

# Notices

of the American Mathematical Society

February 2018

Volume 65, Number 2

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Penrose Conjecture

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Nashville Meeting

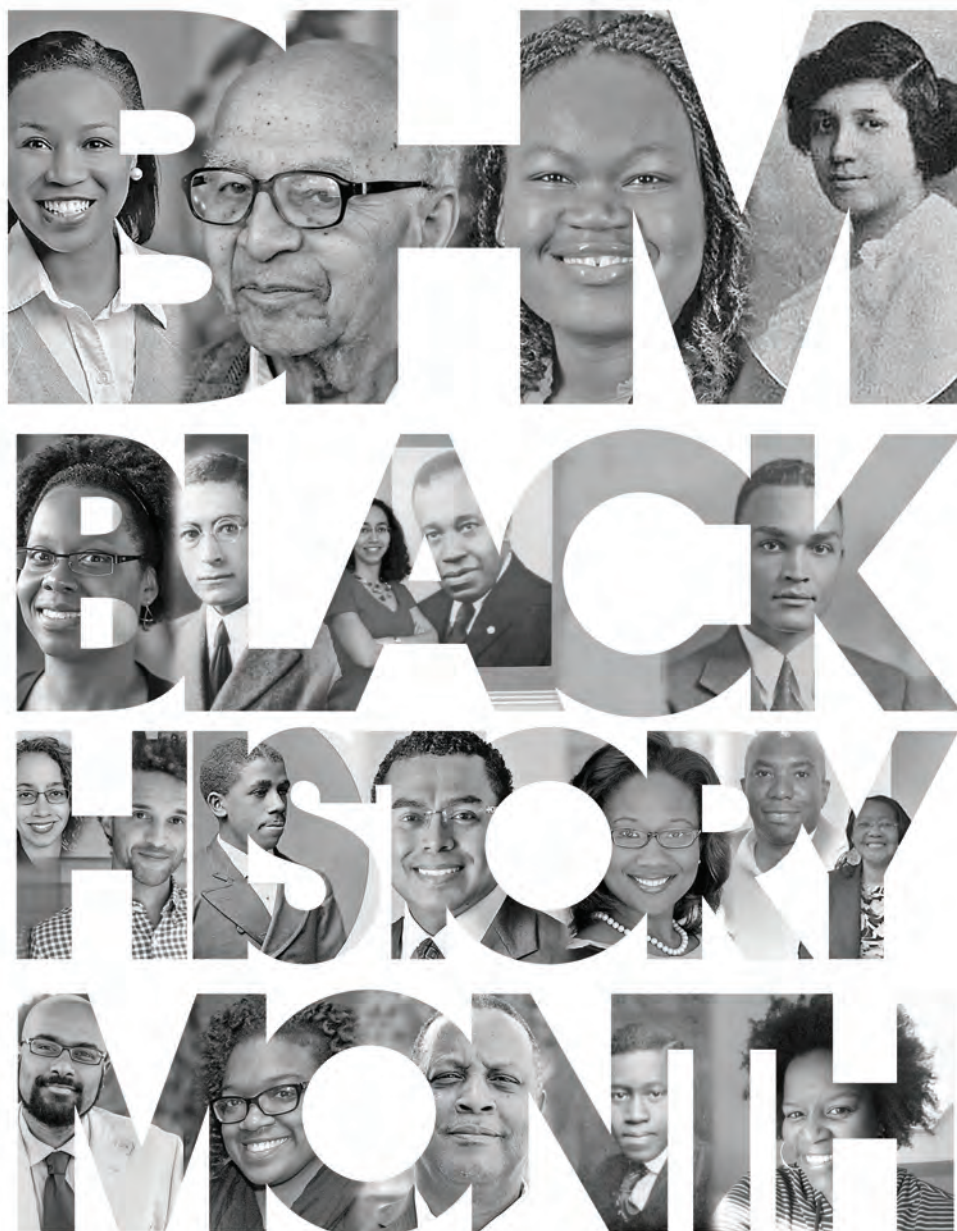
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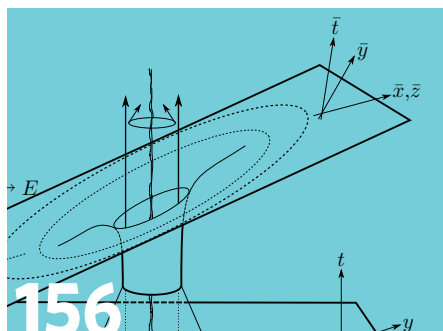


## FEATURED



### In Honor of Black History

by Talithia Williams, Arlie O. Petters, Erica Graham, Raegan Higgins, Candice Price, Shelby Wilson, David Goldberg, Phil Kutzko, Talitha M. Washington, Asamoah Nkwanta, Janet E. Barber, Johnny L. Houston, Edray Herber Goins, and Vernon R. Morris



### Proof of a Null Geometry Penrose Conjecture

by Hubert L. Bray and Henri P. Roesch



### Graduate Student Section

Interview with Chelsea Walton  
Conducted by Alexander Diaz-Lopez  
**WHAT IS...the Amplituhedron?**  
by Jacob Bourjaily and Hugh Thomas  
AMS Graduate Student Blog

Our special section in honor of Black History Month reminds me how much we owe to the noble souls who have done and are doing so much, often under very difficult circumstances, to turn the broad potential of mathematics into a reality. Of course what we owe them is to join the effort. We are happy to report that for *Notices* the number of minority authors rose from 17 (8 percent) in 2015 to 35 (17 percent) in 2017.

This issue also includes an article on the recent proof of a Null Geometry Penrose Conjecture by Hubert L. Bray and Henri P. Roesch, an article on Jacqueline Ferrand by Pierre Pansu, an interview of Chelsea Walton, and "WHAT IS...the Amplituhedron?" by Jacob Bourjaily and Hugh Thomas. Enjoy the month and the math.

—Frank Morgan, Editor-in-Chief

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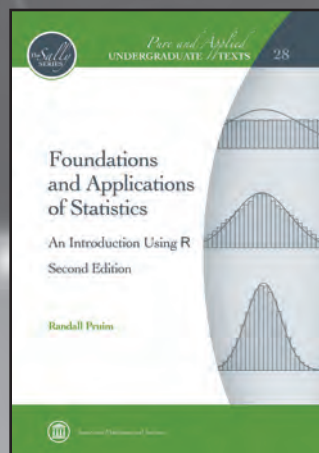


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# Call for Nominations



## LEROY P. STEELE PRIZES

The selection committee for these prizes requests nominations for consideration for the 2019 awards. Further information about the prizes can be found in the November 2017 *Notices* p.1203 (also available at [www.ams.org/steele-prize](http://www.ams.org/steele-prize)).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2019 the prize for Seminal Contribution to Research will be awarded for a paper in mathematics (open subject area).

Further information and instructions for submitting a nomination can be found at the Leroy P. Steele Prizes website: [www.ams.org/steele-prize](http://www.ams.org/steele-prize).

Nominations for the Steele Prizes for Lifetime Achievement and for Mathematical Exposition will remain active and receive consideration for three consecutive years.

For questions contact the AMS Secretary at [secretary@ams.org](mailto:secretary@ams.org).

**The nomination period is February 1, 2018 through March 31, 2018.**

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Applications are invited for a full-time position as an Associate Editor of Mathematical Reviews/MathSciNet®, to commence as early as possible in late spring/early summer 2018. We also welcome applications from suitably qualified candidates interested in spending a sabbatical year from their institution in the position and continuing as an editor working remotely under contract afterwards. The Mathematical Reviews (MR) division of the American Mathematical Society (AMS) is located in Ann Arbor, Michigan, in a beautiful, historic building close to the campus of the University of Michigan. The editors are employees of the AMS; they also enjoy certain privileges at the university. At present, the AMS employs approximately eighty people at Mathematical Reviews, including sixteen mathematical editors. MR's mission is to develop and maintain the MR Database, from which MathSciNet is produced. An Associate Editor is responsible for broad areas of the mathematical sciences. Editors select articles and books for coverage, classify these items, determine the type of coverage, assign selected items for review to reviewers, and edit the reviews on their return.

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The review of the applications will begin on February 12, 2018 and will continue until the position is filled.

Applications can be submitted through MathJobs at [www.mathjobs.org/jobs/jobs/10667](http://www.mathjobs.org/jobs/jobs/10667)

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# In Honor of Black History

On the occasion of Black History Month, February guest editor Talithia Williams presents a special section devoted to historical and contemporary figures and programs in math.



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# An Introduction

*Talithia Williams, Guest Editor*



Black History Month was officially recognized by President Gerald Ford in 1976, when he encouraged the nation to “seize the opportunity to honor the too-often neglected accomplishments of Black Americans in every area of endeavor throughout our history.” In celebration of Black History Month, we’ve chosen to highlight several experiences of African American mathematicians and programs that have worked to broaden participation in mathematics. Organizations such as the National Association of Mathematicians (NAM), the Enhancing Diversity in Graduate Education (EDGE) program, and Math Alliance are examples of go-to organizations for the mathematical development of underrepresented groups. Such organizations have worked to create inclusive mathematical communities and instill a sense of belonging to women and minorities, groups historically mathematically isolated.

If there’s one word that summarizes the theme of this issue, it would be “Belonging.” Arlie O. Petters, widely recognized as the founder of mathematical astronomy, discusses this foundational human need and how a sense of belonging allows all of us to maximize our excellence. Department chairs Asamoah Nkwanta and Janet Barber tell about the renowned genius J. Ernest Wilkins Jr. Johnny L. Houston, NAM co-founder and executive secretary from 1975 until 2000, shares “Ten African American Pioneers

and Mathematicians Who Inspired Me.” Its founders describe the Mathematically Gifted and Black website. Talitha Washington, member of the AMS Council, the AWM executive committee, and the NAM board, pays homage to the men who have nurtured her successful career as a mathematician, professor, and activist. NAM President Edray Herber Goins discusses three questions that have framed his personal journey. Directors David Goldberg and Phil Kutzko discuss the Math Alliance and its extraordinary contributions to the production of minority PhDs.

During the past year, the movie *Hidden Figures* brought visibility to the lives of African American women mathematicians who served as NASA “human computers” in the 1960s. During that same time, Dr. Etta Falconer, the eleventh African American woman to receive a PhD in mathematics, began her tenure at Spelman College, motivating young women of color to be and do more than they dreamed possible in a field where their presence was lacking. Falconer was awarded the AWM Louise Hay Award in 1995 in honor of her outstanding achievements in mathematics education. Her response is a fitting tribute to those that continue to work to create mathematical communities in which all can thrive:

I have devoted my entire life to increasing the number of highly qualified African Americans in mathematics and mathematics related careers. High expectations, the building of self confidence, and the creation of a nurturing environment have been essential components for the success of these students. They have fully justified my beliefs. Perhaps the most rewarding moments have come when younger faculty have undertaken the same goal and have surpassed my efforts—reaching out to the broader community to help minorities and women achieve in mathematics. —Etta Falconer

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# Belonging

Arlie O. Petters

**BHM** Belonging. What a word. What a deep-seated human need. It touches all of us, invoking a strong sense of loyalty and commitment when we are showered by it and feelings of alienation and distrust when it is denied. Julio Frenk, president of the University of Miami, deconstructed this word beautifully [2]:

The components of “belonging” are suggestive: “Be”—as in being—signifies authenticity and freedom from the need to cover aspects of one’s identity. “Longing” reminds us of the profound human yearning to connect with others and be part of something that transcends us.

The times we live in have made this topic even more pressing. I shall discuss it within the context of under-represented minorities and women in the mathematical sciences, though many of the ideas apply more broadly to the human experience. My goal is not to take you on an academic tour of the extensive psychology and sociology research literature on belonging. Instead, I will share some personal reflections and anecdotes intended to provide useful insights into the experiences of marginalized individuals.

## Othering and Such Climatic Joy Killers

I remember being giddy with excitement to attend the welcoming reception for my entering class of math graduate students. I walked into the room and heads turned towards me. Feeling out of place, I walked over to two student-looking faces. One happened to be a fourth-year graduate math student and the other was a first-year like

me. I introduced myself and, because I wanted a quick exit, I asked the more senior student how to get to the main math office. He told me that when I walked out the door, I should make a left, walk down the hallway, make another left, and it would be on my right. “Or, you could tie a rope to the ceiling and swing over to the other side,” he said with a mischievous grin. The first-year student turned red with embarrassment. It did not matter whether the senior student thought of me as a monkey in a tree, Tarzan, or something else; his decision to engage in an unnecessary framing that could provoke a negative stereotype was telling. I quickly responded, “I see that you’re going to be an a\*\*hole,” and I walked out of the room. Here I was, looking forward to being part of a new community of mathematicians and then being made to feel unwelcome at the onset.

I went directly to my apartment and started packing. My mind was racing and I was angry: “To hell with them. They turned around looking at me because I am a person of color. I am leaving this place. To hell with these people.” As I started calming down, a counterintuitive thought occurred to me: “What if the others in the room weren’t like him? What if they turned around and looked at me because they don’t often see someone like me in an entering class and were curious to get to know me? ... If I leave, this guy will win. I refuse to let him win.” My psychological bounce back was that he had brought the fight to me, and I refused to cower in fear or run away in anger. I had briefly allowed him to hijack and taint my perspective. And, even worse, by allowing him to make me angry, I had given him power over me in that moment. Never again. The emotional-intelligence battle was on. Would I have had such a fight-back spirit in the academic sphere if from pre-kindergarten my sense of self had been chipped away, bit by bit, by individual and

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*My mind was racing  
and I was angry.*

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institutional racism? I doubt it. Fortunately, I was raised until the age of fifteen years in Belize by a loving and resilient grandmother who strengthened me internally, fortifying my identity and allowing me to maintain its structural integrity in the face of undermining forces.

I was not naïve about the epiphany that caused me to stay. My hypothesis that most people in the room were not like him needed to be tested. But I had enough internal energy and grit to hold on to it by blind faith in the short term. The energy sustained me through the long hours of hard work needed to perform very well on my homework sets. And the grit enabled me to bear the anxiety that maybe most people in the environment did not really care for my being there and did not think much of me intellectually. In my case, I was fortunate to discover with time that most of the people were not like that graduate student. I had a perceptive and supportive thesis advisor and a positive interaction with the majority of the other math and physics graduate students and faculty. That young man had acted as if he owned the place. To me, he had a warped sense of belonging and entitlement that made him feel confident enough to treat me in a demeaning way without consequences.

I wish I could tell you that my experience was an anomaly. Over the years I have mentored a host of underrepresented minority students and listened to their experiences. They range from regular racial micro-aggression, through “oppressive othering,” to more overt examples, like being the only one not invited to a bus outing organized by fellow math graduate students. A sense of belonging “involves one’s personal belief that one is an accepted member of an academic community whose presence and contributions are valued” [3, p. 701]. This is important not only for the mathematics community but also for education and our society at large. At the convocation for Duke’s entering 2017 undergraduate class, Stephen Nowicki emphasized to our students:

We only learn best from each other and teach each other well if we all feel like we belong. We can only achieve the excellence that lies in the potential of the different people and perspectives, the different aspirations and ideas we’ve brought together at Duke, if everyone feels equally that Duke belongs to them.

There’s another important thing to understand about what it means to belong, which is that “belonging” does not mean “conforming.” ... The excellence of this place emerges from the very different kinds of people who join our community. To diminish those differences through conformity would only diminish our excellence.

If we truly believe that diversity in all its dimensions is a key driver of excellence in our educational institutions

another  
key part of  
belonging...  
the  
opportunity to  
add value

and increases the probability of intellectual breakthroughs, then we cannot ignore the implicit biases directed toward underrepresented minorities and women. Actionable first steps a department can take as part of fostering a welcoming culture are to assign *thoughtfully chosen* mentors to incoming students and faculty; to advocate inclusion, acceptance, and understanding; and to promote effective ways to engage diversity. Imagine for a moment that you are a newcomer. Having someone in your department teach you the ropes and advise you from their own experiences is part of

an onboarding that tells you from the beginning that you matter. Usually it is through such a relationship that your trust in the environment grows. By trust, I mean that you can allow yourself to be intellectually vulnerable without fear that your admission of the need for help or clarity will be attached to your race, ethnicity, gender identity, or social-class history. For example, you can feel secure enough to admit that you have certain gaps in your math background and allow the mentor to assist you with filling them. And you can ask faculty and seminar speakers questions about mathematical issues that are unclear to you.



A diverse group of students exuding a sense of belonging.

### Deficit Thinking: Can They Add Value?

Even when we accept diversity, it is still a problem when we approach it from a deficit-thinking perspective, that is, when we automatically expect lower achievement of students and even faculty from backgrounds that are not the norm. This brings me to an experience when I was inducted into Sigma Xi, the scientific research honor society. I felt so proud. Induction came with a really nice banquet. The guest speaker was the Queen of Carbon Science, Mildred Dresselhaus. At my table was an older Caucasian gentleman who seemed important through my student lens, at least as I inferred from his jacket and tie and the way he took control of our table. Going around the table, he started asking what research we were doing. I could not wait to tell him and the others about my work

in mathematical physics. He asked everyone at the table about their work, except me. He made me feel as if I did not deserve to be in the honor society. It felt as if he were a displeased gatekeeper whose “boundary maintenance” was upset by my being admitted into the society. I had the last laugh, though. After Mildred spoke, I went up to her to introduce myself. She was welcoming and the first thing she wanted to know was, “What are you working on?” Mildred valued me enough intellectually to want to hear about my research.

Belonging involves more than the experience of feeling connected, welcomed, and free to be oneself. As underscored by psychologist Isaac Prilleltensky, another key part of belonging is being given the opportunity to add value. Imagine again that you are a newcomer. Even though everyone in your department is nice to you, if they do not see you as being able to add research value, you will likely not feel a sense of belonging. A simple practical gesture to improve belonging for underrepresented minorities and women in your department would be to talk with them about the mathematical research issues they are currently tackling and perhaps invite them to give a talk.

## No Organization Is Perfect, but Keep Improving

I am sure that Sigma Xi would have been disappointed if they had learned about my induction experience. On August 28, 2017, the society issued its “Statement on Diversity and Recent Events in Charlottesville, Virginia.” It pointed out that in May 1993 its leadership acknowledged that “the rich diversity of contemporary society was not reflected in the composition of the scientific and engineering communities, or in its own membership” and pledged “the Society to set aside barriers encountered by individuals from underrepresented groups who seek to become scientists and engineers, as well as to reach out to all members of our diverse society.”

It is admirable to see the leaders of Sigma Xi acknowledge publicly that diversity within their organization is an important issue to address and then to act accordingly. In fact, some practical interventions on how to address belonging more broadly in one’s department, institution, or organization are given in the intriguing paper [4]. And a very insightful analysis in the college context is in [1], including surveys that probe for belongingness and loneliness.

## It Is about Humility, My Friend

There is a third aspect of belonging, which is important to unearth. It ties in with negative stereotypes about the value underrepresented minorities and women can contribute. To put it candidly, I am talking about views of math ability as innate versus acquirable through hard work, dedication, and practice. And I am not talking about the kind of nutty worldview of white supremacists that still echoes from the rally in Charlottesville, Virginia. I am talking about a subtler culture of how math ability is perceived.

In their revealing paper “Why do women opt out? Sense of belonging and women’s representation in mathematics” the authors state [3, p. 700]:

Sense of belonging to math—one’s feelings of membership and acceptance in the math domain—was established as a new and an important factor in the representation gap between males and females in math.... —the message that math ability is a fixed trait and the stereotype that women have less of this ability than men—worked together to erode women’s, but not men’s, sense of belonging in math. Their lowered sense of belonging, in turn, mediated women’s desire to pursue math in the future and their math grades. Interestingly, the message that math ability could be acquired protected women from negative stereotypes, allowing them to maintain a high sense of belonging in math and the intention to pursue math in the future.

A similar statement about the mathematics representation gap between underrepresented minorities and whites would not come as a surprise. They continued [3, p. 701]:

Females who, despite the stereotype, find themselves in math-related disciplines must now face the “culture of talent” pervading these fields, a culture that may also undermine their sense of belonging. The United States and perhaps Western societies in general often view math ability as a talent, something that one is either born with or not.... In fact, individuals may often console themselves about their mathematics short-comings by falling back on the expression, “I’m not a math person.” Perhaps nowhere is the belief in the fixed nature of math ability more entrenched than within the mathematics community itself, which relies on a ‘talent-driven approach to math.’ ... Research suggests that this mindset about the nature of intelligence as being a fixed trait (an “entity theory”) can undermine achievement in the face of difficulty....

I have personally observed situations and heard anecdotes that are consistent with the above mindset about mathematics. It is then not unexpected to see “deficit thinking” often communicated in the attitudes of some towards underrepresented minorities and women in the mathematical sciences. And it is no surprise to see some marginalized students with high academic achievement in mathematics pursue careers in other fields.

*Emphasize  
that  
mathematics  
can be  
mastered  
through  
effort.*





Students meeting for a tutorial with their instructor.

Every student has the capacity to succeed in mathematics. And, as with any challenging activity, we should purposefully emphasize that mathematics can be mastered through effort. Those who started delving into the subject from childhood, especially under the tutelage of a parent who is a mathematician, will naturally have an advantage in skill. But those who did not get such a head start can still fill their gaps and catch up on the fundamentals if they are deeply committed. Furthermore, the mathematical sciences have varied modes of reasoning, ranging from the algebraic to geometric, to analytic, to topological, to probabilistic, to statistical, to numerical, etc. This does not begin to touch on the interdisciplinary intuition employed in fields like mathematical physics, mathematical biology, mathematical finance, and so on. There is clearly room for a wide array of cognitive skills, and so some people will develop strengths in certain modes and intuitions over other ones. Equally important, we must also allow for the variations in learning styles that come with heterogeneity. Certain people will work best alone, others will excel with one or more co-authors, still others will flourish in large research collaborations. Such variety should be the spice of life in the mathematical sciences. Indeed, we must doggedly preserve this diversity and fight against any unhealthy mindset that promotes exclusion and a sense of superiority.

For many underrepresented minorities and women, the issue of belonging in mathematics has been a continued fundamental challenge. I believe that an integral part of keeping our field vibrant and relevant is for its participants to welcome everyone, knowing that anyone can get better at mathematics through an ample commitment of time and energy by teacher and student. Equally important, one should not only be welcoming at the door but also give people a chance to add value inside. Belonging is indeed a foundational human need, which when nurtured can bring out the best in all of us, enabling our community

**ACKNOWLEDGMENT.** I am indebted to Amir Aazami, Stanley Absher, John Blackshear, Kimberly Blackshear, Linda Burton, Patricia Hull, Elizabeth Petters, Molly Weeks, and Lee Willard for their invaluable feedback. And I especially would like to thank Molly and Linda for guiding me with the research literature on belonging and inequality.

to maximize its excellence. In the end, mathematicians are the custodians of mathematics. The onus is on us.

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### ABOUT THE AUTHOR

**Arlie O. Petters** is an applied mathematician and mathematical physicist at Duke University. His primary research deals with how gravity acts on light. He also has an interest in mathematical finance and co-authored a textbook on the subject. Petters enjoys movies and music and is currently writing a science fiction novel.



Arlie O. Petters

# The Mathematically Gifted and Black Website

*Erica Graham, Raegan Higgins, Candice Price, Shelby Wilson*

**BHM** Mathematically Gifted and Black (MGB)<sup>1</sup> is a website to celebrate the diversity of Black mathematicians, highlighting their contributions to the mathematical sciences and community. First came lathisms.org, a website celebrating Hispanic Heritage Month. Next came the Academy Award-nominated film, *Hidden Figures*, which focused on three Black female mathematicians and their experiences at NASA. With such inspiration, we created a website for Black History Month, February 2017. We reached out to twenty-four Black

*Celebrate  
the diversity  
of Black  
mathematicians.*

mathematicians who are well known for their contributions to research, education, industry, government, academia, and outreach. We also selected an additional four “rising stars” contributing to multiple facets of the mathematical community early in their careers. (See Figures 1–4.) The response was overwhelming, with the website shared on Twitter, Facebook<sup>2</sup>, classroom walls, office doors, and even *The New York Times*.

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<sup>1</sup>[www.mathematicallygiftedandblack.com](http://www.mathematicallygiftedandblack.com)

<sup>2</sup>See the Facebook group “The Network of Minorities in Mathematical Sciences.”

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In selecting the twenty-four Black mathematicians, we relied heavily on our personal networks and on websites such as “Mathematicians of the African Diaspora.” Those featured on the website must have a degree in mathematics and use it in their work. To generate a well-rounded profile group, we sought leaders in research, education, industry, government, academia, and outreach. The academy is represented by professors ranging from research universities to liberal arts institutions. The highest ranks in the federal government are represented as well as business owners, military leaders, and innovators of technology. Higgins noted:

We found it very difficult to find people in leadership roles in academia with math degrees. There are very few deans, provosts, and college or university presidents that fit our criteria. But this was not much of a shock to us. We recognize that there are many barriers that Black mathematicians are still breaking down. What we hope is that we have highlighted some of the future deans, provosts and presidents, those that will break down those barriers.



**Figure 1: Mohamed Omar, assistant professor at Harvey Mudd College, was featured on the website on February 4, 2017. Besides teaching his students, Omar spreads his love and knowledge of mathematics to all via [www.youtube.com/MohamedOmarMath](http://www.youtube.com/MohamedOmarMath).**



**Figure 2: Talea Mayo, assistant professor at the University of Central Florida, was featured on the website on February 11, 2017.**

**Mayo specializes in coastal ocean modeling, with a special interest in hurricane storm surge, work that definitely came in handy this hurricane season.**

A beautiful by-product of the website is showcasing the diversity of paths leading to a mathematical career. Some received degrees from Historically Black Colleges and Universities, while others had only attended predominately white institutions. Many felt that their paths were “nontraditional,” with diverse starting points and one or more hiatuses. Yet, consistent among these varied profiles was an underlying tone valuing good mentorship, provided by those who provided support along the way.

In one conversation about the 2017 profiles, Wilson shared:

One thing I noticed from many of these profiles is the idea that it wasn't a given that they would be good at mathematics, that it would be their career choice. For some, there wasn't this belief in their innate ability to do mathematics. They struggled and sought support, just as many of us do.

So what is next? While the website will continue to spotlight the contributions of a different Black mathematician every day in February, there is a plan to utilize the website in different ways during the rest of the year. We'd like to showcase programs like EDGE and SIAM WCD that support Black mathematicians and create strong research, social, and teaching networks. We'd also like to enhance the historical content on the website, emphasizing the contributions of Black mathematicians to research, education, teaching, outreach, government, and industry.

The power of the personal story is helping people better understand one another and shred stereotypes. The mathematicians spotlighted were able to tell their stories in their own words, to discuss their proudest moments, in mathematics and in life, and to include personal stories of struggle along with inspirational anecdotes. All were allowed to be themselves, unapologetically.



**Figure 3: Ryan Hynd, assistant professor at the University of Pennsylvania, was featured on the website on February 18, 2017. This photo was taken of Hynd while he was visiting MIT as Martin Luther King Visiting Professor.**



**Figure 4: Chelsea Walton, Selma Lee Bloch Brown Assistant Professor at Temple University, in transition to the University of Illinois-Urbana Champaign, was featured on the website February 25, 2017. Walton was awarded a Sloan Fellowship in 2017.**

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Photo of Shelby Wilson courtesy of Shelby Wilson.





**Erica Graham**

### ABOUT THE AUTHORS

**Erica Graham** does research in mathematical biology with applications to endocrinology. Current research includes reproductive hormone regulation and metabolic factors associated with ovulatory dysfunction.



**Raegan Higgins**

**Raegan Higgins's** research is in time scales; her interests focus on oscillation criteria for second-order dynamic equations. Currently, she is studying applications of time scales to biology. Raegan volunteers in Mentor Tech at TTU and serves as president of the Lubbock Alumnae Chapter of Delta Sigma Theta Sorority, Inc.



**Candice Price**

**Candice Price's** service mission statement is to create and contribute to programs that broaden the participation of underrepresented groups by focusing on strong mentoring and research networks.



**Shelby Wilson**

**Shelby Wilson's** research is in the area of mathematical biology and focuses on modeling with applications to medical sciences, immunology, and cancer. In her scant spare time, she loves to read, travel, and eat amazing food.

# The Math Alliance and Its Roots in the African American Community

David Goldberg and Phil Kutzko

**BHM**

The Math Alliance, a national community of math sciences faculty committed to increasing the number of doctoral degrees awarded

in those fields to underrepresented minority students, began in 1995 at the math department at the University of Iowa. In 2016 it moved to the College of Science at Purdue University. Math Alliance Executive Director David Goldberg and Math Alliance Director Phil Kutzko sat down to discuss the history of the Alliance and its relationship to the African American community.

EDITOR'S NOTE. In 2017 the Math Alliance won the AMS Award for Mathematics Programs That Make a Difference.<sup>1</sup> In 2014 Philip Kutzko received the AMS Award for Distinguished Public Service.<sup>2</sup>

**Goldberg:** When you and your colleagues began your initiative at the University of Iowa, I don't think many folks would have bet on its success. And yet, twenty-two years later, thirty-four minority students have received PhDs from your department, including nine African Americans, and the Alliance has exceeded all expectations. How can you explain this?

**Kutzko:** It is actually hard to explain. If you'd have told me back then that a largely white department at a predominantly white university in a definitively white state



would have this kind of success, I would have wondered if we were living in the same country. And yet, if we are going to institutionalize this success we must try to understand the factors that led to it.

One factor was the group of faculty at Iowa who started this initiative. We were about fifteen percent of the faculty, but we were all tenured, and most of us had good research credentials. We counted among our group our chair and our director of graduate studies. So those faculty in our department who were skeptical of what we were trying to do—and there were many—were inclined to let us give it a try. A second factor is that we all *mentored*. In fact, our department had always had a culture of mentoring, although I don't think we called it that or talked about it much before our initiative. These two factors, a critical mass of faculty and a culture of mentoring, formed the basis of our Alliance Graduate Program Groups.

**Goldberg:** Our department at Purdue was watching the developments at Iowa with great interest. There was a small group of Purdue math faculty who were interested in trying to change the demographics of the graduate student population and, more importantly, in trying to improve the experience for underrepresented minority students in our program. We adopted some of the mentoring structures from Iowa, and several faculty engaged in a program to recruit more thoroughly from minority-serving institutions.

**Kutzko:** How did it go?

**Goldberg:** The early results were mixed at best. Our department received its first GAANN grant in 2000 and was successful in attracting six underrepresented minority students to the program. Two of these students received PhDs, but the faculty mentors were not always successful in protecting them from painful experiences. Our colleagues at minority-serving institutions who had entrusted

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<sup>1</sup>See [www.ams.org/publications/journals/notices/201705/rnoti-p476.pdf](http://www.ams.org/publications/journals/notices/201705/rnoti-p476.pdf).

<sup>2</sup>See [www.ams.org/notices/201404/rnoti-p409.pdf](http://www.ams.org/notices/201404/rnoti-p409.pdf).

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*“The Alliance  
has exceeded  
all expectations.”*

happen with us. We had heard of so many departments who had begun to question whether the benefits of sending students to graduate schools at departments like ours was worth the risk.

**Goldberg:** How did you avoid this situation?

**Kutzko:** A lot of it was dumb luck! One of our original group, Herb Hethcote, was a colleague of Carlos Castillo-Chavez, and Carlos began to send us alumni of his MTBI program. Carlos was, even then, an important leader in the work we do, and he was very clear in his expectations of us. Most of our early PhDs were recommended to us by Carlos; eleven of our first twelve PhDs were Latinos.

**Goldberg:** It is interesting that, since that time, about thirty-five percent of your minority PhDs have been African American. Did you do something intentional to increase your numbers of African Americans?

**Kutzko:** Yes, we did, and this led to our original Alliance. The story is a good one. Sometime in 2001, Gene Madison<sup>3</sup> and I were at a social occasion in Iowa City and we were discussing, over a game of Bid Whist,<sup>4</sup> the fact that our minority grad students up to that point had been overwhelmingly Latino. It turned out that another player at our table had gone to Florida Agricultural and Mechanical University (FAMU) and had a sister who was friends with Roselyn Williams, the FAMU math department chair. Gene and I went down to FAMU about two weeks later. We agreed that we would form an Alliance—the *Alliance for the Production of African American PhDs in the Mathematical Sciences*, that Roselyn would recruit representatives of three other Historically Black Colleges and Universities (HBCUs), that Gene and I would reach out to the math sciences departments at the three Iowa State Regents universities, and that we would meet again. We returned to FAMU with seven or eight faculty in the math sciences at all three of the Iowa schools and met there with faculty from our new HBCU partners: Benedict College, Alabama A&M University, and Jackson State University, in addition to FAMU. We came up with a detailed plan and drafted a proposal to the Division of Math Sciences (DMS) at NSF.

their students to us were disappointed. Like many well-meaning departments, we found ourselves more part of the problem than the solution.

**Kutzko:** I always worried that that might

**Goldberg:** I am assuming you were funded.

**Kutzko:** Not at first! Our initial proposal was rejected due to some strongly negative comments about HBCUs on the part of one of the reviewers. However, we were very fortunate that Philippe Tondeur, professor of mathematics at the University of Illinois, was the director of DMS. Tondeur encouraged us to apply again and to make the case for HBCUs. We were funded on our second try.

**Goldberg:** If I am remembering right, both mathematics and statistics were represented in the meeting at FAMU.

**Kutzko:** Yes, that is right. In fact, the Alliance has served all of the mathematical sciences from its very beginning. Dean Isaacson, who was chair of the Iowa State University statistics department, was a leader in the Alliance until his retirement. Kathryn Chaloner (Figure 1), chair of biostatistics at the University of Iowa until her untimely death, headed our Alliance statistics initiative. The biostatistics department provided the Alliance with office space and staff support until its move to Purdue. Leslie McClure, chair of biostatistics at Drexel University, has taken over Kathryn's responsibilities; she is our associate director for statistics.



**Figure 1.** Kathryn Chaloner, founder of the Alliance Statistics Initiative, talks with Professor Joe Omojola.

**Goldberg:** How did the new Alliance go?

**Kutzko:** We had mixed success. Quite a few students participated in the new Alliance each year, but not all of them had the background to enter our doctoral programs. Several of them entered the master's program at the University of Northern Iowa, but some of these students chose to enter doctoral programs in non-math sciences fields or they dropped out altogether. There were some real successes though, and one of them, Reggie McGee, received his PhD at Purdue.

**Goldberg:** Reggie was a really important figure in the transformation of our department, and we were very fortunate that he chose to come to Purdue. I first met Reggie when he was in the Alliance summer program at Iowa, and I happened to be there working with you on a research paper. Then a couple of years later he visited Purdue through the HBI Visitation Program, and we were

<sup>3</sup>Eugene Madison, now retired, was professor of mathematics at the University of Iowa and among the first African Americans to hold such a post. He played a critical role in the UI math department's graduate minority initiative.

<sup>4</sup>For more on this card game popular among African Americans, see the NPR “...Story of Bid Whist” at [www.npr.org/templates/story/story.php?storyId=5069913](http://www.npr.org/templates/story/story.php?storyId=5069913).





**Figure 2.** Reggie McGee started a grad student Twin Day and posed with his office mates Sarah Percival and Kevin Rotz as triplets.

successful in recruiting him to our department. If I remember right, you called me to say we should take him. He forged a community for himself within the department right away, and was quite a cheerleader for people succeeding and working together, for instance on preparing for qualifying exams. He also connected with the wider campus community, and he was very active in the Purdue Black Graduate Student Association. He also became a graduate representative in our department; he represented the grad students to the grad chair (me), and he had a lot of good ideas about making our department a better place. He and his fellow grad reps had a whole series of events and competitions for the week before final exams. One day was Twin Day, where you had to find someone and dress up as twins. Reggie and his office mates actually went as triplets (Figure 2), and it was pretty hilarious. He also was very outspoken about certain conditions for the graduate students. This helped set a tone for how dialog between the faculty and grad students about our climate could move forward.

Reggie worked with Greg Buzzard (now head of our department) on mathematical biology. With Greg, Reggie really blossomed as a researcher. His thesis studied something about modeling certain signal processes between diseased cells. He finished in six years, and then took a postdoc position at MBI.

**Kutzko:** You were fortunate in the make-up of your department during your transition from a more traditional department to one of our strongest Doctoral Program Groups (DPGs).

**Goldberg:** Yes. Rodrigo Bañuelos and Johnny Brown had already been in our department for a long time, and both have been stalwarts in working for diversity in our department and across campus. They have been great role models and mentors not only to minority students but also to majority students and junior faculty. At one point Rodrigo was head and Johnny was associate head, and one external review commented that to their knowledge it was



the first time a Group I department had had underrepresented minority faculty in both of its top two leadership positions. We're also lucky that Greg Buzzard is now our head; he has been very proactive in diversity issues, and supportive of initiatives to improve the climate. He was a big part of making the idea that the Alliance could come to Purdue a reality. We had the challenging task of replacing Billie Townsend (Figure 3) as program manager and were lucky to hire Rebecca Lank. And, of course, neither the transition you asked about nor the Alliance's move to Purdue could have occurred without Edray Goins! Edray joined our faculty in 2004 and, besides making an immediate contribution to our research effort, went about doing a lot of important things which hadn't been done before, such as undergraduate research. In addition to his work at Purdue, he has taken on several national roles, including president of NAM. When the Alliance moved to Purdue, we were really fortunate that Edray agreed to be associate director. He provided new ideas, strengthened ties with the HBCU community, and played a crucial role



**Figure 3.** Billie Townsend (center), retired Alliance program manager, and some Alliance scholars at the 2016 Field of Dreams Conference.

in all developments and decisions. Edray has decided to step away from his work with the Alliance, and he'll be joining the faculty at Pomona College in the fall of 2018.

**Kutzko:** The Alliance has come a long way from its origins as a partnership between four HBCUs and the three Iowa Regents Universities. As our new executive director, how does it look to you now?

**Goldberg:** The Alliance has changed in certain fundamental ways since 2007, when the decision was made to transform the original partnership into a national community. And it changed again in 2016 when we evolved from an NSF-funded project to an independent organization,<sup>5</sup> an organization with the potential to transform the culture and the demographics of our math sciences professions. But what will never change will be our commitment to the African American community.

- We have strong ties with NAM and look forward to working with them in the coming years.
- We have built a strong tie with CAARMS and with its director, Bill Massey, and will work closely with Bill and with the CAARMS network as we continue to build regional alliances in the Northeast.
- We have close relationships with the faculty of several HBCUs including, of course, our original partners Jackson St. and FAMU, but also Southern University at New Orleans, Xavier University of Louisiana, Morehouse College, Spelman College, and Texas Southern University. These and other HBCUs are enrolling an increasing number of their Alliance Scholars each year into our Facilitated Graduate Admissions Process (F-GAP) program and, with the help of F-GAP, more and more of these African American Alliance Scholars are entering

*"Our numbers so far are good."*

graduate programs in the mathematical sciences. (See Figure 4.)

Our numbers so far are good. Over the last two years, twenty-one minority students who started out as Alliance Scholars earned a doctoral degree in a math science. Of these students, almost half (nine) were African Americans. And we have every reason for optimism when we look at our DPGs. In fact, over the past ten years, at least 80 African Americans have been awarded PhDs by these departments, about thirty-eight percent of all doctoral degrees awarded to African Americans in the math sciences during this time.

**Kutzko:** Our colleague, Leslie Hogben, over at Iowa State University has informed me that their math department will be awarding PhDs to four African American mathematicians in spring and summer, 2018, a fitting achievement for one of the founding departments of the

original Alliance.<sup>6</sup>

**Goldberg:** Any final thoughts?

**Kutzko:** I guess we both feel humbled by these accomplishments. I grew up at a time when schools were segregated and I can remember seeing the separate drinking fountains. I remember the deep sense of injustice my parents felt about this and how strongly they expressed it. And I remember your dad and mom and that they had devoted their lives to making the kind of change that we have had the honor to be a part of through the Alliance.

I never thought that, through this work, I would have the opportunity to meet some real heroes, like Bob Moses and our friend Donald Cole, who has played such a critical leadership role in our Alliance community. I never thought I would live to see Barack Obama become President of the United States, and I *certainly* never imagined that I would



**Figure 4.** The Alliance has close relationships with several HBCUs. Left: Texas Southern students, alums, and faculty at the 2014 Field of Dreams Conference: Valerie Tolbert, Marquesha Forman, Aqeeb Sabree, Joan Evans, Franchell Davidson, Roderick Holmes. Right: Professors Lester Jones (left) and Sindhu Unnithan (center) and the Xavier University of Louisiana contingent at the 2014 Field of Dreams Conference.

<sup>5</sup>Information about the history and governance of the Alliance as well as complete data may be found at [mathalliance.org/about-the-math-alliance/](http://mathalliance.org/about-the-math-alliance/).

<sup>6</sup>In 2015, the mathematics department at Iowa State won the AMS Award for an Exemplary Program or Achievement in a Mathematics Department; see [www.ams.org/notices/201505/rnoti-p544.pdf](http://www.ams.org/notices/201505/rnoti-p544.pdf).

get a chance to meet him. But, most of all, I never imagined that there were so many faculty in our professions who felt just as you and I do, who remember how things were and who will give their time and their heart to making sure that our professions provide a welcoming and supportive environment to anyone who feels the unlikely passion that we feel for what everybody else sees as hopelessly abstract.

The Alliance has broken new ground, but there were big steps in this direction before. Among others, we should never forget Raymond Johnson, whose accomplishments at the University of Maryland taught us what might be possible at our Iowa schools and at Purdue. But, again and again, a shift of emphasis in a department or a change of administration at a university or even a couple of retirements have led to a loss of momentum and the process must be begun again. So I would like to end with a plea: This time, let's make it permanent! Let us work together to institutionalize what the Alliance has accomplished in our colleges, and universities, and professional organizations.

EDITOR'S NOTE. Also see the September 2016 *Notices* interview with Goldberg and the December 2017 AMS inclusion/exclusion blog post on the Math Alliance at <http://bit.ly/2ABtLRI>.

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Figure 5 by Donald Cole.

Photo of Phil Kutzko and David Goldberg by Donald Cole.



**Phil Kutzko and David Goldberg**

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### ABOUT THE AUTHORS

**Phil Kutzko and David Goldberg** at the 2017 Field of Dreams Conference. Phil still dreams of beating David at handball, some day.



# Behind Every Successful Woman, There Are a Few Good Men

*Talitha M. Washington*

**BHM** I am fortunate that throughout my mathematical journey, I have had a few good men give me the courage, persistence, and drive to persist in mathematics. I share my story in the hopes that those needing courage receive it and those who can offer help give it. Society may not be just, but each of us can add goodness in all that we do.

As an undergraduate I went to Spelman College, a Black woman's college in Atlanta, Georgia. College was a huge class shock for me, because I grew up in the inner city of Evansville, Indiana, where many of my friends ended up going to jail or prison. At Spelman, I was immediately blessed with not being the only Black female in the class. For the first time in my educational experience, I did not have to be the "negro representative" who is called upon to explain and defend Black people. I did not have to explain my race or why I look this way. Rather, I could simply learn and absorb copious amounts of knowledge centered around my perspective as a Black woman in a multicultural world.

As an undergraduate, I had zero aspirations of going to graduate school. I wanted to work in the business world because that is what I thought I knew. Then along came Jeffrey Ehme (Figure 1), who took me on as a student researcher and in my senior year forced me to apply to graduate school. Had he not made me apply, I would not be here today as a mathematician, and you would not be reading this article.

Life in graduate school at the University of Connecticut was hard. People always asked me what country I was from. That was confusing to me. I later found out that I

*Talitha M. Washington is associate professor of mathematics at Howard University. Her email address is [talitha.washington@howard.edu](mailto:talitha.washington@howard.edu). See a write-up of her talk at the 2017 SIAM annual meeting Hidden Figures event [www.siam.org/meetings/an17/figures.pdf](http://www.siam.org/meetings/an17/figures.pdf).*

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**Figure 1. Jeffrey Ehme took me on as an undergraduate research student and forced me to apply to graduate school.**

McKenna researched. I was more interested in mathematical biology; so I completed my dissertation under Yung-Sze Choi on a partial differential equations model of proteins acting as on/off switches. In 2001, I became the first African American to graduate with a doctorate in mathematics from the University of Connecticut.

Much of my professional career has been guided by Ronald E. Mickens of Clark Atlanta University<sup>1</sup> (Figure 3). I knew him as an undergraduate because Clark Atlanta was literally right across the street from Spelman. I remember that when I was an undergraduate he gave me his book on difference equations, which tended to gather dust over the

was the first US Black to enter the program. Fortunately, I had Joe McKenna (Figure 2). I remember sitting in his office, devastated by the environment, workload, and life. He told me, "You outwork anyone here. You are good." These words encouraged me to work even harder and to see it all through. Fortunately, my Spelman training in proofs and analysis prepared me so well that my analysis preliminary exam was actually fun. At the time, I was not interested in modeling the collapse of the Tacoma Narrows Bridge, which is what

<sup>1</sup> See Mickens's article on "Nonstandard Finite Difference Schemes" in the January 2018 Notices.



**Figure 2.** In graduate school at the University of Connecticut, Joe McKenna told me, “You outwork anyone here. You are good.”

years. Little did I know that a decade later we would do research together. Over the years we have published papers on the construction of nonstandard finite difference schemes. These schemes discretize dynamical systems and maintain dynamical consistency by incorporating features of the dynamical systems into the discretizations. We now talk a couple of times a week about math, life, and what I should be doing and how I should do it. My graduate student at Howard University, Seye Adekanye, chose to work on the development of nonstandard finite difference schemes of the Tacoma Narrows Bridge for his dissertation. For me his work is a beautiful union of both McKenna and Mickens, two men who continue to inspire my mathematics.

At Howard our departmental meetings are held in a room with that grand picture of Elbert Frank Cox (Figure 4). In 1925 Cox became the first Black in the world to earn a doctorate in mathematics. I did not learn about Cox until I was grown, even though we grew up in the same

*“You outwork anyone here. You are good.”*

neighborhood in Evansville, Indiana. Cox was hidden to me. He spent most of his professional career at Howard, as do I. When I sit in departmental meetings, I tend to space out and catch myself staring at his photo and he stares back, encouraging me to continue the pursuit of mathematics and justice.

I’m thinking back to Katherine Johnson, who was featured in the Hollywood movie *Hidden Figures*, surrounded by white males at NASA. Every time they asked her to do something, she outworked them and did more. She was able to do this because of her mathematical research training at West Virginia State University, a Historically Black University. When she entered West Virginia, so did her professor William Claytor, who is also a Howard graduate. She once asked Claytor, “If I got into math, will

anyone hire me?” Claytor replied, “You’ll have to see.... That is your problem.” Claytor knew that she should pursue mathematics. After graduating, the president of her alma mater, John Davis, a Morehouse man, handpicked her to integrate West Virginia public schools in a graduate program, for he also knew she should pursue mathematics. She enrolled in a summer session, but her training in math was so good that no course could duplicate what she already knew from Claytor.



**Figure 3.** Much of my professional career has been guided by Ronald E. Mickens of Clark Atlanta University.



**Figure 4.** At Howard University, I stand by the grand photo of Elbert Frank Cox, the first Black in the world to earn a doctorate in mathematics.

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*so that our children  
can see that they too can  
become mathematicians*

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This past summer while driving to the SIAM meeting in Pittsburgh, Pennsylvania, I passed through West Virginia. I happened to glance over and see a road sign that read “West Virginia State University.” Sitting right beside me was my teenage daughter, snoring away, wearing her *Hidden Figures* T-shirt. As a mathematician, mother, and activist, I hope that we all remain unhidden so that our children can see that they too can become mathematicians. I am deeply thankful for those few good men who helped me along my way so that I could become an unhidden research mathematician.

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Figure 2 courtesy of the University of Connecticut.

Figures 3 and 4 courtesy of Talitha Washington.

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**Talitha M.  
Washington**

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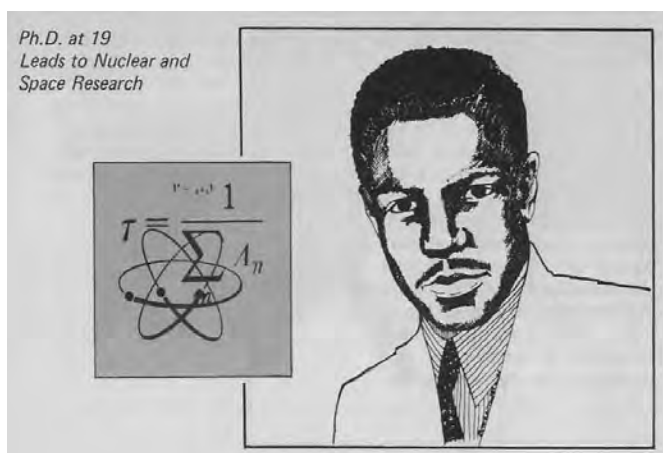
#### ABOUT THE AUTHOR

**Talitha M. Washington** is interested in applications of differential equations to problems in biology and engineering, as well as the development of nonstandard finite difference schemes to numerically solve dynamical systems. Her three teenagers are STEM bound. In her spare time, she enjoys kickboxing, weight- lifting, aerobics, running, and yoga.



# Episodes in the Life of a Genius: J. Ernest Wilkins Jr.

*Asamoah Nkwanta and Janet E. Barber*



**ABSTRACT.** J. Ernest Wilkins Jr. had an IQ of 163 by age nine, earned a college degree at age sixteen, and received his doctorate in mathematics at age nineteen from the University of Chicago. This paper highlights his noteworthy contributions as a mathematician, physicist, engineer, and educator.

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## BHM

### Introduction

By the age of nineteen, J. Ernest Wilkins Jr. had earned a doctorate in mathematics from the University of Chicago. Before being conferred with his PhD, young Wilkins applied to the Institute for Advanced Study (IAS) in Princeton and was invited to become a member shortly thereafter. Over his career, this child prodigy and scientist made notable contributions to mathematics, physics, engineering, and education.

### Child Prodigy with a Big Bang IQ

“One day, I’ll fly to the moon with math.” —Wilkins

In the 1930s Martin D. Jenkins [3], an educational psychologist (and president of Morgan State College, 1948–1970), conducted a study on highly intelligent African American children. Wilkins was classified a genius at age ten with an IQ of 163, surpassing the genius level of 150. Wilkins modestly informed Nkwanta that he was surely case no. 2 (see Table 1) and that he always wondered what happened to case no. 1, the young African American female genius with the IQ of 200.

Jenkins [3, p. 165] wrote:

Finally, these cases bring into sharp focus the limitations which our society places on the development of the highly gifted Negro. These children are nurtured in a culture in which racial inferiority of the Negro is a basic assumption. Consequently, they will experience throughout their lives, educational, social and occupational restrictions which inevitably affect achievement and motivation. ... Some of these individuals will meet frustration and draw away; others will go on to careers of high usefulness and accomplishment.

Though Wilkins was brilliant to genius level and many of his colleagues and mentors wrote letters of recommendation after his tenure with the Institute for Advanced

Case no.	Location	Age	S. Binet IQ	EQ (educ)	Grade	Race
1 Unknown African American (F)	Chicago	9–5	200	158	5 <sup>th</sup>	N
2 J. Ernest Wilkins, Jr. (M)	Chicago	10–6	163	169	8 <sup>th</sup>	NWW
3 Unknown African American (M)	Cincinnati	6–9	173	158	6 <sup>th</sup>	N
4 Unknown African American (M)	Washington	8–5	163	169	6 <sup>th</sup>	NNW

**Table 1. Wilkins's genius IQ of 163 appears in this edited chart entitled "Characteristics of 16 Negro Children of Binet IQ 150 and Above" from Martin D. Jenkins [3]. Under "Race," N denotes Negro, NWW denotes Negro with the possibility of white ancestry, and NNW denotes more Negro ancestry than white ancestry.**

Study, research universities were unwilling to employ an African American. Therefore, most of Wilkins's active scientific career was with industry, the government, and academics working with Historically Black Colleges and

Universities (HBCUs), such as Tuskegee, Howard, and Clark Atlanta.

*He  
persevered.*

As noted, Wilkins did experience his unfair share of racism and discrimination; however, he persevered. At

age twenty, he worked on national projects, training and teaching pre-flight mathematics to hundreds of Tuskegee Airmen in the Army Air Force College Training Program. Between 1941 and 1945 Tuskegee Institute trained over 1,000 African American aviators for the war effort. As his career progressed, he received numerous honors for his work.

### Early Life of a Genius

Wilkins's mother, Lucille B. Wilkins, was an educator who studied mathematics, and his father, J. Ernest Wilkins Sr., was an accomplished attorney, who graduated Phi Beta Kappa with a bachelor's degree in mathematics from the University of Illinois. Wilkins Sr. would eventually work for the White House during the Eisenhower administration. He was the first African American to participate in White House cabinet meetings. He served as assistant secretary of labor in 1954 and as a member of the Civil Rights Commission in 1958.

Wilkins Jr. was a child prodigy. At only a year old, he was able to recite the alphabet, and by the age of five he was able to add, subtract, multiply, and divide. He began school at the age of four. Coached at home by educated parents, Wilkins and his brothers were challenged with intellectual puzzles and math games. A high school teacher who recognized his talents and encouraged his mathematical genius enabled him to graduate from Parker High School in three years at age thirteen.

Wilkins gained fame upon entering the University of Chicago at age thirteen and becoming its youngest student. He participated in the William Lowell Putnam Mathematics Competition. He graduated Phi Beta Kappa

at age sixteen in 1940. He completed his doctoral degree at age nineteen in 1942 with a dissertation titled "Multiple Integral Problems in Parametric Form in the Calculus of Variations," published in *Annals of Mathematics* (1944).

Prior to graduation, Wilkins penned a letter to the mathematician Oswald Veblen of the Institute for Advanced Study in Princeton, requesting permission to enroll there. Wilkins's Chicago dissertation advisor, Magnus Hestenes, followed up with a letter of support. In the letter, though he makes sure to inform Veblen that Wilkins is a good fellow and looks white, Hestenes also writes that "Mr. Wilkins is undoubtedly one of the best, if not the best, student [sic] we have had for a number of years." Wilkins received an offer and attended for three months, between 1942 and 1943. In a reply to Hestenes, Veblen indicated that Wilkins would be the second person of color to attend the Institute: "This year we have had another very good mathematician who came from the University of Illinois."

The first African American member of the IAS was David H. Blackwell, who is the seventh African American to earn a PhD in mathematics, Wilkins the eighth. Veblen suggested that Wilkins write Blackwell

to get suggestions for living arrangements in the Princeton area. Blackwell and Wilkins were contemporaries and attended many of the same mathematical conferences, meetings, and events in their future careers (see Figure 1).

While at the IAS, Wilkins was potentially exposed to the courses, works, and lectures of many internationally recognized mathematicians and physicists, such as Marston Morse, Hermann Weyl, Kurt Gödel, Wolfgang Pauli, Carl Ludwig Siegel, and Albert Einstein. Wilkins and Einstein were housed in the same office building. Wilkins wrote a letter, dated April 13, 1943, to Weyl indicating that he had taken courses in meromorphic functions by Weyl and algebraic geometry by Claude Chevalley. Wilkins wrote that he attended Solomon Lefschetz's seminar on non-linear differential equations and presented a report on a paper,

*"undoubtedly  
one of the best,  
if not the best"*



**Figure 1.** Abdulalim A. Shabazz, David H. Blackwell (the first African American member of the IAS), and Wilkins (the second), at the first CAARMS conference at MSRI in 1995.

“A General Equation for Relaxation Oscillations” by Norman Levison and Oliver K. Smith.

Wilkins later apparently requested a recommendation letter from one of his mathematics professors from Princeton University, IAS affiliate professor Lefschetz, when he was seeking an opportunity with the Navy Department. Lefschetz’s letter, dated November 1, 1943, states that he had “formed the highest opinion of his capacity as a mathematician. He is exceptionally alert and hard working and I am certain would give you entire satisfaction in every way. I strongly recommend him.”

Earlier 1943 letters disclosed that there was back-and-forth correspondence between Wilkins and Morse. In one letter, Morse responded to Wilkins about information on local draft board forms and his position at Tuskegee Institute, as well as about work and education.

### **The Teaching and Working Life of the Genius: Achievements, Honors, and Scientific Service**

Wilkins was a mathematics instructor at the Tuskegee Institute from 1943 to 1944, with work overlapping with industry. In 1944 he returned to the University of Chicago as associate mathematical physicist and as a physicist in its Metallurgical Laboratory, as part of the Manhattan Project, and discovered the Wilkins effect. He collaborated

with the Nobel Prize-winning physicist Eugene Wigner on the Wigner-Wilkins Spectra, estimating the distribution of neutron energies within nuclear reactors.

Starting in 1950, Wilkins worked as a mathematician for the Nuclear Development Corporation of America. In 1956 he was elected Fellow of the American Association for the Advancement of Science and in 1964 Fellow of the American Nuclear Society. He was a visiting lecturer of the Mathematical Association of America from 1963 to 1967. During his remarkable career, Wilkins published many research papers and contributed to and solved many MAA *Monthly* problems.

In 1970 Wilkins was invited by Warren Henry, a Tuskegee Institute and University of Chicago colleague, now a physics professor at Howard University, to come as a distinguished professor of applied mathematical physics. Wilkins accepted, helped establish the doctoral degree program in mathematics, and supervised four doctoral dissertations in physics.

Wilkins was president of the American Nuclear Society from 1974 to 1975. Wilkins became the second African American to be elected to the National Academy of Engineering in 1976.

Wilkins was awarded an achievement medal in 1980 from the US Army for outstanding civilian services, the Quality Engineering for Minorities science award in 1994, and the National Association of Mathematicians (NAM) Lifetime Achievement Award also in 1994. The NAM inscription states:

*In recognition of more than fifty professional years as a world class mathematician, physicist and engineer, gifted teacher and productive scholar it is NAM’s distinct pleasure to name in your honor,*

#### **THE J. ERNEST WILKINS JR. LECTURE**

*To be given annually at the NAM undergraduate MATHFest conference and “Real Zeros of Random Polynomials,” to be the inaugural J. Ernest Wilkins Jr. lecture.*



**Figure 2.** Warren E. Henry, Wilkins, and Henry McBay at the 1996 American Association for the Advancement of Science Meeting, Atlanta, GA.





**Figure 3.** At “The Life and Legacy of J. Ernest Wilkins (1923–2011),” 2017 MathFest. Front row: niece Carolyn M. Wilkins, daughter Sharon Wilkins Hill, organizers Ronald E. Mickens and Talitha Washington. Back row: Nkwanta, organizer Ron Buckmire, former PhD student Cleo Bentley, Robert Fefferman.

Figure 2 shows Wilkins with two other icons in STEM, who were friends and contemporaries: Warren Henry and Henry McBay.

In 1996, Wilkins received the special recognition award from the US Department of Energy. A year later in 1997, he received a Professional Achievement Citation from the University of Chicago Alumni Association. In 2007 he was honored yet again at the University of Chicago in a special ceremony dedicating his portrait and plaque for display in the Eckhart Hall Tea Room.

The 2017 MAA MathFest held a special session on “The Life and Legacy of J. Ernest Wilkins (1923–2011)” (Figure 3). This session shared his impact in nuclear-reactor physics and optics, his plight of being a genius of color, and his admirable influence on the mathematical community.



**Figure 4.** Wilkins with Nkwanta at CAARMS 10 at Lawrence Berkeley National Lab, California, 2004.

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Photo of Janet E. Barber courtesy of Janet Barber.

## ABOUT THE AUTHORS

**Asamoah Nkwanta** encountered Wilkins while an undergraduate at North Carolina Central University in the book *Black Mathematicians and Their Works*, while a graduate student at Howard University at the Institute on the History of Mathematics and its Use in Teaching workshop, and as a professor and research mathematician at several CAARMS conferences.



**Asamoah Nkwanta**

**Janet E. Barber** is a behavioral scientist, sociologist, editor, and writer, who collaborates and works with Nkwanta. Their many research collaborations began at North Carolina Central and Morgan State Universities. Her research and writing involve the study of positive psychology, critical thinking, cultural diversity, and leadership.



**Janet E. Barber**

# Ten African American Pioneers and Mathematicians Who Inspired Me

by Johnny L. Houston

**BHM** This article presents snapshots of the lives of ten African American pioneers and mathematicians who have inspired me in research, teaching, and life. The first five were amazing pioneers, and the second five were extraordinary researchers. At the end of the article are the names of five great African American mathematics teachers. In college I first learned of two of the teachers. In graduate school I learned of a few others. As my career developed, I learned of all of these persons and many more. They inspired me to be the best that I could be in the mathematical sciences community and

*inspired me  
in research,  
teaching, and life*

three degrees in mathematics: A BA from Morehouse College, a MS from Clark Atlanta University, and a PhD from Purdue University. When I entered college I had never traveled outside the state of Georgia. By age 60, I had been to all 50 states and 60 countries on six continents and had produced over twenty publications. I want to share with young mathematicians the inspiration from earlier workers that enriched my life.

## **BENJAMIN BANNEKER (1731–1806) Self-educated**

Benjamin Banneker was born a free Black man and was raised on a farm near Baltimore, MD. Although he period-

ically attended a one-room Quaker schoolhouse, Banneker was largely self-educated and did much of his learning through the voracious reading of borrowed books. He created puzzles for trigonometry that demonstrated his knowledge of logarithms. Banneker also attempted to find the exact lengths of an equilateral triangle inscribed within a circle where the diameter of the circle is known. While still a young man, he built a wooden clock that kept precise time until his death. As early as 1788, Banneker began to make astronomical calculations, and he accurately predicted a solar eclipse that occurred in 1789. In 1791, Banneker helped survey the land that would become Washington, DC. Although a fire at his house on the day of Banneker's funeral destroyed many of his papers and belongings, one of his journals and several of his artifacts were saved and are presently available for public viewing. Many parks, schools, streets, and other entities have been named in his honor after his death. In recent years, a US postage stamp was printed in his honor. Currently the Benjamin Banneker Association, of which I was a member, promotes mathematical development at the high school level.



**Benjamin Banneker: astronomer, mathematician, inventor, surveyor, publisher of almanacs, and author; one of the first recognized multi-talented African American intellectuals in the United States, often considered the first African American mathematician.**

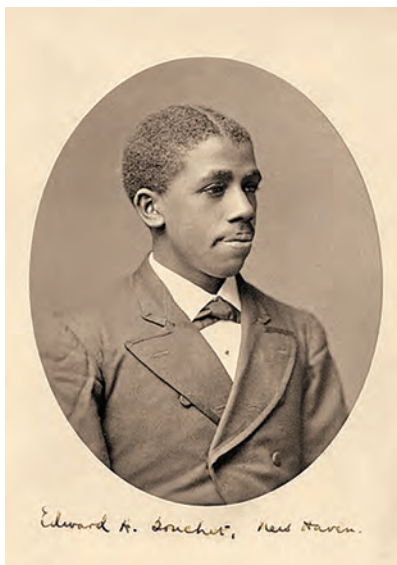
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**EDWARD  
ALEXANDER  
BOUCHET (1852–  
1918) Yale U. BS  
1874, PhD 1876**

Edward Alexander Bouchet was an American physicist and educator. He received a PhD from Yale in 1876, the first by an African American from any American university. Bouchet was unable to find a university teaching position after receiving his PhD because of his race. He moved to Philadelphia in 1876 and took a position at Philadelphia's Institute for Colored Youth (now Cheyney University of Pennsylvania), where he taught science and mathematics for the next twenty-six years. He resigned in 1902, spending the next fourteen years in a variety of jobs. In 1916, illness forced him to move back to his childhood home (New Haven, Connecticut), where he died in 1918. Currently, the American Physical Society confers the Edward A. Bouchet Award on some of the nation's outstanding physicists. Yale University and Howard University founded the Edward A. Bouchet Graduate Honor Society in his name. Yale also gives a Bouchet Leadership Award to help advance diversity in higher education. In 1988 the Edward Bouchet Abdus Salam Institute (EBASI) was created by Abdus Salam, a Nobel Laureate, in Trieste, Italy. One objective of EBASI is to provide for synergistic scientific and technical collaborations between African and American physical scientists. I presented at two EBASI conferences in Africa.



**Edward Alexander Bouchet: The first African American to earn a PhD degree from an American university.**



**Elbert Frank Cox: The first Black person to receive a PhD in mathematics.**

**ELBERT FRANK  
COX (1895–1969)  
Indiana U. BS  
1917, Cornell U.  
PhD 1925**

Elbert Frank Cox in 1925 became the first African American to earn a PhD in mathematics in the United States, as well as the first known Black person to earn a PhD in mathematics in the world. Cox's dissertation advisor, William Lloyd Garrison, urged his student to send his dissertation to a university in another country so that Cox's status would not be disputed. Universities in England and

Germany refused to accept Cox's dissertation (probably because of race), but Japan's Imperial University of San Dei accepted it. Cox inspired many future Black mathematicians and served a 40-year-long teaching career at Howard University and West Virginia State College. The National Association of Mathematicians Cox-Talbot Address is delivered annually at the Joint Mathematics Meetings in his honor, and the Elbert F. Cox Scholarship Fund at Howard University is used to help Black students achieve higher educational goals.

**WALTER  
RICHARD TALBOT  
(1907–1977) U.  
of Pittsburgh BS  
1931, MA 1933,  
PhD 1934**

Walter Richard Talbot accepted an assistant professorship in mathematics in 1934 at Lincoln University in Missouri, moving through the ranks to full professor while also serving as department chair (1940–1963), dean of men (1939–1944), registrar (1946–48), and acting dean of instruction (1955–1957). In 1963 Talbot accepted the positions of professor and department chair of mathematics at Morgan State University, remaining there until he retired in 1977. Talbot



**Walter Richard Talbot: fourth African American to receive a PhD in math, a founder of the National Association of Mathematicians.**



served the MAA in several significant capacities, and he was a major driving force among the seventeen who came together in January 1969 to found the National Association of Mathematicians (NAM). More than any other single individual, it was Talbot's leadership, guidance, organizational skills, and networking skills that raised funds for the establishment of NAM. In 1978, at a luncheon "In Memorial," Morgan State University named a scholarship in his honor. In 1980, NAM honored Talbot with the inauguration of the annual Cox-Talbot Address.



**Euphemia L. Haynes: The first African American woman to earn a PhD in mathematics.**

**EUPHEMIA L. HAYNES<sup>2</sup> (1890–1980) Smith C. BA 1914, U. Chicago MA 1930, Catholic U. PhD 1943**

Martha Euphemia Lofton Haynes was the first Black American woman to earn a PhD in mathematics. Haynes spent all of her professional life in Washington, DC. She was the first woman to chair the DC School Board. Haynes also served as math chair at Dunbar High School and at DC's Teachers College. At Miner Teachers College she established the math department, and at

Howard University she taught part-time as an adjunct professor. She was a prominent figure in bringing about integration in the DC public school system and in the Archdiocesan Council of Catholic Women. Haynes received many awards, including election as Fellow in the American Association for the Advancement of Science in 1962. NAM honored her and two other Black women mathematicians by establishing the Haynes–Granville–Brown Colloquium Lecture Series that is given annually at NAM's national meeting at the JMM. Her highest awards resulted from her humanitarianism and generosity. Upon her death, US\$700,000 was bequeathed to Catholic University in a trust for a chair and student loan fund in her name.

**DUDLEY WELDON WOODARD (1881–1965) Wilberforce U AB 1903, U. Chicago BS 1906, MS 1906, U. Penn PhD 1928**

Dudley Weldon Woodard managed to get his master's thesis and other research papers published in repu-



**Dudley Weldon Woodard: The second African American to earn a PhD in mathematics, established the mathematics MS degree program at Howard University in 1929.**

table mathematics journals. He joined the Howard University faculty in 1920 and retired in 1947. Including positions at Tuskegee and Wilberforce, Woodard taught college-level mathematics for 40 years. In addition to serving as dean of the College of Arts and Science from 1920 to 1929, Woodard established a MS graduate program in mathematics in 1929 and enhanced it by establishing a mathematics library and sponsoring seminars. Woodard advanced the mathematics faculty steadily over a quarter century, including recruiting Elbert Cox. Woodard devoted his entire professional life to the promotion of excellence in mathematics through the advancement of his students, in teaching, and in research. Woodard's students who also earned the PhD degree in mathematics included W. W. S. Claytor, George Butcher, Majorie Lee Brown, and Eleanor Jones. NAM honored Woodard with the naming of the Claytor-Woodard Lecture, given annually at NAM's national meeting at the Joint Mathematics Meetings.

**WILLIAM W. CLAYTOR (1908–1967) Howard U. BS 1929, MS 1930, U. Penn. PhD 1933**

William Waldron Schieffelin Claytor was Dudley Woodard's most promising student at Howard. Thus it was not surprising that William Claytor was recommended for further studies at the University of Pennsylvania. Claytor quickly earned himself an outstanding reputation at U. Penn., where he won the Harrison Fellowship in Mathematics—the



**William W. Claytor: The third African American to earn a PhD in mathematics and a distinguished topologist.**

<sup>2</sup>See the article on Haynes in the October 2017 Notices [www.ams.org/publications/journals/notices/201709/rnoti-p995.pdf](http://www.ams.org/publications/journals/notices/201709/rnoti-p995.pdf).

most prestigious award U. Penn offers. His dissertation under J. R. Kline was also well received by the faculty. After the completion of his PhD, Claytor accepted a teaching position at West Virginia State College, where he remained for three years. In 1937 he was awarded a Rosenwald Fellowship and pursued postdoctoral studies at the University of Michigan, where R. L. Wilder had attracted an able and experienced group of topologists. Kline had written Wilder about Claytor's abilities, and they both were pleased that Claytor was able to join the group at Michigan, which included S. Eilenberg, W. L. Ayers, E. W. Miller, and S. B. Myers. Claytor's published mathematical work, on imbeddability and Peano continua, attracted considerable attention throughout the topological community and still does today. At the end of his postdoctoral studies at Michigan, Claytor was recommended by his colleagues in topology for a faculty position at Michigan. The administration took a firm stand that no such appointment would be made, as did other major research institutions. After this experience Claytor apparently lost interest in doing research at a major university. In 1947 Blackwell invited Claytor to Howard, where he served well until he retired in 1965. In 1980 NAM named the Claytor-Woodard Lecture in his honor.

**DAVID BLACKWELL (1919–2010) U. Illinois AB 1938, MA 1939, PhD 1941**



**David Blackwell: first Black professor at UC Berkeley, first Black American inducted into the National Academy of Sciences (1965).**

David Harold Blackwell took an elementary analysis course as a junior in college and fell in love with mathematics. After earning his PhD in 1941, he held a one-year postdoctoral fellowship at the Institute for Advanced Study in Princeton. Colleagues in Princeton wished to extend Blackwell's appointment at the Institute; however, the president of Princeton organized a great protest. Blackwell then knew that a white university would not hire him. He applied to all 105 Black Colleges in the country. Blackwell joined the faculty at Howard in 1944 as an instructor. By 1947, he was professor and department chair. Blackwell searched for mathematics around Washington and met M.

A. Girschick of the Department of Agriculture, and they became collaborators in many works. Their 1954 book, *Theory of Games and Statistical Decisions*, is now a classic. By 1954 Blackwell had published more than twenty papers. He had spent a couple of summers at the RAND Corporation and was visiting professor of statistics at Stanford University from 1950 to 1951. In 1954 he gave an invited address in probability at the International Congress of Mathematicians in Amsterdam. Immediately afterward he was appointed professor of statistics at UC Berkeley. He served for many years as chair of the statistics department before retiring. He produced numerous publications at UC Berkeley and he received many honors and recognitions, including a number of honorary doctorate degrees. He was elected as the first African American to the National Academy of Sciences. David Blackwell is considered to be a world-class African American mathematician, mechanical engineer, and nuclear scientist. NAM named the summer Blackwell Lecture at the MAA MathFest in his honor. In 2002 the Mathematical Sciences Research Institute in Berkeley and Cornell University established the Blackwell-Tapia Award in honor of Blackwell and Richard A. Tapia, two distinguished mathematical scientists who have been inspirations to more than a generation of African American and Hispanic American students and professionals.

**J. ERNEST WILKINS (1923–2011) U. Chicago BS 1940, MS 1941, PhD 1942**

Jesse Ernest Wilkins, initially frequently referred to by the media as the "Negro genius," was a world-class American scientist—mathematician, mechanical engineer, and nuclear scientist. After earning three degrees in mathematics at U. Chicago, Wilkins earned degrees in mechanical engineering from NYU. Wilkins worked as a contributor to the Manhattan Project during World War II, served several important scientific posts with distinction over a half century, and aided in the recruitment of minority students to take mathematics and science courses. Wilkins wrote almost 100 scientific papers (over 55 in mathematics), earning many awards and much recognition. He was the second African American elected to the National Academy of Engineering (1965). He later served as a faculty member at Howard, assisting the mathematics department in establishing in 1975 the first PhD program in mathematics



**J. Ernest Wilkins Jr.: earned PhD in mathematics at age nineteen.**



at a Historically Black College or University (HBCU). Coming out of one of his many retirements, Wilkins became Distinguished Professor of Mathematics and Physics at Clark Atlanta University in the late 1990s. In his honor, NAM established a J. Ernest Wilkins Lecture, which is given annually at NAM's fall Undergraduate MATHFest.

### ALBERT TURNER BHARUCHA-REID (1927–1985) Iowa St. BS 1949

Albert Turner Bharucha-Reid was an African American research mathematician and probability theorist who worked extensively on probability theory, Markov chains, and statistics. He produced more than 70 papers and six books; his work touched on such diverse fields as economics, physics, and biology. Bharucha-Reid was born Albert Turner Reid in Hampton, Virginia. He studied math and biology at Iowa State University, where he published his first paper. He was a student at the University of Chicago from 1950 to 1953. There he began to focus intensely on statistics and probability. He published eight papers while at Chicago, but he would not do a PhD dissertation because he felt it was a waste of time. In 1954, he married Rodab Phiroze Bharucha, and he legally changed his name to Albert Turner Bharucha-Reid. He taught and lectured in the United States, Europe, and India, holding professorships or research positions at Columbia, UC Berkeley, Oregon, Georgia Tech, Wayne St., the Polish Academy of Science, and Atlanta University (now Clark Atlanta University). In 1970 he was appointed dean of the School of Arts and Sciences at Wayne St., where he graduated his thirteen PhD students. Bharucha-Reid was the editor of about ten international scholarly journals. He bequeathed all of his scientific materials to the Amistad Research Center at Tulane. In May 1984, he was awarded an honorary doctorate of science by Syracuse University. The Citation at Syracuse Commencement stated "We honor you as a founder of Probabilistic Analysis. A mathematician of great distinction, you sensed the importance and glimpsed the possibilities of new developments in your field and then led the way to the applications of new knowledge." This Citation was rather descriptive of much of his life work as a scholar.



**Albert Turner Bharucha-Reid: a world-class African American mathematician who never earned a PhD degree, a great probability theorist who wrote over 70 papers and six books and supervised thirteen PhD students.**

NAM named a lectureship in his honor; it is given annually at NAM's Spring Faculty Research-Teaching Conference.

In closing, I invite you to also read about five great African American mathematics teachers:

1. Clarence F. Stephens
2. Marjorie Lee Browne
3. Claude B. Dansby
4. Abdulalim A. Shabazz
5. Etta Z. Falconer

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Photo of Albert Turner Bharucha-Reid courtesy of Ronald E. Mickens, Executor of the intellectual properties of the late Prof. Albert Turner Bharucha-Reid.

### ABOUT THE AUTHOR

**Johnny L. Houston** is cofounder of NAM, author of *The History of NAM* (2000) and the upcoming *American Mathematicians of Diverse Ethnicities* (2019), and a NAM Lifetime Achievement Awardee (1999).



**Johnny L. Houston**



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# Three Questions: The Journey of One Black Mathematician

*Edray Herber Goins*

**BHM**

AMC's *The Walking Dead* features a leader of a group who asks three questions whenever he meets strangers who might possibly join his group. Well, I am the president of the National Association of Mathematicians (NAM), a professional organization that seeks to increase the public awareness of issues of importance to underrepresented minorities in the mathematical sciences, and I would like to modify these questions and ask you the following:

How many African American mathematicians have you graduated?  
How many African American mathematicians have you hired?  
Why?

Permit me to take you on a journey of personal growth as I have asked these questions of myself while navigating the mathematical professoriate.

## My Undergraduate Days in Pasadena

*She's the terror of Colorado Boulevard | It's The Little Old Lady from Pasadena!*

—"The Little Old Lady from Pasadena," The Beach Boys

When I was an undergraduate at the California Institute of Technology, I spent a lot of time wondering about those who looked like me—about African Americans in mathematics, to be precise. I graduated from a predominantly African American high school in Los Angeles in 1990. Yet fifteen miles away in Pasadena there were only 14 Black students in Caltech's freshman class of nearly 180 students. In fact there were fewer than 50 Black students

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out of nearly 2,000 undergraduate and graduate students. And there were only 2 Black faculty out of nearly 300.

I wanted to know: Why were these numbers so low?

As an African American double-majoring in mathematics and physics, I wanted to know whether the numbers were just as low at other universities. In 1992, during my sophomore year, I found out that Stanford University would host the annual National Conference of Black Physics Students (NCBPS). I decided to attend the conference as the sole representative from Caltech, and I was glad that I did. I learned that I wasn't alone as an African American in mathematics and physics. I made friends at that meeting who remain friends to this day. And I met three African Americans at that conference who would become lifelong mentors: Ronald E. Mickens of Clark Atlanta University, Bill Massey of AT&T Bell Labs, and Sylvester James Gates of the University of Maryland.

But the meeting left me a little confused. There were many capable mathematicians and physicists at this conference—undergraduate students, graduate students, postdoctoral fellows, and faculty. So why were the numbers so small at Caltech?

I asked several faculty about what Caltech had done to recruit faculty over the years. My advisor in physics, Steve Frautschi, told me about the time they tried to hire a young African American physicist from MIT who was a postdoc at Caltech. Unfortunately things didn't work out for several reasons, although he was a pretty successful physicist at the University of Maryland. Frautschi thought it would be great if I met the guy one day. His name was Sylvester James Gates.

So the same person who founded the National Conference of Black Physics Students was almost a Caltech professor? Now I really wanted to learn more.

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*Why were these numbers so low?*

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Figure 1. Grant Venerable (BS 1932) was the first African American undergraduate at Caltech. James Lu Valle (PhD 1940) was the first African American graduate student. Charles McGruder (BS 1965) was the second African American undergraduate.

### My Brief Stint as a Historian

*The great force of history comes from the fact that we carry it within us, are unconsciously controlled by it in many ways, and history is literally present in all that we do.*

—James Baldwin

I decided to spend my senior year at Caltech studying the history of African Americans at Caltech. I wanted to know about the first African American undergraduate and graduate students. I wanted to learn more about their experiences, and whether the numbers were always so small.

I had some rather surprising revelations (see Figure 1).

- The first African American undergraduate at Caltech was Grant D. Venerable; he graduated in 1932 with a Bachelor of Science in Civil Engineering.
- The first African American graduate student at Caltech was James Lu Valle; he graduated in 1940 with a Doctorate in Chemistry. In fact, Lu Valle won a bronze medal in the 1936 Olympics in Berlin—yes, the same Olympics where that other African American, Jesse Owens, won a gold medal. The Graduate Student Union at UCLA is named after him, and Morehouse College recently established a fellowship in his honor using funds from Jim Simons.
- The second African American undergraduate at Caltech was Charles Hosewell McGruder; he graduated in 1965 with a Bachelor of Science in Astronomy. There were no other Black undergraduates after Venerable graduated in 1932 until McGruder enrolled in 1961. As fate would have it, I met McGruder at the National Conference of Black Physics Students in 1998, while I was a graduate student.

I also learned that the number of African American students was not always so small. Longtime Caltech employee Lee Browne spent years recruiting students from Los Angeles to attend Caltech. When affirmative action began in earnest in the early 1970s, Browne successfully recruited dozens of African Americans. In fact, the largest

*I finally felt  
connected  
to a larger  
community.*



Figure 2. Edray Goins and his Black History Month display at Caltech in 1994.

population of Black students was during the mid 1970s! One of those undergraduates was Robert Thornton, who was my mentor through my scholarship with the American

Physical Society (APS). Another was Loretta Carroll, a woman I had known while growing up in Los Angeles. Sadly, I didn't know until I conducted this research that either was a Caltech graduate. In Figure 2 you can see a picture of me next to a prominent display of some of these findings in the Department of Humanities and Social Sciences at Caltech.

With this knowledge, I finally felt connected to a larger community. The numbers of students and faculty were small at Caltech,

but there were many others out there with whom I had a historical connection.

### Time on the Farm

*We hailin from East Oakland, California and, um  
Sometimes it gets a little hectic out there  
But right now, yo, we gonna up you on how we just chill.*

—“'93 Til Infinity,” Souls of Mischief





**Figure 3.** Valerie Nelson, Acha Leke, Edray Goins, Angela Cole, and Duru Ahanotu at Black Graduation at Stanford University in 1999.



**Figure 4.** Edray Goins, Carl Graham, Earl Barnes, Mel Currie, Bill Massey, Ahmad Ridley, and Kathy Lewis at CAARMS in 2000 outside the home of Mel Currie.

The 1992 NCBPS meeting at Stanford convinced me that Stanford was where I should pursue my graduate studies. I was a doctoral student in the mathematics department, although I spent a lot of time in the physics department as well. Not surprisingly, the numbers of African American students and faculty at Stanford were very small. I was the only Black student in the mathematics department, and there were no Black professors.

I received a pleasant surprise in my second year at Stanford. The December 1995 issue of the *Notices* featured a conference for African American mathematicians that had just taken place in Berkeley a few months before. I had mixed feelings: There was a conference for African American Researchers in the Mathematical Sciences (CAARMS)?! How come I didn't know about it until after?!

In the midst of this confusion, I received a phone call one evening. "I'd like to introduce myself," the voice said,

"I am Bill Massey. I started CAARMS." Yes, this was the same person I had met some four years earlier at the NCBPS meeting! "You might be curious to know that I got my doctorate in mathematics from Stanford as well." Massey went on to say there was one other African American to get a doctorate in mathematics from Stanford, so as far as I know there are just three of us now. I may have been the only African American student in mathematics at the time, but I would not be the first African American to graduate from Stanford with a doctorate in mathematics. As proof that I did finish with this degree, Figure 3 shows me and some friends as we marched through Stanford's Black Graduation ceremony.

Once again, I realized that I was part of a larger community than what I was seeing in graduate school. Eventually I would attend many CAARMS conferences and make many more lifelong friends. Figure 4 shows several of us at a later CAARMS in 2000 outside the home of Mel Currie.

### Indiana Wants Me

*I'm goin' back to Indiana | Back to where my baby's from  
I'm goin' back to Indiana | Indiana here I come.*

—"Goin' Back to Indiana," The Jackson 5

By the time I accepted my first tenure-track position at Purdue University, I had nearly resigned myself to the idea that I would just be one of a handful of African Americans in my mathematics department. Still, part of me realized I had to do something to change this.

So I went out to recruit my own African American graduate students. My first graduate student was Kevin Muriithi Mugo, who graduated from Purdue in 2014 (see Figure 5). We met while working together during an NSF-funded REU at Miami University called the Summer Undergraduate Mathematical Sciences Research Institute. I was so impressed by the success Miami University had in recruiting women and underrepresented minorities for their REU that recently I started my own REU titled the Purdue Research in Mathematics Experience (PRiME).



**Figure 5.** Bill Heinzer, Kevin Muriithi Mugo, Sam Wagstaff, Tong Liu, and Edray Goins after Kevin's thesis defense in summer 2014.



## *I'm still looking for answers.*

But I began to wonder much as I had done while an undergraduate: What about the history of African Americans at Purdue? I wanted to know about the first African American graduate students in the mathematics department.

As I tried to ask around Purdue, I realized no one kept records of every African American to come through the math department. Fortunately, one of the first graduates is my mentor as president of NAM, and he had been collecting this information on his own for years! We both discovered that there have been seven African Americans to graduate with a doctorate in mathematics from Purdue:

- Melvin L. Heard, who is now associate professor of mathematics and assistant dean in the College of Liberal Arts at the University of Illinois at Chicago (UIC), finished in 1967 under Robert Arnold Gambill and Merritt S. Webster.
- Benjamin Joseph Martin, who is now head pastor at St. Rest Baptist Church, finished in 1969 under Thomas Wilson Mullikin.
- Johnny L. Houston, who is now emeritus professor of mathematics at Elizabeth City State University (ECSU), finished in 1974 under Eugene V. Schenkman.
- Kathy Marie Lewis, who is now associate professor of mathematics at Morehead State University, finished in 1999 under Carl Cowen.
- Sean Colbert-Kelley, who is now a scientist at the National Institute of Standards and Technology (NIST), finished in 2012 under Daniel Phillips.
- Kevin Muriithi Mugo, finished in 2014 under me.
- Reginald L. McGee, who is now a postdoctoral fellow at the Mathematical Biosciences Institute (MBI), finished in 2015 under Gregory T. Buzzard.

## **Semper Adquirens**

Now that I am a full professor at Purdue University, I spend a lot of time pondering the three questions that began this article. *How many African American mathematicians have you graduated?* My department has graduated seven with doctorates in mathematics, while I have graduated just one. *How many African American mathematicians have you hired?* Sadly none have been hired as tenure-track during my years at Purdue, although we did hire one as a postdoctoral fellow for three years. *Why?* I don't know exactly. I have tried to recruit many, many African American students and faculty to Purdue in the thirteen years I've been on the faculty. I'm still looking for answers.

I'm still amazed that, after all of these years, I look to Jim Gates, Bill Massey, and Ronald E. Mickens for advice and inspiration. As president of NAM, I was honored to have Jim Gates serve on our annual NAM Panel Discussion at the 2017 Joint Mathematics Meetings in Atlanta (see Figure 6). As a director of an NSF-funded REU, I was thrilled to have my own undergraduates meet Bill Massey at the 23rd annual CAARMS at the University of Michigan in 2017. And as a lover of mathematics, I am looking

forward to watching Ronald Mickens speak at the 2018 Joint Mathematics Meetings in San Diego as NAM's Claytor-Woodard Lecturer.

And yes, I'm still asking them the Three Questions.



**Figure 6. Cory Colbert, Jim Gates, Ebony Harvey, and Edray Goins at the 2015 Joint Mathematics Meetings in San Antonio, Texas.**

## **Photo Credits**

Figure 1, photo of Grant Venerable from the California Institute of Technology *Big T*, 1932.

Figure 1, photo of James Lu Valle from Stanford University, *Campus Report*, January 8, 1984.

Figure 1, photo of Charles McGruder from the California Institute of Technology *Big T*, 1965.

Figures 2–6, and the author photo from the personal collection of Edray Goins.

## **ABOUT THE AUTHOR**

**Edray Goins** has been working on properties of Selmer groups for elliptic curves using class groups of number fields, as well as properties of Dessins d'Enfants. He runs an REU that focuses on understanding properties of Belyi maps, and he maintains his own blog about Dessins d'Enfants.



**Edray Goins**

# Personal Conclusions

*Talithia Williams, Guest Editor*



Growing up in Columbus, Georgia, I never dreamed I would one day get a PhD in statistics. My AP Calculus teacher, in a casual conversation after class one day, nonchalantly mentioned that he thought I was good in math and should think about majoring in it. I didn't love math. I didn't love any subject. But my mind has forever attached itself to that memory, not because of the words he shared with me, but because it came from someone who looked so different from me. Mr. Dormon was a middle-aged white man, and my 17-year-old brain saw no reason why he chose to affirm me in that moment. He didn't have anything to gain. But his comment was powerful. *Major in math? When I go to college? Did he really think I could do mathematics?*

I didn't become aware of any African American women mathematicians until I went off to Spelman College, where several were on the faculty. Only later did it occur to me that this was an anomaly. There's a subtle, indescribable confidence that comes from being taught by a person who looks like you. At Spelman, I could be anything. A Sylvia Bozeman. An Etta Falconer. If I failed an exam, it wasn't because I was Black. It wasn't because I was a woman. In fact, from my vantage point, being a Black woman should make me excel in math, just like the faculty that taught me. I chose graduate school in mathematics at Howard University because I wanted that same supportive environment. Two statistics courses later, I found myself loving data.

I transferred to Rice University to finish a PhD in statistics and was the only woman and only African American in my class. The Asian, Indian, Russian, and white male students all had a group. I had...myself. Rarely is one successful in graduate school without a group, so I leaned on my EDGE cohort for support. I'd spent the summer of 2000 learning graduate-level mathematics with a group of

14 women at Bryn Mawr College. They became my virtual group. After finishing my PhD, I excitedly went off to work with the talented students at Harvey Mudd College, eventually becoming the first African American woman to ever receive tenure at the institution.

My decision to attend Spelman, a historically Black college for women, would forever change my attitude towards mathematics. My professors (Sylvia Bozeman, Etta Falconer, Jeffrey Ehme, Colm Mulcahy, Fred Bowers, Nagambal Shah, Yewande Olubummo, Gladys Glass) invited me to the mathematical table and found a place for me in the mathematical community. Although Historically Black Colleges and Universities (HBCUs) enroll only 10 percent of all African American students, they produce over 30 percent of all baccalaureate degrees in mathematics and statistics and produced 32 percent of the African American baccalaureates who eventually earned doctoral degrees in mathematics from 2004 to 2008. Like the Math Alliance, EDGE, and NAM, we must find ways to partner with minority-serving institutions to shepherd today's students into the beautiful world of mathematics.

## Photo Credit

Photo of Talithia Williams courtesy of Harvey Mudd College.

## ABOUT THE EDITOR

Renowned for her popular TED Talk, "Own Your Body's Data," **Talithia Williams** uses statistics to communicate the beauty of mathematics and excite people about data. She is cohost of the upcoming PBS series *NOVA Wonders*, premiering in April 2018, and shares her life with an amazing husband and three energetic boys.



**Talithia Williams**

# The Role of Professional Societies in STEM Diversity

Vernon R. Morris and Talitha M. Washington

EDITOR'S NOTE. This article appeared in the *Journal of the National Technical Association* (NTA), 87(1), 22–31. ISBN-978-0-9993666-0-8. It is reprinted with permission of the NTA.

**ABSTRACT.** The overall percentages of African American scientists indicate underrepresentation in most science, technology, engineering, and mathematics (STEM) disciplines and the percentages appear to be declining over the last three decades [NSF, 2017]. Despite investments in diversity programs, the observable impact on STEM leadership and the demographics of the science and technology workforce remains quite small. This paper will highlight some of the challenges and barriers that many students and professionals who seek to pursue careers in these fields face, and the role of professional societies in either exacerbating the perpetuation of monocultures in the various STEM disciplines or proactively working to eliminate barriers and discrimination. We will present and provide clarity on three common myths that are often articulated in discussions of STEM diversity. We will share insights on how professional societies can directly impact the broadening of participation as well as the persistence of racial groups in the STEM fields and hence, strengthen and sustain the Nation's future workforce.

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## Background

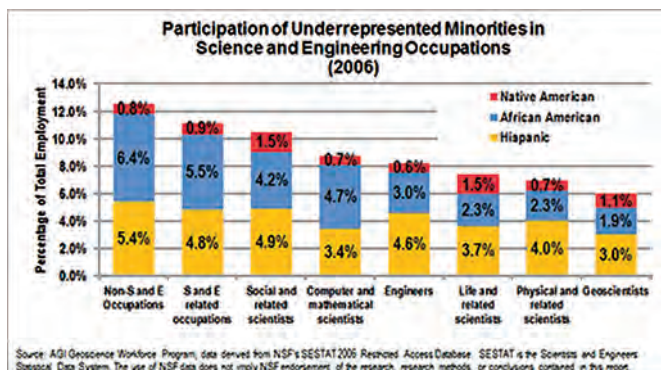
Both authors have been active members of large majority- and minority-based professional scientific organizations for the past twenty years. Within the majority professional societies, we have served on numerous ad hoc organizations and committees committed to addressing access and inclusion issues that are common to both majority and minority organizations. The focus of this article is on racial diversity; in particular, the participation of African Americans in STEM and potential ways scientific organizations can aid in the diversification of STEM.

## What is the current picture?

This paper focuses on the Geosciences and Mathematics disciplines given that they are characterized by severe underrepresentation. A recent *New York Times* article underscored the ineffectiveness of Affirmative Action programs in making a sizeable or sustainable impact on STEM diversity at the top colleges in the Nation over the past 35 years (Ashkenas et al. 2017). Moreover, the article shows trends for participation by African Americans and Hispanics were negative while the trends for Caucasians and Asians were both positive over the same period. Given the demographic patterns among the current and future student populations, this is not just an alarming observation, it is an issue of National Security. Home-grown expertise in STEM undergirds the need for continued advances in cybersecurity, food security, and environmental intelligence National Defense and Homeland Security.

Figure 1 shows the 2006 participation within science and engineering occupations for three of the major ethnic groups considered to be perennially underrepresented in STEM fields. While these data are a decade old, the trends in participation over the past decade have remained negative (NSF 2017). Thus, we can treat these percentages as an effective upper bound to the actual numbers. The total participation of underrepresented minorities in Geosciences and Computer and Mathematical Sciences are 6% and 8.9%, respectively. We note that both of these fields have been identified as the fastest growing jobs in





**Figure 1: Participation of Underrepresented Minorities in Science and Engineering Occupations (AGI Workforce Report 2006).**

the sciences (Eos 2017). According to the 2016 census population estimates, the percentage of Black or African American is 13.3%, Hispanic or Latino is 17.8%, and the percentage of Native American, Alaska Native, Native Hawaiian and Other Pacific Islander is 1.5% (US Census Report 2015). If parity in the fields were achieved, then the percentages in the fields would be a little closer to 32.6%, not 6% or 8.9%. Comparing these percentages for underrepresented groups, the problem of underrepresentation in STEM is clear.

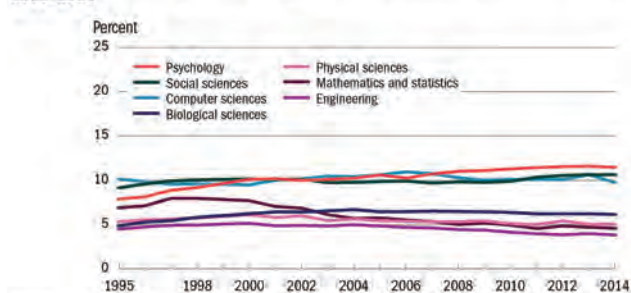
Despite the fact that the total number of Blacks earning bachelor's degrees has increased by 54%, the fraction relative to the total number of those earning a STEM bachelor's degree has only increased from 10% to 11%. Meanwhile, the percentage of STEM degrees earned by Blacks appears to be decreasing. That is, even though there are more Blacks earning bachelor's degrees, fewer of them are earning bachelor's in STEM disciplines (NSF 2017).

Across racial and ethnic groups the recent decadal trend also indicates larger shares of undergraduate degrees and certificates being conferred to female students than to male students (NSF 2017). We recognize that the trend that we see at the postsecondary level is impacted by the rates of minority male incarceration and rates of high school attrition. We remain cognizant of the entire educational experience of minorities in STEM and at the moment focus on the collegiate and professional levels.

In Figure 2, the trendline for Black or African American bachelor's degrees in mathematics and statistics is decreasing over the time interval from 1995 to 2014, from about 7% to less than 5%. As the geosciences is in the physical sciences, Figure 2 indicates that the percentage of Black or African Americans earning bachelor's degrees in this field is relatively flat, around 5%. Note that these percentages are significantly less than the percentages of Black or African Americans in the US population in 2016, which is 13.3% (NSF, 2017).

Nearly 30% of all Black S&E students receiving their doctoral degrees from US universities earned their bachelor's degrees at an HBCU (NSF 2017) even though only 9% of Blacks attended HBCUs in 2015 (Pew Center 2017). HBCUs are even more overrepresented as the "incubators"

**Science and engineering bachelor's degrees earned by blacks or African Americans, by field: 1995-2014**



NOTE: Data not available for 1999.  
Women, Minorities, and Persons with Disabilities in Science and Engineering: 2017

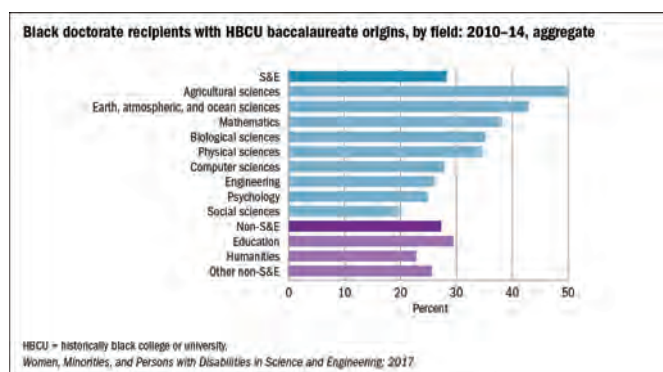
**Figure 2: Science and engineering bachelor's degrees earned by Blacks or African-Americans (NSF 2017).**

of future doctoral degree recipients in mathematics and geosciences (e.g. Earth, Atmospheric and Ocean Sciences). As an example, in 2014, over 40% of the African American students in atmospheric sciences were either educated at an HBCU or earned their terminal degree at an HBCU. As shown in Figure 3, many graduate STEM programs are heavily populated by Blacks who attend HBCUs. However, over the years 2004 to 2014, the percentage of Black or African Americans earning S&E degrees from HBCUs is declining (see Figure 4) which raises great concern about the future of Blacks in STEM.

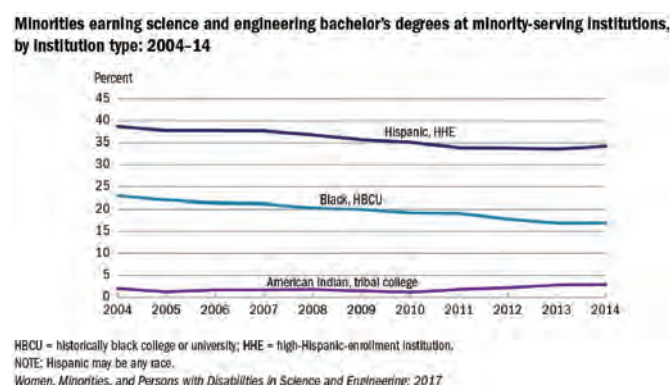
Professional societies often play an intermediary role between students seeking careers and opportunities within the discipline. For the geosciences and mathematical sciences, two disciplines that are among the least diverse by NSF indicators, there are longstanding cultures that have developed within federal agencies, elite academic programs, and amongst private sector stakeholders that tend towards homogenization rather than diversity. We recognize that there are a plethora of ways that diversity in STEM access and inclusion can be improved. This paper focuses on one key element, the scientific professional societies, in an attempt to significantly add to the ongoing holistic strategies that seek to improve retention and growth of diversity in various STEM fields.

## Personal stories

One co-author, Morris, can still remember experiences during the first professional meetings he attended over 25 years ago. He recalls walking anxiously and excitedly through the crowded venues encountering professionals whose papers he had read voraciously, hoping to steal a few moments of their time and garner a prized opinion or word of encouragement. More often than not, they would avert their eyes (even when talking to him), refrain from shaking hands on introductions, or ignore him completely. Often, he has traveled through the crowded meeting halls and instead of being greeted by his peers, they withdraw from him like "the parting of the Red Sea." When these experiences of how our Caucasian attendees respond to visually ethnic minorities in these crowded professional or social spaces are related to African American and Lati-



**Figure 3: Black doctorate recipients who graduated from HBCUs (NSF 2017).**



**Figure 4: Minorities earning science and engineering bachelor's degrees at minority serving institutions (NSF 2017).**

no/a colleagues of all genders, it is often received with affirming and empathizing nods because so many find it to be a common experience. In contrast, he recalls attending meetings of minority professional societies such as the SACNAS Conference where a random lone person would immediately attract other attendees simply seeking to make a scientific connection. Why this same spirit of acceptance cannot be achieved at all meetings is baffling.

The other co-author, Washington, recalls chatting at a breakout table at a conference with her colleague, Dr. Ronald Mickens, Calloway Professor of Physics at Clark Atlanta University. While pontificating about the discretization of dynamical systems, an elderly white woman rolls up in an electric chair. We casually looked over and she asked Professor Mickens, “Do you work here at the hotel?” He replied, “Why do you ask?” Looking around, the lady said, “I need to know where I can put my chair.” We all looked at each other baffled since we were surrounded by other scientists and we had our conference badges proudly on display. Smiling, Mickens turns to the lady and excitedly waves his hand around the room to say, “Anywhere you want.” The lady was pleased and rolled away. We began to try to understand why she thought this man, clearly a conference attendee, a scientist who has published more than 300 scientific research papers and more than 17 books, could be confused for being a hotel worker. Unfor-

tunately, this is the experience of many professionals of color. We are victims of cognitive dissonance—we cannot be scientists and engineers even in spaces occupied by scientists and engineers. It is easier to assume that we are the “help.” In a moment, the sense of belonging and scientific confidence can be shaken down to its core. These assumptions clearly convey who belongs and who does not belong. Why belonging to the scientific academy is assumed to belong to whites only is baffling.

Both of the anecdotes above describe how the monoculture treats or responds to individuals they conceive of as “other.” While individuals of past generations or significant stature can shrug off these incidents, it may be much harder for younger colleagues and certainly for post-secondary students. What often results is the dilemma of the imposter syndrome, which is a concept that describes the sense of alienation that minorities feel when they are subjected to the types of microaggressions described above. Even high-performing and competent students and professionals can suffer from imposter syndrome. That is, they can begin to suspect that their achievements are the result of luck or accident rather than their own talent and actions and see “others” as inherently more worthy.

Social psychologist Amy Cuddy presented a TED talk in which she explains how nonverbal expressions of power and dominance influence how we think and feel about ourselves as well as how others see us (Fiske et al. 2002). She conveys that it can be very difficult for people to see themselves differently from how they think everyone else sees them. Minorities who are affected by imposter syndrome often fear that others will see them as undeserving of their position and presence which then can expose them as imposters, causing them to lose their status (Ewing et al. 2016). Feelings of being an imposter can result in diminished self efficacy, increased cognitive load, and decreased performance. Moreover, these feelings may lead many students and professionals to reconsider a career in S&T.

As authors of this paper, we come together with our shared, painful experiences acquired from participating in STEM professional organizations and conferences, to bring enlightenment to how these organizations can play a pivotal role in enhancing minority participation in STEM. Our collected experience in two different fields has allowed us to synergize and gain a unique perspective that we will share with the hopes that the recommendations we make to professional societies will have a positive impact for all in every facet of the STEM professions.

### Where do professional societies fit in?

The truth of the matter is, we occasionally still encounter conference staff at majority meetings whose demeanors switch from cheerful and friendly to dry and deadpanned when we step forward to pay registration or to ask a simple question. Sadly, this occurs with staffers of all races. Many African American staffers mimic the worst of the behavior that we have seen. It may be worth noting that we consciously try to appear non-threatening at professional meetings by wearing professional attire including

a sport jacket or suit, being reasonably clean-cut, and utilizing proper grammar without Ebonics. Even though we approach others with a polite tone and demeanor, we elicit a reaction as if we do not belong. We do not believe that these types of responses are personally motivated. Rather, these responses emanate from an unconscious confirmation bias, which is where one seeks to confirm one's beliefs or hypotheses, while giving disproportionately less consideration to alternative possibilities. These are common responses towards minority students and professionals from individuals who have had limited exposure or interests in interacting with people from outside their comfort zones. We also believe that these can be serious demotivating factors when it comes to minorities making choices about joining and participating in professional organizations.

In our current "post-racial" America, younger generations have grown up with a Black President of the United States and what appears to be a plethora of equal opportunities and racial equity. The belief in a meritocracy—that if we perform the same, then the reward will also be the same—takes precedence in their outlook. Many students and young professionals have not experienced the regularity of visceral racism of past decades nor are they as sensitive to the systemic and neoliberal racism of the current, post-racial era. This may inhibit their ability to navigate the more nuanced racial terrain. When they enter professional spaces in which they suddenly feel alienated, ignored, or subject to continuous micro-aggressions, they may fall away because of the hurtful, negative interactions and experiences. By maintaining a culture where this sort of treatment is normalized, we may be contributing to the attrition of the student and professional participation in STEM.

To persist as a career minority scientist, one often becomes acclimated to being the only Black person in a room. In retrospect, this acclimation includes being the only minority in a room in addition to being the only minority in a hostile room. Our sense is that today's students coming from Historically Black Colleges, minority serving institutions, or predominantly white institutions may never have experienced the kinds of overt racism that we grew up with in previous eras, nor the racism in its plethora of distinguishable forms that developed as respectability politics became more of the norm. Subtle micro-aggressions can make professional meetings so overwrought with an inclement environment that many do not consider joining or attending on a regular basis. In our generation, we were told "to run faster, jump higher, and work longer and harder to gain the 'same' accolades as our white counterparts." To maintain this level of activity, many often complain of psychological burnout. We have continued to attend and participate in a variety of professional meetings and, unfortunately, the acceptance of diverse participants remains an issue that has yet to be resolved.

Attending a large professional meeting for the first time as a student can be quite daunting for anyone without adequate social networks or the natural ability to

negotiate large groups of professionals angling for career advancement, renewing old acquaintances, and promoting their agendas. Establishing professional connections faced by a young scientist can be especially challenging for a lone African American, Latino/a, Native American, or person from another minority group. This has nothing to do with self-confidence or competence. It has far more to do with how misperceptions and discomfort with interacting with minority students or professionals can lead to unwelcoming and alienating experiences.

### Three common myths

The three myths that we have selected for this article are by no means the only misperceptions limiting progress. These three are selected based on their prominence in the fields with which we are familiar.

*Myth 1:* There is no problem with underrepresentation. The degree of minority representation in the field is consistent with the existing pool of members of that group who have the talent, interest, and/or desire as attracted by market needs.

*Myth 2:* The "white savior model" is the most effective pathway towards success. In other words, majority entities can solve the diversity problem with their superior intellectual capital, post-racial politics, and research on minority issues sans people of color.

*Myth 3:* Diversity is equivalent to having more white females in the organization. Gender balance in a white monoculture simply preserves white privilege and exacerbates racial division and distrust in the long run as discussed in the rich literature of Bell Hooks and Kimberlé Williams Crenshaw (Hooks 1984; Hooks 2000; Crenshaw 2016).

There may be more myths, beliefs, and personal experiences that influence our perceptions and our actions. However, these three myths reflect a broad swath of the perceptual limitations that frame the responses to calls for increased diversity in STEM. Below, we will attempt to undertake a holistic approach to offering suggestions on how these myths can be overturned and a whole new approach to inclusion focusing on professional societies might have a positive impact on the trends discussed in the introduction. Going forward, we need to be aware of our beliefs as well as the beliefs of others so that we can all come together to rally for enhanced inclusivity in STEM. We now outline a few of the barriers that may be used to support and entrench the aforementioned myths in our professional culture. These barriers may represent the support scaffolding for the confirmation bias present in our professional societies. By better understanding these barriers and their underlying subtleties, perhaps we can all work towards making robust strides toward achieving equity in STEM.

### Ten potential barriers linked to the myths

1. **Most people want to avoid conversations about race like the plague.** The tendency to associate racial stereotypes with overt racism (now regarded as a fringe response to diversity in our "post-racial society" despite



recent events) is a common defense mechanism among non-Blacks. Even well-meaning people who often populate diversity committees may be uncomfortable in directly addressing racial issues within the organization. This becomes especially difficult when those exhibiting the biases or denying that structural biases exist are viewed as powerful figures in the organization.

**2. Difficult conversations about racial diversity often get diverted into conversations about gender diversity.**

There are a variety of reasons for this diversion including the fact that diversity can mean a lot of different things to different subgroups/people. This can lead to discussions of alternate types of “isms” and false equivalences (classism, sexism, genderisms, etc.) – all of which are important but shifting focus rarely leads to a solved problem and completely fails to address ongoing biases. For example, stereotypes often present white females as less threatening, more approachable, and possessing a “shared” experience—especially in a racially homogenous context. This does not extend to women of color and men of color, especially those who fail to assimilate to the cultural norms of the society.

**3. The unspoken paradoxes associated with diversity events at professional meetings.** One paradox is that when one calls an event a “diversity event,” then members of the monoculture (and even some from subgroups) may perceive that the event is exclusive and not inclusive to the entire community. Others may be hesitant to enter the space because they interpret diversity as being the “non-white” event and that they might be unwelcome interlopers. As a consequence, the event can take on the appearance of actually being an exclusive event.

**4. Many successful diversity events are attended because they are popular and hence, may attract individuals who do not sincerely support diversity efforts.** As a consequence, the initial motivating focus of the diversity efforts may be derailed and morph into a mission inconsistent with the spirit of the event. This is an example of virtue-signaling, which can be defined as the conspicuous expression of inflated ethical values by an individual performed with the intent of presenting themselves as more enlightened than the broader social group to which they belong.

**5. Basic misconceptions about diversity in an organization.** Many people still do not understand that diversity is not solely about fairness and overcoming personal biases. It is more about benefitting from the full potential of all skilled professionals and becoming a better organization. We must remind ourselves that it is grounded in professional and scientific ethics. Diversity is about making sure that our occupational landscape is inclusive to all persons and ensures opportunities for students from all backgrounds in preparation for careers that may not yet exist.

**6. Deconvolving scientific objectivity: Racism and the post-racial stance.** Many scientists automatically (dogmatically) assume that they are unbiased, objective, and colorblind. That is, they believe that the very nature of their profession may place them beyond decisions based on

racial stereotypes. Unfortunately, our professional arena can be rife with intelligent people in various stages of denial and adamant that no such biases exist. As scientists, we often tend to “dehumanize” ourselves while elevating the science. However, our human tendencies tend to dictate interpersonal interactions that are not included in the scientific method and can impact our judgment when we evaluate the quality of scientific works of others.

**7. Organizations do not naturally diversify.** In stark contrast, as humans, we tend to bend towards nepotism, professional incest, and traditional social networks. Minority students and newcomers can be preferentially excluded when these practices are entrenched. Racial amnesia can color conversations concerning diversity in professional spaces and can lead to the creation of blind spots. We share many ugly realities such as many of those in leadership positions gained their education during the “segregated sixties” or before. Ignoring or choosing not to confront past discrimination inhibits honest, but uncomfortable discussions about systemic biases. Colleagues have related instances during which individuals in leadership positions have intimated a nostalgia for the days when “the troublesome whiners” were not asking for handouts. Actually, nobody is asking for a handout. Diversity efforts may get channeled through standing committees of the professional organization. If these committees have other priorities, they will inevitably seek to either continue their standard modus operandi or argue pragmatism as a reason to fail to pursue diversity issues.

**8. Professional organizations often ask the wrong questions about improving diversity.** Often we get asked, “Why don’t they [minorities] come?” We often get asked this when there has been historically low participation from people of color in scientific events. Our response, “What can you modify to make the event more enticing for them to come? How can we make them feel as welcome as everyone else? How can we demonstrate their tangible benefits for joining?”

**9. Change is hard work.** If what is being done now is not achieving the desired diversity, perhaps something different should be done. Addressing the resistance to change with the direct implications of how a lack of change can continue the plight of having barriers creep into our spaces will hopefully provide sufficient leverage. One has to work hard and consciously (strategically) to ensure that diversity events do not get labeled or perceived as exclusive and, more importantly, meet specific goals.

**10. Racial diversity is not a priority issue of all non-Caucasian professionals.** In fact, many “minorities” are repelled by the notion of diversity because of their own misleading about meritocracy and even basic misunderstandings about diversity. It is important to have people of all backgrounds sharing a common goal and vision of what diversity for the organization means in a practical sense, i.e. beyond a standard diversity statement.

## Potential strategies to enhance diversity in professional organizations

One of the current efforts to address the diversity problem in Geoscience organizations evolved into the Colour of Weather Reception at the American Meteorological Society (AMS) Annual Meetings (Joseph et al. 2008). For the past twenty years, an informal networking reception has been hosted at this meeting focused on making it a more receptive and comfortable space for minority students and young professionals. These receptions grew out of ad hoc gatherings and get-togethers of no more than a handful of like-minded colleagues and grew to incorporate students of color from small meteorology programs at HBCUs and gradually expanded as networks and programs at minority-serving institutions continued to develop. The message at these receptions was never isolation. In fact, it openly encouraged and solicited leadership from the AMS to attend as a critical part of the inclusion process. Over the years, the event grew from ten or fifteen individuals to attendance exceeding two hundred and fifty participants. Colour of Weather has always sought to provide a safe space for provocative, often difficult, but necessary conversations about stereotypes and stereotype threats, overcoming systemic barriers to success, and for making personal contacts that extend beyond typical comfort zones. The two-hour receptions are now structured such that students can maximize professional interactions while engaging decision-makers (leadership from the AMS, federal agencies, private sector, and academic sector) in ice-breakers and activities that address bias and issues of diversity and inclusion as well as areas of common interest in atmospheric sciences, internship and training opportunities, and professional mentoring. The goal is for every minority student in attendance to leave the meeting with a stronger professional network than they possessed when they arrived. It is also designed to lower the activation barrier limiting interactions between students of color and the professional membership. We have been fortunate that the reception has become a regular part of the AMS meeting schedule and one of the highly sought out events of the opening day. Our support and attendance from AMS leadership has been a critical part of the success as has been the commitment of NASA and NOAA leadership. The regular attendance of these representatives has lent credence to the mission of the reception and has inspired even greater attendance from individuals who might have blown off the meeting otherwise.

In early 2017, the AMS President-Elect, Matthew Parker, approached one of the authors with openness and genuine interest to find what is sorely needed to enhance minority participation at AMS. As the Colour of Weather program was drawing to a close, he asked Morris, “How do we catalyze this energy and inclusiveness so that it expands throughout the society? We need these young people precisely because they are the future of this organization. I don’t know how we do it but I’d like to work with you to realize this change.” The openness and honesty that Parker conveyed in admitting that he did not have the answers but he actively sought advice and input was highly commend-

able. This showed true leadership of being able to identify potential cultural barriers and schisms, and seek the input of key members to find solutions so that the organization can thrive for all. By taking on the responsibility of making a change personally and with sincere honesty, solutions can grow in this fertilized land of strategic collaborations.

Given the impending demographic shifts in our Nation, it is in the best interests of all S&T professional societies to decrease the barriers that limit access and full inclusion in STEM. Professional organizations rely on membership. If the demographics of the membership of the professional societies does not evolve with the demographics of our society, then the sustainability of the professional societies may be jeopardized. Leadership in professional societies must also recognize that their voices may have an outsized impact on the culture of S&T workforce. Thus, by taking a definitive and proactive stance they can serve as positive change agents in society. Even though professional societies will not alone be able to completely resolve issues of racial equity in STEM, they may still be able to have a critical role in the solution.

Below, we distill several potential strategies that professional organizations can undertake in efforts to be part of the solution in improving diversity and inclusion in STEM and the S&T workforce.

**1. Racism was not created and cannot be solved by its victims.** Champions who represent the majority population and have significant stature are encouraged to be vocal about inclusion.

**2. Racism in society permeates the scientific fields and professional organizations.** Professional organizations may wish to take a proactive stance in trying to root out both active and passive racism in their meetings and organizational policies and practices. This may include nominations for awardees, memberships on boards, and key committee memberships.

**3. A possible key to fostering inclusion and diversity is helping minority students to establish robust social networks.** These networks allow newcomers of all backgrounds to learn how to navigate in what can often be “hostile” environments for individuals who do not conform to the perceived status quo.

**4. The purpose of diversity events should be recognized and may necessitate invitations to facilitate inclusiveness.** Further, it would be better if successful diversity events could be earmarked by tangible evidence that the organizational culture is changing for the better.

**5. The voice that advocated diversity should come from those who have earned a respected professional reputation.** This reputation can be achieved through traditional markers of academic success so that the voice can overcome skepticism of professional peers and directly address misgivings regarding diversity and exceptionalism. Often, given the current racial climate which can cloud our assessment of others, this voice should come from either a white or Asian male. This is, more voices can bring harmony to STEM for all are needed to push the infrastructure to be more inclusive.

6. **True partnerships between HBCUs as well as African American (e.g., Association of Black Geoscientists), Hispanic, and Native American scientific organizations and larger professional organizations should be forged to enhance participation, inclusion, and recognition.** An example could be reciprocal arrangements involving reduced membership fees for those belonging to minority professional societies. Minority scientists are often financially-stretched by attempting to maintain memberships in multiple organizations. At the same time, they are challenged to maintain time-commitments to committees in both organizations, while maintaining scientific productivity.

7. **Minority students and professionals should be knowledgeable on how to evaluate opportunities.** Their naive enthusiasm could be exploited by diversity programs seeking to fulfill a diversity requirement.

8. **Difficult conversations about the culture and environment of organizations must be undertaken and sustained.** The issue of diversity has been around for some time and as such, will take time and effort to be resolved.

9. **Access to leadership and leadership opportunities is a critical part of the solution.** All need to know and feel that they are a sincere member of the organization as this builds trust among the members that can help propagate the scientific community forward.

10. **Professional societies should not assume what people of color need.** Conversations and information-gathering should be undertaken with various groups or individuals. Finding these groups or individuals may take time and there may need to be some trust developed so that sincere discussions can ensue. Note that this may require a degree of humility and openness to move forward on solutions that each party may not fully understand.

## Conclusion

We do not claim to have all of the answers in this paper. Yet we seek to inspire conversations that can synergize the strengths that spawn from our diversity which broaden our scientific communities. We conclude with the words of Lee Lorch (Case 1996), who continues to speak to all of our scientific societies' conscience.

"Do we care only for those already in the profession? Is the Society willing to accept the present nearly complete exclusion from our mathematical manpower pool of Black America?

Mathematicians and non-mathematicians alike await the answers to these questions. The Council [of all scientific societies] has the task of providing leadership."

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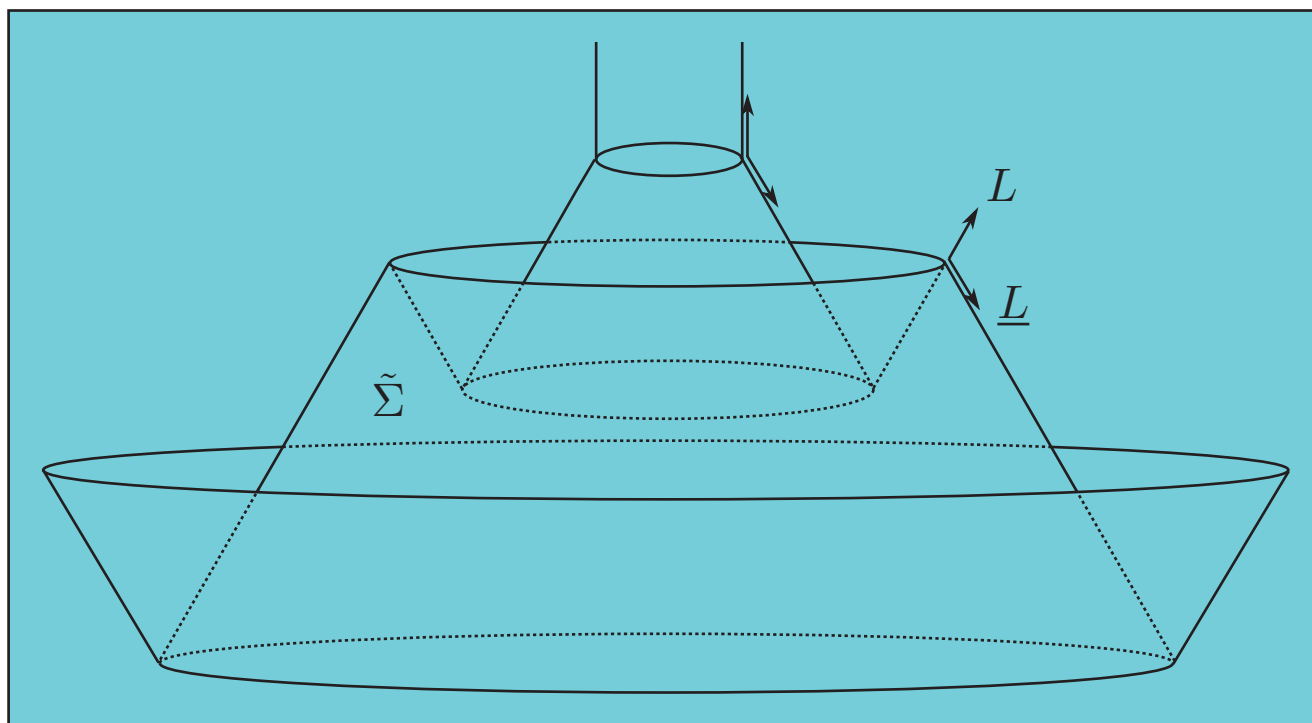
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# Proof of a Null Geometry Penrose Conjecture



Hubert L. Bray and Henri P. Roesch

Beginning in 1973, Roger Penrose wondered about the relation between the mass of the universe and the contributions of black holes ([1], [2]). All we can see are their outer boundaries or “horizons,” the size of which should determine their mass contributions. He conjectured that the total mass of a spacetime containing black hole horizons with combined total area  $|\Sigma|$  should be at least  $\sqrt{|\Sigma|/16\pi}$ . On the one hand, this conjecture is important for physics and our understanding of black holes. On the other hand, Penrose’s physical arguments lead to a fascinating conjecture about the geometry of hypersurfaces in spacetimes. For a spacelike “Riemannian” slice (“the universe at a particular time”) with zero curvature (zero

second fundamental form) the conjecture is known as the *Riemannian Penrose Inequality* and was first proved by Huisken-Ilmanen in 2001 (for one black hole) and then by the first author shortly thereafter, using two different geometric flow techniques. This article concerns a formulation of the conjecture for certain light-cone-like “null” hypersurfaces in spacetimes called the *Null Penrose Conjecture (NPC)*. Over the last ten years, there has been a great deal of progress on the NPC, culminating in a recent proof of the conjecture in a fair amount of generality for smooth null cones by the second author. One surprising fact is that these null hypersurfaces, under physically inspired curvature conditions on the spacetime, have “monotonic” properties (increasing as expected), including cross sectional area, notions of energy, and, as we’ll see with Theorem 1 below, a new notion of mass.

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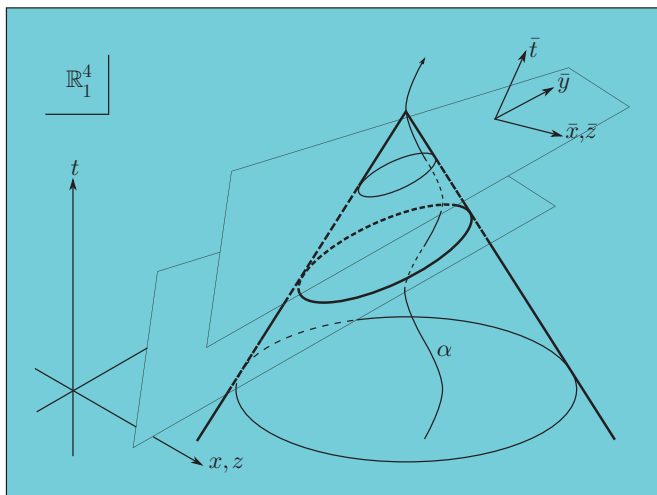
DOI: <http://dx.doi.org/10.1090/noti1629>

## The Null Geometry of Light

The theory of General Relativity emerges from Albert Einstein’s beautiful idea that matter in a physical system curves the intertwining fabric of both space and time. A

spacetime is a four-dimensional manifold with a metric of signature (3,1), meaning that the metric on each tangent plane is isometric to the Minkowski spacetime  $\mathbb{R}_1^4 := (\mathbb{R}^4, -dt^2 + dx^2 + dy^2 + dz^2)$ . The minus sign in the metric implies the existence of null vectors, vectors with zero length (such as  $(1,1,0,0)$ ), even though the vectors themselves are not zero. A null hypersurface is a codimension-one submanifold of a spacetime whose three-dimensional tangent planes are null in one dimension (and hence positive definite in the two other dimensions). For example, if we let  $r = \sqrt{x^2 + y^2 + z^2}$  in the Minkowski spacetime, any translation of the downward cone  $\Lambda := \{t = -r\}$  as depicted in Figure 1, is a null hypersurface.

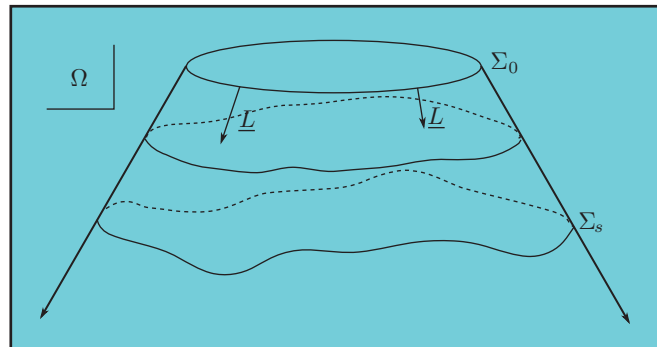
Null geometry is counterintuitive in a number of ways. Since one dimension has zero length, null hypersurfaces have zero volume. Furthermore, since the metric is not invertible, the Riemann curvature tensor of the null hypersurface is not well defined. Also, the vector which is perpendicular to a null hypersurface must also be tangent (hence null). This normal-tangent duality complicates the notion of what the second fundamental form of a null hypersurface should be defined to be (the classical tool for analyzing the “shape” of substructures).



**Figure 1.** The downward light cone  $\Lambda$  of Minkowski is an example of a null hypersurface.

Fortunately, for a conical null hypersurface  $\Omega \cong \mathbb{S}^2 \times \mathbb{R}$  called a *null cone*, where  $\mathbb{S}^2$  accounts for the two positive dimensions,  $\Omega$  can be studied vicariously through the geometry of its spherical cross-sections including their Gauss and Codazzi equations. For our sacrifice in intuition to this normal-tangent duality we do gain some advantages. A direct consequence of a normal vector  $\underline{L}$  being also tangent is that all curves along  $\underline{L}$  must be geodesic. Thus, a null hypersurface can be thought of

*Null geometry  
is  
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ways.*



**Figure 2.** Foliations of  $\Omega$

as a collection of light-rays in the framework of General Relativity. Imagine standing on some 2-sphere  $\Sigma_0$  in a spacetime, for example the surface of the Earth, and collecting all light rays hitting the surface at a particular point in time.

The resulting set constructs (or recovers) a null cone  $\Omega$  reducing the usually complicated system of PDEs associated to flows in spacetimes to an analysis of ODEs. From standard uniqueness of ODEs, any two normal null flows of surfaces off of  $\Sigma_0$  must result in two foliations of the same null cone. Moreover, from standard existence of ODEs, wherever there’s a smooth spacelike 2-sphere  $\Sigma_0$  there will be a null cone off of it, at least in a neighborhood of the sphere.

### The Expanding Null Cone

The thermodynamics of black holes rests upon Hawking’s area theorem (1973), which states that in spacetimes satisfying the Dominant Energy Condition (or DEC) the area of a cross section of a black hole event horizon is nondecreasing. Similarly in our context, we will briefly explore how the DEC, which is a local curvature constraint modelling nonnegative energy, ensures that null cones reaching *null infinity* can only be foliated by spheres of increasing area. To do so, we first need a quantity also needed to state our main result, Theorem 1.

**Definition 1.** For a spacelike 2-sphere  $\Sigma$ , the expansion is given by the inner product of the null-flow vector  $\underline{L}$  with the mean curvature vector  $\vec{H}$ :

$$\sigma := \langle -\vec{H}, \underline{L} \rangle.$$

For a null flow off of  $\Sigma$  along  $\underline{L}$ ,  $\sigma$  comes from  $\frac{d}{ds} dA = \sigma dA$  where  $dA$  is the area form (i.e. an element of area on  $\Sigma$ ). Here the DEC comes into play, by way of the so-called Raychaudhuri equation, ensuring  $d\sigma/ds \leq 0$  along all geodesics. Hence, the only way to get standard asymptotics at infinity, which by any reasonable notion should have expanding 2-spheres (i.e. “ $\sigma(\infty) > 0$ ”), necessarily restricts our choice of  $\Sigma_0$  to have strictly positive expansion. In fact,  $\sigma > 0$  on all of  $\Omega$  enforces that all foliations must have expanding area.

With the null flow vector  $\underline{L}$  having zero length we also forfeit our intuitive notion of measuring the speed of a

flow. However, with an expanding null cone  $\Omega$ ,  $\sigma$  offers a convenient replacement.

Null cones that reach null infinity also have fundamental physical significance beyond the monotonicity of cross-sectional area. In isolated physical systems, such as a cluster of stars or a black hole, we expect the curvature induced by localized matter to settle far away back to flat Minkowski spacetime. In our context, Mars and Soria (2015) introduced the notion of an *asymptotically flat* null cone whereby the geometry of  $\Omega$  approaches that of a downward cone of Minkowski at null infinity in a suitable sense. These asymptotically flat null cones allow us the ability to measure the total energy and mass of a system which we'll need in order to state the NPC.

### Total Energy and Our Main Example

In 1968, Hawking published a new mechanism aimed at capturing the amount of energy in a given region using the curvature of its boundary  $\Sigma$ .

**Definition 2.** *The Hawking Energy is given by*

$$(1) \quad E_H(\Sigma) = \sqrt{\frac{|\Sigma|}{16\pi}} \left( 1 - \frac{1}{16\pi} \int_{\Sigma} \langle \vec{H}, \vec{H} \rangle dA \right).$$

For example, for any cross-section of the downward cone  $\Sigma \hookrightarrow \Lambda := \{t = -r\}$  in Minkowski the Gauss equation identifies a beautifully simple relationship between the intrinsic and extrinsic curvature,  $\mathcal{K} = \frac{1}{4} \langle \vec{H}, \vec{H} \rangle$ , where  $\mathcal{K}$  is the Gauss curvature of  $\Sigma$ . From the Gauss-Bonnet Theorem we therefore conclude that

$$0 = \int \mathcal{K} - \frac{1}{4} \langle \vec{H}, \vec{H} \rangle dA = 4\pi E_H / \sqrt{|\Sigma|/16\pi}.$$

Thus, all cross-sections—no matter how squiggly—envelop matter content of vanishing energy, as expected of a flat vacuum.

For another and our main example of a null cone, we go to the one-parameter family of Schwarzschild spacetimes characterized by the metric

$$-(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\vartheta^2 + (\sin \vartheta)^2 d\varphi^2).$$

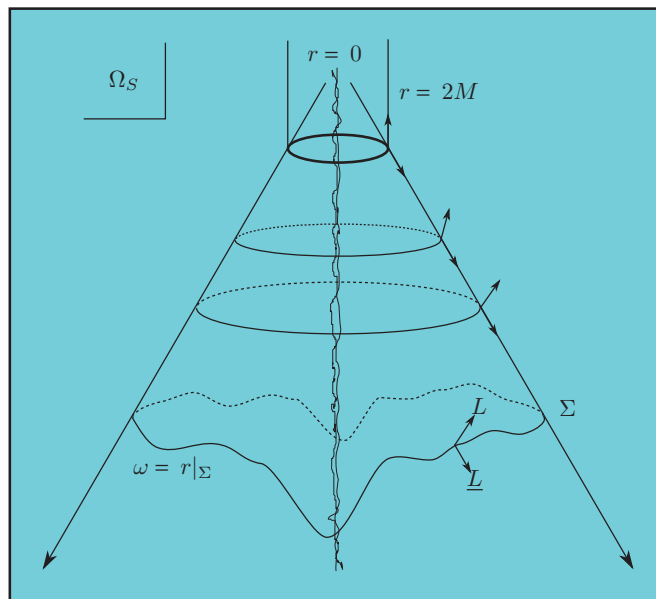
These spacetimes model an isolated black hole with the parameter  $M$  representing total mass. Note from the last two terms of the metric that the coordinate  $r$  has been chosen so that each sphere of fixed  $(t, r)$  is a round sphere of area  $4\pi r^2$ . Also, when  $M = 0$  we recover exactly the Minkowski metric in spherical coordinates. The reader may have noticed the singularities  $r = 0$  and  $r = 2M$ . The singularity at  $r = 0$  is a curvature singularity called the *black hole singularity* giving rise to the isolated black hole. We see this black hole is isolated from the fact that the metric approaches the Minkowski metric for large values of  $r$ . On the other hand, the singularity at  $r = 2M$  is superficial, and can be removed with a change of coordinates. To show this we introduce the ingoing null coordinate  $v$ , whereby

$$dv = dt + \frac{dr}{1 - 2M/r},$$

giving the metric in *ingoing Eddington-Finkelstein* coordinates

$$(2) \quad -(1 - \frac{2M}{r})dv^2 + 2dvdr + r^2(d\vartheta^2 + (\sin \vartheta)^2 d\varphi^2).$$

In the transition from  $M = 0$  to  $M > 0$  the downward cones of Minkowski transition to their spherically symmetric counterparts in Schwarzschild, referred to as the *standard null cones*. In Minkowski,  $\Lambda = \{v = 0\}$  for  $v = t + r$ , from (2) we see the analogous three-dimensional slice  $\Omega_S := \{v = v_0\}$  (i.e.  $dv = 0$ ) inherits the metric  $r^2(d\vartheta^2 + (\sin \vartheta)^2 d\varphi^2)$  assigning positive lengths only for vectors along the two spherical coordinates  $(\vartheta, \varphi)$ , not  $r$ . These coordinates  $(r, \vartheta, \varphi) \in \mathbb{R} \times \mathbb{S}^2$  identify points on the standard null cone  $\Omega_S$  of Figure 3.



**Figure 3.** The Standard Null Cone  $\Omega_S$  of Schwarzschild is the counterpart of  $\Lambda$  in Minkowski.

The  $r$ -coordinate curves are exactly the geodesics that rule  $\Omega_S$ , allowing us to identify any cross-section by simply specifying  $r$  as a function on  $\mathbb{S}^2$ . Similar to Minkowski, the Gauss equation once again simplifies to an intriguingly simple expression for a cross-section  $\Sigma := \{r = \omega(\vartheta, \varphi)\} \hookrightarrow \Omega_S$  ([3])

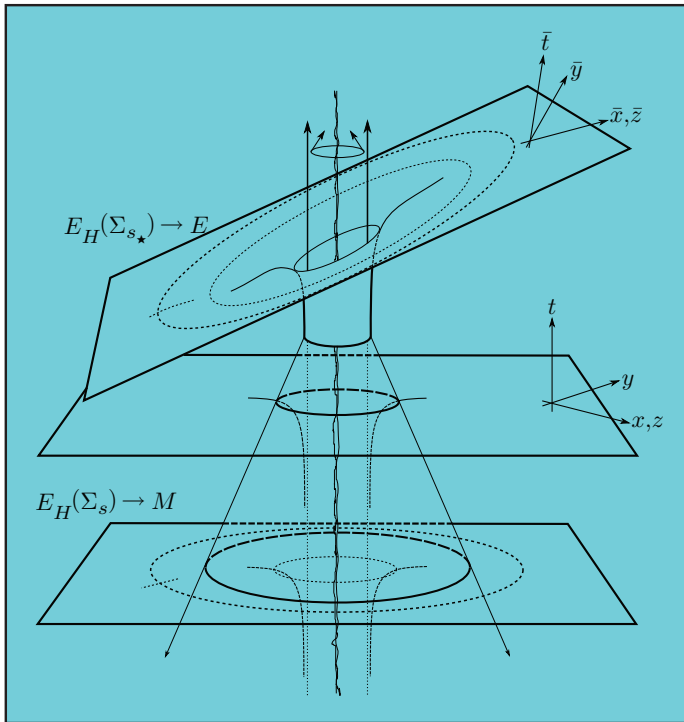
$$(3) \quad \mathcal{K} - \frac{1}{4} \langle \vec{H}, \vec{H} \rangle = \frac{2M}{\omega^3}.$$

$\Sigma$  also inherits the simple metric  $\gamma = \omega^2 \dot{\gamma}$  from  $\Omega_S$ , where  $\dot{\gamma}$  is the standard round metric on a sphere. So from Gauss-Bonnet and (3), we conclude that

$$E_H(\Sigma) = M \sqrt{\int \omega^2 \frac{dS}{4\pi}} \int \frac{1}{\omega} \frac{dS}{4\pi},$$

where  $dS$  is the area form on a round sphere. Some fascinating observations follow from Jensen's inequality. We deduce that  $E_H \geq M$ , the special relativistic notion that the energy of a particle is always bounded below by its mass. Jensen's inequality also ensures that equality





**Figure 4.** Asymptotically round foliations of  $\Omega_S$

is reached only if  $\omega = r_0$ , corresponding to the  $t$ -slice intersections with  $\Omega_S$  (see Figure 4). Moreover, one can show that  $\Sigma$  is a round sphere if and only if

$$\omega(\vartheta, \varphi) = r_0 \frac{\sqrt{1 - |\vec{v}|^2}}{1 - \vec{v} \cdot \vec{n}(\vartheta, \varphi)}$$

for some  $r_0, \vec{v}$  inside the unit ball  $\hat{B}^3 \subset \mathbb{R}^3$ , and  $\vec{n}(\vartheta, \varphi)$  the unit position vector, giving the energy  $E_H(r_0, \vec{v}) = M/\sqrt{1 - |\vec{v}|^2}$ . This is precisely the observed energy of a particle of mass  $M$  traveling at velocity  $\vec{v}$  relative to its observer (with the speed of light set to  $c = 1$ ).

Using Schwarzschild as an example we can also motivate the notion of total energy and mass for an asymptotically flat null cone  $\Omega$ . We start by bringing to the attention of the reader that the intrinsic geometry of  $\Omega_S$  is identical to that of the downward cone in Minkowski. This is evident from (2), since the only component giving rise to the Schwarzschild geometry (beyond Minkowski) is  $dv^2$ , which vanishes for the induced metric on  $\Omega_S$ . So instead, we may actually account for all round spheres of  $\Omega_S$  by intersecting the downward cone of Minkowski by Euclidean hyperplanes (see Figure 1). On the one hand, any family of parallel hyperplanes in Minkowski are given as fixed time slices inside a reference frame (coordinates  $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ ) of an observer traveling at velocity  $\vec{v}$ . On the other hand, the ambient geometry of Schwarzschild settles to that of Minkowski, inheriting such characteristics asymptotically. Therefore, aided by Figure 4, we imagine that an asymptotically round foliation of  $\Omega_S$  is induced by an asymptotically Euclidean slicing of the spacetime (for which  $E_H$  has a verified correlation to total energy).

We conclude that an observer at infinity approximates our black hole to a particle (similarly to  $\alpha$  of Figure 1) of total energy  $M/\sqrt{1 - |\vec{v}|^2}$ . In the general setting, considering an asymptotically flat null cone  $\Omega$ , it follows that  $E_H$  approaches a measure of total energy along an asymptotically round foliation called a *Bondi Energy*,  $E_B$ .

**Definition 3.** For an asymptotically flat null cone  $\Omega$ , minimizing over all Bondi Energies  $E_B$  yields the Bondi Mass  $m_B$ .

Returning to Schwarzschild, we verify that the standard null cone has Bondi Mass

$$m_B(\Omega_S) = \inf_{|\vec{v}| < 1} \frac{M}{\sqrt{1 - |\vec{v}|^2}} = M.$$

### Black Holes and the Null Penrose Conjecture

Further inspection of (2) reveals yet another null cone given by the slice  $\mathcal{H} := \{r = 2M\}$  (see Figure 3). With induced metric  $4M^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$ , positive lengths are once again only assigned to vectors along the spherical coordinates, not  $v$ . In contrast to  $\Omega_S$ , any cross section  $\Sigma \hookrightarrow \mathcal{H}$  has metric  $\gamma = 4M^2\dot{\gamma}$ . Thus, all cross-sections exhibit the exact same area  $16\pi M^2$ . In other words, on any cross-section of  $\mathcal{H}$ , light rays emitted perpendicularly off of the surface remain trapped. Since no material particles travel faster than the speed of light,  $\mathcal{H}$  indicates the hypersurface from which there is no return upon entering, or the *event horizon*. As a result of this trapping, the 2-sphere  $\mathcal{H} \cap \Omega_S$  is the unique cross-section of  $\Omega_S$  satisfying  $\langle \vec{H}, \vec{H} \rangle = 0$ , namely with null mean curvature.

**Definition 4.** A *marginally outer trapped surface (MOTS)*  $\Sigma_0$  in an expanding null cone  $\Omega$  is a surface satisfying the condition

$$\langle \vec{H}, \vec{H} \rangle = 0.$$

With a MOTS  $\Sigma_0$  identifying the presence of a black hole horizon in our context, the Null Penrose Conjecture states that

$$\sqrt{\frac{|\Sigma_0|}{16\pi}} \leq m_B.$$

### Previous Work

Given a spacelike 2-sphere  $\Sigma$  with metric  $\gamma$ , every point on its surface has two positive dimensions in the available four of spacetime spent on tangent vectors. As a result, we can combine the remaining negative and positive dimensions to form a normal null basis  $\{\underline{L}, L\}$ . For a cross-section of a null cone  $\Omega$  we choose  $\underline{L}$  as the normal-tangent to  $\Omega$  (for example, see Figure 3). This finally allows us to introduce some final data for our main Theorem below.

**Definition 5.** For a smooth spacelike 2-sphere  $\Sigma$  of second fundamental form  $\Pi$  and null basis  $\{\underline{L}, L\}$  such that  $\langle \underline{L}, L \rangle = 2$  we define

$$\chi^- := \langle -\Pi, \underline{L}/\sigma \rangle, \quad \zeta(V) := \frac{1}{2} \langle D_V \underline{L}, L \rangle = \tau(V) + V \log \sigma$$

where  $\zeta$  is the *connection 1-form* ( $V$  a tangent vector field of  $\Sigma$ ).

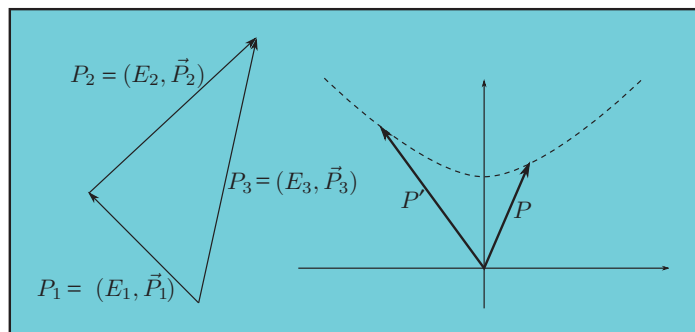
In his PhD thesis, Johannes Sauter (2008) showed for the special class of *shear free* null cones (i.e. satisfying  $\chi^- = \frac{1}{2}\gamma$ ) inside vacuum spacetimes, one is able to solve a system of ODEs to yield explicitly the geometry of  $\Omega$ . This then enables a direct analysis of  $E_H$  at null infinity that allowed Sauter to prove the NPC in this special case. An observation of Christodoulou also shows that  $E_H$  is monotonically increasing along foliations in vacuum if either the *mass aspect function*  $\mu := \mathcal{K} - \frac{1}{4}(\vec{H}, \vec{H}) - \nabla \cdot \zeta$  or the expansion  $\sigma$  remain constant functions on each cross-section. For a black hole horizon  $\Sigma_0$ , we see from (1) that  $E_H(\Sigma_0) = \sqrt{|\Sigma_0|/16\pi}$ , making this observation particularly interesting. If one is successful in interpolating  $E_H$  from the horizon to the Bondi Mass  $m_B$  along one of these flows, the NPC would follow for vacuum spacetimes  $\sqrt{|\Sigma_0|/16\pi} = E_H(\Sigma_0) \leq \lim_{s \rightarrow \infty} m(\Sigma_s) = m_B$ . Sauter was able to show for small perturbations of  $\Omega$  off of the shear free condition, one obtains global existence of either of these flows and that  $E_H$  converges. Unfortunately, one is unable to conclude that the foliating 2-spheres even become round asymptotically let alone  $E_H$  approaching  $m_B$ . In fact, Bergqvist (1997) noticed this exact difficulty had been overlooked in an earlier work of Ludvigsen and Vickers (1983) towards proving the *weak NPC*, namely  $\sqrt{|\Sigma_0|/16\pi} \leq E_B$ .

In 2015, Alexakis was able to prove the NPC for vacuum perturbations of the black hole exterior in Schwarzschild spacetime by successfully using the latter of the two flows in Sauter's thesis. Alexakis was once again afforded an explicit analysis of  $E_H$  at null infinity. Work by Mars and Soria [4] followed soon afterwards in identifying the asymptotically flat condition on  $\Omega$  to maintain an explicit limit of  $\lim_{s \rightarrow \infty} E_H(\Sigma_s)$  along geodesic foliations. In 2016, those authors constructed a new functional on 2-spheres and showed for a special foliation  $\{\Sigma_\lambda\}$  off of the horizon  $\Sigma_0$  called *geodesic asymptotically Bondi* (or GAB) that,  $\sqrt{|\Sigma_0|/16\pi} \leq \lim_{\lambda \rightarrow \infty} E_H(\Sigma_\lambda) < \infty$ . Thus, for GAB foliations that approach round spheres, this reproduces the weak NPC of Bergqvist and of Ludvigsen and Vickers. Unfortunately, as in the aforementioned work of Sauter, Bergqvist, Ludvigsen, and Vickers there is no guarantee of asymptotic roundness.

### Mass Not Energy

These difficulties may very well be symptomatic of the fact that an energy is particularly susceptible to the plethora of ways boosts can develop along any given flow.

We expect an infinitesimal null flow of  $\Sigma$  to gain energy due to an influx of matter analogous to the addition of 4-velocities in Figure 5,  $E_3 = E_1 + E_2$ . However, with energy being a frame dependent measurement and no way to discern a reference frame, we are left at the mercy of distortions along the flow. Without knowing, our measurements could experience either an artificial increase in energy  $P \rightarrow P'$  or decrease  $P' \rightarrow P$ , as depicted in Figure 5. Geometrically, this manifests along the flow in a (local) tilting of  $\Sigma$  (recall Figure 4). From the formula of  $E_H(\Sigma)$  for  $\Sigma \hookrightarrow \Omega_S$ , Jensen's inequality indicates the



**Figure 5. Along a flow in a particular coordinate system, energy is subject to boosts and may increase or decrease.**

existence of many flows with increasing  $E_H$  yet only  $t$ -slice intersections produce the Bondi Mass. Not only is this flow highly specialized, it dictates strong restrictions on our initial choice of  $\Sigma$  from which to begin the flow.

This is not a problem, however, if appealing instead to mass rather than energy since boosts leave mass invariant,  $M^2 = E^2 - |\vec{P}|^2 = (E')^2 - |\vec{P}'|^2 = (M')^2$ . Moreover, by virtue of the Lorentzian triangle inequality (provided all vectors are timelike and either all future or all past-pointing), along any given flow the mass should always increase

$$M_3 = |(E_1 + E_2, \vec{P}_1 + \vec{P}_2)| \geq |(E_1, \vec{P}_1)| + |(E_2, \vec{P}_2)| = M_1 + M_2.$$

One may hope therefore, by appealing to a notion of mass instead of energy, a larger class of flows will arise exhibiting monotonicity of mass and physically meaningful asymptotics.

### Recent Progress from a New Quasi-local Mass

In search of a mass we return to our favorite model spacetime, the Schwarzschild spacetime with *null cones*  $\Omega_S$ . With the tantalizingly simple expression (3), a natural first guess at extracting the black hole mass  $M$  is to take

$$\tilde{m}(\Sigma) = \frac{1}{2} \left( \frac{1}{4\pi} \int (\mathcal{K} - \frac{1}{4}(\vec{H}, \vec{H}))^{\frac{2}{3}} dA \right)^{\frac{3}{2}}.$$

The reason being, irrespective of the cross-section  $\Sigma \hookrightarrow \Omega_S$ , this gives

$$\tilde{m}(\Sigma) = \frac{1}{2} \left( \frac{1}{4\pi} \int \left( \frac{2M}{\omega^3} \right)^{\frac{2}{3}} (\omega^2 dS) \right)^{\frac{3}{2}} = M$$

as desired. Amazingly, Jensen's inequality also ensures that  $\tilde{m} \leq E_H$  whenever the integrand is nonnegative, by Gauss-Bonnet. Unfortunately, upon an analysis of the propagation of this mass on a general null cone, no clear monotonicity properties arise and we're left needing to modify it at the very least. However, on a return to the drawing board  $\Omega_S$ , one finds that all cross-sections also satisfy  $\zeta = d \log \sigma \leftrightarrow \tau = 0$  and a sufficient condition. Inspired by this, the second author [3] put forward the modified *geometric flux*  $\rho$  and mass  $m(\Sigma)$ :

$$\rho := \mathcal{K} - \frac{1}{4}(\vec{H}, \vec{H}) + \nabla \cdot \zeta - \Delta \log \sigma$$

$$m(\Sigma) := \frac{1}{2} \left( \frac{1}{4\pi} \int \rho^{\frac{2}{3}} dA \right)^{\frac{3}{2}}.$$

The first thing we observe is that the previously desired properties  $m(\Sigma) = M$  for  $\Sigma \hookrightarrow \Omega_S$  and  $m \leq E_H$  if  $\rho \geq 0$  are maintained (the second property now following from both the Gauss-Bonnet and the Divergence Theorems). Even better, from a nine-page calculation followed by three different integrations by parts most terms combine to ensure this mass function is nondecreasing in great generality, as summarized by our main theorem.

**Theorem 1** ([3]). *Let  $\Omega$  be a null cone foliated by spacelike spheres  $\{\Sigma_s\}$  expanding along a null flow vector  $\underline{L} = \sigma L^-$  such that  $|\rho(s)| > 0$  for each  $s$ . Then the mass  $m(s) := m(\Sigma_s)$  has rate of change*

$$\frac{dm}{ds} = \frac{(2m)^{\frac{1}{3}}}{8\pi} \int_{\Sigma_s} \frac{\sigma}{\rho^{\frac{1}{3}}} \left( (|\hat{\chi}^-|^2 + G(L^-, L^-)) \left( \frac{1}{4} \langle \vec{H}, \vec{H} \rangle - \frac{1}{3} \Delta \log |\rho| \right) + \frac{1}{2} |\nu|^2 + G(L^-, N) \right) dA,$$

where  $G = \text{Ric} - \frac{1}{2} Rg$  (the Einstein tensor),  $\nu = \frac{2}{3} \hat{\chi}^- \cdot d \log |\rho| - \tau$ , and  $N = \frac{1}{9} |d \log |\rho||^2 L^- + \frac{1}{3} \nabla \log |\rho| - \frac{1}{4} L^+$ .

From the DEC it follows that  $G(L^-, L^-), G(L^-, N) \geq 0$ , so that the convexity conditions

$$(4) \quad \rho > 0$$

$$(5) \quad \frac{1}{4} \langle \vec{H}, \vec{H} \rangle \geq \frac{1}{3} \Delta \log \rho$$

along a foliation  $\{\Sigma_s\}$ , called a *doubly convex foliation*, imply

$$\boxed{\frac{dm}{ds} \geq 0}.$$

So how likely is a foliation to be doubly convex? Well, in the case of a cross-section of  $\Omega_S$  in Schwarzschild, we know (4) holds trivially from (3) since  $\tau = 0$ . It also follows that

$$\frac{1}{4} \langle \vec{H}, \vec{H} \rangle - \frac{1}{3} \Delta \log \rho = \frac{1}{\omega^2} \left( 1 - \frac{2M}{\omega} \right).$$

We conclude that all foliations of  $\Omega_S$  in the black hole exterior ( $\omega \geq 2M$ ) satisfy (4) and (5). A natural question follows as to whether these conditions are physically motivated for more general asymptotically flat null cones.

Having found that this mass functional exhibits somewhat generic monotonicity, our next concern is asymptotic convergence and whether we obtain a physically significant quantity. From the fact that  $m \leq E_H$ , we see that any doubly convex foliation  $\{\Sigma_s\}$  approaching a geodesic foliation of an asymptotically flat null cone  $\Omega$  yields a converging mass, since  $E_H(\Sigma_s)$  converges. However, this convergence is an indirect observation insufficient for a direct analysis of the limit. With a fairly standard strengthening of the decay conditions on the geometry of  $\Omega$ , called *strong flux decay* we're afforded an explicit limit for  $\lim_{s \rightarrow \infty} m(\Sigma_s)$ . Amazingly, this limit is independent of any choice of asymptotically geodesic foliation (as in Schwarzschild), and we conclude with a proof of a Null Penrose Conjecture.

**Theorem 2.** ([3]) *Let  $\Omega$  be an asymptotically flat null cone with strong flux decay in a spacetime satisfying the DEC. Given the existence of an asymptotically geodesic doubly convex foliation  $\{\Sigma_s\}$*

$$\lim_{s \rightarrow \infty} m(\Sigma_s) \leq m_B.$$

*Moreover, in the case that  $\langle \vec{H}, \vec{H} \rangle|_{\Sigma_0} = 0$  (i.e.  $\Sigma_0$  is a horizon) we prove the NPC. Furthermore, if we have the case of equality for the NPC and  $\{\Sigma_s\}$  is a strict doubly convex foliation, then  $\Omega = \Omega_S$ .*

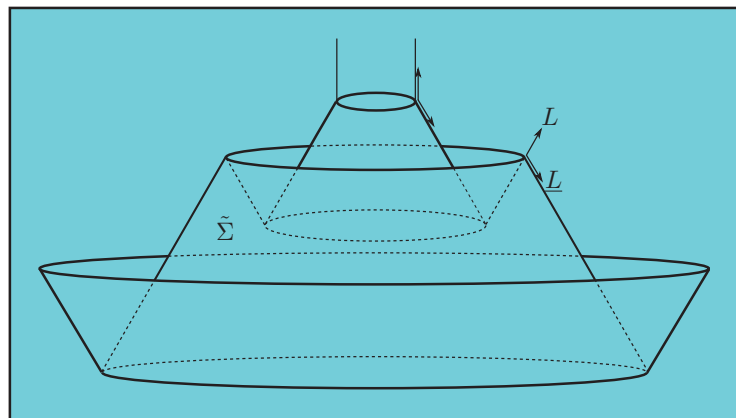
The first part of the theorem follows from the fact that  $m \leq E_H$  and that the limit  $\lim_{s \rightarrow \infty} m(\Sigma_s)$  (being independent of the flow) must therefore bound all Bondi energies from below, hence also  $m_B$ . In the second part, if  $\langle \vec{H}, \vec{H} \rangle = 0$  along with (5), then it's a consequence of the Maximum Principle for elliptic PDE that  $\rho$  must be constant on  $\Sigma_0$  warranting equality in Jensen's inequality. As a result,

$$\sqrt{\frac{|\Sigma_0|}{16\pi}} = E_H(\Sigma_0) = m(\Sigma_0) \leq \lim_{s \rightarrow \infty} m(\Sigma_s) \leq m_B.$$

## Open Problems

An interesting condition resulting in a doubly convex foliation is that  $\rho(s)$  be constant on each of the leaves of a foliation (i.e. on each  $\Sigma_s$ ). As a result, this foliation satisfies  $m(\Sigma_s) = E_H(\Sigma_s)$ , representing a rest-frame flow given that energy equals mass. Studying the existence of this flow is of great interest.

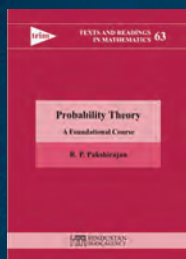
We also invite the reader to recall the dependence of  $\zeta$  and  $\sigma$  on the null basis  $\{\underline{L}, L\}$ . An analogous construction of data under a role reversal between  $\underline{L}$  and  $L$  would result in a new flux function  $\tilde{\rho}$  on our surface  $\Sigma$ . We say a surface  $\tilde{\Sigma}$  is *time-flat* whenever  $\rho = \tilde{\rho}$ . Time-flat surfaces within asymptotically flat null cones are particularly interesting as they serve as pivots where a flow can flip direction or “bounce” as in Figure 6 without causing a discontinuous jump in mass (since  $m = \tilde{m}$ ).



**Figure 6.** On a flat null cone a flow can flip direction or “bounce.”



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This observation is of particular significance in the search for more general foliations that inherit the successes of Theorems 1 and 2. Another objective is to weaken the underlying smoothness assumption associated with null cones in Theorem 2, possibly toward broadening its validity to include focal points or multiple black hole horizons.

These open questions are important not only for understanding the physics of black holes and mass in general relativity but also for expanding our knowledge of null geometry. Since null geometry is not particularly intuitive, physical motivations like these are very useful for providing fascinating conjectures to pursue.

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**Hubert L. Bray**

### ABOUT THE AUTHORS

In 2001, **Hubert Bray** proved the Riemannian Penrose Conjecture for any number of black holes by discovering a new geometric flow that preserves nonnegative scalar curvature.



**Henri P. Roesch**

**Henri P. Roesch** was born in Pretoria, South Africa, and moved to the United States to complete his graduate studies at Duke University.



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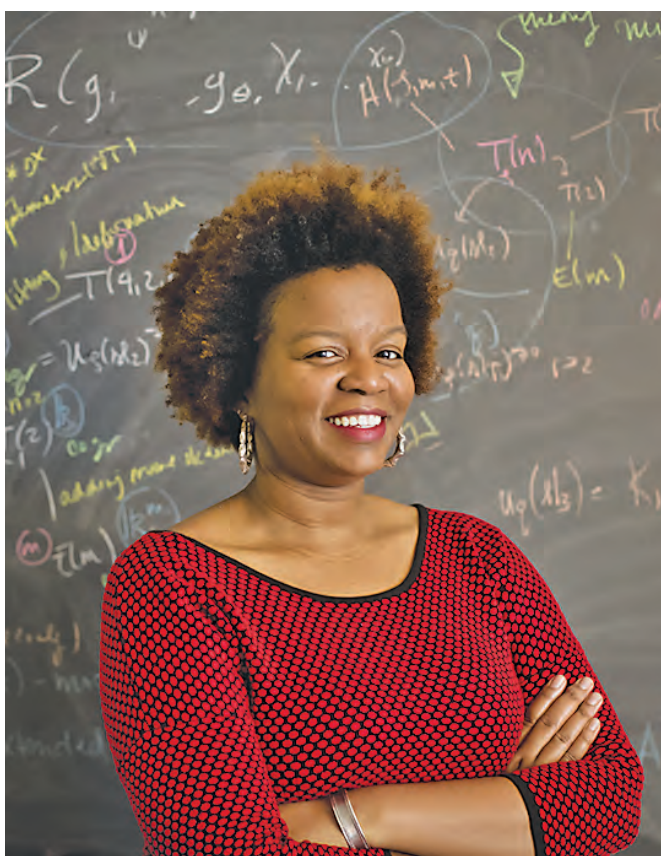
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## Chelsea Walton Interview

*Conducted by Alexander Diaz-Lopez*



Chelsea Walton is assistant professor of mathematics at Temple University. Chelsea works in non-commutative algebra, Hopf algebras, and their relations to other areas of mathematics. In 2017 she received a Sloan Research Fellowship. Her email is [notlaw@temple.edu](mailto:notlaw@temple.edu).

**Diaz-Lopez:** When did you know you wanted to be a mathematician?

**Walton:** I've always loved math. It wasn't just because I had a knack for it; I was drawn to it and always wanted to learn more. I liked identifying patterns and symmetries, and I loved puzzles. But when I was young I didn't know that these were mathematical concepts, as some of these "games" did not involve numbers. Once I discovered in high school that I could have a career in playing such games, and even better, making up new math, I was sold—I was determined to be a mathematician.

**Diaz-Lopez:** Who encouraged or inspired you?

**Walton:** My family has always been encouraging and inspirational. To start, my parents have always trusted my judgment, supported my decisions, and never pushed me into one field or another, although there was that one period when I wanted to be a movie critic, to which my dad replied, "How many Siskels and Eberts do you know?" I couldn't think of any others so I switched back to math,<sup>1</sup> which is ironic because I didn't know any mathematicians when I was growing up! So maybe there was a gentle push there. My paternal grandfather was also very encouraging and liked playing the same games as I did (identify patterns and symmetries, etc.). In fact if circumstances were different back when he was growing up, I reckon that he would have pursued mathematics as a career.

I have also had tons of support from mathematicians throughout my career including undergraduate mentors Jeanne Wald, Joel Shapiro, and Michael Frazier (at Michigan State at the time), grad school mentors at Michigan including Toby Stafford (now at U. Manchester UK) and Karen Smith, and various mentors during my years as a postdoc including James Zhang (U. Washington), Pavel Etingof (MIT), Sarah Witherspoon (Texas A&M), and Ellen Kirkman (Wake Forest).

**Diaz-Lopez:** How would you describe your research to a graduate student?

**Walton:** I'm a noncommutative algebraist and there are many natural collections of objects in the world that

<sup>1</sup>It was actually "mathematical engineering," a term that I made up because I didn't know that "mathematicians" existed.



## *Picking your thesis advisor is the most important decision you will make in grad school.*

algebraic geometry, Poisson geometry, invariant theory, and functional analysis.

More specifically, I'm very much interested in symmetries of noncommutative algebras. Classically, groups suffice as an axiomatization of a given object's symmetries; consider dihedral groups and regular polygons, for instance. But there are many noncommutative algebras that have small automorphism groups. For example, the  $\mathbb{C}$ -algebra generated by two noncommuting variables  $x$  and  $y$  subject to relation  $yx = \pi xy$  admits only linear actions given by scaling, whereas the degree-preserving automorphism group of the commutative polynomial ring  $\mathbb{C}[x, y]$  is all of  $GL_2(\mathbb{C})$ . So a better notion of symmetry of noncommutative algebras is needed, and for this, actions of Hopf algebras are widely accepted to be a good fit. Indeed, an example of a Hopf algebra is a group algebra. I study when such actions exist, and when they do, I sometimes do various things with the actions as one would in the classic setting.

I'm also interested in many other aspects of noncommutative algebra...

**Diaz-Lopez:** *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

**Walton:** This is hard to decide and I'll need more time to answer the first question (and the second depends on the first). My honest answer now is "the theorem that's to come!" I think I'm far too young to reflect upon my mathematical journey in the manner that the question suggests. I'll probably give the same reply 10 years from now, maybe even 20 years from now. I like to focus on learning new math, creating new math, and pushing forward.

**Diaz-Lopez:** *What advice do you have for graduate students?*

**Walton:** Picking your thesis advisor is the most important decision you can make in grad school. I suggest choosing an advisor based on the following criteria.

(1) What type of mathematics they specialize in broadly-speaking (it's not expected, typically, that you can understand their research statement with ease) — you'll need to think about a problem in their area of research rather obsessively so you better like the mathematics that they're doing.

(2) Whether their style of advising is compatible with the way you need to be mentored. Would you like weekly meetings at first? Would you rather be left alone for lon-

ger period of time? Are your methods of communication compatible? Etc.

(3) They respect your plans after grad school and don't fuss about making you a clone of themselves.

Some would disagree with or would downplay (3). However, you may want to pursue a route in the long term that your advisor hasn't explored. You may want a Research I career, or you may want career at a liberal arts institution, or you may want to leave academia altogether after grad school. You may change your mind a lot. A good advisor will roll with it and the mathematics at hand should be the focus of discussions. Only you know what is best to make you happy.

Moreover, your advisor shouldn't be the only person from whom you seek advice. Building up a network of other faculty and grad students with whom you can discuss mathematics and professional development is essential. Don't isolate yourself.

**Diaz-Lopez:** *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

**Walton:** I have a wonderful support network that I reach out to often—the most immediate one includes my husband and my two pups (see Figure 1). Yes, I include the pups—they couldn't care less if I prove a theorem or not. Although some days I think they have a better shot at this than I do! In general, I depend on people who selflessly have my best interests in mind.

It can be discouraging when I have to deal with people with whom I am not on the same page. Some would call these "difficult people," but it could be the case that I'm the difficult one—who knows? In any case, having another ear, like my husband's, or a friend's, to help me navigate these situations is extremely helpful. Fortunately, on most days, I don't feel discouraged because of such a situation.

Figure 1. Walton with one of her dogs, Mr. Mischief Maker.



Figure 1. Walton with one of her dogs, Mr. Mischief Maker.

I actually feel the most discouraged when I'm exhausted and feel like I haven't spent enough time or energy doing math, when I've bitten off more than I can chew with other things that grad students shouldn't worry about. Because there's plenty of time later to worry about conference organizing, grant proposals, and wonderful administrative tasks! Again, having family or friends either help me or provide me with the space to re-prioritize my to-do list is key.

I would not say that I do not get discouraged doing math itself, but it pales in comparison to discouragement occurring in life matters.

**Diaz-Lopez:** *You have won several honors and awards. Which one has been the most meaningful and why?*

**Walton:** Being awarded a Sloan research fellowship was absolutely fantastic—a feather in the cap of not only me but, in my mind, of all (future) Black female mathematicians. Of course, we're not a monolithic group, but when one of us receives a great honor, like Sylvia Bozeman being appointed to President Obama's Committee on the National Medal of Science, or Talithia Williams hosting *NOVA Wonders* on PBS, or Carla Cotwright-Williams serving as an AMS Congressional Fellow, I'm very proud. It's a great personal boost to me and very motivating professionally.

**Diaz-Lopez:** *You have co-organized a BIRS workshop for Women in Noncommutative Algebra and Representation Theory (WINART) and manage the website [women-in-calg-repthy.org](http://women-in-calg-repthy.org). What motivated you to get involved with this and how has it been received? Are there any future activities?*

**Walton:** In short, I decided to co-organize the WINART workshop (which was held at BIRS in April 2016, Figure 2) because I needed it for myself.



**Figure 2: Women in Noncommutative Algebra and Representation Theory workshop at BIRS in April 2016.**

I wanted an experience at a research conference where I was not an outsider gender-wise, and ethnicity-wise (we had a significant number of women of color at the previous WINART workshop). I also thought that other women could benefit from the experience as other participants have in other research collaboration conferences for women at BIRS. Our first event was very successful over-

all and it resulted in several publications, preprints, collaborative works in progress, and even friendships in a short span of time. I learned a great deal of mathematics from being able to work with my collaborators

Adriana Mejía Castaño, Susan Montgomery, Sonia Natale, and María Vega—techniques that I will be able to incorporate into future projects. It was a super stimulating experience and we are in the process of planning an event for 2019, hopefully to be held at BIRS again if our proposal is accepted.

**Diaz-Lopez:** *If you could recommend one book to graduate students, what would it be?*

**Walton:** My advisor, Toby Stafford, raised all of his students on Goodearl and Warfield's *An Introduction to Noncommutative Noetherian Rings*—it's an outstanding text. For graduate students across fields, I would advise reading Terry Tao's article "What Is Good Mathematics?" There's a ton of ways one can be good at math and there isn't a linear hierarchy of X-is-better-than-Y, at least not in my mind. The quicker grad students learn that they are coming in with their own unique bag of tricks, and that there's no point in comparing oneself to others, the better!

**Diaz-Lopez:** *Any final comments or advice?*

**Walton:** Actually, the only person that you should compare yourself with is "you yesterday." Keep pushing to be better, and keep re-defining "better" in the context of what makes you happy!

---

*Keep re-defining  
"better" in the  
context of what  
makes you happy!*

---

## Photo Credits

Interviewee photo courtesy of Ryan S. Brandenburg/Temple University.

Figure 1 courtesy of Chelsea Walton.

Figure 2 courtesy of Banff International Research Station.



**Alexander  
Diaz-Lopez**

## ABOUT THE INTERVIEWER

**Alexander Diaz-Lopez**, having earned his PhD at the University of Notre Dame, is now assistant professor at Villanova University. Diaz-Lopez was the first graduate student member of the *Notices* Editorial Board.

# ? WHAT IS...

## the Amplituhedron?

Jacob Bourjaily and Hugh Thomas

Communicated by Cesar E. Silva

The amplituhedron is a space whose geometric structure conjecturally determines scattering amplitudes in a certain class of quantum field theories. When it was introduced in 2013 by Arkani-Hamed and Trnka [1], *Quanta Magazine*<sup>1</sup> reported that “Physicists have discovered a jewel-like geometric object that dramatically simplifies calculations of particle interactions and challenges the notion that space and time are fundamental components of reality...” In this short article, we will not get as far as challenging the notions of space and time, but we will give some explanation as to the nature and significance of the amplituhedron both in physics and mathematics—where it arises as a natural generalization of the subject of total positivity for Grassmannians.

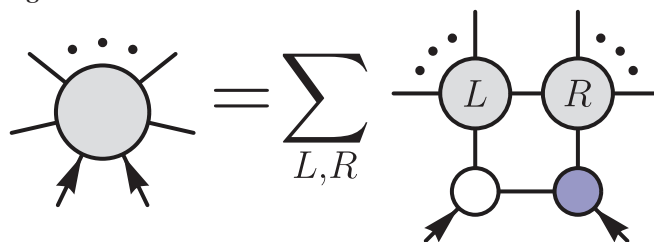
*Scattering amplitudes are the bread and butter of quantum field theory.*

Let us begin with the origin of these ideas in the physics of scattering amplitudes. For any particular quantum field theory, the probability for some number of particles to ‘scatter’ and produce some number of others is encoded by a function called the (*scattering*) *amplitude* for that process. These functions depend on all the observable numbers labeling the states involved in the process—in particular, momenta and helicity. Amplitudes can (often) be determined perturbatively according to the Feynman ‘loop’ expansion. At leading order, they are rational

functions of the momenta; and at the ‘ $\ell$  loop’ order of perturbation, they are integrals over rational forms on the space of  $\ell$  internal loop momenta.

Scattering amplitudes are the bread and butter of quantum field theory; but they are notoriously difficult to compute using the Feynman expansion. Moreover, individual Feynman diagrams depend on many unobservable parameters which can greatly obscure the (often incredible) simplicity of amplitudes’ mathematical structure.

A major breakthrough in physicists’ understanding of (and ability to compute) scattering amplitudes came in 2004 with the discovery of recursion relations for tree-level ( $\ell = 0$ ) amplitudes by Britto, Cachazo, Feng, and Witten [3]. They were first described graphically as in Figure 1:



**Figure 1. Recursion relations for amplitudes, encoded in terms of bi-colored, trivalent graphs.**

The precise meaning of these graphs will not be important to us, but a few aspects are worth mentioning. In Figure 1, amplitudes are represented by grey circles, and any two legs may be chosen for the recursion (indicated by arrows). Thus, the BCFW recursion relations do not provide any particular representation of an amplitude in terms of graphs, but lead to a wide variety of inequivalent representations according to which two legs are chosen at every subsequent order of the recursion. The variety of possible recursive formulae obtained was reminiscent of different ‘triangulations’ of a polytope—an analogy that would be made sharp by the amplituhedron.

For several years after their discovery, the primary importance of the recursion relations followed from

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<sup>1</sup>Natalie Wolchover, *A Jewel in the Heart of Quantum Physics*, Quanta Magazine, September 2013.

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the fact that they yield incredibly more compact—and physically more transparent—representations for tree-level amplitudes. Processes that would have required more Feynman diagrams than atoms in the universe could now be represented by a single diagram. Another surprising feature of these relations was the discovery that individual terms enjoy much more symmetry than anyone had anticipated. These new symmetries were shown to hold for amplitudes, and were later identified with a Yangian Lie algebra.

Another key breakthrough came in 2009, when physicists discovered that the graphs arising in Figure 1 could be related to volume-forms on certain sub-varieties of  $\text{Gr}_{\geq 0}(k, n)$ —the “nonnegative” portion of the Grassmannian of  $k$ -dimensional subspaces of  $\mathbb{R}^n$ , to be defined momentarily. Here,  $n, k$  are determined by the graph:  $n$  is the number of external edges, and  $k = 2n_B + n_W - n_I - 2$  for a graph with  $n_B, n_W$  blue and white vertices, respectively, and  $n_I$  internal edges. (The correspondence between these graphs and the totally nonnegative Grassmannian was fully understood by mathematicians, such as Postnikov, considerably before physicists stumbled upon it independently.) The physical implications of this correspondence, together with its generalization to all orders of perturbation, can be found in [2].

Given the correspondence between the terms generated by BCFW (and its all-orders generalization) and the Grassmannian, it was natural to wonder if a more intrinsically geometric picture existed. Specifically, can the recursion relations be literally viewed as ‘triangulating’ some region in the Grassmannian? And if so, can this space be defined *intrinsically*—without reference to how it should be triangulated? In 2013, Arkani-Hamed and Trnka proposed the amplituhedron as the answer to both questions for the case of amplitudes in planar, maximally supersymmetric Yang-Mills theory [1].

Let us now turn to giving a precise mathematical definition of the amplituhedron, in the 0 loop “tree” case. (The  $\ell$  loop amplituhedron for  $\ell > 0$  has a definition which is similar but somewhat more complicated; we will not discuss it further.) Let  $M \in \text{Mat}(k \times n)$  be a  $k \times n$  matrix of real numbers, with  $k \leq n$ . As we recall from linear algebra, the rows of  $M$  determine a subspace of  $\mathbb{R}^n$  called its *row space*. If the row space is  $k$ -dimensional (the maximum possible), we say that  $M$  has full rank. Two full-rank matrices  $M_1$  and  $M_2$  have the same row space if for some  $K$  in the space of  $k \times k$  invertible matrices,  $K.M_1 = M_2$ . Clearly, any  $k$ -dimensional subspace of  $\mathbb{R}^n$  arises as the row space of some full-rank matrix; so the collection of all  $k$ -dimensional subspaces can be described as  $\text{Mat}_{\text{full}}(k \times n)/GL(k)$ . This is the *Grassmannian*  $\text{Gr}(k, n)$ .

The next ingredient required to define the amplituhedron is the notion of positivity in the Grassmannian. A maximal minor of a matrix  $M \in \text{Mat}(k \times n)$  is the determinant of the  $k \times k$  submatrix obtained from taking a subset of  $k$  of the columns, in the order they appear in the matrix. The totally positive Grassmannian, denoted  $\text{Gr}_{>0}(k, n)$ , is the subspace of the Grassmannian for which

all maximal minors have the same sign. The totally nonnegative Grassmannian,  $\text{Gr}_{\geq 0}(k, n)$ , is defined in the same way, except that we also allow some minors to vanish.

The totally nonnegative Grassmannian was studied by Postnikov as a concrete instance of Lusztig’s theory of total positivity for algebraic groups. It is topologically equivalent to a ball, and has a beautiful cell structure that can be indexed by decorated permutations (permutations of  $S_n$  with one extra bit of information for each fixed point). Importantly, cells in this ‘positroid’ stratification can be endowed with cluster coordinates and a natural volume form.

We now have everything required to define the tree amplituhedron. It is indexed by three integers,  $n, k$ , and  $d$ , and a totally positive  $n \times (d + k)$  matrix  $P$ . From the point of view of physics,  $n$  is the number of particles,  $k$ , defined above in the context of graphs, encodes the total helicity flow,  $d$  is the space-time dimension, and  $P$  encodes the momenta and helicities of the particles. Let  $M \in \text{Gr}_{\geq 0}(k, n)$  vary. The amplituhedron  $\mathcal{A}$ , as defined by Arkani-Hamed and Trnka [1] is simply the total image locus,

$$(1) \quad \mathcal{A} = M.P,$$

inside  $\text{Gr}(k, d + k)$ . Notice that  $M.P$  is a  $k \times (d + k)$  matrix, automatically having full rank, so  $M.P$  really does define a point in  $\text{Gr}(k, d + k)$ .

To help build intuition, it is useful to consider some special cases. If  $n = d + k$ , then the fact that  $P$  is totally positive implies it is invertible, so we may as well take it to be the identity matrix; in this case, the amplituhedron is simply  $\text{Gr}_{\geq 0}(k, d + k)$ . Another simple case arises when  $k = 1$ , for which  $M$  is a totally nonnegative  $1 \times n$  matrix—i.e. a vector of nonnegative numbers, not all zero. In this case,  $\mathcal{A}$  consists of all non-negative linear combinations of the rows of  $P$ , and is thus a convex polytope in projective space. The fact that  $P$  is totally positive means that  $\mathcal{A}$  is what is called a cyclic polytope; these have many interesting extremal properties. In this case, repeatedly applying the BCFW recursion literally triangulates this polytope. A point  $Y$  in projective space determines a hyperplane  $Y^*$  in the dual projective space, and similarly, the amplituhedron, being in this case a polytope in projective space, determines a dual polytope  $\mathcal{A}^*$ . The volume form on  $\mathcal{A}$  at a point  $Y$  is given by the volume of  $\mathcal{A}^*$  when  $Y^*$  is the hyperplane at infinity.

In general, the volume form on  $\mathcal{A}$  can conjecturally be defined by the fact that it has logarithmic singularities on the boundaries of  $\mathcal{A}$ . It can also be calculated explicitly by pushing forward the volume form on BCFW cells. However, it would be desirable to have an explicit, intrinsic description, as given above for  $k = 1$ . This would presumably require an answer to the question, “What is the dual amplituhedron?”—something that we must leave for the future.

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Photo of Hugh Thomas courtesy of Charles Paquette.

### ABOUT THE AUTHORS

Jacob Bourjaily enjoys reading poetry in languages he doesn't know.



Jacob Bourjaily

Hugh Thomas enjoys translating poetry from languages he doesn't know.



Hugh Thomas

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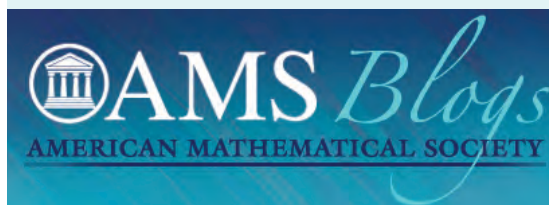
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## Graduate Student Blog *by and for math graduate students*



The AMS Graduate Student Blog, by and for math graduate students, includes puzzles and a variety of interesting columns, such as this one from October 2017. [blogs.ams.org/mathgradblog](http://blogs.ams.org/mathgradblog).

# Machine-Checked Proof

*by Jacob Gross*

*"In my view, the choice between the conventional process by a human referee and computer verification is as evident as the choice between a sundial and an atomic clock in science."*

—Tom Hales

*"The rapid advance of computers has helped dramatize this point, because computers and people are very different. For instance, when Appel and Haken completed a proof of the 4-color map theorem using a massive automatic computation, it evoked much controversy. I interpret the controversy as having little to do with doubt people had as to the veracity of the theorem or the correctness of the proof. Rather, it reflected a continuing desire for human understanding of a proof, in addition to knowledge that the theorem is true."*

—Bill Thurston

A machine-checked proof is a proof written in a piece of software called a 'proof assistant' which ensures the proof complies with the 'axioms of mathematics' and the rules of logic. The question of the significance of computers in proving theorems can be polarizing (and for good reason). The quotations above represent some points of view relevant to this topic. In this post we will try answer three questions:

1. What exactly is a computer-assisted proof?
2. What are the advantages and the drawbacks of using computers to prove theorems?
3. What should an interested person do to start learning to use proof assistants?

...

### ABOUT THE AUTHOR

**Jacob Gross** is a graduate student in geometry at Oxford University, working under the supervision of Dominic Joyce FRS, and supported by the Simons Collaboration on Special Holonomy in Geometry, Analysis, and Physics. His email address is [jacob.gross@maths.ox.ac.uk](mailto:jacob.gross@maths.ox.ac.uk).



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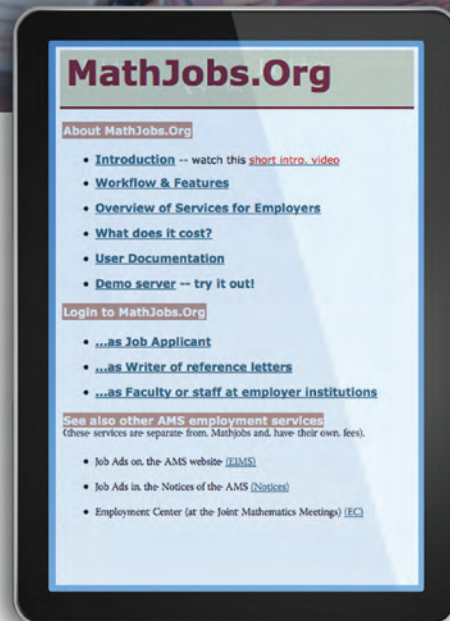
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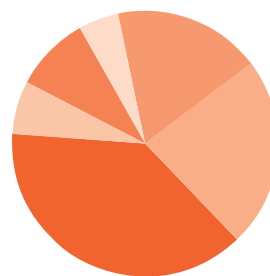
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# Preliminary Report on the 2016–2017 New Doctoral Recipients

*Amanda L. Golbeck, Thomas H. Barr, and Colleen A. Rose*

This report presents a preliminary profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2016 through June 30, 2017. As of November 28, 2017, not all departments have responded to the survey instrument through which data are collected, and a list of the nonresponding departments is presented below.

Data collected to this point show that 1,622 new PhDs were awarded by the 273 mathematical sciences departments that responded in time for this report. Some attributes of the data are:

- 1,148 Males                      · 810 US citizens                      · 88% of those employed are US employed
- 471 Females                      · 800 non-US citizens                      · 12% of those employed are Non-US employed
- 1,260 in Mathematics                      · 362 in Statistics/Biostatistics                      · 52% of those employed in the US are US citizens

Based on the data collected so far, it is likely that the final count of PhDs awarded during 2016–2017 will be slightly lower than the number (1,921) reported for 2015–2016.

Once data collection is concluded, a detailed final report will be published in the August 2018 issue of *Notices*.

## Doctoral Degrees Not Yet Reported

As of press time for this issue of *Notices*, the following departments had not yet responded to the survey. Every effort will be made to collect responses for inclusion in the New Doctoral Recipients Report. In order to be included in the final report, Doctorates Granted survey forms should be sent no later than March 3, 2018. Departments yet to respond can obtain copies of the Doctorates Granted survey forms by visiting [www.ams.org/annual-survey/surveyforms/phds-awarded](http://www.ams.org/annual-survey/surveyforms/phds-awarded), emailing [ams-survey@ams.org](mailto:ams-survey@ams.org), or calling 1-800-321-4267, ext. 4189.

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Columbia University, Applied Mathematics  
Columbia University  
Delaware State University  
Georgia State University  
Howard University  
Rensselaer Polytechnic Institute  
Temple University  
University of California, Merced, School of Natural Sciences  
University of Cincinnati

University of Houston  
University of Pittsburgh  
University of Puerto Rico, Río Piedras  
University of Vermont  
University of Washington  
Utah State University

### Statistics Departments

Carnegie Mellon University  
Colorado State University  
George Washington University  
North Carolina State University

*Amanda L. Golbeck is Associate Dean for Academic Affairs and Professor of Biostatistics in the Fay W. Boozman College of Public Health at University of Arkansas for Medical Sciences. Thomas H. Barr is AMS Special Projects Officer. Colleen A. Rose is AMS Survey Analyst.*

**Statistics Departments**

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 Northwestern University  
 Pennsylvania State University, University Park  
 Rice University  
 Stanford University  
 Temple University  
 University of California, Davis  
 University of California, Los Angeles  
 University of California, Riverside  
 University of California, Santa Barbara  
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# MEMBER SPOTLIGHT

The AMS turns the spotlight on members to share their experiences and how they have benefited from AMS membership. If you are interested in being highlighted or nominating another member for the spotlight, please contact the Membership Department at [membership@ams.org](mailto:membership@ams.org).



## **DARRYL H. YONG**

Professor, Department of  
Mathematics, Harvey Mudd  
College, Claremont, CA.  
AMS member since 1995.

“My co-authors and I are excited that AMS is publishing the [IAS/Park City Mathematics] book series and that it might be a helpful resource to mathematics education faculty, practicing mathematics teachers, and homeschooled families. The work that we do at the Park City Mathematics Institute is unique, and we’ve tried to find a way to take the experience that we provide to mathematics teachers during the three-week program and distill it into a form that is usable by others. I personally find this work very gratifying because I was not trained in these areas of mathematics and the process of putting these materials together allows me to experience mathematics anew with wonder and joy.”



## Add Impact Factors to Journal Backlog Report

Each year, in the month of November, the *Notices* publishes an article on backlog of important mathematical journals. We often need our work to be published in a certain time frame and this information becomes very useful to us in selecting a particular journal. I think that adding impact factors of journals to this article will enhance its value. Certain other analytics may also be added.

—Vivek Sahai  
Lucknow University  
sahai\_vivek@hotmail.com

(Received September 27, 2017)

## Community Outreach a Moral Imperative

Community outreach is one of our greatest responsibilities as professional mathematicians. It is a moral imperative.

I began my involvement ten years ago when my math department at University of North Carolina Asheville reviewed our department goals and felt there was an opportunity for significant improvement in community outreach. I raised my hand to take on that challenge.

I cast a wide net for community partners, because math outreach is really a commitment to building community. After several months of planning, an informal group met on campus to talk about Asheville's math needs, to identify existing resources, and to look for opportunities to create new resources. Attending were math educators from the local school systems, from the community college, and from the private liberal arts school twenty minutes away. There were also representatives from the local hospital, the Chamber of Commerce, the YWCA, the youth prison, the Department of Social Services, and a local tutoring coalition. Colleagues from the math department, engineering, economics, education, and physics joined us, along with two members of the Chancellor's cabinet. We continued meeting quarterly for a year and a half. Many projects flowed from these discussions, including a math science partnership grant, a community Pi Run, math activity booths at street parties, making paper snowflakes at local businesses, starting a math leadership club in public housing, and much more.

By exploring how math connects to each of our organizations, we discovered that math literacy was more important to the whole Asheville community than any one of us realized separately. We thought it was crucial to make these connections more visible. In February 2008, we organized a Math Literacy Summit to be the vehicle

for this message. We identified five themes for the summit that make the connections between math and daily life concrete: workforce development, personal finance, public health, social justice, and the arts. We ended up running the summit four times over the years, impacting hundreds of Asheville educators, parents, students, and business leaders.

Perhaps you have thought about math outreach and dismissed it as too much work. You would be right to think it requires effort, which needs to be around restructuring the expectations of our work environment and professional organizations. In every interaction, we have in our community, on and off campus, in the classroom and in the grocery store, we are unwitting ambassadors of mathematics as a subject and as a profession. We have an obligation to address our role as ambassadors. Most of us have little preparation for conversations about the importance of math and little experience building community relationships. The good news is that we can choose to be intentional about this role as ambassadors.

Math outreach requires creativity, humility, and a professional framework that supports advocacy. Build outreach into your department mission. Train your graduate students in community engagement. In your curriculum, create opportunities for service-learning classes, community-engaged scholarship, public lectures, and a visiting program for public schools. Get started. We often claim math is important to everyone. Let us act like it.

—Samuel R. Kaplan  
Department of Mathematics, UNC Asheville

(Received September 29, 2017)

## Review of *Combinatorial Algebra*...

In his review of my book (*Bulletin AMS*, October, 2017), Laurent Bartholdi made two wrong attributions of results and theorems from my book. Since he made similar wrong statements before, I am sure that these are not misprints. Since this is a review of my book, one might get an impression that the book has wrong attributions too; hence I am writing this letter.

1. Bartholdi writes that Grigorchuk simplified an example by Aleshin. That is not true. The two groups, Aleshin's and Grigorchuk's, were constructed independently in two completely different ways. Moreover Grigorchuk had an uncountable set of groups. The fact that one of these groups is related (and commensurable) to the Aleshin group was discovered later by Merzlyakov. In my book,

there are correct references to Aleshin, Grigorchuk, and Merzlyakov.

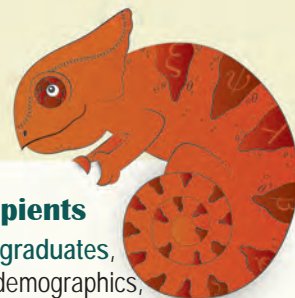
2. Bartholdi writes that in my book, I present Adian's solution of the Burnside problem (existence of finitely generated infinite groups of bounded exponent). That is not true. First of all the problem was solved by Novikov and Adian (that is the order of authors of their paper), and second I present Olshanskii's proof of the Novikov-Adian theorem. I also mention that Olshanskii proved a weaker form of the theorem because his exponent is much higher.

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(Received October 4, 2017)

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## Call for Nominations for CRM Aisenstadt Prize

The Centre de Recherches Mathématiques (CRM) solicits nominations for the André Aisenstadt Mathematics Prize, awarded to recognize talented young Canadian mathematicians. The deadline for nominations is **March 1, 2018**. See: [crm.umontreal.ca/prix/prixAndreAisenstadt/prix\\_attributionAA\\_an.shtml](http://crm.umontreal.ca/prix/prixAndreAisenstadt/prix_attributionAA_an.shtml)

—From a CRM announcement

## Call for Applications for the Sixth Heidelberg Laureate Forum

The Heidelberg Laureate Forum Foundation (HLFF) will hold its sixth forum September 23–28, 2018, bringing together winners of the Abel Prize and the Fields Medal, both in mathematics, as well as the Turing Award and the Nevanlinna Prize, both in computer science. Young researchers interested in attending should submit applications online at [application.heidelberg-laureate-forum.org](http://application.heidelberg-laureate-forum.org) by **February 9, 2018**; see [www.heidelberg-laureate-forum.org/](http://www.heidelberg-laureate-forum.org/) for more information.

—From an HLFF announcement

## News from the Mathematical Biosciences Institute

The Mathematical Biosciences Institute (MBI) Online Colloquium series is available as an online interactive event or through on-demand streaming; individuals can watch talks and ask questions interactively. For more information and to register, see [tinyurl.com/y795yhb1](http://tinyurl.com/y795yhb1).

—From an MBI announcement



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# From Campus to Congress: Navigating Policy, Procedure, and Politics as an AMS Congressional Fellow

*Catherine Paolucci*

Each year, the AMS funds a Congressional Fellowship within the Science and Technology Policy Fellowship program run by the American Association for the Advancement of Science (AAAS). Approximately 35 Congressional Fellows are sponsored by professional organizations in a range of scientific fields, including chemistry, physics, geology, engineering, meteorology, and others. Fellows spend a year working for a member of Congress or a Congressional committee, with the aim of bringing scientific perspectives into the legislative process.

During the 2016–2017 academic year, I had the honor of serving as the 12th AMS Congressional Fellow. The fellowship began with an intensive orientation designed to help bridge the transition from academia to government service. Experts shared insight on all three branches of government, Congressional procedures, legislative processes, the federal budget, and appropriations. A placement process followed, and I was excited to accept a position in the office of Senator Al Franken (MN).

I joined Senator Franken's staff as an Education Policy Fellow, with a specific focus on issues in higher education. Even with a strong education background at both the K-12 level and in higher education, I quickly realized how much there was still to learn about education policy!

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The mutually beneficial nature of the fellowship allowed me to offer insight from the field, while gaining valuable first-hand experience navigating policy, procedure, and politics on Capitol Hill.

As a member of the Senator's staff, my daily responsibilities included researching and analyzing legislation to help advise the Senator, attending briefings on current and emerging issues, writing memos to brief the Senator, supporting him at committee hearings and events, drafting letters on education issues, meeting with constituents and stakeholders, and liaising with the Department of Education. Working for a Senator on the Health, Education, Labor, and Pensions (HELP) Committee also meant collaborating with other HELP Committee members' staff, and gaining first-hand experience with the critical role of Congressional committees.

The Presidential transition played a significant role in shaping a large portion of my work. The first few months of the new Administration were devoted to hearings and votes on cabinet nominees and reviewing regulations that were eligible to be overturned by the Congressional Review Act (CRA). Working for a Senator in the minority meant that much of what we did during this period was reactive. Our efforts were focused on helping the Senator respond to things that were happening in a way that was important to him and his constituents, including nominees for federal roles, executive orders, budget proposals, appropriations, and regulation changes.

The latter months of my fellowship offered more opportunities to help write and introduce legislation, including two new bills supporting career and technical education and teacher education. Through these efforts, I learned

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*The Presidential transition played a significant role in shaping a large portion of my work.*

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what it takes to reach agreement on a bi-partisan bill and how to navigate the politics of policy to help ensure that a bill has the best possible opportunity to become law.

The opportunity to engage directly in so many aspects of Congress meant learning something new every day! The following are some key lessons that capture some of this valuable insight:

### **Lesson #1: Politics and procedure are driving forces in policy.**

As academics, we are trained to carefully craft our research, writing, teaching, and public statements to ensure that they are grounded in theory and informed by research. The biggest misconception I brought to my fellowship was that the same would apply to policy. In Congress, even the most basic ideas are almost always political. I think the hardest lesson was learning just how differently people view and make sense of important issues. Part of this lesson was realizing that clinging to the way I understand something will not allow me to successfully help others to share my vision. Instead, it's often more effective to understand what motivates them, and find a way to frame an issue or proposed solution that aligns with their value system.

In addition, as a mathematics community, we broadly embrace the notion that an argument based on evidence or proof will be sufficiently compelling to those with whom we are discussing an issue. Working with Congressional Fellows from over 15 other scientific fields convinced me that this is also true for the broader scientific community. However, for those outside of our disciplines, and particularly those who are positioned to make decisions impacting the future of our work, the facts must be personalized and presented in way that will resonate. This brings me to my next lesson.

### **Lesson #2: Constituents and key stakeholders in a Member's state or district have their ear.**

As simple as it seems, it is essential to remember that the job of a Member is to represent the people in their state or district. In fact, this is also their key to getting re-elected. So, constituents are often in the best position to influence the policy decisions and priorities of a particular Member. This has two key implications. First, if an issue is important to a national community, localized grassroots efforts in which people engage directly with their own Senators and Representatives can be powerful. Second, when national societies, agencies, or organizations advocate as a voice for their members, it is critical that they customize their message. For instance, when approaching Senators about an issue, it is essential to include details about the issue's specific impact on their state's economy, job market, education system, health care, etc. This is often an effective way to foster bipartisan collaboration.

### **Lesson #3: Committee assignments impact a Member's influence on particular issues.**

While Members have wide-ranging priorities, their capacity to impact an issue area is greater if they are on the

committee that deals with this issue. During my time in Senator Franken's office, his service on the Senate HELP Committee, Judiciary Committee, Energy and Natural Resources Committee, and Indian Affairs Committee placed him at the center of critical conversations about issues in education, healthcare, energy, and the environment. Committee members are the ones who ask critical questions of nominees or witnesses during hearings, and they have the opportunity to shape and amend proposed legislation before sending it out for a full vote.

### **Lesson #4: Congress does a lot more than most people think.**

While passing legislation is one important job of Congress, an equally important part of their job is preventing potentially harmful bills from becoming law. Legislation is often dense and written in legal language that does not make for light reading! Plus, there are thousands of bills introduced each Congress, making it virtually impossible for every Member to know what is in every piece of proposed legislation. This is why it is important for Members to not just hear from staff, constituents, and advocates about the bills they should support, but also hear about any potentially harmful impacts of proposed legislation that may be nuanced or buried within a seemingly good message.

*the notion that an argument based on evidence or proof will be sufficiently compelling*

Members can also be the voice of their constituents through formal letters to the President, federal agencies, the Supreme Court, large corporations, etc. These include individual letters on a local issue, state delegation letters from all House and Senate members from a particular state, committee letters from members of a Senate or House committee, caucus letters from a large portion of the members of a specific party, and bipartisan letters from members on both sides of the aisle. There are also appropriations letters written to the Appropriations Committee with funding requests for federal agencies and programs.

### **Lesson #5: The AMS Congressional Fellowship is a powerful way to enhance your career and bring your professional experience and mathematical perspectives to Congress.**

While opportunities to directly apply mathematical expertise vary by placement, in my position, I found myself constantly drawing on my years of experience working with students, teachers, and other colleagues in academic communities. During my time in the office, Senator Franken cosponsored a broad range of important legislation related to college access and affordability and led the development of new legislation supporting innovations in career and technical education and teacher education. It



## COMMUNICATION

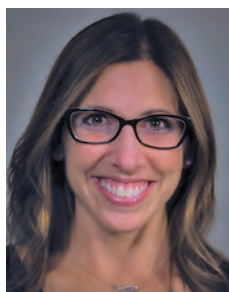
was extremely rewarding to work for a Senator who valued my background and experience and welcomed my voice in conversations about these issues.

In addition to my experience in the Senate, the fellowship also offered opportunities to become a more active and engaged member of the AMS. I am grateful to the AMS for this incredible opportunity and inspired by the AMS's commitment to strengthening the mathematics community's voice on important issues that will affect the future of mathematics research and education.

### Photo Credit

Photo of Catherine Paolucci by Chris Paolucci.

**EDITOR'S NOTE.** Applications for the AMS Congressional Fellowship from September 2018 to August 2019, with a stipend of US\$79,720, are due by February 15, 2018. See [www.ams.org/programs/ams-fellowships/ams-aaas/ams-aaas-congressional-fellowship](http://www.ams.org/programs/ams-fellowships/ams-aaas/ams-aaas-congressional-fellowship).



Catherine Paolucci

### ABOUT THE AUTHOR

**Catherine Paolucci's** research interests support program and policy development for mathematics teacher education. Her work has focused on the evaluation and development of teachers' mathematical knowledge, innovative ways to increase engagement in STEM education, and the role of outreach and social impact in teacher preparation.

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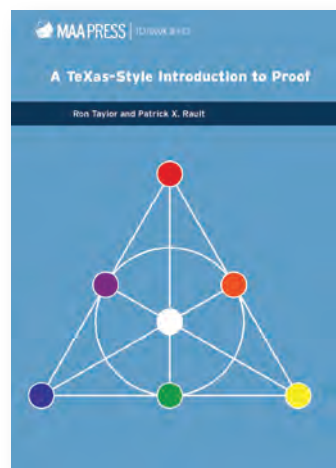
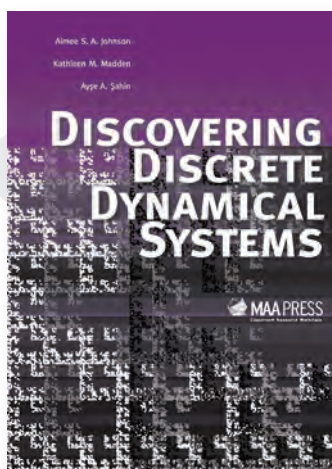
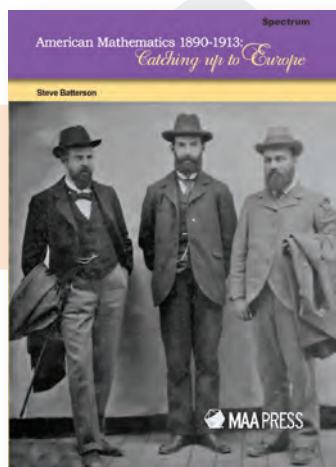
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SOCIETY

In the elections of 2017 the Society elected a president, a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee. The Society also considered two amendments to the AMS bylaws.



Photo courtesy of Brown University

## President

**Jill C. Pipher**  
*Brown University*

President Elect (February 1, 2018–January 31, 2019).  
President (February 1, 2019–January 31, 2021).  
Immediate Past President (February 1, 2021–January 31, 2022).



Photo courtesy of Ken Ono

## Vice President

**Ken Ono**  
*Emory University*

Term is three years  
(February 1, 2018–January 31, 2021).



Photo courtesy of Judy L. Walker

## Trustee

**Judy L. Walker**  
*University of Nebraska at Lincoln*

Term is five years  
(February 1, 2018–January 31, 2023).



## Members at Large of the Council

Terms are three years (February 1, 2018–January 31, 2021).



Photo courtesy of Erika T. Camacho

**Erika T. Camacho**

*Arizona State University*



Photo courtesy of Jennifer Murawski, MSRI

**Victor Reiner**

*University of Minnesota*



Photo courtesy of Brooke Shipley

**Brooke Shipley**

*University of Illinois at Chicago*



Photo courtesy of Bryce Vickmark

**Gigliola Staffilani**

*Massachusetts Institute of Technology*



Photo courtesy of Joseph Rabinoff

**Anthony Várilly-Alvarado**

*Rice University*

## Nominating Committee

Terms are three years (January 1, 2018–December 31, 2020).



Photo courtesy of Melissa Tottman

**Tara S. Holm**

*Cornell University*



Photo courtesy of Alice Silverberg

**Alice Silverberg**

*University of California at Irvine*



Photo courtesy of Shmuel Weinberger

**Shmuel Weinberger**

*University of Chicago*

## Editorial Boards Committee

Terms are three years (February 1, 2018–January 31, 2021).



Photo courtesy of Rod Searcy

**Akshay Venkatesh**  
*Stanford University*



Photo courtesy of Beatrice Farb

**Amie Wilkinson**  
*University of Chicago*

# Proposed Amendments to the AMS Bylaws

According to the Society's Bylaws, amendments are recommended by the AMS Council and approved by the members. To gain approval an amendment must have an affirmative vote of two-thirds of those voting.

The complete Bylaws can be viewed at [www.ams.org/about-us/governance/bylaws](http://www.ams.org/about-us/governance/bylaws).

In the 2017 election, both proposed amendments to the AMS Bylaws passed. In particular, the amendment effectively making all four Associate Secretaries *ex officio* members of Council was adopted. Below are the amendments, as they were proposed.

## First Proposed Bylaws Amendment

The AMS has the following officers: president, immediate past president/president elect, three vice presidents, secretary, four associate secretaries, treasurer, and associate treasurer. The current Bylaws specify that all of the officers are *ex officio* voting members of Council, except the associate secretaries. At any given time, only one of the associate secretaries is a voting member of Council.

The associate secretaries are responsible for all scientific aspects of the AMS meetings program. Each associate secretary arranges two sectional meetings per year. In addition, there is the annual Joint Mathematics Meeting and a yearly Joint International Meeting. The associate secretaries are responsible for these as well, with one associate secretary handling each meeting in a four-year rotation. The associate secretaries work closely with AMS staff, and represent the AMS to members, colleges and universities, and international mathematicians and societies.

The AMS was founded as the New York Mathematical Society in 1888 with the express purpose of holding meetings. It soon began publishing journals as well, with the Bulletin dating from 1891. Today, the AMS has a myriad of activities, but publishing and holding meetings remain as two of its most essential.

The proposed amendment addresses an imbalance in representation on the AMS Council. Currently the chairs of eight AMS journal editorial committees are voting members of the Council. However, a single associate secretary represents meetings of the Society on the Council.

With the proposed amendment, journal editorial committee chairs will have eight votes on Council and the associate secretaries responsible for meetings will have four, a more appropriate ratio. There will be 20 elected members and 15 appointed members, preserving the longstanding AMS policy that the majority of Council be elected members.

This change will allow the Council to benefit fully from the experience and perspectives of the associate secretaries while giving the associate secretaries a role in AMS governance that is commensurate with their contributions to the Society and their status as officers of the AMS.

The Council recommends that Article IV, Section 1 of the AMS Bylaws be changed as indicated below to include all four associate secretaries as *ex officio* members of the Council.

**Current Article IV, Section 1.** The Council shall consist of fifteen members at large and the following *ex officio* members: the officers of the Society specified in Article I, ***except that it shall include only one associate secretary***, the chairman of each of the editorial committees specified in Article III, any former secretary for a period of two years following the terms of office, and members of the Executive Committee (Article V) who remain on the Council by the operation of Article VII, Section 4.

The chairman of any committee designated as a Council member may name a deputy from the committee as substitute. *The associate secretary shall be the one charged with the scientific program of the meeting at which the Council meets except that at a meeting associated with no scientific meeting of the Society the secretary may designate the associate secretary.*

**Proposed Article IV, Section 1.** The Council shall consist of fifteen members at large and the following *ex officio* members: the officers of the Society specified in Article I, the chairman of each of the editorial committees specified in Article III, any former secretary for a period of two years following the terms of office, and members of the Executive Committee (Article V) who remain on the Council by the operation of Article VII, Section 4.

The chairman of any committee designated as a Council member may name a deputy from the committee as substitute.

### Second Proposed Bylaws Amendment

In reviewing the Bylaws in connection with the first proposed amendment, it was observed that the Bylaws are gender-neutral, with the exception of three instances.

The Council recommends the following purely editorial change to the Bylaws: The two appearances of the word “chairman” in Article IV, Section 1, and the one appearance of the word “chairman” in Article VII, Section 2, be changed to “chair.”





## Nominations by Petition

### VICE PRESIDENT OR MEMBER AT LARGE

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2018. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and procedures, which are described below.

### EDITORIAL BOARDS COMMITTEE

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and procedures, described below, should be followed.

### NOMINATING COMMITTEE

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and procedures, described below, should be followed.

### RULES AND PROCEDURES

Use separate copies of the form for each candidate for vice president, member at large, member of the Nominating or Editorial Boards Committees.

1. To be considered, petitions must be addressed to Carla D. Savage, Secretary, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA, and must arrive by 24 February 2018.
2. The name of the candidate must be given as it appears on the "American Mathematical Society" entry in the *Combined Membership List* ([www.ams.org/cml](http://www.ams.org/cml)). If the name does not appear in the list, as in the case of a new AMS member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate's mailing label or by the candidate contacting the AMS headquarters in Providence ([amsmem@ams.org](mailto:amsmem@ams.org)).
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the AMS entry in the *Combined Membership List* and on the mailing lists. No attempt will be made to match variants of names with the form of name in the AMS *CML* entry. A name neither in the *CML* nor on the mailing lists is not that of a member. (Example: The name Carla D. Savage is that of a member. The name C. Savage appears not to be.)
7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

# Nomination Petition for 2018 Election



The undersigned members of the American Mathematical Society propose the name of

\_\_\_\_\_ as a candidate for the position of (check one):

- ☐ **Vice President** (term beginning 02/01/2019)
- ☐ **Member at Large of the Council** (term beginning 02/01/2019)
- ☐ **Member of the Nominating Committee** (term beginning 01/01/2019)
- ☐ **Member of the Editorial Boards Committee** (term beginning 02/01/2019)

of the American Mathematical Society.

Return petitions by February 24, 2018 to:  
Secretary, AMS, 201 Charles Street, Providence, RI 02904-2213 USA

Name and address (printed or typed)

	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

# Inside the AMS

## AMS Committee on Education Meeting



**Katherine F. Stevenson**

In late October of 2017, I attended my first AMS Committee on Education meeting. This annual event is held in Washington, DC, and includes a miniconference on issues broadly related to math education. It strives to include talks by both mathematicians and representatives from related disciplines and national organizations. Here are a few highlights from 2017.

Doug Ensley, Deputy Executive Director of the Mathematical Association of America, discussed two graduate student training programs: CoMinDS and PIC Math programs. The purpose of CoMinDS is to “provide better access to resources for college mathematics instructor development and to create durable versions of the existing informal networks.” The intention is to create an infrastructure within the MAA to facilitate high-quality, teaching-related professional development for graduate students. By contrast, PIC Math strives to prepare mathematical sciences students for industrial careers by engaging them in authentic industrial research problems. The theme of industrial research was picked up later by Dan Spirn, University of Minnesota, who presented on EDT: Math to Industry Boot Camp. That program is “an intense six-week session designed to provide graduate students with training and experience that is valuable for employment outside of academia.”

Representatives of the National Science Foundation (NSF) presented on several programs related to opportunities for graduate training in mathematics, including research grant opportunities. The NSF shared that only 3.5 percent of the NSF Graduate Research Fellowship Program awards go to math and that these grants are distributed by “proposal pressure.” This means that the more math graduate students who apply, the more math grants will be awarded! Here is link for that program: [www.nsfgrfp.org](http://www.nsfgrfp.org).

Finally, several participants presented a forum organized by Su Doree, Augsburg College, on “Being a Professor at a Teaching-Oriented College or University: What Every Graduate Advisor/Director Needs to Know.” This panel

was well received and fielded many questions on this important option for new PhDs.

The meeting was vibrant and informative. If you are a chair or leader in your department, please consider marking your calendars for next October’s meeting. For more information, contact the AMS Washington Office at [amsdc@ams.org](mailto:amsdc@ams.org).

—Katherine F. Stevenson  
California State University

## From the AMS Public Awareness Office

**AMS and Mathematics at the National SACNAS Conference.** The AMS hosted a reception for the mathematics community and an exhibit at the 2017 National SACNAS Conference. See videos and highlights at [www.ams.org/meetings/sacnas2017-mtg](http://www.ams.org/meetings/sacnas2017-mtg).



**AMS Director of Education and Diversity Helen G. Grundman at the 2017 National SACNAS Conference Reception.**

**2016 Annual Report.** A PDF of this report, including Highlights of the Year, Financial Review, Contributors, and Facts & Figures, is at [www.ams.org/about-us/annual-reports/Annual-Report-2016-web.pdf](http://www.ams.org/about-us/annual-reports/Annual-Report-2016-web.pdf).

**Arnold Ross Lecture.** Ken Ono of Emory University gave the 2017 Arnold Ross Lecture, “Why does Ramanujan, ‘The Man Who Knew Infinity,’ matter?” at the Orlando Science Center in Orlando, Florida. Read about the lecture and the game of Who Wants to Be a Mathematician that followed at [www.ams.org/programs/students/wwtbam/ar12017](http://www.ams.org/programs/students/wwtbam/ar12017).



## AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions.

The following is a list of those addresses together with a description of the types of inquiries that should be made through each address:

**abs-coord@ams.org** for questions regarding a particular abstract or abstracts questions in general.

**acquisitions@ams.org** to contact the AMS Acquisitions Department.

**ams@ams.org** to contact AMS Headquarters in Providence, Rhode Island.

**amsdc@ams.org** to reach the AMS Washington Office about government relations and advocacy programs.

**amsfellows@ams.org** to inquire about the Fellows of the AMS.

**amsmem@ams.org** to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.

**ams-mrc@ams.org** for questions about the AMS Mathematics Research Communities.

**ams-simons@ams.org** for information about the AMS Simons Travel Grants Program.

**ams-survey@ams.org** for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.

**bookstore@ams.org** for inquiries related to the online AMS Bookstore.

**captions@ams.org** to submit entries for the Back Page Caption Contest in the *Notices*.

**classads@ams.org** to submit classified advertising for the *Notices*.

**cust-serv@ams.org** for general information about AMS products (including electronic products), to send address changes, or to conduct any general correspondence with the Society's Sales and Member Services Department.

**development@ams.org** for information about charitable giving to the AMS.

**eims-info@ams.org** to request information about Employment Information in the Mathematical Sciences (EIMS). For ad rates and to submit ads go to [eims.ams.org](http://eims.ams.org).

**emp-info@ams.org** for information regarding AMS employment and career services.

**eprod-support@ams.org** for technical questions regarding AMS electronic products and services.

**gradprg-ad@ams.org** to inquire about a listing or ad in the Find Graduate Programs online service.

**mathjobs@ams.org** for questions about the online job application service [Mathjobs.org](http://Mathjobs.org).

**mathcal@ams.org** for questions regarding posting on the Mathematics Calendar.

**mathprograms@ams.org** for questions about the online program application service [Mathprograms.org](http://Mathprograms.org).

**mathrev@ams.org** to send correspondence to Mathematical Reviews related to reviews or other editorial questions.

**meet@ams.org** to request general information about Society meetings and conferences.

**mmsb@ams.org** for information or questions about registration, housing, and exhibits for any Society meetings or conferences (Mathematics Meetings Service Bureau).

**mr-exec@ams.org** to contact the Executive Editor of Mathematical Reviews regarding editorial and related questions.

**mr-librarian@ams.org** to contact Mathematical Reviews regarding the acquisition and cataloging of the mathematical literature indexed in MathSciNet®.

**msn-support@ams.org** for technical questions regarding MathSciNet®.

**notices@ams.org** to send correspondence to the managing editor of the *Notices*.

**notices-ads@ams.org** to submit electronically paid display ads for the *Notices*.

**notices-booklist@ams.org** to submit suggestions for books to be included in the "Bookshelf" section of the *Notices*.

**notices-letters@ams.org** to submit letters and opinion pieces to the *Notices*.

**notices-whatis@ams.org** to comment on or send suggestions for topics for the "WHAT IS...?" column in the *Notices*.

**noti-backpage@ams.org** to submit original short amusing stories, math-related humor, cartoons, and ideas to the Back Page of the *Notices*.

**noti-gradsec@ams.org** to submit ideas to the Graduate Student Section of the *Notices*.

**opportunities@ams.org** for questions about submitting to the Opportunities webpage.

**paoffice@ams.org** to contact the AMS Public Awareness Office.

**president@ams.org** to contact the AMS president.

**prof-serv@ams.org** to send correspondence about AMS professional programs and services.

**promorequests@ams.org** to request AMS giveaway materials such as posters, brochures, and catalogs for use at mathematical conferences, exhibits, and workshops.

**publications@ams.org** to send correspondence to the AMS Publication Division.

**publisher@ams.org** to contact the AMS Publisher.

**pub-submit@ams.org** to submit accepted electronic manuscripts to AMS publications (other than Abstracts). See [www.ams.org/submit-book-journal](http://www.ams.org/submit-book-journal) to electronically submit accepted manuscripts to the AMS book and journal programs.

**reprint-permission@ams.org** to request permission to reprint material from Society publications.

**reviewcopies@ams.org** to request a book for review.

**royalties@ams.org** for AMS authors to direct questions about royalty payments.

**sales@ams.org** to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales and Member Services Department.

## NEWS

**secretary@ams.org** to contact the AMS secretary.

**student-serv@ams.org** for questions about AMS programs and services for students.

**tech-support@ams.org** to contact the Society's typesetting Technical Support Group.

**textbooks@ams.org** to request examination copies or inquire about using AMS publications as course texts.

**webmaster@ams.org** for general information or for assistance in accessing and using the AMS website.

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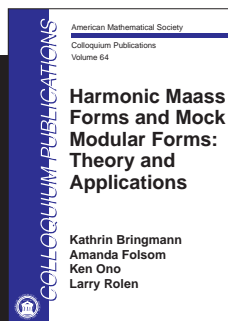
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# Mathematics People

## Tsimerman Receives Aisenstadt Prize



Jacob Tsimerman

JACOB TSIMERMAN of the University of Toronto has been awarded the 2017 André Aisenstadt Prize of the Centre de Recherches Mathématiques (CRM). The prize citation reads as follows: “Jacob Tsimerman is an extraordinary mathematician whose work at the interface of transcendence theory, analytic number theory and arithmetic geometry is remarkable for its creativity and insight.

“Jacob proved the existence of Abelian varieties defined over number fields that are not isogenous to the Jacobian of a curve. This had been conjectured by Katz and Oort and follows from the André-Oort conjecture. In joint work with several collaborators, Jacob established nontrivial bounds for the 2-torsion in the class groups of number fields. For quadratic fields, this can be done by genus theory but the general case was a complete mystery. With Bakker, Jacob has established geometric analogues of the Frey-Mazur uniform boundedness results for elliptic curves over function fields. Their approach has yielded powerful results with methods amenable to far more general applications.

“Among Jacob’s most notable accomplishments are his recent breakthroughs on the André-Oort conjecture. This conjecture about Shimura varieties, at the intersection of diophantine geometry and the arithmetic of automorphic forms, has been a central theme in arithmetic geometry for many years. Jacob already made important progress on it in his thesis, but in the last few years, working together with Pila, he created many of the technical tools for proving the case of the Siegel modular variety. There was still one piece that had to be completed on the size of Galois orbits. Jacob settled this final component in a brilliant short paper which showed that it follows from an average form of the Colmez conjecture. The latter has been proved by Andreatta, Goren, Howard and Madapusi-Perla, and independently by Yuan and Zhang, thus giving a complete unconditional proof of the André-Oort conjecture for this Shimura variety.

“Besides being a brilliant and innovative researcher, Jacob is also an excellent expositor and teacher. Moreover,

he has been active in Math Outreach through his work helping to train the Canadian team for the International Math Olympiad. He is currently the Chair of the Canadian IMO Committee.”

Jacob Tsimerman was born in Kazan, Russia, on April 26, 1988. He received his PhD in 2011 from Princeton University under Peter Sarnak, supported by an AMS Centennial Fellowship. He held a postdoctoral position at Harvard University. In 2014 he was awarded a Sloan Fellowship and joined the faculty at the University of Toronto. He was awarded the SASTRA Ramanujan Prize in 2015. He tells the *Notices*: “I love watching comedy, and Improv in particular, and go to the UCB [Upright Citizens Brigade] Theater in New York as often as I can.”

The Aisenstadt Prize is awarded yearly for outstanding achievement by a young Canadian mathematician no more than seven years past receipt of the PhD.

—From a CRM announcement

## CMS Prizes Given

The Canadian Mathematical Society has awarded several prizes for 2017.



Richard Hoshino

RICHARD HOSHINO of Quest University has received the 2017 Adrien Pouliot Award “for significant and sustained contributions to mathematics education in Canada.” He founded the Nova Scotia High School Math League, an outreach program that reaches thousands of students each year. He also served as the Director of the CMS National Camp for the top grade 9 and 10 high school students in Canada from 1999 to 2004

and led the Dalhousie Math Circles Outreach Program from 2002 to 2004. He has contributed problems to many mathematics competitions and wrote a novel for young people titled *The Math Olympian*. He has given frequent talks at high schools and led professional development workshops for high school math teachers. He is an active member of the Canadian Mathematics Education Study Group.



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Joseph Khoury

at many levels, focused on improving the general community's perception of mathematics and mathematics education." He has served on and chaired a number of CMS committees and has taken on both short-term challenges and long-term projects to benefit the society.



Alan Beardon

forms correspond to the seven classic frieze groups. He is interested in many areas of pure mathematics, and particularly in applying ideas that cross subject boundaries.

JOSEPH KHOURY of the University of Ottawa received the 2017 Graham Wright Award for Distinguished Service. The award recognizes sustained and significant contributions to the Canadian mathematics community, particularly through involvement with CMS. Khoury's contributions to the mathematical community range from committee service to writing books to developing "successful and sustained outreach activities

ALAN BEARDON of the University of Cambridge is the recipient of the 2017 G. de B. Robinson Award of the Canadian Mathematical Society (CMS) for his paper "Non-discrete Frieze groups" published in the *Canadian Mathematical Bulletin* 59 (2016). The paper deals with groups of real isometries which map the real number line to itself, with a particular focus on studying the conjugacy classes of the groups whose discrete

—From a CMS announcement

## Schoen Awarded Hopf and Schock Prizes



Richard Schoen

important mathematical techniques and new methods in the field of differential geometry, which turned out to have dramatic applications in the theory of general relativity, just at the border of maths and physics." The Hopf Prize is awarded every two years at ETH Zurich and honors outstanding scientific achievements in the field of pure

RICHARD SCHOEN of the University of California Irvine has been awarded the 2017 Heinz Hopf Prize of ETH Zurich and the 2017 Rolf Schock Prize for his work in differential geometry and geometric analysis. The citation for the Hopf Prize reads in part: "Richard M. Schoen is a bridge-builder between physics and mathematics, and has enriched the theory of relativity with his proofs and geometric methods." He is "a researcher who has introduced im-

mathematics. Schoen delivered the Hopf Lectures on "How Curvature Shapes Space" in October 2017. The prize carries a cash award of 30,000 Swiss francs (approximately US\$30,000).

The Rolf Schock Prize in Mathematics was awarded to Schoen for "groundbreaking work in differential geometry and geometric analysis, including the proof of the Yamabe conjecture, the positive mass conjecture, and the differentiable sphere theorem." The citation reads in part: "Schoen works in the field of geometric analysis, which he and Shing-Tung Yau founded in the 1970s and 80s. It studies geometry through nonlinear partial differential equations. Development in and around geometric analysis has strikingly transformed large parts of mathematics and been a leading theme for thirty years, including in areas such as gauge theory in 4-dimensional topology (possible structures in space-time), the Floer-Gromov-Witten theory for pseudoholomorphic curves (closely linked to physics' string theory), Ricci and mean curvature flow (proof of the Poincaré's conjecture). Schoen has produced stunning results in this area from the very beginning. His work is characterised by outstanding technical skill and a clear vision of geometric relevance." The Schock Prize carries a cash award of 500,000 Swedish krona (approximately US\$60,000).

Schoen received his PhD in 1977 from Stanford University under Leon Simon and Shing-Tung Yau. He has held positions at the University of California Berkeley, New York University, the University of California San Diego, and Stanford University. His honors include a MacArthur Fellowship (1983), the Bôcher Memorial Prize (1989), the Lobachevsky Prize (2017), and the 2017 Wolf Prize. He is a vice president of the AMS.

—Elaine Kehoe

## Simon Awarded Heineman Prize



Barry Simon

BARRY SIMON of the California Institute of Technology has been awarded the 2018 Dannie Heineman Prize for Mathematical Physics by the American Institute of Physics (AIP) and the American Physical Society (APS). According to the prize citation, Simon was honored "for his fundamental contributions to the mathematical physics of quantum mechanics, quantum field theory, and statistical mechanics, including spectral theory, phase transitions, and geometric phases, and his many books and monographs that have deeply influenced generations of researchers." The citation reads in part: "The accomplishments of Barry Simon's work form a collection of theoretical understandings ranging from anharmonic oscillators to phase transitions, accounting for a

citation that he describes as somewhat of a ‘kitchen sink.’ His mathematical models have deep and fundamental applications to almost all fields of physics from condensed matter to atomic and molecular physics.”

Simon received his PhD in physics from Princeton University in 1970 and has been affiliated with Princeton as well as Caltech throughout his career. He received the Poincaré Prize of the IAMP in 2012, the Bolyai Prize of the Hungarian Academy of Sciences in 2015, and the Leroy P. Steele Prize for Lifetime Achievement from the AMS in 2016. He is coauthor with Mike Reed of the four-volume *Methods of Modern Mathematical Physics*. He is a fellow of the APS, the American Academy of Arts and Sciences, and the AMS and was vice president of the AMS in 1988–1989.

The Heineman Prize recognizes outstanding publications in the field of mathematical physics. The prize carries a cash award of US\$10,000.

*Note.* See the two-part feature on Barry Simon’s work in the August and September 2016 *Notices*.

—From an AIP-APS announcement

## Munshi Awarded Infosys Prize

RITABRATA MUNSHI of the Tata Institute of Fundamental Research and the Indian Statistical Institute has been awarded the 2017 Infosys Science Foundation Prize in Mathematical Sciences “for his outstanding contributions to analytic aspects of number theory. Besides ingenious contributions to the Diophantine problem, he has established important estimates known as sub-convexity bounds for a large class of  $L$ -functions with methods that are powerful and original.” The Infosys Prizes recognize outstanding researchers and scientists in the fields of mathematical sciences, engineering and computer science, humanities, life sciences, physical sciences, and social sciences.

—From an Infosys announcement

## AWM Elects Inaugural Class of Fellows

The Association for Women in Mathematics (AWM) has announced the inaugural class in its new AWM Fellows Program to recognize individuals who have demonstrated a sustained commitment to the support and advancement of women in the mathematical sciences. The inaugural class consists of a group of mathematicians who have shown an unwavering commitment to promoting and supporting women in mathematics. The names and institutions of the new fellows follow.

- RODRIGO BANUELOS, Purdue University
- GEORGIA BENKART, University of Wisconsin, Madison
- LENORE BLUM, Carnegie Mellon
- SYLVIA BOZEMAN, Spelman College

- BETTYE ANNE CASE, Florida State University
- RUTH CHARNEY, Brandeis University
- CAROLYN GORDON, Dartmouth College
- MARY W. GRAY, American University
- HELEN G. GRUNDMAN, AMS and Bryn Mawr College
- RUTH HAAS, University of Hawaii at Manoa
- DEANNA HAUNSPERGER, Carleton College
- RHONDA J. HUGHES, Bryn Mawr College
- TRACHETTE JACKSON, University of Michigan
- NAOMI JOCHNOWITZ, University of Rochester
- LINDA KEEN, City University of New York, Lehman College and the Graduate Center
- CATHY KESSEL, Consultant, Berkeley, California
- BARBARA KEYFITZ, Ohio State University, Columbus
- GENEVIEVE KNIGHT, Coppin State College
- KRISTIN LAUTER, Microsoft Research
- SUZANNE LENHART, University of Tennessee, Knoxville
- JILL P. MESIROV, University of California San Diego
- JAMES MORROW, University of Washington
- JILL PIPHER, Brown University
- JUDITH ROITMAN, University of Kansas
- LINDA P. ROTHSCILD, University of California San Diego
- BHAMA SRINIVASAN, University of Illinois, Chicago
- JEAN E. TAYLOR, Rutgers University and New York University, Courant Institute
- CHUU-LIAN TERNG, University of California Irvine
- MARIEL VAZQUEZ, University of California Davis
- WILLIAM VELEZ, University of Arizona
- SYLVIA M. WIEGAND, University of Nebraska—Lincoln
- CAROL WOOD, Wesleyan University

—From an AWM announcement

## Laflamme Awarded CAP-CRM Prize

RAYMOND LAFLAMME of the University of Waterloo has been awarded the 2017 CAP-CRM Prize in Theoretical and Mathematical Physics by the Canadian Association of Physicists (CAP) and the Centre de Recherches Mathématiques (CRM) “for his groundbreaking contributions on quantum information,” including developing theoretical approaches to quantum error detection and devising and implementing new methods to make quantum information robust against corruption in both cryptographic and computational settings.

—From a CAP-CRM announcement

## Otto Awarded Pascal Medal

FELIX OTTO of the Max Planck Institute has been awarded the 2017 Blaise Pascal Medal in Mathematics of the European Academy of Sciences “in recognition for seminal contributions on stochastic homogenization, calculus

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of variations, functional analysis and applications to thin-film micro magnetism.” The medal recognizes “an outstanding and demonstrated personal contribution to science and technology and the promotion of excellence in research and education.”

—From an EAS announcement

## Lotfi A. Zadeh (1921–2017)

LOTFI A. ZADEH of the University of California Berkeley passed away on September 6, 2017. Zadeh was a pioneer in the field of what he called “fuzzy logic,” as described in his 1965 article, “Fuzzy Sets.” Fuzzy logic was a means of translating vague or ambiguous human concepts into concrete instructions for computers. Today its uses range from industrial processes and economic research to home appliances and consumer electronics. Zadeh was born on February 4, 1921, in Baku, Azerbaijan. His family moved to Iran, where he received a degree in science from the University of Tehran. He earned his master’s in electrical engineering from the Massachusetts Institute of Technology in 1946 and his PhD from Columbia University in 1949. He joined the faculty of Columbia and, with John Ragazzini, developed Z-transformations in discrete time signal processing and analysis. He moved to Berkeley in 1959. Among his many honors and awards are the IEEE Medal of Honor (1995), the Allen Newell Award of the Association for Computing Machinery (2001), the Benjamin Franklin Medal in Electrical Engineering (2009), and the Golden Goose Award (2017). He was a fellow of the Institute of Electrical and Electronics Engineers, the American Academy of Arts and Sciences, and the Association for Computing Machinery, among others, and was a member of the National Academy of Engineering.

—Elaine Kehoe

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# Remembering Donald Sarason (1933–2017)

Communicated by Steven J. Miller

## Stephan Ramon Garcia

Donald Erik Sarason was a leading figure at the intersection of complex analysis and operator theory for several generations. He earned his PhD in 1963 under the direction of Paul Halmos and worked at UC Berkeley from 1964 until his retirement in 2012. Since a complete account of Don's work and his mathematical legacy would run to many pages, we focus here on a few career highlights and personal reminiscences from three of his former students.

Among Don's earlier papers was the 1967 masterpiece "Generalized interpolation in  $H^\infty$ " [1], which laid the ground-

*ground-shaking  
ideas*

work for several decades of research by hundreds of mathematicians worldwide. This remarkable paper brought new techniques to the study of holomorphic interpolation problems and function algebras, while also marking the birth of commutant-lifting theory. We briefly examine the impact and ramifications of these ground-shaking ideas below.

Let  $z_1, z_2, \dots, z_n$  be distinct points in the open unit disk  $\mathbb{D}$  in the complex plane and let  $w_1, w_2, \dots, w_n \in \mathbb{D}$  be arbitrary. The *Nevanlinna-Pick interpolation problem* asks whether there exists a holomorphic function  $f : \mathbb{D} \rightarrow \mathbb{D}$  so that  $f(z_i) = w_i$  for  $i = 1, 2, \dots, n$ . A necessary and sufficient condition for the existence of a solution is that the  $n \times n$  matrix

$$\left[ \frac{1 - \overline{w_j} w_i}{1 - \overline{z_j} z_i} \right]_{i,j=1}^n$$

is positive semidefinite. Don realized that this and the Carathéodory interpolation problem (which asks for a holomorphic  $f : \mathbb{D} \rightarrow \mathbb{D}$  whose first  $n$  Taylor coefficients are prescribed) can be recast in terms of reproducing kernel Hilbert spaces [1]. This perspective permitted a

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Don Sarason in 2003, at his seventieth birthday conference.

unified treatment of a variety of holomorphic interpolation problems, including contemporary multivariate and noncommutative analogues.

Don's work paved the way for deep work on function algebras, which reached new heights in the 1970s. His *Bulletin of the AMS* paper [2] drew attention to the outstanding open problems of the day. Let  $H^\infty$  denote the algebra of bounded analytic functions on  $\mathbb{D}$ , identified with a subspace of  $L^\infty := L^\infty(\mathbb{T})$  by associating each  $f \in H^\infty$  with its almost-everywhere defined boundary function on the unit circle  $\mathbb{T}$ . Of particular interest are the closed algebras that lie between  $L^\infty$  and  $H^\infty$ . R. G. Douglas conjectured that each such algebra is generated by  $H^\infty$  and a set of reciprocals of inner functions (defined below). This became known as the *Douglas problem*. Don made several huge contributions to this area, one of which was the observation that  $H^\infty + C$ , in which  $C = C(\mathbb{T})$  denotes the algebra of continuous complex-valued functions on  $\mathbb{T}$ , is a Douglas algebra. In fact,  $H^\infty + C$  is the smallest norm-closed subalgebra of  $L^\infty$  that properly contains  $H^\infty$ . Don discovered deep connections between the Douglas

problem and L. Carleson's work on the corona problem. This ultimately culminated in the complete description of Douglas algebras by Don's student S.-Y. A. Chang and by D. Marshall in 1976.

The most obvious infinite-dimensional generalization of a Jordan block, the unilateral shift  $(a_0, a_1, a_2, \dots) \mapsto (0, a_0, a_1, \dots)$  on the sequence space  $\ell^2(\mathbb{N})$ , is of fundamental importance. It is unitarily equivalent to the linear operator  $Sf \mapsto zf$  on the Hardy-Hilbert space  $H^2$  of all holomorphic  $f(z) = \sum_{n=0}^{\infty} a_n z^n$  on  $\mathbb{D}$  for which  $\|f\|^2 = \sum_{n=0}^{\infty} |a_n|^2 < \infty$ . A fundamental result of A. Beurling provides an elegant function-theoretic characterization of the nontrivial, closed  $S$ -invariant subspaces of  $H^2$ . They are of the form  $uH^2 = \{uh : h \in H^2\}$ , in which  $u$  is an *inner function*; that is,

$$u(z) = \alpha z^N \left( \prod_{n=1}^{\infty} \frac{|z_n|}{z_n} \frac{z_n - z}{1 - \overline{z_n}z} \right) \exp \left[ - \int_{-\pi}^{\pi} \frac{e^{it} + z}{e^{it} - z} d\mu(t) \right],$$

in which  $\alpha \in \mathbb{T}$ ,  $N \in \{0, 1, 2, \dots\}$ ,  $(z_n)$  is a possibly finite sequence of points in  $\mathbb{D}$  that satisfy  $\sum_{n=1}^{\infty} (1 - |z_n|) < \infty$ , and  $\mu$  is a nonnegative, singular Borel measure on  $\mathbb{T}$ . Inner functions are precisely the elements of  $H^{\infty}$  whose boundary functions have modulus one almost everywhere on  $\mathbb{T}$ .

The so-called *model spaces*  $\mathcal{K}_u = H^2 \ominus uH^2$  are of great importance in function-related operator theory. Don observed that if  $B$  commutes with the compression of  $S$  to  $\mathcal{K}_u$ , then there is a  $\phi \in H^{\infty}$  so that  $\|B\| = \|\phi\|_{\infty}$  and  $B = T_{\phi}|_{\mathcal{K}_u}$  is the restriction of the *Toeplitz operator*  $T_{\phi}f = P(\phi f)$  to  $\mathcal{K}_u$ ; here  $P$  denotes the Riesz projection, the orthogonal projection from  $L^2 := L^2(\mathbb{T})$  onto  $H^2$  [1]. Since  $ST_{\phi} = T_{\phi}S$ , this result was referred to as *commutant lifting*. Don's theorem ultimately led to the general tools developed by Sz. Nagy and Foiaş. "[T]he story of how the author had the good fortune to be able to prove the primordial version of the commutant lifting theorem" is recounted in Don's article [8], written in memory of his advisor, Paul Halmos.

The year 1975 saw the publication of Don's landmark paper "Functions of vanishing mean oscillation" [3]. The space  $BMO$  of functions of bounded mean oscillation had appeared in seminal papers of John-Nirenberg and Fefferman-Stein. A complex-valued function defined on the unit circle has *vanishing mean oscillation* if the average of the absolute value of its difference from its average over an interval tends to zero as the length of the interval shrinks to zero. Thus,  $VMO$  is a subspace of  $BMO$ . Don proved that functions in  $VMO$  can be decomposed as the sum of a function in  $H^{\infty} + C$  and the harmonic conjugate of a function in  $H^{\infty} + C$ . The concept of  $VMO$  has since found wide application in many areas of operator theory, harmonic analysis, and partial differential equations.

In 1978 Don gave a series of ten lectures at Virginia Polytechnic Institute and State University (now Virginia Tech). These lectures tied together classical function theory, contemporary functional analysis, and exciting new discoveries in harmonic analysis, such as Fefferman's duality theorem. His lecture notes, entitled *Function Theory on the Unit Circle* [4], were widely circulated and

extremely influential. Despite never being formally published, these lecture notes are Sarason's sixth most-cited work, according to MathSciNet®.

In the 1990s Don pioneered the abstract treatment of contractive containment of Hilbert spaces and established a fruitful connection between de Branges-Rovnyak spaces and the ranges of certain Toeplitz operators [5]. Using reproducing kernel Hilbert space techniques, he gave elegant new proofs of the Julia-Carathéodory and the Denjoy-Wolff theorems from complex dynamics.

The year 2007 saw the republication, by the AMS, of Don's book *Complex Function Theory* [7], an undergraduate-level textbook known for its rigorous yet approachable development of the subject.

During the period 2007-13 Don focused on the development of truncated Toeplitz operators and their relatives. The seminal paper "Algebraic properties of truncated Toeplitz operators" [6], published in 2007, echoed the title of the famed Brown-Halmos paper from 1963 that largely initiated the study of Toeplitz operators. Don observed many new phenomena that spurred further research and provided operator theorists with a host of new problems and challenges that continue to guide the field.

Although he published many influential papers, perhaps Don's most enduring contributions are his many students. Don had forty PhD students, of whom many are now successful mathematicians; see Table 1 and the annotated picture of Donald Sarason with students and colleagues in D. Cruz-Uribe's section. Although most of his students remained in operator theory or complex analysis, a number of them branched out and became prominent in other areas. We shall hear from three of them: Sun-Yung Alice Chang, a differential geometer; David Cruz-Uribe, a harmonic analyst; and John Doyle, an engineer.

Table 1. Students of Donald Sarason

Lee, Matthew	1969	Laroco Jr., Leonardo	1988
Nash, David	1970	Lotto, Benjamin	1988
Hively, Gregory	1972	Izzo, Alexander	1989
Chang, Sun-Yung	1974	LeBlanc, Emile	1989
Axler, Sheldon	1975	Li, Kin	1989
Gerver, Joseph	1975	McCarthy, John	1989
Cowen, Carl	1976	Crofoot, Robert	1991
Rosenthal, Erik	1977	Davis, Benjamin	1991
Guillory, Carroll	1978	King-Smith, Oliver	1991
Hayashi, Eric	1978	Cruz-Uribe, David	1993
Hoffman, Michael	1979	Sand, Michael	1994
Wolff, Thomas	1979	Silva, Jorge-Nuno	1994
Bravo, Jaime	1980	Marchant, Simon	1995
Budde, Paul	1982	Shapiro, Jonathan	1995
McMillen, William	1983	Chartrand, Rick	1999
Smith, Wayne	1983	Grinshpan, Anatolii	2001
Doyle, John	1984	Serra, Antonio	2002
Hochwald, Scott	1984	Garcia, Stephan	2003
Walsh, Shelley	1984	Neumann, Genevra	2003
Nguyen, Hung	1985	Judson, Zachary	2009



Don Sarason with Sun-Yung Alice Chang, one of his first PhD students, in 1979.

## Sun-Yung Alice Chang

I first met Don Sarason at my preliminary examination in 1971, when I was a second-year graduate student at UC Berkeley. I had arrived in the US from Taiwan the year before and barely spoke English. I was admitted only to the master's degree program at Berkeley; my financial support and admission into the PhD program depended on how well I did on the examination. I was very nervous about it.

The examination was oral, with topics in algebra, analysis, and geometry, each for one hour. Students were questioned on each topic by two faculty members. Partially due to my nervousness, I did rather poorly in the first topic in algebra and I felt enormous pressure to do better in the second topic, which was analysis—my strong suit.

Don Sarason was one of my examiners and posed his questions slowly. He asked me some very basic questions at the beginning, which calmed me down. Later in the exam, he not only asked me to answer the questions but also encouraged me to display more of the in-depth knowledge I had about the topics. After the examination, while I was waiting in the corridor for the outcome, Don walked up to me and told me I did exceptionally well. This gave me the confidence to finish the rest of my examinations.

This was the pattern of the interaction between Don Sarason and me. He later became my PhD advisor. I learned later that Don was an extremely shy person who did not talk much. But during the many occasions that I sat in his office asking questions, he provided me with a lot of guidance. Sometimes he even followed up and passed to me handwritten notes after our meetings. He was always warm and tremendously supportive.

*Sun-Yung Alice Chang is Eugene Higgins Professor of Mathematics at Princeton University. Her email address is chang@math.princeton.edu.*

At the time, Don had a large number of graduate students, as he did throughout his career. We formed learning seminars and studied together. Despite his quiet personality, Don made a conscious effort to provide us with a stimulating environment. He often organized dinners after seminar talks and held frequent Sunday buffets at his home, where we lingered the whole afternoon. Long after I graduated from Berkeley, I continued to attend such gatherings when I visited him. I remember vividly that once I brought my young son with me to one of the lunch buffets; my son became the center of attention at the party, and we had a great time. I also remember the happy time when after a conference I attended a country music concert with Don and his wife, Mary, in Nashville.

During my early career, partially due to the two-body problem (my husband is also a mathematician), I found myself facing a strong constraint of choices in the job market. Once I was very discouraged and talked to Don on the phone. As usual, Don did not speak much, but later he wrote me a two-page letter, telling me, "The basic challenge is the same wherever you are...the most important thing is what you do, not where you are." I was deeply moved by the letter. Later in my career people sometimes asked me if as a woman and a minority I felt discriminated against. I must say that sometimes I am sensitive about the issue, but I always remember that there are some great people and outstanding mathematicians like Don and others who have gone out of their way to help and encourage me. Don was a great advisor, mentor, and friend whom I could lean on. I am sure this is the same role he has played with many of his other students and young colleagues. We all truly miss him.

## David Cruz-Uribe

I first met Donald Sarason in the fall of 1989, when I was a brand-new graduate student at Berkeley. Don was teaching complex analysis. Although I intended to study algebraic number theory, I took his course. He started the semester by introduc-

ing himself and then saying, "Please call me Don." It took me years to say it to his face. Over the course of the semester I grew disillusioned by number theory and increas-

ingly attracted by this soft-spoken but incredibly engaging lecturer. Paul Halmos said: "He is one of the smoothest and clearest lecturers I know and he has an extraordinary sense of time. He knows almost to the minute how long it will take him to explain something." In the spring I took classical harmonic analysis from Don, and late in my first year I asked if I could be his student.

*"He is one of the smoothest and clearest lecturers I know."*

*David Cruz-Uribe is professor of mathematics and department chair at the University of Alabama. His email address is dcruzuribe@ua.edu.*



This was towards the end of a period in which Don produced a quarter of his forty PhD students; see Table 1. Nevertheless, Don seemed happy to take me on as his student. He started me off by having me read his VPI Lecture Notes [4], based on a series of lectures he gave in 1978 at what is now Virginia Tech. Though never published, they were widely circulated and well known to operator theorists of that era. I still have my photocopy, heavily marked up. I would go to Don's office every week and he would patiently answer my questions.

At every level, from introductory calculus to graduate courses and interactions with his students, Don was an amazing teacher. Though a small thing, I was impressed by his ability to draw a perfect, 3-foot diameter circle on the chalkboard with a single stroke. His explanations were always lucid, and his choice of material took students to the heart of the subject. The one exception to this that I can remember was in complex analysis, when we spent the last few weeks grinding through the theory of elliptic functions. After one particularly technical lecture about the intricacies of theta functions, I asked him why he included this material. He laconically replied, "Because I always wanted to learn about them." As his advisor, Paul Halmos, put it, "Don never used eight words when seven would do."

Don was a very careful writer and editor. Every draft of my thesis and of the one small paper I wrote while at Berkeley came back covered with red ink. My mathematical reasoning, of course, was scrutinized, but he also corrected my exposition with an exactitude I had not seen since my high school composition classes. He wanted every sentence to be as clear and as concise as possible. Once, he sent me scrambling to find a mysterious paper after he wrote in the margin, "See Fowler." This was not some unknown mathematician, but rather H. W. Fowler's *A Dictionary of Modern English Usage*, and the reference was to correct a subtle grammatical error.

Don had a very dry sense of humor. When teaching a large calculus course, he would stop in the middle of each lecture and tell a joke. They were usually corny, but his deadpan delivery always evoked laughter. One winter I met him in Sproul Plaza just before the start of classes. We were chatting and I asked if he had done anything interesting over break. He simply responded, "I got married—I guess you know to whom." As a more extended joke, whenever he lectured, he always wore a tie. The ties, however, were ugly and only occasionally went with his outfit. I later learned that he had a huge collection in his office. As he was leaving to go teach he would select one at random and put it on. When I asked him why, he explained that he had gotten a student evaluation that complained his dress was too casual for a professor, so he had gone to a thrift store and bought a bag of ties for a few dollars.

Every student of Don's that I have spoken to agrees that Don was an excellent advisor. Though I complained at the time about his lack of direction, I only realized many years later the confidence that Don had in my ability and the support he gave me to develop as an independent



**Don Sarason (middle) with S. R. Garcia (left) and G. Karaali (right) in Berkeley, May 2004.**

research mathematician. The thesis problem he gave me later became known as the Sarason conjecture on the product of unbounded Toeplitz operators. I suspect that Don had in mind a partial result via classical operator-theoretic methods. However, I soon became interested in a real-variable approach to the problem and showed that it was equivalent to an open question in the study of two-weight norm inequalities. On my own I decided that this was the correct way to approach the problem, or, as Ben Lotto, another of Don's students, put it, I sold out to the dark side of analysis.

I plunged ahead, unfazed (and indeed unaware) that my advisor knew very little about this area and so could provide almost no guidance. I remember one meeting, shortly before I resolved the major sticking point in my thesis, where I outlined what I was trying to prove and all of my failed attempts. In response to my pleas for help, he only responded, "Those are very interesting questions, David." But he carefully read my thesis, and his many marginal comments significantly improved the final results.

My real reward came a decade later, in 2003, at a conference organized in honor of his seventieth birthday. In the decade since getting my PhD, I had made some progress in the study of two-weight norm inequalities, and in my talk I sketched out the partial results as they applied to the Sarason conjecture. As a joke I described my work as the result of an "intellectual falling out" that Don and I had when I was a graduate student. At the end of my talk, Don got up, said, "David, you were right," and then sat down again.



Don Sarason, with students and colleagues, at his seventieth birthday conference in 2003. S.-Y. Alice Chang is in the lower right and S. R. Garcia is in the upper left.

## John Doyle

For a pure mathematician, Don had a surprisingly large impact on control engineering. This connection started in 1978 when I began graduate work at Berkeley. Initially my favorite subject was differential geometry, but when I took functional analysis from Don he definitely became my favorite instructor. He was truly compassionate and took everything very seriously except himself. This showed in his humor and politics, both of which I enjoyed. He was also an avid runner, though our routines were very different, so we didn't train together.

*took everything  
very seriously  
except himself*

At the time, the research frontiers in the new subject of robust control theory began to coincide with pure math in Don's area of operator theory and particularly optimization in  $H^\infty$ . This was particularly relevant to me, as I was commuting between Berkeley and Minneapolis, where Honeywell was the main client of my aerospace consulting business. I planned to focus on pure math

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Pamela Gorkin (left), student of Sheldon Axler (second from left), student of Donald Sarason (second from right), student of Paul Halmos (right) in Lancaster, 1984.

when in Berkeley and I talked to Don briefly and somewhat superficially about some potential research directions. I had some questions about what was known at the time and about what I might study to catch up. Don said he



wasn't sure about my questions and told me to come back in a week after he'd had time to think about it.

When I came back, Don gave me a personalized lecture on his assessment of existing results plus a set of notes with key references that were great starting points. This amazed me, but I learned that this sort of generosity was typical. I don't recall when he became my official advisor, but he immediately helped me get started in important directions.

The pure math side progressed rapidly, building on work by Don and others. My research quickly added a focus on overcoming the apparent computational intractability of the otherwise promising theoretical solutions. This departed somewhat from Don's interest, and our interactions were thus less frequent but always fun.

Don was not thrilled that many initial applications of this work, by others, would likely be to military aerospace systems, but he was always very tolerant of my admittedly hypocritical compartmentalization. It helped that everything was open and unclassified, and ultimately the applications were overwhelmingly academic and commercial, though he was quite reasonably not interested in any of these distinctions. My thesis ended up being more applied and computationally oriented than either of us expected, and it even received the Friedman applied math thesis award. It also formed one of the foundations for what became the Matlab Robust Control Toolbox, which is widely used in academia and industry.

Although Don was a pure mathematician at heart, four of my advisees from Caltech's Control and Dynamical Systems program—Don's academic grandchildren—ultimately became professors in various parts of Berkeley engineering.

### Selected Papers of Donald Sarason

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3. Functions of vanishing mean oscillation, *Trans. Amer. Math. Soc.* **207**:391–405, 1975. MR 0377518
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6. Algebraic properties of truncated Toeplitz operators, *Oper. Matrices* **1**(4):491–526, 2007. MR 2363975
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Photo of Donald Sarason in Stephan Ramon Garcia's section courtesy of Jonathan Shapiro.

Table 1 source: [www.genealogy.ams.org/id.php?id=5021](http://www.genealogy.ams.org/id.php?id=5021).

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# Jacqueline Ferrand and Her Oeuvre

Pierre Pansu

Communicated by Allyn Jackson

Jacqueline Ferrand was born in Alès, in the south of France, one hundred years ago, in 1918. At the time, she was one of the very few women ever admitted to the École Normale Supérieure (ENS), and she succeeded at the (male) *agrégation* in 1939. She immediately won a teaching assistant position at École Normale Supérieure de Jeunes Filles. The director of the school, Mrs. Cotton, was convinced that women should nourish the same intellectual ambition as men, and she reckoned that this brilliant young mathematician would raise the level of teaching at Jeunes Filles to the same level as the ENS. Several people witnessed the energy Jacqueline Ferrand devoted to this task despite poor material conditions. With the same energy, she began to do research, under Arnaud Denjoy's supervision from afar. After her thesis defense, on June 12, 1942, she was awarded the Girbal Barral Prize in 1943 and the Fondation Peccot Prize in 1946. Her career developed quickly: assistant professor in Bordeaux in 1943, professor in Caen in 1945, in Lille in 1948, in Paris from 1956 until her retirement in 1984.

## First Works

Jacqueline Ferrand's thesis deals with boundary values of planar conformal mappings. Since the time of Bernhard Riemann, it has been known that every simply connected domain  $D'$  admits a conformal mapping  $f$  from the unit disk  $D$ , i.e. a geographic map in which angles are preserved. One may view  $f$  as an analytic function of one variable defined on the disk, which is a bijection from  $D$  onto  $D'$ . The issue is whether  $f$  has a limit at every boundary point. The answer is positive if the boundary of

$D'$  is smooth enough. The problem becomes hard if the boundary is irregular.

In 1913, Constantin Carathéodory made a decisive step when he introduced the notion of prime end. Carathéodory considers nested *cuts* in  $D'$ . These are nested sequences of subdomains cut by simple arcs joining two boundary points, and whose lengths tend to 0. Two sequences are equivalent if each can be nested in the other. A *prime end* is an equivalence class of nested cuts. This definition, which involves lengths, is not obviously a conformal invariant. Carathéodory gives a proof of invariance that relies on fine properties of holomorphic functions.

In her thesis, published in the *Annales de l'École Normale Supérieure de Paris* in 1942, Ferrand gives a new proof of conformal invariance of prime ends that highlights the role played by the area of a conformal map  $f$  in estimating lengths of images of almost all curves. Furthermore, the fact that  $f$  is open allows one to pass from almost all curves to all curves, and hence to estimate the modulus of continuity of  $f$ .

Precise estimates lead to sufficient conditions on the domain  $D'$  for the conformal mapping to admit limits along curves contained in the disk and having a higher order contact with the boundary of the disk. Under stronger assumptions, Ferrand shows, in a 1942 paper in the *Bulletin de la Société Mathématique de France*, the existence of *angular derivatives*

$$\lim_{z \rightarrow a} \frac{f(z) - a}{z - a}$$

when  $a$  is a boundary point.

For a conformal (or holomorphic) bijection, the area of the image is given by the Dirichlet integral

$$\int_D |f'(z)|^2 dz.$$

For a harmonic function  $u$  (the real part of a holomorphic function  $f$ ), the Dirichlet integral replaces area and the maximum principle provides an ersatz for the openness of  $f$ . Methods developed for conformal mappings in her 1944 paper in the *SMF Bulletin* hence extend to the boundary study of harmonic and superharmonic functions.

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Jacqueline Ferrand, shown here in a photo dated 1988, was an outstanding French geometer whose works continue to influence research today. She wrote her last mathematical papers at the age of eighty and died in 2014, at the age of ninety-six.<sup>1</sup>

### Preholomorphic Functions

In that same paper, Ferrand introduced a discretization of the notion of holomorphic function. Given  $h > 0$ , she replaces the plane  $\mathbb{C}$  (or a bounded domain  $D'$  of  $\mathbb{C}$ ) with the finite set  $Z_h$  of points of  $D'$  whose real and imaginary parts are integer multiples of  $h$ . Classically, one defines a function  $u$  on  $Z_h$  to be harmonic (Ferrand writes *preharmonic*) if

$$4u(z) = u(z+h) + u(z-h) + u(z+ih) + u(z-ih).$$

Discretizing holomorphicity equations is less classical. Ferrand calls *preholomorphic* a complex-valued function  $f$  that satisfies, for all  $z \in Z_h$ ,

$$f(z+ih) - f(z-h) = i(f(z+h+ih) - f(z)).$$

<sup>1</sup>This photo was taken at École Normale Supérieure and kindly provided to Notices by Ms. Ferrand's children.

The real and imaginary parts of  $f$  are preharmonic functions on sub-lattices

$$Z'_h = \{z \in Z_h; (\operatorname{Re}(z) + \operatorname{Im}(z))/h \text{ is even}\}$$

and

$$Z''_h = \{z \in Z_h; (\operatorname{Re}(z) + \operatorname{Im}(z))/h \text{ is odd}\}.$$

Conversely, every preharmonic function on  $Z_h$  is the real part of a preholomorphic function on  $Z_h$ . Ferrand shows that when the mesh  $h$  tends to 0, preholomorphic functions on  $Z_h$  converge to holomorphic functions on the domain  $D'$ . *A priori* estimates on the modulus of continuity play a crucial role again.

In 1955, Ferrand inferred a pretty and simple proof of the conformal mapping theorem for nonsimply connected domains. *Let  $D'$  be a planar domain whose boundary has at least one isolated connected component that is not a single point. Then there exists an essentially unique conformal mapping of  $D'$  onto a rectangle with segments parallel to one side removed.*

The notion of preholomorphic function has led to numerous developments.

### Group Actions

During a visit to Institute for Advanced Study, Princeton, Ferrand investigated when a Lie algebra action integrates into a Lie group action. The upshot is a functional analytic characterization of completeness of vector fields, which she published in the *SMF Bulletin* in 1942.

*This elegant result does not seem to be well known.*

Let  $\xi$  be a locally Lipschitz vector field with vanishing divergence, on a manifold  $M$  equipped with a volume element  $\omega$ . Then  $\xi$  is complete if and only if the differential operator  $i\xi$  extends to a selfadjoint operator of  $L^2(\omega)$ . Assume that  $\xi$  generates a one-parameter group of isometries of  $M$ . This group is periodic if and only if the operator  $i\xi$  has a closed image.

This elegant result does not seem to be well known.

### Teaching Books

Ferrand's mathematical production decreased between 1958 and 1968. At this time, Jacqueline was the mother of four young children. She put a great deal of effort into her teaching and wrote a series of course notes that, with a lot of work, became textbooks. A basic differential geometry course was published by Masson in 1963. Her freshman courses were published by Armand Colin in 1964 and Dunod in 1967. In the 1970s Dunod published the series of books co-authored with Jean-Marie Arnaudès, which covered the whole curriculum of the first two years of university (including exercises). It is still in use today in the *classes préparatoires aux grandes écoles*, which prepare students for the competitive examinations to enter the École Normale Supérieure.

## Riemannian Geometry

At the end of this period, Ferrand obtained her most famous results, leading to an invitation to the 1974 International Congress of Mathematicians in Vancouver. She solved a problem in Riemannian geometry posed by André Lichnerowicz in 1964, on which several partial answers had been published.

A conformal mapping of an open set of Euclidean space  $\mathbb{R}^n$  is a diffeomorphism whose differential preserves angles. This notion extends to open sets of  $\mathbb{R}^n$  equipped with a Riemannian metric, i.e. an inner product depending on position, to submanifolds of Euclidean spaces, and to abstract Riemannian manifolds. The prototype of a Riemannian manifold is the sphere

$$\{x \in \mathbb{R}^{n+1}; x_0^2 + \cdots + x_n^2 = 1\}.$$

Stereographic projection achieves a conformal diffeomorphism of the sphere, with one point deleted, onto Euclidean space. By transport of structure, Euclidean similarities become conformal diffeomorphisms of the sphere. It follows that the group of conformal self-mappings of the sphere is noncompact, a result that Ferrand published in the *Mémoires de l'Académie Royale Belgique* in 1971.

**Theorem 1.** *If a compact Riemannian manifold  $M$  has a noncompact group of conformal self-mappings, then  $M$  is conformal to the sphere.*

The point is to estimate the modulus of continuity of a conformal mapping, in every dimension. Let us use the four-point conformal invariant Ferrand introduced in her 1973 paper in the *Journal of Differential Geometry*. The proof we are about to give in Euclidean  $n$ -space immediately extends to Riemannian manifolds.

**Definition 2.** *Let  $F_0, F_1$  be disjoint compact connected subsets of  $\mathbb{R}^n$ . The capacity  $\text{cap}(F_0, F_1)$  is the infimum of integrals  $\int |du|^n$  for all smooth functions  $u$  on  $\mathbb{R}^n$  such that  $u = 0$  on  $F_0$  and  $u = 1$  on  $F_1$ .*

Let  $x, y, z, t$  be 4 distinct points of  $\mathbb{R}^n$ . The *Ferrand invariant*  $j(x, y, z, t)$  is the infimum of capacities of pairs  $(F_0, F_1)$  of compact connected sets such that  $F_0$  contains  $x$  and  $z$  and  $F_1$  contains  $y$  and  $t$ .

Using an *a priori* estimate of the modulus of continuity of functions  $u$  that minimize  $\int |du|^n$  (a nonlinear  $n$ -dimensional generalization of her thesis work), Ferrand shows that this infimum is positive. In fact (see the 1991 paper of Ferrand et al. in *Journal d'Analyse Mathématique*, completed by her 1996 paper in the *Pacific Journal of Mathematics*), when  $z$  and  $t$  are fixed,

$$d(x, y) = j(x, y, z, t)^{1/(1-n)}$$

is a distance that defines the usual topology of  $\mathbb{R}^n \setminus \{z, t\}$  and that tends to infinity if, when  $y$  is fixed,  $x$  tends to  $z$ .

To give an idea of how the invariant  $j$  is used, let us show that, in an arbitrary Riemannian manifold, the group  $G$  of conformal mappings that fix three distinct points  $y, z$ , and  $t$  is compact. Indeed,  $G$  acts isometrically for metric  $d$  and fixes  $y$ . Therefore, every sequence of elements of  $G$  has a subsequence that  $C^0$  converges to a homeomorphism. There remains to show that the limit

is a conformal diffeomorphism and that the convergence is in  $C^\infty$ . This is a nonlinear elliptic regularity theorem, due to Frederick Gehring and Yuri Reshetnyak in the Euclidean case, and extended by Ferrand in 1976 to general Riemannian manifolds.

More can be drawn from the  $j$  invariant. If a sequence  $f_k$  of conformal mappings diverges, then for all sequences  $x_k, z_k, t_k$ , if  $f_k(z_k)$  converges to  $z$  and  $f_k(t_k)$  converges to  $t \neq z$ , then  $f_k(x_k)$  converges to  $z$  or to  $t$ . This means that at most two limits are possible,  $z$  and  $t$ . If  $z \neq t$ , then, up to extracting a subsequence,  $f_k$  maps the complement of every neighborhood of  $z$  into arbitrarily small neighborhoods of  $t$ . This implies that the manifold  $M$  is simply connected and that its metric is conformally flat, hence  $M$  is conformal to the sphere.

## Quasiconformal Mappings

How did Ferrand discover her four-point invariants?

The idea of using a conformally invariant metric goes back to Lichnerowicz in 1964. Lichnerowicz uses constant scalar curvature metrics. In the 1960s, this approach was limited due to the incomplete solution of Hideiko Yamabe's problem: existence and uniqueness of a constant scalar curvature metric, conformal to a given Riemannian metric. The essential result obtained since—an analytic analogue of Ferrand's 1971 result mentioned above—is due to Richard Schoen (cf. his contribution to the Do Carmo Jubilee, 1988). Let  $(M, g)$  be a compact Riemannian manifold that is not conformal to the standard sphere. Then the set of constant scalar curvature metrics conformal to  $g$  is compact, up to scaling.

The idea of a naturally invariant metric has been developed with great success by Shoshichi Kobayashi in the holomorphic category (cf. his 1970 book). Although it is likely that Kobayashi's construction may have influenced Ferrand, it differs in an essential way from Ferrand's. The exact transcription of Kobayashi's definition in the conformal category (necessarily limited to conformally flat manifolds) was carried out by Ravi Kulkarni and Ulrich Pinkall in 1994.

Ferrand's inspiration is rather to be found in the theory of quasiconformal mappings launched by Herbert Grötsch in 1928. Here is the definition given by Lars Ahlfors in 1930. A *quadrilateral* is a planar domain delimited by a Jordan curve carrying four marked points. Such a domain admits a conformal mapping onto a rectangle that maps marked points to vertices. This rectangle is unique up to a similitude. The ratio of lengths of two consecutive sides is a conformal invariant of the quadrilateral, called its *modulus*. A homeomorphism  $f$  between planar domains is called *K-quasiconformal* if, for every quadrilateral  $Q$ ,

$$K^{-1} \text{modulus}(Q) \leq \text{modulus}(f(Q)) \leq K \text{modulus}(Q).$$

Note that the modulus of a rectangle is exactly half of the capacity of two opposite sides. In two dimensions, Ferrand's definition is therefore very close to Ahlfors's.

A homeomorphism between planar domains is quasiconformal if and only if it maps small balls to small domains of bounded eccentricity (cf. Jussi Väisälä's 1971



book). This notion makes sense for general metric spaces. Misha Gromov, following G. D. Mostow and Grigori Margulis, has highlighted the role of quasiconformal mappings in geometric group theory: the ideal boundary of a hyperbolic group has a quasiconformal structure. This has motivated works by Juha Heinonen, Pekka Koskela, Adam Koranyi, and Hans-Martin Reimann (1995), where the regularity of quasiconformal mappings is investigated on larger and larger classes of metric spaces. In these works, the key point remains to estimate Ferrand's invariant. To some extent, the notion of a *Loewner space*, abstracted by Heinonen, characterizes spaces whose Ferrand invariant is nontrivial. Ferrand's invariant plays a crucial role in work by Marc Bourdon and Hervé Pajot (2000), who establish the quasi-isometric rigidity of certain hyperbolic buildings, and in Mario Bonk and Bruce Kleiner's 2002 characterization of the standard sphere up to quasiasymmetry.

Gromov has raised the question of whether quasiconformal geometry survives in infinite dimensions. This motivates the search for dimension-independent estimates on Ferrand's invariant.

Ferrand's invariant exploits the conformal invariance of integrals  $\int |du|^n$ . In a sense that we will make precise, these integrals entirely determine the conformal structure. Following Halsey Royden, Ferrand defines a family of commutative Banach algebras attached to an  $n$ -dimensional Riemannian manifold  $M$ . Given  $p > 1$ , let  $A_p(M)$  denote the space of continuous bounded functions on  $M$  whose distributional first derivatives have integrable  $p$ th powers.  $A_p(M)$  is a Banach algebra for the norm  $\|u\|_\infty + \|du\|_p$ .

In a 1973 paper in the *Duke Mathematical Journal*, Ferrand shows that if  $p = n$ , every isomorphism  $A_n(M) \rightarrow A_n(N)$  is induced by a quasiconformal mapping  $M \rightarrow N$ . However, if  $p \neq n$ , every isomorphism  $A_p(M) \rightarrow A_p(N)$  is induced by a bi-Lipschitz homeomorphism. Vladimir Goldshtein and Mati Rubin (1995) have a result that goes in the same direction. Bourdon (2007) has extended these ideas to ideal boundaries of hyperbolic metric spaces.

Ferrand goes one step further. She characterizes maps  $M \rightarrow N$  that map  $A_p(N)$  to  $A_p(M)$ , for  $p > n$ . The case when  $p < n$  was studied by Goldshtein, Leonid Gurov, and Alexandr Romanov in 1995.

### Finite Type Geometric Structures

Lichnerowicz's problem extends to noncompact Riemannian manifolds. Say that a group  $G$  of conformal mappings of a Riemannian manifold  $M$  is *inessential* if it preserves a Riemannian metric  $g'$  conformal to  $g$ . The problem becomes: show that a noncompact Riemannian manifold whose conformal group is essential is conformal to Euclidean space. Dmitry Alekseevski published a solution in 1972. Only in 1992 did Robert Zimmer and Karl Gutscher find a serious gap in Alekseevski's argument.

Here is the context that led Zimmer to study Lichnerowicz's problem. In his 1986 ICM lecture, Zimmer studies actions of noncompact groups on compact manifolds, from the point of view of dynamical systems. Every manifold admits an ergodic action of  $\mathbb{R}$ . However, only very special compact manifolds admit ergodic actions of a semi-simple Lie group (cf. François Labourie's lecture at the 1998 ICM). These groups bear geometry within themselves (Felix Klein's Erlangen program is not far) and carry it to manifolds on which they act. This restricts possibilities for invariant geometric structures.

Let us call an *order  $r$  geometric structure* on a manifold  $M$  the datum of a reduction of the fibre bundle of order  $r$ -jets to an algebraic subgroup of the group  $Gl_n(r)$  of  $r$ -jets of diffeomorphisms fixing the origin in  $\mathbb{R}^n$ . A geometric structure is *of finite type* if every automorphism is determined by its finite jet at some point. For instance, (pseudo-)Riemannian metrics, conformal structures, and projective structures have finite type. Symplectic or complex structures have infinite type.

It is a fascinating problem to decide which manifolds equipped with finite type geometric structures admit noncompact automorphism groups. An important step was made by Gromov in 1988. He shows that if the automorphism group has a dense orbit, this orbit is open. The sought-for objects are thus homogeneous almost everywhere. For instance, it follows that the isometry group of a compact manifold equipped with a real analytic Lorentzian metric is always compact (Giusi d'Ambra 1988). For conformal structures, the problem has been solved in Ferrand's 1971 paper in the *Mémoires de l'Académie Royale Belgique*. Generalizations of this beautiful result have been given by Charles Frances.

Here is the assertion from Alekseevski's paper that attracted Zimmer's attention. If  $G$  is a closed group of automorphisms of a manifold  $M$  equipped with a finite type geometric structure, and if point stabilizers are compact, then  $G$  acts properly on  $M$ . In this generality, this statement is wrong. In the special case of a conformal structure, it is equivalent to the noncompact Lichnerowicz conjecture, which was finally solved in a 1996 paper in *Mathematische Annalen* by Ferrand, who also solved its quasiconformal version in a paper that same year in *Journal d'Analyse Mathématique*. The solution relies on a three-point invariant, obtained from the four-point invariant by letting one point tend to infinity. Other invariants obtained by passing to the limit are discussed in a paper from 1999. After this last text, completed by a survey of the history of Lichnerowicz's conjecture, Ferrand deliberately laid down her mathematical feather, at the age of 80. She passed away on April 26th, 2014, in Sceaux, near Paris.

### Conclusion

Ferrand's works have had a significant influence on several branches of mathematics. However, they are better known abroad than in her home country. She did not try to found

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a school. As she modestly said, she hesitated to draw young mathematicians along paths she thought were not sufficiently promising. Ferrand has had collaborations abroad, especially in Finland where she was held in great esteem. Nevertheless, her intellectual route was mainly a lonely one. The value of her work has been fully recognized only when the mathematical mainstream came close to her. This happened in 1942 at the time of her thesis, in 1969 with Lichnerowicz's problem, and again in 1996. The energy she devoted, at the age of 80, to proving the theorem that provides a definite answer to Lichnerowicz's question, deserves admiration.

*Her intellectual route was mainly a lonely one.*

## Photo Credits

Photo of Jacqueline Ferrand courtesy Jean Lelong.

Photo of Pierre Pansu courtesy of Angela Pasquale.

## ABOUT THE AUTHOR

**Pierre Pansu** is a differential geometer. He first met Jacqueline Ferrand at Arthur Besse's Paris seminar in the early 1980s. His own work being influenced by Ferrand's, he kept in touch with her until 1999.



**Pierre Pansu**

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**Suggested uses** for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.

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Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

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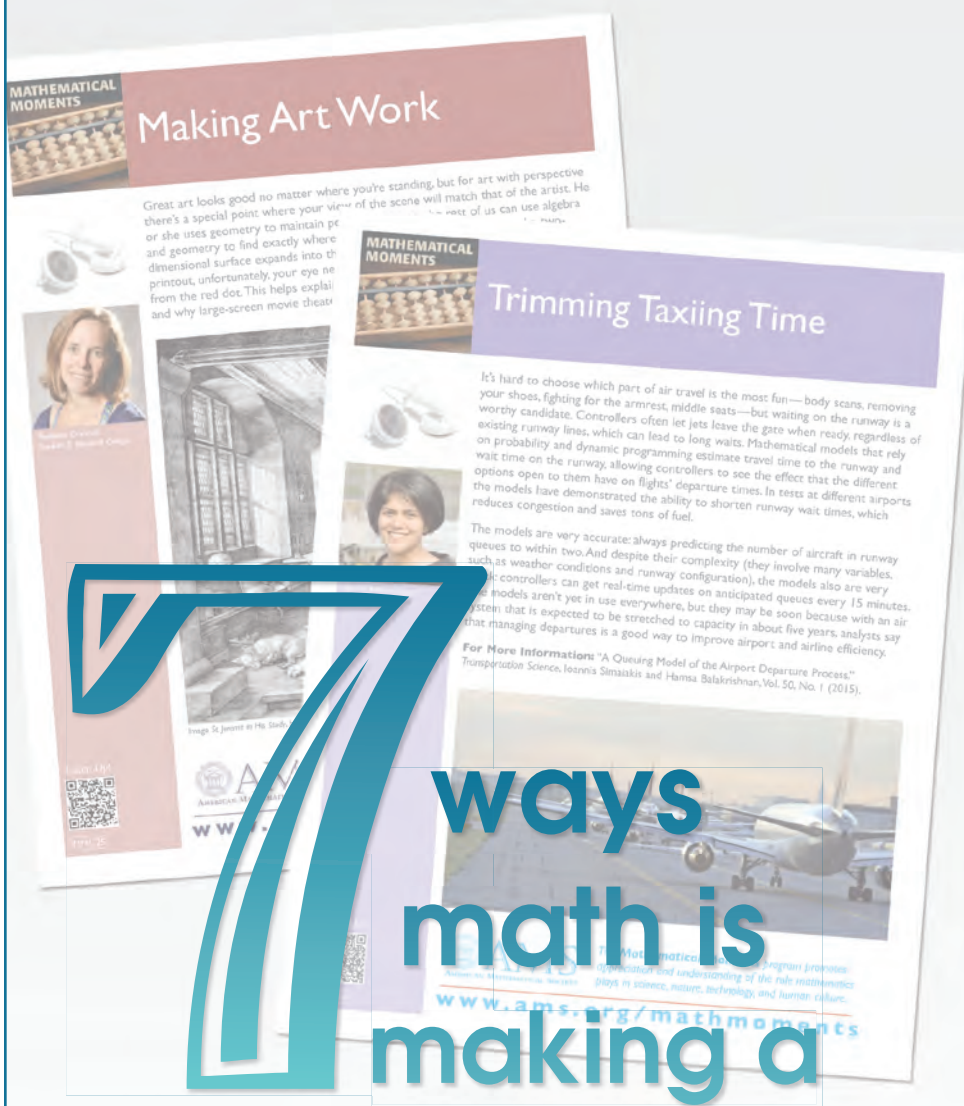
**Upcoming deadlines** for classified advertising are as follows: March 2018—January 2, 2018; April 2018—January 30, 2018; May 2018—March 2, 2018; June/July 2018—April 27, 2018; August 2018—June 6, 2018; June/July 2018—May 16, 2018.

**US laws prohibit** discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

**Situations wanted advertisements** from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

**Submission:** Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to [classads@ams.org](mailto:classads@ams.org). AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

# MATHEMATICAL MOMENTS



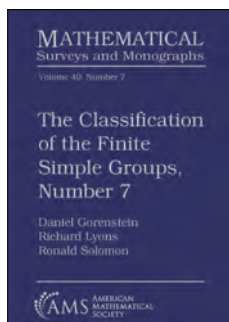
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# New Publications Offered by the AMS

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## Algebra and Algebraic Geometry



### The Classification of the Finite Simple Groups, Number 7

Part III, Chapters 7–11:  
The Generic Case, Stages  
3b and 4a

**Daniel Gorenstein, Richard  
Lyons, Rutgers University,  
Piscataway, NY, and Ronald  
Solomon, The Ohio State  
University, Columbus, OH**

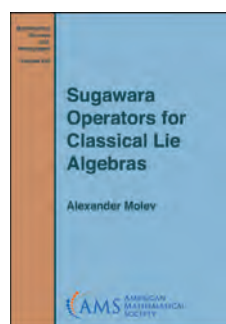
The classification of finite simple groups is a landmark result of modern mathematics. The multipart series of monographs which is being published by the AMS (Volume 40.1–40.7 and future volumes) represents the culmination of a century-long project involving the efforts of scores of mathematicians published in hundreds of journal articles, books, and doctoral theses, totaling an estimated 15,000 pages. This part 7 of the series is the middle of a trilogy (Volume 40.5, Volume 40.7, and forthcoming Volume 40.8) treating the Generic Case, i.e., the identification of the alternating groups of degree at least 13 and most of the finite simple groups of Lie type and Lie rank at least 4. Moreover, Volumes 40.4–40.8 of this series will provide a complete treatment of the simple groups of odd type, i.e., the alternating groups (with two exceptions) and the groups of Lie type defined over a finite field of odd order, as well as some of the sporadic simple groups. In particular, this volume completes the construction, begun in Volume 40.5, of a collection of neighboring centralizers of a particularly nice form. All of this is then applied to complete the identification of the alternating groups of degree at least 13.

The book is suitable for graduate students and researchers interested in the theory of finite groups.

**Contents:** The stages of theorem  $C_7^*$ ; General group-theoretic lemmas; Theorem  $C_7^*$ ; Stage 3b; Theorem  $C_7^*$ , stage 4a: Constructing a large alternating subgroup  $G_0$ ; Properties of  $\mathcal{K}$ -groups; Bibliography; Index.

**Mathematical Surveys and Monographs**, Volume 40

March 2018, 344 pages, Hardcover, ISBN: 978-0-8218-4069-6, LC 994-23001, 2010 *Mathematics Subject Classification*: 20D06, 20D08; 20D05, 20E32, 20G40, **AMS members US\$97.60**, List US\$122, Order code SURV/40.7



### Sugawara Operators for Classical Lie Algebras

**Alexander Molev, University of  
Sydney, Australia**

The celebrated Schur-Weyl duality gives rise to effective ways of constructing invariant polynomials on the classical Lie algebras. The emergence of the theory

of quantum groups in the 1980s brought up special matrix techniques which allowed one to extend these constructions beyond polynomial invariants and produce new families of Casimir elements for finite-dimensional Lie algebras. Sugawara operators are analogs of Casimir elements for the affine Kac-Moody algebras.

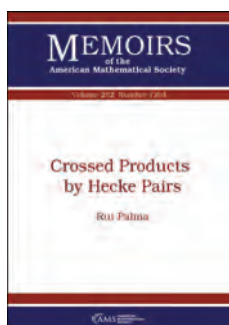
The goal of this book is to describe algebraic structures associated with the affine Lie algebras, including affine vertex algebras, Yangians, and classical  $\mathcal{W}$ -algebras, which have numerous ties with many areas of mathematics and mathematical physics, including modular forms, conformal field theory, and soliton equations. An affine version of the matrix technique is developed and used to explain the elegant constructions of Sugawara operators, which appeared in the last decade. An affine analogue of the Harish-Chandra isomorphism connects the Sugawara operators with the classical  $\mathcal{W}$ -algebras, which play the role of the Weyl group invariants in the finite-dimensional theory.

**Contents:** Idempotents and traces; Invariants of symmetric algebras; Manin matrices; Casimir elements for  $\mathfrak{gl}_N$ ; Casimir elements for  $\mathfrak{o}_N$  and  $\mathfrak{sp}_N$ ; Feigin-Frenkel center; Generators in type A; Generators in types B, C and D; Commutative subalgebras of  $U(\mathfrak{g})$ ; Yangian characters in type A; Yangian characters in types B, C and D; Classical  $\mathcal{W}$ -algebras; Affine Harish-Chandra isomorphism; Higher Hamiltonians in the Gaudin model; Wakimoto modules; Bibliography; Index.

**Mathematical Surveys and Monographs**, Volume 229

March 2018, approximately 305 pages, Hardcover, ISBN: 978-1-4704-3659-9, 2010 *Mathematics Subject Classification*: 17B35, 17B63, 17B67, 17B69, 16S30, **AMS members US\$97.60**, List US\$122, Order code SURV/229





## Crossed Products by Hecke Pairs

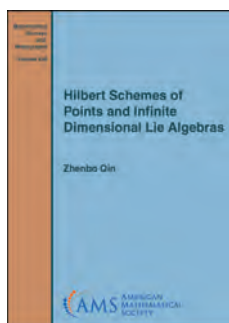
Rui Palma, *University of Oslo, Norway*

*This item will also be of interest to those working in analysis.*

**Contents:** Introduction; Preliminaries; Orbit space groupoids and Fell bundles; \*-Algebraic crossed product by a Hecke pair; Direct limits of sectional algebras; Reduced  $C^*$ -crossed products; Other completions; Stone-von Neumann Theorem for Hecke pairs; Towards Katayama duality; Bibliography; Symbol index; Word index.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1204

March 2018, 141 pages, Softcover, ISBN: 978-1-4704-2809-9, 2010 *Mathematics Subject Classification*: 46L55; 20C08, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1204



## Hilbert Schemes of Points and Infinite Dimensional Lie Algebras

Zhenbo Qin, *University of Missouri, Columbia, MO*

Hilbert schemes, which parametrize subschemes in algebraic varieties, have been extensively studied in algebraic

geometry for the last 50 years. The most interesting class of Hilbert schemes are schemes  $X^{[n]}$  of collections of  $n$  points (zero-dimensional subschemes) in a smooth algebraic surface  $X$ . Schemes  $X^{[n]}$  turn out to be closely related to many areas of mathematics, such as algebraic combinatorics, integrable systems, representation theory, and mathematical physics, among others.

This book surveys recent developments of the theory of Hilbert schemes of points on complex surfaces and its interplay with infinite dimensional Lie algebras. It starts with the basics of Hilbert schemes of points and presents in detail an example of Hilbert schemes of points on the projective plane. Then the author turns to the study of cohomology of  $X^{[n]}$ , including the construction of the action of infinite dimensional Lie algebras on this cohomology, the ring structure of cohomology, equivariant cohomology of  $X^{[n]}$  and the Gromov-Witten correspondence. The last part of the book presents results about quantum cohomology of  $X^{[n]}$  and related questions.

The book is of interest to graduate students and researchers in algebraic geometry, representation theory, combinatorics, topology, number theory, and theoretical physics.

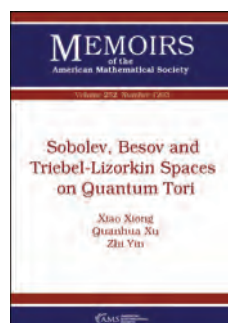
**Contents:** *Hilbert schemes of points on surfaces*: Basic results on Hilbert schemes of points; The nef cone and flip structure of  $(\mathbb{P}^2)^{[n]}$ ; *Hilbert schemes and infinite dimensional Lie algebras*: Hilbert schemes and infinite dimensional Lie algebras; Chern

character operators; Multiple  $q$ -zeta values and Hilbert schemes; Lie algebras and incidence Hilbert schemes; *Cohomology rings of Hilbert schemes of points*: The cohomology rings of Hilbert schemes of points on surfaces; Ideals of the cohomology rings of Hilbert schemes; Integral cohomology of Hilbert schemes; The ring structure of  $H_{\text{orb}}^*(X^{(n)})$ ; *Equivariant cohomology of the Hilbert schemes of points*: Equivariant cohomology of Hilbert schemes; Hilbert/Gromov-Witten correspondence; *Gromov-Witten theory of the Hilbert schemes of points*: Cosection localization for the Hilbert schemes of points; Equivariant quantum operator of Okounkov-Pandharipande; The genus-0 extremal Gromov-Witten invariants; Ruan's Cohomological Crepant Resolution Conjecture; Bibliography; Index.

**Mathematical Surveys and Monographs**, Volume 228

March 2018, 336 pages, Hardcover, ISBN: 978-1-4704-4188-3, LC 2017036491, 2010 *Mathematics Subject Classification*: 14C05, 17B65; 14F43, 14J60, 14N35, 17B69, **AMS members US\$97.60**, List US\$122, Order code SURV/228

## Analysis



## Sobolev, Besov and Triebel-Lizorkin Spaces on Quantum Tori

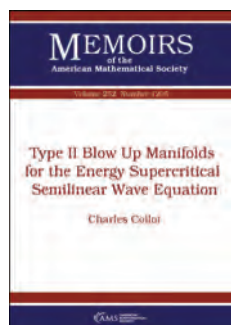
Xiao Xiong, *Wuhan University, China, and Université de Franche-Comté, Besançon, France*, Quanhua Xu, *Wuhan University, China, and Université de Franche-Comté, Besançon, France*, and Zhi Yin, *Wuhan University, China*

**Contents:** Introduction; Preliminaries; Sobolev spaces; Besov spaces; Triebel-Lizorkin spaces; Interpolation; Embedding; Fourier multiplier; Bibliography.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1203

March 2018, 116 pages, Softcover, ISBN: 978-1-4704-2806-8, 2010 *Mathematics Subject Classification*: 46L52, 46L51, 46L87; 47L25, 47L65, 43A99, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1203

## Differential Equations



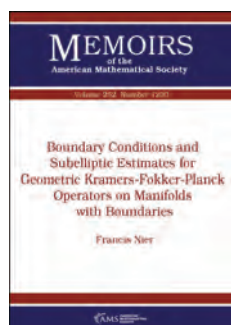
### Type II Blow Up Manifolds for the Energy Supercritical Semilinear Wave Equation

Charles Collot, *Université de Nice Sophia Antipolis, France*

**Contents:** Introduction; The linearized dynamics and the construction of the approximate blow up profile; The trapped regime; End of the proof; Lipschitz aspect and codimension of the set of solutions described by Proposition 3.2; Appendix A. Properties of the stationary state; Appendix B. Equivalence of norms; Appendix C. Hardy inequalities; Appendix D. Coercivity of the adapted norms; Appendix E. Specific bounds for the analysis; Bibliography; Index of the constants.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1205

March 2018, 159 pages, Softcover, ISBN: 978-1-4704-2813-6, 2010 *Mathematics Subject Classification*: 35B44; 35L05, 58B99, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1205



### Boundary Conditions and Subelliptic Estimates for Geometric Kramers-Fokker-Planck Operators on Manifolds with Boundaries

Francis Nier, *Université de Paris, France*

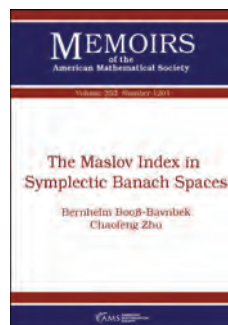
*This item will also be of interest to those working in analysis.*

**Contents:** Introduction; One dimensional model problem; Cuspidal semigroups; Separation of variables; General boundary conditions for half-space problems; Geometric Kramers-Fokker-Planck operator; Geometric KFP-operators on manifolds with boundary; Variations on a theorem; Applications; Appendix A. Translation invariant model problems; Appendix B. Partitions of unity; Acknowledgements; Bibliography.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1200

March 2018, 142 pages, Softcover, ISBN: 978-1-4704-2802-0, 2010 *Mathematics Subject Classification*: 32G15, 35H10, 35H20, 35R01, 47D06, 58J32, 58J50, 58J65, 60H10, 60J65, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1200

## Geometry and Topology



### The Maslov Index in Symplectic Banach Spaces

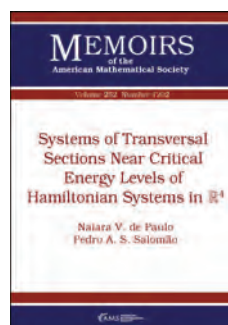
Bernhelm Booß-Bavnbek, *Roskilde University, Denmark*, and Chaofeng Zhu, *Nankai University, Tianjin, People's Republic of China*

**Contents:** Introduction; *Part 1. Maslov index in symplectic Banach spaces*: General theory of symplectic analysis in Banach spaces; The Maslov index in strong symplectic Hilbert space; The Maslov index in Banach bundles over a closed interval; *Part 2. Applications in global analysis*: The desuspension spectral flow formula; Appendix A. Perturbation of closed subspaces in Banach spaces; Bibliography; List of symbols; Index of names/authors; Subject index; Index; Bibliography.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1201

March 2018, 123 pages, Softcover, ISBN: 978-1-4704-2800-6, 2010 *Mathematics Subject Classification*: 53D12; 58J30, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1201

## Mathematical Physics



### Systems of Transversal Sections Near Critical Energy Levels of Hamiltonian Systems in $\mathbb{R}^4$

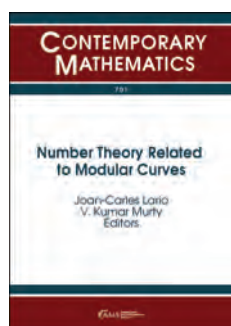
Naiara V. de Paulo, *Cidade Universitária, São Paulo, Brazil*, and Pedro A. S. Salomão, *Cidade Universitária, São Paulo, Brazil*

**Contents:** Introduction; Proof of the main statement; Proof of Proposition 2.1; Proof of Proposition 2.2; Proof of Proposition 2.8; Proof of Proposition 2.9; Proof of Proposition 2.10-i; Proof of Proposition 2.10-ii; Proof of Proposition 2.10-iii; Appendix A. Basics on pseudo-holomorphic curves in symplectizations; Appendix B. Linking properties; Appendix C. Uniqueness and intersections of pseudo-holomorphic curves; References.

**Memoirs of the American Mathematical Society**, Volume 252, Number 1202

March 2018, 105 pages, Softcover, ISBN: 978-1-4704-2801-3, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/252/1202

## Number Theory



### Number Theory Related to Modular Curves

Momose Memorial Volume

**Joan-Carles Lario**, *Universitat Politècnica de Catalunya, Barcelona, Spain*, and **V. Kumar Murty**, *University of Toronto, Ontario, Canada*, Editors

This volume contains the proceedings of the Barcelona-Boston-Tokyo Number Theory Seminar, which was held in memory of Fumiyuki Momose, a distinguished number theorist from Chuo University in Tokyo.

Momose, who was a student of Yasutaka Ihara, made important contributions to the theory of Galois representations attached to modular forms, rational points on elliptic and modular curves, modularity of some families of Abelian varieties, and applications of arithmetic geometry to cryptography.

Papers contained in this volume cover these general themes in addition to discussing Momose's contributions as well as recent work and new results.

**Contents:** **T. Saito**, An overview of the mathematical work of Fumiyuki Momose; **R. Burko**,  $p$ -adic point counting on singular superelliptic curves; **K. Arai**, A note on algebraic points on Shimura curves; **T. Chatterjee**, **M. R. Murty**, and **S. Pathak**, A vanishing criterion for Dirichlet series with periodic coefficients; **N. Hashizume**, **F. Momose**, and **J. Chao**, On implementation of GHS attack against elliptic curve cryptosystems over cubic extension fields of odd characteristic; **V. K. Murty**, Arithmetic twists and Abelian extensions; **Y. Gon** and **T. Oda**, An explicit integral representation of Siegel-Whittaker functions on  $\mathrm{Sp}(2, \mathbb{R})$  for the large discrete series representations; **M. R. Murty**, Transcendental numbers and special values of Dirichlet series; **J. C. Lario** and **A. Somoza**, The Sato-Tate conjecture for a Picard curve with complex multiplication (with an Appendix by F. Fité); **F. Bars**, On quadratic points of classical modular curves; **M. Derickx**, **B. Mazur**, and **S. Kamienny**, Rational families of 17-torsion of elliptic curves over number fields; **C. Castaño-Bernard**, A refinement of a conjecture of Gross, Kohnen, and Zagier.

**Contemporary Mathematics**, Volume 701

February 2018, approximately 228 pages, Softcover, ISBN: 978-1-4704-1991-2, 2010 *Mathematics Subject Classification*: 11G18, 14G05, 14G50, 11F41, 11R37, 11M99; 14G35, 11G40, 11G30, **AMS members US\$93.60**, List US\$117, Order code CONM/701

## New AMS-Distributed Publications

### Algebra and Algebraic Geometry



### Diophantine Applications of Geometric Invariant Theory

**Marco Maculan**, *Institut Mathématique de Jussieu, Université Pierre et Marie Curie, Paris, France*

The author presents a proof of Roth's theorem (and some more recent variants) based on geometric invariant theory. A crucial role is played by a formula of Burnol-Zhang. The formula is studied in detail and is linked to Berkovich's  $p$ -adic analytic geometry and a conjecture of Bost on the tensor product of Hermitian vector bundles on a number field.

*This item will also be of interest to those working in number theory.*

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

**Mémoires de la Société Mathématique de France**, Number 152

October 2017, 149 pages, Softcover, ISBN: 978-2-85629-865-7, 2010 *Mathematics Subject Classification*: 14L24, 14G40, 11K60, 14G22, **AMS members US\$41.60**, List US\$52, Order code SMFMEM/152



# Geometry and Topology



## The Realization Space of an Unstable Coalgebra

**Georg Biedermann**, *LAGA, Institut Galilée, Université Paris 13, Villetaneuse, France*,  
**Georgios Raptis**, *Universität Regensburg, Germany*, and  
**Manfred Stelzer**, *Institut für Mathematik, Universität Osnabrück, Germany*

Unstable coalgebras over the Steenrod algebra form a natural target category for singular homology with prime field coefficients. The realization problem asks whether an unstable coalgebra is isomorphic to the homology of a topological space. The authors study the moduli space of such realizations and give a description of this in terms of cohomological invariants of the unstable coalgebra. This is accomplished by a thorough comparative study of the homotopy theories of cosimplicial unstable coalgebras and of cosimplicial spaces.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

**Astérisque**, Number 393

February 2018, 143 pages, Softcover, ISBN: 978-2-85629-868-8, 2010 *Mathematics Subject Classification*: 55S10, 55S35, 55N10, 55P62, **AMS members US\$41.60**, List US\$52, Order code AST/393

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# MEETINGS & CONFERENCES OF THE AMS

## FEBRUARY TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: [www.ams.org/meetings/](http://www.ams.org/meetings/).

**Important Information About AMS Meetings:** Potential organizers, speakers, and hosts should refer to page 88 in the January 2018 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

**Abstracts:** Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of  $\text{\LaTeX}$  is

necessary to submit an electronic form, although those who use  $\text{\LaTeX}$  may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in  $\text{\LaTeX}$ . Visit [www.ams.org/cgi-bin/abstracts/abstract.pl/](http://www.ams.org/cgi-bin/abstracts/abstract.pl/). Questions about abstracts may be sent to [abs-info@ams.org](mailto:abs-info@ams.org). Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

## MEETINGS IN THIS ISSUE

### 2018

January 10-13	San Diego, California	p. 215
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### 2019

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### 2020

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### 2021

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See [www.ams.org/meetings/](http://www.ams.org/meetings/) for the most up-to-date information on the meetings and conferences that we offer.

## ASSOCIATE SECRETARIES OF THE AMS

**Central Section:** Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: [benkart@math.wisc.edu](mailto:benkart@math.wisc.edu); telephone: 608-263-4283.

**Eastern Section:** Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: [steve.weintraub@lehigh.edu](mailto:steve.weintraub@lehigh.edu); telephone: 610-758-3717.

**Southeastern Section:** Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: [brian@math.uga.edu](mailto:brian@math.uga.edu); telephone: 706-542-2547.

**Western Section:** Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: [lapidus@math.ucr.edu](mailto:lapidus@math.ucr.edu); telephone: 951-827-5910.

# Meetings & Conferences of the AMS

**IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS:** AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See [www.ams.org/meetings/](http://www.ams.org/meetings/).

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

## San Diego, California

*San Diego Convention Center Marriott  
Marquis San Diego Marina*

**January 10–13, 2018**

*Wednesday – Saturday*

### Meeting #1135

*Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).*

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: October 2017

Program first available on AMS website: November, 2017

Issue of *Abstracts*: Volume 39, Issue 1

### Deadlines

For organizers: Expired

For abstracts: Expired

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgts/national.html](http://www.ams.org/amsmtgts/national.html).*

### Joint Invited Addresses

**Gunnar Carlsson**, Stanford University, *Topological Modeling of Complex Data* (AMS-MAA Invited Address).

**Moon Duchin**, Tufts University, *Political Geometry: Voting districts, "compactness," and ideas about fairness* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

**André Neves**, University of Chicago, *Wow, so many minimal surfaces!* (AMS-MAA Invited Address).

**Jill Pipher**, Brown University, *Nonsmooth boundary value problems* (AWM-AMS Noether Lecture).

### AMS Invited Addresses

**Federico Ardila**, San Francisco State University, *Algebraic structures on polytopes*.

**Robert L. Bryant**, Duke University, *The concept of Holonomy—its history and recent developments* (AMS Retiring Presidential Address).

**Ruth Charney**, Brandeis University, *Searching for hyperbolicity*.

**Cynthia Dwork**, Harvard University, *Privacy in the land of plenty* (AMS Josiah Willard Gibbs Lecture).

**Dana Randall**, Georgia Institute of Technology, *Emergent phenomena in random structures and algorithms*.

**Edriss S. Titi**, Texas A&M University; and The Weizmann Institute of Science, *The Navier-Stokes, Euler and related equations*.

**Avi Wigderson**, Institute for Advanced Study, *Alternate Minimization and Scaling algorithms: theory, applications and connections across mathematics and computer science* (AMS Colloquium Lectures: Lecture I).

**Avi Wigderson**, Institute for Advanced Study, *Proving algebraic identities* (AMS Colloquium Lectures: Lecture II).

**Avi Wigderson**, Institute for Advanced Study, *Proving analytic inequalities* (AMS Colloquium Lectures: Lecture III).

**Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.**

*A Showcase of Number Theory at Liberal Arts Colleges*, **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

*Accelerated Advances in Mathematical Fractional Programming*, **Ram Verma**, International Publications USA, and **Alexander Zaslavski**, Israel Institute of Technology.

*Advances in Applications of Differential Equations to Disease Modeling*, **Libin Rong**, Oakland University, **Elissa**



**Schwartz**, Washington State University, and **Naveen K. Vaidya**, San Diego State University.

*Advances in Difference, Differential, and Dynamic Equations with Applications*, **Elvan Akin**, Missouri University S&T, and **John Davis**, Baylor University.

*Advances in Operator Algebras*, **Marcel Bischoff**, Vanderbilt University, **Ian Charlesworth**, University of California, Los Angeles, **Brent Nelson**, University of California, Berkeley, and **Sarah Reznikoff**, Kansas State University.

*Advances in Operator Theory, Operator Algebras, and Operator Semigroups*, **Asuman G. Aksoy**, Claremont McKenna College, **Zair Ibragimov**, California State University, Fullerton, **Marat Markin**, California State University, Fresno, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

*Algebraic, Analytic, and Geometric Aspects of Integrable Systems, Painlevé Equations, and Random Matrices*, **Vladimir Dragovic**, University of Texas at Dallas, **Anton Dzhamay**, University of Northern Colorado, and **Sevak Mkrtchyan**, University of Rochester.

*Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology*, **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

*Alternative Proofs in Mathematical Practice*, **John W. Dawson, Jr.**, Pennsylvania State University, York.

*Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems*, **John R. Graef**, University of Tennessee at Chattanooga, **Gangaram S. Ladde**, University of South Florida, and **Aghalaya S. Vatsala**, University of Louisiana at Lafayette.

*Analysis of Nonlinear Partial Differential Equations and Applications*, **Tarek M. Elgindi**, University of California, San Diego, and **Edriss S. Titi**, Texas A&M University and Weizmann Institute of Science.

*Applied and Computational Combinatorics*, **Torin Greenwood**, Georgia Institute of Technology, and **Jay Pantone**, Dartmouth College.

*Arithmetic Dynamics*, **Robert L. Benedetto**, Amherst College, **Benjamin Hutz**, Saint Louis University, **Jamie Juul**, Amherst College, and **Bianca Thompson**, Harvey Mudd College.

*Beyond Planarity: Crossing Numbers of Graphs (a Mathematics Research Communities Session)*, **Axel Brandt**, Davidson College, **Garner Cochran**, University of South Carolina, and **Sarah Loeb**, College of William and Mary.

*Bifurcations of Difference Equations and Discrete Dynamical Systems*, **Arzu Bilgin** and **Toufik Khyat**, University of Rhode Island.

*Boundaries for Groups and Spaces*, **Joseph Maher**, CUNY College of Staten Island, and **Genevieve Walsh**, Tufts University.

*Combinatorial Commutative Algebra and Polytopes*, **Robert Davis**, Michigan State University, and **Liam Solus**, KTH Royal Institute of Technology.

*Combinatorics and Geometry*, **Federico Ardila**, San Francisco State University, **Anastasia Chavez**, MSRI and University of California, Davis, and **Laura Escobar**, University of Illinois Urbana—Champaign.

*Commutative Algebra in All Characteristics*, **Neil Epstein**, George Mason University, **Karl Schwede**, University of Utah, and **Janet Vassilev**, University of New Mexico.

*Computational Combinatorics and Number Theory*, **Jeremy F. Alm**, Lamar University, and **David Andrews** and **Rob Hochberg**, University of Dallas.

*Connections in Discrete Mathematics: Graphs, Hypergraphs, and Designs*, **Amin Bahmanian**, Illinois State University, and **Theodore Molla**, University Illinois Urbana—Champaign.

*Differential Geometry*, **Vincent B. Bonini** and **Joseph E. Borzellino**, Cal Poly San Luis Obispo, **Bogdan D. Suceava**, California State University, Fullerton, and **Guofang Wei**, University of California, Santa Barbara.

*Diophantine Approximation and Analytic Number Theory in Honor of Jeffrey Vaaler*, **Shabnam Akhtari**, University of Oregon, **Lenny Fukshansky**, Claremont McKenna College, and **Clayton Petsche**, Oregon State University.

*Discrete Dynamical Systems and Applications*, **E. Cabral Balreira**, **Saber Elaydi**, and **Eddy Kwessi**, Trinity University.

*Discrete Neural Networking and Applications*, **Murat Adivar**, Fayetteville State University, **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

*Dynamical Algebraic Combinatorics*, **James Propp**, University of Massachusetts, Lowell, **Tom Roby**, University of Connecticut, **Jessica Striker**, North Dakota State University, and **Nathan Williams**, University of California Santa Barbara.

*Dynamical Systems with Applications to Mathematical Biology*, **Guihong Fan**, Columbus State University, **Jing Li**, California State University Northridge, and **Chunhua Shan**, University of Toledo.

*Dynamical Systems: Smooth, Symbolic, and Measurable (a Mathematics Research Communities Session)*, **Kathryn Lindsey**, Boston College, **Scott Schmieding**, Northwestern University, and **Kurt Vinhage**, University of Chicago.

*Emergent Phenomena in Discrete Models*, **Dana Randall**, Georgia Institute of Technology, and **Andrea Richa**, Arizona State University.

*Emerging Topics in Graphs and Matrices*, **Sudipta Mallik**, Northern Arizona University, **Keivan Hassani Monfared**, University of Calgary, and **Bryan Shader**, University of Wyoming.

*Ergodic Theory and Dynamical Systems—to Celebrate the Work of Jane Hawkins*, **Julia Barnes**, Western Carolina University, **Rachel Bayless**, Agnes Scott College, **Emily Burkhead**, Duke University, and **Lorelei Koss**, Dickinson College.

*Extremal Problems in Approximations and Geometric Function Theory*, **Ram Mohapatra**, University of Central Florida.

*Financial Mathematics, Actuarial Sciences, and Related Fields*, **Albert Cohen**, Michigan State University, **Nguyet Nguyen**, Youngstown State University, **Oana Mocioalca**, Kent State University, and **Thomas Wakefield**, Youngstown State University.

*Fractional Difference Operators and Their Application*, **Christopher S. Goodrich**, Creighton Preparatory School, and **Rajendra Dahal**, Coastal Carolina University.

*Free Convexity and Free Analysis*, **J. William Helton**, University of California, San Diego, and **Igor Klep**, University of Auckland.

*Geometric Analysis*, **Davi Maximo**, University of Pennsylvania, **Lu Wang**, University of Wisconsin-Madison, and **Xin Zhou**, University of California Santa Barbara.

*Geometric Analysis and Geometric Flows*, **David Glickenstein**, University of Arizona, and **Brett Kotschwar**, Arizona State University.

*History of Mathematics*, **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Clemency Montelle**, University of Canterbury, **Daniel Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College.

*Homotopy Type Theory (a Mathematics Research Communities Session)*, **Simon Cho**, University of Michigan, **Liron Cohen**, Cornell University, and **Edward Morehouse**, Wesleyan University.

*If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences*, **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

*Interactions of Inverse Problems, Signal Processing, and Imaging*, **M. Zuhair Nashed**, University of Central Florida, **Willi Freeden**, University of Kaiserslautern, and **Otmar Scherzer**, University of Vienna.

*Markov Chains, Markov Processes and Applications*, **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

*Mathematical Analysis and Nonlinear Partial Differential Equations*, **Hongjie Dong**, Brown University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

*Mathematical Fluid Mechanics: Analysis and Applications*, **Zachary Bradshaw** and **Aseel Farhat**, University of Virginia.

*Mathematical Information in the Digital Age of Science*, **Patrick Ion**, University of Michigan, **Olaf Teschke**, zbMath Berlin, and **Stephen Watt**, University of Waterloo.

*Mathematical Modeling and Analysis of Infectious Diseases*, **Kazuo Yamazaki**, University of Rochester.

*Mathematical Modeling of Natural Resources*, **Shandelle M. Henson**, Andrews University, and **Natali Hritonenko**, Prairie View A&M University.

*Mathematical Modeling, Analysis and Applications in Population Biology*, **Yu Jin**, University of Nebraska-Lincoln, and **Ying Zhou**, Lafayette College.

*Mathematical Problems in Ocean Wave Modeling and Fluid Mechanics*, **Christopher W. Curtis**, San Diego State University, and **Katie Oliveras**, Seattle University.

*Mathematical Relativity and Geometric Analysis*, **James Diltz** and **Michael Holst**, University of California, San Diego.

*Mathematics Research from the SMALL Undergraduate Research Program*, **Colin Adams**, **Frank Morgan**, and **Cesar E. Silva**, Williams College.

*Mathematics of Gravitational Wave Science*, **Andrew Gillette** and **Nikki Holtzer**, University of Arizona.

*Mathematics of Quantum Computing and Topological Phases of Matter*, **Paul Bruillard**, Pacific Northwest National Laboratory, **David Meyer**, University of California San Diego, and **Julia Plavnik**, Texas A&M University.

*Metric Geometry and Topology*, **Christine Escher**, Oregon State University, and **Catherine Searle**, Wichita State University.

*Modeling in Differential Equations - High School, Two-Year College, Four-Year Institution*, **Corban Harwood**, George Fox University, **William Skerbitz**, Wayzata High School, **Brian Winkel**, SIMIODE, and **Dina Yagodich**, Frederick Community College.

*Multi-scale Modeling with PDEs in Computational Science and Engineering: Algorithms, Simulations, Analysis, and Applications*, **Salim M. Haidar**, Grand Valley State University.

*Network Science*, **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng Shi**, University of North Carolina at Chapel Hill.

*New Trends in Celestial Mechanics*, **Richard Montgomery**, University of California Santa Cruz, and **Zhifu Xie**, University of Southern Mississippi.

*Nilpotent and Solvable Geometry*, **Michael Jablonski**, University of Oklahoma, **Megan Kerr**, Wellesley College, and **Tracy Payne**, Idaho State University.

*Noncommutative Algebras and Noncommutative Invariant Theory*, **Ellen Kirkman**, Wake Forest University, and **James Zhang**, University of Washington.

*Nonlinear Evolution Equations of Quantum Physics and Their Topological Solutions*, **Stephen Gustafson**, University of British Columbia, **Israel Michael Sigal**, University of Toronto, and **Avy Soffer**, Rutgers University.

*Novel Methods of Enhancing Success in Mathematics Classes*, **Ellina Grigorieva**, Texas Womans University, and **Natali Hritonenko**, Prairie View A&M University.

*Open and Accessible Problems for Undergraduate Research*, **Michael Dorff**, Brigham Young University, **Alison Henrich**, Seattle University, and **Nicholas Scoville**, Ursinus College.

*Operators on Function Spaces in One and Several Variables*, **Catherine Bénéteau**, University of South Florida, and **Matthew Fleeman** and **Constanze Liaw**, Baylor University.

*Orthogonal Polynomials and Applications*, **Abey Lopez-Garcia**, University of South Alabama, and **Xiang-Sheng Wang**, University of Louisiana at Lafayette.

*Orthogonal Polynomials, Quantum Probability, and Stochastic Analysis*, **Julius N. Esunge**, University of Mary Washington, and **Aurel I. Stan**, Ohio State University.

*Quantum Link Invariants, Khovanov Homology, and Low-dimensional Manifolds*, **Diana Hubbard**, University

of Michigan, and **Christine Ruey Shan Lee**, University of Texas at Austin.

*Quaternions*, **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

*Recent Trends in Analysis of Numerical Methods of Partial Differential Equations*, **Sara Pollock**, Wright State University, and **Leo Rebholz**, Clemson University.

*Research by Postdocs of the Alliance for Diversity in Mathematics*, **Aloysius Helminck**, University of Hawaii - Manoa, and **Michael Young**, Iowa State University.

*Research from the Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics*, **Michael Ferrara**, University of Colorado Denver, **Leslie Hogben**, Iowa State University, **Paul Horn**, University of Denver, and **Tyrrell McAllister**, University of Wyoming.

*Research in Mathematics by Early Career Graduate Students*, **Michael Bishop**, **Marat Markin**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

*Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs*, **Tamas Forgacs**, CSU Fresno, **Darren A. Narayan**, Rochester Institute of Technology, and **Mark David Ward**, Purdue University (AMS-MAA-SIAM).

*Set Theory, Logic and Ramsey Theory*, **Andrés Caicedo**, Mathematical Reviews, and **José Mijares**, University of Colorado, Denver (AMS-ASL).

*Set-theoretic Topology (Dedicated to Jack Porter in Honor of 50 Years of Dedicated Research)*, **Nathan Carlson**, California Lutheran University, **Jila Niknejad**, University of Kansas, and **Lynne Yengulalp**, University of Dayton.

*Special Functions and Combinatorics (in honor of Dennis Stanton's 65th birthday)*, **Susanna Fishel**, Arizona State University, **Mourad Ismail**, University of Central Florida, and **Vic Reiner**, University of Minnesota.

*Spectral Theory, Disorder and Quantum Physics*, **Rajinder Mavi** and **Jeffery Schenker**, Michigan State University.

*Stochastic Processes, Stochastic Optimization and Control, Numerics and Applications*, **Hongwei Mei**, University of Central Florida, **Zhixin Yang** and **Quan Yuan**, Ball State University, and **Guangliang Zhao**, GE Global Research.

*Strengthening Infrastructures to Increase Capacity Around K-20 Mathematics*, **Brianna Donaldson**, American Institute of Mathematics, **William Jaco** and **Michael Oehrtman**, Oklahoma State University, and **Levi Patrick**, Oklahoma State Department of Education.

*Structure and Representations of Hopf Algebras: a Session in Honor of Susan Montgomery*, **Siu-Hung Ng**, Louisiana State University, and **Lance Small** and **Henry Tucker**, University of California, San Diego.

*Theory, Practice, and Applications of Graph Clustering*, **David Gleich**, Purdue University, and **Jennifer Webster** and **Stephen J. Young**, Pacific Northwest National Laboratory.

*Topological Data Analysis*, **Henry Adams**, Colorado State University, **Gunnar Carlsson**, Stanford University,

and **Mikael Vejdemo-Johansson**, CUNY College of Staten Island.

*Topological Graph Theory: Structure and Symmetry*, **Jonathan L. Gross**, Columbia University, and **Thomas W. Tucker**, Colgate University.

*Visualization in Mathematics: Perspectives of Mathematicians and Mathematics Educators*, **Karen Allen Keene**, North Carolina State University, and **Mile Krajcevski**, University of South Florida.

*Women in Symplectic and Contact Geometry and Topology*, **Bahar Acu**, Northwestern University, **Ziva Myer**, Duke University, and **Yu Pan**, Massachusetts Institute of Technology (AMS-AWM).

## Columbus, Ohio

Ohio State University

March 17–18, 2018

Saturday – Sunday

### Meeting #1136

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: December 2017

Program first available on AMS website: January 31, 2018

Issue of *Abstracts*: Volume 39, Issue 2

### Deadlines

For organizers: Expired

For abstracts: January 22, 2018

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html).*

### Invited Addresses

**Aaron Brown**, University of Chicago, *Title to be announced.*

**Tullia Dymarz**, University of Wisconsin-Madison, *Title to be announced.*

**June Huh**, Institute for Advanced Study, *Title to be announced.*

### Special Sessions

*If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).*

*Advances in Integral and Differential Equations* (Code: SS 26A), **Jeffrey T. Neugebauer**, Eastern Kentucky University, and **Min Wang**, Rowan University.

*Algebraic Coding Theory and Applications* (Code: SS 27A), **Heide Gluesing-Luerssen**, University of Kentucky, **Christine A. Kelley**, University of Nebraska-Lincoln, and **Steve Szabo**, Eastern Kentucky University.



*Algebraic Combinatorics: Association Schemes, Finite Geometry, and Related Topics* (Code: SS 15A), **Sung Y. Song**, Iowa State University, and **Bangteng Xu**, Eastern Kentucky University.

*Algebraic Curves and Their Applications* (Code: SS 17A), **Artur Elezi**, American University, **Monika Polak**, Maria Curie-Skłodowska University (Poland) and University of Information Science and Technology (Mac), and **Tony Shaska**, Oakland University.

*Algebraic and Combinatorial Aspects of Tropical Geometry* (Code: SS 11A), **Maria Angelica Cueto**, Ohio State University, **Yoav Len**, University of Waterloo, and **Martin Ulirsch**, University of Michigan.

*Algebraic, Combinatorial, and Quantum Invariants of Knots and Manifolds* (Code: SS 6A), **Cody Armond**, Ohio State University, Mansfield, **Micah Chrisman**, Monmouth University, and **Heather Dye**, McKendree University.

*Analytical and Computational Advances in Mathematical Biology Across Scales* (Code: SS 30A), **Veronica Ciocanel** and **Alexandria Volkening**, Mathematical Biosciences Institute.

*Categorical, Homological and Combinatorial Methods in Algebra (Celebrating the 80th birthday of S. K. Jain)* (Code: SS 28A), **Pedro A. Guil Asensio**, University of Murcia, **Ivo Herzog**, Ohio State University, **Andre Leroy**, University of Artois, and **Ashish K. Srivastava**, Saint Louis University.

*Coherent Structures in Interfacial Flows* (Code: SS 14A), **Benjamin Akers** and **Jonah Reeger**, Air Force Institute of Technology.

*Commutative and Combinatorial Algebra* (Code: SS 18A), **Jennifer Biermann**, Hobart and William Smith Colleges, and **Kuei-Nuan Lin**, Penn State University, Greater Allegheny.

*Convex Bodies in Algebraic Geometry and Representation Theory* (Code: SS 20A), **Dave Anderson**, Ohio State University, and **Kiumars Kaveh**, University of Pittsburgh.

*Differential Equations and Applications* (Code: SS 8A), **King-Yeung Lam** and **Yuan Lou**, Ohio State University, and **Qiliang Wu**, Michigan State University.

*Function Spaces, Operator Theory, and Non-Linear Differential Operators* (Code: SS 21A), **David Cruz-Uribe**, University of Alabama, and **Oswaldo Mendez**, University of Texas.

*Geometric Methods in Shape Analysis* (Code: SS 10A), **Sebastian Kurtek** and **Tom Needham**, Ohio State University.

*Graph Theory* (Code: SS 5A), **John Maharry**, Ohio State University, **Yue Zhao**, University of Central Florida, and **Xiangqian Zhou**, Wright State University.

*Homological Algebra* (Code: SS 4A), **Ela Celikbas** and **Olgur Celikbas**, West Virginia University.

*Homotopy Theory* (Code: SS 29A), **Ernest Fontes**, **John E. Harper**, **Crichton Ogle**, and **Gabriel Valenzuela**, Ohio State University.

*Lefschetz Properties* (Code: SS 24A), **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

*Mathematical Modeling of Neuronal Networks* (Code: SS 36A), **Janet Best**, Ohio State University, **Alicia Prieto**

**Langarica**, Youngstown State University, and **Pamela B. Pyzza**, Ohio Wesleyan University.

*Multiplicative Ideal Theory and Factorization (in honor of Tom Lucas retirement)* (Code: SS 7A), **Evan Houston**, University of North Carolina, Charlotte, and **Alan Loper**, Ohio State University.

*Noncommutative Algebra and Noncommutative Algebraic Geometry* (Code: SS 16A), **Jason Gaddis**, Miami University, and **Robert Won**, Wake Forest University.

*Nonlinear Evolution Equations* (Code: SS 9A), **John Holmes** and **Feride Tiglay**, Ohio State University.

*Nonlinear Waves and Patterns* (Code: SS 19A), **Anna Ghazaryan**, Miami University, **Stephane Lafortune**, College of Charleston, and **Vahagn Manukian** and **Alin Pogan**, Miami University.

*Parameter Analysis and Estimation in Applied Dynamical Systems* (Code: SS 35A), **Adriana Dawes**, The Ohio State University, and **Reginald L. McGee**, Mathematical Biosciences Institute.

*Probabilistic and Extremal Graph Theory* (Code: SS 32A), **Louis DeBiasio** and **Tao Jiang**, Miami University.

*Probability in Convexity and Convexity in Probability* (Code: SS 2A), **Elizabeth Meckes**, **Mark Meckes**, and **Elisabeth Werner**, Case Western Reserve University.

*Quantum Symmetries* (Code: SS 3A), **David Penneys**, The Ohio State University, and **Julia Plavnik**, Texas A & M University.

*Recent Advances in Approximation Theory and Operator Theory* (Code: SS 1A), **Jan Lang** and **Paul Nevai**, The Ohio State University.

*Recent Advances in Finite Element Methods for Partial Differential Equations* (Code: SS 31A), **Ching-shan Chou**, **Yukun Li**, and **Yulong Xing**, The Ohio State University.

*Recent Advances in Packing* (Code: SS 23A), **Joseph W. Iverson**, University of Maryland, **John Jasper**, South Dakota State University, and **Dustin G. Mixon**, The Ohio State University.

*Recent Development of Nonlinear Geometric PDEs* (Code: SS 12A), **Bo Guan**, Ohio State University, **Qun Li**, Wright State University, **Xiangwen Zhang**, University of California, Irvine, and **Fangyang Zheng**, Ohio State University.

*Several Complex Variables* (Code: SS 13A), **Liwei Chen**, **Kenneth Koenig**, and **Liz Vivas**, Ohio State University.

*Stochastic Analysis in Infinite Dimensions* (Code: SS 22A), **Parisa Fatheddin**, Air Force Institute of Technology, and **Arnab Ganguly**, Louisiana State University.

*Structure and Representation Theory of Finite Groups* (Code: SS 33A), **Justin Lynd**, University of Louisiana at Lafayette, and **Hung Ngoc Nguyen**, University of Akron.

*Symmetry in Differential Geometry* (Code: SS 34A), **Samuel Lin**, Dartmouth College, **Barry Minemyer**, Bloomsburg University, and **Ben Schmidt**, Michigan State University.

*The Mathematics of Phylogenetics* (Code: SS 25A), **Colby Long**, Mathematical Biosciences Institute.

*Topology and Geometry in Data Analysis* (Code: SS 37A), **Sanjeevi Krishnan** and **Facundo Memoli**, Ohio State University.

# Nashville, Tennessee

*Vanderbilt University*

**April 14–15, 2018**

*Saturday – Sunday*

## Meeting #1138

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: February 2018

Program first available on AMS website: February 22, 2018

Issue of *Abstracts*: Volume 39, Issue 2

## Deadlines

For organizers: Expired

For abstracts: February 13, 2018

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtg/sectional.html](http://www.ams.org/amsmtg/sectional.html).*

## Invited Addresses

**Andrea Bertozzi**, University of California Los Angeles, *Geometric graph-based methods for high dimensional data*. (Erdős Memorial Lecture).

**J. M. Landsberg**, Texas A & M University, *On the geometry of matrix multiplication*.

**Jennifer Morse**, University of Virginia, *Combinatorics, computing, and K-schur functions*.

**Kirsten Wickelgren**, Georgia Institute of Technology, *An Arithmetic Count of the Lines on a Smooth Cubic Surface*.

## Special Sessions

*If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).*

*Advances in Operator Algebras* (Code: SS 7A), **Scott Atkinson**, **Dietmar Bisch**, **Vaughan Jones**, and **Jesse Peterson**, Vanderbilt University.

*Algebraic Geometry, Representation Theory, and Applications* (Code: SS 21A), **Shrawan Kumar**, University of North Carolina at Chapel Hill, **J. M. Landsberg**, Texas A&M University, and **Luke Oeding**, Auburn University.

*Boundaries and Non-positive Curvature in Group Theory* (Code: SS 15A), **Spencer Dowdall** and **Matthew Hallmark**, Vanderbilt University, and **Michael Hull**, University of Florida.

**CANCELLED:** *Difference Equations and Applications* (Code: SS 2A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton, Ohio.

*Commutative Algebra* (Code: SS 8A), **Florian Enescu** and **Yongwei Yao**, Georgia State University.

*Evolution Equations and Applications* (Code: SS 14A), **Marcelo Disconzi**, **Chenyun Luo**, **Giusy Mazzone**, and **Gieri Simonett**, Vanderbilt University.

*Function Spaces and Operator Theory* (Code: SS 9A), **Cheng Chu** and **Dechao Zheng**, Vanderbilt University.

*Harmonic Analysis, Functional Analysis, and Their Applications* (Code: SS 11A), **Akram Aldroubi** and **Keaton Hamm**, Vanderbilt University, **Michael Worthington**, Georgia Institute of Technology, and **Alex Powell**, Vanderbilt University.

*Hermitian Geometry* (Code: SS 18A), **Mehdi Lejmi**, Bronx Community College of CUNY, and **Rares Rasdeaconu** and **Ioana Suvaina**, Vanderbilt University.

*Interactions between Geometry, Group Theory and Dynamics* (Code: SS 13A), **Jayadev Athreya**, University of Washington, and **Caglar Uyanik** and **Grace Work**, Vanderbilt University.

*Macdonald Polynomials and Related Structures* (Code: SS 23A), **Jennifer Morse**, University of Virginia, and **Dan Orr** and **Mark Shimozone**, Virginia Polytechnic Institute and State University.

*Mathematical Chemistry* (Code: SS 10A), **Hua Wang**, Georgia Southern University.

*Matroids and Related Structures* (Code: SS 5A), **Carolyn Chun**, United States Naval Academy, **Deborah Chun** and **Tyler Moss**, West Virginia University Institute of Technology, and **Jakayla Robbins**, Vanderbilt University.

*Partial Differential Equations and New Perspective of Variational Methods* (Code: SS 16A), **Abbas Moameni**, Carleton University, **Futoshi Takahashi**, Osaka City University, **Michinori Ishiwata**, Osaka University, and **Craig Cowen**, University of Manitoba.

*Probabilistic Models in Mathematical Physics* (Code: SS 6A), **Robert Buckingham**, University of Cincinnati, **Seung-Yeop Lee**, University of South Florida, and **Karl Liechty**, DePaul University.

*Quantization for Probability Distributions and Dynamical Systems* (Code: SS 1A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

*Random Discrete Structures* (Code: SS 22A), **Lutz P Warnke**, Georgia Institute of Technology, and **Xavier Pérez-Giménez**, University of Nebraska-Lincoln.

*Recent Advances in Mathematical Biology* (Code: SS 12A), **Glenn Webb** and **Yixiang Wu**, Vanderbilt University.

*Recent Advances on Complex Bio-systems and Their Applications* (Code: SS 17A), **Pengcheng Xiao**, University of Evansville.

*Recent Progress and New Directions in Homotopy Theory* (Code: SS 20A), **Anna Marie Bohmann**, Vanderbilt University, and **Kirsten Wickelgren**, Georgia Institute of Technology.

*Selected Topics in Graph Theory* (Code: SS 3A), **Songling Shan**, Vanderbilt University, and **David Chris Stephens** and **Dong Ye**, Middle Tennessee State University.

*Structural Graph Theory* (Code: SS 4A), **Joshua Fallon**, Louisiana State University, and **Emily Marshall**, Arcadia University.

*Tensor Categories and Diagrammatic Methods* (Code: SS 19A), **Marcel Bischoff**, Ohio University, and **Henry Tucker**, University of California San Diego.

### Erdos Memorial Lecture

The Erdos Memorial Lecture will be given by **Andrea Bertozzi**, University of California, Los Angeles. The title of her talk is *Geometric graph-based methods for high dimensional data*. The lecture will be given on Saturday, April 14th.

There will be a reception hosted by Vanderbilt University Department of Mathematics following the Erdos Memorial Lecture.

### Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include state and local tax (15.25%) and occupancy tax (\$2.50 per room per night). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at Vanderbilt University** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

**Homewood Suites by Hilton Nashville Vanderbilt** (.3 miles from Butterick Hall on campus), 2400 West End Ave., Nashville, TN 37212; Tel. 615-340-8000; [homewood-suites3.hilton.com/en/hotels/tennessee/homewood-suites-by-hilton-nashville-vanderbilt-tn-BNAVHWH/index.html?WT.mc\\_id=zDA01MB20LG34YX](http://homewood-suites3.hilton.com/en/hotels/tennessee/homewood-suites-by-hilton-nashville-vanderbilt-tn-BNAVHWH/index.html?WT.mc_id=zDA01MB20LG34YX). Rates are **US\$229** per night for a studio suite with one king bed. Rate includes complimentary hot breakfast and Wi-Fi. Self-Parking is **US\$25** per night. Homewood Suites offers complimentary shuttle service within 2 miles of the hotel on a first come, first serve basis. The deadline for reservations at this rate is **March 13, 2018**.

**Holiday Inn Nashville Vanderbilt** (.6 miles from Butterick Hall on campus), 2613 West End Ave., Nashville, TN 37203; Tel 615-327-4707; [https://www.ihg.com/holidayinn/hotels/us/en/reservation/book?qGrpCd=AMS&qGRM=0&qBrS=hi.ex.rs.ic.cp.in.sb.cw.cv.6c.vn.ul.ki.sp.nd.ct&qRpn=1&srb\\_u=1&qCoMy=032018&qCoD=15&qRms=1&qRef=df&qCiD=13&qCiMy=032018&qShp=1&qWch=0&qRtP=6CBARC&qS1H=bnavb&qAdlt=1&qHt1C=bnavb&qSrt=sBR&qSmP=3&qPSt=0&qRpp=20&qChld=0&qRRSrt=rt&method=roomRate&icdv=99801505#roomraterstitle](https://www.ihg.com/holidayinn/hotels/us/en/reservation/book?qGrpCd=AMS&qGRM=0&qBrS=hi.ex.rs.ic.cp.in.sb.cw.cv.6c.vn.ul.ki.sp.nd.ct&qRpn=1&srb_u=1&qCoMy=032018&qCoD=15&qRms=1&qRef=df&qCiD=13&qCiMy=032018&qShp=1&qWch=0&qRtP=6CBARC&qS1H=bnavb&qAdlt=1&qHt1C=bnavb&qSrt=sBR&qSmP=3&qPSt=0&qRpp=20&qChld=0&qRRSrt=rt&method=roomRate&icdv=99801505#roomraterstitle). Rates are **US\$179** per night for a guest room with one king bed or two queen beds. This rate includes free Wi-Fi. Parking at the Holiday Inn Nashville Vanderbilt is **US\$22** per day for self-parking and **US\$25** per day for valet. The deadline for reservations at this rate is **February 27, 2018**.

**Hilton Garden Inn Nashville/Vanderbilt** (1.1 miles from Butterick Hall on campus), 1715 Broadway, Nashville, TN 37203; Tel 615-369-5900; [hiltongardeninn.hilton.com/en/gi/groups/personalized/B/BNANVGI-AMS18-20180413/index.jhtml?WT.mc\\_id=POG](http://hiltongardeninn.hilton.com/en/gi/groups/personalized/B/BNANVGI-AMS18-20180413/index.jhtml?WT.mc_id=POG). Rates are **US\$199** for a room with a king bed. The Hilton Garden Inn offers complimentary local shuttle service and free Wi-Fi. A discounted self parking rate of **\$18** per night is available to guests attending the AMS sectional meeting at Vanderbilt. Valet parking is **US\$29** per night. The deadline for reservations at this rate is **March 13, 2018**.

**Home 2 Suites by Hilton Nashville** (1.2 miles from Butterick Hall on campus), 1800 Division Street, Nashville, TN 37203; 615-254-2170; [home2suites.hilton.com/en/ht/groups/personalized/B/BNAHTHT-AMS-20180413/index.jhtml?WT.mc\\_id=POG](http://home2suites.hilton.com/en/ht/groups/personalized/B/BNAHTHT-AMS-20180413/index.jhtml?WT.mc_id=POG). Rates are **US\$189** per night for a studio suite with a queen bed. Rates include complimentary breakfast and Wi-Fi. The special rate for self parking at Home 2 Suites is **US\$14** per night. The deadline for reservations at this rate is **March 13, 2018**.

**Food Services**

**On Campus:** There are many dining options on campus within walking distance of the meeting. The hours of the on-campus restaurants vary on the weekends. Please see a complete listing here: [campusdining.vanderbilt.edu/locations/](http://campusdining.vanderbilt.edu/locations/)

**Off Campus:** Nashville offers many dining options of all types of cuisine. There several coffee shops and restaurants within a few blocks of Butterick Hall and Stevenson Center. For more information on dining throughout Nashville, please visit, [www.visitmusiccity.com/visitors/food](http://www.visitmusiccity.com/visitors/food). And for more information on visiting Nashville in general, please visit [www.visitmusiccity.com/Visitors](http://www.visitmusiccity.com/Visitors).

### Food Services

Some of the nearby off-campus dining options include:

- *Bruegger's Bagels*, 422 21st Ave S; [brueggers.com](http://brueggers.com)
- *Panera*, 406 21st Ave S; [panerabread.com](http://panerabread.com)
- *San Antonio Taco Co*, 416 21st Ave S; [thesatco.com](http://thesatco.com)
- *Starbucks*, 402 21st Ave S; [starbucks.com](http://starbucks.com)
- *Mello Mushroom*, 212 21st Ave S; [mellowmushroom.com](http://mellowmushroom.com)
- *South Street Restaurant*, 907 20th Ave S; [southstreet-nash.com](http://southstreet-nash.com)
- *Le Sel*, 1922 Adelicia St; [leselnashville.com](http://leselnashville.com)
- *Union Common*, 1929 Broadway; [unioncommon.com](http://unioncommon.com)
- *Ruth's Chris Steakhouse*, 2100 West End Ave; [ruth-schris.com](http://ruth-schris.com)
- *Tavern*, 1904 Broadway; [mstreetnashville.com](http://mstreetnashville.com)

### Registration and Meeting Information

**Advance Registration:** Advance registration for this meeting opens on **January 30, 2018**. Advance registration fees are **US\$61** for AMS members, **US\$90** for nonmembers, and **US\$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing [mmsb@ams.org](mailto:mmsb@ams.org). The deadline to cancel is the first day of the meeting.

**On-site Information and Registration:** The registration desk, AMS book exhibit and coffee service will be located in the lobby on the ground level of Butterick Hall. Special Sessions and Contributed Paper Sessions will take place



## MEETINGS & CONFERENCES

in Butterick Hall and Stevenson Center, Building 1, and the Invited Addresses will take place in Stevenson Center, Building 4, Room 309. For information on building locations, a campus map is available at [www.vanderbilt.edu/map/location-category/academic/](http://www.vanderbilt.edu/map/location-category/academic/). A map of Stevenson Center can be found at [www.vanderbilt.edu/chemistry/location.php](http://www.vanderbilt.edu/chemistry/location.php). All of the Stevenson Center buildings are connected underground.

The registration desk will be open on Saturday, April 14, 7:30 am–4:00 pm and Sunday, April 15, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

### Other Activities

**Book Sales:** Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on-site orders! AMS members receive 40% off list price. Nonmembers receive a 25% discount.

**Membership Activities:** Join us for appetizers at our Welcome Reception on Friday, April 13, 5:00–7:00 pm in the Music Square Room at the Hilton Garden Inn, 1715 Broadway, Nashville, TN. Please sign up for this event, at no charge, on the meeting registration form. The deadline to sign up for this reception will be **Tuesday, April 10, 2018**.

During the meeting, stop by the AMS Membership Exhibit to learn about the benefits of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more! Complimentary Refreshments will be served courtesy of the AMS Membership Department.

**AMS Editorial Activity:** An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

### Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at [mmsb@ams.org](mailto:mmsb@ams.org) or [meet@ams.org](mailto:meet@ams.org).

### AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the

free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

### Local Information and Maps

This meeting will take place at Vanderbilt University in Nashville, TN. A campus map can be found at [www.vanderbilt.edu/map/location-category/academic/](http://www.vanderbilt.edu/map/location-category/academic/). Information about the Vanderbilt University Department of Mathematics can be found at [as.vanderbilt.edu/math/](http://as.vanderbilt.edu/math/). Please visit the Vanderbilt University website at [www.vanderbilt.edu/](http://www.vanderbilt.edu/).

Please watch the AMS website at [www.ams.orgmeetings/sectional/sectional.html](http://www.ams.orgmeetings/sectional/sectional.html) for additional information on this meeting.

### Parking

The public parking closest to Butterick Hall and Stevenson Center is Wesley Place Parking, which is located at the corner of 21st Avenue S and Scarritt Place. The map of visitor parking can be found at [cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf](http://cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf).

### Travel

Vanderbilt University is located in Nashville, Tennessee. **By Air:** The Nashville International Airport is located at One Terminal Drive, Nashville, Tennessee, 37214. Tel: (615) 275-1675 Website: [www.flynashville.com](http://www.flynashville.com). Nashville International Airport is approximately a 12 mile drive from the meeting at Vanderbilt University.

The cost for an Uber from the airport to the meeting is approximately US\$25–\$35 each way.

**To drive from the Nashville International Airport to the meeting at Vanderbilt University:**

Directions from the airport to Wesley Place parking garage: Take I-40 West toward Nashville. Follow the signs for I-24 West/I-40 West, then I-65 South/I-40 West. Exit at Broadway. At the base of the ramp, go straight through the traffic light at McGavock Street, and turn left onto Broadway. Stay left. Continue on Broadway to 16th Avenue, where the road splits. Veer left onto Broadway. Broadway will become 21st Avenue South. As you pass the traffic light at Grand Avenue, the Vanderbilt main campus will be on your right. Turn left at Scarritt Place (first traffic light after Grand Avenue). The entrance to the Wesley Place parking garage will be on your right, 1/2 block past the intersection. Public parking spaces are 52–170. [cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf](http://cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf)

**By Train:** Amtrak does not have a train station in Nashville. One can take a Greyhound bus from Nashville to the Memphis Amtrak station. Please contact Amtrak at Tel: 800-USA-RAIL, website: [www.amtrak.com](http://www.amtrak.com)

**By Bus:** The Greyhound Bus station located at 709 5th Ave S, Nashville, TN 37203 is about 2.5 miles from the meeting at Vanderbilt University. Please contact

Greyhound at: 1-800-231-2222; or on the web at: [www.greyhound.com](http://www.greyhound.com).

#### By Car:

#### Getting to Vanderbilt University via the Interstates

**From the north:** Take I-65 to I-40 west and then look for I-40 east to exit 209B. Turn right on Broadway. Continue on Broadway to 16th Avenue, where the road splits. Veer left onto Broadway. Broadway will become 21st Avenue South. As you pass the traffic light at Grand Avenue, the Vanderbilt main campus will be on your right. Turn left at Scarritt Place (first traffic light after Grand Avenue). The entrance to the Wesley Place parking garage will be on your right, 1/2 block past the intersection. Public parking spaces are 52-170. [cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf](http://cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf)

**From the east or south:** Take I-40 west to exit 209A. Turn left on Broadway. Continue on Broadway to 16th Avenue, where the road splits. Veer left onto Broadway. Broadway will become 21st Avenue South. As you pass the traffic light at Grand Avenue, the Vanderbilt main campus will be on your right. Turn left at Scarritt Place (first traffic light after Grand Avenue). The entrance to the Wesley Place parking garage will be on your right, 1/2 block past the intersection. Public parking spaces are 52-170. [cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf](http://cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf)

**From the west:** Take I-40 east to exit 209B. Turn right on Broadway (US 70S). Continue on Broadway to 16th Avenue, where the road splits. Veer left onto Broadway. Broadway will become 21st Avenue South. As you pass the traffic light at Grand Avenue, the Vanderbilt main campus will be on your right. Turn left at Scarritt Place (first traffic light after Grand Avenue). The entrance to the Wesley Place parking garage will be on your right, 1/2 block past the intersection. Public parking spaces are 52-170. [cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf](http://cpc-fis.vanderbilt.edu/pdf/VisitorCampusMap.pdf)

Vanderbilt University is located a mile and a half southwest of downtown Nashville and is approximately 4 hours from Atlanta, 3 hours from Birmingham, 2 1/2 hours from Knoxville, 3 hours from Louisville, and 3 hours from Memphis.

**Car Rental:** Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at [www.hertz.com](http://www.hertz.com), click on the box "Enter a discount or promo code", and type in our convention number (CV): CV#04N30008. You can also call Hertz directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

#### Local Transportation

Nashville and Vanderbilt University are serviced by *MTA bus service*. Please find information including schedules and maps on the website: [www.nashvillemta.org/Nashville-MTA-Maps-and-Schedules.asp](http://www.nashvillemta.org/Nashville-MTA-Maps-and-Schedules.asp)

#### Weather

Weather in April in Nashville is typically mild. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

#### Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtags #AMSmtg.

#### Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at [travel.state.gov/content/travel/en.html](http://travel.state.gov/content/travel/en.html). If you need a preliminary conference invitation in order to secure a visa, please send your request to [mac@ams.org](mailto:mac@ams.org).

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

- \* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

- \* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

- \* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

- \* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

- \* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

- \* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

# Portland, Oregon

Portland State University

April 14–15, 2018

Saturday – Sunday

## Meeting #1137

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: February 2018

Program first available on AMS website: February 15, 2018

Issue of *Abstracts*: Volume 39, Issue 2

## Deadlines

For organizers: Expired

For abstracts: February 6, 2018

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtg/sectional.html](http://www.ams.org/amsmtg/sectional.html).*

## Invited Addresses

**Sándor Kovács**, University of Washington, Seattle, *Title to be announced.*

**Elena Mantovan**, California Institute of Technology, *Title to be announced.*

**Dimitri Shlyakhtenko**, University of California, Los Angeles, *Title to be announced.*

## Special Sessions

*If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).*

*Algebraic Geometry and its Connections* (Code: SS 9A), **Sándor Kovács**, University of Washington, Seattle, and **Karl Schwede**, University of Utah, Salt Lake City.

*Algebraic Topology* (Code: SS 23A), **Angélica Osorno**, Reed College, and **Dev Sinha**, University of Oregon.

*Algebraic and Combinatorial Structures in Knot Theory* (Code: SS 3A), **Allison Henrich**, Seattle University, **Inga Johnson**, Willamette University, and **Sam Nelson**, Claremont McKenna College.

*Automorphisms of Riemann Surfaces and Related Topics* (Code: SS 14A), **S. Allen Broughton**, Rose-Hulman Institute of Technology, **Mariela Carvacho**, Universidad Tecnica Federico Santa Maria, **Anthony Weaver**, Bronx Community College, the City University of New York, and **Aaron Wootton**, University of Portland.

*Biomathematics - Progress and Future Directions* (Code: SS 4A), **Hannah Callender Highlander**, University of Portland, **Peter Hinow**, University of Wisconsin - Milwaukee, and **Deena Schmidt**, University of Nevada, Reno.

*Commutative Algebra* (Code: SS 5A), **Adam Boocher**, University of Utah, and **Irena Swanson**, Reed College.

*Complex Analysis and Applications* (Code: SS 11A), **Malik Younsi**, University of Hawaii Manoa.

*Differential Geometry* (Code: SS 19A), **Christine Escher**, Oregon State University, and **Catherine Searle**, Wichita State University.

*Forest Modeling* (Code: SS 20A), **Gatzliolis Demetrios**, Pacific Northwest Research Station, US Forest Service, and **Nikolay Strigul**, Washington State University, Vancouver.

*General Relativity and Geometric Analysis* (Code: SS 13A), **Paul T. Allen**, Lewis & Clark College, **Jeffrey Jauregui**, Union College, and **Iva Stavrov Allen**, Lewis & Clark College.

*Geometric Measure Theory and Partial Differential Equations* (Code: SS 12A), **Mark Allen**, Brigham Young University, and **Spencer Becker-Kahn** and **Mariana Smit Vega Garcia**, University of Washington.

*Inverse Problems* (Code: SS 2A), **Hanna Makaruk**, Los Alamos National Laboratory (LANL), and **Robert Owczarek**, University of New Mexico, Albuquerque & Los Alamos.

*Mock Modular and Quantum Modular Forms* (Code: SS 24A), **Holly Swisher**, Oregon State University, and **Stephanie Treneer**, Western Washington University.

*Modeling, Analysis, and Simulation of PDEs with Multiple Scales, Interfaces, and Coupled Phenomena* (Code: SS 17A), **Yekaterina Epshteyn**, University of Utah, and **Malgorzata Peszynska**, Oregon State University.

*Moduli Spaces* (Code: SS 21A), **Renzo Cavalieri**, Colorado State University, and **Damiano Fulghesu**, Minnesota State University Moorhead.

*Motivic homotopy theory* (Code: SS 6A), **Daniel Dugger**, University of Oregon, and **Kyle Ormsby**, Reed College.

*Noncommutative Algebraic Geometry and Related Topics* (Code: SS 16A), **Jesse Levitt**, University of Southern California, **Hans Nordstrom**, University of Portland, and **Xinting Wang**, Temple University.

*Nonsmooth Optimization and Applications (Dedicated to Prof. B. S. Mordukhovich on the occasion of his 70th birthday)* (Code: SS 7A), **Mau Nam Nguyen**, Portland State University, **Hung M. Phan**, University of Massachusetts Lowell, and **Shawn Xianfu Wang**, University of British Columbia.

*Numerical Methods for Partial Differential Equations* (Code: SS 22A), **Brittany A. Erickson** and **Jeffrey S. Owall**, Portland State University.

*Pattern Formation in Crowds, Flocks, and Traffic* (Code: SS 1A), **J. J. P. Veerman**, Portland State University, **Alethea Barbaro**, Case Western Reserve University, and **Bassam Bamieh**, UC Santa Barbara.

*Recent Advances in Actuarial Mathematics* (Code: SS 18A), **Sooie-Hoe Loke**, Central Washington University, and **Enrique Thomann**, Oregon State University.

*Spectral Theory* (Code: SS 8A), **Jake Fillman**, Virginia Tech, and **Milivoje Lukic**, Rice University.

*Teaching and Learning in Undergraduate Mathematics* (Code: SS 15A), **Natalie LF Hobson**, Sonoma State University, and **Elise Lockwood**, Oregon State University.

*Wavelets, Frames, and Related Expansions* (Code: SS 10A), **Marcin Bownik**, University of Oregon, and **Darrin Speegle**, Saint Louis University.



## Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the Oregon state hotel tax (13.3%), Tourism Improvement District assessment fee (2%), local taxes and other hotel fees may apply. Participants must state that they are with the **American Mathematical Society's (AMS) Spring Western Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

**Portland Marriott City Center**, 520 SW Broadway, Portland, OR 97205; (503) 226-6300; [www.marriott.com/hotels/travel/pdxct-portland-marriott-city-center/](http://www.marriott.com/hotels/travel/pdxct-portland-marriott-city-center/). Rates are **US\$159** per night for a standard room. Amenities include fitness center, coin operated laundry facilities on-site and complimentary wireless internet in lobby and public areas. The current valet parking rate is **US\$45** daily. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about a 7-minute drive to campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 2, 2018**.

**Residence Inn Portland Downtown/RiverPlace**, 2115 SW River Pkwy, Portland, OR 97201; (503) 552-9500; [www.marriott.com/hotels/hotel-photos/pdxri-residence-inn-portland-downtown-riverplace/](http://www.marriott.com/hotels/hotel-photos/pdxri-residence-inn-portland-downtown-riverplace/). Rates are **US\$165** per night for a room. Amenities include complimentary hot breakfast, free high-speed internet, fitness center and indoor pool. The current parking rate is **US\$35** a day and guests in the AMS block receive **US\$10** off prevailing rates. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. This property is located within walking distance to PSU or 3 stops on the Portland Streetcar Line (across street from hotel). The deadline for reservations at a reduced rate is by **March 14, 2018**.

**Residence Inn Portland Downtown Lloyd Center**, 1710 NE Multnomah Street, Portland, OR 97232; (503) 288-1400; [www.marriott.com/hotels/travel/pdxlc-residence-inn-portland-downtown-convention-center/](http://www.marriott.com/hotels/travel/pdxlc-residence-inn-portland-downtown-convention-center/). Rates are **US\$159** per night for a room. Amenities include complimentary hot breakfast, free high-speed internet, fitness center and seasonal outdoor pool. The current parking rate is **US\$15** a day and guests in the AMS block receive **US\$10** off prevailing rates. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about a 11-minute drive from the campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your

reservation. The deadline for reservations at a reduced rate is **March 14, 2018**.

**Hotel Rose**, 50 SW Morrison St, Portland, OR 97204; (503) 221-0711; [www.hotelroseportland.com/](http://www.hotelroseportland.com/). Rates are **US\$139** per night for a king bed or double bed room. Amenities include high-speed wireless internet, custom yellow beach cruisers to explore the city, unlimited in-room bottled water, fitness center and afternoon delights in the lobby with local coffee, and signature pineapple cupcakes. The amenity fee of **US\$10** plus tax per night will be waived for guests in the AMS block. The current self-parking rate is **US\$25** per night. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located approximately 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **February 13, 2018**.

**AC Hotel Portland Downtown**, 888 SW 3rd Ave, Portland, OR 97204; (503) 223-2100; [achotels.marriott.com/hotels/ac-hotel-portland-downtown](http://achotels.marriott.com/hotels/ac-hotel-portland-downtown). Rates are **US\$139** per night for a standard king bed room. Amenities include fitness center and free high-speed internet. This hotel offers a limited number of parking spaces available on a first come, first served basis. Current valet parking rates are **US\$18** daily and **US\$40** overnight with unlimited in/out privileges. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 14, 2018**.

**DoubleTree by Hilton**, 1000 NE Multnomah St, Portland, OR 97232; (503) 281-6111; [doubletree3.hilton.com/en/hotels/oregon/doubletree-by-hilton-hotel-portland-RLLC-DT/index.html](http://doubletree3.hilton.com/en/hotels/oregon/doubletree-by-hilton-hotel-portland-RLLC-DT/index.html). Rates are **US\$155** per night for a standard queen bed room. A charge of **US\$15** per person/per room will be assessed for triple and quadruple occupancy. Amenities include seasonal outdoor pool, 24-hour fitness center, 24-hour business center, on-site restaurants, and location adjacent to MAX light rail with access to many downtown businesses. The current on-site self parking rate is **US\$15** per day and the current valet parking rate is **US\$25** per day. These rates are subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about a 11-minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 14, 2018**.

**Mark Spencer Hotel**, 409 SW 11th Ave, Portland, OR 97205; (503) 224-3293; [www.markspencer.com/](http://www.markspencer.com/). Rates are **US\$120** per night for a studio room or **US\$130** for a 1 bedroom room. Amenities include complimentary Wi-Fi, fitness room and evening wine tasting reception held daily from 5:00 pm to 6:00 pm. The current self-parking rate

is US\$25 per day. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about 10-minutes from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 13, 2018**.

**Hotel Modera**, 515 SW Clay St, Portland, OR 97201; (877) 484-1084; [www.hotelmodera.com/](http://www.hotelmodera.com/). Rates are US\$179 for a premiere king room. Amenities fee of US\$20 plus tax per night includes two bottles of water, US\$15 food and beverage credit, day passes to *24 Hour Fitness* (located one block away), use of cruiser bikes, and business center with complimentary printing. The current valet parking rate is US\$38 per night, per vehicle. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 14, 2018**.

**Hampton Inn & Suites by Hilton-Pearl District**, 354 NW 9th Ave, Portland, OR 97209; (503) 222-5200; [hamptoninn3.hilton.com/en/hotels/oregon/hampton-inn-and-suites-portland-pearl-district-PDXPDHX/index.html](http://hamptoninn3.hilton.com/en/hotels/oregon/hampton-inn-and-suites-portland-pearl-district-PDXPDHX/index.html). Rates are US\$189 for a standard 2 queen or single king room. Amenities include coin laundry, business center, pool and snack shop. This property is located about a 10-minute drive from campus. The current valet parking rate is US\$39 a night. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 13, 2018**.

**Portland Marriott Waterfront**, 1401 SW Naito Pkwy, Portland, OR 97201; (503) 226-7600; [www.marriott.com/hotels/travel/pdxor-portland-marriott-downtown-waterfront/](http://www.marriott.com/hotels/travel/pdxor-portland-marriott-downtown-waterfront/). Rates are US\$159 for a studio king room. Amenities include fitness facility and complimentary Wi-Fi access in public areas. The current valet parking fee is US\$39 a night. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 23, 2018**.

**Portland Residence Inn Downtown Pearl District**, 1150 NW 9th Ave, Portland, OR 97209; (503) 220-1339; [www.marriott.com/hotels/travel/pdxpd-residence-inn-portland-downtown-pearl-district/](http://www.marriott.com/hotels/travel/pdxpd-residence-inn-portland-downtown-pearl-district/). Rates are US\$159 for a studio king room. Amenities include free breakfast, free high-speed internet, grocery shipping service, coin operated laundry, and newspaper in lobby. The current valet parking rate is US\$39 a day. This rate is subject to change and the parking fee in effect during the dates of your stay will be charged. This property is located about a 10-minute drive from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 23, 2018**.

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### Food Services

**On Campus:** The below restaurants are scheduled to be open on Saturdays and Sundays. Please visit [dineon-campus.com/PSU](http://dineon-campus.com/PSU) for the most up-to-date list of university restaurants and hours.

*Victor's Dining Center*, open 11:00 am to 7:00 pm on Saturdays and Sundays; all-you-care-to-eat dining hall serving grilled favorites, baked entrees, pizza, pasta and more.

*Smith's Place*, first floor of the Student Union; open 7:00 am to 10:00 pm on Saturdays and 9:00 am to 6:00 pm on Sundays; serves specialty coffee, salads, wraps and sandwiches.

*Branford's Bean*, Miller Library; open 10:30 am to 6:00 pm on Saturdays and noon to 11:00 pm on Sundays; serves specialty coffees, on-the-go sandwiches, salads and parfaits.

*The Meetro*, alongside King Albert Residence Hall; open 5:00 pm to 10:00 pm on Sunday; serves Mediterranean food from Bashas, espresso and freshly brewed coffee, on-the-go sandwiches and snacks.

*Lincoln Station Grill*, University Place Hotel; open 3:30 pm to 9:30 pm on Saturdays and Sundays; serving Northwest regional craft beers, hand-crafted cocktails and a comprehensive dinner menu.

**Off Campus:** America's best food city, per the *Washington Post*, offers farm-to-table dining, innovative food carts, acclaimed craft beer, stellar coffee and more. For an interactive directory of downtown restaurants and attractions visit [www.travelportland.com/directory/region/downtown/](http://www.travelportland.com/directory/region/downtown/).

Some options for coffee include:

*40 LBS Coffee Bar*, 824 S.W. 2nd Ave, Portland; (503) 329-5782; [www.40lbscoffee.com/](http://www.40lbscoffee.com/); a true Portland coffee bar experience.

*Stumptown Coffee Roasters*, 128 S.W. Third Ave, Portland; (503) 295-6144; [www.stumptowncoffee.com/](http://www.stumptowncoffee.com/); coffee house emphasizing direct trade and highlights the different flavor profiles of coffee varietals.

*Case Study Coffee*, 802 S.W. 10th Ave, Portland; (503) 477-8221; [www.casestudycoffee.com/](http://www.casestudycoffee.com/); roasted-on-site beans expertly prepared in coffee and espresso drinks.

Some options for downtown dining include:

*Alderman's Portland Tavern*, 71 S.W. Second Ave, Portland; (971) 229-1657; [www.aldermanspdx.com/](http://www.aldermanspdx.com/); serving local brews, classic cocktails and happy hour specials.

*Alto Bajo*, 310 S.W. Stark St, Portland; (971) 222-2100; [h/altobajopdx.com/](http://h/altobajopdx.com/); modern Mexican restaurant with a distinct culinary perspective on traditional Mexican dishes.

*Aquariva*, 0470 S.W. Hamilton Court, Portland; (503) 802-5850; [www.riversedgehotel.com/experiences/](http://www.riversedgehotel.com/experiences/);

serving a collection of locally sourced comfort food with outdoor patio and bar.

*Cafe Yumm!*, 1806 S.W. 6th Ave, Portland; (614) 261-4686; [www.cafeyumm.com/portland](http://www.cafeyumm.com/portland); uniquely Northwest menu features vegetarian and vegan options with organic ingredients.

*Bistro by Truss*, 1401 S.W. Naito Pkwy, Portland; (503) 226-7600; [www.travelportland.com/directory/bistro-truss/](http://www.travelportland.com/directory/bistro-truss/); open for lunch and dinner, the restaurant also offers a selection of eight local and seasonal beers on tap, as well as a coffee bar in the mornings with breakfast sandwiches and Starbucks coffee.

*Fogo de Chao Brazilian Steak House*, 930 S.W. Sixth Ave, Portland; (503) 241.0900; [fogodechao.com/location/portland](http://fogodechao.com/location/portland); menu includes delectable cuts of grilled meats carved table side as well as a variety of traditional Brazilian side dishes and desserts.

## Registration and Meeting Information

**Advance Registration:** Advance registration for this meeting opens on **January 30, 2018**. Advance registration fees will be **US\$61** for AMS members, **US\$90** for nonmembers, and **US\$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing [mmsb@ams.org](mailto:mmsb@ams.org). The deadline to cancel is the first day of the meeting.

**On-site Information and Registration:** The registration desk, AMS book exhibit, and coffee service will be located in L101, the Hoffman Hall main lecture hall lobby. The Invited Address lectures will be located in room 109, the Hoffman Hall main lecture hall. Special Sessions and Contributed Paper Sessions will take place in the nearby classrooms. Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at [www.pdx.edu/campus-map](http://www.pdx.edu/campus-map).

The registration desk will be open on Saturday, April 14, 7:30 am to 4:00 pm and Sunday, April 15, 8:00 am to 12:00 pm. The same fees listed above apply for on-site registration and are payable with cash, check or credit card.

## Other Activities

**Book Sales:** Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on-site orders! AMS members receive 40 percent off list price. Nonmembers receive a 25 percent discount. AMS Members receive additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

**Complimentary Coffee** will be served courtesy of the AMS Membership Department.

**AMS Editorial Activity:** An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

## Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at [mmsb@ams.org](mailto:mmsb@ams.org) or [meet@ams.org](mailto:meet@ams.org).

## AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

## Local Information and Maps

This meeting will take place on the campus of the Portland State University. A campus map can be found at [www.pdx.edu/campus-map](http://www.pdx.edu/campus-map). Information about the Portland State University Mathematics Department can be found at [www.pdx.edu/math/home](http://www.pdx.edu/math/home). Please visit the university website at [www.pdx.edu/](http://www.pdx.edu/) for additional information on the campus.

Please watch the AMS website at [www.ams.org/meetings/sectional/sectional.html](http://www.ams.org/meetings/sectional/sectional.html) for additional information on this meeting.

## Parking

Portland State University offers several visitor parking options on campus. Hourly pay-by-plate parking is available to any guest, student, or employee of the University. Payment is required all hours, all days, except for official university holidays. Self-service pay stations do not provide change. Visit [www.pdx.edu/transportation/sites/www.pdx.edu.transportation/files/17-18%20Hourly%20%26%20Daily%20Parking%20Handout.pdf](http://www.pdx.edu/transportation/sites/www.pdx.edu.transportation/files/17-18%20Hourly%20%26%20Daily%20Parking%20Handout.pdf) for a campus parking map of all hourly & visitor parking locations and rates.

The main meeting area in Hoffman Hall is conveniently located two blocks from Parking Structure 3 on SW 12th. The lower floor is public parking and the fee is currently **US\$2.50** per hour or **US\$10** per day, **US\$6** on Saturdays and Sundays. Public parking is payable by credit card at kiosks or by cash at an attended booth. There are also a number of **US\$2** per hour, 5 hour max metered parking spots on 12th Avenue.



Additional information about parking on campus can be found at [www.pdx.edu/transportation/visitors-0](http://www.pdx.edu/transportation/visitors-0). Please call the PSU Transportation and Parking Office at (503) 725-3442 or email [psupark@pdx.edu](mailto:psupark@pdx.edu) with parking inquiries.

### Travel

This meeting will take place on the main campus of the Portland State University located in Portland, Oregon.

#### By Air:

The Portland International Airport (PDX) is about 20 minutes from the Portland State University and is the most convenient air travel choice. Please note, the drive could take longer during rush hour traffic. Please visit the airport web site for a list of airlines and lists of cities with daily direct flights; [www.flypdx.com/PDX](http://www.flypdx.com/PDX).

There are several options available for transportation to and from the airport.

Commercial and transportation network vehicles are available on the lower roadway outside of baggage claim on islands 1, 2 and 3. Please see the lower roadway map for specific locations at <https://popcdn.azureedge.net/pdfs/Upper-Lower%20Roadway%20as%20of%20Nov%209%202017.pdf>.

On airport rental car companies include *Avis*, *Dollar*, *Enterprise*, *Hertz* and *National*. For phone numbers and further information visit [www.flypdx.com/PDX/GroundTransportation](http://www.flypdx.com/PDX/GroundTransportation).

*MAX Red Line* light rail service connects Portland International Airport, E/NE Portland, Portland City Center and Beaverton. The trip to/from downtown Portland takes about 38 minutes and costs **US\$2.50** for adults. The first train of the day arrives at PDX at 4:57 am on weekdays and weekends. The last train departs PDX at 11:49 pm, daily. The MAX station and ticket machines are located near baggage claim on the lower level. For the MAX system map and schedules visit [trimet.org/schedules/maxredline.htm](http://trimet.org/schedules/maxredline.htm).

Visit the *TriMet* trip planner at [trimet.org/#/planner](http://trimet.org/#/planner) to plan your trip to campus ahead of time. Simply enter Portland State University as your destination and you will get step-by-step transit directions with available bus or train options.

#### By Bus:

For a list of *TriMet* bus stops near Portland State University visit [trimet.org/ride/stop\\_select\\_form.html](http://trimet.org/ride/stop_select_form.html). The closest bus stop is SW 6th & Mill Northbound served by bus lines 8,9,17 and 66.

*Portland Streetcar* operates three lines around 16-miles of track in Portland's Central City. Visit the Ride Guide at [portlandstreetcar.org/ride-guide/how-to-ride](http://portlandstreetcar.org/ride-guide/how-to-ride) for step-by-step instructions for using the streetcar. For schedules and maps visit <https://portlandstreetcar.org/schedules>.

Also, visit the interactive campus map at [map.pdx.edu/#/map?zoom=17&lat=45.5112589&lng=-122.6838359&b=road&l=](http://map.pdx.edu/#/map?zoom=17&lat=45.5112589&lng=-122.6838359&b=road&l=) for details on where *TriMet* bus, *MAX light rail*, and *Portland Streetcar* stops are located on campus.

#### By Car:

**From Portland International Airport:** Head northwest on NE Airport Way and use the left lane to stay on NE Airport Way. Take a slight left to stay on NE Airport Way and continue straight to stay on NE Airport Way. Keep right to stay on NE Airport Way and use the right 2 lanes to turn slightly right onto the Interstate 205 S ramp to Interstate 84/Portland/Salem. Merge onto I-205 S and take exit 21B to merge onto I-84 W/US-30 W toward Portland. Use the left 2 lanes to take the exit toward Salem. Merge onto I-5 S. Use the left lane to keep left at the fork, continue on I-405 N and follow signs for Beaverton/Interstate 405/US 26. Take exit 1C to merge onto SW 6th Ave. Merge onto SW 6th Ave and turn left onto SW Mill St. Turn left onto SW Broadway and destination will be on the right.

**Car Rental:** Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at [www.hertz.com](http://www.hertz.com), click on the box "I have a discount", and type in our convention number (CV): CV#04N30008. You can also call Hertz directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

### Local Transportation

The PSU campus is a hub of bus, lightrail, and streetcar lines that get you here from any part of the city.

Visit the interactive campus map at [map.pdx.edu/#/map?zoom=17&lat=45.5112589&lng=-122.6838359&b=road&l=](http://map.pdx.edu/#/map?zoom=17&lat=45.5112589&lng=-122.6838359&b=road&l=) for details on where *TriMet* bus, *MAX light rail*, and *Portland Streetcar* stops are located on campus.

**Bike Share:** The *BIKE TOWN* system is designed with convenience in mind, for quick trips with no need to wait for traffic or transit. Just join and choose your plan, unlock a bike using your 6-digit account number or Member Card and your 4-digit PIN and place the U-lock in the holster before you take off. To end your ride, lock your bike to any *BIKE TOWN* station. You can also lock it to a public rack nearby for a small fee. Pricing details and sign up information can be found at [www.biketownpdx.com/pricing](http://www.biketownpdx.com/pricing).

**Other options:** Other local transportation options include *Uber*; [www.uber.com](http://www.uber.com) and *Lyft*; [www.lyft.com](http://www.lyft.com).

### Weather:

Portland averages a daily maximum temperature for April that's between 59 and 65 °F (15 to 18 °C). The minimum temperature usually falls between 41 and 45 °F (5 to 7 °C). The days at Portland warm most quickly during April. Attendees are advised to dress in layers and prepare for rain.

### Social Networking:

Attendees and speakers are encouraged to use the hashtag #AMSmtg to tweet about the meeting.

## Information for International Participants:

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at [travel.state.gov/content/travel/en.html](http://travel.state.gov/content/travel/en.html). If you need a preliminary conference invitation in order to secure a visa, please send your request to [cro@ams.org](mailto:cro@ams.org).

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

\* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

\* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

\* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

\* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

\* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

\* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

# Boston, Massachusetts

*Northeastern University*

**April 21–22, 2018**

*Saturday – Sunday*

## Meeting #1139

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: February 2018

Program first available on AMS website: March 1, 2018

Issue of *Abstracts*: Volume 39, Issue 2

## Deadlines

For organizers: Expired

For abstracts: February 20, 2018

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgsectional.html](http://www.ams.org/amsmtgsectional.html).*

## Invited Addresses

**Jian Ding**, University of Pennsylvania, *Random walk, random media and random geometry*.

**Edward Frenkel**, University of California, Berkeley, *Imagination and knowledge* (Einstein Public Lecture in Mathematics).

**Valentino Tosatti**, Northwestern University, *Metric limits of Calabi-Yau manifolds*.

**Maryna Viazovska**, École Polytechnique Fédérale de Lausanne, *Title to be announced*.

## Special Sessions

*If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).*

*Algebraic Number Theory* (Code: SS 35A), **Michael Bush**, Washington and Lee University, **Farshid Hajir**, University of Massachusetts, and **Christian Maire**, Université Bourgogne Franche-Comté.

*Algebraic Statistics* (Code: SS 33A), **Kaie Kubjas** and **Elina Robeva**, Massachusetts Institute of Technology.

*Algebraic, Geometric, and Topological Methods in Combinatorics* (Code: SS 21A), **Florian Frick**, Cornell University, and **Pablo Soberón**, Northeastern University.

*Algorithmic Group Theory and Applications* (Code: SS 26A), **Delaram Kahrobaei**, City University of New York, and **Antonio Tortora**, University of Salerno.

*Analysis and Geometry in Non-smooth Spaces* (Code: SS 5A), **Nageswari Shanmugalingam** and **Gareth Speight**, University of Cincinnati.

*Arithmetic Dynamics* (Code: SS 1A), **Jacqueline M. Anderson**, Bridgewater State University, **Robert Benedetto**, Amherst College, and **Joseph H. Silverman**, Brown University.

*Arrangements of Hypersurfaces* (Code: SS 2A), **Graham Denham**, University of Western Ontario, and **Alexander I. Suciu**, Northeastern University.

*Combinatorial Aspects of Nilpotent Orbits* (Code: SS 15A), **Anthony Iarrobino**, Northeastern University, **Leila Khatami**, Union College, and **Juliana Tymoczko**, Smith College.

*Combinatorial Representation Theory* (Code: SS 41A), **Laura Colmenarejo**, York University, **Ricky Liu**, North Carolina State University, and **Rosa Orellana**, Dartmouth College.

*Connections Between Trisections of 4-manifolds and Low-dimensional Topology* (Code: SS 32A), **Jeffrey Meier**,

## MEETINGS & CONFERENCES

University of Georgia, and **Juanita Pinzon-Caicedó**, North Carolina State University.

*Discretization in Geometry and Dynamics* (Code: SS 36A), **Richard Kenyon**, **Wai Yeung Lam**, and **Richard Schwartz**, Brown University.

*Dynamical systems, Geometric Structures and Special Functions* (Code: SS 23A), **Alessandro Arsie**, University of Toledo, and **Oksana Bihun**, University of Colorado, Colorado Springs.

*Effective Behavior in Random Environments* (Code: SS 25A), **Jessica Lin**, McGill University, and **Charles Smart**, University of Chicago.

*Ergodic Theory and Dynamics in Combinatorial Number Theory* (Code: SS 7A), **Stanley Eigen** and **Daniel Glasscock**, Northeastern University, and **Vidhu Prasad**, University of Massachusetts, Lowell.

*Extremal Graph Theory and Quantum Walks on Graphs* (Code: SS 13A), **Sebastian Cioabă**, University of Delaware, **Mark Kempton**, Harvard University, **Gabor Lippner**, Northeastern University, and **Michael Tait**, Carnegie Mellon University.

*Facets of Symplectic Geometry and Topology* (Code: SS 3A), **Tara Holm**, Cornell University, **Jo Nelson**, Columbia University, and **Jonathan Weitsman**, Northeastern University.

*Geometries Defined by Differential Forms* (Code: SS 34A), **Mahir Bilen Can**, Tulane University, **Sergey Grigorian**, University of Texas Rio Grande Valley, and **Sema Salur**, University of Rochester.

*Geometry and Analysis of Fluid Equations* (Code: SS 28A), **Robert McOwen** and **Peter Topalov**, Northeastern University.

*Geometry of Moduli Spaces* (Code: SS 10A), **Ana-Marie Castravet** and **Emanuele Macrì**, Northeastern University, **Benjamin Schmidt**, University of Texas, and **Xiaolei Zhao**, Northeastern University.

*Global Dynamics of Real Discrete Dynamical Systems* (Code: SS 30A), **M. R. S. Kulenović** and **O. Merino**, University of Rhode Island.

*Harmonic Analysis and Partial Differential Equations* (Code: SS 29A), **Donatella Danielli**, Purdue University, and **Irina Mitrea**, Temple University.

*Homological Commutative Algebra* (Code: SS 11A), **Sean Sather-Wagstaff**, Clemson University, and **Oana Veliche**, Northeastern University.

*Hopf Algebras, Tensor Categories, and Homological Algebra* (Code: SS 8A), **Cris Negron**, Massachusetts Institute of Technology, **Julia Plavnik**, Texas A&M, and **Sarah Witherspoon**, Texas A&M University.

*Mathematical Perspectives in Quantum Information Theory* (Code: SS 24A), **Aram Harrow**, Massachusetts Institute of Technology, and **Christopher King**, Northeastern University.

*Mathematical Problems of Relativistic Physics: Classical and Quantum* (Code: SS 37A), **Michael Kiessling** and **A. Shadi Tahvildar-Zadeh**, Rutgers University.

*Modeling of Biological Processes* (Code: SS 38A), **Simone Cassani** and **Sarah Olson**, Worcester Polytechnic Institute.

*New Developments in Inverse Problems and Imaging* (Code: SS 9A), **Ru-Yu Lai**, University of Minnesota, and **Ting Zhou**, Northeastern University.

*Noncommutative Algebra and Representation Theory* (Code: SS 22A), **Van C. Nguyen**, Hood College, and **Alex Martsinkovsky** and **Gordana Todorov**, Northeastern University.

*Nonlinear Reaction-Diffusion Equations and Their Applications* (Code: SS 31A), **Nsoki Mavinga** and **Quinn Morris**, Swarthmore College.

*Nonlinear and Stochastic Partial Differential Equations and Applications* (Code: SS 19A), **Nathan Glatt-Holtz** and **Vincent Martinez**, Tulane University, and **Cecilia Mondaini**, Texas A&M University.

*Numerical Methods and Applications* (Code: SS 16A), **Vera Babenko**, Ithaca College.

*Optimization Under Uncertainty* (Code: SS 40A), **Yingdong Lu** and **Mark S. Squillante**, IBM Research.

*Polytopes and Discrete Geometry* (Code: SS 6A), **Gabriel Cunningham**, University of Massachusetts, Boston, **Mark Mixer**, Wentworth Institute of Technology, and **Egon Schulte**, Northeastern University.

*Regularity of PDEs on Rough Domains* (Code: SS 14A), **Murat Akman**, University of Connecticut, and **Max Engelstein**, Massachusetts Institute of Technology.

*Relations Between the History and Pedagogy of Mathematics* (Code: SS 20A), **Amy Ackenberg-Hastings**, and **David L. Roberts**, Prince George's Community College.

*Singularities of Spaces and Maps* (Code: SS 4A), **Terence Gaffney** and **David Massey**, Northeastern University.

*The Analysis of Dispersive Equations* (Code: SS 39A), **Marius Beceanu**, University at Albany, and **Andrew Lawrie**, Massachusetts Institute of Technology.

*The Gaussian Free Field and Random Geometry* (Code: SS 12A), **Jian Ding**, University of Pennsylvania, and **Vadim Gorin**, Massachusetts Institute of Technology.

*Topics in Qualitative Properties of Partial Differential Equations* (Code: SS 27A), **Changfeng Gui**, University of Texas at San Antonio, **Changyou Wang**, Purdue University, and **Jiuyi Zhu**, Louisiana State University.

*Topics in Toric Geometry* (Code: SS 17A), **Ivan Martino**, Northeastern University, and **Emanuele Ventura**, Texas A&M University.

*Topology of Biopolymers* (Code: SS 18A), **Erica Flapan**, Pomona College, and **Helen Wong**, Carleton College.

## Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the tax of 14.45% per room (5.7% MA State Tax, 6% City Tax, and 2.75% (CCF) tax). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at Northeastern University** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**



**Boston Marriott Copley Place**, 110 Huntington Ave, Boston, MA, 02116; (617) 236-5000, [www.marriott.com/hotels/travel/bosco-boston-marriott-copley-place/](http://www.marriott.com/hotels/travel/bosco-boston-marriott-copley-place/). Rates are US\$249 per night for a guest room, this rate is applicable for single or double occupancy. To reserve a room at these rates online please use this booking link: [aws.passkey.com/e/49559410](http://aws.passkey.com/e/49559410). To reserve a room over the phone please contact the hotel directly at 877-901-2079 and identify the block of rooms for the **American Mathematical Society Spring Sectional Meeting**. Amenities include complimentary high-speed wireless Internet access in sleeping rooms, fitness center, 24-hour business center, *Champions* on-site restaurant (offering room service), *Connexion Lounge*, and self-parking by the hour is available off-site for US\$40 daily and valet parking is available for US\$60 daily. Check-in is at 4:00 pm, check-out is at noon. This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 23, 2018**.

**The Colonnade Hotel**, 120 Huntington Ave, Boston, MA, 02116; (617) 424-7000, (800) 962-3030, [www.colonnadehotel.com/](http://www.colonnadehotel.com/). Rates are US\$269 per night for a guest room with single or double occupancy; US\$299 for a guest room with triple occupancy; and US\$329 for a guest room with quad occupancy. To reserve a room at these rates online please use this booking link: [protect-us.mimecast.com/s/Y1D9BRszkLMUA?domain=gc.synxis.com](http://protect-us.mimecast.com/s/Y1D9BRszkLMUA?domain=gc.synxis.com). To reserve a room over the phone please contact the hotel directly at (800) 962-3030 and identify the block of rooms for the **AMS Spring Sectional Meeting**. Amenities include complimentary high-speed wired and wireless Internet access in sleeping rooms, fitness center, 24-hour business center, *Brasserie JO* on-site restaurant (offering room service), in-room safes and mini fridges, and self-parking by the hour (rate varies) and US\$48 overnight, in a covered lot. This property is pet-friendly. Check-in is at 3:00 pm, check-out is at noon. This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 30, 2018**.

**Courtyard by Marriott Brookline**, 40 Webster St, Brookline, MA, 02446; (617) 734-1393, [www.marriott.com/hotels/travel/bosbl-courtyard-boston-brookline/](http://www.marriott.com/hotels/travel/bosbl-courtyard-boston-brookline/). Rates are US\$239 per night for a guest room with single or double occupancy with one king bed. To reserve a room at this rate online please use this booking link: [www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkData=American%20Mathematical%20Society%5EBOSBL%60AMSAMSA%60239%60USD%60false%602%604/20/18%604/23/18%603/21/18&app=resvlink&stop\\_mobi=yes](http://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkData=American%20Mathematical%20Society%5EBOSBL%60AMSAMSA%60239%60USD%60false%602%604/20/18%604/23/18%603/21/18&app=resvlink&stop_mobi=yes). To reserve a room over the phone please use the Marriott Reservations line at (866) 296-2296 and identify the block of rooms for the **American Mathematical Society Room Block**. Amenities include complimentary high-speed wireless Internet access in sleeping rooms,

fitness center, indoor swimming pool, 24-hour business center, *the Bistro* on-site restaurant, mini fridges, and self-parking is available at a discounted rate of US\$35 per night. Check-in is at 3:00 pm, check-out is at noon. This property is located approximately 2.6 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 21, 2018**.

**Midtown Hotel**, 220 Huntington Ave, Boston, MA, 02116; (617) 262-8739; [www.midtownhotel.com/](http://www.midtownhotel.com/). Rates are US\$189 per night for a guest room with single or double occupancy in a room with two double or one king bed. To reserve a room at this rate online please visit the property's website at [www.midtownhotel.com/](http://www.midtownhotel.com/) and type in the group block code: **ESSMNEU**. Be sure to use the box for *group code*, not *promo code*. To reserve a room over the phone please contact the hotel directly at (800) 343-1177 and identify the block of rooms for the **Eastern Spring Sectional Meeting**. Amenities include complimentary high-speed wireless Internet access in sleeping rooms, pull-out sleeper sofas, in-room microwaves and mini fridges, on property hair salon, and on-site parking at a rate of US\$29 daily. This property is pet-friendly. Check-in is at 3:00 pm, check-out is at noon. This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **April 6, 2018**.

**Residence Inn by Marriott Boston Back Bay/ Fenway**, 125 Brookline Ave, Boston, MA, 02215; (617) 236-8787, [www.marriott.com/hotels/travel/bosfn-residence-inn-boston-back-bay-fenway/](http://www.marriott.com/hotels/travel/bosfn-residence-inn-boston-back-bay-fenway/). Rates are US\$259 per night for a studio suite with a city view, this rate is applicable for single or double occupancy. To reserve a room at this rate online please use this booking link: [protect-us.mimecast.com/s/kJNDBRUmOGVUw?domain=marriott.com](http://protect-us.mimecast.com/s/kJNDBRUmOGVUw?domain=marriott.com). To reserve a room over the phone please contact the hotel directly at 800-331-3131 and identify the block of rooms for the **American Mathematical Society**. Amenities include complimentary high-speed wireless Internet access in sleeping rooms, complimentary breakfast buffet, fitness center, heated indoor pool, 24-hour business center, fully equipped kitchen with full sized appliances, and valet parking is available for US\$48 daily in underground garage. This property is pet-friendly with a US\$150 fee for pets. Check-in is at 4:00 pm, check-out is at noon. This property is located approximately 1 mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 20, 2018**.

**Sheraton Boston Hotel**, 39 Dalton St., Boston, MA; (617) 834-2231, [www.sheratonbostonhotel.com/](http://www.sheratonbostonhotel.com/). Rates are US\$259 per night for a guest room with single or double occupancy; US\$299 for a guest room with triple occupancy; and US\$339 for a guest room with quad occupancy. To reserve a room over the phone please contact

## MEETINGS & CONFERENCES

the Starwood Central Reservations Office at 888-627-7054 and identify the block of rooms for the **AMS Eastern Spring Sectional Meeting at Northeastern**. This property has indoor access to the Prudential Center and Copley Place shopping centers. Amenities include complimentary high-speed wireless Internet access for SPG members booking online, fitness center, 24-hour business center, indoor-outdoor pool, on-site *Green Tangerine Spa and Salon*, on-site dining options include *SideBar* and *Apropos*, and self-parking is available for **US\$42** daily and valet parking is available for **US\$58** daily. This property offers a discounted rate on shuttle service to Logan International Airport; inquire with the hotel. Check-in is at 3:00 pm, check-out is at noon. This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 23, 2018 at 5:00 pm EST**.

**Westin Copley Place**, 10 Huntington Avenue, Boston, MA; (617) 262-9600, [www.westincopleyplaceboston.com/](http://www.westincopleyplaceboston.com/). Rates are **US\$309** per night for a guest room with single or double occupancy; **US\$339** for a guest room with triple occupancy; and **US\$369** for a guest room with quad occupancy. To reserve a room over the phone please contact the Starwood Central Reservations Office at (888) 627-7216 and identify the block of rooms for the **AMS Eastern Spring Sectional Room Block**. This property has indoor access to the Prudential Center and Copley Place shopping centers. Amenities include complimentary high-speed wireless Internet access for SPG members, fitness center, 24-hour business center, in-room safe, mini fridges, on-site *Gretacole Spa*, on-site dining options include *Bar 10*, *The Huntington*, *Fogo de Chão Brazilian Steakhouse*, and *ShabuMaru*, and valet parking is available for **US\$61** daily. This property is pet-friendly. Check-in is at 3:00 pm, check-out is at noon. This property is located less than .5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 23, 2018 at 5:00 pm EST**.

### Food Services

**On Campus:** There are various options which are anticipated to be open on campus during the meeting. The hours of operation are listed below and are subject to change.

#### Curry Student Center:

- *On the Go*, Saturday 9:00 am–9:00 pm ; Sunday 11:00 am–9:00 pm (grab-and-go selections)
- *UBurger*, Saturday 11:00 am–7:00 pm ; Sunday noon–7:00 pm (burgers)
- *Popeye's Louisiana Kitchen*, Saturday 10:30 am–8:00 pm ; Sunday noon–7:00 pm (fried chicken and sides)
- *Starbucks*, Saturday 8:00 am–9:00 pm ; Sunday 10:00 am–6:00 pm (coffee, beverages, pastries, light snacks)

#### Nearby on Campus:

- *Argo Tea*, Sunday 1:00 pm–8 pm (teas, beverages, and light fare)

**Off Campus:** Downtown Boston is a major metropolitan area with array of dining options throughout the Back

Bay and Fenway neighborhoods. For more information on dining throughout the neighborhoods please visit, [www.bostonusa.com/](http://www.bostonusa.com/)

Some dining options nearby to the Main Campus include:

- *Amelia's Taqueria*, 309 Huntington Ave, at Gainsborough St, Boston (617) 266-0040, serving lunch and dinner; Mexican fare.
- *Au Bon Pain*, 369 Huntington Ave, (617) 578-0711, Saturday and Sunday, 8:00 am–10:00 p.m; serving bakery items, soups, salads, sandwiches, coffees, and beverages.
- *B Good*, 359 Huntington Ave, Boston (857) 277-1693, Saturday 11:00 am–10:00 pm and Sunday, 11:00 a.m–9:00 p.m; locally sourced, sustainable sandwiches, salads, burgers, smoothies.
- *California Pizza Kitchen*, Prudential Center, 800 Boylston St, Boston, (617) 247-0888, Saturday 11:00 am–11:00 p.m; Sun 11:00 am–10:00 pm.; serving pizza, salads, California cuisine.
- *Chicken Lou's*, 50 Forsyth St, Boston (617) 859-7017, Saturday 7:30 am–2:30 pm, closed Sunday; family owned diner-style restaurant with 25 year history.
- *5 Napkin Burger*, Prudential Center, 800 Boylston St, Boston, (617) 375-2277, Saturday 11:00 am–12:00 am and Sunday 11:00 am–11:00 pm; full-service burger restaurant.
- *Mumbai Spice*, 251 Massachusetts Ave, Boston, (857) 350-4305, serving lunch and dinner; Indian and Vegetarian cuisine.
- *Pho Basil*, 177A Massachusetts Ave, Boston (617) 262-5377, Saturday and Sunday 11:30 am–10:00 pm; Thai and Vietnamese cuisine.
- *Qdoba Mexican Grill*, 393 Huntington Ave, Boston, (617) 450-0910, Saturday and Sunday 11:00 am - 9:00 pm ; Chain restaurant serving made to order Mexican fare.
- *Summer Shack Boston*, 50 Dalton St., Boston, (617) 867-9955. Serving brunch and dinner; specializing in seafood and New England style dishes.
- *Symphony Sushi*, 45 Gainsborough St, Boston, (617) 262-3888, Saturday 11:30 am–10:15 pm and Sunday, 12:00 p.m–10:00 pm; Sushi and Japanese fare.

### Registration and Meeting Information

**Advance Registration:** Advance registration for this meeting will open on **January 30th**. Advance registration fees are **US\$61** for AMS members, **US\$90** for nonmembers, and **US\$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing [mmsb@ams.org](mailto:mmsb@ams.org). The deadline to cancel is the first day of the meeting.

**On-site Information and Registration:** The registration desk, AMS book exhibit, AMS membership exhibit, and coffee service will be located in the Indoor Quad Pit, in the Curry Student Center. The Invited Addresses will be held in the same building, in the Ballroom, located one floor up from the Indoor Quad Pit. The AMS Einstein Public Lecture in Mathematics will take place in the Blackman

Auditorium in Ell Hall. Special Sessions and Contributed Paper Sessions will take place in Shillman Hall, Ryder Hall and Behrakis Health Sciences Center. These buildings are located around Centennial Common, approximately a five-minute walk from the Curry Student Center and Ell Hall.

For additional information on building locations and a printable campus map, visit [www.northeastern.edu/campusmap/printable/campusmap15.pdf](http://www.northeastern.edu/campusmap/printable/campusmap15.pdf).

The registration desk will be open on Saturday, April 21, 7:30 am–4:00 pm and Sunday, April 22, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

### Other Activities

**Book Sales:** Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on-site orders! AMS members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

**AMS Editorial Activity:** An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

**Membership Activities:** Join us for appetizers at our Welcome Reception on **Friday, April 20th**. Please sign up for this event, at no charge, on the meeting registration form. The deadline to sign up for this reception will be **Tuesday, April 17, 2018**. Please visit the AMS website at [www.ams.org/meetings/sectional/sectional.html](http://www.ams.org/meetings/sectional/sectional.html) and reference the page for the Northeastern University Meeting for more details about this event.

During the meeting, stop by the AMS Membership Exhibit to learn about the benefits of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

Complimentary Refreshments will be served courtesy of the AMS Membership Department.

### AMS Einstein Public Lecture in Mathematics

The Einstein Public Lecture will be given by **Edward Frenkel**, University of California, Berkeley. The title of his talk is *Imagination and knowledge*. The lecture will be given on Saturday, April 21, at 5:15 pm, in the Blackman Auditorium located in Ell Hall.

A reception in conjunction with this lecture, and hosted by the Northeastern Mathematics Department and the AMS, will take place in the Ballroom in the Curry Student Center. The AMS thanks our hosts for their gracious hospitality.

### Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at [mmsb@ams.org](mailto:mmsb@ams.org) or [meet@ams.org](mailto:meet@ams.org).

### AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity, or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

### Local Information and Maps

This sectional will take place on the Main Campus of Northeastern University. A campus map can be found at [www.northeastern.edu/neuhome/about/maps.html](http://www.northeastern.edu/neuhome/about/maps.html). Information about the Northeastern University Mathematics Department can be found at [cos.northeastern.edu/mathematics/](http://cos.northeastern.edu/mathematics/). For additional information about the University please visit the Northeastern University website at [www.northeastern.edu/](http://www.northeastern.edu/).

The meeting will occupy space in the Curry Student Center, Ell Hall, Shillman Hall, Ryder Hall and Behrakis Health Sciences Center. For a printable version of the University map please visit [www.northeastern.edu/campusmap/printable/campusmap15.pdf](http://www.northeastern.edu/campusmap/printable/campusmap15.pdf).

### Parking

Northeastern recommends parking facilities for visitors at the Renaissance Parking Garage. Please select the “Maps and Directions” link for driving directions to these parking locations.

General parking is available at the Renaissance Parking Garage, 835 Columbus Avenue. There is an hourly fee to park in this facility; rates start at US\$12 for one hour. A weekend guest rate may be available, please inquire at the garage. This garage is not affiliated with the University.

### Travel

Northeastern University is located in the heart of the city of Boston. Logan International Airport is the closest airport to the University. The most common types of transportation used from the airport, are taxis, shuttles, and the public transportation system called the “T”.

**By Air:** *Logan International Airport* (BOS) is the closest airport to Northeastern University. Logan Airport is located two miles from the city center and approximately 9 miles from the University. Most Boston and Cambridge hotels are within a 5 mile drive from the airport.

The Logan Express bus offers a service from all terminals to the Back Bay’s Copley Square and the Hynes Con-



vention Center. The bus service will run on a 20-minute schedule (departures occur at :00, :20, :40 past the hour) with departures running between 6:00 am and 10:00 pm each day. A one-way fare costs US\$7.50 and is payable via credit/debit card only, paid upon bus entry.

Taxis can be found at the ground transportation area outside each terminal. Fare to downtown Boston will be approximately US\$35 depending on traffic and the time of day.

The campus can also be reached via mass transit on the MBTA "T" system. Shuttle buses are available to pick up passengers in front of the airline terminals and bring them to the airport subway stop on the Blue Line. Take the Blue Line subway inbound to Government Center Station. At Government Center change to the Green Line utilizing an "E" train. Take the *Northeastern* stop. For more information on alternate ground transportation please visit, [www.massport.com/massport/gtu/Pages/default.aspx](http://www.massport.com/massport/gtu/Pages/default.aspx).

**By Train:** The Boston region is served by Amtrak and by the MBTA commuter trains. Reservations can be made on Amtrak at [www.amtrak.com](http://www.amtrak.com). Information about regional trains on the MBTA commuter lines can be found at [www.mbta.com/schedules/commuter-rail](http://www.mbta.com/schedules/commuter-rail). If traveling by rail, you will arrive at South Station, located at Summer Street & Atlantic Avenue, or North Station, located at 135 Causeway Street. Some commuter trains make an additional stop at the Ruggles station on the Orange Line, please reference the MBTA website for details. From South Station you can board the MBTA "T" Red Line and transfer to the green line at Park St. Station utilizing an "E" train to travel to the campus and then take the *Northeastern* stop. From North Station you can board the Green Line directly, looking for an "E" train and utilize the *Northeastern* stop. Both stations also offer taxis and buses. Ruggles Station, on the orange line, is located adjacent to the Northeastern campus.

Please reference the *Local Transportation* listing below for walking directions from these stations to campus.

**By Bus:** The Boston region is served several regional bus lines including *Bolt Bus*, *Greyhound*, and *Peter Pan*. All intercity/interstate buses depart from and arrive at South Station. Ticket counters are located on the third level of the Transportation Center. For information, call the South Station Bus Terminal at 617-737-8040 or visit [www.southstation.net/regional-bus](http://www.southstation.net/regional-bus). The MBTA's Red and Silver Lines stop at South Station, connecting travelers to the Orange and Green lines to access Northeastern's campus. For ground transportation options via these subway lines please see the *Local Transportation* listing below.

**By Car:** All directions to Northeastern will direct drivers to a final destination of the Renaissance Parking Garage.

From the north (via Route I-93 or Route 1):

Take the Storrow Drive exit, and proceed to the Fenway exit. Follow signs for Boylston Street inbound, and bear right onto Westland Avenue. Turn right onto Massachusetts Avenue, proceed to the third traffic light, and turn right onto Columbus Avenue. The Renaissance Parking Garage is at 835 Columbus Avenue.

**From the west (via Route I-90, Massachusetts Turnpike):**

Take Exit 22 (Copley Square), and bear right. Proceed to the first traffic light, and turn right onto Dartmouth Street. Take the next right onto Columbus Avenue. The Renaissance Parking Garage is at 835 Columbus Avenue.

**From the west (via Route 9):**

Proceed east on Route 9; it will become Huntington Avenue. Turn right onto Ruggles Street. At the fourth traffic light, turn left onto Tremont Street. At the second set of lights, turn left onto Melnea Cass Boulevard, and then turn left onto Columbus Avenue. The Renaissance Parking Garage is at 835 Columbus Avenue.

**From the south (via I-93, Route 3):**

Take Exit 18 (Massachusetts Avenue/Roxbury/Frontage Road). Turn left at the third light, staying in one of the two left lanes. Proceed straight onto Melnea Cass Boulevard. Continue for approximately two miles and turn left onto Columbus Avenue. The Renaissance Parking Garage is at 835 Columbus Avenue.

### Local Transportation

**Car Rental:** Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at [www.hertz.com](http://www.hertz.com), click on the box "I have a discount", and type in our convention number (CV): 04N30008. You can also call Hertz directly at 800-654-2240 (US and Canada) or 405-749-4434 (other countries). At the time of your reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available. Meeting rates include unlimited mileage and are subject to availability. Advance reservations are recommended, blackout dates may apply.

**Taxi Service:** Licensed, metered taxis are available throughout Boston. To find information on authorized cab associations please visit [bpdnews.com/authorized-taxi-companies/](http://bpdnews.com/authorized-taxi-companies/). Both *Lyft* and *Uber* also operate in the Boston area.

**Bus and Subway Service:** The Massachusetts Bay Transportation Authority (MBTA, known as the "T" in Boston), offers access to Northeastern University. The meeting will be held on the Main Campus of Northeastern University. This campus is accessible by subway via the Green Line of the MBTA. From downtown Boston, take an "E" train outbound to the Northeastern stop, the first stop above ground. Upon exiting the station cross Huntington Avenue in the direction of the Krentzman Quadrangle. The campus can also be reached from downtown via the Orange Line by taking any train going outbound to Forest Hills and getting off at Ruggles Station. From Ruggles Station, exit the station toward Forsyth St.

Commuter rail lines connect with the Orange Line at Ruggles Station, Back Bay Station, and North Station. More information can be found here [www.mbta.com/](http://www.mbta.com/).

A one-way ticket costs US\$2.75, a one-day pass costs US\$12, and a seven-day pass costs US\$21.25. Any amount of money for one-way fares or passes (one-day, seven-day) can be purchased as a CharlieCard or CharlieTicket

at fare vending machines found at all subway stations. CharlieCards and CharlieTickets can be used for subway and bus fares.

For additional information about bus fares and schedules please visit [www.mbta.com/schedules/bus](http://www.mbta.com/schedules/bus).

### Weather

The average high temperature for April is approximately 58 °F and the average low is approximately 40 °F. Rain is common in Boston for this time of year. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

### Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtags #AMSmtg.

### Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at [travel.state.gov/content/travel/en.html](http://travel.state.gov/content/travel/en.html). If you need a preliminary conference invitation in order to secure a visa, please send your request to [mac@ams.org](mailto:mac@ams.org).

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

- \* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

- \* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

- \* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

- \* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

- \* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

- \* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

## Shanghai, People's Republic of China

*Fudan University*

**June 11–14, 2018**

*Monday – Thursday*

### Meeting #1140

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: April 2018

Program first available on AMS website: Not applicable

Issue of *Abstracts*: Not applicable

### Deadlines

For organizers: To be announced

For abstracts: To be announced

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgs/internmtgs.html](http://www.ams.org/amsmtgs/internmtgs.html).*

### Invited Addresses

**Yu-Hong Dai**, Academy of Mathematics and System Sciences, *Title to be announced.*

**Kenneth A. Ribet**, University of California, Berkeley, *Title to be announced.*

**Richard M. Schoen**, University of California, Irvine, *Title to be announced.*

**Sijue Wu**, University of Michigan, *Title to be announced.*

**Chenyang Xu**, Peking University, *Title to be announced.*

**Jiangong You**, Nankai University, *Title to be announced.*

## Newark, Delaware

*University of Delaware*

**September 29–30, 2018**

*Saturday – Sunday*

### Meeting #1141

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: June 2018

Program first available on AMS website: August 9, 2018

Issue of *Abstracts*: Volume 39, Issue 3

### Deadlines

For organizers: February 28, 2018

For abstracts: July 31, 2018

The scientific information listed below may be dated.  
For the latest information, see [www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html).

## Invited Addresses

**Leslie Greengard**, New York University, *Title to be announced*.

**Elisenda Grigsby**, Boston College, *Title to be announced*.

**Davesh Maulik**, Massachusetts Institute of Technology, *Title to be announced*.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).

*Applied Algebraic Topology* (Code: SS 2A), **Chad Giusti**, University of Delaware, and **Gregory Henselman**, Princeton University.

*Convex Geometry and Functional Inequalities* (Code: SS 3A), **Mokshay Madiman**, University of Delaware, **Elisabeth Werner**, Case Western Reserve University, and **Artem Zvavitch**, Kent State University.

*Operator and Function Theory* (Code: SS 4A), **Kelly Bickel**, Bucknell University, **Michael Hartz**, Washington University, St. Louis, **Constanze Liaw**, University of Delaware, and **Alan Sola**, Stockholm University.

*Recent Advances in Nonlinear Schrödinger Equations* (Code: SS 1A), **Alexander Pankov**, Morgan State University, **Junping Shi**, College of William and Mary, and **Jun Wang**, Jiangsu University.

# Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

## Meeting #1143

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: July 2018

Program first available on AMS website: August 30, 2018

Issue of *Abstracts*: Volume 39, Issue 4

## Deadlines

For organizers: March 20, 2018

For abstracts: August 21, 2018

The scientific information listed below may be dated.  
For the latest information, see [www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html).

## Invited Addresses

**Elena Fuchs**, University of Illinois Urbana—Champaign, *Title to be announced*.

**Andrew Putman**, University of Notre Dame, *Title to be announced*.

**Charles Smart**, University of Chicago, *Title to be announced*.

## Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).

*Cluster Algebra, Poisson Geometry, and Related Topics* (Code: SS 9A), **Eric Bucher**, Michigan State University, and **Maitreyee Kulkarni** and **Bach Nguyen**, Louisiana State University.

*From Hyperelliptic to Superelliptic Curves* (Code: SS 6A), **Tony Shaska**, Oakland University, **Nicola Tarasca**, Rutgers University, and **Yuri Zarhin**, Pennsylvania State University.

*Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday* (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

*Interactions between Algebra, Machine Learning and Data Privacy* (Code: SS 3A), **Jonathan Gryak**, University of Michigan, **Kelsey Horan**, CUNY Graduate Center, **Delaram Kahrobaei**, CUNY Graduate Center and New York University, **Kayvan Najarian** and **Reza Soroushmehr**, University of Michigan, and **Alexander Wood**, CUNY Graduate Center.

*Large Cardinals and Combinatorial Set Theory* (Code: SS 10A), **Andres E. Caicedo**, Mathematical Reviews, and **Paul B. Larson**, Miami University.

*Probabilistic Methods in Combinatorics* (Code: SS 7A), **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University, and **David Galvin**, University of Notre Dame.

*Random Matrix Theory Beyond Wigner and Wishart* (Code: SS 2A), **Elizabeth Meckes** and **Mark Meckes**, Case Western Reserve University, and **Mark Rudelson**, University of Michigan.

*Representations of Reductive Groups over Local Fields and Related Topics* (Code: SS 8A), **Anne-Marie Aubert**, Institut Mathématiques de Jussieu, Paris Rive Gauche, **Jessica Fintzen**, IAS, University of Michigan, University of Cambridge, and **Camelia Karimianpour**, University of Michigan.

*Self-similarity and Long-range Dependence in Stochastic Processes* (Code: SS 4A), **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

*Structured Homotopy Theory* (Code: SS 5A), **Thomas Fiore**, University of Michigan, Dearborn, **Po Hu** and **Dan Isaksen**, Wayne State University, and **Igor Kriz**, University of Michigan.



# San Francisco, California

*San Francisco State University*

**October 27–28, 2018**

*Saturday – Sunday*

## Meeting #1144

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: July 2018

Program first available on AMS website: September 6, 2018

Issue of *Abstracts*: Volume 39, Issue 4

## Deadlines

For organizers: March 27, 2018

For abstracts: August 28, 2018

*The scientific information listed below may be dated.  
For the latest information, see [www.ams.org/amsmtg/sectional.html](http://www.ams.org/amsmtg/sectional.html).*

## Invited Addresses

**Srikanth B. Iyengar**, University of Utah, *Title to be announced.*

**Sarah Witherspoon**, Texas A&M University, *Title to be announced.*

**Abdul-Aziz Yakubu**, Howard University, *Title to be announced.*

## Special Sessions

*If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).*

*Coupling in Probability and Related Fields* (Code: SS 3A), **Sayan Banerjee**, University of North Carolina, Chapel Hill, and **Terry Soo**, University of Kansas.

*Homological Aspects of Noncommutative Algebra and Geometry* (Code: SS 2A), **Dan Rogalski**, University of California San Diego, **Sarah Witherspoon**, Texas A&M University, and **James Zhang**, University of Washington, Seattle.

*Mathematical Biology with a focus on Modeling, Analysis, and Simulation* (Code: SS 1A), **Jim Cushing**, The University of Arizona, **Saber Elaydi**, Trinity University, **Suzanne Sindi**, University of California, Merced, and **Abdul-Aziz Yakubu**, Howard University.

# Fayetteville, Arkansas

*University of Arkansas*

**November 3–4, 2018**

*Saturday – Sunday*

## Meeting #1142

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: July 2018

Program first available on AMS website: August 16, 2018

Issue of *Abstracts*: Volume 39, Issue 3

## Deadlines

For organizers: April 3, 2018

For abstracts: September 4, 2018

*The scientific information listed below may be dated.  
For the latest information, see [www.ams.org/amsmtg/sectional.html](http://www.ams.org/amsmtg/sectional.html).*

## Invited Addresses

**Mihalis Dafermos**, Princeton University, *Title to be announced.*

**Jonathan Hauenstein**, University of Notre Dame, *Title to be announced.*

**Kathryn Mann**, University of California Berkeley, *Title to be announced.*

# Baltimore, Maryland

*Baltimore Convention Center, Hilton  
Baltimore, and Baltimore Marriott Inner  
Harbor Hotel*

**January 16–19, 2019**

*Wednesday – Saturday*

## Meeting #1145

*Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).*

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2018

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

## Deadlines

For organizers: April 2, 2018

For abstracts: To be announced

## Call for Proposals

Steven H. Weintraub, Associate Secretary responsible for the AMS program at the 2019

Joint Mathematics Meetings (to be held from Wednesday, January 16 through Saturday, January 19, 2019, in Baltimore, MD) solicits proposals for Special Sessions for this meeting. Each proposal must include:

1. the name, affiliation, and email address of each organizer, with one organizer designated as the contact person for all communication about the session;

2. the title and a brief (two or three paragraph) description of the topic of the proposed special session;

3. a sample list of speakers whom the organizers plan to invite. (It is not necessary to have received confirmed commitments from these potential speakers.)

Organizers are encouraged to read the AMS Manual for Special Session Organizers

at: [www.ams.org/meetings/meet-specialsession-manual](http://www.ams.org/meetings/meet-specialsession-manual) in its entirety.

Proposals for AMS Special Sessions should be sent by email to Prof. Weintraub ([shw2@lehigh.edu](mailto:shw2@lehigh.edu)) and must be received by the deadline for organizers, April 1, 2018. Late proposals will not be considered. No decisions will be made on Special Session proposals until after the submission deadline has passed.

Special Sessions will in general be allotted between 5 and 10 hours in which to schedule speakers. To enable maximum movement of participants between sessions, organizers must schedule each speaker for either a 20-minute talk, 5-minute discussion, and 5-minute break; or a 45-minute talk, 10-minute discussion, and 5-minute break. Any combination of 20-minute and 45-minute talks is permitted, but all talks should begin and end at the scheduled time.

The number of Special Sessions on the AMS program at the Joint Mathematics Meetings is limited, and not all proposals can be accepted. Please be sure to submit as detailed a proposal as possible for review by the Program Committee. We aim to notify organizers whether their proposal has been accepted by May 1, 2018.

## Auburn, Alabama

*Auburn University*

**March 15–17, 2019**

*Friday – Sunday*

### Meeting #1146

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

## Deadlines

For organizers: To be announced

For abstracts: To be announced

## Honolulu, Hawaii

*University of Hawaii at Manoa*

**March 22–24, 2019**

*Friday – Sunday*

### Meeting #1147

Central Section

Associate secretaries: Georgia Benkart and Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

## Deadlines

For organizers: May 15, 2018

For abstracts: January 22, 2019

*The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html).*

## Invited Addresses

**Barry Mazur**, Harvard University, *Title to be announced* (Einstein Public Lecture in Mathematics).

**Aaron Naber**, Northwestern University, *Title to be announced*.

**Deanna Needell**, University of California, Los Angeles, *Title to be announced*.

**Katherine Stange**, University of Colorado, Boulder, *Title to be announced*.

**Andrew Suk**, University of Illinois at Chicago, *Title to be announced*.

## Hartford, Connecticut

*University of Connecticut Hartford  
(Hartford Regional Campus)*

**April 13–14, 2019**

*Saturday – Sunday*

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

## Deadlines

For organizers: September 13, 2018

For abstracts: To be announced

## Quy Nhon City, Vietnam

*Quy Nhon University*

**June 10–13, 2019**

*Monday – Thursday*

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

### Deadlines

For organizers: To be announced

For abstracts: To be announced

## Binghamton, New York

*Binghamton University*

**October 12–13, 2019**

*Saturday – Sunday*

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

### Deadlines

For organizers: March 12, 2019

For abstracts: To be announced

## Gainesville, Florida

*University of Florida*

**November 2–3, 2019**

*Saturday – Sunday*

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

### Deadlines

For organizers: To be announced

For abstracts: To be announced

## Denver, Colorado

*Colorado Convention Center*

**January 15–18, 2020**

*Wednesday – Saturday*

*Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)*

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2019

Program first available on AMS website: November 1, 2019

Issue of *Abstracts*: To be announced

### Deadlines

For organizers: April 1, 2019

For abstracts: To be announced

## Washington, District of Columbia

*Walter E. Washington Convention Center*

**January 6–9, 2021**

*Wednesday – Saturday*

*Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).*

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: October 2020

Program first available on AMS website: November 1, 2020

Issue of *Abstracts*: To be announced

### Deadlines

For organizers: April 1, 2020

For abstracts: To be announced



"One day, I'll fly to the moon with math." —J. Ernest Wilkins Jr., who earned a PhD at Chicago in 1942 at age 19, as quoted in the Black History Month section (page 135).

## Complaints



*Drawn by Michael Berg.*

"If I had to live my life again, I wouldn't do anything else. I love mathematics."

—Marjorie Lee Browne, early proponent of math education  
for everyone, especially school teachers.

[massivesci.com/articles/marjorie-lee-browne-african-american-math-pioneer/](https://massivesci.com/articles/marjorie-lee-browne-african-american-math-pioneer/)

**What crazy things happen to you?** Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: [noti-backpage@ams.org](mailto:noti-backpage@ams.org).

# IN THE NEXT ISSUE OF NOTICES



## MARCH 2018...

The history of women in mathematics in the US started more than 130 years ago.

Generally, the women who earned doctorates in mathematics before WWII did their work with distinguished advisors, but had difficulties when it came to finding jobs.

After WWII, the percentage of women earning PhDs in mathematics dropped dramatically and only returned to its earlier levels in the 1980s.



Margaret A. Readdy

Contemporary women mathematicians come from a variety of backgrounds, entering the American workforce from every continent. Within the US women from diverse social, economic, and racial backgrounds have slowly found their footing and risen to the forefront in mathematics. On the occasion of **Women's History Month**, guest editors Margaret A. Readdy and Christine Taylor take this opportunity to honor some of these women.



Christine Taylor

## Go on your own journey or guide others on theirs!



“...[T]hese problems were carefully and coherently sequenced to unfold an interesting mathematical story complete with plot twists and turns, ultimately building to a satisfying resolution of real mathematical substance...”

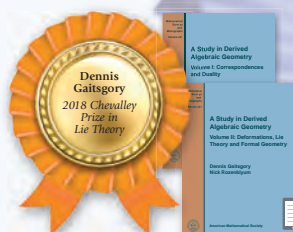
—Thomas Dick, *The College Mathematics Journal*

To learn more about IAS/PCMI—The Teacher Program Series, visit [bookstore.ams.org/SSTP](http://bookstore.ams.org/SSTP).

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# The AMS Presents *Prize-Winning Authors*



## **A Study in Derived Algebraic Geometry** Volumes I and II

Dennis Gaitsgory, *Harvard University, Cambridge, MA*, and  
Nick Rozenblyum, *University of Chicago, IL*

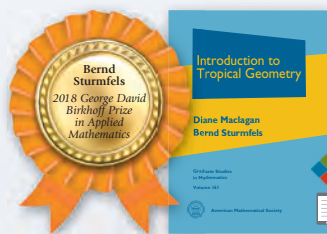
This two-volume monograph explores various topics in algebraic

geometry in the context of derived algebraic geometry.

**Part 1: Mathematical Surveys and Monographs**, Volume 221; 2017; 533 pages; Hardcover; ISBN: 978-1-4704-3569-1; Order code SURV/221.1

**Part 2: Mathematical Surveys and Monographs**, Volume 221; 2017; 436 pages; Hardcover; ISBN: 978-1-4704-3570-7; Order code SURV/221.2

**Set: Mathematical Surveys and Monographs**, Volume 221; 2017; approximately 1016 pages; Hardcover; ISBN: 978-1-4704-3568-4; List US\$220; AMS members US\$176; Order code SURV/221



## **Introduction to Tropical Geometry**

Diane Maclagan, *University of Warwick, Coventry, United Kingdom*, and Bernd Sturmfels, *University of California, Berkeley, CA*

Who should read the book?  
Everybody who wants to learn

what tropical geometry can be about...The result is a straight path to tropical geometry via combinatorial commutative algebra and polyhedral combinatorics. In this way, the book by Maclagan and Sturmfels will become a standard reference in the field for years to come.

—M. Joswig, *Jahresber Dtsch Math-Ver*

This book offers a self-contained introduction to tropical geometry, suitable as a course text for beginning graduate students.

**Graduate Studies in Mathematics**, Volume 161; 2015; 363 pages; Hardcover; ISBN: 978-0-8218-5198-2; List US\$79; AMS members US\$63.20; Order code GSM/161

◆ = Textbook   ◆ = Applied Mathematics   📄 = Available in eBook Format



## **Knots, Molecules, and the Universe**

An Introduction to Topology

Erica Flapan, *Pomona College, Claremont, CA*

[T]his is a wonderful introduction to geometry and topology and their applications to the sciences. The book contains a

unique collection of topics that might entice young readers to continue their academic careers by learning more about the world of mathematics.

—Claus Ernst, *Zentralblatt MATH*

An elementary introduction to geometric topology and its applications to chemistry, molecular biology, and cosmology, this book does not assume any mathematical or scientific background.

2016; 386 pages; Hardcover; ISBN: 978-1-4704-2535-7; List US\$69; AMS members US\$55.20; Order code MBK/96



AMS / MAA Press

## **The Riemann Hypothesis**

Roland van der Veen and Jan van de Craats

Originating from an online course for mathematically talented secondary school students, this book is a first

exploration of the fascinating, unknown world of the Riemann Hypothesis.

**Anneli Lax New Mathematical Library**, Volume 46; 2015; 144 pages; Softcover; ISBN: 978-0-88385-650-5; List US\$45; Individual member US\$33.75; Order code NML/46

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