutions—all, themselves, historically channels for wealthy patronage—eventually became the dominant sites and funders of mathematical scholarship. So long as publication and travel were relatively small parts of such scholarship, this arrangement suited mathematicians' needs well enough. But by the early twentieth century mathematicians were publishing and traveling much more than before and across greater distances. They needed new organizations and new sponsors to support their work.

Such were the rationales for mathematicians' first professional societies, many of which date to the latter part of the nineteenth century and the start of the twentieth.

IAS mathematician Marston Morse, who served as AMS president during 1941–1942, approached the Rockefeller Foundation with the argument that his discipline was "unique… as having no natural sources of support."

The AMS originated in 1888 on the heels of corresponding societies in Europe such as the London Mathematical Society (1865) and Société Mathématique de France (1872). The Mathematical Association of America (MAA) entered the scene at the close of 1915. These societies drew their support principally from their members' universities (both directly and by way of members' dues), national governments, and private sponsorship.

Following the First World War, American mathematicians had some limited success securing corporate sponsorship for their work, for instance from the American Telephone & Telegraph Company, and considerably greater success courting major philanthropies such as the Rockefeller Foundation and Carnegie Corporation of New York. Yet these relationships tended to be piecemeal and tenuous. American mathematicians had successfully bid to host the 1924 International Congress of Mathematicians but ended up ceding the congress to Toronto after finding themselves unable to secure the needed financial backing. (John Charles Fields, on the other hand, managed to find enough money for the Toronto meeting that it concluded with a modest surplus, which provided the seed money for what became the Fields Medals.)

The Americans tried again to host an International Congress in 1940. When preliminary fundraising efforts again fell short, Institute for Advanced Study mathematician Marston Morse approached the Rockefeller Foundation with the argument that his discipline was "unique…as having no natural sources of support." Indeed, when it came to major donors, Rockefeller and Carnegie were the only relatively sure bets, and no grant was assured. To grow, American mathematics would need new constituencies and new sources of funding.

Preparing for War

The AMS suspended plans for the envisioned 1940 International Congress of Mathematicians following the German invasion of Poland in 1939. As war threatened to engulf Europe and beyond, American mathematicians thought back to their experiences of the Great War. Some concluded that a lack of coordination among American mathematicians had restricted their contributions to the previous war effort. Without such coordination, military leaders would have a hard time learning where mathematicians were needed and where the needed mathematicians could be found.

A new joint AMS-MAA War Preparedness Committee aimed to provide this coordination by synthesizing the lessons from the last war and positioning mathematicians for a new conflict that seemed sure to draw American involvement sooner or later. Marston Morse was appointed chair, and a subcommittee chaired by Dunham Jackson focused on mathematical research. The subcommittee included Marshall Stone, whose two-year presidency of the AMS would fall in the middle of the United States' official engagement in World War II. From start to finish, Stone advocated formal mathematical coordination with particular force and frequency. In a summer 1940 missive on "the organizational aspects of the research problems of national defense," Stone articulated three purposes for the subcommittee. First and most urgent was to find an efficient means to join together current "technical problems and competent mathematicians" who could solve them. Second, the US would need to make much greater use of mathematical techniques than it currently did. The third, long-range, goal was to make war service pay off for the US mathematical profession even after the war's end.

Marshall Stone argued that math needed propaganda.

"If mathematics is to be brought to bear upon our defense problems in full measure," Stone then asserted, "we shall have to organize and conduct propaganda to this end." He anticipated an uphill struggle. The subcommittee would have to confront "not only the appalling limitations of our military officers, but also the general American attitude of antagonism to theory in general and to mathematical refinements in particular and the abysmal ignorance of the majority of intelligent Americans concerning the uses of mathematics."

Enter Mina Rees

The US military officially entered the war as 1941 drew to a close. The next year the AMS and MAA responded by dissolving the War Preparedness Committee and appointing a new War Policy Committee, with Marshall Stone (soon to be AMS president) as chair and Marston Morse (who was just finishing his own term as AMS president) in a supporting role. Soon, leaders from academia, philanthropy, and the military drew on approaches from their respective fields to develop for the US government a system for identifying problems and contracting them out to academic research groups. This formed the basis for a massive system of contracts that would support advanced mathematical research and training after the war, as well as the postwar system of government grants familiar to many mathematicians today. But mathematics awareness remained the exclusive province of a narrow elite.

Richard Courant, one of the academic leaders who helped to craft that system, used his wartime government connections both during and after the war to build his institute at New York University into one of the world's leading centers of mathematics. Perhaps even more important for postwar mathematics in and beyond the United States, however, was Courant's close associate Mina Rees. Although she had earned a PhD under Leonard Dickson from the University of Chicago, Rees's prospects within the mathematics profession were limited by widespread institutional sexism. At Courant's urging, Rees was appointed as technical aide to the main government clearinghouse for coordinating mathematicians' war service. There, she facilitated the broad array of contracts by gathering information, assessing outcomes, and making needed connections.

Stone, in 1944, expressed his frustration that the government "would display considerable reluctance to call on the leaders of our profession." In his view, mathematicians could and should have done much more to dedicate themselves wholly to the war effort. He himself set out immediately after the conclusion of his term as AMS president on a mission classified top secret to advise and assess Allied signal intelligence in India, Burma, and China in the first part of 1945.

Mathematicians in the US concluded the war with a range of views of their relative success or failure. While they did not lay claim to a breakthrough on the scale of the Manhattan Project as physicists would, mathematicians contributed to a great many of the United States' decisive wartime innovations in weaponry, aeronautics, provisioning, communications and intelligence, and other areas.

Marshall Stone, who served as AMS president during 1943–1944, focused his efforts on a few key men at the top.

A Few Key Men at the Top

Marshall Stone's dim view of the public appreciation for mathematics led him to focus on a few "key men at the top" rather than aim to convince the masses or even the much smaller mass of officers and policymakers. Among those key men were Harvard president James Bryant Conant, chair of the National Defense Research Committee, and Frank Jewett, chairman of the Board of Directors of Bell Laboratories and president of the National Academy of Sciences. Referring to the commonplace characterization of the Great War as the chemist's war, Conant famously quipped on the front page of *Chemical & Engineering* News in November 1941,"This is a physicist's war rather than a chemist's." According to AMS secretary Roland Richardson, when Conant shared the view with Jewett the latter shot back that "It may be a war of physics, but the physicists say it is a war of mathematics." At least one key man got the message.

Further Reading: Michael J. Barany, "Remunerative combinatorics: Mathematicians and their sponsors in the mid-twentieth century," in *Mathematical Cultures: The London Meetings 2012–2014*, edited by Brendan Larvor (Basel: Birkhäuser, 2016), pp. 329–346; preprint online at `mbarany.com/publications.html`.

Coordination for a Growing Discipline

In 1946 Rees took over the mathematics arm of the Office of Naval Research (ONR), helping the navy to become a leading funder of research and publication in both pure and applied mathematics. One of her earliest efforts was to forge a partnership with the AMS's *Mathematical Reviews* to translate new Russian mathematical works into English. This and related undertakings reinforced the United States as an international clearinghouse for postwar mathematics, allowing American institutions to assume a lasting dominance in the discipline.

The ONR led the way for a wide range of government-funded research programs associated with other branches of the military and various civilian offices. These eventually included the National Science Foundation, founded in 1950 after years of debate informed by wartime and early postwar military-sponsored science. ONR and other contracts funded faculty, students, seminars, and publications on a wide scale in the first postwar decades. They supported visiting researchers from abroad and allowed US mathematicians to disseminate their work rapidly and efficiently. These new funding sources allowed the mathematics profession to grow quickly—so quickly, in fact, that US mathematicians could turn from worrying about finding enough sources of support to worrying about finding enough people to make good use of that support.

Over the years, as new mathematical organizations sprang up and the mathematical profession collectively faced new funding and policy issues, the AMS and MAA continued to adapt and reconfigure a series of joint undertakings aimed at the kind of mathematics awareness pursued in the 1940s.[1] Early postwar joint committees

[1] *For a blow-by-blow account of the various joint committees, see Everett Pitcher,* A History of the Second Fifty Years: American Mathematical Society, 1939–1988 *(Providence: American Mathematical Society, 1988), pp. 273–285.*

Mina Rees, pictured here around 1966 with Leon Henkin, had an enormous impact in her position as head of the mathematics arm of the Office of Naval Research.

Mina Rees

In recognition of Rees's contributions to the mathematics profession, the AMS Council adopted a resolution in 1953 asserting:

Under her guidance, basic research in general, and especially in mathematics, received the most intelligent and wholehearted support. No greater wisdom and foresight could have been displayed and the whole postwar development of mathematical research in the United States owes an immeasurable debt to the pioneering work of the Office of Naval Research and to the alert, vigorous and farsighted policy conducted by Miss Rees. (Quoted from the *Bulletin of the AMS*, March 1954.)

Rees continued after 1953 to serve as an administrator and advisor for a wide range of important boards and institutions, fostering the development of pure and applied mathematics in and beyond the United States.

Further reading: Judy Green, Jeanne LaDuke, Saunders MacLane, and Uta C. Merzbach, "Mina Spiegel Rees (1902–1997)," *Notices of the American Mathematical Society* **45**, (1998) no. 7, 866–873; Amy Shell-Gellasch, *In Service to Mathematics: The Life and Work of Mina Rees* (Boston: Docent Press, 2011).

laid the groundwork for the International Mathematical Union and advocated for the new NSF and a mathematics division in the National Research Council. The 1970s-era Joint Projects Committee in Mathematics eventually grew into the Joint Policy Board for Mathematics, which currently spearheads the annual Mathematics Awareness Month (in 2017 the name became Mathematics and Statistics Awareness Month).

A New Wave

If more people saw how important math was to their lives, the thinking since the 1980s has been, more people would participate in shaping that mathematics in the future. In practice, it has not always worked out that way. National and even international campaigns for mathematics education have put the power of mathematics on public display, but for many there remain significant barriers to success in the mathematical profession.

The first wave of mathematics awareness took place at a time when American mathematicians were relatively isolated from many areas of policy and the economy and correspondingly lacked obvious places to turn for resources. The second wave responded to a different problem: the need for a public posture that would assure a supply of mathematicians in the future. The Mathematics Awareness Weeks, and later Months, addressed that need by displaying the relevance of mathematics to modern life and to society at large.

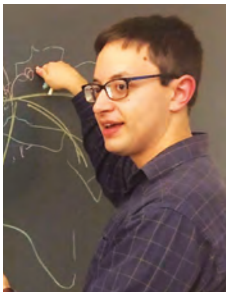
We are now seeing a third wave of mathematics awareness.

We are now starting to see a third wave of mathematics awareness, focused less on what mathematics can do for war-making policy elites or for everyday citizens and more on the mathematicians themselves who create and apply that mathematics and train future generations in their field. Efforts to highlight women and minority pioneers in the discipline and to support underrepresented groups may yet make headway where other approaches have fallen short. The history of mathematics awareness shows that by banding together and working systematically, mathematics organizations can make real changes in how important constituencies view and engage the discipline. The history also shows that mathematics awareness can take many forms, each reflecting the priorities and blind spots of its time.

Photo Credits

Photo of Mina Rees is from the Marion Walter Photograph Collection, 1952–1980s, undated, Archives of American Mathematics, e_math_01146, Dolph Briscoe Center for American History, University of Texas at Austin.

Michael J. Barany, photo by Yana Stainova.



Michael J. Barany

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Michael J. Barany is a historian of modern mathematics, currently at the Dartmouth College Society of Fellows. He recently completed his PhD at Princeton University on the topic of the globalization of the mathematics profession in the mid-twentieth century, with a focus on Laurent Schwartz's theory of distributions. His work on topics ranging from blackboards to "primitive" counting to the Fields Medal has appeared in the *Notices of the AMS*, *New Scientist*, *The New York Times*, and other publications not beginning with N (including twice in the *Best Writing on Mathematics* anthology).

IMAGINARY—a How-to Guide for Math Exhibitions

Andreas Daniel Matt

Not so long ago, mathematicians debated the question of *why* they should communicate about their subject to a broad public. Today, we seem to have collected enough good reasons why and can pass on to the next question: *How* can we communicate about mathematics to a broad public?

In this article I offer answers to this question in the form of a hands-on guide to planning and organizing interactive mathematics exhibitions. The guide draws on experience from almost ten years of organizing such exhibitions in more than fifty countries, under the umbrella of a project called IMAGINARY. Mathematics and Statistics Awareness Month, celebrated in the US each year in April, provides the perfect occasion to jump in and organize a math exhibition. IMAGINARY can provide a wealth of ideas and resources to get you started.

Math Communication on the Rise

All over the globe, new ways of communicating about mathematics are mushrooming.

New math museums are popping up. Mathematicians produce movies, mobile apps, 3-dimensional prints, and visualizations. They give talks, conduct workshops, visit schools, and write popular books. Math communication conferences are being held on all continents. Mathematical research projects often include outreach components, and math research centers are hiring dedicated outreach mathematicians. More and more professional math communicators are earning a living through this work. There are even independent math communication companies and organizations. A new booming field is arising!

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DOI: http://dx.doi.org/10.1090/noti1508

IMAGINARY

IMAGINARY started as a mathematics traveling exhibition for the German Year of Mathematics in 2008. It was a project of the Mathematisches Forschungsinstitut Oberwolfach (MFO), initiated and organized by former MFO director Gert-Martin Greuel and me. The exhibition contained a collection of state-of-the-art mathematical visualizations and interactive programs. One, called SURFER, allows one to experience and explore the relationship between formulas and forms [1].

The exhibition was a big success. With increasing interest from universities, research institutes, and schools, we decided to offer the exhibition through an open license, called an "open science exhibition," which operates in a way similar to open software [2]. Everybody can copy IMAGINARY exhibits and create their own IMAGINARY exhibitions. So far, IMAGINARY exhibitions and events have been staged in collaboration with local partners (mainly mathematicians) in more than one hundred fifty cities, fifty-two countries, and twenty-five languages.

An algebraic geometry exhibit in collaboration with IMAGINARY at the Museum of Mathematics in New York City.

IMAGINARY exhibition in Hannover, Germany, 2009.

There are more than ten permanent IMAGINARY museum installations, for example, at the Deutsches Museum in Munich, one of the world's oldest and largest science and technology museums, and the Museum of Mathematics in New York City.

More than 2.5 million people have visited an IMAGINARY exhibition, and many more have downloaded modules and background material from the website `www.imaginary.org`. This platform hosts information about mathematics outreach events and includes computer programs, picture galleries, hands-on exhibits, films, and texts, as well as fully developed exhibitions. The platform also allows people to add new exhibits, and this has stimulated a large international community and network for knowledge transfer in mathematics.

Examples of collaborations stimulated by IMAGINARY include exhibitions in ten Russian cities on the occasion of the German-Russian Year of Education, Science and Innovation, 2011–2012. Since 2010, institutions in such countries as Spain, Belgium, the Netherlands, Uruguay, France, Taiwan, and South Korea have adopted the IMAGINARY idea, established local teams, and organized nationwide traveling exhibitions. These efforts are continuing in 2017 and 2018 and will be further extended with new exhibits and activities.

IMAGINARY exhibition in Montevideo, Uruguay, 2015.

A collaboration with the African Institute for Mathematical Sciences (AIMS) was launched in 2014, focusing on mathematics popularization on the African continent. It began with an IMAGINARY event in Dar-Es-Salaam, Tanzania, and continued with workshops, exhibitions, and road shows in Cape Town, South Africa, and in Dakar/M'Bour, Senegal. Further exhibitions are planned in Cape Verde, Mozambique, Democratic Republic of Congo, and Rwanda.

IMAGINARY has grown into a worldwide network of partners and individuals, with many international exhibitions and related events. In this way IMAGINARY has become a hub for transferring mathematical knowledge to society. Three key features of the project stand out: (1) it is close to mathematics research and mathematicians; (2) it is international and multilingual; and (3) it is free, open access, and open source. In addition, IMAGINARY offers physical and interactive content (online and offline) and tries to be as appealing, aesthetic, and advanced in its design as possible. All IMAGINARY activities are highly interactive and participative, involving mathematical and non-mathematical audiences in the creation of exhibitions.

IMAGINARY activities have been organized in more than fifty countries.

Recently, IMAGINARY was organized as an independent nonprofit organization, with support by the Leibniz Association and the Klaus Tschira Foundation. The MFO remains a shareholder of the new organization, which is managed by all former project members.

Stage a Mathematics Exhibition

To get started planning a mathematics exhibition, you must resolve three main issues: the venue, the budget, and the exhibits. We recommend starting by selecting the venue. The venue often decides the dates of the exhibition and implies the type of audience. It also sets the exhibition space and thus the number of exhibits needed and the budget required. Venues should be attractive and easy to reach and should suit your target audience. They can be colleges or universities, art galleries, schools, public squares, train stations, shopping malls, bank foyers, etc. Often the venues are provided for free in return for co-sponsorship of the exhibition.

The budget for the whole exhibition could vary from US$2,000 to US$200,000 and depends mainly on whether you use paid personnel or volunteers and on the quantity and quality of the exhibits. Usually the costs are covered partly by the host organization—for example, your college or university—in collaboration with outside partners or sponsors. Typical sponsors are banks, telecommunications firms, high tech companies, and foundations and organizations that promote science and education. Sometimes national or local governments contribute funds.

We classify four forms of exhibits:

- Program: an interactive software exhibit, often used on a touch screen
- Hands-on: a physical exhibit that can be manipulated or a 3-dimensional printed sculpture
- Film: a video that can be shown in an exhibition, for example, in a dedicated cinema room
- Image gallery: a collection of images accompanied by mathematical descriptions.

Ideally, an exhibition lets visitors immerse themselves in a new world. On the IMAGINARY platform you can find more than one hundred exhibits ready to be used in your exhibition. Many of them cover current research topics.


3D-printed algebraic surface at the NIMS-IMAGINARY exhibition during the ICM2014 in Seoul, South Korea.

One example of an exhibit is SURFER, a real-time ray-tracer of algebraic surfaces. Visitors can enter or alter a polynomial equation in three variables and will see immediately the resulting zero-set, an algebraic surface. They can rotate the surface and apply colors. The program has built-in galleries of interesting surfaces, such as surfaces with many


SURFER exhibit on a touch screen at IMAGINARY Belgrade, 2012.


IMAGINARY exhibition at Heidelberg, Germany, with pictures printed in high quality by Luc Benard, Uli Gaenshirt, and Richard Palais, 2014.


IMAGINARY image gallery on quasiperiodic tilings by Uli Gaenshirt at an exhibition in Istanbul coordinated by IMAGINARY Turkey, 2015.

Interactive exhibit "Sphere of the Earth" with explainer at the MPE exhibition, Imperial College London, 2015.

singularities, as well as explanations. Visitors can print their creations and add them to a user gallery at the exhibition and/or take the images home. Image galleries, video animations, and 3-dimensional printed sculptures of algebraic surfaces make great accompaniments for SURFER. The program can be used to stage online image competitions held parallel to the exhibition and can also be used in schools after the exhibition is over.

A special type of exhibit is the collection of "snapshots of modern mathematics" (see also [3]). These are short papers written by participants of workshops held at the MFO. The idea is to explain as accessibly and understandably as possible suitable topics from their research areas. The snapshots are then edited and shared under an open license. IMAGINARY offers a touch screen station to show these short texts during an exhibition, with an option to print them or e-mail them. By giving a glimpse into ongoing current research, they can give mathematical depth to your exhibition.

Once the venue, budget, and exhibits are set, you step into production and media work. To produce the exhibits, you either have to hire a company or buy the technology yourself. Often colleges and universities already have touch screens and computers that you can borrow. We recommend printing images in high quality and mounting them using a gallery system rather than affixing poster prints to walls. Media work can be carried out through your institution's public relations office or with assistance of IMAGINARY's media channels.

During the exhibition, guided tours can be offered, and you should arrange for tutors to be available on-site to explain the mathematical content. This makes a huge difference! Human explainers, who are often motivated students, inspire the visitors, and together they can explore the mathematics behind the exhibits. IMAGINARY offers special tutorials and training material for tutors.

Once the exhibition is over, the exhibits can be permanently installed in your institution or a local science museum.

AIMS-IMAGINARY exhibition and workshop for math communicators, Cape Town, 2014.

Interactive exhibit Dune Ash at the AIMS-IMAGINARY exhibition in M'bour in Senegal, 2015. It simulates, in real time, the dispersion of volcanic ash from a user-chosen volcano location in the presence of a wind field that is drawn by hand.

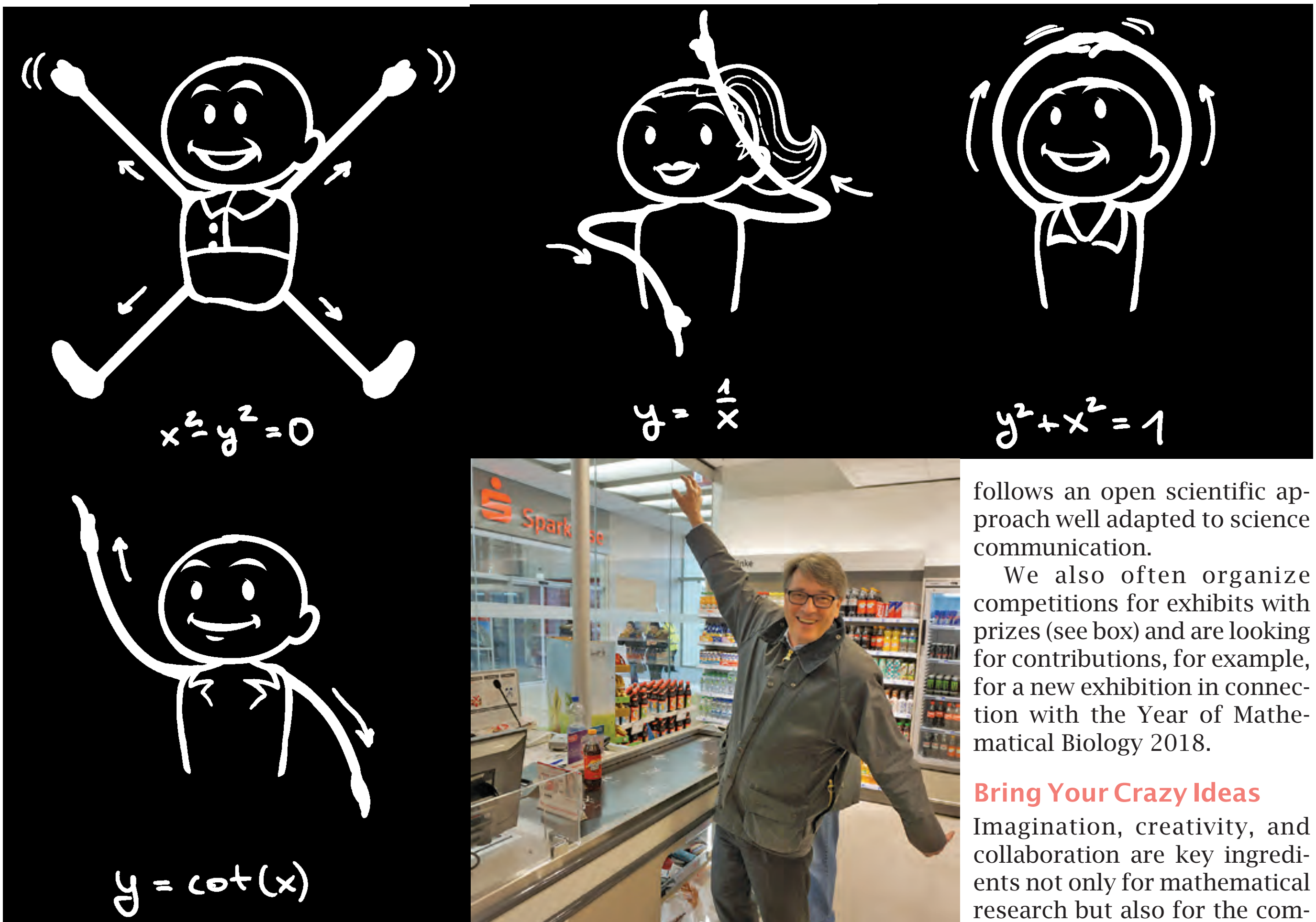
Hands-on exhibit on cartography of the MPE exhibition, Imperial College London, 2015.

Create a Mathematics Exhibit

An exhibition should always include local content. This means that you or your partner institution should try whenever possible to add your own ideas and your own flair to the exhibition. For example, you can prepare or enhance mathematical visualizations, films, or software simulations for the exhibition.

We have collected many hints about how mathematical knowledge can be transferred to an exhibit. Tools are available to create images in high resolution; movies can be made with digital cameras, via stop motion, or through animation; and 3-dimensional prints can be made at your institution or ordered online in very high quality at affordable prices.

You can then share your exhibit under an open license, which means that your exhibit can then be duplicated at other exhibitions. This way, the collection of available content is constantly changing and evolving, and we all benefit from each other's ideas. In this sense, IMAGINARY



Dancing functions on the conveyor belts of the two supermarket chains in the Mathematikon mall, 2016.

follows an open scientific approach well adapted to science communication.

We also often organize competitions for exhibits with prizes (see box) and are looking for contributions, for example, for a new exhibition in connection with the Year of Mathematical Biology 2018.

Bring Your Crazy Ideas

Imagination, creativity, and collaboration are key ingredients not only for mathematical research but also for the communication of mathematics. To successfully reach and involve a large audience, we recommend thinking "outside the box." Crazy ideas are welcome!

In IMAGINARY activities, we have seen many creative inventions, which are often initiated in collaboration with artists, designers, musicians, or even chefs—

Join the Mathematics of Planet Earth Competition!

Participate in the new competition of exhibition modules for Mathematics of Planet Earth (MPE). You can submit exhibits designed to showcase the ways in which the mathematical sciences can be useful in tackling our world's problems. The best modules will receive prizes and will enrich the successful MPE exhibition. There is a special prize for Africa-related exhibits.

The competition is organized by UNESCO, the International Mathematical Union, the International Commission on Mathematical Instruction, and IMAGINARY.

1st prize: US$5,000
2nd prize: US$2,000
3rd prize: US$1,000
Special prize for African topic: US$2,000
Submission Deadline: **June 30, 2017**

More information at `www.imaginary.org/mpe-competition`.

Picture from the exhibition "The Taste of Mathematics," 2012.

people who visited our exhibitions and got inspired. One example started with the question, How does math taste? Michelin-starred chef José Carlos García teamed up with mathematician Mercedes Siles Molina and photographer Pedro Reyes Dueñas (all three from Malaga) to compare the creative process of cooking with the creative process of doing mathematics. The results are pictures of a mathematical menu and an exhibition plus documentation of the worlds of cooking and doing math. They were inspired by the IMAGINARY exhibition organized in Spain by the Real Sociedad de Matemática Española.

What are the best venues for exhibitions? Surprising venues, like shopping malls, where you would not expect to encounter math, work wonderfully. A prime example is the mathematics-themed shopping mall called Mathematikon Shops that opened in Heidelberg in February 2016. In collaboration with mathematicians from fourteen countries, IMAGINARY equipped the shopping mall—including the conveyor belts, the parking lot, and the restrooms—with mathematical images, puzzles, sculptures, and a huge multitouch interactive station [4].

Mathematician Cédric Villani exploring augmented reality for math museums in the holo-math.com project, 2016.

How can new technology be used in mathematics museums? The Holo-Math project initiated by Cédric Villani, director of the Institut Henri Poincaré, in collaboration with IMAGINARY and tech partners, explores how augmented reality can be used to explain and explore mathematical insights. Museum visitors are equipped with HoloLens devices and introduced to mathematically augmented worlds. The first episode of a Holo-Math adventure on Brownian motion is now being finalized.

Stay Connected

If you want to be informed about current mathematics exhibitions or trends in mathematics communication, please join the Mathematics Communication Network (`imaginary.org/network`). You can subscribe to its newsletter and participate in joint projects and conferences. To find out about international mathematics outreach projects, including museums, exhibitions, and events, please visit `wikimathcom.imaginary.org`, a WikiPage dedicated exclusively to mathematics communication.

If you want to organize an IMAGINARY exhibition and would like support, feel free to contact me. If you organize an exhibition, please announce your event to the online listing of all exhibitions at `imaginary.org/events`. There you can also find out about all current exhibitions and museum installations, including ones happening in the coming months in Brazil, Germany, the Netherlands—and maybe a place near you!

Credits

Photos of the IMAGINARY exhibition in Hannover, the world map, the 3D-printed algebraic surface in Seoul, the exhibition in Belgrade, the image gallery in Istanbul, the exhibition and workshop in Cape Town, the interactive exhibit in M'bour, the hands-on exhibit in London, the algebraic geometry exhibit in New York, the math dance in Heidelberg, and the Holo-Math project are courtesy of IMAGINARY.

Photo of the IMAGINARY exhibition in Montevideo courtesy of Matu Tejera.

Photo of the IMAGINARY exhibition in Heidelberg courtesy of Heidelberg Laureate Forum Foundation.

Photo from the exhibition "The Taste of Mathematics" courtesy of Pedro Reyes Dueñas.

Illustrations of dancing functions courtesy of Michael Gralmann.

Photo of Andreas Daniel Matt courtesy of Andreas Daniel Matt.

ACKNOWLEDGMENT. For feedback and assistance with the article: Bianca Violet, Nadja Pernat, Gert-Martin Greuel, and Christian Stussak.

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Andreas Daniel Matt

ABOUT THE AUTHOR

A co-founder of IMAGINARY, **Andreas Daniel Matt** worked as mathematics communicator at the Mathematisches Forschungsinstitut Oberwolfach from 2007 until 2016. He studied mathematics and computer science and did his PhD in mathematics in machine learning at the University of Innsbruck and the University of Buenos Aires. He curated the MiMa, a museum for minerals and mathematics in Oberwolfach, and is a member of the Raising Public Awareness Committee of the European Mathematical Society. He has received several prizes for his outreach activities, including the Media Prize of the German Mathematical Society in 2013 (together with Gert-Martin Greuel) for the IMAGINARY project.

A National Mathematics Festival and a Movement

David Eisenbud and Kirsten Bohl

Note: The opinions expressed here are not necessarily those of Notices. *Responses on the* Notices *webpage are invited.*

We write this in the last days of 2016, an unsettling and divisive year on many fronts. There's one continuing trend that readers of this column will probably all agree is good: the remarkable increase in public attention paid to mathematics.

By the time you read this, the cherries will be blooming in Washington, DC, where more than thirty thousand children and adults will gather on April 22 at the Washington Convention Center for a day of lectures, puzzles, games, mathletic events, art-making, and performances—that's right, for the second National Math Festival, organized by MSRI in cooperation with the Mathematical Sciences Research Institute in Berkeley in cooperation with the Institute for Advanced Study in Princeton, with MoMath, the National Museum of Mathematics.

We are struck by how different the national backdrop is from the first time in 2015. More and more, our math and science organizations are taking lead roles in encouraging the public to take joy in, understand more deeply, and perhaps feel a deeper and more positive personal connection with mathematics. It would be hard to make a systematic list of all that's happening—that's certainly beyond our capacity and beyond the space for this essay! But here are a few morsels:

Let's keep one another informed about the good things we notice.

- the weekly "Varsity Math" puzzle column by MoMath in the *Wall Street Journal*;
- the proliferation of math festivals around the world, including in Mexico City and Rio de Janeiro;
- the creation of Math Outreach International (born in Banff) and the likewise international *MATRIX* and *MOVES* conferences;
- the declaration of Global Math Week October 10–17, 2017, and its ambition to make short, engaging math videos entirely visual to obviate the trickiness of language differences;
- the expansion of Bob Moses's Algebra Project and the Flagway Game from their firm successes in inner cities of the United States to small villages in the Irish countryside;
- Math Circles reaching a national spotlight in *The Atlantic* magazine;
- the nearly two-million-strong Numberphile YouTube channel, which has occasioned many, many self-reports of turnarounds in feelings about math (and maths);
- the awards accruing to the *Navajo Math Circles* movie;
- the nascent Mathical Book Prize for young children to teens;
- the international IMAGINARY mathematics exhibitions, described in the previous *Notices* article, page 368.

It's a heady world these days and one that's different from that of 2015.

As we write this, Francis Su has just launched his MathFeed app, aggregating news, social media, puzzle columns, book reviews, and more about mathematics,

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DOI: http://dx.doi.org/10.1090/noti1501

math education, and related fields. As more of the rising tide of information around us begins to celebrate mathematics in myriad and surprising ways, let's keep one another informed about the good things we notice. At MSRI we will keep developing the More Math! section of the `NationalMathFestival.org` website, where we are curating recreational math resources year-round. What will 2017 bring? It's safe to wager we can expect more good news!

When April comes around each year, let's make an even bigger splash with Math Awareness Month. The public understanding of math serves us all, and we are pleased and proud to be shoulder to shoulder with the AMS, AWM, NAM, MAA, SACNAS, and SIAM* to kick up a mathy fuss in fresh and interesting ways. With the Association of Science-Technology Centers, this year MSRI will host Giant SOMA Cube workshops at many dozens of science museums around the country as an extension of the National Math Festival.

If we keep to the trend, the next National Math Festival after this April will be in 2019. We can only guess what the world might be saying and doing and feeling and thinking about mathematics by then, but we do know we want to be in the company of others of goodwill, excitement, and a growth mindset to find out.

**Association for Women in Mathematics, National Association of Mathematicians, Mathematical Association of America, Society for Advancement of Chicanos/Hispanics and Native Americans in Science, Society for Industrial and Applied Mathematics.*

Photo Credits

Photos of David Eisenbud and Kirsten Bohl are courtesy of Cliff Stoll.

David Eisenbud

ABOUT THE AUTHORS

David Eisenbud loves mathematical fun of all kinds—from core research to balloons—and is delighted to see many nonmathematicians reacting in the same way when they get the chance.

Kirsten Bohl

Kirsten Bohl is happiest in her work at MSRI when surrounded by balloons, children, and mathematics—not necessarily in that order.

OPINION

A Great Time to Be a Statistician

Ronald L. Wasserstein

Note: The opinions expressed here are not necessarily those of Notices.
Responses on the Notices *webpage are invited.*

Taking my cue from David Eisenbud and Kirsten Bohl, as I write this I am sitting at my desk just a few miles south of where the cherry blossoms will be harbingers of springtime. However, looking out my window on a chilly January day, barren trees and salty streets say nothing but winter, winter, winter.

As you read this, however, it is Mathematics and Statistics Awareness Month, and it is a great time to make students aware of the opportunities afforded them through study of and careers in math and stats. Our research reveals that when today's high school and college students think about what they would like to do, they want to do work that can make a difference in the world, work that will challenge their intellect and satisfy their curiosity, work that is enjoyable, and (naturally) work that pays well. It is hard to beat mathematics and statistics for jobs that fit these aspirations.

And the word is getting out! Four of the top ten highest rated jobs in the current `CareerCast.com` survey are directly related to our professions. Similarly, four of the top ten best graduate degrees as ranked by *Fortune* magazine are in mathematics and statistics. `USNews.com` ranks "statistician" as number four in its list of twenty-five best jobs. This kind of recognition shows up in other rankings as well, and is being reflected in student degree choices. Since 2010, according to IPEDS data, the number of mathematics bachelors degrees awarded has grown by 28 percent, and in statistics by a whopping 170 percent. (That's starting from a smaller base, but still very encouraging.)

The mathematics community, as David and Kirsten have pointed out, is doing wonderful things to promote the importance of mathematics. At the end of this brief column, I say a quick word about one of them. The statistics community is also hard at work to increase student awareness of the value of basic statistical literacy and the potential for careers in statistics. Our biggest investment is an outreach initiative called "*This* Is Statistics" (`www.thisisstatistics.org`), and we're very encouraged with the results of this program as it enters its third year. Thousands of students are learning that statistics—the science of learning from data, and of measuring, controlling, and communicating uncertainty—is an excellent career choice.

While 2016 has been labeled the "worst year ever,"[1] an assertion that, historical considerations aside, almost any mathematician or statistician would dispute on extreme value theory grounds alone, it was a good year for statistics. ASA Director of Science Policy Steve Pierson has documented this well in his blog "Nine Recognitions for Statistics in 2016."[2]

Of these nine, a highlight for me was the announcement of the first recipient of the International Prize in Statistics. The contributions of Sir David Cox[3] have impacted

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DOI: http://dx.doi.org/10.1090/noti1505

[1] `https://www.nytimes.com/2016/12/28/opinion/2016-worst-year-ever.html?_r=1`

[2] `community.amstat.org/blogs/steve-pierson/2016/12/29/nine-recognitions-for-statistics-in-2016`

[3] `www.worldofstatistics.org/files/2016/10/WOS_newsletter_10202016_SPECIAL.pdf`

the well-being of most of us, so it was fitting that Dr. Cox is the winner of the statistics equivalent of top prizes in other fields, such as the Abel Prize in mathematics. These prizes serve to remind the public that the work of statisticians and mathematicians may be very abstract but it often impacts how we live.

In this "post-truth world," mathematicians and statisticians everywhere ought to engage the public to increase understanding in and trust of our fields. It is encouraging to see how much great work in the area of public engagement is already being done by our communities.

One sterling example of public engagement is the National Museum of Mathematics, which my ASA colleague Donna LaLonde and I visited last year. We spent part of our time viewing the exhibits, and part of our time just watching visitors engage with them. In the building that day were a large group of middle school-aged children. In our observation it wasn't obvious that a single child learned anything specific about mathematics, but it was clear that they all came away with the idea that mathematics is pretty cool. That's an idea very much worth planting, and we are excited to work with you to grow awareness and enthusiasm for mathematics and statistics.

Photo Credit

Photo of Ronald L. Wasserstein and family is courtesy of Ronald L. Wasserstein.

Ronald L. Wasserstein

ABOUT THE AUTHOR

Ron Wasserstein and family (Peterson, Abner, Rose, and Sherry) enjoying the cherry blossoms in Washington, DC.

WHAT IS...

Tropical Geometry?

Eric Katz

Communicated by Cesar E. Silva

This note was written to answer the question, What is tropical geometry? That question can be interpreted in two ways: Would you tell me something about this research area? and Why the unusual name 'tropical geometry'? To address the second question, tropical geometry is named in honor of Brazilian computer scientist Imre Simon. This naming is complicated by the fact that he lived in São Paolo and commuted across the Tropic of Capricorn. Whether his work is tropical depends on whether he preferred to do his research at home or in the office.

> *Tropical geometry transforms questions about algebraic varieties into questions about polyhedral complexes.*

The main goal of tropical geometry is transforming questions about algebraic varieties into questions about polyhedral complexes. A process called *tropicalization* attaches a polyhedral complex to an algebraic variety. The polyhedral complex, a combinatorial object, encodes some of the geometry of the original algebraic variety. There are other ways of constructing similar polyhedral complexes, and the polyhedral complexes that arise can be studied in their own right. We will discuss three approaches: the synthetic, the valuative, and the degeneration-theoretic.

Synthetic Approach

Tropical geometry originally arose from considerations of tropical algebra, itself motivated by questions in computer science. Here, tropical geometry can be considered as algebraic geometry over the tropical semifield $(\mathbb{R}\cup\{\infty\}, \oplus, \otimes)$ with operations given by

$$a \oplus b = \min(a,b), \quad a \otimes b = a+b.$$

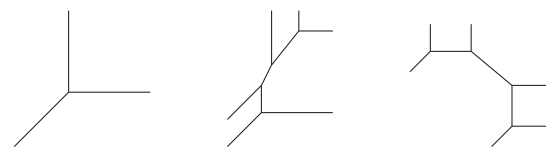

Figure 1. Tropical curves, such as this tropical line and two tropical conics, are polyhedral complexes.

One can then find tropical analogues of classical mathematics and define tropical polynomials, tropical hypersurfaces, and tropical varieties. For example, a degree 2 polynomial in variables x, y would be of the form

$$\min(a_{20}+2x, a_{11}+x+y, a_{02}+2y, a_{10}+x, a_{01}+y, a_{00})$$

for constants $a_{ij} \in \mathbb{R}\cup\{\infty\}$. The zero locus of a tropical polynomial is defined to be the set of points where the minimum is achieved by at least two entries. In the above example, it would be the set of points (x,y) where there are distinct indices $(i_1,j_1),(i_2,j_2)$ such that

$$a_{i_1j_1}+i_1x+j_1y = a_{i_2j_2}+i_2x+j_2y \le a_{ij}+ix+jy$$

for all pairs (i,j). These objects do not look like their classical counterparts and instead are polyhedral complexes of differing combinatorial types. For example, Figure 1 shows a tropical line and two tropical curves cut out by degree 2 polynomials in x and y.

Recall that a polyhedral complex in $\mathbb{R}^n$ is a union of polyhedra (that is, sets cut out by linear equations and inequalities) such that any set of polyhedra intersects in a common (possibly empty) face of each member. More is true about the polyhedral complexes that arise in this fashion; they are tropical varieties, which are defined to be integral, weighted, balanced polyhedral complexes. Here *integral* means that their defining linear equations and inequalities have integer coefficients; *weighted* means that their top-dimensional polyhedra are assigned positive

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DOI: http://dx.doi.org/10.1090/noti1507

integer weight (in the above example, all weights are 1); and *balanced* means that the weights around a codimension 1 face satisfy a particular balancing relation, which in the 1-dimensional case says that the primitive integer vectors along edges emanating from a given vertex, multiplied by the weight of the edges, sum to zero. This can be thought of as a zero tension condition as in physical dynamics.

A combinatorial object encodes some geometry of the algebraic variety.

Tropical geometry's first major result was Grigory Mikhalkin's proof (2005) that the number of plane curves of degree d and genus g passing through $3d-1+g$ points in general position could be computed by counting tropical curves with multiplicity. This led to the definition of an abstract tropical curve as a graph equipped with additional data. Much of the early development of tropical geometry involved finding tropical analogues of theorems about algebraic curves and their enumerative geometry.

Valuation-Theoretic Approach

Another approach to tropical geometry, which was described in an unpublished manuscript of Mikhail Kapranov from the early 1990s but dates back to work of George Bergman (1971) and Robert Bieri and J. R. J. Groves (1984), is to define a tropical variety as a shadow of an algebraic variety. We will first discuss a more familiar, analytic version of this approach involving logarithmic limit sets. For $t>0$, consider the map $\mathrm{Log}_t\colon (\mathbb{C}^*)^n \to \mathbb{R}^n$ given by

$$(z_1,\dots,z_n)\mapsto (\log_t(|z_1|),\dots,\log_t(|z_n|)).$$

The image of an algebraic subvariety $X\subset(\mathbb{C}^*)^n$ is called an *amoeba*. Under the Hausdorff limit, $\lim_{t\to\infty}\mathrm{Log}_t(X)$, it becomes a piecewise linear object called a polyhedral fan. When one studies a family of varieties $X_t\subset(\mathbb{C}^*)^n$ parameterized by t and considers $\lim_{t\to\infty}\mathrm{Log}_t(X_t)$, one obtains a richer polyhedral complex.

For the algebraic approach, let $\mathbb{K}$ be an algebraically closed field equipped with a nontrivial non-Archimedean valuation $v\colon \mathbb{K}^*\to G\subseteq\mathbb{R}$ where G is an additive subgroup of $\mathbb{R}$. Here $\mathbb{K}^*=\mathbb{K}\backslash\{0\}$ is the set of units in $\mathbb{K}$. The valuation is said to be non-Archimedean if

$$\begin{aligned} v(xy) &= v(x)+v(y),\\ v(x+y) &\geq \min(v(x),v(y)). \end{aligned}$$

An example to keep in mind is $\mathbb{K}=\mathbb{C}\{\{t\}\}$, the field of formal Puiseux series, which is the algebraic closure of the field of Laurent series. Here, an element $x\in\mathbb{K}$ can be written as a formal power series with complex coefficients and fractional exponents with bounded denominator:

$$x=\sum_{i=l}^{\infty} c_i t^{i/N},\ c_l\neq 0.$$

The valuation is defined by $v(x)=l/N$, the smallest exponent with nonzero coefficient.

An algebraic torus $(\mathbb{K}^*)^n$ is the Cartesian product of finitely many copies of $\mathbb{K}^*$ and should be thought of as analogous to $(S^1)^n$. A subvariety of $(\mathbb{K}^*)^n$ is the common zero set of a system of Laurent polynomial equations in the coordinates $x_1,x_2,\dots,x_n$. The tropicalization of X is defined to be $\mathrm{Trop}(X)=\overline{v(X)}$, the topological closure of the image of X under the product of valuation maps $v\colon(\mathbb{K}^*)^n\to\mathbb{R}^n$. This approach does specialize to the synthetic approach, a fact that we illustrate with an example. Consider the subvariety X in $(\mathbb{K}^*)^2$ defined by $x+y+1=0$. For a point (x,y) to belong to X, what must be true of its valuations? Let us express the defining equation of X in terms of the Puiseux series of x, y, and 1:

$$\sum_{i=l}^{\infty} c_i t^{i/N}+\sum_{i=m}^{\infty} d_i t^{i/N}+t^0=0.$$

For this equality to be satisfied, the coefficients of any exponent must sum to zero. In particular, the coefficients of the smallest power of t must sum to 0, so there must be at least two such nonzero coefficients. This tells us that $\min(v(x),v(y),v(1))$ must be achieved by at least two entries. It follows that $\mathrm{Trop}(X)$ is contained in the tropical hypersurface of $x\oplus y\oplus 0$ (where $0=v(1)$). It is a theorem of Kapranov that the reverse containment is true, in fact, for all hypersurfaces.

This valuative approach allows one to speak of tropical varieties, not just tropical hypersurfaces. The tropical varieties that arise are integral, weighted, balanced polyhedral complexes by a result of David Speyer (2005). Moreover, they capture some of the geometry of the original variety X by reflecting X's class in intersection theory and, under suitable smoothness conditions, some of X's cohomology. In this sense, $\mathrm{Trop}(X)$ is a shadow of X. Computing tropicalizations of algebraic varieties is an interesting problem making use of Gröbner basis techniques.

Degeneration-Theoretic Approach

Tropical geometry is very closely related to the study of degenerations of algebraic varieties. In the classical situation, one may have a family of varieties $Y_t\subset(\mathbb{C}^*)^n$ depending on a parameter t varying in a small disc $\mathbb{D}$ around the origin. This arises, for instance, if one is given an algebraic variety $\mathcal{Y}\subset(\mathbb{C}^*)^n\times\mathbb{D}$ with projection $p\colon(\mathbb{C}^*)^n\times\mathbb{D}\to\mathbb{D}$. The varieties Y_t are fibers of the projection restricted to $\mathcal{Y}$. One usually studies semistable families so that $\mathcal{Y}$ is nonsingular, the fibers Y_t for $t\neq 0$ are smooth, and the central fiber Y_0 has very mild (so-called normal crossing) singularities. The irreducible components of Y_0 may intersect in a combinatorially interesting fashion encoded by a polyhedral complex Γ, called the dual complex. The central fiber Y_0 is called a degeneration of the generic fiber Y_t for $t\neq 0$.

There is a purely algebraic analogue of the study of families of algebraic varieties over a disc, the study of schemes over a valuation ring. Here, the field $\mathbb{K}$ is the algebraic analogue of the field of germs of analytic functions near the origin on a punctured disc $\mathbb{D}^*$. A variety X over $\mathbb{K}$ is analogous to a family of varieties

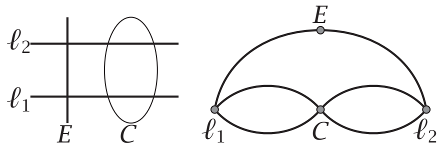

Figure 2. Associated to a curve (left) is its dual graph (right).

over the punctured disc. Under suitable conditions, one may extend X to a semistable scheme $\mathcal{X}$ over $\mathcal{O}$, the valuation ring of $\mathbb{K}$. The scheme $\mathcal{X}$ is analogous to an extension of the family over the disc. The reduction X_0 of $\mathcal{X}$ to the residue field $\mathbf{k}$ of $\mathcal{O}$ is analogous to the central fiber of the family. Under conditions on X introduced by Jenia Tevelev, $\mathrm{Trop}(X)$ is very closely related to the dual complex. The complex $\mathrm{Trop}(X)$ encodes some of the combinatorics of the components of X_0. In that sense, it is not surprising that $\mathrm{Trop}(X)$ would reflect geometric properties of X. There is a certain tension between algebraic geometry and combinatorics in tropical geometry: if the combinatorics of $\mathrm{Trop}(X)$ are simple, the algebraic geometry of the components is likely to be complicated and rich; if the components of the degeneration are simple, the combinatorics of $\mathrm{Trop}(X)$ are rich and capture the geometry of X.

One can try to make tropical geometry more intrinsic by studying the combinatorics of degenerations of abstract varieties X over $\mathbb{K}$. This approach has been developed furthest in the case of curves and has led to the theory of linear systems on graphs as pioneered by Matthew Baker and Sergei Norine. We will work with an example borrowed from the work of Baker to illustrate this theory. We examine a family of curves in $\mathbb{P}^2$ parameterized by t: consider the family $\mathcal{X}$ of quartic curves in $\mathbb{P}^2 \times \mathbb{D}$ defined by $F((X,Y,Z),t) = 0$ with

$$F((X,Y,Z),t) = (X^2 - 2Y^2 + Z^2)(X^2 - Z^2) + tY^3Z.$$

When $t \neq 0$ is small, this defines a smooth plane quartic. When $t = 0$, the curve is the union of a conic C and two lines ℓ_1 and ℓ_2. The total space of the family is singular but can be made nonsingular by blowing up the intersection point of the two lines. This introduces a new component of X_0, which is a rational curve E. The curve X_0 has at worst nodal singularities, meaning that near the intersection of components, the curve locally looks like $xy = 0$.

In Figure 2, the central fiber of the resulting family is pictured on the left. One may form its dual graph Γ by associating a vertex to each irreducible component of X_0 and associating an edge to each nodal singularity of X_0. This gives the dual graph pictured on the right.

One can define divisors on the dual graph as formal integer combinations of vertices of Γ. There is a notion of specializing a divisor $\mathcal{D}$ from the generic fiber X of $\mathcal{X}$ to a divisor D on the dual graph Γ. In fact, one can work out a rich combinatorial theory of divisors on graphs. Here, linear equivalence of divisors is generated by chip-firing moves on graphs, which have been studied in other contexts. One can define the rank $r_\Gamma(D)$ of the linear system associated with the divisor. By a semicontinuity argument, this rank provides an upper bound for the dimension of the linear system on X containing D. With this bound, one is able to use combinatorial methods to prove strong results in the theory of algebraic curves as described in the survey [1]. Recent work of Dustin Cartwright extends this theory to higher dimensions.

Research in Tropical Geometry

Research in tropical geometry is heading in several different directions. The foundations of tropical geometry are undergoing continual revision and are not yet settled. Applications of tropical geometry to enumerative geometry are still being uncovered, many of them in the direction of mirror symmetry. Tropical geometry also provides a hands-on way of studying Berkovich spaces, a theory of analytic geometry over complete non-Archimedean fields. Tropical techniques are now being employed in computational algebraic geometry. There have been many new applications of the theory of linear systems on graphs to algebraic curves. Careful use of degeneration methods has led to results in Diophantine geometry, a branch of number theory. Tropical geometry also allows one to apply geometrically motivated techniques to purely combinatorial objects through associated tropical varieties, bringing powerful new techniques to combinatorics and resolving old problems. Tropical varieties have even been studied in their own right, as they are combinatorially interesting objects.

Further Reading

[1] M. Baker and D. Jensen, Degeneration of linear series from the tropical point of view and applications, *Proceedings of the Simons Symposium on Nonarchimedean Geometry*, Simons Symposia, Springer, 2016, 365–433.

[2] D. Maclagan and B. Sturmfels, *Introduction to Tropical Geometry*, Amer. Math. Soc., Providence, RI, 2015. MR3287221

Editor's Note

See also "What is a tropical curve?" by Grigory Mikhalkin in the April 2007 *Notices*. Other related columns are "What is an amoeba?" by Oleg Viro (September 2002) and "What is a Gröbner basis?" by Bernd Sturmfels (November 2005).

Photo Credit

Photo of Eric Katz is courtesy of Joseph Rabinoff.

Eric Katz

ABOUT THE AUTHOR

Eric Katz's mathematical interests include combinatorial algebraic geometry and number theory. He likes to ride bikes, run, and make outlandish statements.

Underrepresented Students in Topology and Algebra Research Symposium (USTARS)

Candice Price

Communicated by Alexander Diaz-Lopez

We wanted to create a space where any graduate student would feel safe presenting for the first time.

For the past six years, Underrepresented Students in Topology and Algebra Research Symposium (USTARS) has met every April and has grown into a three-day event featuring thirty-minute research talks given by graduate students in the fields of topology and algebra, an early career faculty workshop, a professional development session, and a poster session for invited undergraduates. It also now includes two Distinguished Graduate Student Awards, given to a graduate student in each of topology and algebra, and an early-career faculty speaker presenting on their mathematical journey. Graduate students at all stages of their research careers are invited to attend USTARS and discuss techniques and concepts that they are exploring as part of the process of producing their graduate theses and, in turn, gain new insight into their research from students and professors at other institutions who may view the problem from a different perspective. Additionally, USTARS provides a venue for mentorship and potential collaboration. Faculty who attend are encouraged to help undergraduate and transitional graduate students find research areas and to urge students to meet and network with people interested in their areas of research. But, most of all, it provides a place for underrepresented students to showcase their research to a diverse group of mathematicians in a supportive environment.

USTARS Locations

- 2011: University of Iowa
- 2012: University of Iowa
- 2013: Purdue University
- 2014: University of California, Berkeley
- 2015: Florida Gulf Coast University
- 2016: Sam Houston State University
- 2017: Amherst College

Twenty undergraduates, forty-two graduate students, five postdocs, and eleven faculty attended USTARS 2014.

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DOI: http://dx.doi.org/10.1090/noti1504

USTARS Organizing Committee: Syvillia Averett, Garrett Jones, Candice Price, Erik Insko, Kathy McElroy, Shannon Talbott, and Jeannine Abiva.

USTARS 2014 participants: Amanda Ruiz, Candice Price, Emillie Davie Lawrence, Federico Ardila, Dagan Karp, and Mohamed Omar.

The History of USTARS

The idea for USTARS was born in November of 2010 when, as a fifth-year graduate student at the University of Iowa, I attended the student-run Binghamton University Graduate Conference in Algebra and Topology. Inspired by the inclusiveness and organization of the conference, I reached out to three of my fellow classmates at the University of Iowa—Syvillia Averett, Carlos De la Mora, and Erik Insko—and asked a simple question: "Would you like to plan a conference for graduate students focusing on algebra and topology?" They cautiously said yes. Together, with the support of faculty member Julianna Tymoczko, we decided to plan a conference run by graduate students for graduate students that showcased and connected underrepresented mathematicians.

We wanted an event to which we could invite our friends, a possible collaborator, or even someone whose work we admired and wanted to get to know. We wanted to create a space where any graduate student would feel safe presenting for the first time. These were our first goals. What happened next is USTARS transformed into something much more than that: it has turned into what a recent USTARS participant called "a family gathering." This symposium creates a space where participants can network and exchange ideas in an environment full of diverse backgrounds and experiences, something rarely experienced as a graduate student of color. On April 1–2, 2011, sixty-eight participants attended the NSF-funded USTARS at the University of Iowa.

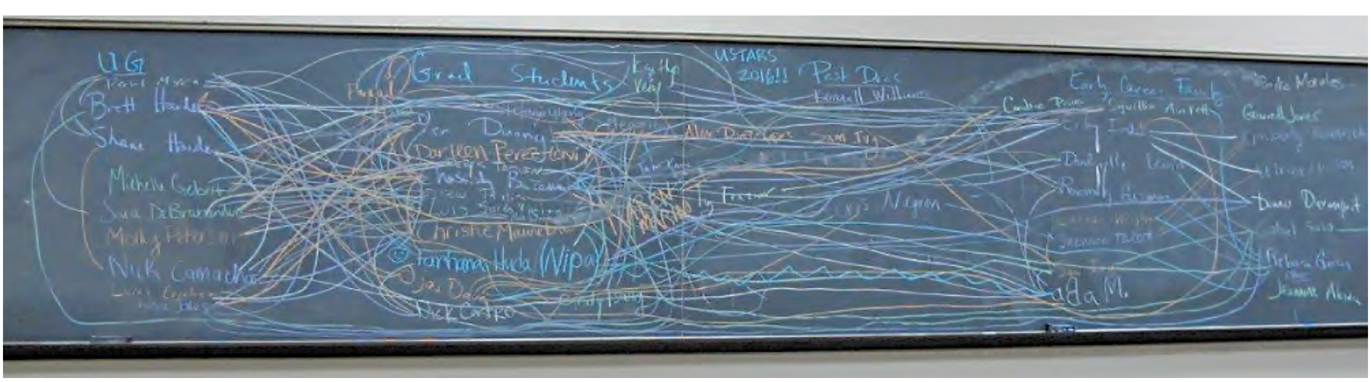
In the fall of 2011, Jeannine Abiva, Garrett Jones, and Shannon Talbott, all graduate students at UIowa, joined Syvillia, Erik, and me to make up the current group of USTARS organizers.

In 2013 we founded the USTARS Advisory Board. The board consists of faculty who have demonstrated an impact in addressing issues of underrepresentation in mathematical sciences. But more than that, they are also people the organizers of USTARS have leaned on, learned from, and been led by at various times in their journey.

The USTARS Invited Volume, posted at arXiv.org, is open to contributions from any participant of USTARS. The articles are peer-reviewed by USTARS participants and then posted on the arXiv to receive more feedback from the larger math community before possibly being submitted for publication. This opportunity was one that we as organizers wish we had when we were graduate students.

In 2015 we prepared a five-year review to discuss the influence of USTARS on its participants and to express our goals for the future of USTARS. We sent surveys to all participants of USTARS asking about how attending USTARS influenced their lives. The report can be found at `www.ustars.org`.

In 2016, we hosted a pre-USTARS workshop for early career faculty. Twelve past USTARS graduate student participants, who are now faculty, came together to USTARS.



Network of USTARS 2016 participants.

The Impact of USTARS

To assess the impact of USTARS, we survey participants every year. Below are some of the testimonials we received:

> *I met three of my current collaborators at USTARS meetings.*
>
> *USTARS allowed me to present my research for the first time in a friendly setting.*
>
> *I really liked the family environment and feel of the symposium. It is probably the best conference I have ever attended.*
>
> *It is important to see and meet people who have a similar background and have similar career goals. It enhances the sense of belonging and increases motivation. The Questions & Answers panel discussions at the end of each conference are very helpful in guiding students in the first steps to take and knowing about the possible directions and common pitfalls.*

The impact that USTARS has on the organizers is also tremendous. Not only do the six of us get to work together on a program we are all so passionate about, it allows us to stay connected as friends that started graduate school together.

One collaboration team that materialized at past USTARS conferences includes Alexander Diaz-Lopez, Pamela E. Harris, Erik Insko, Mohamed Omar, and Darleen Perez-Lavin. Their joint work has resulted in two published articles and two preprints in the areas of representation theory and combinatorics over the past three years, and in the spring of 2016 four of them proved the peak polynomial positivity conjecture posed by Sara Billey, Krzysztof Burdzy, and Bruce Sagan in 2013. Mohamed Omar, one of those participants, stated that USTARS "has been one of the best scholastic interactions in terms of fruitfulness and feeling a sense of belonging."

The Future of USTARS

The seventh meeting of USTARS will be hosted by the mathematics and statistics department at Amherst College, March 31–April 2, 2017. The theme for the 2017 USTARS Early Career Faculty Development Workshop will be "Writing throughout Your Career." The growth of USTARS continues to amaze me. Our next goals are to enhance the sustainability of USTARS by partnering with nonprofit organizations and to create post-USTARS support.

Photo Credits

Photos are courtesy of the USTARS Media Group.

Acknowledgment. Thank you to the National Science Foundation, National Security Agency, Joanna Kania-Bartoszynska, USTARS Advisory Board, Margaret Owens, Kathy McElroy, David Eisenbud, Colette Patt, Alejandra Alvarado, Pamela E. Harris, Carlos De la Mora, Elizabeth Sterba, Jean Tashima, Del Insko, Tanya Moore, Julianna Tymoczko, Maggy Tomova, RB McGee, and all USTARS participants.

ABOUT THE AUTHOR

Candice Price, along with some colleagues, founded USTARS in 2010. Her service mission statement is to create and contribute to programs that broaden the participation of underrepresented groups by focusing on strong mentoring and research networks.

Candice Price

COMMUNICATION

Strengthening Doctoral Programs in Mathematics Education: A Continuous Process

Barbara Reys and Robert Reys

Editor's note: *An article "Math PhD Careers…" in the March* Notices *described a model program preparing young mathematicians for a broader range of careers. The article "Disruptions of the Academic Math Employment Market" appeared in the October 2016* Notices.

ABSTRACT. *We offer some ideas for improving doctoral preparation based on a survey of faculty members from twenty-three different institutions.*

There have been many calls to examine doctoral preparation in the United States [1], [2]. These calls have been initiated by professional organizations and by the Carnegie Foundation and have led to timely and practical suggestions for change [3], [4]. The Carnegie Foundation initiative addresses a wide range of disciplines, including doctoral preparation in mathematics and education [5].

From 2010 to 2014, 129 different institutions in the United States produced at least one doctoral graduate in mathematics education [6]. These institutions differ greatly in the number of faculty in mathematics education, the number of full-time doctoral students, the number and types of doctoral courses and experiences offered, and the institutional resources available to support doctoral candidates [7].

The purpose of this article is not to detail the content of any specific doctoral program in mathematics education, but to offer some ideas regarding improvement efforts from a recent survey of twenty-three institutions.[1] Over the past fifteen years these institutions have been involved in improving their doctoral programs in mathematics education with support from the Centers for Learning and Teaching (CLT) initiative of the National Science Foundation. Faculty members at the CLT institutions were asked to identify unique features of their doctoral preparation that might be of interest to faculty members involved in doctoral preparation at other institutions. This article provides a summary of their responses.

Each CLT institution was engaged in improving its doctoral preparation (in quality and quantity). While these efforts clearly led to an increase in the number of graduates [6], the extent to which particular features improved the quality of graduates (in terms of impact on the field) is much harder to document. Anecdotal evidence suggests that many of the CLT doctoral graduates are conducting research and publishing their work in high-impact research journals at a higher rate than is typical of early-career graduates. However, more systematic attention is needed to document trends. In this article we summarize some unique features of the CLT doctoral programs that survey respondents believe set their programs apart and likely

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DOI: http://dx.doi.org/10.1090/noti1499

[1] *City University of New York, Michigan State University, Ohio University, Penn State University, Rutgers University, University of Arizona, University of California–Berkeley, University of California–Los Angeles, University of California–Santa Cruz, University of Delaware, University of Georgia, University of Illinois–Chicago, University of Kentucky, University of Louisville, University of Maryland, University of Michigan, University of Missouri, University of New Mexico, University of Pennsylvania, University of Tennessee, University of Wisconsin, West Virginia University, and Western Michigan University.*

increase the quality and productivity of graduates. These features are grouped around four themes.

Theme 1. *Establishing and articulating the nature, content, and specific expectations of the doctoral program.* Given the uniqueness of most doctoral programs in mathematics education, it is essential to make public to prospective and current students the specific program requirements. This can be done through a variety of means, including a handbook, website, and/or induction seminars.

- Some institutions touted the number of mathematics education courses (as many as eight) available for doctoral students, including courses focusing on mathematics assesment, mathematics curriculum, mathematics teaching, learning mathematics, and technology in mathematics education.
- Some institutions offer unique pathways toward a doctorate in mathematics education. One program is completely personalized, requiring only two one-credit courses, with the remainder determined by the student's committee in collaboration with the student. Other programs are designed to integrate mathematics education into other disciplines, such as intersecting mathematics with language and culture. Of the integrated programs, STEM was the most frequently cited pathway, although the composition of STEM programs and the extent to which mathematics education is reflected varied greatly. Each of the integrated programs touted the opportunities to work with outstanding faculty members representing a variety of disciplines.
- Some institutions reported a special focus/theme embedded within their doctoral program. Examples included: urban education, rural education, social justice/equity/diversity, mathematics curriculum, and mathematics teaching. Establishing and promoting such specialty niches is consistent with the recommendation for creating intellectual communities that would allow doctoral students to choose institutions aligned with their particular interests [8]. Several institutions focus on preparing collegiate teachers of mathematics. These doctoral programs are housed in mathematics departments and require a substantial amount of graduate-level work in mathematics.

Theme 2. *Aggressively recruiting doctoral candidates.* Obtaining a steady stream of new, high-quality doctoral students is an essential component in shaping and improving programs. Strategies for identifying and attracting potential candidates that some institutions have used successfully include:

- Recruiting through word of mouth by alumni (i.e., asking alumni to be ambassadors for their doctoral program).
- Advertising in professional journals and booths at professional meetings to alert people to their doctoral program.
- Conducting a letter-writing campaign every year to colleges and universities within the region that do not have doctoral programs in mathematics education, encouraging faculty to invite and direct their best students with interests in mathematics education to consider entering their doctoral program.

Participants in the Third International Center for the Study of Mathematics Curriculum Conference on Mathematics Curriculum at the University of Chicago in 2014 (left to right, showing graduating institution and current employment): Chris Engledowl (University of Missouri, current doctoral student), Nevels Nevels (University of Missouri, Hazlewood School District), Dawn Teuscher (University of Missouri, Brigham Young University), Karen Fonkert (Western Michigan University, Charleston Southern University), Shannon Dingman (University of Missouri, University of Arkansas), Nicole Fonger (Western Michigan University, postdoc at University of Wisconsin), Becky Darrough (University of Missouri, Austin Peay University), Amanda Thomas (University of Missouri, University of Nebraska), Lorraine Males (Michigan State University, University of Nebraska), A. J. Edson (Western Michigan University, Michigan State University).

- Contacting local school districts to recruit experienced mathematics teachers, including a specific effort to recruit urban and rural mathematics teachers who may not have considered entering a doctoral program.

Once strong candidates are identified, strategies are utilized to help them make a commitment to doctoral preparation, including:

- Talking with prospective candidates and highlighting the job opportunities along with the shortages of doctorates in mathematics education for faculty positions in institutions of higher education.
- Providing information about financial support available to them (scholarships, fellowships, assistantships, and fringe benefits, such as medical insurance, travel support, office space, and computers).
- Inviting the top-rated applicants to campus for one- or two-day visits. This costs between \$500 and \$1,000 per candidate and has served as an excellent investment of resources. These visits provide an opportunity for the potential applicant to meet and engage in conversations with faculty and learn about current interests and research opportunities. The applicants also meet with current doctoral students so they learn about the culture and environment

of the mathematics education program. Only a limited number of new doctoral students can be supported each year, and these visits allow wise decisions as to which of the applicants should be awarded the largest scholarships and most prodigious fellowships.

Theme 3. *Establishing a professional community that includes doctoral students.* It is important to help new students acclimate to the doctoral program and also to provide continuous opportunities for every doctoral student to bond and grow and interact with a community of professionals including faculty, staff, and other doctoral students throughout the doctoral program. Strategies to establish and nurture such a community include:

• Organizing a seminar for first-year doctoral students to help them acclimate to their new environment, learn about institutional resources (library, health center, etc.), and get acquainted with faculty members and become knowledgeable about some of their expertise.

• Arranging for every student to be involved in a research study that must be completed by the end of the second year. This early immersion in research is facilitated by faculty-run research groups in an apprenticeship mode. This active engagement in research may be done independently or in a cohort, but the research study or studies are monitored by a faculty member regularly involved in the doctoral program.

• Scheduling a weekly seminar that doctoral students participate in throughout their program. Every doctoral student must make at least two presentations in a weekly seminar. Their presentations may take different forms, such as soliciting suggestions for a potential research project, providing a report of research in progress, sharing a draft of a manuscript being prepared for publication consideration, or highlighting a research presentation that has been or will be made at a professional meeting.

Enjoying a barbecue during a CSMC Research Conference in 2007 (left to right, showing graduating institution and current employment): Amanda Thomas (University of Missouri, University of Nebraska); Eileen Olson, spouse of Travis Olson (University of Missouri, UNLV); Jill Newton (Michigan State University, Purdue University); Karen Fonkert (Western Michigan University, Charleston Southern University), Jeff Shih (UCLA, UNLV); Dawn Teuscher (University of Missouri, Brigham Young University); Ryan Nivens (University of Missouri, Eastern Tennessee University).

Continual review and improvement is critical to growth in both quantity and quality.

Theme 4. *Preparing for a career in higher education.* Regardless of the research emphasis of an institution, teaching is a common expectation of mathematics educators entering positions in higher education. Another widespread need is preparing proposals for funding. Strategies used to support this preparation include:

• Improving instruction by having doctoral students serve as teaching assistants (TAs) within undergraduate or graduate K–12 teacher preparation courses. As part of these assistantships, they meet regularly (e.g., weekly) with other TAs and faculty to share challenges faced in the classes they are teaching and to discuss research-based evidence to improve their collective practice [9], [10]. At some institutions, TAs teach from a common set of shared, detailed lesson plans and meet weekly to debrief and to plan for the following week. Everyone is engaged in research and development that aims to improve the lessons and the overall learning and teaching of mathematics. This teach-research-teach model provides an algorithm for doctoral graduates to continue to reflect upon and improve their teaching throughout their careers.

• Giving students first-hand experience in preparing grant proposals. Doctoral students are engaged in preparing at least one proposal for funding. This may include, but is not limited to, searching for appropriate funding agencies, securing proposal guidelines, and conceptualizing a study, as well as writing and critically reviewing drafts of a proposal. In some cases, it has provided opportunities for faculty members and doctoral students to visit funding agencies to meet and talk with program officers about forthcoming proposals.

Where to from here? A major takeaway from the survey of CLT doctoral programs in mathematics education is that continual review and improvement is critical to growth in both quantity and quality. This article is an effort to share suggestions that might be useful in developing and improving doctoral preparation. While the focus has been on mathematics education, it is certainly possible that some of these suggestions are generic and would be of interest to mathematicians involved in PhD programs in mathematics.

One message from the Carnegie Foundation efforts is that doctoral programs need to be examined and reviewed regularly [3]. It could be argued that a doctoral program should be critically examined every five to ten years. If it has not changed during that time, then it is probably time to make some changes. Toward that end, it is our hope that some grist from this paper will be useful to doctoral faculty members looking for fresh ideas to strengthen their doctoral programs.

ACKNOWLEDGMENT. Research for this article was funded by the National Science Foundation under grant No. 1434442. However, the opinions expressed are the authors' and do not reflect any endorsement by the National Science Foundation.

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THE AUTHORS

Robert and Barbara Reys

CALL FOR NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2018 awards, which will be presented at the Joint Mathematics Meetings in San Diego, CA, in January 2018. Information about past recipients of these prizes may be found in the April 2014 (pp. 399–410), April 2015 (pp. 423–428), and April 2016 (pp. 422–437) issues of the *Notices*, and at www.ams.org/profession/prizes-awards/prizes.

ALBERT LEON WHITEMAN MEMORIAL PRIZE

The Albert Leon Whiteman Memorial Prize now is awarded every third year, for notable exposition on the history of mathematics. The ideas expressed and understandings embodied in that exposition should reflect exceptional mathematical scholarship.

AWARD FOR DISTINGUISHED PUBLIC SERVICE

This award, which is made every two years, recognizes a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years.

CHEVALLEY PRIZE IN LIE THEORY

The Chevalley Prize prize, awarded in even-numbered years, recognizes notable work in Lie Theory published during the preceding six years; a recipient should be at most twenty-five years past the PhD.

NOMINATION PERIOD: MARCH 1–JUNE 30, 2017.

FRANK NELSON COLE PRIZE IN ALGEBRA

The Cole Prize in Algebra, which recognizes a notable paper in algebra published during the preceding six years, is awarded every three years. To be eligible, papers must be either authored by an AMS member or published in a recognized North American journal.

GEORGE DAVID BIRKHOFF PRIZE IN APPLIED MATHEMATICS

The George David Birkhoff Prize is awarded jointly by the AMS and SIAM for an outstanding contribution to applied mathematics in its highest sense. The award was first made in 1968 and now is presented every third year.

LEVI L. CONANT PRIZE

The Levi L. Conant Prize is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

Further information about AMS prizes can be found at the Prizes and Awards website:
www.ams.org/profession/prizes-awards/prizes

Further information and instructions for submitting a nomination can be found at the prize nominations website: **www.ams.org/profession/prizes-awards/nominations**

For questions contact the AMS Secretary at secretary@ams.org

Nomination Period: March 1–June 30, 2017.

Mathematics People

Darmon Awarded 2017 CRM-Fields-PIMS Prize


Henri Darmon

Henri Darmon of McGill University has been awarded the 2017 CRM-Fields-PIMS Prize. The prize citation reads: "Professor Darmon is one of the leading number theorists of his generation. He has an extraordinary record of deep and highly influential contributions to the arithmetic theory of elliptic curves, including his recent breakthrough on the Birch and Swinnerton-Dyer Conjecture. He has also been an exceptional mentor to students and an exemplary citizen of the mathematical community." Darmon received his PhD in mathematics from Harvard University in 1991 under the supervision of Benedict Gross. He received the 1998 Coxeter-James Prize of the Canadian Mathematical Society, the 2008 John L. Synge Award of the Royal Society of Canada, and the 2017 Cole Prize in Number Theory of the AMS. For the AMS he gave invited lectures at the annual joint meetings in Orlando (1996) and San Antonio (2015), and he wrote an article on "Andrew Wiles's Marvelous Proof" for the March 2017 *Notices*. He is a member of the Royal Society of Canada.

—From a CRM-Fields-PIMS announcement

PECASE Awards Announced


Emily Fox

Two young researchers who work in the mathematical sciences have received Presidential Early Career Awards for Scientists and Engineers (PECASE). PECASE is the US government's highest award for scientists and engineers in the early stages of their research careers and who show the potential for exceptional leadership.

Emily Fox of the University of Washington was honored for her "groundbreaking work in large-scale Bayesian modeling and computational approaches to time series and longitudinal data analysis…and for outstanding outreach and mentoring of women in computer science and statistics."

Jacob Fox of Stanford University was honored "for his work on extending the regularity method to sparse graphs and hypergraphs…and for taking a leadership role in the combinatorics community, mentoring high school students in the MIT PRIMES activity, training graduate students and serving as adviser for MIT undergraduate students."

—From a White House announcement

Sacks Prizes Awarded


Omer Ben-Neria


Martino Lupini

The Association for Symbolic Logic (ASL) has awarded the Gerald Sacks Prize for 2015 to Omer Ben-Neria of the University of California Los Angeles and Martino Lupini of the California Institute of Technology. Ben-Neria was honored for his thesis "The Possible Structure of the Mitchell Order," in which he "proved the remarkable result that, under suitable large cardinal assumptions on the cardinal κ, every well-founded partial order of cardinality κ can be realized as the Mitchell order of κ in some forcing extension. The Prizes and Awards Committee noted that the proof is a tour de force combination of sophisticated forcing techniques with the methods of inner model theory." Lupini was honored for his thesis "Operator Algebras and Abstract Classification," which "includes a beautiful result establishing a fundamental dichotomy in the classification problem for the automorphisms of a separable unital C^*-algebra up to unitary equivalence, as well as a proof that the Gurarij operator space is unique, homogeneous, and universal among separable 1-exact operator spaces. The Prizes and Awards Committee noted that his thesis exhibits a high level of originality, as well as technical sophistication, in a broad spectrum of areas of logic and operator algebras." Lupini tells the *Notices*: "I would like to have a pet capybara, but I do not own a swamp. Some people mistake me for a panda." The Sacks Prize is awarded for the most outstanding doctoral dissertation in mathematical logic.

—From an ASL announcement

Charles L. Epstein and François Trèves Receive Bergman Prize

Charles L. Epstein

François Trèves

Charles L. Epstein of the University of Pennsylvania and François Trèves of Rutgers University have received the 2016 Bergman Prize. Established in 1988, the prize recognizes mathematical accomplishments in the areas of research in which Stefan Bergman worked. Epstein and Trèves will each receive US$12,000, which is one-half of the 2016 income from the Stefan Bergman Trust.

Citation: Epstein

Charles L. Epstein is awarded the Bergman Prize for his fundamental contributions to the theory of embeddability and stability of 3-dimensional Cauchy-Riemann (CR) structures. His extended series of papers on this subject includes his important early work with Burns in the 1980s, including their introduction of what is now called the Burns-Epstein invariant, his deep work on a relative index on the space of embeddable CR structures, his remarkable results with Henkin on the delicate structure of embeddable deformations of embeddable 3-dimensional CR structures, as well as his many other papers on the subject, on his own and with other collaborators. His other outstanding accomplishments include his incisive results on subelliptic boundary problems for Spin$_{\mathbb{C}}$-Dirac operators, leading to a proof of the Atiyah-Weinstein conjecture. Finally, Epstein is commended for the extensive range of his work in many parts of mathematical analysis.

Biographical Sketch: Epstein

Charles L. Epstein received his SB in mathematics from the Massachusetts Institute of Technology in 1978 and his PhD from New York University in 1983. Peter D. Lax directed his PhD thesis, "Geometrically Periodic Hyperbolic 3-Manifolds," which won the K. O. Friedrichs Prize of the Courant Institute. Epstein was then a National Science Foundation postdoctoral fellow and instructor at Princeton University; his postodoctoral mentor was William P. Thurston. While at Princeton, J. J. Kohn and a remarkable group of junior faculty colleagues and visitors (all of whom went on to win the Bergman Prize) introduced Epstein to the subject of several complex variables. In 1985, he joined the faculty at the University of Pennsylvania, where he currently holds the Thomas A. Scott Chair in Mathematics. He has held visiting appointments at the Universität Göttingen, the Institut des Hautes Études Scientifiques in Bures-sur-Yvette, the Eidgenössisches Technische Hochschule in Zurich, Universität Bern, Université Paris VI, the Institute for Advanced Study in Princeton, the Department of Radiology at the Hospital of the University of Pennsylvania, and New York University. He would like to acknowledge the important contributions of his collaborators—Daniel M. Burns Jr., John Bland, Richard Melrose, and Gennadi M. Henkin, as well as his friend László Lempert—to the work for which he has been awarded the Bergman Prize.

Epstein has also worked in spectral theory, hyperbolic geometry, univalent function theory, microlocal analysis, and index theory. For more than a decade, he has worked on a range of problems in medical imaging, image analysis, computational electro-magnetics, numerical analysis, and population genetics. In 2007 he founded the Graduate Group in Applied Mathematics and Computational Science at the University of Pennsylvania, which he continues to chair. He was a Sloan Foundation Fellow in 1988–1990. He is a Fellow of the AMS and the American Association for the Advancement of Science and serves on the Scientific Advisory Boards of the Math-Physics Division of the Simons Foundation in New York and of Brown University's Institute for Computational and Experimental Research in Mathematics. He and his wife Jane have two children.

Citation: Trèves

François Trèves is awarded the Bergman Prize for his many fundamental contributions to several complex variables and partial differential equations. This includes his work on the analytic hypoellipticity of pseudodifferential operators with double characteristics, which implies the analytic hypoellipticity of the Bergman projection on real analytic strictly pseudoconvex domains (the latter result was also proved independently by David Tartakoff using different methods). That in turn has had many applications to function theory in several complex variables. Both alone and in collaboration with Boutet de Monvel, Trèves proved many other pivotal results about C^∞ and analytic hypoellipticity of differential and pseudodifferential operators. The Baouendi-Trèves Approximation Theorem has had enormous impact on Cauchy-Reimann (CR) geometry. One consequence is the identification of the submanifolds on which CR functions are determined, which resolves a fundamental uniqueness question; another significant application is to the CR extension problem, which is a crucial ingredient in the determination of the local hull of holomorphy of a given CR submanifold. In a seminal work, with many far-reaching new ideas, Baouendi, Jacobowitz, and Trèves proved the real-analyticity of CR diffeomorphisms between real analytic CR manifolds under very general hypotheses. Together, Nirenberg and Trèves made groundbreaking contributions to the theory of local solvability of linear partial differential operators. Their work inspired a long stream of research by many of the most prominent mathematicians in the subject, leading eventually to Dencker's resolution of the Nirenberg-Trèves "Condition ψ conjecture."

Trèves is the author of several important and extensively used monographs. His insights continue to influence the research of many mathematicians.

Biographical Sketch: Trèves

François Trèves was born in Brussels, Belgium, in 1930, to Italian parents. He is an Italian citizen and became a US citizen in 1972. He did his graduate studies at Université Paris IV-Sorbonne, writing his thesis under the supervision of Laurent Schwartz. Trèves received the Doctorat d'Étatés Sciences degree in 1958.

Not being a French citizen, he could not at that time get a position in France. He came to the United States and was an assistant professor at the University of California Berkeley (1958–1960), an associate professor at Yeshiva University (1960–1963), and a professor at Purdue University (1964–1965 and 1967–1969). During 1965–1967, he was a lecturer at the Sorbonne. In 1970, he became a professor at Rutgers University, where he held the Robert Adrain Chair of Mathematics from 1984 until his retirement in 2005.

Trèves held a Sloan Fellowship during 1960–1964. He received the 1972 Chauvenet Prize of the Mathematical Association of America for his article "On local solvability of linear partial differential equations," which appeared in the *Bulletin of the AMS* in 1970. In 1991, he received the AMS Leroy P. Steele Prize for Mathematical Exposition for his two-volume work *Pseudodifferential and Fourier Integral Operators* (Plenum Press, 1980). He received the *Laurea Specialistica Honoris Causa* from the University of Pisa in 2004. He is Fellow of the AMS and a foreign member of the Brazilian Academy of Sciences.

Trèves has collected butterflies across the tropics and is currently writing his eighteenth book about mathematical analysis.

About the Prize

The Bergman Prize honors the memory of Stefan Bergman, best known for his research in several complex variables, as well as the Bergman projection and the Bergman kernel function that bear his name. A native of Poland, he taught at Stanford University for many years and died in 1977 at the age of eighty-two. He was an AMS member for thirty-five years. When his wife died, the terms of her will stipulated that funds should go toward a special prize in her husband's honor.

The AMS was asked by Wells Fargo Bank of California, the managers of the Bergman Trust, to assemble a committee to select recipients of the prize. In addition the Society assisted Wells Fargo in interpreting the terms of the will to assure sufficient breadth in the mathematical areas in which the prize may be given. Awards are made every one or two years in the following areas: (1) the theory of the kernel function and its applications in real and complex analysis; and (2) function-theoretic methods in the theory of partial differential equations of elliptic type with attention to Bergman's operator method.

The previous Bergman Prize winners are:

- David W. Catlin (1989)
- Steven R. Bell and Ewa Ligocka (1991)
- Charles Fefferman (1992)
- Yum Tong Siu (1993)
- John Erik Fornæss (1994)
- Harold P. Boas and Emil J. Straube (1995)
- David E. Barrett and Michael Christ (1997)
- John P. D'Angelo (1999)
- Masatake Kuranishi (2000)
- László Lempert and Sidney Webster (2001)
- M. Salah Baouendi and Linda Preiss Rothschild (2003)
- Joseph J. Kohn (2004)
- Elias M. Stein (2005)
- Kengo Hirachi (2006)
- Alexander Nagel and Stephen Wainger (2007–2008)
- Ngaiming Mok and Duong H. Phong (2009)
- Gennadi Henkin (2011)
- David Jerison and John M. Lee (2012)
- Xiaojun Huang and Steve Zelditch (2013)
- Sławomir Kołdziej and Takeo Ohsawa (2014).
- Eric D. Bedford and Jean-Pierre Demailly (2015).

On the selection committee for the 2016 prize were:

- Xiaojun Huang
- Rafe Mazzeo (Chair)
- Anna Mazzucato.

—*Allyn Jackson*

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Photo of Martino Lupini is courtesy of Mathematisches Forschungsinstitut Oberwolfach.

Mathematics Opportunities

Math in Moscow Scholarship Program

The Math in Moscow program at the Independent University of Moscow is a semester-long, mathematically intensive program (in English) in the Russian tradition of teaching mathematics—the emphasis being on problem solving rather than memorizing theorems. With funding from the NSF, the AMS awards five scholarships each semester to US students. The deadlines for applications for the scholarship program are **March 30** for the fall and **September 30** for the spring semester. Information and application forms are available at www.mccme.ru/mathinmoscow. Application forms for the AMS scholarships are available at www.ams.org/programs/travel-grants/mimoscow.

—AMS Membership and Programs Department

*NSF Enriched Doctoral Training in the Mathematical Sciences

The Enriched Doctoral Training in the Mathematical Sciences Program of the National Science Foundation (NSF) supports efforts to enrich research training in the mathematical sciences at the doctoral level by preparing PhD students to recognize and find solutions to mathematical challenges arising in other fields and in areas outside today's academic setting. The deadline for proposals is **July 12, 2017**. See www.nsf.gov/pubs/2014/nsf14589/nsf14589.htm.

—From a DMS announcement

*NSF-CBMS Regional Conferences 2017

With NSF support, the Conference Board of the Mathematical Sciences (CBMS) will hold seven Regional Research Conferences during the summer of 2017. Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focused area. Support for about thirty participants is provided for each conference.

May 22–26, 2017: Sparse Approximation and Signal Recovery Algorithms. Anna C. Gilbert, lecturer. New Mexico State University. Organizers: Joseph D. Lakey, Jameson Cahill, and Nicholas Michalowski. See www.math.nmsu.edu/activities/cbms2017/cbms2017.html .

June 4–9, 2017: Nonlocal Dynamics: Theory, Computation, and Applications. Qiang Du, lecturer. Illinois Institute of Technology. Organizers: Jinqaio Duan and Xiaofan Li. See math.iit.edu/nonlocaldynamics.html.

June 12–17, 2017: Topological Data Analysis: Theory and Applications.Vin de Silva, lecturer. Macalester College. Organizers: Lori B. Ziegelmeier, Matthew Richey, and Matthew L. Wright. See pages.stolaf.edu/tda-conference/ .

July 24–28, 2017: Tensors and Their Uses in Approximation Theory, Quantum Information Theory, and Geometry. J. M. Landsberg, lecturer. Auburn University. Organizer: Luke Oeding. See www.auburn.edu/~lao0004/cbms.html.

July 31–August 4, 2017: Topological and Geometric Methods in Quantum Field Theory. Dan Freed, lecturer. Montana State University. Organizers: David Ayala and Ryan E. Grady. See www.math.montana.edu/cbms/.

**The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at:* www.nsf.gov/dms *and for the Directorate of Education and Human Resources at* www.nsf.gov/dir/index.jsp?org=ehr. *To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at* www.nsf.gov/mps/dms/about.jsp.

August 14–18, 2017: Bayesian Modeling for Spatial and Spatio-temporal Data. Alan E. Gelfand, lecturer. University of California Santa Cruz. Organizers: Athanasios Kottas, Rajarshi Guhaniyogi, and Bruno Sanso. See https://cbms.soe.ucsc.edu/home.

August 28–September 1, 2017: Dyson-Schwinger Equations, Topological Expansions and Random Matrices. Alice Guionnet, lecturer. Columbia University. Organizers: Ivan Corwin and Yi Sun. See www.math.columbia.edu/department/probability/seminar/guionnet.html.

—From a CBMS announcement

*Call for Proposals for 2018 NSF-CBMS Regional Conferences

The NSF-CBMS Regional Research Conferences in the Mathematical Sciences are a series of five-day conferences, each of which features a distinguished lecturer delivering ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. Proposals should address the unique characteristics of the NSF-CBMS conferences, which can be found at www.cbmsweb.org/NSF/2018_call.htm. The deadline for full proposals is **April 28, 2017**.

—From a CBMS announcement

AWM Gweneth Humphreys Award

The Association for Women in Mathematics awards the Gweneth Humphreys Award annually to a mathematics teacher who has encouraged female undergraduate students to pursue mathematical careers and/or the study of mathematics at the graduate level. The deadline for nominations is **April 30, 2017**. See https://sites.google.com/site/awmmath/programs/humphreys-award or e-mail awm@awm-math.org.

—From an AWM announcement

Call for Applications for Rosenthal Prize

The National Museum of Mathematics awards the annual Rosenthal Prize for innovative mathematics teaching in upper elementary and middle schools. The deadline for applications is **May 24, 2017**. See momath.org/rosenthal-prize/.

—From a Museum of Mathematics announcement

Call for Nominations for the 2019 ICIAM Prizes

The ICIAM Prize Committee for 2019 calls for nominations for the five ICIAM Prizes to be awarded in 2019 (the Collatz Prize, the Lagrange Prize, the Maxwell Prize, the Pioneer Prize, and the Su Buchin Prize). Nominations are welcome from every part of the world.

A nomination should take into account the specifications for a particular prize; see www.iciam.org/iciam-prizes for additional information.

All nominations should be made electronically at https://iciamprizes.org/.

The deadline for nominations is **July 15th, 2017**.

—From an ICIAM announcement

Call for Nominations for the Ostrowski Prize, 2017

The aim of the Ostrowski Foundation is to promote the mathematical sciences. Every second year it provides a prize for recent outstanding achievements in pure mathematics and in the foundations of numerical mathematics. The value of the prize for 2017 is 100.000 Swiss francs (approximately US$99,000).

The jury invites nominations for candidates for the 2017 Ostrowski Prize. Nominations are due by **May 15, 2017**.

See https://www.ostrowski.ch/index_e.php for further details.

—From a CMI announcement

News from IPAM

The Institute for Pure and Applied Mathematics (IPAM) is an NSF math institute located at the University of California, Los Angeles. IPAM offers programs that encourage collaboration across disciplines and between two areas of mathematics. IPAM holds long programs (three months) and workshops (three to five days) throughout the academic year for junior and senior mathematicians and scientists who work in academia, research laboratories, and industry. In the summer, IPAM offers industrial research programs in Los Angeles, Hong Kong, and Berlin for undergraduate and graduate students.

IPAM seeks program proposals from the math and science communities. **Please send your idea for a workshop, long program, or summer school to** director@ipam.ucla.edu.

IPAM's upcoming programs are listed below. Please go to www.ipam.ucla.edu/programs for detailed information on each program, and to find application and registration forms.

On April 27–28, 2017, IPAM will host the *National Meeting of Women in Financial Mathematics.*

The spring 2017 long program *Computational Issues in Oil Field Applications* began on March 20. You may register online for one of the following workshops:

- Workshop I: Multiphysics, Multiscale, and Coupled Problems in Subsurface Physics. April 3–7, 2017
- Workshop II: Full Waveform Inversion and Velocity Analysis. May 1–5, 2017
- Workshop III: Data Assimilation, Uncertainty Reduction, and Optimization for Subsurface Flow. May 22–26, 2017

The fall 2017 long program *Complex High-Dimensional Energy Landscapes* is still accepting applications. You may also register or apply for funding for one of the following workshops online.

- Complex High-Dimensional Energy Landscapes Tutorials: September 11–15, 2017
- Workshop I: Optimization and Optimal Control for Complex Energy and Property Landscapes: October 2–6, 2017
- Workshop II: Stochastic Sampling and Accelerated Time Dynamics on Multidimensional Surfaces: October 16–20, 2017
- Workshop III: Surrogate Models and Coarsening Techniques: October 30–November 3, 2017
- Workshop IV: Uncertainty Quantification for Stochastic Systems and Applications: November 13–17, 2017

IPAM's spring 2018 long program is *Quantitative Linear Algebra*. More information on these programs, including a schedule of workshops and application and registration forms, is available online.

- Quantitative Linear Algebra Tutorials: March 19–23, 2018
- Workshop I: Expected Characteristic Polynomial Techniques and Applications: April 9–13, 2018
- Workshop II: Approximation Properties in von Neumann Algebras and Ergodic Theory: April 30–May 5, 2018
- Workshop III: Random Matrices and Free Probability Theory: May 14–18, 2018

IPAM is offering the following *Winter Workshops* in 2018. You may apply for support or register for each workshop online.

- Algorithmic Challenges in Protecting Privacy for Biomedical Data: January 10–12, 2018
- New Methods for Zimmer's Conjectures: January 22–26, 2018
- New Deep Learning Techniques: February 5–9, 2018

Finally, IPAM will be sponsoring the second *Latino/as in the Mathematical Sciences Conference* on March 8–10, 2018. More information will be available in the coming months.

—*IPAM announcement*

AMERICAN MATHEMATICAL SOCIETY

CALL FOR NOMINATIONS

NEW

ULF GRENANDER PRIZE IN STOCHASTIC THEORY AND MODELING

This prize was established in 2016 by colleagues of Ulf Grenander (1923–2016).

Professor Grenander was an influential scholar in stochastic processes, abstract inference, and pattern theory. He published landmark works throughout his career, notably his 1950 dissertation, "Stochastic Processes and Statistical Interference" at Stockholm University, *Abstract Inference*, and seminal works in pattern theory, *General Pattern Theory* and *Pattern Theory: From Representation to Inference*. A long-time faculty member of Brown University's Division of Applied Mathematics, Grenander received many honors. He was a fellow of the American Academy of Arts and Sciences, the National Academy of Sciences and was a member of the Royal Swedish Academy of Sciences.

The Grenander Prize recognizes exceptional theoretical and applied contributions in stochastic theory and modeling. It is awarded for seminal work, theoretical or applied, in the areas of probabilistic modeling, statistical inference, or related computational algorithms, especially for the analysis of complex or high-dimensional systems.

The prize amount is $5,000 and the prize is awarded every three years. The first award will be made in 2018.

Further information about AMS prizes can be found at the Prizes and Awards website:
www.ams.org/profession/prizes-awards/prizes

Further information and instructions for submitting a nomination can be found at the prize nominations website:
www.ams.org/profession/prizes-awards/nominations

For questions contact the AMS Secretary at secretary@ams.org

Nomination Period: March 1–June 30, 2017

AMS
American Mathematical Society
www.ams.org

Inside the AMS

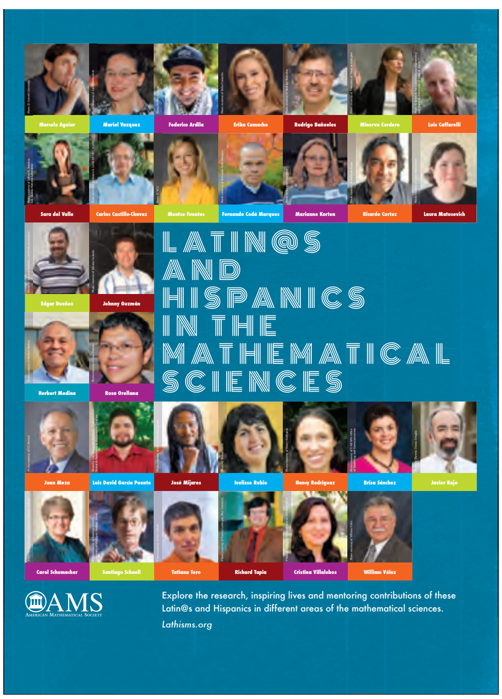
From the AMS Public Awareness Office

JMM 2017 Highlights

See photos from the 2017 Joint Mathematics Meetings—from invited addresses, the Joint Prize Session, public events at Mathemati-Con, and more—at `www.ams.org/jmm2017-highlights`.

Latin@s and Hispanics in the Mathematical Sciences Poster

This poster shows thirty-one Latinas, Latinos, and Hispanics in different areas of the mathematical sciences



and points to the `Lathisms.org` website, where readers can explore the research, inspiring lives, and mentoring contributions of the individuals.

Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

"News from the Road"

Robin Marek

AMS Director of Development, Robin Marek, often meets with mathematics scholars and industry professionals in planned visits around the country. During 2016 Robin met one-on-one or in small groups with mathematicians and related professionals in Arizona (Glendale, Mesa, Phoenix, Tempe, Tucson), New Mexico (Albuquerque, Las Cruces, Los Alamos, Santa Fe), New York City, North Carolina (Burlington, Carrboro, Chapel Hill, Davidson, Durham, Greensboro, Mooresville, Raleigh), Omaha, and Philadelphia. In her meetings with over seventy people she learned first-hand about the needs of mathematics professionals in these regions and the efficacy of AMS programs for them. She also gathered feedback and ideas for new ways the Society might serve and advocate for the mathematics community and reported these suggestions and invaluable information to appropriate AMS directors, staff, and governance.

If you would like to have Robin come to a discussion in your area, please contact her at `robin.marek@ams.org`.

Deaths of AMS Members

Clarence M. Ablow, of Tustin, California, died on February 9, 2016. Born on November 6, 1919, he was a member of the Society for 74 years.

Hans Heinrich Brungs, of Canada, died on February 4, 2016. Born on December 28, 1939, he was a member of the Society for 31 years.

Mark Alan Cooper, of Clinton, New Jersey, died on February 21, 2016. Born on March 8, 1950, he was a member of the Society for 39 years.

Frederick J. Fuglister, of Wausau, Wisconsin, died on March 6, 2015. Born on February 14, 1940, he was a member of the Society for 50 years.

Uwe R. Helmke, professor, University of Wurzburg, died on March 1, 2016. Born on April 26, 1952, he was a member of the Society for 20 years.

AMERICAN MATHEMATICAL SOCIETY

CALL FOR NOMINATIONS

NEW

BERTRAND RUSSELL PRIZE OF THE AMS

The Bertrand Russell Prize of the AMS was established in 2016 by Thomas Hales.

The prize looks beyond the confines of our profession to research or service contributions of mathematicians or related professionals promoting good in the world. It recognizes the various ways that mathematics furthers fundamental human values. The mission of the American Mathematical Society includes:

- Promoting the uses of mathematical research
- Advancing the status of the profession of mathematics
- Fostering an awareness and appreciation of mathematics and its connections to other disciplines and everyday life

This prize is designed to promote these goals. Mathematical contributions that further world health, our understanding of climate change, digital privacy, or education in developing countries, are some examples of the type of work that might be considered for the prize.

The US$5,000 prize is awarded every three years. The first award will be made in 2018.

Further information about AMS prizes can be found at the Prizes and Awards website:
www.ams.org/profession/prizes-awards/prizes

Further information and instructions for submitting a nomination can be found at the prize nominations website:
www.ams.org/profession/prizes-awards/nominations

For questions contact the AMS Secretary at secretary@ams.org

Nomination Period: March 1–June 30, 2017

AMS
AMERICAN MATHEMATICAL SOCIETY
www.ams.org

MATHEMATICS CALENDAR

This section contains new announcements of worldwide meetings and conferences of interest to the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. New announcements only are published in the print Mathematics Calendar featured in each *Notices* issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable). A second announcement will be published only if there are changes or necessary additional information. Asterisks (*) mark those announcements containing revised information.

In general, print announcements of meetings and conferences carry only the date, title and location of the event.

The complete listing of the Mathematics Calendar is available at: www.ams.org/meetings/calendar/mathcal

All submissions to the Mathematics Calendar should be done online via: www.ams.org/cgi-bin/mathcal/mathcal-submit.pl

Any questions or difficulties may be directed to mathcal@ams.org.

March 2017

*23 – April 7 **Noncommutative Geometry Festival in Shanghai**
Location: *Fudan University, Shanghai, China.*
URL: www.connes70.fudan.edu.cn

25 – 26 **Potential Theory and Arithmetic Dynamics, a Conference in Honor of Robert Rumely.**
Location: *University of Georgia, Athens, GA.*
URL: research.franklin.uga.edu/agant/content/agant-rtg

31 – 31 **Columbia-Princeton Probability Day 2017**
Location: *Princeton University, Princeton, NJ.*
URL: orfe.princeton.edu/conferences/cp17

April 2017

1 – June 30 **MOOC: Data Science and Predictive Analytics**
Location: *Virtual Massive Open Online Course (MOOC).*
URL: www.socr.umich.edu/people/dinov/2017/Spring/DSPA_HS650

7 – 9 **Eastern Illinois Integrated Conference in Geometry, Dynamics, and Topology**
Location: *Charleston, IL.*
URL: ux1.eiu.edu/~gdt/2017.html

17 – 20 **Young Researcher Workshop on Differential Geometry in Minkowski Space**
Location: *Granada, Spain.*
URL: www.ugr.es/~rcamino/minkowski/index.html

May 2017

8 – 11 **International Conference in Approximation Theory**
Location: *Georgia Southern University, Savannah, GA.*
URL: sites.google.com/a/georgiasouthern.edu/approximation-conf

*11 – 13 **International Conference on Mathematics and Mathematics Education (ICMME-2017)**
Location: *Harran University, Sanliurfa, Turkey.*
URL: theicmme.org I

15 – 17 **2017 Southeastern Probability Conference**
Location: *Duke University, Durham, North Carolina.*
URL: sites.duke.edu/sepc

15 – 18 **Harmonic Map Workshop**
Location: *Aber Wrac'h, Brest, France.*
URL: www.math.univ-brest.fr/perso/eric.loubeau/hmw-brest17.html

17 – 19 **Nonlinear Diffusion and Free Boundary Problems. A Conference on the Occasion of the Seventieth Anniversary of Juan Luis Vázquez.**
Location: *Universidad Autonoma de Madrid, Campus de Cantoblanco, 28049 Madrid, Spain.*
URL: verso.mat.uam.es/~jlv70

25 – 27 **Workshop on Nonlinear Analysis on the Occasion of the 65th Birthday of Patrizia Pucci**
Location: *Babes-Bolyai University, Cluj-Napoca, Romania.*
URL: www.cs.ubbcluj.ro/nonlinear-analysis-workshop-65th-birthday-of-patrizia-pucci

29 – June 2 **Representation Theory at the Crossroads of Modern Mathematics. Conference in Honor of Alexandre Kirillov.**
Location: *University of Reims, France.*
URL: reims.math.cnrs.fr/pevzner/aak81.html

31 – June 5 **Algebraic and Geometric Methods of Analysis**
Location: *Odessa National Academy of Food Technologies, Odessa city, Ukraine.*
URL: www.imath.kiev.ua/~topology/conf/agma2017

June 2017

4 – 10 **Geometric Topology in Cortona**
Location: *Palazzone, Cortona, Italy.*
URL: www.dm.unipi.it/~martelli/Cortona2017/Cortona.html

5 – 9 **Mathematical Aspects of the Physics with Non-self-adjoint Operators**
Location: *CIRM, Marseille, France.*
URL: www.ujf.cas.cz/NSAatCIRM

5 – 10 **Summer School on Modern Knot Theory: Aspects in Algebra, Analysis, Biology, and Physics**
Location: *University Freiburg, Germany.*
URL: home.mathematik.uni-freiburg.de/cookies17

12 – 12 **EPCO 2017—Portuguese Meeting on Optimal Control**
Location: *ISEG, Universidade de Lisboa, Lisbon, Portugal.*
URL: epco2017.weebly.com

12 – 16 **23rd Rolf Nevanlinna Colloquium**
Location: *Mathematics Institute/FIM ETH Zürich, Switzerland.*
URL: www.math.ethz.ch/fim/conferences/nevanlinna.html.

13 – 17 **6th Cornell Conference on Analysis, Probability, and Mathematical Physics on Fractals**
Location: *Cornell University, Ithaca, New York.*
URL: www.math.cornell.edu/~fractals

15 – 16 **Control and Dispersion of Waves**
Location: *Laboratoire de Mathématiques, Université de Versailles Saint-Quentin, Université Paris-Saclay.*
URL: lmv.math.cnrs.fr/evenements-scientifiques/controle-et-dispersion-des-ondes

26 – 30 **Algebraic Analysis and Representation Theory**
Location: *RIMS, Kyoto University, Kyoto, Japan.*
URL: sites.google.com/site/kashiwara2017

28 – July 2 **The 4th Conference of the Mathematical Society of the Republic of Moldova (CMSM4), Devoted to the Centennial of Vladimir Andrunachievici (1917-1997)**
Location: *Moldova State University and Academy of Sciences of Moldova, Chisinau, Republic of Moldova.*
URL: cmsm4.math.md

July 2017

1 – December 31 **Fields Institute Major Thematic Program on Geometric Analysis**
Location: *Fields Institute, Toronto, Ontario, Canada.*
URL: www.fields.utoronto.ca/activities/17-18/geometricanalysis

3 – 7 **Persistent Homology, Summer School**
Location: *Fac. Sc and CRMEF Rabat, Morocco.*
URL: sites.google.com/site/persistenthomology

10 – 21 **SMS 2017 Summer School: Contemporary Dynamical Systems**
Location: *Centre de recherches mathématiques, Montréal, Canada.*
URL: www.crm.umontreal.ca/sms/2017/index_e.php

17 – 21 **International Meeting in Commutative Algebra and its Related Areas, A Conference to Celebrate Professor Aron Simis Seventy-fifth Birthday**
Location: *ICMC□USP, São Carlos, Brazil.*
URL: simcara.wixsite.com/imcara

24 – 27 **13th Algebraic Hyperstructures and its Applications Conference**
Location: *Yildiz Technical University, İstanbul, Turkey.*
URL: www.aha2017.yildiz.edu.tr

24 – 28 **International Conference on Difference Equations and Applications (ICDEA 2017)**
Location: *West University of Timişoara, Timişoara, Romania.*
URL: icdea2017.uvt.ro

August 2017

14 – 18 **International Workshop on Operator Theory and its Applications (IWOTA 2017)**
Location: *TU Chemnitz, Chemnitz, Germany.*
URL: www.tu-chemnitz.de/mathematik/iwota2017

15 – 18 **6th International Eurasian Conference on Mathematical Sciences and Applications**
Location: *Budapest, Hungary.*
URL: www.iecmsa.org

22 – 24 **Caucasian Mathematics Conference (CMC)-II**
Location: *Department of Mathematics, Faculty of Sciences, Yuzuncu Yil University, 65080, Van, Turkey.*
URL: www.euro-math-soc.eu/cmc

September 2017

1 – August 31, 2018 **EPSRC-Warwick Symposium on Geometry, Topology and Dynamics in Low Dimensions**
Location: *Mathematics Institute, University of Warwick, Coventry CV8 1PR, United Kingdom.*
URL: www2.warwick.ac.uk/fac/sci/maths/research/events/2017-18/symposium

3 – 9 **Rationality, Stable Rationality and Birationally Rigidity of Complex Algebraic Varieties**
Location: *C.I.S.M., Udine, Italy.*
URL: rational.dimi.uniud.it

4 – 7 **Differential Equations and Applications**
Location: *Brno University of Technology, Faculty of Mechanical Engineering, Brno, Czech Republic.*
URL: diffeqapp.fme.vutbr.cz/index.html

11 – 15 **Introduction to Geometry, Dynamics, and Moduli in Low Dimensions**
Location: *Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom.*
URL: www2.warwick.ac.uk/fac/sci/maths/research/events/2017-18/symposium/igdm

12 – 15 **Complex High-Dimensional Energy Landscapes Tutorials**
Location: *Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA.*
URL: www.ipam.ucla.edu/eltut

14 – 17 **The 25th Conference on Applied and Industrial Mathematics CAIM 2017**
Location: *University "Alexandru Ioan Cuza" of Iasi, Iasi, Romania.*
URL: www.romai.ro/conferintele_romai/caim2017_en.html

18 – 22 **Geometric Topology in Low Dimensions**
Location: *Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom.*
URL: www2.warwick.ac.uk/fac/sci/maths/research/events/2017-18/symposium/gtld

25 – 29 **AIM Workshop: Boltzmann Machines**
Location: *American Institute of Mathematics, San Jose, CA.*
URL: `aimath.org/workshops/upcoming/boltzmann`

October 2017

2 – 5 **VI Congress of Turkic World Mathematical Society (TWMS 2017)**
Location: *L. N. Gumilyov Eurasian National University, Astana, Kazakhstan.*
URL: `www.twms-astana-2017.kz`

4 – 6 **Analysis and PDE**
Location: *Leibniz Universität Hannover, Hannover, Germany.*
URL: `www.math-conf.uni-hannover.de/anapde17.html`

9 – 13 **AIM Workshop: Sparse Domination of Singular Integral Operators**
Location: *American Institute of Mathematics, San Jose, CA.*
URL: `aimath.org/workshops/upcoming/sparsedomop`

30 – November 3 **AIM Workshop: Singular Geometry and Higgs Bundles in String Theory**
Location: *American Institute of Mathematics, San Jose, CA.*
URL: `aimath.org/workshops/upcoming/singularhiggs`

November 2017

6 – 24 **ICTS Program: Geometry, Groups and Dynamics (GGD)-2017**
Location: *ICTS Bangalore, India.*
URL: `www.icts.res.in/program/ggd2017`

13 – 17 **Workshop IV: Uncertainty Quantification for Stochastic Systems and Applications**
Location: *Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.*
URL: `www.ipam.ucla.edu/elws4`

19 – 22 **Groups, Group Rings, and Related Topics - 2017 (GGRRT 2017)**
Location: *The Oceanic Hotel, Khorfakan, United Arab Emirates.*
URL: `conferences.uaeu.ac.ae/ggrrt2017`

27 – 30 **ICTS discussion meeting: Surface Group Representations and Geometric Structures**
Location: *ICTS Bangalore, India.*
URL: `www.icts.res.in/discussion-meeting/SGGS2017`

30 – December 2 **Conference on Matrix and Functional Analysis**
Location: *National Institute of Technology, Jalandhar-144011, Punjab, India.*
URL: `matrixandfunctionalanalysis.in`

December 2017

7 – 8 **Workshop on Integrable Systems**
Location: *University of Sydney, Australia.*
URL: `wp.maths.usyd.edu.au/igs/workshops/integrable-systems-2017/`

11 – 15 **Computation in geometric topology**
Location: *Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom.*
URL: `www2.warwick.ac.uk/fac/sci/maths/research/events/2017-18/symposium/cgt`

15 – 19 **The 22nd Asian Technology Conference in Mathematics (ATCM 2017)**
Location: *Chung Yuan Christian University, Chungli, Taiwan.*
URL: `atcm.mathandtech.org`

January 2018

4 – 6 **2nd International Conference on Modern Mathematical Methods and High Performance Computing in Science & Technology (M3HPCST-2018)**
Location: *Department of Applied Mathematics Inderprastha Engineering College 63 Site IV, Sahibabad Industrial Area, Surya Nagar Flyover Road Sahibabad, Ghaziabad-U.P, PIN Code-201010, India.*
URL: `www.ipec.org.in/m3hpcst-2018`

22 – 26 **Introductory Workshop: Enumerative Geometry Beyond Numbers**
Location: *Mathematical Sciences Research Institute, Berkeley, CA.*
URL: `www.msri.org/workshops/815`

March 2018

19 – 23 **Workshop on Teichmuller Dynamics**
Location: *Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom.*
URL: `www2.warwick.ac.uk/fac/sci/maths/research/events/2017-18/symposium/td`

August 2019

12 – December 13 **Holomorphic Differentials in Mathematics and Physics (HDMP)**
Location: *Mathematical Sciences Research Institute, Berkeley, CA.*
URL: `www.msri.org/programs/310`

Classified Advertising

Positions available, items for sale, services available, and more

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at The Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment, and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the honorary director, Professor Zhiming Ma is the director of the Advisory Board. Professor William Y. C. Chen serves as the director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

To apply, send your resume to `zhangry@tju.edu.cn`.

For more information, please visit `www.cam.tju.edu.cn` or contact Ms. Debbie Renyuan Zhang at `zhangry@tju.edu.cn`, telephone: 86-22-2740-5389.

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SOUTH AFRICA

Gottfried Wilhelm Leibniz Basic Research Institute Researchers Wanted

The Gottfried Wilhelm Leibniz Basic Research Institute, which operates as a PBO, invites exceptional researchers in mathematics and/or quantum physics to apply for shorter or longer term support. Contact `eerosinger@hotmail.com`.

00007

Classified Advertising

CONSULTANT SOUGHT

To Help Prepare Paper, "A Solution to the $3x+1$ Problem," for Publication

Considerable prestige is awaiting the mathematician who helps me prepare the paper for publication, and who writes the editor stating the mathematician's opinion that my solution is correct. (My degree is in computer science, and most of my career has been spent as a researcher in the computer industry.)

I will be glad to offer shared-authorship for contribution to the content.

A brief summary of the Solution—i.e., of a proof of the $3x+1$ Conjecture—is available in Appendix E of the paper.

The paper is on the website occampress.com.

Peter Schorer, peteschorer@gmail.com.

00005

PUBLICATIONS FOR SALE

The New Mathematical Science
By Dr. Mehran Basti

Differential Equations and Polynomials,
Volumes 1-3
Abstract available at
www.infinitypublishing.com

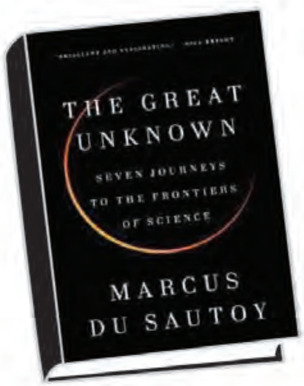
Additional Work: ***DNA of Mathematics***
friesenpress.com

00006

BOOKSHELF

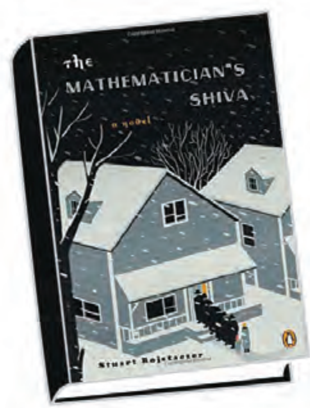
New and Noteworthy Titles on Our Bookshelf

April 2017



The Great Unknown: Seven Journeys to the Frontiers of Science, by Marcus du Sautoy (Penguin Group Viking, April 2017).

Marcus du Sautoy, in addition to being a mathematics professor at Oxford University, is the Simonyi Professor for the Public Understanding of Science. In this latter capacity, he has contributed to raising public awareness of mathematics in a multitude of ways, such as appearing in television and radio broadcasts, serving as a math consultant for theatrical productions, curating and presenting musical concerts, and even writing and performing in his own play about mathematics called *X & Y*. He has also pursued the more-traditional path to reaching the public, namely, writing popular books. One of these, titled *Symmetry: A Journey into the Patterns of Nature* (Harper, March 2008), tells the story of the classification of finite simple groups and was reviewed by Brian Blank in the February 2011 issue of the *Notices*. His latest book, *The Great Unknown* (originally published in the UK as *What We Cannot Know*), draws on many of his wide-ranging interests. The subject of the book is "what we know we cannot know"—that is, scientific and mathematical questions that we know can never be answered. Du Sautoy discusses many examples in a variety of areas such as particle physics, brain science, cosmology, biology, and mathematics. Not attempting to be definitive, the book instead provides philosophical reflections on the limits of knowledge. In a review on the web site *Undark*, John Durant, director of the MIT Museum, wrote of the book: "Du Sautoy is an eminently enjoyable interpreter of science; and his, like others', is a distinctive view of the place of science in human affairs."



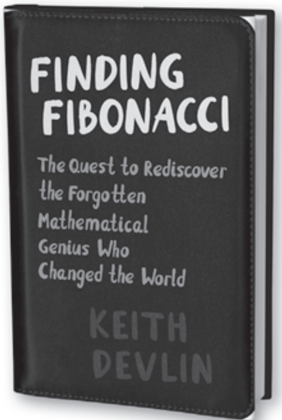
The Mathematician's Shiva, by Stuart Rojstaczer (Penguin Random House, September 2014).

The central character of the novel *The Mathematician's Shiva* is Rachela, a Polish Jew who was educated in Moscow—she was a student of Kolmogorov—and is now a mathematician at the University of Wisconsin. Her son narrates the story, which mainly centers on inter-relationships within her family—her father, husband, brother, and (semi-)adopted Russian daughter, who is a ballet dancer. Rachela's death is the beginning of the novel. She is the greatest mathematician of the age, and people suspect that she might have in hand a solution to the Navier-Stokes problem. The book's only explanation of the problem is that it has something to do with turbulence. The mathematics is not important in the book, except as a vehicle for Rachela's singular genius and a metaphor for aspects of her life (turbulent, rigorous). Perhaps it is also supposed to supply a reason for the personality quirks of the mathematical horde that descends on Madison for her funeral/shiva. Together, the mathematicians take the opportunity to mount an attack on Navier-Stokes for the week. They fail. But the deceased has already succeeded, in secret, and the climax of the novel is her husband's posthumous presentation of her work at the 107th Annual Meeting of the AMS in Boston!

Suggestions for the BookShelf can be sent to `notices-booklist@ams.org`.

We try to feature items of broad interest. Appearance of a book in the *Notices* BookShelf does not represent an endorsement by the *Notices* or by the AMS. For more, visit the AMS Reviews webpage `www.ams.org/news/math-in-the-media/reviews`.

New from Princeton



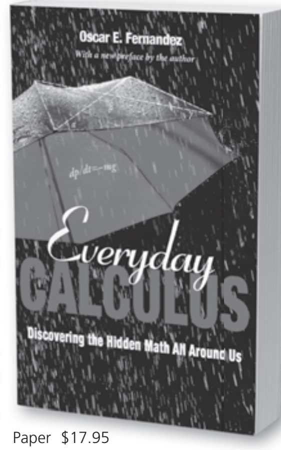
Cloth $29.95

Finding Fibonacci

The Quest to Rediscover the Forgotten Mathematical Genius Who Changed the World

Keith Devlin

"Lovers of history, travel, and mathematics alike will relish this journey through time to ancient worlds, as master expositor Keith Devlin navigates Italy to uncover the beginnings of modern math. Fascinating!"
—Danica McKellar, *New York Times* bestselling author of *Math Doesn't Suck*



Paper $17.95

Everyday Calculus

Discovering the Hidden Math All around Us

Oscar E. Fernandez

"Fernandez's witty, delightful approach makes for a winning introduction to the wonderland of math behind the scenes of everyday life."
—*Publishers Weekly*

"A delightful read. . . . A triumph in the pursuit of the lofty goal of comprehending the world."
—*San Francisco Book Review*

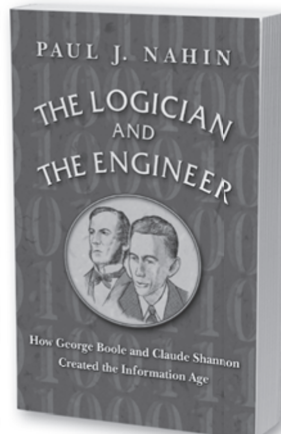
Paper $17.95

The Golden Ticket

P, NP, and the Search for the Impossible

Lance Fortnow

"You will love this book. It's completely accessible and captures the thrill, potential, and heartbreak of an edgy mathematical problem in terms that nonmathematicians will appreciate. After reading *The Golden Ticket*, I sort of hope P isn't NP after all."
—Vint Cerf, Internet Pioneer



Paper $17.95

The Logician and the Engineer

How George Boole and Claude Shannon Created the Information Age

Paul J. Nahin

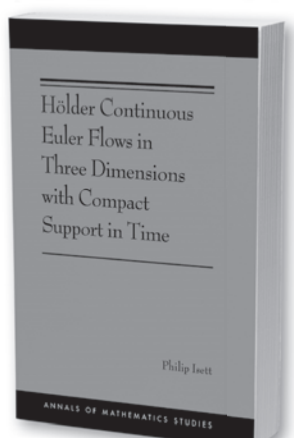
"Meshing logic problems with the stories of two extraordinary men. . . . Paul Nahin fashions a tale of innovation and discovery. Alongside a gripping account of how Shannon built on Boole's work, Nahin explores others key to the technological revolution, from Georg Cantor to Alan Turing."
—*Nature*

Paper $32.95

The Best Writing on Mathematics 2016

Edited by Mircea Pitici

This annual anthology brings together the year's finest mathematics writing from around the world. Featuring promising new voices alongside some of the foremost names in the field, *The Best Writing on Mathematics 2016* makes available to a wide audience many articles not easily found anywhere else.



Cloth $165.00
Paper $75.00

Hölder Continuous Euler Flows in Three Dimensions with Compact Support in Time

Philip Isett

Lars Onsager conjectured in 1949 that weak solutions to the incompressible Euler equations might fail to conserve energy if their spatial regularity was below 1/3-Hölder. In this book, Philip Isett uses the method of convex integration to achieve the best-known results regarding nonuniqueness of solutions and Onsager's conjecture.

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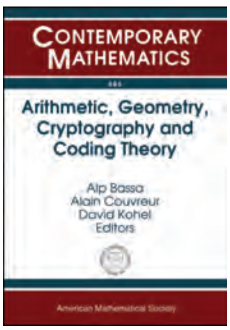
Algebra and Algebraic Geometry



Arithmetic, Geometry, Cryptography and Coding Theory

Alp Bassa, *Bogazici University, Istanbul, Bebek, Turkey*, **Alain Couvreur**, *Ecole Polytechnique, Palaiseau, France*, and **David Kohel**, *Aix-Marseille Université, France*, Editors

This volume contains the proceedings of the 15th International Conference on Arithmetic, Geometry, Cryptography, and Coding Theory (AGCT), held at the Centre International de Rencontres Mathématiques in Marseille, France, from May 18–22, 2015.

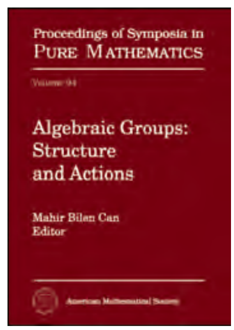
Since the first meeting almost 30 years ago, the biennial AGCT meetings have been one of the main events bringing together researchers interested in explicit aspects of arithmetic geometry and applications to coding theory and cryptography. This volume contains original research articles reflecting recent developments in the field.

This item will also be of interest to those working in applications.

Contents: N. Anbar, **P. Beelen**, and **N. Nguyen**, The exact limit of some cubic towers; **S. Dib** and **F. Rodier**, Error-correction capability of Reed-Muller codes; **Y. Aubry** and **A. Iezzi**, Optimal and maximal singular curves; **E. Férard**, An infinite class of Kasami functions that are not APN infinitely often; **R. Lercier** and **M. Olive**, Covariant algebra of the binary nonic and the binary decimic; **S. Ballet**, **J. Pieltant**, **M. Rambaud**, and **J. Sijsling**, On some bounds for symmetric tensor rank of multiplication in finite fields; **S. Haloui**, Codes from Jacobian surfaces; **E. Nart** and **C. Ritzenthaler**, A new proof of a Thomae-like formula for non hyperelliptic genus 3 curves; **M. Datta** and **S. R. Ghorpade**, Remarks on the Tsfasman-Boguslavsky Conjecture and higher weights of projective Reed-Muller codes; **J. P. Hansen**, Secret sharing schemes with strong multiplication and a large number of players from toric varieties; **V. Vitse**, Field extensions and index calculus on algebraic curves.

Contemporary Mathematics, Volume 686

April 2017, approximately 196 pages, Softcover, ISBN: 978-1-4704-2810-5, 2010 *Mathematics Subject Classification:* 11T71, 11G20, 11G25, 14G15, 14H40, 94A60, 94B27, **AMS members US$88.80**, List US$111, Order code CONM/686



Algebraic Groups: Structure and Actions

Mahir Bilen Can, *Tulane University, New Orleans, LA*, Editor

This volume contains the proceedings of the 2015 Clifford Lectures on Algebraic Groups: Structures and Actions, held from March 2–5, 2015, at Tulane University, New Orleans, Louisiana.

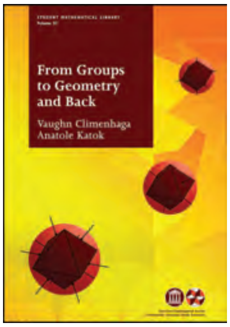
This volume consists of six articles on algebraic groups, including an enhanced exposition of the classical results of Chevalley and Rosenlicht on the structure of algebraic groups; an enhanced survey of the recently developed theory of pseudo-reductive groups; and an exposition of the recently developed operational K-theory for singular varieties. In addition, there are three research articles containing previously unpublished foundational results on birational automorphism groups of algebraic varieties; solution of Hermite-Joubert problem over p-closed fields; and cohomological invariants and applications to classifying spaces.

The old and new results presented in these articles will hopefully become cornerstones for the future development of the theory of algebraic groups and applications. Graduate students and researchers working in the fields of algebraic geometry, number theory, and representation theory will benefit from this unique and broad compilation of fundamental results on algebraic group theory.

Contents: D. Anderson, Computing torus-equivariant K-theory of singular varieties; **J. Blanc**, Algebraic structures of groups of birational transformations; **M. Brassil** and **Z. Reichstein**, The Hermite-Joubert problem over p-closed fields; **M. Brion**, Some structure theorems for algebraic groups; **B. Conrad** and **G. Prasad**, Structure and classification of pseudo-reductive groups; **A. S. Merkurjev**, Invariants of algebraic groups and retract rationality of classifying spaces.

Proceedings of Symposia in Pure Mathematics, Volume 94

April 2017, 294 pages, Hardcover, ISBN: 978-1-4704-2601-9, LC 2016021970, 2010 *Mathematics Subject Classification:* 12F10, 14C15, 14C35, 14C40, 14E07, 14E08, 14J70, 14L15, 14L30, 14M17, 14M25, 20G15, **AMS members US$100.80**, List US$126, Order code PSPUM/94



From Groups to Geometry and Back

Vaughn Climenhaga, *University of Houston, TX*, and **Anatole Katok**, *Pennsylvania State University, University Park, PA*

Groups arise naturally as symmetries of geometric objects, and so groups can be used to understand geometry and topology. Conversely, one can study abstract groups by using geometric techniques and ultimately by treating groups themselves as geometric objects. This book explores these connections between group theory and geometry, introducing some of the main ideas of transformation groups, algebraic topology, and geometric group theory.

The first half of the book introduces basic notions of group theory and studies symmetry groups in various geometries, including Euclidean, projective, and hyperbolic. The classification of Euclidean isometries leads to results on regular polyhedra and polytopes; the study of symmetry groups using matrices leads to Lie groups and Lie algebras.

The second half of the book explores ideas from algebraic topology and geometric group theory. The fundamental group appears as yet another group associated to a geometric object and turns out to be a symmetry group using covering spaces and deck transformations. In the other direction, Cayley graphs, planar models, and fundamental domains appear as geometric objects associated to groups. The final chapter discusses groups themselves as geometric objects, including a gentle introduction to Gromov's theorem on polynomial growth and Grigorchuk's example of intermediate growth.

The book is accessible to undergraduate students (and anyone else) with a background in calculus, linear algebra, and basic real analysis, including topological notions of convergence and connectedness.

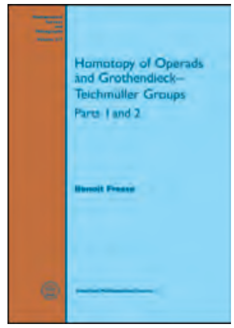
This book is a result of the MASS course in algebra at Penn State University in the fall semester of 2009.

This item will also be of interest to those working in geometry and topology.

Contents: Elements of group theory; Symmetry in the Euclidean world: Groups of isometries of planar and spatial objects; Groups of matrices: Linear algebra and symmetry in various geometries; Fundamental group: A different kind of group associated to geometric objects; From groups to geometric objects and back; Groups at large scale; Hints to selected exercises; Suggestions for projects and further reading; Bibliography; Index.

Student Mathematical Library, Volume 81

May 2017, approximately 433 pages, Softcover, ISBN: 978-1-4704-3479-3, LC 2016043600, 2010 *Mathematics Subject Classification:* 20-01, 51-01; 20F65, 22E40, 22F50, 51M05, 51M10, 54H15, 57M10, 57M60, **All Individuals US$46.40**, List US$58, Institutional member US$46.40, Order code STML/81



Homotopy of Operads and Grothendieck–Teichmüller Groups

Parts 1 and 2

Benoit Fresse, *Université de Lille 1, Villeneuve d'Ascq, France*

The Grothendieck–Teichmüller group was defined by Drinfeld in quantum group theory with insights coming from the Grothendieck program in Galois theory. The ultimate goal of this book set is to explain that this group has a topological interpretation as a group of homotopy automorphisms associated to the operad of little 2-discs, which is an object used to model commutative homotopy structures in topology.

The first part of this two-part set gives a comprehensive survey on the algebraic aspects of this subject. The book explains the definition of an operad in a general context, reviews the definition of the little discs operads, and explains the definition of the Grothendieck–Teichmüller group from the viewpoint of the theory of operads. In the course of this study, the relationship between the little discs operads and the definition of universal operations associated to braided monoidal category structures is explained. Also provided is a comprehensive and self-contained survey of the applications of Hopf algebras to the definition of a rationalization process, the Malcev completion, for groups and groupoids.

Most definitions are carefully reviewed in the book; it requires minimal prerequisites to be accessible to a broad readership of graduate students and researchers interested in the applications of operads.

The ultimate goal of the second part of the book is to explain that the Grothendieck–Teichmüller group, as defined by Drinfeld in quantum group theory, has a topological interpretation as a group of homotopy automorphisms associated to the little 2-disc operad. To establish this result, the applications of methods of algebraic topology to operads must be developed. This volume is devoted primarily to this subject, with the main objective of developing a rational homotopy theory for operads.

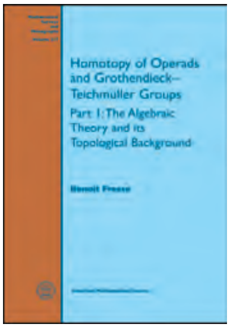
The book starts with a comprehensive review of the general theory of model categories and of general methods of homotopy theory. The definition of the Sullivan model for the rational homotopy of spaces is revisited, and the definition of models for the rational homotopy of operads is then explained. The applications of spectral sequence methods to compute homotopy automorphism spaces associated to operads are also explained. This approach is used to get a topological interpretation of the Grothendieck–Teichmüller group in the case of the little 2-disc operad.

This volume is intended for graduate students and researchers interested in the applications of homotopy theory methods in operad theory. It is accessible to readers with a minimal background in classical algebraic topology and operad theory.

Each volume in this set is sold separately. For a description of each volume, see the New Publication entries that follow.

Mathematical Surveys and Monographs, Volume 217

Set: May 2017, approximately 1278 pages, Hardcover, ISBN: 978-1-4704-3480-9, LC 2016032055, 2010 *Mathematics Subject Classification:* 55P48, 18G55, 55P10, 55P62, 57T05, 20B27, 20F36, **AMS members US$200**, List US$250, Order code SURV/217



Homotopy of Operads and Grothendieck–Teichmüller Groups

Part 1: The Algebraic Theory and its Topological Background

Benoit Fresse, *Université de Lille 1, Villeneuve d'Ascq, France*

The Grothendieck–Teichmüller group was defined by Drinfeld in quantum group theory with insights coming from the Grothendieck program in Galois theory. The ultimate goal of this book is to explain that this group has a topological interpretation as a group of homotopy automorphisms associated to the operad of little 2-discs, which is an object used to model commutative homotopy structures in topology.

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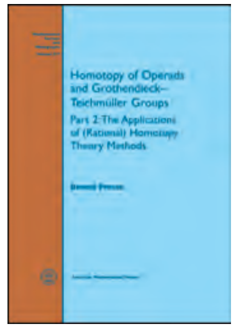
Most definitions are carefully reviewed in the book; it requires minimal prerequisites to be accessible to a broad readership of graduate students and researchers interested in the applications of operads.

Contents: *From operads to Grothendieck–Teichmüller groups. The general theory of operads:* The basic concepts of the theory of operads; The definition of operadic composition structures revisited; Symmetric monoidal categories and operads; *Braids and E_n-operads:* The little discs model of E_n-operads; Braids and the recognition of E_2-operads; The magma and parenthesized braid operators; *Hopf algebras and the Malcev completion:* Hopf algebras; The Malcev completion for groups; The Malcev completion for groupoids and operads; *The operadic definition of the Grothendieck–Teichmüller group:* The Malcev completion of the braid operads and Drinfeld's associators; The Grothendieck–Teichmüller group; A glimpse at the Grothendieck program; *Appendices:* Trees and the construction of free operads; The cotriple resolution of operads; Glossary of notation; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 217

May 2017, approximately 563 pages, Hardcover, ISBN: 978-1-4704-3481-6, LC 2016032055, 2010 *Mathematics Subject Classification:* 55P48; 18G55, 55P10, 55P62, 57T05, 20B27, 20F36, **AMS members US$108**, List US$135, Order code SURV/217.1



Homotopy of Operads and Grothendieck–Teichmüller Groups

Part 2: The Applications of (Rational) Homotopy Theory Methods

Benoit Fresse, *Université de Lille 1, Villeneuve d'Ascq, France*

The ultimate goal of this book is to explain that the Grothendieck–Teichmüller group, as defined by Drinfeld in quantum group theory, has a topological interpretation as a group of homotopy automorphisms associated to the little 2-disc operad. To establish this result, the applications of methods of algebraic topology to operads must be developed. This volume is devoted primarily to this subject, with the main objective of developing a rational homotopy theory for operads.

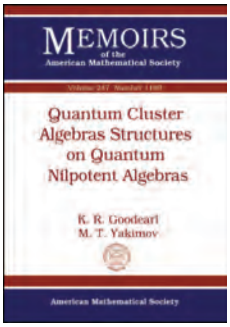
The book starts with a comprehensive review of the general theory of model categories and of general methods of homotopy theory. The definition of the Sullivan model for the rational homotopy of spaces is revisited, and the definition of models for the rational homotopy of operads is then explained. The applications of spectral sequence methods to compute homotopy automorphism spaces associated to operads are also explained. This approach is used to get a topological interpretation of the Grothendieck–Teichmüller group in the case of the little 2-disc operad.

This volume is intended for graduate students and researchers interested in the applications of homotopy theory methods in operad theory. It is accessible to readers with a minimal background in classical algebraic topology and operad theory.

Contents: *Homotopy theory and its applications to operads. General methods of homotopy theory:* Model categories and homotopy theory; Mapping spaces and simplicial model categories; Simplicial structures and mapping spaces in general model categories; Cofibrantly generated model categories; *Modules, algebras, and the rational homotopy of spaces:* Differential graded modules, simplicial modules, and cosimplicial modules; Differential graded algebras, simplicial algebras, and cosimplicial algebras; Models for the rational homotopy of spaces; *The (rational) homotopy of operads:* The model category of operads in simplicial sets; The homotopy theory of (Hopf) cooperads; Models for the rational homotopy of (non-unitary) operads; The homotopy theory of (Hopf) Λ-cooperads; Models for the rational homotopy of unitary operads; *Applications of the rational homotopy to E_n-operads:* Complete Lie algebras and rational models of classifying spaces; Formality and rational models of E_n-operads; *The computation of homotopy automorphism spaces of operads:* Introduction to the results of the computations for the E_n-operads; *The applications of homotopy spectral sequences:* Homotopy spsectral sequences and mapping spaces of operads; Applications of the cotriple cohomology of operads; Applications of the Koszul duality of operads; *The case of E_n-operads:* The applications of the Koszul duality for E_n-operads; The interpretation of the result of the spectral sequence in the case of E_2-operads; *Conclusion: A survey of further research on operadic mapping spaces and their applications:* Graph complexes and E_n-operads; From E_n-operads to embedding spaces; *Appendices:* Cofree cooperads and the bar duality of operads; Glossary of notation; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 217

May 2017, approximately 715 pages, Hardcover, ISBN: 978-1-4704-3482-3, LC 2016032055, 2010 *Mathematics Subject Classification:* 55P48; 18G55, 55P10, 55P62, 57T05, 20B27, 20F36, **AMS members US$108**, List US$135, Order code SURV/217.2



Quantum Cluster Algebras Structures on Quantum Nilpotent Algebras

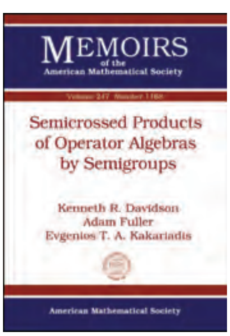
K. R. Goodearl, *University of California, Santa Barbara*, and **M. T. Yakimov**, *Louisiana State University, Baton Rouge*

Contents: Introduction; Quantum cluster algebras; Iterated skew polynomial algebras and noncommutative UFDs; One-step mutations in CGL extensions; Homogeneous prime elements for subalgebras of symmetric CGL extensions; Chains of mutations in symmetric CGL extensions; Division properties of mutations between CGL extension presentations; Symmetric CGL extensions and quantum cluster algebras; Quantum groups and quantum Schubert cell algebras; Quantum cluster algebra structures on quantum Schubert cell algebras; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 247, Number 1169

April 2017, 119 pages, Softcover, ISBN: 978-1-4704-3694-0, 2010 *Mathematics Subject Classification:* 16T20; 13F60, 17B37, 14M15, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1169

Analysis



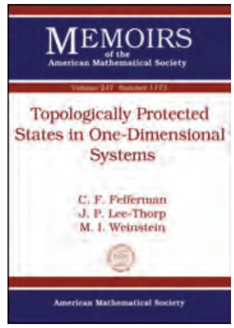
Semicrossed Products of Operator Algebras by Semigroups

Kenneth R. Davidson, *University of Waterloo, ON, Canada*, **Adam Fuller**, *Ohio University, Athens*, and **Evgenios T. A. Kakariadis**, *Newcastle University, Newcastle upon Tyne, UK*

Contents: Introduction; Preliminaries; Semicrossed products by abelian semigroups; Nica-covariant semicrosssed products; Semicrossed products by non-abelian semigroups; Bibliography.

Memoirs of the American Mathematical Society, Volume 247, Number 1168

April 2017, 97 pages, Softcover, ISBN: 978-1-4704-2309-4, 2010 *Mathematics Subject Classification:* 47A20, 47L25, 47L65, 46L07, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1168



Topologically Protected States in One-Dimensional Systems

C. F. Fefferman, *Princeton University, New Jersey*, **J. P. Lee-Thorp**, *Columbia University, New York, NY*, and **M. I. Weinstein**, *Columbia University, New York, NY*

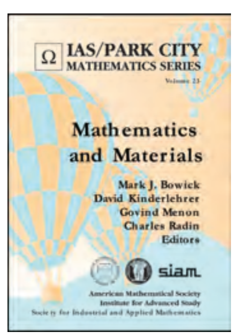
This item will also be of interest to those working in mathematical physics.

Contents: Introduction and outline; Floquet-Bloch and Fourier analysis; Dirac points of 1D periodic structures; Domain wall modulated periodic Hamiltonian and formal derivation of topologically protected bound states; Main Theorem—Bifurcation of topologically protected states; Proof of the Main Theorem; Appendix A. A variant of Poisson summation; Appendix B. 1D Dirac points and Floquet-Bloch eigenfunctions; Appendix C. Dirac points for small amplitude potentials; Appendix D. Genericity of Dirac points - 1D and 2D cases; Appendix E. Degeneracy lifting at Quasi-momentum zero; Appendix F. Gap opening due to breaking of inversion symmetry; Appendix G. Bounds on leading order terms in multiple scale expansion; Appendix H. Derivation of key bounds and limiting relations in the Lyapunov-Schmidt reduction; References.

Memoirs of the American Mathematical Society, Volume 247, Number 1173

April 2017, 118 pages, Softcover, ISBN: 978-1-4704-2323-0, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1173

Applications



Mathematics and Materials

Mark J. Bowick, *Syracuse University, NY*, **David Kinderlehrer**, *Carnegie Mellon University, Pittsburgh, PA*, **Govind Menon**, *Brown University, Providence, RI*, and **Charles Radin**, *University of Texas at Austin, TX*, Editors

Articles in this volume are based on lectures presented at the Park City summer school on "Mathematics and Materials" in July 2014. The central theme is a description of material behavior that is rooted in statistical mechanics. While many presentations of mathematical problems in materials science begin with continuum mechanics, this volume takes an alternate approach. All the lectures present unique pedagogical introductions to the rich variety of material behavior that emerges from the interplay of geometry and statistical mechanics. The topics include the order-disorder transition in many geometric models of materials including nonlinear elasticity, sphere packings, granular materials, liquid crystals, and the emerging field of synthetic self-assembly. Several lectures touch on discrete geometry (especially packing) and statistical mechanics.

The problems discussed in this book have an immediate mathematical appeal and are of increasing importance in applications, but are not as widely known as they should be to mathematicians interested in materials science. The volume will be of interest to graduate students and researchers in analysis and partial differential equations, continuum mechanics, condensed matter physics, discrete geometry, and mathematical physics.

This volume is a co-publication of the AMS, IAS/Park City Mathematics Institute, and Society for Industrial and Applied Mathematics (SIAM).

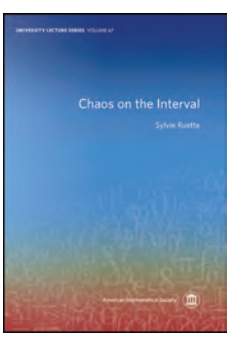
Titles in this series are co-published with the Institute for Advanced Study/Park City Mathematics Institute. Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price. *NOTE: This discount does not apply to volumes in this series co-published with the Society for Industrial and Applied Mathematics (SIAM).*

Contents: **V. Elser**, Three lectures on statistical mechanics; **H. Cohn**, Packing, coding, and ground states; **A. A. Lee** and **D. Frenkel**, Entropy, probability and packing; **M. P. Brenner**, Ideas about self assembly; **P. Palffy-Muhoray**, **M. V. Pevnyi**, **E. G. Virga**, and **X. Zheng**, The effects of particle shape in orientationally ordered soft materials; **R. Kotecký**, Statistical mechanics and nonlinear elasticity; **P. Bella**, **A. Giunti**, and **F. Otto**, Quantitative stochastic homogenization: Local control of homogenization error through corrector.

IAS/Park City Mathematics Series, Volume 23

May 2017, approximately 354 pages, Hardcover, ISBN: 978-1-4704-2919-5, LC 2016030010, 2010 *Mathematics Subject Classification:* 82B05, 35Q70, 82B26, 74N05, 51P05, 52C17, 52C23, **AMS members US$83.20**, List US$104, Order code PCMS/23

Differential Equations



Chaos on the Interval

Sylvie Ruette, *Université Paris-Sud, Orsay, France*

The aim of this book is to survey the relations between the various kinds of chaos and related notions for continuous interval maps from a topological point of view. The papers on this topic are numerous and widely scattered in the literature; some of them are little known, difficult to find, or originally published in Russian, Ukrainian, or Chinese. Dynamical systems given by the iteration of a continuous map on an interval have been broadly studied because they are simple but nevertheless exhibit complex behaviors. They also allow numerical simulations, which enabled the discovery of some chaotic phenomena. Moreover, the "most interesting" part of some higher-dimensional systems can be of lower dimension, which allows, in some cases, boiling it down to systems in dimension one.

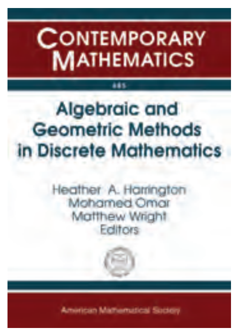
Some of the more recent developments such as distributional chaos, the relation between entropy and Li-Yorke chaos, sequence entropy, and maps with infinitely many branches are presented in book form for the first time. The author gives complete proofs and addresses both graduate students and researchers.

Contents: Notation and basic tools; Links between transitivity, mixing and sensitivity; Periodic points; Topological entropy; Chaos in the sense of Li-Yorke, scrambled sets; Other notions related to Li-Yorke pairs: Generic and dense chaos, distributional chaos; Chaotic subsystems; Appendix: Some background in topology; Bibliography; Notation; Index.

University Lecture Series, Volume 67

April 2017, 215 pages, Softcover, ISBN: 978-1-4704-2956-0, LC 2016042280, 2010 *Mathematics Subject Classification:* 37E05, **AMS members US$43.20**, List US$54, Order code ULECT/67

Discrete Mathematics and Combinatorics



Algebraic and Geometric Methods in Discrete Mathematics

Heather A. Harrington, *University of Oxford, United Kingdom*, **Mohamed Omar**, *Harvey Mudd College, Claremont, CA*, and **Matthew Wright**, *St. Olaf College, Northfield, MN*, Editors

This volume contains the proceedings of the AMS Special Session on Algebraic and Geometric Methods in Applied Discrete Mathematics, held on January 11, 2015, in San Antonio, Texas.

The papers present connections between techniques from "pure" mathematics and various applications amenable to the analysis of discrete models, encompassing applications of combinatorics, topology, algebra, geometry, optimization, and representation theory. Papers not only present novel results, but also survey the current state of knowledge of important topics in applied discrete mathematics.

Particular highlights include: a new computational framework, based on geometric combinatorics, for structure prediction from RNA sequences; a new method for approximating the optimal solution of a sum of squares problem; a survey of recent Helly-type geometric theorems; applications of representation theory to voting theory and game theory; a study of fixed points of tensors; and exponential random graph models from the perspective of algebraic statistics with applications to networks.

This volume was written for those trained in areas such as algebra, topology, geometry, and combinatorics who are interested in tackling problems in fields such as biology, the social sciences, data analysis, and optimization. It may be useful not only for experts, but also for students who wish to gain an applied or interdisciplinary perspective.

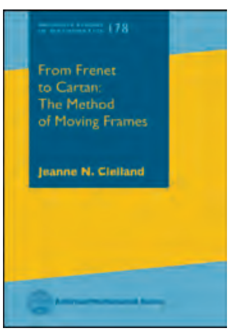
This item will also be of interest to those working in applications.

Contents: **H. Abo**, **A. Seigal**, and **B. Sturmfels**, Eigenconfigurations of tensors; **A. A. Ahmadi** and **G. Hall**, Sum of squares basis pursuit with linear and second order cone programming; **N. Amenta**, **J. A. De Loera**, and **P. Soberón**, Helly's theorem: New variations and applications; **K.-D. Crisman** and **M. E. Orrison**, Representation theory of the symmetric group in voting theory and game theory; **R. Davidson**, **J. Rusinko**, **Z. Vernon**, and **J. Xi**, Modeling the distribution of distance data in Euclidean space; **E. Drellich**, **A. Gainer-Dewar**, **H. A. Harrington**, **Q. He**, **C. Heitsch**, and **S. Poznanović**, Geometric combinatorics and computational molecular biology: Branching polytopes for RNA sequences; **D. Haws**, **J. Cussens**, and **M. Studený**, Polyhedral approaches to learning Bayesian networks; **C. J. Hillar** and **S. E. Marzen**, Neural network coding of natural images with applications to pure mathematics; **B. Kuture**, **O. Leong**, **C. Loa**, **M. Sondjaja**, and **F. E. Su**, Proving Tucker's Lemma with a volume argument; **C. O'Neill** and **R. Pelayo**, Factorization invariants in numerical monoids; **S. Petrović**, A survey of discrete methods in (algebraic) statistics for networks.

Contemporary Mathematics, Volume 685

April 2017, 278 pages, Softcover, ISBN: 978-1-4704-2321-6, LC 2016042006, 2010 *Mathematics Subject Classification:* 00B20, 13P25, 20C30, 46N30, 51D20, 52B05, 62-07, 62P10, 65C60, 91B12, **AMS members US$88.80**, List US$111, Order code CONM/685

Geometry and Topology



From Frenet to Cartan: The Method of Moving Frames

Jeanne N. Clelland, *University of Colorado, Boulder, CO*

The method of moving frames originated in the early nineteenth century with the notion of the Frenet frame along a curve in Euclidean space. Later, Darboux expanded this idea to the study of surfaces. The method was brought to its full power in the early twentieth century by Elie Cartan, and its development continues today with the work of Fels, Olver, and others.

This book is an introduction to the method of moving frames as developed by Cartan, at a level suitable for beginning graduate students familiar with the geometry of curves and surfaces in Euclidean space. The main focus is on the use of this method to compute local geometric invariants for curves and surfaces in various 3-dimensional homogeneous spaces, including Euclidean, Minkowski, equi-affine, and projective spaces. Later chapters include applications to several classical problems in differential geometry, as well as an introduction to the nonhomogeneous case via moving frames on Riemannian manifolds.

The book is written in a reader-friendly style, building on already familiar concepts from curves and surfaces in Euclidean space. A special feature of this book is the inclusion of detailed guidance regarding the use of the computer algebra system Maple® to perform many of the computations involved in the exercises.

An excellent and unique graduate level exposition of the differential geometry of curves, surfaces and higher-dimensional submanifolds of homogeneous spaces based on the powerful and elegant method of moving frames. The treatment is self-contained and illustrated through a large number of examples and exercises, augmented by Maple code to assist in both concrete calculations and plotting. Highly recommended.

—Niky Kamran, McGill University

The method of moving frames has seen a tremendous explosion of research activity in recent years, expanding into many new areas of applications, from computer vision to the calculus of variations to geometric partial differential equations to geometric numerical integration schemes to classical invariant theory to integrable systems to infinite-dimensional Lie pseudo-groups and beyond. Cartan theory remains a touchstone in modern differential geometry, and Clelland's book provides a fine new introduction that includes both classic and contemporary geometric developments and is supplemented by Maple symbolic software routines that enable the reader to both tackle the exercises and delve further into this fascinating and important field of contemporary mathematics.

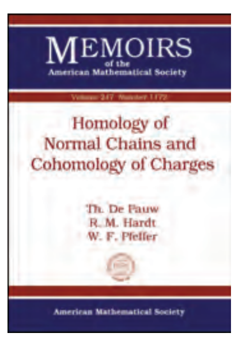
Recommended for students and researchers wishing to expand their geometric horizons.

—Peter Olver, University of Minnesota

Contents: *Background material:* Assorted notions from differential geometry; Differential forms; *Curves and surfaces in homogeneous spaces via the method of moving frames:* Homogeneous spaces; Curves and surfaces in Euclidean space; Curves and surfaces in Minkowski space; Curves and surfaces in equi-affine space; Curves and surfaces in projective space; *Applications of moving frames:* Minimal surfaces in $\mathbb{E}^3$ and $\mathbb{A}^3$; Pseudospherical surfaces in Bäcklund's theorem; Two classical theorems; *Beyond the flat case: Moving frames on Riemannian manifolds:* Curves and surfaces in elliptic and hyperbolic spaces; The nonhomogeneous case: Moving frames on Riemannian manifolds; Bibliography; Index.

Graduate Studies in Mathematics, Volume 178

April 2017, 414 pages, Hardcover, ISBN: 978-1-4704-2952-2, LC 2016041073, 2010 *Mathematics Subject Classification:* 22F30, 53A04, 53A05, 53A15, 53A20, 53A55, 53B25, 53B30, 58A10, 58A15, **AMS members US$58.40**, List US$73, Order code GSM/178



Homology of Normal Chains and Cohomology of Charges

Th. De Pauw, *Université Denis Diderot, Paris, France*, **R. M. Hardt**, *Rice University, Houston, TX*, and **W. F. Pfeffer**, *University of California, Davis*

This item will also be of interest to those working in analysis.

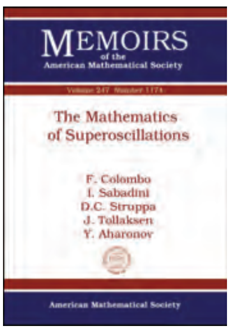
Contents: Introduction; Notation and preliminaries; Rectifiable chains; Lipschitz chains; Flat norm and flat chains; The lower semicontinuity of slicing mass; Supports of flat chains; Flat chains of finite mass; Supports of flat chains of finite mass;

Measures defined by flat chains of finite mass; Products; Flat chains in compact metric spaces; Localized topology; Homology and cohomology; q-bounded pairs; Dimension zero; Relation to the Čech cohomology; Locally compact spaces; References.

Memoirs of the American Mathematical Society, Volume 247, Number 1172

April 2017, 115 pages, Softcover, ISBN: 978-1-4704-2335-3, 2010 *Mathematics Subject Classification:* 49Q15, 55N35, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1172

Mathematical Physics



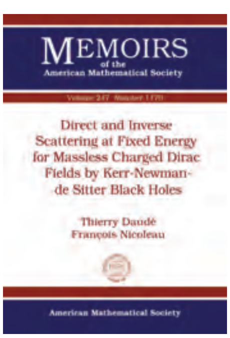
The Mathematics of Superoscillations

F. Colombo, *Politecnico di Milano, Italy*, **I. Sabadini**, *Polytechnic Institute of Milan, Italy*, **D. C. Struppa**, *Chapman University, Orange, CA*, **J. Tollaksen**, *Chapman University, Orange, CA*, and **Y. Aharonov**, *Chapman University, Orange, CA*

Contents: Introduction; Physical motivations; Basic mathematical properties of superoscillating sequences; Function spaces of holomorphic functions with growth; Schrödinger equation and superoscillations; Superoscillating functions and convolution equations; Superoscillating functions and operators; Superoscillations in $SO(3)$; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 247, Number 1174

April 2017, 107 pages, Softcover, ISBN: 978-1-4704-2324-7, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1174



Direct and Inverse Scattering at Fixed Energy for Massless Charged Dirac Fields by Kerr-Newman-de Sitter Black Holes

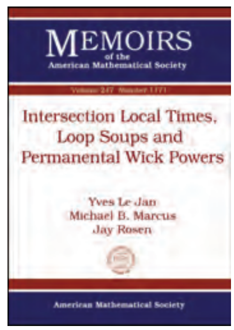
Thierry Daudé, *Université de Cergy-Pontoise, France*, and **François Nicoleau**, *Université de Nantes, France*

Contents: Introduction; Kerr-Newman-de-Sitter black holes; The massless charged Dirac equation; The direct scattering problem; Uniqueness results in the inverse scattering problem at fixed energy; The angular equation and partial inverse result; The radial equation: complexification of the angular momentum; Large z asymptotics of the scattering data; The inverse scattering problem; Appendix A. Growth estimate of the eigenvalues $\mu_{kl}(\lambda)$; Appendix B. Limiting Absorption principles and scattering theory for H_0 and H; Bibliography.

Memoirs of the American Mathematical Society, Volume 247, Number 1170

April 2017, 113 pages, Softcover, ISBN: 978-1-4704-2376-6, 2010 *Mathematics Subject Classification:* 81U40, 35P25; 58J50, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1170

Probability and Statistics



Intersection Local Times, Loop Soups and Permanental Wick Powers

Yves Le Jan, *Université Paris-Sud, Orsay, France*, **Michael B. Marcus**, *City College, CUNY, New York, NY*, and **Jay Rosen**, *College of Staten Island, CUNY, New York, NY*

Contents: Introduction; Loop measures and renormalized intersection local times; Continuity of intersection local time processes; Loop soup and permanental chaos; Isomorphism Theorem I; Permanental Wick powers; Poisson chaos decomposition, I; Loop soup decomposition of permanental Wick powers; Poisson chaos decomposition, II; Convolutions of regularly varying functions; References.

Memoirs of the American Mathematical Society, Volume 247, Number 1171

April 2017, 78 pages, Softcover, ISBN: 978-1-4704-3695-7, 2010 *Mathematics Subject Classification:* 60K99, 60J55; 60G17, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/247/1171

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Instanton Moduli Spaces and $\mathcal{W}$-Algebras

Alexander Braverman , *Brown University, Providence, RI, and University of Toronto and Perimeter Institute of Theoretical Physics, Waterloo, Ontario, Canada*, **Michael Finkelberg**, *National Research University Higher School of Economics, Moscow, Russia*, and **Hiraku Nakajima**, *Kyoto University, Japan*

The authors describe the (equivariant) intersection cohomology of certain moduli spaces ("framed Uhlenbeck spaces") together with some structures on them (e.g.,the Poincaré pairing) in terms of representation theory of some vertex operator algebras ("$\mathcal{W}$-algebras").

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 385

December 2016, 128 pages, Softcover, ISBN: 978-2-85629-848-0, 2010 *Mathematics Subject Classification:* 14C05, 14D21, 14J60, **AMS members US$41.60**, List US$52, Order code AST/385

Quantizations of Conical Symplectic Resolutions

Tom Braden, *University of Massachusetts, Amherst, MA*, **Anthony Licata**, *Australian National University, Canberra, Australia*, **Nicholas Proudfoot**, *University of Oregon, Eugene, OR*, and **Ben Webster**, *University of Virginia, Charlottesville, VA*

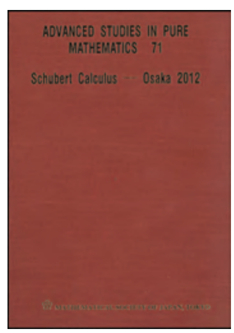
The authors re-examine some topics in representation theory of Lie algebras and Springer theory in a more general context, viewing the universal enveloping algebra as an example of the section ring of a quantization of a conical symplectic resolution. Many familiar features from the classical theory survive, including analogues of Beilinson-Bernstein localization and Bernstein-Gelfand-Gelfand category $\mathcal{O}$.

This item will also be of interest to those working in geometry and topology.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 384

December 2016, 179 pages, Softcover, ISBN: 978-2-85629-845-9, 2010 *Mathematics Subject Classification:* 53D55, 16G99, 14M99, 17B10, **AMS members US$48**, List US$60, Order code AST/384



Schubert Calculus Osaka 2012

Hiroshi Naruse, *University of Yamanashi, Japan*, **Takeshi Ikeda**, *Okayama University of Science, Japan*, **Mikiya Masuda**, *Osaka City University, Japan*, and **Toshiyuki Tanisaki**, *Osaka City University, Japan*, Editors

This volume contains the proceedings of the 5th MSJ Seasonal Institute on Schubert Calculus, held at Osaka City University, from September 17–27, 2012. It is recommended for all researchers and graduate students who are interested in Schubert calculus and its many connections and applications to related areas of mathematics, such as geometric representation theory, combinatorial aspects of algebraic varieties arising in Lie theory, and equivariant topology.

Alain Lascoux, one of the pioneers of modern Schubert calculus and a contributor to this volume, passed away during the time of the editing process of the proceedings. This volume is dedicated to him.

This item will also be of interest to those working in discrete mathematics and combinatorics.

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Advanced Studies in Pure Mathematics, Volume 71

December 2016, 518 pages, Hardcover, ISBN: 978-4-86497-038-9, 2010 *Mathematics Subject Classification:* 14N15; 05E05, 05E10, 20G05, **AMS members US$123.20**, List US$154, Order code ASPM/71

Finite Groups: An Introduction

Jean-Pierre Serre, *College of France, Paris, France*

This is a hardcover version—with some revisions—of a previously distributed book (INPR/99, ISBN: 978-1-57146-320-3).

Finite group theory is remarkable for the simplicity of its statements—and the difficulty of their proofs. It is essential in several branches of mathematics, notably number theory.

This book is a short introduction to the subject, written both for beginners and for mathematicians at large. There are ten chapters. Each chapter is followed by a series of exercises.

A publication of International Press of Boston, Inc. Distributed worldwide by the American Mathematical Society.

International Press of Boston, Inc.

December 2016, 190 pages, Hardcover, ISBN: 978-1-57146-327-2, 2010 *Mathematics Subject Classification:* 20-XX; 20-01, **AMS members US$63.20**, List US$79, Order code INPR/100

MEETINGS & CONFERENCES OF THE AMS

APRIL TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: **www.ams.org/meetings/**.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 75 in the January 2017 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit **www.ams.org/cgi-bin/abstracts/abstract.pl/**. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

MEETINGS IN THIS ISSUE

See **www.ams.org/meetings/** *for the most up-to-date information on these conferences.*

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `www.ams.org/meetings/`.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Bloomington, Indiana

Indiana University

April 1–2, 2017

Saturday – Sunday

Meeting #1127

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2017
Program first available on AMS website: February 23, 2017
Issue of *Abstracts*: Volume 38, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Ciprian Demeter, Indiana University, Bloomington, *Decouplings and applications: a journey from continuous to discrete*.

Sarah C. Koch, University of Michigan, *Postcritical sets in complex dynamics*.

Richard Evan Schwartz, Brown University, *Modern scratch paper: Graphical explorations in geometry and dynamics* (Einstein Public Lecture in Mathematics).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic and Enumerative Combinatorics with Applications, **Saúl A. Blanco**, Indiana University, and **Kyle Peterson**, DePaul University.

Analysis and Numerical Computations of PDEs in Fluid Mechanics, **Gung-Min Gie**, University of Louisville, and **Makram Hamouda** and **Roger Temam**, Indiana University.

Analysis of Variational Problems and Nonlinear Partial Differential Equations, **Nam Q. Le** and **Peter Sternberg**, Indiana University.

Automorphic Forms and Algebraic Number Theory, **Patrick B. Allen**, University of Illinois at Urbana-Champaign, and **Matthias Strauch**, Indiana University Bloomington.

Commutative Algebra, **Ela Celikbas** and **Olgur Celikbas**, West Virginia University.

Computability and Inductive Definability over Structures, **Siddharth Bhaskar**, **Lawrence Valby**, and **Alex Kruckman**, Indiana University.

Dependence in Probability and Statistics, **Richard C. Bradley** and **Lanh T. Tran**, Indiana University.

Differential Equations and Their Applications to Biology, **Changbing Hu**, **Bingtuan Li**, and **Jiaxu Li**, University of Louisville.

Discrete Structures in Conformal Dynamics and Geometry, **Sarah Koch**, University of Michigan, and **Kevin Pilgrim** and **Dylan Thurston**, Indiana University.

Extremal Problems in Graphs, Hypergraphs and Other Combinatorial Structures, **Amin Bahmanian**, Illinois State University, and **Theodore Molla**, University of Illinois Urbana-Champaign.

Financial Mathematics and Statistics, **Ryan Gill**, University of Louisville, **Rasitha Jayasekera**, Butler University, and **Kiseop Lee**, Purdue University.

Fusion Categories and Applications, **Paul Bruillard**, Pacific Northwest National Laboratory, and **Julia Plavnik** and **Eric Rowell**, Texas A&M University.

Harmonic Analysis and Partial Differential Equations, **Lucas Chaffee**, Western Washington University, **William Green**, Rose-Hulman Institute of Technology, and **Jarod Hart**, University of Kansas.

Homotopy Theory, **David Gepner**, Purdue University, **Ayelet Lindenstrauss** and **Michael Mandell**, Indiana University, and **Daniel Ramras**, Indiana University-Purdue University Indianapolis.

Model Theory, **Gabriel Conant**, University of Notre Dame, and **Philipp Hieronymi**, University of Illinois Urbana Champaign.

Multivariate Operator Theory and Function Theory, **Hari Bercovici**, Indiana University, **Kelly Bickel**, Bucknell University, **Constanze Liaw**, Baylor University, and **Alan Sola**, Stockholm University.

Network Theory, **Jeremy Alm** and **Keenan M.L. Mack**, Illinois College.

Nonlinear Elliptic and Parabolic Partial Differential Equations and Their Various Applications, **Changyou Wang**, Purdue University, and **Yifeng Yu**, University of California, Irvine.

Probabilistic Methods in Combinatorics, **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University.

Probability and Applications, **Russell Lyons** and **Nick Travers**, Indiana University.

Randomness in Complex Geometry, **Turgay Bayraktar**, Syracuse University, and **Norman Levenberg**, Indiana University.

Representation Stability and its Applications, **Patricia Hersh**, North Carolina State University, **Jeremy Miller**, Purdue University, and **Andrew Putman**, University of Notre Dame.

Representation Theory and Integrable Systems, **Eugene Mukhin**, Indiana University,Purdue University Indianapolis, and **Vitaly Tarasov**, Indiana University, Purdue University Indianapolis.

Self-similarity and Long-range Dependence in Stochastic Processes, **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

Spectrum of the Laplacian on Domains and Manifolds, **Chris Judge** and **Sugata Mondal**, Indiana University.

Topics in Extremal, Probabilistic and Structural Graph Theory, **John Engbers**, Marquette University, and **David Galvin**, University of Notre Dame.

Topological Mathematical Physics, **E. Birgit Kaufmann** and **Ralph M. Kaufmann**, Purdue University, and **Emil Prodan**, Yeshiva University.

Pullman, Washington

Washington State University

April 22–23, 2017

Saturday – Sunday

Meeting #1128

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2017
Program first available on AMS website: March 9, 2017
Issue of *Abstracts*: Volume 38, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hitrik, University of California, Los Angeles, *Spectra for non-self-adjoint operators and integrable dynamics*.

Andrew S. Raich, University of Arkansas, *Closed Range of the Cauchy-Riemann Operator on Domains in* $\mathbb{C}^n$.

Daniel Rogalski, University of California, San Diego, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Analysis on the Navier-Stokes equations and related PDEs, **Kazuo Yamazaki**, University of Rochester, and **Litzheng Tao**, University of California, Riverside.

Analytic Number Theory and Automorphic Forms, **Steven J. Miller**, Williams College, and **Sheng-Chi Liu**, Washington State University.

Clustering of Graphs: Theory and Practice, **Stephen J. Young** and **Jennifer Webster**, Pacific Northwest National Laboratory.

Combinatorial and Algebraic Structures in Knot Theory, **Sam Nelson**, Claremont McKenna College, and **Allison Henrich**, Seattle University.

Combinatorial and Computational Commutative Algebra and Algebraic Geometry, **Hirotachi Abo**, **Stefan Tohaneanu**, and **Alexander Woo**, University of Idaho.

Commutative Algebra, **Jason Lutz** and **Katharine Shultis**, Gonzaga University.

Early Career Research and Exposition: Posters and Discussions, **Enrique Alvarado** and **Yunfeng Hu**, Washington State University, and **Kevin R Vixie**, Washington State University and Sailfan Research.

Fixed Point Methods in Differential and Integral Equations, **Theodore A. Burton**, Southern Illinois University in Carbondale.

Geometric Measure Theory and it's Applications, **Kevin R. Vixie**, Washington State University.

Geometry and Optimization in Computer Vision, **Bala Krishnamoorthy**, Washington State University, and **Sudipta Sinha**, Microsoft Research, Redmond, WA.

Internships and Postgraduate Positions in Industry and the National Labs with Discussion, **Laramie Paxton**, Washington State University, **Justin Theriot**, EMSI, and **Kevin R Vixie**, Washington State University and Sailfan Research.

Inverse Problems, **Hanna Makaruk**, Los Alamos National Laboratory (LANL), and **Robert Owczarek**, University of New Mexico, Albuquerque & Los Alamos.

Mathematical Modeling of Forest and Landscape Change, **Demetrios Gatziolis**, US Forest Service, and **Nikolay Strigul**, Washington State University Vancouver.

Mathematical and Computational Neuroscience, **Alexander Dimitrov**, Washington State University Vancouver, **Andrew Oster**, Eastern Washington University, and **Predrag Tosic**, Washington State University.

Microlocal Analysis and Spectral Theory, **Michael Hitrik**, University of California, Los Angeles, and **Semyon Dyatlov**, Masssachusetts Institute of Technology.

Noncommutative Algebraic Geometry and Related Topics, **Daniel Rogalski**, University of California, San Diego, and **James Zhang**, University of Washington.

Partial Differential Equations and Applications, **V. S. Manoranjan**, **C. Moore**, **Lynn Schreyer**, and **Hong-Ming Yin**, Washington State University.

Recent Advances in Applied Algebraic Topology, **Henry Adams**, Colorado State University, and **Bala Krishnamoorthy**, Washington State University.

Recent Advances in Optimization and Statistical Learning, **Hongbo Dong**, **Bala Krishnamoorthy**, **Haijun Li**, and **Robert Mifflin**, Washington State University.

Recent Advances on Mathematical Biology and Their Applications, **Robert Dillon** and **Xueying Wang**, Washington State University.

Several Complex Variables and PDEs, **Andrew Raich** and **Phillip Harrington**, University of Arkansas.

Theory and Applications of Linear Algebra, **Judi McDonald** and **Michael Tsatsomeros**, Washington State University.

Undergraduate Research Experiences in the Classroom, **Heather Moon**, Lewis-Clark State College.

New York, New York

Hunter College, City University of New York

May 6–7, 2017

Saturday – Sunday

Meeting #1129

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: March 2017
Program first available on AMS website: March 29, 2017
Issue of *Abstracts*: Volume 38, Issue 2

Deadlines

For organizers: Expired
For abstracts: March 21, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jeremy Kahn, City University of New York, *Title to be announced.*

Fernando Coda Marques, Princeton University, *Title to be announced.*

James Maynard, Magdalen College, University of Oxford, *Title to be announced* (Erdős Memorial Lecture).

Kavita Ramanan, Brown University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis and Numerics on Liquid Crystals and Soft Matter (Code: SS 16A), **Xiang Xu**, Old Dominion University, and **Wujun Zhang**, Rutgers University.

Applications of Modular Forms (Code: SS 35A), **Cormac O'Sullivan** and **Karen Taylor**, Bronx Community College, City University of New York.

Applications of Network Analysis, in Honor of Charlie Suffel's 75th Birthday (Code: SS 18A), **Michael Yatauro**, Pennsylvania State University-Brandywine.

Asymptotic Properties of Discrete Dynamical Systems (Code: SS 29A), **Ann Brett**, Johnson and Wales University, and **M. R.S. Kulenović** and **O. Merino**, University of Rhode Island.

Automorphic Forms and Arithmetic (Code: SS 22A), **Jim Brown**, Clemson University, and **Krzysztof Klosin**, Queens College, City University of New York.

Banach Space Theory and Metric Embeddings (Code: SS 10A), **Mikhail Ostrovskii**, St John's University, and **Beata Randrianantoanina**, Miami University of Ohio.

Bases in Hilbert Function Spaces (Code: SS 30A), **Azita Mayeli**, City University of New York, and **Shahaf Nitzan**, Georgia Institute of Technology.

Cluster Algebras in Representation Theory and Combinatorics (Code: SS 6A), **Alexander Garver**, Université du Quebéc à Montréal and Sherbrooke, and **Khrystyna Serhiyenko**, University of California at Berkeley.

Cohomologies and Combinatorics (Code: SS 15A), **Rebecca Patrias**, Université du Québec à Montréal, and **Oliver Pechenik**, Rutgers University.

Common Threads to Nonlinear Elliptic Equations and Systems (Code: SS 14A), **Florin Catrina**, St. John's University, and **Wenxiong Chen**, Yeshiva University.

Commutative Algebra (Code: SS 1A), **Laura Ghezzi**, New York City College of Technology-CUNY, and **Jooyoun Hong**, Southern Connecticut State University.

Computability Theory: Pushing the Boundaries (Code: SS 9A), **Johanna Franklin**, Hofstra University, and **Russell Miller**, Queens College and Graduate Center, City University of New York.

Computational and Algorithmic Group Theory (Code: SS 7A), **Denis Serbin** and **Alexander Ushakov**, Stevens Institute of Technology.

Cryptography (Code: SS 3A), **Xiaowen Zhang**, College of Staten Island and Graduate Center-CUNY.

Current Trends in Function Spaces and Nonlinear Analysis (Code: SS 2A), **David Cruz-Uribe**, University of Alabama, **Jan Lang**, The Ohio State University, and **Osvaldo Mendez**, University of Texas at El Paso.

Differential and Difference Algebra: Recent Developments, Applications, and Interactions (Code: SS 12A), **Omár León-Sanchez**, McMaster University, and **Alexander Levin**, The Catholic University of America.

Dynamical Systems (Code: SS 31A), **Marian Gidea**, Yeshiva University, **W. Patrick Hooper**, City College of New York and the City University of New York, and **Anatole Katok**, Pennsylvania State University.

Ehrhart Theory and its Applications (Code: SS 25A), **Dan Cristofaro-Gardiner**, Harvard University, **Quang-Nhat Le**, Brown University, and **Sinai Robins**, University of São Paulo.

Euler and Related PDEs: Geometric and Harmonic Methods (Code: SS 27A), **Stephen C. Preston**, Brooklyn College, City University of New York, and **Kazuo Yamazaki**, Kazuo Yamazaki, University of Rochester.

Finite Fields and their Applications (Code: SS 23A), **Ricardo Conceicao** and **Darren Glass**, Gettysburg College, and **Ariane Masuda**, New York City College of Technology, City University of New York.

Geometric Function Theory and Related Topics (Code: SS 19A), **Sudeb Mitra**, Queens College and Graduate Center-CUNY, and **Zhe Wang**, Bronx Community College-CUNY.

Geometry and Topology of Ball Quotients and Related Topics (Code: SS 5A), **Luca F. Di Cerbo**, Max Planck Institute, Bonn, and **Matthew Stover**, Temple University.

Hydrodynamic and Wave Turbulence (Code: SS 11A), **Tristan Buckmaster**, Courant Institute of Mathematical Sciences, New York University, and **Vlad Vicol**, Princeton University.

Infinite Permutation Groups, Totally Disconnected Locally Compact Groups, and Geometric Group Theory (Code: SS 4A), **Delaram Kahrobaei**, New York City College of Technology and Graduate Center-CUNY, and **Simon Smith**, University of Lincoln, U.K..

Invariants in Low-dimensional Topology (Code: SS 24A), **Abhijit Champanerkar**, College of Staten Island and The Graduate Center, City University of New York, and **Anastasiia Tsvietkova**, Rutgers University, Newark.

Invariants of Knots, Links and 3-manifolds (Code: SS 26A), **Moshe Cohen**, Vassar College, **Ilya Kofman Kofman**, College of Staten Island and The Graduate Center, City University of New York, and **Adam Lowrance**, Vassar College.

Mathematical Phylogenetics (Code: SS 34A), **Megan Owen**, City University of New York, and **Katherine St. John**, Lehman College, City University of New York, and American Museum of Natural History.

Model Theory: Algebraic Structures in "Tame" Model Theoretic Contexts (Code: SS 28A), **Alfred Dolich**, Kingsborough Community College and The Graduate Center, City University of New York, **Michael Laskowski**, University of Maryland, and **Mahmood Sohrabi**, Stevens Institute of Technology.

Nonlinear and Stochastic Partial Differential Equations: Theory and Applications in Turbulence and Geophysical Flows (Code: SS 8A), **Nathan Glatt-Holtz**, Tulane University, **Geordie Richards**, Utah State University, and **Xiaoming Wang**, Florida State University.

Numerical Analysis and Mathematical Modeling (Code: SS 32A), **Vera Babenko**, Ithaca College.

Operator Algebras and Ergodic Theory (Code: SS 17A), **Genady Grabarnik** and **Alexander Katz**, St John's University.

Qualitative and Quantitative Properties of Solutions to Partial Differential Equations (Code: SS 20A), **Blair Davey**, The City College of New York-CUNY, and **Nguyen Cong Phuc** and **Jiuyi Zhu**, Louisiana State University.

Recent Developments in Automorphic Forms and Representation Theory (Code: SS 21A), **Moshe Adrian**, Queens College-CUNY, and **Shuichiro Takeda**, University of Missouri.

Representation Spaces and Toric Topology (Code: SS 13A), **Anthony Bahri**, Rider University, and **Daniel Ramras** and **Mentor Stafa**, Indiana University-Purdue University Indianapolis.

Topological Dynamics (Code: SS 33A), **Alica Miller**, University of Louisville.

Montréal, Quebec Canada

McGill University

July 24–28, 2017

Monday – Friday

Meeting #1130

The second Mathematical Congress of the Americas (MCA 2017) is being hosted by the Canadian Mathematical Society (CMS) in collaboration with the Pacific Institute for the Mathematical Sciences (PIMS), the Fields Institute (FIELDS), Le Centre de Recherches Mathématiques (CRM), and the Atlantic Association for Research in the Mathematical Sciences (AARMS).

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Deadlines

For organizers: Expired

For abstracts: March 31, 2017

Denton, Texas

University of North Texas

September 9–10, 2017

Saturday – Sunday

Meeting #1131

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2017
Program first available on AMS website: July 27, 2017
Issue of *Abstracts*: Volume 38, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 18, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mirela Çiperiani, University of Texas at Austin, *Title to be announced*.

Adrianna Gillman, Rice University, *Title to be announced*.

Kevin Pilgrim, Indiana University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Combinatorics of Flag Varieties (Code: SS 11A), **Martha Precup**, Northwestern University, and **Edward Richmond**, Oklahoma State University.

Analysis and PDEs in Geometry (Code: SS 20A), **Stephen McKeown**, Princeton University.

Applicable and Computational Algebraic Geometry (Code: SS 17A), **Eric Hanson**, Texas Christian University, and **Frank Sottile**, Texas A&M University.

Banach Spaces and Applications (Code: SS 9A), **Pavlos Motakis**, Texas A&M University, and **Bünyamin Sari**, University of North Texas.

Combinatorial/Geometric/Probabilistic Methods in Group Theory (Code: SS 19A), **Rostislav Grigorchuk** and **Volodymyr Nekrashevych**, Texas A&M University, **Dmytro Savchuk**, University of South Florida, and **Zoran Sunic**, Texas A&M University.

Combinatorics and Representation Theory of Reflection Groups: Real and Complex (Code: SS 14A), **Elizabeth Drellich**, Swarthmore College, and **Drew Tomlin**, Hendrix College.

Commutative Algebra (Code: SS 10A), **Jonathan Montano**, University of Kansas, and **Alessio Sammartano**, Purdue University.

Differential Equation Modeling and Analysis for Complex Bio-systems (Code: SS 8A), **Pengcheng Xiao**, University of Evansville, and **Honghui Zhang**, Northwestern Polytechnical University.

Differential Geometry of Smooth and Discrete Surfaces in Euclidean and Lorentz Spaces (Code: SS 18A), **Barbara Shipman**, University of Texas at Arlington, and **Patrick Shipman**, Colorado State University.

Dynamics, Geometry and Number Theory (Code: SS 1A), **Lior Fishman** and **Mariusz Urbanski**, University of North Texas.

Fractal Geometry and Ergodic Theory (Code: SS 5A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Generalizations of Graph Theory (Code: SS 22A), **Nathan Reff**, SUNY Brockport, and **Lucas Rusnak** and **Piyush Shroff**, Texas State University.

Geometric Combinatorics and Combinatorial Commutative Algebra (Code: SS 16A), **Anton Dochtermann** and **Suho Oh**, Texas State University.

Homological Methods in Commutative Algebra (Code: SS 15A), **Peder Thompson**, Texas Tech University, and **Ashley Wheeler**, University of Arkansas.

Invariants of Links and 3-Manifolds (Code: SS 7A), **Mieczyslaw K. Dabkowski** and **Anh T. Tran**, The University of Texas at Dallas.

Lie algebras, Superalgebras, and Applications (Code: SS 3A), **Charles H. Conley**, University of North Texas, **Dimitar Grantcharov**, University of Texas at Arlington, and **Natalia Rozhkovskaya**, Kansas State University.

Mathematical and Computational Biology (Code: SS 21A), **Rajeev K. Azad**, University of North Texas, and **Brandilyn Stigler**, Southern Methodist University.

Noncommutative and Homological Algebra (Code: SS 4A), **Anne Shepler**, University of North Texas, and **Sarah Witherspoon**, Texas A&M University.

Nonlocal PDEs in Fluid Dynamics (Code: SS 12A), **Changhui Tan**, Rice University, and **Xiaoqian Xu**, Carnegie Mellon University.

Numbers, Functions, Transcendence, and Geometry (Code: SS 6A), **William Cherry**, University of North Texas, **Mirela Çiperiani**, University of Texas Austin, **Matt Papanikolas**, Texas A&M University, and **Min Ru**, University of Houston.

Real-Analytic Automorphic Forms (Code: SS 2A), **Olav K Richter**, University of North Texas, and **Martin Westerholt-Raum**, Chalmers University of Technology.

Recent Progress on Hyperbolic Conservation Laws (Code: SS 23A), **Ilija Jegdic**, Texas Southern University, and **Katarina Jegdic**, University of Houston, Downtown.

Topics Related to the Interplay of Noncommutative Algebra and Geometry (Code: SS 13A), **Richard Chandler**, University of North Texas at Dallas, **Michaela Vancliff**, University of Texas at Arlington, and **Padmini Veerapen**, Tennessee Technological University.

Buffalo, New York

State University of New York at Buffalo

September 16–17, 2017

Saturday – Sunday

Meeting #1132

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2017
Program first available on AMS website: August 3, 2017
Issue of *Abstracts*: Volume 38, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 25, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Inwon Kim, University of California at Los Angeles, *Title to be announced*.

Govind Menon, Brown University, *Title to be announced*.

Bruce Sagan, Michigan State University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advanced Techniques in Graph Theory (Code: SS 9A), **Sogol Jahanbekam** and **Paul Wenger**, Rochester Institute of Technology.

Algebraic Topology (Code: SS 17A), **Claudia Miller**, Syracuse University, and **Inna Zakharevich**, Cornell University.

Automorphic Forms and L-functions (Code: SS 14A), **Mahdi Asgari**, Oklahoma State University, and **Joseph Hundley**, University at Buffalo-SUNY.

CR Geometry and Partial Differential Equations in Complex Analysis (Code: SS 5A), **Ming Xiao**, University of Illinois at Urbana-Champaign, and **Yuan Yuan**, Syracuse University.

Cohomology, Deformations, and Quantum Groups: A Session Dedicated to the Memory of Samuel D. Schack (Code: SS 6A), **Miodrag Iovanov**, University of Iowa, **Mihai D. Staic**, Bowling Green State University, and **Alin Stancu**, Columbus State University.

Geometric Group Theory (Code: SS 4A), **Joel Louwsma**, Niagara University, and **Johanna Mangahas**, University at Buffalo-SUNY.

High Order Numerical Methods for Hyperbolic PDEs and Applications (Code: SS 2A), **Jae-Hun Jung**, University at Buffalo-SUNY, **Fengyan Li**, Rensselaer Polytechnic Institute, and **Li Wang**, University at Buffalo-SUNY.

Infinite Groups and Geometric Structures: A Session in Honor of the Sixtieth Birthday of Andrew Nicas (Code: SS 7A), **Hans Boden**, McMaster University, and **David Rosenthal**, St. John's University.

Knots, 3-manifolds and their Invariants (Code: SS 15A), **William Menasco** and **Adam Sikora**, University at Buffalo-SUNY, and **Stephan Wehrli**, Syracuse University.

Nonlinear Dispersive Partial Differential Equations (Code: SS 18A), **Santosh Bhattrai**, Trocaire College, and **Sharad Silwal**, Jefferson College of Health Sciences.

Nonlinear Evolution Equations (Code: SS 16A), **Marius Beceanu**, SUNY Albany, and **Dan-Andrei Geba**, University of Rochester.

Nonlinear Partial Differential Equations Arising from Life Science (Code: SS 8A), **Junping Shi**, College of William and Mary, and **Xingfu Zou**, University of Western Ontario.

Nonlinear Wave Equations, Inverse Scattering and Applications. (Code: SS 1A), **Gino Biondini**, University at Buffalo-SUNY.

Polynomials in Enumerative, Algebraic, and Geometric Combinatorics (Code: SS 13A), **Robert Davis** and **Bruce Sagan**, Michigan State University.

Recent Advancements in Representation Theory (Code: SS 12A), **Yiqiang Li**, University at Buffalo-SUNY, and **Gufang Zhao**, University of Massachusetts.

Recent Progress in Geometric Analysis (Code: SS 11A), **Ovidiu Munteanu**, University of Connecticut, **Terrence Napier**, Lehigh University, and **Mohan Ramachandran**, University at Buffalo.

Structural and Chromatic Graph Theory (Code: SS 10A), **Hong-Jian Lai**, **Rong Luo**, and **Cun-Quan Zhang**, West Virginia University, and **Yue Zhao**, University of Central Florida.

p-adic Aspects of Arithmetic Geometry (Code: SS 3A), **Ling Xiao**, University of Connecticut, and **Hui June Zhu**, University at Buffalo-SUNY.

Orlando, Florida

University of Central Florida, Orlando

September 23–24, 2017

Saturday – Sunday

Meeting #1133

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: June 2017
Program first available on AMS website: August 10, 2017
Issue of *Abstracts*: Volume 38, Issue 4

Deadlines

For organizers: Expired
For abstracts: August 1, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Christine Heitsch, Georgia Institute of Technology, *Title to be announced.*

Jonathan Kujawa, University of Oklahoma, *Title to be announced.*

Christopher D Sogge, Johns Hopkins University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Curves and their Applications (Code: SS 3A), **Lubjana Beshaj**, The University of Texas at Austin.

Applied Harmonic Analysis: Frames, Samplings and Applications (Code: SS 6A), **Dorin Dutkay**, **Deguang Han**, and **Qiyu Sun**, University of Central Florida.

Categorical Methods in Representation Theory (Code: SS 11A), **Brian Boe**, University of Georgia, **Jonathan Kujawa**, University of Oklahoma, and **Daniel K. Nakano**, University of Georgia.

Commutative Algebra: Interactions with Algebraic Geometry and Algebraic Topology (Code: SS 1A), **Joseph Brennan**, University of Central Florida, and **Alina Iacob** and **Saeed Nasseh**, Georgia Southern University.

Differential Equations in Mathematical Biology (Code: SS 12A), **Andrew Nevai**, **Yuanwei Qi**, and **Zhisheng Shuai**, University of Central Florida.

Fractal Geometry, Dynamical Systems, and Their Applications (Code: SS 4A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Global Harmonic Analysis and its Applications (Code: SS 10A), **Christopher Sogge** and **Yakun Xi**, Johns Hopkins University, and **Steve Zelditch**, Northwestern University.

Graph Connectivity and Edge Coloring (Code: SS 5A), **Colton Magnant**, Georgia Southern University.

Nonlinear Dispersive Equations (Code: SS 7A), **Benjamin Harrop-Griffiths**, New York University, **Jonas Lührmann**, Johns Hopkins University, and **Dana Mendelson**, University of Chicago.

Operator Algebras and Related Topics (Code: SS 8A), **Zhe Liu**, University of Central Florida.

Progress in Fixed Point Theory and Its Applications (Code: SS 9A), **Clement Boateng Ampadu**, Boston, MA, and **Buthinah A. Bin Rehash** and **Afrah A. N. Abdou**, King Abdulaziz University, Saudi Arabia.

Structural Graph Theory (Code: SS 2A), **Martin Rolek**, **Zixia Song**, and **Yue Zhao**, University of Central Florida.

Symplectic and Contact Topology and Dynamics (Code: SS 13A), **Basak G/"urel**, University of Central Florida.

Riverside, California

University of California, Riverside

November 4–5, 2017

Saturday – Sunday

Meeting #1134

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2017
Program first available on AMS website: September 21, 2017
Issue of *Abstracts*: Volume 38, Issue 4

Deadlines

For organizers: April 14, 2017
For abstracts: September 12, 2017

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Paul Balmer, University of California, Los Angeles, *An invitation to tensor-triangular geometry.*

Pavel Etingof, Massachusetts Institute of Technology, *Double affine hecke algebras and their applications.*

Monica Vazirani, University of California, Davi, *Combinatorics, categorification, and crystals.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry (Code: SS 9A), **Humberto Diaz**, **Jose Gonzalez**, and **Ziv Ran**, University of California, Riverside.

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 3A), **Patricia Cahn**, Smith College, and **Sam Nelson**, Claremont McKenna College.

Analysis and Geometry of Fractals (Code: SS 6A), **Erin Pearse**, California Polytechnic State University, and **Goran Radunovic**, University of California, Riverside.

Applied Category Theory (Code: SS 4A), **John Baez**, University of California, Riverside.

Combinatorial Representation Theory (Code: SS 5A), **Vyjayanthi Chari**, University of California, Riverside, and **Maria Monks Gillespie** and **Monica Vazirani**, University of California, Davis.

Combinatorial aspects of the polynomial ring (Code: SS 1A), **Sami Assaf** and **Dominic Searles**, University of Southern California.

Preparing Students for American Mathematical Competitions (Code: SS 7A), **Adam Glesser**, **Phillip Ramirez**, and **Bogdan D. Suceava**, California State University, Fullerton.

Rational Cherednik Algebras and Categorification (Code: SS 8A), **Pavel Etingof**, Massachusetts Institute of Technology, and **Ivan Losev**, Northeastern University.

Ring Theory and Related Topics (Celebrating the 75th Birthday of Lance W. Small) (Code: SS 2A), **Jason Bell**, University of Waterloo, **Ellen Kirkman**, Wake Forest University, and **Susan Montgomery**, University of Southern California.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Meeting #1135

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Issue of *Abstracts*: Volume 39, Issue 1

Deadlines

For organizers: April 1, 2017
For abstracts: September 26, 2017

Request for Proposals for AMS Special Sessions at the Joint Mathematics Meetings in San Diego, CA, January 10--13, 2018

The AMS solicits proposals for AMS Special Sessions at the 2018 Joint Mathematics Meetings to be held Wednesday, January 10, through Saturday, January 13, 2018 in San Diego, CA. Each proposal must include:

1. the name, affiliation, and e-mail address of each organizer, with one organizer designated as the contact person for all communication about the session;
2. the title and a brief (no longer than one or two paragraphs) description of the topic of the proposed special session;
3. the primary two-digit MSC (Mathematics Subject Classfication) number for the topic—see `www.ams.org/mathscinet/msc/msc2010.html`.
4. a sample list of speakers whom the organizers plan to invite. (It is not necessary to have received confirmed commitments from these potential speakers.)

Organizers are strongly encouraged to consult the *AMS Manual for Special Session Organizers* at: `www.ams.org/meetings/specialsessionmanual.html`.

Proposals for AMS Special Sessions should be sent by e-mail to AMS Associate Secretary Georgia Benkart (`benkart@math.wisc.edu`) by April 7, 2017. Late proposals will not be considered. No decisions will be made on Special Session proposals until after the submission deadline has passed.

Special Sessions will be allotted between five and ten hours in which to schedule speakers. To enable maximum movement of participants between sessions, organizers must schedule session speakers for either (a) a twenty-minute talk with five-minute discussion and five-minute break or (b) a forty-five-minute talk with five-minute discussion and ten-minute break. Any combination of -minute and forty-five-minute talks is permitted, but all talks must begin and end at the scheduled time.

The number of Special Sessions in the AMS program at the Joint Mathematics Meetings is limited, and because of the large number of high-quality proposals, not all can be accepted. Please be sure to submit as detailed a proposal as possible for review by the Program Committee. Organizers of proposals will be notified whether their proposal has been accepted by May 12, 2017. Additional instructions and the session's schedule will be sent to the contact organizers of the accepted sessions shortly after that deadline.

Columbus, Ohio

Ohio State University

March 17–18, 2018

Saturday – Sunday

Meeting #1136

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: January 31, 2018
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 15, 2017
For abstracts: January 22, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Probability in Convexity and Convexity in Probability (Code: SS 2A), **Elizabeth Meckes**, **Mark Meckes**, and **Elisabeth Werner**, Case Western Reserve University.

Recent Advances in Approximation Theory and Operator Theory (Code: SS 1A), **Jan Lang** and **Paul Nevai**, The Ohio State University.

Nashville, Tennessee

Vanderbilt University

April 14–15, 2018

Saturday – Sunday

Meeting #1138

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: February 22, 2018
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 14, 2017
For abstracts: February 13, 2018

Portland, Oregon

Portland State University

April 14–15, 2018

Saturday – Sunday

Meeting #1137

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: February 15, 2018

Deadlines

For organizers: September 14, 2017
For abstracts: February 6, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Sándor Kovács, University of Washington, Seattle, *Title to be announced.*

Elena Mantovan, California Institute of Technology, *Title to be announced.*

Dimitri Shlyakhtenko, University of California, Los Angeles, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Inverse Problems (Code: SS 2A), **Hanna Makaruk**, Los Alamos National Laboratory (LANL), and **Robert Owczarek**, University of New Mexico, Albuquerque & Los Alamos.

Pattern Formation in Crowds, Flocks, and Traffic (Code: SS 1A), **J. J. P. Veerman**, Portland State University, **Alethea Barbaro**, Case Western Reserve University, and **Bassam Bamieh**, UC Santa Barbara.

Boston, Massachusetts

Northeastern University

April 21–22, 2018

Saturday – Sunday

Meeting #1139

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: March 1, 2018
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 21, 2017
For abstracts: February 20, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Arithmetic Dynamics (Code: SS 1A), **Jacqueline M. Anderson**, Bridgewater State University, **Robert Benedetto**, Amherst College, and **Joseph H. Silverman**, Brown University.

Arrangements of Hypersurfaces (Code: SS 2A), **Graham Denham**, University of Western Ontario, and **Alexander I. Suciu**, Northeastern University.

Facets of Symplectic Geometry and Topology (Code: SS 3A), **Tara Holm**, Cornell University, **Jo Nelson**, Columbia University, and **Jonathan Weitsman**, Northeastern University.

Singularities of Spaces and Maps (Code: SS 4A), **Terence Gaffney** and **David Massey**, Northeastern University.

Shanghai, People's Republic of China

Fudan University

June 11–14, 2018
Monday – Thursday

Meeting #1140
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Newark, Delaware

University of Delaware

September 29–30, 2018
Saturday – Sunday

Meeting #1141
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 9, 2018
Issue of *Abstracts*: To be announced

Deadlines
For organizers: February 28, 2018
For abstracts: July 31, 2018

Fayetteville, Arkansas

University of Arkansas

October 6–7, 2018
Saturday – Sunday

Meeting #1142
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 16, 2018
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 6, 2018
For abstracts: August 7, 2018

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018
Saturday – Sunday

Meeting #1143
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 30, 2018
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 20, 2018
For abstracts: August 21, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

San Francisco, California

San Francisco State University

October 27–28, 2018
Saturday – Sunday

Meeting #1144
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2018
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 27, 2018
For abstracts: August 28, 2018

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019

Friday – Sunday

Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Denver, Colorado

Colorado Convention Center

January 15–18, 2020

Wednesday – Saturday

Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: November 1, 2019
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2019
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021

Wednesday – Saturday

Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2020
For abstracts: To be announced

The AMS Graduate Student Blog

Talk that matters to mathematicians.

From "**Things You Should Do Before Your Last Year**"…

Write stuff up. Write up background, write down little ideas and bits of progress you make. It's difficult to imagine that these trivial, inconsequential bits will make it to your dissertation. But recreating a week's/month's worth of ideas is way more time-consuming than just writing them down now. Or better yet, TeX it up.

From "**The Glory of Starting Over**"…

What I would recommend is not being too narrowly focused, but finding a few things that really interest you and develop different skillsets. Make sure you can do some things that are abstract, but also quantitative/programming oriented things, because this shows that you can attack a problem from multiple angles. In my experience, these two sides also serve as nice vacations from each other, which can be important when you start to work hard on research.

From "**Student Seminar**"…

A talk can be too short if not enough material is introduced to make it interesting, but in research level talks, the last third of the talk (approximately) is usually very technical and usually only accessible to experts in the field. I will avoid going into details that are not of general interest and I plan to present more ideas than theorems. The most important thing when giving any talk is to know your audience.

AMS Grad Blog

Advice on careers, research, and going the distance … by and for math grads.

mathgradblog.williams.edu

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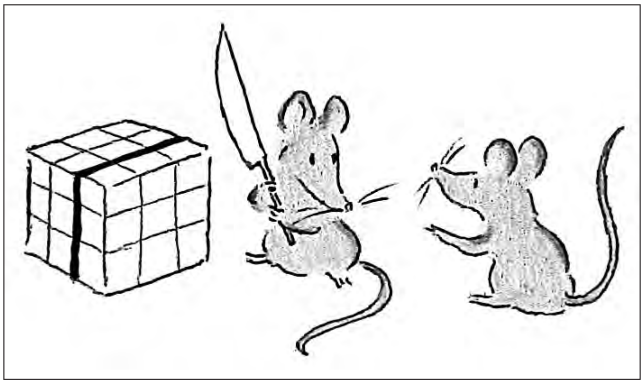
The January Contest Winner Is...

Mason Porter, who receives our book award.

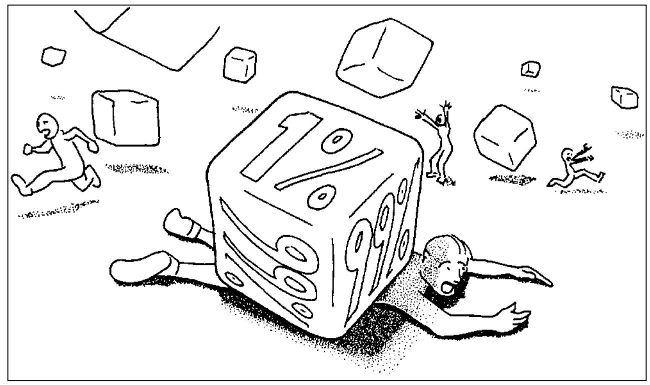


Courtesy of Tadashi Tokieda.

"Natasha, I think you're probably going to be better at equitable cake cutting than you are at solving a Rubik's® Cube."

The April 2017 Caption Contest:

What's the Caption?



Drawn by James F. Bredt, © Frank Morgan.

Submit your entry to `captions@ams.org` by April 25. The winning entry will be posted here in the June/July 2017 issue.

"With regard to the computer I have heard over and over again the saying: Whether mathematicians like it or not, the computer is here to stay. *I do not agree with that formulation. We like the computer and we use it. But today I find it important to turn that phrase around and say:* Whether the computer likes it or not, mathematics is here to stay."

—Beno Eckmann, 1995 ICM, Zurich

"TWITTER MATHEMATICS" created by Li Zhou

If AB and CD are opposite sides of a parallelogram, then AB#CD. (The Twitter # sign combining parallel | | and equal =.)

What crazy things happen to you? Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: **noti-backpage@ams.org**.

IN THE NEXT ISSUE OF NOTICES

MAY 2017...

Mathematical Congress of the Americas 2017 Invited Speakers Lecture Sampler:

- Andrew Granville, "The Benefits of an Alternative Approach to Analytic Number Theory"
- Yuval Peres, "Two Surprising Appearances of Potential Theory"

Calls for Nominations on the following prizes:

- AMS-SIAM George David Birkhoff Prize in Applied Mathematics
- Chevalley Prize in Lie Theory
- Frank Nelson Cole Prize in Algebra
- Levi L. Conant Prize
- Albert Leon Whiteman Memorial Prize

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