LETTERS TO THE EDITOR

More on Mathematics and the Military

As a mathematician and a veteran, I read Thomas Gruber's letter to the editor (January 2018) decrying the influence of the military in mathematics with interest, and I thought it might be useful to add a different perspective to the conversation. My own decision to serve was based on a belief that the dictum *Si vis pacem, para bellum*[1] is correct and just as relevant now as when first penned. I served with men and women from all walks of life, of every race, religion, and background. Their motivation was almost always selfless: to protect the weak and free the oppressed; to stand watch in the night so that others could sleep without fear. I think that our increasingly polarized country could learn much from the military; I still remember my drill sergeant in basic training saying "The only color in the Army is green," and it was true.

Entering a PhD program after spending 4 years in the Army was tough, and brings me to my main point: military service makes returning to graduate school challenging. Many professors are unaware that veterans are the most underrepresented group in academia.

Before leaving University of Illinois last year, I worked with the veterans center to set up a mentoring program for student veterans. In the course of collecting data, I found that of 1900 tenure stream faculty, only 6 were veterans, and of the 800 faculty in STEM, I was the sole veteran. This is an underrepresentation in STEM faculty by a factor of over fifty times. So I'll close with a challenge for the AMS (and for all of us): How can we bring more of those who have served our country into the faculty ranks?

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*We invite readers to submit letters to the editor at notices-letters@ams.org.

1 (Schenck) *"If you want peace, prepare for war."*

Beurling Archive on the Web

Arne Beurling[1] (1905–1986) was one of the most influential mathematicians in the area of harmonic and complex analysis. During WWII Beurling cracked a German cryptomachine, the Siemens Geheimschreiber, which was a long kept secret.[2] In 1954 Beurling became Permanent Member and Professor at the Institute for Advanced Study.

Beurling was very selective in what he published and was known for keeping quite a few of his discoveries private. He always required that his work should have a final elegant form and left a considerable amount of unpublished notes and manuscripts. Some of them were included in his *Collected Works*, but some were not.

Several years ago, Beurling's grandchildren donated his Nachlass to Uppsala University Library. On the suggestion of Lennart Carleson, we selected portions to post at www.math.uu.se/beurling. They include:

Beurling's handwritten notebooks;
Notes of Beurling's seminar talks in Uppsala (1938–1951);
Preparatory notes for courses;
Several unpublished manuscripts;
Short notes on various topics.

Apparently, this material includes results that were never published and might be new even now. We believe that it may be of interest for scholars and students of complex and harmonic analysis.

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1 (Benedicks and Sodin) *See the obituary by Ahlfors and Carleson,* Acta Math. *161 (1988) and the recollections by Ahlfors, Kjellberg, and Wermer in* Math. Intelligencer *15:3 (1993).*

2 (Benedicks and Sodin) *See Beckman's* Codebreakers: Arne Beurling and the Swedish Crypto Program During World War II, *Amer. Math. Soc., 2002.*

Notices Reprint Omits Important Mathematical Background of 2016 Nobel Prize in Physics

We are writing about an article on the 2016 Nobel Prize in Physics, which appeared in the *Notices of the American Mathematical Society*, **64**, Number 6, 557–567, (2017). The work for which this prize was awarded has an important mathematical component. We thus applaud the decision by the editors of the *Notices* to publish an article about it. However, we are dismayed by some aspects of the presentation.

The article reprinted in the *Notices* was compiled by the "Class for Physics of the Royal Swedish Academy of Sciences"; (names of authors are not listed). It describes the groundbreaking work of F. Duncan Haldane, J. Michael Kosterlitz, and David J. Thouless on a "topological phase transition," the so-called Kosterlitz-Thouless transition, and on "topological states of matter." It was excerpted from a longer article authored by the Academy. Unfortunately, the *Notices* chose to selectively include only *nine* of *fifty-five* references in the original article. We would like to know why the editors of the *Notices* chose to eliminate so many references to important work, and why they did not include any references to mathematical results that had already strangely been missing in the original article released by the Royal Swedish Academy.

We wonder whether the article published in the *Notices* was refereed. It appears that knowledgeable mathematicians were not contacted in this matter. Although the article may be well suited to a physics audience, it neglects to mention a significant body of mathematical research closely related to the work of Haldane, Kosterlitz, and Thouless, some of which we have been involved in. We believe that the editors of the *Notices* should have consulted the mathematical physics community before publication of this article. Addition of a mathematical perspective would have enriched the article and made it more relevant for the readership of the *Notices*. It might also have inspired further mathematical research. Below, we include references to some of the important mathematical work related to the 2016 Nobel Prize that we feel are useful to a mathematical readership,

I. Papers on Spin Chains:

1) E. H. Lieb, T. D. Schultz, and D. C. Mattis, Two Soluble Models of an Antiferromagnetic Chain, *Ann. Phys.* **16**, 407 (1961).

The authors prove that the spin-1/2 chain is gapless.

2) I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Rigorous Results on Valence-Bond Ground States in Antiferromagnets, *Phys. Rev. Lett.* **59**, 799 (1987).

This work is mentioned in the *Notices*, but no reference is provided.

3) T. Kennedy and H. Tasaki. Hidden $Z_2 \times Z_2$ symmetry breaking in Haldane-gap antiferromagnets, *Phys. Rev. B*, **45**, 304 (1992).

This paper is cited in the original article of the Academy.

Also, A. Polyakov's fundamental prediction of the gap in the two-dimensional classical Heisenberg model should have been mentioned in connection with Haldane's work on spin chains: A. Polyakov, *Phys. Lett.* **59B**, 79 (1975).

The results in this paper are closely related to the gap in the integer spin chain. It is cited in the original article of the Academy.

II. Papers on Phase Transitions:

4) J. Fröhlich, B. Simon, and T. Spencer, Phase Transitions and Continuous Symmetry Breaking, *Phys. Rev. Lett.* **36**, 804 (1976). Details appear in: Infrared Bounds, Phase Transitions and Continuous Symmetry Breaking, *Commun. Math. Phys.* **50**, 79 (1976).

This work contains the first proof of "infrared bounds" and applies them to prove the existence of phase transitions accompanied by continuous symmetry breaking in classical spin systems.

5) F. J. Dyson, E. H. Lieb, and B. Simon, Phase transitions in quantum spin systems with isotropic and nonisotropic interactions, *J. Stat. Phys.* **18**, 335 (1978). See also: T. Kennedy, E. H. Lieb, B. Shastry, Existence of Néel Order in Some Spin-1/2 Heisenberg Antiferromagnets, *J. Stat. Phys.* **53**, 1019 (1988).

In these papers, symmetry breaking and Néel order are established for anti-ferromagnetic quantum magnets.

6) J. Fröhlich and T. Spencer, The Kosterlitz-Thouless Transition in the Two-Dimensional Plane Rotator and Coulomb Gas, *Phys. Rev. Lett.* **46**, 1006 (1981). Details appear in: The Kosterlitz-Thouless-Transition in Two-Dimensional Abelian Spin Systems and the Coulomb Gas, *Commun. Math. Phys.* **81**, 527 (1981).

This work contains the first rigorous proof of existence of the Kosterlitz-Thouless transition; (it actually settled a controversy on this question).

A more detailed analysis of this transition appears in: P. Falco, Kosterlitz-Thouless Transition Line for the Two-Dimensional Coulomb Gas, *Commun. Math Phys.* **312**, 559–609 (2012).

III. Papers on topological states of matter:

7) D. J. Thouless, Mahito Kohmoto, MP Nightingale, and M Den Nijs, Quantized Hall conductance in a two-dimensional periodic potential, *Phys. Rev. Lett.* **49**, 405 (1982).

The relation of this work to homotopy groups of certain natural vector bundles is pointed out in: J. E. Avron, R. Seiler, and B. Simon, Homotopy and quantization in condensed matter physics, *Phys. Rev. Lett.* 51, 51 (1983).

8) B. Simon, Holonomy, the Quantum Adiabatic Theorem, and Berry's Phase, *Phys. Rev. Lett.* **51**, 2167 (1983).

This article establishes a connection between Berry's work and that of Thouless et al. quoted above. This connection allows the author to use Berry's ideas to interpret the integers of Thouless et al. in terms of eigenvalue degeneracies.

9) Jean Bellissard, Noncommutative Geometry and Quantum Hall Effect, in: *Proc. of ICM'94*, S. D. Chatterji (ed.), Basel, Boston, Berlin, Birkhäuser Verlag 1995.

J. E. Avron, R. Seiler, B. Simon, Charge deficiency, charge transport and comparison of dimensions, *Commun. Math. Phys.* **159**, 399 (1994).

Berry's phase for fermions (which has a quarternionic structure) was studied in: J. Avron, L. Sadun, J. Segert, and B. Simon, Chern numbers and Berry's phases in Fermi systems, *Commun. Math. Phys.* **124**, 595 (1989).

10) J. Fröhlich and U.M. Studer, Gauge Invariance and Current Algebra in Non-Relativistic Many-Body Theory, *Rev. Mod. Phys.* 65, 733 (1993).

To our knowledge, the "spin Hall effect" in time-reversal invariant topological insulators with chiral edge spin currents has been described in this paper for the first time.

11) J. Fröhlich et al., The Fractional Quantum Hall Effect, Chern-Simons Theory, and Integral Lattices, in: *Proc. of ICM'94,* S.D. Chatterji (ed.), Basel, Boston, Berlin: Birkhäuser Verlag 1995. J. Fröhlich, U.M. Studer, and E. Thiran, Quantum Theory of Large Systems of Non-Relativistic Matter, in: *Proc. of Les Houches LXII, Fluctuating Geometries in Statistical Mechanics and Field Theory,* F. David, P. Ginsparg and J. Zinn-Justin (eds.), Amsterdam: Elsevier Science 1995.

In these papers (and refs. to original papers given therein), results on the Fractional Quantum Hall Effect and other phenomena related to topological states of matter are described. Topological Chern-Simons (field) theory and current algebra are applied to problems in condensed matter physics.

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