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Notices

of the American Mathematical Society

June/July 2017
Volume 64, Number 6

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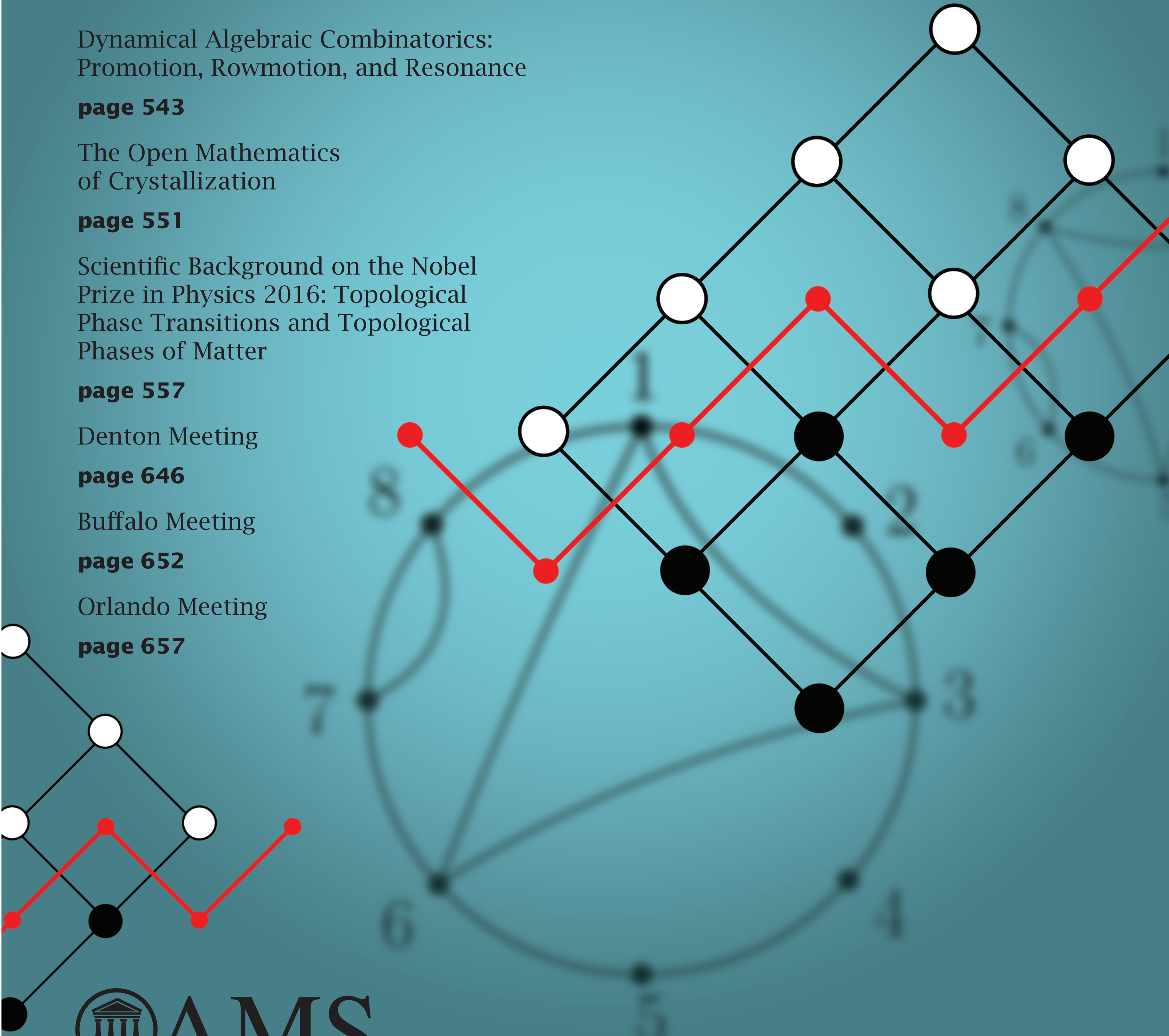

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Cover artwork from Jessica Striker's feature on algebraic combinatorics (see page 543).

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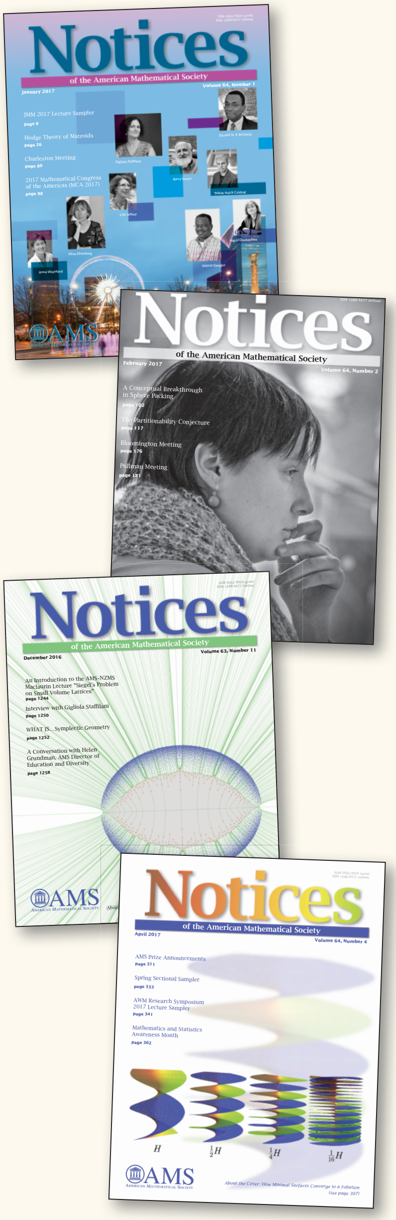
Call for Applications & Nominations

Chief Editor of the *Notices*



Applications and nominations are invited for the position of Chief Editor of the *Notices of the American Mathematical Society*, to commence with the January 2019 issue. The Society seeks an individual with strong mathematical research experience, broad mathematical interests, and a commitment to communicating mathematics to a diverse audience at a wide range of levels. The applicant must demonstrate excellent written communication skills.

The Chief Editor has editorial responsibility for a major portion of the *Notices* within broad guidelines. The goal of the *Notices* is to serve all mathematicians by providing a lively and informative magazine containing exposition about mathematics and mathematicians, and information about the profession and the Society.

The Chief Editor is assisted by a board of Associate Editors, nominated by the Chief Editor, who help to fashion the contents of the *Notices* and solicit material for publication. Some writing, and all publication support, will be provided by AMS staff. The Chief Editor will operate from her or his home base. Compensation will be negotiated for this half-time position and local part-time secretarial support will be provided. In order to begin working on the January 2019 issue, some editorial work would begin in early 2018.

Nominations and applications (including curriculum vitae) should be sent to the Chair of the Search Committee, Executive Director Catherine A. Roberts, at `exdir@ams.org`. Confidential inquiries may also be sent directly to Catherine A. Roberts or to any other member of the Search Committee (David Jerison, Mary Pugh, Kenneth Ribet, or Carla Savage).

To receive full consideration, nominations and applications should be sent on or before **September 15, 2017**.

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of the American Mathematical Society

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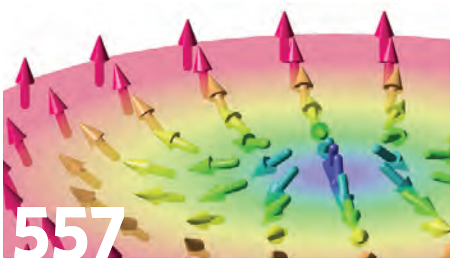

557 **Scientific Background on the Nobel Prize in Physics 2016: Topological Phase Transitions and Topological Phases of Matter**
Compiled by the Class for Physics of the Royal Swedish Academy of Sciences

Whether on the road or at home, enjoy the summer features on "Dynamical Algebraic Combinatorics" and "The Open Mathematics of Crystallization" (related to the 2016 Nobel Prize in Physics, also featured). We're also highlighting a review of Hidden Figures, *the movie about the amazing role of black women at NASA during the Space Race. But first this month we have some interesting Letters to the Editor, and we hope readers will write more and participate in our online commentary on our webpage* `www.ams.org/notices`. *—**Frank Morgan, Editor-in-Chief***

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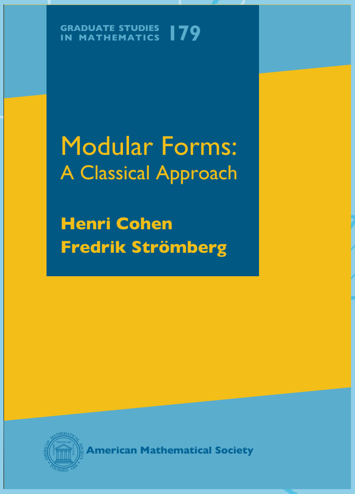

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American Mathematical Society

Notices
of the American Mathematical Society

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For back issues see **www.ams.org/backvols.** Note: Single issues of the *Notices* are not available after one calendar year.

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LETTERS TO THE EDITOR

Women's History Month: Don't Put a Man on the Cover

There should not be a man on the cover of the Women's History Month issue of the *Notices* [March 2017 issue].

I have been told that this choice was made to avoid giving the impression that a woman was featured due to her gender and not her mathematics. To avoid tokenizing us you have erased us, and indicated "Women's History Month" in the margin.

A moment's thought produces many alternatives. Feature a figure or formula or diagram from an influential paper by a woman, or an illustration of a woman's theorem. Feature several historical women in mathematics, or several living women in math with a group interview inside, or some of the young women that have won the Sloan recently. And do this throughout the year. Take a break from men on the cover. Give women some visibility.

> *To avoid tokenizing us you have erased us, and indicated "Women's History Month" in the margin.*

Give our mathematics some visibility. Show young people the powerful work that has been done, and continues to be done, by women. Marina Ratner's theorems on unipotent flows, Alice Roth's approximation theorems, Karen Uhlenbeck's compactness theorems, Maryam Mirzakhani's volume formula, Mary Ellen Rudin's counterexamples, Karen Vogtmann's work on Outer Space, Alice Chang's work in analysis, Ivelisse Rubio Canabal's work in finite fields, Karen Smith's work on tight closure, Suzanne Weekes's work in applied mathematics, Amie Wilkinson's work in dynamics, Melanie Matchett Wood's work in number theory and algebraic geometry, Mariel Vazquez's work in mathematical biology, Nathalie Wahl's work in algebraic topology, Chelsea Walton's work in noncommutative algebra, Katie Mann's work in topology, Lillian Pierce's work in number theory, Minerva Cordero's work in finite geometries, Ingrid Daubechies's work in wavelets, Xenia de la Ossa's work in mathematical physics,… .

Marginalized people are frequently and systematically erased the way that women were this March in the *Notices,* the way that black people were erased in the February issue.

Next year's March issue would benefit from a discussion of sexism in academia. It would be a good follow-up to a February issue addressing racism.

—**Autumn Kent**
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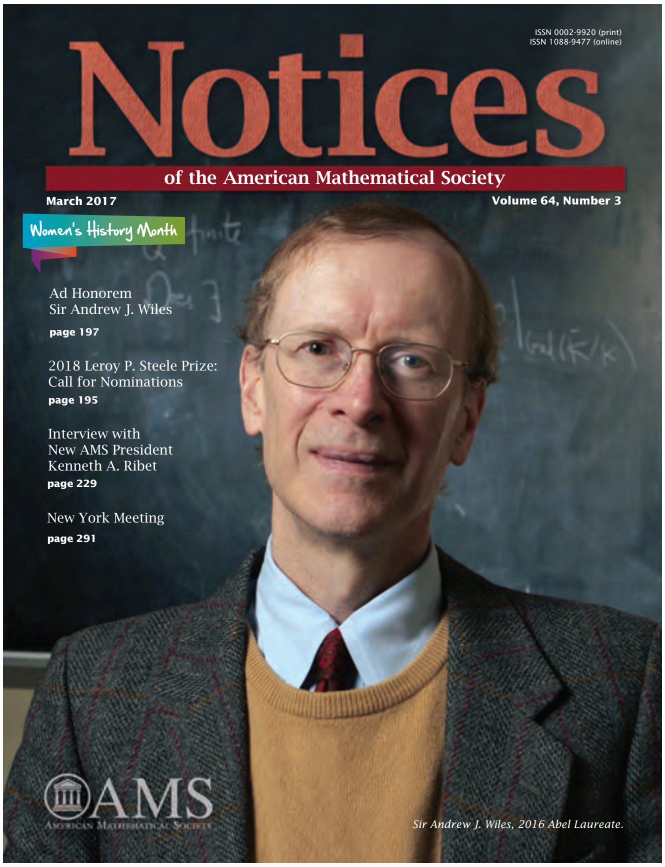

The March 2017 cover.

Survey on Math Post-docs

In connection with my article on “Disruptions of the Academic Math Employment Market” in the October 2016 *Notices*, I report on a pilot survey about math post-docs sent to chairs of 14 doctoral mathematics departments in October 2016. Most were at public universities in the northeastern quadrant of the US. Respondents were promised anonymity to encourage frankness. Eleven replies provided data for 162 mathematicians leaving research post-doc positions in 2012–13, 2013–14, or 2014–15.

- 41 (25%) obtained tenure-track positions in doctoral departments
- 36 (22%) moved to another post-doc position
- 14 (9%) moved to tenure-track jobs in the US at bachelors or masters institutions
- 14 (9%) moved to non-US academic institutions (tenure status and title unknown)
- 15 (9%) took full-time non-tenure-track academic jobs
- 13 (8%) moved to business, industry, or government
- 29 (18%) were reported in the category “other/unknown”

Assuming optimistically that two-thirds of those moving to an additional post-doc eventually find a tenure track job in a doctoral university, the pipeline from post-docs to academic research careers appears to leak. Better tracking by post-docs’ employers might give a more encouraging picture. Certainly, post-docs who move into business, industry, and government contribute to the nation’s STEM workforce.

The flow from one post-doc to another may result from the increasing number of post-doctoral positions since the economic downturn of 2008, the decreasing number of tenure-eligible jobs in doctoral departments, and the increasing reliance of academic institutions on full-time, non-tenure-track doctoral mathematicians as teaching-intensive faculty. The heavy teaching loads of most (certainly not all) such faculty are not conducive to research.

Some post-docs in this study were already in a second post-doc and moved to a third post-doc. This may reflect the attraction of geographic mobility, or the unavailability of attractive tenure-track jobs. An employment pattern of 6 years or more of post-doctoral support is not generally attractive to early career mathematicians (male as well as female) concerned about stability and work-life balance.

Methodology. Results of a pilot survey are not conclusive but should spur discussion and further study. For this survey, a research post-doc position was defined to be a multi-year but non-renewable position intended to support the transition from directed thesis research to an independent research program. The definition explicitly included not only positions formally called post-docs but also positions serving the same purposes under other titles such as named instructorships, named assistant professorships, and certain one-year positions anticipating renewals for up to three years.

The survey was not sent to directors of research centers. It is unclear how many departments reported on post-docs located in centers. In most cases, the department filled out an on-line survey. In one case, no survey was returned, but the names of research post-docs were available on the department’s website for survey personnel to track using search engines. In another case, the responding department filled out most sections of the survey but provided names of research post-docs for survey personnel to track. Too little data was available to compare the recent situation with that a decade earlier.

References. A fuller account of my survey, including prior jobs and preparation provided for future careers, is posted at `arXiv.org`. Consistent results from an AMS survey on job placements of post-docs moving in 2015 will appear in the *2015 CBMS Statistical Abstracts*. See also the article on “Math PhD Careers: New Opportunities Emerging Amidst Crisis” in the March 2017 *Notices* (`www.ams.org/publications/journals/notices/201703/rnoti-p260.pdf`).

—**Amy Cohen**
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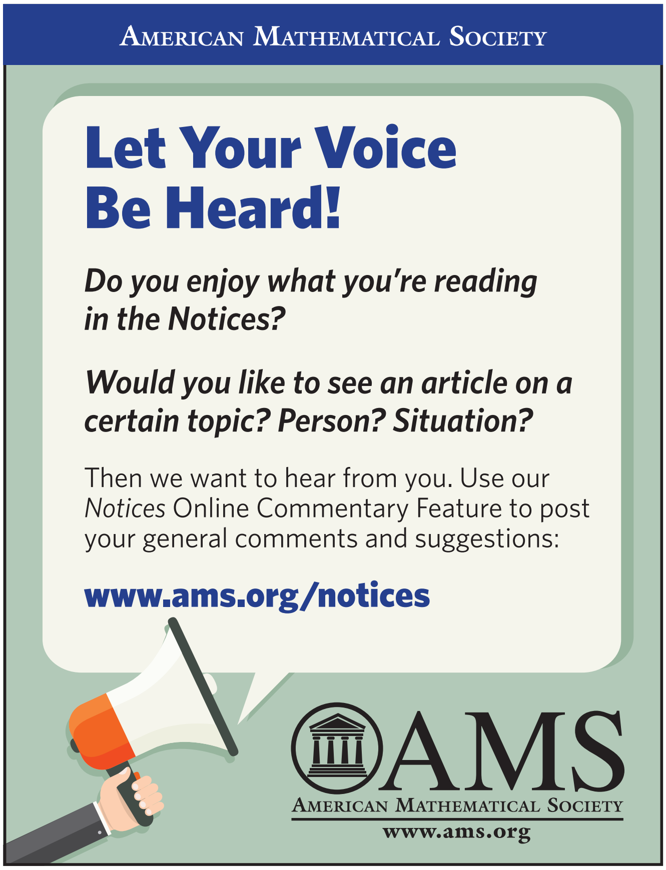

Dynamical Algebraic Combinatorics: Promotion, Rowmotion, and Resonance

Jessica Striker

Communicated by Benjamin Braun

Abstract. Dynamical algebraic combinatorics studies actions on objects important in algebraic combinatorics, with particular focus on actions with surprisingly nice properties. We discuss recent research in this emerging area, including actions on order ideals and tableaux that exhibit the *cyclic sieving, homomesy,* and *resonance* phenomena.

Dynamical Algebraic Combinatorics

Mathematical inquiry often begins with the study of *objects* (numbers, shapes, variables, matrices, ideals, metric spaces, …) and the question, "What are the objects like?" It then moves to the study of *actions* (functions, rotations, reflections, multiplication, derivatives, shifts, …) and the question, "How do the objects behave?" The study of actions in various mathematical contexts has been extremely fruitful; consider the study of metric spaces through the lens of dynamical systems or the study of symmetries arising from group actions. For objects and actions arising from algebraic combinatorics, we call this study *dynamical algebraic combinatorics.*

Let g be a bijective action on a finite set X. Such an action breaks the space X into *orbits.* Often, the study of the orbits of g provides insight into the structure of the objects in X, revealing hidden symmetries and connections. One typically first seeks to understand the *order* n of the action and then finds interesting properties the action exhibits. One surprisingly ubiquitous property is the *cyclic sieving phenomenon* [ReStWh04], which occurs when the evaluation of a *generating function* for X at the primitive nth root of unity ζ^d (where $\zeta = e^{2\pi i/n}$) counts the number of elements of X fixed under g^d.

For example, let X be the set of binary words composed of two zeros and two ones. Let g be the cyclic shift that acts by moving the first digit to the end of the word. Each word has four digits, so g is of order four. Consider the *inversion number statistic* $\text{inv}(w)$ of $w \in X$, which equals the number of pairs (i, j) with $i < j$ such that the ith digit of w is 1 and the jth digit is 0. See Figure 1 for the orbits of X under g and the inversion numbers of each binary word.

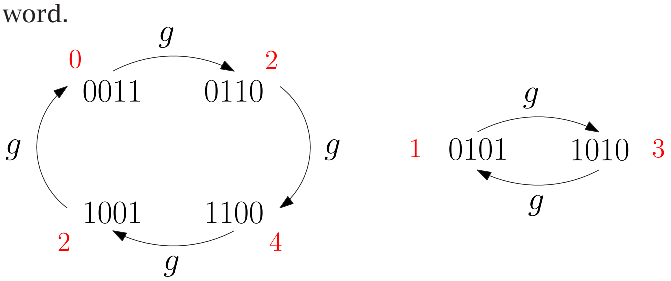

Figure 1. The orbits of binary words of length four with two ones under a cyclic shift; the inversion numbers corresponding to the binary words are shown in red.

The inversion number statistic determines the generating function

$$X(q) = \sum_{w\in X} q^{\text{inv}(w)} = 1 + q + 2q^2 + q^3 + q^4. \tag{1}$$

Since g is of order four, $\zeta = e^{2\pi i/4} = i$. One may check using Figure 1 and (1) that $X(i^d)$ equals the number of elements of X fixed by g^d, thus this is an instance of the cyclic sieving phenomenon. For example, $X(i^1) = 1 + i + 2i^2 + i^3 + i^4 = 1 + i + 2(-1) + (-i) + 1 = 0$, and zero

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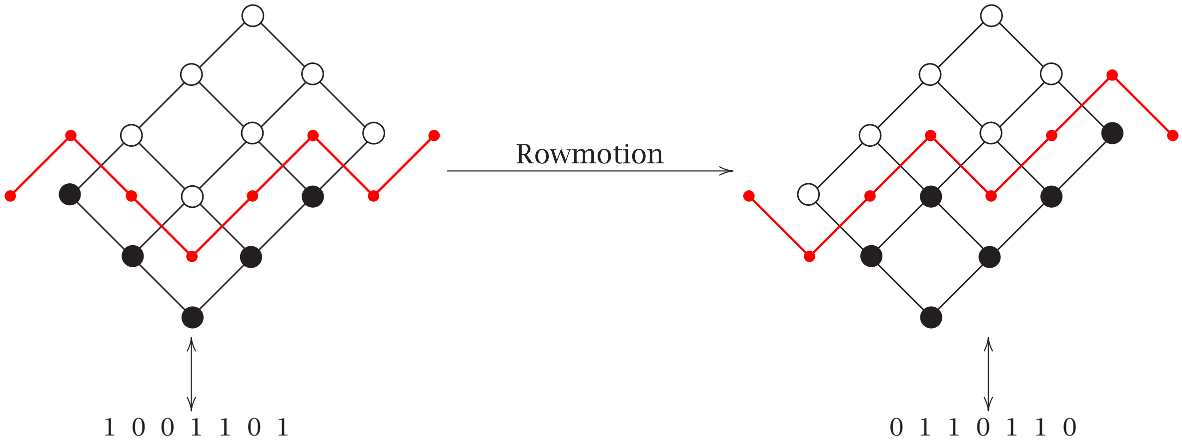


Figure 2. An order ideal in the product of chains poset 3×4 and its image under rowmotion. The red paths are *boundary paths* that divide the order ideal from the rest of the poset; the binary words corresponding to these paths are shown beneath.

elements of X are fixed under g^1. $X(i^2) = X(-1) = 2$, and two elements are fixed under g^2.

Another interesting property actions in dynamical algebraic combinatorics often exhibit is the *homomesy phenomenon* [PrRo15], in which the average value of a statistic on every g-orbit equals the global average value of that statistic. Each orbit in Figure 1 has an average inversion number of 2. Thus, the inversion number statistic on X is *homomesic* with respect to g. Note that both cyclic sieving and homomesy still hold in the more general case in which X is the set of binary words of length n with k ones [ReStWh04], [PrRo15].

In Figure 1, our action was visibly cyclic, so its order was clear from the definition. In the coming sections, we discuss more complicated combinatorial actions whose orders are difficult to predict. We will see examples in which an action g on a large combinatorial set X has a relatively small order; this indicates the elements of X have hidden cyclic symmetry such that g is a rotation in disguise. We will also see examples in which g is not of small order, but rather exhibits *resonance*, meaning that g maps to an underlying cyclic action with small order. We discuss examples of such actions on *order ideals* and *tableaux* and then give a surprising relation between them, which we found via this study of dynamics.

Rowmotion on Order Ideals

As our first example, let our set X be the *order ideals* of a *poset*.

Definition 1. A **poset** is a **p**artially **o**rdered **set**. The set of **order ideals** $J(P)$ of a poset P is the set of all subsets $I \subseteq P$ such that if $y \in I$ and $z \leq y$, then $z \in I$.

Specifically, we will take X to be the set of order ideals $J(P)$ for P the product of chains poset $\mathbf{a} \times \mathbf{b}$. That is, for a natural number a, $\mathbf{a} = \{1, 2, \ldots, a\}$ and the product partial order on $\mathbf{a} \times \mathbf{b}$ is $(x, y) \leq (x', y')$ if and only if $x \leq x'$ and $y \leq y'$. See Figure 2 for an example. An easy counting argument shows the number of order ideals in $J(\mathbf{a} \times \mathbf{b})$ is given by the binomial coefficient $\binom{a+b}{a}$, since these order ideals are in bijection with binary words with a zeros and b ones via the *boundary path* that separates

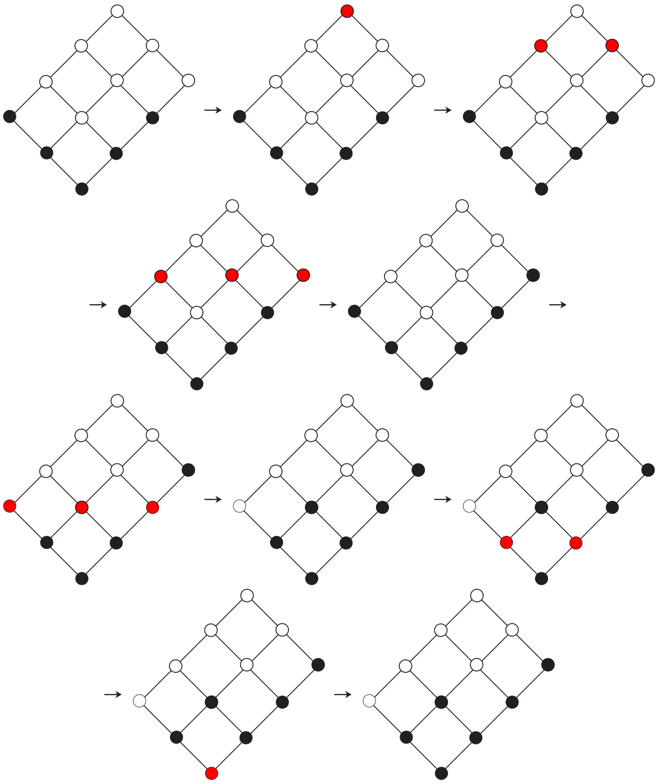


Figure 3. Rowmotion from Figure 2 computed by toggling rows from top to bottom. The elements that are being toggled at each step are shown in red, and at any step in which the toggles act nontrivially, the order ideal resulting from those toggles is shown.

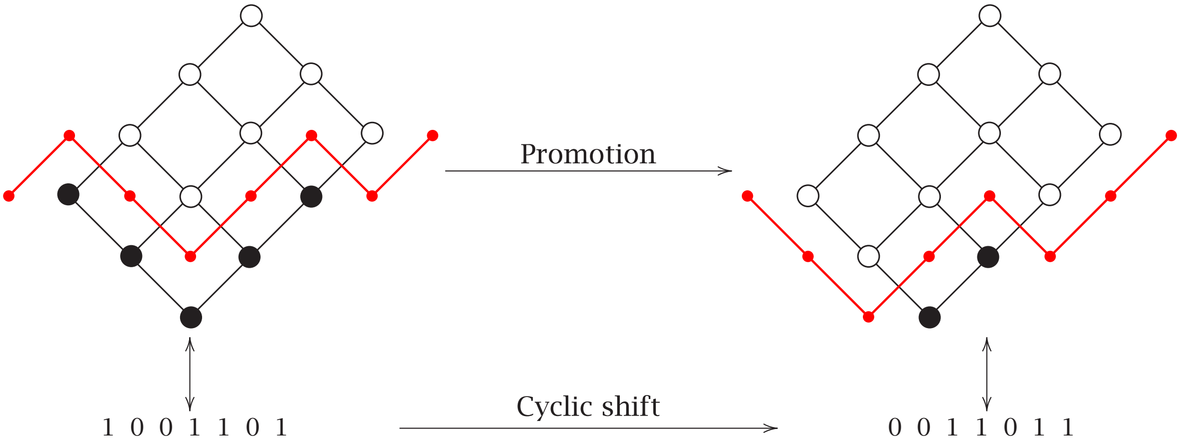


Figure 4. An order ideal in $J(3 \times 4)$ and its image under promotion. Promotion acts as a cyclic shift on the binary word corresponding to the boundary path.

the order ideal from the rest of the poset (where a one indicates an up-step and a zero indicates a down-step); see Figure 2.

Our action g will be the following; see Figure 2.

Definition 2. Let P be a poset, and let $I \in J(P)$. Then **rowmotion** on I is the order ideal generated by the minimal elements of $P \backslash I$.

The order of rowmotion on general posets is neither well behaved nor predictable. But on $J(\mathbf{a} \times \mathbf{b})$ its order is, surprisingly, much smaller than $\binom{a+b}{a}$.

Theorem 3 ([BrSc74])**.** *The order of rowmotion on $J(\mathbf{a} \times \mathbf{b})$ is $a+b$.*

To see why the order of a global action such as rowmotion is surprisingly small, it often helps to interpret the action as a composition of local involutions and work within the group generated by those involutions. In the case of rowmotion, we call this group the *toggle group*.

Definition 4. For each element $e \in P$ define its **toggle** $t_e : J(P) \to J(P)$ as follows.

$$t_e(I) = \begin{cases} I \cup \{e\} & \text{if } e \notin I \text{ and } I \cup \{e\} \in J(P) \\ I \backslash \{e\} & \text{if } e \in I \text{ and } I \backslash \{e\} \in J(P) \\ I & \text{otherwise} \end{cases}$$

The **toggle group** $T(J(P))$ is the subgroup of the symmetric group $S_{J(P)}$ generated by $\{t_e\}_{e \in P}$.

Theorem 5 ([CaFo95])**.** *Given any poset P, rowmotion is the toggle group element that toggles the elements of P from top to bottom (in the reverse order of any linear extension).*

For example, in Figure 3, we recompute rowmotion on the order ideal from Figure 2 by toggling from top to bottom by rows.

In joint work with Nathan Williams, we found a toggle group action conjugate to rowmotion, which we called *promotion*, defined for $J(\mathbf{a} \times \mathbf{b})$ as toggling the elements from left to right (we will soon make this more precise). We showed rowmotion and promotion are conjugate actions in the toggle group of *any* ranked poset; this implies they are *equivariant*, or have the same orbit structure.

Theorem 6 ([StWi12])**.** *In any ranked poset, there is an equivariant bijection between the order ideals under rowmotion (toggle top to bottom by rows) and promotion (toggle left to right by columns).*

By the bijection to binary words, promotion on $J(\mathbf{a} \times \mathbf{b})$ is a cyclic shift of a binary word of length $a+b$; each toggle swaps adjacent letters, resulting in the first letter swapping all the way through the word to the end; see Figure 4. Therefore, by the above theorem, the cyclic nature of promotion on $J(\mathbf{a} \times \mathbf{b})$ gives a satisfying explanation for the order of rowmotion on $J(\mathbf{a} \times \mathbf{b})$. Furthermore, since binary words under a cyclic shift exhibit the cyclic sieving phenomenon (as discussed in Section 1), so does rowmotion on $J(\mathbf{a} \times \mathbf{b})$.

Corollary 7 ([StWi12])**.** *There is an equivariant bijection between the order ideals of $\mathbf{a} \times \mathbf{b}$ under rowmotion and binary words of length $a+b$ with b ones under rotation. The cyclic sieving phenomenon follows.*

Rowmotion on these order ideals also exhibits the homomesy phenomenon.

Theorem 8 ([PrRo15])**.** *The order ideal cardinality statistic in $J(\mathbf{a} \times \mathbf{b})$ exhibits homomesy (orbit-average = global-average) with respect to rowmotion or promotion.*

It is natural to ask whether similar results hold on posets constructed as products of more than two chains.

Theorem 9 ([CaFo95])**.** *The order of rowmotion on $J(\mathbf{a} \times \mathbf{b} \times \mathbf{2})$ is $a+b+1$.*

Again, as a corollary of Theorem 6, we used the toggle group to explain this result by showing promotion on $J(\mathbf{a} \times \mathbf{b} \times \mathbf{2})$ is a cyclic action on combinatorial objects in bijection with these order ideals, which also exhibit the cyclic sieving phenomenon; see Figure 5.

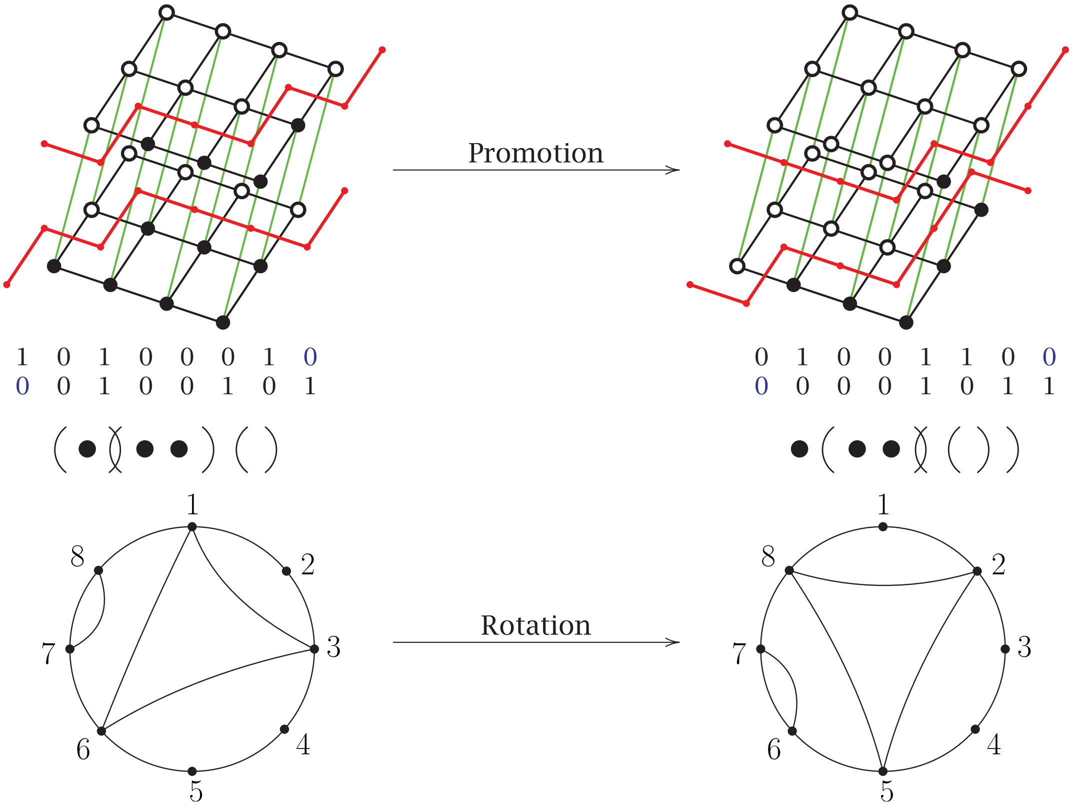


Figure 5. An order ideal in $J(4\times 3\times 2)$ and its image under promotion, along with the boundary paths for each layer and the corresponding *boundary path matrices*, which transform via the parenthesizations shown to the given noncrossing partitions. Promotion on $J(\mathbf{a}\times\mathbf{b}\times 2)$ rotates the corresponding noncrossing partition.

Theorem 10 ([StWi12])**.** *There is an equivariant bijection between $J(\mathbf{a}\times\mathbf{b}\times\mathbf{2})$ under rowmotion and noncrossing partitions of $a+b+1$ into $b+1$ blocks under rotation. The cyclic sieving phenomenon follows.*

Homomesy also holds in this case.

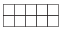
Theorem 11 ([Vo17])**.** *The order ideal cardinality statistic in $J(\mathbf{a}\times\mathbf{b}\times\mathbf{2})$ exhibits homomesy with respect to rowmotion or promotion.*

The above theorems may lead one to guess that rowmotion on $J(\mathbf{a}\times\mathbf{b}\times\mathbf{c})$ is of order $a+b+c-1$ and exhibits the cyclic sieving and homomesy phenomena. This is not true in general. For example, when $a=b=c=3$, the order of rowmotion is 8, but the generating function no longer exhibits the cyclic sieving phenomenon. For $a=b=3$, $c=4$, the cardinality statistic is no longer homomesic. When $a=b=c=4$, the order of rowmotion is not $a+b+c-1=11$, but rather 33. Similar computations for many other values of a,b,c show that the orbits of rowmotion on $J(\mathbf{a}\times\mathbf{b}\times\mathbf{c})$ are each of cardinality a *multiple of a divisor of* $a+b+c-1$. This is an admittedly fuzzy notion, which we will make more precise in the last section of this article. But first, we move to our second example of an action on *standard Young tableaux*.

Promotion on Standard Young Tableaux

As our second example, let our set X be composed of the following well-loved objects in algebraic combinatorics and representation theory.

Definition 12. A **standard Young tableau** of partition shape λ is a bijective filling of λ with the numbers $\{1,2,\ldots,n\}$, where n is the number of boxes in λ, such that labels strictly increase from left to right across rows and from top to bottom down columns. Let $SYT(\lambda)$ denote the set of standard Young tableaux of shape λ.

See Figures 6 and 7 for examples of tableaux in $SYT($  $) = SYT(2\times 5)$. $SYT(\lambda)$ is enumerated by the famous Frame–Robinson–Thrall hook formula: $\prod_{x\in\lambda}\frac{n!}{h(x)}$, where x ranges over all boxes in the diagram of λ. $h(x)$ is the *hook number* of x, that is, the number of boxes in λ in the same row as x and to its right, plus the number of boxes in the same column as x and below, plus one (for x itself).

Our action g will be M.-P. Schützenberger's *promotion* operator (see Figure 6). In general, this is a different action from the toggle-promotion defined in the previous section. A relation between these actions will be made clear later in this section and the next.

Definition 13. Given a partition λ, **promotion** on $T\in SYT(\lambda)$ is the product $\rho_{n-1}\cdots\rho_2\rho_1(T)$ of the

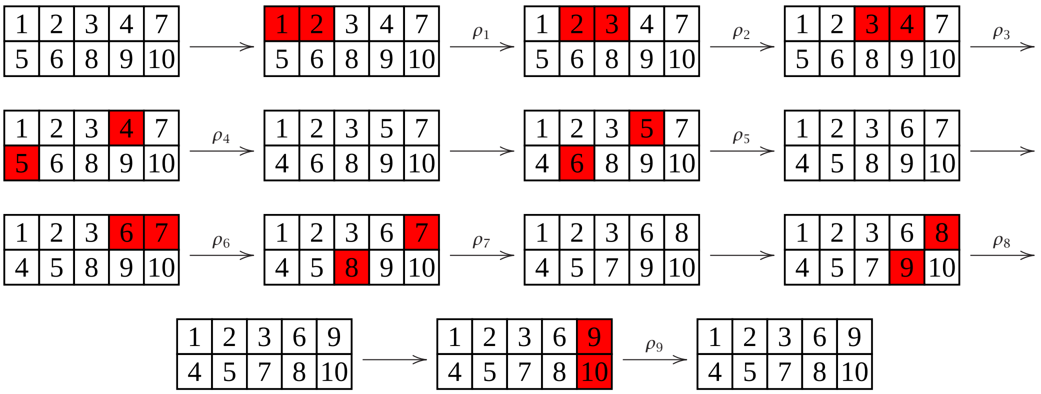


Figure 6. Promotion computed as the product $\rho_9 \cdots \rho_2 \rho_1(T)$ of Bender–Knuth involutions. The entries that are being acted on at each step are shown in red, and at any step in which the involution acts nontrivially, the resulting tableau is shown.

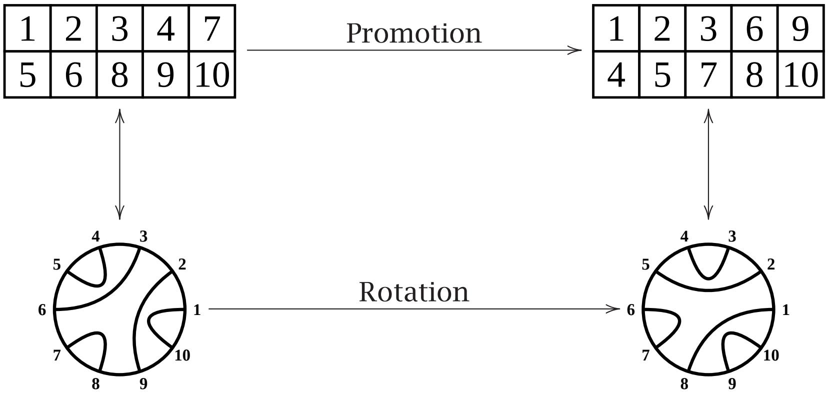
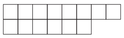


Figure 7. Promotion on $SYT(2 \times b)$ is in equivariant bijection with rotation on noncrossing matchings of $2b$ points. Each top row tableau entry is the smaller number in its matching pair.

Bender–Knuth involutions ρ_i, where each ρ_i swaps i and $i+1$ *if possible.*

Promotion on standard Young tableaux in an $a \times b$ rectangle has the unexpected property that all the orbit sizes are *divisors* of the maximum value ab.

Theorem 14 ([Hai92]). *Promotion on $SYT(a \times b)$ is of order ab.*

For example, by the hook formula, $|SYT(5 \times 7)| = \frac{35!}{1^1 2^2 3^3 4^4 5^5 6^5 7^5 8^4 9^3 10^2 11} = 278{,}607{,}172{,}289{,}160$, but promotion is of order $7 \cdot 5 = 35$. Rectangles are among a few special shapes for which promotion is of a nice order. Promotion on tableaux of general shape does not have a nice order; for example, promotion on the partition  has order 7,554,844,752.

Even more surprisingly, $SYT(a \times b)$ under promotion exhibits the cyclic sieving phenomenon, which means the evaluation of the *q-analogue* of the hook length formula for $SYT(a \times b)$ at the (ab)th root of unity $(e^{2\pi i/ab})^d$ counts the number of $T \in SYT(a \times b)$ such that $\text{Promotion}^d(T) = T$.

Theorem 15 ([Rh10]). *Promotion on $SYT(a \times b)$ exhibits the cyclic sieving phenomenon.*

Proofs of this result illuminated connections to representation theory and geometry. In the cases $a = 2, 3$, there are proofs that proceed by giving a bijection to other combinatorial objects (*noncrossing matchings* and *webs*, respectively) that sends promotion to *rotation* [PePyRh09]; see Figure 7. There are also homomesy results in this and more general settings; see [BlPeSa16].

Promotion on two-row rectangular tableaux $SYT(2 \times b)$ is equivalent to toggle-promotion on the *type A positive root poset* [StWi12], which is why we gave the name promotion to the action of toggling from left to right. A few years later, this name was further validated by the result that *hyperplane-toggle promotion* on $J(\mathbf{a} \times \mathbf{b} \times \mathbf{c})$ is equivalent to *K-theoretic promotion* on *increasing tableaux* [DiPeSt17]; we explain this in the next section.

Resonance on Orbits of Increasing Tableaux and Plane Partitions

For our final example, we take our set X to be *increasing tableaux*, objects that have appeared in various contexts within algebraic combinatorics, in particular, in relation to *K-theoretic Schubert calculus of Grassmannians*.

Definition 16. An **increasing tableau** of partition shape λ is a filling of λ with positive integers such that labels strictly increase from left to right across rows and from top to bottom down columns. Let $\mathrm{Inc}^q(\lambda)$ denote the set of all increasing tableaux of shape λ with entries at most q.

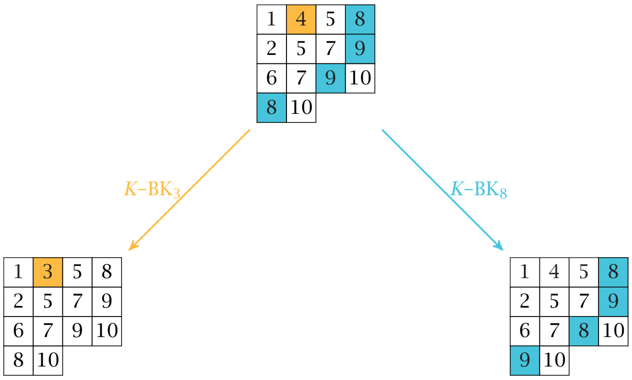


Figure 8. The action of some K-Bender–Knuth involutions on an increasing tableau in $\mathrm{Inc}^{10}(4,4,4,2)$.

See Figures 8 and 9 for examples. Note that $\mathrm{Inc}^n(\lambda)$ (where n is the number of boxes in λ) contains $SYT(\lambda)$ as a subset; increasing tableaux differ from standard Young tableaux in that there may be repeated and/or missing numbers in the filling.

Our action g will be *K-promotion* (see Figure 8), which was originally defined globally using *K-jeu-de-taquin*; we give here our equivalent definition from [DiPeSt17]. Note that if an increasing tableau is a standard Young tableau, K-Bender–Knuth involutions are equivalent to Bender–Knuth involutions.

Definition 17. Given a partition λ and a natural number q, **K-promotion** on $T \in \mathrm{Inc}^q(\lambda)$ is the product $K\text{–BK}_{q-1} \cdots K\text{–BK}_2\, K\text{–BK}_1(T)$ of the *K-Bender–Knuth involutions* $K\text{–BK}_i$, where each $K\text{–BK}_i$ increments i and/or decrements $i+1$ *wherever possible.*

For $\mathrm{Inc}^q(a \times b)$ under K-promotion, it is no longer true that orbit sizes are divisors of the maximum value q, but rather, they are *multiples of divisors* of q. For example, $\mathrm{Inc}^{11}(4 \times 4)$ has orbits of size 11 and 33. With Kevin Dilks and Oliver Pechenik, we made this observation more precise and gave this phenomenon the name *resonance*.

Definition 18 ([DiPeSt17]). Suppose g is a cyclic group action on a set X, c a cyclic group action of order ω acting nontrivially on a set Y, and $f : X \to Y$ a surjection. We say the triple $(X, \langle g \rangle, f)$ exhibits **resonance with frequency** ω if, for all $x \in X$, $c \cdot f(x) = f(g \cdot x)$, that is, the following diagram commutes:

$$\begin{tikzcd} X \arrow[r, "g\cdot"'] \arrow[d, "f"] & X \arrow[d, "f"] \\ Y \arrow[r, "c\cdot"'] & Y \end{tikzcd}$$

Resonance is a first step in understanding dynamics of more complicated combinatorial objects, since it is an analogue of an action having a nice order. Resonance is a *pseudo-periodicity* property, in that the resonant frequency ω is generally less than the order of g. (If ω equals the order of g, this is an instance of *trivial resonance*.) For non-trivial resonance, we also need a surjective map f to a simpler underlying set Y on which g corresponds to a cyclic action c with smaller order ω. In the second theorem below, this map is the **binary content vector**, defined on an increasing tableau $T \in \mathrm{Inc}^q(\lambda)$ as the sequence $\mathrm{Con}(T) = (a_1, a_2, \ldots, a_q)$, where $a_i = 1$ if i is an entry of T and $a_i = 0$ if it is not. See Figure 9.

Theorem 19 ([DiPeSt17]). *For a certain map f, $(J(\mathbf{a} \times \mathbf{b} \times \mathbf{c}), \langle \mathrm{Rowmotion} \rangle, f)$ exhibits resonance with frequency $a + b + c - 1$.*

Theorem 20 ([DiPeSt17]). $(\mathrm{Inc}^q(\lambda), \langle K\text{-promotion} \rangle, \mathrm{Con})$ *exhibits resonance with frequency q.*

The uncanny similarity of the objects and actions in these theorems when $\lambda = a \times b$ and $q = a + b + c - 1$ led to a nonobvious connection between them.

Theorem 21 ([DiPeSt17]). $\mathrm{Inc}^{a+b+c-1}(a \times b)$ *under K-promotion is in equivariant bijection with $J(\mathbf{a} \times \mathbf{b} \times \mathbf{c})$ under rowmotion.*

This proof relies on the following extension of Theorem 6 from the 2- to n-dimensional lattice. For this generalization, we introduced and developed the machinery of *affine hyperplane toggles* and *n-dimensional lattice projections*. We obtained a large family of toggle group actions $\{\mathrm{Pro}_{\pi,v}\}$, whose orbit structures are equivalent to that of rowmotion. That is, given a poset P, we project the elements in a way consistent with the covering relations into the n-dimensional lattice by the map π. Then $\mathrm{Pro}_{\pi,v}$ is the toggle group action that toggles elements as it sweeps through $\pi(P)$ by an affine hyperplane in the direction determined by the vector v.

Theorem 22 ([DiPeSt17]). *Let P be a finite poset with an n-dimensional lattice projection π. Let v and w be length n vectors with entries in $\{\pm 1\}$. Then there is an equivariant bijection between $J(P)$ under $\mathrm{Pro}_{\pi,v}$ and $J(P)$ under $\mathrm{Pro}_{\pi,w}$.*

It is not hard to show that $\mathrm{Pro}_{\pi,(1,1,\ldots,1)}$ is rowmotion, so this theorem says rowmotion and $2^n - 1$ other promotions have the same orbit structure. For π the natural three-dimensional embedding, $\mathrm{Pro}_{\pi,(1,1,-1)}$ on $J(\mathbf{a} \times \mathbf{b} \times \mathbf{c})$ is equivalent to K-promotion on $\mathrm{Inc}^{a+b+c-1}(a \times b)$, which

$$\begin{tikzcd}
\begin{array}{|c|c|c|c|}\hline 1&2&4&7\\\hline 3&5&6&8\\\hline 5&7&8&10\\\hline 7&9&10&12\\\hline\end{array} \arrow[r, "K\text{-Promotion}"] \arrow[d, "\text{Con}"] & \begin{array}{|c|c|c|c|}\hline 1&3&5&6\\\hline 2&4&7&9\\\hline 4&6&9&11\\\hline 6&8&11&12\\\hline\end{array} \arrow[d, "\text{Con}"] \\
(1,1,1,1,1,1,1,1,1,1,0,1) \arrow[r, "\text{Cyclic shift}"] & (1,1,1,1,1,1,1,1,1,0,1,1)
\end{tikzcd}$$

Figure 9. An increasing tableau in $\mathrm{Inc}^{12}(4\times 4)$ and its image under K-promotion, along with the map to the binary content of each. K-promotion cyclically shifts the binary content vector.

when combined with the above theorem gives the proof of Theorem 21, showing that toggle-promotion and tableaux-promotion coincide on a much larger set than we had previously known.

Conclusion

We have given a flavor of dynamical algebraic combinatorics through examples of combinatorial objects with actions that have small order and other nice properties, such as cyclic sieving and homomesy. Most of the objects and actions that behave nicely in these three respects are, in some sense, planar. These phenomena often still occur in objects that are only *slightly* three-dimensional, since such objects may often be mapped bijectively to planar objects, as in Theorem 10. But once such maps are no longer possible, these properties tend to no longer hold. We have seen examples of higher-dimensional combinatorial objects with natural actions that no longer have a nice order, but rather exhibit resonance. We hope that further study of resonance in such combinatorial dynamical systems will uncover more interesting properties and surprising connections.

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Credits

Figure 7 is courtesy of Nathan Williams.
Figure 8 is courtesy of Oliver Pechenik.
Photo of Jessica Striker is courtesy of Ryan Striker.

ABOUT THE AUTHOR

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Jessica Striker is an assistant professor at North Dakota State University. A cycling enthusiast, Jessica may be spotted on a tandem or cargo bike for weekend outings with her husband and two children, on a fat tire bike for winter commuting, or on a ride share bike for sightseeing while on mathematical travels.

Editor's Note. Related to Striker's article are two previous pieces that appeared in the *Notices*: "What is Cyclic Sieving?" by Victor Reiner, Dennis Stanton, and Dennis White, February 2014; and "What is a Young Tableau?" by Alexander Yong, February 2007.

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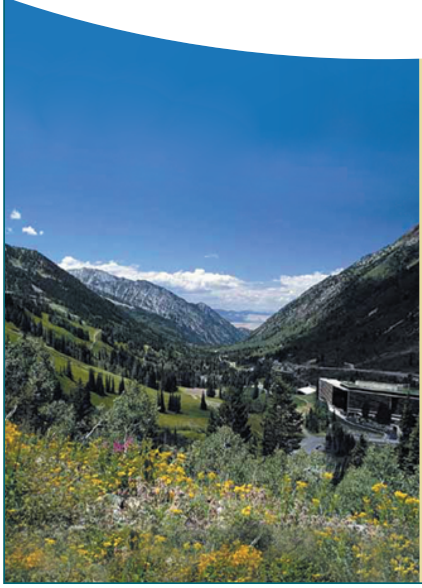

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The Open Mathematics of Crystallization

Charles Radin

Ammonium-cobalt-sulfate crystal.

Abstract. We discuss open mathematical questions in modelling the transition from water to ice and such phenomena.

Charles Radin is professor of mathematics at the University of Texas at Austin. His e-mail address is `radin@math.utexas.edu`.

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A drop of water suspended in a bath of oil is round because of the pull of its surface tension. The spherical shape is not a surprise because of the symmetry of all the forces on it. However, if the drop of water is then frozen into ice its surface becomes polyhedral, which suggests that somehow the symmetry decreases. The decrease is eloquent evidence of something not visible to the eye: the crystalline configuration of the water molecules in the water after it turns to ice (see Figure 1). Slightly different situations create facets in salt, quartz, and many other minerals, as in Figure 2. Our goal is to give an overview

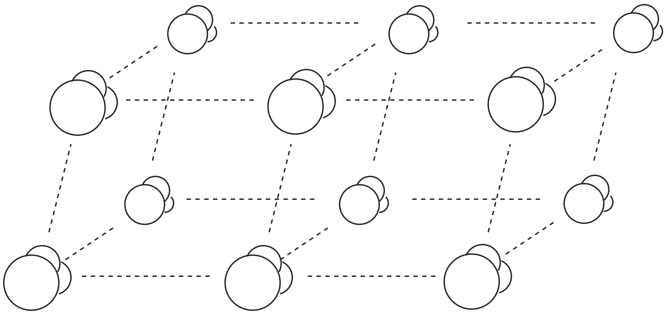

Figure 1. When water turns to ice it assumes a crystalline configuration.

of several open problems connected by a common origin of "minimization of surface tension," the mathematics coming from probability and the calculus of variations. The narrative will fork several times, the various directions leading to open problems, some of which are old and famous, and will use intuitive arguments where there are gaps in the mathematics.

The creation of polyhedral shapes for the boundary of crystalline collections of molecules is widely believed to follow as a straightforward exercise in statistical physics. Disclaimer: no one has actually been able to *prove* that crystalline configurations follow automatically from the standard statistical physics model at low but nonzero temperatures, let alone the minimal surface tension shape boundary which should then follow! Still, the difficulty of such proofs is believed to be "merely technical," and the basic structure of this theory for the study of bulk matter has been universally accepted for about one hundred years, during which our focus has gradually shifted to less familiar material phenomena such as superconductivity and superfluidity. In the midst of studying these more exotic behaviors, physicists became interested in various types of two-dimensional matter, for instance graphene, which is a sheet of carbon atoms bound together in a hexagonal lattice, sometimes just one atom thick. The 2016 Nobel Prize in Physics (see p.557) was awarded for developing topological methods to study exotic behavior in two-dimensional materials and in 2010 was given for experimental developments on graphene. This article will analyze the dusty old phenomena of crystallization and faceting, but for the more interesting two-dimensional matter. In two dimensions some basics are unknown, not only in the mathematical modelling but even in the experimental physics it should reproduce.

We begin with the more familiar three-dimensional situation before modifying for two dimensions.

First of all, by going to a model at the molecular level we will have to replace the macroscopic notion of surface tension, which makes sense as a (dissipative) force on the water drop as a whole but not as a force between individual water molecules. At the molecular level it is appropriate and traditional to reexpress the effect of surface tension in terms of the change in the energy of the water molecules *due to the presence of their interface with the oil.* We want to know how the so-called ideal "Wulff shape" for our water drop is thereby determined by the interacting water molecules, why that shape changes with temperature, and how that shape reveals the hidden molecular structure.

Figure 2. The ammonium-cobalt-sulfate crystal assumes a faceted polyhedral shape due to the underlying crystal structure.

To give some context, we note that the basic analysis of the solid, liquid, and gas phases of matter, and of the transformations between such phases is part of thermodynamics developed in the nineteenth century. Thermodynamics focuses on energy and models how substances change when energy flows between materials due to a variety of possible processes, including heating/cooling. Scientists became increasingly convinced that the laws of thermodynamics should be straightforward consequences of the interactions of constituent particles (atoms), a prescient conviction since the atomic structure of matter was not convincingly demonstrated until much later. The new, deeper analysis of energy in materials is called (equilbrium) statistical mechanics, and we will discuss such an analysis for the freezing of two-dimensional matter, the polygonal shape of such frozen matter.

The formalism of statistical mechanics is surprisingly flexible and is easily adapted to particles in any spatial dimension, so we will start with the familiar dimension three. To understand through statistical mechanics why a water drop freezes and develops facets, one might reasonably start with the nuclei and electrons making up all the atoms in water. The energy of such a system of particles consists of their individual kinetic energies plus the potential energy associated with the forces of interaction; the only forces they can feel are the well-known electromagnetic forces associated with their charges. (Gravity can be ignored in the modelling, if not in experiment!) It turns out to be hopeless to analyze the freezing of matter in this way. Instead one normally uses a simplified, or toy, model of the material in which the nuclei and electrons are assumed to have "previously" formed molecules, which then interact through some artificial force law, which, to capture realistic influences, would consist of a strong repulsion at small separation and weak attraction out to some larger separation. For instance we could assume the force was infinite and repulsive for separation less than 1 and constant and attactive for separations between 1 and 3: hard balls that attract for some distance. More specifically, one would get such a force as the negative gradient of the following potential energy function of the separation r between any particle pair:

$$\phi(r) = \begin{cases} +\infty, & \text{if } r < 1; \\ r - 3, & \text{if } 1 \le r \le 3; \\ 0, & \text{if } r > 3. \end{cases} \tag{1}$$

This is a reasonable approach for many materials of interest (though not for metals, among others), and for simplicity we use this traditional approach. It has the advantage that it removes any necessity for quantum mechanics, which disappears into the formation of the molecules. One thus imagines the material to be made of molecules modelled as small impenetrable unit volume balls which attract one another out to some small multiple of their radii. The fact then, which is at least counterintuitive if not mind boggling, is that a *large* collection of such interacting balls would *automatically* freeze into a crystalline configuration when energy is removed, i.e. when cooled, and do so at some infinitely precise temperature, which is 0° Celsius for water!

> *molecules modelled as small impenetrable unit volume balls*

We will define the formalism and sketch first how it is used to analyze a water drop in an oil bath and then the differences in two dimensions, our real focus. As its name suggests, statistical mechanics uses probabilistic tools, though from it we expect to get information about the polyhedral shape of a crystal as in Figure 2, which is as nonrandom as anything in nature. Technically, probability is used to model the precise state of $\approx 10^{23}$ molecules, and the randomness disappears when one considers macroscopic quantities such as the shape of a chunk of ice.

Here is the framework. Imagine a large number N of point particles, the centers of the molecules, in a cubic box B_V of volume V in d-dimensional space, $d = 2$ or 3, and interacting via (1). The 'state' of the j-th particle consists of its position x_j in B_V and its velocity v_j in $\mathbb{R}^d$, and therefore the state of the collection of N particles is a point $(\bar{x}, \bar{v})$ in the $2dN$-dimensional space $(B_V)^N \times \mathbb{R}^{dN} \subset \mathbb{R}^{dN} \times \mathbb{R}^{dN}$. The basic structure of statistical mechanics is a parametric family of probability distributions on the space of states of the many-particle system, the parameters being whatever thermodynamic quantities are needed to fully specify the large-scale behavior of the material. For simple materials there is a variety of alternative parameter pairs such as (pressure, temperature) or (temperature, mass density), and for each choice one has a corresponding 2-parameter family of probability distributions, called an ensemble. (Water freezes at 0° Celsius at sea level, but this varies slightly with altitude, so temperature is clearly insufficient to specify the state.) Though they differ for small systems, for macroscopic systems the various ensembles are all fully equivalent, amounting to changes of variables affected by so-called Legendre transforms.

To define the pressure/temperature or (P, T) ensemble for N particles, we first confine the particle positions within the cubic box B_V and give the configurations the relative probability density $e^{-E_N/T}\,dx dv$, where $E_N = E_N(\bar{x}, \bar{v})$ is the total energy of the particles. (Note that the lower the temperature, the more the distribution is concentrated on small values of energy.) Then we sum or integrate over all volumes V with a weighting factor $e^{-PV/T}$; high pressure concentrates the distribution on small volumes.

Putting this all together and normalizing the probability distribution, the average of a function g of positions and velocities of the particles, for instance E_N, is given by

$$\begin{aligned} \langle g \rangle_{P,T} &= \frac{1}{Z_N(P,T)} \\ &\times \int_0^\infty \left[\int_{(B_V)^N \times \mathbb{R}^{dN}} g(\bar{x}, \bar{v}) e^{-E_N(\bar{x},\bar{v})/T} d\bar{x}\, d\bar{v} \right] e^{-PV/T} dV, \end{aligned} \tag{2}$$

where $Z_N(P, T)$ is the normalization constant or partition function:

$$Z_N(P,T) = \int_0^\infty \left[\int_{(B_V)^N \times \mathbb{R}^{dN}} e^{-E_N(\bar{x},\bar{v})/T} d\bar{x}\, d\bar{v} \right] e^{-PV/T} dV. \tag{3}$$

That's it. For dimension $d = 3$ and sufficiently large N this is the standard starting point for understanding everything we would like to know at the macroscopic level about matter in thermal equilibrium, from the relationship of the familiar solid, liquid, and gas phases in Figure 3 to properties such as the coefficient of thermal expansion as a function of temperature and pressure. For instance, the transitions in Figure 3 correspond to notable changes in the Gibbs free energy $G_N(P, T) = (-T)\ln[Z_N(P, T)/N!]$. Consequences obtained from this model are rarely straightforward or mathematically rigorous. It is nevertheless generally regarded as highly reliable by physicists because of a variety of nonrigorous evidence, including computer simulation and predictions verified by experiment. For the remainder of this article we will concentrate on the role that the box B_V plays in the ensemble and will make free use of the nonrigorous intuition and results of this physical theory until a point where we can state some clean mathematical open problems.

We choose mass, time, and length units so that the total energy $E_N = E_N(\bar{x}, \bar{v})$ is the sum of the following kinetic and potential energies of the system:

$$\begin{aligned} E_N^{kin} &= E_N^{kin}(\bar{v}) = \sum_{j=1}^{N} v_j^2/2, \\ E_N^{pot} &= E_N^{pot}(\bar{x}) = \sum_{j,k=1}^{N} \phi(|x_j - x_k|). \end{aligned} \tag{4}$$

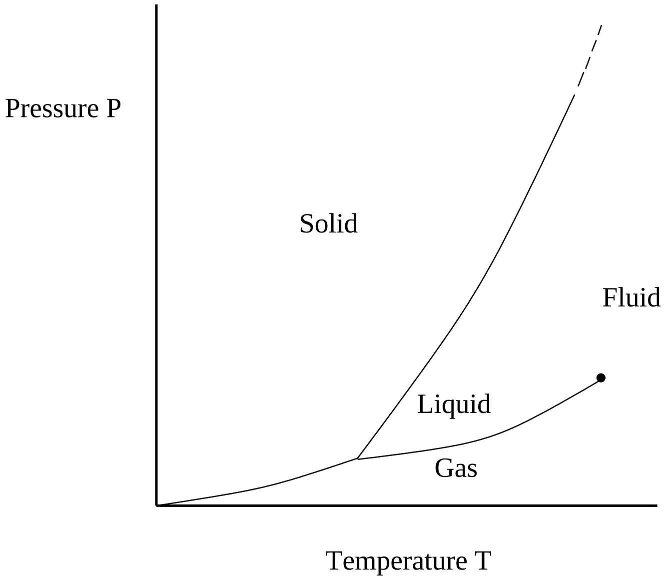

Figure 3. Given an interaction potential ϕ, the model determines a so-called phase diagram showing the separation between phases in terms of temperature and pressure.

Note that the only adjustable element of the model is the interaction (1), which is used in the potential energy $E_N^{pot}(\bar{x})$, part of the total energy $E_N(\bar{x}, \bar{v})$, and that our choice of (1) makes sense in any dimension d. There turns out to be profound differences with dimension in the predictions of the model, which we discuss after introducing a new twist.

Recall that we have used a *cube* B_V of volume V as a container of the particles. We did not emphasize the shape because it has been proven that the predictions of bulk properties from such models, obtained from asymptotically large systems, are in fact independent of the shape of the container. But we are interested in the boundary shape of our water drop, which is *not* a bulk property, i.e. not proportional to N. With this in mind it will be useful to consider containers with a variety of shapes and introduce a new variable S, the fixed shape of our variable-volume container; we normalize S to have unit volume. So now our (P, T) ensemble also depends on S.

Trying to model the surface shape of a water drop adds another complication: the environment of the drop, oil in this case. Clearly the features of the water/oil contact are very relevant. If the oil is replaced by butyl alcohol, in which water dissolves to a noticable extent, the notion of interface is complicated by having a significant fraction of molecules of alcohol in the water and a significant fraction of water molecules in the alcohol. To get a simpler example we need something like motor oil, in which water is immiscible and which produces a very sharply defined interface between the water and oil. By modelling the oil as we have done the interface cannot affect the energy of the water molecules, except perhaps by the shape of the interface, which is very useful, as we will see. We recall from (4) that the average total energy, $\langle E_N \rangle_{P,T}$, has both kinetic and potential portions:

$$\langle E_N \rangle_{P,T} = \langle E_N^{kin} \rangle_{P,T} + \langle E_N^{pot} \rangle_{P,T} \ , \tag{5}$$

and from (2) we see that

$$\langle E_N^{kin} \rangle_{P,T} = \frac{\int_0^\infty E_N^{kin}(\bar{v}) e^{-E_N^{kin}(\bar{v})/T} d\bar{v}}{\int_0^\infty e^{-E_N^{kin}(\bar{v})/T} d\bar{v}}, \tag{6}$$

which is not affected by the shape of the container. So to minimize the effect of the boundary the drop must attain that surface shape S which minimizes the effect on the average potential energy, $\langle E_N^{pot} \rangle_{P,T}$.

Intuitively, the presence of the boundary is felt directly only by those water molecules near the boundary; instead of being fully surrounded by water molecules, those near the boundary are "missing" some neighbors. From (1) each contribution to the potential energy is nonpositive, so the presence of a boundary *increases* the average potential energy, and minimizing its effect means choosing that shape S which minimizes $\langle E_N^{pot} \rangle_{P,T}$.

We need to confront a serious complication.

Here we need to confront a serious complication. Minimizing $\langle E_N^{pot} \rangle_{P,T}$ for finite N might produce one or more rather complicated surfaces $S_N(P, T)$; what we want is the Wulff shape $S_W(P, T)$, the $\lim_{N\to\infty} S_N(P, T)$. As we will see, the latter may even be one or more accumulation points rather than a limit, but in any case in order to prove anything one must make some useful choice of a space of surfaces, with a topology on it in which we supposedly take our limits. A natural approach would be to limit ourselves to star-shaped surfaces containing unit volume, or even convex star-shaped surfaces, which are describable by a radial coordinate as a (piecewise smooth) function of angle (point on the unit sphere). And to reduce redundancy one could take the quotient of such surfaces by Euclidean congruence. Such matters are easy. It is much harder to choose a topology in which one can prove existence and uniqueness of our limits. Our optimization problem is entering the domain of the calculus of variations but with a quantity to minimize, $\langle E_N^{pot} \rangle_{P,T}$, which has the special probabilistic features of statistical mechanics. Such a calculus has not been worked out yet.

Ignoring that thorny "technical" issue, at least for now, we note that from the formula for $\langle E_N^{pot} \rangle_{P,T}$ and from our expectation that for (P, T) in the fluid phase our probability distribution will, for large N, be concentrated on highly disordered configurations of molecules, simple symmetry considerations should then imply that the limit of the minimum of $\langle E_N^{pot} \rangle_{P,T}$ is attained at that $S = S_W(P, T)$ which minimizes the surface area for fixed volume, namely the sphere. This is in satisfying agreement with the classical intuitive picture, which ignored molecules.

The more interesting situation occurs when (P, T) is in the solid phase. We note from (2) that for fixed $P > 0$, as T

decreases to 0 the (P,T) ensemble becomes concentrated on those configurations $\bar{x}$ which minimize

$$E_N^{pot}(\bar{x}) = \sum_{j,k=1}^{N} \phi(|x_j - x_k|), \tag{7}$$

which is crystalline for our very simple (1). In more detail, if the boundary shape S in our ensemble were a sphere, those potential crystalline configurations would be oriented equiprobably in every direction; the breaking of symmetry comes about by the *minimization of* $\langle E_N^{pot}\rangle_{P,T}$ *by a surface* S *which is not rotationally symmetric.*

It is expected that this effect is retained as T increases from 0 until the point where the transition occurs to a fluid phase. The optimizing Wulff shape, $S_W(P,T)$, is presumably polyhedral in the solid phase, though none of this has been proven of course.

This is the standard picture in three dimensions: very pretty, indeed impressive, but no surprises for physicists. It does pose important challenges for mathematicians to justify all the "we expect"s, of course. But that is not the focus of this article. Rather, at this point we continue to draw a bit more from the nonrigorous theory by noting a significant difference in two dimensions from the Mermin–Wagner theorem, which proves that the solid state *cannot be crystalline in two dimensions!* In two dimensions it is not hard to show that the minimum of the potential energy from (1) is crystalline in the strict sense that the particles must sit on a triangular lattice; the Mermin–Wagner theorem shows that this orderly behavior is at least partly destroyed at any $P, T > 0$. Symmetry principles have been useful in the standard physics of three-dimensional materials but cannot be as useful here. The 2016 physics Nobel award was given for developing a picture of phases in two-dimensional systems distinguished by topological features rather than by symmetry, but it is not clear that even this would help for our simple model of fluid and solid phases in two dimensions.

Going beyond the solid phase in two dimensions, there has been little analysis of what to expect for the more complicated notion of the optimal Wulff shape $S_W(P,T)$ for a finite piece of the system in the solid phase. Determining the Wulff shape or even determining the structure of solid configurations for our model is presumably very hard. But even without solving either of these one might be able to solve the following, which would still be of real interest:

Problem: *Prove, in two dimensions, whether or not there is some region of* (P,T) *values throughout which the optimal Wulff shape* $S_W(P,T)$ *is not the circle.*

The circle is presumably the Wulff shape for sufficiently small P and large T, so a positive solution to the problem would imply that one could "see" the effect of atomic structure in two-dimensional materials at a *macroscopic* scale, as in the facets of salt grains. And we note that a positive solution might be obtainable using nonoptimal shapes, avoiding the need to prove the existence of an optimal Wulff shape, "solid phase," or other nonrigorous terms we used as a guide.

We have shown how minimizing surface tension leads to interesting physics and mathematics when analyzed in two dimensions rather than the standard three dimensions. We conclude by noting a different nontraditional path concerning Wulff shapes that also leads to interesting mathematics, namely, if one chooses the parameters (P,T) precisely on the phase transition curves in Figure 3. This is easy to understand in terms of our water drop suspended in oil. Assume fixed P higher than the gas/liquid transition curve, and slowly decrease T, starting from large T, by pulling energy out of the water, i.e. by cooling it. When T reaches the water/ice transition curve we start to change some water to ice, but the temperature does not actually change until *all* the water has become ice, and then the temperature starts to decrease again. The delicate situation where there is ice surrounded by water is reminiscent of our water drop surrounded by oil, but the conditions are subtly different. The ice/water interface is maintained by molecules moving back and forth between the water and the ice, equilibrium achieved by having the two back-and-forth rates being the same. This is very different from the water/oil interface, where we reasonably modelled the oil by an impenetrable but very flexible wall. Intuitively, the boundary between the ice and water is fuzzier, and it is harder to make sense of a "boundary" between the ice and the water.

A similar analysis can be performed for (P,T) on the interface between water vapor and liquid water. This time the typical molecular configurations are isotropic in both phases, so the gas/liquid interface should still be spherical, but again it would be fuzzy and hard to make sense of, especially near the high temperature endpoint of the transition between gas and liquid in Figure 3. This situation has been analyzed quite carefully in a model where the particles can only live on the vertices of a square lattice in the plane. In that model, the lattice gas version of the so-called ferromagnetic Ising model, there is only a gas/liquid transition, so the goal is to clarify the interface between coexisting liquid and vapor in the presence of the fuzziness. Overcoming this complication, which we gladly ignored for the water/oil situation, was a tour-de-force in the probability subfield of limit shapes.

> *There is an unlimited amount of wonderful mathematics in such settings.*

To summarize, we have considered minimization of surface tension for large collections of interacting molecules in two extreme cases: where the collection is in contact with a material with which it interacts minimally, such as water in contact with motor oil, and where it is in one of two coexisting phases, such as liquid water in contact with water vapor, where there is significant interaction. This leaves out most of chemistry, where abutting materials can do all sorts of complicated things, such as iron rusting in air. There is an unlimited amount of wonderful mathematics in such settings, which is largely unexplored *for its mathematics*, especially in two dimensions, where even the traditional guide of symmetry is missing.

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Charles Radin's training was in physics, and his research is in mathematical physics, in particular the study of the solid/fluid phase transition.

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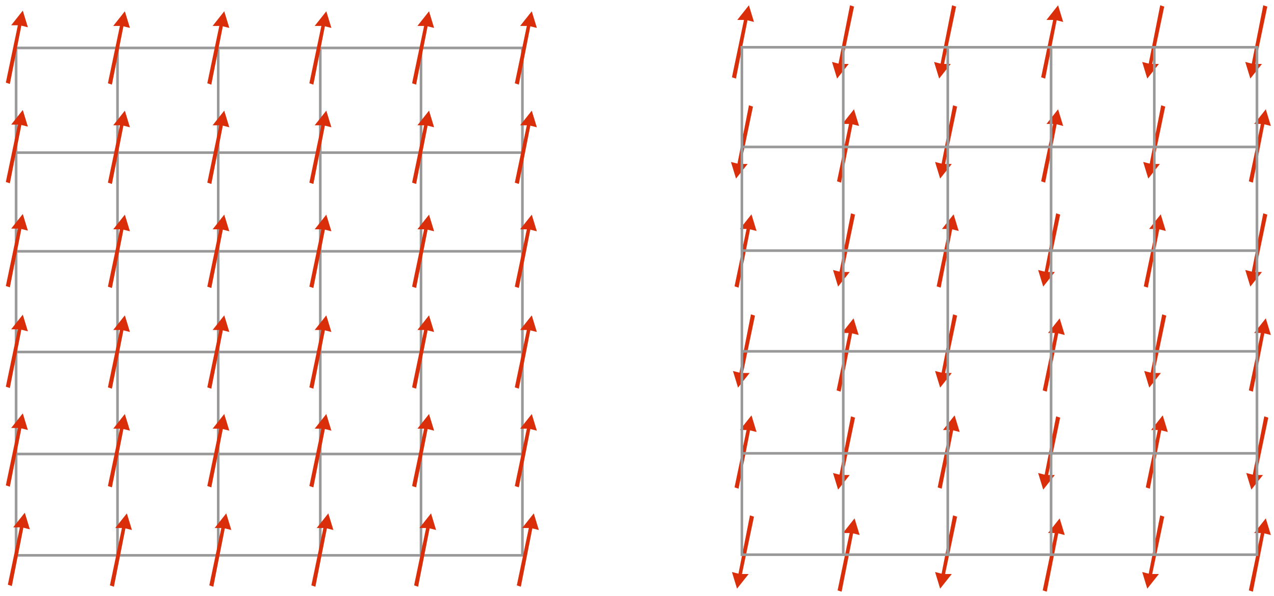

Ferromagnet

Antiferromagnet

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Introduction

In 1972 J. Michael Kosterlitz and David J. Thouless identified a completely new type of phase transition in two-dimensional systems where topological defects play a crucial role [35], [36]. Their theory applied to certain kinds of magnets and to superconducting and superfluid films, and has also been very important for understanding the quantum theory of one-dimensional systems at very low temperatures.

In the early 1980s David J. Thouless and F. Duncan M. Haldane developed theoretical methods to describe phases of matter that cannot be identified by their pattern of symmetry breaking. In a 1982 paper, David Thouless and his collaborators Mahito Kohmoto, Peter Nightingale, and Marcel den Nijs, explained the very precise quantization of the Hall conductance in two-dimensional electron gases using topological concepts [51]. In 1983 Duncan Haldane derived a theory for spin chains that incorporated effects of topology in a crucial way [21], [23]. Based on this he predicted that chains with integer and half-integer spins should be qualitatively different, and this totally unexpected effect was later confirmed by experiments.

In the following, we shall first provide some background material to put the achievement of this year's Laureates in context, and then describe the discoveries themselves.

Background

Crystalline solids are a very important class of materials in which the atoms are arranged in periodic patterns. These patterns can be classified by their symmetries; the science of crystallography, based on observations of macroscopic crystals, predated the X-ray diffraction studies that allowed the positions of the atoms to be mapped in detail. The latter was a revolutionary development that was rewarded with two consecutive Nobel Prizes, in 1914 to Max von Laue and in 1915 to William and Lawrence Bragg.

When a liquid solidifies into a crystal, it changes from a phase which is, on macroscopic scales, invariant under both translations and rotations, to a phase where these continuous symmetries are broken down to a finite symmetry group characteristic of the crystal. Another example of such a *phase transition* occurs when a ferromagnet is cooled below the Curie temperature, and the atomic magnetic moments, or the spins, line up and give rise to a net magnetization. This is illustrated in Figure 1.

The study of magnetism has been very important for our understanding of the role of symmetry in physics. Using new experimental techniques, hidden patterns of symmetry were discovered. For example, there are magnetic materials where the moments form a chequerboard pattern where the neighbouring moments are anti-parallel, see Figure 1.

In spite of not having any net magnetization, such *antiferromagnets* are nevertheless ordered states, and the pattern of microscopic spins can be revealed by neutron scattering. The antiferromagnetic order can again be understood in terms of the associated symmetry breaking.

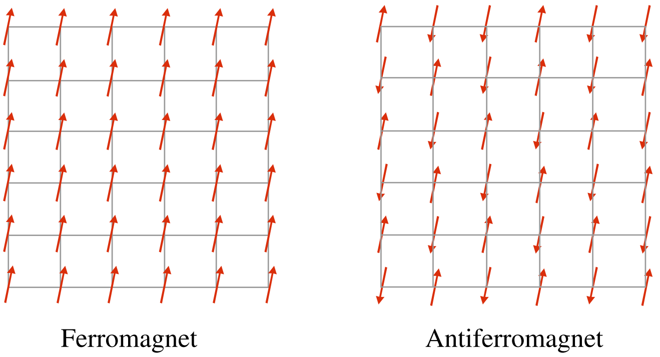

Figure 1. Schematic picture of ferro- and antiferromagnets. The chequerboard pattern in the antiferromagnet is called a Néel state.

In a mathematical description of ferromagnetism, the important variable is the magnetization, $\vec{m}_i = \mu \vec{S}_i$, where μ is the magnetic moment and $\vec{S}_i$ the spin on site i. In an *ordered phase*, the average value of all the spins is different from zero, $\langle \vec{m}_i \rangle \neq 0$. The magnetization is an example of an *order parameter*, which is a quantity that has a nonzero average in the ordered phase. In a crystal it is natural to think of the sites as just the atomic positions, but more generally one can define "block spins" which are averages of spins on many neighbouring atoms. The "renormalization group" techniques used to understand the theory of such aggregate spins are crucial for understanding phase transitions, and resulted in a Nobel Prize for Ken Wilson in 1982.

It is instructive to consider a simple model, introduced by Heisenberg, that describes both ferro- and antiferromagnets. The Hamiltonian is

$$(1) \qquad H_F = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - \mu \sum_i \vec{B} \cdot \vec{S}_i$$

where the spins are defined on lattice sites i and $\langle ij \rangle$ denotes nearest neighbours. The constant J determines the strength of the magnetic interaction, and μ is the magnetic moment of the atoms. If $J > 0$ the energy is lowest when all the spins are aligned in the direction of the external magnetic field $\vec{B}$, which explicitly breaks the symmetry under rotations and singles out a direction. When the magnetic field is zero, the spins are still aligned, but in an arbitrary direction. So in spite of the model being isotropic, the lowest energy state is not—the rotational symmetry is *spontaneously broken*.

Taking $J < 0$ favors the "chequerboard" *Néel state*, named after Louis Néel (Nobel Prize 1970), shown in the right panel of Figure 1, so in this case (1) describes an antiferromagnet where the order parameter is the *staggered magnetization*, which is defined so that it is constant in the Néel state. In a chain, this amounts to defining $\vec{m}_i = \mu(-1)^i \vec{S}_i$ where the integer i numbers the sites on the chain as illustrated in Figure 2.

Note that in spite of ferromagnetism and antiferromagnetism being quantum mechanical effects at the atomic level, they can nevertheless be described by the classical model in (1).

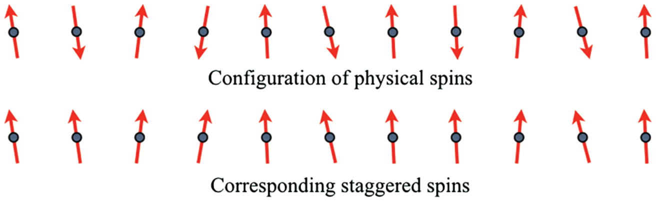

Figure 2. Illustration of the concept of staggered magnetization. The upper figure shows the physical spins in an antiferromagnetic state, while the equivalent staggered spins in the lower figure are ferromagnetically ordered.

However, it was realized rather early on that at low temperatures there are macroscopic effects, both in liquid helium and in ordinary metals, that can not be understood in terms of classical physics. The discovery of superconductivity by Kamerlingh Onnes (Nobel Prize 1913) and of superfluid helium II by Pyotr Kapitsa (Nobel Prize 1978) firmly established the existence of superfluid phases of matter. The common feature of these phases is "condensation" which is most simply understood in a gas of noninteracting bosons. Here one can show that below the *Bose–Einstein condensation* temperature, a macroscopic number of particles will populate the lowest quantum mechanical energy eigenstate, and these particles form a *condensate*. The case of the superconductor is somewhat more complicated. What condenses here are bosonic *Cooper pairs* of electrons, as explained by the BCS theory of superconductivity (Nobel Prize 1972 to John Bardeen, Leon Cooper and John Schrieffer).

These condensates can also be described by an order parameter which, in a sense, can be thought of as a "macroscopic wave function," ψ, for the bosons, or for the Cooper pairs. In 1950, well before the advent of the microscopic BCS theory, Ginzburg and Landau proposed a theory for the order parameter that describes the phase transitions between the normal and superconducting phases, and some ten years later, the corresponding theory for the normal to superfluid transition in a gas of bosons, was given by Gross and Pitaevskii.

Although the order parameter ψ for a superfluid is a classical variable, it differs in a crucial way from the order parameter $\vec{m}$ for a magnet, in that it is a complex number, where the phase of this complex number is a memory of its quantum mechanical origin. A phase ordered state, or a condensate, now amounts to having $\langle\psi\rangle \neq 0$ which means that the phase is constant, or slowly varying, in the whole system. This property is often referred to as *phase rigidity*. Since the phase of a quantum mechanical wave function is related to currents, variations in the phase of the order parameters correspond to "supercurrents" that flow without any resistance.

An important part of the Ginzburg–Landau (GL) theory is the potential function for the order parameter

$$V(\rho) = -\frac{\mu}{2}|\psi|^2 + \frac{\lambda}{4}|\psi|^4, \tag{2}$$

where μ is the chemical potential, or the cost of creating a Cooper pair, and where λ is the strength of the short range repulsion between the pairs. In GL theory these are both phenomenological parameters. Depending on whether μ is negative or positive, the potential will have a minimum either at $|\psi| = 0$ or at $|\psi| = \sqrt{\mu/\lambda}$. Thus, if μ depends on the temperature such that it becomes positive below some *critical temperature* T_c, this will correspond to a phase transition into a superconducting state.

A very important insight about superconductors was due to Abrikosov, who studied in detail the response of a GL superconductor to a magnetic field. It was already known from the work of Meissner and Ochsenfeld that superconductivity does not mix well with magnetism. When a material such as lead is placed in a magnetic field, and then is cooled below T_c, the magnetic field is expelled from the bulk of the material and only penetrates a thin region of depth λ_L close to the surface. This is called the *Meissner effect*, and the *London length* λ_L is typically 50–500 nm. A superconductor with this response to magnetic fields is said to be of type I. What Abrikosov found was that not all superconductors act like this. In a "type II" superconductor a sufficiently strong field will penetrate the (still superconducting) material, but not as a homogeneous field; instead it is in the form of *Abrikosov vortices*, which are thin magnetic flux tubes with a diameter $\sim \lambda_L$. Only weak magnetic fields are totally expelled and a full Meissner effect is recovered. Alexei Abrikosov, Vitaly Ginzburg and Anthony Leggett shared the 2003 Nobel Prize for their work on superconductivity and superfluidity.

There is a striking similarity between a two-dimensional magnet and certain superconducting or superfluid films. The magnetization is a vector that normally can point in any direction but in certain magnets, the spins are constrained to lie in a plane, say the xy-plane, where they are free to rotate. In such an "easy-plane" magnet the direction of the magnetization is determined by a single angle, θ denoting the rotation around the z-axis. It will be important in the following that there are configurations of such planar, or XY, spins that are *topologically distinct*. This is illustrated in the left panel of Figure 3, which shows a vortex configuration. The vortex is a *topological*

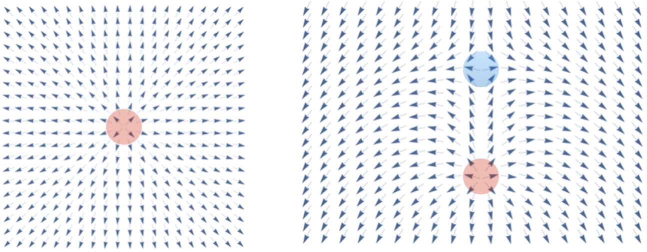

Figure 3. To the left a single vortex configuration, and to the right a vortex-antivortex pair. The angle θ is shown as the direction of the arrows, and the cores of the vortex and antivortex are shaded in red and blue respectively. Note how the arrows rotate as you follow a contour around a vortex.

defect that cannot be transformed into the ground state where all the spins are aligned, by a continuous rotations of the spins.[1]

The right panel shows a vortex anti-vortex configuration, which *can* be smoothly transformed to the ground state. In the next section we shall expand on this and classify the configurations by their vorticity, which is a *topological invariant*.

The complex order parameter for a superconductor or superfluid can be expressed as, $\psi = \sqrt{\rho_s}\, e^{i\theta}$, where ρ_s is the *superfluid density* and θ the phase. In a superfluid state, the important thermal fluctuations are only in the phase and are hence again described by a single angle, θ, just as in the easy plane magnet. An important concept in the general theory of phase transitions is that of *universality class*. What determines the universality class of a transition is the dimension of the system and the nature of the order parameter. Since the planar magnet and a superfluid both have order parameters described by a single angle, they belong to the same class and can be described by the same effective theory.

A simple model that describes both these systems is the XY-model defined by the Hamiltonian,

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) \tag{3}$$

where $\langle ij \rangle$ again denotes nearest neighbours and the angular variables, $0 \leq \theta_i < 2\pi$ can denote either the direction of an XY-spin or the phase of a superfluid. We shall discuss this model in some detail below.

Although the GL and BCS theories were very successful in describing many aspects of superconductors, as were the theories developed by Lev Landau (Nobel Prize 1962), Nikolay Bogoliubov, Richard Feynman, Lars Onsager and others for the Bose superfluids, not everything fit neatly into the Landau paradigm of order parameters and spontaneous symmetry breaking. Problems occur in low-dimensional systems, such as thin films or thin wires. Here, the thermal fluctuations become much more important and often prevent ordering even at zero temperature. The exact result of interest here is due to Wegner, who showed that there cannot be any spontaneous symmetry breaking in the XY-model at finite temperature.

So far we have discussed phenomena that can be understood using classical concepts, at least as long as one accepts that superfluids are characterised by a complex phase. There are however important macroscopic phenomena that cannot be explained without using quantum mechanics. To find the ground state of a quantum many-body problem is usually very difficult, but there are some important examples where solutions to simplified problems give deep physical insights. Electromagnetic response in crystalline materials is an example that is of central importance for this year's Nobel Prize.

That certain crystals are metals, and others are insulators can often be understood from the solution of the Schrödinger equation for a single electron in a periodic lattice potential. The crucial simplification neglects the electron-electron interaction, so that the ground state is obtained simply by filling the lowest energy levels. The important result is that the energy spectrum is not continuous, but forms bands of allowed energies with forbidden gaps in between. In an insulator, the itinerant electrons completely fill a number of bands and it takes significant energy to generate a current. In a metal, the highest populated band is only partially filled, and there are low energy excitations that allow for conduction.

> *Band theory was much richer than expected.*

This simple picture was challenged by the 1980 discovery by Klaus von Klitzing (Nobel Prize 1985) of the integer quantum Hall effect. This was the first discovery of a topological quantum liquid, with many more to come, and it demonstrated that band theory was much richer than expected.

The classical example of a phase transition is a system going from a disordered phase to an ordered phase as the temperature is lowered below a critical value. More recently, the phase transition concept has been extended to quantum systems at zero temperature. A quantum system can undergo a radical change of its ground-state as a parameter in its Hamiltonian, such as pressure, magnetic field or impurity concentration, is tuned through a critical value, and such a *quantum phase transition* signals the change from one state of matter to another. This insight has provided an important link between statistical mechanics, quantum many-body physics and high energy physics, and these fields now share a large body of theoretical techniques and results.

The Kosterlitz–Thouless Phase Transition

We already mentioned that the thermal fluctuations prevent ordering of XY-spins in two dimensions. A more precise statement is based on the large distance behaviour of the spin-spin correlation function. In three dimensions a calculation gives,

$$\lim_{r\to\infty} \langle e^{i(\theta(\vec{r})-\theta(\vec{0}))} \rangle = \begin{cases} c_1 & T < T_c \\ c_2\, e^{-r/\xi} & T > T_c \end{cases}$$

where ξ is the correlation length, $r = |\vec{r}|$, and c_1 and c_2 are constants. At precisely the critical temperature $T = T_c$ the correlation falls as a power *i.e.* $\sim r^{-(1+\eta)}$, signalling a *critical behaviour*. The constant η is an example of a critical exponent which is characteristic of a *universality class*, which can encompass many different systems that all behave in a similar way close to the phase transition.

To see what happens in two dimensions, we take the continuum limit of the Hamiltonian (3), to get

$$H_{XY} = \frac{J}{2} \int d^2 r\, (\vec{\nabla}\theta(\vec{r}))^2 . \tag{4}$$

---

[1] *We assume that the lattice spacing is small so it makes sense to use a continuum description, where the lattice site is replaced by a position $\vec{r}$. With a continuous rotation we mean a transformation $\vec{S}(\vec{r}) \to \vec{S}'(\vec{r}) = \mathcal{R}(\vec{r})\vec{S}(\vec{r})$ where the matrix $\mathcal{R}(\vec{r})$ is a continuous function of $\vec{r}$.*

A simplification is to extend the range of the angular variable to $-\infty < \theta < \infty$ to get a free field Hamiltonian and thus Gaussian fluctuations, and a direct calculation using a short distance cutoff a gives

$$\langle e^{i(\theta(\vec{r})-\theta(\vec{0}))}\rangle \sim \left(\frac{a}{r}\right)^{\frac{k_B T}{2\pi J}} . \tag{5}$$

This is a power law even at high temperatures, where an exponential fall-off would be expected. Kosterlitz and Thouless [35], [36] resolved the apparent paradox by showing that there is indeed a finite temperature phase transition, but of a new and unexpected nature where the vortex configurations play an essential role. One year before the work of Kosterlitz and Thouless, Vadim Berezinskii (died in 1980) also recognized the importance of vortex excitations in the XY-model. He understood that they could drive a phase transition, but did not correctly describe the nature of this finite temperature transition, which we therefore will refer to as the "KT-transition".

The glitch in the argument leading to (5) is that the periodic, or U(1), nature of θ cannot be ignored, since that amounts to neglecting vortex configurations. A vortex like the one in Figure 3 is characterised by a nonzero value of the vorticity,

$$\nu = \frac{1}{2\pi}\oint_C d\vec{r}\cdot\vec{\nabla}\theta(\vec{r}) \tag{6}$$

where C is any curve enclosing the centre position of the vortex. The integral measures the total rotation of the spin vector along the curve, so after dividing with 2π, ν is simply the number of full turns it makes when circling the vortex. From this, we also understand that there can also be antivortices, where the spin rotates in the opposite direction as seen in the right panel in Figure 3. For a rotationally symmetrical vortex with $\nu = \pm 1$ it follows from (6) that $|\vec{\nabla}\theta(\vec{r})| = 1/r$, so the energy cost for a single vortex becomes,

$$E_\nu = \frac{J}{2}\int d^2r\left(\frac{1}{r}\right)^2 = J\pi\ln\frac{L}{a} \tag{7}$$

where L is the size of the system, and a a short distance cutoff that can be thought of as the size of the vortex core. So for a large system, the energy cost for a single vortex is very large, and cannot be excited by thermal fluctuations. This seems to imply that vortices can be neglected, but we shall see that this is not the case.

We can understand the essence of this new type of *topological phase transition* by a quite simple thermodynamic argument. Although the energy of a single vortex diverges as $\ln L$, this is not true for vortex-antivortex pairs since they have zero total vorticity. The energy required to create such a pair is $J2\pi\ln r/a$ where r is the separation between the vortices. Such pairs can thus be thermally excited, and the low temperature phase will host a gas of such pairs. The insight by Kosterlitz and Thouless was that at a certain temperature T_{KT} the pairs will break up into individual vortices. It is this vortex pair unbinding transition that will take the system to a high temperature phase with exponentially decaying correlations.

The vortices and anti-vortices act as if they were two point particles with charges +1 and -1 interacting with a $1/r$ force. Since this corresponds to the Coulomb interaction in two dimensions, the physics of the topological defects is just like the physics of a two-dimensional neutral Coulomb gas. Thouless' and Kosterlitz's heuristic entropy-energy balance argument for the unbinding transition is as follows: The free energy for a single vortex is

$$F = E - TS = J\pi\ln\left(\frac{L}{a}\right) - Tk_B\ln\left(\frac{L^2}{a^2}\right) \tag{8}$$

where k_B is Boltzmann's constant, and where the entropy is calculated assuming that there are L^2/a^2 possible positions for a vortex with area a^2. At the critical temperature $T_{KT} = J\pi/2k_B$ the energy exactly balances the entropy, so we can expect the transition to a phase of free vortices.

In contrast to usual continuous phase transitions, the KT-transition does not break any symmetry, something that was completely new and unexpected. In their 1973 paper [36] Kosterlitz and Thouless both explained the physics behind the transition and fully recognized its importance. The next important contribution to the theory was Kosterlitz' derivation of the *Kosterlitz renormalization group equations* and his analysis of the associated flow.

completely new and unexpected

The Kosterlitz–Thouless topological model of a phase transition in two dimensions has been used to explain experiments with many different types of physical systems. Examples include: very thin films of superfluid ^{4}He that form naturally on a solid substrate , disordered thin films of superconductors, artificial planar arrays of superconducting tunnel junctions and wire networks, granular films of superconductors, melting of two-dimensional solids.

An important idea due to Nelson and Kosterlitz that is used in the analysis of superfluid and superconducting films is that of a "universal jump" in the superfluid density, which occurs at the critical temperature of the phase transition. At the critical temperature, the superfluid density jumps from zero to a universal value predicted by the Kosterlitz–Thouless theory. Bishop and Reppy [**?**] measured this jump by monitoring the resonant frequency and quality factor of a torsional oscillator with a large spiral surface of Mylar plastic. When the system is cooled below the critical temperature, a thin superfluid film forms on the Mylar substrate and the added mass of this film causes a sudden jump in the period of torsional oscillation. Bishop and Reppy compared their results with other experiments by Rudnick, as well as by Hallock and by Mochel, which measured the temperature dependence of the velocity of a particular "third sound" surface wave in liquid helium. See Figure 4 (following page). All results were consistent with the prediction of a universal jump at the KT-transition.

Beasley, Mooij and Orlando predicted that the KT-transition would also be visible in thin films of superconductors if they were made to be sufficiently disordered.

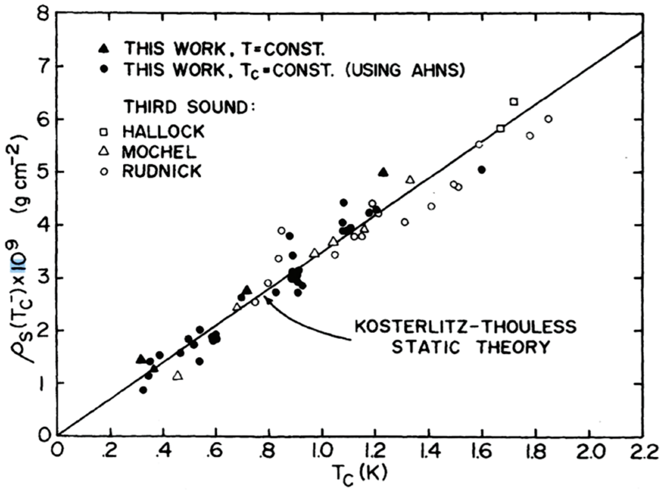

Figure 4. At the Kosterlitz–Thouless transition the superfluid density and critical temperature are predicted to have a linear relation depending only on fundamental constants $\rho(T_c) = T_c \frac{2}{\pi}\frac{m^2 k_B}{\hbar^2}$.

The jump in the density of superconducting electrons appears in analysis of the nonlinear current-voltage characteristics of the thin superconducting film as it is cooled through the superconducting transition. Early experiments on thin disordered superconducting films were consistent with this picture. Some of the most elegant experiments and analysis of superconducting systems detect the phase transition as a change in the complex impedance of the two-dimensional superconductor as it is cooled through the KT-transition[28]. See Figure 5.

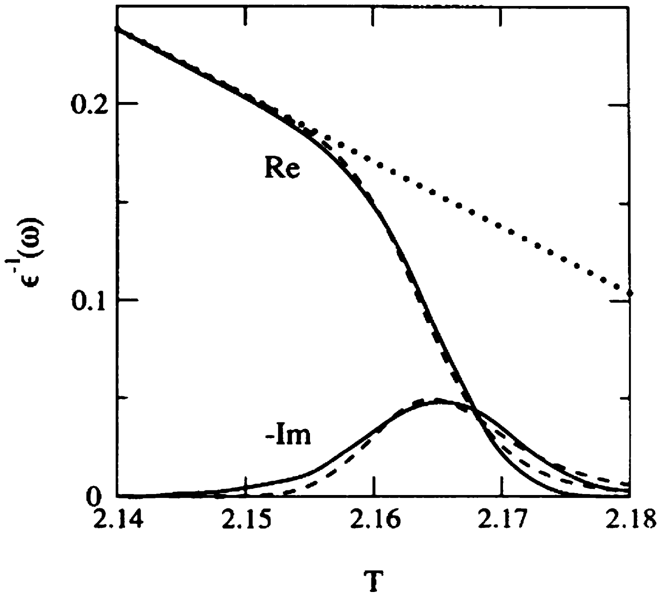

Figure 5. Solid lines: The measured real and imaginary parts of the linear response an AC circuit containing a two-dimensional wire network of superconductors as it is cooled through the Kosterlitz–Thouless transition (data from [28]). Dashed lines: The theory of Minnhagen is fit to the data. A narrow temperature range reveals a drop in the superfluid density $\sim$ $\mathrm{Re}(\epsilon(\omega))$ and a peak in the disssipation $\sim$ $\mathrm{Im}(\epsilon(\omega))$ caused by vortex-unbinding around the transition temperature.

Quantum Hall Conductance and Topological Band Theory

The discovery of the integer quantum Hall effect was a milestone in the understanding of the phases of matter. The Hall conductance in a very clean (mobility about $10^5 cm^2/Vs$) two-dimensional electron gas cooled below 2 K and subjected to a perpendicular magnetic field of the order of 15 T, was observed to obey the relation

$$\sigma_H = n\frac{e^2}{h}, \tag{9}$$

where n is an integer, to a precision of less than one part in 10^9. This finding was simply astonishing, given the much larger relative variations in magnetic field, impurity concentration and temperature.[2]

Using an ingenious thought experiment, Robert Laughlin (Nobel Prize for the fractional quantum Hall effect in 1998) gave an argument based on gauge invariance to explain the exactness of the Hall conductance. However, deeper understanding of the Hall response in real crystalline materials was missing, and a straightforward application of Laughlin's reasoning led to an apparent paradox. In resolving these problems, Thouless *et al.* derived a new formula for σ_H that turned out to have far reaching consequences [51]. Here we present a much simplified version of their argument that stresses the relevance of topology.

If we neglect electron–electron interactions, and replace the lattice potential with a smeared-out positive background charge, we only have charged particles moving independently in a magnetic field. This problem was solved by Landau, who showed that the energy is quantized as $E_n = (n + \frac{1}{2})\hbar\omega_c$ with the cyclotron frequency $\omega_c = eB/m$. Each such "Landau level" has a macroscopic degeneracy such that there is one quantum state for every unit $\phi_0 = h/e = 2\pi/e$ of magnetic flux. There are many ways to label the degenerate states and a convenient one for our purposes is to use a crystal momentum. The reason for this is that even in a homogenous magnetic field the symmetry under translations is broken, and the relevant symmetry group is that of finite "magnetic translations". These translations span a lattice such that there is a unit magnetic flux through each lattice cell. The particular expressions for the magnetic translation operators are not important for our discussion, nor is the precise choice of the lattice. What is important, however, is that just as for an ordinary crystal lattice, we can label the eigenfunctions in a band, here the Landau level, with a vector in reciprocal space. We say that the crystal

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[2] *The measurement is so precise that the Klitzing constant,* $R_K = h/e^2 = 25812.807557(18)\Omega$ *is now the definition of electrical resistance.*

momentum, $\vec{k}$, takes values in the first Brillouin zone, and we denote the wave functions in the n^{th} Landau level by $u_{\vec{k},n}(\vec{r})$.

It is now an exercise in linear response theory to derive an expression for the conductance in terms of the wave functions; the final formula becomes

$$\sigma_H = \frac{e^2}{2\pi h}\sum_n \int_{\vec{k}\in BZ} d^2k\,\mathcal{B}(\vec{k},n) \tag{10}$$

where the *Berry field strength*, $\mathcal{B}$ can be calculated from the *Berry potential*, which is given in terms of the wave functions by

$$\mathcal{A}_j(\vec{k},n) = i\langle u_{\vec{k},n}|\partial_{k_j}|u_{\vec{k},n}\rangle\,. \tag{11}$$

Just as in ordinary electromagnetism, the field strength is related to the vector potential by,

$$\mathcal{B}(\vec{k},n) = \partial_{k_x}\mathcal{A}_y(\vec{k},n) - \partial_{k_y}\mathcal{A}_x(\vec{k},n)\,. \tag{12}$$

The integral in (10) is of a (fictitious) magnetic field over a closed surface, so, in analogy with the case of a magnetic monopole in ordinary electromagnetism, it is expected to be quantized. A detailed analysis in fact shows that,

$$\frac{1}{2\pi}\int_{Bz}\mathcal{B}(\vec{k},n) = C_1(n)\,, \tag{13}$$

where the first *Chern number*, C_1, is known from the mathematics of fibre bundles to be an integer. This explains why the conductance is quantized, and why it is insensitive to perturbations such as disorder or interactions between particles — the Chern number is a *topological invariant*. It can only be an integer! Using the explicit wave functions for the Landau problem, one can explicitly calculate the integrals in (10), and show that $C_1 = 1$ for any filled Landau level.

The original paper by Thouless *et al.* also included the effect of a lattice potential. The calculations become more complicated, since in that case the Landau levels split into subbands, but the result is the same—the conductance is still given by the formula (9). In later papers, the relation to the mathematics of fibre bundles was established, and Niu, Thouless, and Wu gave a derivation that also applied to systems with impurities.

The great importance of the Thouless *et al.* result is that it opens up the possibility of having a Hall conductance even in the absence of a magnetic field. It would, however, be another six years before this conceptually very important step was taken by Haldane, who realized that the important thing is to break the invariance under time-reversal and have an energy band, which does not have to be a Landau level, with a nonzero Chern number. He considered a tight-binding model of fermions on a hexagonal lattice and introduced hopping between both nearest and next-nearest neighbours.

In a lattice model, magnetic flux is incorporated by making the hopping matrix elements complex, and Haldane picked the phases to give fluxes with alternating signs giving zero flux in each unit cell. This configuration still breaks the invariance under time reversal, which is necessary to have a Hall effect. This phase of matter described by Haldane is now called a *Chern insulator*, and twenty-five years later, in 2013, a quantized Hall effect was observed in thin films of Cr-doped (Bi,Sb)$_2$Te$_3$ at zero magnetic field, thus providing the first experimental detection of this phase of matter. In Figure 6 we see a clear plateau in the Hall resistance ρ_{yx} at a density (regulated by the gate voltage) corresponding to a filled band. The later development of *topological band theory* will be discussed in the concluding section.

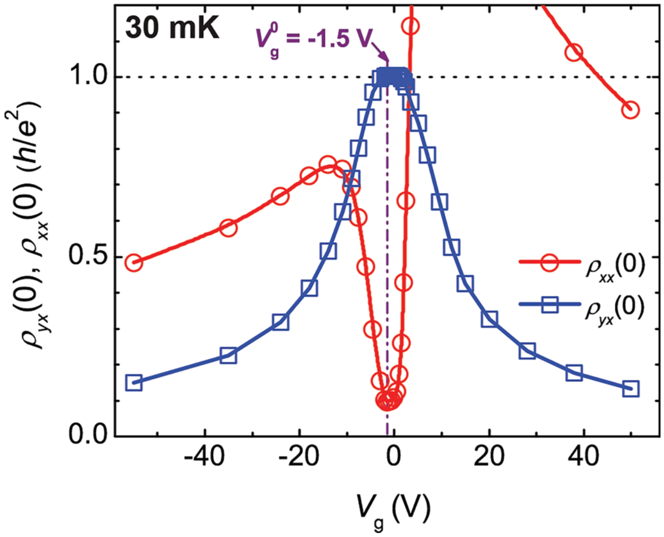

Figure 6. The blue curve shows the Hall resistance ρ_{yx} as a function of the gate voltage at zero magnetic field. Note the plateau at $V_g = 0$, which is the point corresponding to a filled band.

Quantum Spin Chains and Symmetry-Protected Topological Phases of Matter

One dimensional systems, such as spin chains, or electrons moving in thin wires, are radically different from their relatives in higher dimensions. The reason for this is that both thermal and quantum fluctuations are much more important and prevent most of the symmetry-breaking patterns that characterise phases in higher dimension. A lot of important work in the 1960s and 1970s had established quite a complete and coherent picture of both quantum and classical one-dimensional systems. In the quantum case there are various transformations, both in the continuum and on the lattice, that map seemingly very different systems into each other. An example is the Jordan–Wigner transformation that maps the Heisenberg chain of spin 1/2's to (spin less) lattice fermions, with nearest neighbour interactions. Since the spin chain can be solved exactly using the Bethe ansatz techniques, this also provides a solution for the fermion model, which is but an example of the many cross-connections between different one-dimensional models.

The antiferromagnetic Heisenberg chain, illustrated in Figure 2, is for spin 1/2 described by (1) with $\vec{S}_i = \frac{\hbar}{2}\vec{\sigma}_i$ (with $\vec{\sigma}$ the Pauli matrices), and the Bethe ansatz solution shows that it is gapless. Although there were no proofs, it was commonly believed that the same would be true for Heisenberg chains of higher spins. In two papers from 1983 [21], [23], Haldane applied

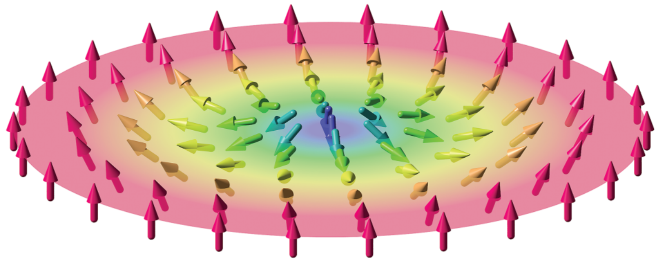

Figure 7. A field configuration with winding number $Q = 1$. Notice that no continuous change in the spin directions can transform this to a configuration where all spins point upwards.

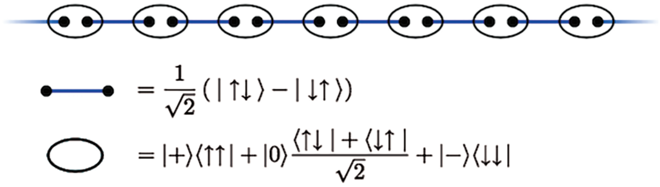

Figure 8. Graphical representation of the AKLT ground state. The black dots denote auxiliary spin 1/2 "particles," the ovals project on spin 1, and the lines mean that two spins 1/2 form a singlet state. Since two of the four spin 1/2 on two adjacent sites form a singlet, the maximal total spin is 1, so the projection on spin 2 gives zero. Since this is true for all pairs, this state is clearly an eigenstate of H_{AKLT}. Also note the unpaired spins at the end of the chain, which are the fractionalized edge modes.

new mathematical techniques to the problem, revealing a fundamental difference between chains with integer and half integer spins, leading him to the famous "Haldane conjecture" that half-integer chains are gapless while the integer ones are gapped.

The key idea was to derive an effective model that describes the low momentum excitations.[3]

Assuming the spins to be large, Haldane derived the following action integral for the continuum limit of the antiferromagnetic spin chain,

$$S_{NLS} = \frac{1}{2g}\int dt dx \left(\frac{1}{v}(\partial_t \vec{n})^2 - v(\partial_x \vec{n})^2\right) \tag{14}$$

where $\vec{n}(\vec{x}, t)$ is a unit vector describing the slowly varying part of the staggered spin field, v is the spin wave velocity and $g = 2/S$ the coupling constant. This is the $O(3)$ nonlinear sigma model, which at the time was already well understood. Although naively the theory has no mass, it was known that because of strong quantum fluctuations a mass scale is dynamically generated[4] so, according to this line of argument, all spin chains should be gapped. This is in apparent contradiction to the spin 1/2 chain being gapless. Haldane pointed out that there are large fluctuations that contribute very differently depending on the value of the spin.

One way to understand this difference is to notice that a direct derivation of the action integral will, in addition to (14), also give a *topological θ-term*

$$S_{top} = i\frac{\theta}{4\pi}\int d^2x\, \vec{n} \cdot (\partial_1 \vec{n} \times \partial_2 \vec{n})\,, \tag{15}$$

where $\theta = 2\pi S$, and where we use the Euclidean space coordinates $(x^1, x^2) = (it, x)$ appropriate for a path integral treatment.[5] Although this term does not contribute to the equations of motion, it is nevertheless important. To see this we first notice that, for any smooth field configuration, the *winding number*

$$Q = \frac{1}{4\pi}\int d^2x\, \vec{n} \cdot (\partial_1 \vec{n} \times \partial_2 \vec{n})\,, \tag{16}$$

is an integer. The geometric significance of this winding number is explained in Figure 7.

In a path integral where one sums over all possible spin configurations to calculate various quantities, such as the partition function,

$$Z(g) = \int \mathcal{D}[\vec{n}]\, e^{-(S_{NLS}+S_{top})} \tag{17}$$

there will be a phase factor $e^{i2\pi SQ}$. It follows that for integer spins this factor is always 1, and we conclude that the chain is gapped. For half-integer chains the problem is much more complicated. The large fluctuations responsible for generating the mass gap typically have nonzero winding numbers and, because of the sign, $(-1)^Q$, there may be important cancellations. Thus, although the argument based on the behaviour of the sigma model works for the integer spin chains, it breaks down in the half-integer case. This observation, together with the spin 1/2 chain being gapless, provides a motivation for the Haldane conjecture. Note that the most surprising result—that the integer spin chain is gapped—is natural in the language of the sigma model, while it was harder to understand what happens in the half-integer case. Only later was it proven that the $\theta = \pi$ sigma model really is gapless.

We now complement the above rather abstract argument, which also relied on the assumption of large S, with a description of a very instructive and exactly solvable model for $S = 1$, which is a close cousin of the Heisenberg model in (1). The Hamiltonian for this AKLT chain, named

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[3] *How this is done in detail is well explained in the textbooks by Auerbach and Fradkin.*

[4] *Note that large S corresponds to a small coupling g, which effectively suppresses the contributions to the path integral from configurations with large fluctuations. A renormalization group analysis shows that the theory is asymptotically free, meaning that the coupling constant grows at small momenta, very similar to QCD. It is these strong quantum fluctuations that drive the mass generation.*

[5] *In the original work* [23] *another line of reasoning, also employing topological concepts, was used to reach the same conclusion.*

For the derivation of the θ-term and references to the original papers, see [19].

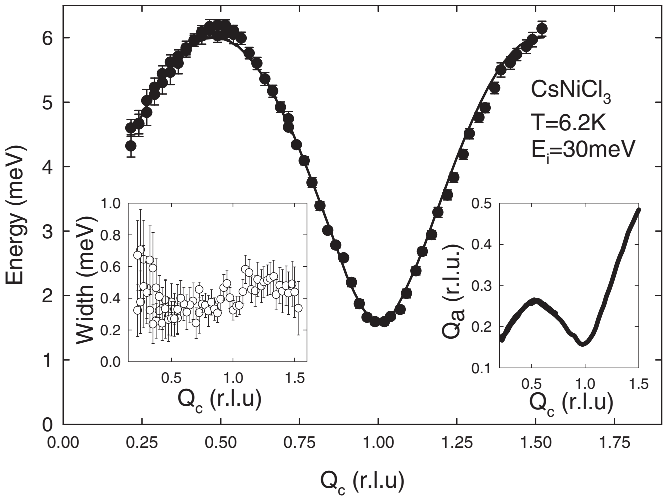

Figure 9. The graph in the middle shows the energy of a spin excitation in a spin 1 chain as a function of momenta close to the Néel point $Q_c = 1$, which corresponds to a π phase difference between the Ni spins along the chains; the Haldane gap is clearly visible.

after its inventors Ian Affleck, Tom Kennedy, Elliott Lieb and Hal Tasaki, is

(18)

$$H_{AKLT} = \sum_i \left[\frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{6} \left(\vec{S}_i \cdot \vec{S}_{i+1} \right)^2 + \frac{1}{3} \right] = \sum_i P_2(\vec{S}_i + \vec{S}_{i+1})$$

where $\vec{S}_i$ is a spin 1 operator at the lattice site i, and P_2 projects on the subspace corresponding to spin 2 on two adjacent lattice sites. To find the ground state, we imagine that each link in the chain hosts two auxiliary spin 1/2 that are projected to a spin 1. As explained in Figure 8, forming a spin singlet at each link in the chain gives an eigenstate of the Hamiltonian with zero energy. Since the Hamiltonian is a sum of projectors, the ground state energy has to be nonnegative and we conclude that we have constructed a ground state of the full interacting model. From the figure we also see that there are two "unpaired" spin 1/2 degrees of freedom at the two ends of the chain, which is an example of *quantum number fractionalization*, since the original degrees of freedom were spin 1! One can show that the unpaired spins give rise to a double degeneracy of the ground state, but the most striking property of the AKLT chain is that it has a *Haldane gap*, as was shown analytically in a later article by the same authors.

The existence of the *Haldane phase* has been confirmed both by experiments and by numerical simulations. The first experiment on $CsNiCl_3$ was done by Buyers *et al.*, and in Figure 9 we show results from a more recent experiment [31].

Later work has greatly deepened our understanding of the Haldane phase of the Heisenberg antiferromagnetic chain. Although there is no local order parameter, it is sometimes possible to characterise it by a nonlocal *string order parameter* introduced earlier in the context of statistical mechanics. To define a distinct phase of matter, it is important that the characteristic properties are not destroyed by small perturbations. For the Haldane phase this was investigated in 2009 by Gu and Wen who showed that as long as certain symmetry properties are respected by the perturbations, the phase remains intact, and in 2010 the entanglement properties of the state was studied in detail by Pollmann *et al.*. Thus the Haldane phase was identified as the charter member of a class often referred to as *symmetry protected topological states*, and we shall provide some other examples below.

Some Recent Developments

In the previous sections we described the fundamental work that is rewarded with this year's Nobel Prize. We shall now try to give an impressionistic view of some of the exciting research directions and results that have followed.

What States of Matter are There?

This is an ancient question. The simple schoolbook answer is solids, liquids and gases, but we know that this simple division is far too crude. We have already mentioned different kinds of magnets and superfluids, but there are many more examples. Most real materials are in fact much more complex than homogeneous crystals, liquids or gases; plasmas, liquid crystals polymers and gels are but a few examples. In spite of this enormous variety there are many important phenomena that can be understood in the simple context of crystalline materials with impurities and lattice defects. The theory of electronic bands explains why materials are conductors, isolators or semiconductors, and how they conduct heat and respond to magnetic fields. The discovery of the quantum Hall effect, and the subsequent development of topological band theory, has opened new and unexpected vistas where deep theoretical insights have developed in parallel with search for applications in electronics and quantum information science.

The most spectacular results have been the predictions.

Perhaps the most spectacular results so far have been the predictions, and later experimental discoveries, of topological insulators in both two and three dimensions. These states of matter are, just like the Haldane chain, examples of symmetry protected topological phases, as are the topological superconductors. Another example that has recently gained a lot of attention is the "Kitaev chain," in its various incarnations. These are the simplest examples of materials harboring *Majorana modes*, which can be thought of as a fractionalized qubit. This observation has generated lots of excitement because of the possibility of topological quantum computation. A more recent, but equally fascinating set of materials are the Weyl semi metals, which were experimentally discovered as recently as 2015.

There is now a topological classification of gapped phases of free fermions in any dimension, and many efforts are being made to classify interacting phases. Important tools in this effort to enumerate and classify phases of matter are different measures of quantum entanglement, such as entanglement entropy and entanglement spectra. One distinguishes between short-range entangled states, such as the symmetry protected states, the integer quantum Hall states and the Chern insulator, and states with long range entanglement such as the fractional Hall liquids or the putative *spin liquids*. The long-range entangled states are characterised by having fractionalized excitations in the bulk, the typical example being the fractionally charged quasiparticles in the Laughlin quantum Hall liquids. The hunt for spin liquids, in both two and three dimensions, is very much at the forefront of current research, as is the attempt to realize states where the quasiparticles have nonabelian fractional statistics.

Quantum Simulations, and Artificial States of Matter

In our discussion of the KT phase transition, we stressed universality, meaning that the same model Hamiltonian describes critical phenomena in very different physical systems. This universality is however to a very small *critical region* in the vicinity of the transition temperature. There is however another strategy that allows the use of the same Hamiltonian for different systems in wider parameter ranges. The basic idea goes back to Feynman, who pointed out that one could hope to solve very hard quantum problems by designing a *quantum simulator*.

Such a simulator is itself a quantum system with many degrees of freedom, but it should be well controlled and designed to embody the important aspects of the physical system one is attempting to simulate. More precisely, this means having the correct degrees of freedom and the correct interactions. Cold atomic gases have turned out to provide a perfect platform for obtaining this. An important tool in these experiments is the optical lattices that are formed as an interference pattern by intersecting laser beams.

An example of this is the observation of the KT-transition in a layered Bose–Einstein condensate of ^{87}Ru atoms. At low temperatures one observes coherence effects characteristic of the phase with power law correlations, and at higher temperatures one sees free vortices.

We have already mentioned that the KT theory also describes the quantum phase transition in the one-dimensional XY universality class. Imaginary time provides the extra dimension, and the control parameter analogous to temperature is the ratio of two energy scales in the Hamiltonian. In this way, the KT-transition forms the basis for understanding how a one-dimensional chain of Josephson tunnel junctions undergoes a zero-temperature transition from superconducting to insulating behaviour as the Josephson coupling between junctions is tuned. The same model was later realized using ultra cold atomic gases trapped to form discrete lattices, and also here one could observe the KT-transition.

Both fermionic and bosonic atoms can be trapped in optical lattices, similarly to electrons in a crystal lattice. This also makes it possible to engineer topologically nontrivial bands, and we mention two recent examples.

In a 2014 experiment, the group led by Immanuel Bloch managed to design a lattice with topologically nontrivial bands, similar to the ones studied in the famous paper by Thouless *et al.* [51]. These bands were then populated by a gas of cold bosonic ^{87}Rb atoms, and using an intricate measuring procedure, they could experimentally determine the Chern number for the lowest band to $C_1^{exp} = 0.99(5)$.

Also in 2014, the group led by Tilman Esslinger made an experiment with cold ^{40}K atoms in an optical lattice to simulate the precise model proposed by Haldane in 1988. This shows that reality sometimes surpasses dreams. At the end of his paper Haldane wrote: "While the particular model presented here, is unlikely to be directly physically realizable, it indicates…." What he could not imagine was that 25 years later, new experimental techniques would make it possible to create an *artificial state of matter* that would indeed provide that "unlikely" realization.

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We created an online CV Training on our website, where we provide tips and examples of CVs and encourage graduate students to send us their CVs so we can all have the opportunity to review one another's.

A success story or a time where things didn't go according to plan:
Recently, we were planning on having an invited speaker who is also an alumnus of our program, and, unfortunately, he got sick the day before! Luckily our graduate students are flexible, and we rescheduled for the end of March.

What members enjoy most about the Chapter:
Probably the pizza! Just kidding. I think they enjoy the various opportunities we provide to network within our department and the time we get to spend away from research and getting to know one another on a personal level.

AUBREY KEMP

Chapter president, from Snellville, GA, attended Georgia College in Milledgeville, GA, for undergraduate degree.

How did you become interested in mathematics?

I seemed to be naturally good at it, and with time came to love it in high school. It was an automatic decision that it would be my major in college.

Where do you see yourself in ten years?

I am anticipated to graduate next year and want to teach at a university. As I am applying for jobs, I hope to have the opportunity to move somewhere in the country outside Georgia and get to know a new place and new school! In ten years, I could still be at that university or have moved to another city!

How has being the Student Chapter president assisted you with leadership training for your future?

Before being president of this chapter, I doubted myself in terms of belonging to the program. Once I had the opportunity to get to know others and work on my confidence with delegating tasks to board members, I really felt as though I appreciated my colleagues and fellow graduate students more, and I think they felt the same about me. It will help with confidence in the workplace in the future and having to adapt to new circumstances.

What are your interests?

Academically, I research in Collegiate Mathematics Education. Specifically, how students come to understand certain challenging topics in college math courses. Outside school, I love hanging out with friends, running and exercising, and being outside.

What is your strongest quality?

I think my strongest quality is making connections with people. At first I am a little shy, but once I start a conversation, I always enjoy getting to know someone and their story, and I find that people appreciate that.

Who or what motivates you to be a better person?

There are so many people in my program that motivate and challenge me academically to be the best student, teacher, and researcher I can be. Specifically, many of the board members of this chapter, other colleagues, and my advisor. I also have a wonderful family and friend group outside the school that support me in all that I do and only have words of encouragement for me. I think the biggest motivator I have is myself, though. Once I decide I want to do something, I don't let myself quit, although that thought crossed my mind when I was under a lot of stress a few years ago.

What is the best advice you can give to someone pursuing a degree in mathematics?

Obviously, mathematics is an incredibly hard field to go into, but it is even harder to stay in and complete. My biggest advice is to know that things will get hard at some point and you may do worse than you expect at some point, but mathematics is all about the struggle. What matters the most is how to bounce back from the times that you are struggling. In fact, I believe the following quote I came across the other day (unknown author) summarizes it perfectly: "Good mathematics is not about how many answers you know, it's about how you behave when you don't know."

What do you want to be remembered for?

I would love to be remembered as someone who brought people together and encouraged people to enjoy life, even if you are spending a lot of time researching, or studying, or anything else that may be time consuming. The most important thing in life in being able to do the things you enjoy doing (including mathematics).

To start your own AMS Graduate Student Chapter or for more information about the program, please email the Membership Department at **membership@ams.org**.

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What Is Inquiry-Based Learning?

Dana C. Ernst, Angie Hodge, and Stan Yoshinobu

> "*The objective in mathematics is not to obtain the highest ranking, the highest 'score,' or the highest number of prizes and awards; instead, it is to increase understanding of mathematics (both for yourself, and for your colleagues and students), and to contribute to its development and applications. For these tasks, mathematics needs all the good people it can get.*"
>
> —Terence Tao [1]

Abstract. Inquiry-based learning (IBL) can manifest itself differently in various classroom settings. Historically, IBL was most often implemented in proof-based courses. Lately it can be found in university mathematics classrooms at all levels. This paper provides an overview of IBL followed by three detailed examples of what IBL can look like in particular classroom settings.

Communicated by Harriet Pollatsek

Introduction

The main goal of this article is to address the question, "What is inquiry-based learning today?" IBL is a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups. In this article, we describe the core principles of IBL and provide three specific but representative examples of IBL classrooms from our perspective: upper-division proof-based mathematics courses, the calculus sequence, and mathematics for elementary teachers. Active, student-centered pedagogies such as IBL exist in a dynamic landscape, so describing teaching methods that are constantly evolving is a challenging and slippery task. We are still developing and trying to understand the variety of IBL methods, when and where they are applicable, and identifying best practices. Here we share some of the commonly used examples and core ideas that drive instructor decisions.

Prospective elementary school teachers at Cal Poly San Luis Obispo discuss problems and prepare to present their work.

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Thinking, experimenting, asking questions, working with ideas, and all that goes into building knowledge are what we aim for as mathematicians. We share these objectives broadly, no matter what we teach or how. Indeed, active learning strategies have gained significant endorsements in recent years, most notably by the Conference Board of the Mathematical Sciences this year [2]. Although a discussion of the evidence in support of IBL is beyond the scope of this article, much work has been done on the effectiveness of IBL. Readers wanting to learn more about this research can start with David Bressoud's overview of IBL [3] and Kogan and Laursen's work [4].

Core Principles and the Big Tent

IBL is a form of active learning that comes in many shapes and sizes. Common to these many forms of IBL are two principles, the "twin pillars" that education research [5] has shown to be at the core of most implementations of IBL:

1. Deep engagement in rich mathematics.
2. Opportunities to collaborate (in some form).

Deep engagement in rich mathematics is the first pillar and is an encompassing phrase, indicating that students are actively and intentionally working on challenging (to them) mathematics problems. It means that the students themselves do a significant portion of the development of mathematical ideas, which are more sophisticated than rote skill-level exercises (although rote skill work is valued and developed). Typically, students do not know the answer or method ahead of time, and the questions generally require grappling with mathematical ideas before arriving at a solution to the problem.

Collaboration is the second pillar and can come in many forms. The most common is structured group work. Students are asked questions and work collaboratively as a team to think through mathematical ideas. Students' thinking benefits from verbalizing their thoughts, and, in doing so, students learn how to effectively communicate mathematics orally and in written form.

Collaboration can take other forms besides group work. For example, in an upper-division course with a focus on proof, students often present their proofs to the entire class. The class peer-reviews the proof, discussing its features such as validity, techniques, and coherence. Hence, class discussion is a class-wide collaboration, moderated by the instructor. In this case the class works together to validate and understand the meaning of proof.

Class discussion is a class-wide collaboration.

The existence of the many varieties of IBL is a natural result of the broad and changing landscape of college mathematics education. Class size, prior experiences of students, course topic, and other factors significantly affect an instructor's choices when implementing IBL. An instructor's level of skill and experience with IBL also plays a role in teaching decisions about the structure of a class. These factors and more combine to suggest that varied and disparate factors influencing teaching decisions have given rise to a great variety of IBL under one "big tent." Thus IBL classes can look very different and yet still have in common the twin pillars of deep engagement in rich mathematics and opportunities to collaborate.

IBL Example 1: Upper-Division, Proof-Based Courses

Proof-based courses are a natural setting for IBL. In fact, there is a long tradition of using IBL in these courses, where class size and content pressure are typically minimized when compared to other courses we teach. Students in IBL proof-based courses are asked to develop the fundamental concepts and to produce the proofs of the important theorems. This may require abandoning a traditional textbook in favor of a customized sequence of tasks that meets the students where they are mathematically and is designed to guide them on a journey of mathematical discovery. As opposed to completing exercises after an instructor has covered the relevant material, students decipher definitions, explore examples, make conjectures, and prove theorems. The intention in IBL courses is for the students to make sense of and solve core exercises and problems as they progress through the course.

After each class meeting, students are assigned problems to work on outside of class. Each student is expected to come to the next class meeting prepared to present and discuss their proposed solutions or proofs. In our experience, a typical assignment will consist of roughly 5–15 problems. Each batch of problems is meant to do some subset of the following:

- Introduce a new topic
- Develop intuition about a concept
- Synthesize ideas from a few concepts
- Make a conjecture
- Prove a theorem
- Get practice doing routine or non routine problems

On a typical day, most of the class time is devoted to student presentations. The purpose of the presentations is to drive classroom discussion. In many cases, the most productive presentations are ones that are interestingly wrong, since this creates opportunities for students to engage in meaningful discourse. Implementation of the student presentations can take several forms, such as these scenarios:

1. An individual student acts as a spokesperson for her or his small group and presents a group's proposed solution.
2. An individual presents her solution.

In practice, an instructor is likely to utilize a mixture of both styles when managing student presentations throughout the semester, adjusting for various factors such as difficulty of the material.

The first scenario can play out as follows. Suppose a class of thirty students has an assignment consisting of ten problems that is due the next class meeting. The assignment includes a mixture of exercises related to the relevant concepts and proofs of related theorems. Students are expected to answer all of the problems to the best of their ability before the start of class. Collaboration of peers is encouraged, but students should write up their own solutions independently. As students enter the classroom, they each grab a colored marker pen that they are encouraged to use to annotate their assignment during class. The purpose of the colored pen is so that the instructor can distinguish the work that a student did before class from the work done during class.

Students are arranged in small groups and each group is assigned one of the problems. The group's task is to arrive at consensus on a solution or proof for their assigned problem. The first phase of class (roughly 10–20 minutes of a 50-minute class) is devoted to small group discussion. In a classroom with ample board space, the small groups can spread out around the room and write their solutions on the board. Otherwise, each group can write their solution on a sheet of paper that can later be displayed using a document camera. While the groups are working, the instructor can float around the classroom listening to student conversation, providing gentle nudges and encouragement as needed, and assisting when necessary.

The next phase of class provides an opportunity for a spokesperson for each group to present their proposed solution, or possibly partial solution. During each presentation, students in the audience are asked to validate the ideas presented. It is vital that the classroom environment provide a safe atmosphere where students are comfortable sharing and critiquing ideas respectfully.

One of the main roles for IBL instructors is to manage class discussions, ensuring that discussions are fruitful. The skills required to do this are essential and are distinct from lecturing skills. IBL instructors face a range of in-class decisions when managing discussions. When students get stuck or need assistance when presenting, IBL instructor options include (but are not limited to) tabling a discussion until the next class meeting; writing a lemma on the board, providing a middle step for the current problem; asking students to work in small groups to offer suggestions; and offering instructor hints or insights. Choices depend on a variety of factors and are situational. Each strategy has strengths and weaknesses, and choices are based on maximizing student learning.

IBL Example 2: Calculus

The calculus sequence stands as either a gateway or a gatekeeper for many of the STEM disciplines. Bressoud, Rasmussen et al. in the Mathematical Association of America's Calculus Study determined that there are seven common characteristics of successful calculus programs. Among those seven are "construction of challenging and engaging courses" and "use of student centered pedagogies and active learning strategies." IBL addresses both of these characteristics but is often dismissed in the calculus sequence due to time and coverage constraints. We offer a sample of useful IBL strategies for calculus courses. Some or all of these strategies could be used for any lower-division mathematics courses, where class sizes are often large and one must cover a certain amount of material over the semester. In this section we describe three strategies: student presentations, IBL worksheets, and TACTivities.

Calculus II students at the University of Nebraska Omaha engage in active learning worksheets.

Everything said about presentations and group work in a proofs course also applies to the calculus sequence.

Presentations. A typical calculus course often begins with the instructor answering student homework questions. In an IBL calculus class, homework is still addressed, but instead of having students watch an instructor write out solutions to homework problems, we can engage the students in this process. Allow the students to come to class and post problems on the board that they would like to see worked out. If they do not have any suggestions, then have a handful of problems that you want to be sure all students have thought about carefully. Put these problems on the board with space for solutions to be written out. Instead of the instructor writing the answers, have the students write out their solutions (or even their partial solutions). Then have the students present these solutions to the class. The instructor moderates the discussions and assists with communication of the ideas. Taking time to have students present their proposed solutions helps develop many important communication skills as well as engaging students in the learning process.

IBL worksheets. While teaching new material, carefully designed day-to-day tasks will help ensure material is still covered that is required for the course. To create worksheets (or problem sets) to take the place of lectures, one must determine what the students can figure out on their own and what they will need guidance on. For instance, students may not be able to discover the Fundamental Theorem of Calculus on their own, but they may be able to develop intuition about why it works with a carefully selected set of steps guiding them. They may also need one example showing them how to use the Fundamental Theorem of Calculus, but not all of the steps or "tricks." IBL calculus worksheets often have problem sets that are designed so that the students have to figure out the ideas themselves once they have an understanding of the basics. Fill-in-the-blank two-column proofs have also been used successfully in calculus courses to include rigor without the instructor providing the entire proof. For example, one may provide the step in the proof and ask for the reason from the student. One may also give a hint in one column and have the student complete the step. This fill-in-the-blank method also works well with new concepts, when students need hints/scaffolding to complete the first problem on the worksheet. Subsequent problems in the set have few or no hints.

Tactile learning activities (TACTivities) are another way to engage calculus students. These activities are ones in which there are literally moving pieces. They can be used to introduce a new topic, but they are even better for reviewing or enhancing skills without "drill and kill" worksheets. For example,[1] students may be given graphs of several functions and their derivatives. These graphs are on small cards, usually laminated or printed on card stock. The students only have graphs of the functions, without equations. The students then have to match each function graph to its derivative graph by sorting out these cut-out pieces. Another type is a "domino TACTivity." The dominoes are great for reviewing concepts. For example, students are given a set of cards that look like dominoes with one half of each domino having an integral on it and the other half having an antiderivative on it. The cards are mixed up and students must sort the cards so that each integral is matched with its corresponding antiderivative. Once the cards make a complete "circle" the activity is complete. This TACTivity works for anything that comes in pairs.

Worksheets/problem sets and TACTivities are designed to be completed in groups of two to four students. Each student fills out her/his worksheet as notes for the course to encourage both collaboration and individualized learning. Students enjoy working together and often bond with other students while discussing/arguing peacefully about how to solve hard problems or complete the TACTivities. Student presentations can be given on problems that students are having trouble with or to help students see multiple solution paths.

IBL Example 3: Mathematics for Elementary Teachers

Elementary school teachers teach mathematics to our children, developing the foundational knowledge of mathematics young children carry with them the rest of their lives. Future elementary school teachers require a deep, sophisticated understanding of school mathematics and development of children's mathematical thinking. IBL math courses provide a framework for instructors to accomplish the critically important learning objectives for prospective elementary school teachers. The example below is one of a family of IBL courses for future elementary school teachers.

The main components of this IBL math course for elementary teachers are

1. Authentic IBL mathematical experiences
2. Readings about problem solving and learning school mathematics
3. Video lesson study and projects

The most important component is providing an authentic IBL mathematical experience. In courses for future elementary school teachers, students redevelop fundamental mathematical structures that children must learn from an advanced, teacher's perspective. Future teachers remember how to do arithmetic, but they may not fully understand why things work the way they do. Courses for future elementary teachers can illuminate the deeper ideas and mathematical structures via IBL methods, where students are given problems involving learning how and why school mathematics works the way it does.

Teaching is a profession with specific skills and practices that need to be learned.

Example: Students investigate why an algorithm for comparing fractions works. The algorithm is to find which fraction is larger, a/b or c/d, by cross-multiplying and comparing ad versus bc. The larger side indicates the larger fraction. Students start by studying equivalent fractions and more generally the notion of equivalence (that two objects can be equal yet have nonidentical expressions). As they work through the set of problems, key ideas appear, and students find a pathway of ideas and techniques that enables them to answer the original question, eventually justifying the algorithm as really a shortcut to comparing numerators of fractions with the same denominator.

To a mathematician this fact may be obvious. To future teachers, it likely is not obvious. Many of them have only seen or memorized the algorithm. It is hard enough to teach well something you understand, much less something you have only memorized.

IBL math activities are the heart of the class and comprise the bulk of the class time and assignments. To explicitly connect their mathematical work to school mathematics, students are assigned readings, and a few classes are used for video lesson study. What this involves is studying videos of children doing mathematics and learning how the mathematics, student thinking, and teaching decisions are interconnected. These videos present children's mathematical thinking in an inquiry-based learning environment, where children are given mathematical tasks, work on them, and share their findings. The prospective teachers evaluate the mathematical thinking in evidence in the videos and how tasks were given to students.

How to Get Involved or Get Started

Learning to use IBL is nontrivial. The start-up time and pitfalls for beginning IBL instructors can be significant. Other factors in IBL teaching include obtaining student buy-in, building a safe learning environment, managing group work, making changes in assessment, managing the classroom when students are presenting material, harnessing mistakes or productive failure, and creating appropriate mathematical tasks. Learning these is best done with support.

The IBL community supports both new and experienced IBL instructors. The IBL community is open to everyone and there exist several ways to join the community,

---

[1] *Many examples of active learning materials for calculus, including TACTivities, can be found at* `math.colorado.edu/activecalc`.

including the Academy of Inquiry Based Learning,[2] programs by the Educational Advancement Foundation,[3] the Discovering the Art of Mathematics project,[4] *Journal of Inquiry-Based Learning in Mathematics*,[5] and the recently formed IBL Special Interest Group of the Mathematical Association of America.[6]

Teaching is a profession with specific skills and practices that need to be learned and developed. Just as research shows there is no math gene, likewise there is no teaching gene. Mathematics instructors at all levels can learn to engage students in the process of doing mathematics, and we invite those who are interested to join us and give IBL a try!

2`www.inquirybasedlearning.org`

[3] `eduadvance.org`

[4] `www.artofmathematics.org`

[5] `www.jiblm.org`

[6] `www.maa.org/community/sigmaas`

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Photo Credits

Photo of prospective elementary school teachers is courtesy of Stan Yoshinobu.

Photo of Calculus II students is courtesy of the University of Nebraska Omaha.

ABOUT THE AUTHORS

Dana C. Ernst was the 2016 recipient of the MAA Southwest Section Teaching Award and has been incorporating IBL in his teaching since 2009. In addition to mathematics and teaching, his passions include cycling and rock climbing.

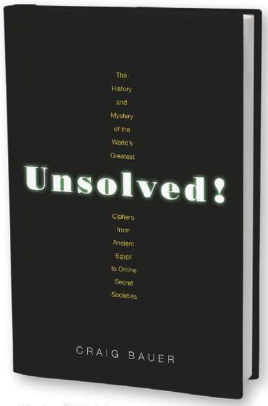
Dana C. Ernst

Angie Hodge is the PI of an NSF Noyce grant to develop expertise in IBL in pre-service mathematics teachers. She travels all over the world to run ultramarathons, including the Leadville 100-mile run.

Angie Hodge

Stan Yoshinobu is director of the Academy of Inquiry Based Learning. He has taught a variety of courses via IBL for more than a decade. Away from his office he enjoys spending time with his family, photography, travel, and hiking.

Stan Yoshinobu



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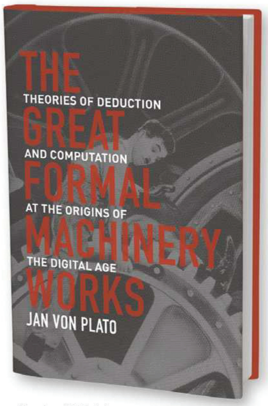

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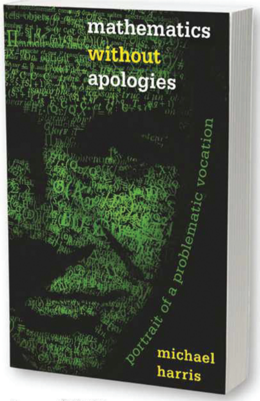

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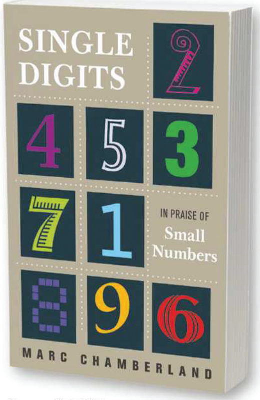

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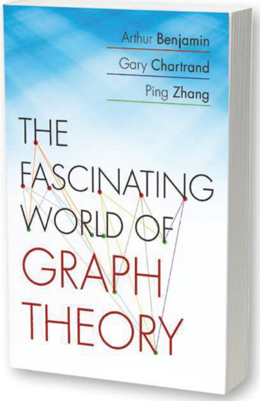

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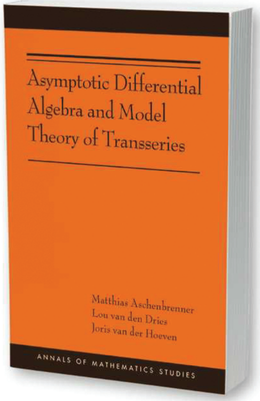
—*SIAM Review*



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THE GRADUATE STUDENT SECTION

Lucas Sabalka Interview

Conducted by Melinda Lanius

Communicated by Alexander Diaz-Lopez

Lucas Sabalka is at Ocuvera (see `www.ocuvera.com`).

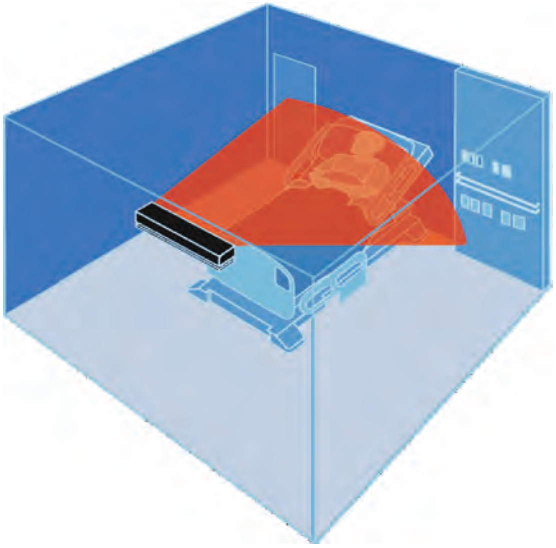
***Lanius**: When and how did you know you wanted to be a mathematician?*

Sabalka: I have always been drawn to mathematics and science. I would play with science-based toys and kits and watch shows like *Square One* and *3-2-1 Contact* as a child. When I got to high school, I joined the Math Club, and that clinched my career path. My mentor and the organizer of the Math Club is Bill Rogge, an energetic and passionate mathematics enthusiast, who shared his infectious joy for puzzles, logic, and mathematics, and shaped my view of the beauty of mathematics. I voraciously consumed mathematics, attending the Canada/USA Mathcamp three times in high school, participating in the Penn State Mathematics Advanced Study Semesters as a college sophomore, and completing my undergraduate mathematics requirements in my first three semesters.

***Lanius**: Who encouraged or inspired you?*

Sabalka: My path has been most influenced by strong mentors. My mother was always supportive of me and guided me to who I am. My high school teacher, Bill Rogge, was a large influence.

My undergraduate thesis advisors, Susan Hermiller and John Meakin, showed me the appeal of geometric group theory, an intriguing overlap between algebra and topology. Dr. Hermiller especially helped me choose my



The Ocuvera system allows hospitals to automatically detect patient behaviors, such as agitation, that are correlated with increased risk of falling. This image shows a representation of Ocuvera deployed in a hospital room.

DOI: http://dx.doi.org/10.1090/noti1541

graduate school and career path. I went to the University of Illinois at Urbana-Champaign for my graduate work in order to work with Ilya Kapovich, who was a great advisor. My postdoctoral advisors, Misha Kapovich and Ross Geoghegan, were also encouraging. Dr. Geoghegan helped me navigate the academic job market and supported me in transitioning from academia to industry.

***Lanius**: How would you describe your work to a graduate student?*

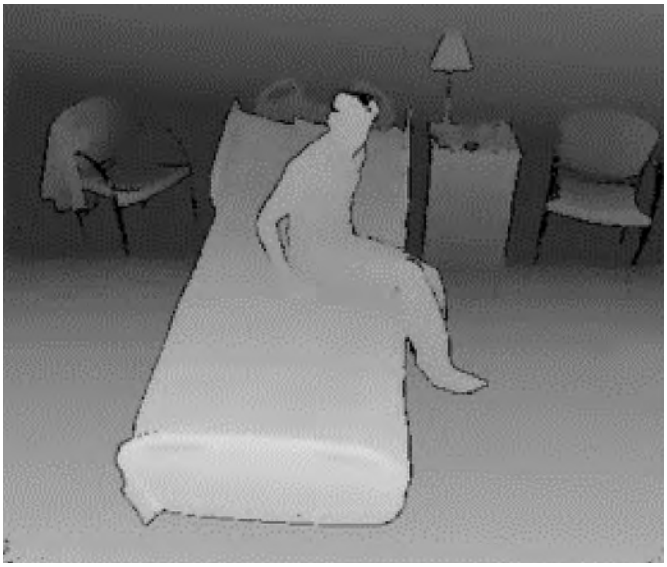
Sabalka: I am currently working on a project called Ocuvera. Ocuvera is a technology-based fall-prevention product for hospitals. Falls are one of the top preventable medical problems today, especially for older adults. Ocuvera attempts to reduce fall risk by using a three-dimensional camera, placed in hospital rooms, to automatically monitor patient behavior. If behavior is detected that is indicative of elevated risk of falling, such as agitation, beginning to get out of bed, or getting out of bed, the system detects this behavior and alerts hospital personnel. Personnel are able to immediately view what is happening in the room, talk to the patient via two-way audio communication, and go to the room to mitigate the cause for the risky behavior.

I work on a technology-based fall-prevention product for hospitals.

I am part of a three-person team, with Josh Brown Kramer and Ben Rush, tasked with implementing the computer-vision components of the Ocuvera system. The raw three-dimensional signal comes off the camera as a two-dimensional array of distances. Our job is in part to determine what portion of that signal represents specific objects, such as the floor, a wall, a table, or a hospital bed. This information can change over time if the camera is bumped, a table is moved, or the bed is raised. We monitor the scene for movement and track objects resembling people. Our system can track an arbitrary number of people. We identify which person is the patient, and we monitor that person for signals correlated with elevated risk of falls, such as sitting up or showing agitation. We use many mathematical tools, including geometry, probability theory, statistics, mathematical morphology, image manipulation, decision-forest-based machine learning, convolutional neural networks, and image filtering.

***Lanius**: Do you have a favorite past project you could share with us?*

Sabalka: At Ocuvera, I am most proud of my work on image filtering. We use an affordable, commercially available three-dimensional camera based on the time-of-flight of infrared light from the camera to the scene and back. The raw signal has significant noise, including variance in depth, missing values, and visual artifacts. My colleagues and I observed that the variation is largely predictable for a given pixel over time, so incorporating a time-based element can significantly improve image fidelity. This is useful for both image quality and video compression. The biggest challenge is in smoothing pixels over time but still faithfully representing objects in motion: discerning between motion and noise, handling missing information in the presence of motion, and showing small movements accurately. Creating the real-time algorithm to implement this smoothing has been my favorite project.



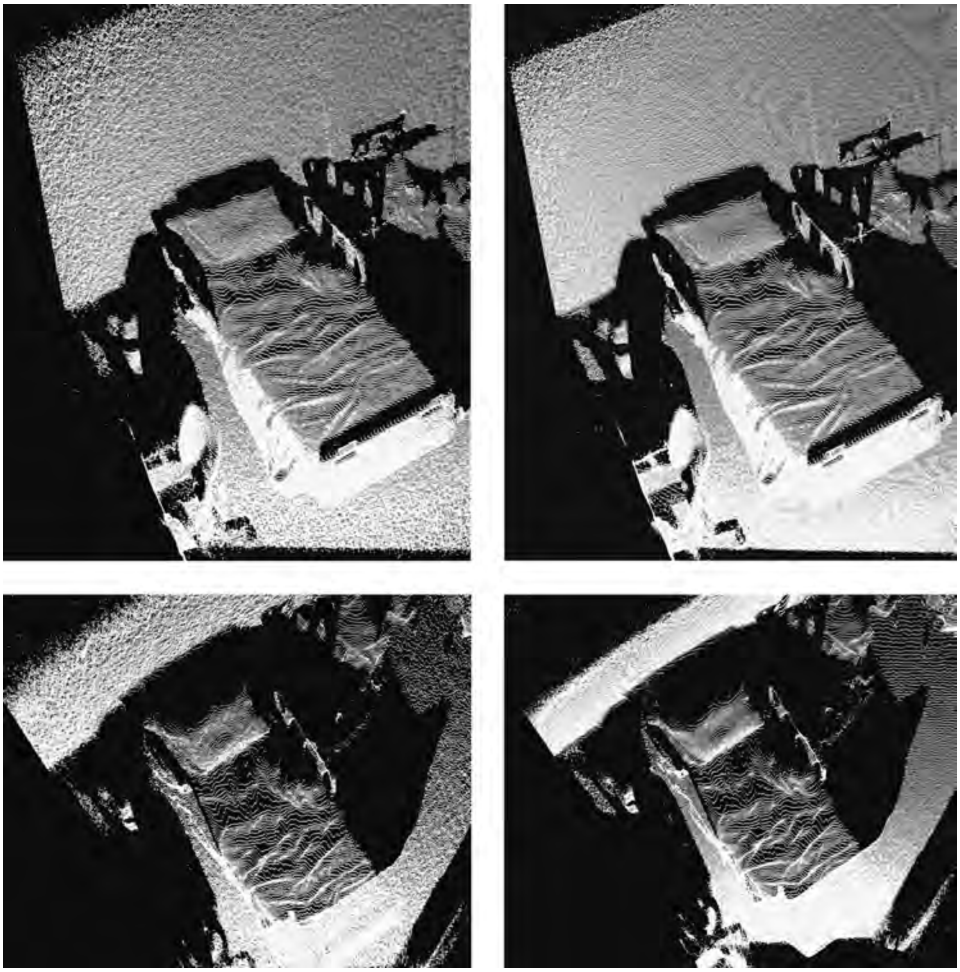
This image shows a visual representation of a depth frame: an array of distances to points in the scene from the plane going through the camera and perpendicular to the focal axis of the camera. Shown here is a typical situation of someone in a hospital setting about to get out of bed.

***Lanius**: What is a typical workday like?*

Sabalka: I have a roughly 9-5 job, where by "roughly" I mean I have flexible hours: some days I get in before 8 and stay until after 6; others I'll work 9:30-4:30. I usually start the day by catching up on email and checking on the results of the previous day's test runs: we have a cluster of about 60 computers that calculate our machine-learned algorithms and test our latest improvements. We have a daily ten-minute team meeting that we call "stand up" because the idea is if everyone is standing then they'll be more succinct. We have an open office plan, so many times per day I will be asked to help a colleague with a problem or review a particular piece of code or algorithm, and reciprocally I'll ask for help. Most of my time is spent implementing and testing my various ideas, though a fair bit of time is spent just thinking, in front of my computer or with a piece of paper or at a whiteboard. I am coding probably a bit over half of my day.

***Lanius**: What is the work culture like at Ocuvera and Nebraska Global?*

Sabalka: Nebraska Global is an incubator company that helped found Ocuvera, and it is connected to about a dozen small companies with a total of around 75 employees. I have worked with two of these sis-



These two pairs of images show point clouds encoded in a depth frame. Both images are from the same overhead perspective and of the same scene of a bed. The camera captured this data from a position higher than the bed. On the left in each image pair is the unsmoothed frame, that is, the raw data from the camera. On the right in each pair is the smoothed frame, which has been processed to reduce noise and increase fidelity to actual distances: note especially the back wall and floor.

ter companies, but I have always primarily been with Ocuvera. It was started because of the success of EliteForm, a company harnessing three-dimensional cameras in athletic training space. I was brought on to work on Ocuvera for my knowledge of geometry and experience with programming. I have less control over the broad project that I'm working on, but I still largely dictate where my energies are best directed, and I find almost every problem I've been tasked with engaging. I set my hours and pace, and in return I am expected to show reasonable progress towards my goals.

Making the decision to step into the unknown was intense and personal.

Nebraska Global companies employ a variety of people. My coauthor Josh Brown Kramer was a professor of mathematics and is now a coworker. Our CEO dropped out of high school in the 1980s to found his first very successful tech company. Most employees have a background in software engineering, especially software design. We also have people from a variety of backgrounds who do business development, project management, sales, tech support, human resources, hardware—many roles you would find in a medium-sized company.

Lanius: *How do you balance career and outside interests (e.g. hobbies, family time, vacations)?*

Sabalka: One benefit of my particular job is that it typically stops when I leave work. I may toy with a problem at night, or log a few hours on a weekend, but I find myself largely free for outside interests after 5 pm. Vacation at my work is flexible and not closely tracked, though it's important not to abuse that privilege. Volunteerism is strongly encouraged, including during work hours, so I donate 5-15 hours per week both inside and outside of work hours to charity: I use my speaking skills developed from lecturing to talk to thought leaders and community members about solutions to global warming.

Lanius: *Are there any speed bumps in your journey that you could share with us?*

Sabalka: Choosing to move from academia to industry was a difficult one for me. The transition itself was not nearly as hard as I feared, but making the decision to step into the unknown, after having worked for 14 years to get my tenure-track research position, was intense and personal. My wife and I had many long discussions about what we wanted both near term and long term, what were the risks were, and what the costs and benefits were. Industry is better for me personally. In retrospect, I should have had more confidence in my ability to adapt, but change is scary.

Lanius: *What advice do you have for graduate students?*

Sabalka: I chose my graduate school to work with my doctoral advisor, and I'm grateful I did. I highly recommend seeking out someone you work well with. Aside from choosing an advisor, I recommend considering early what your post-graduation options are. Would you be comfortable in a teaching college? Doing software engineering? Working on Wall Street? Do you handle well the self-driven nature and pressures of research? You do not have to commit to any single career path, but you may want to keep more doors open, especially if one job market tightens before you graduate. Consider choosing an advisor or dissertation topic that opens more options. If you are thinking about industry, try to find an internship over a summer, both to see if it's right for you and to have the experience. If computer programming or financial quantitative analysis are options, you might look at courses in those areas. Even if you aren't considering non-academic jobs, a supporting skill like computer programming can be useful.

> *A degree in mathematics is a degree in advanced problem solving.*

Lanius: *If you could recommend one book to graduate students, what would it be?*

Sabalka: My personal favorite is Hatcher's *Algebraic Topology*. I'm a bit biased here, but I find thinking topologically/geometrically/physically is a vital perspective in many areas.

Lanius: *Any final comments or advice?*

Sabalka: Mathematicians don't just do mathematics. Higher mathematics is about problem solving, logical reasoning, and creative thinking. If you are considering non-academic careers, it is important to gain knowledge or experience to make you competitive, but don't sell yourself short in the job market: a degree in mathematics is a degree in advanced problem solving.

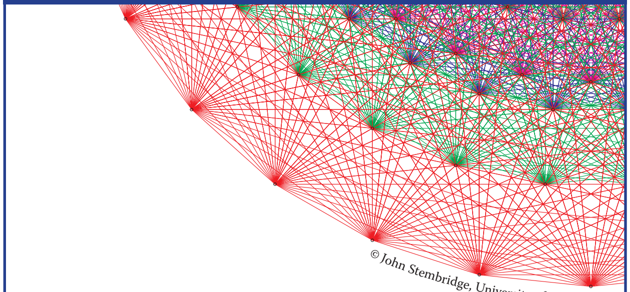
Photo Credits

Interviewee photo courtesy of Lucas Sabalka.
Computer vision images courtesy of Ocuvera, LLC.
Interviewer photo courtesy of Melinda Lanius.

Melinda Lanius

ABOUT THE INTERVIEWER

Melinda Lanius, a Wellesley College graduate, is currently earning her PhD in mathematics at the University of Illinois at Urbana–Champaign.



?WHAT IS...

a Generalised Mean-Curvature Flow?

Hui Yu

Communicated by Cesar E. Silva

Editor's Note: The Graduate Student section welcomes submissions that, like the present "WHAT IS...?" column, are written by students. See "For Authors" on the *Notices* website, `www.ams.org/notices/noticesauthors`.

To untie a shoelace by pulling both ends of the string, shrinking the bunny ears until they disappear, gives great intellectual pleasure. Mathematically, we unknotted an unknot by decreasing the length of the curve contained in the knot. This procedure turns out to be a rather effective way to simplify geometries. It is one of the reasons why geometers like Richard Hamilton study the so-called *curve-shortening flow*, where an initial curve is continuously deformed so that its length decreases over time.

The mean-curvature flow enjoys nice properties.

The way to do this is to move or flow the curve normal to itself at a rate proportional to its curvature. There is a similar flow as in Figure 1 to reduce the area of a $(d-1)$-dimensional surface in $\mathbb{R}^d$, at a rate proportional to the *mean curvature*, the mean of the curvatures of 1-dimensional slices by orthogonal planes.

Apart from shortening curves, the mean-curvature flow enjoys other nice properties, such as *locality*, since the motion of a point depends only on an infinitesimal neighbourhood, and *rigid-motion invariance*, since "length" is invariant under rigid-motions. Other properties are more subtle. If you know that the mean curvature is the Laplacian of the function that measures distances from points to the manifold, then you can see that the motion is driven by a parabolic equation. This parabolicity gives rise to the *inclusion principle*, which says that if we deform two domains $U \subset V$ by letting their respective boundaries follow the mean-curvature flow, then these domains remain ordered. This is a consequence of the comparison principle for parabolic equations. It is also a consequence of geometry: If V failed to contain U at some time, then there would be a critical moment when U is still inside V but their boundaries touch at a certain point. Doodle a bit with curves, and you see that in this situation, to reduce "length," the boundaries are instantly pulled away from each other. Conclusion: U remains inside V.

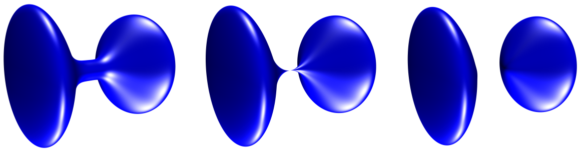


Figure 1. The surface of a dumbbell develops a singularity before turning into two disconnected spherical shapes.

All these nice properties do not make the mean-curvature flow perfect. One major flaw is that it is not well defined globally in time. After convex surfaces shrink into points, the flow is no longer defined. But this example does not capture the worst behaviour, because the object itself disappears after the critical point. Think of a dumbbell, as in Figure 1. If the plates are too big compared to the neck, the dumbbell turns into two smaller plates connected by a one-dimensional "string." Again the flow ceases to be well defined afterwards.

But what we have called a flaw is actually something that makes the mean-curvature flow analytically interesting.

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We can look for generalised/weak versions of the flow that continue the motion after the critical time. This search for generalised mean-curvature flows has become one of the central themes in analysis in the past few decades and leads to a lot of exciting mathematics.

A manifold is the zero set of the function that measures distances between any point and the manifold. This distance function satisfies a parabolic equation in the mean-curvature flow. Therefore, one natural way to get a generalised mean-curvature flow is to study generalised solutions to such equations, and then define their zero level sets to be solutions to the generalised flow. This so-called *level-set method*, as described in the December 2016 *Notices* cover story by Colding and Minicozzi, reduces the notion of a weak geometric flow to the notion of a weak solution to parabolic equations. Luckily, the viscosity solution provides exactly what we need. In fact, much of the theory of viscosity solutions is inspired by this approach to a generalised mean-curvature flow.

> *The search for generalised mean-curvature flows leads to a lot of exciting mathematics.*

Another approach takes advantage of the inclusion principle. Suppose our manifold is the boundary of some domain U. Then the inclusion principle dictates that any domain initially containing U continues to do so in the future. Although U might be a very rough set where the smooth mean-curvature flow does not exist, we can always find nice sets containing U. Starting from these nice sets we do have flows, and we might simply define a generalised mean-curvature flow of U to be the intersection of all flows starting from these nice sets. This method of *minimal barriers* might remind the reader of the method of Perron, whereby one constructs solutions to elliptic equations as least supersolutions.

A third approach, which goes back to the curve-shortening property, begins by defining a weak notion of the length of curves or the area of surfaces. The hope is that, using this generalised length/area, we would be able to consider the flow for much rougher objects. Such generalised length/area is provided by the theory of sets of finite perimeter from geometric measure theory. At each instant of time, we look for a set of finite perimeter that most efficiently reduces this generalised length/area and define this set to be the solution to the generalised flow. This gives rise to the notion of *flat flows*.

A related approach follows from the observation that the length/area can be seen as the limit of the Ginzburg-Landau energy functional when a certain parameter ϵ goes to 0. For each positive ϵ, we have a globally well-defined gradient flow for this energy. We simply define our generalised mean-curvature flow as the limit as ϵ goes to zero of this gradient flow. This is called the *phase field* approach.

The literature on generalised mean-curvature flows is particularly rich, containing many methods beyond the scope of a two-page article. To earn the label "generalised mean-curvature," however, a flow needs to satisfy several requirements: firstly, it should be globally defined; secondly, it should be consistent with smooth flows as long as the latter exist; and lastly, it should inherit certain key features of the smooth flows.

These naive-looking requirements sometimes lead to challenging problems that have inspired a great deal of research. Because generalised mean-curvature flows provide a paradigm for the study of all kinds of geometric flows, results in this area are of especially wide interest.

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Image in Figure 1 by Oliver Knill, Harvard University.
Photo of Hui Yu by Yunan Yang, PhD student at UT Austin.

ABOUT THE AUTHOR

Hui Yu is interested in the theory of elliptic partial differential equations and calculus of variations as well as geometric flows. When he is not busy with moving surfaces, he likes to play with curvatures of the strings of the classical guitar.

Hui Yu

THE GRADUATE STUDENT SECTION

AMS Blogs
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Graduate Student Blog
by and for math graduate students

The AMS Graduate Student Blog, by and for math graduate students, includes puzzles and a variety of interesting columns such as the one from March 2017 excerpted below.
`blogs.ams.org/mathgradblog`.

"On 'Imposter Syndrome'"

by Tom Gannon, University of Texas at Austin

Here's how it happens: You're in graduate school and were one of the best people in your major from your school. … It's really fun, being with people who are just as excited about math as you are. But then, a horrible thing happens. Someone, in conversation, mentions something you don't know. And not only that, but the way they talk about it suggests that anyone who knows anything about anything knows what they're talking about. Or maybe, in an even worse turn of events, this person is a professor. What are you going to do?

I've pieced together advice I've gotten from various people and compiled it here. Being a math person, I have decided to divide it into two cases (although half of the problem with imposter syndrome is actually not knowing which case you fall into).

Case 1: You are not the person that knows the least in the room.

Case 2: You really are the person that knows the least in the room.

Everyone needs to cross the bridge at some point in their life where they learn that they're not the best. Hopefully these tips lead you to recognize and start to get over your imposter syndrome just a little more easily.

Photo Credit

Photo of Tom Gannon is courtesy of Tom Gannon.

ABOUT THE AUTHOR

Tom Gannon is a first-year graduate student at the University of Texas at Austin.

Tom Gannon

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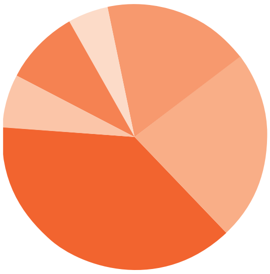


Report on 2015–2016 Academic Recruitment, Hiring, and Attrition

Amanda L. Golbeck, Thomas H. Barr, and Colleen A. Rose

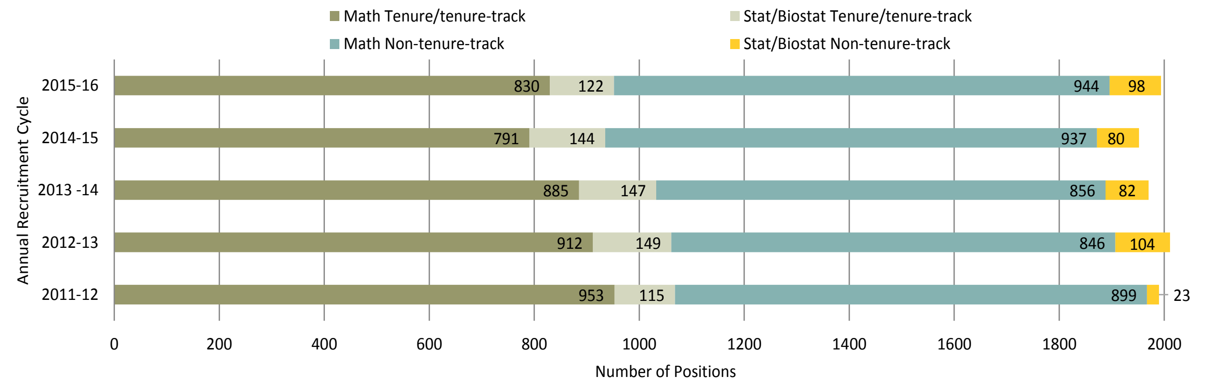
Each year in academic mathematical sciences departments around the United States, there are searches for new full-time faculty members, and a subset of those positions are filled. The hiring infuses into the profession a new cohort of mathematical scientists actively engaged in research, teaching, and service. At the same time, others retire, take jobs outside of academe, or die, and this process removes a segment of the population of academic mathematical scientists. This report provides a snapshot of that process to aid in understanding the current status of indicators such as: hiring rates; and distributions of gender, position type, and prior experience. Along with current data, the report provides historical context to aid the reader in discerning trends and patterns. For further details, including all tables generated to prepare this report, please see `www.ams.org/annual-survey.`

A total of 958 mathematical sciences departments participated in this survey. This report is based on the completed questionnaires received from a subset of these departments, specifically the 574 departments that reported they were recruiting to fill doctoral tenure-track and non-tenure-track positions during the academic year 2015–2016 for employment beginning in the fall of 2016. An additional 50 departments (8 Math Doctoral, 8 Stat/Biostat, 8 Masters, and 26 Bachelors) reported conducting recruitment and hiring during this time but did not return a completed questionnaire and were not included in the analysis.

Overview of Recruitment

This year's data show an overall increase of 2% in the number of positions under recruitment. The Masters and Bachelors Groups were the biggest contributors to the increase, up 11%—offset by the decline reported by the Doctoral Math (6%) and Stat/Biostat (2%) Groups. The Doctoral Math Group reported increases in only the number of open

Figure R.1: Positions Under Recruitment in Mathematical Sciences



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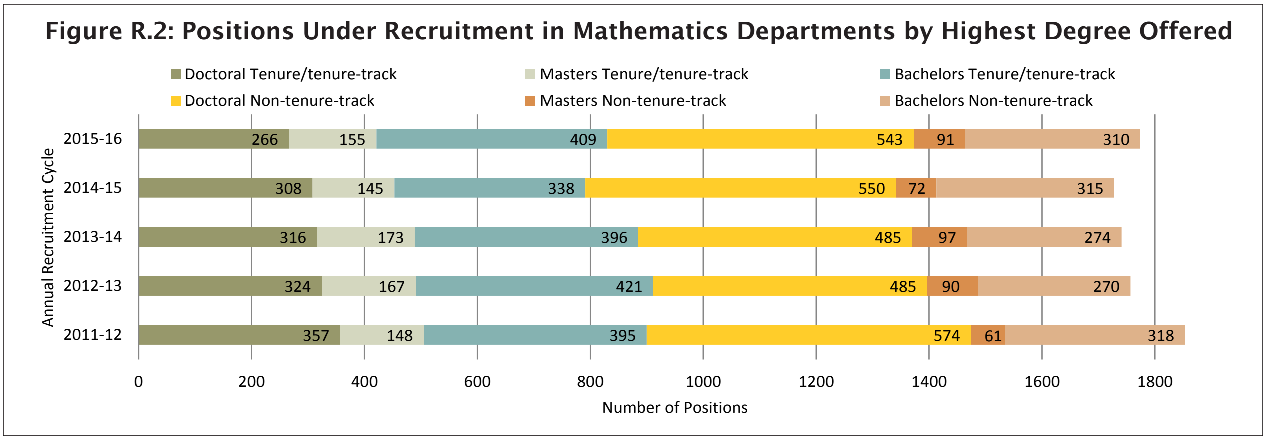


temporary non-tenure-track appointments, while the Stat/Biostat group reported an increase in the overall number of open non-tenure-track positions.

During the 2015–16 academic year, the estimated number of full-time positions under recruitment in mathematical sciences departments was 1,994. This figure breaks down as follows: 830 tenure-track mathematics positions, 944 non-tenure-track mathematics positions, 122 tenure-track statistics or biostatistics positions, and 98 non-tenure-track statistics or biostatistics positions. See Figure R.1 for comparisons. In the period from 2011 to 2016, the overall percentage of positions under recruitment that were tenure-track ranged from 48% to 53%, with the highest percentages in 2011–12 and 2012–13 of this range of time.

- In the 2015–2016 cycle:
    - The estimated number of positions under recruitment was 1,994; this figure represents a slight increase from last year's estimate of 1,952 positions.
    - Females account for 32% of those hired; this is up from 29% for 2014–2015.
    - Since 2011–12 recruitment has decreased 4% in Math and increased 59% in Stat/Biostat.
- Tenure-track positions under recruitment:
    - Open tenure-track positions increased 2% overall from last year.
    - 48% (952) of all positions under recruitment were tenure-track. Of these 952 positions, 88% (833) were open to new PhDs, and 21% (198) were at the rank of associate/full professor.
- Non-tenure-track positions under recruitment:
    - Non-tenure-track positions increased 2% overall, up to 1,042 from 1,017 the previous year.
    - 52% (1,042) of all positions under recruitment were non-tenure-track.

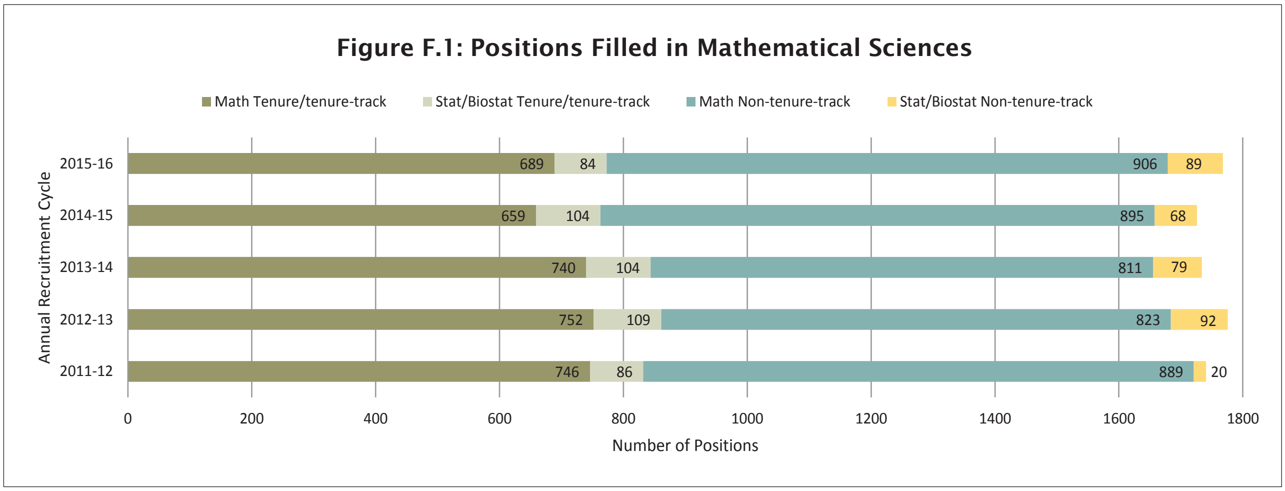
In Math, the number of positions under recruitment (1,774) in 2015–16 is comparable with that for 2014–2015 (1,728) and is up after dropping for three consecutive years. See Figure R.2. Over the period since 2005–06 recruitment in Doctoral departments has increased by 16%, in Masters departments decreased by 30%, and in Bachelors departments increased by 4%. In the same ten-year period, the net number of mathematics positions under recruitment has decreased by 4%.

In Stat/Biostat, the number of positions under recruitment was 220, a 2% decrease from 2014–15. The trend over the past few years has been downward, although reviewers should keep in mind that numbers are small.

Positions Filled

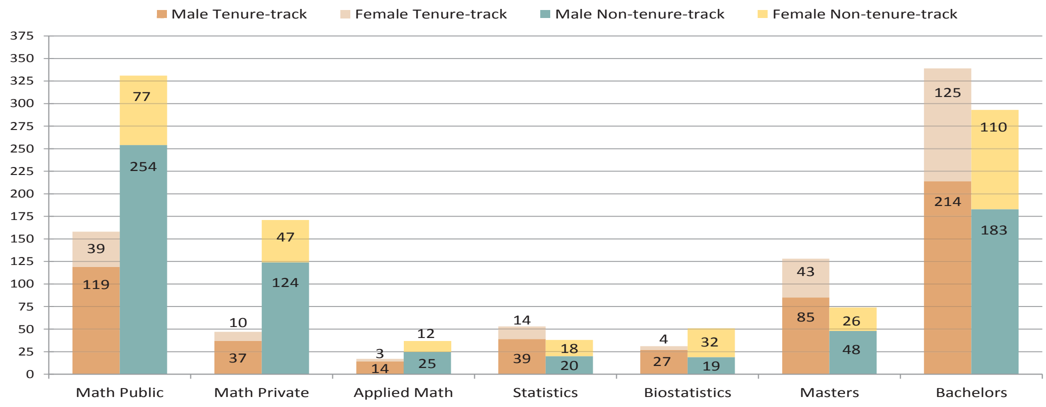
A total of 1,768 full-time positions in Mathematical Sciences were filled during the 2015–16 academic cycle, 1,595 from Mathematics Departments and 173 from Statistics or Biostatistics. Figure F.1 gives a breakdown. The total for Math is down 2% from the 2011–12 cycle. For Stat/Biostat, the number of filled positions is up 63% from 2011–12. One interesting feature in these data is that the success rate for filling mathematical sciences tenure-track positions over the period 2009–2016 is about 81%, whereas the success rate for non-tenure-track is about 96%.

Figure F.2 gives a breakdown on hiring by gender and department grouping. Percentages generally are obtained by comparison with Figure R1. Here are further highlights and comparisons from the data:



- Overall features of hires in mathematical sciences:
    - Females hold 32% (560) of positions filled.
    - Of all hires, 44% (773) were tenure-track; females constitute 31% (238) of these.
    - Of all hires, 56% (995) were non-tenure track; females constitute 32% (322) of these.
- Math and Stat/Biostat breakdown:
    - In Math overall, 1,595 of 1,774 positions (90%) were filled; 31% of Math positions were filled by females.
    - In Stat/Biostat, 173 of 220 positions (79%) were filled; 39% of Stat/Biostat positions were filled by females.
- Tenure-track hires in mathematical sciences:
    - Of the tenure-track positions under recruitment, 81% (773) were filled.
    - Of tenure-track positions filled, 75% (580) were filled by doctoral faculty (excluding new PhDs). Of these positions filled by doctoral faculty, 28% went to females. In comparison with 2014–2015, all groups except Public Small, Applied, Masters, and Bachelors reported decreases in tenure-track hires of doctoral faculty.
    - Of the 25% of tenure-track hires who were new PhDs, 40% were female.
    - Of tenure-track hires, 32% (244) had a non-tenure-track position in 2014–2015; of these individuals, 20% were female.
    - Of tenure-track hires, 26% (202) held a postdoc last year, and 34% of these postdocs were female.
- Non-tenure-track hires in mathematical sciences:
    - Of the 1,042 non-tenure-track positions under recruitment, 95% were filled. In comparison to 2014–2015, all groups except Math Public Large, Math Public Small, and Statistics reported increased hiring of non-tenure-track faculty.
    - Of non-tenure-track hires, 44% (454) were filled by doctoral faculty (excluding new PhDs); 28% of these doctoral faculty hires were female.

Figure F.2: Gender of Tenure-track and Non-tenure-track Hires by Department Grouping

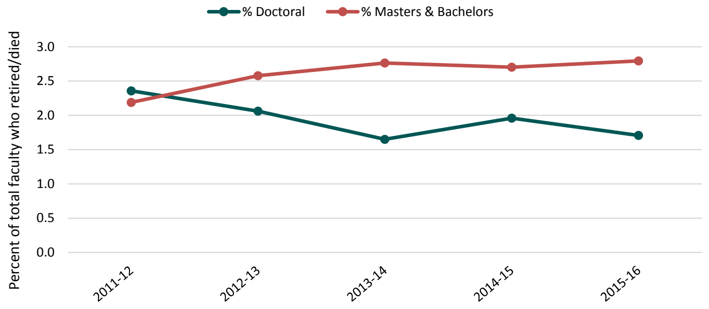


  - Of non-tenure-track hires, 43% (428) were filled by new PhDs; 31% of these new PhD hires were female.
  - Of non-tenure-track hires, 11% (113) were filled by non-doctoral faculty; 57% of these non-doctoral hires were female. Over half of these non-doctoral, non-tenure-track hires were in Bachelors departments.
  - Of non-tenure-track hires, 25% (253) are temporary (one-year); 28% of these temporary hires are female. About half of all temporary hires were in Bachelors departments.
  - Of non-tenure-track hires, 36% (362) were in postdoctoral positions; 23% of these postdocs were female.
- Female hires (see Figure F.2):
  - Of all hires, 32% (560) were female; of these women, Bachelors departments hired 42%, and Doctoral Math departments hired 34%.
  - In the Doctoral Math Group, female hires increased 6% over the past year to 188.
  - All groups except Math Public Large, Math Public Small, and Biostatistics reported increases in the number of female hires over 2014–2015.
  - Over the past year, the number of females hired into tenure-track positions remained essentially unchanged at 238; the number hired into non-tenure-track positions decreased by 3% to 259.
  - Females accounted for 31% of all tenure-track and 32% of all non-tenure track hires; 2014–2015 these percentages were, respectively, 31% and 27%.

Faculty Attrition

Figure A.1 shows rates of attrition from deaths and retirements among full-time faculty numbers for the academic years 2011–12 through 2015–16. On average over the period shown, the percentage of faculty in doctoral departments retiring or dying each year is about 1.9%, and in Masters and Bachelors departments that percentage is about 2.6%.

Figure A.1: Percentage Full-time Faculty Died/Retired*



* The percentage of full-time faculty who died or retired is the number of faculty who died or retired at some point during the academic year (September 1 through August 31) divided by the number of full-time faculty at the start of the academic year.

During the same period, in the respective groups, the percentages of tenured faculty who retired averaged 3.5% for Doctoral Math departments, 3.9% for Bachelors and Masters, and 2.9% for Stat/Biostat. The majority of individuals who are reported by their department as retiring are, in fact, members of the tenured faculty. For instance, data collected for 2011–15 indicate that approximately 86% of those retiring were tenured.

Here are a few other highlights for the attrition data from the 2015–16 cycle in comparison with the previous year:

- Overall retirements by tenured faculty decreased by 5% to 430
- Deaths and retirements decreased by 4% to 565
- Overall retirements (515) break down by departmental grouping as follows:
  - 48% (247) were from Bachelor
  - 30% (154) were from Doctoral Math
  - 18% (93) were from Masters
  - 4% (21) were from Stat/Biostat

Department Grouping Response Rates

In this report, *Mathematical Sciences* departments are those in four-year institutions in the US that refer to themselves with a name that incorporates (with a few exceptions) "Mathematics" or "Statistics" in some form. For instance, the term includes, but is not limited to, departments of "Mathematics," "Mathematical Sciences," "Mathematics and Statistics," "Mathematics and Computer Science," "Applied Mathematics," "Statistics," and "Biostatistics." Also, *Mathematics* (*Math*) refers to departments that (with exceptions) have "mathematics" in the name; *Stat/Biostat* refers to departments that incorporate (again, with exceptions) "statistics" or "biostatistics" in the name but do not use "mathematics." The streamlining of language here militates against the possible objection to foreshortening the full subject names.

Starting with reports on the 2012 AMS-ASA-IMS-MAA-SIAM Annual Survey of the Mathematical Sciences, the Joint Data Committee implemented a new method for grouping doctorate-granting Mathematics departments. These departments are first grouped into those at public institutions and those at private institutions. These groups are further subdivided based on the size of their doctoral program as reflected in the average annual number of PhDs awarded between 2000 and 2010, based on their reports to the Annual Survey during that period.

For further details on the change in the doctoral department groupings, see the article in the October 2012 issue of *Notices of the AMS* at `www.ams.org/journals/notices/201209/rtx120901262p.pdf`.

Math Public Large consists of departments with the highest annual rate of production of PhDs, ranging between 7.0 and 24.2 per year.
Math Public Medium consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 6.9 per year.
Math Public Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.
Math Private Large consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 19.8 per year.
Math Private Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.
Applied Mathematics consists of doctoral-degree-granting applied mathematics departments.
Statistics consists of doctoral-degree-granting statistics departments.
Biostatistics consists of doctoral-degree-granting biostatistics departments.
Masters contains US departments granting a master's degree as the highest graduate degree.
Bachelors contains US departments granting a baccalaureate degree only.
Doctoral Math contains all US math public, math private, and applied math mathematics departments granting a PhD as the highest graduate degree.
Mathematics (Math) contains all US Math Public, Math Private, and Applied Math, Masters, and Bachelors Groups above.
Stat/Biostat contains all doctoral-degree-granting statistics and biostatistics departments.

Listings of the actual departments that compose these groups are available on the AMS website at `www.ams.org/annual-survey/groups`.

Response Rates by Survey Groups

Faculty Recruitment & Hiring Response Rates*

Group	Received (%)
Math Public Large	20 of 26 with 20 recruiting (77%)
Math Public Medium	36 of 40 with 29 recruiting (90%)
Math Public Small	58 of 64 with 41 recruiting (91%)
Math Private Large	20 of 24 with 17 recruiting (83%)
Math Private Small	21 of 29 with 16 recruiting (72%)
Applied Math	20 of 23 with 16 recruiting (87%)
Statistics	43 of 59 with 28 recruiting (73%)
Biostatistics	33 of 46 with 24 recruiting (72%)
Masters	113 of 176 with 67 recruiting (64%)
Bachelors	519 of 1021 with 220 recruiting (51%)
Total	908 of 1512 with 574 recruiting (60%)

* Doctoral programs that do not formally "house" faculty and their salaries are excluded from this survey.

Other Information

The interested reader may view additional details on the results of this survey and prior year trends by visiting the AMS website at `www.ams.org/annual-survey`.

Acknowledgements

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires. Comments or suggestions regarding this Survey Report may be e-mailed to the committee at `ams-survey@ams.org`.

A Vision for CBMS

David M. Bressoud

After almost a third of a century in which their names have become synonymous with the Conference Board of the Mathematical Sciences (CBMS), Ron Rosier and Lisa Kolbe retired from their respective positions as director and administrative coordinator on December 31, 2016. Our community owes them a tremendous debt of gratitude for their tireless work on behalf of the professional societies in the mathematical sciences, work that was always done with professionalism and style.

As the new director of CBMS, my first task is to maintain and continue to strengthen the projects that they have nurtured. The CBMS Executive Committee and I also see this as an opportune moment to build on the work begun in the *Common Vision* report ([1], [2]).

What Is CBMS?

Most mathematicians know CBMS either through the NSF-CBMS Regional Research Conferences, the Regional Conference Series monographs, or the *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States* that CBMS publishes, jointly with the AMS, every five years. Less well known are its monograph series in Issues in Mathematics Education, the *Mathematical Education of Teachers* (MET II) report, the CBMS national forums on issues of mathematics education, or the statements issued jointly by the presidents of the member societies in support of the Common Core State Standards for Mathematics (2013), the Collective Effort to Improve the First Two Years of College Math (2015), or the Statement on Active Learning (2016). Information on all of these can be found on the CBMS website, `cbmsweb.org`.

CBMS grew out of informal meetings of the presidents of the societies in the mathematical sciences and was formally incorporated in 1960. Its core purpose is to provide a forum for these presidents to meet on a regular basis to share information about issues of common concern and to coordinate efforts. From its earliest days, it has taken on the role of providing a home for initiatives that are supported by multiple societies. The NSF-CBMS Regional Research Conferences was one of the first such efforts. Begun in 1969, the seven conferences being offered this summer will bring the 49-year total to 365 such five-day conferences and 221 monographs published by the AMS, SIAM, or IMS-ASA. Today, CBMS brings together 17 organizations. Among them are the major societies at the post-secondary and research level, those that operate primarily within K–12, those that focus on issues of underrepresented groups, and those with less academically focused interests. A complete list of the member societies (with acronyms spelled out) is given at the end of this article.

CBMS is a skeletal organization, staffed by just two quarter-time employees, now Kelly Chapman and me. While it has long shared MAA's facilities in Washington, DC, the lean structure and the ability to draw on the resources of the member societies has enabled us to move its office to the campus of Macalester College in Saint Paul, MN, while continuing to hold the meetings of the society presidents in the Washington, DC area, currently at the American Statistical Association's headquarters in Alexandria, VA.

CBMS is uniquely positioned to address issues of transition.

The Opportunity

CBMS is interested in all aspects of the profession. In education, the member societies span K–12, undergraduate, graduate, and workplace concerns. As such, CBMS is uniquely positioned to address issues of transition: high school to college, two-year to four-year college, undergraduate to workplace, undergraduate to graduate, and graduate to both academic and non-academic employment. While these are all important, much of my own focus, as well as that of the member societies, has been on high school to college, grades 11 to 14.

This is a uniquely opportune moment to work on these issues, following on the heels of several converging efforts: the *Common Vision* report, the Statement on Active

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Learning endorsed by the presidents of the CBMS member societies, the work by many of the member societies on the first two years of undergraduate instruction, and the involvement of the mathematics research community through TPSE Math (Transforming Post-Secondary Education in Mathematics), whose executive director, William (Brit) Kirwan, currently sits as chair of CBMS.

An Agenda

The immediate task before the societies is to transform their statements of support into concrete actions that will help bring about needed changes. I recognize two fronts along which this work must progress:

Research to help us understand our present situation and the true difficulties that must be tackled, especially with regard to encouraging and supporting women, students from underresourced schools, first-generation college students, and students from underrepresented minorities. We know much more about effective mathematics education and how to increase access than we did just ten years ago, but there are still huge gaps in our knowledge. As a first step, CBMS is now exploring the formation of a **Research Advisory Group** to catalog the broad spectrum of data and research that can inform mathematics education at all levels, identify the most important research questions that need to be answered, and look at how we can get relevant information into the hands of change leaders as we help them understand how to analyze and use this information.

Resources and Support for departments that recognize a need for change but require help understanding what they can and should do and how to accomplish these actions. The assistance that departmental leadership will need includes:

1. Information about what is happening at other institutions and within various networks across the country.
2. Guidance on how to evaluate the effectiveness of their own programs. This includes descriptions of the nature of data that should be collected and the various instruments that can be used to better understand what is happening within their own programs.
3. Networks of mutual support connecting comparable institutions that can share their experiences with obstacles and successes. This must include a structure that facilitates and maintains these networks. The experience of PULSE (Partnership for Undergraduate Life Science Education) provides one possible model.
4. Suggestions for and assistance in developing programs for graduate students heading into academia so that they are prepared to be effective teachers.
5. Regular national meetings of department chairs and departmental leaders to learn of recent developments, resources that have become available, and how other departments are facing issues of mutual concern.

Our agenda is ambitious but critically important. We live in a challenging time when major shifts in how we understand and conduct mathematics education are under way. Without active involvement by the professional societies, change will be shaped by those outside our community. As an organization that fosters dialogue and cooperation among the societies, CBMS is uniquely positioned to help us take charge of our own future by identifying, promoting, and encouraging best practices across all of education in the mathematical sciences.

References

[1] T. Holm and K. Saxe, A Common Vision for undergraduate mathematics, *Notices Amer. Math. Soc.*, June/July 2016.

[2] K. Saxe and L. Braddy, *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*, MAA, Washington, DC, 2015. Available at `www.maa.org/programs/faculty-and-departments/common-vision`.

The Societies of CBMS:

AMATYC	American Mathematical Association of Two-Year Colleges
AMS	American Mathematical Society
AMTE	Association of Mathematics Teacher Educators
ASA	American Statistical Association
ASL	Association for Symbolic Logic
AWM	Association for Women in Mathematics
ASSM	Association of State Supervisors of Mathematics
BBA	Benjamin Banneker Association
IMS	Institute of Mathematical Statistics
INFORMS	Institute for Operations Research and the Management Sciences
MAA	Mathematical Association of America
NAM	National Association of Mathematicians
NCSM	National Council of Supervisors of Mathematics
NCTM	National Council of Teachers of Mathematics
SIAM	Society for Industrial and Applied Mathematics
SOA	Society of Actuaries
TODOS	TODOS: Mathematics for ALL

Photo Credit

Photo of David M. Bressoud is courtesy of Jan Bressoud.

David M. Bressoud

ABOUT THE AUTHOR

David M. Bressoud is a Returned Peace Corps Volunteer (West Indies, 1971–73), who is most proud of his collaborations with George Andrews, his work with the AP Calculus program, and his successes at weaving the history of mathematics into his classes and textbooks.

Yves Meyer Awarded Abel Prize

The Norwegian Academy of Science and Letters has awarded the 2017 Abel Prize to YVES MEYER of École Normale Supérieure Paris-Saclay, France, "for his pivotal role in the development of the mathematical theory of wavelets." The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6,000,000 Norwegian krone (approximately US$715,000).

Citation

Fourier analysis provides a useful way of decomposing a signal or function into simply-structured pieces such as sine and cosine waves. These pieces have a concentrated frequency spectrum but are very spread out in space. Wavelet analysis provides a way of cutting up functions into pieces that are localized in both frequency and space. Yves Meyer was the visionary leader in the modern development of this theory, at the intersection of mathematics, information technology, and computational science.

The history of wavelets goes back over a hundred years, to an early construction by Alfréd Haar. In the late 1970s the seismologist Jean Morlet analyzed reflection data obtained for oil prospecting and empirically introduced a new class of functions, now called "ondelettes" or "wavelets," obtained by both dilating and translating a fixed function.

In the spring of 1985, Yves Meyer recognized that a recovery formula found by Morlet and Alex Grossmann was an identity previously discovered by Alberto Calderón. At that time, Yves Meyer was already a leading figure in the Calderón–Zygmund theory of singular integral operators. Thus began Meyer's study of wavelets, which in less than ten years would develop into a coherent and widely applicable theory.

The first crucial contribution by Meyer was the construction of a smooth orthonormal wavelet basis. The existence of such a basis had been in doubt. As in Morlet's construction, all of the functions in Meyer's basis arise by translating and dilating a single smooth "mother wavelet," which can be specified quite explicitly. Its construction, though essentially elementary, appears rather miraculous.

Yves Meyer

Stéphane Mallat and Yves Meyer then systematically developed multiresolution analysis, a flexible and general framework for constructing wavelet bases, which places many of the earlier constructions on a more conceptual footing. Roughly speaking, multiresolution analysis allows one to explicitly construct an orthonormal wavelet basis from any bi-infinite sequence of nested subspaces of $L^2(R)$ that satisfy a few additional invariance properties. This work paved the way for the construction by Ingrid Daubechies of orthonormal bases of compactly supported wavelets.

In the following decades, wavelet analysis has been applied in a wide variety of arenas as diverse as applied and computational harmonic analysis, data compression, noise reduction, medical imaging, archiving, digital cinema,

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deconvolution of the Hubble space telescope images, and the recent LIGO detection of gravitational waves created by the collision of two black holes.

Yves Meyer has also made fundamental contributions to problems in number theory, harmonic analysis, and partial differential equations, on topics such as quasicrystals, singular integral operators, and the Navier-Stokes equations. The crowning achievement of his pre-wavelets work is his proof, with Ronald Coifman and Alan McIntosh, of the L^2-boundedness of the Cauchy integral on Lipschitz curves, thus resolving the major open question in Calderón's program. The methods developed by Meyer have had a long-lasting impact in both harmonic analysis and partial differential equations. Moreover, it was Meyer‘s expertise in the mathematics of the Calderón–Zygmund school that opened the way for the development of wavelet theory, providing a remarkably fruitful link between a problem set squarely in pure mathematics and a theory with wide applicability in the real world.

Biographical Sketch

Yves Meyer, professor emeritus at the École Normale Supérieure Paris-Saclay in France, proves that, in contrast to what F. Scott Fitzgerald said about American lives, in mathematics a life can indeed have a second act, and perhaps even several more. Having made important contributions in the field of number theory early in his career, Meyer's boundless energy and curiosity prompted him to work on methods for breaking down complex mathematical objects into simpler wavelike components—a topic called harmonic analysis. This led him, in turn, to help construct a theory for analyzing complicated signals, with important ramifications for computer and information technologies. Then he moved on again to tackle fundamental problems in the mathematics of fluid flow.

That tendency to cross boundaries was with him from the start. Born on July 19, 1939, of French nationality, he grew up in Tunis on the North African coast. "The Tunis of my childhood was a melting pot where people from all over the Mediterranean had found sanctuary," he said in a 2011 interview. "As a child I was obsessed by the desire of crossing the frontiers between these distinct ethnic groups."

Meyer entered the elite École Normale Supérieure de la rue d'Ulm in Paris in 1957, coming in first in the entrance examination. "If you enter ENS-Ulm, you know that you are giving up money and power," he later said. "It is a choice of life. Your life will be devoted to acquiring and transmitting knowledge."

After graduating, Meyer completed his military service as a teacher in a military school. But despite his deep commitment to education and his students, he wasn't suited to the role. "A good teacher needs to be much more methodical and organized than I was," he admits. Moreover, he was uncomfortable with being the one who was "always right." "To do research," Meyer has said, "is to be ignorant most of the time and often to make mistakes." Nevertheless, he feels his experience of high school teaching shaped his life: "I understood that I was more happy to share than to possess."

He joined the University of Strasbourg as a teaching assistant, and in 1966 he was awarded a PhD there—officially under Jean-Pierre Kahane, but Meyer asserts that, like some others in France at that time, he essentially supervised himself. He became a professor of mathematics first at the Université Paris-Sud (as it is now known), then the École Polytechnique and the Université Paris-Dauphine. He moved to the ENS Cachan (recently renamed the ENS Paris-Saclay) in 1995, where he worked at the Centre of Mathematics and Its Applications (CMLA) until formally retiring in 2008. He is still an associate member of the research centre.

Yves Meyer's work has, in the most general terms, been concerned with understanding mathematical functions with complex and changing forms: a character that can be described by so-called partial differential equations. Fluid flow, for example, is described by a set of such equations called the Navier–Stokes equations, and in the 1990s Meyer helped to elucidate particular solutions to them—a topic that ranks among the biggest challenges in mathematics.

Meyer's interest in what might be called the structures and regularities of complicated mathematical objects led him in the 1960s to a theory of "model sets": a means of describing arrays of objects that lack the perfect regularity and symmetry of crystal lattices. This work, which arose from number theory, provided the underpinning theory for materials called quasicrystals, first identified in metal alloys in 1982 but prefigured by quasiregular tiling schemes identified by mathematical physicist Roger Penrose in 1974. The discovery of quasicrystals by materials scientist Dan Shechtman earned him the 2011 Nobel Prize in chemistry. Meyer has sustained his interest in quasicrystals, and, together with Basarab Matei, in 2010 he helped to elucidate their mathematical structure.

In the 1970s Meyer made profound contributions to the field of harmonic analysis, which seeks to decompose complex functions and signals into components made of simple waves. Along with Ronald Coifman and Alan McIntosh, he solved a long-standing problem in the field in 1982 by proving a theorem about a construction called the Cauchy integral operator. This interest in harmonic decomposition led Meyer into wavelet theory, which enables complex signals to be "atomized" into a kind of mathematical particle called a wavelet.

Wavelet theory began with the work of, among others, physics Nobel laureates Eugene Wigner and Dennis Gabor, geophysicist Jean Morlet, theoretical physicist Alex Grossmann, and mathematician Jan-Olov Strömberg. During a conversation over the photocopier at the Ecole Polytechnique in 1984, Meyer was handed a paper on the subject by Grossmann and Morlet and was captivated. "I took the first train to Marseilles, where I met Ingrid Daubechies, Alex Grossmann, and Jean Morlet," he says. "It was like a fairy tale. I felt I had finally found my home."

From the mid-1980s, in what he called a "second scientific life," Meyer, together with Daubechies and Coifman, brought together earlier work on wavelets into a unified picture. In particular, Meyer showed how to relate Grossmann and Morlet's wavelets to the work of Argentinian mathematician Alberto Calderón, which had supplied the basis for some of Meyer's most significant contributions to harmonic analysis. In 1986 Meyer and Pierre Gilles Lemarié-Rieusset showed that wavelets may form mutually independent sets of mathematical objects called orthogonal bases.

Coifman, Daubechies, and Stéphane Mallat went on to develop applications to many problems in signal and image processing. Wavelet theory is now omnipresent in many such technologies. Wavelet analysis of images and sounds allows them to be broken down into mathematical fragments that capture the irregularities of the pattern using smooth, "well-behaved" mathematical functions. This decomposition is important for image compression in computer science, being used, for example, in the JPEG 2000 format. Wavelets are also useful for characterizing objects with very complex shapes, such as so-called multifractals, and Meyer says that they prompted his interest in the Navier–Stokes equations in the mid-1990s.

In the past twenty years Meyer's passion for the structure of oscillating patterns has led him to contribute to the success of the Herschel deep-space telescope mission, and he is working on algorithms to detect cosmic gravitational waves. Meyer's contribution to image processing is also wide ranging. In 2001 he proposed a mathematical theory to decompose any image into a "cartoon" and a "texture." This "cartoon plus texture" algorithm is now routinely used in criminal investigations to extract digital fingerprints from a complex background.

In such ways, Meyer's work has a relevance extending from theoretical areas of mathematics such as harmonic analysis to the development of practical tools in computer and information science. As such, it is a perfect example of the claim that work in pure mathematics often turns out to have important and useful real-world applications.

Meyer is a member of the French Academy of Science and an honorary member of the American Academy of Arts and Sciences. He was elected a Fellow of the AMS in 2012. His previous prizes include the Salem Prize (1970) and the Gauss Prize (2010), the latter awarded jointly by the International Mathematical Union and the German Mathematical Society for advances in mathematics that have had an impact outside the field. The diversity of his work, reflected in its broad range of application, reflects his conviction that intellectual vitality is kept alive by facing fresh challenges. He has been quoted as saying that when you become too much an expert in a field, then you should leave it—but he is wary of sounding arrogant here. "I am not smarter than my more stable colleagues," he says simply. "I have always been a nomad—intellectually and institutionally."

Some feel that Meyer has not yet had the recognition his profound achievements warrant, perhaps because he has been so selfless in promoting the careers of others and in devoting himself to mathematical education as well as research. "The progress of mathematics is a collective enterprise," he has said. "All of us are needed."

He has inspired a generation of mathematicians who have gone on to make important contributions in their own right. His collaborator on wavelet theory Stéphane Mallat calls him a "visionary" whose work cannot be labeled either pure or applied mathematics, nor computer science, either, but simply "amazing." His students and colleagues speak of his insatiable curiosity, energy, generosity, and openness to other fields. "You must dig deeply into your own self in order to do something as difficult as research in mathematics," Meyer claims. "You need to believe that you possess a treasure hidden in the depths of your mind, a treasure which has to be unveiled."

AMS President Kenneth A. Ribet said, "On behalf of the American Mathematical Society, it is my great pleasure to congratulate Professor Yves Meyer, recipient of the 2017 Abel Prize. Professor Meyer has been a visionary in a broad range of fields, including number theory and differential equations. His fundamental work in the theory of wavelets has transformed the world of signal processing and has led to a myriad of practical applications."

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized research scientists in the field of mathematics. The Committee is appointed for a period of two years. The members of the current committee are:

- John Rognes (Chair)
- Marta Sanz-Solé
- Luigi Ambrosio
- Marie-France Vignéras
- Ben J. Green

Previous recipients of the Abel Prize are:

- Jean-Pierre Serre (2003)
- Michael Atiyah and I. M. Singer (2004)
- Peter Lax (2005)
- Lennart Carleson (2006)
- S. R. S. Varadhan (2007)
- John G. Thompson and Jacques Tits (2008)
- Mikhail L. Gromov (2009)
- John Tate (2010)
- John Milnor (2011)
- Endre Szemerédi (2012)
- Pierre Deligne (2013)
- Yakov Sinai (2014)
- John F. Nash Jr. and Louis Nirenberg (2015)
- Andrew J. Wiles (2016)

—From an announcement of the
Norwegian Academy of Science and Letters

Photo Credit

The photo of Yves Meyer is ©B. Eymann—Académie des sciences.

Mathematics People

Takeda Awarded 2017–2018 Centennial Fellowship

Shuichiro Takeda

The AMS has awarded its Centennial Fellowship for 2017–2018 to SHUICHIRO TAKEDA. Takeda's research focuses on automorphic forms and representations of p-adic groups, especially from the point of view of the Langlands program. He will use the Centennial Fellowship to visit the National University of Singapore and work with Wee Teck Gan during the academic year 2017–2018.

Takeda obtained a bachelor's degree in mechanical engineering from Tokyo University of Science, master's degrees in philosophy and mathematics from San Francisco State University, and a PhD in 2006 from the University of Pennsylvania. After postdoctoral positions at the University of California at San Diego, Ben-Gurion University in Israel, and Purdue University, since 2011 he has been assistant and now associate professor at the University of Missouri at Columbia.

The Fellowship carries a stipend of US$91,000, an expense allowance of US$9,100, and a complimentary Society membership for one year.

Please note: Information about the competition for the 2018–2019 AMS Centennial Fellowships will be published in the "Mathematics Opportunities" section of an upcoming issue of the *Notices*.

—*Allyn Jackson*

Borcea Awarded Kovalevsky Lectureship

Liliana Borcea

LILIANA BORCEA of the University of Michigan has been chosen as the AWM-SIAM Sonia Kovalevsky Lecturer by the Association for Women in Mathematics (AWM) and the Society for Industrial and Applied Mathematics (SIAM). She was honored "for her distinguished scientific contributions to the mathematical and numerical analysis of wave propagation in random media, array imaging in complex environments, and inverse problems in high-contrast electrical impedance tomography, as well as model reduction techniques for parabolic and hyperbolic partial differential equations."

Borcea received her PhD from Stanford University and has since spent time at the California Institute of Technology, Rice University, the Mathematical Sciences Research Institute, Stanford University, and the École Normale Supérieure, Paris. Currently Peter Field Collegiate Professor of Mathematics at Michigan, she is deeply involved in service to the applied and computational mathematics community, in particular on editorial boards and as an elected member of the SIAM Council.

The Sonia Kovalevsky Lectureship honors significant contributions by women to applied or computational mathematics.

—*From an AWM announcement*

Pardon Receives Waterman Award

John Pardon

JOHN PARDON of Princeton University has been named the recipient of the Alan T. Waterman Award of the National Science Foundation (NSF) for "revolutionary, groundbreaking results in geometry and topology." The award is the nation's highest honor for scientists and engineers younger than thirty-five. It consists of a five-year grant worth US$1 million.

The prize citation reads: "Pardon is a Clay Research Fellow and professor of mathematics at Princeton University. His research focuses on geometry and topology, the study of properties of shapes that are unaffected by deformations, such as stretching or twisting. He is known for solving problems that stumped other mathematicians for decades and generating solutions that provide new tools for geometric analysis.

"In 2013, Pardon published a solution to the Hilbert-Smith conjecture, a mathematical proposition involving the actions of groups of 'manifolds' in three dimensions. Manifolds include spheres and doughnut-shaped objects.

"The conjecture originates from one of the twenty-three problems published in 1900 by German mathematician David Hilbert, which helped guide the course of twentieth-century mathematics. American topologist Paul Althaus Smith proposed a stronger version of the problem in 1941. This problem has connections to many other areas of mathematics and physics. Pardon's publication was notable for proving this long-standing conjecture, a major achievement in mathematics.

"Prior to that publication, as a senior undergraduate at Princeton, Pardon answered a question posed in 1983 by Russian mathematician Mikhail Gromov regarding 'knots,' mathematical structures that resemble physical knots, but are closed, instead of having any ends.

"Gromov's question involved a special class of knots called 'torus knots.' He asked whether these knots could be tied without altering or distorting their topology. Pardon figured out a way to use the distortion between two properties of knots—their intrinsic and extrinsic distances—to control their topology. He showed that torus knots are limited by their geometric properties, and can be tied without altering their topology.

"Pardon's solution has important applications in fluid dynamics and electrodynamics, calculating forces involved in aircraft movement, predicting weather patterns, determining the flow of liquids through water treatment plant pipelines, determining the flow of electrical charges, and more."

Pardon received his PhD in 2015 from Stanford University under the direction of Yakov Eliashberg. He was the recipient of an NSF Graduate Research Fellowship and of the Morgan Prize in 2012.

—From a National Science Foundation announcement

Gordon Awarded AWM Michler Prize

Julia Gordon

Julia Gordon of the University of British Columbia has been named the recipient of the 2017–2018 Ruth I. Michler Memorial Prize of the Association for Women in Mathematics (AWM). Gordon was selected to receive the Michler Prize because of her "wide range of mathematical talents" and the connection of her work with the research of several Cornell faculty members. Gordon's research is in the areas of representation theory of p-adic groups and of motivic integration.

Gordon received her PhD in 2003 from the University of Michigan, Ann Arbor, under the direction of Thomas C. Hales. She has been a postdoctoral fellow at the University of Toronto and at the Fields Institute for Research in Mathematical Sciences and spent a year at the Institute for Advanced Study.

The Michler Prize grants a midcareer woman in academia a residential fellowship in the Cornell University mathematics department without teaching obligations.

—From an AWM announcement

Ribet Awarded Brouwer Medal

Kenneth A. Ribet

Kenneth A. Ribet of the University of California Berkeley and president of the AMS has been awarded the 2017 Brouwer Medal by the Royal Dutch Mathematical Society (KWG). According to the prize citation, he was honored "for his contributions to number theory, in particular for the groundbreaking work in which he applies methods of algebraic geometry to number theoretical problems. This work later became of decisive importance for the proof of Fermat's Last Theorem." The prize is awarded every three years to a mathematician of international renown.

—From a KWG announcement

Clay Research Awards Presented

The Clay Mathematics Institute (CMI) has made a number of Research Awards for 2017.

Aleksandr Logunov of Tel Aviv University and Chebyshev Laboratory, St. Petersburg State University, and Eugenia Malinnikova of the Norwegian University of Science and Technology have received a Clay Research Award "in recognition of their introduction of a novel geometric combinatorial method to study doubling properties of solutions to elliptic eigenvalue problems." According to the prize citation, this work "has led to the solution of long-standing problems in spectral geometry, for instance the optimal lower bound on the measure of the nodal set of an eigenfunction of the Laplace-Beltrami operator in a compact smooth manifold (Yau and Nadirashvili's conjectures)."

Jason Miller of Cambridge University and Scott Sheffield of the Massachusetts Institute of Technology have received a Clay Research Award "in recognition of their groundbreaking and conceptually novel work on the geometry of the Gaussian free field and its application to the solution of open problems in the theory of two-dimensional random structures." The prize citation reads: "The two-dimensional Gaussian free field (GFF) is a classical and fundamental object in probability theory and


Jason Miller


Scott Sheffield

field theory. It is a random and Gaussian generalized function h defined in a planar domain D. Despite its roughness and the fact that it is not a continuous function, it possesses a spatial Markov property that explains why it is the natural counterpart of Brownian motion when the time-line is replaced by the two-dimensional set D. Miller and Sheffield have studied what can be viewed as level-lines of h and more generally flow lines of the vector fields $\exp(iah)$, where a is any given constant. This framework, which they call imaginary geometry, allows them to embed many Schramm-Loewner Evolutions within a given GFF. A detailed study of the way in which the flow lines interact and bounce off each other allowed Miller and Sheffield to shed light on a number of open questions in the area and to pave the way for further investigations involving new random growth processes and connections with quantum gravity."


Maryna Viazovska

Maryna Viazovska of Princeton University and École Polytechnique Fédérale de Lausanne has received a Clay Research Award "in recognition of her groundbreaking work on sphere-packing problems in eight and twenty-four dimensions. In particular, her innovative use of modular and quasimodular forms, which enabled her to prove that the E_8 lattice is an optimal solution in eight dimensions." The prize citation reads in part: "The result had been suggested by earlier work of Henry Cohn and Noam Elkies, who had conjectured the existence of a certain special function that would force the optimality of the E_8 lattice through an application of the Poisson summation formula. Viazovska's construction of the function involved the introduction of unexpected new techniques and establishes important connections with number theory and analysis. Her elegant proof is conceptually simpler than that of the corresponding result in three dimensions."

The awards will be presented at the 2017 Clay Research Conference at the University of Oxford in September 2017.

Note: See the feature story on Viazovska and her work in the February 2017 *Notices*.

—From a CMI announcement

Simons Fellows in Mathematics

The Simons Foundation Mathematics and Physical Sciences (MPS) division supports research in mathematics, theoretical physics, and theoretical computer science. The MPS division provides funding for individuals, institutions, and science infrastructure. The Fellows Program provides funds to faculty for up to a semester-long research leave from classroom teaching and administrative obligations. The mathematical scientists who have been awarded Simons Fellowships for 2017 are:

- Matthew Baker, Georgia Institute of Technology
- David Ben-Zvi, University of Texas at Austin
- Mladen Bestvina, University of Utah
- Lewis Bowen, University of Texas at Austin
- Tobias Colding, Massachusetts Institute of Technology
- Panagiota Daskalopoulos, Columbia University
- Aleksandar Donev, New York University
- Zeev Dvir, Princeton University
- Ezra Getzler, Northwestern University
- Anna Gilbert, University of Michigan
- Florian Herzig, University of Toronto
- John Imbrie, University of Virginia
- Jeff Kahn, Rutgers, The State University of New Jersey
- Jeremy Kahn, Brown University
- Michael Kapovich, University of California, Davis
- Boris Khesin, University of Toronto
- Kay Kirkpatrick, University of Illinois at Urbana—Champaign
- Nitu Kitchloo, Johns Hopkins University
- Alex Kontorovich, Rutgers, The State University of New Jersey
- Svitlana Mayboroda, University of Minnesota
- Chikako Mese, Johns Hopkins University
- Tomasz Mrowka, Massachusetts Institute of Technology
- Camil Muscalu, Cornell University
- Irina Nenciu, University of Illinois at Chicago
- Thomas Nevins, University of Illinois at Urbana—Champaign
- Hee Oh, Yale University
- Julia Pevtsova, University of Washington
- Andrei Rapinchuk, University of Virginia
- Daniel Ruberman, Brandeis University
- Mark Rudelson, University of Michigan
- Thomas Scanlon, University of California, Berkeley
- Natasa Sesum, Rutgers, The State University of New Jersey
- Gigliola Staffilani, Massachusetts Institute of Technology
- Nicolas Templier, Cornell University
- Benedek Valkó, University of Wisconsin—Madison
- András Vasy, Stanford University
- Alexander Volberg, Michigan State University
- Sijue Wu, University of Michigan
- Wei Zhang, Massachusetts Institute of Technology
- Maciej Zworski, University of California, Berkeley

—From a Simons Foundation announcement

NEWS

Spohn Receives Max Planck Medal

Herbert Spohn

Herbert Spohn of Technical University Munich has been awarded the 2017 Max Planck Medal for his "important contributions to statistical physics regarding the transition from microscopic physics to macroscopic phenomena." The prize citation reads in part: "His seminal contributions include the derivation of kinetic and diffusive behavior on the basis of classical and quantum many-body systems, the hydrodynamic limit for stochastic interacting particle systems, and the fluctuation behavior of surface growth models. Spohn has worked with great innovativeness and independence. His research has a major impact on the field of statistical mechanics of non-equilibrium systems."

His awards include the 2011 Dannie Heineman Prize for Mathematical Physics, the 2011 Leonard Eisenbud Prize for Mathematics and Physics of the AMS, the 2014 Cantor Medal, and the 2015 Henri Poincaré Prize.

—From a German Physical Society announcement

Prizes of the Canadian Mathematical Society

Robert McCann

The Canadian Mathematical Society (CMS) has awarded a number of prizes for 2017.

Robert McCann of the University of Toronto has been awarded the Jeffery-Williams Prize for Research Excellence. According to the prize citation, "McCann is an internationally recognized expert in applied mathematics at the forefront of the development of the theory and applications of optimal transportation. Together with his collaborators and peers worldwide, he has led a renaissance in the theory of optimal transportation, helping to transform it into one of the most vibrant and exciting areas in mathematics today." McCann received his PhD from Princeton University in 1994 for his thesis introducing the concept of displacement convexity under the supervision of Elliot H. Lieb. His honors include the Monroe H. Martin Prize (2001) and the Coxeter-James Prize of the CMS (2005). He is an elected member of the Royal Society of Canada (2014) and the Fields Institute of Mathematics (2015). McCann and his wife, Carolyn, live in Toronto, where, he says, they live in a home "originally built to accommodate the gardener of Casa Loma, a turn-of-the-century castle and folly which is now a Toronto tourist attraction. It was the site of the secret laboratory in which sonar was developed during the Second World War."

Stephanie van Willigenburg

Stephanie van Willigenburg of the University of British Columbia has been awarded the 2017 Krieger-Nelson Prize for her outstanding research contributions. The prize recognizes outstanding research by a woman mathematician. The citation reads in part: "Professor van Willigenburg is a leading expert in algebraic combinatorics, a vibrant area of mathematics that connects with many other fields of study, including representation theory, algebraic geometry, mathematical physics, topology, and probability. Her research and subsequent discoveries have focused on Schur functions, skew Schur functions, and quasisymmetric Schur functions, central topics within the field of algebraic combinatorics." She received her PhD from the University of St. Andrews, Scotland, in 1998. She has been a postdoctoral fellow at York University and a visiting assistant professor at Cornell University before joining the faculty at the University of British Columbia. She is a cofounder and organizer of the Algebraic Combinatorixx workshops at the Banff International Research Station to foster mentoring, collaborations, and networking for women in algebraic combinatorics and related areas. Van Willigenburg and her husband, medieval historian Niall Christie, live in "Hollywood North" (i.e., Vancouver). They have sung on movie soundtracks when not in the classroom.

Sabin Cautis

Sabin Cautis of the University of British Columbia has been awarded the 2017 Coxeter-James Prize for outstanding contributions to mathematical research. According to the prize citation, he "is a leader in the new and rapidly developing field of categorification as it relates to geometric representation theory, algebraic geometry, mathematical physics and low-dimensional topology. Categorification is a search for deeper structure behind invariants in algebra and topology." Cautis received his PhD from Harvard University in 2006 under the supervision of Joe Harris. He taught at Rice University, Columbia University, and the University of California before joining the University of British Columbia in 2013. He was an Alfred P. Sloan Fellow in 2011–2013 and received the André-Aisenstadt Prize in 2014.

Note: See van Willigenburg's short article in the April 2016 *Notices*.

—From CMS announcements

Resende and Žilinskas Awarded Carathéodory Prize

Mauricio Resende of Amazon.com, Inc., and Antanas Žilinskas of the Institute of Mathematics and Informatics, Vilnius University, Lithuania, have been awarded the 2017 Constantin Carathéodory Prize of the International Society of Global Optimization for fundamental contributions to theory, algorithms, and applications of stochastic global optimization. Resende was honored for his major work in combinatorial optimization, analysis of very large and massive databases, and metaheuristics. Žilinskas was recognized for his fundamental work on statistical models in global optimization and developing powerful Bayesian algorithms. The prize is awarded biennially to an individual (or a group) for fundamental contributions to theory, algorithms, and applications of global optimization. The prize carries a cash award of US$2,000 and a certificate.

—International Society of Global Optimization

Bender Awarded Heineman Prize

Carl M. Bender

Carl M. Bender of Washington University in St. Louis has been awarded the 2017 Dannie Heineman Prize for Mathematical Physics "for developing the theory of PT symmetry in quantum systems and sustained seminal contributions that have generated profound and creative new mathematics, impacted broad areas of experimental physics, and inspired generations of mathematical physicists." His research interests are in developing techniques such as asymptotics, perturbation theory, semiclassical methods, differential equations, complex variable theory, numerical methods, and combinatorics for solving difficult mathematical problems arising in theoretical physics. Bender tells the *Notices*: "I love music and play several instruments seriously, such as the clarinet, and I also love chess. I enjoy traveling to conferences, universities, and laboratories and meeting new and interesting people. I also love teaching very much and enjoy preparing students for mathematical competitions such as the Putnam exam."

The Heineman Prize is awarded annually in recognition of outstanding publications in the field of mathematical physics. The prize consists of US$10,000 and a certificate. It was established by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Physical Society and the American Institute of Physics.

—From a Heineman Foundation announcement

Putnam Prizes Awarded

The winners of the seventy-seventh William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America (MAA) and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded both to individuals and to teams.

The five highest ranking individuals each received a cash award of US$2,500. Listed in alphabetical order, they are:

- Joshua D. Brakensiek, Carnegie Mellon University
- Dong Ryul Kim, Harvard University
- Thomas E. Swayze, Carnegie Mellon University
- Samuel Zbarsky, Carnegie Mellon University
- Yunkun Zhou, Massachusetts Institute of Technology

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with members listed in alphabetical order) were:

- Carnegie Mellon University (Joshua D. Brakensiek, Thomas E. Swayze, Samuel Zbarsky)
- Princeton University (Eric D. Schneider, Zhuo Qun Song, Xiaoyu Xu)
- Harvard University (Pakawut Jiradilok, Dong Ryul Kim, David W. Stoner)
- Massachusetts Institute of Technology (Robert C. Shen, David H. Yang, Yunkun Zhou)
- Stanford University (Jie Jun Ang, Huy T. Pham, Albert R. Zhang)

The first-place team receives an award of US$25,000, and each member of the team receives US$1,000. The awards for second place are US$20,000 and US$800; for third place, US$15,000 and US$600; for fourth place, US$10,000 and $400; and for fifth place, US$5,000 and US$200.

Simona Diaconu of Princeton University was awarded the Elizabeth Lowell Putnam Prize for outstanding performance by a woman in the competition. She received an award of US$1,000.

—From an MAA announcement

Tapia Receives AAAS Public Engagement Award

Richard Tapia of Rice University has been named the recipient of the 2016 Public Engagement with Science Award of the American Association for the Advancement of Science (AAAS) for his "remarkable career blending world-class scholarship, admirable mentoring and profound contributions to science, technology, engineering and mathematics education and public engagement." He has devoted much time and effort to inspiring and encouraging women, minorities, and young people from

Richard Tapia

economically challenged communities to achieve in mathematics and science and has served as a model for other mathematicians in public engagement. Tapia received his PhD in 1967 from the University of California Los Angeles and taught there, as well as at the University of Wisconsin, before joining the faculty at Rice University. He has also been associated with the Baylor College of Medicine and the University of Houston. Among his many awards and honors is the 2011 National Medal of Science and the 2004 Distinguished Public Service Award of the AMS. He, along with David Blackwell, have been honored with a conference named after them, as well as the Blackwell–Tapia Prize, which honors a mathematician who has made significant contributions to research and to addressing the problem of underrepresentation of minorities in mathematics.

Tapia is the son of Mexican immigrants and was the first in his family to attend college. He and his twin brother, Robert, loved drag racing and worked on cars throughout high school. Tapia has used his knowledge of muscle cars and drag racing to connect with youth from communities underrepresented in the sciences and has delivered a popular talk titled "Math at Top Speed: Exploring and Breaking Myths in the Drag Racing Folklore" at many universities and professional conferences.

—From an AAAS announcement

Reiman and Williams Awarded 2016 von Neumann Theory Prize

Martin Reiman

Martin I. Reiman of Columbia University and Ruth J. Williams of the University of California San Diego have been awarded the John von Neumann Theory Prize for 2016. The citation reads in part: "The award recognizes seminal research contributions that Marty Reiman and Ruth Williams have made, over the past several decades, to the theory and applications of 'stochastic networks/systems' and their 'heavy traffic approximations.' These profound contributions have been and have further led to breakthroughs in stochastic operations research in general, and queueing theory in particular. Their analysis of complex stochastic networks under conditions of heavy traffic has not only led to the discovery and rigorous articulations of properties of the networks and penetrating insights into the operational laws of real-world systems they model, but also led to deep theoretical developments in the study of reflected diffusions."

About Reiman's work, the citation goes on to say, "Reiman's research is characterized by deep intuition and penetrating understanding of the physical and mathematical laws that govern the systems that he studies.… In Reiman's work, one sees real inventiveness combined with strong mathematical and expository skills, supported by a solid command of several distinct application domains. His research has influenced and inspired work by the very best people in stochastic OR, including several previous winners of the von Neumann Theory Prize." Williams's research, according to the citation, "is characterized by its mathematical depth and elegance. She has greatly influenced researchers in operations research, stochastic processes and mathematics, doing so through survey lectures and articles that are exemplary in clarity and insight. Her expositions have introduced the field to researchers and described challenging open problems and directions, which have spurred further research."

Ruth J. Williams

Martin Reiman received his PhD from Stanford University and was associated with Bell Laboratories until 2015. He has been associate editor of *Mathematics Operations Research* and of the *Annals of Applied Probability*. Ruth Williams received her PhD from Stanford University in 1983. She is a past president of the Institute of Mathematical Statistics and is a Fellow of the AMS, the American Academy of Arts and Sciences, and the American Association for the Advancement of Science, as well as a member of the National Academy of Sciences. She tells the *Notices*: "I grew up in Australia, where I received an excellent grounding in mathematics. I enjoy working on theoretical problems motivated by applications. For relaxation, I enjoy spending time outdoors, especially hiking."

The John von Neumann Theory Prize is awarded annually to a scholar (or scholars in the case of joint work) who has made fundamental, sustained contributions to theory in operations research and the management sciences. It is the highest prize given in the field. It carries a cash award of US$5,000.

—From an INFORMS announcement

Needell and Ward Awarded IMA Prize

Deanna Needell of Claremont McKenna College and Rachel Ward of the University of Texas at Austin have been awarded the 2016 IMA Prize in Mathematics and Its Applications. The prize citation reads in part: "While Needell is recognized for her contributions to sparse approximation, signal processing, and stochastic optimization, and Ward is recognized for her contributions to

Deanna Needell

Rachel Ward

the mathematics of machine learning and signal processing, much of their research overlaps. Their 2013 joint paper, 'Stable image reconstruction using total variation minimization,' was published in the *SIAM Journal on Imaging Sciences*. Their work has applications in medical imaging such as magnetic resonance imaging (MRI) scans, as well as in sensor and distributed networks, statistical problems, compression, and image processing problems."

Needell began her undergraduate studies as a veterinary science major. She tells the *Notices*: "After working as a vet tech, however, I realized it wasn't for me and eventually changed my major seven times before discovering my passion for mathematics. Now my love for animals resides in our rescue dog named 'Jacobi.'" Ward tells the *Notices*: "I had my first baby … (a girl named Mara) just a couple months after this prize was announced."

The prize is awarded annually by the Institute for Mathematics and Its Applications (IMA) to an individual or individuals within ten years of the PhD who have made a transformative impact on the mathematical sciences and their applications. The prize carries a cash award of US$3,000.

—From an IMA announcement

Berners-Lee Receives ACM Turing Award

Tim Berners-Lee of the Massachusetts Institute of Technology and the University of Oxford has been named the recipient of the 2016 A. M. Turing Award of the Association for Computing Machinery (ACM). He was honored "for inventing the World Wide Web, the first web browser, and the fundamental protocols and algorithms allowing the Web to scale." The Turing Award carries a cash prize of US$1,000,000.

—From an ACM announcement

Guggenheim Fellowship Awards to Mathematical Scientists

The John Simon Guggenheim Memorial Foundation has announced the names of 173 scholars, artists, and scientists who were selected as Guggenheim Fellows for 2017. Selected as fellows in mathematics and statistics were:

- David Blei, Columbia University
- Hee Oh, Yale University
- Gigliola Staffilani, Massachusetts Institute of Technology

Guggenheim Fellows are appointed on the basis of impressive achievement in the past and exceptional promise for future accomplishments.

—From a Guggenheim Foundation announcement

ANZIAM Prizes Awarded

Australia and New Zealand Industrial and Applied Mathematics (ANZIAM), a division of the Australian Mathematical Society, has awarded medals for 2017 to two mathematical scientists. Kate Smith-Miles of Monash University has been awarded the 2017 E. O. Tuck Medal. According to the prize citation, she "has developed a broad toolkit of mathematical techniques, as well as the language and communication skills necessary to collaborate with researchers and industry partners, on problems from fields as diverse as manufacturing design, epidemiology, neural prosthetics, computer vision, finance, and stem cell modelling." The Tuck Medal is a midcareer award given for outstanding research and distinguished service to the field of applied mathematics. Alys Clark of the University of Auckland was awarded the 2017 J. H. Michell Medal. According to the prize citation, she "works at the interface between mathematical modeling and the biological sciences with research interests in modeling the transport of nutrients in the complex and heterogeneous structures of the lungs, placenta and ovaries to guide clinicians in making medical decisions." The medal recognizes an outstanding young researcher in applied/industrial mathematics.

—From an ANZIAM announcement

Balaguer Prize Awarded

Antoine Chambert-Loir of Université Paris–Diderot Paris 7, Johannes Nicaise of Imperial College London, and Julien Sebag of Université Rennes 1 have been awarded the 2017 Ferran Sunyer i Balaguer Prize for their joint monograph, *Motivic Integration*. The prize is awarded by the Ferran Sunyer i Balaguer Foundation for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics, in which the applicant has made important contributions. It carries a cash award of 15,000 euros (approximately US$16,000). The winning monograph will be published in Birkhäuser's series Progress in Mathematics.

—From a Balaguer Foundation announcement

2017 AAAS Fellows Elected

The American Academy of Arts and Sciences (AAAS) has elected 188 new fellows and 40 foreign honorary members for 2017. Following are the new members whose work involves the mathematical sciences.

- Michael Aizenman, Princeton University
- Manjul Bhargava, Princeton University
- Christopher D. Hacon, University of Utah
- Robert V. Kohn, New York University
- Maryam Mirzakhani, Stanford University
- Vera Serganova, University of California Berkeley

Elected as a foreign honorary member was Johannes Sjöstrand, Université de Bourgogne.

—From an AAAS announcement

Watson Fellowship Awarded

Dina Sinclair, a senior math major at Harvey Mudd College, has been awarded a T. J. Watson Fellowship for her project, "High School Math Contests: Gender, Culture, and Access." Sinclair will travel to seven countries to learn "how logistical and social choices affect student perception of math contests." The Thomas J. Watson Fellowship is a one-year grant of US$30,000 for purposeful, independent exploration outside the United States, awarded to graduating seniors nominated by one of forty partner colleges.

—From a Watson Foundation announcement

SIAM Fellows Elected

The Society for Industrial and Applied Mathematics has elected its class of fellows for 2017. Their names and institutions follow.

- Zhaojun Bai, University of California, Davis
- Peter Benner, Max Planck Institute for Dynamics of Complex Technical Systems
- Angelika Bunse-Gerstner, Universität Bremen
- Emmanuel Candés, Stanford University
- Rama Cont, Imperial College London
- Ricardo Cortez, Tulane University
- Lieven De Lathauwer, KU Leuven
- Bart De Moor, KU Leuven
- Andreas Griewank, Yachay Tech University
- Helge Holden, Norwegian University of Science and Technology
- Panayotis Kevrekidis, University of Massachusetts, Amherst
- Vipin Kumar, University of Minnesota
- Karl Kunisch, Karl Franzens Universität Graz
- Monique Laurent, Centrum Wiskunde and Informatica
- Mark A. Lewis, University of Alberta
- Lois Curfman McInnes, Argonne National Laboratory
- Igor Mezic, University of California, Santa Barbara
- Michael Kwok-Po Ng, Hong Kong Baptist University
- James Renegar, Cornell University
- Andrew J. Sommese, University of Notre Dame
- Joel H. Spencer, Courant Institute of Mathematical Sciences
- Gabor Stepan, Budapest University of Technology and Economics
- Daniel B. Szyld, Temple University
- Jean E. Taylor, Courant Institute of Mathematical Sciences and Rutgers University
- Marc Teboulle, Tel Aviv University
- J. A. C. Weideman, Stellenbosch University
- Carol S. Woodward, Lawrence Livermore National Laboratory
- Kevin Zumbrun, Indiana University

—From a SIAM announcement

Regeneron Science Talent Search

Three young scientists whose work involves the mathematical sciences are among the top winners in the 2017 Regeneron Science Talent Search.

Aaron Yeiser, eighteen, of Schwenksville, Pennsylvania, was awarded second place and US$175,000 for his development of a new mathematical method for solving partial differential equations on complicated geometries. Arjun Ramani, eighteen, of West Lafayette, Indiana, received third-place honors and US$150,000 for blending the mathematical field of graph theory with computer programming to answer questions about networks. Laura Pierson, seventeen, of Oakland, California, received the sixth-place award and US$80,000 for her use of theoretical algebra to study the representation theory of mathematically symmetric groups.

The Regeneron (formerly Intel) Science Talent Search is the United States' oldest and most prestigious science and mathematics competition for high school seniors. It is administered by the Society for Science and the Public.

—From a Society for Science and the Public announcement

Hertz Foundation Fellows Announced

The Fannie and John Hertz Foundation has announced the awarding of twelve fellowships for graduate work in science and mathematics. Each fellow receives five full years of support toward their PhD degrees. The two new fellows in the mathematical sciences are Linus Hamilton of the Massachusetts Institute of Technology and Hannah Larson of Harvard University.

—From a Hertz Foundation announcement

*NSF Graduate Research Fellowships Awarded

The National Science Foundation (NSF) has awarded a number of Graduate Research Fellowships for fiscal year 2017. Further awards may be announced later in the year.

This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US$30,000 per year for a maximum of three years of full-time graduate study. Information about the solicitation for the 2018 competition will be published in the "Mathematics Opportunities"' section of an upcoming issue of the *Notices*.

Following are the names of the awardees in the mathematical sciences selected so far in 2017, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

- Dylan Airey (University of Texas at Austin), University of Texas at Austin
- Colin C. Aitken (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- Ayah K. Almousa (University of Wisconsin–Madison), Cornell University
- Ethan E. Alwaise (Emory University), Emory University
- Benjamin E. Anzis (University of Idaho), University of Idaho
- Adam A. Atanas (Harvard University), Harvard University
- Jess Banks (Oberlin College), University of California, Berkeley
- Robert J. Baraldi (North Carolina State University), University of Washington
- Rajendra Beekie (University of Minnesota–Twin Cities), University of Minnesota–Twin Cities
- Daniel C. Bourgeois (Louisiana State University and Agricultural and Mechanical College), Louisiana State University and Agricultural and Mechanical College
- Elizabeth C. Chase (University of North Carolina at Chapel Hill), University of North Carolina at Chapel Hill
- Katherine G. Christianson (Columbia University), University of California, Berkeley
- Olivia J. Chu (New York University), Princeton University
- Kyle R. Conniff (Saint Norbert College), University of California, Irvine
- Jane I. Coons (State University of New York College at Geneseo), North Carolina State University
- Katherine L. Cordwell (University of Maryland), University of Maryland
- Colin Defant (University of Florida), University of Florida
- Calvin Deng (Harvard University), Harvard University
- Derek T. Driggs (University of Colorado at Boulder), University of Colorado at Boulder
- Patrick Eastham (Florida State University), Florida State University
- Matthew S. Farrell (Cornell University), University of Washington
- Yakir M. Forman (Yeshiva University), Yeshiva University
- Ian Francis (University of Tennessee, Knoxville), University of Tennessee, Knoxville
- Peter J. Geldermans (Purdue University), William Marsh Rice University
- Tyler Genao (Florida Atlantic University), Florida Atlantic University
- Jonathan Gerhard (James Madison University), James Madison University
- Claudio J. Gonzales (New Mexico Institute of Mining and Technology), University of Chicago
- Elizabeth Greco (Kenyon College), Cornell University
- Araminta Gwynne (Northwestern University), Northwestern University
- Lily Hu (Harvard College), Harvard University
- Melissa N. Jay (Colorado College)
- Edna L. Jones (Rose-Hulman Institute of Technology), Rutgers University
- Anya Katsevich (University of North Carolina at Chapel Hill), University of North Carolina at Chapel Hill
- Lea Kenigsberg (State University of New York at Stony Brook), State University of New York at Stony Brook
- Brianna G. Kozemzak (Saint Mary's College), Saint Mary's College
- Vivian Z. Kuperberg (Cornell University), Cornell University
- Taylor M. Lagler (Millersville University), University of North Carolina at Chapel Hill
- Hannah K. Larson (Harvard University), Harvard University
- Kathy Y. Li (Harvard University)
- Runjing Liu (Duke University), University of California, Berkeley
- Jaime G. Lopez (Arizona State University), Arizona State University
- Wesley Maddox (Case Western Reserve University), Case Western Reserve University
- Holly B. Mandel (Yale University), Rutgers University
- Kaitlyn Martinez (Colorado College), Colorado School of Mines
- Jacob M. McNamara (Harvard University), Harvard University
- Emili V. Moan (Winthrop University), North Carolina State University
- Gwyneth Moreland (University of Michigan, Ann Arbor), University of Michigan, Ann Arbor
- Jessica K. Nadalin (University of California Berkeley), Boston University
- Seth V. Neel (Harvard University), University of Pennsylvania

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The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at `www.nsf.gov/dms` *and for the Directorate of Education and Human Resources at* `www.nsf.gov/dir/index.jsp?org=ehr`. *To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at* `www.nsf.gov/mps/dms/about.jsp`.

- Khanh P. Nguyen (University of Houston), University of Houston
- Michelle Nuno (University of California Riverside), University of California Irvine
- Colin Okasaki (Harvey Mudd College), Harvey Mudd College
- Omomayowa Olawoyin (University of Texas at Arlington), University of Texas at Arlington
- Yujia Pan (University of Chicago), University of Michigan, Ann Arbor
- Peter S. Park (Princeton University), Princeton University
- Gregory J. Parker (Harvard College), Harvard College
- Nicole Pashley (Queen's University), Harvard University
- Danilo T. Perez (University of Puerto Rico, Cayey), University of Puerto Rico, Cayey
- Alex Pieloch (Duke University), Duke University
- Kim C. Raath (Arkansas State University), William Marsh Rice University
- Eugene Rabinovich (Duke University), University of California, Berkeley
- Bradley A. Rava (University of Southern California), Yale University
- Catherine E. Ray (George Mason University), University of Chicago
- Diego A. Rojas (University of Florida), University of Florida
- Adrienne Sands (Washington University), University of Minnesota–Twin Cities
- Rebecca Santorella (The College of New Jersey), The College of New Jersey
- Mark A. Sellke (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- Susannah Shoemaker (Pomona College), Princeton University
- Elizabeth Spencer (University of Maryland, College Park), Boston University Sargent College
- Ashvin A. Swaminathan (Harvard College), Harvard University
- Jonathan B. Tidor (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- Kimberly T. Truong (Brown University), Brown University
- Sahana Vasudevan (Harvard University), Harvard University
- Andres R. Vindas Melendez (University of California Berkeley), San Francisco State University
- George Y. Wang (University of Southern California), University of Pennsylvania
- Max H. Weinreich (Yale University)
- Robert M. Weylandt (Princeton University), William Marsh Rice University
- Yuval Wigderson (Princeton University), Stanford University
- Samuel Zbarsky (Carnegie Mellon University), Carnegie Mellon University
- Albert Zhang (Stanford University), Stanford University
- Robin Zhang (Stanford University), Stanford University

—NSF announcement

Ludvig Faddeev (1934–2017)


Ludvig Faddeev

Ludvig Faddeev, eminent Russian theoretical physicist and mathematician, was famous for his contributions to the quantum mechanical three-body problem and for his work on the quantization of non-abelian gauge field theories. He was one of the scientists to bridge the gap between mathematics and physics.

Faddeev served the International Mathematical Union for twelve years. As president (1987–1990), he worked tirelessly on improving cooperation among mathematicians from all regions of the world and fostering good relations with other scientists. Very recently, he was deeply involved in the Russian bid for a 2022 International Congress of Mathematicians in St. Petersburg.

For many years, Faddeev was head of the St. Petersburg Department of the Steklov Institute of Mathematics of the Russian Academy of Sciences, and he was founder of the Euler International Mathematical Institute in St. Petersburg. He received the 2006 Henri Poincaré Prize and the 2008 Shaw Prize.

—Adapted from IMU-Net Newsletter No. 82

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AMERICAN MATHEMATICAL SOCIETY

PUSHING LIMITS

From West Point to Berkeley & Beyond

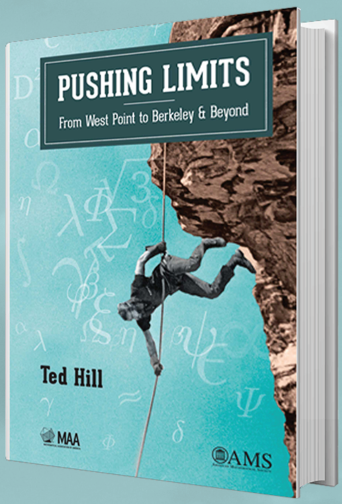
Pushing Limits

From West Point to Berkeley & Beyond

Ted Hill, *Georgia Tech, Atlanta, GA, and Cal Poly, San Luis Obispo, CA*

Recounting the unique odyssey of a noted mathematician who overcame military hurdles at West Point, Army Ranger School, and the Vietnam War, this is the tale of an academic career as noteworthy for its offbeat adventures as for its teaching and research accomplishments.

This book is co-published with the Mathematical Association of America. 2017; approximately 334 pages; Hardcover; ISBN: 978-1-4704-3584-4; List US$45; AMS members US$36; Order code MBK/103



... captivating memoir reveals an intriguing character who is part Renaissance Man, part Huckleberry Finn. Fast-paced and often hilarious ... provides some penetrating and impious insights into some of our more revered institutions.

—Rick Atkinson, three-time Pulitzer Prize winner,
***author of* The Long Gray Line**

Ted Hill is the Indiana Jones of mathematics. A West Point graduate, [he] served in Vietnam, swam with sharks in the Caribbean, and has resolutely defied unreasoned authority. With this same love of adventure, he has confronted the sublime challenges of mathematics. Whether it's discovering intellectual treasures or careening down jungle trails, this real life Dr. Jones has done it all.

—Michael Monticino, professor of mathematics and
special assistant to the president, U. North Texas

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Joseph B. Keller (1923–2016)

Communicated by Stephen Kennedy

Editor's Note: Alice S. Whittemore, George Papanicolaou, Donald S. Cohen, L. Mahadevan, and Bernard J. Matkowsky have kindly contributed to this memorial article.

Alice S. Whittemore

Joe Keller (right) with his younger brother Herbert, who also became a mathematician, in their childhood home of Paterson, NJ, 1930.

Joseph Bishop Keller was an applied mathematician of world renown, whose research interests spanned a wide range of topics, including wave propagation, semi-classical mechanics, geophysical fluid dynamics, operations research, finance, biomechanics, epidemiology, biostatistics, and the mathematics of sports. His work combined a love of physics, mathematics, and natural phenomena with an irrepressible curiosity to pursue explanations of practical and often playful enigmas.

Keller was born in Paterson, New Jersey in 1923. His father emigrated from Russia in flight from anti-Semitic pogroms and sold liquor wholesale during Prohibition, later opening a bar. His mother, who emigrated from England where her family had fled Russia, did the bookkeeping for the bar. Joe's father challenged his two sons (Joe and Herbert) with math puzzles at dinner. Both boys became mathematicians.

Joe received his bachelor's degree from New York University in 1943. He was an instructor in physics at Princeton during 1943–1944 and then became a research assistant in the Columbia University Division of War Research during 1944–1945. After receiving his PhD from NYU in 1948, he joined the faculty there and participated in the building of the Courant Institute of Mathematical Sciences. In 1979, he joined Stanford University where he served as an active member of the departments of mathematics and mechanical engineering.

In 1974, when Joe was based at the Courant Institute, he was asked by Donald Thomsen, the founder of the SIAM Institute for Mathematics and Society (SIMS), to oversee a SIMS transplant fellowship at NYU Medical Center. The mission of SIMS was to help research mathematicians apply their training to societal problems by temporarily transplanting them from their theoretical academic environments to settings engaged with applied problems. Joe agreed to Thomsen's request and promptly arranged an initial meeting with the prospective transplant, me. My research goals were to explore biomedical problems involving epidemiology and biostatistics, rather than the problems in group theory with which I had been struggling. During this two-year fellowship, Joe provided characteristically supportive and inspiring mentoring at weekly blackboard sessions.

Keller on his 1943 graduation from New York University.

We were intrigued by the unknown biological mechanisms underlying the formation of cancers and the role of environmental exposures (like cigarette smoke) in causing them. Several investigators had proposed quantitative theories of carcinogenesis in attempts to explain the temporal behavior of cancer occurrence in humans and laboratory rodents exposed to carcinogens. The theories involved the transformation of normal cells to malignancy and the subsequent proliferation of malignant cells to form a detectable tumor.

A major puzzle was why the incidence of many cancers increases with the fifth or sixth power of an individual's age. For example, if I'm twice as old as you, my cancer risks are 32 or 64 times yours. To account for this puzzle, investigators proposed that a normal cell

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becomes malignant after suffering multiple sequential mutations. Once transformed, it proliferates more rapidly than normal cells until its progeny form a detectable tumor. Exposures to carcinogens increase cancer incidence by increasing the common rate at which the mutations occur. This theory explains the steep rise in incidence with age, but it conflicts with the linear or quadratic dependence of incidence on dose of carcinogens. To avoid this conflict, Armitage and Doll instead assumed that the sequential mutations occur at different rates, only some of which are affected by a given carcinogen. Although this multistage theory accounts for some of the cancer data, it has some biological defects. The main defect is the lack of any direct experimental evidence that cancer occurs in more than two stages. This led Armitage and Doll to modify the theory so that only two stages were needed, but cells in the intermediate stage could multiply more rapidly than normal cells, providing an increasing supply of partially transformed cells awaiting a final transition to malignancy.

[Problems] must be neither too difficult nor too mathematically trivial.

These and other theories had been presented in a wide variety of medical journals with varying degrees of mathematical rigor, and their predictions were compared to observed patterns of cancer incidence in epidemiological or experimental data. Joe quickly saw the utility of a review and synthesis of the many different theories, using a common framework of stochastic equations to describe the rates of cell transformation and tumor growth in our joint 1978 SIAM Review paper on “Quantitative Theories of Carcinogenesis.”

The modified two-stage theory and its subsequent extensions form the basis for much of our current understanding of how genetic and non-genetic factors cause human cancers. They explain certain enigmas regarding the roles of cigarette smoking in lung cancer, of mammographic density in breast cancer, and of genetics in colorectal cancer and retinoblastoma (a malignancy of the eye). For example, they explain why lung cancer incidence rates are proportional to the fourth power of smoking duration but only to the square of smoking rate (packs per day) and why the lung cancer incidence rates for ex-smokers never drop to the rates of lifelong nonsmokers.

Mathematician and future wife Alice Whittemore with Joe Keller in NJ, 1975.

Joe’s catholic curiosity about all natural phenomena included a broad range of biological, biomechanical, and bio-mathematical enigmas. In addition to his work on lung cancer in smokers and skin tumors in mice, he wrote about the mechanisms underlying breathing attacks in asthmatics, vision in kittens, running in athletes, crawling in worms, genetics in families, hearing in humans, and leukemia in children. This work, together with his major contributions in mathematical physics and applied mechanics, has earned him some of the world’s highest scientific honors, including the National Medal of Science and foreign membership in the Royal Society of London.

When asked how he selected problems to work on, Joe replied that he needed to understand the phenomenon underlying the problem and he needed to see that it had a mathematical aspect. Moreover he needed to see that its

Whittemore and Keller on a long-distance hiking trail (Grande Randonnée) in France, circa 1995.

solution could be enlightening and significant. And, like Goldilock's chair, it must be neither too difficult nor too mathematically trivial. Joe loved working with students and colleagues, many of whom approached him for help in solving problems they had encountered in their own work, and these associations led to fruitful collaborations.

One evening soon after the start of my transplant fellowship at NYU, Joe and I were both hungry at the conclusion of our weekly blackboard session, so we decided to grab a bite at a nearby Chinese restaurant. My mathematical education continued at dinner, but now the subject was inverse problems. Joe explained that in a typical mathematical problem, you are given a question for which you must provide an answer. In an inverse problem, however, you are given the answer and your job is to provide a corresponding question. For example, you might be asked to provide a question to which the answer is "1 and -1," and your question might be, "What are the roots of the equation $x^2 - 1 = 0$?" To further clarify the concept, he then gave me several other answers in need of corresponding questions. When he asked me to provide a question whose answer was "Dr. Livingston I presume," I suggested a question based on Stanley's search for Livingston near the Nile. He promptly informed me that, while that question was acceptable, the optimal question was "And what is your full name, Dr. Presume?"

That evening, neither of us knew that we were starting a life of work, fun, and happiness together that would last until his death 42 years later.

George Papanicolaou

I first met Joe Keller in September 1965 when I arrived at the Courant Institute as a graduate student interested in applied mathematics. Joe was giving a one-semester course in methods of theoretical physics, and I took it, along with some other first-year graduate courses in mathematics. What was different about Joe's teaching was that it presented mathematics as empowering, even in this basic class, and not as an edifice to be maintained and enriched for its own sake. He had an outward oriented view of mathematics and an infectious confidence of what could be done with a deeper understanding of formulations and methodology, which fit well the problems under consideration, often coming from outside mathematics. His viewpoint was somewhere between mathematics, physics, and engineering, because he cared about methods, their analysis and scope, but he also cared about the emerging results and their interpretation, and the potential impact they could have.

In the second half of the 1960s Joe had already branched into many different research areas, quite distinct from diffraction theory, which was where the bulk of his research was in the 50s and early 60s, culminating in his geometrical theory of diffraction. This theory was a brilliant synthesis of high-frequency asymptotic analysis, the careful use of the very few exact solutions of diffraction problems that were available, and a consistent geometric interpretation of the wave components that contributed to the overall wave field. The insight that his geometrical theory of diffraction provided came from its elegant conceptual simplicity. From the location and geometry of corners, edges, and other features of the scattering environment, one could in principle write down the high-frequency form of the field anywhere, but this would require numerical computations except in relatively simple cases. Scattering phenomena in the high-frequency regime even today are difficult to analyze with direct numerical computations because the region of interest could extend over thousands or millions of wavelengths in a three-dimensional setting.

There was something special about Joe Keller and seminars.

I learned diffraction theory and uniform asymptotic methods by going to the Friday afternoon applied mathematics seminar at Courant. This was a more general theory that could, in diffraction for example, express the field across a shadow boundary in a way that was almost exact near it. It was a lot more elaborate and less geometrical than Joe's original geometrical theory of diffraction. But it was also much closer to a complete mathematical theory, something that the theory of Fourier integral operators and microlocal analysis began to develop in the 1970s and later. Joe was deeply interested in this, and some of his results on uniform asymptotic methods are still the best available today, but he was already moving into nonlinear waves, random media, and many other areas. He was much more interested in new horizons where asymptotic methods could be used in a transformative way.

There was something special about Joe Keller and seminars. He was a very good listener and very quickly got to the essential point of what was being presented ahead of everyone else in the room, including often the speaker. In anything that had to do with waves, asymptotics, and related areas his comments and questions during the seminars made a huge difference to me at the time, and no doubt to others, because I saw where the fault lines were in the methodology and the theory. It is hard to get that by reading papers or listening to a polished presentation at a seminar, except when Joe Keller is in the room. After Joe left Courant in the 1970s to go to Stanford, we at Courant worked hard to keep the Friday seminar going in the tradition that Joe had started, and I think we did well in several areas. When I joined him at Stanford in the early 1990s, the first thing I realized was that Joe's reputation as a special participant at seminars was firmly established. And that was not just at the Friday applied math seminar that he had transplanted at Stanford, but also in materials science, in applied physics, and especially in the fluid dynamics seminar. His active participation in the geophysical fluid dynamics program every summer at Woods Hole is

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Joe, here at a Stanford seminar in 2003, was a very good listener and very quickly got to the essential point of what was being presented ahead of everyone else in the room.

fondly remembered by generations of graduate students for his deeply knowledgeable comments that kept things sharply focused. At the fluids seminar at Stanford, Joe's comments and questions were expected, especially when the speaker was a bit obscure or too fast. And, of course, Joe would have no patience with pretentious speakers as his not always diplomatic comments indicated, to the delight of the regulars at the seminar.

I worked with Joe on waves in random media, which is a field that was very much influenced by his thinking. Wave propagation in inhomogeneous media had received attention in the early part of the twentieth century, and even earlier by Maxwell and others, but it really became important after the Second World War, because of sonar and to a lesser extent radar, as well as seismic exploration. It had already been rather well developed to address the passage of light through the atmosphere, motivated by astronomy and astrophysics. This was done with radiative transport theory, which was phenomenological and unrelated to Maxwell's theory. Joe formulated clearly the mathematical aspects of waves in random media, including the identification of regimes for different types of phenomena depending on the several length scales and other parameters, somewhat like the dimensionless formulation of fluid dynamics. Throughout the 60s and early 1970s he lectured often on this topic and his seminars were very well received.

He moved on to many other research areas: nonlinear waves, various fluid dynamics problems including lubrication theory, the effective properties of materials with and without variational principles, homogenization theory in materials, effective boundary conditions for numerical computations, mathematical biology, and even American options in financial mathematics. His contributions have had an enormous and lasting impact in applied mathematics.

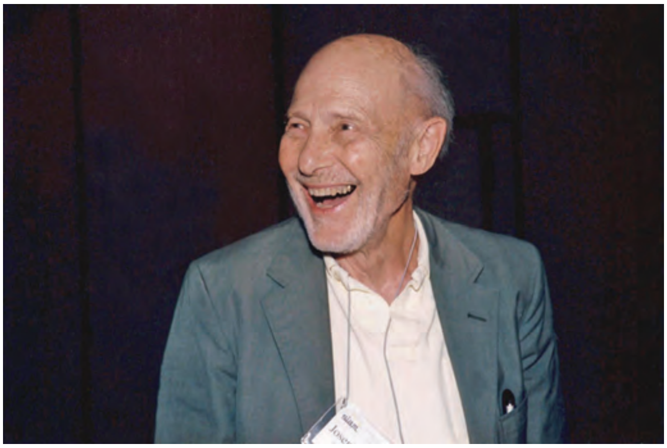
Donald S. Cohen

I was a graduate student at New York University (NYU) (1959–1962) and then a post-doc part time between NYU and Columbia University (1963–1965). The NYU group was not known as the Courant Institute at that time, and they were housed in two old buildings near Washington Square in New York City.

Richard Courant had succeeded in bringing a few refugees from Nazi Germany to New York to attempt to continue the great Göttingen tradition, and while I was there Courant, K. O. Friedrichs, Fritz John, Wilhelm Magnus, and J. J. Stoker were senior people still lecturing and teaching courses. Young people, slightly older than I, including Cathleen Morawetz, Jürgen Moser, Peter Lax, Louis Nirenberg, Paul Garabedian, Harold Grad, and Joe Keller, were then creating the reputations for which they later received many prestigious awards. I took courses from several of them and listened to many lectures from the others.

Twentieth-century physics produced important and difficult systems of differential and integral equations, and much of mathematics at Göttingen was devoted to attempts to understand their solutions. Theory and techniques from many parts of analysis were studied in these attempts. That same philosophy dominated NYU. Functional analysis, algebra, topology, geometry, and approximate methods were also considered with that goal in mind. It was a fantastic place to be.

One of the younger stars-to-be was Joe Keller. He insisted on being called Joe by everyone. Reputation and respect were derived from his command of mathematics and the way he ran his group. He was playful, could be chided and equally chided back, but it was clear that he was in command. His domain was the seventh floor of a small old building at 25 Waverly Place. His door in the middle of a short hallway was always open, and so were those of the surrounding grad students and post-docs. This was prior



Joe was playful, could be chided and equally chided back.

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to the existence of the desktop computer and the internet, so people were physically present, and the excitement for mathematics was everywhere. Joe was easily approachable, and he was often in the various offices where he supervised research in a wide variety of areas.

An important part of Joe's existence at that time was the weekly basketball seminar involving both those with very little ability as well as some with very serious ability. As in all his endeavors Joe was very competitive. He himself was only moderately skilled, but he was extraordinarily aggressive, and he didn't mind getting physically abused, as long as it was clean, legal basketball. The real purpose of the seminar was to have a good time and then go to Wah Kee Chinese Restaurant in New York City's Chinatown. Joe always collected the check, told each of us how much to pay, and then paid the total, remarking that his service charge was to supplement his salary. I have no doubt that he personally paid a good portion of the meals for those of us who were grad students.

Joe was partially bald and, for a long while, had a full bushy beard. One morning he appeared clean shaven and said that a total stranger told him that he had his head on upside down, thus the shave. (It was well known that one of Courant's admonitions was to tell a story as it should be told rather than how it actually happened.)

I never took a course that he taught nor read more than a few parts of some of his papers. Nevertheless, Joe did more to shape my early outlook and development than anyone else. He often came into my office for a few hours late in the afternoon, wondering about some problem then on his mind. Almost everything he saw suggested a mathematical problem to him. The question was how to formulate tractable problems from which answers could be extracted to give reasonable explanations of the phenomena. I was a physics major as an undergraduate and had learned a great deal of classical physics at both Brown and Cornell before I found my intellectual home at NYU. Seeking dimensionless groupings and looking at equations that replaced the basic general electromagnetic equations or those from fluid or solid mechanics was just done by a physicist, and ad hoc "laws" (optimally conservation laws) were accepted as answers. I did this when I could (not very often as a grad student), and it clearly was unacceptable to Joe, who wanted to know where these things came from by some rational process applied to rigorously derived theory. Joe's goal was to make the nature of approximations rationally follow from his manipulations and to be able to give explanations with the meaning of the approximations clear. Moreover, he wanted to answer deeper questions when no theory of any kind existed. In those days much of his work was to eventually provide a beautiful theory of high frequency diffraction of both penetrable and impenetrable bodies and through homogeneous and inhomogeneous media. He worked at the blackboard until nothing more seemed promising. Watching all this, done by an exceptionally gifted researcher, was a wonderful education.

Joe wanted to know everything that was being done by everyone. In addition to my selection of graduate courses, one day he suggested that I go to a series of lectures by Friedrichs on spectral theory of operators in Hilbert space. He and I both attended; the attendees consisted of almost all the faculty and a few grad student and post docs. We also went to lectures by Nirenberg on L^p estimates for solutions and their derivatives of elliptic partial differential equations satisfying general boundary conditions, and lectures by Moser (who was visiting) on what would later become part of KAM theory and the difficulty of small divisors for quasiperiodic orbits in dynamical systems. Joe studied and knew more pure mathematics than is generally known about him (he taught the graduate course on topology the year before I arrived); he was able to rapidly assimilate the pertinent ideas and how they would be useful to him for some of the many problems he had in mind.

One Monday morning after he asked several of us gathered in an office what we had done during the weekend, he was asked what he had done. He replied simply, "Oh, I got married." That was his somewhat playful announcement of his marriage to Evelyn Fox, a post-doc who was never a part of the physics-oriented research done by the rest of us and whom none of us knew that Joe was courting.

After that, Joe, for obvious reasons, spent less time in his office. There was a new building and a new name was given to the department. The grad students called the new building the Courant Hilton to contrast its elegance with the dilapidated conditions of the two old buildings. The senior professors occupied large corner offices, and their people were scattered throughout the building. I thought that the exciting atmosphere of 25 Waverly Place disappeared, and Joe often told me that it was significantly different for him.

After that, I saw Joe many times throughout the years at Caltech, where I've been for over fifty years and Joe's brother Herb is a professor, and at Los Alamos where I consulted. We always talked in depth about math and physics, and his curiosity and intensity never diminished. Everything he looked at seemed to suggest something needing investigation, and when he presented his results, as he often did in talks, the extraordinary depth and originality of his investigations became apparent.

My persistent memory of him at all ages is of the young Joe Keller, mentally and physically very active, often playful, telling terrible jokes involving what he called inverse problems, and deeply interested in learning new things and solving extremely difficult problems with a true teacher's desire to lecture on the results, thereby continually educating succeeding generations of interested people. Part of this group of those fortunate to have interacted with him constitutes what some have called the Keller School.

L. Mahadevan

Applied mathematics in the middle of the twentieth century was the intellectual continuation of nineteenth-century natural philosophy, which included mechanics, thermodynamics, optics, hydrodynamics, and electromagnetism. After the Second World War, and particularly with the dawn of the Space Age, the subject blossomed to include the creation of new mathematical tools to solve analytically intractable problems approximately and the application of mathematical ideas creatively to engineering, physics, biology, and beyond.

While it is difficult to imagine any individual excelling in both of these domains, Joseph Keller, perhaps the pre-eminent applied mathematician of this era, did just this. He was recognized for his foundational mathematical contributions in the domains of asymptotic analysis, perturbation methods, and hybrid numerical-analytical methods, and their deployment over a very wide range of areas, including wave propagation, quantum, statistical, and continuum mechanics, and transport phenomena in both deterministic and stochastic settings.

Following his PhD in 1948 at New York University, Keller thrived in the mid-century intellectual ferment at the Institute for Mathematical Sciences, which his mentor Richard Courant had set up. Over nearly three decades, his work expanded from initial studies in wave propagation to include the entire gamut of natural philosophy, quantum and statistical mechanics, and applications to engineering. He spent the last three and a half decades of his life at Stanford, where he expanded his interests further into engineering and biology, with occasional forays into medicine, sports, and finance. His celebrated studies in these fields led to many honors and have been written about in depth by others here and elsewhere.

> *[He could formulate a question] when others did not even realize that there was something to be asked.*

In addition to the specific problems that he illuminated and the techniques that he created, there are a number of scientific and mathematical themes that appear repeatedly in his work: his exquisite taste in questions and problems, his use of analogies to illuminate problems in one area with ideas from another, and a deep physical intuition for and mathematical economy in creating and using techniques to solve problems. Surely others will see more and different threads in the rich tapestry that he wove, but the following vignettes of his approach to applied mathematics-as-a-science might serve to open a window into how he thought.

An enduring hallmark of his style was an ability to formulate a tractable mathematical question in any subject, often when others did not even realize that there was something to be asked. This led to a wide range of papers in which he explored such problems as the conditions, often for a fair coin toss (only possible for a coin spinning about a diameter, sometimes referred to as the Keller flip, and asymptotically correct in the limit of large angular and vertical velocities, *Amer. Math. Monthly*, 1986), the number of shuffles needed to mix a deck of cards (seven, deduced using a simple argument that complemented earlier work by Persi Diaconis), the mechanics of impacting rigid bodies in the presence of friction (that can lead to very counter-intuitive motions such as the reverse bounce, *J. App. Mech.*, 1986), and a correction to Archimedes' principle for buoyant objects [3] (that accounted for the effects of surface tension, *Phys. Fluids*, 1998), among others. A guiding principle in many of these analyses is that there are fruits aplenty at the fertile boundary between two scientific fields and sometimes quite literally between two media. Keller had a keen eye for how to spot and pick these fruits!

The all-too-human need to optimize served as a wellspring of problems that he dipped into repeatedly. Likely inspired by his work on inverse scattering problems in electromagnetism, Keller studied many optimal design, control, and strategy problems in engineering, mechanics, and physiology. For example, he provided the solution to a problem first posed by Lagrange—the shape of the strongest column [1], given its volume and length—by posing it in terms of an eigenvalue problem for a particular Sturm-Liouville operator and using isoperimetric inequalities to derive the result that a triangular cross-section is best. Later, others refined and generalized this, with continuing implications for structural optimization. Keller also calculated optimal strategies for running a race (for short distances, an anaerobic strategy is best, but once the distance is larger than about 300m, one must switch to an aerobic strategy by accelerating to the maximum speed as quickly as possible and then staying at that speed, coasting past the finish line with no energy left, *Phys. Today* 1973), or for maximizing longevity (some, but not too much caloric restriction and exercise is good, and Keller practiced as he wrote). He also worked out strategies for ranking baseball teams (in an early precursor to Google's page-rank algorithm, for a Christmas lecture in the 1970s at New York University, he showed how the Perron-Frobenius theorem could be used to guarantee the existence of a ranking vector based on the relative strengths of the teams involved), and for best inspection practices in a factory (formulated and solved as a variational problem). In each case, he was able to get to the mathematical essence of a problem that not only illuminated its specific origin, but also its broader ramifications.

In addition to adroitly exploiting the no-man's-land between domains, Keller was a master of using analogies to bridge fields, aided by a combination of physical intuition and mathematical expertise. In one of his most cited works in the area of semi-classical mechanics that straddles the quantum and the classical [2], Keller resolved a puzzle linking different frameworks of quantum mechanics. Using an analogy between the high-frequency limit of the reduced wave equation and the Schrödinger equation, he showed how to solve problems in one domain using knowledge from the other, elaborated on by him and Sol Rubinow (pictured), and many others later. In a different setting, recognizing the inherent linearity and analogies between the equations of electrostatics, slow fluid flow,

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Joe Keller (right) and Solomon Rubinow, pictured here in Woods Hole MA, mid-1960s.

and elastostatics, Keller realized how the presence of a small parameter (due to geometry, as in narrow slits; due to large contrast in properties, as in dielectric mixtures, suspensions, etc.) makes the problem of determining the effective properties amenable to analysis using the theory of harmonic functions. This allowed him to deduce a theorem for the effective conductivity of composites [3], and variations on this idea over the following decades have been the source of much elaboration in applied mathematics in the context of homogenization electromagnetism theory—a subject that deals with the statistically averaged properties of materials, with implications for engineering.

The virtue of concision seems to have been another (perhaps unspoken) theme. Indeed, a number of his papers were no more than a page or two, had few references, and yet packed an impact. In one such exemplar (*Amer. J. Phys.*, 1959), he showed that the large amplitude motion of a string that has been stretched to many times its rest length can be described by a linear wave equation, and later generalized it to finite deformed continua. Interestingly, this has a macroscopic realization in a toy—a helical spring called a Slinky®, and a microscopic realization in highly stretched polymers. In another short paper, half a page long, he discussed how to reconcile the transition from one power law to another in turbulent boundary layer flow using a soluble differential equation (*Phys. Fluids*, 2002). And in a four-paragraph paper in *Theoretical Population Biology* (2004), he tackled the link between mortality rate and age, showing how a simple model can explain its initial increase followed by saturation in old age!

Keller was happiest when discussing a new problem or solution and wore his fame lightly. Although he received many prestigious awards, he got a particular pleasure from two Ig Nobel Prizes for "research that makes you laugh, and then makes you think." Very likely, after him, the prestige of the Ig Nobel went up! The first was for explaining the teapot effect (shared with J. M. Vanden-Broeck) and the second was for explaining the dynamics of ponytails (shared with R. Ball, R. Goldstein, and P. Warren, who calculated their shape). He came to the ceremony, wizard-like, wearing a pony-tailed fez to explain his idea and enjoyed the riotous ceremony, paper planes and all, as the author can attest to. And what exactly did he do and why should one care?

Anyone who has poured tea from a kettle knows to be wary of the dribble along the spout that can ruin the rest of the afternoon. Most onlookers asked to explain this effect will mumble something about surface tension. Inspired by experiments of the rheologist Marcus Reiner (who poured colored tea underwater, where interfacial forces are unimportant but the effect persists), Keller wrote a note (Teapot effect, *J. Appl Phys.*) in the 1950s about how inertial effects (and Bernoulli's principle) can explain this phenomenon, and later worked out a more complete theory. Some sixty years later, likely inspired by the swaying ponytails of runners in front of him during his hikes, he asked why a ponytail swings from side to side while the head bobs up and down? The key is an instability of a flexible string forced periodically and vertically at its boundary. Keller showed ("Ponytail motion," *SIAM J. Appl. Math.*) that under some fairly general assumptions, it is possible to derive a Hill equation for this phenomenon, which arises generically in the theory of parametrically driven oscillators, in celestial mechanics and a variety of other situations (and is the theoretical basis for the Nobel Prize winning ion-trap of Paul and Dehmelt). This insight allowed Keller to deduce the conditions for instability and show that for a ponytail bobbing at a frequency of a few hertz, the most unstable length is about 25 cm. Test it yourself if your hair is long enough!

His eclectic interests in science along with a warm and friendly demeanor made him easily approachable and an inspiration to all. He was particularly encouraging of young mathematicians and scientists, and mentored many both formally and informally. At the Woods Hole Oceanographic Institution, where he was on the summer faculty at the Geophysical Fluid Dynamics Program for more than fifty years, it was commonplace to see him on the porch every afternoon, with students and colleagues who sought him out as a universal consultant. His scientific legacy—an unquenchable curiosity about nature, and a humility embodied in the belief that every problem is worth thinking about, and learning from—will last.

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Bernard J. Matkowsky

When I attended college in the 1950s, very few programs in Applied Mathematics existed, and those were largely unknown. While I liked problems in engineering and science, I wasn't satisfied with the methods employed to analyze them. I preferred the approach of mathematicians, though

I still wanted to be involved with problems in science and engineering. It wasn't until I was a graduate student in electrical engineering in 1960 that some professors suggested that I might find what I was looking for at the Institute of Mathematical Sciences attached to NYU. One professor went even further, suggesting that I work only with Joe Keller. Fortunately for me, I followed this advice and am grateful to this day for having done so. Joe has had a profound influence on, and has served as an inspiration to, me as well as to generations of students at NYU and at Stanford who are part of the so-called Keller School of Applied Mathematics.

As a student I read as many of Keller's papers as I could get hold of. Of course I read his papers on the geometrical theory of diffraction. I read his papers on the asymptotic solution of eigenvalues, which was related to my thesis work. I read his papers on boundary layer problems, as well as his work on perturbation of nonlinear boundary value problems and bifurcation theory and a host of others. I learned from them all. Though unrelated to my thesis, one perhaps lesser-known paper nevertheless made a particularly strong impression on me.

In science and engineering a number of different theories based on different mathematical models were often proposed to explain a given phenomenon. However, it wasn't always clear which model was appropriate under what conditions. Some models were postulated in an ad-hoc manner, some were based on simplifying assumptions, while others were purported to be "approximations" to a more general model, though they were not derived in a systematic manner, nor was it always clear how the different models were related to one another. In the purported approximation of one by another, simpler, model some terms were retained, while others, though possibly of comparable size, were discarded. Needless to say, these approaches were not very satisfying, especially to a young student.

In "A Theory of Thin Jets," Keller (with his PhD student Mortimer Weitz) considered the problem of jet flow, specifically, the problem of determining the shape of the jet and the velocity distribution within it. The theory of jets is based on the equations of hydrodynamics, though only a limited number of problems were successfully treated this way. More general problems were treated with the simpler hydraulic theory, in which both the pressure and the velocity on each cross section are assumed to be constant, though these assumptions are incompatible with the equations of hydrodynamics. Thus, hydraulic theory is based on different, approximate equations. Joe writes: "Two questions which immediately arise are: What is the relationship between the two theories and How can the results of Hydraulic Theory be improved? In this paper we answer these questions by presenting a method of solution of the hydrodynamic problem as a series in powers of the jet thickness divided by some typical length of the jet (epsilon), i.e., an asymptotic expansion in epsilon. The first term of this solution is found to be the solution given by hydraulic theory, thus answering the first question. The higher order terms of the series yield corrections to the hydraulic theory, thus answering the second question."

Joe's paper was a revelation to me, not only for presenting a nice solution to the given problem, but more importantly, for presenting a systematic, rational approach to a general question that had long troubled me. I have employed this approach to analyze a variety of problems in various fields in the years since.

He had an uncanny ability to describe real world problems by simple yet realistic models.

Joseph Bishop Keller was the foremost contemporary creator of mathematical techniques to solve problems in science and engineering. He earned this reputation by his outstanding research contributions to both mathematical methodology and a wide variety of areas of application. Through his own work, as well as that of his students and other scientists with whom he interacted, he had a profound influence on the way that problems are formulated and solved mathematically. Joe combined unmatched creativity in the development of mathematical methods with very deep physical insight. He had an uncanny ability to describe real world problems by simple yet realistic models, to solve those mathematical problems by sophisticated techniques (many of which he himself created), and then to explain the results and their consequences in simple terms. He was a master of asymptotics and a virtuoso in showing how to adapt ideas found useful in one area to others. His work is characterized by originality, depth, breadth, and elegance, and the results he obtained have sustained importance. We briefly describe certain highlights.

One of Joe's most outstanding contributions is the Geometrical Theory of Diffraction (GTD), which he originated for solving problems of wave propagation. He began thinking about such problems during World War II while working on sonar for the Columbia University Division of War Research. GTD is an important extension of the Geometrical Theory of Optics (GTO), in which wave propagation is described by rays. The extension includes phenomena such as diffraction and the occurrence of signals where GTO predicts none. Joe developed a systematic way to treat high-frequency wave propagation and thus derived and solved the equations determining the rays, the paths along which signals propagate, as well as those governing how signals propagate along the rays. He predicts what happens as the rays encounter obstacles or inhomogeneities of the medium in which they travel. Prior to Joe's work, only a few isolated problems were solved and understood, and there was no general theory for the solution

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of more complex and technologically important problems. Now there exist books devoted to Joe's theory. Engineers and scientists employ his systematic theory to this day. Indeed, his theory is indispensable for those working on radar, antenna design, and general high-frequency systems in complicated environments. His theory has been and is still applied to a wide variety of other problems in which signals are transmitted by waves. Such problems occur in acoustics (as in sonar), elastodynamics (as in quantitative non-destructive testing), and seismic exploration for oil, to name but a few. It is commonplace in all these fields to see articles that read, "we employ Keller's method to…"

Joe also showed that his methods for wave propagation could be extended to other classes of problems, such as semi-classical mechanics. In this fundamental and penetrating work, Joe generalized work of Planck, Bohr, Sommerfeld, Wilson, Einstein, and Brioullin to derive the correct quantization rules for non-separable systems, thus yielding results valid in any coordinate system. His results, referred to as the Einstein-Brioullin-Keller (EBK) quantization rules, are currently employed by many chemical physicists and other scientists. In his work on semi-classical quantization he introduced an important measure corresponding to the number of times a closed curve passes through a caustic surface. This measure, later generalized to curves on Lagrangian manifolds by Maslov, is referred to as the Keller-Maslov index. This index was subsequently extended by Joe to eigenvalue problems in bounded domains, not necessarily associated with quantum mechanics, but governed by general systems of partial differential equations.

Joe's work has stimulated a vast literature in both the United States and abroad, not only in many areas of science and engineering where his methods and results are routinely employed, but also in the mathematics community, where his work has been taken up by pure mathematicians. For example, his work has been the impetus for a number of developments in the theory of Fourier integral operators and Lagrangian manifolds.

In addition, Joe often opened up directions of investigation by considering problem areas, that were then enthusiastically taken up by the research community. His pioneering work on the evolution of singularities of nonlinear wave equations is one such example, as is his work on bifurcation theory and nonlinear eigenvalue problems, to which scant attention was paid until the notes of his seminar appeared, and which is now one of the hottest topics of investigation by both pure and applied mathematicians.

Joe also considered problems of wave propagation through heterogeneous, turbulent, or random media, involving the transmission of signals through media such as the atmosphere and oceans, in which fluctuations occur due to the properties of the medium. He originated two methods that are very widely used. The first is the smoothing method, for problems involving small amplitude variations. The second is a multiple scale method, for problems corresponding to rapidly varying coefficients. The second method is capable of dealing with fluctuations that are not small in size, but rather small in scale. This theory, since taken up by others, now known as the theory of homogenization, has had volumes written on it. In each case, Joe showed how to systematically replace the fluctuating coefficients by effective coefficients, which are appropriate averages of the fluctuating coefficients. He then extended the work to show how to systematically derive effective equations for all sorts of problems, not necessarily associated with wave propagation. These include problems of composite media and problems of determining the large-scale macroscopic behavior of a medium that exhibits small-scale microscopic heterogeneity. His work was characterized by a simple formulation, which overcame the nonuniformities restricting earlier theories.

Joe, pictured here at the South Street Seaport, New York, 1990s, began thinking about problems in wave propagation during World War II while working on sonar for the Columbia University Division of War Research.

No stranger to national service, Joe worked on many problems related to national security, and he served on various advisory boards, national panels, and committees. After his work on sonar for the Columbia University Division of War Research, he worked on problems of underwater explosions, in order to predict the shock wave and water waves to be expected at the Bikini atomic bomb tests. At the time there was concern about producing a tsunami that might devastate Japan and other Pacific countries. His analysis showed there was no such danger. He also spent time at Argonne and Los Alamos national laboratories, studying hydrogen bomb explosions. In the early 1950s he served, with Von Neumann, on a committee on underwater atomic bombs for the Air Force Special Weapons Project (AFSWP), to consider the effects of A-bomb explosions on ships and submarines. He headed another project on A-bomb explosions for the AFSWP. During the late 1960s he was a member of JASON, a group of high-level consultants to the Defense Department and other governmental agencies on scientific and technical matters. He served as consultant to AFSWP on other projects, to the US Naval Air Development Center, to the US Army Chemical Corps, and to Argonne, Brookhaven, and Los Alamos national laboratories.

Bernard J. Matkowsky was a 1966 PhD graduate of the Courant Institute of Mathematical Sciences with Joe Keller. This photo dates from 1986, when Keller was awarded the Nemmers Prize from Northwestern University.

For more on Joseph Keller, see the interview with Keller in the August 2004 issue of Notices (`www.ams.org/notices/200407/fea-keller.pdf`) *and his Google scholar profile* (`https://scholar.google.com/citations?user=Pbn6aU8AAAAJ&hl=en`)

Joe was a teacher and expositor *par excellence*. He twice received the MAA's Lester Ford Award for outstanding expository writing. He received awards from all three major US mathematical societies, from various engineering societies, and from national scientific societies in the United States and abroad. The approximately 60 PhD students and numerous post-doctoral associates whom he has trained, now successful applied mathematicians in their own right, further attest to Joe's impact.

Finally, there is Joe Keller the man. Countless numbers of mathematicians, engineers, and scientists have come to him through the years to benefit from his acumen and understanding. To each he listened patiently, contributed helpful insights, and offered words of advice and encouragement. For us he was simply "Joe," teacher, colleague, and friend. The world has lost a giant. He will be sorely missed; his legacy endures.

Photo Credits

The first 7 photos in this article are courtesy of Alice S. Whitemore.

The final photo in this article, of Keller and Matkowsky, is courtesy of Bernard J. Matkowsky.

A Conversation with Mu-Fa Chen

Davar Khoshnevisan and Edward Waymire

Mu-Fa Chen is among China's most distinguished and influential living resident probabilists. He and his student F. K. Wang found sharp estimates for the eigenvalues of the Laplacian on a manifold in terms of the Ricci curvature and diameter by probabilistic methods, as explained in his 2005 Springer monograph *Eigenvalues, Inequalities, and Ergodic Theory*. Having spent his entire career in China, Professor Chen's invited address at the May 16, 2016 Frontier Probability Days Conference, held at the University of Utah, presented a welcome opportunity to interview him.

Mu-Fa Chen, official portrait as a member of Academy of Science, 2006, Beijing.

Mu-Fa Chen: I grew up in the countryside, in a very small village of about eighty people in southern China. I had not seen a train and never rode a bus; we only rode bicycles.

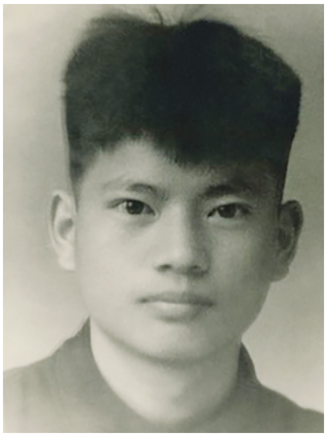
Khoshnevisan/Waymire: *What do you remember about your school?*

Mu-Fa Chen: I was especially lucky because, during my time, two very good teachers—one from Shaman University and the other from Tsinghua University—moved to my middle school.

In the beginning, I studied mathematics by myself. I wanted to create something. I did not know how to learn mathematics, so I just looked for exercises. I did exercises everyday.

Khoshnevisan/Waymire: *At what age was this?*

Mu-Fa Chen: At fourteen, I think. Then I read some popular little books written by famous Chinese mathematicians. I spent about one or two years reading these small books. Then one day the teacher who came from Tsinghua University told me, "No, no, you cannot just read these things. You should study calculus." So, I started to learn calculus by myself. I spent two or more years studying calculus and algebra in middle school. So I am lucky because of my teachers.



Chen graduated from high school in 1965.

Khoshnevisan/Waymire: *Geometry also?*

Mu-Fa Chen: Yes. In middle school we had so much homework that we had to plan our time carefully in order to get everything done. Even now, I am surprised when I recall that period; not only middle school, but also other periods, for example when I was in the Guizhou province for six years.

I wrote two Masters theses, learned from Loéve's book, went to more than fifty factories, taught middle school, translated two books, and wrote several articles. All in six years.

Khoshnevisan/Waymire: *You taught yourself how to read Loéve's book?*

Mu-Fa Chen: Yes. My teacher suggested that I read Loéve's book because I did not know much probability. My teacher was a probabilist, so he suggested to me to first read half of the first volume by William Feller, *Introduction to Probability Theory*. That was in my sophomore year, and my teacher was my advisor. I spent three months reading this and when I went to the Guizhou province I found out that I needed more training. So I wrote to my teacher to ask for his help.

It was a strange situation, and you may not be able to understand it. After a few months, the Cultural Revolution

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That's the reason I can keep doing mathematics, because the people need it.

happened. We had little freedom, and I could not talk with my teacher for several years.

Khoshnevisan/Waymire: *What age were you at this time?*

Mu-Fa Chen: Nineteen, I think. We went to work at a factory and slept in the same room, forty people at a time. My teacher was there sleeping by me. Even then we could not talk. We had no such freedom.

Khoshnevisan/Waymire: *You mean that you could not talk about mathematics?*

Mu-Fa Chen: We could not talk about anything except in special cases. For example, when there was nobody around us, we could say hello to each other. When I moved to the Guizhou province I wanted to further my studies. So I wrote a letter to my advisor, asking for advice. My advisor went to the old bookshops and bought about fifteen books for me, maybe for about two dollars. Many scientists could not work at that time. Therefore, they sold their books to the bookstores. Everything was very cheap. One of those books was the third edition of Loéve's book. I spent two and a half years reading that book.

Khoshnevisan/Waymire: *Do you know S. S. Chern from Berkeley?*

Mu-Fa Chen: Yes. One day S. S. Chern suggested that I go to Berkeley to study with Jack Kiefer. Kiefer is one of the founders of optimization. I had written four papers on that topic. So Chern suggested to me to study optimization with Kiefer. But, during that period, I was on my way to probability theory, and could not change.

Khoshnevisan/Waymire: *Was your interest in optimization related to your work in the factories?*

Mu-Fa Chen: Yes. Two months before I left Beijing for Guizhou, I attended a talk by K. Hua, a top mathematician in China. K. Hua went to a factory and talked about optimization to the workers there. One of my classmates asked me to attend this talk. I attended that talk but it surprised me so much. Hua gave a large number of examples of how mathematics improved the working conditions, the working results, and so on. So when I went to the Guizhou province, maybe the next day, I tried to find out which factories were interested in this approach. The workers asked me to teach them mathematics because I was a teacher. They would drive to my home, because I walked, and bring me to their homes; a very lovely story in that period. That's the reason I can keep doing mathematics, because the people need it.

Khoshnevisan/Waymire: *So what was the next phase of your life after this?*

Mu-Fa Chen: After I finished my study of Loéve's book, I was lucky to find a new paper by Zhen-Ting Hou, a probabilist from the Hunan province, not far from Guizhou. In 1974, Hou published a uniqueness criterion for Q-processes of time-continuous Markov chains in the nonconservative case. The problem in the conservative case was solved by G. E. H. Reuter in 1957. In 1975 I contacted Hou, and then I started to study Markov chains. One year later I got a chance to visit him. I spent two months in Changsha. Every day we studied Kai-Lai Chung's book *Markov Chains with Stationary Transition Probabilities*, but not inside a room. We went out on a hill, you see. So every day we studied Chung's book out on a hill in secret, because there was not much freedom. Then I translated about half of Chung's book into Chinese.

Khoshnevisan/Waymire: *Was Chung visiting China in that period?*

Mu-Fa Chen: His visit was around 1977. After the Cultural Revolution people were looking for new research directions. Chung introduced us to a new direction by Dobrushin's group, called Random Fields. I returned to Beijing in 1978 and started our study of Random Fields for nearly one semester. Then we saw several articles by Tom Liggett, and we learned of Spitzer's ideas on interacting particle systems. Particle systems were more close to

Chen in the Rocky Mountains, Boulder, Colorado, 1982.

us because we were working on time-continuous Markov chains. That was the beginning step. From that time on I finally had the working environment that I needed to fully devote myself to mathematical research.

Khoshnevisan/Waymire: *Did people from Frank Spitzer's group visit you in China?*

Mu-Fa Chen: Yes. Spitzer visited me for forty-five days in 1984, though he was not well at that time. Maybe we asked him to lecture too much. To this Spitzer would joke and say, "I have now become a lecturing machine!"

Khoshnevisan/Waymire: *Did you know R. L. Dobrushin's group as well?*

Mu-Fa Chen: Certainly. Dobrushin visited us for forty-five days in 1988. I also visited him, and his group, and I now have a lot of Russian books.

Khoshnevisan/Waymire: *Do you read Russian also?*

Mu-Fa Chen: Yes, but not so much now. I recall that there was very heavy snow in the winter when I visited Moscow. Dobrushin spent half a day with me going to shops to buy a lot of books. Finally, he took me to a restaurant where he was very proud to say that it was the first private restaurant in Moscow. We even had a joint research project and the members of the groups visited each other for some years afterwards to work on the project.

Whenever I met someone I asked for advice.

Khoshnevisan/Waymire: *Was your first visit to Dobrushin in the 1970s?*

Mu-Fa Chen: Actually, it was several years later, at the end of 1988. I returned to the university only in 1978.

Khoshnevisan/Waymire: *You mentioned G. E. H. Reuter. Did you meet him as well?*

Mu-Fa Chen: Yes, I met Reuter at the International Conference on Probability Theory in Cambridge in April 1987. Reuter had made great contributions to the theory of Markov chains. I asked him, and also David Kendall, how to do research. I asked everybody questions, because, you see, I had to teach myself. So whenever I met someone I asked for advice. When I asked how to do research Reuter answered that his supervisor Littlewood told him not to read anything before doing research. I often feel that I have not listened to him enough, but I have remembered his advice.

Khoshnevisan/Waymire: *It seems to us that you absorbed many of those influences, and then added to them in your work. Even in your lecture today, we saw that. From optimization all the way through Markov chains to quantum field theory, interacting particle systems, and random fields.*

Mu-Fa Chen: Yes. That is because I was not well educated by the university system. Somehow I had more freedom to go from here to there to ask questions.

Khoshnevisan/Waymire: *In your own mathematical work, is there something special that you feel happiest about?*

Mu-Fa Chen: Yes. I will give you two examples. The first example is our study of infinite-dimensional mathematics. We started from local, finite-dimensional mathematics. We had many models from nonequilibrium statistical mechanics in mind, at least sixteen models. First, we had to prove the uniqueness of local processes; physicists are not interested in this [Laughs]. Then, the question was to prove uniqueness for high-dimensional Markov chains. The only known theory was based on solving a homogeneous equation. The process is unique when the equation has only the trivial [zero] bounded solution. In high dimensions, this equation consists of infinitely many variables. I had no idea how to solve such an equation. So I spent five years on this problem. In the end I found a beautiful sufficient condition which covers all the examples or models that I had in mind. I also proved that the condition is necessary in all computable cases. I didn't know about the more general case. That was in 1983, when I returned from the United States to China. Surprisingly, a few months ago, a researcher from the Netherlands proved that my sufficient condition is also necessary for a large class of Markov chains. That's the first example.

The second example comes from my study of the first nontrivial eigenvalue. I published the first paper in 1991. At the time, one could compute precisely the principal eigenvalue of the generator of a Markov chain in only two or three examples. If you take a look at this paper and compare it with what I talked about today, you will see how far we have come since then. The four volumes of the collections of papers in my homepage[2] record the long journey that we have traveled in the past twenty-five years.

Also, regarding the geometric case, some time ago I might have given a talk here on Riemannian geometry. With my former student, Feng-Yu Wang, we found a new variational formula for the first nontrivial eigenvalue of Laplacian on a Riemannian manifold. This is exciting because people have worked on such problems for many years, and our formula includes most of the previously-known results.

Khoshnevisan/Waymire: *How did you get interested in that particular problem? Were you reading S.T. Yau's work?*

Mu-Fa Chen: Yes. At the beginning, I wanted to describe phase transition using the first nontrivial eigenvalue. I learned this idea from Tom Liggett, and also from Richard Holley and Daniel Stroock.

Khoshnevisan/Waymire: *Did Holley and Stroock also come to China?*

Mu-Fa Chen: Yes. Stroock was my supervisor when I visited the States. From him, I learned Malliavin calculus, large deviations, and many other things. He has visited China many times. I also arranged the visits of Tom Liggett, Rick Durrett, and Frank Spitzer. During 1988–1989, in particular, there was a special year dedicated to probability and statistics at Nankai University at S. S. Chern's Institute. At that time, Holley and Stroock were in China. We benefitted a great deal from the interactions with American probabilists.

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[2] `math0.bnu.edu.cn/~chenmf`

Chen being interviewed by Khoshnevisan and Waymire at the Frontier Probability Days Conference, Salt Lake City, 2016.

Khoshnevisan/Waymire: *In particular, at that time, did you know geometry? Did you have to learn geometry?*

Mu-Fa Chen: Geometry was popular in China because of S. S. Chern. So I did learn the fundamental points of the theory. One geometer came to my university to give a short course. I attended his course; that was the first time I learned some geometry. The second time was when I visited Daniel Stroock here. He also gave some lectures on geometry. All this happened in the early 1980s. After that, I studied geometry selectively.

I learned from the third chapter of a book by S. T. Yau on Riemannian geometry. That chapter concentrated on the first eigenvalue and I wanted to borrow tools from it. Somehow, for reasons of luck, we went the opposite direction, using coupling techniques to recover all of these results. I kept thinking that there was something left to improve upon, some places to go further. But we didn't know where to go. This was a challenging problem until one day I realized that we could find a good way to mimic the eigenfunction. This turned out to be important. That was my initial contribution to the eigenvalue problem until today. After we found out how to mimic the eigenfunction, the story was complete. Since I could not solve this problem for a long time, I got very tired, and sometimes was tempted to publish just what I had. But I ignored it and went to smoke a cigarette instead. I was a smoker at that time. It calmed me when I smoked. Unexpectedly, one day after having a smoke, suddenly a new idea came. Be careful, this by no means encourages smoking! In fact, I quit smoking years ago [Laughs]. During that time, my student F. Y. Wang was in the UK. When he came back, I told him the idea in ten minutes or so. Sometimes people say that it is the coupling and the distance method; we needed to understand how to choose the distance. In other words, we needed to understand how to mimic the eigenfunction and that is the key point.

Khoshnevisan/Waymire: *I have heard that you have also translated a lot of mathematics into Chinese.*

Mu-Fa Chen: Yes. But we did not publish much of that. We just mimeographed those notes and distributed them to our colleagues. I still have some copies.

Khoshnevisan/Waymire: *Was that a lot of work?*

Mu-Fa Chen: Certainly. This was from 1972 to 1976. In the next few years, around 1978, even in the early 1980s, we still often copied papers by hand. The copy machine was not popular until maybe the end of the 1980s.

Khoshnevisan/Waymire: *You seem to have a good facility for language. Did you have that from childhood?*

Mu-Fa Chen: Yes. In middle school and the first two years of university I only learned Russian. The foreign language of the whole country was Russian at that time. At a teacher's suggestion, I started to learn English. I had asked my teacher whether it was okay for me to do research in mathematics without learning English, and he said no without hesitation [Laughs]. So I had to learn English by myself, privately. At first I borrowed an English mathematics book. I checked every word using a dictionary, letter by letter. Since I didn't know how to pronounce the words, I simply memorized the letters in each word. After I learned two to three hundred words it became too difficult to continue in this way. So I asked the same teacher whether I could learn English without having to speak it. He immediately replied, "No, that is impossible." [Laughs], I could not remember what I had learned. So I borrowed a self-learning English book. That book taught me to speak English with the help of Chinese spelling. So every day I learned English using Chinese spelling. After many years, when I taught at a middle school, every day I read Loéve's book in English.

An English teacher at the school was surprised because no one else was learning science in English. She asked me to read a paragraph to her. After I read a paragraph, she said that she did not understand any of my words [Laughs]. After some years, I entered the university as a graduate student, and studied English regularly as a foreign language. In the first class, since the teacher had seen my homework…I told you that I was good at writing…the teacher thought that maybe my English was very good. So the teacher asked me a question. The question was, "What's your name?" I could not understand that question [Laughs]. It is very different after gaining some experience. For example, I spent fifteen months in the States. I also spent one year in the UK. My language improved a lot after that but still it is very limited.

Khoshnevisan/Waymire: *Where in the States were you?*

Mu-Fa Chen: Colorado. I suppose that I have not traveled enough in the States. And this is my third visit to Utah, so thank you very much.

Khoshnevisan/Waymire: *We should thank you. We certainly hope that you will visit us many more times.*

Mu-Fa Chen: Thank you so much.

Photo Credits

All photos courtesy of Mu-Fa Chen.

FILM REVIEW

Hidden Figures

Reviewed by Fern Y. Hunt

Editor's Note: At the Joint Mathematics Meetings in Atlanta in January, a panel was held about the movie *Hidden Figures*. Featured on the panel was the author of the book *Hidden Figures*, Margot Lee Shetterley, as well as one of the women profiled in her book, Christine Darden. The article "Spotlight Shines on *Hidden Figures* at JMM," by Alexandra Branscombe, appeared in the February/March 2017 issue of *MAA FOCUS*.

Hidden Figures
Twentieth Century Fox Films, 2016
Director: Theodore Melfi
Screenplay by: Allison Schroeder and Theodore Melfi
Based on the book by Margot Lee Shetterly

Ask an American of a certain age when America was great and many if not most would point to the years 1940–1965. In 1940, unprepared and still shaken by depression, our country waged total war against fascism and emerged confident and dominant both economically and technologically. The airplanes of WWII, the missiles and rockets of the postwar period, and the development of space flight culminating in the landing of American astronauts on the moon in 1969 were powerful and visible demonstrations of this ascent. That mathematics played an indispensable supporting role in these developments is well known to readers of the *Notices*. Less well known is that before electronic computers were available, women were the human beings carrying out the required computations. And until the 2016 book *Hidden Figures* by Margot Lee Shetterly, and the release of the currently popular movie of the same name, few knew that dozens of black women worked as human computers, computer scientists, engineers, and mathematicians at the Langley Memorial Aeronautical Laboratory, the principal federal government institution tasked with the development and testing of aircraft. Later, Langley became the command center for NASA's human space flight program. During this time many of these women made significant technical contributions while experiencing and overcoming the dual humiliations of sex discrimination and racial segregation.

> *The movie deftly captures the emotional tone and drama of the characters' lives.*

The movie portrays three of these extraordinary people: Dorothy Vaughan, the supervisor of the pool of "colored" human computers in the West Building; Mary Jackson, who broke the colored glass ceiling and achieved professional status as an engineer; and Katherine Johnson, the mathematician whose orbital calculations at a critical point in John Glenn's Friendship 7 flight are the dramatic focus of the movie. For her work on orbital trajectories and the application of celestial navigation to space flight, Johnson was awarded the Presidential Medal of Freedom in 2015.

The film set largely in the early 1960s is based on, but is not entirely coincident with, Shetterly's nonfiction book. It nevertheless deftly captures the emotional tone and drama of these characters' lives during a complex and very important period of American history. That it manages to do so in a way that is attractive to the general movie-going audience is a notable achievement. The film appealed to me as a viewer because it starts to answer my question "How did they do it?" in scenes that at once showcase these women's talent, daring, and humanity. Often the portrayal is lighthearted. For example, when we first see Dorothy Vaughan, she is lodged underneath her stalled car on the side of the road correctly diagnosing the cause of the problem. Her passengers Mary and Katherine look on, and a police cruiser suddenly approaches them.

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[1] *Fern Y. Hunt is a research mathematician in the Applied and Computational Mathematics Division of the National Institute of Standards and Technology. Her e-mail address is* `fern.hunt@nist.gov`.

Women "computers" working at NASA made a substantial contribution to the effort to the effort to launch astronaut John Glenn into orbit. The movie *Hidden Figures* tells their remarkable story.

Katherine G. Johnson (portrayed by Taraji P. Henson) makes one of many key contributions to the effort to send John Glenn into orbit.

Mathematicians will also enjoy seeing references to real mathematics. For example, I saw the polar form equation for conic sections on the blackboard, and we learn that Katherine's numerical calculation of the required position and velocity for the safe transition of a space capsule from an orbital trajectory to a reentry path back to earth employed Euler's method. Technical consultant Rudy Horne of Morehouse College and the team of NASA consultants should be commended. One caveat: The film elides the systemic, institutional character of the prejudices the women faced. This is perhaps inevitable given the constraints of a Hollywood dramatization—a more faithful recounting should be handled in a future documentary. For example, colored signs for bathrooms in the West Building were maintained because they were required to be there by Virginia state law (see Shetterly's book, pages 43–44). They did not disappear because one of them was taken down in righteous anger by a NASA manager. Rather, cold war abroad and more importantly civil rights victories, both legal and in the streets, eventually forced an end to the practice. Meanwhile, Katherine Johnson (defiantly?) used the unlabeled white bathrooms while working in the East Building. Those who want the full history and context for the events portrayed in the movie could do no better than read the meticulously researched and beautifully rendered account in Shetterly's book.

Acknowledgments: Thanks to Dr. Bonita Saunders of the National Institute of Standards and Technology and Professor Rudy Horne of the Mathematics Department of Morehouse College for their comments on earlier drafts of this review.

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Reviewer photo is courtesy of Fern Y. Hunt.
Photo of women "computers" is courtesy of TM & ©2017 Twentieth Century Fox Film Corporation.
Photo of blackboard work is by Hopper Stone. Courtesy of TM & ©2017 Twentieth Century Fox Film Corporation.

ABOUT THE REVIEWER

Fern Hunt is a mathematician at the National Institute of Standards and Technology. Her research interest lies in the application of probability to networks, dynamical systems, and biology. In 2000, she received the Arthur Flemming Award for Outstanding Federal Service.

Fern Y. Hunt

Inside the AMS

Fan China Exchange Program Grants Awarded

The Society's Fan China Exchange Program awards grants to support collaborations between Chinese and US or Canadian researchers. Institutions in the United States or Canada apply for the funds to support a visitor from China or vice versa. This funding is made possible through a generous gift made to the AMS by Ky and Yu-Fen Fan in 1999. The 2017 grants follow.

Jinqiao Duan from Illinois Institute of Technology was awarded US$5,000 to support a visit by Wei Wang from Nanjing University. In addition, US$1,000 will be awarded to Nanjing University after the visit is completed.

Qin Sheng from Baylor University was awarded US$3,600 to support a visit by Yufeng Xu from Central South University. In addition, US$1,000 will be awarded to Central South University after the visit is completed.

Wujie Shi from Chongqing University of Arts and Sciences was awarded US$2,500 to support a visit by Yong Yang from Texas State University.

For information about the Fan China Exchange Program, visit the website `www.ams.org/programs/travel-grants/china-exchange/china-exchange` or contact the AMS Membership and Programs Department, e-mail: `chinaexchange@ams.org`, telephone: 401-455-4170 (within the US call 800-321-4267, ext. 4170).

—AMS Membership and Programs Department

Epsilon Awards Announced

The AMS has chosen 17 summer mathematics programs to receive Epsilon grants for 2017. These summer programs give students a chance to see aspects of mathematics that they may not see in school and allow them to share their enthusiasm for mathematics with like-minded students.

The programs that received Epsilon grants for 2017 are:

All Girls/All Math Summer Camp, University of Nebraska, Lincoln, Yu Jin, director

Bridge to Enter Advanced Mathematics (BEAM), Bard College, Annandale-on-Hudson; one other to be announced), Daniel Zaharopol, director

Camp Euclid, Euclid Lab (online), David Gay, director

2016 BEAM program participants and staff.

Canada/USA Mathcamp, University of Puget Sound, Tacoma, Washington, Marisa Debowsky, director

GirlsGetMath@ICERM 2017, Brown University, Providence, Rhode Island, Brendan Edward Hassett, director

GirlsGetMath@Rochester, University of Rochester, Rochester, New York, Amanda M. Tucker, director

Hampshire College Summer Studies in Mathematics, Hampshire College, Amherst, Massachusetts, David C. Kelly, director

MathILy (serious Mathematics Infused with Levity), Bryn Mawr College, Bryn Mawr, Pennsylvania, Sarah-Marie Belcastro, director

MathILy-Er, Willamette University, Salem, Oregon, Jonah K. Ostroff, director

MathPath, Mount Holyoke College, South Hadley, Massachusetts, Stephen B. Maurer, director

Mathworks Honors Summer Math Camp, Texas State University, San Marcos, Max Warshauer/Michelle Pruett, directors

Michigan Math and Science Scholars (MMSS), University of Michigan, Ann Arbor, Doreen J. Fussman, director

PROMYS (Program in Mathematics for Young Scientists), Boston University, Boston, Massachusetts, Glenn Stevens, director

PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayaguez, Luis F. Caceres, director

Research Science Institute (RSI), Massachusetts Institute of Technology, Cambridge, Charles M. Farmer, director

SigmaCamp, Silver Lake Camp and Conference Center, Sharon, Connecticut, Alexander Kirillov, director

Summer Institute for Mathematics at UW, University of Washington, Seattle, Ron Irving, director

—AMS announcement

Erdős Memorial Lecture

The Erdős Memorial Lecture is an annual invited address named for the prolific mathematician Paul Erdős (1913–1996). The lectures are supported by a fund created by Andrew Beal, a Dallas banker and mathematics enthusiast. The Beal Prize Fund (`www.ams.org/profession/prizes-awards/ams-supported/beal-prize`), now US$1,000,000, is being held by the AMS until it is awarded. An AMS-appointed committee, the Beal Prize Committee, will recommend awarding this prize for either a proof or a counterexample of the Beal Conjecture published in a refereed and respected mathematics publication. At Mr. Beal's request, the interest from the fund is used to support the Erdős Memorial Lecture.

The 2017 Erdős Memorial Lecturer was James Maynard of Magdalen College, who spoke at the 2017 Spring Eastern Sectional Meeting at Hunter College, City University of New York, in May 2017.

The 2018 Erdős Memorial Lecture will be held during the 2018 Spring Southeastern Sectional Meeting at Vanderbilt University, Nashville, Tennessee, April 14–15, 2018. The lecturer will be Andrea Bertozzi of the University of California Los Angeles.

—AMS announcement

New AMS Blog from the Director of the AMS Washington Office

Wonder what is going on in federal policymaking that affects the mathematical community?

According to a McKinley survey done as part of recent AMS strategic planning efforts, only 58 percent of AMS member respondents even know that the AMS Washington Office—which provides an important link between the federal government and the mathematical community—exists; yet 95 percent, when asked about future priorities of the AMS, list increased advocacy as a most important priority (62 percent) or as a somewhat important priority (33 percent).

Director of the AMS Washington Office Karen Saxe has started a new blog called *Capital Currents*, intended to keep the mathematics community up to date on activities of the AMS Washington Office and on activities in Congress and the federal agencies that affect the mathematical community. Ideally, AMS members would know where this office is and what it can do for them, feel comfortable dropping in, and have a clear understanding of its role in Washington, DC.

At the time of this writing, there have been three blog posts. The first is a brief primer about the office and our work. One of the goals for the office is to launch a grassroots advocacy program which will engage more members with our advocacy work. The second post provides such an opportunity, encouraging members (and explaining why it is important) to write their congressional delegations about National Science Foundation funding for the next year. Funds for the NSF and other agencies are appropriated every year in Congress, and the third post walks readers through a very basic outline of this process.

Future posts will include more on the federal budget and on legislative matters of relevance to AMS members and to the broader scientific and higher-education communities; pieces meant to educate readers about the workings of Congress, its members, and committees; and suggestions for engaging with Congress and other policymakers. Check it out, and subscribe to follow at `blogs.ams.org/capitalcurrents`.

—AMS Washington Office

AMS Staff Support Local K–12 Mathematics Education

In April 2017, for the third year in a row, AMS staff collectively donated US$1,000 to each of three nonprofit mathematics tutoring agencies in the regional areas of AMS offices: The Family Learning Institute (FLI) in Ann Arbor, MI; Rhode Island Tutorial and Educational Services (RITES) in Pawtucket, RI; and For Love of Children (FLOC) in Washington, DC. Additionally, over US$500 was given to the AMS mathematics game, Who Wants to Be a Mathematician, to be used for schools in Michigan, Rhode Island, or Washington, DC, that need assistance to ensure student participation in the game, such as bus rental or substitute teacher expenses.

Started as part of the AMS 125th anniversary, the AMS Staff Fund is overseen by a committee of staff members. The committee has chosen to support K–12 mathematics programs in the regional areas where AMS staff members live and work. Donations to FLI, RITES, and FLOC help students who face economic need, academic need, or both. Since 2013, AMS employees have donated over US$19,000 to these nonprofits. The gifts are made during April to celebrate Mathematics and Statistics Awareness Month. Employee philanthropy has resulted in staff earning a place on the AMS Donor Wall of Honor each of the past four years.

—From the AMS Development Office

Deaths of AMS Members

Choong Yun Cho, of Portland, Oregon, died on March 16, 2014. No date of birth could be found. He was a member of the Society for 42 years.

Ruth M. Davis, of McLean, Virginia, died on March 28, 2012. Born on November 19, 1928, she was a member of the Society for 60 years.

Marj Enneking, of Portland, Oregon, died on March 13, 2016. Born on June 21, 1941, he was a member of the Society for 50 years.

Fritz Gackstatter, of Germany, died on March 6, 2016. Born on November 20, 1941, he was a member of the Society for 31 years.

Hillel Gauchman, of Chicago, Illinois, died on March 18, 2016. Born on July 14, 1937, he was a member of the Society for 41 years.

Verena Huber-Dyson, of Bellingham, Washington, died on March 12, 2016. Born on May 6, 1923, she was a member of the Society for 66 years.

Jerome Karle, of Washington, DC, died on June 6, 2013. Born on June 18, 1918, he was a member of the Society for 56 years.

Solomon A. Marcus, of Romania, died on March 17, 2016. Born on March 1, 1925, he was a member of the Society for 25 years.

Brian J. McCartin, of Flint, Michigan, died on January 29, 2016. Born on August 26, 1951, he was a member of the Society for 37 years.

Hyo Chul Myung, professor, Korea Institute for Advanced Study, died on February 11, 2010. Born on June 15, 1937, he was a member of the Society for 42 years.

David C. Newell, of Long Beach, California, died on August 18, 2015. Born on August 31, 1940, he was a member of the Society for 47 years.

A. G. Davis Philip, of Schenectady, New York, died on March 28, 2016. Born on January 1, 1929, he was a member of the Society for 25 years.

Lloyd S. Shapley, professor, University of California Los Angeles, died on March 12, 2016. Born on June 2, 1923, he was a member of the Society for 66 years.

Roy D. Sheffield, of Oxford, Mississippi, died on May 4, 2014. Born on September 5, 1922, he was a member of the Society for 55 years.

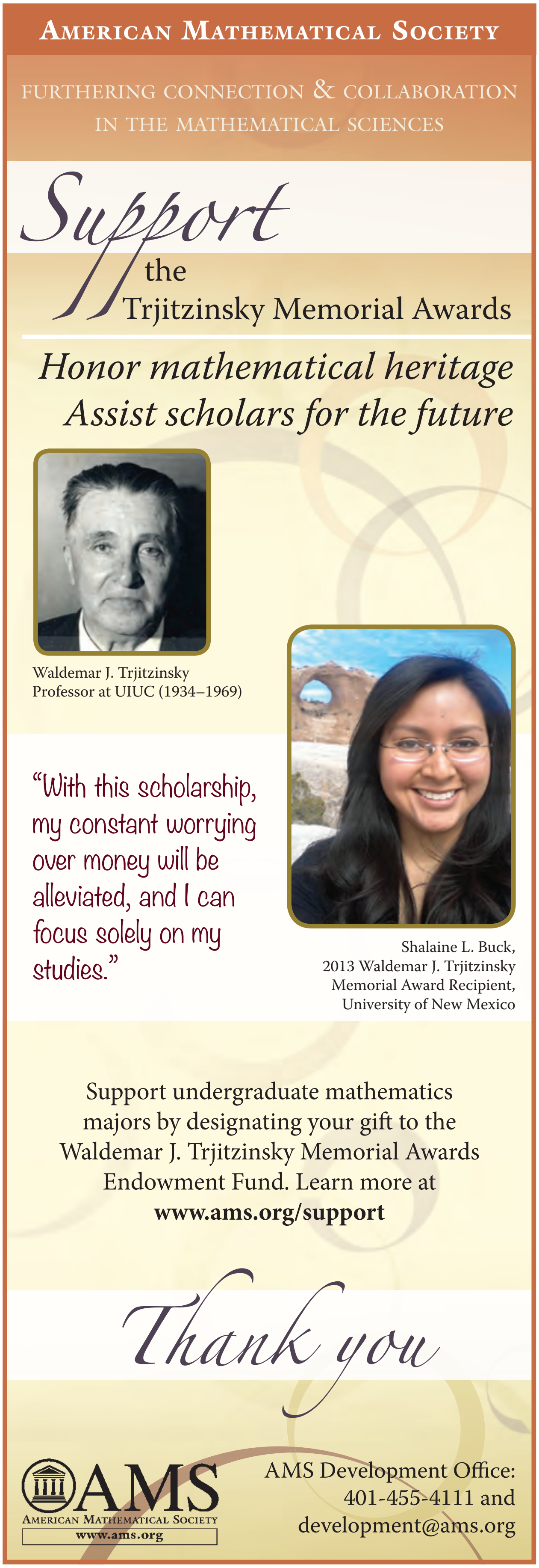
Al Shenk, of Solana Beach, California, died on July 15, 2015. Born on January 8, 1938, he was a member of the Society for 39 years.

Robert Silverman, of Yellow Springs, Ohio, died on March 5, 2016. Born on October 23, 1928, he was a member of the Society for 57 years.

Robert J. Wisner, of Las Cruces, New Mexico, died on October 29, 2015. Born on January 18, 1925, he was a member of the Society for 27 years.

Photo Credit

Photo of BEAM program participants and staff courtesy of BEAM.



AMS Governance

2017 Council

OFFICERS

President	**Kenneth A. Ribet** (1 Feb 2017 – 31 Jan 2019)
Immediate Past President	**Robert L. Bryant** (1 Feb 2017 – 31 Jan 2018)
Vice Presidents	**David Jerison** (1 Feb 2017 – 31 Jan 2020)
	Carlos E. Kenig (1 Feb 2015 – 31 Jan 2018)
	Richard Schoen (1 Feb 2016 – 31 Jan 2019)
Secretary	**Carla D. Savage** (1 Feb 2013 – 31 Jan 2019)
Associate Secretaries	**Georgia Benkart** (1 Feb 2010 – 31 Jan 2020)
	Brian D. Boe (1 Feb 2013 – 31 Jan 2019)
	Michel L. Lapidus (1 Feb 2002 – 31 Jan 2020)
	Steven H. Weintraub (1 Feb 2009 – 31 Jan 2019)
Treasurer	**Jane M. Hawkins** (1 Feb 2011 – 31 Jan 2019)
Associate Treasurer	**Zbigniew H. Nitecki** (1 Feb 2012 – 31 Jan 2020)

REPRESENTATIVES OF COMMITTEES

Bulletin of the AMS	**Susan J. Friedlander** (1 Jul 2005 – 31 Jan 2021)
Colloquium Publications	**Peter Sarnak** (1 Feb 2014 – 31 Jan 2021)
Executive Committee	**Jesús A. De Loera** (1 Mar 2015 – 28 Feb 2019)
	Jennifer Taback (1 Feb 2016 – 31 Jan 2018)
Journal of the AMS	**Sergey V. Fomin** (1 July 2013 – 31 Jan 2021)
Mathematical Reviews®	**Andreas J. Frommer** (1 Feb 2017 – 31 Jan 2019)
Mathematical Surveys and Monographs	**Michael A. Singer** (1 Feb 2015 – 31 Jan 2018)
Mathematics of Computation	**Susanne C. Brenner** (1 Feb 2012 – 31 Jan 2020)
Proceedings of the AMS	**Ken Ono** (1 Feb 2010 – 31 Jan 2018)
Transactions and Memoirs	**Alejandro Adem** (1 Feb 2013 – 31 Jan 2021)

MEMBERS AT LARGE

Matthew Baker (1 Mar 2015 – 31 Jan 2018)
Henry Cohn (1 Feb 2016 – 31 Jan 2019)
Alicia Dickenstein (1 Feb 2016 – 31 Jan 2019)
Nathan M. Dunfield (1 Feb 2017 – 31 Jan 2020)
Erica Flapan (1 Feb 2016 – 31 Jan 2019)
Edward Frenkel (1 Feb 2015 – 31 Jan 2018)
Pamela Gorkin (1 Feb 2015 – 31 Jan 2018)
Gregory F. Lawler (1 Feb 2017 – 31 Jan 2020)
Wen-Ching Winnie Li (1 Feb 2015 – 31 Jan 2018)
Anna Mazzucato (1 Feb 2016 – 31 Jan 2019)
Irina Mitrea (1 Feb 2017 – 31 Jan 2020)

Mary Pugh (1 Feb 2015 – 31 Jan 2018)
Alan William Reid (1 Feb 2016 – 31 Jan 2019)
Ravi Vakil (1 Feb 2017 – 31 Jan 2020)
Talitha M. Washington (1 Feb 2017 – 31 Jan 2020)

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Alejandro Adem (1 Mar 2016 – 29 Feb 2020)
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Jennifer Taback (1 Feb 2016 – 28 Feb 2018)
Ravi Vakil (1 Mar 2017 – 28 Feb 2021)

2017 Trustees

Ralph L. Cohen (1 Feb 2017 – 31 Jan 2022)
Jane M. Hawkins, *ex officio—Treasurer*
Bryna Kra (1 Feb 2016 – 31 Jan 2021)
Robert K. Lazarsfeld (1 Feb 2014 – 31 Jan 2019)
Zbigniew H. Nitecki, *ex officio—Associate Treasurer*
Kenneth A. Ribet *ex officio—President*
Joseph H. Silverman (1 Feb 2015 – 31 Jan 2020)
Karen Vogtmann (1 Feb 2008 – 31 Jan 2018)

Society Governance

The American Mathematical Society has a bicameral governance structure consisting of the Council (created when the Society's constitution was ratified in December 1889) and the Board of Trustees (created when the Society was incorporated in May 1923). These bodies have the ultimate responsibility and authority for representing the AMS membership and the broader mathematical community, determining how the AMS can best serve their collective needs, and formulating and approving policies to address these needs. The governing bodies determine what the Society does and the general framework for how it utilizes its volunteer, staff, and financial resources.

The Governance Leadership consists of the Officers (President, three Vice Presidents, Secretary, four Associate Secretaries, Treasurer, and Associate Treasurer), the Council, Executive Committee of the Council, and Board of Trustees.

The Council formulates and administers the scientific policies of the Society and acts in an advisory capacity to the Board of Trustees. Council Meetings are held twice a year (January and the spring).

The Board of Trustees receives and administers the funds of the Society, has full legal control of its investments and properties, and conducts all business affairs of the Society. The Trustees meet jointly with the Executive Committee of the Council twice a year (May and November) at ECBT Meetings.

The Council and Board of Trustees are advised by nearly 100 Committees, including five Policy Committees (Education, Meetings and Conferences, Profession, Publications, and Science Policy) and over 20 Editorial Committees for the various journals and books it publishes.

The Council and Board of Trustees are also advised by the Executive Director and the Executive Staff, who are responsible for seeing that governance decisions are implemented by the Society's 210 staff members.

Learn more at `www.ams.org/about-us/governance`.

Mathematics Opportunities

Listings for upcoming math opportunities to appear in Notices may be submitted to `notices@ams.org`*.*

Bertrand Russell Prize of the AMS

The AMS announces the Bertrand Russell Prize honoring research or service contributions of mathematicians or related professionals that promote good in the world. It recognizes the various ways that mathematics furthers fundamental human values; our understanding of climate change, digital privacy, or education in developing countries are some examples of the type of work that might be considered. The initial prize of US$5,000 will be awarded in 2018 and then every three years. The deadline for nominations is **June 30, 2017**. See `www.ams.org/profession/prizes-awards/russell-prize`.

—AMS announcement

Ulf Grenander Prize in Stochastic Theory and Modeling

The AMS announces the Ulf Grenander Prize recognizing exceptional theoretical and applied contributions in stochastic theory and modeling. It is awarded for seminal work in probabilistic modeling, statistical inference, or related computational algorithms, especially for the analysis of complex or high-dimensional systems. The first prize of US$5,000 will be awarded in 2018 and will be given every three years. The deadline for nominations is **June 30, 2017**. See `www.ams.org/profession/prizes-awards/ams-prizes/grenander-prize`.

—AMS announcement

Call for Nominations for Taussky-Todd Lecture

The Olga Taussky-Todd Lecture is held every four years at the International Congress on Industrial and Applied Mathematics (ICIAM). The lecture honors a woman who has made outstanding contributions in applied mathematics and/or scientific computation. Nominations are sought for the lecture at the Congress in Valencia, Spain, July 15–19, 2019. The deadline for nominations is **September 30, 2017**. See the website `www.iciam.org/news/17/3/17/olga-taussky-todd-lecture-2019-call-nominations`.

—From an ICIAM announcement

Call for Nominations for the SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology & Research Academy (SASTRA) is seeking nominations for the 2017 SASTRA Ramanujan Prize, awarded to a mathematician not exceeding the age of thirty-two for outstanding contributions in an area of mathematics influenced by the late Indian mathematical genius Srinivasa Ramanujan. It carries a cash prize of US$10,000. The deadline for nominations is **July 31, 2017**. See the website `qseries.org/sastra-prize/nominations-2017.html`.

—Krishnaswami Alladi, University of Florida

Travel Grants for ICM 2018

The organizing committee for the International Congress of Mathematicians (ICM), through the Open Arms Program, will grant 550 travel awards to mathematicians from developing countries for ICM 2018. Two hundred of these grants will be awarded to mathematicians from Latin American countries other than Brazil. Other eligible regions include Eastern Europe, Africa, and the Asia and Pacific region. The deadline for applications for the grants is **July 20, 2017**. For more details, see the website `www.icm2018.org/portal/en/travel-grants-program`.

—From an ICM 2018 announcement

*NSF Mathematical Sciences Postdoctoral Research Fellowships

The National Science Foundation (NSF) solicits proposals for the Mathematical Sciences Postdoctoral Research Fellowships. The deadline for full proposals is **October 18, 2017**. See `www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301&org=NSF&sel_org=NSF&from=fund`.

—From an NSF announcement

*NSF CAREER Awards

The National Science Foundation (NSF) solicits proposals for the Faculty Early Career Development Awards. The deadline for full proposals is **July 19, 2017**. See `www.nsf.gov/funding/pgm_summ.jsp?pims_id=503214`.

—From an NSF announcement

Call for Nominations for 2018 Heineman Prize

The American Physical Society and the American Institute of Physics are seeking nominations for the 2018 Dannie Heineman Prize for Mathematical Physics. The deadline is **June 30, 2017**. See `www.aps.org/programs/honors/prizes/heineman.cfm`.

—From an APS announcement

Call for Nominations for Balaguer Prize

The Ferran Sunyer i Balaguer Foundation seeks submissions for the 2018 Balaguer Prize. The prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics in which the applicant has made important contributions. The deadline is **November 30, 2017**. See `ffsb.espais.iec.cat/en/the-ferran-sunyer-i-balaguer-prize/`.

—Fundació Ferran Sunyer i Balaguer announcement

IMA Prize in Mathematics and Its Applications

The Institute for Mathematics and Its Applications (IMA) awards the annual Prize in Mathematics and its Applications to an individual who has made a transformative impact on the mathematical sciences and their applications. The deadline for nominations is **July 21, 2017**. See `www.ima.umn.edu/prize/`.

—From an IMA announcement

News from MSRI

With support from the National Science Foundation, the National Security Agency, academic departments, private foundations, and individuals, the Mathematical Sciences Research Institute (MSRI) will hold a number of workshops during the fall of 2017.

Established researchers, postdoctoral fellows, and graduate students are invited to apply for funding. MSRI actively seeks to achieve diversity in its workshops. Thus a strong effort is made to remove barriers that hinder equal opportunity, particularly for those groups that have been historically underrepresented in the mathematical sciences. MSRI has a resource to assist visitors with finding child care. Contact Sanjani Varkey at `sanjani@msri.org`.

The workshops are as follows:

August 17–August 18, 2017: Connections for Women: Geometry and Probability in High Dimensions. `www.msri.org/workshops/808`

August 21–August 25, 2017: Introductory Workshop: Phenomena in High Dimensions. `www.msri.org/workshops/809`

August 31–September 1, 2017: Connections for Women Workshop: Geometric and Topological Combinatorics. `www.msri.org/workshops/812`

September 5–September 8, 2017: Introductory Workshop: Geometric and Topological Combinatorics. `www.msri.org/workshops/813`

October 9–October 13, 2017: Geometric and Topological Combinatorics: Modern Techniques and Methods. `www.msri.org/workshops/819`

November 13–November 17, 2017: Geometric Functional Analysis and Applications. `www.msri.org/workshops/811`

November 29–December 1, 2017: Women in Topology. `www.msri.org/workshops/797`

—From an MSRI announcement

**The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at:* `www.nsf.gov/dms` *and for the Directorate of Education and Human Resources at* `www.nsf.gov/dir/index.jsp?org=ehr`*. To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at* `www.nsf.gov/mps/dms/about.jsp`*.*

Classified Advertisements

Positions available, items for sale, services available, and more

NEBRASKA

Milton Mohr Professor of Mathematics

The Department of Mathematics at the University of Nebraska-Lincoln invites applications for the Milton Mohr Professor of Mathematics, at the Associate Professor or Full Professor level, to begin in August 2018. The ideal candidate will have a strong, internationally recognized research program in mathematics, a demonstrated ability to attract external funding, and a strong record of mentoring Ph.D. students and postdocs. To be considered for the position, applicants must complete the Faculty/Administrative application at `http://employment.unl.edu`, requisition # F_160191. In addition, applicants must also submit a cover letter, a curriculum vitae, and the names and contact information of three references. Materials may be submitted through `mathjobs.org` or via email to `hiring@math.unl.edu`. Review of applications will begin October 1, 2017 and continue until the position is filled. For more information about this position, please go to: `www.math.unl.edu/department/jobs/`. The University of Nebraska Lincoln is committed to a pluralistic campus community through affirmative action, equal opportunity, work life balance, and dual careers. See `www.unl.edu/equity/notice-nondiscrimination`.

00009

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at The Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment, and strong research support. We are prepared to make quick and competitive offers to self-motivated, hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the honorary director, Professor Zhiming Ma is the director of the Advisory Board. Professor William Y. C. Chen serves as the director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

To apply, send your resume to `zhangry@tju.edu.cn`.

For more information, please visit `www.cam.tju.edu.cn` or contact Ms. Debbie Renyuan Zhang at `zhangry@tju.edu.cn`, telephone: 86-22-2740-5389.

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Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.

The 2017 rate is $3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: April 2017—February 3, 2017; May 2017—March 9, 2017; June/July 2017—May 9, 2017; August 2017—June 6, 2017; September 2017—July 7, 2017; October 2017—August 4, 2017; November 2017—September 5, 2017; December 2017—September 28, 2017.

US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

SOUTH AFRICA

Gottfried Wilhelm Leibniz Basic Research Institute Researchers Wanted

The Gottfried Wilhelm Leibniz Basic Research Institute, which operates as a PBO, invites exceptional researchers in mathematics and/or quantum physics to apply for shorter or longer term support. Contact `eerosinger@hotmail.com`.

00007

READERS SOUGHT

Of Possible Second Proof of the $3x+1$ Conjecture

This is the first possible proof I have been able to discover that is based on the 1-tree (the tree in which level n is the set of all odd, positive integers that map to 1 in n iterations of the $3x+1$ function).

The possible proof is less than a page, with another two pages of supporting material.

I welcome comments by insightful readers. See “Possible Proof of Conjecture Based on 1-Tree”, p. 31, in “A Solution to the $3x+1$ Problem”, on `occampress.com`.

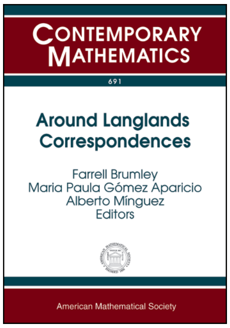
Peter Schorer, `peteschorer@gmail.com`.

00010

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to `www.ams.org/bookstore-email`.

Algebra and Algebraic Geometry



Contemporary Mathematics
691
Around Langlands Correspondences
Farrell Brumley
Maria Paula Gómez Aparicio
Alberto Mínguez
Editors
American Mathematical Society

Around Langlands Correspondences

Farrell Brumley, *Université Paris 13, Villetaneuse, France*, **Maria Paula Gómez Aparicio**, *Université Paris-Sud 11, Orsay, France*, and **Alberto Mínguez**, *Université Pierre et Marie Curie, Paris, France*, Editors

This volume contains the proceedings of the international conference "Around Langlands Correspondences", held from June 17–20, 2015, at Université Paris Sud in Orsay, France.

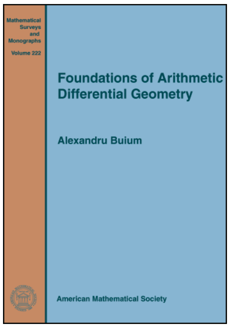
The Langlands correspondence (nowadays called the usual Langlands correspondence), conjectured by Robert Langlands in the late 1960s and early 1970s, has recently seen some new mysterious generalizations: the modular Langlands correspondence, the p-adic Langlands correspondence, and the geometric Langlands correspondence, the last of which seems to share deep connections with the Baum-Connes conjecture.

The aim of this volume is to present, through a mix of research and expository articles, some of the fascinating new directions in number theory and representation theory arising from recent developments in the Langlands program. Special emphasis is placed on nonclassical versions of the conjectural Langlands correspondences, where the underlying field is no longer the complex numbers.

Contents: N. Abe, Change of weight theorem for pro-p-Iwahori Hecke algebras; **A.-M. Aubert**, **P. Baum**, **R. Plymen**, and **M. Solleveld**, Conjectures about p-adic groups and their noncommutative geometry; **I. Chatterji**, Introduction to the Rapid Decay property; **T. Crisp** and **N. Higson**, A second adjoint theorem for $\mathrm{SL}(2,\mathbb{R})$; **J.-F. Dat**, A functoriality principle for blocks of p-adic linear groups; **A. David**, Poids de Serre dans la conjecture de Breuil–Mézard; **N. Imai** and **T. Tsushima**, Affinoids in Lubin-Tate surfaces with exponential full level two; **J. Lin**, An automorphic variant of a conjecture of Deligne; **C. Moeglin** and **D. Renard**, Paquets d'Arthur des groupes classiques complexes; **A. Moussaoui**, Proof of the Aubert-Baum-Plymen-Solleveld conjecture for split classical groups; **E. Nagel**, From crystalline to unitary representations; **A. Stasinski**, Representations of GL_N over finite local principal ideal rings: An overview; **J. Tymoczko**, The geometry and combinatorics of Springer fibers.

Contemporary Mathematics, Volume 691

July 2017, 376 pages, Softcover, ISBN: 978-1-4704-3573-8, LC 2016049805, 2010 *Mathematics Subject Classification:* 20G25, 22E50, 11S37, 19L47, 22E45, 11F55, 11R39, 20C33, 11F85, 11F67, **AMS members US$88.80**, List US$111, Order code CONM/691



Mathematical Surveys and Monographs
Volume 222
Foundations of Arithmetic Differential Geometry
Alexandru Buium
American Mathematical Society

Foundations of Arithmetic Differential Geometry

Alexandru Buium, *University of New Mexico, Albuquerque, NM*

The aim of this book is to introduce and develop an arithmetic analogue of classical differential geometry. In this new geometry the ring of integers plays the role of a ring of functions on an infinite dimensional manifold. The role of coordinate functions on this manifold is played by the prime numbers. The role of partial derivatives of functions with respect to the coordinates is played by the Fermat quotients of integers with respect to the primes. The role of metrics is played by symmetric matrices with integer coefficients. The role of connections (respectively curvature) attached to metrics is played by certain adelic (respectively global) objects attached to the corresponding matrices.

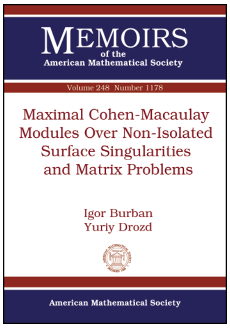
One of the main conclusions of the theory is that the spectrum of the integers is "intrinsically curved"; the study of this curvature is then the main task of the theory. The book follows, and builds upon, a series of recent research papers. A significant part of the material has never been published before.

This item will also be of interest to those working in number theory.

Contents: Algebraic background; Classical differential geometry revisited; Arithmetic differential geometry: Generalities; Arithmetic differential geometry: The case of GL_n; Curvature and Galois groups of Ehresmann connections; Curvature of Chern connections; Curvature of Levi-Cività connections; Curvature of Lax connections; Open problems; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 222

June 2017, 344 pages, Hardcover, ISBN: 978-1-4704-3623-0, LC 2016056302, 2010 *Mathematics Subject Classification:* 11E57, 11F85, 14G20, 53C21, **AMS members US$99.20**, List US$124, Order code SURV/222



MEMOIRS of the American Mathematical Society
Volume 248 Number 1178
Maximal Cohen-Macaulay Modules Over Non-Isolated Surface Singularities and Matrix Problems
Igor Burban
Yuriy Drozd
American Mathematical Society

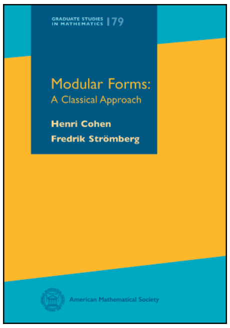
Maximal Cohen-Macaulay Modules Over Non-Isolated Surface Singularities and Matrix Problems

Igor Burban, *Universität zu Köln, Germany*, and **Yuriy Drozd**, *National Academy of Sciences, Kyiv, Ukraine*

Contents: Generalities on maximal Cohen–Macaulay modules; Category of triples in dimension one; Main construction; Serre quotients and proof of Main Theorem; Singularities obtained by gluing cyclic quotient singularities; Maximal Cohen–Macaulay modules over $k[[x,y,z]]/(x^2+y^3-xyz)$; Representations of decorated bunches of chains–I; Maximal Cohen–Macaulay modules over degenerate cusps–I; Maximal Cohen–Macaulay modules over degenerate cusps–II; Schreyer's question; Remarks on rings of discrete and tame CM–representation type; Representations of decorated bunches of chains–II; References.

Memoirs of the American Mathematical Society, Volume 248, Number 1178

June 2017, 114 pages, Softcover, ISBN: 978-1-4704-2537-1, 2010 *Mathematics Subject Classification:* 16G50, 16G60, 13C14, 13H10, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/248/1178



GRADUATE STUDIES IN MATHEMATICS 179
Modular Forms:
A Classical Approach
Henri Cohen
Fredrik Strömberg
American Mathematical Society

Modular Forms

A Classical Approach

Henri Cohen, *Université Bordeaux, France*, and **Fredrik Strömberg**, *University of Nottingham, United Kingdom*

The theory of modular forms is a fundamental tool used in many areas of mathematics and physics. It is also a very concrete and "fun" subject in itself and abounds with an amazing number of surprising identities.

This comprehensive textbook, which includes numerous exercises, aims to give a complete picture of the classical aspects of the subject, with an emphasis on explicit formulas. After a number of motivating examples such as elliptic functions and theta functions, the modular group, its subgroups, and general aspects of holomorphic and nonholomorphic modular forms are explained, with an emphasis on explicit examples. The heart of the book is the classical theory developed by Hecke and continued up to the Atkin–Lehner–Li theory of newforms and including the theory of Eisenstein series, Rankin–Selberg theory, and a more general theory of theta series including the Weil representation. The final chapter explores in some detail more general types of modular forms such as half-integral weight, Hilbert, Jacobi, Maass, and Siegel modular forms.

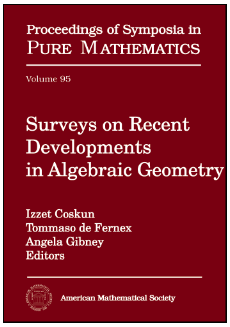
Some "gems" of the book are an immediately implementable trace formula for Hecke operators, generalizations of Haberland's formulas for the computation of Petersson inner products, W. Li's little-known theorem on the diagonalization of the *full* space of modular forms, and explicit algorithms due to the second author for computing Maass forms.

This book is essentially self-contained, the necessary tools such as gamma and Bessel functions, Bernoulli numbers, and so on being given in a separate chapter.

Contents: Introduction; Elliptic functions, elliptic curves, and theta function; Basic tools; The modular group; General aspects of holomorphic and nonholomorphic modular forms; Sets of 2×2 integer matrices; Modular forms and functions on subgroups; Eisenstein and Poincaré series; Fourier coefficients of modular forms; Hecke operators and Euler products; Dirichlet series, functional equations, and periods; Unfolding and kernels; Atkin–Lehner–Li theory; Theta functions; More general modular forms: An introduction; Bibliography; Index of notation; General index.

Graduate Studies in Mathematics, Volume 179

August 2017, approximately 699 pages, Hardcover, ISBN: 978-0-8218-4947-7, LC 2016052884, 2010 *Mathematics Subject Classification:* 11Fxx, 11F03, 11F11, 11F20, 11F25, 11F27, 11F30, 11F33, 11F37, 11F41, 11F50, **AMS members US$75.20**, List US$94, Order code GSM/179



Proceedings of Symposia in PURE MATHEMATICS
Volume 95
Surveys on Recent Developments in Algebraic Geometry
Izzet Coskun
Tommaso de Fernex
Angela Gibney
Editors
American Mathematical Society

Surveys on Recent Developments in Algebraic Geometry

Izzet Coskun, *University of Illinois at Chicago, IL*, **Tommaso de Fernex**, *University of Utah, Salt Lake City, UT*, and **Angela Gibney**, *University of Georgia, Athens, GA*, Editors

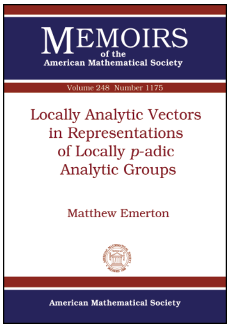
The algebraic geometry community has a tradition of running a summer research institute every ten years. During these influential meetings a large number of mathematicians from around the world convene to overview the developments of the past decade and to outline the most fundamental and far-reaching problems for the next. The meeting is preceded by a Bootcamp aimed at graduate students and young researchers. This volume collects ten surveys that grew out of the Bootcamp, held July 6–10, 2015, at University of Utah, Salt Lake City, Utah.

These papers give succinct and thorough introductions to some of the most important and exciting developments in algebraic geometry in the last decade. Included are descriptions of the striking advances in the Minimal Model Program, moduli spaces, derived categories, Bridgeland stability, motivic homotopy theory, methods in characteristic p and Hodge theory. Surveys contain many examples, exercises and open problems, which will make this volume an invaluable and enduring resource for researchers looking for new directions.

Contents: B. Lehmann, A snapshot of the Minimal Model Program; **Z. Patakfalvi**, **K. Schwede**, and **K. Tucker**, Positive characteristic algebraic geometry; **O. Tommasi**, The geometry of the moduli space of curves and abelian varieties; **J. Huizenga**, Birational geometry of moduli spaces of sheaves and Bridgeland stability; **E. Clader**, Gromov-Witten theory: From curve counts to string theory; **D. Chen**, Teichmüller dynamics in the eyes of an algebraic geometer; **A. Auel** and **M. Bernardara**, Cycles, derived categories, and rationality; **C. Robles**, Degenerations of Hodge structure; **D. Erman** and **S. V. Sam**, Questions about Boij-Söderberg theory; **B. Antieau** and **E. Elmanto**, A primer for unstable motivic homotopy theory.

Proceedings of Symposia in Pure Mathematics, Volume 95

July 2017, approximately 374 pages, Hardcover, ISBN: 978-1-4704-3557-8, 2010 *Mathematics Subject Classification:* 14H10, 14E30, 14E08, 14D07, 14N35, 14J60, 14G17, 13D02, 19D06, 37D40, **AMS members US$100.80**, List US$126, Order code PSPUM/95



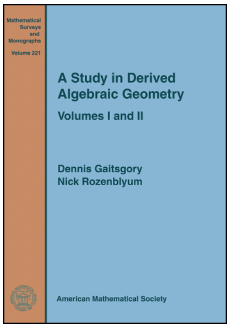
Locally Analytic Vectors in Representations of Locally p-adic Analytic Groups

Matthew J. Emerton, *University of Chicago, IL*

Contents: Introduction; Non-archimedean functional analysis; Non-archimedean function theory; Continuous, analytic, and locally analytic vectors; Smooth, locally finite, and locally algebraic vectors; Rings of distributions; Admissible locally analytic representations; Representations of certain product groups; Bibliography.

Memoirs of the American Mathematical Society, Volume 248, Number 1175

June 2017, 158 pages, Softcover, ISBN: 978-0-8218-7562-9, 2010 *Mathematics Subject Classification:* 22-XX, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/248/1175



A Study in Derived Algebraic Geometry

Volumes I and II

Dennis Gaitsgory, *Harvard University, Cambridge, MA*, and **Nick Rozenblyum**, *University of Chicago, IL*

Derived algebraic geometry is a far-reaching generalization of algebraic geometry. It has found numerous applications in various parts of mathematics, most prominently in representation theory. This two-volume monograph develops generalization of various topics in algebraic geometry in the context derived algebraic geometry.

Volume 1 presents the theory of ind-coherent sheaves, which are a "renormalization" of quasi-coherent sheaves and provide a natural setting for Grothendieck-Serre duality as well as geometric incarnations of numerous categories of interest in representation theory.

Volume 2 develops deformation theory, Lie theory and the theory of algebroids in the context of derived algebraic geometry. To that end, it introduces the notion of inf-scheme, which is an infinitesimal deformation of a scheme and studies ind-coherent sheaves on inf-schemes. As an application of the general theory, the six-functor formalism for D-modules in derived geometry is obtained.

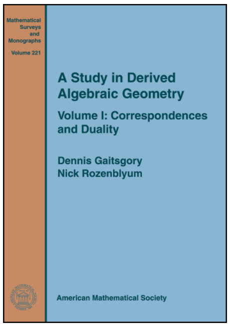
Contents: *Contents for Volume I: Preliminaries:* Introduction; Some higher algebra; Basics of derived algebraic geometry; Quasi-coherent sheaves on prestacks; *Ind-coherent sheaves:* Introduction; Ind-coherent sheaves on schemes; Ind-coherent sheaves as a functor out of the category of correspondences; Interaction of Qcoh and IndCoh; *Categories of correspondences:* Introduction; The $(\infty, 2)$-category of correspondences; Extension theorems for the category of correspondences; The (symmetric) monoidal structure on the category of correspondences; $(\infty, 2)$-*categories:* Introduction; Basics of 2-categories; Straightening and Yoneda for $(\infty, 2)$-categories; Adjunctions in $(\infty, 2)$-categories; Bibliography; Index of notations; Index; *Contents for Volume II: Inf-schemes:* Introduction; Deformation theory; Ind-schemes and inf-schemes; Ind-coherent sheaves on ind-inf-schemes; An application: Crystals; *Formal geometry:* Introduction; Formal moduli; Lie algebras and co-commutative co-algebras; Formal groups and Lie algebras; Lie algebroids; Infinitesimal differential geometry; Bibliography; Index of notations; Index.

Mathematical Surveys and Monographs, Volume 221

Volume 1: July 2017, approximately 557 pages, Hardcover, ISBN: 978-1-4704-3569-1, 2010 *Mathematics Subject Classification:* 14-02, 14A20, 14F05, 18G55, 18D05, Order code SURV/221.1

Volume 2: July 2017, approximately 463 pages, Hardcover, ISBN: 978-1-4704-3570-7, 2010 *Mathematics Subject Classification:* 14-02, 14A20, 14F05, 14D15, 14F10, 18G55, Order code SURV/221.2

Set: July 2017, approximately 1016 pages, Hardcover, ISBN: 978-1-4704-3568-4, **AMS members US$176**, List US$220, Order code SURV/221



A Study in Derived Algebraic Geometry

Volume I: Correspondences and Duality

Dennis Gaitsgory, *Harvard University, Cambridge, MA*, and **Nick Rozenblyum**, *University of Chicago, IL*

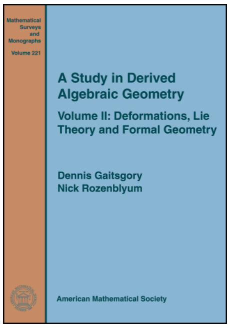
Derived algebraic geometry is a far-reaching generalization of algebraic geometry. It has found numerous applications in various parts of mathematics, most prominently in representation theory. This volume develops the theory of ind-coherent sheaves in the context of derived algebraic geometry. Ind-coherent sheaves are a "renormalization" of quasi-coherent sheaves and provide a natural setting for Grothendieck-Serre duality as well as geometric incarnations of numerous categories of interest in representation theory.

This volume consists of three parts and an appendix. The first part is a survey of homotopical algebra in the setting of ∞-categories and the basics of derived algebraic geometry. The second part builds the theory of ind-coherent sheaves as a functor out of the category of correspondences and studies the relationship between ind-coherent and quasi-coherent sheaves. The third part sets up the general machinery of the $(\infty, 2)$-category of correspondences needed for the second part. The category of correspondences, via the theory developed in the third part, provides a general framework for Grothendieck's six-functor formalism. The appendix provides the necessary background on $(\infty, 2)$-categories needed for the third part.

Contents: *Preliminaries:* Introduction; Some higher algebra; Basics of derived algebraic geometry; Quasi-coherent sheaves on prestacks; *Ind-coherent sheaves:* Introduction; Ind-coherent sheaves on schemes; Ind-coherent sheaves as a functor out of the category of correspondences; Interaction of Qcoh and IndCoh; *Categories of correspondences:* Introduction; The $(\infty, 2)$-category of correspondences; Extension theorems for the category of correspondences; The (symmetric) monoidal structure on the category of correspondences; *$(\infty, 2)$-categories:* Introduction; Basics of 2-categories; Straightening and Yoneda for $(\infty, 2)$-categories; Adjunctions in $(\infty, 2)$-categories; Bibliography; Index of notations; Index.

Mathematical Surveys and Monographs, Volume 221

July 2017, approximately 557 pages, Hardcover, ISBN: 978-1-4704-3569-1, 2010 *Mathematics Subject Classification:* 14-02, 14A20, 14F05, 18G55, 18D05, **AMS members US$99.20**, List US$124, Order code SURV/221.1



A Study in Derived Algebraic Geometry

Volume II: Deformations, Lie Theory and Formal Geometry

Dennis Gaitsgory, *Harvard University, Cambridge, MA*, and **Nick Rozenblyum**, *University of Chicago, IL*

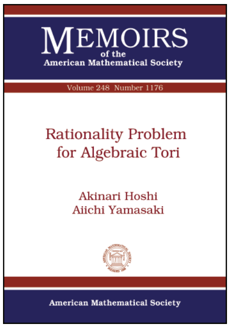
Derived algebraic geometry is a far-reaching generalization of algebraic geometry. It has found numerous applications in other parts of mathematics, most prominently in representation theory. This volume develops deformation theory, Lie theory and the theory of algebroids in the context of derived algebraic geometry. To that end, it introduces the notion of inf-scheme, which is an infinitesimal deformation of a scheme and studies ind-coherent sheaves on such. As an application of the general theory, the six-functor formalism for D-modules in derived geometry is obtained.

This volume consists of two parts. The first part introduces the notion of ind-scheme and extends the theory of ind-coherent sheaves to inf-schemes, obtaining the theory of D-modules as an application. The second part establishes the equivalence between formal Lie group(oids) and Lie algebr(oids) in the category of ind-coherent sheaves. This equivalence gives a vast generalization of the equivalence between Lie algebras and formal moduli problems. This theory is applied to study natural filtrations in formal derived geometry generalizing the Hodge filtration.

Contents: *Inf-schemes:* Introduction; Deformation theory; Ind-schemes and inf-schemes; Ind-coherent sheaves on ind-inf-schemes; An application: Crystals; *Formal geometry:* Introduction; Formal moduli; Lie algebras and co-commutative co-algebras; Formal groups and Lie algebras; Lie algebroids; Infinitesimal differential geometry; Bibliography; Index of notations; Index.

Mathematical Surveys and Monographs, Volume 221

August 2017, approximately 463 pages, Hardcover, ISBN: 978-1-4704-3570-7, 2010 *Mathematics Subject Classification:* 14-02, 14A20, 14F05, 14D15, 14F10, 18G55, **AMS members US$99.20**, List US$124, Order code SURV/221.2



Rationality Problem for Algebraic Tori

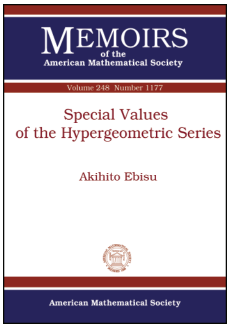
Akinari Hoshi, *Niigata University, Japan*, and **Aiichi Yamasaki**, *Kyoto University, Japan*

Contents: Introduction; Preliminaries: Tate cohomology and flabby resolutions; CARAT ID of the $\mathbb{Z}$-classes in dimensions 5 and 6; Krull-Schmidt theorem fails for dimension 5; GAP algorithms: the flabby class $[M_G]^{fl}$; Flabby and coflabby G-lattices; $H^1(G, [M_G]^{fl}) = 0$ for any Bravais group G of dimension $n \leq 6$; Norm one tori; Tate cohomology: GAP computations; Proof of Theorem 1.27; Proof of Theorem 1.28; Proof of Theorem 12.3; Application of Theorem 12.3; Tables for the stably rational classification of algebraic k-tori of dimension 5; Bibliography.

Memoirs of the American Mathematical Society, Volume 248, Number 1176

June 2017, 215 pages, Softcover, ISBN: 978-1-4704-2409-1, 2010 *Mathematics Subject Classification:* 11E72, 12F20, 13A50, 14E08, 20C10, 20G15, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/248/1176

Analysis



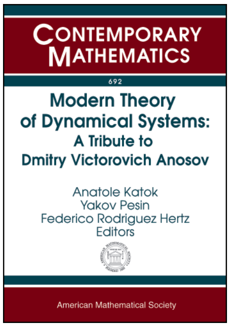
Special Values of the Hypergeometric Series

Akihito Ebisu, *Hokkaido University, Sapporo, Japan*

Contents: Introduction; Preliminaries; Derivation of special values; Tables of special values; Appendix A. Some hypergeometric identities for generalized hypergeometric series and Appell-Lauricella hypergeometric series; Acknowledgments; Bibliography.

Memoirs of the American Mathematical Society, Volume 248, Number 1177

June 2017, 96 pages, Softcover, ISBN: 978-1-4704-2533-3, 2010 *Mathematics Subject Classification:* 33C05; 13P10, 13P15, 33C20, 33C65, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/248/1177


CONTEMPORARY MATHEMATICS
692
Modern Theory of Dynamical Systems: A Tribute to Dmitry Victorovich Anosov
Anatole Katok
Yakov Pesin
Federico Rodriguez Hertz
Editors
American Mathematical Society

Modern Theory of Dynamical Systems

A Tribute to Dmitry Victorovich Anosov

Anatole Katok, *Pennsylvania State University, University Park, PA*, **Yakov Pesin**, *Pennsylvania State University, University Park, PA*, and **Federico Rodriguez Hertz**, *Pennsylvania State University, University Park, PA*, Editors

This volume is a tribute to one of the founders of modern theory of dynamical systems, the late Dmitry Victorovich Anosov.

It contains both original papers and surveys, written by some distinguished experts in dynamics, which are related to important themes of Anosov's work, as well as broadly interpreted further crucial developments in the theory of dynamical systems that followed Anosov's original work.

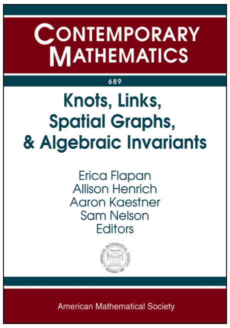
Also included is an article by A. Katok that presents Anosov's scientific biography and a picture of the early development of hyperbolicity theory in its various incarnations, complete and partial, uniform and nonuniform.

Contents: A. Katok, Dmitry Viktorovich Anosov: His life and mathematics; **M. Brin** and **Y. Pesin**, D. V. Anosov and our road to partial hyperbolicity; **V. Afraimovich** and **L. Bunimovich**, Escape from large holes in Anosov systems; **F. Béguin**, **S. Crovisier**, and **T. Jäger**, A dynamical decomposition of the torus into pseudo-circles; **A. Dudko** and **R. Grigorchuk**, On irreducibility and disjointness of Koopman and quasi-regular representations of weakly branch groups; **B. Fayad** and **M. Saprykina**, Isolated elliptic fixed points for smooth Hamiltonians; **T. Fisher**, **T. Petty**, and **S. Tikhomirov**, Nonlocally maximal and premaximal hyperbolic sets; **J. Franks**, Rotation numbers for S^2 diffeomorphisms; **A. Gorodetski** and **Y. Pesin**, Path connectedness and entropy density of the space of hyperbolic ergodic measures; **V. Grines** and **E. Zhuzhoma**, Around Anosov-Weil theory; **Y. Ilyashenko** and **I. Shilin**, Attractors and skew products; **M. Jakobson**, Thermodynamic formalism for some systems with countable Markov structures; **A. Katok** and **F. Rodriguez Hertz**, Non-uniform measure rigidity for $\mathbb{Z}^k$ actions of symplectic type; **S. E. Newhouse**, On a differentiable linearization theorem of Philip Hartman; **M. Ratner**, Time change invariants for measure preserving flows; **A. M. Stepin** and **I. V. Tsylin**, Spectral boundary value problems for Laplace-Beltrami operator: Moduli for continuity of eigenvalues under domain deformation; **M. Viana** and **J. Yang**, Measure-theoretical properties of center foliations.

Contemporary Mathematics, Volume 692

July 2017, approximately 322 pages, Softcover, ISBN: 978-1-4704-2560-9, 2010 *Mathematics Subject Classification:* 37Bxx, 37Cxx, 37Dxx, 37Exx, 37Gxx, 37Jxx, **AMS members US$88.80**, List US$111, Order code CONM/692

Geometry and Topology


CONTEMPORARY MATHEMATICS
689
Knots, Links, Spatial Graphs, & Algebraic Invariants
Erica Flapan
Allison Henrich
Aaron Kaestner
Sam Nelson
Editors
American Mathematical Society

Knots, Links, Spatial Graphs, and Algebraic Invariants

Erica Flapan, *Pomona College, Claremont, CA*, **Allison Henrich**, *Seattle University, WA*, **Aaron Kaestner**, *North Park University, Chicago, IL*, and **Sam Nelson**, *Claremont McKenna College, CA*, Editors

This volume contains the proceedings of the AMS Special Session on Algebraic and Combinatorial Structures in Knot Theory and the AMS Special Session on Spatial Graphs, both held from October 24–25, 2015, at California State University, Fullerton, CA.

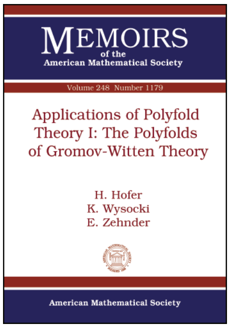
Included in this volume are articles that draw on techniques from geometry and algebra to address topological problems about knot theory and spatial graph theory, and their combinatorial generalizations to equivalence classes of diagrams that are preserved under a set of Reidemeister-type moves.

The interconnections of these areas and their connections within the broader field of topology are illustrated by articles about knots and links in spatial graphs and symmetries of spatial graphs in S^3 and other 3-manifolds.

Contents: J. H. Przytycki, The first coefficient of Homflypt and Kauffman polynomials: Vertigan proof of polynomial complexity using dynamic programming; **M. Elhamdadi** and **J. Kerr**, Linear Alexander quandle colorings and the minimum number of colors; **W. E. Clark** and **M. Saito**, Quandle identities and homology; **E. Denne**, **M. Kamp**, **R. Terry**, and **X. Zhu**, Ribbonlength of folded ribbon unknots in the plane; **H. A. Dye**, Checkerboard framings and states of virtual link diagrams; **M. Chrisman** and **A. Kaestner**, Virtual covers of links II; **E. Flapan**, **T. W. Mattman**, **B. Mellor**, **R. Naimi**, and **R. Nikkuni**, Recent developments in spatial graph theory; **T. W. Mattman**, **C. Morris**, and **J. Ryker**, Order nine MMIK graphs; **A. Kawauchi**, A chord graph constructed from a ribbon surface-link; **T. W. Mattman** and **M. Pierce**, The K_{n+5} and $K_{3^2,1^n}$ families and obstructions to n-apex; **A. Ishii** and **S. Nelson**, Partially multiplicative biquandles and handlebody-knots; **A. Henrich** and **L. H. Kauffman**, Tangle insertion invariants for pseudoknots, singular knots, and rigid vertex spatial graphs.

Contemporary Mathematics, Volume 689

May 2017, 189 pages, Softcover, ISBN: 978-1-4704-2847-1, LC 2016042011, 2010 *Mathematics Subject Classification:* 05C10, 57M15, 57M25, 57M27, **AMS members US$88.80**, List US$111, Order code CONM/689



MEMOIRS of the American Mathematical Society
Volume 248 Number 1179
Applications of Polyfold Theory I: The Polyfolds of Gromov-Witten Theory
H. Hofer
K. Wysocki
E. Zehnder
American Mathematical Society

Applications of Polyfold Theory I: The Polyfolds of Gromov-Witten Theory

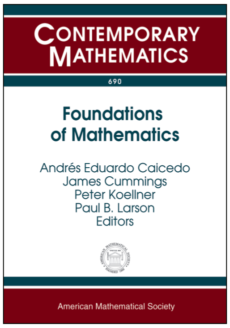
H. Hofer, *Institute for Advanced Study, Princeton, New Jersey*, **K. Wysocki**, *Penn State University, State College, Pennsylvania*, and **E. Zehnder**, *ETH-Zurich, Switzerland*

Contents: Introduction and main results; Recollections and technical results; The polyfold structures; The nonlinear Cauchy-Riemann operator; Appendices; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 248, Number 1179

June 2017, 218 pages, Softcover, ISBN: 978-1-4704-2203-5, 2010 *Mathematics Subject Classification:* 58B99, 58C99, 57R17, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/248/1179

Logic and Foundations



CONTEMPORARY MATHEMATICS
690
Foundations of Mathematics
Andrés Eduardo Caicedo
James Cummings
Peter Koellner
Paul B. Larson
Editors
American Mathematical Society

Foundations of Mathematics

Andrés Eduardo Caicedo, *Mathematical Reviews, Ann Arbor, MI*, **James Cummings**, *Carnegie Mellon University, Pittsburgh, PA*, **Peter Koellner**, *Harvard University, Cambridge, MA*, and **Paul B. Larson**, *Miami University, Oxford, OH*, Editors

This volume contains the proceedings of the Logic at Harvard conference in honor of W. Hugh Woodin's 60th birthday, held March 27–29, 2015, at Harvard University. It presents a collection of papers related to the work of Woodin, who has been one of the leading figures in set theory since the early 1980s.

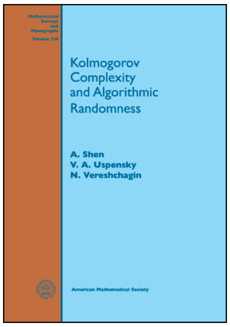
The topics cover many of the areas central to Woodin's work, including large cardinals, determinacy, descriptive set theory and the continuum problem, as well as connections between set theory and Banach spaces, recursion theory, and philosophy, each reflecting a period of Woodin's career. Other topics covered are forcing axioms, inner model theory, the partition calculus, and the theory of ultrafilters.

This volume should make a suitable introduction to Woodin's work and the concerns which motivate it. The papers should be of interest to graduate students and researchers in both mathematics and philosophy of mathematics, particularly in set theory, foundations and related areas.

Contents: **H. G. Dales**, Norming infinitesimals of large fields; **T. A. Slaman** and **M. I. Soskova**, The enumeration degrees: Local and global structural interactions; **A. S. Kechris**, **M. Sokić**, and **S. Todorcevic**, Ramsey properties of finite measure algebras and topological dynamics of the group of measure preserving automorphisms: Some results and an open problem; **A. E. Caicedo** and **J. Hilton**, Topological Ramsey numbers and countable ordinals; **V. Gitman** and **J. D. Hamkins**, Open determinacy for class games; **M. Malliaris** and **S. Shelah**, Open problems on ultrafilters and some connections to the continuum; **P. D. Welch**, Obtaining Woodin's cardinals; **R. Schindler**, Woodin's axiom (*), or Martin's maximum, or both?; **G. Sargsyan**, Translation procedures in descriptive inner model theory; **S. Cramer**, Implications of very large cardinals; **J. T. Moore**, What makes the continuum $\aleph_2$; **P. Maddy**, Set-theoretic foundations.

Contemporary Mathematics, Volume 690

May 2017, 322 pages, Softcover, ISBN: 978-1-4704-2256-1, LC 2016043599, 2010 *Mathematics Subject Classification:* 03E55; 03E60, 03E57, 03E45, 03E35, 03E15, 00A30, 03D03, **AMS members US$88.80**, List US$111, Order code CONM/690



Kolmogorov Complexity and Algorithmic Randomness
A. Shen
V. A. Uspensky
N. Vereshchagin
American Mathematical Society

Kolmogorov Complexity and Algorithmic Randomness

A. Shen, *LIRMM CRNS, Université de Montpellier, France*, **V. A. Uspensky**, *Lomonosov Moscow State University, Russia*, and **N. Vereshchagin**, *Lomonosov Moscow State University, Russia*

Looking at a sequence of zeros and ones, we often feel that it is not random, that is, it is not plausible as an outcome of fair coin tossing. Why? The answer is provided by algorithmic information theory: because the sequence is compressible, that is, it has small complexity or, equivalently, can be produced by a short program. This idea, going back to Solomonoff, Kolmogorov, Chaitin, Levin, and others, is now the starting point of algorithmic information theory.

The first part of this book is a textbook-style exposition of the basic notions of complexity and randomness; the second part covers some recent work done by participants of the "Kolmogorov seminar" in Moscow (started by Kolmogorov himself in the 1980s) and their colleagues.

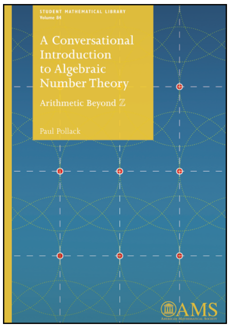
This book contains numerous exercises (embedded in the text) that will help readers to grasp the material.

Contents: Plain Kolmogorov complexity; Complexity of pairs and conditional complexity; Martin-Löf randomness; A priori probability and prefix complexity; Monotone complexity; General scheme for complexities; Shannon entropy and Kolmogorov complexity; Some applications; Frequency and game approaches to randomness; Inequalities for entropy, complexity, and size; Common information; Multisource algorithmic information theory; Information and logic; Algorithmic statistics; Complexity and foundations of probability; Four algorithmic faces of randomness; Bibliography; Name index; Subject Index.

Mathematical Surveys and Monographs, Volume 220

August 2017, approximately 499 pages, Hardcover, ISBN: 978-1-4704-3182-2, LC 2016049293, 2010 *Mathematics Subject Classification:* 68Q30, 03D32, 60A99, 97K70, **AMS members US$99.20**, List US$124, Order code SURV/220

Number Theory



A Conversational Introduction to Algebraic Number Theory

Arithmetic Beyond $\mathbb{Z}$

Paul Pollack, *University of Georgia, Athens, GA*

Gauss famously referred to mathematics as the "queen of the sciences" and to number theory as the "queen of mathematics". This book is an introduction to **algebraic** number theory, meaning the study of arithmetic in finite extensions of the rational number field $\mathbb{Q}$. Originating in the work of Gauss, the foundations of modern algebraic number theory are due to Dirichlet, Dedekind, Kronecker, Kummer, and others. This book lays out basic results, including the three "fundamental theorems": unique factorization of ideals, finiteness of the class number, and Dirichlet's units theorem. While these theorems are by now quite classical, both the text and the exercises allude frequently to more recent developments.

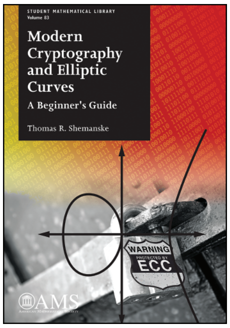
In addition to traversing the main highways, the book reveals some remarkable vistas by exploring scenic side roads. Several topics appear that are not present in the usual introductory texts. One example is the inclusion of an extensive discussion of the theory of elasticity, which provides a precise way of measuring the failure of unique factorization.

The book is based on the author's notes from a course delivered at the University of Georgia; pains have been taken to preserve the conversational style of the original lectures.

Contents: Getting our feet wet; Cast of characters; Quadratic number fields: First steps; Paradise lost — and found; Euclidean quadratic fields; Ideal theory for quadratic fields; Prime ideals in quadratic number rings; Units in quadratic number rings; A touch of class; Measuring the failure of unique factorization; Euler's prime-producing polynomial and the criterion of Frobenius–Rabinowitsch; Interlude: Lattice points; Back to basics: Starting over with arbitrary number fields; Integral bases: From theory to practice, and back; Ideal theory in general number rings; Finiteness of the class group and the arithmetic of $\overline{\mathbb{Z}}$; Prime decomposition in general number rings; Dirichlet's units theorem, I; A case study: Units in $\mathbb{Z}[\sqrt[3]{2}]$ and the Diophantine equation $X^3 - 2Y^3 = \pm 1$; Dirichlet's units theorem, II; More Minkowski magic, with a cameo appearance by Hermite; Dedekind's discriminant theorem; The quadratic Gauss sum; Ideal density in quadratic number fields; Dirichlet's class number formula; Three miraculous appearances of quadratic class numbers; Index.

Student Mathematical Library, Volume 84

September 2017, approximately 311 pages, Softcover, ISBN: 978-1-4704-3653-7, 2010 *Mathematics Subject Classification:* 11R04, 11R11, 11R27, 11R29, 11R32, **AMS members US$41.60**, List US$52, Order code STML/84



Modern Cryptography and Elliptic Curves

A Beginner's Guide

Thomas R. Shemanske, *Dartmouth College, Hanover, NH*

This book offers the beginning undergraduate student some of the vista of modern mathematics by developing and presenting the tools needed to gain an understanding of the arithmetic of elliptic curves over finite fields and their applications to modern cryptography. This gradual introduction also makes a significant effort to teach students how to produce or discover a proof by presenting mathematics as an exploration, and at the same time, it provides the necessary mathematical underpinnings to investigate the practical and implementation side of elliptic curve cryptography (ECC).

Elements of abstract algebra, number theory, and affine and projective geometry are introduced and developed, and their interplay is exploited. Algebra and geometry combine to characterize congruent numbers via rational points on the unit circle, and group law for the set of points on an elliptic curve arises from geometric intuition provided by Bézout's theorem as well as the construction of projective space. The structure of the unit group of the integers modulo a prime explains RSA encryption, Pollard's method of factorization, Diffie–Hellman key exchange, and ElGamal encryption, while the group of points of an elliptic curve over a finite field motivates Lenstra's elliptic curve factorization method and ECC.

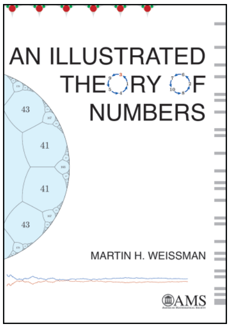
The only real prerequisite for this book is a course on one-variable calculus; other necessary mathematical topics are introduced on-the-fly. Numerous exercises further guide the exploration.

This item will also be of interest to those working in applications.

Contents: Three motivating problems; Back to the beginning; Some elementary number theory; A second view of modular arithmetic: $\mathbb{Z}_n$ and U_n; Public-key cryptography and RSA; A little more algebra; Curves in affine and projective space; Applications of elliptic curves; Deeper results and concluding thoughts; Answers to selected exercises; Bibliography; Index.

Student Mathematical Library, Volume 83

August 2017, approximately 261 pages, Softcover, ISBN: 978-1-4704-3582-0, LC 2017001191, 2010 *Mathematics Subject Classification:* 11-01, 68-01, 11Axx, 14G50, 11T71, 68P25, 11Y05, 91A60, 11G05, 81P68, **AMS members US$41.60**, List US$52, Order code STML/83



An Illustrated Theory of Numbers

Martin H. Weissman, *University of California, Santa Cruz, CA*

An Illustrated Theory of Numbers gives a comprehensive introduction to number theory, with complete proofs, worked examples, and exercises. Its exposition reflects the most recent scholarship in mathematics and its history. Almost 500 sharp illustrations accompany elegant proofs, from prime decomposition through quadratic reciprocity. Geometric and dynamical arguments provide new insights, and allow for a rigorous approach with less algebraic manipulation. The final chapters contain an extended treatment of binary quadratic forms, using Conway's topograph to solve quadratic Diophantine equations (e.g., Pell's equation) and to study reduction and the finiteness of class numbers.

Data visualizations introduce the reader to open questions and cutting-edge results in analytic number theory such as the Riemann hypothesis, boundedness of prime gaps, and the class number 1 problem. Accompanying each chapter, historical notes curate primary sources and secondary scholarship to trace the development of number theory within and outside the Western tradition.

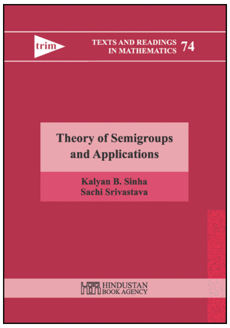
Requiring only high school algebra and geometry, this text is recommended for a first course in elementary number theory, and to all mathematicians seeking a fresh perspective on an ancient subject.

Contents: Seeing arithmetic; *Foundations:* The Euclidean algorithm; Prime factorization; Rational and constructible numbers; Gaussian and Eisenstein integers; *Modular arithmetic:* The modular worlds; Modular dynamics; Assembling the modular worlds; Quadratic residues; *Quadratic forms:* The topograph; Definite forms; Indefinite forms; Index of theorems; Index of terms; Index of names; Bibliography.

August 2017, approximately 321 pages, Hardcover, ISBN: 978-1-4704-3493-9, 2010 *Mathematics Subject Classification:* 11A05, 11A07, 11A15, 11A41, 11A51, 11D04, 11D09, 11E16, 11E41, **AMS members US$55.20**, List US$69, Order code MBK/105

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Theory of Semigroups and Applications

Kalyan B. Sinha, *Jawaharlal Nehru Centre for Advanced Scientific Research, Bangladore, India*, and **Sachi Srivastava**, *University of Delhi South Campus, New Delhi, India*

This book combines the spirit of a textbook with that of a monograph on the topic of semigroups and their applications. It is expected to have potential readers across a broad spectrum that includes operator theory, partial differential equations, harmonic analysis, probability and statistics, and classical and quantum mechanics.

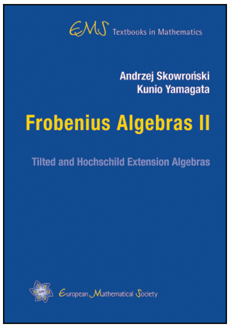
A reasonable amount of familiarity with real analysis, including the Lebesgue-integration theory, basic functional analysis, and bounded linear operators is assumed. However, any discourse on a theory of semigroups needs an introduction to unbounded linear operators, some elements of which have been included in the Appendix, along with the basic ideas of the Fourier transform and of Sobolev spaces. Chapters 4–6 contain advanced material not often found in textbooks but which have many interesting applications, such as the Feynman–Kac formula, the central limit theorem, and the construction of Markov semigroups. The exercises are given in the text as the topics are developed so that interested readers can solve these while learning that topic.

This item will also be of interest to those working in analysis.

A publication of Hindustan Book Agency; distributed within the Americas by the American Mathematical Society. Maximum discount of 20% for all commercial channels.

Hindustan Book Agency

April 2017, 180 pages, Hardcover, ISBN: 978-93-86279-63-7, 2010 *Mathematics Subject Classification:* 47D06, **AMS members US$30.40**, List US$38, Order code HIN/73



Frobenius Algebras II

Tilted and Hochschild Extension Algebras

Andrzej Skowroński, *Nicolaus Copernicus University, Toruń, Poland*, and **Kunio Yamagata**, *Tokyo University of Agriculture and Technology, Japan*

This is the second of three volumes which will provide a comprehensive introduction to the modern representation theory of Frobenius algebras. The first part of the book is devoted to fundamental results of the representation theory of finite dimensional hereditary algebras and their tilted algebras, which allow the authors to describe the representation theory of prominent classes of Frobenius algebras.

The second part is devoted to basic classical and recent results concerning the Hochschild extensions of finite dimensional algebras by duality bimodules and their module categories. Moreover, the shapes of connected components of the stable Auslander-Reiten quivers of Frobenius algebras are described.

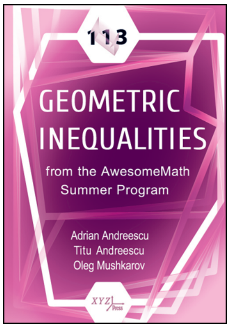
The only prerequisite for this volume is a basic knowledge of linear algebra and some results of the first volume. It includes complete proofs of all results presented and provides a rich supply of examples and exercises.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Textbooks in Mathematics, Volume 19

May 2017, 629 pages, Hardcover, ISBN: 978-3-03719-174-3, 2010 *Mathematics Subject Classification:* 16-01, 13E10, 15A63, 15A69, 16Dxx, 16E10, 16E30, 16E40, 16G10, 16G20, 16G60, 16G70, 16S50, 16S70, 18A25, 18E30, 18G15, **AMS members US$62.40**, List US$78, Order code EMSTEXT/19

Math Education



113 Geometric Inequalities from the AwesomeMath Summer Program

Adrian Andreescu, *University of Texas at Dallas, Richardson, TX*, **Titu Andreescu**, *University of Texas at Dallas, Richardson, TX*, and **Oleg Mushkarov**, *Bulgarian Academy of Sciences, Institute of Mathematics, Sofia, Bulgaria*

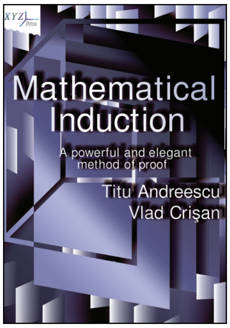
For curious readers looking to sharpen their arsenal of mathematical strategies on the Olympiad level, *113 Geometric Inequalities from the AwesomeMath Summer Program* is a valuable addition. This problem-solving methodology prompts key ideas in other domains such as calculus or complex numbers as the solutions are usually nonstandard in a geometric sense. Nevertheless, the book encourages readers to try their hand at these types of inequalities. The goal is to consolidate mathematical reasoning while providing exposure to a broad range of problems, all teeming with insightful inequality-type solutions.

This item will also be of interest to those working in general interest.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 23

December 2016, 202 pages, Hardcover, ISBN: 978-0-9799269-8-3, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$47.96**, List US$59.95, Order code XYZ/23



Mathematical Induction: A Powerful and Elegant Method of Proof

Titu Andreescu, *University of Texas at Dallas, Richardson, TX*, and **Vlad Crişan**, *University of Göttingen, Germany*

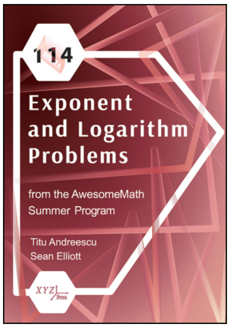
This book serves as a very good resource and teaching material for anyone who wants to discover the beauty of Induction and its applications, from novice mathematicians to Olympiad-driven students and professors teaching undergraduate courses. The authors explore 10 different areas of mathematics, including topics that are not usually discussed in an Olympiad-oriented book on the subject. Induction is one of the most important techniques used in competitions and its applications permeate almost every area of mathematics.

This item will also be of interest to those working in general interest.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 25

March 2017, 432 pages, Hardcover, ISBN: 978-0-9968745-9-5, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$55.96**, List US$69.95, Order code XYZ/25



114 Exponent and Logarithm Problems from the AwesomeMath Summer Program

Titu Andreescu, *University of Texas at Dallas, Richardson, TX* and **Sean Elliott**

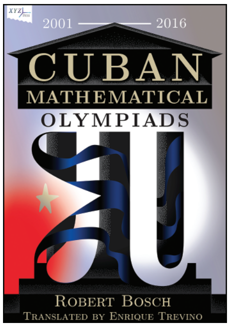
This book covers the theoretical background of exponents and logarithms, as well as some of their important applications. Starting from the basics, the reader will gain familiarity with how the exponential and logarithmic functions work and will then learn how to solve different problems with them. The authors give the readers the opportunity to test their understanding of the topics discussed by exposing them to 114 carefully chosen problems whose full solutions can be found at the end of the book.

This item will also be of interest to those working in general interest.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 24

December 2016, 150 pages, Hardcover, ISBN: 978-0-9968745-6-4, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$39.96**, List US$49.95, Order code XYZ/24



Cuban Mathematical Olympiads

Robert Bosch

Translated by Enrique Treviño

This books stems from the desire to publish Cuban National Mathematical Olympiad problems with elegant solutions and illustrations. It encompasses all problems from the 2001 to 2016 Olympiads, except for 2002, with thorough, in-depth solutions. This work is much more than a compilation of problems; it is a meticulous exposition with complete solutions to every problem, allowing readers to grasp the problem-solving techniques discussed.

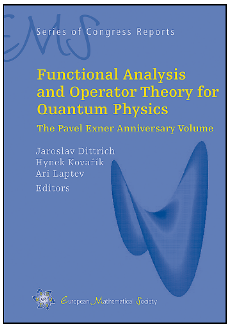
This item will also be of interest to those working in general interest.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 26

March 2017, 203 pages, Hardcover, ISBN: 978-0-9968745-4-0, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$43.96**, List US$54.95, Order code XYZ/26

Mathematical Physics



Functional Analysis and Operator Theory for Quantum Physics

The Pavner Exner Anniversary Volume

Jaroslav Dittrich, *Czech Academy of Sciences, Rez-Prague, Czech Republic*, **Hynek Kovařík**, *Università degli Studi di Brescia, Italy*, and **Ari Laptev**, *Imperial College London, United Kingdom*, Editors

This volume is dedicated to Pavel Exner on the occasion of his 70th anniversary. It collects contributions by numerous scientists with expertise in mathematical physics, and, in particular, in problems arising from quantum mechanics.

The questions addressed in the contributions cover a wide range of topics. A lot of attention is paid to differential operators with zero range interactions, which are often used as models in quantum mechanics. Several authors consider problems related to systems with mixed-dimensions such as quantum waveguides, quantum layers and quantum graphs. Eigenvalues and eigenfunctions of Laplace and Schrödinger operators are discussed too, as well as systems with adiabatic time evolution.

Although most of the problems treated in the book have a quantum mechanical background, some contributions deal with issues which go well beyond this framework: for example, the Cayley–Hamilton theorem, approximation formulae for contraction semigroups or factorization of analytic operator-valued Fredholm functions. As for the mathematical tools involved, the book provides a wide variety of techniques from functional analysis and operator theory.

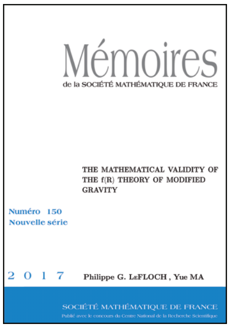
Altogether, the volume presents a collection of research papers which will be of interest to any active scientist working in one of the above mentioned fields.

This item will also be of interest to those working in differential equations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Series of Congress Reports, Volume 12

May 2017, 597 pages, Hardcover, ISBN: 978-3-03719-175-0, 2010 *Mathematics Subject Classification:* 81Q37, 81Q35, 35P15, 35P25, **AMS members US$94.40**, List US$118, Order code EMSSCR/12



Mémoires
de la SOCIÉTÉ MATHÉMATIQUE DE FRANCE

THE MATHEMATICAL VALIDITY OF
THE f(R) THEORY OF MODIFIED
GRAVITY

Numéro 150
Nouvelle série

2017

Philippe G. LeFLOCH , Yue MA

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du Centre National de la Recherche Scientifique

The Mathematical Validity of the $f(R)$ Theory of Modified Gravity

Philippe G. LeFloch, *Université Pierre et Marie Curie, Paris, France*, and **Yue Ma**, *Xi'an Jiaotong University, China*

This monograph solves the Cauchy problem for the $f(R)$ theory of modified gravity, which generalizes Einstein's theory. In the Einstein–Hilbert functional, the spacetime scalar curvature R is replaced by a nonlinear function $f(R)$. The field equations are of order four in the derivatives of the metric.

In this pioneering work, the authors provide a rigorous validation of this theory by analyzing the singular convergence of $f(R)$ toward R.

This item will also be of interest to those working in differential equations.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

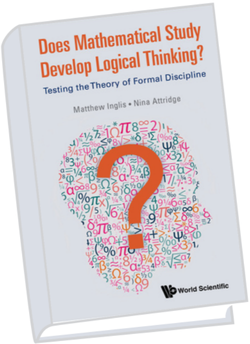
Mémoires de la Société Mathématique de France, Number 150

January 2017, 119 pages, Softcover, ISBN: 978-2-85629-849-7, 2010 *Mathematics Subject Classification:* 83C05, 35L15, 83C99, **AMS members US$41.60**, List US$52, Order code SMFMEM/150

BOOKSHELF

New and Noteworthy Titles on Our Bookshelf

June/July 2017

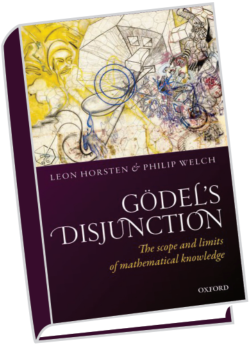


Does Mathematical Study Develop Logical Thinking?: Testing the Theory of Formal Discipline, by Matthew Inglis and Nina Attridge (World Scientific, February 2017).

One reason mathematics occupies such a central position in education is the widespread belief that studying mathematics doesn't just endow students with skills specific to the subject. It also helps them develop general abilities to think and reason logically. This belief, called the "Theory of Formal Discipline (TFD)," was for a long time invoked to justify teaching Latin, which was also thought to develop general brainpower. But does the TFD really stand up to scrutiny? This is the question this book sets out to explore. The authors point out that, during the 20th century, a large body of research in educational psychology has concluded that the TFD is "largely false." This has led to a "strange situation" regarding the TFD: "large numbers of mathematics graduates and mathematicians are apparently convinced that studying mathematics develops the general thinking skills of logical reasoning and problem solving, but throughout most of the twentieth century psychologists were apparently convinced that this could not be the case." The authors present a series of studies that attempt to resolve this conundrum. They find that while the TFD is consistent with the results of their studies, the picture is more complex than commonly assumed. In particular, the results are sometimes better explained by what they call the "filtering hypothesis." This hypothesis says that study of mathematics acts as a filter that retains students who are good at logical reasoning. In other words, it's exactly those students with good logical reasoning skills who persist in taking math beyond the required courses; their superior skills are then chalked up to all the math they have taken. Trying to carefully tease out the various factors at play, the authors offer a nuanced picture of the reliability and limitations of the TFD.

Suggestions for the BookShelf can be sent to `notices-booklist@ams.org`.

We try to feature items of broad interest. Appearance of a book in the *Notices* BookShelf does not represent an endorsement by the *Notices* or by the AMS. For more, visit the AMS Reviews webpage `www.ams.org/news/math-in-the-media/reviews`.



Gödel's Disjunction: The Scope and Limits of Mathematical Knowledge, edited by Leon Horsten and Philip Welch (Oxford University Press, August 2016).

"I want to start by quarrelling with Sir Roger Penrose." These words appear in a 2003 paper by Oxford philosopher John Lucas, whose lively and longstanding debate with Roger Penrose drew in many thinkers from philosophy, mathematics, physics, and other subjects. The nub of the debate is the topic of this book, namely, Gödel's disjunction, which Kurt Gödel stated in his Gibbs Lecture, presented at an AMS meeting in December 1951 in Providence, RI. The lecture initially went unpublished but eventually appeared in 1995 in a collection of his work edited by Solomon Feferman. Here is Gödel's disjunction, in Gödel's own words: "Either ... the human mind ... infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable diophantine problems." The introduction of *Gödel's Disjunction* provides this alternate statement: "Either the human mind cannot be captured by an algorithm, or there exist absolutely undecidable propositions." Gödel asserted in the Gibbs lecture that this disjunction is a "mathematically established fact." But, as Peter Koellner asks in the paper he contributed to this volume: "Which is it? Is it the case that the mind cannot be mechanized? Or is it the case that mathematical truth outstrips human reason? Or is it perhaps both?" These are the questions that inspired not only the Lucas-Penrose debate but a great deal of scholarly thought. *Gödel's Disjunction* contains papers that grew out of two workshops held in 2012 and 2013 at the University of Bristol, in which participants sought to develop insights into the two disjuncts and, as the introduction states, "to come closer to deciding the truth value of either of them." Interestingly, Gödel accepted as true the first disjunct—the mind is not like a machine—and he hoped to refute the second disjunct that there exist absolutely undecidable propositions. Emil Post, who independently thought about such matters in the 1920s, took the opposite view. As Graham Leach-Krouse writes in his contribution to the volume, "Post, on the basis of a rejection of [the first disjunct], apparently hoped to prove [the second]." Containing essays at a range of technical levels, this thought-provoking volume holds appeal for experts as well as more-general readers.

MATHEMATICS CALENDAR

This section contains new announcements of worldwide meetings and conferences of interest to the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. New announcements only are published in the print Mathematics Calendar featured in each *Notices* issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable). A second announcement will be published only if there are changes or necessary additional information. Asterisks (*) mark those announcements containing revised information.

In general, print announcements of meetings and conferences carry only the date, title and location of the event.

The complete listing of the Mathematics Calendar is available at: `www.ams.org/meetings/calendar/mathcal`

All submissions to the Mathematics Calendar should be done online via: `www.ams.org/cgi-bin/mathcal/mathcal-submit.pl`

Any questions or difficulties may be directed to `mathcal@ams.org`.

May 2017

1 – July 25 **Call for papers for August 2017 Issue.**
Location: *Kumasi, Ghana.*
URL: `pubs.jomaar.org`

6 – November 23 **Open International Call Veras Mathematic World 2017 (VMW'17)**
Location: *Centre of Mathematic & Design, Faculty of Architecture, Design and Urbanism of the University of Buenos Aires.*
URL: `verasmathematicworld.org`

29 – June 2 **CRC 1114 Spring School: Methods for Particle Systems with Multiple Scales**
Location: *WIAS, Mohrenstr. 39, 10117 Berlin, Germany.*
URL: `www.wias-berlin.de/workshops/MPSMS17`

June 2017

10 – 11 **Fifth Annual Conference for the Exchange of Mathematical Ideas**
Location: *Department of Mathematics, University of Northern Iowa, Cedar Falls, IA, USA.*
URL: `noetherian.net/conference`

23 – 23 **Colloquium "Celebrating Contributions of Antonio Campillo to Mathematics"**
Location: *Valladolid, Spain.*
URL: `www.singacom.uva.es/ColoquioCampillo2017`

26 – 30 **Boston University - Keio University Workshop**
Location: *Boston University, Boston, MA.*
URL: `math.bu.edu/keio2017`

29 – 30 **Journées palaisiennes de combinatoire additive / Palaiseau Days on Additive Combinatorics**
Location: *Ecole polytechnique, Palaiseau, France.*
URL: `www.cmls.polytechnique.fr/perso/plagne/JPCA.htm`

July 2017

17 – 19 **International Conference in Number Theory and Applications 2017 (ICNA2017)**
Location: *Kasetsart University, Bangkok, Thailand.*
URL: `maths.sci.ku.ac.th/icna2017/main.html`

17 – 21 **Enumerative Geometry, Mirror Symmetry, and Physics**
Location: *University of Illinois at Urbana-Champaign, Urbana, Illinois.*
URL: `www.math.uiuc.edu/~nevins/EGMSP.html`

31 – August 4 **GAP XV "Quantization" State College (Penn State)**
Location: *The Pennsylvania State University, State College, Pennsylvania.*
URL: `www.geometryandphysics.org/gapxv/`

August 2017

8 – 10 **The 42nd Sapporo Symposium on Partial Differential Equations**
Location: *7-310, 7-219/220, Faculty of Science Bld. #7, Hokkaido University, Sapporo, Japan.*
URL: `www.math.sci.hokudai.ac.jp/sympo/sapporo/program170808_en.html`

11 – 13 **International Conference on Research and Education in Mathematics (ICREM)**
Location: *Bandung Institute of Technology, Bandung, West Java, Indonesia.*
URL: `icrem.math.itb.ac.id`

12 – 19 **The XI Americas Conference on Differential Equations and Nonlinear Analysis**
Location: *University of Alberta, Edmonton, Canada.*
URL: `sites.ualberta.ca/~mathirl/AmericasXI.html`

20 – January 31, 2018 **Jean-Morlet whole semester on Tiling and Discrete Geometry (Shigeki Akiyama and Pierre Arnoux)**
Location: *CIRM Luminy, Marseille.*
URL: `akiyama-arnoux.weebly.com`

21 – 24 **SUMMER SCHOOL IN QUANTITATIVE RISK MANAGEMENT**
Location: *Room Beethoven at the CIRANO, 14th floor, 1130, rue Sherbrooke ouest, Montréal (métro Peel), Canada.*
URL: `qrmtutorial.org/news/37-summer-school-in-montreal-2017`

24 – 31 **6th Workshop on Fourier Analysis and Related Fields**
Location: *The Hungarian Academy of Sciences local Headquarters, Pécs, Hungary.*
URL: `www.fourier2017.ttk.pte.hu`

27 – September 2 **XII International Symposium on Generalized Convexity and Monotonicity**
Location: *Hajduszoboszlo, Hungary.*
URL: `gcm.up.krakow.pl`

28 – 31 **Environmental Risk Modeling and Extreme Events**
Location: *Centre de recherches mathématiques Université de Montréal Pavillon André-Aisenstadt 2920, Chemin de la tour, 5th floor Montréal (Québec) H3T 1J4 CANADA.*
URL: `www.crm.umontreal.ca/2017/Extreme17`

28 – September 6 **Summer school on geometrical methods in control theory and mathematical physics**
Location: *Wisła, Poland.*
URL: `baltinmat.diffiety.org/events/event/summer-school-on-geometrical-methods-in-control-theory-and-mathematical-physics/`

29 – 31 **Nonstandard growth phenomena 2017**
Location: *University of Turku, Turku, Finland.*
URL: `sites.google.com/site/nonstandardgrowth`

September 2017

14 – 15 **SPIFEC 2017: 1st European Workshop on Security and Privacy in Fog and Edge Computing (In conjunction with ESORICS 2017)**
Location: *Norway, Oslo.*
URL: `www.nics.uma.es/pub/spifec`

16 – 19 **Joint International Meeting of the German Mathematical Society and the Romanian Mathematical Society**
Location: *”Ovidius” University of Constanta, Romania.*
URL: `imar.ro/~imar/2017/DFG/description.php.html`

19 – 22 **Advances in Mathematics and Theoretical Physics**
Location: *Accademia Nazionale dei Lincei Via della Lungara, 230 I-00165 Rome, Italy.*
URL: `www.mat.uniroma2.it/tlc/17SIMP/index.php?p=home`

25 – 28 **Measurement and Control of Systemic Risk**
Location: *Centre de recherches mathématiques Université de Montréal Pavillon André-Aisenstadt 2920, Chemin de la tour, 5th floor Montréal (Québec) H3T 1J4 CANADA.*
URL: `www.crm.umontreal.ca/2017/Financier17`

October 2017

2 – 5 **Dependence Modeling Tools for Risk Management**
Location: *Centre de recherches mathématiques Université de Montréal Pavillon André-Aisenstadt 2920, Chemin de la tour, 5th floor Montréal (Québec) H3T 1J4 CANADA.*
URL: `www.crm.umontreal.ca/2017/Dependence17`

2 – 6 **Applications of the UCT for C*-algebras**
Location: *Department of Mathematical Sciences, University of Copenhagen, Copenhagen, Denmark.*
URL: `www.math.ku.dk/conferences/uct`

27 – 29 **No Boundaries: Groups in Algebra, Geometry, and Topology (A Celebration of the Mathematical Contributions of Benson Farb)**
Location: *University of Chicago.*
URL: `noboundaries.uchicago.edu`

November 2017

20 – 22 **International Conference on Analysis and its Applications (ICAA-2017)**
Location: *Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India.*
URL: `icaa2017.com`

20 – 24 **Tiling Dynamical System**
Location: *CIRM Luminy, Marseille.*
URL: `akiyama-arnoux.weebly.com/school.html`

25 – 26 **2nd International Conference for Computational Physics, Mathematics and Applications (ICCPMA 2017)**
Location: *Zurich, Switzerland.*
URL: `www.iccpma.iisrcrcc.com`

December 2017

4 – 8 **Future Directions in Representation Theory**
Location: *University of Sydney, Sydney, Australia.*
URL: `sites.google.com/site/ausreptheory/conference2017`

4 – 8 **Tilings and Recurrence**
Location: *CIRM Luminy, Marseille.*
URL: `akiyama-arnoux.weebly.com/conference.html`

21 – 23 **11th International Conference of IMBIC on "Mathematical Sciences for Advancement of Science and Technology (MSAST 2017)"**
Location: *IMBIC, Salt Lake City, Kolkata, India. Venue: Indismart Hotel, Salt Lake, Kolkata, website: indismart.in/*
URL: `imbic.org/forthcoming.html`

January 2018

8 – 13 **Singularities and Algebraic Geometry**
Location: *Da Nang, Vietnam.*
URL: `mim.hus.vnu.edu.vn/singularities-and-algebraic-geometry`

18 – 19 **Connections for Women: Enumerative Geometry Beyond Numbers**
Location: *Mathematical Sciences Research Institute, Berkeley, CA.*
URL: `www.msri.org/workshops/814`

February 2018

6 – 8 **3rd International Conference on Mathematical Sciences and Statistics (ICMSS2018)**
Location: *Putrajaya, Malaysia.*
URL: `einspem.upm.edu.my/icmss2018`

MEETINGS & CONFERENCES OF THE AMS

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The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: **www.ams.org/meetings/**.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 75 in the January 2017 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of $\LaTeX$ is necessary to submit an electronic form, although those who use $\LaTeX$ may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in $\LaTeX$. Visit **www.ams.org/cgi-bin/abstracts/abstract.pl/**. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

MEETINGS IN THIS ISSUE

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January 6–9	Washington, DC	p. 670

See **www.ams.org/meetings/** *for the most up-to-date information on these conferences.*

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `www.ams.org/meetings/`.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Montréal, Quebec Canada

McGill University

July 24–28, 2017

Monday – Friday

Meeting #1130

The second Mathematical Congress of the Americas (MCA 2017) is being hosted by the Canadian Mathematical Society (CMS) in collaboration with the Pacific Institute for the Mathematical Sciences (PIMS), the Fields Institute (FIELDS), Le Centre de Recherches Mathématiques (CRM), and the Atlantic Association for Research in the Mathematical Sciences (AARMS).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: January 23, 2017
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For abstracts: Expired

Denton, Texas

University of North Texas

September 9–10, 2017

Saturday – Sunday

Meeting #1131

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2017
Program first available on AMS website: July 27, 2017
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 38, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 18, 2017

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mirela Çiperiani, University of Texas at Austin, *Title to be announced.*

Adrianna Gillman, Rice University, *Title to be announced.*

Kevin M. Pilgrim, Department of Mathematics, Indiana University, Bloomington, IN, *Semigroups of branched mapping classes: dynamics and geometry.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Combinatorics of Flag Varieties (Code: SS 11A), **Martha Precup**, Northwestern University, and **Edward Richmond**, Oklahoma State University.

Analysis and PDEs in Geometry (Code: SS 20A), **Stephen McKeown**, Princeton University.

Applicable and Computational Algebraic Geometry (Code: SS 17A), **Eric Hanson**, Texas Christian University, and **Frank Sottile**, Texas A&M University.

Banach Spaces and Applications (Code: SS 9A), **Pavlos Motakis**, Texas A&M University, and **Bönyamin Sari**, University of North Texas.

Combinatorial/Geometric/Probabilistic Methods in Group Theory (Code: SS 19A), **Rostislav Grigorchuk** and **Volodymyr Nekrashevych**, Texas A&M University,

Dmytro Savchuk, University of South Florida, and **Zoran Sunic**, Texas A&M University.

Combinatorics and Representation Theory of Reflection Groups: Real and Complex (Code: SS 14A), **Elizabeth Drellich**, Swarthmore College, and **Drew Tomlin**, Hendrix College.

Commutative Algebra (Code: SS 10A), **Jonathan Montano**, University of Kansas, and **Alessio Sammartano**, Purdue University.

Differential Equation Modeling and Analysis for Complex Bio-systems (Code: SS 8A), **Pengcheng Xiao**, University of Evansville, and **Honghui Zhang**, Northwestern Polytechnical University.

Differential Geometry of Smooth and Discrete Surfaces in Euclidean and Lorentz Spaces (Code: SS 18A), **Barbara Shipman**, University of Texas at Arlington, and **Patrick Shipman**, Colorado State University.

Dynamics, Geometry and Number Theory (Code: SS 1A), **Lior Fishman** and **Mariusz Urbanski**, University of North Texas.

Fractal Geometry and Ergodic Theory (Code: SS 5A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Generalizations of Graph Theory (Code: SS 22A), **Nathan Reff**, SUNY Brockport, and **Lucas Rusnak** and **Piyush Shroff**, Texas State University.

Geometric Combinatorics and Combinatorial Commutative Algebra (Code: SS 16A), **Anton Dochtermann** and **Suho Oh**, Texas State University.

Homological Methods in Commutative Algebra (Code: SS 15A), **Peder Thompson**, Texas Tech University, and **Ashley Wheeler**, University of Arkansas.

Integrable Systems and Applications (Code: SS 24A), **Baofeng Feng**, The University of Texas Rio Grande Valley, and **Akif Ibragimov** and **Magdalena Toda**, Texas Tech University.

Invariants of Links and 3-Manifolds (Code: SS 7A), **Mieczyslaw K. Dabkowski** and **Anh T. Tran**, The University of Texas at Dallas.

Lie Algebras, Superalgebras, and Applications (Code: SS 3A), **Charles H. Conley**, University of North Texas, **Dimitar Grantcharov**, University of Texas at Arlington, and **Natalia Rozhkovskaya**, Kansas State University.

Mathematical and Computational Biology (Code: SS 21A), **Rajeev K. Azad**, University of North Texas, and **Brandilyn Stigler**, Southern Methodist University.

Noncommutative and Homological Algebra (Code: SS 4A), **Anne Shepler**, University of North Texas, and **Sarah Witherspoon**, Texas A&M University.

Nonlocal PDEs in Fluid Dynamics (Code: SS 12A), **Changhui Tan**, Rice University, and **Xiaoqian Xu**, Carnegie Mellon University.

Numbers, Functions, Transcendence, and Geometry (Code: SS 6A), **William Cherry**, University of North Texas, **Mirela Çiperiani**, University of Texas Austin, **Matt Papanikolas**, Texas A&M University, and **Min Ru**, University of Houston.

Real-Analytic Automorphic Forms (Code: SS 2A), **Olav K. Richter**, University of North Texas, and **Martin Westerholt-Raum**, Chalmers University of Technology.

Recent Progress on Hyperbolic Conservation Laws (Code: SS 23A), **Ilija Jegdic**, Texas Southern University, and **Katarina Jegdic**, University of Houston, Downtown.

Topics Related to the Interplay of Noncommutative Algebra and Geometry (Code: SS 13A), **Richard Chandler**, University of North Texas at Dallas, **Michaela Vancliff**, University of Texas at Arlington, and **Padmini Veerapen**, Tennessee Technological University.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the Texas state hotel tax (13%) and hotel fees may apply. Participants must state that they are with the **American Mathematical Society's (AMS) Fall Central Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

SpringHill Suites by Marriott Denton, 1434 Centre Pl Dr, Denton, TX 76205; (940) 383-4100; `www.marriott.com/hotels/fact-sheet/travel/dfwsd-springhill-suites-denton/`. Rates are **US$119** per night for a single occupancy, king room. Amenities include complimentary Wi-Fi in guest rooms; complimentary on-site parking; shuttle service to campus; newspaper and coffee in lobby. Rooms are furnished with a mini-fridge and coffee maker. This property is about a 5 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is 5:00 pm CST, **August 11, 2017.**

Courtyard by Marriott Denton, 2800 Colorado Blvd, Denton, TX 76210; (940) 382-4600; `www.marriott.com/hotels/hotel-photos/dfwde-courtyard-denton/.html`. Rates are **US$119** per night for single or double occupancy rooms. Amenities include complimentary on-site parking; 24-hour market stocked and convenient for snacks or meals on the go; free Wi-Fi; courtesy shuttle service to the University of North Texas campus; the latest news, weather and local information via the GoBoard; Bistro-Eat. Drink. Connect., which provides guests with healthy food, beverages, and Starbucks items during the morning breakfast and evening dinner service with cocktails; fitness center; indoor pool; 24-hour business center with boarding pass printing. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at a reduced rate is by 5:00 pm CST, **August 9, 2017.**

Best Western Premier Crown Chase Inn & Suites, 2450 Brinker, Denton, Texas 76208; (940) 387-1000; `https://www.bestwestern.com/en_US/book/hotels-in-denton/best-western-premier-crown-chase-`

`inn-suites/propertyCode.44648`.html. Rates for king and double queen rooms are **US$119** per night. Amenities include complimentary full breakfast buffet; exercise facilities; wireless internet connection in public areas; hot tub; cocktail lounge with light fare; outdoor pool; mircrowave, coffee maker and mini-frige in rooms.This property is about a 10 minute drive from the campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at a reduced rate is **August 18, 2017.**

Comfort Inn, 4050 Mesa Dr, Denton, TX 76207; (940) 320-5150; `https://www.choicehotels.com/texas/denton/comfort-inn-hotels/txa96`. Rates are **US$79** per night for a single king bed or **US$83** per night for two queen beds. Amenities include free WiFi; indoor heated pool and hot tub; exercise room; business center and guest laundry facilities. This property is about a 15 minute drive from the University of North Texas. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Thursday, August 10, 2017.**

Days Inn, Denton, 4211 I-35 Frontage Rd, Denton, TX 76207; (940) 600-1527; `https://www.wyndhamhotels.com/days-inn/denton-texas/days-inn-denton/overview`. Rates are **US$59.99** per night for a room with a single bed or **US$64.99** for a room with double beds; this rate is applicable for single or double occupancy. Amenities include business center; fitness center; free breakfast; free WiFi; laundry services; on-site parking and outdoor pool. This property is located approximately 2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation.The deadline for reservations at this rate is **Friday, August 11, 2017.**

La Quinta Inn & Suites, 4465 N Interstate 35, Denton, TX 76207; (940) 808-0444; `www.laquintadentonuniversitydr.com/`. Rates are **US$109** per night for a room with a single king or double queen beds; this rate is applicable for single or double occupancy. Amenities include free Bright Side Breakfast®, a daily assortment of waffles, hot and cold cereals, breads and muffins, fresh fruit, and all-you-can-drink coffee and juice; full business center; indoor swimming pool and Jacuzzi-style spa; free Wi-Fi access and free premium cable channels on a flat-screen television. This property is located approximately 2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Friday, August 11, 2017.**

Best Western Plus Denton Inn & Suites, 2910 W University Dr, Denton, TX 76201; (940) 591-7726; `bestwesterntexas.com/hotels/best-western-plus-denton-inn-and-suites`. Rates are **US$85.99** per night for a room with a single king or **US$93.99** per night for a room with double queen beds; this rate is applicable for single or double occupancy. Amenities include exercise facilities; outdoor pool; complimentary full breakfast including scrambled eggs, bagels, assorted pastries, juices and much more from 6:00am to 9:00am; guest laundry; free parking; business center and high-speed Internet access. This property is about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Thursday, August 24, 2017.**

Homewood Suites by Hilton, 2907 Shoreline Dr, Denton, TX 76201; (940) 382-0420; `homewoodsuites.hilton.com/en/hw/groups/personalized/F/FTWDEHW-AMS-20170906/index.jhtml?WT.mc_id=POG`. Rates are **US$119** per night for a king stuio suite or king one bedroom suite; this rate is applicable for single or double occupancy. Amenities include free full hot breakfast on weekend days from 7:00am–10:00am; complimentary Wi-Fi; fitness and business centers; outdoor pool and courtyard with sports court and BBQ grills. This property is about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Thursday, August 11, 2017.**

Fairfield Inn and Suites, 2900 W University Dr, Denton, TX 76201; (940) 384-1700; `www.marriott.com/hotels/travel/dfwdn-fairfield-inn-and-suites-denton/`. Rates are **US$99** per night for a room with a king bed or double queen beds; this rate is applicable for single or double occupancy. Amenities include free cable TV with HBO; mini-fridge and microwave in every room; LCD TVs in all rooms and complimentary hot breakfast. This property is about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Friday, August 25, 2017.**

Holiday Inn Express and Suites, Denton, 4485 N Interstate 35, Denton, TX 76207; (940) 808-0600; `https://www.ihg.com/holidayinnexpress/hotels/us/en/denton/dtowu/hoteldetail`. Rates are **US$119** per night for a room with double queen beds; this rate is applicable for single or double occupancy. Amenities include business center; free high-speed Internet; complimentary Express Start breakfast bar; heated indoor pool; indoor whirlpool; complimentary fitness center and on-site guest self-laundry facilities. This property is about a 10 minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **Monday, August 7, 2017.**

Food Services

On Campus: The University Union is the place on campus to relax and enjoy a variety of foods that will satisfy any taste and budget. Hours listed below are subject to change.

Located on the first floor of the University Union, *The Corner Store* serves grab-and-go salads, sandwiches, snacks, and homemade Scrappy's Ice Cream and is open from 11:00 am to 5:00 pm on Saturdays. *Which Wich i*s located near *The Corner Store* offering customized fresh-made sandwiches or nutritious Bowlwich salads and is open 11:00 am to 3:00 pm on Saturdays. *Jamba Juice* is located on the first floor in the *Barnes and Noble* and offers a wide selection of delicious, nutritious smoothies, freshly squeezed juices, and energy bowls packed with wholesome ingredients on Saturdays from 11:00 am to 3:00 pm.

Located on the second floor of the University Union, *Burger King* serves the famous Whopper, fries, shakes, and assorted breakfast items on Saturdays from 11:00 am to 9:00 pm. *Chick-fil-A* is nearby for fried or grilled sandwiches and waffle fries on Saturdays from 11:00 am to 9:00 pm. *Starbucks* serves rich brewed flavor of hot coffees and teas or iced coffees and specialty drinks on Saturdays from 9:00 am to 5:00 pm.

Fuzzy's Taco Shop is located on the second floor of the University Union, in the sports bar atmosphere of *The Syndicate,* and serves uniquely flavorful Baja-style tacos and spicy sides. *Fuzzy's Taco Shop* is open on Saturday from 10:00 am to midnight and Sunday from noon to midnight.

The University has two resident dining halls where the public is welcome to purchase dining passes for approximatley **US$7.50** plus taxes at either *Bruce Cafeteria,* open Saturday, or *Kerr Cafeteria*, open Sunday.

Bruce Cafeteria is centrally located on the corner of Chestnut and Avenue C. It features a fantastic salad bar with fresh fruit and veggies, freshly made deli sandwiches, an amazing selection of diverse hot entrees, healthy vegetarian options, hand tossed pizza and freshly prepared pasta. For breakfast, lunch and dinner hours visit `www.dining.unt.edu/brucehall.`

Kerr Cafeteria is located on Maple Street between Avenues A and B in Kerr Hall and is the University's largest dining facility. It features two salad bars, multiple entrée lines, a burger grill and a taco bar. The cafeteria features unique custom-designed food lines modeled after the historic Denton Square. It's also adorned with graphics featuring famous UNT alumni and prominent Texans, and a history of Denton, including historical photographs. For breakfast, lunch and dinner hours visit `www.dining.unt.edu/kerrhall.`

Off Campus: Denton's Historic Downtown Square is located about 10 blocks from campus and is characterized by its beautiful courthouse and light-drenched lawn with numerous restaurants. The corner of Hickory and Fry Streets near the campus also has a cluster of restaurants. More information on restaurants and local attractions in the Denton area can be found at `www.discoverdenton.com/what-to-do/`.

Some options for coffee include:

Aura Coffee, 1306 W Hickory St, Denton; (940) 565-1860; small shop serving espresso, lattes, smoothies and breakfast bites.

Cultivar Coffee, 235 W Hickory St, Denton; (940) 380-9319; `www.cultivarcoffee.com`; a micro-roaster and coffee bar in North Texas serving handcrafted espresso drinks and expertly brewed single-origin coffees.

Loco Cafe, 603 N Locust St, Denton; (940) 387-1413; `www.locodenton.com`; serving breakfast and lunch 7 days a week with biscuits baked daily, customized coffee with fresh flavors, loose tea, fresh sqeezed juice, handmade lemonade and limeade.

Some options nearby for dining include:

Potbelly Sandwich Shop, 1216 W Hickory St, Denton; (940) 297-1287; `www.potbelly.com`; a retro-style counter-serve chain known for made-to-order toasted sandwiches, salads and baked goods.

Chipotle Mexican Grill, 1224 W Hickory St, Denton; (940) 808-1073; `www.chipotle.com`; fast-food chain offering Mexican fare, including design-your-own burritos, tacos and bowls.

Crooked Crust, 101 Ave A, Denton; (940) 565-5999; `www.crookedcrust.com`; funky pizzeria serving hoagies, beer and build-your-own pies with unlimited toppings.

Fry Street Public House, 125 Ave A, Denton; (940) 323-9600; `www.frystreetpublichouse.com`; a retro-modern style pub serving delicious 'Pub Grub' 12:00 pm to 9:00 pm seven days a week.

Salata, 1200 W Hickory St, Denton; (940) 435-0831; `www.salata.com`; counter-serve chain offering made-to-order salads featuring signature dressings, plus wraps and soups.

Cool Beans, 1210 W Hickory St, Denton; (940) 382-7025; `www.coolbeansdenton.com`; bar and grill featuring occasional live music on its rooftop deck plus beer, whiskey and American grub.

Jimmy John's, 107 Ave A, Denton; (940) 484-5466; `www.jimmyjohns.com`; counter-serve chain specializing in sub and club sandwiches, plus signature potato chips.

Sushi Cafe and Bar, 1115 W Hickory St, Denton; (940) 380-1030; Asian restaurant with fresh sushi, a variety of sake at the bar and other Japanese classics.

Registration and Meeting Information

Advance Registration: Advance registration for this meeting opens on **July 3, 2017.** Advance registration fees will be **US$61** for AMS members, **US$90** for nonmembers, and **US$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing `mmsb@ams.org`. The deadline to cancel is the first day of the meeting.

On-site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located in the Business Leadership Building (BLB) Atrium. The Invited Addresses, Special Sessions and Contributed Paper Sessions will take place in classrooms which will be in the Business Leadership Building (BLB). Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at `https://it.unt.edu/sites/default/files/untcampusmap-fall2016.pdf`.

The registration desk will be open on Saturday, September 9, 7:30 am–4:00 pm. and Sunday, September 10, 8:00 am-12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

Other Activities

Book Sales: Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on-site orders! AMS members receive 40 percent off list price. Nonmembers receive a 25 percent discount. Not a member? Ask a representative about the benefits of AMS membership.

Complimentary Coffee will be served courtesy of AMS membership services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at mmsb@ams.org or meet@ams.org.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Local Information and Maps

This meeting will take place on the campus of University of North Texas. A campus map can be found at `https://it.unt.edu/sites/default/files/untcampusmap-fall2016.pdf`. Information about the University of North Texas Mathematics Department can be found at `math.unt.edu/`. Please visit the University of North Texas website at `https://www.unt.edu/` for additional information on the campus.

Please watch the AMS website at `www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting.

Parking

Attendees electing to stay at off campus hotels may park their car on campus in the public Highland Street Garage by the new Business Leadership Building or the Union Circle Garage, located at Welch and Union Circle by the University Union. The Highland Street Garage only accepts credit cards on weekends at a rate of **US$2** per hour, with a daily maximum of **US$16**. Parking in the Union Circle Garage requires downloading the Parkmobile App for payment.

Visitors may also park at parking meters, which are located on Highland Street and West Sycamore Street. The meters located on Fry Street, north of campus, have a one hour time limit. Meter parking is **US$2.25** per hour. The meters DO NOT accept silver coinage. You may only pay using the Parkmobile App. Read more about Parkmobile here `transportation.unt.edu/news.`

A campus parking map is available at `transportation.unt.edu/sites/default/files/UNT_Campus_Parking.pdf`. Questions regarding parking for visitors should be directed to the Parking and Transportation Office at (940) 565-3020. Additional information about parking on campus can be found here `transportation.unt.edu/parking`.

Travel

This meeting will take place on the main campus of University of North Texas, located in the heart of Denton, Texas.

By Air: The Dallas/Fort Worth International Airport (DFW) is served by most major airlines. Please visit the Dallas/Fort Worth International Airport web site for a list of airlines and lists of cities with daily direct flights; `www.dfwairport.com/airlines/index.php`.

There are several options available for transportation to and from the airport.

Super Shuttle offers van rides and car service from the Dallas/Fort Worth International Airport to University of North Texas with prices ranging from **US$54** to **US$110** per passenger each way and should be booked in advance at `https://www.supershuttle.com/locations/dallas-dfw-dal/`.

Taxi service is available on the upper level of Dallas/Fort Worth International Airport Terminals A, B, C and E, and on the lower level of Terminal D. An Airport Guest Assistant will be available at the taxi stands between 8:00 am and midnight. Between midnight and 8:00 am, dial (972) 574-5878 to request a taxi. For more information on taxi service, including approximate fares to various locations, please visit `https://www.dfwairport.com/transport/taxis/`.

Rental cars are also available at Dallas/Fort Worth International Airport. The Rental Center can be accessed by shuttle buses located at the lower level of each terminal. Car rental shuttle buses run 24 hours a day. For more details on the various car rental companies available at the airport, please visit `https://www.dfwairport.com/transport/`.

By Train: At the airport the DART Rail Station is located at Terminal A, Lower Level Curb, Entry A-10. A two-hour Local DART pass is **US$2.50**, a local day pass is **US$5** and is good for unlimited rides on all DART buses, DART Rail and TRE trains between Union Station and CentrePort/DFW Airport until 3:00 am the next day. The DART public transit connects to the DCTA train to Denton.

DART Rail Orange Line operates seven days a week to and from the DFW Airport Station, located at DFW Airport's Terminal A, Lower Level Curb, Entry A-10. Trains arrive at DFW regularly from 3:50 am to 1:19 am daily. Schedules for the Dallas Orange Line are available at `www.dart.org/riding/dartrailorangeline.asp`.

From the DART Rail Orange Line transfer to the DART Rail Green Line at Bachman Station and then continue to Denton on the A-train at the Trinity Mills station. The A-train schedule, fares and route map are available at `https://www.dcta.net/routes-schedules/a-train/a-train-schedule`. Please plan your trip by viewing the DART rail system map at `https://www.dart.org/maps/printrailmap.asp`.

From Dallas: take DART's Green Line to the Trinity Mills station in Carrollton. At the Trinity Mills station transfer to the A-train* with a simple cross-platform transfer. Take the A-train* to the *Downtown Denton Transit Center* located near the heart of Denton. For the A-train schedule please visit `https://www.dcta.net/routes-schedules/a-train/a-train-schedule`. For the Green Line schedule please visit `https://www.dart.org/riding/dartrailgreenline.asp`.

From Ft. Worth: take the TRE to Victory Station in Dallas. At Victory Station take DART's Green Line headed to Carrollton. Get off the Green Line train at the Trinity Mills station. At the Trinity Mills station transfer to the A-train* with a simple cross-platform transfer. Take the A-train* to the *Downtown Denton Transit Center* located near the heart of Denton.

**Please note, the A-train does not run on Sundays.For information on the DART and the A-train, please visit* `https://www.dcta.net/`.

By Bus: The Fort Worth Transportation Authority (FWTA) and the Denton County Transportation Authority (DCTA), have started a new bus service that connects Fort Worth and Denton with stops on the Alliance Heritage Parkway. The *North Texas Xpress Bus Service* makes round trips each weekday between approximately 5:15 am and 9:30 pm. For a map, schedule and fare information please visit `https://www.dcta.net/routes-schedules/other-dcta-services/north-texas-xpress`.

By Car:

From Dallas: Head North on 35 E N and merge with 35 N.

From Ft. Worth: From the Fort Worth area, head North on 35 W N and merge with 35 N

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): CV#04N30007. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

Local Transportation

Walking, biking and personal cars are recommended to get around campus and Denton.

By Bus: The University of North Texas *Campus Cruiser* provides convenient local transportation around campus. Please visit `transportation.unt.edu/transit/maps-routes/campus-cruiser` for the full area map with all bus stops. The *Campus Cruiser* operates Saturdays from 8:30 am to 6:46 pm and Sundays 8:30 am to 8:30 pm.

The *Campus Cruiser* route services on and off campus stops during the week and on weekends. Stops include Uptown Apartments, Victory Hall, & the GAB. Sunday service is provided by UNT mini shuttle buses. The *Campus Cruiser* will pick up and drop off passengers at any of the UNT bus stops on the route. *Please note, the Campus Cruiser does not operate on Fridays.*

Weather:

The average high temperature for September is approximately 88 degrees Fahrenheit and the average low is approximately 65 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking:

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants:

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `https://travel.state.gov/content/travel/en.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `cro@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Buffalo, New York

State University of New York at Buffalo

September 16–17, 2017

Saturday – Sunday

Meeting #1132

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2017
Program first available on AMS website: August 3, 2017
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 38, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 25, 2017

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Inwon Kim, University of California at Los Angeles, *Capillary drops on rough surfaces.*

Govind Menon, Brown University, *Building polyhedra by self-assembly.*

Bruce E. Sagan, Michigan State University, *The protean chromatic polynomial.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advanced Techniques in Graph Theory (Code: SS 9A), **Sogol Jahanbekam** and **Paul Wenger**, Rochester Institute of Technology.

Algebraic Topology (Code: SS 17A), **Claudia Miller**, Syracuse University, and **Inna Zakharevich**, Cornell University.

Automorphic Forms and L-functions (Code: SS 14A), **Mahdi Asgari**, Oklahoma State University, and **Joseph Hundley**, University at Buffalo-SUNY.

CR Geometry and Partial Differential Equations in Complex Analysis (Code: SS 5A), **Ming Xiao**, University of Illinois at Urbana-Champaign, and **Yuan Yuan**, Syracuse University.

Cohomology, Deformations, and Quantum Groups: A Session Dedicated to the Memory of Samuel D. Schack (Code: SS 6A), **Miodrag Iovanov**, University of Iowa, **Mihai D. Staic**, Bowling Green State University, and **Alin Stancu**, Columbus State University.

Geometric Group Theory (Code: SS 4A), **Joel Louwsma**, Niagara University, and **Johanna Mangahas**, University at Buffalo-SUNY.

High Order Numerical Methods for Hyperbolic PDEs and Applications (Code: SS 2A), **Jae-Hun Jung**, University at Buffalo-SUNY, **Fengyan Li**, Rensselaer Polytechnic Institute, and **Li Wang**, University at Buffalo-SUNY.

Infinite Groups and Geometric Structures: A Session in Honor of the Sixtieth Birthday of Andrew Nicas (Code: SS 7A), **Hans Boden**, McMaster University, and **David Rosenthal**, St. John's University.

Knots, 3-manifolds and their Invariants (Code: SS 15A), **William Menasco** and **Adam Sikora**, University at Buffalo-SUNY, and **Stephan Wehrli**, Syracuse University.

Nonlinear Dispersive Partial Differential Equations (Code: SS 18A), **Santosh Bhattrai**, Trocaire College, and **Sharad Silwal**, Jefferson College of Health Sciences.

Nonlinear Evolution Equations (Code: SS 16A), **Marius Beceanu**, SUNY Albany, and **Dan-Andrei Geba**, University of Rochester.

Nonlinear Partial Differential Equations Arising from Life Science (Code: SS 8A), **Junping Shi**, College of William and Mary, and **Xingfu Zou**, University of Western Ontario.

Nonlinear Wave Equations, Inverse Scattering and Applications. (Code: SS 1A), **Gino Biondini**, University at Buffalo-SUNY.

Polynomials in Enumerative, Algebraic, and Geometric Combinatorics (Code: SS 13A), **Robert Davis** and **Bruce Sagan**, Michigan State University.

Recent Advancements in Representation Theory (Code: SS 12A), **Yiqiang Li**, University at Buffalo-SUNY, and **Gufang Zhao**, University of Massachusetts.

Recent Progress in Geometric Analysis (Code: SS 11A), **Ovidiu Munteanu**, University of Connecticut, **Terrence Napier**, Lehigh University, and **Mohan Ramachandran**, University at Buffalo.

Structural and Chromatic Graph Theory (Code: SS 10A), **Hong-Jian Lai**, **Rong Luo**, and **Cun-Quan Zhang**, West

Virginia University, and **Yue Zhao**, University of Central Florida.

p-adic Aspects of Arithmetic Geometry (Code: SS 3A), **Liang Xiao**, University of Connecticut, and **Hui June Zhu**, University at Buffalo-SUNY.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the tax of 13.75% per room (8.75% NY State Sales Tax and 5% Erie County Hotel Tax). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at SUNY Buffalo** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Buffalo Marriott Niagara, 1340 Millersport Highway, Amherst, NY, 14221; (716) 689-6900 ,`http://www.marriott.com/hotels/travel/bufny-buffalo-marriott-niagara/`. Rates are US$97 per night for a guest room with one king bed or two double beds; this rate is applicable for single or double occupancy. To reserve a room at this rate contact Marriott reservations at (800) 228-9290 or (716) 689-6900. Amenities include free WiFi in sleeping rooms, fitness center with indoor/outdoor pool, business center, *BlueFire Bar & Grille* on site restaurant (offering room service), complimentary airport shuttle upon request, and a gift shop serving Starbucks coffee daily. Check-in is at 4:00 pm, check-out is at 12:00 pm This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 25, 2017.**

Candlewood Suites Buffalo/Amherst, 20 Flint Road, Amherst, NY, 14226; (716) 688-2100, `http://cwsbuffaloamherst.com/`. Rates are US$107 per night for a guest room with one queen bed or two queen beds, this rate is applicable for single or double occupancy. Each guest room features a fully equipped kitchen – including refrigerator, dishwasher, microwave and stove top, plus a variety of dishes, cookware and utensils. Amenities include free WiFi in sleeping rooms, *Candlewood Gym* (fitness center), business center, and a convenience store on property. This is a pet-friendly property. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately .2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 25, 2017.**

Comfort Inn University, 1 Flint Road, Amherst, NY, 14226; (716) 688-0811, `https://www.choicehotels.com/new-york/amherst/comfort-inn-hotels/ny293` Rates are US$109 per night for a guest room with two queen beds, this rate is applicable for single or double occupancy. Amenities include free WiFi in sleeping rooms, fitness center, indoor pool, business center, laundry facilities, and complimentary hot breakfast. This is a pet-friendly property. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately .2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 25, 2017.**

Doubletree by Hilton Buffalo-Amherst, 10 Flint Road, Amherst, NY, 14226; (716) 689-4296, `http://doubletree3.hilton.com/en/hotels/new-york/doubletree-by-hilton-hotel-buffalo-amherst-BUFFLDT/index.html`. Rates are US$107 per night for a guest room with one king bed or two double beds, this rate is applicable for single or double occupancy. To reserve a room at this rate contact the hotel and please reference the group code *AMS*. Amenities include free WiFi in sleeping rooms, fitness center with indoor pool, business center, *Cappuccino's Sidewalk Café* on site restaurant (offering room service), *Lobby Bar*, daily complimentary airport shuttle (7:00 am–11:00 pm), and a gift shop on property. Check-in is at 3:00 pm, check-out is at 12:00 pm. This property is located approximately .2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 25, 2017.**

Motel 6 Buffalo Amherst, 4400 Maple Road, Amherst,14226; (716) 834-2231, `http://www.motel6.com/en/motels.1298.html`. Rates are dependent upon occupancy in room. Rooms with one queen bed are available for US$69.99 for single occupancy, US$75.99 for double occupancy and US$78.99 for triple occupancy. Rooms with two queen beds are available for US$69.99 for single occupancy, US$75.99 for double occupancy, US$78.99 for triple occupancy and US$81.99 for quad occupancy. Children stay free. To reserve a room at this rate contact Motel 6 reservations at (800) 544-4866 and reference *American Mathematical Society*. Amenities include WiFi in sleeping rooms for a nominal fee, and free morning coffee daily. This is a pet friendly property. Check-in is at 4:00 pm, check-out is at 12:00 pm. This property is located approximately .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 16, 2017.**

Motel 6 Hamburg, 5245 Camp Road, Hamburg, 14221; (716) 648-2000, `https://www.motel6.com/en/motels.4793.html`. Rates are dependent upon occupancy in room. Rooms with one queen bed are available for US$69.99 for single occupancy, US$75.99 for double occupancy and US$78.99 for triple occupancy. Rooms with two queen beds are available for US$69.99 for single occupancy, US$75.99 for double occupancy, US$78.99 for triple occupancy and US$81.99 for quad occupancy. Children stay free. To reserve a room at this rate contact Motel 6 reservations at (800) 544-4866 and reference *American Mathematical Society*. Amenities include WiFi in sleeping rooms for a nominal fee, and free morning coffee daily. This is a pet friendly property. Check-in is at 4:00 pm, check-out is at 12:00 pm. This property is located approximately .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 16, 2017.**

Staybridge Suites Buffalo-Amherst, 1290 Sweet Home Road, Amherst, 14228; (716) 276-8750, `https://www.ihg.com/staybridge/hotels/us/en/amherst/bufrr/hoteldetail`. Rates are US$119 per night for a guest room with one king bed or two queen beds,this rate is applicable for single or double occupancy. Each guest room features a fully equipped kitchen—including refrigerator, dishwasher, microwave and stove top, plus a variety of dishes, cookware and utensils. Amenities include free WiFi in sleeping rooms, fitness center, indoor saltwater pool, business center, and a complimentary hot breakfast. This is a pet-friendly property. Check-in is at 3:00 pm, check-out is at 12:00 pm. This property is located approximately 1 mile from campus by car and .3 miles from campus walking distance. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 11, 2017.**

Wyndham Garden Buffalo Williamsville, 5195 Main Street, Williamsville, 14221; (716) 689-9600 `https://www.wyndhamhotels.com/wyndham-garden/williamsville-new-york/wyndham-garden-williamsville-buffalo/overview`. Rates are US$119 per night for a guest room with one king bed or two queen beds, this rate is applicable for single or double occupancy. Amenities include free WiFi in sleeping rooms, fitness center, indoor pool, *MTK Prime* on site restaurant (offering room service), lounge in lobby, on property spa *Tres Auraé Spa*, and a gift shop. Check-in is at 3:00 pm, check-out is at 12:00 pm. This property is located approximately 3.3 miles from campus (6 minutes driving time). Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 11, 2017.**

Food Services

On Campus: There are various options which are anticipated to be open on campus during the meeting. The hours of operation listed below are for both Saturday and Sunday, unless otherwise indicated and are subject to change.

Student Union:

Moe's Southwest Grill, 11:00 am–3:30 pm (Mexican -Tex Mex)

Tim Hortons, 8:00 am–6:00 pm (Coffee, bakery, sandwiches)

The Commons, (located across the street from the Student Union at 520 Lee Entrance.):

Bollywood Bistro, 11:00 am–8 pm (Indian)

Chick Mex Grill, noon–9 pm (Mexican, fried chicken and seafood)

Dancing Chopstics, 11:30 am–9 pm (Japanese)

Korean Express, noon–9 pm (Korean)

Rachel's Mediterranean, 11:00 am–6:00 pm (salads, wraps, and rice bowls)

Starbucks, 7:00 am–11:00 pm (Coffee, bakery, sandwiches)

Subway, 10:00 am–9:00 pm (fast food sandwiches)

Young Chow, noon–9:00 pm (Chinese)

Capen Hall:

Whispers Cafe, Saturday, 9:00 am–4:00 pm, Sunday, noon–8:00 pm (coffee, sandwiches, snacks and salads)

Off Campus: The Buffalo-Niagara region is a major metropolitan area with array of dining options throughout the area. For more information on dining throughout the region please visit, `http://www.visitbuffaloniagara.com/eat-drink/`.

Some dining options nearby to the North Campus include:

Amherst Pizza and Ale House, 55 Crosspoint Parkway, Amherst, (716)625-7100, Saturday 11:00 am–1:00 am and Sunday, 12:00 pm–9:00 pm (pizza, wings and a large selection of beers)

Black Forest Adler, 2447 Niagara Falls Blvd, Amherst, (716) 564-2447, Saturday 4:00 pm–9:00 pm and Sunday, 4:00 pm–8:00 pm (German, European Cuisine)

Brickhouse Tavern + Tap, 4120 Maple Road, Amherst, (716) 834-1490, Saturday 10:30 am–1:00 am and Sunday, 10:30 am– 11:00 pm (serving brunch, lunch and dinner; New American cuisine and pub fare)

Buffalo Roadhouse Grill, 1980 Niagara Falls Blvd, Tonawanda, (716) 692-7999, Saturday and Sunday, 11:00 am–10:00 pm (American bar and steakhouse.)

Duff's, 3651 Sheridan Dr, Amherst, (716) 834-6234, Saturday 11:00 am–12:00 am and Sunday, 12:00 pm–10:00 pm (American, bar, specializing in wings)

The Grapevine Restaurant, 2545 Niagara Falls Blvd, Amherst, (716) 691-7799, Saturday 11:30 am–11:00 pm and Sunday, 10:00 am–9:00 pm (Italian, American, Gluten Free Options)

Jasmine Thai Restaurant, 1330 Niagara Falls Blvd, Tonawanda, (716) 838-3011, Saturday and Sunday, 12:00 pm–9:30 pm (Asian, Thai, vegetarian friendly options)

Joe's Crab Shack, 4125 Maple Rd, Amherst, (716) 836-473, Saturday 11:00 am–12:00 am and Sunday, 11:00 am–11:00 pm (Chain restaurant serving crab and seafood°

Olive Garden, 3951 Maple Rd, Amherst, (716) 837-5211, Saturday 11:00 am–11:00 pm and Sunday, 11:00 am–10:00 pm (Chain restaurant serving Italian dishes)

Pi Craft, 1750 Niagara Falls Blvd, Tonawanda, (716) 833-5300, Saturday and Sunday, 12:00 pm–9:00 pm (Handcrafted pizza, focaccia sandwiches and salads)

Red Pepper Chinese-Vietnamese, 3910 Maple Road, Amherst, (716) 831-3878, Saturday 12:00 p.m. - 11:00 p.m. and Sunday, 12:00 pm–10:00 pm (Chinese and Vietnamese cuisine)

Ted's Hot Dogs, 2351 Niagara Falls Blvd, Amherst, (716) 691-3761, Saturday and Sunday, 10:30 am–10:30 pm (charcoal cooked hot dogs and hamburgers)

800 Maple, 800 Maple Rd, Williamsville, (716) 688-5800, Saturday 5:00 pm–11:00 pm and Sunday, 5:00 pm–9:00 pm (wood fired brick oven pizza, pasta, fresh seafood and steaks)

Registration and Meeting Information

Advance Registration: Advance registration for this meeting will open on **July 3rd.** Advance registration fees are US$61 for AMS members, US$90 for nonmembers, and US$10 for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing `mmsb@ams.org`. The deadline to cancel is the first day of the meeting.

On site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located on the second floor of Knox Lecture Hall. The Invited Addresses will be held in the same building in Knox 104. Special Sessions and Contributed Paper Sessions will take place in NSC (Natural Sciences Complex), O'Brian Hall, and Norton Hall. Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at `http://www.buffalo.edu/content/dam/www/shared_assets/campus_maps/NorthCampus_bus-routes-opt.pdf`.

The registration desk will be open on Saturday, September 16, 7:30 am–4:00 pm and Sunday, September 17, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

There will be a **reception** for participants on Saturday evening hosted by the University. Please watch for more details on this event on the AMS website and at the registration desk on site at the meeting. The AMS thanks our hosts for their gracious hospitality.

Other Activities

Book Sales: Stop by the on site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership. Enjoy **Complimentary coffee** served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

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Local Information and Maps

This meeting will take place on the North Campus of the University at Buffalo, the State University of New York. A campus map can be found at `http://www.buffalo.edu/home/visiting-ub/CampusMaps.html`. Information about the University at Buffalo, SUNY Department of Mathematics can be found at `http://www.buffalo.edu/cas/math.html`. For additional information about the university please visit the University at Buffalo, SUNY website at `https://www.buffalo.edu/`.

Please watch the AMS website at `www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting.

Parking

Parking is free on campus after 3:00 p.m. on Friday and all day Saturday and Sunday. At these times visitors may park in any lot on campus.

The map located at `http://www.buffalo.edu/content/dam/www/shared_assets/campus_maps/NorthCampus_bus-routes-opt.pdf` depicts all parking on the North Campus. The meeting will take place in Knox Hall, NSC (Natural Sciences Complex), O'Brian Hall, and Norton Hall, which are labeled 22, 15, 11, and 21 respectively. There are a number of options for lots located close by to all of these locations.

Travel

The North Campus of the University at Buffalo, SUNY is the largest of the three UB campuses and is located just outside of Buffalo in suburban Amherst, NY.

By Air: *Buffalo Niagara International Airport* (BUF) is the closest airport to the University. It is located approximately 8 miles (15 minutes drive) away from campus in Buffalo, NY and offers over 100 direct flights each day from six air carriers: *United*, *American*, *Delta*, *Jet Blue*, *Southwest*, and *Vacation Express*.

There are a variety of ground transportation options to travel from the Buffalo Niagara International Airport to UB.

Taxi fare is approximately US$30 and should take approximately 15 minutes. There are also a number of shuttle operators servicing the airport, rates will vary depending upon the service and number of passengers.

For a list of taxi and shuttle vendors please visit the airport website at `http://buffaloairport.com/Ground/Shuttle.aspx` .

NFTA-Metro Bus and Rail also services the airport. Regular Metro Bus service runs through the Buffalo Niagara metropolitan area. One-way fare for this bus line is US$2. For information on schedules please visit `http://metro.nfta.com/` .

There a number of car rental vendors located on site at the Buffalo Niagara International Airport. More information on rental cars can be found here `http://buffaloairport.com/Ground/Rental.aspx` . Please see below (**By Car**) for driving directions from the airport.

By Train: Amtrak services the Buffalo Niagara region with two train stations: one located in downtown Buffalo at Exchange Street, near the light rail line, and a second in the Buffalo suburb of Depew, a 20 minute drive from UB's North Campus. For information on schedules please visit Amtrak at `https://www.amtrak.com/home` .

By Bus: Bus service to Buffalo is offered through three large carriers: *Greyhound, Megabus* and *Trailways*, with stops in downtown Buffalo and at the Buffalo Niagara International Airport. Many buses utilize the Metropolitan Transportation Center Station located on the southeast corner of North Division and Ellicott Streets in Downtown Buffalo.

For more information on *Greyhound* please call 800-231-2222 or visit `https://www.greyhound.com/.` For information on Trailways please visit `http://www.trailways.com/` or call (877) 908-9330. For details on *Megabus* please visit `https://us.megabus.com/.`

By Car: When driving to the North Campus please use this address to navigate with GPS: Flint Entrance, Amherst, NY 14260

From the West: Take the I-90 East until you reach the I-290 (past the Buffalo airport exit). Take the I-290 West (toward Niagara Falls) to Exit 5B (Millersport Highway, Route 263). Get off exit and merge into traffic. Go through first light (Marriott is on your left), and merge into right lane. Enter campus at the Flint Road exit.

From the East: Take the I-90 West until you reach the I-290 (just past tolls, follow State University signs). Take the I-290 West (toward Niagara Falls) to Exit 5B (Millersport Highway, Route 263). Get off exit and merge into traffic. Go through first light (Marriott is on your left), and merge into right lane. Enter campus at the Flint Road exit.

From the Buffalo Niagara Airport: Exit the airport following the signs leading to Route 33 West toward Buffalo. Route 33 (Kensington Expressway) begins at the West Gate of the Buffalo Niagara Airport. Take Route 33 West to the I-90 East to the I-290 West. Take the I-290 West (toward Niagara Falls) to Exit 5B (Millersport Highway, Route 263). Get off exit and merge into traffic. Go through first light (Marriott is on your left), and merge into right lane. Enter campus at the Flint Road exit.

From the South: Take I-79 North to Erie, PA. Take the I-90 East until you reach the I-290 (past the Buffalo airport exit). Take the I-290 West (toward Niagara Falls) to Exit 5B (Millersport Highway, Route 263). Get off exit and merge into traffic. Go through first light (Marriott is on your left), and merge into right lane. Enter campus at the Flint Road exit.

From the North: Take the QEW south to St. Catharines. Follow Highway 405 to the Queenston-Lewiston Bridge and continue east over the bridge (toll) into New York state. Once across bridge take the I-190 South. Follow I-190 South until you reach the I-290 East and proceed until Exit 5B (Millersport Highway, Route 263). Get off exit and merge into traffic. Go through first light (Marriott is on your left), and merge into right lane. Enter campus at the Flint Road exit. If taking the QEW to the Peace Bridge, take the 190 North to the I-290 East and follow the same directions as above paragraph.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "Enter a discount or promo code", and type in our convention number (CV): CV#04N30007. You can also call Hertz directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus (please see directions included above under the heading **By Car**).

Local Transportation

The Buffalo Niagara region offers **public transportation** in the forms of Metro Buses and the Metro Rail. For more information on the NFTA-Metro or to plan your trip please visit `http://metro.nfta.com/`.

Metro tickets and passes are valid on Metro Buses and Metro Rail. Pass must be retained for duration of trip. Standard fare is US$2 for a full fare or US$1 for a reduced fare (children aged 5–11 or seniors 65 and older), valid for a single ride. A day pass is available for service on the day of purchase (5:00 am–2:00 am); a full fare is US$5 and a reduced fare is US$2.50.

Taxi Service is available in the Buffalo Niagara region. Two vendors familiar with the campus are:

Buffalo Transportation, (716) 877-5600, `http://www.buffalotransportation.com/` .

Liberty Cab, (716) 877-7111, `http://www.libertycab.com/` .

Weather

Weather in September in Buffalo is typically pleasant and mild. The average high temperature is approximately 75 degrees Fahrenheit and the average low is approximately 50 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtags #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at `https://travel.state.gov/content/travel/en.html` . If you need a preliminary conference invitation in order to secure a visa, please send your request to `mac@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Orlando, Florida

University of Central Florida, Orlando

September 23–24, 2017

Saturday – Sunday

Meeting #1133

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: June 2017
Program first available on AMS website: August 10, 2017
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 38, Issue 4

Deadlines

For organizers: Expired
For abstracts: August 1, 2017

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Christine Heitsch, Georgia Institute of Technology, *Strings, trees, and RNA folding*.

Jonathan Kujawa, University of Oklahoma, *Realizing the spectrum of tensor categories*.

Christopher D. Sogge, Johns Hopkins University, *On the concentration of eigenfunctions*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Dirac Equations, Variational Inequalities, Sequence Spaces and Optimization (Code: SS 21A), **Ram N. Mohapatra**, University of Central Florida, and **Turhan Koprubasi**, Kastamonu University (Turkey).

Algebraic Curves and their Applications (Code: SS 3A), **Lubjana Beshaj**, The University of Texas at Austin.

Applied Harmonic Analysis: Frames, Samplings and Applications (Code: SS 6A), **Dorin Dutkay**, **Deguang Han**, and **Qiyu Sun**, University of Central Florida.

Categorical Methods in Representation Theory (Code: SS 11A), **Brian Boe**, University of Georgia, **Jonathan Kujawa**, University of Oklahoma, and **Daniel K. Nakano**, University of Georgia.

Commutative Algebra: Interactions with Algebraic Geometry and Algebraic Topology (Code: SS 1A), **Joseph Brennan**, University of Central Florida, and **Alina Iacob** and **Saeed Nasseh**, Georgia Southern University.

Complex Analysis, Harmonic Analysis, and Approximation Theory (Code: SS 15A), **Alexander V. Tovstolis**, University of Central Florida, and **John Paul Ward**, North Carolina A&T State University.

Differential Equations in Mathematical Biology (Code: SS 12A), **Andrew Nevai**, **Yuanwei Qi**, and **Zhisheng Shuai**, University of Central Florida.

Fractal Geometry, Dynamical Systems, and Their Applications (Code: SS 4A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Global Harmonic Analysis and its Applications (Code: SS 10A), **Christopher Sogge** and **Yakun Xi**, Johns Hopkins University, and **Steve Zelditch**, Northwestern University.

Graph Connectivity and Edge Coloring (Code: SS 5A), **Colton Magnant**, Georgia Southern University.

Mathematics of Biomolecules: Discrete, Algebraic, and Topological (Code: SS 20A), **Natasha Jonoska**, University of South Florida, and **Christine Heitsch**, Georgia Institute of Technology.

Modern Statistical Methods for Structured Data (Code: SS 17A), **Marianna Pensky**, University of Central Florida.

Nonlinear Dispersive Equations (Code: SS 7A), **Benjamin Harrop-Griffiths**, New York University, **Jonas Lührmann**, Johns Hopkins University, and **Dana Mendelson**, University of Chicago.

Nonlinear Elliptic Partial Differential Equations (Code: SS 16A), **Luis E. Silvestre**, University of Chicago, and **Eduardo V. Teixeira**, University of Central Florida.

Operator Algebras and Related Topics (Code: SS 8A), **Zhe Liu**, University of Central Florida.

Progress in Fixed Point Theory and Its Applications (Code: SS 9A), **Clement Boateng Ampadu**, Boston, MA, and **Buthinah A. Bin Rehash** and **Afrah A. N. Abdou**, King Abdulaziz University, Saudi Arabia.

Recent Developments in Integral Geometry and Tomography (Code: SS 14A), **Alexander Katsevich**, **Alexander Tovbis**, and **Alexandru Tamasan**, University of Central Florida.

Stochastic Analysis and Applications (Code: SS 19A), **Hongwei Long**, Florida Atlantic University, and **Jiongmin Yong**, University of Central Florida.

Structural Graph Theory (Code: SS 2A), **Martin Rolek**, **Zixia Song**, and **Yue Zhao**, University of Central Florida.

Symplectic and Contact Topology and Dynamics (Code: SS 13A), **Basak Gürel**, University of Central Florida, and **Viktor Ginzburg**, University of California, Santa Cruz.

Trends in Applications of Functional Analysis in Computational and Applied Mathematics (Code: SS 18A), **M. Zuhair Nashed**, University of Central Florida.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include Florida tax (6%) and occupancy tax (6.5%). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at the University of Central Florida** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

DoubleTree by Hilton Orlando East - UCF Area (1.3 miles from Classroom Building II on campus), 12125 High Tech Ave., Orlando, FL 32817; 407-275-9000; `doubletree3hilton.com`. Rates are US$124 per night for a guest room with one king bed or two double beds. Please use this link for making reservations online: `http://group.doubletree.com/americanmathematicalsociety`. Rate includes full hot breakfast buffet, high speed wireless internet and self-parking. The deadline for reservations at this rate is **September 1, 2017.**

Courtyard by Marriott East/UCF Area (1.6 miles from Classroom Building II on campus), 12000 Collegiate Way Orlando, 32817 `marriott.com/mcoce`. Rates are US$129 per night for a guest room with one king bed or two queen beds. Please use this link for making reservations online: `http://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkData=American%20Math%20Society%5EMCOCE%60AMSAMSA%7CAMSAMSB%60129%60USD%60false%604%609/22/17%609/24/17%609/8/17&app=resvlink&stop_mobi=yes`. This rate includes breakfast, high speed internet access and complimentary parking. The deadline for reservations at this rate is **September 1, 2017.**

Towneplace Suites (1.8 miles from Classroom Building II on campus), 11801 High Tech Avenue Orlando, FL 32817 , 407-243-6100; marriott.com/mcots. Rates are US$139 for a studio suite with a queen bed plus a pull out sofa. This rate includes breakfast, high speed internet access and complimentary parking. Please use this link for making online reservations: `http://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkData=AMS-American%20Math%20Society%20Conference%5EMCOTS%60MSCAMSC%7CMSCBMSC%60139.00%60USD%60false%601%609/22/17%609/24/17%609/1/17&app=resvlink&stop_mobi=yes`.

The deadline for reservations at this rate is **September 1, 2017.**

Residence Inn (1.9 miles from Classroom Building II on campus), 11651 University Blvd., Orlando, FL 32817; 407-513-9000; `marriott.com/mcore`. Rates are US$149 per night for a one-bedroom or studio suite. Rates include breakfast, high speed internet access and complimentary parking. Please use this link for online reservations: `http://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkData=AMS-American%20Math%20Society%20Conference%5EMCORE%60MSCAMSC%7CMSCBMSC%60149.00%60USD%60false%601%609/22/17%609/24/17%609/1/17&app=resvlink&stop_mobi=yes`.

The deadline for reservations at this rate is **September 1, 2017.**

LaQuinta, (2 miles from Classroom Building II on campus) 11805 Research Parkway, Orlando, FL 32826; 407-737-6075; `laquintaorlandoucf.com/`. Rates are US$99 per night for a room with two double beds. This rate includes complimentary breakfast, internet access and free parking. Please make your reservation by calling 407-737-6075.

The deadline for reservations at this rate is **September 1, 2017.**

Food Services

On Campus: There are many dining options on campus within walking distance of the meeting. The hours of the on-campus restaurants vary on the weekends. Please see a complete listing here: `http://map.ucf.edu/locations/food/`

Off Campus: Orlando offers many dining options of all types of cuisine. There a quite a few restaurants within a couple of miles from the UCF campus. For more information on dining throughout Orlando please visit, `http://www.visitorlando.com/things-to-do/restaurants/`. And for more information on visiting Orlando in general please visit, `www.visitorlando.com/`.

Some of the nearby off-campus dining options include:

Applebee's, 12103 Collegiate Way `http://restaurants.applebees.com/fl/orlando/12103-collegiate-way.html`

Zaxby's, 11554 University Blvd, `https://www.zaxbys.com`

Firehouse Subs, 12075a Collegiate Way `http://locations.firehousesubs.com/locations/fl/ucf`

4 Rivers Smokehouse, 11764 University Boulevard `https://4rsmokehouse.com/`

Registration and Meeting Information

Advance Registration: Advance registration for this meeting opens on **July 3.** Advance registration fees are US$61 for AMS members, US$90 for nonmembers, and US$10 for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing `mmsb@ams.org`. The deadline to cancel is the first day of the meeting.

On site Information and Registration: The registration desk, AMS book exhibit and coffee service, as well as the Invited Addresses will be located in Classroom Building II. Special Sessions and Contributed Paper Sessions will take place in Classroom Buildings I and II. For information on building locations, a campus map is available at `https://map.ucf.edu/printable/`.

The registration desk will be open on Saturday, September 23, 7:30 am–4:00 pm and Sunday, September 24, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

Other Activities

Book Sales: Stop by the on site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at `mmsb@ams.org` or `meet@ams.org`.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Local Information and Maps

This meeting will take place at the University of Central Florida in Orlando, FL. A campus map can be found at `https://map.ucf.edu/printable/`. Information about the University of Central Florida Department of Mathematics can be found at `https://sciences.ucf.edu/math/`. Please visit the University of Central Florida website at `http://www.ucf.edu`.

Please watch the AMS `website at www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting.

Parking

Parking for this meeting is available in Parking Garage H, immediately north of Conference Building II (12650 Gemini Blvd North, Orlando, FL 32816). There is no charge for parking in Garage H on the weekends.

Travel

The University of Central Florida is a located in metropolitan Orlando.

By Air: The Orlando International Aiport is located at One Jeff Fuqua Boulevard, Orlando, Florida 32827. Tel: (407) 825-2001 Website: `https://orlandoairports.net/`.

Orlando International Aiport is approximately a 20 mile drive from the meeting at the University of Central Florida.

It is strongly recommended to rent a car to get to the meeting from the airport as public transportation will take over 2 hours with several bus transfers.

The cost for an Uber from the aiport to the meeting is $54–$69 each way.

To drive from the Orlando International Airport to the meeting at University of Central Florida:

Take 417 North to University Blvd. Exit east onto University Blvd. Continue east on University Blvd. for approximately two miles and it will take you to the main entrance to the campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

By Train: Amtrak has a station approximately 18 miles from the meeting at the University of Central Florida. Orlando Florida Station Building, which has a waiting room, is located at 1400 Sligh Bouldevard, Orlando, FL 32806. Please contact Amtrack at Tel: 800-USA-RAIL, website: `www.amtrak.com`.

By Bus: The Greyhound Bus station located at 555 North John Young Parkway is about 18 miles from the meeting at the University of Central Florida. Please contact Greyhound Tel: 1-800-231-2222; Website: `www.greyhound.com`.

By Car:

From The North:

Option 1 – Take the Florida Turnpike South to 408 east. Take 408 east to 417 North toward Sanford. Exit off of 417 to University Blvd. East. Take University Blvd 2 miles and it will take you to the main entrance of the campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

Option 2 – Take I-95 South to SR 50 West. Take SR 50 to Alafaya Trail. Make a right hand turn on Alafaya Trail. Continue on Alafaya Trail to University Blvd. Make a right turn on University Blvd. and you are on campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

From The South and Melbourne Area:

Option 1 – Take the Florida Turnpike north and exit onto I-4 East. Continue until you reach 408 east. Take 408 east to 417 North toward Sanford. Exit off of 417 to University Blvd East. Continue on University Blvd. for 2 miles and it will take you to the main entrance of the campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

Option 2 – Take I-95 north to 520 West. Continue West and the road will merge with SR 50. Take SR 50 West to Alafaya Trail. Make a right turn on Alafaya Trail. Continue for approximately 2 miles to University Blvd. Make a right turn on University Blvd. and you will be on campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

From The East, Titusville and Daytona:

Option 1 – Take Highway 50 West to Alafaya Trail. Turn right on Alafaya Trail, go north for two miles. Turn right onto University Blvd. and you are on campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

Option 2 – Take I-4 West to 417 South. Exit off of 417 at University Blvd. East. Continue on University Blvd East and it will take you to the main entrance of the campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

From The Tampa Area: Take I-4 east to 408 east toward Titusville. Take 408 to 417 North toward Sanford. Exit off of 417 at University Blvd.East. Continue going on University Blvd and it will take you to the main entrance of the campus. Make a left turn at the first light, Gemini Blvd West. Continue on Gemini Blvd West, bearing right, Gemini Blvd West will turn into Gemini Blvd North. Parking Garage H will be on your right at 12650 Gemini Blvd North. Classrooms I & II will be just beyond the Parking Garage H, `https://map.ucf.edu/printable/`.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "Enter a discount or promo code", and type in our convention number (CV): CV#04N30007. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

Local Transportation

Orlando and the University of Central Florida area are serviced by Linx bus service. Please find information including schedles and maps on the website: `http://www.golynx.com/maps-schedules/`.

Weather

Weather in September in Orlando is typically in the high 80s/low 90s Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtags #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at `https://travel.state.gov/content/travel/en.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `mac@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
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- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Riverside, California

University of California, Riverside

November 4–5, 2017

Saturday – Sunday

Meeting #1134

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2017
Program first available on AMS website: September 21, 2017
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 38, Issue 4

Deadlines

For organizers: Expired
For abstracts: September 12, 2017

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Paul Balmer, University of California, Los Angeles, *An invitation to tensor-triangular geometry*.

Pavel Etingof, Massachusetts Institute of Technology, *Double affine hecke algebras and their applications*.

Monica Vazirani, University of California, Davi, *Combinatorics, categorification, and crystals*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Operator Algebras (Code: SS 13A), **Michael Hartglass**, UC Riverside, Santa Clara University, and **Chenxu Wen** and **Feng Xu**, University of California, Riverside.

Algebraic Geometry (Code: SS 9A), **Humberto Diaz**, **Jose Gonzalez**, and **Ziv Ran**, University of California, Riverside.

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 3A), **Patricia Cahn**, Smith College, and **Sam Nelson**, Claremont McKenna College.

Analysis and Geometry of Fractals (Code: SS 6A), **Erin Pearse**, California Polytechnic State University, **Goran Radunovic**, University of California, Riverside, and **Tim Cobler**, Fullerton College, California.

Applied Category Theory (Code: SS 4A), **John Baez**, University of California, Riverside.

Characteristics of a Successful Mathematics Gateway Program (Code: SS 12A), **Sara Lapan**, University of California, Riverside, **Jeff Meyer**, California State University, San Bernardino, and **David Weisbart**, University of California, Riverside.

Combinatorial Aspects of the Polynomial Ring (Code: SS 1A), **Sami Assaf** and **Dominic Searles**, University of Southern California.

Combinatorial Representation Theory (Code: SS 5A), **Vyjayanthi Chari**, University of California, Riverside, and **Maria Monks Gillespie** and **Monica Vazirani**, University of California, Davis.

Conservation Laws, Nonlinear Waves and Applications (Code: SS 18A), **Geng Chen**, University of Kansas, **Tien Khai Nguyen**, North Carolina State University, and **Qingtian Zhang**, University of California, Davis.

Foundations of Quantum Theory (Code: SS 26A), **Jukka Virtanen**, University of California, Los Angeles, and **David Weisbart**, University of California, Riverside.

Generalized Geometry (Code: SS 16A), **Daniele Grandini**, Virginia State University, and **Yat-Sun Poon**, University of California, Riverside.

Geometric Analysis (Code: SS 24A), **Zhiqi Lu**, University of California, Irvine, **Jie Qing**, University of California, Santa Cruz, **Guofang Wei**, University of California, Santa Barbara, and **Qi Zhang**, University of California, Riverside.

Geometric Partial Differential Equations and their Applications (Code: SS 29A), **Po-Ning Chen**, University of California, Riverside, **Henri Roesch**, Duke University, and **Richard M. Schoen** and **Xiangwen Zhang**, University of California, Irvine.

Homotopy Theory (Code: SS 28A), **Jonathan Beardsley**, University of Washington.

Mathematical Fluid Mechanics (Code: SS 27A), **James P Kelliher** and **Lizheng Tao**, University of California, Riverside.

Model Theory (Code: SS 14A), **Artem Chernikov**, University of California, Los Angeles, and **Isaac Goldbring**, University of California, Irvine.

Non-Commutative Birational Geometry, Cluster Structures and Canonical Bases (Code: SS 19A), **Arkady Berenstein**, University of Oregon, Eugene, **Jacob Greenstein**, University of California, Riverside, and **Vladimir Retakh**, Rutgers University.

Nonlinear Elliptic Differential and Integral Equations (Code: SS 25A), **Mathew Gluck**, University of Oklahoma, and **John Villavert**, University of Texas, Rio Grande Valley.

Particle Methods and Nonlocal Partial Differential Equations (Code: SS 23A), **Katy Craig**, University of California, Santa Barbara, and **Franca Hoffman**, University of Cambridge.

Preparing Students for American Mathematical Competitions (Code: SS 7A), **Adam Glesser**, **Phillip Ramirez**, and **Bogdan D. Suceava**, California State University, Fullerton.

Random Matrices: Theory and Applications (Code: SS 20A), **Ioana Dumitriu**, University of Washington, and **Thomas Trogdon**, University of California, Irvine.

Random and Deterministic Dynamical Systems (Code: SS 15A), **Nicolai Haydn**, University of Southern California, Los Angeles.

Rational Cherednik Algebras and Categorification (Code: SS 8A), **Pavel Etingof**, Massachusetts Institute of Technology, and **Ivan Losev**, Northeastern University.

Research in Mathematics by Early Career Graduate Students (Code: SS 22A), **Michael Bishop**, **Stefaan Delcroix**, **Marat Markin**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Riemannian Manifolds of Non-Negative Sectional Curvature (Code: SS 21A), **Owen Dearricott**, University of Melbourne, and **Fernando Galaz-Garcia**, Karlsruhe Institute of Technology.

Ring Theory and Related Topics (Celebrating the 75th Birthday of Lance W. Small) (Code: SS 2A), **Jason Bell**, University of Waterloo, **Ellen Kirkman**, Wake Forest University, and **Susan Montgomery**, University of Southern California.

Several Complex Variables (Code: SS 10A), **Bingyuan Liu** and **Bun Wong**, University of California, Riverside.

Stochastic and Multi-scale Models in Mathematical Biology, Analysis and Simulations (Code: SS 17A), **Mark Alber**, University of California, Riverside, and **Bjorn Birnir**, University of California, Santa Barbara.

Tensor Categories: Bridging Algebra, Topology, and Physics (Code: SS 11A), **Paul Bruillard**, Pacific Northwest National Laboratory, **Julia Plavnik**, Texas A&M University, and **Henry Tucker**, University of California, San Diego.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Meeting #1135

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 1

Deadlines

For organizers: Expired
For abstracts: September 26, 2017
The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/national.html`.

AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as pos-

sible via the abstract submission form found at `http://jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm`.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

A Showcase of Number Theory at Liberal Arts Colleges (Code: SS 53A), **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

Accelerated Advances in Mathematical Fractional Programming (Code: SS 12A), **Ram Verma**, International Publications USA, and **Alexander Zaslavski**, Israel Institute of Technology.

Advances in Applications of Differential Equations to Disease Modeling (Code: SS 56A), **Libin Rong**, Oakland University, **Elissa Schwartz**, Washington State University, and **Naveen K. Vaidya**, University of Missouri - Kansas City.

Advances in Difference, Differential, and Dynamic Equations with Applications (Code: SS 20A), **Elvan Akin**, Missouri University S&T, and **John Davis**, Baylor University.

Advances in Operator Algebras (Code: SS 58A), **Marcel Bischoff**, Vanderbilt University, **Ian Charlesworth**, University of California, Los Angeles, **Brent Nelson**, University of California, Berkeley, and **Sarah Reznikoff**, Kansas State University.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups (Code: SS 42A), **Asuman G. Aksoy**, Claremont McKenna College, **Zair Ibragimov**, California State University, Fullerton, **Marat Markin**, California State University, Fresno, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

Algebraic, Analytic, and Geometric Aspects of Integrable Systems, Painlevé Equations, and Random Matrices (Code: SS 75A), **Vladimir Dragovic**, University of Texas at Dallas, **Anton Dzhamay**, University of Northern Colorado, and **Sevak Mkrtchyan**, University of Rochester.

Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology (Code: SS 66A), **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

Alternative Proofs in Mathematical Practice (Code: SS 11A), **John W. Dawson, Jr.**, Pennsylvania State University, York.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems (Code: SS 13A), **John R. Graef**, University of Tennessee at Chattanooga, **Gangaram S. Ladde**, University of South Florida, and **Aghalaya S. Vatsala**, University of Louisiana at Lafayette.

Analysis of Nonlinear Partial Differential Equations and Applications (Code: SS 85A), **Tarek M. Elgindi**, University of California, San Diego, and **Edriss S. Titi**, Texas A&M University and Weizmann Institute of Science.

Applied and Computational Combinatorics (Code: SS 55A), **Torin Greenwood**, Georgia Institute of Technology, and **Jay Pantone**, Dartmouth College.

Arithmetic Dynamics (Code: SS 41A), **Robert L. Benedetto**, Amherst College, **Benjamin Hutz**, Saint Louis University, **Jamie Juul**, Amherst College, and **Bianca Thompson**, Harvey Mudd College.

Beyond Planarity: Crossing Numbers of Graphs (a Mathematics Research Communities Session) (Code: SS 15A), .

Bifurcations of Difference Equations and Discrete Dynamical Systems (Code: SS 61A), **Arzu Bilgin** and **Toufik Khyat**, University of Rhode Island.

Boundaries for Groups and Spaces (Code: SS 7A), **Joseph Maher**, CUNY College of Staten Island, and **Genevieve Walsh**, Tufts University.

Combinatorial Commutative Algebra and Polytopes (Code: SS 1A), **Robert David**, Michigan State University, and **Liam Solus**, KTH Royal Institute of Technology.

Combinatorics and Geometry (Code: SS 70A), **Federico Ardila**, San Francisco State University, **Anastasia Chavez**, MSRI and University of California, Davis, and **Laura Escobar**, University of Illinois Urbana Champaign.

Commutative Algebra in All Characteristics (Code: SS 67A), **Neil Epstein**, George Mason University, **Karl Schwede**, University of Utah, and **Janet Vassilev**, University of New Mexico.

Computational Combinatorics and Number Theory (Code: SS 76A), **Jeremy F. Alm**, Illinois College, and **David Andrews** and **Rob Hochberg**, University of Dallas.

Connections in Discrete Mathematics: Graphs, Hypergraphs, and Designs (Code: SS 80A), **Amin Bahmanian**, Illinois State University, and **Theodore Molla**, University Illinois Urbana-Champaign.

Differential Geometry (Code: SS 28A), **Vincent B. Bonini** and **Joseph E. Borzellino**, Cal Poly San Luis Obispo, **Bogdan D. Suceava**, California State University, Fullerton, and **Guofang Wei**, University of California, Santa Barbara.

Diophantine Approximation and Analytic Number Theory in Honor of Jeffrey Vaaler (Code: SS 29A), **Shabnam Akhtari**, University of Oregon, and **Lenny Fukshansky**, Claremont McKenna College.

Discrete Dynamical Systems and Applications (Code: SS 51A), **E. Cabral Balreira**, **Saber Elaydi**, and **Eddy Kwessi**, Trinity University.

Discrete Neural Networking and Applications (Code: SS 6A), **Murat Adivar**, Fayetteville State University, **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

Dynamical Algebraic Combinatorics (Code: SS 68A), **James Propp**, University of Massachusetts, Lowell, **Tom Roby**, University of Connecticu, **Jessica Striker**, North Dakota State University, and **Nathan Williams**, University of California Santa Barbara.

Dynamical Systems: Smooth, Symbolic, and Measurable (a Mathematics Research Communities Session) (Code: SS 16A), .

Emergent Phenomena in Discrete Models (Code: SS 82A), **Dana Randall**, Georgia Institute of Technology, and **Andrea Richa**, Arizona State University.

Emerging Topics in Graphs and Matrices (Code: SS 60A), **Sudipta Mallik**, Northern Arizona University, **Keivan Hassani Monfared**, University of Calgary, and **Bryan Shader**, University of Wyoming.

Ergodic Theory and Dynamical Systems (Code: SS 71A), **Julia Barnes**, Western Carolina University, **Rachel Bayless**, Agnes Scott College, **Emily Burkhead**, Duke University, and **Lorelei Koss**, Dickinson College.

Extremal Problems in Approximations and Geometric Function Theory (Code: SS 81A), **Ram Mohapatra**, University of Central Florida.

Financial Mathematics, Actuarial Sciences, and Related Fields (Code: SS 48A), **Albert Cohen**, Michigan State University, **Nguyet Nguyen**, Youngstown State University, **Oana Mocioalca**, Kent State University, and **Thomas Wakefield**, Youngstown State University.

Fractional Difference Operators and Their Application (Code: SS 59A), **Christopher S. Goodrich**, Creighton Preparatory School, and **Rajendra Dahal**, Coastal Carolina University.

Free Convexity and Free Analysis (Code: SS 21A), **J. William Helton**, University of California, San Diego, and **Igor Klep**, University of Auckland.

Geometric Analysis (Code: SS 86A), **Davi Maximo**, University of Pennsylvania, **Lu Wang**, University of Wisconsin-Madison, and **Xin Zhou**, University of California Santa Barbara.

Geometric Analysis and Geometric Flows (Code: SS 54A), **David Glickenstein**, University of Arizona, and **Brett Kotschwar**, Arizona State University.

History of Mathematics (Code: SS 50A), **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Clemency Montelle**, University of Canterbury, **Daniel Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College.

Homotopy Type Theory (a Mathematics Research Communities Session) (Code: SS 14A), .

If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences (Code: SS 25A), **Edray Goins** and **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

Interactions of Inverse Problems, Signal Processing, and Imaging (Code: SS 36A), **M. Zuhair Nashed**, University of Central Florida, **Willi Freeden**, University of Kaiserslautern, and **Otmar Scherzer**, University of Vienna.

Markov Chains, Markov Processes and Applications (Code: SS 27A), **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Mathematical Analysis and Nonlinear Partial Differential Equations (Code: SS 33A), **Hongjie Dong**, Brown University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

Mathematical Fluid Mechanics: Analysis and Applications (Code: SS 38A), **Zachary Bradshaw** and **Aseel Farhat**, University of Virginia.

Mathematical Information in the Digital Age of Science (Code: SS 83A), **Patrick Ion**, University of Michigan, **Olaf Teschke**, zbMath Berlin, and **Stephen Watt**, University of Waterloo.

Mathematical Methods in Genomics (Code: SS 4A), **David Koslicki**, Oregon State University.

Mathematical Modeling and Analysis of Infectious Diseases (Code: SS 65A), **Kazuo Yamazaki**, University of Rochester.

Mathematical Modeling of Natural Resources (Code: SS 39A), **Shandelle M. Henson**, Andrews University, and **Natali Hritonenko**, Prairie View A&M University.

Mathematical Modeling, Analysis and Applications in Population Biology (Code: SS 47A), **Yu Jin**, University of Nebraska-Lincoln, and **Ying Zhou**, Lafayette College.

Mathematical Problems in Ocean Wave Modeling and Fluid Mechanics (Code: SS 49A), **Christopher W. Curtis**, San Diego State University, and **Katie Oliveras**, Seattle University.

Mathematical Relativity and Geometric Analysis (Code: SS 72A), **James Dilts** and **Michael Holst**, University of California, San Diego.

Mathematics Research from the SMALL Undergraduate Research Program (Code: SS 73A), **Colin Adams**, **Frank Morgan**, and **Cesar E. Silva**, Williams College.

Mathematics of Gravitational Wave Science (Code: SS 40A), **Andrew Gillette** and **Nikki Holtzer**, University of Arizona.

Mathematics of Quantum Computing and Topological Phases of Matter (Code: SS 26A), **Paul Bruillard**, Pacific Northwest National Laboratory, **David Meyer**, University of California San Diego, and **Julia Plavnik**, Texas A&M University.

Modeling in Differential Equations - High School, Two-Year College, Four-Year Institution (Code: SS 22A), **Corban Harwood**, George Fox University, **William Skerbitz**, Wayzata High School, **Brian Winkel**, SIMIODE, and **Dina Yagodich**, Frederick Community College.

Multi-scale Modeling with PDEs in Computational Science and Engineering:Algorithms, Simulations, Analysis, and Applications (Code: SS 37A), **Salim M. Haidar**, Grand Valley State University.

Network Science (Code: SS 31A), **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng Shi**, University of North Carolina at Chapel Hill.

New Trends in Celestial Mechanics (Code: SS 10A), **Richard Montgomery**, University of California Santa Cruz, and **Zhifu Xie**, University of Southern Mississippi.

Nilpotent and Solvable Geometry (Code: SS 32A), **Michael Jablonski**, University of Oklahoma, **Megan Kerr**, Wellesley College, and **Tracy Payne**, Idaho State University.

Noncommutative Algebras and Noncommutative Invariant Theory (Code: SS 24A), **Ellen Kirkman**, Wake Forest University, and **James Zhang**, University of Washington.

Nonlinear Evolution Equations of Quantum Physics and Their Topological Solutions (Code: SS 34A), **Stephen Gustafson**, University of British Columbia, **Israel Michael Sigal**, University of Toronto, and **Avy Soffer**, Rutgers University.

Novel Methods of Enhancing Success in Mathematics Classes (Code: SS 35A), **Ellina Grigorieva**, Texas Womans University, and **Natali Hritonenko**, Prairie View A&M University.

Open and Accessible Problems for Undergraduate Research (Code: SS 18A), **Michael Dorff**, Brigham Young University, **Allison Henrich**, Seattle University, and **Nicholas Scoville**, Ursinus College.

Operators on Function Spaces in One and Several Variables (Code: SS 45A), **Catherine Bénéteau**, University of South Florida, and **Matthew Fleeman** and **Constanze Liaw**, Baylor University.

Orthogonal Polynomials and Applications (Code: SS 17A), **Abey Lopez-Garcia**, University of South Alabama, and **Xiang-Sheng Wang**, University of Louisiana at Lafayette.

Orthogonal Polynomials, Quantum Probability, and Stochastic Analysis (Code: SS 8A), **Julius N. Esunge**, University of Mary Washington, and **Aurel I. Stan**, Ohio State University.

Quantum Link Invariants, Khovanov Homology, and Low-dimensional Manifolds (Code: SS 64A), **Diana Hubbard**, University of Michigan, and **Christine Ruey Shan Lee**, University of Texas at Austin.

Quaternions (Code: SS 23A), **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

Recent Trends in Analysis of Numerical Methods of Partial Differential Equations (Code: SS 2A), **Sara Pollock**, Wright State University, and **Leo Rebholz**, Clemson University.

Research by Postdocs of the Alliance for Diversity in Mathematics (Code: SS 62A), **Aloysius Helminck**, University of Hawaii - Manoa, and **Michael Young**, Iowa State University.

Research from the Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics (Code: SS 69A), **Michael Ferrara**, University of Colorado Denver, **Leslie Hogben**, Iowa State University, **Paul Horn**, University of Denver, and **Tyrrell McAllister**, University of Wyoming.

Research in Mathematics by Early Career Graduate Students (Code: SS 46A), **Michael Bishop**, **Marat Markin**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 19A), **Tamas Forgacs**, CSU Fresno, **Darren A. Narayan**, Rochester Institute of Technology, and **Mark David Ward**, Purdue University (AMS-MAA-SIAM).

Set Theory, Logic and Ramsey Theory (Code: SS 5A), **Andrés Caicedo**, Mathematical Reviews, and **José Mijares**, University of Colorado, Denver (AMS-ASL).

Set-theoretic Topology (Dedicated to Jack Porter in honor of 50 years of dedicated research) (Code: SS 43A), **Nathan Carlson**, California Lutheran University, **Jila Niknejad**, University of Kansas, and **Lynne Yengulalp**, University of Dayton.

Special Functions and Combinatorics (in honor of Dennis Stanton's 65th birthday) (Code: SS 9A), **Susanna Fishel**, Arizona State University, **Mourad Ismail**, University of Central Florida, and **Vic Reiner**, University of Minnesota.

Spectral Theory, Disorder and Quantum Physics (Code: SS 57A), **Rajinder Mavi** and **Jeffery Schenker**, Michigan State University.

Stochastic Processes, Stochastic Optimization and Control, Numerics and Applications (Code: SS 78A), **Hongwei Mei**, University of Central Florida, **Zhixin Yang** and **Quan Yuan**, Ball State University, and **Guangliang Zhao**, GE Global Research.

Strengthening Infrastructures to Increase Capacity Around K-20 Mathematics (Code: SS 74A), **Brianna Donaldson**, American Institute of Mathematics, and **William Jaco** and **Michael Oehrtman**, Oklahoma State University.

Structure and Representations of Hopf Algebras: a session in honor of Susan Montgomery (Code: SS 30A), **Siu-Hung Ng**, Louisiana State University, and **Lance Small** and **Henry Tucker**, University of California, San Diego.

Theory, Practice, and Applications of Graph Clustering (Code: SS 63A), **David Gleich**, Purdue University, and **Jennifer Webster** and **Stephen J. Young**, Pacific Northwest National Laboratory.

Topological Data Analysis (Code: SS 84A), **Henry Adams**, Colorado State University, **Gunnar Carlsson**, Stanford University, and **Mikael Vejdemo-Johansson**, CUNY College of Staten Island.

Topological Graph Theory: Structure and Symmetry (Code: SS 3A), **Jonathan L. Gross**, Columbia University, and **Thomas W. Tucker**, Colgate University.

Visualization in Mathematics: Perspectives of Mathematicians and Mathematics Educators (Code: SS 52A), **Karen Allen Keene**, North Carolina State University, and **Mile Krajcevski**, University of South Florida.

Women in Symplectic and Contact Geometry and Topology (Code: SS 44A), **Bahar Acu**, Northwestern University, **Ziva Myer**, Duke University, and **Yu Pan**, Massachusetts Institute of Technology (AMS-AWM).

Columbus, Ohio

Ohio State University

March 17–18, 2018

Saturday – Sunday

Meeting #1136

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: January 31, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: August 15, 2017
For abstracts: January 22, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Aaron Brown, University of Chicago, *Title to be announced.*

Tullia Dymarz, University of Wisconsin-Madison, *Title to be announced.*

June Huh, Institute for Advanced Study, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Combinatorics: Association Schemes, Finite Geometry, and Related Topics (Code: SS 15A), **Sung Y. Song**, Iowa State University, and **Bangteng Xu**, Eastern Kentucky University.

Algebraic and Combinatorial Aspects of Tropical Geometry (Code: SS 11A), **Maria Angelica Cueto**, Ohio State University, **Yoav Len**, University of Waterloo, and **Martin Ulirsch**, University of Michigan.

Algebraic, Combinatorial, and Quantum Invariants of Knots and Manifolds (Code: SS 6A), **Cody Armond**, Ohio State University, Mansfield, **Micah Chrisman**, Monmouth University, and **Heather Dye**, McKendree University.

Coherent Structures in Interfacial Flows (Code: SS 14A), **Benjamin Akers** and **Jonah Reeger**, Air Force Institute of Technology.

Differential Equations and Applications (Code: SS 8A), **King-Yeung Lam** and **Yuan Lou**, Ohio State University, and **Qiliang Wu**, Michigan State University.

Geometric Methods in Shape Analysis (Code: SS 10A), **Sebastian Kurtek** and **Tom Needham**, Ohio State University.

Graph Theory (Code: SS 5A), **John Maharry**, Ohio State University, **Yue Zhao**, University of Central Florida, and **Xiangqian Zhou**, Wright State University.

Homological Algebra (Code: SS 4A), **Ela Celikbas** and **Olgur Celikbas**, West Virginia University.

Multiplicative Ideal Theory and Factorization (in honor of Tom Lucas retirement) (Code: SS 7A), **Evan Houston**, University of North Carolina, Charlotte, and **Alan Loper**, Ohio State University.

Nonlinear Evolution Equations (Code: SS 9A), **John Holmes** and **Feride Tiglay**, Ohio State University.

Probability in Convexity and Convexity in Probability (Code: SS 2A), **Elizabeth Meckes**, **Mark Meckes**, and **Elisabeth Werner**, Case Western Reserve University.

Quantum Symmetries (Code: SS 3A), **David Penneys**, The Ohio State University, and **Julia Plavnik**, Texas A & M University.

Recent Advances in Approximation Theory and Operator Theory (Code: SS 1A), **Jan Lang** and **Paul Nevai**, The Ohio State University.

Recent Development of Nonlinear Geometric PDEs (Code: SS 12A), **Bo Guan**, Ohio State University, **Qun Li**, Wright State University, **Xiangwen Zhang**, University of California, Irvine, and **Fangyang Zheng**, Ohio State University.

Several Complex Variables (Code: SS 13A), **Liwei Chen**, **Kenneth Koenig**, and **Liz Vivas**, Ohio State University.

Portland, Oregon

Portland State University

April 14–15, 2018

Saturday – Sunday

Meeting #1137

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: February 15, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: September 14, 2017
For abstracts: February 6, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Sándor Kovács, University of Washington, Seattle, *Title to be announced.*

Elena Mantovan, California Institute of Technology, *Title to be announced.*

Dimitri Shlyakhtenko, University of California, Los Angeles, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Inverse Problems (Code: SS 2A), **Hanna Makaruk**, Los Alamos National Laboratory (LANL), and **Robert Owczarek**, University of New Mexico, Albuquerque & Los Alamos.

Pattern Formation in Crowds, Flocks, and Traffic (Code: SS 1A), **J. J. P. Veerman**, Portland State University, **Alethea Barbaro**, Case Western Reserve University, and **Bassam Bamieh**, UC Santa Barbara.

Nashville, Tennessee

Vanderbilt University

April 14–15, 2018
Saturday – Sunday

Meeting #1138

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: February 22, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: September 14, 2017
For abstracts: February 13, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Andrea Bertozzi, University of California Los Angeles, *Title to be announced* (Erdős Memorial Lecture).

J. M. Landsberg, Texas A & M University, *Title to be announced.*

Jennifer Morse, University of Virginia, *Title to be announced.*

Kirsten Wickelgren, Georgia Institute of Technology, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Difference Equations and Applications (Code: SS 2A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton, Ohio.

Quantization for Probability Distributions (Code: SS 1A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Selected Topics in Graph Theory (Code: SS 3A), **Songling Shan**, Vanderbilt University, **David Chris Stephens**, Middle Tennessee State University, and **Dong Ye**, Middle Tennessee Stae University.

Boston, Massachusetts

Northeastern University

April 21–22, 2018
Saturday – Sunday

Meeting #1139

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: March 1, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 2

Deadlines

For organizers: September 21, 2017
For abstracts: February 20, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Jian Ding, University of Chicago, *Title to be announced.*

Edward Frenkel, University of California, Berkeley, *Title to be announced* (Einstein Public Lecture in Mathematics).

Valentino Tosatti, Northwestern University, *Title to be announced.*

Maryna Viazovska, École Polytechnique Fédérale de Lausanne, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Analysis and Geometry in Non-smooth Spaces (Code: SS 5A), **Nageswari Shanmugalingam** and **Gareth Speight**, University of Cincinnati.

Arithmetic Dynamics (Code: SS 1A), **Jacqueline M. Anderson**, Bridgewater State University, **Robert Benedetto**, Amherst College, and **Joseph H. Silverman**, Brown University.

Arrangements of Hypersurfaces (Code: SS 2A), **Graham Denham**, University of Western Ontario, and **Alexander I. Suciu**, Northeastern University.

Ergodic Theory and Dynamics in Combinatorial Number Theory (Code: SS 7A), **Stanley Eigen**, Northeastern University, **Daniel Glasscock**, Ohio State University, and **Vidhu Prasad**, University of Massachusetts, Lowell.

Facets of Symplectic Geometry and Topology (Code: SS 3A), **Tara Holm**, Cornell University, **Jo Nelson**, Columbia

University, and **Jonathan Weitsman**, Northeastern University.

Geometry of Moduli Spaces (Code: SS 10A), **Ana-Marie Castravet** and **Emanuele Macrí**, Northeastern University, **Benjamin Schmidt**, University of Texas, and **Xiaolei Zhao**, Northeastern University.

Homological Commutative Algebra (Code: SS 11A), **Sean Sather-Wagstaff**, Clemson University, and **Oana Veliche**, Northeastern University.

Hopf Algebras, Tensor Categories, and Homological Algebra (Code: SS 8A), **Cris Negron**, Massachusetts Institute of Technology, **Julia Plavnik**, Texas A&M, and **Sarah Witherspoon**, Texas A&M University.

New Developments in Inverse Problems and Imaging (Code: SS 9A), **Ru-Yu Lai**, University of Minnesota, and **Ting Zhou**, Northeastern University.

Polytopes and Discrete Geometry (Code: SS 6A), **Gabriel Cunningham**, University of Massachusetts, Boston, **Mark Mixer**, Wentworth Institute of Technology, and **Egon Schulte**, Northeastern University.

Singularities of Spaces and Maps (Code: SS 4A), **Terence Gaffney** and **David Massey**, Northeastern University.

Shanghai, Peoples' Republic of China

Fudan University

June 11–14, 2018

Monday – Thursday

Meeting #1140

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Yu-Hong Dai, Academy of Mathematics and System Sciences, *Title to be announced.*

Kenneth A. Ribet, University of California, Berkeley, *Title to be announced.*

Richard M. Schoen, University of California, Irvine, *Title to be announced.*

Sijue Wu, University of Michigan, *Title to be announced.*

Chenyang Xu, Peking University, *Title to be announced.*

Jiangong You, Nankai University, *Title to be announced.*

Newark, Delaware

University of Delaware

September 29–30, 2018

Saturday – Sunday

Meeting #1141

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 9, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: February 28, 2018
For abstracts: July 31, 2018

Invited Addresses

Leslie Greengard, New York University, *Title to be announced.*

Elisenda Grigsby, Boston College, *Title to be announced.*

Davesh Maulik, Massachusetts Institute of Technology, *Title to be announced.*

Fayetteville, Arkansas

University of Arkansas

October 6–7, 2018

Saturday – Sunday

Meeting #1142

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 16, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: March 6, 2018
For abstracts: August 7, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mihalis Dafermos, Princeton University, *Title to be announced.*

Jonathan Hauenstein, University of Notre Dame, *Title to be announced.*

Kathryn Mann, University of California Berkeley, *Title to be announced.*

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

Meeting #1143

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 30, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: March 20, 2018
For abstracts: August 21, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Elena Fuchs, University of Illinois Urbana-Champaign, *Title to be announced.*

Andrew Putman, University of Notre Dame, *Title to be announced.*

Charles Smart, University of Chicago, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

San Francisco, California

San Francisco State University

October 27–28, 2018

Saturday – Sunday

Meeting #1144

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2018
Program first available on AMS website: September 6, 2018
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: March 27, 2018
For abstracts: August 28, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Srikanth B. Iyengar, University of Utah, *Title to be announced.*

Sarah Witherspoon, Texas A&M University, *Title to be announced.*

Abdul-Aziz Yakubu, Howard University, *Title to be announced.*

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Auburn, Alabama

Auburn University

March 15–17, 2019

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019

Friday – Sunday
Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: May 15, 2018
For abstracts: January 22, 2019

Denver, Colorado

Colorado Convention Center

January 15–18, 2020

Wednesday – Saturday
Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: November 1, 2019
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2019
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021

Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2020
For abstracts: To be announced

William Benter Prize in Applied Mathematics 2018

Call for NOMINATIONS

The Liu Bie Ju Centre for Mathematical Sciences of City University of Hong Kong is inviting nominations of candidates for the William Benter Prize in Applied Mathematics, an international award.

The Prize

The Prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, financial, and engineering applications.

It will be awarded to a single person for a single contribution or for a body of related contributions of his/her research or for his/her lifetime achievement.

The Prize is presented every two years and the amount of the award is US$100,000.

Nominations

Nomination is open to everyone. Nominations should not be disclosed to the nominees and self-nominations will not be accepted.

A nomination should include a covering letter with justifications, the CV of the nominee, and two supporting letters. Nominations should be submitted to:

Selection Committee
c/o Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong
Tat Chee Avenue
Kowloon
Hong Kong

Or **by email to: lbj@cityu.edu.hk**

Deadline for nominations: 30 September 2017

Presentation of Prize

The recipient of the Prize will be announced at the **International Conference on Applied Mathematics 2018** to be held in summer 2018. The Prize Laureate is expected to attend the award ceremony and to present a lecture at the conference.

The Prize was set up in 2008 in honor of Mr William Benter for his dedication and generous support to the enhancement of the University's strength in mathematics. The inaugural winner in 2010 was George C Papanicolaou (Robert Grimmett Professor of Mathematics at Stanford University), and the 2012 Prize went to James D Murray (Senior Scholar, Princeton University; Professor Emeritus of Mathematical Biology, University of Oxford; and Professor Emeritus of Applied Mathematics, University of Washington), the winner in 2014 was Vladimir Rokhlin (Professor of Mathematics and Arthur K. Watson Professor of Computer Science at Yale University). The winner in 2016 was Stanley Osher, Professor of Mathematics, Computer Science, Electrical Engineering, Chemical and Biomolecular Engineering at University of California (Los Angeles).

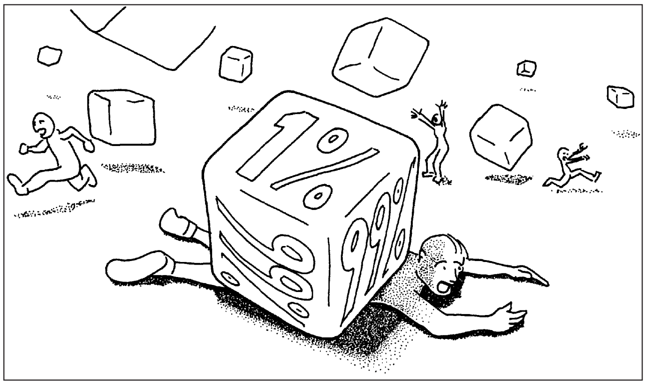
The Liu Bie Ju Centre for Mathematical Sciences was established in 1995 with the aim of supporting world-class research in applied mathematics and in computational mathematics. As a leading research centre in the Asia-Pacific region, its basic objective is to strive for excellence in applied mathematical sciences. For more information about the Prize and the Centre, please visit ***http://www.cityu.edu.hk/lbj/***

香港城市大學
City University of Hong Kong

LIU BIE JU CENTRE FOR MATHEMATICAL SCIENCES

The April Contest Winner Is...

Jim Ferry, who receives our book award.

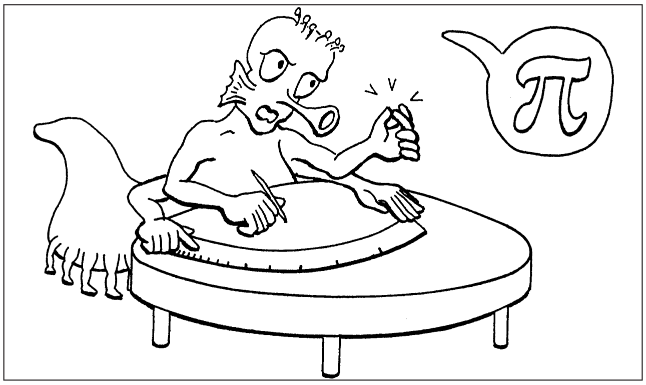


Drawn by James F. Bredt, © Frank Morgan.

Quantum Economics: God plays
unfair dice with the universe.

The June/July 2017 Caption Contest:

What's the Caption?



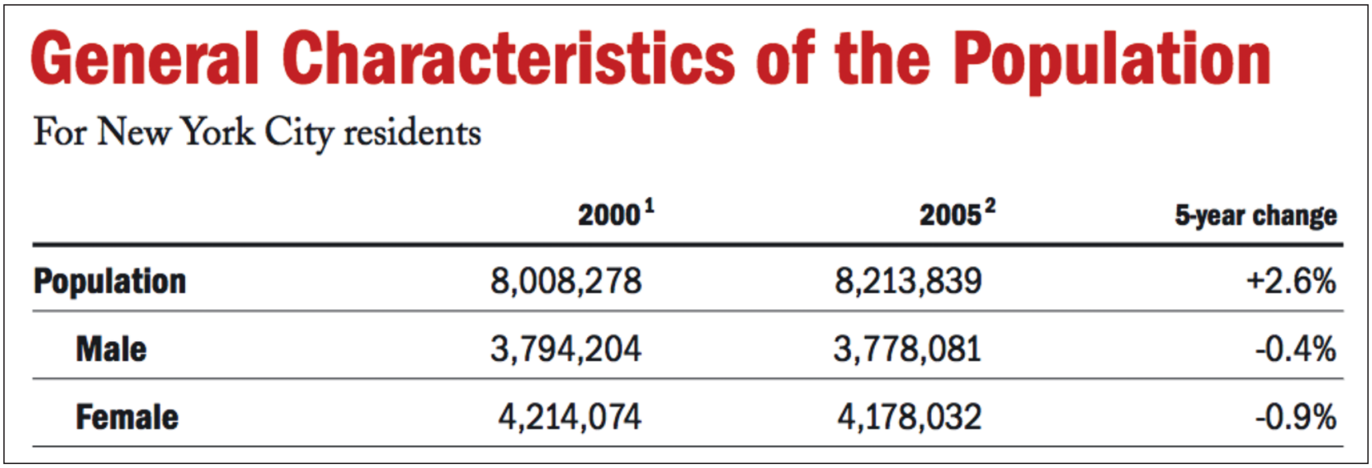
Drawn by James F. Bredt, © Frank Morgan.

Submit your entry to `captions@ams.org`
by July 25. The winning entry will be
posted here in the October 2017 issue.

Most cited 2015 publication (75 hits as of January 6, 2017):

Larry Guth and Nets Hawk Katz, On the Erdős distinct distances problem in the plane, *Ann. of Math*. [From MathSciNet®.]

QUESTIONABLE MATHEMATICS



General Characteristics of the Population

For New York City residents

	2000[1]	2005[2]	5-year change
Population	8,008,278	8,213,839	+2.6%
Male	3,794,204	3,778,081	-0.4%
Female	4,214,074	4,178,032	-0.9%

Crain's New York Business Special Issue, Vol. XXIII, July 2–8, 2007.

Michael Levitan submitted this impossible example of population increasing with the number of males and females both decreasing, well beyond the effect of a growing number of persons that might choose other categories. The footnotes (not shown here) explain that conflicting bases were used for population and gender.

What crazy things happen to you? Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: **`noti-backpage@ams.org`**.

IN THE NEXT ISSUE OF NOTICES

AUGUST 2017…

The US Travel Ban and How It Has Affected Mathematicians: Nima Rasekh, Beheshteh Tolouei Rakhshan, Camelia Karimianpour, and Hamed Razavi share their stories with *Notices*.

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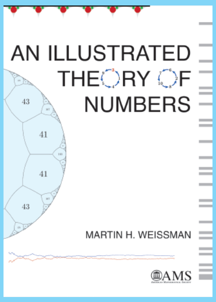
AMERICAN MATHEMATICAL SOCIETY

New from the
AMS



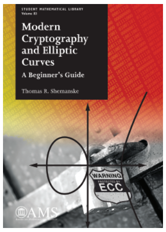
An Illustrated Theory of Numbers ◆

Martin H. Weissman, *University of California, Santa Cruz, CA*

This is a meticulously written and stunningly laid-out book influenced not only by the classical masters of number theory like Fermat, Euler, and Gauss but also by the work of Edward Tufte on data visualization. Assuming little beyond basic high school mathematics, the author covers a tremendous amount of territory, including topics like Ford circles, Conway's topographs, and Zolotarev's lemma which are rarely seen in introductory courses. All of this is done with a visual and literary flair which very few math books even strive for, let alone accomplish.

—Matthew Baker, Georgia Institute of Technology

2017; approximately 321 pages; Hardcover; ISBN: 978-1-4704-3493-9; List US$69; AMS members US$55.20; Order code MBK/105



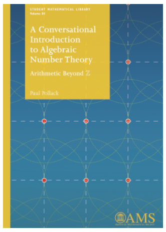
Modern Cryptography and Elliptic Curves ◆ ❂

A Beginner's Guide

Thomas R. Shemanske, *Dartmouth College, Hanover, NH*

This gradual introduction to modern cryptography and elliptic curves offers the beginning undergraduate student some of the vista of modern mathematics and presents the tools needed to gain an understanding of the arithmetic of elliptic curves over finite fields and their applications to modern cryptography.

Student Mathematical Library, Volume 83; 2017; approximately 261 pages; Softcover; ISBN: 978-1-4704-3582-0; List US$52; AMS members US$41.60; Order code STML/83



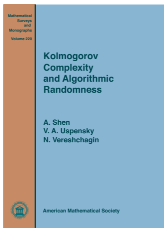
A Conversational Introduction to Algebraic Number Theory ◆

Arithmetic Beyond $\mathbb{Z}$

Paul Pollack, *University of Georgia, Athens, GA*

Written in a conversational style, this introduction to algebraic number theory lays out basic results in the form of three classical "fundamental theorems": unique factorization of ideals, finiteness of the class number, and Dirichlet's unit theorem, while also frequently mentioning recent developments within the field.

Student Mathematical Library, Volume 84; 2017; approximately 311 pages; Softcover; ISBN: 978-1-4704-3653-7; List US$52; AMS members US$41.60; Order code STML/84

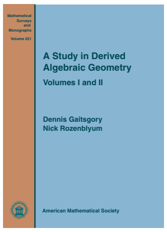


Kolmogorov Complexity and Algorithmic Randomness ◆ ❂

A. Shen, *LIRMM CRNS, Université de Montpellier, France*, **V. A. Uspensky**, *Lomonosov Moscow State University, Russia*, and **N. Vereshchagin**, *Lomonosov Moscow State University, Russia*

Aiming to explore algorithmic information theory, the first part of this book is a textbook-style exposition of the basic notions of complexity and randomness, while the second part covers some recent work done by participants of the "Kolmogorov seminar" in Moscow.

Mathematical Surveys and Monographs, Volume 220; 2017; approximately 499 pages; Hardcover; ISBN: 978-1-4704-3182-2; List US$124; AMS members US$99.20; Order code SURV/220



A Study in Derived Algebraic Geometry

Volumes I and II

Dennis Gaitsgory, *Harvard University, Cambridge, MA*, and **Nick Rozenblyum**, *University of Chicago, IL*

This two-volume monograph explores various topics in algebraic geometry in the context of derived algebraic geometry.

Part 1: **Mathematical Surveys and Monographs**, Volume 221; 2017; approximately 553 pages; Hardcover; ISBN: 978-1-4704-3569-1; Order code SURV/221.1

Part 2: **Mathematical Surveys and Monographs**, Volume 221; 2017; approximately 463 pages; Hardcover; ISBN: 978-1-4704-3570-7; Order code SURV/221.2

Set: **Mathematical Surveys and Monographs**, Volume 221; 2017; approximately 1016 pages; Hardcover; ISBN: 978-1-4704-3568-4; List US$220; AMS members US$176; Order code SURV/221

◆ = Textbook ❂ = Applied Mathematics

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