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Graduate Student Blog
by and for math graduate students

The AMS Graduate Student Blog, by and for graduate students, includes puzzles and a variety of interesting columns, such as the one from February 2018 excerpted here. `blogs.ams.org/mathgradblog`.

Introduction to Ideal Class Groups

by Tom Gannon, University of Texas at Austin

Algebraic number theory is a really interesting subject, but unlike some other subjects, it's not 100% clear what objects people study. This post provides an introduction to the class group of a finite dimensional field extension of $\mathbb{Q}$, an object often used in modern number theory....

One of the first cool facts about this is that the class group is always a finite group! This also develops the subject of class field theory, the study of Galois extensions of $\mathbb{Q}$ whose Galois groups are abelian over $\mathbb{Q}$. This can be used to prove the Kronecker-Weber theorem, which says that for any abelian extension $K/\mathbb{Q}$, there is a cyclotomic field containing K. In short – the class group of a number field is a rich object worth studying!

Photo Credit

Photo of Tom Gannon by Rachel Schlossman.

ABOUT THE AUTHOR

Tom Gannon is a second-year graduate student at the University of Texas at Austin. When not doing mathematics, Tom can be found in the kitchen cooking new recipes, or can be found in the kitchen cooking old ones. His email address is `gannonth@math.utexas.edu`.

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