

Notices

of the American Mathematical Society

August 2018

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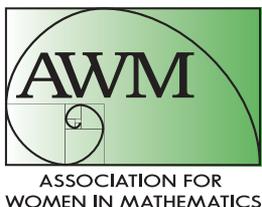
Call for Nominations

AWM–AMS NOETHER LECTURE

The Association for Women in Mathematics (AWM) established the Emmy Noether Lectures in 1980 to honor women who have made fundamental and sustained contributions to the mathematical sciences. In April 2013 this one-hour expository lecture was renamed the AWM–AMS Noether Lecture. The first jointly sponsored lecture was held in January 2015 at the Joint Mathematics Meetings (JMM) in San Antonio, Texas. Emmy Noether was one of the great mathematicians of her time, someone who worked and struggled for what she loved and believed in. Her life and work remain a tremendous inspiration.

The mathematicians who have given the Noether Lectures in the recent past include: Jill C. Pipher, Lisa Jeffrey, Karen E. Smith, Wen-Ching Winnie Li, Georgia Benkart, Raman Parimala, and Barbara Keyfitz. Additional past Noether lecturers can be found at <https://sites.google.com/site/awmmath/programs/noether-lectures/noether-lecturers>.

The letter of nomination should include a one-page outline of the nominee's contribution to mathematics, giving four of her most important papers and other relevant information. Nominations must be submitted by October 15, 2018, and will be held active for three years.



The nomination procedure is described here:

<https://www.sites.google.com/site/awmmath/programs/noether-lectures>.

If you have questions, call 401-455-4042
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Notices

of the American Mathematical Society



August 2018

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The Mathematics of Cathleen Synge Morawetz

Christina Sormani, Editor



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—Frank Morgan, Editor-in-Chief

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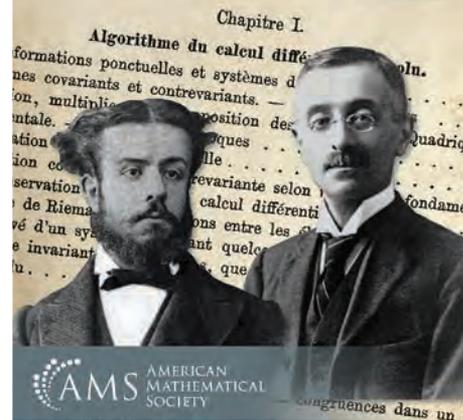
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EINSTEIN'S ITALIAN MATHEMATICIANS

RICCI, LEVI-CIVITA, AND THE BIRTH OF GENERAL RELATIVITY

Judith R. Goodstein



AVAILABLE IN
EBOOK FORMAT

Galileo said that mathematics is the language of nature. Einstein might have found himself mute when it came to describing gravity if it weren't for the mathematics of covariant derivatives developed by Galileo's countrymen Gregorio Ricci-Curbastro and Tullio Levi-Civita. Judy Goodstein tells their stories and their connection to Einstein with clarity and grace in a most readable book.

—Barry Simon, California Institute of Technology

This volume chronicles the lives and intellectual contributions of Italian mathematician Gregorio Ricci and his brilliant student Tullio Levi-Civita, including letters, interviews, memoranda, and other personal and professional papers, to tell the remarkable, little-known story of how two Italian academicians, of widely divergent backgrounds and temperaments, came to provide the indispensable mathematical foundation—today known as the tensor calculus—for general relativity.

2018; approximately 227 pages; Softcover; ISBN: 978-1-4704-2846-4; List US\$35; AMS members US\$28; MAA members US\$31.50; Order code MBK/113

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The Mathematics of Cathleen Synge Morawetz

Communicated by Christina Sormani



Morawetz, an avid sailor, invited us all to “sail with her, near the speed of sound.”

Irene Gamba and Christina Sormani

Introduction

In this memorial we celebrate the mathematics of Cathleen Synge Morawetz (1923–2017). She was awarded the National Medal of Science in 1998 “for pioneering advances in partial differential equations and wave propagation resulting in applications to aerodynamics, acoustics and optics.” In 2004 she won the Steele Prize for lifetime achievement and in 2006 she won the Birkhoff Prize “for her deep and influential work in partial differential equations, most notably in the study of shock waves, transonic flow, scattering theory, and conformally invariant estimates for the wave equation.”

As it is impossible to review all her profound contributions to pure and applied mathematics, we have chosen instead to present some of her most influential work in depth. Terence Tao presents the Morawetz Energies and Morawetz Inequalities, which are ubiquitous in the analysis of nonlinear wave equations. Leslie Greengard and Tonatiuh Sánchez-Vizuet have written about her work on scattering theory. Kevin R. Payne describes the importance of her early work on transonic flows which both provided a new understanding of mixed-type partial differential equations and led to new methods of efficient aircraft design. In this introduction, we provide a little history about her career, and we close the article with a quote of hers thanking one of the mathematicians who supported her the most when she was young.

Morawetz was encouraged to study mathematics by her mother and a family friend, Cecilia Kreger, who was a mathematics professor at the University of Toronto. Her father, who was also a mathematician at Toronto did not encourage her pursuit of mathematics, but did encourage her to be “ambitious.” Morawetz graduated with a bachelors in mathematics at Toronto in 1945 and completed her masters at MIT the following year.

In 1946 Morawetz was hired at New York University to edit the manuscript “Supersonic Flow and Shock Waves” by Richard Courant and Kurt Otto Friedrichs. She described this later in life as “an invaluable and immersive learning experience.” Upon completing her doctorate in 1951 with Friedrichs, Morawetz first accepted a research associate position at MIT. However she quickly returned to New York University, where she stayed for the remainder of her career. Originally hired as research associate, she became assistant professor in 1957. At that time Courant also hired other NYU graduates to join the faculty, including

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Morawetz was awarded the National Medal of Science in 1998 “for pioneering advances in partial differential equations and wave propagation.”

Harold Grad, Anneli Cahn Lax, then Peter Lax and Louis Nirenberg. They remained close friends throughout her life. Morawetz was tenured in 1960 and earned a full professorship in 1965, the year before being awarded her first of two Guggenheim Fellowships. She was a Gibbs Lecturer in 1981, gave an invited address for SIAM in 1982, and was Noether Lecturer in 1983 and 1988. She served as the director of the Courant Institute at NYU from 1984 to 1988.

Morawetz was an astounding mentor and a dedicated coauthor. Irene Gamba worked with her in 1992–1994 as an NSF postdoctoral fellow. She writes:

Our discussions lasted for endless hours and were most illuminating and prolific. They culminated with two joint publications related to the approximation to transonic flow problems, and the life changing opportunity of joining the faculty as an assistant professor in the fall of 1994. She was an extraordinary role model for me.



Cathleen Morawetz and fellow New York University PhD, Harold Grad, back at NYU on the faculty (1964).

Morawetz collaborated often with younger mathematicians, including Gregory Kriegsmann, Walter Strauss, Alvin Bayliss, Kevin Payne, Susan Friedlander, Jane Gilman, and James Ralston. Among her doctoral students were Christian Klingenberg and Leslie Sibner.

Morawetz was elected president of the American Mathematical Society in 1993. At that time funding in core mathematics was under threat, the US government was shut down twice, the job market for new doctorates in mathematics was terrible, and universities were reconsidering the importance of having research mathematicians teaching their mathematics courses. We are facing these same difficulties today and can learn from her example.



Morawetz with Irene Gamba on the day Morawetz gave her Noether Lecture at the International Congress of Mathematicians in 1998. Their work was an extraordinary leap into an area that today remains quite unexplored.

As AMS president, Morawetz joined forces with the SIAM president, Margaret Wright, to defend the funding of both pure and applied mathematics. "Together, they formulated carefully worded statements for Congress and

agency leaders, always stressing (equally) the remarkable track record of useful mathematics as well as the unexpected benefits that consistently emerge from undirected basic research."¹ Their work led eventually towards the creation of the NSF DMS Grants for Vertical Integration of Research and Education (VIGRE) "to increase the number of well-prepared US citizens, nationals, and permanent residents who pursue careers in the mathematical sciences." This program provided funding for postdocs, graduate students, and undergraduates engaged in research with one another and has directly influenced the careers of many young mathematicians.



Morawetz with Bella Manel (l) and Christina Sormani (r) at NYU in 1996. Manel received her doctorate at NYU in 1939.

Morawetz was a powerful leader, a wonderful mentor, and an amazing mathematician. All of us that were fortunate enough to be influenced by her aura through her ninety-four years of life can admire her relentless pursuit of excellence. Perhaps we too can strive to make a difference.

See Also

Morawetz's retiring AMS presidential address: <https://www.ams.org/notices/199901/morawetz.pdf>

Morawetz's work on the board of JSTOR: <https://www.ams.org/notices/199806/comm-jstor.pdf>

Happy 91st, Cathleen Syngé Morawetz <https://www.ams.org/notices/201405/rnoti-p510.pdf>

¹Quote taken from a *SIAM Memorial of Morawetz* by Margaret Wright and John Ewing.

Terence Tao

Morawetz Inequalities

Cast a stone into a still lake. There is a large splash, and waves begin radiating out from the splash point on the surface of the water. But, as time passes, the amplitude of the waves decays to zero.

This type of behavior is common in physical waves, and also in the partial differential equations used in mathematics to model these waves. Let us begin with the classical wave equation

$$(1) \quad -\partial_{tt}u + \Delta u = 0,$$

where $u : \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{R}$ is a function of both time $t \in \mathbb{R}$ and space $x \in \mathbb{R}^3$, which is a simple model for the amplitude of a wave propagating at unit speed in three dimensional space; here

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

denotes the spatial Laplacian. One can verify that one has the family of explicit solutions

$$(2) \quad u(t, x) = \frac{F(t + |x|) - F(t - |x|)}{|x|}$$

to (1) for any smooth, compactly supported function $F : \mathbb{R} \rightarrow \mathbb{R}$, where

$$|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

denotes the Euclidean magnitude of a position $x \in \mathbb{R}^3$. The dispersive nature of this equation can be seen in the observation that the amplitude

$$\sup_{x \in \mathbb{R}^3} |u(t, x)|$$

of such solutions decays to zero as $t \rightarrow \pm\infty$, whilst other quantities such as the energy

$$\int_{\mathbb{R}^3} \frac{1}{2} |\partial_t u(t, x)|^2 + \frac{1}{2} |\nabla u(t, x)|^2 dx$$

stay constant in time (and in particular do not decay to zero).

The wave equation can be viewed as a special case of the more general linear Klein-Gordon equation

$$(3) \quad -\partial_{tt}u + \Delta u = m^2 u,$$

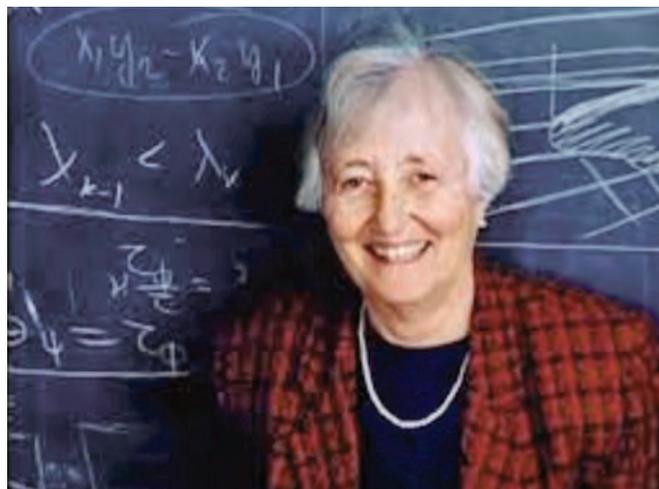
where $m \geq 0$ is a constant. Even more important is the linear Schrödinger equation, which we will normalize here as

$$(4) \quad i\partial_t u + \frac{1}{2}\Delta u = 0,$$

where the unknown field $u : \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{C}$ is now complex-valued. There are also nonlinear variants of these equations, such as the nonlinear Klein-Gordon equation

$$(5) \quad -\partial_{tt}u + \Delta u = m^2 u + \lambda |u|^{p-1} u,$$

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Morawetz worked to change the way we think about partial differential equations.

and the nonlinear Schrödinger equation

$$(6) \quad i\partial_t u + \frac{1}{2}\Delta u = \lambda |u|^{p-1} u,$$

where $\lambda = \pm 1$ and $p > 1$ are specified parameters. There are countless other further variations (both linear and nonlinear) of these dispersive equations, such as Einstein's equations of general relativity, or the Korteweg-de Vries equations for shallow water waves.



Figure 1. Dispersion is illustrated in this numerical simulation of the Klein-Gordon equation implemented by Brian Leu, Albert Liu, and Parth Sheth using XSEDE when they were undergrads at U Michigan in 2013. For a video see www-personal.umich.edu/~brianleu.

An important way to capture dispersion mathematically is through the establishment of dispersive inequalities that assert, roughly speaking, that if a solution u to

one of these equations is sufficiently localized in space at an initial time, $t = 0$, then it will decay as $t \rightarrow \infty$. (If a solution u is not localized enough in space initially, it does not need to decay; consider for instance the traveling wave solution

$$u(t, x) = F(t - x_1)$$

to the wave equation (1). This decay has to be measured in suitable function space norms, such as the $L_x^\infty(\mathbb{R}^3)$ norm.

One can represent any solution u to the linear Schrödinger equation explicitly in terms of the initial data $u(0)$ by the formula

$$u(t, x) = \frac{1}{(2\pi it)^{3/2}} \int_{\mathbb{R}^3} e^{-i|x-y|^2/2t} u(0, y) dy$$

for all $t \neq 0$ and $x \in \mathbb{R}^3$, where the quantity $(2\pi it)^{3/2}$ is defined using a suitable branch cut. From the triangle inequality, this immediately gives the dispersive inequality

$$(7) \quad \|u(t)\|_{L_x^\infty(\mathbb{R}^3)} \leq \frac{1}{(2\pi|t|)^{3/2}} \|u(0)\|_{L_x^1(\mathbb{R}^3)}.$$

If the solution is initially spatially localized in the sense that the L^1 norm

$$\|u(0)\|_{L_x^1(\mathbb{R}^3)}$$

is finite, then the solution $u(t)$ decays uniformly to zero as $t \rightarrow \pm\infty$. A similar (but slightly more complicated) dispersive inequality can also be obtained for solutions to the linear Klein-Gordon equation (3).

On the other hand, solutions to the linear Schrödinger Equation (4) satisfy the pointwise mass conservation law

$$(8) \quad \partial_t |u|^2 = \sum_{j=1}^3 \partial_{x_j} \text{Im}(\bar{u} \partial_{x_j} u).$$

From this, one can easily derive conservation of the spacial L^2 norm of the solution:

$$\|u(t)\|_{L_x^2(\mathbb{R}^3)} = \|u(0)\|_{L_x^2(\mathbb{R}^3)}.$$

In particular, the L^2 norm of the solution will stay constant in time, rather than decay to zero.

To reconcile this fact with the dispersive estimate, we observe that solutions to dispersive equations such as linear Schrödinger equation spread out in space as time goes to infinity (much as the ripples on a pond do), allowing the L^∞ norm of such a solution to go to zero even while the L^2 norm stays bounded away from zero. As mentioned earlier, this effect can also be seen for the wave equation (1).

The above analysis of the linear Schrödinger equation relied crucially on having an explicit fundamental solution at hand. What happens if one works with nonlinear (and not completely integrable) equations, such as (5) or (6), in which no explicit and tractable formula for the solution is available? For linear equations (such as the wave or Schrödinger equation outside of an obstacle, or in the presence of potentials or magnetic fields) one can still hope to use methods from spectral theory to understand the long-time behavior (as is done for instance in the famous RAGE theorem of Ruelle (1969),

Amrein-Georgescu (1973), and Enss (1977)). However, such methods are absent for nonlinear equations such as (5) or (6), particularly when dealing with solutions that are too large for perturbative theory to be of much use.



Cathleen Synge Morawetz in 1964.

Recall that in 1961, Morawetz proved the decay of solutions to the classical wave equation in the presence of a star-shaped obstacle. Morawetz used the “Friedrichs *abc* method,” in which one multiplied both sides of a PDE such as (5) or (6) by a multiplier

$$a\partial_t u + b \cdot \nabla u + cu$$

for well chosen functions a, b, c , integrated over a space-time domain, and rearranging using integration by parts and omitting some terms of definite sign, obtained a useful integral inequality. The key discovery of Morawetz (a version of which first appeared in work of Ludwig) was that this method was particularly fruitful when the multiplier was equal to the radial derivative

$$\frac{x \cdot \nabla u}{|x|}$$

of the solution (in some cases one also adds a lower order term $\frac{u}{|x|}$).

In 1968, Morawetz applied this technique to study solutions u to the nonlinear Klein-Gordon equation (5), assuming one is in the nonfocusing case with $\lambda = m = +1$. (In the *focusing* case $\lambda = -1$, the equation (5) admits “soliton” solutions that are stationary in time and thus do not disperse.) By using a multiplier of the above form, Morawetz obtained an inequality of the form

$$(9) \quad \int_{\mathbb{R}} \int_{\mathbb{R}^3} U(t, x) dx dt \leq CE(u(0)),$$

where

$$U(t, x) = \frac{|u(t, x)|^2 + |u(t, x)|^{p+1}}{|x|}.$$

Here the constant C depends only on the exponent p and $E(u(0))$ is the energy:

$$E(u(0)) = \int_{\mathbb{R}^3} \frac{1}{2} |\nabla u(0, x)|^2 + \frac{1}{2} |\partial_t u(0, x)|^2 + \frac{1}{(p+1)} |u(0, x)|^{p+1} dx.$$

This type of estimate is now known as a *Morawetz inequality*. The key point here is that the left-hand side of the Morawetz inequality in (9) contains an integration over the entire time domain \mathbb{R} (as opposed to a time integral over a bounded interval). It immediately rules out soliton-type solutions that move at bounded speed (as this would make the left-hand side of (9) infinite). It forces some time-averaged decay of the solution near the spatial origin $x = 0$. For instance, it is immediate from (9) that

$$\frac{1}{T} \int_0^T \int_K |u(t, x)|^2 dx dt \rightarrow 0$$

as $T \rightarrow \infty$ for any compact spatial region $K \subset \mathbb{R}^3$.



Cathleen Morawetz and Walter Strauss in 2008.

Once one has some sort of decay estimate for a dispersive equation, it is often possible to “bootstrap” the estimate to obtain additional decay estimates. For instance one might use the decay estimate one already has to bound the right-hand side of a nonlinear PDE such as (5) or (6), and then solve the associated (inhomogeneous) linear PDE to obtain a new decay estimate for the solution.

An early result of this type was developed by Morawetz and Strauss in 1975. They showed that for any finite

energy solution u to the nonlinear Klein-Gordon equation (5) with $\lambda = m = +1$ and $p = 3$, the solution decays like a solution to the linear Klein-Gordon equation (3). More precisely, there exist finite solutions u_+, u_- to (3) such that $u(t) - u_+(t)$ (resp. $u(t) - u_-(t)$) goes to zero in the energy norm as $t \rightarrow +\infty$ (resp. $t \rightarrow -\infty$). This can be developed further into a satisfactory *scattering theory* for such equations, which among other things gives a continuous *scattering map* from u_- to u_+ or vice versa. See Figure 2.

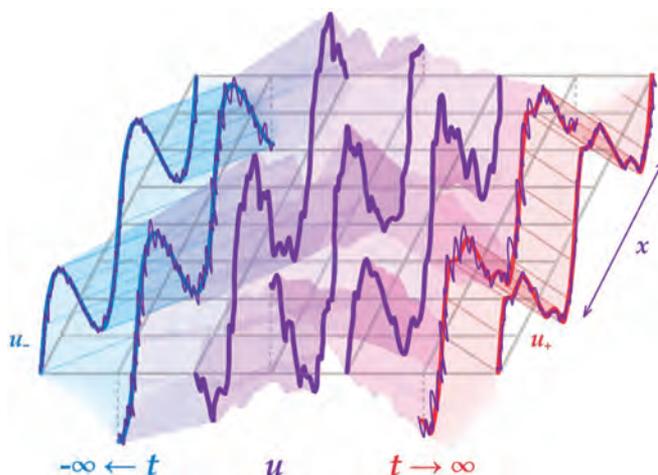


Figure 2. Here we see u_- on the left in blue and u_+ on the right in red, with u in purple approximating u_- as $t \rightarrow -\infty$ and approximating u_+ as $t \rightarrow \infty$.

In the decades since Morawetz’s pioneering work, many additional Morawetz inequalities have been developed. For instance, in 1978, Lin and Strauss developed Morawetz inequalities for the nonlinear Schrödinger equation, and Morawetz herself discovered further such estimates for the wave equation outside of an obstacle. In more recent years, “interaction Morawetz inequalities” were introduced, which could control correlation quantities such as

$$(10) \quad \int_{\mathbb{R}} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \frac{|u(t, x)|^2 |u(t, y)|^p}{|x - y|} dx dy dt$$

for solutions u to the nonlinear Schrödinger equation (6).

One way to view Morawetz inequalities is as an assertion of monotonicity of the *radial momentum*, which takes the form

$$\int_{\mathbb{R}^3} (\partial_t u) \left(\frac{x}{|x|} \cdot \nabla u \right) dx$$

for wave or Klein-Gordon equations, and

$$\int_{\mathbb{R}^3} \text{Im}(\bar{u}) \frac{x}{|x|} \cdot \nabla u dx$$

for Schrödinger equations. Informally, this quantity is expected to be positive when waves propagate away from the origin, and negative when they propagate towards the origin. The intuition is that while waves can sometimes propagate towards the origin, eventually they will move past the origin and begin radiating away from the origin. However, in the absence of focusing mechanisms (such

as a negative sign $\lambda = -1$ in the nonlinearity), the reverse phenomenon of outward net radial momentum being converted to inward net radial momentum cannot occur. Thus the radial momentum is always expected to be increasing in time.

On the other hand, under hypotheses such as finite energy, this radial momentum should be bounded.

The Morawetz inequalities are indispensable.

So by the fundamental theorem of calculus, the time derivative of the radial momentum should have a bounded integral in time. Intuitively, one expects this time derivative to be large when the solution has a strong presence near the origin, but not when the solution is far away from the origin. Far from the origin the radial vector field

$$\frac{\mathbf{x}}{|\mathbf{x}|} \cdot \nabla$$

behaves like a constant, and the radial momentum approaches a fixed coordinate of the total momentum. This explains why Morawetz inequalities tend to involve factors such as $\frac{1}{|\mathbf{x}|}$ that localize the estimate to near the origin.

The Morawetz inequalities are indispensable as an ingredient in controlling the long-time behavior of solutions to a wide array of dispersive defocusing equations, including a number of *energy-critical* or *mass-critical* equations in which the analysis is particularly delicate and interesting; see for instance the texts [1], [4], [3] for detailed coverage of these topics. They have also been successfully applied to many equations in general relativity (such as Einstein's equations for gravitational fields), for instance to analyze the asymptotic behaviour around a black hole. The fundamental tools that Morawetz has introduced to the field of dispersive equations will certainly underlie future progress in this field for decades to come.

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Leslie Greengard and Tonatiuh Sánchez-Vizuet

Cathleen Morawetz and the Scattering of Acoustic Waves

Cathleen Morawetz was a force at the Courant Institute when one of us (L.G.) arrived as a postdoctoral fellow. It was the last year of her directorship, but she made the time to welcome all newcomers. Her generosity of spirit was unmatched — she encouraged young people in every discipline, and her humour and enthusiasm were infectious.

When she began to study the decay properties of acoustic waves after impinging on an obstacle, essentially no general results were available. To understand the relevant issues, let us begin with the formulation of the problem in terms of the governing linear, scalar wave equation in \mathbb{R}^3 , with a forcing term which is turned on for a finite time:

$$(11) \quad u_{tt}(\mathbf{x}, t) = \Delta u(\mathbf{x}, t) + f(\mathbf{x}, t).$$

Here, Δu is the Laplacian operator acting on the scalar function $u(\mathbf{x}, t)$ and $f(\mathbf{x}, t)$ is nonzero only in the finite time interval $0 \leq t \leq T$. We assume that we have zero initial (Cauchy) data at time $t = 0$:

$$u(\mathbf{x}, 0) = 0 \quad \text{and} \quad u_t(\mathbf{x}, 0) = 0.$$

We also assume that $f(\mathbf{x}, t)$ is a smooth, compactly supported and square integrable function in space-time, such as

$$(12) \quad f(\mathbf{x}, t) = W(\|\mathbf{x} - \mathbf{x}_0\|) W\left(\frac{2t - T}{T}\right),$$

where $\mathbf{x}_0 \in \mathbb{R}^3$ and $W(x)$ is a standard C^∞ bump function such as

$$W(x) = \begin{cases} e^{-\frac{1}{1-x^2}} & \text{for } |x| < 1; \\ 0 & \text{otherwise.} \end{cases}$$

Then, it is well known that

$$(13) \quad u(\mathbf{x}, t) = \frac{1}{4\pi} \int_{B_{\mathbf{x}_0}(1)} \frac{f(\mathbf{x}', t - \|\mathbf{x} - \mathbf{x}'\|)}{\|\mathbf{x} - \mathbf{x}'\|} d\mathbf{x}',$$

where $B_{\mathbf{x}_0}(1)$ denotes the unit ball centered at \mathbf{x}_0 . From this formula it is clear that at any point \mathbf{x} in space, the solution first becomes nonzero at $t = d_{min}$, where d_{min} is the distance from \mathbf{x} to the closest point in $B_{\mathbf{x}_0}(1)$. It vanishes identically at \mathbf{x} as soon as $t > T + d_{max}$, where d_{max} is the distance from \mathbf{x} to the farthest point in $B_{\mathbf{x}_0}(1)$.

Suppose now that, rather than propagating in free-space, the outgoing spherical wavefront emanating from \mathbf{x}_0 hits an object as in Figure 3. That is, we assume there is a “sound-soft,” smooth, bounded obstacle Ω with boundary $\partial\Omega$, which is at some distance from $B_{\mathbf{x}_0}(1)$, so that $\Omega \cap B_{\mathbf{x}_0}(1) = \emptyset$. Then, using the language of scattering theory, the total acoustic field is given by

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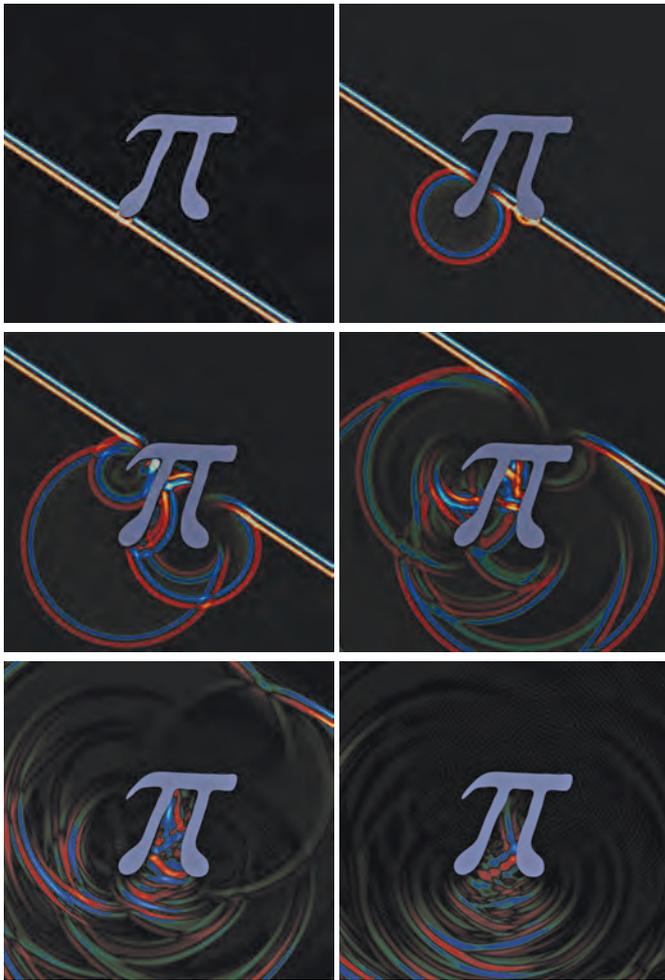


Figure 3. The evolution of an acoustic wave impinging upon a non-star shaped object, π , at times $t = 1, 4, 8, 13, 19, 26$ implemented using a high-order integral equation solver. A video of the simulation may be found at <https://cims.nyu.edu/~tonatiuh/morawetz.html>.

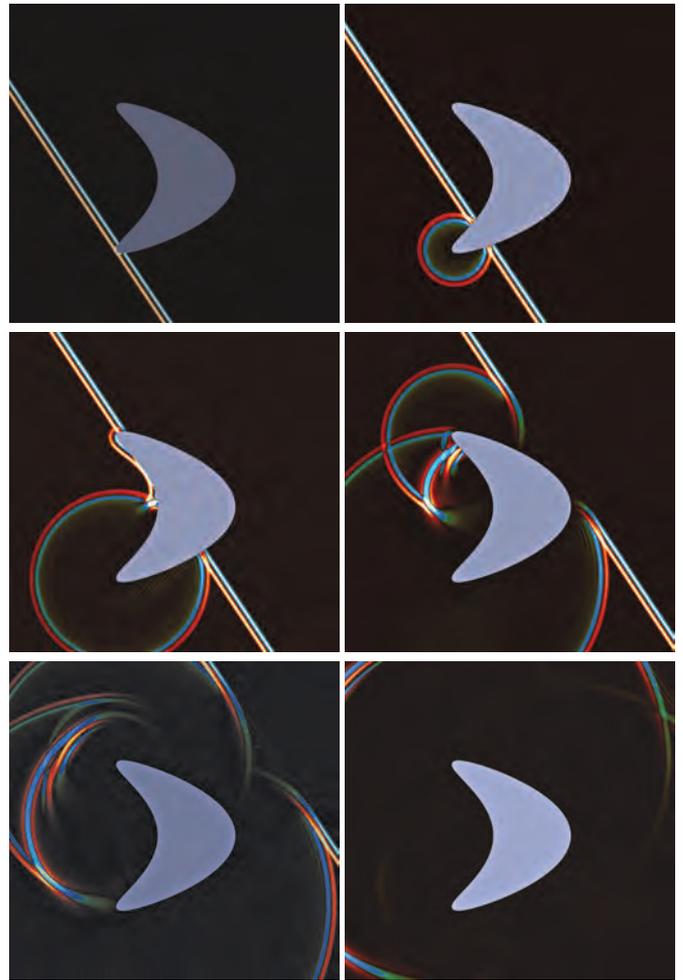


Figure 4. The evolution of an acoustic wave being scattered off a star shaped obstacle at the same sequence of times as in Figure 3. In the final panel, the scattered wave has almost completely left the simulation region, in contrast with the final panel of Figure 3.

$u(\mathbf{x}, t) + u^{scat}(\mathbf{x}, t)$, where the scattered field satisfies the homogeneous wave equation

$$u_{tt}^{scat}(\mathbf{x}, t) - \Delta u^{scat}(\mathbf{x}, t) = 0$$

for $t > 0$, with initial data

$$u^{scat}(\mathbf{x}, 0) = 0 \quad \text{and} \quad u_t^{scat}(\mathbf{x}, 0) = 0$$

and Dirichlet boundary conditions

$$u^{scat}(\mathbf{x}, t) = -u(\mathbf{x}, t)$$

for $\mathbf{x} \in \partial\Omega$.

Let \mathbf{y} denote some fixed point away from both the ball $B_{\mathbf{x}_0}(1)$ and the obstacle Ω . The question is: can one prove that the scattered field decays at \mathbf{y} , and if so, at what rate? Very little progress had been made on this question until 1959, when Wilcox published a short note showing that in the case of a spherical obstacle, an exact solution could be expressed in terms of spherical harmonics. From

this, Wilcox was able to conclude that the solution decays exponentially fast. While an important step, his result yielded no suggestion as to how to proceed in the general case.

In 1961, Morawetz [4] made a critical step forward. She showed that if the reflecting obstacle is star-shaped, then the solution to the wave equation decays like $t^{-1/2}$. A region Ω is said to be star-shaped if there exists a point $\mathbf{p} \in \Omega$, such that for all $\mathbf{x} \in \Omega$, the line segment from \mathbf{p} to \mathbf{x} is contained in Ω . The object in Figure 3, for example, is not star shaped, while the object in Figure 4 is. It is perhaps surprising that for non star-shaped obstacles, very little is understood to the present day.

The qualitative difference in the behavior of waves reflecting from obstacles that are not star-shaped and those that are is illustrated in Figures 3 and 4. In the

first three panels of each figure, as the incoming wave hits the object, the scattered wave is clearly visible, with energy propagating outwards in all directions. In the next three panels, more of the energy is carried away. In Figure 3, some of the energy remains behind for quite some time, and in the last panel a significant amount of energy has focused in a small neighborhood. In Figure 4, the energy has propagated outward without significant concentration and appears to decay much more rapidly.

Remark. The simulations in these figures are actually for the two-dimensional wave equation, with an incoming plane wave of the form

$$u^{inc}(\mathbf{x}, t) = \chi(s/\alpha) \sin^3(s/\alpha), \quad s := \mathbf{x} \cdot \mathbf{d} - t.$$

The unit vector \mathbf{d} points in the direction in which the wave propagates, $\chi(\cdot)$ is a smooth approximation to the characteristic function of the interval $[0, 2\pi]$, and α is a scaling factor that has the effect of shrinking (if $\alpha < 1$) or dilating (if $\alpha > 1$) the wave profile. In two dimensions, waves do *not* decay exponentially fast, even in the absence of a scatterer, but the focusing/trapping effect caused by nonstar shaped obstacles is similar.



Peter Lax with Cathleen Morawetz at the 2008 Conference on Nonlinear Phenomena in Mathematical Physics: Dedicated to Cathleen Synge Morawetz on her 85th Birthday.

Morawetz’s writing style was very much that of a storyteller. To get a sense of that, here is the beginning of the proof of the main theorem in her 1961 paper [4]:

The proof is based on energy identities, i.e. quadratic integral relations satisfied by all solutions. This is one of the most powerful tools for getting estimates for solutions of elliptic, hyperbolic or mixed equations. The most familiar identity of this kind for the wave equation is obtained by multiplying $u_{tt} = \Delta u$ by u_t and integrating in the slab $0 \leq t \leq t_1$; the resulting integral identity satisfies the conservation of energy. Here we use another multiplier in the place of u_t introduced by Protter for another purpose. The significance of

using alternative multipliers has been frequently emphasized by Friedrichs and is often referred to as Friedrichs’s *a, b, c*-method. The multiplier here is

$$(14) \quad xu_x + yu_y + zu_z + tu_t + u$$

and from the resulting identity we conclude that all the energy is carried outward.

In truth, Morawetz was being overly modest. It was her keen insight that allowed for the selection of a multiplier which would yield the desired result. The power and generality of this approach led to breakthroughs in many wave propagation problems, with the state of the art collected in Morawetz’s 1966 monograph “Energy identities for the wave equation,” originally released as a Courant Institute technical report.

A second major step forward in understanding the decay of waves scattered from star-shaped obstacles came in 1963, in joint work with Lax and Phillips. They showed that, in fact, such solutions decay exponentially (as they do for a sphere), not just as $t^{-1/2}$. The proof relies on an observation of Lax and Phillips that there is a function $Z(t)$ which satisfies the semigroup property

$$(15) \quad Z(t + s) = Z(t)Z(s),$$

and whose norm controls the decay of the solution. In this context, Morawetz’s 1961 paper shows that for some time $t = \tau$, $|Z(\tau)|$ has decayed to less than one:

$$|Z(\tau)| < 1 = e^{-\alpha} \text{ for some } \alpha > 0.$$

That is enough to guarantee exponential decay! One simply writes

$$t = n\tau + t_1 \text{ where } t_1 < \tau,$$

from which

$$\begin{aligned} |Z(t)| &= |Z(t_1)| | [Z(\tau)]^n | \\ &\leq |Z(t_1)| e^{-\alpha n} = Ce^{-\alpha t/\tau}. \end{aligned}$$

Nontrapping Objects

One of the features of star-shaped objects is that rays impinging on them cannot be trapped. A ray here is the path taken by an infinitely thin beam of light which reflects from the surface according to geometrical optics. For a complicated scatterer, one can imagine that a ray could undergo successive bounces without escaping from the convolutions of the surface $\partial\Omega$ in any finite time interval (see Figure 5).

This situation was studied by Morawetz, Ralston, and Strauss in their 1977 article, where they proved a remarkable extension of Morawetz’s earlier results; if the object Ω does not trap rays, then the scattered wave decays exponentially. The proof involves the introduction of an *escape function* (a generalization of the geometric intuition of an “escape path of finite length”) and a different multiplier from that in Morawetz’s 1961 paper. The use of such *Morawetz multipliers* is now ubiquitous in the analysis of PDEs.

Denoting by S a sphere which contains the smooth scatterer Ω , considering $\mathbf{x} \in S \setminus \Omega$, and letting ξ be a unit

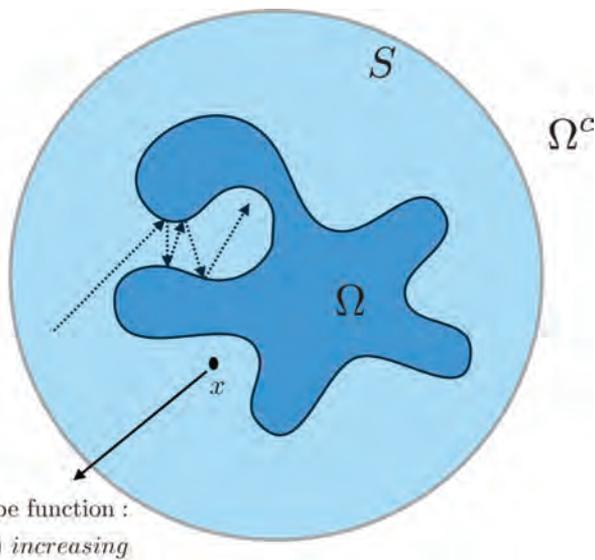


Figure 5. In 1977, Morawetz, Ralston, and Strauss generalized the class of scatterers for which decay results could be proven. They showed, in three dimensions, that if the object Ω does not trap rays, then the local energy of the wave must decay exponentially. Star-shaped objects are a subset of this much larger class. The path taken by a ray is depicted, reflecting from the surface each time according to geometrical optics.

vector in \mathbb{R}^3 , $p(\mathbf{x}, \xi)$ is said to be an escape function if it is real-valued, C^∞ , and, informally speaking, “strictly increasing along rays, ξ being the ray direction at \mathbf{x} .” Rays are said to be *not trapped* if the total path length in $S \setminus \Omega$ is bounded and waves are said to be *not trapped* if the local energy in $S \setminus \Omega$ decays to zero uniformly. Without entering into details, Morawetz, Ralston, and Strauss showed (1) that if rays are not trapped, then there exists an escape function and (2) that if there exists an escape function, then waves are not trapped, from which the result follows.

Geometric Optics and Frequency Domain Analysis

In the study of linear wave propagation, much of our understanding comes from the frequency domain — that is, analyzing the Fourier transform of the wave equation (11):

$$(16) \quad -k^2 U(\mathbf{x}, k) - \Delta U(\mathbf{x}, k) = F(\mathbf{x}, k).$$

Depending on the context, this is referred to as the Helmholtz or reduced wave equation. In 1968, Morawetz, together with Don Ludwig, began an investigation of exterior scattering from star-shaped surfaces in the frequency domain [5]. Two major results were presented there. First, they provided a key proof of the well-posedness of the scattering problem for sound-soft boundaries (homogeneous Dirichlet boundary conditions) with respect to the boundary data and forcing term $F(\mathbf{x}, k)$ in (16). They also introduced what are now called Morawetz identities for the Helmholtz equation. Second, they showed that the formulas produced by the theory of geometrical optics are

asymptotic to the exact solution. The relevant asymptotic regimes are illustrated in Figure 6.

Without entering into technical details, geometrical optics is based on expanding the incoming and scattered waves in terms of a series in inverse powers of the wavenumber k about the point \mathbf{x}_0 (see Figure 6). Ludwig had earlier proposed an expansion for the penumbra region as well. Morawetz and Ludwig showed that all of these expansions are truly asymptotic: to the solution in the illuminated region and penumbra, and asymptotically zero in the deep shadow.

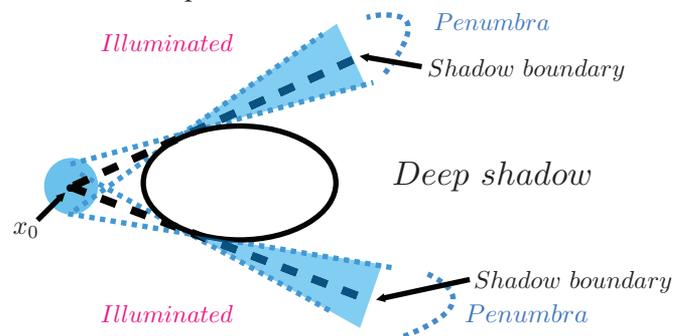


Figure 6. The asymptotic regimes for geometrical optics. For a fixed point \mathbf{x}_0 , the two tangent lines to the scatterer define the *shadow boundary* (dashed black lines), which separates the illuminated region from the shadow region. The *penumbra* is a neighborhood of the shadow boundary, formed by the union of all shadow boundaries of spherical waves with centers in a neighborhood of \mathbf{x}_0 . (Adapted from [5]).

Although Morawetz herself did little numerical computation, her analytic work (especially on multipliers) has played a major role in the design of numerical methods. We cannot do justice to the literature here, but refer the reader to three recent papers: one on eigenvalue computation, one on frequency domain scattering, and one on time-domain integral equations [1–3]. We have only been able to scratch the surface of her legacy in this note. Her contributions are profound and deep, and have changed the way we think about partial differential equations. She was a wonderful friend and colleague and is greatly missed.

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Kevin R. Payne

Transonic Flow and Mixed Type Partial Differential Equations

The work of Cathleen Morawetz on transonic fluid flow and the underlying PDEs of mixed elliptic-hyperbolic type spanned her career. Here we describe her earliest work. Beginning in the mid 1950s, Morawetz began working on transonic flow problems through her interactions with Kurt O. Friedrichs and Lipman Bers. This problem area was ripe for the unique blend of joyous ingenuity and practical tenacity which characterized her approach to happily doing mathematics in order to say something about a real world problem. Morawetz quickly made a name for herself by giving a mathematical answer to an important engineering question in transonic airfoil design.



Morawetz in 1958.

Morawetz periodically returned to this area with bursts of productivity that resulted in fundamental contributions over the next five decades. The photo of Morawetz in 1958 shows the happy face that Morawetz would display when

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discussing what interested her most. It was with the same gentle smile and glint in the eyes that she might also show her warm toughness and attachment to physical relevance when liquidating a night’s calculations of a collaborator with a phrase like: “You know, the solutions should not really behave this way. Let’s change the equation.”

What is Transonic Flow About?

In aerodynamics, a basic question is: *How does one fly at a relatively high speed, with relatively low cost and relatively low ecological damage?* In Morawetz’s 1982 article in the *Bulletin of the AMS*, she described the problem as follows. The science of flight depends on the relative speed of the aircraft with respect to the speed of sound in the surrounding air. At relatively low speeds, the *subsonic range*, one can “sail” by designing wings to “get as much as possible of a free ride” from the wind. At very high speeds, the *supersonic range*, one needs “rocket propulsion” to overcome the *drag* produced by *shocks* that invariably form (the sonic boom). The goal of studying transonic flow is to find a compromise which allows for “sailing” efficiently “near the speed of sound.” Shocks produce drag, which increases fuel consumption and hence increases cost. As seen in Figure 16, shocks (colored red) begin to appear on airfoils in wind tunnels when the upstream velocity is below, but near the speed of sound.

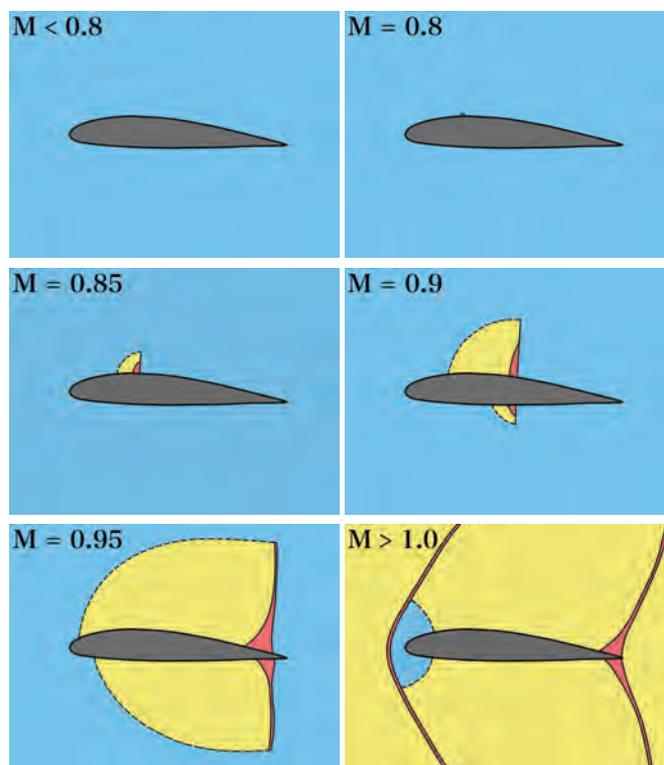


Figure 7. As wind tunnel speed increases from subsonic (blue regime, Mach $M < 1$) to supersonic (yellow regime, Mach $M > 1$), some supersonic shock (in red) appears over the wing already at Mach $M = .85$.

The 2-D irrotational, stationary, compressible and isentropic flow of air about a profile \mathcal{P} is governed by an equation for the potential $\phi(x, y)$ whose gradient is the velocity field of the fluid with variable density ρ :

$$(17) \quad (c^2 - \varphi_x^2) \varphi_{xx} - 2\varphi_x \varphi_y \varphi_{xy} + (c^2 - \varphi_y^2) \varphi_{yy} = 0.$$

The natural boundary condition is to have normal derivative

$$(18) \quad \frac{\partial \varphi}{\partial n} = 0 \text{ on } \partial \mathcal{P}.$$

The nature of the flow is determined by the local Mach number $M = q/c$ where $q = |\nabla \varphi|$ is the flow speed and $c > 0$ is the local speed of sound defined by $c^2 = \partial p / \partial \rho$, where the adiabatic pressure density relation in air is $p = p(\rho) \sim \rho^\gamma$ with $\gamma \approx 1.4$. Observe that equation (17) is of the form

$$A\varphi_{xx} - 2B\varphi_{xy} + C\varphi_{yy}.$$

It is *elliptic* when

$$AC - B^2 > 0,$$

which occurs at points where the flow is subsonic ($q < c$). It is *hyperbolic* when

$$AC - B^2 < 0,$$

which occurs at points where the flow is supersonic ($q > c$) (see Figure 7). A transonic flow happens when there are both sub- and supersonic regions and the equation (17) is of mixed elliptic-hyperbolic type.

The presence of shocks in supersonic regions corresponds to drastic changes in air density and pressure coming from the compressibility, and these large pressure changes propagate at supersonic speeds, resulting in a shock wave which typically has a small but finite thickness. In Figure 7, the shock wave region is depicted in red. The velocity field $\nabla \varphi$ governed by (17) will experience jump discontinuities as one crosses the shock wave. One can use the presence of such discontinuities to detect the presence of shocks. The mathematical description of shocks requires a separate analysis of entropy effects, where equation (17) has broken down.

The Transonic Controversy

By the time of the Third International Congress for Applied Mechanics in 1930, a lively debate centered around the question: *Do transonic flows about a given airfoil always, never, or sometimes produce shocks?* In particular, *is it possible to design a viable airfoil capable of shock-free flight at a range of transonic speeds?* Contrasting evidence was presented at the congress which led many aerodynamicists to take opposing views. G.I. Taylor presented convergent Rayleigh series expansions for the velocity potential of some smooth transonic flows, while A. Busemann presented the results of wind tunnel experiments that indicated the presence of a lot of shocks. World War II moved attention to rocket propulsion. An answer would await the work of Morawetz in the 1950s. It was a case of “mathematics coming to the rescue.”

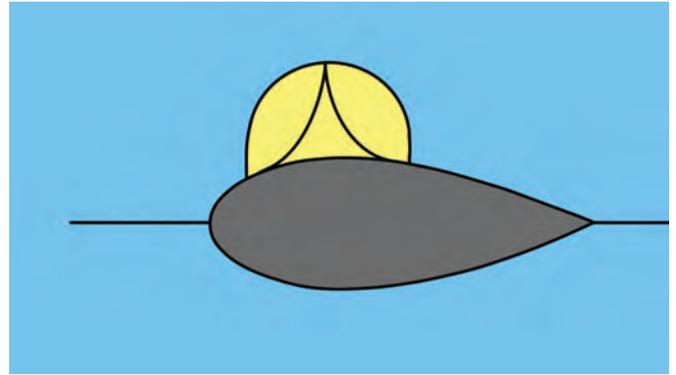


Figure 8. Morawetz’s theorem proved that any perturbation of the wing inside the yellow supersonic regime creates shocks.

Morawetz’s Answer to the Transonic Controversy

In a series of papers published in 1956–58 in *Comm. Pure Appl. Math.*, Morawetz gave a mathematical answer by proving that shock-free transonic flows are unstable with respect to arbitrarily small perturbations in the shape of the profile. Her theorem says that even if one can design a viable profile capable of a shock-free transonic flow, imperfection in its construction will result in the formation of shocks at the design speed.

Theorem. *Let φ be a transonic solution to (17)-(18) with continuous velocity field $\nabla \varphi$ and fixed speed q_∞ at infinity about a symmetric profile \mathcal{P} as in Figure 8. For an arbitrary perturbation $\tilde{\mathcal{P}}$ of \mathcal{P} along an arc inside the supersonic region attached to the profile which contains the point of maximum speed in the flow, there is NO continuous $\nabla \tilde{\varphi}$ solving the corresponding problem (17)-(18) with $\tilde{\mathcal{P}}$.*

Morawetz’s proof involved two major steps. First, she determined the correct boundary value problem satisfied the perturbation of the velocity potential in the *hodograph plane* where a *hodograph transformation* linearizes the PDE (17) and sends the known profile exterior into an unknown domain. Then, using carefully tailored integral identities, she proved a uniqueness theorem for regular solutions of the transformed PDE with data prescribed on only a proper subset of the transformed boundary profile, which says that the transformed problem is *overdetermined* and no regular solutions exist. Morawetz extended this result to include fixed profiles but finite perturbations in q_∞ , and the extension to non symmetric profiles was carried out by L. Pamela Cook (*Indiana Univ. Math. J.*, 1978).

Engineering Impact

While Morawetz’s work left open the theoretical possibility of a *perfect transonic airfoil* capable of shock-free flight over a small range of transonic speeds, imperfection in its construction means the search for it is futile. Instead engineers must calibrate wing design to *minimize shock strength* over a useful range of transonic speeds.

Beginning in the early 1960s with the work of H.H. Pearcy and later R.R. Whitcomb on *supercritical airfoils*, transonic airfoil design paid close attention to the impact of Morawetz's findings. In the midst of the energy crisis of the 1970s, this direction of research exploded as part of the field of *computational fluid dynamics*. The *type-dependent difference scheme* of E.M. Murman and J.D. Cole (1971), the *complex characteristic method* of P. Garabedian and D. Korn (1971), and the *rotated difference scheme* of A. Jameson (1974) were some of the milestones in the economically viable calculation of steady transonic flows and codes for transonic airfoil design.



Morawetz with Paul Garabedian, whose complex characteristic method with D. Korn applied Morawetz's work to computational fluid dynamics.

Mathematical Impact

Cathleen Morawetz's early work on transonic flow both transformed the field of mixed type partial differential equations and served as excellent publicity for mathematics. Commenting on the transonic controversy in 1955, the celebrated aerodynamicist Theodore von Kármán observed: "... the mathematician may exactly prove existence and uniqueness of solutions in cases where the answer is evident to the physicist or engineer... On the other hand, if there is really serious doubt about the answer, the mathematician is of little help." Morawetz's surprising theorem on the nonexistence of smooth flows was a cheerful response to von Kármán's well-intentioned challenge.

Having settled the engineering question about the "exceptional nature" of shock-free transonic flows, Morawetz turned to related questions: *Can one prove robust existence theorems for weak shock solutions? Can one "contract" a weak shock to a sonic point on the profile?* The first question was supported by work of Garabedian-Korn in 1971, which demonstrated that small perturbations of continuous flows can have only weak shocks. The second question was inspired by the thinking of K.G. Guderley in the 1950s. Morawetz took two very different approaches to such questions.

Taking a singular perturbation with a hodograph transformation, the questions reduce to proving the existence of weak solutions to the Dirichlet problem for *linear* mixed type equations on domains Ω in the hodograph plane:

$$(19) \quad K(\sigma)\psi_{\theta\theta} + \psi_{\sigma\sigma} = f \quad \text{in } \Omega$$

$$(20) \quad \text{and } \psi = 0 \quad \text{on } \partial\Omega,$$

where $K(\sigma) \sim \sigma$ as $\sigma \rightarrow 0$. Here ψ is the stream function of the flow, σ is a logarithmic rescaling of the flow speed which is sonic at $\sigma = 0$, and θ is the flow angle. For special domains, Morawetz [*Comm. Pure Appl. Math.* 1970] proved the surprising result of the existence of a unique weak solution to the problem.

Inspired by the differencing method of Jameson, Morawetz introduced an *artificial viscosity* parameter ν into the *nonlinear* potential equation by replacing the (inviscid) Bernoulli law

$$\rho = \rho_B(|\nabla\varphi|)$$

with a first order PDE which retards the density ρ . An ambitious program ensued in order to prove the existence of weak solutions to the inviscid problem as a weak limit of viscous solutions. Powerful but delicate tools in the application of the *compensated compactness method* of F. Murat, L. Tartar, and R. Di Perna were applied with success to complete parts of the program in Morawetz [*Comm. Pure Appl. Math.* 1985, 1991] and Gamba-Morawetz [*Comm. Pure Appl. Math.* 1996].

The Legacy of Cathleen Morawetz



Cathleen Morawetz with her family in 1958 when she solved the transonic controversy.

During the period 1952–2007, Morawetz produced 22 deep research papers and 10 survey papers on transonic flow and mixed type partial differential equations. She was an exemplary figure of the applied mathematician "who proves theorems to solve problems." Morawetz discovered and implemented a wide variety of tools to handle the complexity of mixed type PDEs. She developed energy methods and important identities by the skillful and ingenious use of multiplier methods championed by K. Friedrichs [*Comm. Pure Appl. Math.* 1958] and found surprising maximum principles which were calibrated to invariances in the equation.

The legacy of Cathleen Morawetz includes her dedication to the proposition that "there is no such thing as

a distant relative,” which she applied to every part of her well-lived life. Her grace, warmth and generosity to generations of mathematicians working in the area will be long remembered. She was a truly inspirational figure who invited us all to “sail with her, near the speed of sound.”²

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Closing Thoughts



Cathleen Syngé Morawetz with Richard Courant.

We close this article with a quote by Cathleen Syngé Morawetz. Upon receiving the Birkhoff Prize in 2006, she said:

There are many, many people whom I would have liked to thank for helping me over the years, but I would not have room for their names on this

²Quotes in this paragraph are from Kevin Payne’s talk “Cathleen’s mathematics: transonic flow” and Nancy Morawetz’s talk, respectively, at the Courant Institute “Celebration in Honor of Cathleen Syngé Morawetz” on November 17, 2017.

page. But one person stands out for supporting and encouraging me when I was between the crucial professional ages of twenty-three and thirty-five. I worked part-time on my PhD, part-time as a postdoc, and I had four children. That person was Richard Courant, the creator of the Courant Institute at New York University, where I have been a professor ever since.

It is truly rare for any department to support a woman’s career in this way: with part-time research-associate positions and a long term commitment that does not require the woman to relocate every few years to eventually obtain a tenure track position. Many women leave academia after completing their doctorates, switching to jobs in industry, while others land in teaching positions and never have the opportunity to develop a research career. It is a great loss of talent. Imagine a world in which Morawetz had never developed her paramount results on transonic flow models, functional inequalities and scattering theory. Imagine a world in which more women’s research were supported as well as hers was. It would be a better place.

ACKNOWLEDGMENTS. We would like to thank Cathleen Morawetz’s son, John, and also the head of the Courant Institute Library, Carol Hutchins, for finding many wonderful photos and researching their dates and locations. We regret that we could only include a few here. We would also like to thank Penelope Chang for volunteering as an artist for the *AMS Notices*.

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Irene Martinez Gamba

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Kevin R. Payne is an AMS Fellow. His research centers on the use of energy methods and maximum principles for PDE of mixed elliptic-hyperbolic and degenerate types. He will be forever thankful to Cathleen Morawetz for getting him interested in such problems during a post-doc at the Courant Institute and for the honor of a happy collaboration over many years.

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Tonatiuh Sánchez-Vizuet

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Christina Sormani

Terence Tao is a Fields Medalist and a MacArthur Fellow. He has completed groundbreaking research in harmonic analysis, partial differential equations, algebraic combinatorics, arithmetic combinatorics, geometric combinatorics, compressed sensing, and analytic number theory.



Terence Tao

Call for Applications

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ICM 2018 LECTURE SAMPLER



From left to right: Gil Kalai, Sanjeev Arora, Catherine Goldstein, Sylvia Serfaty, Gregory F. Lawler.

In this sampler, the speakers below have kindly provided introductions to their Plenary Lectures for the International Congress of Mathematicians 2018 (ICM 2018) taking place August 1–9 in Rio de Janeiro.

ICM 2018

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Gil Kalai

Three Puzzles on Mathematics, Computation, and Games

The theory of computing and computer science as a whole are precious resources for mathematicians.

They bring new questions, new profound ideas, and new perspectives on classical mathematical objects, and serve as new areas for applications of mathematics and of mathematical reasoning.

In my lecture I will talk about three mathematical puzzles involving mathematics and computation (and, at times, other fields) that have preoccupied me over the years.

Puzzle 1: What is the Explanation for the Success of the Simplex Algorithm?

Linear programming is the problem of maximizing a linear function ϕ subject to a system of linear inequalities. The set of solutions for the linear inequalities is a convex polytope P (which can be unbounded). The simplex algorithm was developed by George Danzig. Geometrically it can be described by moving from one vertex to a neighboring vertex so as to improve the value of the objective function.

The simplex algorithm is one of the most successful mathematical algorithms. The explanation of this success is an applied, vaguely stated question, which is connected with computers. The problem has strong relations to the study of convex polytopes, which fascinated mathematicians from ancient times, and which served as a starting point for my own research.

Two important elements in studying the mathematics of linear programming are abstractions and reductions. Various abstract forms of linear programming, convex polytopes and other objects play an important role, and there are very clever and important reductions from one set of problems and objects to others.

If I needed to choose the single most important mathematical explanation for the success of the simplex algorithm, my choice would point to a theorem about another algorithm. I would choose Khachiyan's 1979 theorem asserting that there is a polynomial-time algorithm for linear programming. (Or briefly $LP \in P$.) Khachiyan's theorem refers to the ellipsoid method, and the answer is given in the language of computational complexity, a language that was not at all available when the question was originally raised. A more recent explanation is the

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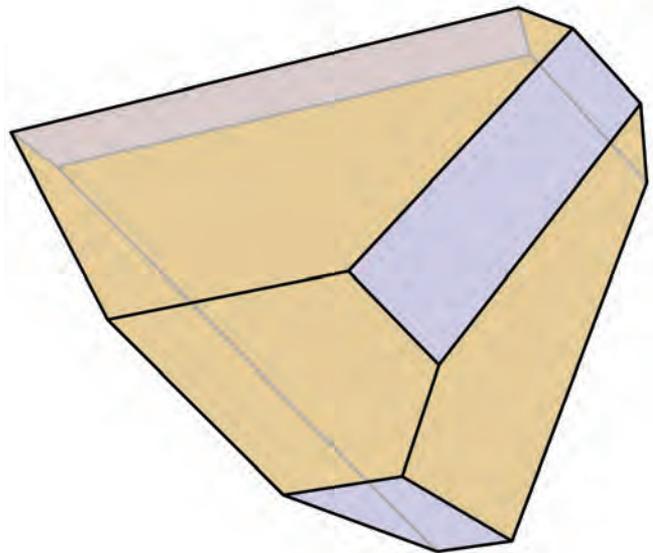


Figure 1. The associahedron (Stasheff polytope).

work of Spielman and Teng asserting that a certain pivot rule requires an expected polynomial number of steps for a random perturbation of every linear programming problem.

In the lecture I will concentrate on the study of diameter of graphs of polytopes and the discovery of randomized subexponential variants of the simplex algorithm, and mention recent advances: The disproof of the Hirsch conjecture by Santos and the connection between linear programming and stochastic games leading to subexponential lower bounds proved by Friedman, Hansen, and Zwick, for certain randomized pivot rules for the simplex algorithm.

Puzzle 2: What are Methods of Election that are Immune to Errors in the Counting of Votes?

The second puzzle can be seen in the context of understanding and planning of electoral methods. We all remember the sight of vote recounts in Florida in the 2000 US presidential election.

Is the American electoral system, based on electoral votes, inherently more susceptible to mistakes than the majority system? And what is the most stable method? Together with Itai Benjamini and Oded Schramm I investigated these and similar problems. We asked the following question: given that there are two candidates and each voter chooses at random and with equal probability (independently) between them, what is the stability of the outcome, when in the vote-counting process one percent of the votes is counted incorrectly? The mathematical jargon for these errors is “noise.” We defined a measure



Figure 2. The US 2000 presidential elections raised questions about whether the Electoral College system is more susceptible to mistakes than popular vote.

of noise sensitivity of electoral methods and found that weighted majority methods are immune to noise: when the probability of error is small, the chances that the elections' outcome will be affected diminish. We also showed that every stable to noise method is close in some mathematical sense to a weighted majority method. In later work, O'Donnell, Oleszkiewicz, and Mossel showed that the majority system is most stable to noise among all non-dictatorial methods.

Our work was published in 1999, a year before the question appeared in the headlines in the US presidential election, and it did not even deal with the subject of elections.¹ We were interested in understanding the problem of planar percolation, a mathematical model derived from statistical physics. In our article we showed that if we adopt an electoral system based on the model of percolation, this method will be very sensitive to noise. This insight is of no use at all in planning good electoral methods, but it makes it possible to understand interesting phenomena in the study of percolation.

After the US presidential election in 2000 we tried to understand the relevance of our model and the concepts of stability and noise in real-life elections: is the measure for noise stability that we proposed relevant, even though

¹Here one can witness a secret of our trade. Mathematicians recycle their models, and the same model can be used for very different purposes.

the basic assumption that each voter randomly votes with equal probability for one of the candidates is far from realistic? The attempt to link mathematical models to questions about elections (and, more generally, to social science) is fascinating and complicated, and a true pioneer in this study was the Marquis de Condorcet, a mathematician and philosopher, a democrat, a human rights advocate, and a feminist who lived in France in the eighteenth century. One of Condorcet's findings, often referred to as Condorcet's paradox, is that when there are three candidates, the majority rule can sometimes lead to cyclic outcomes, and it turns out that the probability for cyclic outcomes depends on the stability to noise of the voting system. We will discuss the connection of noise stability to computational complexity and mainly to the important theory of hardness of approximations and PCP.

Puzzle 3: Are Quantum Computers Possible?²

A quantum computer is a hypothetical physical device that exploits quantum phenomena such as interference and entanglement in order to enhance computing power. The study of quantum computation combines fascinating physics, mathematics, and computer science. In the 1990s, Peter Shor discovered that quantum computers would make it possible to perform certain computational tasks hundreds of orders of magnitude faster than ordinary computers and, in particular, would break most of today's encryption methods. At that time, the first doubts about the model were raised: quantum systems are of a noisy and unstable nature. Peter Shor himself found a key to a possible solution to the problem of noise: quantum error-correcting codes and quantum fault-tolerance.

In the mid-1990s, three groups of researchers studied the noisy quantum computer model and showed that noisy quantum computers still make it possible to perform all miracles of universal quantum computing, as long as engineers succeeded in lowering the noise level below a certain threshold.

One possibility, which reflects widespread opinion, is that the construction of quantum computers is possible, that the remaining challenge is essentially of an engineering nature, and that such computers will be built in the coming decades. Moreover, people expect to build in the next few years quantum codes of the quality required for quantum fault-tolerance, and to demonstrate the concept of "quantum computational supremacy" on quantum computers with fifty qubits. A second possibility, which expresses my position, is that it will not be possible to construct quantum codes that are required for quantum computation, nor will it be possible to demonstrate quantum computational superiority in other quantum systems. Let me explain why.

My analysis is based on the same model of noise that led researchers in the 1990s to optimism about quantum computation, and it points to the need for different analyses

²See Kalai's article on "The Quantum Computer Puzzle" in the May 2016 Notices.

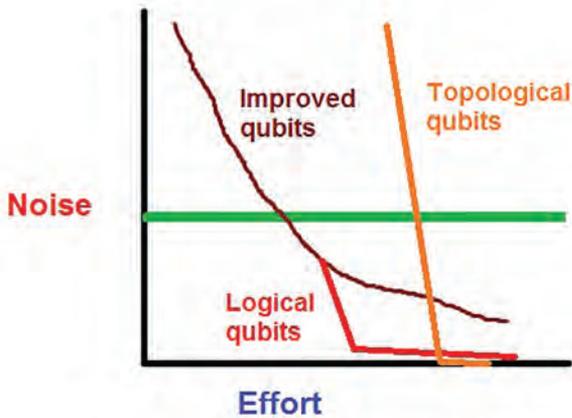


Figure 3. It is commonly believed that by putting more effort into creating qubits the noise level can be pushed down to as close to zero as we want. Once the noise level is small enough and crosses the green threshold line, quantum error correction allows logical qubits to reduce the noise even further with a small amount of additional effort. Very high quality topological qubits are also expected.

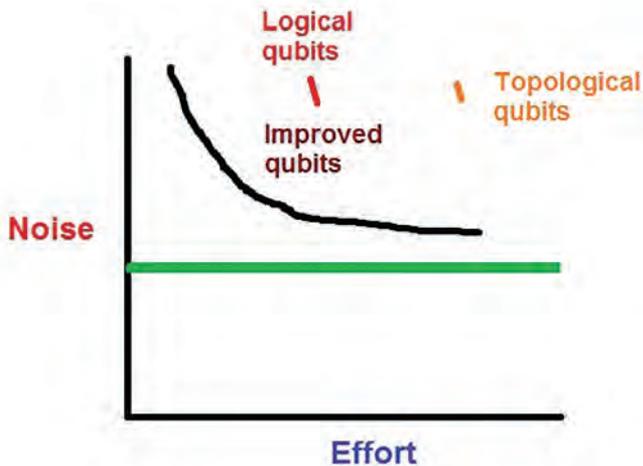


Figure 4. My analysis gives good reasons to expect that we will not be able to reach the green line and that all attempts for good quality logical and topological qubits will fail.

on different scales. My analysis shows that noisy quantum computers on the small scale (a few dozen qubits) express such a primitive computational power that it will not allow the creation of quantum codes that are required as building blocks for quantum computers on a higher scale. A reader may ask: how does the noisy quantum world allow classical information and classical computation? The difference between classical and quantum

The connection and tension between the pure and the applied, between models and reality, and between foundations and engineering.

information is related to puzzle 2. Encoding using repetitions and decoding using the majority rule (or related techniques) enable classical information and calculation: the majority method corrects errors and is stable to noise, and therefore can be implemented in a very primitive computational system. This enables, on larger scales, classical information and computation.

Conclusion

The lecture deals with three fascinating puzzles on mathematics and computation telling a story of pure and applied mathematics, theoretical computer science, physics and social sciences, as well as games of various kinds.

All our three puzzles involve the study of beautiful underlying mathematical objects, and deep mathematical methods, and also demonstrate the connection and tension between the pure and the applied, between models and reality, and between foundations and engineering.

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Gil Kalai

ABOUT THE AUTHOR

Many members of Gil Kalai's family are artists, and sometimes he thinks about mathematics as a form of art.

Sanjeev Arora

Toward Mathematical Understanding of Deep Learning

Deep learning, the modern name for learning using neural networks, has in recent years become the dominant paradigm of machine learning. Performance on tasks such as image recognition and game-playing now exceeds human performance.

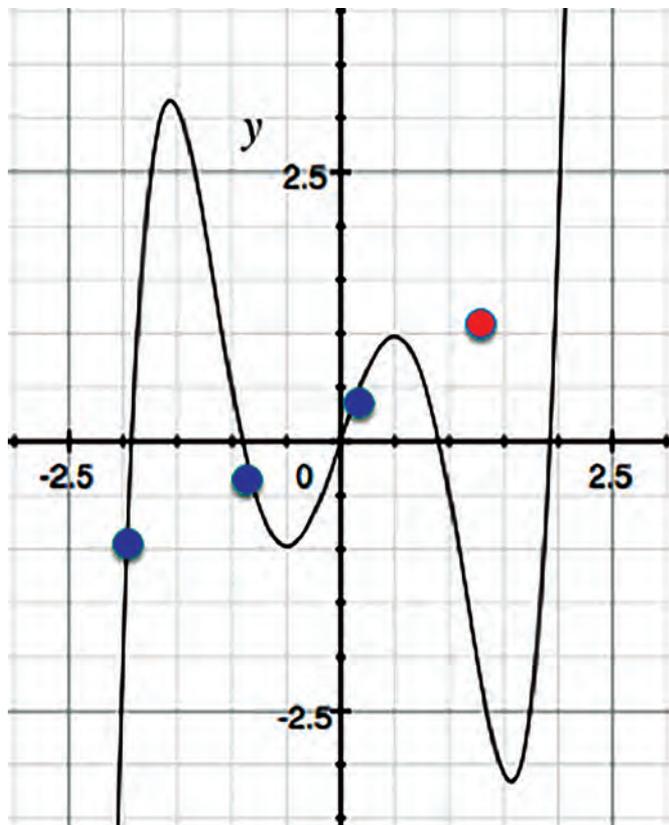


Figure 1. If one tries to fit a degree 5 polynomial to three blue data points, then there is no reason such a fitted polynomial will predict the value of some previously unseen fourth red point. This classic intuition suggests that deep nets with too many parameters would overfit to the data, but in practice they do not.

At a basic level deep learning—like many other machine learning algorithms—can be seen as fitting a model to a data. This is quite analogous to classical statistical techniques such as regression, but with far more trainable parameters and more complex

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optimization. For example, vision tasks may involve models with tens of millions of parameters and nonconvex optimization objective.

Mathematical understanding of this field is still in its infancy, and my talk will be a survey of the nascent efforts to develop such understanding. Some of the new mathematical results concern properties of optimization algorithms: when/how do they find reasonable solutions? Other results concern generalization: why do the trained nets perform well on unseen data? This is a nontrivial question because usually the net has far more parameters than the number of training examples. Remember the overfitting issue in modeling, as in Figure 1. If one tries to fit a degree 5 polynomial to three data points, then there is no reason such a fitted polynomial will predict the value of some previously unseen fourth point. Thus clearly the optimization algorithm somehow implicitly cuts down the effective number of parameters in the net, but it is unclear how this happens. Recent results begin to cast some light on this mystery.

Next, there is the question of what kinds of functionalities can or cannot be expressed by deep nets of a certain size and depth. This includes the ability of deep nets to model interesting distributions of images, using ideas such as dueling deep nets (GANs).

The talk will be largely self-contained. I have also prepared a short article for the ICM proceedings, which surveys the main conceptual frameworks of machine learning. I will also put that article on arXiv and on my webpage.

Image Credits

Figure 1 by Editors.

Author photo courtesy of Sanjeev Arora.

ABOUT THE AUTHOR

Sanjeev Arora has lived in Princeton, New Jersey, for twenty-four years. His interests outside work include listening to classical music and jazz, cooking, travel, and photography.



Sanjeev Arora

Catherine Goldstein

On the History of Mathematical Concepts

A key issue in my field, history of mathematics, is to describe and understand the development of mathematics. The focus might be on the shaping of concepts, of results, of applications, but also on ways and practices of thinking.

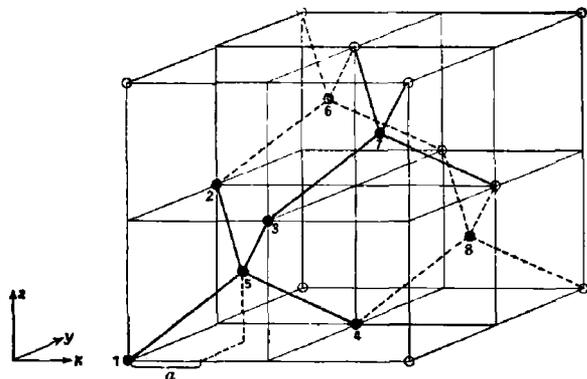


Fig. 1.

Figure 1. Hermitian forms appear in a 1914 paper by Max Born on the elasticity theory of the diamond, long before their entry with the same person into quantum theory.

One main advance of the last decades is a much more acute awareness of the richness of mathematical activities and of the various layers of these activities. In a mathematical result many aspects interfere. Who were the mathematicians involved, perhaps academic mathematicians, or engineers, or scribes, or lawyers, or even amateurs? What was their training? What were their sources? What were their mathematical priorities, whether specific applications or effective results or englobing conceptualizations? What were their arguments and proofs? What were their ways of transmitting their findings? Recent focus has been on collective issues such as the history of mathematical journals or societies, but also on the ways figures and formulas intervene in mathematical texts and how they help or hamper the creation and diffusion of mathematical ideas. Such a focus favors interactions between historians of mathematics and general historians, sociologists, linguists, and other specialists of the human sciences. The understanding of the cultural variety encapsulated in a single mathematical text or concept has just begun to be explored.

On the other hand, historians of mathematics have been much more reflexive about our own terminology and arguments. What do we mean precisely by such things as a “discipline” or a “mathematical school” or even the “reception” of a result or

domain? This is particularly important if we want to write a long-term history of the development of a concept or a theorem or a method, using our new understanding of how a piece of mathematics, born in a particular environment, has been partially taken over, partially rediscovered, or contextualized afresh. The textual, technical, cultural, social, economical, and logical strata all have and their different scales and rhythms of change to be taken into account if we want to understand how mathematical constructs evolve.

In my ICM talk, I intend to explain and illustrate this concretely, using the simple, but historically intriguing example of Hermitian forms, arising from the classical arithmetical question concerning the decomposition of any integer as a sum of four squares and becoming within a few decades a common tool in geometry, algebra, and physics.

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Catherine Goldstein

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Catherine Goldstein is particularly interested in the history of number theory, in women in mathematics, and in the role of observation for mathematicians. She has recently coedited a book on mathematics and WWI (*A War of Guns and Mathematics*, AMS, 2014) and another on the history of combinatorics and Delannoy numbers (*Les travaux combinatoires en France (1870–1914) et leur actualité*, PULIM, 2017).

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Sylvia Serfaty

Systems with Coulomb Interaction

What do vortices in superconductors, eigenvalues of random matrices, and optimal interpolation points on spheres have in common? They all consist of points that repel one another via a logarithmic interaction, i.e. have an energy of the form

$$(0.1) \quad \mathcal{H}_N(x_1, \dots, x_N) = \sum_{1 \leq i \neq j \leq N} g(x_i - x_j),$$

where $g(x) = -\log|x|$. It is natural to generalize this to the Coulomb interaction $g(x) = |x|^{2-d}$ in dimension $d \geq 3$ and to add a possible confinement energy $\sum_{i=1}^N V(x_i)$, where V grows sufficiently fast at infinity. One is interested in understanding the large N behavior of such systems. This can be in the setting of minimizers of such energies, of the time-evolutions of such systems, but also of states with temperature. When the temperature is $\frac{1}{\beta}$, the probability density of observing the system in the configuration (x_1, \dots, x_N) is given by the Gibbs measure

$$(0.2) \quad d_{\{N,\beta\}}(x_1, \dots, x_N) = \frac{1}{Z_{N,\beta}} e^{-\beta \mathcal{H}_N(x_1, \dots, x_N)} dx_1 \dots dx_N,$$

where $Z_{N,\beta}$ is a normalizing constant to make the total probability 1. Motivation comes for instance from statistical mechanics, random matrix theory, quantum mechanics, condensed matter physics, fluid mechanics, approximation theory, and even geometry.

Simulations of realizations of (0.2) for the logarithmic interaction in two dimensions with a quadratic confining potential are shown in Figure 1.

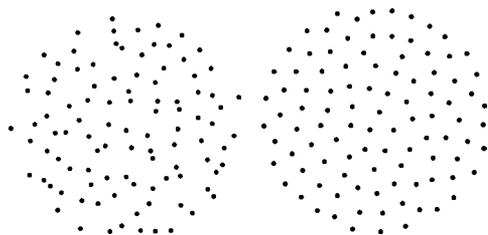


Figure 1. Configurations of $N = 100$ points tend to be less ordered at higher temperatures ($1/\beta = 1/5$ on the left) than at lower temperatures ($1/\beta = 1/400$ on the right). Simulation with $V(x) = |x|^2$ by Thomas Leblé.

They seem to indicate that the macroscopic distribution of the particles does not depend much on the temperature, while the microscopic patterns strongly depend on it. On the left, the temperature is larger and the system seems quite disordered, while on the right, the temperature is much smaller and the system seems more ordered at the microscopic level. Can this be proven rigorously? Is there a phase transition happening between a “liquid state” and a “solid state”? Is there a limiting pattern that the

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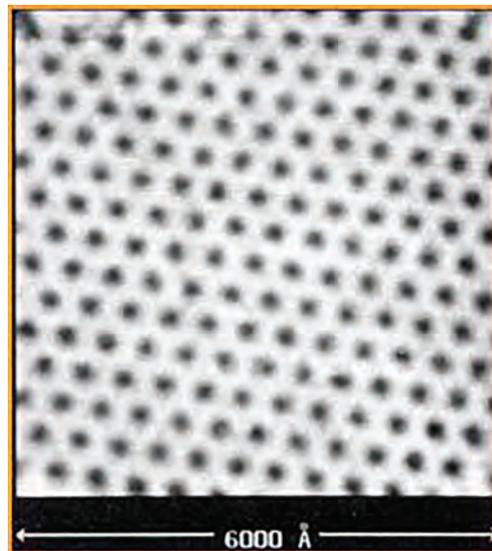


Figure 2. Vortices in superconductors form perfect triangular Abrikosov lattices, which are also optimal packings.

points want to form as temperature goes to 0? Is that the same as the patterns that energy minimizers would form?

A hint is provided by the behavior of vortices in superconductors, which can be directly connected to minimizers of (0.1): they form perfect triangular lattices, called Abrikosov lattices in physics, as in Figure 2. How can one mathematically explain the formation of these triangular lattices? The triangular lattice is also the solution of the best-packing problem in two dimensions and of other minimization questions related to modular functions. A 2007 conjecture of H. Cohn and A. Kumar³ says that the triangular lattice should be *universally minimizing* in the sense that it should minimize all interactions with kernels which are *completely monotone* (i.e. satisfy $(-1)^k g^{(k)} \geq 0$), indicating that this is part of a broader phenomenon. One can also wonder about higher dimensions: what would play the analogous role of the triangular lattice? Many of these questions are actually still open, but certain qualitative properties of these large N limit configurations can be rigorously established.

³H. Cohn, A. Kumar, *Universally optimal distribution of points on spheres*. J. Amer. Math. Soc. **20** (2007), no. 1, 99-148, Conjecture 9.4.

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ABOUT THE AUTHOR

Sylvia Serfaty is interested in developing analysis tools to understand problems from physics. She has extensively studied the Ginzburg–Landau model of superconductivity and this has led her to questions on the statistical mechanics of Coulomb-type systems. She has been splitting her career between Paris and New York.



Sylvia Serfaty

Gregory F. Lawler

Loops and Loop-Erased Random Walk

Random fractal structures arise in statistical mechanical models viewed at or near a phase transition. A challenge for mathematics is to describe the random curves $\gamma : [0, \infty) \rightarrow \mathbb{R}^d$ that arise in the continuum limit. Usual calculus defines smooth curves in terms of derivatives. At the other extreme, “(completely) random continuous” motion is modeled by random walk or Brownian motion, as in Figure 1. One can combine completely random and smooth motions using stochastic calculus. The curves we are studying are more complicated than these because of their strong interaction with the past. For example, many of the curves are forbidden to visit any place that has already been visited.

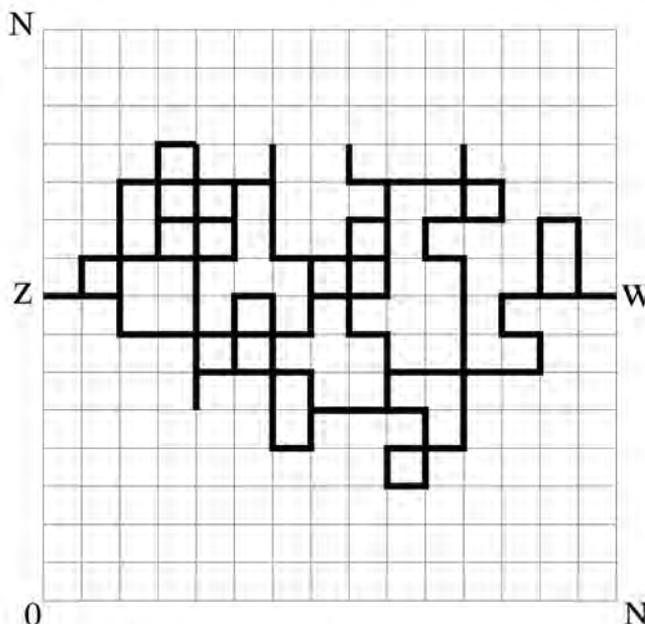


Figure 1. A random walk from z to w in the square typically has self-intersections and loops.

To try to define such curves, one often discretizes by giving a precise definition on each discrete scale with the hope of being able to take a limit. For curves with self repulsion, the most interesting dimensions are $d = 2, 3$. For $d = 1$ “self-avoidance” means that curves can only go in a single direction, and for $d \geq 4$ it turns out that completely random paths do not want to have self-intersections. Roughly speaking, this is because the path of a random walk has “fractal dimension two.” This follows from the central limit theorem, the most fundamental fact in probability.

There have been many major advances in the last twenty years towards the rigorous understanding of two-dimensional critical

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The AMS Graduate Student Blog

Talk that matters to mathematicians.

From **"Things You Should Do Before Your Last Year"** ...

Write stuff up. Write up background, write down little ideas and bits of progress you make. It's difficult to imagine that these trivial, inconsequential bits will make it to your dissertation. But recreating a week's/month's worth of ideas is way more time-consuming than just writing them down now. Or better yet, TeX it up.



From **"The Glory of Starting Over"** ...

What I would recommend is not being too narrowly focused, but finding a few things that really interest you and develop different skillsets. Make sure you can do some things that are abstract, but also quantitative/programming oriented things, because this shows that you can attack a problem from multiple angles. In my experience, these two sides also serve as nice vacations from each other, which can be important when you start to work hard on research.



From **"Student Seminar"** ...

A talk can be too short if not enough material is introduced to make it interesting, but in research level talks, the last third of the talk (approximately) is usually very technical and usually only accessible to experts in the field. I will avoid going into details that are not of general interest and I plan to present more ideas than theorems. The most important thing when giving any talk is to know your audience.





Rochelle Gutiérrez Interview

Conducted by Melinda Lanius and Simone Sisneros-Thiry

Communicated by Alexander Diaz-Lopez



Rochelle Gutiérrez is professor in the Department of Curriculum and Instruction at University of Illinois at Urbana-Champaign. Rochelle's research focuses on equity issues in mathematics education. She has published more than 20 journal articles, has co-authored more than 15 books, and in 2018 has given more than 10 talks related to the topic "Rehumanizing mathematics."

Lanius & Sisneros-Thiry: *When and how did you know you wanted to study mathematics education?*

Gutiérrez: In my senior year of college, I was a biology major, had already taken my MCATs, was working at Stanford Hospital, and applying to medical schools. I was focused and knew exactly where I was going. But, at Stanford, they want you to be well rounded, so they make you take all of these courses in areas you might not normally study, such as ethics, society, and art. For every one of those mandatory areas outside of science, I found ways to make them work for my very narrow-minded view of only wanting things that would help me become a better doctor. For example, for the course in ethics, I took bioethics. But in order to graduate I still needed one last course to fulfill the art requirement. My advisor had heard of a good professor in art education, Elliot Eisner, who offered a course on the Artistic Development of the Child. I wanted to become a pediatrician and thought maybe I could have my patients draw their pain. While I was in that course, I remember fighting a lot with the professor. He was world-renowned and one of the founders of the field, so other students just looked at me wondering how I could argue with someone so esteemed. But there were views discussed that didn't map onto my experiences or my family's, and I wasn't worried about getting a bad grade because I was planning to attend medical school and was really just checking off a box to graduate. After many arguments and many office hours, that professor and I became close. He respected that I stood my ground on things, especially providing views that other students hadn't heard (at the time Stanford was three percent students of color). Toward the end of the course, he said to me, "You know, Rochelle, you could become a great doctor. You are analytic, meticulous, and passionate. But, why not try something really difficult...like education?"

It should have been obvious. I had always loved puzzles, logic problems, and finding patterns all around me, especially in nature. In my free time, I was tutoring other kids in mathematics, working in programs like Mathematics Engineering Science Achievement (similar to Upward

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THE GRADUATE STUDENT SECTION

Bound) and Migrant Education at Stanford. After that conversation with Eisner, I worked in the José Valdés Summer Math Institute. But the message we receive in society about what's important or requires real intelligence to succeed isn't generally about education. Here was a professor challenging me to rethink what I was passionate about and to apply my expertise there. Many people I meet often wonder, well, then, why not go into *science* education if you already had a degree in biology? For me, it came down to understanding the power of mathematics as a gatekeeper in society, seeing that you cannot even enter the field of science without doing well in mathematics. So, that became one of my life goals—to change both how we think of education and how mathematics operates in society.

Lanius & Sisneros-Thiry: *How would you describe your work to a mathematics graduate student?*

Gutiérrez: My work requires becoming multilingual across fields so I can speak to lots of people. I often have one foot in one discipline, such as biology or education, and another foot in a different discipline, such as critical ethnic studies or mathematics. It is similar to mathematicians who do work in number theory and then ask how that might relate to topology; now just imagine that you expand that to a discipline outside of mathematics. My work requires staying abreast of literature in many rapidly changing fields.

In addressing problems in mathematics education, I'm always looking for new patterns, new ways of putting things together. Finding new angles on a problem often requires different tools. Often those tools don't yet exist, and I need to develop them by pulling things from different disciplines. The work I do is like weaving a tapestry. Each thread has a purpose and provides structure for the others.

For over 20 years, I've been committed to addressing equity, especially as it relates to issues of identity and power for people of color, women, and people living in areas of poverty. I'm not just interested in getting more diverse peoples to enter mathematics and do well in it, to "play the game," but to think about how such people will ask different questions, will have different goals, and will ultimately change mathematics and how we practice it, will "change the game." Lately, that work has shifted to what I call *rehumanizing mathematics*, which aims to capture more of the connections with emotions, the body, and our relationships with each other on this planet. In some ways, it's simply an expansion of the four dimensions of equity that I have written about before: access, achievement, identity, and power. However, while those four dimensions tended to relate exclusively to students and teachers, rehumanizing mathematics also takes up how everyday citizens think about, practice, and are affected by mathematics.

Lanius & Sisneros-Thiry: *What was your career trajectory? How did you end up working in the College of Education at the University of Illinois at Urbana-Champaign?*

Gutiérrez: When I graduated from the University of Chicago, I had several job offers. The University of Illinois was the most prestigious in terms of research. So, I figured

I'd start there and quickly move back to California as soon as I could. I grew up in the Bay Area, and as a Chicana my body needs sun and fresh fruit. I never thought I'd stay at the University of Illinois. I've had a number of job offers at excellent institutions over the years, but I keep coming back to the fact that the University of Illinois is set up in ways—working groups, funding, campus culture—that support interdisciplinary work.

Lanius & Sisneros-Thiry: *Do you teach? What type of courses do you teach?*

Gutiérrez: First, I LOVE to teach and feel so lucky that it is a part of my job!! There is no better feeling than helping someone come to that Aha moment of figuring out something that was previously puzzling, making a new connection, or rethinking something they *thought* they understood or believed to be true and grappling with that uncertainty. Teaching allows me to stay young because it's always a conversation between you and your students. I plan my course, but it's just a plan. There's always part of that plan that gets changed along the way—new things I need to learn, new sensibilities I need to develop, new material that changes how we think about our field, new things my students will teach me. At the undergraduate level, for the past 22 years, I have been teaching mathematics methods courses for students who are mathematics majors and who want to become high school teachers. A few years ago, I developed a new course called Social Justice, Schooling, and Society that has a pretty large enrollment (about 200 students) because it fulfills the intensive writing requirement. The course enrolls students from majors ranging from education to urban and regional planning to mechanical engineering. It makes me smile to think that I now teach a course that may change someone's view about education just as Eisner did for me! My graduate seminars range in topics from urban education to mathematics, science, and engineering to sociopolitical perspectives in mathematics and science education. In that latter course, I have doctoral students in mathematics, physics, biology, and mathematics and science education, along with practicing teachers in the local area. We apply my *abuela* (grandmother) rule, which means that when you talk, you need to use language that your grandmother would understand. Students create social justice projects that have immediate impact, such as new colloquia or working groups in their home departments, summer camps for girls, or advanced mathematics courses for locally incarcerated men.

Lanius & Sisneros-Thiry: *Do you have a professional support network? How did you make connections with a support system or community?*

Gutiérrez: My professional network includes faculty in Latina/Latino Studies, mathematics, anthropology, and other departments on campus. I also find that my interactions with graduate students serve as an amazing support system. They always push me to think about things differently and to read broadly to keep up with their areas of research.

I've recently become involved in Science for the People, a professional group of scientists that began in the 1970s

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ematics as norm by which we measure people in society. However, that message got twisted through social media and I was reported to have claimed, “Teaching mathematics is racist.” It just goes to show what a crazy moment we are in with social media right now. Many mathematicians and scholars in mathematics education supported me as I navigated the hate mail and trolls on Twitter. You can read a bit about it at ed-osprey.gsu.edu/ojs/index.php/JUME/ and see some of the timeline and how it relates to other mathematicians and mathematics education scholars who have come under attack, as well as find some resources for supporting colleagues under attack at <https://mathedcollective.wordpress.com/>. Although the attack was meant to divide us, I am pleased that we came together and created alliances and structures for supporting each other and launching some important conversations about our future.

Lanius & Sisneros-Thiry: *What advice do you have for graduate students?*

Gutiérrez: Your advisor, in some ways, will become your parent in the sense that this person will model for you how to move through the world as a professional; in a sense, they will “raise” you. Although not all children end up becoming their parents, most do not end up falling far from the tree. So, when looking for an advisor, ask yourself if you can imagine yourself becoming like this person? How does this person interact with students? How does this person interact with their network? What kinds of jobs do this advisor’s students obtain upon graduating? What is the relationship of the advisor to their students after they graduate?

I always tell my graduate students that the best dissertation is one that only you could write. You need to bring your full self to the dissertation process. It is extremely difficult to contribute something new to the field if you are trying to be someone other than yourself. I would avoid writing a thesis that you think your advisor wants you to write or solving a problem that is meaningless to you but your advisor or others want you to solve. You need to learn how to figure out what makes a good problem, and that involves developing some intuition and learning to trust your gut at times. If you are always waiting for others to find a good problem for you, you may find yourself feeling lost after graduate school, when you are on your own. Sometimes solving a problem will require that you use new tools or break with rules that have been followed in the past. Be open to that.

Lanius & Sisneros-Thiry: *If you could recommend one book to mathematics graduate students, what would it be?*

Gutiérrez: One book I very much enjoyed reading was *Pi in the Sky: Counting, Thinking, and Being* by John D. Barrow. It covers the history of counting throughout the world, and it includes a discussion of the question that many mathematicians like to ask: Is mathematics invented or discovered?

Lanius & Sisneros-Thiry: *What should a mathematics graduate student do if they want to learn more about your work or want to be more involved in mathematics education in general?*

Gutiérrez: To become more involved in the mathematics education community, consider attending a mathematics education conference like the Critical Issues in Mathematics Education workshop at the Mathematical Sciences Research Institute (MSRI) or the annual meeting of the International Group for the Psychology of Mathematics Education (the North American chapter offers its own version of the conference every year). MSRI has archived on its website¹ the plenaries of its CIME workshops with detailed notes. So, you can get a sense of the kinds of issues discussed at their meetings.

To learn more about my work, you might consider listening to my podcast on “Equity in the Mathematics Classroom.”² It is an interview with me about my research over the years. If anything seems interesting from that podcast, you could reach out to me at rg1@illinois.edu and I’d be more than happy to suggest a paper to read or a video to listen to. My more recent work on rehumanizing mathematics has been featured at MSRI, and you could find a video on their website.³

Lanius & Sisneros-Thiry: *Any final comments or advice?*

Gutiérrez: Do what you love, not what you think others will value. If you are really passionate about something, you’ll find that you always have the desire to learn more about that and to have really important insights that others might not come to naturally.

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Melinsa Lanius

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Simone Sisneros-Thiry

¹ See for example <https://www.msri.org/workshops/877>.

² <https://tinyurl.com/yb6nck8j>.

³ <https://www.msri.org/workshops/877/schedules/23134>.

WHAT IS...

Riemannian holonomy?

Jacob Gross

Communicated by Cesar E. Silva

Berger's classification of Riemannian holonomy groups is a strong organizing principle in differential geometry. It tells us about exceptional geometric structures, existing only in certain dimensions, that occupy central roles in physics, and generally serves as a road map for some research trends in algebraic and symplectic geometry.

To each Riemannian manifold (M, g) there are associated parallel transport maps P_γ that move vectors along a path $\gamma : I \rightarrow M$ such that the motion looks parallel from the metric's point of view. Fixing a point p in M the *holonomy group* is the group, written $\text{Hol}_p(g)$, of parallel transport maps around loops based at p , as in Figure 1.

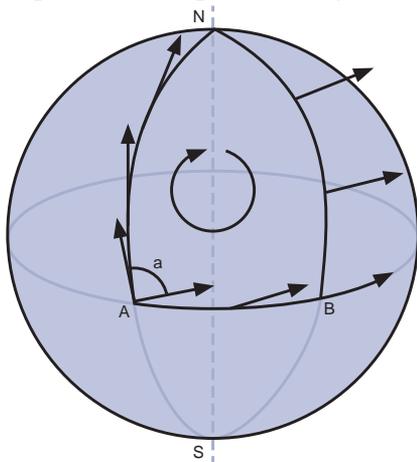


Figure 1. The holonomy group results from parallel-translating a vector around all loops from a fixed point A. On the round sphere, it includes all rotations, but on some manifolds, it is a smaller subgroup.

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If M is path-connected then $\text{Hol}_p(g)$ is, up to conjugacy, independent of p . One should always regard the holonomy group as coming with a natural representation: the inclusion $\text{Hol}_p(g) \rightarrow \text{GL}(T_p M)$. In 1926 Élie Cartan observed that the holonomy group acts reducibly if and only if the metric is (locally) a product metric. This reduces the problem of classifying holonomy groups of Riemannian manifolds to the problem of classifying holonomy groups of irreducible Riemannian manifolds. Cartan also wrote down all the holonomy groups of so-called ‘symmetric spaces.’ A *symmetric space* is a Riemannian manifold M such that, at each $p \in M$, the geodesic reflection s_p is an isometry. Euclidean spaces \mathbb{R}^n , spheres S^n , and hyperbolic spaces \mathbb{H}^n are all examples of symmetric spaces.

For (simply-connected) irreducible nonsymmetric Riemannian manifolds, Marcel Berger wrote down a list of all possible holonomy groups.

Theorem (Berger, 1955). *Let (M, g) be a simply-connected, irreducible, nonsymmetric Riemannian manifold. Let $n = \dim M$. Then the holonomy group $\text{Hol}(g)$ of (M, g) is either*

- $\text{SO}(n)$,
- $\text{U}(m)$ with $n = 2m$ and $m \geq 2$,
- $\text{SU}(m)$ with $n = 2m$ and $m \geq 2$,
- $\text{Sp}(m)$ with $n = 4m$ and $m \geq 2$,
- $\text{Sp}(m)\text{Sp}(1)$ with $n = 4m$ and $m \geq 2$,
- G_2 with $n = 7$,
- $\text{Spin}(7)$ with $n = 8$, or
- $\text{Spin}(9)$ with $n = 16$.

As it turns out, there are no irreducible nonsymmetric Riemannian manifolds with holonomy group equal to $\text{Spin}(9)$. In 1968 Alexeevsky eliminated $\text{Spin}(9)$ from this list. All other entries on Berger’s list, however, do occur as the holonomy group of some irreducible nonsymmetric Riemannian manifold, although it took some time to realize this.

Manifolds with holonomy contained in $\text{U}(m)$ are called *Kähler*, manifolds with holonomy contained in $\text{SU}(m)$ are called *Calabi–Yau*, and those with holonomy contained in $\text{Sp}(m)$ are called *hyperkähler*.

Berger's list, in a sense, echoes the classification of real normed division algebras.

Theorem (Dickson). *There are exactly four real normed division algebras: the real numbers \mathbb{R} , the complex numbers \mathbb{C} , the quaternions \mathbb{H} , and the octonions \mathbb{O} .*

Manifolds with special holonomy are important in physics.

Each group in Berger's list is a group whose elements are automorphisms/isometries of a vector space over some real division algebra. For example $\mathrm{SO}(m)$ (resp. $\mathrm{SU}(m)$) is a group of automorphisms of \mathbb{R}^m (resp. \mathbb{C}^m). In this sense, $\mathrm{Sp}(m)$ and $\mathrm{Sp}(m)\mathrm{Sp}(1)$ holonomies are quaternionic geometries, while G_2 holonomy and $\mathrm{Spin}(7)$ holonomy are octonionic geometries; G_2 is the group of automorphisms of \mathbb{O} and $\mathrm{Spin}(7)$ is the group of isometries of the octonions \mathbb{O} generated by left multiplication by unit length imaginary octonions.

Manifolds with special holonomy are important in physics. One reason is that a so-called "parallel spinor field" is required for the equations of supersymmetry to work. On a general Riemannian manifold, the parallel tensors determine the holonomy group. On a spin manifold, the holonomy group determines the parallel spinors. For this reason, manifolds with special holonomy groups (especially $\mathrm{SU}(3)$ and G_2) can be useful in theoretical physics.

Manifolds with holonomy $\mathrm{SU}(3)$, Calabi-Yau 3-folds, form so-called "string compactifications" in ten-dimensional supersymmetric string theories. This means that, in such a string theory, the universe is locally modeled on $\mathbb{R}^{(1,3)} \times X$, where $\mathbb{R}^{(1,3)}$ denotes Minkowski spacetime and X is a Calabi-Yau manifold of 6 real dimensions. In M -theory, the universe is supposed to have 11 dimensions and to be locally modelled on $\mathbb{R}^{(1,3)} \times X$, where X is a compact (singular) seven-dimensional manifold with holonomy G_2 .

G_2 and $\mathrm{Spin}(7)$ manifolds are rather unlike Calabi-Yau manifolds, however, in that it is notoriously hard to write down examples. To illustrate the difficulty: finding a metric with holonomy G_2 amounts to solving a simultaneous system of forty-nine nonlinear PDEs. Bryant wrote down the first metrics with holonomy G_2 and with holonomy $\mathrm{Spin}(7)$ in 1987. These metrics were not complete. Later Bryant and Salamon constructed complete noncompact examples of manifolds with holonomy G_2 and $\mathrm{Spin}(7)$. In 1993 Joyce constructed the first examples of compact 7-manifolds with holonomy G_2 and of compact 8-manifolds with holonomy $\mathrm{Spin}(7)$. Since then other examples of compact manifolds with holonomy G_2 have been constructed by Corti-Haskins-Pacini-Nördstrom, Joyce-Karigiannis, and Kovalev.

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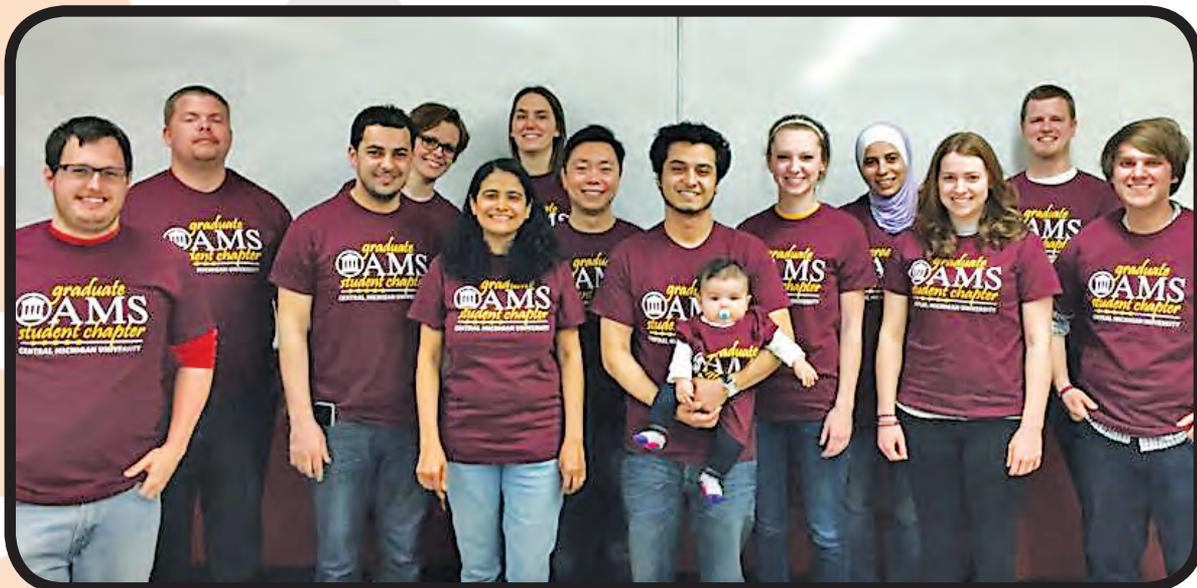
Jacob Gross

ABOUT THE AUTHOR

When not doing mathematics, **Jacob Gross** trains in acrobatics and in break dancing.

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THANK YOU

Snapshots of Mathematics in Sub-Saharan Africa

Edited by Allyn Jackson and Carol Shubin

Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.

—David Hilbert

Mathematics is on the rise in Sub-Saharan Africa, where a critical mass of researchers and educators is developing. In October 2003, the *Notices* carried the article “The Challenge of Strengthening Mathematics in Africa” by Phillip A. Griffiths and Wandera Ogana.¹ Since that time, there have been dramatic increases in the number of people engaged in mathematics education and research through research centers, conferences, international partnerships, collaborations, and exchanges. Many entities have contributed to these efforts, including African governments, government agencies of other countries, universities, international foundations, and individuals. Tremendous optimism and great potential coexist alongside serious political, social, and cultural challenges.²

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¹www.ams.org/notices/200309/commentary.pdf

²*See the two reports prepared by the International Mathematical Union: Mathematics in Africa: Challenges and Opportunities, prepared by the IMU for the John Templeton Foundation, 2009, <https://bit.ly/2JszQkt>; and Mathematics in Africa: Challenges and Opportunities, prepared for the International Congress of Mathematicians in Seoul, Korea August 13–21, 2014, <https://bit.ly/2HtL00u>.*

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This article provides snapshots of experiences of several mathematicians who have done research and teaching in Sub-Saharan Africa. While each snapshot encapsulates the writer’s personal viewpoint on mathematics in Africa, there is a common theme to them all: The interest in mathematics is there, but Africans need more opportunities and resources to develop the continent’s talent base.

The purpose of this article is to inspire more mathematicians to help in the effort to build African mathematics. There are many ways to do this. You can spend a sabbatical term teaching, give a mini-course, attend a conference, encourage your university to provide tuition waivers or scholarships for African students, or consider inviting an African faculty member to your institution. You can contribute to funds for graduate student travel. You can be a mentor or publish an article with an African researcher in an African journal. You can support travel grant programs or the African Math Olympiad. Many suggestions are outlined in the sidebar at the end of this article.

Try Africa. It could change your life—and the lives of others.

Augustin Banyaga

My Life and My Contributions to Building African Mathematics

I was born in 1947 and raised in a village near Kigali, the Rwandan capital. At that time Rwanda was a Belgian colony. My parents were farmers, and my father occasionally worked as a bricklayer on construction sites. My parents had no formal education. But I was able to go to an elementary school in Kigali, walking several miles there and back home each day. There were no books to read. After school

I spent the rest of the day helping with household work, like going to the river to bring water for the family. I used my free time walking and dreaming on my family's banana plantation. I was fond of nature: I collected dried flowers, butterflies, and beautiful quartz stones. I was fascinated by symmetry, which is the foundation of geometry. Maybe my love of symmetries and hence of geometry came from these early experiences. They were more important than what I was taught at school.

After these happy years, I was admitted to a high school located about thirty miles from the village, where I studied classics (Latin and Greek). I finished high school at a different institution near Kigali, where the main emphasis was on Latin and sciences. The mathematics curriculum was okay but not great. I did very well and finished first in my class in 1966. The following year, I received a government scholarship to go to Geneva, Switzerland, to study mining engineering. I didn't like my geology classes, but I loved the freshman linear algebra course, which was taught by Professor André Haefliger. Meeting this famous mathematician was the turning point of my life. Thanks to him, my heart completely turned to mathematics. I changed my major to mathematics and performed very well, finishing the equivalent of the bachelor's degree in 1971, in record time. Professor Haefliger agreed to be my doctoral advisor, and I finished my PhD thesis in 1976. My area of research was and remains in differential geometry/topology. I was the first Rwandan to hold a PhD in mathematics.

After my PhD, I wanted to go back to my native Rwanda. I applied for a position at the unique university in the country, the National University of Rwanda (NUR). After several months without a reply, I sent an application for a visiting position to the Institute of Advanced Study in Princeton. I got an immediate positive answer and went to the IAS in 1977.

I have always wanted to do mathematics in Africa. When I applied for the position at NUR, there were no Rwandan mathematicians. The university was functioning with expatriate faculty who held only bachelor's or master's degrees. This is why the NUR neglected my application: I was perceived as a threat. Ironically, at the same time they were rejecting me, Harvard University offered me a Benjamin Peirce Assistant Professorship. I was at Harvard from 1978 until 1982 and then moved to Boston University. In

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1984 I joined the faculty of my present institution, Penn State University.

In 1981, 1982, and 1983, I visited NUR as a UNESCO consultant and started a mathematics PhD program there. The program functioned for only a couple of years before dying off. One of my students from this program, Philippe Rukimbira, came to Penn State to finish his PhD under my supervision and is now a professor at Florida International University. In 1979 I took a leave of absence from Harvard to continue teaching in my PhD program in Rwanda. I asked people at NUR if it would be possible for me to maintain half a position in Rwanda, and the other half abroad. The answer was no. I then decided to settle in the US. I became a US citizen in 1995.

That same year I reconnected with Africa at the Fourth African Mathematical Union Pan-African Congress of Mathematics, in Ifrane, Morocco. There I met Jean-Pierre Ezin, a mathematician from Benin, who was the director of the Institut de Mathématiques et Sciences Physiques (IMSP) in Benin (see Figure 1). Professor Ezin invited me to the IMSP, and since then I have visited almost every year for over twenty years and even spent a sabbatical leave there.

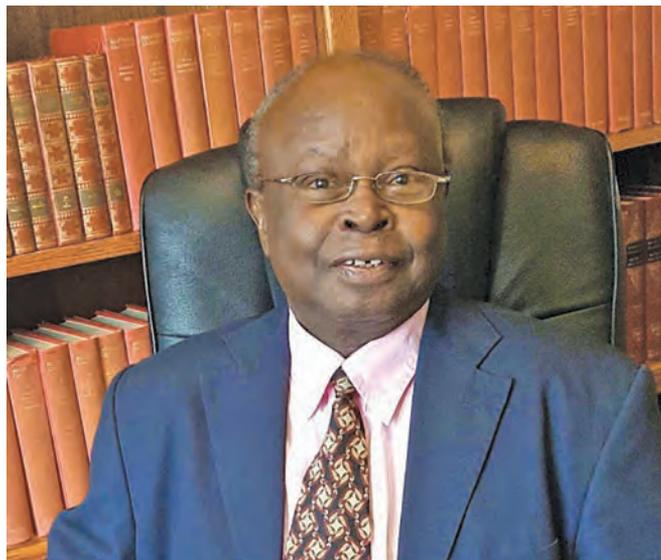
The IMSP is a small graduate school that recruits students from all over Africa. Admission to the IMSP is

extremely competitive: only excellent and highly motivated students get in. Since its founding in the 1980s, the IMSP has been supported by the Abdus Salam International Center for Theoretical Physics in Trieste, an organization that aims to build scientific research in the developing world. More recently, the IMSP won an award from the World Bank to host a "center of excellence." Since 1995, I have been a member of the IMSP Scientific Committee.

Over the course of my visits to IMSP, I gave sixteen graduate minicourses, of two to three weeks each, in my area of research: symplectic geometry, contact geometry, and Morse theory. Teaching

at IMSP was a very enjoyable and rewarding experience. I had classes of fewer than 15 students, all of them very bright and highly motivated, comparable to the best students one gets in top universities in the US. Some of them finished their PhDs under my supervision. This was a difficult task because of the distance—I was at Penn State and they were in Africa—and we often had to work together over the internet. I published joint papers with some of my former students, as well as a book.

Over the last twenty years, I have also taught and done research at other African universities, including University



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of Botswana, University of Dakar, Universities of Yaounde and Dschang in Cameroon, University of Niamey in Niger, and University of Congo-Brazzaville. I have also visited AIMS Senegal, where I gave lectures and seminars and directed master's theses.



Figure 1: Pictured here is the Institut de Mathématiques et Sciences Physique (IMSP) in Benin. A small graduate school in mathematics that recruits students from all over Africa, IMSP recently won a World Bank award to host a “center of excellence.”

I have been involved in other efforts to build mathematics in Africa, including creating research networks that sponsored conferences and workshops and serving on the selection committees for the African Mathematics Millennium Science Initiative (founded by Phillip Griffiths of the IAS) and for the Simons Foundation's Africa Mathematics Project. For many years, I have been on the editorial board of *Afrika Matematika* (the journal of the African Mathematical Union), the *African Diaspora Journal of Mathematics*, and the *African Annals of Mathematics*.

Around the time I left Rwanda in 1967, many countries in Black Africa were asserting their independence from colonizers. They were more focused on political reorganization than on economic reforms. By the time I reconnected with Africa in 1995, all the governments were trying to improve their economic situations. They had begun to realize that economic progress is built on technology, which depends on engineering and ultimately on science and mathematics.

Africa possesses tremendous potential for developing outstanding mathematicians and producing significant advances in research. The continent still needs much sustained help from mathematicians outside of Africa who can help native students to develop their talent and to build a mathematical life for themselves on African soil.

Wilfrid Gangbo



Wilfrid Gangbo

Building Mathematics in Benin

I was born in the West African nation of Benin and attended kindergarten through high school there. Benin is known for its rigor in education, which is viewed as a stepping stone into the upper middle class in a country that was not blessed, as were other African countries, with mining resources. In the 1970s, there was great confusion about the importance of intellectuals in Benin, which had opted for a socialist government. Nevertheless,

partly due to French influence, mathematics continued to be recognized as an important subject.

In the capital city of Porto-Novo, the school students who were the best in mathematics were treated like celebrities and praised by the population, including by those who never made it to high school. In the 1980s, the leading Porto-Novo high school, Lycée Behanzin, had many more high school students studying mathematics and physics than any other subject. One spectacular high school mathematics student was known in Porto-Novo as “Dr. Cosine.” Many of us were motivated to uncover his strategy for success in mathematics.

In the 1970s and 1980s, Benin was full of excellent high school mathematics teachers, such as Dieudonné Quenum, who seemed to know the secret of how to instill mathematics in the heads of his students. Even people who were never in his class were fascinated by his fame, which is still talked about in some circles. My highly qualified K-11 mathematics teacher, Daniel Tetelin, who left Benin when relations with France were souring, taught us that you had to write mathematics with the greatest rigor if you wanted to pass his class. His influence drew several of us to the world of mathematics—where we have since been trapped! The ability to write detailed proofs and read mathematics carefully became a strength and made mathematics a fascination.

After those years, Benin faced a desert crossing, a devastation for mathematics. Starting in 2000, the number of students attending K-12 rose greatly, but the number specializing in mathematics rose very little, from 1364 in 2001 to just 1603 today. By contrast, the number specializing in biology rose from around 11,000 to nearly 51,000. Nowadays many mathematics instructors in Benin high schools were not undergraduate mathematics majors.

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*[With help]
we can
continue
to build
mathematics
in Africa.*

Benin decided to search for remedies to these problems and—along with some other African countries such as Cameroon and Senegal—has witnessed a positive slope in mathematical development. Although nobody has discovered a recipe to keep the curve concave up, we are hopeful that, with the help of the international mathematical community, we can continue to build mathematics in Africa.

There are several initiatives that deserve support and development; here are two examples.

Senior mathematicians in Ivory Coast had the bright and original idea to launch a “Miss Mathematics” competition, with a monetary award. The IMSP (Institut de Mathématiques et de Sciences Physiques) in Porto-Novo decided in 2008 to create its own Miss Mathematics, which is now a national competition targeting K-9 female students who have shown superior performance in mathematics. IMSP is proud to report that today the proportion of female students in mathematics has greatly increased all over Benin. Happily, the nation of Senegal has also decided to initiate a Miss Mathematics competition.

In 2013, a team of mathematicians from IMSP won a World Bank competition to create a Center of Excellence in Mathematics. This extraordinary experiment is in its infancy and needs much constructive criticism from experienced people. Benin is fortunate to have supporters such as John Ball, Irene Gamba, Nassif Ghoussoub, Cédric Villani, and many others. Since 2014, a large number of mathematicians have spent time teaching mini-courses and participating in workshops funded by the center. Not only have students learned a great deal from the lectures by visitors, but also, and perhaps more importantly, they have been immersed in the international mathematics culture.

People often ask me, “Why do you need to invite the very best mathematicians to workshops or to teach mini-courses in Africa?” I often answer, “Would Zygmund have discovered Calderón if he had not traveled to Argentina?” There is no scientifically recognized educational method to elevate the level of mathematics in a country. However, we have seen that exposing students to the very best mathematicians has had a tremendous impact in many parts of the world.

Florian Luca

Fun and Adventure Teaching Math in Africa

In 2013, I taught a three-week course in number theory at AIMS in Senegal. The course was not easy, and the pace was intense. The forty students met five days per week, two hours per day for a total of thirty hours in the three-week period.

Students come to AIMS from all over Africa, mostly from French-speaking West African countries. Occasionally, there are students from Egypt, Sudan, or other Arabic-speaking countries or students from Eastern African countries who speak English but almost no French. In addition to courses, each student has to write a master’s essay in order to graduate. I had two students interested in writing an essay with me, Bernadette Faye from Senegal and Taoufiq Damir from Morocco. Instead of giving them something to read, I involved them in some nice elementary number theory projects. The students worked very hard and learned a variety of techniques and some elementary algebraic number theory as well as gaining some basic acquaintance with multiplicative number theory and sieves. The projects led to papers coauthored by the three of us together with Dr. Amadou Tall, the local adviser of the two students, in *Fibonacci Quarterly* and *Integers*. Both students went on to pursue PhDs. Since this first experience, I have returned to teach in the AIMS network every year.



Florian Luca (fourth from left) with students at AIMS Senegal in 2013.

Traveling to and living in Africa has its challenges. For example, for my first AIMS Senegal trip, I read on the web page of the Senegalese Embassy that citizens of the European Union do not need visas. I hold a Romanian passport, but at that time Senegal was not aware that my home country was a member of the European Union. As a

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consequence, I was denied boarding my flight from Washington, DC, to Johannesburg because it had a stopover in Dakar, Senegal. If you are kicked off a plane for not having a visa for your destination country, maybe Washington, DC, is the best place for it to happen, since every nation has consular representation there. So I asked the airline to move me to the next day's flight and ventured out into the January snow in T-shirt and shorts (I was traveling between Mexico and Senegal so did not have a coat). The next day I visited the Senegalese Embassy, convinced them to give me a visa, and made the flight that evening.

Every time I have traveled to Africa my luggage has arrived at least one week after I did, so for days I'd function modulo two, with the green T-shirt on even days and the blue T-shirt on odd days, washing them every evening and letting them dry in the hot African nights. While in Ghana in 2013, unwisely having made the decision not to take my malaria pills—believing that malaria is something that happens only to other people—I got malaria and had to be treated at the local hospital in Cape Coast. The staff of AIMS Ghana took good care of me then and during all my trips to AIMS Ghana in various ways, including quarreling with the local traffic police and recovering from them my driver's license, which had been confiscated at a police roadblock because it was in a language other than English.

Such stories show that life in Africa poses challenges, but that is part of the experience of teaching and living there. I hope my story will convince you that there is great fun and great adventure to be had by teaching in Africa and involving students in your research projects.

Mathematicians of the African Diaspora

In 1997, Scott Williams of the University of New York at Buffalo launched a website called *Mathematicians of the African Diaspora*.^{*} The home page of the site says that Williams started the site in order to counter negative stereotype about black people “by exhibiting the accomplishments of the peoples of Africa and the African Diaspora within the Mathematical Sciences.” The site supplies biographical information about hundreds of mathematicians of African descent.

www.math.buffalo.edu/mad

Nancy Ann Neudauer

Learning to Eat Soup with My Hands

Four years ago I had never been to Africa. I had never planned to go to Africa. Now I can't seem to stop going to Africa—I've been eleven times to six countries, with plans to visit more. How did this happen? I was first invited by Rob Beezer of the University of Puget Sound, who had taught several courses at AIMS, to design and teach a course with him at AIMS South Africa. Rather than restrict my involvement to a single visit, he suggested an ambitious plan whereby I'd apply to the Fulbright Specialist Roster and return during my sabbatical the following year to teach at two AIMS centers. I did return on sabbatical, teaching courses at three of the AIMS centers, and went again later that year to supervise thesis projects. Four years and three Fulbright Specialist grants later, I'd taught at five of the six existing centers. Sometimes you take a step in a direction that unexpectedly changes your course.



Nancy Ann Neudauer (fourth from left) gave a course at AIMS in South Africa in January 2014. Here she poses with some of her students, as well as the tutor for the course (third from left), Martha Kamkuemah, who is from Namibia and is currently working on her PhD at Stellenbosch University.

Established in Cape Town in 2003, AIMS has expanded to include centers in Cameroon, Ghana, Rwanda, Senegal, and Tanzania. Students come to AIMS from all over Africa for an intensive, one-year structured master's program. All of the courses are taught by visiting lecturers, with topics and expertise changing from year to year. The goal is to prepare the students for further graduate study in Europe, the United States, Canada, and Africa. The guiding philosophy is that Africa cannot be just consumers of mathematics; Africa must be producers of mathematics.

What is it like for an AIMS lecturer? The lecturers live and eat with the students. Someone new is always arriving

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*a revolving
door of
researchers
and lecturers
from around
the world*

or departing, so there is an incredibly vibrant atmosphere—a revolving door of researchers and lecturers from around the world, in addition to dedicated teams of permanent staff and directors. All classes are three weeks, with students taking two at a time. The first few blocks focus on basic skills, such as problem solving, advanced linear algebra, and TeX. This is followed by six blocks in which the instructor is expected to take the students from the basic foundations of a field to research-level problems in just three weeks. Finally, the students spend several months conducting independent research and writing a thesis. While all of the students have a bachelor's degree in a mathematical science, this is broadly defined, so many have neither seen formal mathematics nor ever written a proof.

Beezer and I designed and taught a course called “Designs, Matroids, and Graphs,” which I have since refined and taught at several AIMS centers. We had to think radically differently about what we were teaching, not having a full semester to work through a book or slowly build up ideas. We needed to pare down to the material that was essential to get the students to a point where they could understand advanced results and open problems. Introducing students to new areas of mathematics and getting them up to current problems in several fields, in just three weeks, was a challenge for us as well as them!

I can't talk about Africa without mentioning the food. Yams, cassava, or maize, pounded to a powder and cooked into a mass—*ugali* in Tanzania, *fufu* in Ghana, *couscous* in Cameroon, and *pap* in South Africa—are universal staples that I learned to employ to pick up bits of chicken or fish heads. I even learned to scoop up soup with my hands. The students are always surprised, exclaiming, “You're eating with your hands! Like we do.” I was amazed that this extremely high-carbohydrate diet, with very little protein, fresh vegetables, or fruit, could sustain the students in the intensive and demanding AIMS program. While there are some common themes to food in Africa, there are many regional differences. One of the few complaints I ever heard from the students—despite the frequent lack of internet, power, and water, despite the heat, despite the lack of books and resources we take for granted—was about the food. It was not like the food in their home countries.

What are the challenges? Everyday things we take for granted are either absent or severely compromised, but cannot be used as excuses to stop studying.

Electricity: We watched the students practice their presentations at the board illuminated by only one dim light in the back of the room because of problems with the wiring. There are often scheduled—and unscheduled—power outages.

Internet: On the days it works, it is excruciatingly slow.

Water: I wondered at first what the large vat of water in my bathroom was for. And I wondered about the bucket. I soon found out that the water from the taps frequently doesn't flow—and was not for drinking. Living for a month at a time in places where 30 percent of the population makes less than US\$1.25 a day and the GDP is under US\$3,000.00, yet everyone needs to buy drinking water and other necessities, helped me to understand how incredibly adaptable and resilient human beings are. The students epitomize this.

What keeps me coming back? This has been one of my most rewarding professional experiences. I was motivated by a professional duty and the opportunity to work with graduate students, which was new for me since my own school has only undergraduates. I also hoped to have a positive impact on the student experience. I had not anticipated that the more profound impact would be on me. I am part of an ever-expanding faculty of visiting AIMS lecturers who work tirelessly to develop the initiative in Africa, at home, and worldwide. Connections formed in this network have led to visits to my own university and to research collaborations. Principally, however, it is the students who draw me back. They have such energy and enthusiasm for learning and are very appreciative of the work I do. I leave feeling invigorated both personally and professionally. An unexpected benefit is the degree to which this undertaking has informed my own teaching, as well as my research. Teaching students who are desperate to learn but for whom one cannot assume a standard background in mathematics has helped me distill precisely what is needed to explain a result. I supervise students for whom English may be a third language. Teaching them to write mathematics well has improved my own writing, and I am much better and faster at reading and editing student work. Finally, the more I visit Africa, the more I appreciate how important female role models are. Young women and girls need to know what is possible.

*I wondered
how I might
change the
world. Each
year I teach,
I realize this
happens one
experience at a
time.*

When I was young, I wondered how I might change the world. Each year I teach, I realize this happens one experience at a time.

Carol Shubin

Into Africa



Carol Shubin in Kigali, Rwanda, holding her son James Wolff while sitting with friends Professors Paul Mwangi and Maina Maringa.

I am a professor at California State University Northridge, a large regional comprehensive university in the northern suburbs of Los Angeles. By 2006, I had wrapped up an educational grant from NASA and completed some work of my late husband, Thomas Wolff of Caltech. I wanted to do something different. I began considering “exotic” teaching locations. While searching the web, I came across a post from 1994 by Dino J. Lorenzini, Distinguished Research Professor of Mathematics at the University of Georgia, in which he stated:

The University [of Rwanda] reopened in April 1995, nine months after the end of the genocide. Before the genocide, the department of mathematics had a staff of 17. In September 1995, the staff was reduced to two graduate students, the other members being either dead, in exile or refugee camps, or in jail. The teaching was done by these graduate students and by temporary visitors from Universities in neighboring countries. The University was eager for academics of all countries to come teach at the University, and will accommodate any schedule.

I might not be Angelina Jolie, but I can teach math. I decided to find out more about Rwanda, a landlocked country known as “the land of a thousand hills.” I was very impressed by the Rwanda Vision 2020 plan. The country strongly supports education and the development of STEM enterprises and is committed to gender equity.

I received a Fulbright scholarship to teach at the Kigali Institute of Science and Technology (KIST), now called College of Science and Technology - University of Rwanda (see Figure 2). My plan was to try to set up a version of boot-camp style courses that I had run with my NASA grant during 2000–2005. These three-week intensive courses on applied math topics included speaker identification, diffusion tensor imaging, analysis of the Northridge earth-



Figure 2: Pictured here is the Math Department, University of Rwanda (formerly KIST), in Kigali, Rwanda. For six months, Carol Shubin and two colleagues from her home department lived in Kigali and taught at KIST.

quake’s GPS data, solar disk data, electrophoresis gel data, and satellite trajectory data. I figured that the US has more data than can be analyzed, and Africa has the talent for the work. With all the available freeware, the cost would be minimal. So off I went.

For six months in 2007, I lived in Kigali with my children (then in the fourth and sixth grades); a CSUN graduate student, Jennifer Wright; and two CSUN math faculty, Werner Horn and Susan Taylor, who were on sabbatical leave. There was a need for basic instruction, so I found myself teaching several large sections of calculus. Some of the large classrooms had small blackboards and poor acoustics and lighting, making it quite difficult for students to follow the lectures.

The hardships many of the students endured to get to KIST impressed me greatly. Although a large percentage of them had been educated in French, they were expected to take courses in English with little preparation. Most of the students had received their entire education, from K-12 to college, without ever possessing a textbook. Nonetheless, I gave the same calculus course that I give at CSUN, and the students did about as well. In fact, they had stronger algebraic skills.

The following year I received a grant from the African Mathematics Millennium Science Initiative to come back. I had designed several online courses, and five KIST students tested two of them. Unfortunately, there were a lot of problems with students being able to access the internet to complete the courses. I ended up sending them money so that they could complete the work at internet cafes. I am still in touch with some of my KIST students who went on to earn advanced degrees in South Africa. I helped one student pay for college at a private university in Ohio.

Not only was the teaching experience very intense, but there is a strong bonding with other visiting instructors. Cornell math department chair Ravi Ramakrishna and I

had many lively discussions over dinner about our observations and ideas. Following his three-month stint at KIST in 2009, he became a Tours of Purpose volunteer in Uganda, teaching a math class at Makerere University (see Figure 3). He also gave two three-week courses at AIMS South Africa. He too found the experience addictive.

In 2011, Irina Zlotnikova, my former officemate at KIST, came to visit CSUN. Her interest was in programs that focused on poverty eradication and education in Africa using information and communications technology. She had an opportunity to visit our College of Education and interact with math faculty. Later Irina invited me to the Nelson Mandela African Institution of Science and Technology (NM-AIST) in Arusha, Tanzania (2014). I was extremely impressed with the institute and enjoyed giving some talks.



Figure 3: Ravi Ramakrishna lecturing in Uganda. Ramakrishna is chair of the mathematics department and Stephen H. Weiss Presidential Fellow at Cornell University. Following his three-month stint at KIST in Kigali, Rwanda, in 2009, he became a Tours of Purpose volunteer in Uganda, teaching a math class at Makerere University. He also gave two three-week courses at AIMS South Africa.

During a 2014 visit to the NM-AIST, I connected with Eunice Mureithi from the University of Dar es Salaam and Josephine Kagunda from the University of Nairobi. They were active in organizing the CIMPA³ Research School on Mathematical Modeling and Analysis of Complex Systems, held in Kenya during summer 2015. Although I was not able to attend, CSUN's College of Mathematics and Science and provost's office donated US\$5000 for students to attend the program. In 2016, CSUN hosted Josephine Kagunda for three months. She worked on several papers and co-taught a senior-level seminar on disease modeling. During summer 2016, CSUN students began collaborating with University of Nairobi students on several projects stemming from the CSUN seminar. In August 2016, I vis-



In August 2016 Wandera Ogana (left) talks with Josephine Kagunda (right), who like Ogana is in the School of Mathematics at the University of Nairobi, and student, Marilyn Ronoh.

Wandera Ogana, Chair of IMU Commission for Developing Countries

Wandera Ogana, professor of mathematics at the University of Nairobi, has been on the Commission for Developing Countries of the International Mathematical Union since 2011 and is currently serving a four-year term as CDC chair. His extensive work on building mathematics in Africa includes serving as secretary/executive director of the African Mathematics Millennium Science Initiative.

Born in Kenya in 1946, Ogana received his bachelor's degree in mathematics from the University of East Africa in Nairobi in 1970. After a year teaching secondary school, he went to Stanford University, where he received his PhD in applied mechanics in 1975, with a dissertation about transonic flows written under the direction of John Spreiter. After a year as a postdoctoral researcher at the NASA Ames Center in California, Ogana returned to Kenya to join the faculty of the University of Nairobi.

Ogana's research centers on computational fluid dynamics, particularly the application of boundary element methods to solving transonic flow problems. He has also developed mathematical models in ecology, wind energy assessment, and the impact of climate change on infectious diseases. He is a Fellow of the African Academy of Sciences and of the Kenya National Academy of Sciences and is Vice President (East Africa) of the African Mathematical Union.

Ogana has an unusual sideline: He writes fiction. His first novel, *Hand of Chance*, written while he was still an undergraduate, was published in 1970. He has since written three other works of fiction, the latest being *A Family Affair*, published by the Jomo Kenyatta Foundation in 2011.

³Centre International de Mathématiques Pures et Appliquées

ited the University of Nairobi, where graduate students are mentoring our undergraduates' projects.

Mathematics is on the rise in Africa. I feel revitalized to participate in even a small way. I am so grateful to many people who have opened my mind and heart.

Cédric Villani



Cédric Villani

Falling in Love with Africa

My interest in mathematics in Africa started with my friend and fellow contributor here Wilfrid Gangbo, a brilliant African mathematician who is now a professor at the University of California, Los Angeles. I first met him in 1998 in Atlanta, when he was at Georgia Tech. He arranged for me to spend a semester teaching in Atlanta, which was a very important step in my career. Usually when we speak about north-south cooperation, we think of

north helping south. In my case, the beginning of my career was strongly helped by this African mathematician.

In 2010, I attended a conference Wilfrid had organized in Benin. At that time, I had heard about AIMS, and as director of the Institut Henri Poincaré, I had been asked to participate. My reply was, "Let's be cautious, we don't know what this is..." But as soon as I returned from the Benin conference, my reaction was: "Math in Africa—I have to do something. It will be complicated, but it is important. It is a mission." Soon after that, I began traveling regularly to Africa. I have made eight trips to Senegal, where I participated in the launching of the AIMS center in M'Bour, as well as four trips to Cameroon, three to Benin, and one to Rwanda.

With Africa, there is a "before" you have been there and an "after." Africa has a special atmosphere. Time flows in a different manner from Europe or the US. The contacts are closer, there is a special way people talk and interact. If you are open to it, you are likely, as I did, to immediately fall in love with the whole continent.

Developing mathematics in Africa is a long-term goal. There is an appetite for mathematics, and the students are eager to learn. In a few decades, I believe Africa will

Cédric Villani was director of the Institut Henri Poincaré until 2017, when he was elected as a member of the French National Assembly.

be a reservoir of students who want to improve their status through study and knowledge. For this to happen, infrastructure is crucial. Setting up internet connections is not sufficient. Facilities are needed, but so are local human networks. You need competent administrators who will make the enterprise grow and who will keep it in resonance with local conditions.

Africa is an enormously diverse continent, with many different cultures and educational systems. Some countries have huge numbers of students, and others have very few. But they all face the problem of identifying and nurturing their best students and giving them access to opportunities.

When I have taught in Africa, I can stay for a maximum of two weeks, but the courses often last longer. So I have always paired with a local professor who continued the course after I left. In the case of Senegal, I was paired with Diaraf Seck. We became close friends—we got along so well, in fact, that we made a MOOC together on differential equations. I was based in Paris and he was based in Dakar, so we had to coordinate the recording very carefully. We would mix the videos in Paris and get help from coordinators in other places. It was very complicated, but we loved it. For various reasons it did not have as much impact as we hoped, but it was a great experience.

Each time I come back to Africa, I am happy to see my friends again. My work with AIMS has been a real source of personal enrichment for me. If you measure the impact in terms of, for example, the number of the students you taught who were able to get into good PhD programs, the numbers are always small. But in terms of how good you feel, this work is extremely rewarding. When you meet people who participate in these adventures and have hope, it is really heart-warming.

Some of the students I have worked with have touched me deeply. One of them, Merlin Mouafo Wouodjie, is now doing his PhD in Germany. Due to a medical accident with a childhood vaccination, he is disabled—his arm and leg on one side do not function properly. He comes from an extremely religious background and signs all his email with

"God bless you." He is very gentle and kind, and, far from complaining about his disabilities, he defies them—crouching down to talk to students at the desks, or enthusiastically throwing himself towards the ball during collective volleyball games. He was absolutely the brightest student his univer-

sity had ever seen, and he was always hardworking. He comes from a very poor family and could not afford to study continuously, so he would return home every year to work, and then the next year he would continue his studies. He's a typical example of a person who, given the right opportunities, could be an international-level researcher.

The goal of AIMS is to create opportunities for exactly that kind of student. At the beginning, some mathematicians and institutions viewed AIMS with suspicion,

You are likely, as I did, to immediately fall in love with the whole continent.

and some wondered about the emphasis on physics and applied mathematics. I understand the concerns but am still an enthusiastic fan of AIMS. Whatever criticisms there have been can be taken constructively and will help improve the project. I have been helping AIMS as much as I can in the past few years and am now a member of the scientific board.

To build a network that can identify excellent students takes years and requires subtlety and insight. Cultural and linguistic concerns have to be taken into consideration. There are hundreds, if not thousands, of dialects in Africa. And then there are the communication languages, mainly French and English, which were mostly imposed by the colonizers. There are big differences in the culture of the colonizers, and this influenced the teaching of mathematics in Africa. You can do a respectable career throughout Africa speaking only French, but as soon as you go to the international level, English prevails. The linguistic issue of French versus English has to be taken very seriously in the development of math in Africa, at least for some time to come.

Among the institutions that are well trusted in Africa is the Abdus Salam International Center for Theoretical Physics, based in Trieste, which is launching a new branch in Rwanda called the East African Institute for Fundamental Research. Another is Centre International de Mathématiques Pures et Appliquées (CIMPA), which was founded in Nice in 1978. CIMPA funds local mathematics initiatives in developing countries across the world, including Africa.

For those who are interested in helping the development of mathematics in Africa: Do it! Africa is like no other place. World society has a big stake in Africa's political, economic, and technological development. It's vitally important that science and mathematics take their part in that construction.

The single biggest mistake we can make is to see helping Africa as a charity act: "I am rich. Let me give you a little bit of my wealth." The right attitude is: "We are all in the same boat. I can do good for you, you can do good for me. We have a lot to share." I strongly advise mathematicians to participate in existing initiatives, such as AIMS, or the World Bank centers, or the projects funded by the Simons Foundation. These initiatives need to be reinforced and to gain in efficiency and recognition. Systemic thinking is important.

The politics of many countries in Europe and other parts of the world has become much too inward looking. This makes it all the more urgent that we who work in mathematics, science, and culture take up our duty to try to bring diverse people of the world together for the common good.

Michel Waldschmidt

Teaching in Africa

I taught mathematics to African students in various circumstances, including through research schools of the Centre International de Mathématiques Pures et Appliquées (CIMPA) and through African Mathematical Schools, which are jointly supported by CIMPA and the African Mathematical Union. These courses took place in Algeria, Morocco, Mauritania, Mali, Ivory Coast, Benin, Senegal, Congo Kinshasa, Cabo Verde, and Ghana.



Standing seventh from the right (in yellow shirt) is Michel Waldschmidt at Ghana-AIMS.

I often meet the same students from Africa at different places, including CIMPA Research Schools and African Mathematical Schools. These students are among the ones who are most enthusiastic about learning and improving their skills. In Africa, as in other places, one cannot avoid being asked, "What is this useful for?" This question might be asked any time, for instance right after the teacher has completed a proof of a beautiful result that he or she considers a work of art!

Most students have a weak background but are eager to learn. A motivation for many of them is to get a scholarship and pursue their studies abroad, usually in the US. Very often, they will end up as a teacher in a small college in the US and give up research. However, some of them come back and contribute to the development of mathematics in their home countries.

Through AIMS, students attend courses on a wide variety of topics, not only mathematics; this is good for enlarging the scope of their interests and for giving them a broader perspective of mathematics and related topics. They come to consider mathematics as a part of science, which is related to many different subjects. However, they gain only a superficial knowledge of each topic. A better goal might be to improve their background, which often

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AIMS is good for enlarging the scope of their interests and for giving them a broader perspective of mathematics and related topics.

during these three weeks I taught only what I had prepared for the first week: I needed to explain basic notions that the students had never heard of before. During the first of each of these three-week periods, the students are supposed to attend all courses on various unrelated topics, including not only mathematics, but also topics ranging from theoretical physics to management. For the last two weeks they select some topics with tutorial sessions, where they have to solve exercises. Each course, except the first one, starts with a short quiz of some ten minutes, which I find a good idea for the students as well as for the tutors, who can assess what the students have understood. The fact that there is no diploma at the end is coherent with the whole process. The AIMS centers play an important role in the development of mathematics in Africa. I wish something similar could be launched in Asia, where there is also a strong need to support students who wish to become mathematicians.

is not very strong. At the end of their stay at AIMS they write a memoir, which is very often for them the first experience of writing a piece of mathematical work that is not an exam or homework. The teachers play a fundamental role in the process and are one of the main components of the AIMS centers.

The teachers visit AIMS for a period of three weeks. My own experience at AIMS in M'Bour, Senegal, is that

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Noted Resources for Sidebar (facing page)

- ¹www.cies.org/programs
- ²www.mathunion.org/cdc/volunteer-lecturer-program
- ³www.nexteinsteinstein.org
- ⁴www.cimpa.info/en
- ⁵ammsi.or.ke/available-opportunities
- ⁶www.mathunion.org/cdc/grants/imu-simons-african-fellowship-program
- ⁷<https://sites.google.com/site/awmmath/programs/travel-grants/mathematics-travel-grants>
- ⁸<https://www.carnegie.org>
- ⁹*Africa, Near East, and South Asia Regional Opportunities*, <https://www.nsf.gov/od/oise/anesa.jsp>
- ¹⁰www.mathunion.org/cdc/grants/african-diaspora-mathematicians-program-admp
- ¹¹friends-imu.org/graidd
- ¹²friends-imu.org/graidd-donation
- ¹³<https://www.lms.ac.uk/grants/mentoring-african-research-mathematics>
- ¹⁴imomath.com/index.php?option=Pa&mod=23&ttt=Pan-African
- ¹⁵www.iie.org/Programs/Carnegie-African-Diaspora-Fellowship-Program
- ¹⁶africanwomeninmath.org

Ways to Get Involved

There are many ways mathematicians at all stages of their careers can contribute to the development of mathematics in Africa. Here are some suggestions, assembled by Carol Shubin and Wandera Ogana. Ogana is professor of mathematics at the University of Nairobi and chair of the Commission for Developing Countries (CDC) of the International Mathematical Union (IMU).

Teach for a semester: Traditional ways include taking a sabbatical or applying for a *Fulbright Fellowship*¹ to teach at an African university.

Teach in a summer school or intensive course: The *IMU Volunteer Lecturer Program*² brings lecturers to mathematics departments in universities in the developing world, to teach intensive 3–4 week courses in mathematics at the advanced undergraduate or master's level. Courses for a Master of “Mathematics and Informatics” or similar subjects are also admissible. The lecturer receives funding for all living expenses, including travel.

The African Institute for Mathematical Sciences (AIMS) has launched the *AIMS-Next Einstein Initiative*,³ a pan-African network that offers post-baccalaureate training in the mathematical sciences to African students. AIMS centers are located in South Africa, Cameroon, Ghana, Rwanda, Senegal, and Tanzania. The academic year at AIMS centers consists of three phases: the skills, review, and research phases, each subdivided into modular three-week blocks. To teach a three-week bootcamp-style course, one can submit a structured master's course proposal to AIMS.

The *Centre International de Mathématiques Pures et Appliquées (CIMPA)*,⁴ founded in France in 1978, is a nonprofit organization that aims to build mathematics research in developing countries. CIMPA supports about twenty Research Schools per year, offering research-level courses in mathematics and short-term thematic programs.

Travel Grants: The *African Mathematics Millennium Science Initiative (AMMSI)*⁵ supports travel grants and scholarships for African master's and doctoral students to study abroad.

The *IMU-Simons Africa Fellowship Program*⁶ supports research sabbaticals for mathematicians who are from developing countries in Africa and are employed in Africa. The fellowships support travel to internationally recognized mathematical centers worldwide for collaborative research. The grants are for a limited period and cover travel and basic living costs.

The *Association for Women in Mathematics*⁷ awards travel grants on a competitive basis; they can be used for research travel anywhere, including Africa.

Grant Opportunities: The *Carnegie Corporation of New York*,⁸ a grant-making foundation, supports higher education and research in Africa, particularly Sub-Saharan Africa, with the aim of fostering Africa's knowledge-based, globally competitive economy. Carnegie funds scholars, academic institutions, and continental networks.

The *National Science Foundation's ANESA program*⁹ provides funding to build research collaborations with African scientists and engineers and to give US students research experiences in Africa. Participation of junior investigators from both the US and the host country is strongly encouraged. Support is available to all disciplines funded by the NSF.

Activities of the IMU: The *African Diaspora Mathematicians Program*¹⁰ of the IMU's Commission for Developing Countries taps the expertise of Diaspora mathematicians, in a more formal manner, in order to provide additional staff and support to mathematics departments.

The *Graduate Research Assistantships in Developing Countries (GRAID)*,¹¹ recently launched by the IMU, allows graduate students to study mathematics full-time when no other fellowships are available. The effort is funded by donations to the Friends of the IMU.¹²

The *IMU's Mentoring African Research in Mathematics (MARM)*¹³ program links African academics with their counterparts in the United Kingdom and Europe, via professional mentoring partnerships. The program is implemented through collaboration of the IMU, the London Mathematical Society, and the African Mathematics Millennium Science Initiative.

Other ways to connect: The *Pan-African Mathematics Olympiads*¹⁴ are events of the African Mathematics Union, organized each year in an African country. Promising students who are less than twenty years old are invited to compete.

The *Carnegie African Diaspora Fellowship Program*¹⁵ allows institutions in several African countries to host an African-born scholar to work on projects in research collaboration, graduate student teaching/mentoring, and curriculum development.

The *African Women in Mathematics Association*¹⁶ was established in 2013 to promote women in mathematics and encourage girls to study the subject.

You don't need to work through a foundation or an agency: You can reach out to African colleagues on an individual, informal basis and ask how you can help.



LETTER TO THE EDITOR

Women Underrepresented in Mathematics? Not Everywhere!

I was moved when I saw the name of Mary Dolciani on page 304 of the March issue of the *Notices*. We were colleagues at Hunter College and the Graduate Center of the City University of New York in the 1970s and early 1980s, and I remember her as a wonderful person: highly sensitive to the students, open to the world, friendly to all colleagues, sometimes looking so happy as if she were in her twenties.

I never realized she had been a kind of pioneer among US female mathematicians; I myself never questioned the place of women in mathematics. I graduated from the University of Coimbra, in Portugal, in 1962. We were 11 males and 16 females. In Portugal, mathematics attracted more girls than boys. Nowadays, the math department in Coimbra has more women than men at the top faculty level: six females and five males as full professors, plus three females and one male as associate professors holding a degree which is the equivalent of the German *Habilitation*; not to forget two emeritus professors, one male and one female!

My comment: In the absence of discrimination women mathematicians flourish!

—*J. M. S. Simões-Pereira*
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(Received April 15, 2018)



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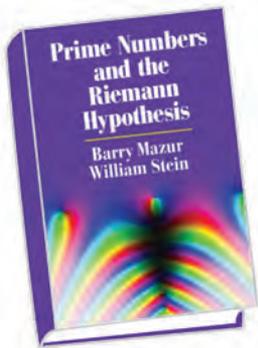
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Prime Numbers and the Riemann Hypothesis

Donal O'Shea

Communicated by Harriet Pollatsek

Prime Numbers and the Riemann Hypothesis

Barry Mazur and William Stein

Cambridge University Press; 1st edition (April 11, 2016)

150 pages, \$20.49

ISBN-13: 978-1-1074-9943-0

Introduction

The centenary of Hilbert's problems and the announcement of the Clay Institute's millennial prize problems resulted in a number of very good trade books about the Riemann Hypothesis. In their book *Prime Numbers and the Riemann Hypothesis*, Barry Mazur and William Stein set themselves [p. vii] a different goal:

A reader of these books will get a fairly rich picture of the personalities engaged in the pursuit [of the Riemann Hypothesis], and of related mathematical and historical issues. That is *not* the mission of [this book]. We aim—instead—to explain, in a manner as direct as possible and with the least mathematical background required, what this problem is all about and why it is so important.

Mazur and Stein are distinguished mathematicians who have made significant contributions in areas the Riemann Hypothesis touches. Both care deeply about teaching. Mazur writes like an angel, and Stein is the

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founder of SageMath, a powerful open-source mathematical software system, and CoCalc. So, if anyone could write an account of this sort, it is they. Nonetheless, the goal they set themselves is audacious and highly non-trivial. Even today, Riemann's paper is tough going for those without specialized knowledge in the area. Writing mathematically meaningful, but readily comprehensible, text about a highly technical subject for serious readers is an undertaking not to be underestimated. When that subject lies deep enough to have consequences in many other areas, that difficulty is compounded.

Mazur and Stein succeed brilliantly. The first half of their book assumes no knowledge of calculus, the next third requires some knowledge of real differential calculus, and the last sixth some complex analysis. I cannot think of an interested reader who will not profit from this little book. It will fascinate anyone who enjoys mathematics. It provides amateurs and students with an attractive and accessible entry to a compelling, but difficult realm of mathematics. Professional mathematicians and mathematical scientists will love it—so will electrical engineers and computer scientists. It deserves to become a classic.

Riemann and His Hypothesis

In 1859 at age thirty-two, Bernhard Riemann was appointed to the chair at the University of Göttingen previously held by his teachers Gauss and Dirichlet. The death of Dirichlet, to whom Riemann was very close, had left the chair vacant. For Riemann, the appointment would have been bittersweet. He desperately needed the income and recognition, but Dirichlet was one of his closest friends, and one of the few who understood him and his mathematics. Shortly afterwards the Prussian Academy of Sciences in Berlin elected Riemann as a corresponding member, and Kronecker encouraged

him to submit a paper describing his research. Whether by design or serendipity, Riemann had been exploring some ideas that had intensely interested Gauss and Dirichlet. The resulting hastily-written, eight-page paper introduced the complex-valued function now known as the *Riemann zeta function* and related it to the number $\pi(X)$ of prime numbers less than or equal to a given quantity X . It introduced a host of astonishing new ideas and techniques. As Selberg [2] points out, the paper was clearly a research note sketching some ideas to which Riemann intended to return. There were holes that needed to be filled and Riemann's mathematical notebooks subsequently made it clear that Riemann had more material to present. Riemann would die seven years later, without returning to the subject. At the time, the paper had little discernible impact, but in time it would completely transform analytic number theory and reverberate across mathematics.

Thirty-five years later, Riemann's paper was better understood. The Prime Number Theorem, originally conjectured by Gauss and equivalent to the statement that no zeroes of the zeta function have real part equal to 1, was established independently by Hadamard and de la Vallée Poussin in 1896. But a statement, now known as the *Riemann Hypothesis*, that Riemann conjectured would give much better information, remains unsolved to this day. Hilbert included it as the eighth problem among his list of twenty-three problems for the twentieth century. After giving a nod to then-recent progress on the distribution of prime numbers, he writes [2, p. 456]

It still remains to prove the correctness of an exceedingly important statement of Riemann, viz., that the zero points of the function $\zeta(s)$ defined by the series

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \dots$$

all have the real part $\frac{1}{2}$, except the well-known negative integral zeros. As soon as this proof has been successfully established, the next problem would consist in testing more exactly Riemann's infinite series for the number of primes below a given number and, especially, to decide whether the difference between the number of primes below a number x and the integral logarithm of x does in fact become infinite of an order not greater than $\frac{1}{2}$ in x . Furthermore, we should determine whether the occasional condensation of prime numbers which has been noticed in counting primes is really due to those terms in Riemann's formula which depend upon the first complex zeroes of the function $\zeta(s)$.

In the late 1990s, the Clay Mathematical Institute surveyed a representative group of mathematicians as to what they considered the most pressing open problems for the new millennium. There were, of course, differences of opinion, but all approached included two problems on their lists, the Poincaré Conjecture (now solved) and the Riemann Hypothesis. Both were among

the seven "millennial" problems for the solution of each of which the Institute offered a one million dollar prize.

The Book

The authors begin by sorting their readership into three groups: 1) interested individuals who may not have had a calculus course, but who are comfortable with the notion of a graph of a function; 2) interested individuals who know some differential calculus; 3) interested individuals who know a little complex analysis. They then order their material accordingly. Part I, about seventy pages of the book, addresses the first group. Parts II and III, about forty pages in total, address the second group. Part IV, the shortest at twenty-five pages, addresses the third group, as do the ten pages of endnotes (which are terrific).

The first part opens with a careful discussion of primes, sieves, the frequency of primes, and the behavior of $\pi(X)$ as X grows larger. The prose is spare, but engaging and informal. In addition to photographs of individuals (regrettably all male, all but one white), and excerpts from notebooks (see Figure 3), there are many tables and lots of well-chosen graphs (most generated by SageMath, with code available online). The function $\text{Li}(X)$ is simply defined as the area from 2 to X under the graph of $1/\log(x)$. No fuss, no over-explanation, no integral signs. The authors zoom out of the graph of the step function $\pi(X)$, looking at it over larger and larger intervals. The reader clearly sees the graph that becomes a smooth curve at the resolution of the human eye and that the graph of $\frac{X}{\log(X)}$ appears to diverge from $\pi(X)$ faster than $\text{Li}(X)$.

Tables are used to show the distinction between going to infinity at the same rate (as many leading digits as desired can be made to coincide) and to within square root error (first half of leading digits coincide). A wonderful discussion that links square root error to random walks follows. This is done by simulating and plotting lots of random walks on the line (see Figure 1), and it firmly establishes that square root error is the gold standard: the best that one can expect.

The authors present their first formulation of the Riemann Hypothesis as the statement that Gauss's logarithmic integral $\text{Li}(X)$ is a square-root-close approximation to $\pi(X)$. This, of course, is the second part of Hilbert's statement of the Riemann Hypothesis, and their discussion allows the reader to appreciate the contrast between the Prime Number Theorem and the Riemann Hypothesis.

From here, the account pivots to a discussion of the information contained in $\pi(X)$, and in the graphs of related functions that contain the same information, which the authors refer to as carpentry on the staircase of primes. In particular, the authors consider the step function $\psi(X)$ that increases not just at prime numbers by 1, but at all prime powers by the logarithm of the prime, and provide a second restatement of the Riemann Hypothesis as the assertion that $\psi(X)$ is square-root-close to $y = X$, the graph of which is the straight line in the first quadrant at 45 degrees off the X -axis.

The discussion about preservation of information and the utility of different representations of functions leads

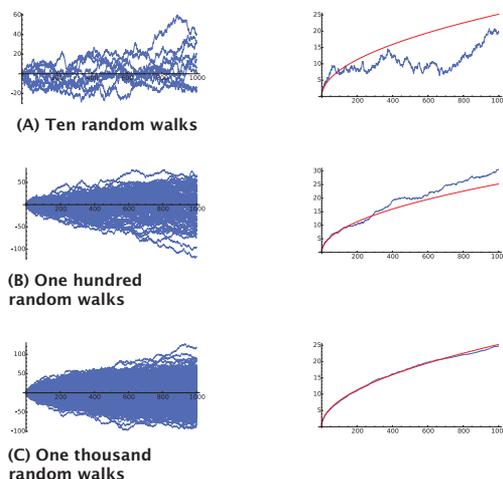


Figure 1. A wonderful discussion links square root error to random walks on the line. The figures on the left plot random walks starting at the origin; the blue curves on the right plot the average distance from the origin after X steps of the random walks on the left, and the red line is the graph of $\sqrt{\frac{2}{\pi}} \cdot \sqrt{X}$.

naturally to a discussion of how the spectrum (that is, the collection of frequencies), amplitudes, and phases of a trigonometric sum more efficiently encode the information in the function than does sampling points on its graph. These discussions lead in turn, and seemingly ineluctably, to the notion of capturing the information in the graph of $\pi(X)$ by using something akin to Fourier analysis.

So ends Part I. It tells a complete and satisfying story, and a reader with no knowledge of calculus who leaves off at this point will have learned a great deal.

Parts II and III are aimed at readers with some, but minimal, differential calculus, and continue the information theme. The authors use the same pedagogical techniques as in the first section (many of which are nowadays used in reform calculus texts). They begin with a brief introduction to generalized functions, starting with the Dirac delta function, introduced as the limit of the derivatives of bump functions approximating a step function with one jump discontinuity. Since they assume a little familiarity with calculus, they are able to be precise about convergence. They discuss Fourier transforms of distributions with discrete support, transforms of trigonometric sums, and spike values. All this is done with lots of graphs.

With this in hand, they move to two trigonometric series that are related to one another in that the frequencies (the logarithms of prime powers) of the first are the spike values (the points of divergence) of the second, and vice versa the spike values of the first are the frequencies (or “prime spectrum”) of the other. The first infinite sum comes right out of information in $\pi(X)$, and the graphs of the trigonometric sums approximating the series are computed, so the reader can see the graphs of these sums beginning to spike at the spike values.

There are many details here, and one of the delightful things is that the authors guide readers through the

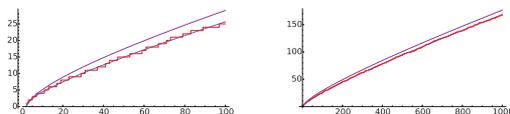
thicket, pointing out the essentials, and indicating what is central and what are conveniences. The calculus that the authors ask of their readers is not a facility with computation, but that the reader be comfortable with the key ideas of derivative and definite integral. For readers who have forgotten most of their calculus, the book will underscore the utility of those key concepts. There are many references to the literature and to sites where computations of the spectrum have been done, and I look forward to returning to the book to follow up on these.

The authors had clearly intended [p. 113] to provide a third restatement of the Riemann Hypothesis in terms of the concepts they developed in Parts II and III, but somewhere in the course of the many revisions of the book, this restatement went missing. Although this tacitly leaves an exercise for the reader, the omission is unfortunate as the restatement would have given the reader a sense of closure.

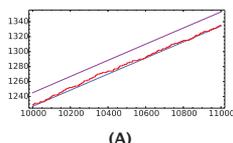
The book closes with Part IV, a beautiful short section, aimed at the reader with some knowledge of complex analysis. The authors return to Riemann’s paper and present Riemann’s formula for $\pi(X)$. Here again, the authors plot the graphs of the function $R(X)$ that Riemann used (see Figure 2), and the reader sees how much better Riemann’s successive approximations are to $\pi(X)$ than $\text{Li}(X)$.

The authors [p. 124] make the crucial point that the error terms can be explicitly computed once one knows the zeroes of the zeta function, and provide the “fourth” and final restatement of the Riemann Hypothesis as the conjecture that all non-trivial zeroes of the zeta function have imaginary part equal to $\frac{1}{2}$. This, of course, is the famous version that Hilbert states, and with which we are all familiar. The authors go on to write:

That a simple geometric property of these zeroes (lying on a line!) is directly equivalent to such profound (and more difficult to express) regularities among prime numbers suggests that these



(A) Comparisons of $\text{Li}(X)$ (top), $\pi(X)$ (middle), and $R(X)$ (bottom, computed using 100 terms on the left and 1000 terms on the right).



(B) Closeup comparison of $\text{Li}(X)$ (top), $\pi(X)$ (middle), and $R(X)$ (bottom, computed using 11000 terms).

Figure 2. Riemann’s initial approximation to $\pi(X)$ is much better than $\text{Li}(X)$.

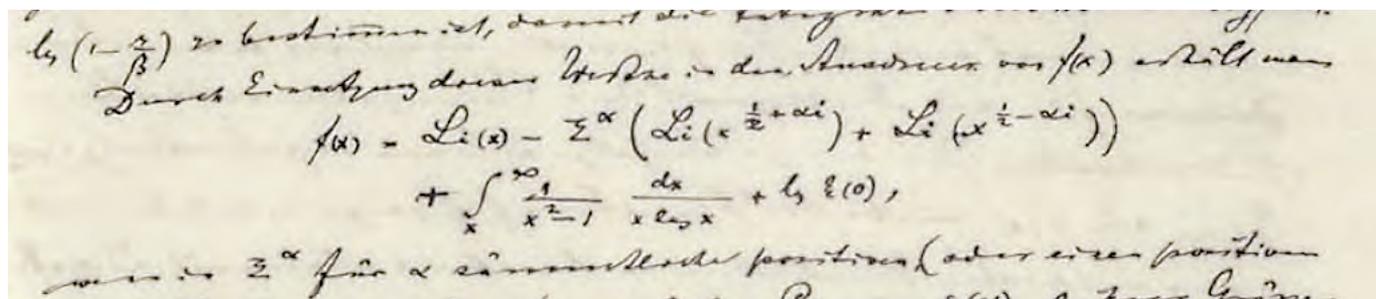


Figure 3. Riemann’s exact formula for $\pi(X)$.

zeroes and the parade of Riemann’s corrections governed by them—when we truly comprehend their message—may have lots more to teach us, may eventually allow us a more powerful understanding of arithmetic. ... Are the primes themselves no more than an epiphenomenon, behind which there lies, still veiled from us, a yet-to-be-discovered, yet-to-be-hypothesized, profound conceptual key to their perplexing orneriness?

The final chapter in Part IV exemplifies what is so special about the book. The authors want the reader to understand that the Riemann zeta function is one of a family of functions that arise in many other contexts. In these last few pages of the book, they could have been forgiven for going wild and citing loads of interesting, inscrutable examples. They do not. Rather, they concentrate on a single example that their hypothetical reader will understand, the distribution of Gaussian primes. They do not take shortcuts. They define norms, Gaussian integers and units, and then Gaussian primes. They plot the distributions of Gaussian primes $a + bi$ in the first quadrant of the plane with $a, b \leq 10$, and then $a, b \leq 100$. They then look at the staircase of primes (the graph of the number of Gaussian primes with norm less than or equal to X), and produce the graphs for norms up to 10, 100, 1000, 10000. In the process of zooming out, one sees the graphs approximating a smooth curve, and one

has no difficulty believing their assertion that there is an analogue of the zeta function in this context. Only then do they mention that there is a larger story, the Grand Riemann Hypothesis, and give the example where zeta-type functions count solutions of polynomials of several variables in finite fields of characteristic p .

Seldom have I, as a reader, felt so respected. This is superb teaching, and beautiful writing.

Conclusion

The authors say that they wrote the book over a period of ten years, getting together for one week during the summers. Nonetheless, the book itself is highly disciplined. It is short and very attractively laid out. The four parts consist of a number of short chapters, which are perfect for sporadic reading.

The authors have given the mathematical community a great gift. Buy the book for yourselves, your friends, and your students.

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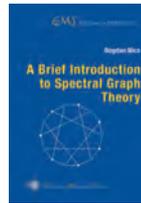
ABOUT THE AUTHOR

Donal O’Shea came to New College after thirty-two years at Mount Holyoke College. He is author or co-author of a number of books including *The Poincare Conjecture*; *Ideals, Varieties and Algorithms*; *Using Algebraic Geometry*; *Calculus in Context*; and *Laboratories in Mathematical Experimentation*.



Donal O’Shea

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A Brief Introduction to Spectral Graph Theory

Bogdan Nica, *McGill University, Montreal, Canada*

Spectral graph theory starts by associating matrices to graphs—notably, the adjacency matrix and the Laplacian matrix. The general theme is then, first, to compute or estimate the eigenvalues of such matrices, and, second, to relate the eigenvalues to structural properties of graphs. As it turns out, the spectral perspective is a powerful tool. Some of its loveliest applications concern facts that are, in principle, purely graph theoretic or combinatorial. This text is an introduction to spectral graph theory, but it could also be seen as an invitation to algebraic graph theory.

EMS Textbooks in Mathematics, Volume 21; 2018; 168 pages; Hardcover; ISBN: 978-3-03719-188-0; List US\$48; AMS members US\$38.40; Order code EMSTEXT/21



Geometric and Topological Aspects of Coxeter Groups and Buildings

Anne Thomas, *University of Sydney, Australia*

Coxeter groups are groups generated by reflections. They appear throughout mathematics. Tits developed the general theory of Coxeter groups in order to develop the theory of buildings. Buildings have interrelated algebraic, combinatorial and geometric structures and are powerful tools for understanding the groups which act on them. These notes focus on the geometry and topology of Coxeter groups and buildings, especially nonspherical cases. The emphasis is on geometric intuition, and there are many examples and illustrations.

Zurich Lectures in Advanced Mathematics, Volume 24; 2018; 160 pages; Softcover; ISBN: 978-3-03719-189-7; List US\$39; AMS members US\$31.20; Order code EMSZLEC/24

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SUSAN SCHWARTZ WILDSTROM

Mathematics Teacher, Walt Whitman High School
in Bethesda, MD.
AMS member since 2000.

"I am a high school mathematics teacher and a member of the AMS. I have become an active supporter since the AMS has become more involved in pre-college activities. These initiatives include financial support for many excellent "math camps" through the Epsilon Fund; Who Wants To Be A Mathematician contests at JMM and other events; and the AMS booth at the USA Science and Engineering Festival, which for the past couple of fairs has featured a popular curve-stitching activity that I designed and managed for them."



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Free and Fair Open Access Journals: Flipping, Fostering, Founding

Mark C. Wilson

Communicated by Harriet Pollatsek

Note: The opinions expressed here are not necessarily those of Notices.

ABSTRACT. We make a case for free fair open access journals, which can be obtained by flipping existing journals, fostering volunteer-based journals, or founding new journals.

There is widespread and longstanding dissatisfaction with the current state of scholarly publishing. Space constraints here preclude full discussion—I recommend the online forum <https://gitlab.io> as the best entry point for those interested in more information and action. Recent developments in internet-based publishing technology, and increased emphasis on open access by research funders, give some cause for optimism.

Perhaps the most frequently voiced criticism is the high and rapidly rising cost of journal subscriptions. The consolidation of journal publishing in the last few decades has led to dominance by a few publishers such as Elsevier and Springer, who have exerted their power in a dysfunctional market. There is no true price competition because journal articles are not substitutable for one another, and there is little incentive to compete on quality (“repu-

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tation” and quality of editorial and journal processes are definitely not the same thing). Even when libraries want to cut a subscription, the pricing of bundles (so-called “Big Deals”) by the large publishers often makes it pointless. The historical overview [1] explains well how we got into the current highly suboptimal situation.

Even though more and more money is being spent, those without subscriptions are still denied access. For various good reasons, including public funders wanting more impact of the research they fund and authors wanting more citations, the model of open access to research publications has been gaining substantial ground in the last few years. Publishers have responded to pressure from research funders by offering open access journals (this is known as “Gold OA” as opposed to “Green OA,” which involves authors self-archiving).

These are usually funded by author publication charges (APCs) on the order of several hundred to several thousand US dollars/euros per article. Examples in mathe-

“Reputation” and quality of editorial and journal processes are definitely not the same thing.

mathematics include Cambridge University Press's *Forum of Mathematics* and the American Mathematical Society's *Proceedings of the American Mathematical Society, Series B* and *Transactions of the American Mathematical Society, Series B*. Publishers such as Elsevier and Springer are fond of the per-article open access model ("hybrid journals"), where APCs are often \$3000 or more, and even for fully OA journals their APCs are usually well over \$1000. The mathematics journals above charge closer to the latter figure, but this is still considerably more than the proper price of commercial production, usually estimated at less than \$500 per article (an upper bound on the charges of several new full-service commercial publishers such as Ubiquity Press).

Gold OA with direct author charges may work in highly funded disciplines where the publication charge is a small fraction of the grant funding needed to do the research in the first place, but is opposed by researchers in many less expensive disciplines, including mathematics: for example, in a 2016 survey of 1000 mathematicians [2], about a quarter of respondents opposed author charges in principle, no matter what their level. Furthermore there has been a recent cooling of enthusiasm by funders for the Gold model. It does not exert any downward pressure on prices, and it is wasteful of public money. There is still no good incentive for publishers to compete genuinely on quality—they simply run down the accumulated "capital" of reputation developed by the research community.

—

*A third
[model] exists:
journals
controlled by
the scholarly
community.*

—

An alternative, historically earlier, open access model is based on noncommercial journals, usually hosted by academics on department webpages, often using free open source software such as Public Knowledge Project's Open Journal Systems. Such journals typically have zero charge for authors, are often small, and run on a very low budget and dedicated volunteer time. The term "Diamond OA" is sometimes used.

The Diamond model can be seen as wasteful of the time of highly skilled researchers, sustainability can be a problem, and innovation can be difficult owing to lack of resources. For example, websites of such journals are often very basic, with no thought of mobile-phone-friendly articles. Mathematics is relatively well endowed with such journals. Examples surviving for over two decades include *Theory and Application of Categories*, *Electronic Journal of Combinatorics*, *New York Journal of Mathematics*, and *Electronic Journal of Differential Equations*. Robert Rosebrugh's description [3] gives a good idea of what is involved.

A third way exists: journals controlled by the scholarly community, with publishers competing on price and service, and with the relatively low production costs gladly paid for by libraries. This has been formalized recently via the Fair Open Access Principles, and is the main focus of this article.

The Fair Open Access Principles are:

- 1) The journal has a transparent ownership structure, and is controlled by and responsive to the scholarly community.
- 2) Authors of articles in the journal retain copyright.
- 3) All articles are published open access and an explicit open access license is used.
- 4) Submission and publication is not conditional in any way on the payment of a fee from the author or their employing institution, or on membership of an institution or society.
- 5) Any fees paid on behalf of the journal to publishers are low, transparent, and in proportion to the work carried out.

For clarification and explanation of the reasoning behind these principles, see: www.fairopenaccess.org.

Diamond open access journals often fail to satisfy the second half of (3), and sometimes fail (2) and/or (1). Subscription-reallocation schemes such as high energy physics' SCOAP3 do not address (1) or (5) properly, and have led so far to payments to publishers such as Elsevier and Springer of more than twice the basic market rate per article. Principle 1 is key—once ownership of a journal is ceded to a for-profit publisher, there is little chance of a good overall outcome.

Wholesale conversion of the journal system to the Fair OA model can occur via a combination of three main approaches. *Flipping* refers to converting an existing journal from subscription to Fair Open Access. Another way involves *fostering* the existing volunteer-based journals by adding money for extra support services. Finally, we can *found* new Fair OA journals, which can make use of modern tools to operate efficiently. All three approaches have been used recently in mathematics, as we now show by examples.

The editorial board of *Journal of Algebraic Combinatorics* resigned en masse in 2017 and set up the journal (under the name *Algebraic Combinatorics*) at Centre Mersenne (algebraic-combinatorics.org). Funding has been provided by Foundation Compositio Mathematica and French and German library organizations. Note the change in name of the journal, which is typical with flipping. A journal represents a community, is a brand, and is also a legal entity. The first two are typically determined by the composition of the editorial board and the papers published in the journal, while the third typically is under control of the publisher. The survey [2] showed that peer review quality and editorial board research record are seen as massively more important than the identity of the publisher. The outcome of mathematics journal defections from commercial publishers is very positive: the old journal usually ceases to publish within a few years, and the new one's reputation increases [4].

Another recent example is the major linguistics journal *Lingua*, whose board defected from Elsevier in 2015 and founded *Glossa*, currently published by Ubiquity Press. However to my knowledge *J. Algebraic Combinatorics* is the only journal in mathematics that has taken this route. Several others have successfully declared independence from their commercial publisher [5], but all continued with a subscription model rather than OA.

The Free Journal Network (freejournals.org) aims to nurture an ecosystem of existing independent open access journals. This involves setting up a way for them to share information, best practices, and logistical support while preserving their independence. It also provides a higher profile for the journals and assists in improving their offerings (for example, listing in DOAJ, assigning DOIs). The Directory of Open Access Journals (doaj.org) has reasonably strict requirements for membership, which rule out obviously substandard or predatory journals. However it does not require Principles 1, 4, or 5. Some well-known open access mathematics journals are not (yet) members of DOAJ, and membership in the latter will likely become a requirement for FJN membership from 2019.

Recent examples of scholar-founded journals complying with the Fair Open Access Principles and where publishing is outsourced include *Discrete Analysis* (from 2016, using Scholastica), *Epijournal de Géométrie Algébrique* (from 2017, using Episciences), and *Annales Henri Lebesgue* (from 2018, using Mersenne). These all run on a low cost model, involving small subsidies from academic institutions, charitable donations, and volunteer work. Two use the “arXiv overlay” idea, making article submission almost trivial.

All three methods are important in advancing the cause of Fair Open Access and have different advantages. Flipping generates substantial publicity and puts direct pressure on the large commercial publishers to yield market power. It has a big payoff, if the reputation and skills developed by the old journal can be transferred to the new one. Fostering existing reputable volunteer-based journals is a way of quickly creating a sustainable alternative to the large commercial publishers. Founding new journals allows the community to set up journals that satisfy all the Fair Open Access Principles by design and that may include many other innovations in editorial policy, website design, etc.

Of course, each method has its associated negatives. Flipping may be considered to be confrontational, and involves many perceived risks for the editors. Fostering already existing journals and founding new ones present a much less immediate challenge to the power of the legacy journals. For new journals, gaining reputation can be a long process.

Some people express concern that new non-commercial journals will not be able to assure long-term access to the results they contain. It is important to note that this problem can also occur with large commercial publishers. For example, back-issues of Springer’s *K-Theory* disappeared for years (see <https://gitlab.com/publishing-reform/discussion/issues/22>). More positively, organizations like LOCKSS are designed to preserve journal

articles, and as mentioned above, Diamond OA journals have already existed stably for 25 years. Because papers are open access, they may be freely shared, copied, and collected, which makes losing them less likely than losing subscription content.

Flipping to Fair Open Access has not been tried in an organized manner on a large scale until recently. The new organizations LingOA, MathOA, PsyOA, and the overarching Fair OA Alliance have been set up to facilitate conversion of journals, by finding funding, giving legal and logistical advice, and acting as an intermediary between editorial boards and publishers. The main visible success to date in mathematics is *Algebraic Combinatorics*, but intensive work is continuing.

Existing Fair OA journals have not been organized until very recently, and this work is still in its early stages—the Free Journal Network was begun officially in February 2018. At time of writing it has 16 member journals in mathematics, including several of the scholar-run journals mentioned above.

To my knowledge there has been no large-scale attempt to start new journals conforming to the Fair Open Access Principles. Of course, lack of funding may be an issue, although in mathematics at least, such journals can be run on very low budgets: essentially zero if volunteer time by editors and basic IT support by universities are not counted, and well under \$100 per article otherwise, at least with the modern trend not to provide major copyediting.

If the journal publication system were designed from scratch by the research community, it would presumably conform closely to the Fair Open Access Principles. Getting from where we are now to a much superior system will be much easier with substantial community support. I urge all readers to make clear their support for the Fair Open Access Principles.

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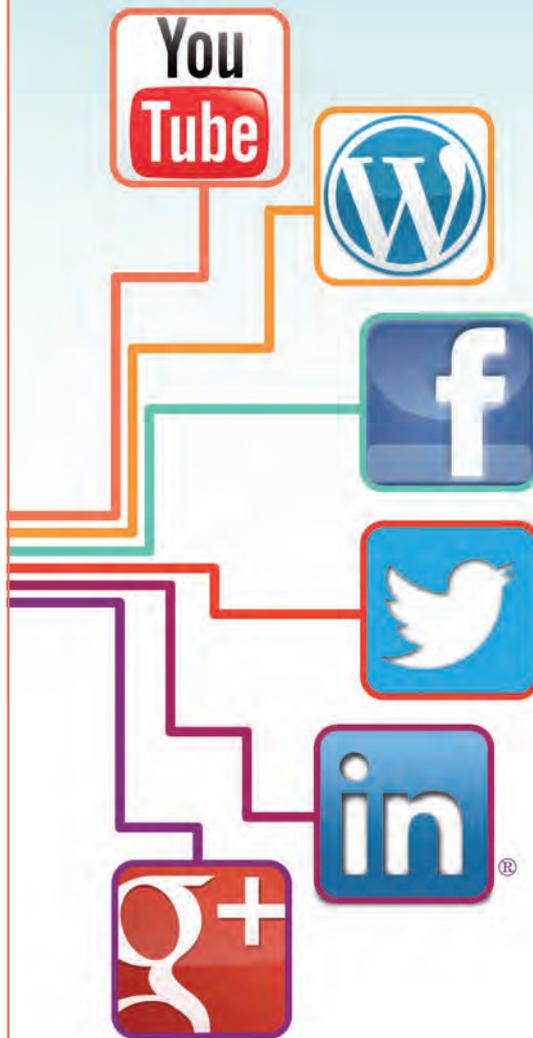


Mark C. Wilson

ABOUT THE AUTHOR

Mark C. Wilson received a PhD in mathematics from University of Wisconsin-Madison. His research interests involve analytic combinatorics, social choice theory, and network science. He is a founding board member of MathOA and signatory of the Elsevier boycott. Once scientific publishing utopia is reached, he will concentrate more on his research, singing, and foreign languages.

AMS on Social Networks



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Students' Combinatorial Reasoning: Counting Processes and Sets of Outcomes

Elise Lockwood

Communicated by Benjamin Braun

My research focuses on undergraduate students' reasoning about combinatorics, and this note specifically concerns "counting problems" – determining the cardinalities of sets that satisfy certain conditions. These problems involve a wide variety of reasoning skills, but my major findings all elaborate a central theme:

Students should focus on the sets of outcomes they are trying to count, not just the counting process.

Although it might be clear to mathematicians that counting is fundamentally focused on sets of outcomes, this is not always how students view counting; nor is it always how students are taught to count in university courses. My goal here is to share highlights from my research on teaching discrete mathematics, combinatorics, and probability.

In 2013 I [1] introduced a model of students' combinatorial reasoning with three interconnected components: *formulas/expressions*, *counting processes*, and *sets of outcomes* (Figure 1). *Formulas/expressions* are mathematical expressions that yield numerical values that are often considered the "answer" to a counting problem, for example $\binom{8}{5} \cdot \binom{4}{3}$ or $\frac{9!}{2!} - 8$. *Counting processes* are the enumeration processes in which a counter engages as they solve a counting problem. For example, a counting process to enumerate all arrangements of the letters A, B, C, and

D might involve using the multiplication principle in a four-stage process of first considering which letter can be in the first position, then considering which letter can be second, then third, and then fourth. *Sets of outcomes* are the collections of objects being counted, and these can often be organized in a way that reflects the counting process used.

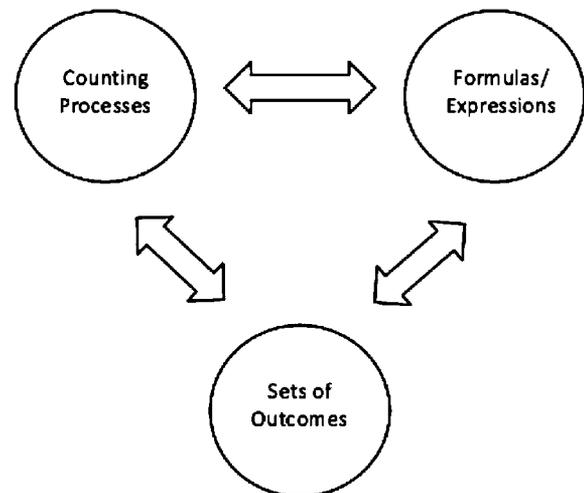


Figure 1: Lockwood's model of students' combinatorial thinking has three interconnected components [1,5].

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In my own experience, it has been surprising to see how frequently students attempt to solve counting problems without giving explicit attention to what they are trying to count. I use the phrase *set-oriented perspective toward counting* to refer to the “way of thinking about counting that involves attending to sets of outcomes as an intrinsic component of solving counting problems” [2, p. 31]. My research suggests that by explicitly teaching students to develop a set-oriented perspective toward counting, instructors can achieve the following three goals.

1. Help students reason about common problematic issues that arise in counting, particularly determining whether or not order matters and identifying/correcting overcounting. Sets of outcomes can help to illuminate for students common errors in solely process-based reasoning. For example, the phrase “order matters” (which is not always well understood by students) means that one needs to consider the nature of the outcomes, whether the problem is counting *sets* or *sequences*, for example. For problems whose outcomes are sets, order “does not matter” because different arrangements of elements within the sets do not create distinct outcomes. Students who focus on the nature of what is being counted to determine whether order matters may gain a better understanding of their solution methods than those who merely attempt to fit given formulas to certain kinds of problems.

Students can also use the relationship between counting processes and sets of outcomes to reason about overcounting. To help students see this, instructors can give problems in which overcounting naturally occurs and have students reflect on how counting processes generate too many outcomes. The following problem (adapted from Tucker [4]) is a favorite of mine to introduce how a counting process can overcount a set of outcomes.

How many ways are there to form a three-letter sequence using the letters a, b, c, d, e, f :

- (a) without repetition and containing the letter e ?
- (b) with repetition and containing at least one e ?

Part (b) has a common incorrect answer of $3 \cdot 6 \cdot 6$. In order to explain why this answer does not work, students need to very clearly understand how the counting process reflected by $3 \cdot 6 \cdot 6$ generates too many outcomes. Focusing on the relationship between counting processes and sets of outcomes can help to illuminate the phenomenon of overcounting.

2. Help students understand the relationship between counting processes and sets of outcomes through the use of explicit listing. We want students to be able to explicitly generate and structure sets of outcomes to explain their reasoning. B. Gibson and I [3] found a positive correlation between listing outcomes and solving problems correctly among novice undergraduates. Our findings suggest that creating explicit lists is a worthwhile activity that grounds students’ thinking and activity in sets of outcomes, even though students might find it mundane. Students will necessarily need to move on to more efficient methods and formulas, but initial listing activity can help

them orient themselves to what they are trying to count. Furthermore, by understanding the relationship between counting processes and sets of outcomes, students can become more flexible counters, who are willing to consider alternate processes and problem-solving approaches.

3. Help students justify key formulas. Too often we present students with formulas for basic combinatorial operations, and they struggle to make sense of them even when a combinatorial explanation has been provided. This is indicative of a broader phenomenon in mathematics in which students may not understand formulas, but it feels especially relevant in combinatorics, where a few key formulas form the basis for much of counting. My research has found that students can make sense of, justify, and even develop formulas if they are given opportunities to reason about counting processes and sets of outcomes. Specifically, one idea is for instructors to give students a number of problems that involve different simple combinatorial operations (such as some permutation problems and certain combination problems), without first having presented formulas. Then, have students solve then categorize those problems and articulate what each kind of problem counts. Through this activity, students can engage in problem solving, reflect on their processes, and consider the nature of what they are trying to count. Students can either then generalize a formula themselves, or, if they are then presented with a formula, they can have a better understanding of why a formula makes sense, why an identity holds, or why a solution is correct. This development of student knowledge based on problem-solving stands in stark contrast to a scenario in which students are provided with a formula and then asked to apply that formula repeatedly.

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ABOUT THE AUTHOR

Elise Lockwood loves research and feels fortunate to be able to teach students how to count. She recently received an NSF Early Career award to study students' combinatorial reasoning in computational settings. She enjoys running, cooking, traveling, collecting *Rocket Raccoon* comics, and playing with her Ragdoll cats, Nicodemus and Sebastian.



Elise Lockwood

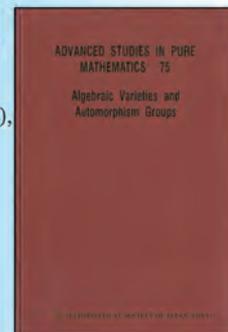
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Report of the Executive Director: State of the AMS, 2017



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Catherine A. Roberts, Executive Director

The American Mathematical Society continues to advance research and to create connections in the mathematics community. Notable highlights of the year include:

- The acquisition of the Mathematical Association of America (MAA) book program. Over 450 titles have been added to our rich publication program of books, journals, and MathSciNet®. In addition, the AMS published 78 new books, and we are seeing sales of our eBooks continue to grow.
- Extensive efforts went into creating a new logo and tagline:



- Over 6,100 people attended the Joint Mathematics Meetings in Atlanta. Almost a third of these attendees were students.
- In 2017, sixty-five new mathematical scientists from around the world were named Fellows of the American Mathematical Society. Among the many prizes and awards in 2017, the Leroy P. Steele Prize for Lifetime Achievement went to James G. Arthur.
- We launched our new Membership Department with Megan Turcotte as its Director.
- The National Science Foundation awarded us \$297,000 for travel grants to US mathematicians participating in the International Congress of Mathematicians in 2018.
- The AMS regularly sponsors a mathematician each year as a Congressional Fellow. Typically, only one or two of the over 300 scientists serving in this way are mathematicians. Our Office of Government Relations is pleased to let our community know that in 2017 there were five mathematicians (including our own AMS Fellow, Margaret Callahan) serving as Congressional Fellows.

Strategic Plan 2016–2020

How can the AMS best serve our members and the larger mathematics community? This question helped drive the formulation of a new AMS Strategic Plan. Just as the previous Strategic Plan from 1991 led to many effective changes, including the establishment of five policy committees to streamline our governance structure, our new plan offers a framework to move the Society forward with six overarching initiatives. The plan was approved by the Executive Committee and Board of Trustees in November 2015 and by the Council in January 2016.

Changes will continue to be rolled out as we implement this thoughtful and ambitious plan. Here are some highlights from 2017 in each of the six major initiatives:

1. Advocacy, Awareness, and Visibility

The AMS is a volunteer-driven organization with a spectacular range of valuable programs, publications, and services. We want to increase members' awareness of and participation in our offerings, including connecting people more efficiently with helpful resources, helping professionals become involved in advancing public policy around mathematics, and promoting a broader public appreciation for mathematics.

Our Strategic Plan calls for the creation of new and consistent branding across the AMS for its publications, programs, and services. A working group of our AMS president, representatives from our Board of Trustees, Council, and AMS staff collaborated with a design firm to update and improve our visual presence and messaging. Tremendous efforts are underway in our Computer Services Division to update our website with the new branding. The public spaces in our Providence headquarters were updated in 2017 with this branding and also now display the *Concinnitas* art prints, generously donated to us by David Mumford.

On the advocacy front, software was purchased to launch an AMS Grassroots Advocacy Network in 2018.

2. Develop and promote a coherent portfolio of programs, meetings, publications, and professional services

We are taking stock of what we do, evaluating it in terms of efficacy, content, and costs, to craft a portfolio that delivers on our mission for all mathematics professionals. We are identifying ways in which existing programs can reinforce one another, and determining whether new programs are needed to complement ones that already exist. In 2017, we concluded the initial phase of a systemic assessment of our meetings, programs, and activities. The Math in Moscow scholarship program will end after the final scholarships to support students in spring 2018 are granted. The AMS Activity Groups were terminated due to lack of interest.

3. Diversity and Inclusion

In 2016, Helen G. Grundman became our first Director of the newly established Department of Education and Diversity. Her initial focus is on graduate education in the mathematical sciences, preparation of students entering graduate programs, mentoring for success in graduate school, and the promotion of diversity and inclusiveness at the graduate level. This office joined several other organizations in organizing the highly successful panel “The Mathematics and Mathematicians Behind Hidden Figures” at the 2017 Joint Mathematics Meetings.

4. Mathematical Reviews/MathSciNet

Research mathematicians recognize the value provided by MathSciNet, our online database of the mathematical literature containing expert reviews and additional resources. The AMS will ensure that the Mathematical Reviews database evolves with the changing needs of researchers and with advances in technology. In order to integrate MathSciNet into the daily habits of mathematicians, we are improving the user interface, creating new features and tools, and building partnerships with valued resources such as the arXiv and MathOverflow.

5. Membership Development

To be successful, we need to serve each rising generation of mathematics professionals and to keep our communication channels open in order to hear what the membership needs. In 2017, we established a Membership Department and hired Megan Turcotte as its first Director.

The AMS supports you and all mathematicians at many stages of your careers. Our unified presence helps advocate for funding, policies, and programming, as well as improving collaboration across fields of study. The AMS helps us connect as a community of mathematicians through our meetings and conferences, with programs like the Mathematics Research Communities (MRC), and with our many excellent publications. We develop and maintain products like MathSciNet, MathJobs.Org, and MathJax™ to support your work.

We have a presence in Washington, DC, to advocate for NSF funding and other matters that impact mathematics. We also honor mathematicians through the AMS Fellows and many prizes and awards. With the generous help of donors, we established new awards in 2017, including the Joan and Joseph Birman Fellowship for Women Scholars and the Mary P. Dolciani Prize for Excellence in Research. We also received a generous challenge grant that led us to begin fundraising for a new Campaign for the Next Generation. You'll hear more about all these exciting opportunities in the coming months.

6. Publishing

There are more than a dozen projects underway as part of our implementation of the AMS Strategic Plan within Publishing. To accommodate the growth in the volume of research literature, the AMS is striving

to publish more high quality content through our book and journal programs. In 2017, we acquired the MAA book program, which adds over 450 new titles to our library.

We are always working to develop innovative tools for research and teaching. With blogs, eBooks, and other emerging forms of publishing, we continue to enrich the professional work of mathematicians. For example, in January 2017, we launched AMS Open Math Notes—ams.org/open-math-notes—to host freely available course notes and syllabi from undergraduate and graduate mathematics courses. We also modified the online version of the *Notices* to create a more dynamic and streamlined look.

Washington, DC, Office is now the AMS Office of Government Relations

The AMS Office of Government Relations advocates for science and mathematics at the national level. Staff members and member volunteers represent the interests of the mathematical community to federal agencies, legislative offices, and other science policy groups, coalitions, and professional organizations. We aim to strengthen the public perception of the significance of mathematics to impact science policy considerations. If you are interested in being a part of our grassroots advocacy efforts, keep an eye out for announcements in the coming months.

Karen Saxe started in January 2017 as the new Associate Executive Director for the Division of Government Relations. This division organized several events in 2017, including the 20th annual AMS Department Chairs Workshop, meetings of the AMS Committee on Science Policy and the AMS Committee on Education, and panels at the Joint Mathematics Meetings. This year, we sponsored an exhibit entitled “Berry Smart: Mathematics for Food and Water Security” at the Coalition for National Science Foundation Exhibition presented by Lea Jenkins (Clemson University). Our Office of Government Relations joined with the Mathematical Sciences Research Institute to host two Congressional lunch briefings on Capitol Hill for members of Congress and their staff. In June, David Donoho (Stanford University) presented “Blackboard to Bedside: How High-Dimensional Geometry is Transforming the MRI Industry” and in December, Shafi Goldwasser (MIT) presented “Cryptography: How to Enable Privacy in a Data-Driven World.”

The AMS selects and sponsors one Congressional Fellow in concert with the American Association for the Advancement of Science (AAAS). This year’s fellow is Margaret Callahan (PhD from Case Western Reserve University) who is working in the office of Senator Amy Klobuchar (MN). We also select and support one Mass Media Science & Engineering Fellow through AAAS. In 2017, we selected Ben Thompson, a PhD student at Boston University who spent the summer at Voice of America.

Publishing

Mathematics professionals need access to high-quality mathematics; publishing is thus an essential component of the AMS mission. The AMS publishing program is multi-faceted and consists primarily of our book program, our journal program, and the Mathematical Reviews database, which feeds our online tool, MathSciNet.

As mentioned above, the big news in 2017 was our acquisition of the MAA book program. The MAA has long-published high-quality books. Their acquisitions editor, Stephen Kennedy, is now bringing new MAA manuscripts to the AMS. Additionally, we published 78 new AMS books. We encourage our members to discuss new book or journal ideas with us, as publishing with the AMS generates income to the Society used to support our conferences, programs, and services. Our production staff provide excellent author support and our books, as you surely know, are priced reasonably.

AMS journals remain a highly respected venue for the publication of important mathematics research. We continue to publish two Gold Open Access journals, *Proceedings of the AMS, Series B* and *Transactions of the AMS, Series B*. We are actively addressing backlog issues and are seeing some improvements.

As the research literature grows, we strive to maintain our Mathematical Reviews database so it can continue to serve as a valuable resource to the mathematics community. This year, we added 133,581 bibliographic items and 105,563 reviews to the database that feeds MathSciNet. As the volume of mathematics literature continues to grow, the AMS is increasingly cautious about adding new journals for coverage. This will help preserve the excellent quality of this important research tool.

The AMS continues to expand its promotion and marketing to help mathematicians access our strong publishing program’s products. In 2017, Marketing became a stand-alone department and received new resources to help support these efforts.

Meetings and Professional Services

The importance of gathering with colleagues to share ideas and new research cannot be overestimated. AMS professional meetings, programs, and services support the continuing professional development of our membership and the mathematics community at large. We run a rich constellation of mathematical programs, such as the tenth year of our Mathematics Research Communities (MRC) program. This program helps early-career mathematicians launch their research programs by fostering collaborations during week-long workshops. Three MRC conferences served early-career mathematicians last year.

The workshops in 2017 were “Homotopy Type Theory”; “Beyond Planarity: Crossing Numbers of Graphs”; and “Dynamical Systems: Smooth, Symbolic, and Measurable.” We also provide support for several programs benefiting the entire mathematics community. For example, we conduct the survey for the Conference Board of the Mathematical Sciences, an umbrella organization of seventeen professional societies in the United States.

FROM THE AMS SECRETARY

The AMS writes grant proposals to external agencies and in 2017 received continued support of the AMS-Simons Travel Grants. The MRC program is funded, in part, by the National Science Foundation. Additionally, a new grant from the Simons Foundation is supporting ongoing enhancements to MathJax, an open-source cross-browser JavaScript library that displays mathematical notation in web browsers.

In addition to running our recurring meetings and conferences, such as the Joint Mathematics Meetings (held in Atlanta in January 2017) and eight regional Sectional meetings, we also provided financial and operational support for the MRC workshops in Snowbird, UT. We note that the MRC program will be held in Rhode Island in 2018.

Our Public Awareness Office maintained and expanded its activities to promote mathematics and to promote the Society and its programs. We continued to bring our popular *Who Wants to Be a Mathematician?* game to new audiences. We produced the printed Calendar of Mathematical Imagery and new Mathematical Moments.

AMS Operations

The AMS has over 200 employees working in Rhode Island, Michigan, and Washington, DC. All of our efforts are facilitated by professionals who help us support our human resources and manage our fiscal health, investments, and budget. In light of the upcoming implementation of the European Union's General Data Protection Regulation, staff have received legal advice and professional training to help ensure we stay on the leading edge of security. I am reminded daily how the tremendous efforts of AMS staff and volunteer leaders come together to create the wonderful world of AMS activities.

To Our Members and Supporters

If you are reading this you are part of our mathematics community and hopefully a member of the AMS. The AMS is responding to evolution in the mathematics landscape, and I am pleased to have you join in our important work.

There are many ways to get involved—for example, you could volunteer to serve on a committee at

www.ams.org/about-us/governance/committees/comm-all.htm or contact our publisher, Sergei Gelfand (sxg@ams.org), with your ideas for new books. Our Public Awareness Office might be able to help publicize your math event on AMS social media (contact paoffice@ams.org or tag your posts with [#amermathsoc](https://twitter.com/amermathsoc)). If you are a member, let us know about any professional accomplishments!

Please drop in to visit us at our Providence, RI, headquarters or in our Ann Arbor, MI, (Mathematical Reviews) or Washington, DC, (Government Relations) locations. Visit our refreshed website at www.ams.org for more details on anything described in this report.

Thank you for all the ways in which you advance mathematics.

—Catherine A. Roberts
Executive Director
April 2018



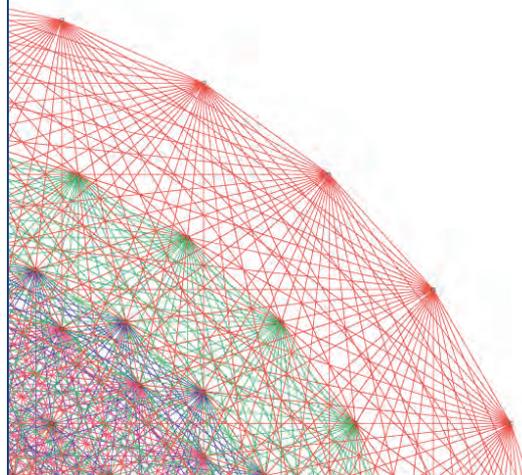
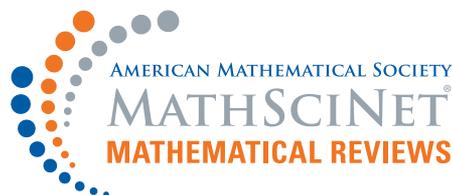
Beyond Reviews

A blog about MathSciNet®

From the Executive Editor of
Mathematical Reviews,
Edward Dunne

A blog created to highlight the innovative features of MathSciNet. Updates will include particularly informative reviews and will discuss tips and tricks for navigating MathSciNet, all with the goal of being helpful to users both old and new.

blogs.ams.org/beyondreviews



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From the AMS Secretary

ATTENTION ALL AMS MEMBERS

Voting Information for 2018 AMS Election

AMS members who have chosen to vote online will receive an email message on or shortly after August 20, 2018, from the AMS Election Coordinator, Survey & Ballot Systems.

The From line will be "AMS Election Coordinator", the Sender email address will be noreply@directvote.net, and the Subject line will be "AMS 2018 Election—login information below". If you use a spam filter you may want to use the above address or subject information to configure your spam filter to ensure this email will be delivered to you.

The body of the message will provide your unique voting login information and the address (URL) of the voting website.

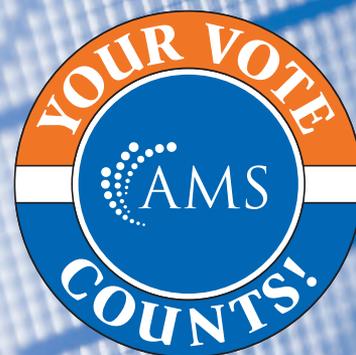
AMS members who have chosen to vote by paper should expect to receive their ballot by the middle of September. Unique voting login information will be printed on the ballot should you wish to vote online.

At midnight (US Eastern Time) on November 2, 2018, the website will stop accepting votes. Paper ballots received after this date will not be counted.

Additional information regarding the 2018 AMS Election is available on the AMS website: www.ams.org/election-info or by contacting the AMS: election@ams.org, 800-321-4267 (US & Canada), 401-455-4033 (worldwide).

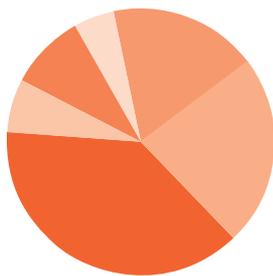
Thank you and . . . please remember to vote.

Carla D. Savage



To learn more visit:

www.ams.org/election-info



2017-2018 Faculty Salaries Report

Amanda L. Golbeck, Thomas H. Barr, and Colleen A. Rose

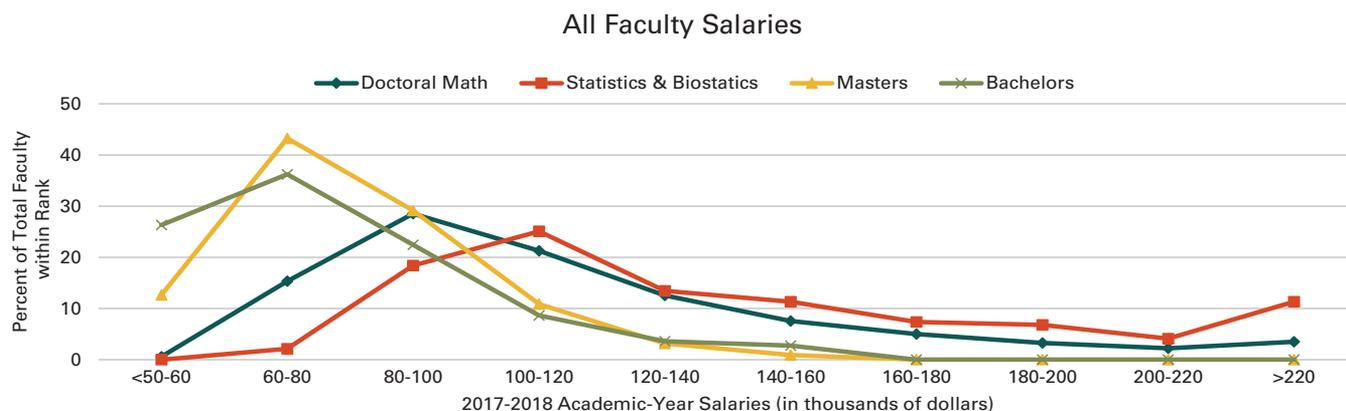
This salary report is one part of the Annual Survey of Mathematical Sciences, a nation-wide survey administered by the AMS on behalf of the American Statistical Association (ASA), the Institute for Mathematical Statistics (IMS), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM). It provides a look at the salaries of faculty in the Mathematical Sciences in the US by rank in several different department groupings based on discipline, highest degree offered, and graduate counts. The graphs here are identified by those group names, and the group definitions are given at the end of the report.

Departments were asked to report for each rank the number of tenured and tenure-track faculty whose 2017-18 academic-year salaries fell within given salary intervals. Reporting salary data in this fashion ensures confidentiality of individual responses, though it does mean that the quartiles reported in the tables are approximations. The quartiles reported have been estimated assuming that the density over each interval is uniform.

Note: In the graphics for all faculty salaries, and the Masters and Bachelors Groups, the percentages scales range from 0 to 50, while the scales for all other groups is 0 to 100.

Faculty Salary Reports from prior years are at www.ams.org/annual-survey/salaries. Interpretation of historical trends should be made with care. For instance, one factor influencing year to year changes in the mean reported salaries may be differences in the set of responding departments within the groups.

The first graphic below provides a coarse comparative view of faculty salaries among four broad groups: departments whose highest degree is a (1) PhD in mathematics (including applied mathematics departments), (2) PhD in statistics or biostatistics, (3) masters degree in mathematics, and (4) bachelors degree in mathematics. In the remainder of this report, salary distributions are broken down within finer departmental categories and by faculty rank.

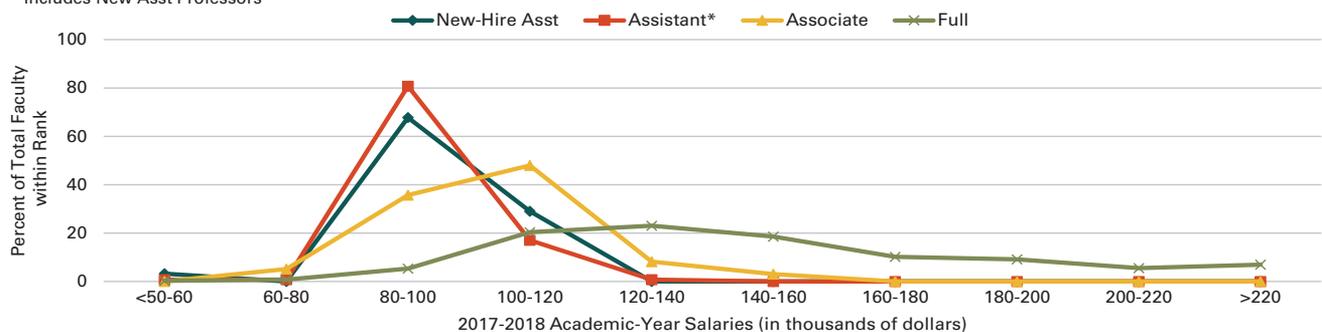


Amanda L. Golbeck is Associate Dean for Academic Affairs and Professor of Biostatistics in the Fay W. Boozman College of Public Health at University of Arkansas for Medical Sciences. Thomas H. Barr is AMS special projects officer. Colleen A. Rose is AMS survey analyst.

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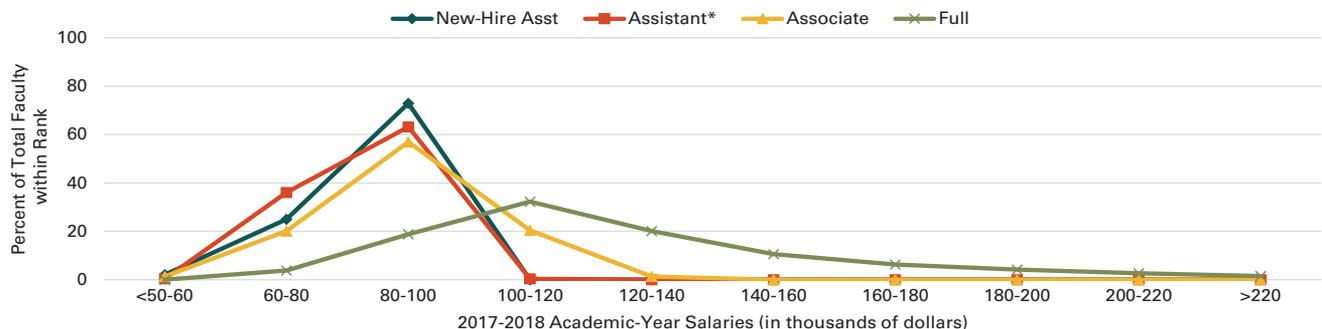
Math Public Large Group Faculty Salaries							
21 responses out of 26 departments (81%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	31	91,100	93,900	101,000	93,358	88,201
	Male	24	91,100	93,900	101,400	93,096	88,832
	Female	7	91,300	93,800	95,000	94,257	86,150
Assistant Professor*	All	135	90,600	93,400	98,100	93,525	90,230
	Male	106	90,700	93,500	98,400	93,849	90,209
	Female	29	90,200	93,100	97,200	92,342	90,293
Associate Professor	All	196	91,800	101,500	111,900	102,487	97,877
	Male	152	91,100	101,500	112,600	102,297	98,039
	Female	44	95,400	101,600	110,500	103,144	97,402
Full Professor	All	777	118,700	140,400	173,500	150,030	144,661
	Male	698	118,800	140,100	174,500	150,617	145,099
	Female	79	117,300	142,100	157,700	144,846	140,581

* Includes New Asst Professors



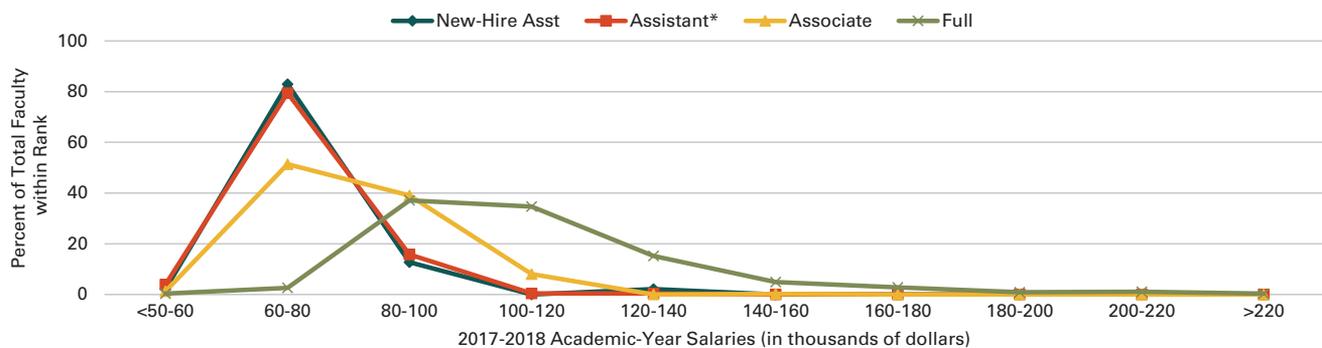
Math Public Medium Group Faculty Salaries							
36 responses out of 40 departments (90%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	48	79,100	84,100	88,300	81,591	81,135
	Male	35	80,800	84,600	88,500	82,501	80,271
	Female	13	73,100	81,700	88,100	79,143	83,267
Assistant Professor*	All	258	77,700	82,500	88,300	82,416	80,982
	Male	187	78,200	82,800	88,400	82,796	81,419
	Female	71	76,500	81,600	88,100	81,417	79,899
Associate Professor	All	309	81,300	89,400	98,800	90,222	89,848
	Male	245	81,000	89,000	98,100	89,684	88,374
	Female	64	82,800	91,300	100,600	92,280	94,155
Full Professor	All	691	100,700	116,900	140,200	124,180	120,906
	Male	615	101,000	117,700	140,600	124,737	121,531
	Female	76	97,800	104,100	135,000	119,669	114,804

* Includes New Asst Professors



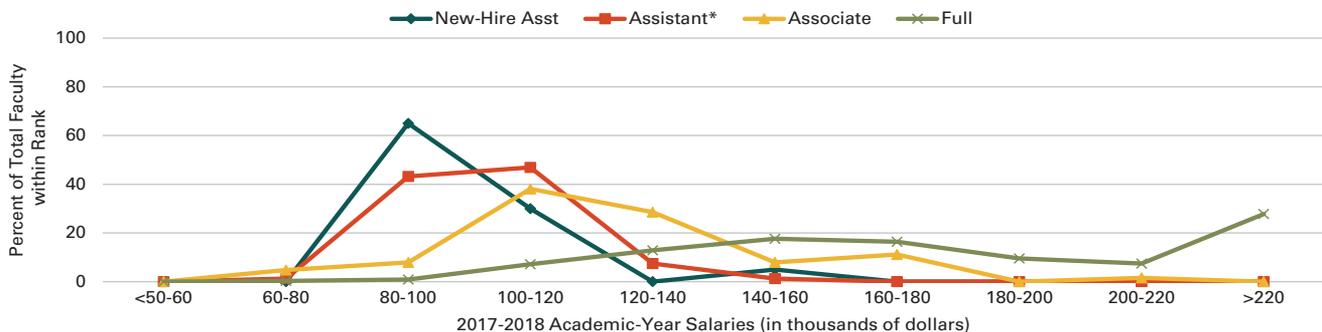
Math Public Small Group Faculty Salaries							
49 responses out of 68 departments (72%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	47	71,200	74,700	78,500	75,050	68,552
	Male	37	71,600	75,300	78,500	75,709	68,747
	Female	10	67,500	73,100	78,300	72,610	68,245
Assistant Professor*	All	253	68,700	73,700	78,400	73,518	72,019
	Male	174	68,500	74,000	78,300	73,580	71,720
	Female	79	69,100	73,200	78,600	73,380	72,750
Associate Professor	All	397	72,900	79,300	86,900	80,957	80,415
	Male	295	73,200	79,800	87,500	81,356	80,966
	Female	102	72,200	77,800	84,500	79,802	79,052
Full Professor	All	566	92,400	102,400	120,400	110,163	116,357
	Male	481	92,800	103,000	122,800	111,491	120,080
	Female	85	91,500	99,400	104,700	102,646	96,558

* Includes New Asst Professors



Math Private Large Group Faculty Salaries							
16 responses out of 24 departments (67%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	20	92,500	97,500	103,100	94,575	99,841
	Male	16	91,500	96,100	98,900	94,781	88,832
	Female	4	111,700	102,500	152,500	93,750	86,150
Assistant Professor*	All	81	93,900	101,000	111,600	100,837	97,863
	Male	62	94,000	100,400	104,700	101,436	90,209
	Female	19	92,500	102,900	115,000	98,881	90,293
Associate Professor	All	63	102,200	118,800	135,000	121,560	119,792
	Male	57	102,300	120,800	137,000	122,584	98,039
	Female	6	101,700	112,500	125,000	111,825	97,402
Full Professor	All	335	144,500	173,300	221,100	186,825	178,954
	Male	304	145,200	174,200	221,000	187,410	145,099
	Female	31	138,000	165,000	222,000	181,095	140,581

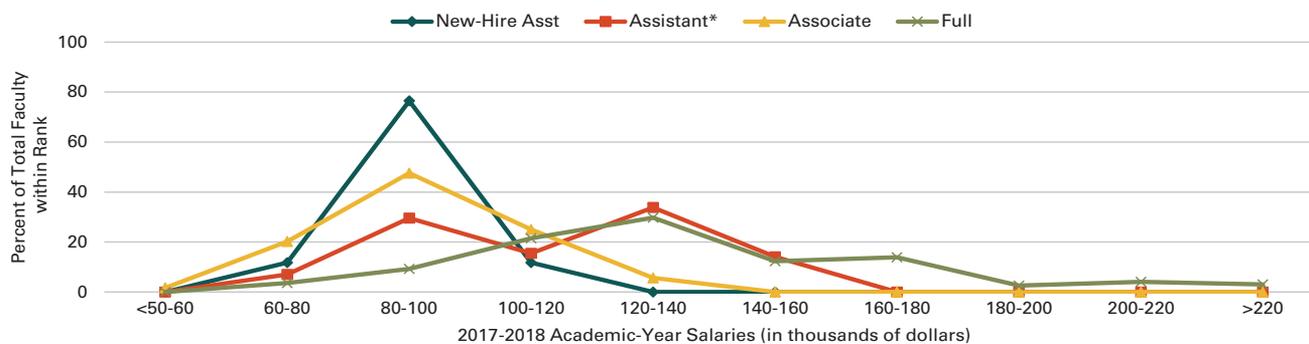
* Includes New Asst Professors



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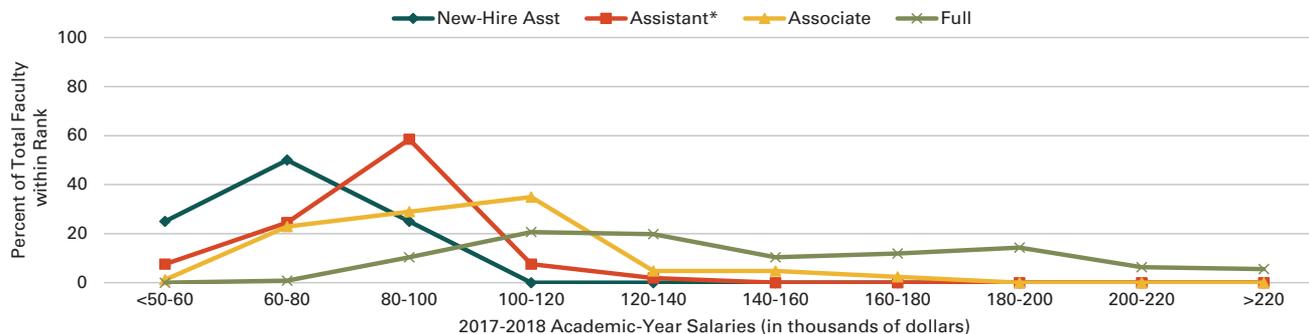
Math Private Small Group Faculty Salaries							
19 responses out of 29 departments (66%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	17	85,200	87,300	89,300	87,116	82,600
	Male	10	85,700	87,500	89,300	86,763	86,333
	Female	7	82,500	87,000	89,000	87,621	77,000
Assistant Professor*	All	71	80,400	86,500	90,700	85,779	84,908
	Male	54	81,100	86,900	91,700	86,280	85,529
	Female	17	77,500	85,700	88,900	84,186	83,188
Associate Professor	All	124	81,900	91,700	101,800	92,447	89,683
	Male	98	82,900	92,100	102,100	92,788	89,582
	Female	26	80,800	88,100	101,000	91,161	90,074
Full Professor	All	195	112,700	130,700	157,500	128,166	122,420
	Male	176	112,600	131,000	157,700	138,517	132,750
	Female	19	113,300	126,700	155,000	130,407	128,027

* Includes New Asst Professors



Applied Mathematics Group Faculty Salaries							
14 responses out of 23 departments (61%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	4	71,300	76,300	96,300	75,500	85,611
	Male	4	71,300	76,300	96,300	75,500	88,832
	Female	0	-	-	-	-	86,150
Assistant Professor*	All	53	75,500	92,100	97,800	86,198	85,570
	Male	44	71,900	91,500	97,700	84,949	90,209
	Female	9	86,300	95,300	98,100	92,305	90,293
Associate Professor	All	83	80,600	98,500	112,700	98,925	94,766
	Male	73	81,300	101,300	113,900	100,802	98,039
	Female	10	78,300	81,800	96,700	85,220	97,402
Full Professor	All	126	114,400	138,300	182,000	147,856	149,211
	Male	112	114,600	140,600	182,800	148,153	145,099
	Female	14	113,300	131,700	165,000	145,476	140,581

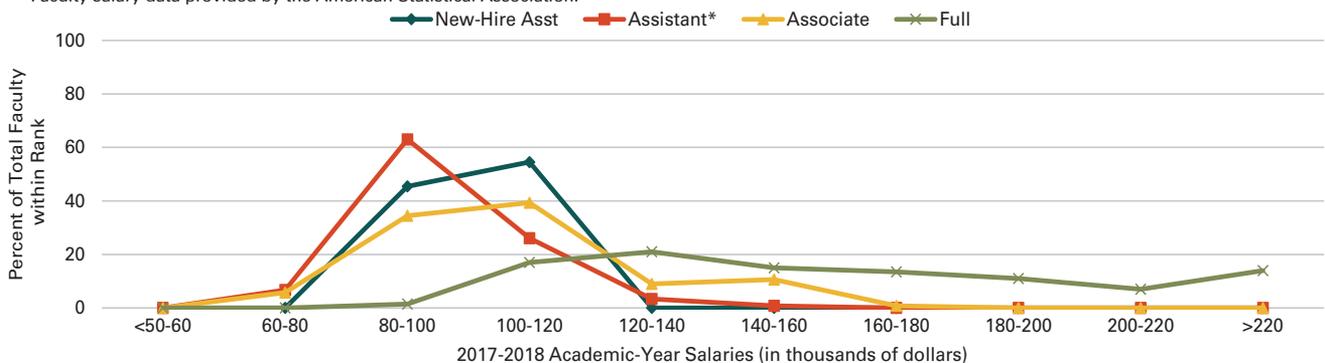
* Includes New Asst Professors



Statistics Group Faculty Salaries**							
26 responses out of 59 departments (44%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	11	93,800	100,800	103,300	99,178	107,069
	Male	10	93,800	100,400	102,500	98,096	105,888
	Female	1	112,500	115,000	0	110,000	110,614
Assistant Professor*	All	119	90,600	95,600	101,500	96,376	93,342
	Male	80	91,100	96,300	101,900	96,921	93,609
	Female	39	89,200	94,200	99,500	95,260	92,688
Associate Professor	All	122	93,900	101,700	115,800	106,375	115,774
	Male	83	93,900	102,300	118,800	108,230	116,697
	Female	39	93,300	99,500	105,000	102,428	113,005
Full Professor	All	200	125,400	152,500	191,400	132,696	127,997
	Male	164	127,400	155,000	190,500	161,825	155,729
	Female	36	120,700	137,000	201,700	158,088	135,550

* Includes New Asst Professors

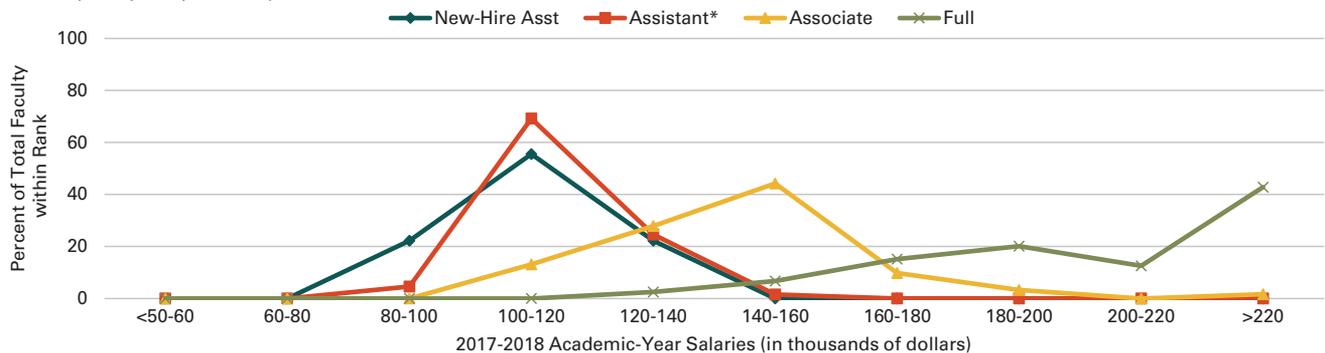
**Faculty salary data provided by the American Statistical Association.



Biostatistics Group Faculty Salaries**							
17 responses out of 46 departments (37%)							
Rank	2017-2018						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	9	100,500	103,000	121,700	110,885	138,083
	Male	6	98,300	102,500	115,000	109,674	135,241
	Female	3	101,700	103,300	125,000	113,308	143,767
Assistant Professor*	All	65	102,000	104,500	121,100	112,283	119,654
	Male	48	102,000	104,500	122,300	112,685	120,321
	Female	17	102,300	119,000	117,500	111,147	117,987
Associate Professor	All	61	131,300	143,200	153,500	143,070	136,634
	Male	40	132,300	142,800	155,000	142,400	134,892
	Female	21	127,000	143,600	148,600	144,347	141,705
Full Professor	All	119	181,100	205,800	224,200	218,676	203,724
	Male	97	184,200	208,200	224,400	224,407	200,282
	Female	22	175,000	193,800	223,300	193,406	221,670

* Includes New Asst Professors

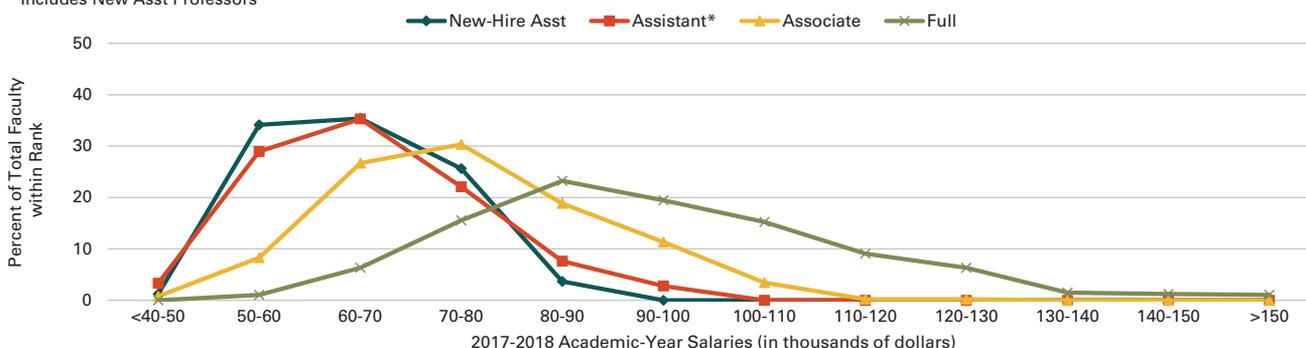
**Faculty salary data provided by the American Statistical Association.



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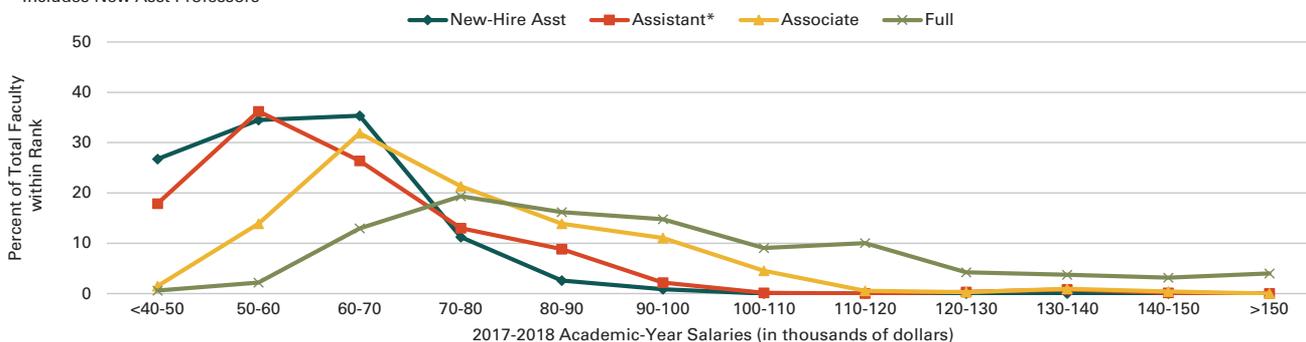
Masters Group Faculty Salaries							
88 responses out of 178 departments (49%)							
Rank	2017-18						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	82	57,000	63,500	71,400	63,542	68,698
	Male	49	56,600	63,300	72,800	63,748	69,026
	Female	33	57,500	63,500	70,400	63,237	68,002
Assistant Professor*	All	394	58,200	64,400	73,400	65,808	50,025
	Male	247	58,200	64,300	73,000	65,573	65,935
	Female	147	58,000	64,300	74,200	66,202	65,245
Associate Professor	All	521	65,400	74,900	83,700	75,432	74,909
	Male	358	64,600	75,500	84,100	75,565	74,868
	Female	163	67,000	74,100	82,900	75,139	75,007
Full Professor	All	663	80,400	91,800	105,600	94,226	96,123
	Male	520	81,100	92,400	107,100	95,043	97,010
	Female	143	80,200	89,200	102,200	91,256	92,804

* Includes New Asst Professors



Bachelors Group Faculty Salaries							
278 responses out of 1018 departments (27%)							
Rank	2017-18						2016-17
	Gender	No. Reported	Q1	Median	Q3	Mean	Mean
New Asst Professors	All	116	52,900	60,100	66,700	59,464	61,809
	Male	69	52,800	59,400	65,300	59,436	69026
	Female	47	52,700	61,300	67,900	59,505	68002
Assistant Professor*	All	599	53,700	60,600	70,400	63,239	62,679
	Male	358	53,200	60,100	69,400	62,922	65,935
	Female	241	54,500	61,500	71,400	63,710	65,245
Associate Professor	All	750	62,900	71,100	84,000	74,576	71,994
	Male	503	62,900	71,000	83,400	74,621	74,868
	Female	194	55,100	61,600	72,700	74,483	75,007
Full Professor	All	827	74,600	89,400	110,300	95,063	92,096
	Male	618	74,900	90,200	110,200	95,800	97,010
	Female	209	73,500	86,300	110,800	92,884	92,804

* Includes New Asst Professors



Departmental Groupings

In this report, *Mathematical Sciences* departments are those in four-year institutions in the US that refer to themselves with a name that incorporates (with a few exceptions) “Mathematics” or “Statistics” in some form. For instance, the term includes, but is not limited to, departments of “Mathematics,” “Mathematical Sciences,” “Mathematics and Statistics,” “Mathematics and Computer Science,” “Applied Mathematics,” “Statistics,” and “Biostatistics.” Also, *Mathematics (Math)* refers to departments that (with exceptions) have “mathematics” in the name; *Stat/Biostat* refers to departments that incorporate (again, with exceptions) “statistics” or “biostatistics” in the name but do not use “mathematics.”

Starting with reports on the 2012 AMS-ASA-IMS-MAA-SIAM Annual Survey of the Mathematical Sciences, the Joint Data Committee implemented a new method for grouping doctorate-

granting Mathematics departments. These departments are first grouped into those at public institutions and those at private institutions. These groups are further subdivided based on the size of their doctoral program as reflected in the average annual number of PhDs awarded between 2000 and 2010, based on their reports to the Annual Survey during that period. These groupings are listed below.

For further details on the change in the doctoral department groupings, see the article in the October 2012 issue of *Notices of the AMS* at www.ams.org/journals/notices/201209/rtx120901262p.pdf.

Math Public Large consists of departments with the highest annual rate of production of PhDs, ranging between 7.0 and 24.2 per year.

Math Public Medium consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 6.9 per year.

Math Public Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.

Math Private Large consists of departments with an annual rate of production of PhDs, ranging between 3.9 and 19.8 per year.

Math Private Small consists of departments with an annual rate of production of PhDs of 3.8 or less per year.

Applied Mathematics consists of doctoral-degree-granting applied mathematics departments.

Statistics consists of doctoral-degree-granting statistics departments.

Biostatistics consists of doctoral-degree-granting biostatistics departments.

Masters consists of US departments granting a master’s degree as the highest graduate degree.

Bachelors consists of US departments granting a baccalaureate degree only.

Doctoral Math consists of all US math public, math private, and applied math mathematics departments granting a PhD as the highest graduate degree.

Mathematics (Math) consists of all US Math Public, Math Private, and Applied Math, Masters, and Bachelors Groups above.

Stat/Biostat consists of all doctoral-degree-granting statistics and biostatistics departments.

Listings of the actual departments that compose these groups are available on the AMS website at www.ams.org/annual-survey/groups.

Obtain a Special Faculty Salaries Analysis

Each year AMS provides a limited number of special faculty salary analyses to departments requesting them. These reports are based on data gathered through the Survey and provide more nuanced comparisons with similar institutions than is possible with the Faculty Salaries Report. In order to receive a special analysis, your department must have responded to the most recent Faculty Survey.

Send a list of your peer institutions (a minimum of 12 institutions is required) to ams-survey@ams.org along with the date by which the analysis is needed. (If not enough of your peer group

have responded to the salary survey, you will be asked to provide additional institutions.) A minimum of two weeks is needed to complete a special analysis.

The analysis produced includes a listing of your peer group institutions along with their salary survey response status; a summary table including the rank (assistant, associate, and full professor); the number reported in each rank; the 1st quartile, median, 3rd quartile, and mean salaries for each along with bar graphs.

Acknowledgments

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of

these departments for the quality of its information. On behalf of the Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

Inside the AMS

AMS Congressional Fellow Announced



James Ricci

JAMES RICCI has been awarded the 2018–2019 AMS Congressional Fellowship. Ricci is currently an assistant professor at Daemen College, teaching courses in math and computer science. He received his PhD in mathematics from Wesleyan University.

Ricci was part of an effort with Daemen’s Natural Sciences Department to address the retention issues of educationally disadvantaged students. The team created a multidisciplinary freshman course focused on sparking interest in STEM for students whose minimal mathematics backgrounds did not support their chosen majors. He used a similar approach to redesign an introductory computer science course, incorporating current topics such as cybersecurity, cryptocurrencies, artificial intelligence, and net neutrality into class discussions in order to address topics in a more meaningful and engaging way.

The Congressional Fellowship program is administered by the American Association for the Advancement of Science (AAAS) and provides an opportunity for scientists and engineers to learn about federal policy making while contributing their knowledge and analytical skills to the process. Fellows spend a year working on the staff of a member of Congress or a congressional committee, working as a special legislative assistant in legislative and policy areas requiring scientific and technical input. The fellowship program includes an orientation on congressional and executive branch operations and a year-long professional development program.

The Fellowship is designed to provide a unique public policy learning experience that demonstrates the value of science-government interaction and brings a technical background and external perspective to the decision-making process in Congress.

For more information on the AMS-AAAS Congressional Fellowship, go to bit.ly/AMSCongressionalFellowship.

—AMS Office of Government Relations

Listen Up!—Podcasts

AMS Public Awareness Officer Mike Breen interviews mathematicians as part of the Mathematical Moments program. Hear “Scoring with New Thinking,” with Andy Andres (Boston University), “Unscrambling Eggs,” with Mary Caswell Stoddard (Princeton University), “Giving Currency to Not Using Currency,” with Peter J. Denning (Naval Postgraduate School), “Countermanding Gerrymandering,” with Moon Duchin (Tufts University), and more at <https://www.ams.org/mathmoments>.

AMS Blog Posts to Revisit for the New Academic Year



Benjamin Braun

• “Advice for New Doctoral Advisors,” “Mathematical Culture beyond the Classroom” and “To Active Learning and Beyond: Attending to Student Thinking AND Student Experience in Active-Learning Math Classes” by Benjamin Braun, University of Kentucky, founder and editor-in-chief of the *On Teaching and Learning Mathematics* blog, who retired from the blog in June. We thank him for his vision and enlightening posts at

<https://blogs.ams.org/matheducation/author/ben-braun>.

- “Teaching What You (Really) Don’t Know” by Sara Malec: <https://blogs.ams.org/phdplus/2018/04/30/teaching-what-you-really-dont-know>
 - “Radical Notation” by Anna Haensch: <https://blogs.ams.org/blogonmathblogs/2018/04/09/radical-notation>
 - “NSF is Taking Action against Sexual Harassment in Science” by Karen Saxe: <https://blogs.ams.org/capitalcurrents/2018/04/24/nsf-is-taking-action-against-sexual-harassment-in-science>
- and more at <https://blogs.ams.org>.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
paoffice@ams.org

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Photo of Benjamin Braun courtesy of Benjamin Braun.

Deaths of AMS Members

JAMES H. ABBOTT, of Plano, Texas, died on July 19, 2015. Born on August 19, 1924, he was a member of the Society for 61 years.

ROY L. ADLER, of Chappaqua, New York, died on July 26, 2016. Born on February 22, 1931, he was a member of the Society for 59 years.

FRANK W. ANDERSON, professor, University of Oregon, died on February 5, 2016. Born on February 5, 1928, he was a member of the Society for 62 years.

HUBERT BERENS, professor, University of Erlangen-Nurnberg, died on February 9, 2015. Born on May 6, 1936, he was a member of the Society for 46 years.

JONATHAN BORWEIN, professor, University of Newcastle, died on August 2, 2016. Born on May 20, 1951, he was a member of the Society for 26 years.

J. W. S. CASSELS, of the United Kingdom, died on July 27, 2015. Born on July 11, 1922, he was a member of the Society for 46 years.

E. W. CHENEY, of Austin, Texas, died on July 13, 2016. Born on June 28, 1929, he was a member of the Society for 63 years.

ROMAN J. DWILEWICZ, professor, Missouri University of Science & Technology, died on July 29, 2016. Born on November 6, 1949, he was a member of the Society for 27 years.

ELMER EISNER, of Houston, Texas, died on August 2, 2016. Born on March 8, 1919, he was a member of the Society for 56 years.

EUGENIA E. FITZGERALD, of Nashua, New Hampshire, died on March 17, 2016. Born on November 8, 1925, she was a member of the Society for 39 years.

MELVIN D. GEORGE, of Columbia, Missouri, died on April 25, 2016. Born on February 13, 1936, he was a member of the Society for 59 years.

ANTHONY A. GIOIA, of Augusta, Michigan, died on January 4, 2016. Born on April 7, 1934, he was a member of the Society for 54 years.

NEIL E. GRETSKY, of Colorado Springs, Colorado, died on September 21, 2015. Born on March 17, 1941, he was a member of the Society for 50 years.

JOHN W. HARDY JR., of Bakersfield, California, died on August 17, 2014. Born on April 21, 1927, he was a member of the Society for 63 years.

DAVID LEE HILLIKER, of Costa Mesa, California, died on February 1, 2012. Born on September 1, 1935, he was a member of the Society for 44 years.

MELVIN F. JANOWITZ, of Haworth, New Jersey, died on December 8, 2015. Born on May 8, 1929, he was a member of the Society for 53 years.

JOHN F. KURTZKE JR., of Notre Dame, Indiana, died on February 28, 2013. Born on March 28, 1951, he was a member of the Society for 38 years.

NORMAN M. MARTIN, of Austin, Texas, died on July 13, 2016. Born on January 16, 1924, he was a member of the Society for 67 years.

EDWARD B. MCLEOD JR., of Long Beach, California, died on December 20, 2014. Born on July 25, 1924, he was a member of the Society for 61 years.

MARVIN G. MUNDT, professor, Valparaiso University, died on May 6, 2013. Born on April 5, 1933, he was a member of the Society for 52 years.

DONALD E. SANDERSON, of Torrey, Utah, died on January 17, 2016. Born on February 4, 1926, he was a member of the Society for 66 years.

SOL SCHWARTZMAN, of Providence, Rhode Island, died on January 30, 2016. Born on October 17, 1926, he was a member of the Society for 26 years.

ATWELL R. TURQUETTE, of Champaign, Illinois, died on December 14, 2014. Born on July 14, 1914, he was a member of the Society for 77 years.

ROGER P. WARE, of Lincoln, California, died in June, 2016. Born on April 2, 1942, he was a member of the Society for 48 years.

WILLIAM C. WATERHOUSE, of State College, Pennsylvania, died on June 26, 2016. Born on December 31, 1941, he was a member of the Society for 51 years.

JOHN A. WINN JR., of Pittsfield, Massachusetts, died on April 3, 2016. Born on April 8, 1945, he was a member of the Society for 37 years.

PAUL M. YOUNG, of Manhattan, Kansas, died on July 20, 2016. Born on February 13, 1916, he was a member of the Society for 70 years.

PAWEL DOMANSKI, professor, Adam Mickiewicz University, died on August 4, 2016. Born on June 5, 1959, he was a member of the Society for 26 years.

WACLAW SZYMANSKI, professor, West Chester University of Pennsylvania, died on August 7, 2016. Born on December 12, 1949, he was a member of the Society for 31 years.

ALAN G. MCINTOSH, professor, Australian National University, died on August 8, 2016. Born on January 17, 1942, he was a member of the Society for 50 years.

NICHOLAS SCHEALL, professor, University Center, Michigan, died on August 18, 2016. Born on May 19, 1978, he was a member of the Society for 13 years.

Mathematics Opportunities

Listings for upcoming math opportunities to appear in Notices may be submitted to notices@ams.org.

Call for Nominations for 2019 Abel Prize

The Norwegian Academy of Science and Letters awards the Abel Prize annually to recognize outstanding scientific work in mathematics, including mathematical aspects of computer science, mathematical physics, probability, numerical analysis and scientific computing, statistics, and also applications in the sciences. Nominations are due by **September 15, 2018**. See www.abelprize.no/c53676/artikkel/vis.html?tid=53705.

—Norwegian Academy of Science and Letters

Call for Nominations for AWM Falconer Lectureship

The Association for Women in Mathematics (AWM) and the Mathematical Association of America (MAA) annually present the Etta Z. Falconer Lecture at MathFest to honor women who have made distinguished contributions to the mathematical sciences or mathematics education. The deadline for nominations is **September 1, 2018**. See <https://sites.google.com/site/awmmath/programs/falconer-lectures>.

—From an AWM announcement

Call for Nominations for AWM Schafer Prize

The Association for Women in Mathematics (AWM) calls for nominations for the Alice T. Schafer Mathematics Prize to be awarded to an undergraduate woman for excellence in mathematics. The nominee must be an undergraduate when nominated. The deadline is **October 1, 2018**. See <https://sites.google.com/site/awmmath/programs/schafer-prize>.

—From an AWM announcement

Call for Nominations for Gerald Sacks Prize

The Association for Symbolic Logic (ASL) invites nominations for the Gerald Sacks Prize for the most outstanding doctoral dissertation in mathematical logic. The deadline is **September 30, 2018**. See www.aslonline.org/info-prizes.html or www.aslonline.org/Sacks_nominations.html.

—From an ASL announcement

*Research Experiences for Undergraduates

The Research Experiences for Undergraduates (REU) program supports student research in areas funded by the National Science Foundation (NSF) through REU sites and REU supplements. See www.nsf.gov/funding/pgm_summ.jsp?pims_id=5517. The deadline date for proposals from institutions wishing to host REU sites is **August 22, 2018**. Dates for REU supplements vary with the research program (contact the program director for more information). Students apply directly to REU sites; see www.nsf.gov/crssprgm/reu/list_result.jsp?unitid=5044 for active REU sites.

—From an NSF announcement

*The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at: www.nsf.gov/dms and for the Directorate of Education and Human Resources at www.nsf.gov/dir/index.jsp?org=ehr. To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at www.nsf.gov/mps/dms/about.jsp.

*Joint DMS/NIGMS Initiative to Support Research at the Interface of the Biological and Mathematical Sciences

The National Science Foundation (NSF) and the National Institute of General Medical Sciences (NIGMS) at the National Institutes of Health (NIH) support research in mathematics and statistics on questions in the biological and biomedical sciences. The application period is **September 1–18, 2018**. For more information see https://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5300.

—From an NSF announcement

Call for Nominations for Joseph F. Traub Prize

The Joseph F. Traub Prize for Achievement in Information-Based Complexity is given for work done in a single year, a number of years, or a lifetime. The deadline for nominations for 2019 is **March 31, 2019**. See <https://www.journals.elsevier.com/journal-of-complexity/awards/joseph-f-traub-prize>.

—Joseph F. Traub Prize Award Committee

IPAM Call for Proposals

The Institute for Pure and Applied Mathematics (IPAM) seeks program proposals from the mathematical, statistical, and scientific communities for long programs and workshops, to be reviewed at IPAM's Science Advisory Board meeting in November. For more information, go to www.ipam.ucla.edu/propose-a-program or contact the IPAM Director at director@ipam.ucla.edu. For all proposals, the inclusion of women and members of underrepresented minorities as speakers, organizers, or participants is required.

—Stacey Beggs, Assistant Director, IPAM

News from AIM

The American Institute of Mathematics (AIM) is accepting applications for the following scientific programs.

Focused Workshop Program: These week-long workshops are distinguished by their specific mathematical goals. These may involve making progress on a significant unsolved problem or examining the convergence of two distinct areas of mathematics. Researchers may apply to attend an upcoming AIM workshop or may propose one. The deadline is **November 1, 2018**. A list of upcoming

workshops is available at www.aimath.org/workshops/upcoming/.

SQuaREs Program: Structured Quartet Research Ensembles (SQuaREs) brings together groups of four to six researchers for a week of focused work on a specific research problem, returning for up to three consecutive years. The deadline for applications is **November 1, 2018**. See www.aimath.org.

—AIM announcement

MSRI News

MSRI invites applications for Research Professors, Research Members, and Postdoctoral Fellows in the following programs:

Holomorphic Differentials in Mathematics and Physics (August 12–December 13, 2019); **Microlocal Analysis** (August 12–December 13, 2019); **Quantum Symmetries** (January 21–May 29, 2020); and **Higher Categories and Categorification** (January 21–May 29, 2020).

Research Professorships are intended for senior researchers who will be making key contributions to a program, including the mentoring of postdoctoral fellows, and who will be in residence for three or more months. Application deadline: **October 1, 2018**.

Research Memberships are intended for researchers who will be making contributions to a program and who will be in residence for one or more months. Application deadline: **December 1, 2018**.

Postdoctoral Fellowships are intended for recent PhDs. Application deadline: **December 1, 2018**.

More information can be found at www.msri.org/application.

—MSRI announcement

News from IPAM

The Institute for Pure and Applied Mathematics (IPAM) offers programs that encourage collaboration across disciplines and between two areas of mathematics. IPAM holds long programs (three months) and workshops (three to five days) throughout the academic year for junior and senior mathematicians and scientists who work in academia, research laboratories, and industry. In the summer, IPAM offers industrial research programs in multiple locations for undergraduate and graduate students and a one- to three-week “summer school” for graduate students and recent PhDs.

IPAM seeks proposals for workshops, long programs, and summer schools. Proposals should be sent to director@ipam.ucla.edu by **October 1, 2018**. See www.ipam.ucla.edu/propose-a-program.

Following are IPAM's upcoming programs. For complete information and application and registration forms, see www.ipam.ucla.edu/programs.

On August 27–29, 2018, IPAM will offer an exploratory workshop on “Quantum Computing Materials Challenges.” Register online.

The fall 2018 long program, “Science at Extreme Scales: Where Big Data Meets Large-Scale Computing,” is no longer accepting applications for long-term visitors, but researchers may register or apply for funding for one of the following one-week workshops online.

- **September 13–18, 2018:** Science at Extreme Scales: Where Big Data Meets Large-Scale Computing Tutorials
- **September 24–28, 2018:** Workshop I: Big Data Meets Large-Scale Computing
- **October 15–19, 2018:** Workshop II: HPC and Data Science for Scientific Discovery
- **November 5–9, 2018:** Workshop III: HPC for Computationally and Data-Intensive Problems
- **November 26–30, 2018:** Workshop IV: New Architectures and Algorithms

Following are the winter workshops in 2019. Apply for travel support or register online.

- **January 7–11, 2019:** Analysis and Geometry of Random Shapes
- **January 28–February 1, 2019:** Computational Challenges in Gravitational Wave Astronomy
- **February 11–15, 2019:** Operator Theoretic Methods in Dynamic Data Analysis and Control
- **February 19–21, 2019:** Braids, Resolvent Degree and Hilbert’s 13th Problem
- **February 25–March 1, 2019:** Autonomous Vehicles

Applications for long-term visitors are being accepted for the spring 2019 long program, “Geometry and Learning from Data in 3D and Beyond.” You may also register or apply for funding for one of the following one-week workshops online.

- **March 12–15, 2019:** Geometry and Learning from Data Tutorials
- **April 1–5, 2019:** Workshop I: Geometric Processing
- **April 15–19, 2019:** Workshop II: Shape Analysis
- **April 29–May 3, 2019:** Workshop III: Geometry of Big Data
- **May 20–24, 2019:** Workshop IV: Deep Geometric Learning of Big Data and Applications

An industrial short course on “Deep Learning and the Latest AI Algorithms” will be held on October 1–2, 2018. The course is designed for industry professionals who want to get started in deep learning and apply deep learning techniques and artificial intelligence to their research projects. The instructor is Xavier Bresson of Nanyang Technological University, Singapore. The course is limited to twenty participants; early registration is encouraged. You may register for the short course online.

— From an IPAM announcement

Call for Proposals for the 2020 AMS Short Courses

The AMS Short Course Subcommittee invites submissions of preliminary proposals for Short Courses to be offered on January 13–14, 2020, in coordination with the 2020 Joint Mathematics Meetings in Denver, Colorado. Members of the mathematical community are also welcome to suggest names of colleagues as potential organizers.

Preliminary proposals may be as short as one page, and suggestions and questions are welcome. Proposals should be sent via email to the Associate Executive Director (aed-mps@ams.org) with a cc to Robin Hagan Aguiar (rha@ams.org).

A short course typically incorporates a sequence of survey lectures and other activities focused on a single theme of applied mathematics. The Subcommittee is also interested in proposals that go beyond the traditional course in methodology and subject matter. Proposers might be interested in a webinar format or other mechanisms for reaching an audience that extends beyond those at the JMM site, or they may want to appeal to mathematicians who are considering careers in business, industry, government, and nonprofit sectors that utilize mathematical training and experience.

For full consideration, 2020 Short Course proposals should be submitted by **December 18, 2018**. More detailed guidance on proposals is available at www.ams.org/meetings/short-courses/2019call.



MATHEMATICAL SCIENCE OPPORTUNITIES FROM THE AMS

The AMS Online Opportunities Page provides another avenue for the math community to **Announce and Browse:**

- **Calls for fellowship appointments**
- **Prize and award nominations**
- **Grant applications**
- **Meeting and workshop proposals**

search now!

www.ams.org/opportunities

Mathematics People

Mickens Awarded Blackwell–Tapia Prize



Ronald E. Mickens

RONALD E. MICKENS of Clark Atlanta University has been awarded the 2018 Blackwell–Tapia Prize. The prize recognizes a mathematician who has contributed significantly to research in his or her field of expertise and who has served as a role model for mathematical scientists and students from underrepresented minority groups or has contributed in other significant ways to addressing the problem of the

underrepresentation of minorities in math.

The prize citation reads in part: “Mickens’ mathematical reach extends across multiple disciplines and has a significant global impact. He is well known for his contributions in multiple areas of applied mathematics generally related to the solution of differential equations, in particular, the areas of nonstandard finite differences (NSFD) and nonlinear oscillations. In fact, he created the field of NSFD which seeks to discretize dynamical systems while retaining properties of the system. Many researchers around the world have extended Mickens’ pioneering work on NSFDs to a plethora of systems.

“Mickens’ interest and engagement in issues around the underrepresentation of people of African American and Latinx descent in mathematics is sustained, significant, and substantial. He has been unearthing, celebrating, and publicizing the achievements of Black scientists for more than four decades. For example, he was elected a charter Fellow of the National Society of Black Physicists in 1992 and received the Edward Bouchet Award for Excellence in Research from the National Conference of Black Physics Students in 2004. His book *Edward Bouchet: The First African American Doctorate* was published in 2002 and is an important contribution to the history of the participation of African Americans in STEM fields.”

Mickens received his PhD in theoretical physics from Vanderbilt University in 1968. He taught physics at Fisk University from 1970 to 1982 and at Atlanta University from 1982 to 1990 before joining the faculty at Clark Atlanta, a Historically Black College and University. He received a Ford Foundation Postdoctoral Fellowship for

Minorities in 1980 and has been active in mentoring Ford Fellows of all disciplines. He is the recipient of the Edward Bouchet Award from the American Physical Society (APS, 2008) and is an elected Fellow of the APS. He tells the *Notices*: “My research engagement is with ‘physical mathematics,’ i.e., the creation, understanding, and application of techniques related to mathematics which can be used to model, analyze, and provide insightful understandings of systems based in the physical universe.”

The prize will be presented at the Blackwell–Tapia Conference and Award Ceremony at the Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University in November 2018.

Previous recipients of the prize are:

- Mariel Vazquez (2016)
- Jacqueline Hughes–Oliver (2014)
- Ricardo Cortez (2012)
- Trachette Jackson (2010)
- Juan Meza (2008)
- William Massey (2006)
- Rodrigo Bañuelos (2004)
- Arlie Petters (2002)

—From a National Blackwell–Tapia Committee announcement

Bergner Awarded Michler Prize



Julie Bergner

JULIE BERGNER of the University of Virginia has been awarded the 2018–2019 Ruth I. Michler Memorial Prize of the Association for Women in Mathematics (AWM). She was selected for her “proposed project to connect some of her recent work with the research of Cornell faculty member Inna Zakharevich, including simultaneous developments by both women (and their respective coauthors) on algebraic K -theory

constructions.” Bergner’s research has been in the areas of homotopy theory. Her proposed research will bring together several facets of her work: the theoretical framework of homotopical categories and generalizations, the realization of 2-Segal spaces as a form of algebraic K -theory, and looking at derived Hall algebras as algebraic

homotopical categories. Bergner received her PhD in 2005 from the University of Notre Dame under the direction of William Dwyer. She taught at the University of California Riverside before joining the University of Virginia.

The Michler Prize grants a mid-career woman in academia a residential fellowship in the Cornell University mathematics department without teaching obligations.

—From an AWM announcement

Prizes of the Mathematical Society of Japan

The Mathematical Society of Japan (MSJ) has announced a number of prizes for 2018.



Yoshikata Kida

YOSHIKATA KIDA of the University of Tokyo has been awarded the MSJ Spring Prize “for his outstanding contributions to the study of discrete groups and ergodic theory.” The Spring Prize and the Autumn Prize are the most prestigious prizes awarded by the MSJ to its members. The Spring Prize is awarded to those of under the age of forty who have obtained outstanding mathematical results.

The Algebra Prizes have been awarded to TAKAYUKI HIBI of Osaka University for computational commutative algebra and combinatorics; to SATOSHI NAITO of the Tokyo Institute of Technology for representation theory of quantum affine algebras; and to KANETOMO SATO of Chuo University for a new cohomology theory for arithmetic schemes and its applications.

The Outstanding Paper Prizes, given for papers published in the *Journal of the Mathematical Society of Japan*, were awarded to the following: JONATHAN BENNETT of the University of Birmingham, NEAL BEZ of Saitama University, and CHRIS JEAVONS and NIKOLAOS PATTAKOS, both of the University of Birmingham, for their paper “On Sharp Bilinear Strichartz Estimates of Ozawa-Tsutsumi Type,” *Journal of the MSJ* 69 (2017), no. 2; to TOSHIYUKI TANISAKI of Osaka City University for “Modules over Quantized Coordinate Algebras and PBW-Bases,” *Journal of the MSJ* 69 (2017), no. 3; and to YASUNORI MAEKAWA of Tohoku University and JONAS SAUER of Technische Universität Darmstadt for their paper, “Maximal Regularity of the Time-Periodic Stokes Operator on Unbounded and Bounded Domains,” 69 (2017), no. 4.

—From MSJ announcements

Chen Awarded CAIMS-Fields Industrial Mathematics Prize



Zhangxing (John) Chen

ZHANGXING (JOHN) CHEN of the University of Calgary has been awarded the 2017 CAIMS-Fields Industrial Mathematics Prize of the Canadian Applied and Industrial Mathematics Society (CAIMS) and the Fields Institute “for his seminal contributions to industrial and applied mathematics, computational science, and modeling of flow in porous media.” Chen’s group uses modeling and simulation to develop new, more economical,

and more sustainable ways to recover heavy oil and oil sands resources. Chen has led many collaborative projects with such industrial partners as Suncor, Nexen Energy, Petróleos Mexicanos, China National Petroleum Corporation, and Computer Modeling Group. His research has had major impact on practical applications in the oil and energy sectors. He has published sixteen books and more than 500 refereed journal papers covering such a range of problems as existence-uniqueness theory, finite element approximations, homogenization, and parallel algorithms, with applications to multiphase porous media flow, CO₂ sequestration, and semiconductor device simulations. He has supervised more than 100 graduate students and postdocs in industrial applications of mathematics and advanced computing algorithms.

Chen tells the *Notices*: “My favorite hobby is watching sports (NBA, NFL, and NHL) and playing sports (soccer). I am still playing soccer games and organizing soccer games to play with my grad students regularly, at least one game per week.”

—From a CAIMS-Fields announcement

Traub Prize for Achievement in Information-Based Complexity

PAWEŁ PRZYBYŁOWICZ of AGH University of Science and Technology has been awarded the 2018 Joseph F. Traub Prize for Achievement in Information-Based Complexity. He will receive a cash prize of US\$3,000 and a plaque at the UMI-SIMAI-PTM Conference in September 2018 in Wrocław, Poland.

—Joseph F. Traub Prize Committee announcement

Pevtsova Receives PIMS Education Prize



Julia Pevtsova

JULIA PEVTSOVA of the University of Washington has been awarded the 2018 PIMS Education Prize of the Pacific Institute for the Mathematical Sciences (PIMS). According to the prize citation, she has “an astonishing record of local K–12 outreach,” including starting a program for fourth and fifth graders called Math Challenge, running a Math Circle for seventh to ninth graders, and holding Math Hours and Math Hour Olympiads. Math Hour is a series of monthly lectures given by faculty members and open to a wide audience. The prize recognizes individuals in Western Canada and Washington State who have played a major role in encouraging activities that enhance public awareness and appreciation of mathematics, as well as fostering communication among various groups concerned with mathematical education at all levels.

—From a PIMS announcement

NCTM Lifetime Achievement Awards

The National Council of Teachers of Mathematics (NCTM) has chosen two educators to receive Lifetime Achievement Awards for 2018. They are CAROLE E. GREENES of Arizona State University and WILLIAM R. SPEER of the University of Nevada, Las Vegas.

The prize citation for Greenes states: “Throughout her distinguished career, Dr. Carole Greenes has dedicated herself to excellence in mathematics education. Known for her generous, creative, and encouraging approach, she has made a long-lasting impact on the field.” Greenes has been professor of mathematics education at Boston University and dean of the School of Educational Innovation at Arizona State and is currently director of the PRIME Center at Arizona State. She is a member of the Massachusetts Mathematics Educators Hall of Fame and has received the Ross Taylor/Glenn Gilbert National Leadership Award in Mathematics Education, the Arizona Copper Apple Award in Leadership in Mathematics, and the Alfred D. Wilde Award.



William R. Speer

According to his prize citation, Speer “has spent most of his life working to improve mathematics education in more than fifteen different roles over the years at institutions across the country. He is known for his tireless efforts and contributions to advancements within the field.” Speer received his PhD from Kent State University. He was a member of the 1991 writing team that helped create NCTM’s Professional Standards for Teaching Mathematics. He served as the editor of the IDEAS section for *Arithmetic Teacher* and editor of the INVESTIGATIONS section of *Teaching Children Mathematics*, and he was the general editor for the final series of NCTM Yearbooks. He was also the recipient of two Fulbright Awards. Among his honors are Outstanding Service in Math Education Awards from both the Ohio Council and the Nevada Council and the University of Nevada, Las Vegas, Distinguished Teaching Award. He is past president of the Ohio Council of Teachers of Mathematics, the Ohio Mathematics Education Leadership Council, the Nevada Mathematics Council, and the Nevada Association of Teacher Educators. He tells the *Notices*, “I have traveled extensively to nearly 100 countries and lectured in 36 throughout my career. All the while, my wanderlust has not been evidenced by transient school affiliation, having only served as a professor at Bowling Green State University (for 25 years) and the University of Nevada Las Vegas (for 22 years), with brief sabbaticals at Northern Arizona University, the College of the Bahamas and the University of West Indies.”

—From NCTM announcements

Raymond J. Carroll Young Investigator Award in Statistics

ERIC B. LABER of North Carolina State University has been awarded the 2017 Raymond J. Carroll Young Investigator Award in Statistics for his research, which includes the areas of reinforcement learning, data-driven decision making, causal inference, statistical computing, optimization, empirical processes, and bootstrap. The award is presented biannually by the department of statistics at Texas A&M University to an outstanding young researcher in statistical science.

—From a Texas A&M announcement

Borodin, Corwin, and Ferrari Receive Inaugural Alexanderson Award

ALEXEI BORODIN of the Massachusetts Institute of Technology, IVAN CORWIN of Columbia University, and PATRIK FERRARI of the University of Bonn have been chosen to receive the first Alexanderson Award, given by the American Institute of Mathematics (AIM). They were honored for their article, “Free Energy Fluctuations for Directed Polymers in Random Media in $1 + 1$ Dimensions,” *Communications in Pure and Applied Mathematics* 67 (2014).

The prize citation reads, “This work began during the October 2011 AIM workshop ‘The Kardar-Parisi-Zhang equation and universality class.’ The paper concerns the extreme behavior of certain models for polymers—long chains of molecules that occur in nearly every manufactured product. The authors discovered that the behavior of such polymers is governed by universal laws, including the Tracy-Widom distribution and the Kardar-Parisi-Zhang equation. These results may have implications in the fields of physics, engineering, materials science, biology, ecology, and other areas.”

The new award is given in honor of Gerald Alexanderson, professor of mathematics at Santa Clara University and founding chair of AIM’s Board of Trustees. His leadership has extended nationally, contributing to the work of both the Mathematical Association of America (MAA) and the American Mathematical Society (AMS). The Alexanderson Award recognizes outstanding research articles arising from AIM research activities that have been published within the past few years.

—From an AIM announcement

AWM Essay Contest Winners Announced

The Association for Women in Mathematics (AWM) has announced the winners of its 2018 essay contest, “Biographies of Contemporary Women in Mathematics.” The grand prize was awarded to JACOB SLAUGHTER of Thetford Academy, Thetford, Vermont, for the essay “Running the Numbers,” about Rosa Orellana of Dartmouth College. This essay also won first place in the high school category and will be published in the *AWM Newsletter*. First place in the undergraduate category was awarded to FRANCESCA PARIS of Williams College for the essay “Dr. Bhramar Mukherjee: Balance and Big Data,” about Bhramar Mukherjee of the University of Michigan. First place in the middle school category was awarded to BRONWEN ROOSA of Harvard-Westlake Middle School, Los Angeles, for the essay “Excellence = Math Circle²” about Olga Radko of the University of California Los Angeles/Los Angeles Math Circle.

—From an AWM announcement

MathWorks Math Modeling (M3) Challenge

The 2018 MathWorks Math Modeling (M3) Challenge (formerly called Moody’s Mega Math Challenge) was held in New York City in April 2018. This year’s challenge was to use mathematical modeling to recommend solutions to the issue of food security in the United States.

The Challenge Champions Team Prize of US\$20,000 in scholarship money was awarded to a team from Los Altos High School, Los Altos, California. The team members were RYAN HUANG, MICHAEL VRONSKY, DANIEL WANG, JUSTIN YU, and JOANNE YUAN. They were coached by Carol Evans.

The First Runner-Up Team Prize of US\$15,000 in scholarship money went to a team from Marvin Ridge High School in Waxhaw, North Carolina. The team members were TYLER BOLO, ANDREW CLAXTON, DANIEL HALLER, JAINITH PATEL, and GEORGE RATEB. They were coached by Robin Filter.

The Third Place Team Prize of US\$10,000 in scholarship money was awarded to a team from Pine View School in Osprey, Florida. The team members were CHLOE HARRIS, DYLAN HULL, TRISTAN LEE, ZOE McDONALD, and SARAH MIHM. They were coached by Mark Mattia.

Finalist Team Prizes of US\$5,000 were awarded to three teams. The team from Adlai E. Stevenson High School in Lincolnshire, Illinois, consisted of ALBERT CAO, ANDREW HWANG, DEEPAK MOPARTHI, JOSHUA YOON, and HAoyang YU. Their coach was Paul Kim. The team from High Technology High School in Lincroft, New Jersey, consisted of ERIC CHAI, KYLE LUI, STEVEN LIU, ADITHYA PARAMASIVAM, and YIHAN WU, and they were coached by Raymond Eng. The team from Middlebury Union High School in Middlebury, Vermont, consisted of JANET BARKDOLL, EZRA MARKS, BASTIAAN PHAIR, JULIAN SCHMITT, and LAURA WHITLEY; their coach was Perry Lessing.

The M3 Challenge invites teams of high school juniors and seniors to solve an open-ended, realistic, challenging modeling problem focused on real-world issues. The top five teams receive awards ranging from US\$5,000 to US\$20,000 in scholarship money. The competition is sponsored by MathWorks, a developer of computing software for engineers and scientists, and is organized by the Society for Industrial and Applied Mathematics (SIAM).

—From a MathWorks/SIAM announcement

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Photo of Ronald E. Mickens courtesy of Ronald E. Mickens.

Photo of Julie Bergner courtesy of Julie Bergner.

Photo of Yoshikata Kida by Makoto Yamashita.

Photo of Zhangxing (John) Chen courtesy of Zhangxing Chen.

Photo of Julia Pevtsova courtesy of Julia Pevtsova.

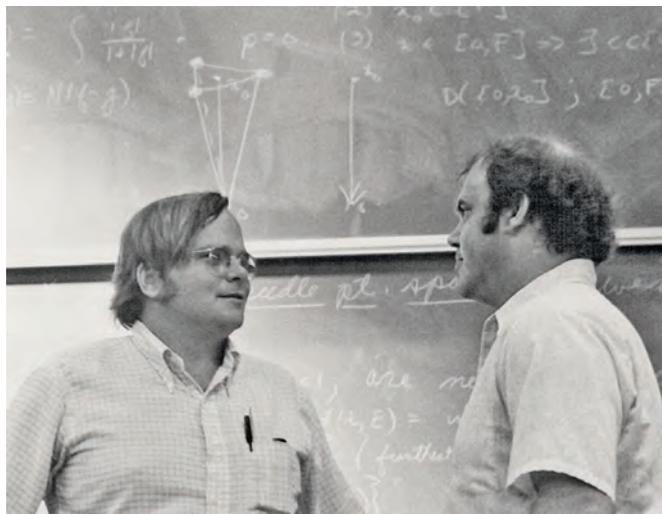
Photo of William R. Speer by Jeff Speer.

Tribute to Joe Diestel (1943–2017)

An excellent mathematician specializing in Banach space and measure theory, Joe Diestel made a lasting, distinctive contribution as a proselytizer of university-level mathematics in general and functional analysis in particular. He organized important conferences and was able to overcome prevailing Cold War sentiment to help organize East-West meetings on functional analysis during the 1960s and 1970s. Joe's own lectures were magnetic in holding one's interest, in large part because of his evident love for the subject. Joe made no bones about the hard work that was required of him in order to understand the work of masters like Grothendieck and Pettis. One of his messages to all was that there is no shame in spending many hours to understand a difficult idea. His baseball-infused, often elegant lectures reflected the benefit of this approach.

It is Joe's books that will be his most significant legacy. In 1968 Joe got his PhD at Catholic University under Victor Michael Bogdan. In 1975 he published the Springer *Lecture Notes Geometry of Banach Spaces—Selected Topics* and in 1977 *Vector Measures* with J. Jerry Uhl. Four other books appeared in due course. All are typical of Joe: reader-friendly but uncompromising and written at a high level for serious students and lovers of functional analysis.

—Richard Aron



Joe's influential books include *Vector Measures* with Jerry Uhl, pictured here with Joe in the mid-1970s at an informal analysis seminar at Kent State University.

Seán Dineen

Joe was passionate about everything: his family, friends, mathematics, social gatherings, Banach spaces, Kent State University, vector measures, baseball, tensor products, traveling, Grothendieck's inequality, practical jokes and stories. He was even passionate about the fact that he never learned to drive. He was serious about everything and, yet, everything was fun to Joe. To Joe there was a tale behind every theorem, a person behind every example, a community behind every mathematical theory, interconnections between any two mathematical areas. Whatever the social or scientific occasion, he delighted in telling the tale to whomever would listen. Joe roamed the world as an ambassador extraordinaire, explaining to the mildly initiated the works of Grothendieck, James, Pełczyński, Pettis, and Pisier. Today we are lucky he recognized and developed his gift as a communicator and that he chose to share his many insights with us.

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Joe was always working hard to make Kent State a leading place for functional analysis and recruiting the best graduate students. Here he is pictured with some of his doctoral students. Front row: Barbara Faires, Jeffrey Connors, Joe, Rajappa Asthagiri, Emily Sprague, Berhani Ghaim, and Mienie De Kock. Back row: Qingying Bu, Patrick Dowling, Tom Barton, John Alexopoulos, Paul Abraham, and Chris Lennard.

Joe had an uncanny ability to empathize with his audience whether it was a first-year class, a group of mature academics from diverse disciplines, a social gathering in a pub, or a room full of union activists. Formally, he shared his gift by writing those wonderful books, by organizing conferences, by attending conferences, by giving endless seminars, by arranging many visits to Kent State University, and by being a very nice person to everyone he met. He enticed many a budding mathematician to follow him into analysis.

In 1976 Joe decided that he would spend a year abroad. On hearing that Ireland was due a Fulbright Fellowship in the sciences, he applied and as a result visited University College Dublin for the academic year 1977–1978. I met him for the first time when he arrived in Ireland and we shared an office, F209, during his sabbatical year. Joe made himself at home in Dublin very rapidly. Almost immediately most of the mathematics faculty were swept up in Joe's enthusiastic seminars—he gave at least three every week. At the time, he was writing chapters in what later became *Sequences and Series*. In those far-off days before h-numbers, email, mobile phones, citation indices, rankings, and rampant managerialism, our university had well-attended faculty meetings, and academics from across the disciplines had more than a nodding acquaintance with one another. It was the custom in those days to gather in the Common Room for some socializing. Joe, with his endless stories and anecdotes and willingness to engage anyone on any topic, soon got to know at least half the academics in the university. Under his tutelage, the mathematics faculty became very adept at the game of darts.

Joe's lecturing style was lively. He kept his audience in suspense by promising dramatic revelations later. He made claims, sometimes outlandish; e.g., *for fifteen years nobody understood the proof of this theorem*, that were delivered with such conviction that we immediately believed them. His elementary courses were very well attended because of their entertainment value. I was very impressed to overhear, while in a lunch queue one day, a student of his from a very basic course repeat an entertaining story about Pełczyński at a conference in South Africa. Regardless of what mathematics these students retained after their college days, they at least left with the opinion that mathematicians were interesting and normal and that they had a sense of humor.

Joe's writing style was original, lively, entertaining, and a refreshing contrast to what was accepted as standard mathematical prose. Of course, he included what would ordinarily be regarded as the main results of whatever theory he was discussing, but additionally he gave insights that would rarely if ever appear in print elsewhere. He resurrected forgotten proofs, he compared different proofs, he presented and analyzed special cases, he pointed out key lemmata and crucial turning points, and he showed the relevance, and occasionally the irrelevance, of results to other parts of analysis and mathematics. Joe was able to appreciate that the isolated scholar might wonder why the precise value of Grothendieck's constant K_G was important and so, for the benefit of all isolated scholars, he proceeded to tell the world that it was only in discussing the failure of the von Neumann–Andô Inequalities that the estimate $K_G > 1$ was ever used.

Guillermo P. Curbera

Joe played around with words, and it's not difficult to find examples. A random opening of his books revealed within two minutes the following examples. He did not avoid complicated computations, but when they were difficult or long or tedious he took it personally and called them *grubby manipulations* and *gruesome calculations*. Joe liked to find direct proofs of simple cases of important results—this was one of his approaches to understanding mathematics. Afterwards he would consider whether this led to a new proof. Sometimes it did, but at times his new proof might turn out to be more complicated than the standard proof. To explain on one occasion that such an approach was possible but not advisable he wrote: *But it should also be clear that such a procedure would inevitably lead us to countenance considerably convoluted combinatorial contortions.*

Vitali Milman

Joe Diestel was the most American among all American mathematicians I knew. He liked baseball and beer. He was an excellent organizer and a very clever and intelligent person. Joe organized and led regular bi-weekly meetings in Kent for the “Ohio” group of functional analysis, and many experts visited these meetings from all over the world. They continued for decades.

Joe organized two huge conferences in geometric functional analysis, in 1979 and 1985. Many hundreds of people participated in them.

Per Enflo

The first time I met Joe Diestel was in April 1972, at a conference in Baton Rouge. I immediately liked him, and I liked the way he talked about the development of Banach space theory. It was passionate, with many ideas of what ought to be done. With the years Joe became a dear friend and a colleague at Kent State University. He was an excellent mathematician, and he was always working hard and very successfully to make Kent State a leading place for functional analysis, arranging conferences, bringing in the best researchers, and recruiting the best graduate students. I admired his way of always standing up for high quality, his integrity, and his constant and courageous fight against mediocrity. I miss him a lot.

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I met Joe Diestel for the first time in the spring of 1989. He was giving a colloquium talk in the Universidad Complutense de Madrid. Not long before I had started working in my PhD on a problem that he had posed. I was struck by his vivid style of lecturing and the passion he placed on the technical details of a proof. When I visited Kent State University for three months, I was placed with Joe in his tiny office, with no windows and 4- or 5-foot-tall piles of math papers on the floor. There was his famous refrigerator, filled with beer to treat visitors. And there I discovered Joe's other passion, baseball.

Years later, when I started directing my first PhD student, with a tone of solemnity Joe gave a piece of advice: he said there is always a moment when the advisor, or the coach, has to tap the student, or baseball player, on the shoulder and say, “Well done, kid.” And he remarked, “It should never be too early, neither too late; just in the right moment.”

That is how I got to understand the link between Joe's two passions. In both, mathematics and baseball, it is crucial the human factor: there are people behind, people who work, people who dream, people who fight. What Joe appreciated were those efforts and dreams via their outcome: beautiful mathematical proofs in one case, spectacular baseball moments in the other. Joe's passion was what people create with their efforts. Joe's passion was people.

Photo Credits

Photo of Joe and Jerry Uhl courtesy of the Kent State University Banach Center Archives.

Photo of Joe and doctoral students courtesy of Mary Kellermann.

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Clarence F. Stephens (1917–2018)

Johnny L. Houston



“More than fifty years ago, I arrived at the conclusion that every college student who desired to learn mathematics could do so. I spent my entire professional life believing that this was the case.”—Clarence Stephens, June 20, 1997

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Honors Received by Clarence Stephens

PhD in mathematics (ninth African American, 1943)

Ford Fellowship at the Institute for Advanced Study in Princeton, where he worked alongside Albert Einstein (1953)

Honored for distinguished service to education by Governor J. Millard Tawes of Maryland (1962) and by Governor Mario Cuomo of New York (1987)

Inducted into the National Museum of American History at the Smithsonian Institution (1983)

Three honorary doctorates: from Johnson C. Smith University (1954), Chicago State University (1990), and SUNY (1996)

Seaway Section of MAA names their teaching award after Stephens (2003)

National Association of Mathematicians Lifetime Achievement Award (1998)

MAA Gung and Hu Award for Distinguished Service to Mathematics (2003)

100th birthday symposium at SUNY Potsdam (2017)

NAM Centenarian Award and naming of Stephens annual teaching award in his honor (2018)

Clarence F. Stephens was a nationally acclaimed master teacher of collegiate mathematics and creator of the Morgan-Potsdam Model and the “Potsdam Miracle”: for several years in the 1980s, while Stephens was mathematics department chair at SUNY Potsdam, over 20 percent of all graduates were mathematics majors (as compared to the national average of about 1 percent), without sacrificing rigor. In 1985, over 25 percent of all graduates and over 40 percent of the honors students were mathematics majors [1–5].

Clarence Francis Stephens, Sr., was the fifth of six children (three girls and three boys) born to Sam Stephens (a chef and railroad worker) and Jeannette Morehead Stephens in Gaffney, South Carolina, on July 24, 1917. Sadly,

his mother died when Clarence was two, his father died when he was eight, and all six children went to live with their maternal grandmother, who died when Clarence was ten. The three boys went to Harbinger Institute, a boarding school in Immo, South Carolina, where they worked on the farm in summer to pay for their schooling in winter. All three boys attended Johnson C. Smith University and majored in math. Clarence graduated in 1938 and began graduate study in mathematics that fall at the University of Michigan. Stephens received his MS in 1939 and his PhD in 1943 on nonlinear difference equations.

After a tour as a teaching specialist in the US Navy (1942–1946), he joined the faculty of Prairie View A&M University. In 1947, the President of Morgan State University sent Stephens an invitation to join Morgan’s math faculty. One of Stephens’ main reasons for going to Morgan was that he would be near John Hopkins University, a major research institution. Stephens’ focus at that time was on research. Research articles by Stephens had been printed in AMS publications while he was at Morgan, prior to his going to Princeton to study.

While at Morgan, Stephens became appalled at what a poor job was being done, in general, to teach and inspire students to learn mathematics. He then completely changed his focus to that of being a master teacher. He remained at Morgan until 1962. Prior to his arrival at Morgan, no student from Morgan had earned a graduate degree in the mathematical sciences. Many of his students went on to earn doctoral degrees in the mathematical sciences, including Earl Barnes, Vassily Cateforis, Earl Embree, Gloria Ford Gilmer, Arthur Grainger, Charles Moore, Sylvester Reese, Robert Smith, and Scott Williams.

In 1962 Stephens accepted an appointment as professor of mathematics and department chair at the State University of New York (SUNY) at Geneseo, and in 1969 he joined SUNY at Potsdam, serving as professor and department

chair until his retirement in 1987. During his tenure at Potsdam the department became nationally known as a model of teaching excellence. For several years, the program was among the top producers of undergraduate mathematics majors in the country.

Stephens discovered at a very early age that he could learn mathematics with very little help from his teachers. This ability to read mathematics with understanding and to enjoy it for its intrinsic beauty accounts for much of his success in becoming a mathematician. His teaching techniques consisted mainly of developing these abilities in students. He realized that a student who can study independently and find joy in discovering new ideas already has much of what is required for success in mathematics. Stephens recognized that this is not an easy task. He stated [2, p. 297]:

From my experiences at Morgan and Potsdam I knew that it was difficult to establish favorable academic environments, so that any college student who desired to do so could learn and enjoy learning mathematics. To establish a successful program depended on *creative thinking, time, and place*.

In his passionate work with students, Clarence Stephens showed us all what is possible with the right combination of these ingredients.

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Photo courtesy of SUNY Potsdam.

Clarence Stephens Aphorisms [5]

Believe in your students—everyone CAN do mathematics, philosophy, art, literature, etc.

Know your students—their names, what they know, their hopes and fears.

Don’t say *this is easy*, or *this is trivial*.

Go fast slowly (most teachers go slow fast).

It is very difficult to learn how to solve problems by watching someone else do it or by reading a book; the best method to learn to solve problems is by solving a lot of problems yourself.

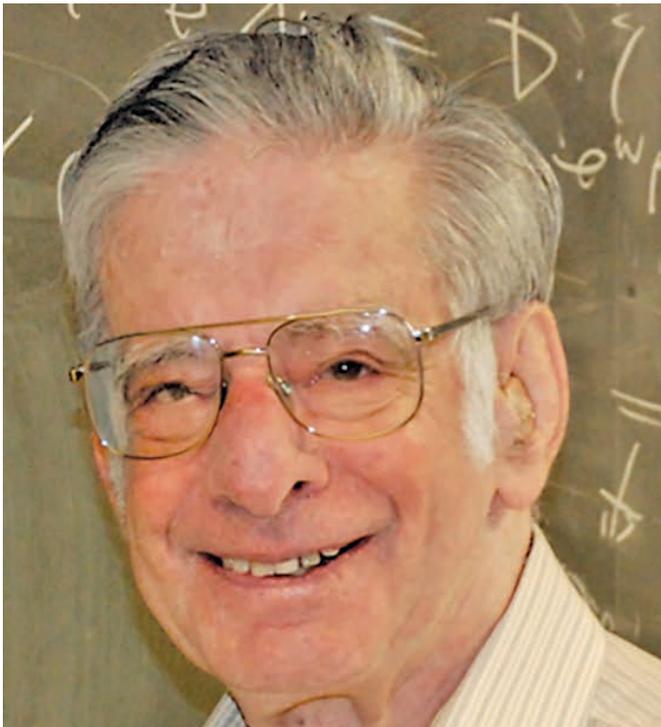
High standards do not mean having unrealistic expectations so students feel that they have failed.

Write tests carefully—know what your average student can do and what your best student can do.

And most of all—teach the students you have, not the ones you wish you had.

Hans F. Weinberger (1928–2017)

Don Aronson, Peter Olver, and Fadil Santosa, Coordinating Editors



Hans Felix Weinberger passed away at the age of 88 in Durham, North Carolina, on September 15, 2017. He was a member of the University of Minnesota School of Mathematics for thirty-seven years and played a vital role in bringing the School to its present eminence and in the establishment of the NSF Institute for Mathematics and its Applications (IMA) on the Minneapolis campus. He

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specialized in the study of various aspects of the analysis of partial differential equations, including estimation of eigenvalues, isoperimetric inequalities, and the maximum principle. More recently he turned his attention to mathematical biology.

Hans was born in Vienna, Austria. In the 1930s it became clear that, with the rise of Hitler in Germany and his influence on Austria, the situation for Jews in Austria was becoming perilous. The Weinberger family immigrated to the US in the fall of 1938. Hans was an excellent student and graduated from high school at the age of 16. He was very interested in science and became a finalist in the Westinghouse Science Talent Search in 1945. In 1950 at the age of 21 he received an ScD in mathematics at Carnegie Institute of Technology under Richard Duffin, who at the same time also supervised the work of Raoul Bott. Another classmate was the legendary John Nash, who was Hans's roommate for one semester. An account of their relationship can be found in Sylvia Nasar's book *A Beautiful Mind*.

Hans's earliest work, at the University of Maryland, was on variational methods for eigenvalue approximation, often in collaboration with Larry Payne. In addition, Hans also worked on isoperimetric problems, properties of solutions of hyperbolic and elliptic partial differential equations, and strength of materials.

In 1960 Hans joined the faculty of the University of Minnesota, where he served as department head and supervised nine PhD students, including Bert Hubbard, Roger Lui, John Osborne, and Jianzhong Zu. He wrote or coauthored more than 140 research papers, the last appearing in 2015.

Hans published three influential books: the CBMS monograph *Variational Methods for Eigenvalue Approximation* based on his lectures at Vanderbilt University; the monograph *Maximum Principles in Differential Equations* with Murray Protter; and the widely used textbook *A First Course in Partial Differential Equations*. In 1986 Hans was elected to membership in the American Academy of Arts and Sciences and was a member of the inaugural class of Fellows of the American Mathematical Society. He was Managing Editor of the *Bulletin of the AMS* from 1972 to 1977.

In 1979 Hans, George Sell, and Willard Miller submitted a proposal to establish the Institute for Mathematics and

its Applications (IMA) at the University of Minnesota, and Hans served as its first director. Hans was very much a hands-on director, attending almost all lectures and collaborating with visitors and postdocs.



Weinberger and George Sell in 1982 at the IMA, which they founded with Willard Miller.

Avner Friedman, who succeeded Hans as IMA director, said on his passing: “In addition to being a marvelous mathematician, Hans was a kind person, always seeing the good things in people, always considerate. I was fortunate to have known him and to have had him as a very close friend. He will be missed.”

Hans was modest and unassuming, but he had prodigious mathematical talent. His office door was always open, and he encouraged students, colleagues, and visitors to come in to discuss their current work. He was very quick at seeing the essence of any problem and often was able to offer extremely helpful comments and suggestions. He was, in sum, an ideal colleague.

Mark Lewis

Hans was a great teacher for me. Since first reading his work I have been in awe of him; he thought about mathematics in such a powerful way. I continually learned from him. He was always a gentleman, keeping a twinkle in his eye and a smile on his face as he dissected what I thought were some of my perfectly good proofs. He was perceptive. I remember him apparently sleeping at mathematics seminars only to come up with a very insightful question right at the end of the lecture.

Hans contributed immensely to our understanding of nonlinear reaction-diffusion systems. He coauthored a beautiful paper with Don Aronson (1975), which establishes the rate of spread of locally introduced populations, also of great interest to ecologists. Hans generalized the results of the 1975 paper to general discrete-time recursions and non-diffusive motion. This gave rise to a

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monumental 1982 paper that was, however, no easy read. Nonetheless it turned out to have broad-reaching scientific impacts due to the generality of the results.

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Bingtuan Li

I first met Hans in the fall of 1998 when I was a postdoc with Mark Lewis participating in the year-long program Mathematics in Biology at the IMA. I always saw Hans at seminars, sitting in the front row and asking tough and piercing scientific questions, some of which were apparently outside the scope of mathematics. He effusively engaged in all the program activities, interacting with visitors and postdocs during discussion meetings and coffee breaks. When my postdoc mentor Mark Lewis introduced me to Hans, it marked a turning point in my academic life.

Mark had noticed that in a two-species Lotka-Volterra-type competition model, the spreading speed at which a species spreads into the equilibrium distribution of its rival may not be linearly determined and sought to address the problem analytically. The problem was related to Hans’s work in the 1970s and 1980s. We had regular lively discussions, which were full of questions, ideas, answers, and, most of all, fun. As I had nearly zero background about spreading dynamics of spatial-temporal systems, understanding the meeting discussions was a constant challenge for me. Hans was patient and helpful when I asked questions, no matter how simplistic. Besides getting direct help from Hans, I also read his papers with Don Aronson published in 1970s and his remarkable 1982 paper. This paper is very abstract and not easy to read, but nonetheless written beautifully, with precision, elegance, and power. Hans and Mark initiated a research plan for studying spatial dynamics of general cooperative systems. Previous work from Roger Lui, Hans’s PhD student, focused on cooperative systems with two equilibria, which extended the results from Hans’s 1982 paper. Lui’s results showed the uniqueness of spreading speed. Hans designed a comprehensive framework for analyzing more general systems with more than two equilibria, where multiple spreading speeds are identified, spreading speeds as lowest speeds of traveling waves are characterized, and linear determinacy is studied. It works for many spatial-temporal equations, including reaction-diffusion equations and integro-difference equations. I learned a great deal from Hans that year through his wonderful academic and personal mentorship.

Since leaving the IMA, I have continued learning from and working with Hans and Mark on wave propagation problems. Hans has given me his advice, thoughts, and comments on various problems as well as his encourage-

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ment and support. Very often I would send him an email at night, only to see his reply early morning the next day. His 1982 paper has been a source of ideas and inspiration for me to think about new problems as well as an influence on countless researchers in the field. He dedicated his life to his research and his love of mathematics and to his students, mentoring a new generation of curious and passionate mathematicians. Hans will be forever missed.

Howard Levine

During my graduate student years, my PhD advisor, Larry Payne, gave me two sets of University of Minnesota lecture notes by Hans Weinberger to read. One of them became the basis for “Variational Methods for Eigenvalue Approximation” *CBMS Notes*, #15, SIAM, 1972. These notes were extremely well written and very enlightening. I hoped that someday I would have a chance to meet their author.

When I showed up to teach at Minnesota in 1969, I saw that I was down to teach something called “mathematical fluid dynamics.” My heart sank. I knew almost nothing about fluid mechanics beyond the fact that water and oil do not mix. When I told Hans, who was head, of my ignorance of the subject, he said, “Don’t worry. You’ll learn.” That was the kind of guy he was, a Renaissance mathematician.

Once, after a particularly boring colloquium during which Hans was dozing, he asked the speaker what I considered a very deep question, which the speaker seemed pleased to answer. After the talk I asked him if he followed the talk, and he replied that he did not. So I asked him why he asked a question, and such a very good one at that. “Howard,” he said, “asking a question is just being polite to our guest.”

At a personal level, his office door was always open, both literally and figuratively. When I was in the department or, later, visiting IMA, I knew that if I had a scientific question, Hans would be a welcoming person to ask. Sometimes I would ask him a question by snail mail. I would invariably receive either a written reply or else a phone call. One of these exchanges led to our joint paper, “Inequalities between Dirichlet and Neumann Eigenvalues.” I proposed extending a result of Payne’s for such inequalities for convex domains in the plane to convex domains in several dimensions. I had a plausible argument for doing this but it wasn’t plausible enough for Hans. He then proceeded to give a rigorous argument, extending the results to certain wide classes of nonconvex domains. Some people might have just published these results under their own name, perhaps acknowledging the source of the conjecture, but not Hans. “You proposed the problem; you should share in the result,” he told me. He was not only a gentleman, but a real mensch!

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Catherine Bandle

The first time I encountered the name of Hans Weinberger was during my master’s thesis. In those days I studied his paper on the second eigenvalue of a free membrane. By a clever trick, using Browder’s fixed-point theorem, he showed that the ball has the largest second eigenvalue among all domains of the same volume. This short paper reflects the art and the originality of Weinberger’s mathematics. Fascinated by this paper I then continued to read his other papers on eigenvalue estimates and isoperimetric inequalities. Many of them were written in collaboration with L. Payne. They contributed significantly to the development of the isoperimetric inequalities in mathematical physics, an area that started to become popular after the pioneering work of Pólya and Szegő. By now several of Weinberger’s results belong to the classical inventory of analysis.

Unique in Weinberger’s work is his deep understanding of the geometry hidden in the classical partial differential equations. This together with his physical intuition enabled him to discover many important properties of the spectrum of second and fourth order partial differential equations, and to tackle open problems in a very original and elegant way.

Needless to say how pleased I was when, during a visit at the ETH in Zürich, I had the occasion to meet him personally. I got to know him as a gentle and modest person. His open-mindedness to things other than mathematics impressed me greatly at that time. During a trip with him and [his wife] Laura to some medieval Swiss town he showed great interest in cultural matters. This impression was confirmed at a later meeting in Minneapolis where Laura and Hans brought me to a performance of Tchaikovsky’s *1812 Overture*.

Nanako Shigesada

I met Hans Weinberger only twice.

My first encounter was in the early 1980s, when he was visiting Kyoto University. I still remember that I was so tense in front of him, because I majored in physics and had little professional knowledge of mathematics. In response to his friendly inquiry about my research, I somehow explained that I was working on an RD model for the spatial segregation of interacting species, a well-known phenomenon in ecology. At that time, I never supposed that we would later collaborate three times as co-authors.

Our second encounter was in April 1999 at an IMA workshop. When I stood on the stage as the second speaker on the first day, I was surprised to find Hans sitting in the first row and looking towards me. My talk was titled “Biological invasions into periodically fragmented environments: Reaction-diffusion models.” It demonstrated numerically the pattern and the ray speed of the range expansion of species introduced at a point in a stripe-shaped habitat.

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During a coffee-break the next day, Hans approached and handed me a paper napkin on which he had scribbled an equation that contained enlightening suggestions for my work instantly recognizable to me. Retrospectively, this was the start of our collaboration.

One year later, Hans sent me his own draft on traveling waves in a periodically varying habitat. Although it was tough mathematically, I was able to grasp its fundamental importance after prolonged reading. For my part, I had heuristically derived a formula for the ray speed in a stripe-shaped habitat, in collaboration with my students Noriko Kinezaki, Kohkichi Kawasaki, and Fugo Takasu. When we sent our manuscript to Hans, he immediately noticed an apparent discrepancy between his ray speed formula and ours. To our great relief, however, careful cross-examination revealed that both formulas were essentially equivalent to each other. After many more rounds of such lively exchanges, our article was finalized. We offered Hans co-authorship of the article because of his indispensable contributions. However, he cordially declined our offer, saying, "If I were to become an author, I would be inclined to push for a number of changes to turn this very nice exposition into a piece of rigorous mathematics. I think this would lose your audience, and so it is not a good idea." I was impressed with Hans's strict attitude toward science. Eventually, both parties published their own separate articles (Weinberger, 2002 and Kinezaki et al., 2003).

Subsequently, my interest shifted gradually from RD models to integro-difference models that can incorporate the life history of organisms involving non-overlapping generations and various types of dispersal kernels. In 2007, I sent Hans an article on an ID version of the RD model written by Kawasaki and myself, to which he responded, "I think there are some useful ideas, and I would be happy to collaborate with you on their development." We accepted his offer with deep gratitude. From that time, he sent us an updated draft incorporating new ideas almost every week. As a consequence of these exchanges, we were able to publish this work in 2008.

In 2009, we collaborated with Hans again, primarily at his initiative, to publish an article on an RD model for a partially cooperative 2-species system.

Our third and last collaboration with Hans was in 2014–2015, this time starting with my proposal. In spatially varying habitats, organisms tend to show directed movements (taxis) toward more favorable patches. We extended our previous reaction-diffusion (RD) model to an advection reaction-diffusion (ARD) equation in which taxis is incorporated in an advection term. For this work, Hans provided mathematical proofs of the existence of the travelling wave and its speed formula as presented in the appendix of our article published in 2015.

We had fruitful interactions with Hans for 18 years, and exchanged more than 400 emails. Tracing back through these emails made me realize how sincerely and patiently Hans had been trying to convey the role and potential of mathematics to us. Hans was truly a gentleman with the warmest heart.

Roger Lui

Hans was my PhD adviser. The first problem he had me look at was generalizing the results of the famous 1937 Kolmogorov–Petrovsky–Piskunov paper from Heaviside initial data to general initial data, but before long Hans told me that the problem had been solved by Uchiyama in Japan. Hans then asked me to look at the multi-dimensional case. That turned out to be an incredibly hard problem and I believe the problem, convergence to traveling front, is still open. I was not able to solve the problem, and I even came up with a wrong proof. Hans was very forgiving and patient with me. We had weekly meetings, and I remember that during one of the meetings, Hans told me something I remember even today—don't keep repeating the same idea (if it does not work). After I was unable to make any progress on the problem for a long time, Hans told me to look at another problem. At that time, Hans had just developed a discrete-time continuous space population genetic model, which he believed was under more realistic assumptions than the Fisher's equa-



Weinberger with PhD student Roger Lui and John Osborn at the conference celebrating Hans's 80th birthday in October 2008.

tion. He wanted me to prove for his model what KPP and Uchiyama proved for the Fisher-KPP equation. I was able to complete the first half of the problem, monotone initial data, relatively quickly, but I was stuck on the second half, initial data with compact support, for a long time, until one afternoon I tried estimating the relative difference between two functions rather than their absolute difference. That was the key idea I was missing. The rest was easy and I wrote up a 40-page note and left it in Hans's mailbox. In his office, Hans asked me to go over the proof on the board. Hans finally agreed that the proof was correct. Then I asked Hans to give me another problem to work on. He hesitated for a while and said I didn't have to do another problem. That was one of the happiest days in my life.

Roger Lui is professor of mathematical sciences at Worcester Polytechnic Institute. His email address is r1ui@wp1.edu.

Hans had a profound influence on my professional life, especially the way I do mathematics. I tend not to work in one area of mathematics for a long time, and I try to adopt his style of writing, always very clear and enjoyable to read. Hans often looked at a problem and saw what the key point was; I try to do that also.

Willard Miller Jr.

Hans Weinberger was a pillar of support and leadership for the Minnesota School of Mathematics. He is my model for how one should live as a mathematician. He was a brilliant researcher, well known for his work on the maximum principle for partial differential equations, and a dedicated teacher with very high standards, but also dedicated to service and leadership for the department and to the profession. He had the unique ability to engage profitably with everyone, not just academics. Privately, he was very modest.

Hans had a great reputation among researchers in the engineering departments at Minnesota for his ability to interact with their faculty and students on their problems. For many years the course based on his textbook *A First Course in Partial Differential Equations* was required for undergraduates in the department of chemical engineering.



At the 30th anniversary celebration of the IMA in July 2012, from right to left: Weinberger (first director), Avner Friedman (second director), Willard Miller (co-founder), Douglas Arnold, and Fadil Santosa.

My first close interaction with Hans occurred in 1978. I was newly appointed as department head, and George Sell had just returned to the school after a year as rotator in the math division of the National Science Foundation, with the news that the division was about to announce a national competition for a mathematics institute. We decided to prepare a proposal for an applied mathematics-oriented institute at Minnesota, based on the strong research connections between our department, the engineering departments, and industry in the Twin Cities area. However, we needed a leader to assume the duties

Willard Miller Jr. is professor emeritus of mathematics at the University of Minnesota. His email address is miller@ima.umn.edu.

as director. We approached all of the leading researchers in the department but were turned down by everyone, except Hans. Most of those we contacted considered the task of preparing a proposal too time consuming with little chance of success. Hans agreed, for the good of the school. He had the vision to refine our idea of an applied mathematics institute to that of an institute on the interface between mathematics, the other sciences, engineering, and industry, where mathematicians would interact with researchers from these other disciplines to solve mutual problems. All fields of mathematics would be involved, not just traditional applied mathematics, but the emphasis would be on the outreach of mathematics to attack problems in other fields. Hans took the leadership to write these critical parts of our proposal and his vision and reputation carried the day. Ultimately, our proposal was accepted and the Institute for Mathematics and its Applications (IMA) was born with Hans as its founding director, 1982–1987, and George as associate director. It continues to this day, with many successes. The task of creating an institute from scratch was very demanding and lots of teamwork was involved, with difficulties to be overcome along the way and many stops and starts. Once I said that we were like the Three Musketeers. George said, no, that we were more like the Three Amigos.

Catherine Weinberger, Sylvia Weinberger Hewitt, and Ralph Weinberger

Our father's description of how he became a mathematician was simple: "I was a physics major, but I kept breaking things." His public biography explains that he was a finalist in the Westinghouse Science Talent Search, which led to a college scholarship, and that he completed his doctorate at a young age. But it does not clarify that he never mentioned this personal history to his own children. We only learned these details from his mother after we were grown. When queried, his explanation for his young graduation age was that most of the college-aged men were away at war, so the colleges were recruiting promising sixteen- and seventeen-year-olds and running straight through the summer to train more scientists as quickly as possible. While we were growing up, all we knew was that Dad was a mathematician, as were most of his friends.

Our childhood family life was very different from the never-ending workweek now common among scientists. Although Mom would often observe that Dad was still working in his head, he came home at 6, and he was completely a family man during the evenings and weekends. Nearly every day we would all five sit down to dinner, lovingly prepared by our mother and reliably served at 6:15 pm. Dad used this family time to teach us table manners and to show us that our experiences and opinions mattered. We would share conversation and laughter, then clear the table and do something relaxing together as a family: watch TV, do jigsaw puzzles, or play board games. Dad would often keep us company while we practiced our

Catherine Weinberger, Sylvia Weinberger Hewitt, and Ralph Weinberger are Hans Weinberger's children.

musical instruments. On weekends we usually went on outings to a park or lake for a picnic and a swim, sometimes with other mathematician families or kids from the neighborhood. In the winter a group of mathematicians would take all the kids to the skating rink on Sunday while the moms had some time off. We also remember the adults-only parties the mathematicians shared, with waves of laughter rolling upstairs after we had been put to bed. On a day-to-day basis, we kids did not really understand what the mathematicians did at work, other than scribble on napkins and notepads in a foreign language.



Hans with his wife Laura at the IMA in 2002 and with his children on a camping trip in 1973.

As we grew, summer vacations often involved travel to conferences, camping along the way once we were old enough to pitch tents and wash dishes. When we arrived, some of our friends and their parents would be there, too. As teenagers, the other kids stopped coming and we three siblings kept ourselves amused by challenging each other to try to get the Russian mathematicians to smile and practicing our mathematician impersonations. Ralph particularly remembers a joint conference of mathematicians and biologists in Heidelberg—shortly before the inception of the IMA—where the children were encouraged by Peter and Peggy Rejto to sit in on Dad’s talk. In that talk, Dad broke a problem down to its simplest essentials and made mathematical points that even we teenagers could understand. Afterward, Don Aronson teased, “So, now you know.” It seemed as if everywhere we went, from Berkeley, California, to Cortona, Italy, his mathematician friends were there, too. Three times, we accompanied him for a year away from Minnesota while he visited at NYU, the University of Arizona, and Stanford. But Minnesota remained his home, and his Minnesota colleagues were his extended family.

An unwavering proponent of public schools, he was appreciative of all forms of education. He often spoke about the industrial arts training he received soon after arriving in the US. He carried those lessons with him, building cabinets in the basement, replacing flooring and masonry, and fixing anything that needed repair including plumbing and electrical work, with his son as apprentice. When a bike needed attention, Dad was there to teach us how to fix it ourselves. Later, if a headlight went out while we were borrowing the car, we were expected to drive to the store, purchase a replacement, and figure out how to install it by reading the instructions on the box. His

expectations of independence and self-sufficiency began early in our lives, and served us all well.

Our dad built a lot of things in his life, but the most enduring was his family. His world revolved around our mother, his life partner and soul mate. His children, grandchildren, great-grandchild, and in-laws will always savor the memory of his welcoming embrace.

When we were in our late twenties and early thirties, our mother used to wonder what dad would do when he retired, because he had no “hobbies.” She could see how much he enjoyed spending time with his young grandchildren but thought he would need something else to help fill his retirement days. In fact, mathematics would continue to fill his mind, notepads, computer, and (with technical assistance from a graduate student grandchild) his account on a Cloud-based platform, until his very last days.

Photo Credits

Photo of Weinberger family camping courtesy of Laura Weinberger.

All other article photos are courtesy of the photo archives of the Institute for Mathematics and its Applications (IMA).

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AMS AMERICAN MATHEMATICAL SOCIETY

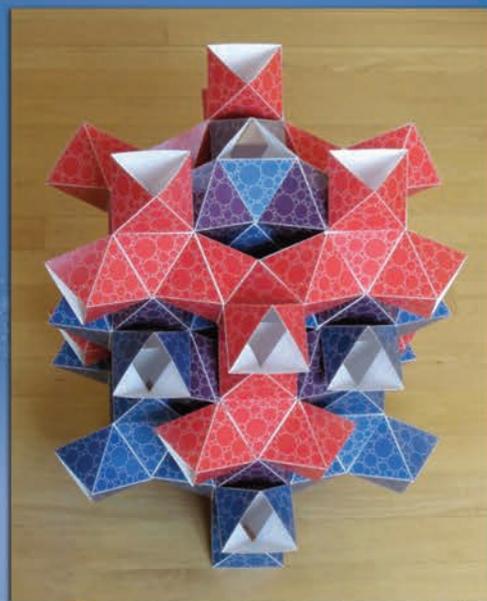
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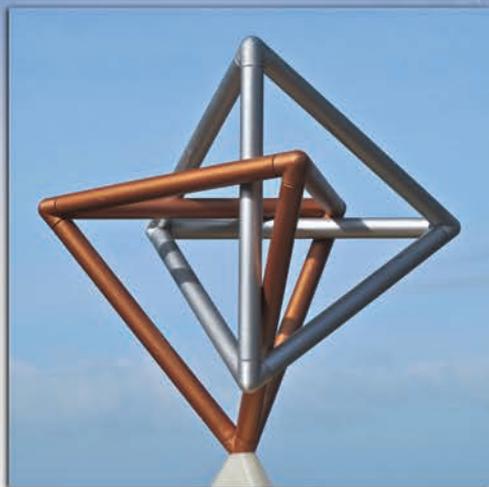
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The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, Oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Dutch artist M.C. Escher represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works.

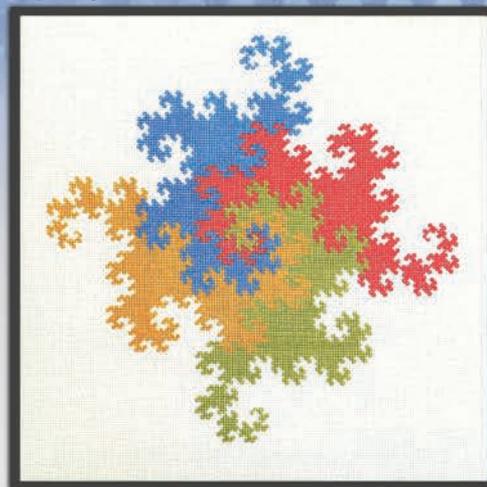
Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics—origami, computer-generated landscapes, tessellations, fractals, anamorphic art, and more.



"A Fractal Circle Pattern on the $\{3,12\}$ Polyhedron,"
by Doug Dunham, *University of Minnesota - Duluth*



"Linked Tetra Frames," by Carlo Séquin,
University of California, Berkeley

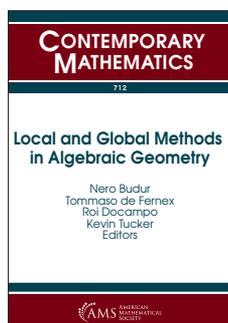


"Heighway Dragon Tiling," by Larry Riddle,
Agnes Scott College, Decatur, GA

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Algebra and Algebraic Geometry



Local and Global Methods in Algebraic Geometry

Nero Budur, *KU Leuven, Belgium*,
Tommaso de Fernex, *University
of Utah, Salt Lake City, UT*,
Roi Docampo, *University of
Oklahoma, Norman, OK*, and
Kevin Tucker, *University of
Illinois at Chicago, IL*, Editors

This volume contains the proceedings of the conference Local and Global Methods in Algebraic Geometry, held from May 12–15, 2016, at the University of Illinois at Chicago, in honor of Lawrence Ein's 60th birthday.

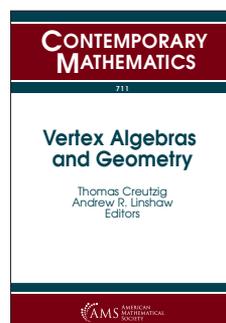
The articles cover a broad range of topics in algebraic geometry and related fields, including birational geometry and moduli theory, analytic and positive characteristic methods, geometry of surfaces, singularity theory, hyper-Kähler geometry, rational points, and rational curves.

Contents: **R. Lazarsfeld**, Some remarks on the work of Lawrence Ein; **A. Calabri** and **C. Ciliberto**, Contractible curves on a rational surface; **F. Catanese**, On the canonical map of some surfaces isogenous to a product; **C. Ciliberto**, **F. Flamini**, **C. Galati**, and **A. L. Knutsen**, Degeneration of differentials and moduli of nodal curves on $K3$ surfaces; **I. Coskun** and **J. Huizenga**, Weak Brill-Noether for rational surfaces; **R. Datta** and **K. E. Smith**, Excellence in prime characteristic; **M. González Villa**, **A. Libgober**, and **L. Maxim**, Motivic zeta functions and infinite cyclic covers; **C. Hacon**, **M. Popa**, and **C. Schnell**, Algebraic fiber spaces over abelian varieties: Around a recent theorem by Cao and Păun; **S. Ishii** and **W. Niu**, A strongly geometric general residual intersection; **J. Kollár**, Quadratic solutions of quadratic forms; **S. J. Kovács**, Non-Cohen-Macaulay canonical singularities; **N. Mok**, Full cones swept out by minimal rational curves on irreducible Hermitian symmetric spaces as examples of varieties underlying geometric substructures; **M. Mustață** and **Y. Nakamura**, A boundedness conjecture for minimal log discrepancies on a fixed germ;

E. Sernesi, The Wahl map of one-nodal curves on $K3$ surfaces; **Y.-T. Siu**, Skoda's ideal generation from vanishing theorem for semipositive Nakano curvature and Cauchy-Schwarz inequality for tensors; **C. Voisin**, Hyper-Kähler compactification of the intermediate Jacobian fibration of a cubic fourfold: The twisted case.

Contemporary Mathematics, Volume 712

August 2018, 368 pages, Softcover, ISBN: 978-1-4704-3488-5, LC 2018003650, 2010 *Mathematics Subject Classification*: 13A35, 13C40, 14B05, 14D06, 14G05, 14J25, 14J60, 32S55, 53B21, **AMS members US\$93.60**, List US\$117, Order code CONM/712



Vertex Algebras and Geometry

Thomas Creutzig, *University of
Alberta, Edmonton, AB, Canada*,
and **Andrew R. Linshaw**,
University of Denver, CO, Editors

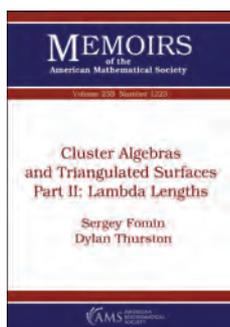
This book contains the proceedings of the AMS Special Session on Vertex Algebras and Geometry, held from October 8–9, 2016, and the mini-conference on Vertex Algebras, held from October 10–11, 2016, in Denver, Colorado.

The papers cover vertex algebras in connection with geometry and tensor categories, with topics in vertex rings, chiral algebroids, the Higgs branch conjecture, and applicability and use of vertex tensor categories.

Contents: **F. Malikov**, Strongly homotopy chiral algebroids; **T. Arakawa**, Associated varieties and Higgs branches (a survey); **G. Mason**, Vertex rings and their Pierce bundles; **T. Creutzig** and **A. R. Linshaw**, Cosets of the $\mathcal{W}^k(\mathfrak{sl}_4, f_{\text{subreg}})$ -algebra; **J. Yang**, A sufficient condition for convergence and extension property for strongly graded vertex algebras; **J. Auger** and **M. Rupert**, On infinite order simple current extensions of vertex operator algebras.

Contemporary Mathematics, Volume 711

August 2018, approximately 174 pages, Softcover, ISBN: 978-1-4704-3717-6, LC 2018003590, 2010 *Mathematics Subject Classification*: 81R10, 17B69, **AMS members US\$93.60**, List US\$117, Order code CONM/711



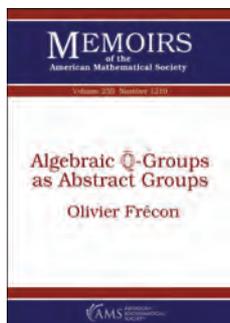
Cluster Algebras and Triangulated Surfaces Part II: Lambda Lengths

Sergey Fomin, *University of Michigan, Ann Arbor, MI*, and Dylan Thurston, *Indiana University, Bloomington, IN*

Contents: Introduction; Non-normalized cluster algebras; Rescaling and normalization; Cluster algebras of geometric type and their positive realizations; Bordered surfaces, arc complexes, and tagged arcs; Structural results; Lambda lengths on bordered surfaces with punctures; Lambda lengths of tagged arcs; Opened surfaces; Lambda lengths on opened surfaces; Non-normalized exchange patterns from surfaces; Laminations and shear coordinates; Shear coordinates with respect to tagged triangulations; Tropical lambda lengths; Laminated Teichmüller spaces; Topological realizations of some coordinate rings; Principal and universal coefficients; Appendix A. Tropical degeneration and relative lambda lengths; Appendix B. Versions of Teichmüller spaces and coordinates; Bibliography.

Memoirs of the American Mathematical Society, Volume 255, Number 1223

August 2018, 101 pages, Softcover, ISBN: 978-1-4704-2967-6, 2010 *Mathematics Subject Classification*: 13F60; 30F60, 57M50, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1223



Algebraic $\overline{\mathbb{Q}}$ -Groups as Abstract Groups

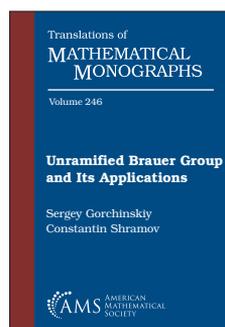
Olivier Frécon, *Laboratoire de Mathématiques et Applications, Université de Poitiers, France*

This item will also be of interest to those working in logic and foundations.

Contents: Introduction; Background material; Expanded pure groups; Unipotent groups over \mathbb{Q} and definable linearity; Definably affine groups; Tori in expanded pure groups; The definably linear quotients of an ACF-group; The group D_G and the Main Theorem for $K = \mathbb{Q}$; The Main Theorem for $K \neq \mathbb{Q}$; Bi-interpretability and standard isomorphisms; Acknowledgements; Bibliography; Index of notations; Index.

Memoirs of the American Mathematical Society, Volume 255, Number 1219

August 2018, 99 pages, Softcover, ISBN: 978-1-4704-2923-2, 2010 *Mathematics Subject Classification*: 20F11; 03C60, 14L17, 20E36, 20G15, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1219



Unramified Brauer Group and Its Applications

Sergey Gorchinskiy, *Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia*, and Constantin Shramov, *Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia*

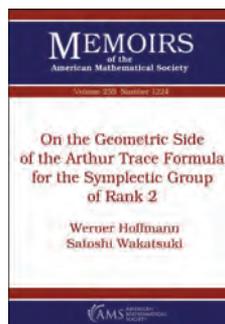
This book is devoted to arithmetic geometry with special attention given to the unramified Brauer group of algebraic varieties and its most striking applications in birational and Diophantine geometry. The topics include Galois cohomology, Brauer groups, obstructions to stable rationality, Weil restriction of scalars, algebraic tori, the Hasse principle, Brauer-Manin obstruction, and étale cohomology. The book contains a detailed presentation of an example of a stably rational but not rational variety, which is presented as series of exercises with detailed hints. This approach is aimed to help the reader understand crucial ideas without being lost in technical details. The reader will end up with a good working knowledge of the Brauer group and its important geometric applications, including the construction of unirational but not stably rational algebraic varieties, a subject which has become fashionable again in connection with the recent breakthroughs by a number of mathematicians.

This item will also be of interest to those working in number theory.

Contents: *Preliminaries on Galois cohomology:* Group Cohomology; Galois Cohomology; *Brauer group:* Brauer Group of a Field; Residue Map on a Brauer Group; *Applications to rationality problems:* Example of a Unirational Non-rational Variety; Arithmetic of Two-dimensional Quadratics; Non-rational Double Covers of \mathbb{P}^3 ; Weil Restriction and Algebraic Tori; Example of a Non-rational Stably Rational Variety; *Hasse principle and its failure:* Minkowski-Hasse Theorem; Brauer-Manin Obstruction; Étale Cohomology; Bibliography; Index.

Translations of Mathematical Monographs, Volume 246

August 2018, 200 pages, Hardcover, ISBN: 978-1-4704-4072-5, 2010 *Mathematics Subject Classification*: 16K50, 14E08; 14M20, 14G05, 20J06, 12G05, **AMS members US\$100.80**, List US\$126, Order code MMONO/246



On the Geometric Side of the Arthur Trace Formula for the Symplectic Group of Rank 2

Werner Hoffmann, *Universität Bielefeld, Germany*, and Satoshi Wakatsuki, *Institute of Science and Engineering, Kanazawa University, Japan*

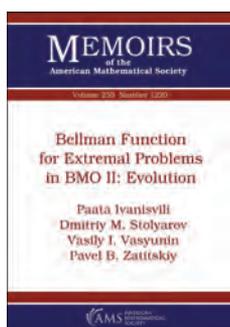
This item will also be of interest to those working in number theory.

Contents: Introduction; Preliminaries; A formula of Labesse and Langlands; Shintani zeta function for the space of binary quadratic forms; Structure of $\mathrm{GSp}(2)$; The geometric side of the trace formula for $\mathrm{GSp}(2)$; The geometric side of the trace formula for $\mathrm{Sp}(2)$; Appendix A. The group $\mathrm{GL}(3)$; Appendix B. The group $\mathrm{SL}(3)$; References.

Memoirs of the American Mathematical Society, Volume 255, Number 1224

August 2018, 88 pages, Softcover, ISBN: 978-1-4704-3102-0, 2010 *Mathematics Subject Classification*: 11F72, 11S90; 11R42, 11E45, 22E30, 22E35, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1224

Analysis



Bellman Function for Extremal Problems in BMO II: Evolution

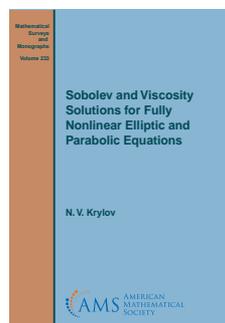
Paata Ivanisvili, *Kent State University, OH*, **Dmitriy M. Stolyarov**, *St. Petersburg State University, Russia*, **Vasily I. Vasyunin**, *Steklov Mathematical Institute, Russian Academy of Sciences, St. Petersburg, Russia*, and **Pavel B. Zatitskiy**, *St. Petersburg State University, Russia*

Contents: Introduction; Setting and sketch of proof; Patterns for Bellman candidates; Evolution of Bellman candidates; Optimizers; Related questions and further development; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 255, Number 1220

August 2018, 136 pages, Softcover, ISBN: 978-1-4704-2954-6, 2010 *Mathematics Subject Classification*: 42B35, 26D07, 52A10, 35E10, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1220

Differential Equations



Sobolev and Viscosity Solutions for Fully Nonlinear Elliptic and Parabolic Equations

N. V. Krylov, *University of Minnesota, Minneapolis, MN*

This book concentrates on first boundary-value problems for fully nonlinear second-order uniformly elliptic

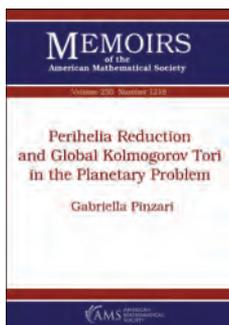
and parabolic equations with discontinuous coefficients. We look for solutions in Sobolev classes, local or global, or for viscosity solutions. Most of the auxiliary results, such as Aleksandrov's elliptic and parabolic estimates, the Krylov-Safonov and the Evans-Krylov theorems, are taken from old sources, and the main results were obtained in the last few years.

Presentation of these results is based on a generalization of the Fefferman-Stein theorem, on Fang-Hua Lin's like estimates, and on the so-called "ersatz" existence theorems, saying that one can slightly modify "any" equation and get a "cut-off" equation that has solutions with bounded derivatives. These theorems allow us to prove the solvability in Sobolev classes for equations that are quite far from the ones which are convex or concave with respect to the Hessians of the unknown functions. In studying viscosity solutions, these theorems also allow us to deal with classical approximating solutions, thus avoiding sometimes heavy constructions from the usual theory of viscosity solutions.

Contents: Bellman's equations with constant "coefficients" in the whole space; Estimates in L_p for solutions of the Monge-Ampère type equations; The Aleksandrov estimates; First results for fully nonlinear equations; Finite-difference equations of elliptic type; Elliptic differential equations of cut-off type; Finite-difference equations of parabolic type; Parabolic differential equations of cut-off type; A priori estimates in C^α for solutions of linear and nonlinear equations; Solvability in $W_{p,loc}^2$ of fully nonlinear elliptic equations; Nonlinear elliptic equations in $C_{loc}^{2+\alpha}(\Omega) \cap C(\bar{\Omega})$; Solvability in $W_{p,loc}^{1,2}$ of fully nonlinear parabolic equations; Elements of the $C^{2+\alpha}$ -theory of fully nonlinear elliptic and parabolic equations; Nonlinear elliptic equations in $W_p^2(\Omega)$; Nonlinear parabolic equations in $W_p^{1,2}$; $C^{1+\alpha}$ -regularity of viscosity solutions of general parabolic equations; $C^{1+\alpha}$ -regularity of L_p -viscosity solutions of the Isaacs parabolic equations with almost VMO coefficients; Uniqueness and existence of extremal viscosity solutions for parabolic equations; Appendix A. Proof of Theorem 6.2.1; Appendix B. Proof of Lemma 9.2.6; Appendix C. Some tools from real analysis; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 233

September 2018, 456 pages, Hardcover, ISBN: 978-1-4704-4740-3, 2010 *Mathematics Subject Classification*: 35-02; 35J60, 35K55, **AMS members US\$97.60**, List US\$122, Order code SURV/233



Perihelia Reduction and Global Kolmogorov Tori in the Planetary Problem

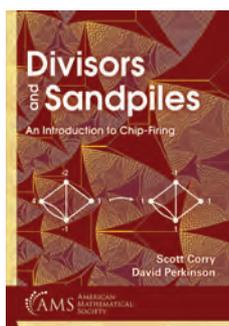
Gabriella Pinzari, *Università di Napoli, Italy*

Contents: Background and results; Kepler maps and the Perihelia reduction; The P-map and the planetary problem; Global Kolmogorov tori in the planetary problem; Proofs; Appendix A. Computing the domain of holomorphy; Appendix B. Proof of Lemma 3.2; Appendix C. Checking the non-degeneracy condition; Appendix D. Some results from perturbation theory; Appendix E. More on the geometrical structure of the P-coordinates, compared to Deprit's coordinates; Bibliography.

Memoirs of the American Mathematical Society, Volume 255, Number 1218

August 2018, 92 pages, Softcover, ISBN: 978-1-4704-4102-9, 2010 *Mathematics Subject Classification*: 34C20, 70F10, 37J10, 37J15, 37J40; 34D10, 70F07, 70F15, 37J25, 37J45, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1218

Discrete Mathematics and Combinatorics



Divisors and Sandpiles

An Introduction to Chip-Firing

Scott Corry, *Lawrence University, Appleton, WI*, and David Perkinson, *Reed College, Portland, OR*

Divisors and Sandpiles provides an introduction to the combinatorial theory

of chip-firing on finite graphs. Part 1 motivates the study of the discrete Laplacian by introducing the dollar game. The resulting theory of divisors on graphs runs in close parallel to the geometric theory of divisors on Riemann surfaces, and Part 1 culminates in a full exposition of the graph-theoretic Riemann-Roch theorem due to M. Baker and S. Norine. The text leverages the reader's understanding of the discrete story to provide a brief overview of the classical theory of Riemann surfaces.

Part 2 focuses on sandpiles, which are toy models of physical systems with dynamics controlled by the discrete Laplacian of the underlying graph. The text provides a careful introduction to the sandpile group and the abelian sandpile model, leading ultimately to L. Levine's threshold density theorem for the fixed-energy sandpile Markov chain. In a precise sense, the theory of sandpiles is dual to the theory of divisors, and there are many beautiful connections between the first two parts of the book.

Part 3 addresses various topics connecting the theory of chip-firing to other areas of mathematics, including the matrix-tree theorem, harmonic morphisms, parking functions, M -matrices, matroids, the Tutte polynomial, and simplicial homology. The text is suitable for advanced undergraduates and beginning graduate students.

This item will also be of interest to those working in algebra and algebraic geometry and probability and statistics.

Contents: *Divisors:* The dollar game; The Laplacian; Algorithms for winning; Acyclic orientations; Riemann-Roch; *Sandpiles:* The sandpile group; Burning and duality; Threshold density; *Topics:* Trees; Harmonic morphisms; Divisors on complete graphs; More about sandpiles; Cycles and cuts; Matroids and the Tutte polynomial; Higher dimensions; *Appendices:* Appendix A; Appendix B; Glossary of symbols; Bibliography; Index.

August 2018, 329 pages, Softcover, ISBN: 978-1-4704-4218-7, LC 2018008412, 2010 *Mathematics Subject Classification*: 05C25, 05C50, 05C99, **AMS members US\$63.20**, List US\$79, Order code MBK/114



Extremal Problems for Finite Sets

Peter Frankl, *Rényi Institute, Budapest, Hungary*, and Norihide Tokushige, *Ryukyu University, Okinawa, Japan*

One of the great appeals of Extremal Set Theory as a subject is that the statements are easily accessible without a lot of mathematical background, yet the proofs

and ideas have applications in a wide range of fields including combinatorics, number theory, and probability theory. Written by two of the leading researchers in the subject, this book is aimed at mathematically mature undergraduates, and highlights the elegance and power of this field of study.

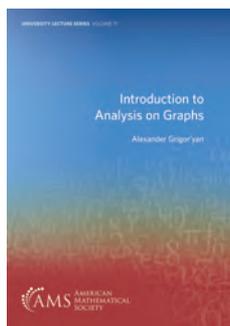
The first half of the book provides classic results with some new proofs including a complete proof of the Ahlswede-Khachatrian theorem as well as some recent progress on the Erdős matching conjecture. The second half presents some combinatorial structural results and linear algebra methods including the Deza-Erdős-Frankl theorem, application of Rödl's packing theorem, application of semidefinite programming, and very recent progress (obtained in 2016) on the Erdős-Szemerédi sunflower conjecture and capset problem. The book concludes with a collection of challenging open problems.

Contents: Introduction; Operations on sets and set systems; Theorems on traces; The Erdős-Ko-Rado theorem via shifting; Katona's circle; The Kurskal-Katona theorem; Kleitman theorem for no s pairwise disjoint sets; The Hilton-Milner theorem; The Erdős matching conjecture; The Ahswede-Khachatrian theorem; Pushing-pulling method; Uniform measure versus product measure; Kleitman's correlation inequality; r -cross union families; Random walk method; L -systems; Exponent of $(10, \{0, 1, 3, 6\})$ -system; The Deza-Erdős-Frankl theorem; Füredi's structure theorem; Rödl's packing theorem; Upper bounds using multilinear polynomials; Application to discrete geometry; Upper bounds using inclusion matrices; Some algebraic constructions for L -systems; Oddtown and eventown problems; Tensor product method; The ratio bound; Measures of cross independent sets; Application of semidefinite programming; A cross intersection

problem with measures; Capsets and sunflowers; Challenging open problems; Bibliography; Index.

Student Mathematical Library, Volume 86

August 2018, 224 pages, Softcover, ISBN: 978-1-4704-4039-8, 2010 *Mathematics Subject Classification*: 05-01, 05D05, **All Individuals US\$41.60**, List US\$52, Institutional member US\$41.60, Order code STML/86



Introduction to Analysis on Graphs

Alexander Grigor'yan,
University of Bielefeld, Germany

A central object of this book is the discrete Laplace operator on finite and infinite graphs. The eigenvalues of the discrete Laplace operator have long been used in graph theory as a convenient tool for understanding the structure of

complex graphs. They can also be used in order to estimate the rate of convergence to equilibrium of a random walk (Markov chain) on finite graphs. For infinite graphs, a study of the heat kernel allows to solve the type problem—a problem of deciding whether the random walk is recurrent or transient.

This book starts with elementary properties of the eigenvalues on finite graphs, continues with their estimates and applications, and concludes with heat kernel estimates on infinite graphs and their application to the type problem.

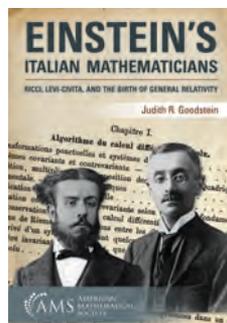
The book is suitable for beginners in the subject and accessible to undergraduate and graduate students with a background in linear algebra I and analysis I. It is based on a lecture course taught by the author and includes a wide variety of exercises. The book will help the reader to reach a level of understanding sufficient to start pursuing research in this exciting area.

Contents: The Laplace operator on graphs; Spectral properties of the Laplace operator; Geometric bounds for the eigenvalues; Eigenvalues on infinite graphs; Estimates of the heat kernel; The type problem; Exercises; Bibliography; Index.

University Lecture Series, Volume 71

September 2018, 168 pages, Softcover, ISBN: 978-1-4704-4397-9, 2010 *Mathematics Subject Classification*: 05C50, 05C63, 05C76, 05C81, 60J10, **AMS members US\$39.20**, List US\$49, Order code ULECT/71

General Interest



Einstein's Italian Mathematicians

Ricci, Levi-Civita, and the Birth of General Relativity

Judith R. Goodstein, *California Institute of Technology, Pasadena, CA*

In the first decade of the twentieth century as Albert Einstein began

formulating a revolutionary theory of gravity, the Italian mathematician Gregorio Ricci was entering the later stages of what appeared to be a productive if not particularly memorable career, devoted largely to what his colleagues regarded as the dogged development of a mathematical language he called the absolute differential calculus. In 1912, the work of these two dedicated scientists would intersect—and physics and mathematics would never be the same. *Einstein's Italian Mathematicians* chronicles the lives and intellectual contributions of Ricci and his brilliant student Tullio Levi-Civita, including letters, interviews, memoranda, and other personal and professional papers, to tell the remarkable, little-known story of how two Italian academicians, of widely divergent backgrounds and temperaments, came to provide the indispensable mathematical foundation—today known as the tensor calculus—for general relativity.

A wonderfully written chronicle of the lives of two great mathematicians and how their work shaped Einstein's masterpiece as well as ushering in new fields of mathematics. The book is also an intriguing and insightful portrait of Italy during the period from Italian independence in 1870 until the onset of World War II.

—**Gino Segre**, *Physics Department, University of Pennsylvania*

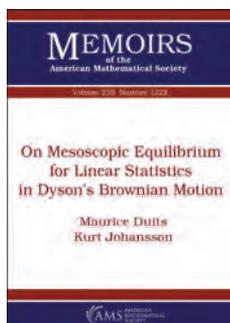
Galileo said that mathematics is the language of nature. Einstein might have found himself mute when it came to describing gravity if it weren't for the mathematics of covariant derivatives developed by Galileo's countrymen Gregorio Ricci-Curbastro and Tullio Levi-Civita. Judy Goodstein tells their stories and their connection to Einstein with clarity and grace in a most readable book.

—**Barry Simon**, *California Institute of Technology*

Contents: The Ricci of Lugo; The making of a mathematician; Munich; Padua; Math and marriage; A promotion that wasn't; The absolute differential calculus; The alter ego; Intermezzo; The indispensable mathematical tool; "Write to me next time in Italian"; Parallel displacements; From Ricci's absolute differential calculus to Einstein's theorem for general relativity; T. Levi-Civita, "Gregorio Ricci-Curbastro"; Obituary of Tullio Levi-Civita; Selected references; Notes; Index.

August 2018, approximately 227 pages, Softcover, ISBN: 978-1-4704-2846-4, 2010 *Mathematics Subject Classification*: 01-02, 01A55, 01A60, 01A70, 83-03, **AMS members US\$28**, List US\$35, Order code MBK/113

Mathematical Physics



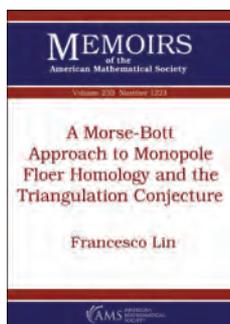
On Mesoscopic Equilibrium for Linear Statistics in Dyson's Brownian Motion

Maurice Duits, *Royal Institute of Technology, Stockholm, Sweden*, and Kurt Johansson, *Royal Institute of Technology, Stockholm, Sweden*

Contents: Introduction; Statement of results; Proof of Theorem 2.1; Proof of Theorem 2.3; Asymptotic analysis of K_n and R_n ; Proof of Proposition 2.4 ; Proof of Lemma 4.3; Random initial points; Proof of Theorem 2.6: the general case; Appendix A. Appendix; Bibliography.

Memoirs of the American Mathematical Society, Volume 255, Number 1222

August 2018, 114 pages, Softcover, ISBN: 978-1-4704-2964-5, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1222



A Morse-Bott Approach to Monopole Floer Homology and the Triangulation Conjecture

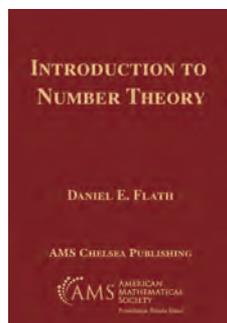
Francesco Lin, *Massachusetts Institute of Technology, Cambridge, MA*

Contents: Introduction; Basic setup; The analysis of Morse-Bott singularities; Floer homology for Morse-Bott singularities; Pin(2)-monopole Floer homology; Bibliography.

Memoirs of the American Mathematical Society, Volume 255, Number 1221

August 2018, 162 pages, Softcover, ISBN: 978-1-4704-2963-8, **Individual member US\$46.80**, List US\$78, Institutional member US\$62.40, Order code MEMO/255/1221

Number Theory



Introduction to Number Theory

Daniel E. Flath

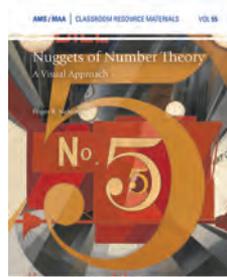
Growing out of a course designed to teach Gauss's *Disquisitiones Arithmeticae* to honors-level undergraduates, Flath's *Introduction to Number Theory* focuses on Gauss's theory of binary quadratic forms. It is suitable for use as a textbook in a course or self-study by advanced undergraduates or graduate students who possess a basic familiarity with abstract algebra. The text treats a variety of topics from elementary number theory including the distribution of primes, sums of squares, continued fractions, the Legendre, Jacobi and Kronecker symbols, the class group and genera. But the focus is on quadratic reciprocity (several proofs are given including one that highlights the $p - q$ symmetry) and binary quadratic forms. The reader will come away with a good understanding of what Gauss intended in the *Disquisitiones* and Dirichlet in his *Vorlesungen*. The text also includes a lovely appendix by J. P. Serre titled $\Delta = b^2 - 4ac$.

The clarity of the author's vision is matched by the clarity of his exposition. This is a book that reveals the discovery of the quadratic core of algebraic number theory. It should be on the desk of every instructor of introductory number theory as a source of inspiration, motivation, examples, and historical insight.

Contents: Prime numbers and unique factorization; Sums of two squares; Quadratic reciprocity; Indefinite forms; The class group and genera; $\Delta = b^2 - 4ac^*$; Tables; Errata to "Introduction to number theory"; Bibliography; Subject index; Notation index.

AMS Chelsea Publishing, Volume 384

August 2018, 212 pages, Hardcover, ISBN: 978-1-4704-4694-9, LC 2018014214, 2010 *Mathematics Subject Classification*: 11-01, **AMS members US\$42.40**, List US\$53, Order code CHEL/384.H



Nuggets of Number Theory

A Visual Approach

Roger B. Nelsen, *Lewis & Clark College, Portland, OR*

Nuggets of Number Theory will attract fans of visual thinking, number theory, and surprising connections. This book contains hundreds of visual explanations of results from elementary number theory. Figurate numbers and Pythagorean triples feature prominently, of course, but there are also proofs of Fermat's Little and Wilson's Theorems. Fibonacci and perfect numbers, Pell's equation, and continued fractions all find visual representation in this charming collection. It will be a rich source of visual inspiration for anyone teaching, or learning, number theory and will provide endless pleasure to those interested in looking at number theory with new eyes. Author Roger Nelsen is a long-time contributor of "Proofs Without Words"

in the MAA's *Mathematics Magazine* and *College Mathematics Journal*. This is his twelfth book with MAA Press.

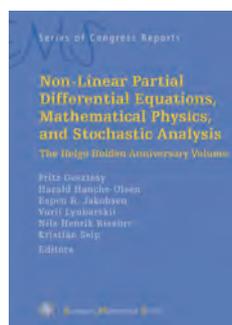
Contents: Figurate numbers; Congruence; Diophantine equations; Pythagorean triples; Irrational numbers; Fibonacci and Lucas numbers; Perfect numbers; Solutions to the exercises; Bibliography; Index.

Classroom Resource Materials, Volume 55

September 2018, 153 pages, Softcover, ISBN: 978-1-4704-4398-6, LC 2018000043, 2010 *Mathematics Subject Classification*: 11-01; 11A07, 11B39, 11D04, **Individual member US\$33.75**, List US\$45, Institutional member US\$36, Order code CLRM/55

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Non-Linear Partial Differential Equations, Mathematical Physics, and Stochastic Analysis

The Helge Holden Anniversary Volume

Fritz Gesztesy, *Baylor University, Waco, Texas*, **Harald Hanche-Olsen**, **Espen R. Jakobsen**, **Yurii Lyubarskii**, *Norwegian University of Science and Technology, Trondheim, Norway*, **Nils Henrik Risebro**, *University of Oslo, Norway*, and **Kristian Seip**, *Norwegian University of Science and Technology, Trondheim, Norway*, Editors

This volume is dedicated to Helge Holden on the occasion of his 60th anniversary. It collects contributions by numerous scientists with expertise in non-linear partial differential equations (PDEs), mathematical physics, and stochastic analysis, reflecting to a large degree Helge Holden's longstanding research interests.

Accordingly, the problems addressed in the contributions deal with a large range of topics, including, in particular, infinite-dimensional analysis, linear and nonlinear PDEs, stochastic analysis, spectral theory, completely integrable systems, random matrix theory, and chaotic dynamics and sestina

poetry. They represent to some extent the lectures presented at the conference *Non-linear PDEs, Mathematical Physics and Stochastic Analysis*, held at the Norwegian University of Science and Technology, Trondheim, July 4–7, 2016.

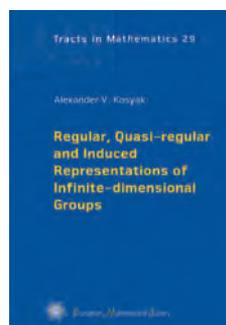
The mathematical tools involved draw from a wide variety of techniques in functional analysis, operator theory, and probability theory. This collection of research papers will be of interest to any active scientist working in one of the above-mentioned areas.

This item will also be of interest to those working in differential equations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Series of Congress Reports, Volume 14

June 2018, 436 pages, Hardcover, ISBN: 978-3-03719-186-6, 2010 *Mathematics Subject Classification*: 15B52, 35J10, 35L65, 35Q41, 35Q51, 35Q53, 37K10, 42B20, 46N20, 46N30, 46T12, 47B36, 47F05, 60H20, 68N30, 76S05; 33C45, 35A01, 35A02, 35L80, 37D45, 39A12, 47A10, 47N20, 60B20, **AMS members US\$94.40**, List US\$118, Order code EMSSCR/14



Regular, Quasi-regular and Induced Representations of Infinite-dimensional Groups

Alexander V. Kosyak, *National Academy of Science of Ukraine, Kiev, Ukraine*

Almost all harmonic analysis on locally compact groups is based on the existence (and uniqueness) of a Haar measure. Therefore, it is very natural to attempt a similar construction for non-locally compact groups. The essential idea is to replace the non-existing Haar measure on an infinite-dimensional group by a suitable quasi-invariant measure on an appropriate completion of the initial group or on the completion of a homogeneous space.

The aim of the book is a systematic development, by example, of noncommutative harmonic analysis on infinite-dimensional (non-locally compact) matrix groups. We generalize the notion of regular, quasiregular and induced representations for arbitrary infinite-dimensional groups. The central idea to verify the irreducibility is the Ismagilov conjecture. The author also extends the Kirillov orbit method for the group of upper triangular matrices of infinite order.

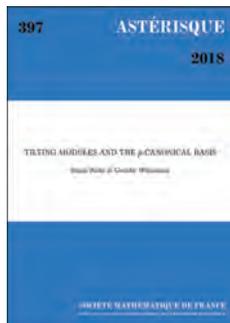
In order to make the content accessible to a wide audience of nonspecialists, the exposition is essentially self-contained and very few prerequisites are needed. The book is aimed at graduate and advanced undergraduate students, as well as mathematicians looking for an introduction to representations of infinite-dimensional groups.

This item will also be of interest to those working in probability and statistics.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Tracts in Mathematics, Volume 29

June 2018, 587 pages, Hardcover, ISBN: 978-3-03719-181-1, 2010 *Mathematics Subject Classification*: 22E66, 22E65; 60B15, 28C20, **AMS members US\$94.40**, List US\$118, Order code EMSTM/29



Tilting Modules and the p -Canonical Basis

Simon Riche, *Université Clermont Auvergne, Clermont-Ferrand, France*, and **Geordie Williamson**, *University of Sydney, Australia*

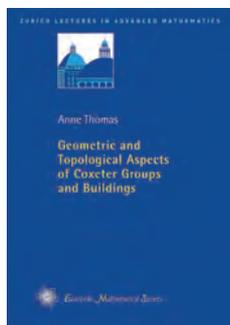
In this book, the authors propose a new approach to tilting modules for reductive algebraic groups in positive characteristic. They conjecture that translation functors give an action of the (diagrammatic) Hecke category of the affine Weyl group on the principal block. Their conjecture implies character formulas for the simple and tilting modules in terms of the p -canonical basis, as well as a description of the principal block as the antispherical quotient of the Hecke category. The authors prove their conjecture for $GL_n(k)$ using the theory of 2-Kac-Moody actions.

Finally, the authors prove that the diagrammatic Hecke category of a general crystallographic Coxeter group may be described in terms of parity complexes on the flag variety of the corresponding Kac-Moody group.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 397

May 2018, 184 pages, Softcover, ISBN: 978-2-85629-880-0, 2010 *Mathematics Subject Classification*: 17B10, 20G15, 14F05, **AMS members US\$53.60**, List US\$67, Order code AST/397



Geometric and Topological Aspects of Coxeter Groups and Buildings

Anne Thomas, *University of Sydney, Australia*

Coxeter groups are groups generated by reflections. They appear throughout mathematics. Tits developed the general theory of Coxeter groups in order to develop the theory of buildings. Buildings have interrelated algebraic, combinatorial and geometric structures and are powerful tools for understanding the groups which act on them.

These notes focus on the geometry and topology of Coxeter groups and buildings, especially nonspherical cases. The emphasis is on geometric intuition, and there are many examples and illustrations. Part I describes Coxeter groups and their geometric realizations, particularly the Davis complex, and Part II gives a concise introduction to buildings.

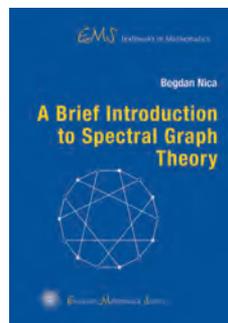
This book will be suitable for graduate students and researchers in geometric group theory, as well as algebra and combinatorics. The assumed background is basic group theory, including group actions, and basic algebraic topology, together with some knowledge of Riemannian geometry.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Zurich Lectures in Advanced Mathematics, Volume 24

June 2018, 160 pages, Softcover, ISBN: 978-3-03719-189-7, 2010 *Mathematics Subject Classification*: 20F55; 20E42, 51E24, 57M07, **AMS members US\$31.20**, List US\$39, Order code EMSZLEC/24

Discrete Mathematics and Combinatorics



A Brief Introduction to Spectral Graph Theory

Bogdan Nica, *McGill University, Montreal, Canada*

Spectral graph theory starts by associating matrices to graphs—notably, the adjacency matrix and the Laplacian matrix. The general theme is then, first, to compute or estimate the eigenvalues of such matrices, and, second, to relate

the eigenvalues to structural properties of graphs. As it turns out, the spectral perspective is a powerful tool. Some of its loveliest applications concern facts that are, in principle, purely graph theoretic or combinatorial.

This text is an introduction to spectral graph theory, but it could also be seen as an invitation to algebraic graph theory. The first half is devoted to graphs, finite fields, and how they come together. This part provides an appealing motivation and context for the second spectral half. The text is enriched by many exercises and their solutions.

The target audience is students at the upper undergraduate level and above. The book only assumes a familiarity with linear algebra and basic group theory. Graph theory, finite fields, and character theory for abelian groups receive a concise overview and render the text essentially self-contained.

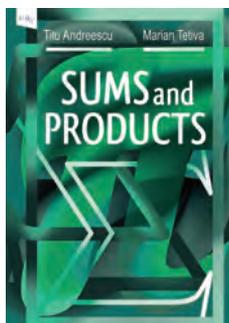
This item will also be of interest to those working in number theory.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Textbooks in Mathematics, Volume 21

June 2018, 168 pages, Hardcover, ISBN: 978-3-03719-188-0, 2010 *Mathematics Subject Classification*: 05-01, 05C50; 05C25, 11T24, 15A42, **AMS members US\$38.40**, List US\$48, Order code EMSTEXT/21

Math Education



Sums and Products

Titu Andreescu, *University of Texas at Dallas, TX*, and **Marian Tetiva**, *Rosca Codreanu National College, Barlad, Romania*

The main areas covered are:
(1) telescoping sums and products in algebra and trigonometry; (2) the use of complex numbers and de Moivre's formula; (3) Abel's summation formula;

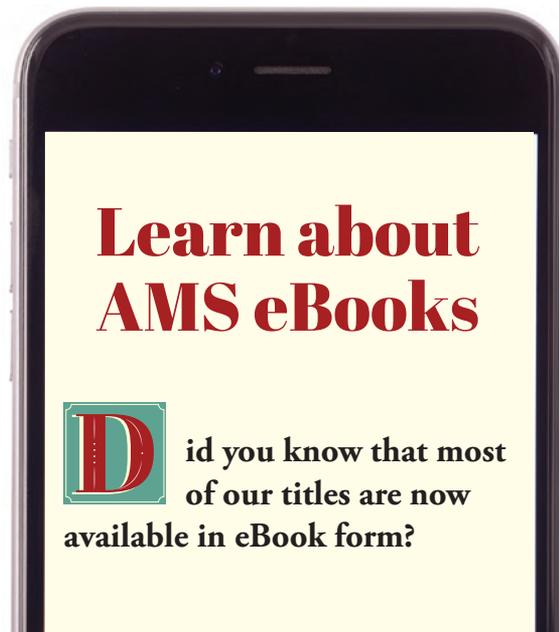
(4) mathematical induction; (5) combinatorial identities; and (6) multiplicative functions and the use of Möbius function.

The theory is presented together with rich examples. At the end, readers are invited to solve problems from a list of 125 questions divided into three levels.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 31

April 2018, 340 pages, Hardcover, ISBN: 978-0-9993428-1-7, 2010 *Mathematics Subject Classification*: 00A05, 00A07, 97U40, 97D50, **AMS members US\$47.96**, List US\$59.95, Order code XYZ/31



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The University of Maine at Machias, a member of the University of Maine System, sits on the Gulf of Maine, surrounded by rivers, forests, fishing villages, and blueberry barrens. For more information about the University visit www.machias.edu.

The University of Maine at Machias is an EEO/AA employer. Women and diverse candidates and others who can help us support a diverse campus community are encouraged to apply.

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Review of applications will begin immediately. To ensure full consideration, materials should be submitted by August 15, 2018. Materials must be submitted via "Apply for Position" at <https://maine.hiretouch.com/>.

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CHINA

Southern University of Science and Technology (SUSTech) Faculty Positions of Mathematics The Department of Mathematics

The Department of Mathematics at Southern University of Science and Technology (SUSTech) is founded in 2015 with a dual mission of creating a first-class research and education organization for mathematics and providing service courses in support of other academic departments at SUSTech. We currently have 36 full-time faculty members, including 6 Chair Professors & 7 Full Professors, 3 Associate Professors, 12 Assistant Professors, and 8 teaching faculty members. Research interests of the faculty members cover a broad array of Mathematics including Pure Mathematics, Com-

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.

The 2018 rate is \$3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

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There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: September 2018—June 28, 2018; October 2018—July 27, 2018; November 2018—August 29, 2018; December 2018—September 21, 2018.

US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

putational and Applied Mathematics, Probability and Statistics, and Financial Mathematics.

Call for Applications

We invite applications for full-time faculty positions at all ranks and in all areas of Mathematics, including Financial Mathematics and Statistics. SUSTech has a tenure system. Qualified candidates may apply for appointments with tenure.

Candidates should have demonstrated excellence in research and a strong commitment to teaching. A doctoral degree is required at the time of appointment. A candidate for a senior position must have an established record of research and teaching, and a track-record in securing external funding.

To apply, please visit www.mathjobs.org and look up our job ad for instructions. For an informal discussion about applying to one of our positions, please contact Ms. Xianghui Yu, the Secretary of Department of Mathematics, by phone +86-755-88018703 or email: yuxh@sustc.edu.cn.

SUSTech offers competitive salaries, fringe benefits including medical insurance, retirement and housing subsidy, which are among the best in China. Salary and rank will be commensurate with qualifications and experiences of an appointee.

About the University

Established in 2012, SUSTech is a public institution funded by Shenzhen, a city with a designated special economic zone status in Southern China bordering Hong Kong. As one of China's key gateways to the world, Shenzhen is the country's fastest-growing city in the past three decades. From a small fishing village 30 years ago to a modern city with a population of over 10 million, the city has become the high-tech and manufacturing hub of southern China. It is home to the world's third-busiest container port and the fourth-busiest airport on the Chinese mainland. Being a picturesque coastal city, Shenzhen is also a popular tourist destination.

SUSTech is a pioneer in higher education reform in China. Its mission is to become a globally recognized institution that excels in research and promotes innovation, creativity and entrepreneurship. Ninety percent of SUSTech faculty members have overseas work experiences, and sixty percent studied or worked in top 100 universities in the world. The languages of instruction are English and Chinese. Sitting on five hundred acres of subtropical woodland with hills, rivers and a natural lake in Nanshan District of Shenzhen, the SUSTech campus is a beautiful place for learning and research.

The prosperity of Shenzhen is built on innovations and entrepreneurship of its citizens. The city has some of China's most successful high-tech companies such as Huawei and Tencent. SUSTech strongly supports innovations and entrepreneurship, and provides funding for promising initiatives. The university encourages candidates with intention and experience on entrepreneurship to apply.

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is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

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For more information, please visit cam.tju.edu.cn or contact Ms. Erica Liu at mathjobs@tju.edu.cn, telephone: 86-22-2740-6039.

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Tianjin University, China Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture,

MEETINGS & CONFERENCES OF THE AMS

AUGUST TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: www.ams.org/meetings.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 88 in the January 2018 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX is

necessary to submit an electronic form, although those who use \LaTeX may submit abstracts with such codings, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX . Visit www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

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See www.ams.org/meetings for the most up-to-date information on the meetings and conferences that we offer.

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www.ams.org/meetings/.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Newark, Delaware

University of Delaware

September 29–30, 2018

Saturday – Sunday

Meeting #1141

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: June 2018

Program first available on AMS website: August 9, 2018

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: Expired

For abstracts: July 31, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Leslie Greengard, New York University, *Linear and nonlinear inverse problems in imaging*.

Elisenda Grigsby, Boston College, *Braids, surfaces, and homological invariants*.

Davesh Maulik, Massachusetts Institute of Technology, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Numerical Approximation of Partial Differential Equations (Code: SS 8A), **Constantin Bacuta** and **Jingmei Qiu**, University of Delaware.

Applied Algebraic Topology (Code: SS 2A), **Chad Giusti**, University of Delaware, and **Gregory Henselman**, Princeton University.

Billiard Dynamics: Standard and Alternative Collision Models (Code: SS 15A), **Tim Chumley**, Mount Holyoke College, **Chris Cox**, University of Delaware, and **Renato Feres**, Washington University in St. Louis.

Commutative Algebra (Code: SS 19A), **Ela Celikbas**, West Virginia University, **Sema Gunturkun**, University of Michigan, and **Oana Veliche**, Northeastern University.

Convex Geometry and Functional Inequalities (Code: SS 3A), **Mokshay Madiman**, University of Delaware, **Elisabeth Werner**, Case Western Reserve University, and **Artem Zvavitch**, Kent State University.

Fixed Point Theory with Application and Computation (Code: SS 7A), **Clement Boateng Ampadu**, Boston, MA, **Penumarthy Parvateesam Murthy**, Guru Ghasidas Vishwavidyalaya, Bilaspur, India, **Naeem Saleem**, University of Management and Technology, Lahore, Pakistan, **Yaé Ulrich Gaba**, Institut de Mathématiques et de Sciences Physiques (IMSP), Porto-Novo, Bénin, and **Xavier Udo-utun**, University of Uyo, Uyo, Nigeria.

Graph Theory (Code: SS 12A), **Sebastian M. Cioabă**, University of Delaware, **Brian Kronenthal**, Kutztown University of Pennsylvania, **Felix Lazebnik**, University of Delaware, and **Wing Hong Tony Wong**, Kutztown University of Pennsylvania.

Interplay between Analysis and Combinatorics (Code: SS 5A), **Mahya Ghandehari** and **Dominique Guillot**, University of Delaware.

Modern Quasiconformal Analysis and Geometric Function Theory (Code: SS 6A), **David Herron**, University of Cincinnati, and **Yuk-J Leung**, University of Delaware.

Nonlinear Water Waves and Related Problems (Code: SS 9A), **Philippe Guyenne**, University of Delaware.

Operator and Function Theory (Code: SS 4A), **Kelly Bickel**, Bucknell University, **Michael Hartz**, Washington University, St. Louis, **Constanze Liaw**, University of Delaware, and **Alan Sola**, Stockholm University.

MEETINGS & CONFERENCES

Probability, Combinatorics, and Statistical Mechanics (Code: SS 18A), **Nayantara Bhatnagar** and **Douglas Rizzolo**, University of Delaware.

Quantum Correlation Sets in Quantum Information Theory (Code: SS 13A), **Elie Alhajar** and **Travis B. Russell**, US Military Academy.

Recent Advances in Nonlinear Schrödinger Equations (Code: SS 1A), **Alexander Pankov**, Morgan State University, **Junping Shi**, College of William and Mary, and **Jun Wang**, Jiangsu University.

Recent Analytic and Numeric Results on Nonlinear Evolution Equations (Code: SS 10A), **Xiang Xu**, Old Dominion University, and **Wujun Zhang**, Rutgers University.

Representations of Infinite Dimensional Lie Algebras and Applications (Code: SS 16A), **Marco Aldi**, Virginia Commonwealth University, **Michael Penn**, Randolph College, and **Juan Villarreal**, Virginia Commonwealth University.

Stochastic Processes in Mathematical Biology (Code: SS 14A), **Yao Li**, University of Massachusetts Amherst, and **Abhyudai Singh**, University of Delaware.

The Mathematics of Swimmers and Active Particles (Code: SS 11A), **Louis Rossi**, University of Delaware, and **Enkeleida Lushi**, Flatiron Institute.

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

Meeting #1143

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: July 2018

Program first available on AMS website: August 30, 2018

Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired

For abstracts: August 21, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Elena Fuchs, University of Illinois Urbana-Champaign, *Title to be announced.*

Andrew Putman, University of Notre Dame, *Title to be announced.*

Charles Smart, University of Chicago, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the ab-

stract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Commutative Algebra (Code: SS 15A), **Jack Jeffries**, University of Michigan, **Linquan Ma**, Purdue University, and **Karl Schwede**, University of Utah.

Advances on Analytical and Geometric Aspects of Differential Equations (Code: SS 24A), **Alessandro Arsie**, **Chunhua Shan**, and **Ekaterina Shemyakova**, University of Toledo.

Analytical and Numerical Aspects of Turbulent Transport (Code: SS 23A), **Michele Coti Zelati**, Imperial College London, and **Ian Tobasco** and **Karen Zaya**, University of Michigan.

Aspects of Geometric Mechanics and Dynamics (Code: SS 13A), **Anthony M Bloch** and **Marta Farre Puiggali**, University of Michigan.

Bio-inspired Mechanics and Propulsion (Code: SS 16A), **Silas Alben**, University of Michigan, and **Longhua Zhao**, Case Western Reserve University.

Canonical Operators in Several Complex Variables and Related Topics (Code: SS 21A), **David Barrett** and **Luke Edholm**, University of Michigan, and **Yunus Zeytuncu**, University of Michigan, Dearborn.

Cell Motility: Models and Applications (Code: SS 20A), **Magdalena Stolarska**, University of St. Thomas, and **Nicoleta Tarfulea**, Purdue University Northwest.

Cluster Algebra, Poisson Geometry, and Related Topics (Code: SS 9A), **Eric Bucher**, Michigan State University, and **Maitreyee Kulkarni** and **Bach Nguyen**, Louisiana State University.

Combinatorics in Algebra and Algebraic Geometry (Code: SS 14A), **Zachary Hamaker**, **Steven Karp**, and **Oli-ver Pechenik**, University of Michigan.

Commutative Algebra and Complexity (Code: SS 32A), **Harm Derksen**, **Francesca Gandini**, and **Visu Makam**, University of Michigan.

Commutative Ring Theory (Code: SS 22A), **Joe Stickles**, Millikin University, and **Darrin Weber**, University of Evansville.

Ergodic and Topological Quantum Systems (Code: SS 28A), **Matthew Cha**, **Ilya Kachkovskiy**, and **Shiwen Zhang**, Michigan State University.

Extensions-Interpolation-Shape Matching in R^d , Symmetry-Invariance, Algorithms and Related Topics (Code: SS 11A), **Steven Damelin**, American Mathematical Society, and **Nir Sharon**, Princeton University.

From Hyperelliptic to Superelliptic Curves (Code: SS 6A), **Tony Shaska**, Oakland University, **Nicola Tarasca**, Rutgers University, and **Yuri Zarhin**, Pennsylvania State University.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

Interactions between Algebra, Machine Learning and Data Privacy (Code: SS 3A), **Jonathan Gryak**, University of

Michigan, **Kelsey Horan**, CUNY Graduate Center, **Delaram Kahrobaei**, CUNY Graduate Center and New York University, **Kayvan Najarian** and **Reza Soroushmehr**, University of Michigan, and **Alexander Wood**, CUNY Graduate Center.

Large Cardinals and Combinatorial Set Theory (Code: SS 10A), **Andres E. Caicedo**, Mathematical Reviews, and **Paul B. Larson**, Miami University.

Mathematics of the Genome (Code: SS 30A), **Anthony Bloch**, **Daniel Burns**, and **Indika Rajapakse**, University of Michigan.

Modern Trends in Integrable Systems (Code: SS 12A), **Deniz Bilman**, **Peter Miller**, **Michael Music**, and **Guilherme Silva**, University of Michigan.

Multiplicities and Volumes: An Interplay Among Algebra, Combinatorics, and Geometry (Code: SS 19A), **Federico Castillo**, University of Kansas, and **Jonathan Montaño**, New Mexico State University.

New Trends in Numerical Methods for Partial Differential Equations: Theory and Applications (Code: SS 17A), **Fatih Celiker**, Wayne State University.

Nonlocality in Models for Kinetic, Chemical, and Population Dynamics (Code: SS 25A), **Christopher Henderson**, University of Chicago, **Stanley Snelson**, Florida Institute of Technology, and **Andrei Tarfulea**, University of Chicago.

Probabilistic Methods in Combinatorics (Code: SS 7A), **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University, and **David Galvin**, University of Notre Dame.

Random Matrix Theory Beyond Wigner and Wishart (Code: SS 2A), **Elizabeth Meckes** and **Mark Meckes**, Case Western Reserve University, and **Mark Rudelson**, University of Michigan.

Recent Advances in Nonlinear PDE (Code: SS 31A), **Jessica Lin**, McGill University, and **Russell Schwab**, Michigan State University.

Recent Developments in Discontinuous Galerkin Methods for Differential Equations (Code: SS 34A), **Mahboub Bacouch**, University of Nebraska at Omaha.

Recent Developments in Mathematical Analysis of Some Nonlinear Partial Differential Equations (Code: SS 18A), **Mimi Dai**, University of Illinois at Chicago.

Recent Developments in the Mathematics of Tomography and Scattering (Code: SS 26A), **Shixu Meng**, University of Michigan, and **Yang Yang**, Michigan State University.

Recent Trends on Local, Nonlocal and Fractional Partial Differential Equations (Code: SS 27A), **Pablo Raúl Stinga**, Iowa State University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

Representations of Reductive Groups over Local Fields and Related Topics (Code: SS 8A), **Anne-Marie Aubert**, Institut Mathématiques de Jussieu, Paris Rive Gauche, **Jessica Fintzen**, IAS, University of Michigan, University of Cambridge, and **Camelia Karimianpour**, University of Michigan.

Self-similarity and Long-range Dependence in Stochastic Processes (Code: SS 4A), **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

Structured Homotopy Theory (Code: SS 5A), **Thomas Fiore**, University of Michigan, Dearborn, **Po Hu** and **Dan**

Isaksen, Wayne State University, and **Igor Kriz**, University of Michigan.

The Mathematics of Decisions, Elections, and Games (Code: SS 29A), **Michael A. Jones**, Mathematical Reviews, and **David McCune**, William Jewell College.

Topics in Graph Theory, Hypergraphs and Set Systems (Code: SS 33A), **John Engbers**, Marquette University, and **Cliff Smyth**, University of North Carolina, Greensboro.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the Michigan state hotel tax (11%), local taxes and hotel fees may apply. Participants must state that they are with the **American Mathematical Society's (AMS) Fall Central Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Wyndham Garden Hotel Ann Arbor, 2900 Jackson Ave., Ann Arbor, MI 48103; (734) 665-4444; <https://www.wyndhamhotels.com/wyndham-garden/ann-arbor-michigan/wyndham-garden-ann-arbor/overview?CID=LC:GN::GGL:RIO:National:36846&iata=00065402>. To book online visit the link provided, use the "special rate" drop down menu, select "block code" and enter "AMS". Rates are **US\$84** per night for a room. The group rate includes deluxe continental breakfast, fitness center, indoor pool, free Wi-Fi access and free parking. This property is located about 2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 18, 2018**.

Comfort Inn & Suites University South, 3501 South State St., Ann Arbor, MI, 48108; (734) 761-8838 ; <https://www.choicehotels.com/michigan/ann-arbor/comfort-inn-hotels/mi229>. Rates are **US\$98** per night for a single king or double queen guest room. Amenities include free Wi-Fi, free parking, free hot breakfast, computer and printer in lobby, indoor pool, hot tub and fitness center. This property is located about 5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual reservations can be cancelled without penalty up to October 1, 2018. Cancellations after this date will be charged one night room and tax. A cancellation number should be obtained from the reservations agent for reference. The deadline for reservations at a reduced rate is by **October 1, 2018**.

Courtyard by Marriott Ann Arbor, 3205 Boardwalk, Ann Arbor, MI 48108; (734) 995-5900; https://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkIdData=American%20Mathematical%20Society%5Earbch%60AMSAMS%60134.00%60USD%60false%602%6010/19/18%6010/22/18%609/19/18&app=resvlink&stop_mobi=yes. Rates are

MEETINGS & CONFERENCES

US\$134 per night for a king bed guest room with pullout sofa. Amenities include free high-speed internet, fitness center, complimentary self-parking and pool. This property is located about 3 miles from the campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. All individual room reservations must be cancelled by October 16, 2018 at 12:01am. In the event individual room reservations are cancelled less than 3 days prior to arrival, the individual must pay the hotel a cancellation fee equal to the first night's room and tax. Cancellation includes length of stay reduction within the designated 3 days prior to arrival and are also subject to the above stated cancellation fee. The deadline for reservations at a reduced rate is **September 19, 2018**.

Hilton Garden Inn Ann Arbor, 1401 Briarwood Cir., Ann Arbor, MI, 48108; (734) 327-6400; https://secure3.hilton.com/en_US/gi/reservation/book.htm?inputModule=HOTEL&ctyhocn=ARBGIGI&spec_plan=FCSM&arrival=20181019&departure=20181022&cid=OM,W,W,HILTONLINK,EN,DirectLink&fromId=HILTONLINKDIRECT. Rates are **US\$159** per night for a single king bed or two queen bed guest room. Amenities include complimentary Wi-Fi, a 24-hour business center, complimentary fitness center, an on-site full-service restaurant, on-site convenience store, coin laundry and lounge. This property is located approximately 3.5 miles from campus. Complimentary self-parking is available. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual reservations may be cancelled up to 48 hours prior to arrival without penalty. Cancellation within 48 hours prior to arrival may be subject to one night's rate plus tax to be charged to the credit card on file. A no-show reservation will be subject to one night's rate plus tax to be charged to the credit card on file. The deadline for reservations at this rate is **September 19, 2018**.

Hampton Inn Ann Arbor South, 925 Victors Way, Ann Arbor, MI, 48108; (734) 665-5000; group.hamptoninn.com/Fall-Cntrl-Sectional-Mting-2018. Rates are **US\$119** per night for a standard guest room. Rate includes hot breakfast buffet, free Wi-Fi, heated indoor pool, fitness center and free self-parking. This property is located about 2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual cancellations must be made 48 hours prior to arrival to avoid a cancellation fee. The deadline for reservations at this rate is **September 19, 2018**.

Ann Arbor Regent Hotel and Suites, 2455 Carpenter Rd., Ann Arbor, MI 48108; (734) 973-6100; <https://annarborregent.com/>. Rate is **US\$129** per night for a two queen bed guest room. To book online go to the website above then enter your dates, click 'make a reservation' and enter group code "FCSM2018". Amenities include free self-parking, complimentary hot breakfast buffet, same business-day dry cleaning, indoor pool and hot tub, fitness center and complimentary Wi-Fi throughout the property. This property is located about 4 miles from campus. Can-

cellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 21, 2018**.

The Fairfield Inn Ann Arbor, 3285 Boardwalk Dr., Ann Arbor, MI 48108; (734) 995-5200; https://www.marriott.com/meeting-event-hotels/group-corporate-travel/groupCorp.mi?resLinkIdData=American%20Mathematical%20Society%20-%20Fall%20Central%20Sectional%20Meeting%202018%5Earbfi%60AMSAMS%7CAMSAMSB%60119.00%60USD%60false%604%6010/19/18%6010/22/18%6009/27/2018&app=resvlink&stop_mobi=yes. Rates are **US\$119** per night for a room with either single king bed or two double beds. Amenities include complimentary hot breakfast, indoor pool, 24 hour fitness center, complimentary high-speed internet and 24 hour Corner Market. This property is located about 3 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual room reservations must be cancelled by Tuesday, October 16, 2018 at 12:01am. In the event individual room reservations are cancelled less than 3 days prior to arrival, the individual must pay the hotel a cancellation fee equal to the first night's room and tax. Cancellation includes length of stay reduction within the designated 3 days prior to arrival and are also subject to the above stated cancellation fee. The deadline for reservations at this rate is **September 19, 2018**.

Holiday Inn, 3600 Plymouth Rd., Ann Arbor, MI 48105; (734) 769-9800; https://www.ihg.com/holidayinn/hotels/us/en/ann-arbor/arbnc/hoteldetail?qAdlt=1&qBrs=6c.hi.ex.rs.ic.cp.in.sb.cw.cv.ul.vn.ki.va.sp.nd.ct&qChld=0&qFRA=1&qGRM=0&qGrpCd=AMS&qIta=99801505&qPst=0&qRRSrt=rt&qRef=df&qRms=1&qRpn=1&qRpp=20&qSHp=1&qSmP=3&qSrt=sBR&qWch=0&srb_u=1&icdv=99801505&setPMCookies=true. Rates are **US\$114** for a room. To book online use the booking link above and scroll down to select your check-in and check-out dates. The group code will automatically populate once you click on 'check availability'. Amenities include free self-parking, complimentary Wi-Fi, indoor/outdoor heated swimming pool, fitness center, tennis and basketball court and complimentary shuttle service to and from Ann Arbor destinations. This property is located about 4 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Cancellations must be received by October 18, 2018 to avoid a charge of one night room and tax. The deadline for reservations at this rate is **October 5, 2018**.

The Kensington Hotel, 3500 S. State St., Ann Arbor, MI 48108; (734) 761-7800; <https://www.kcourtaa.com/>. Rates are **US\$129** for a standard guest room. Amenities include free self-parking, complimentary Viennese Coffee Bar, complimentary Wi-Fi access, heated indoor pool, fitness center and hot tub. This property is located about 7 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure

to check when you make your reservation. The deadline for reservations at this rate is **September 19, 2018**.

The Inn at the Michigan League, 911 N. University Ave., Ann Arbor, MI 48104; (734) 764-3177; <https://unions.umich.edu/league/inn/>. Rates are **US\$145** for single occupancy or **US\$155** for double occupancy. Included in the rate is one parking pass per guest room, free Wi-Fi access and a **US\$4.50** food coupon for each night to be used in one of the hotel's food establishments. This property is located on the university campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 20, 2018**.

Housing Warning: Please beware of aggressive housing bureaus that target potential attendees of a meeting. They are sometimes called “room poachers” or “room-block pirates” and these companies generally position themselves as a meeting's housing bureau, convincing attendees to unknowingly book outside the official room block. They call people who they think will more likely than not attend a meeting and lure them with room rates that are significantly less than the published group rate—for a limited time only. And people who find this offer tempting may hand over their credit card data, believing they have scored a great rate and their housing is a done deal. Unfortunately, this often turns out to be the start of a long, costly nightmare.

These housing bureaus are not affiliated with the American Mathematical Society or any of its meetings, in any way. The AMS would never call anyone to solicit reservations for a meeting. The only way to book a room at a rate negotiated for an AMS Sectional Meeting is through AMS Sectional Meetings pages. The AMS cannot be responsible for any damages incurred as a result of hotel bookings made with unofficial housing bureaus.

Food Services

On Campus: South University (AKA South U) is filled with a wide range of food including burritos, sushi, bi bim bop, subs, pizza, and more. Not to mention that many places on South U are open till as late as 4:00 am! For a quicker bite to eat or a late night craving, South U has you covered. The other area on campus is State Street. Here you will find many quick and easy foods under **US\$10**. This street has food that ranges from Mediterranean, to Mexican, to Italian, and much more. State Street is a popular lunch destination among many students, faculty, and visitors.

Good Time Charley's, 1140 S. University Ave.; (734) 668-8411; a University of Michigan campus tradition since 1979! Full menu and full bar with nightly specials.

Tios Mexican Cafe and Tequila Bar, 401 E. Liberty St.; (734) 761-6650; open 11:00 am to 2:00 pm on Saturday and Sunday; serves late-night eats, a massive hot sauce collection & homemade salsa.

Frita Batidos, 117 W. Washington St.; (734) 761-2882; open 11:00 am to midnight on Saturday and Sunday; serves colorful Cuban street food & tropical cocktails.

Zingerman's Roadhouse, 2501 Jackson Ave.; open 9:00 am to 11:00 pm on Saturday and 9:00 am to 9:00 pm on Sunday; offers hearty American comfort fare, local draft beers & a gourmet food shop.

Ama Bistro Family Restaurant, 215 S. State St.; (734) 780-7202; open 8:00 am to 10:00 pm on Saturday and 8:00 am to 4:00 pm on Sunday; elevated diner fare including french toast, burgers & Albanian stews.

Brown Jug, 1204 S. University Ave.; (734) 761-3355; open 11:00 am to 2:00 am on Saturday and noon to 2:00 am on Sunday; serves breakfast all day, full bar, pizza, spinach pie, lasagna, plus daily specials.

Cottage Inn, 2900 S. State St.; (734) 663-4500; open 10:00 am to 1:00 am on Saturday and 10:00 am to midnight on Sunday; decadent, delicious, delivery-ready gourmet pizza.

Romano's Macaroni Grill, 3010 S. State St.; (734) 663-4433; open 11:00 am to 10:00 pm on Saturday and Sunday; lively Italian chain with an open kitchen, known for small plates, pizzas, pastas & wine.

Beanster's Cafe, 911 N. University (Michigan League Ground level); (734) 764-3586; quick service cafe with sandwiches and coffee.

U-go's Express, 911 N. University (Michigan League Ground level); (734) 764-3586; bottled beverages, assorted snacks and ready-to-eat food.

Off Campus: Downtown Ann Arbor is a great place for a more quiet and sit-down type restaurant. There are a few areas in Downtown which are famous for delicious dinners, such as Main Street, Liberty Street, and Kerrytown. Main Street is packed with restaurants of all different kinds, like seafood, sushi, artisan, bar food and more. In the heart of Ann Arbor, this is the place to get the best of the best restaurants the city has to offer. Additionally, Liberty Street, which intersects Main Street, is home to a wide array of restaurants which can be seen filled with people past 6:00 pm every night of the week. Be sure to call ahead, because these places fill up quickly. Finally, Kerrytown is a unique experience due to its wide range of options, which include home grown, vegan, locally sourced, and more. Please go to the Visit Ann Arbor site for a complete restaurant guide at <https://www.visitannarbor.org/eat>.

Some options for coffee include:

Roasting Plant Coffee, 312 S. State St.; (734) 263-1881; <https://roastingplant.com/>; open 8:00 am to 6:00 pm Saturday and Sunday; serving coffee brewed to order from your choice of single origin beans or custom blend.

lab, 505 E. Liberty St.; (734) 827-2233; labcafe.tumblr.com/; open 9:00 am to 8:00 pm on Saturday and Sunday; light-filled cafe known for pour-over cups & colorful bakery fare showcases local art & musicians.

Starbucks, 222 S. State St.; (734) 623-8067; Seattle-based coffeehouse chain known for its signature roasts, light bites and Wi-Fi availability.

Some options for downtown dining include:

The Chop House Ann Arbor, 322 S. Main St.; (734) 669-9977; open for dinner on Saturdays and Sundays; offers premium steak, wine & interactive tablet menus.

Afternoon Delight Cafe, 251 E. Liberty St.; (734) 665-7513; www.afternoonlightcafe.com/; specializing in fresh, homemade meals for breakfast and lunch with brunch on weekends.

BD's Mongolian Barbeque, 200 S. Main St.; (734) 913-0999; <https://www.gomongo.com/>; all-you-can-eat, create-your-own stir-fry.

Isalita, 341 E. Liberty St.; (734) 213-7400; www.isalita.com/; flavor-packed menu of snacks, small plates, tacos and other delicious bites inspired mainly by popular Mexican street foods.

Bar Louie, 401 E. Liberty St.; (734) 794-3000; cool neighborhood restaurant & bar serving sandwiches, appetizers, pastas, entrees and a wide assortment of interesting beers, micro brews and wines by the glass or bottle.

Registration and Meeting Information

Advance Registration: Advance registration for this meeting opens on **August 1, 2018**. Advance registration fees will be **US\$63** for AMS members, **US\$95** for nonmembers, and **US\$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing mmsb@ams.org. The deadline to cancel is the first day of the meeting.

On-site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located in Haven Hall lobby. The Invited Address lectures will be located in Angell Hall in Auditorium B. Special Sessions and Contributed Paper Sessions will take place in the nearby classrooms. Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at <https://maps.studentlife.umich.edu/>.

The registration desk will be open on Saturday, October 20, 7:30 am to 4:00 pm and Sunday, October 21, 8:00 am to 12:00 pm. The same fees listed above apply for on-site registration and are payable with cash, check or credit card.

Other Activities

Book Sales: Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS and MAA members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Membership Activities: On Saturday, October 20, stop by the AMS Membership Exhibit to learn about the benefits

of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

Complimentary Refreshments will be served courtesy in part by the AMS Membership Department.

Math Reviews Open House: All attendees at the AMS Fall Central Sectional Meeting are invited to an Open House hosted by the AMS at the Mathematical Reviews Building. You will have the opportunity to meet some of the people who put together this impressive database of the mathematics literature. The building has an interesting history, having been originally built in 1902 as a brewery, then served later as an ice cream plant, then as a camera factory. Snacks and refreshments will be served. The event will take place Saturday, October 20 from 12:30 to 2:00 pm at 416 Fourth Street in Ann Arbor, Michigan.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at mmsb@ams.org or meet@ams.org.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Harassment is a form of misconduct that undermines the integrity of AMS activities and mission.

The AMS will make every effort to maintain an environment that is free of harassment, even though it does not control the behavior of third parties. A commitment to a welcoming environment is expected of all attendees at AMS activities, including mathematicians, students, guests, staff, contractors and exhibitors, and participants in scientific sessions and social events. To this end, the AMS will include a statement concerning its expectations towards maintaining a welcoming environment in registration materials for all its meetings, and has put in place a mechanism for reporting violations. Violations may be reported confidentially and anonymously to 855-282-5703 or at www.mathsociety.ethicspoint.com. The reporting

mechanism ensures the respect of privacy while alerting the AMS to the situation. Violations may also be brought to the attention of the coordinator for the meeting (who is usually at the meeting registration desk), and that person can provide advice about how to proceed.

For AMS policy statements concerning discrimination and harassment, see the AMS Anti-Harassment Policy at see the www.ams.org/about-us/governance/policy-statements/anti-harassment-policy.

Questions about this welcoming environment policy should be directed to the AMS Secretary at www.ams.org/about-us/governance/sec-contact.

Local Information and Maps

This meeting will take place on the *University of Michigan* campus. A campus map can be found at <https://maps.studentlife.umich.edu/>. Information about the *University of Michigan Mathematics Department* can be found at <https://lsa.umich.edu/math>. Please visit the university website at <https://www.umich.edu> for additional information on the campus.

Please watch the AMS website at www.ams.org/meetings/sectional/sectional.html for additional information on this meeting.

Parking

If you are unfamiliar with Ann Arbor, please be aware that parking may be challenging. We recommend arriving early so that you may find parking and make your way to the meeting spaces.

The *City of Ann Arbor Parking Structure* on Maynard St. between William St. and Liberty St. is a convenient parking structure near the *Huetwell Visitors Center* which allows for all-day parking. The cost is approximately US\$1.40/hour. The structure is located at 324 Maynard Street in Ann Arbor, MI 48104.

Public parking meters are enforced Monday through Saturday, 8:00 am to 6:00 pm. The cost is US\$1.70 per hour. These meters are free on evenings, Sundays and all federal holidays observed by city employees. To download the Ann Arbor Field Guide to Parking please visit www.a2dda.org/wp-content/uploads/Field-Guide-Updated-3-26-2018.pdf.

For parking information please contact *Republic Parking* at (734) 761-7235 or visit www.a2dda.org/transportation/.

Travel

This meeting will take place on the main campus of the *University of Michigan* located in Ann Arbor, Michigan.

By Air:

Detroit Metropolitan International Airport (DTW) is only 20 miles from Ann Arbor, about a 45 minute drive, and is the most convenient air travel choice. Please note, the drive could take longer during rush hour traffic. DTW has two terminals, *North* and *McNamara*. For a list of which airlines serve each terminal visit <https://www.metroairport.com/flights/airline-directory>. Please visit the

airport web site for a list of airlines and lists of cities with daily direct flights; <https://www.metroairport.com/>

There are several options available for transportation to and from the airport.

To get to the airport, you can use *AirRide*, which is available for transportation to the airport from Ann Arbor. This shuttle runs 13 times during the day and picks up at *Blake Transit Center* and the *Kensington Court Hotel*. At DTW pick up and drop off is at the south end of the *Ground Transportation Center* for the *McNamara Terminal* and at the north end of the *Ground Transportation Center* at Stall #1 for the *North Terminal*. A one way ticket on *AirRide* is US\$15, however, if a reservation is made in advance the fare is reduced to US\$12. Please visit www.theride.org/Services/Airport-Service for more information and to make reservations.

From the *McNamara Terminal* taxi services dispatch in the middle of the *Ground Transportation Center*, located on Level 4. Upon arrival in Detroit, and once all baggage has been claimed, follow signs to ground transportation. From the *North Terminal* taxi services dispatch from the upper level of the *Ground Transportation Center*. Upon arrival in Detroit, and once all baggage has been claimed, follow signs to ground transportation on Level 4.

Rideshare passengers can only meet their driver at the terminal's *Ground Transportation Center* within the designated rideshare area. Once across the skybridge, "rideshare" signs will guide passengers to the designated rideshare areas of *McNamara* or *North Terminal's Ground Transportation Center*. From the *McNamara Terminal* the rideshare area is located on Level 4 of the parking garage. From the *North Terminal* the rideshare is located on Level 1 of the parking garage.

To find the rental cars follow signs to ground transportation and take a courtesy shuttle to the rental car campus, located at the Northeast corner of the airport along Lucas Drive. Rental car shuttle bus stops can be found in the *Ground Transportation Centers* of both the *McNamara Terminal Parking Garage* and the *Big Blue Deck* of the *North Terminal*.

Phones for contacting each agency can be found in baggage claim and the *Ground Transportation Center* located at each terminal. Rental car desks are not located inside the terminals. For a rental car directory please visit <https://www.metroairport.com/parking/rental-cars>.

By Bus:

A *Greyhound* station is located at 325 Depot Street in Ann Arbor, about 1 mile from campus. For tickets and travel information visit their website at www.greyhound.com/default.aspx.

By Car:

From Windsor, Detroit and Airports:

- I-94 West to State St. (exit 177)
- North (right) on State St. to E. William St.
- West (left) on E. William St., one block to Maynard St.

MEETINGS & CONFERENCES

- North (right) on Maynard St. to public parking structure.

From Toledo and South:

- US 23 North to I-94 West (Exit 35)
- I-94 West to State St. (Exit 177)
- North (right) on State St. to E. William St.
- West (left) on E. William St., one block to Maynard St.
- North (right) on Maynard St. to public parking structure.

From Jackson and West:

- I-94 East to State St. (Exit 177)
- North (left) on State St. to E. William St.
- West (left) on E. William St., one block to Maynard St.
- North (right) on Maynard St. to public parking structure.

From Flint and North:

- US 23 South to M-14 West (exit 45)
- M-14 West to Downtown Ann Arbor (exit 3); exit turns into Main St.
- Take Main St. to William St.
- East (left) on William St., 6 blocks to Maynard St.
- North (left) on Maynard St. to public parking structure.

Car Rental: *Hertz* is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount", and type in our convention number (CV): CV#04N30008. You can also call *Hertz* directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other *Hertz* rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

Local Transportation

Walking, biking and personal cars are recommended to get around campus and Ann Arbor.

By Bus: Downtown Ann Arbor is accessible by public transit operated by *TheRide*. *TheRide* operates public transportation services in the greater Ann Arbor-Ypsilanti area. For information on bus routes, schedules and security information please visit www.theride.org/How-to-Ride. For a bus trip planner please visit www.theride.org/Plan-My-Trip.

Seasonal On-Street Bike Parking: On-street bike racks accommodate up to fourteen bikes bringing an additional ninety-eight spots to the downtown neighborhoods. Locations include:

Zingerman's Deli - 419 Detroit Street
People's Food Co-op - 216 N. Fourth Ave.
Spencer - 113 E. Liberty Street
Bivouac - 336 S. State Street
Ann Arbor District Library - 343 S. Fifth Ave.
Bill's Beer Garden - 218 S. Ashley Street
Mighty Good Coffee - 217 N. Main

Other options: Other local transportation options include *Uber*; www.uber.com and *Lyft*; www.lyft.com.

Weather

The average weather in October in Ann Arbor, Michigan is characterized by decreasing daily high temperatures, with daily high temperatures decreasing by 12°F, from 67°F to 55°F, rarely falling below 42°F or exceeding 79°F. Attendees are advised to wear coats and layered clothing. The weather can be unpredictable so umbrellas are also recommended.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at <https://travel.state.gov/content/travel/en.html>. If you need a preliminary conference invitation in order to secure a visa, please send your request to cro@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

- * Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

- * Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

- * Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

- * Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

- * If travel plans will depend on early approval of the visa application, specify this at the time of the application;

- * Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

San Francisco, California

San Francisco State University

October 27–28, 2018

Saturday – Sunday

Meeting #1144

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: July 2018

Program first available on AMS website: September 6, 2018

Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired

For abstracts: August 28, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Srikanth B. Iyengar, University of Utah, *Title to be announced*.

Sarah Witherspoon, Texas A&M University, *Derivatives, derivations, and Hochschild cohomology*.

Abdul-Aziz Yakubu, Howard University, *Population cycles in discrete-time infectious disease models*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups (Code: SS 14A), **Asuman G. Aksoy**, Claremont McKenna College, **Michael Hartglass**, Santa Clara University, **Zair Ibragimov**, California State University, Fullerton, and **Marat Markin**, California State University, Fresno.

Algebraic Geometry (Code: SS 21A), **Emily Clader** and **Dustin Ross**, San Francisco State University, and **Mark Shoemaker**, Colorado State University.

Analysis and Geometry of Fractals (Code: SS 7A), **Kyle Hambrook**, University of Rochester, **Chun-Kit Lai**, San Francisco State University, and **Sze-Man Ngai**, Georgia Southern University.

Applied Harmonic Analysis: Frame Theory and Applications (Code: SS 9A), **Chun-Kit Lai** and **Shidong Li**, San Francisco State University.

Big Data and Statistical Analytics (Code: SS 17A), **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Combinatorial and Categorical Aspects of Representation Theory (Code: SS 10A), **Nicholas Davidson** and **Jonathan Kujawa**, University of Oklahoma, and **Robert Muth**, Tarleton State University.

Coupling in Probability and Related Fields (Code: SS 3A), **Sayan Banerjee**, University of North Carolina, Chapel Hill, and **Terry Soo**, University of Kansas.

Geometric Analysis (Code: SS 8A), **Ovidiu Munteanu**, University of Connecticut, and **David Bao**, San Francisco State University.

Geometric Methods in Hypercomplex Analysis (Code: SS 13A), **Paula Cerejeiras**, Universidade de Aveiro, **Matvei Libine**, Indiana University, Bloomington, and **Mihaela B. Vajiac**, Chapman University.

Geometric and Analytic Inequalities and their Applications (Code: SS 4A), **Nicholas Brubaker**, **Isabel M. Serrano**, and **Bogdan D. Suceavă**, California State University, Fullerton.

Homological Aspects in Commutative Algebra and Representation Theory (Code: SS 5A), **Srikanth B. Iyengar**, University of Utah, and **Julia Pevtsova**, University of Washington.

Homological Aspects of Noncommutative Algebra and Geometry (Code: SS 2A), **Dan Rogalski**, University of California San Diego, **Sarah Witherspoon**, Texas A&M University, and **James Zhang**, University of Washington, Seattle.

Markov Processes, Gaussian Processes and Applications (Code: SS 18A), **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Mathematical Biology with a focus on Modeling, Analysis, and Simulation (Code: SS 1A), **Jim Cushing**, The University of Arizona, **Saber Elaydi**, Trinity University, **Suzanne Sindi**, University of California, Merced, and **Abdul-Aziz Yakubu**, Howard University.

Mathematical Methods for the study of the Three Dimensional Structure of Biopolymers (Code: SS 22A), **Javier Arsuaga** and **Mariel Vazquez**, University of California Davis, Davis, and **Robin Wilson**, Cal Poly Pomona.

Noncommutative Geometry and Fundamental Applications (Code: SS 12A), **Konrad Aguilar**, Arizona State University, and **Federic Latremoliere**, University of Denver.

Nonlocal PDEs via Harmonic Analysis (Code: SS 20A), **Tadele Mengesha**, University of Tennessee, Knoxville, and **Armin Schikorra**, University of Pittsburgh.

Probabilistic and Statistical Problems in Stochastic Dynamics (Code: SS 16A), **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Research in Mathematics by Early Career Graduate Students (Code: SS 11A), **Michael Bishop**, **Marat Markin**, **Jenna Tague**, and **Khang Tran**, California State University, Fresno.

Social Change In and Through Mathematics and Education (Code: SS 19A), **Federico Ardila** and **Matthias Beck**, San Francisco State University, **Jamylle Carter**, Diablo Valley Community College, and **Kimberly Seashore**, San Francisco State University.

Statistical and Geometrical Properties of Dynamical Systems (Code: SS 6A), **Joanna Furno** and **Matthew Nicol**,

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University of Houston, and **Mariusz Urbanski**, University of North Texas.

Topics in Operator Theory: CANCELLED (Code: SS 15A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include the California state hotel tax (12.7%), local taxes and hotel fees may apply. Participants must state that they are with the **American Mathematical Society's (AMS) Fall Western Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

AC Hotel San Francisco Airport/Oyster Point Waterfront, 1333 Veterans Blvd., South San Francisco, CA 94080; (650) 742-9211; achotels.marriott.com/hotels/ac-hotel-san-francisco-airport. Rates are **US\$229** per night for a standard guest room. Amenities include AC Lounge, Wi-Fi everywhere, complimentary parking, 24-hour fitness center access and AC Kitchen with European-inspired breakfast. This property is located about 12 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 26, 2018**.

DoubleTree by Hilton San Francisco Airport, 835 Airport Blvd., Burlingame, CA 94010; (650) 344-5500; doubletree3.hilton.com/en/hotels/california/doubletree-by-hilton-hotel-san-francisco-airport-SFOAODT/index.html. Rates are **US\$199** per night for single or double occupancy in a non-smoking king or non-smoking guest room with two double beds. Discounted parking of **US\$10** per day is available for room block overnight guests. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Amenities include complimentary internet in the guest rooms, complimentary shuttle to and from SFO airport, complimentary trolley to downtown Burlingame (based on schedule), refrigerator in each room and warm chocolate chip cookie at check-in. This property is located about 14 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at a reduced rate is by **October 5, 2018**.

Hampton by Hilton San Francisco Downtown, 942 Mission St., San Francisco, CA 94103; (415) 546-3110; hamptoninn3.hilton.com/en/hotels/california/hampton-inn-san-francisco-downtown-convention-center-SFOCCHX/index.html. Rates are **US\$269** per night for a standard guest room. Amenities include complimentary hot breakfast offered daily, fitness center, 24-hour Suite Shop and complimentary Wi-Fi in all guest rooms. This property is located about 10 miles from the

campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Valet parking is available for an overnight fee of **US\$53** plus tax per vehicle per night. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellations on or after September 26, 2018 will incur a cancellation fee of one night room and tax. No modification can be made to the reservation after the cancellation date. A reduction in length of stay will incur a penalty of one night room and tax. The deadline for reservations at a reduced rate is **August 26, 2018**.

Harbor Court Hotel, 165 Steuart St., San Francisco, CA 94105; (415) 882-1300; <https://www.harborcourthotel.com>. Rates are **US\$209** per night for single or double occupancy guest room. Amenities include complimentary Wi-Fi, hosted evening wine hour, morning coffee and tea. This property is located approximately 10 miles from campus. The following hotel charges are separate and distinct from, and in addition to, the room rate and taxes. Hotel charges include portage charges in the amount of **US\$6.50** per bag, round trip; special room delivery service in the amount of **US\$6.40** per delivery; overnight parking for guests is **US\$60** per day, plus taxes, with in and out privileges; **US\$10** amenity fee per person per day. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. A night of room and tax will be assessed for departure before the confirmed departure date. The deadline for reservations at this rate is **September 11, 2018**.

Hotel Focus SFO, 111 Mitchell Ave., South San Francisco, CA 94080; (650) 877-0770; <https://hotelfocussfo.com>. Rates are **US\$189** per night for a standard guest room. Amenities include complimentary continental breakfast, free Wi-Fi, free airport shuttle and IMediaCast panels and charging stations in each guest room. This property is located about 10 miles from campus. Guest parking is **US\$12** per night plus tax per vehicle per night. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual reservations require a first night's room and tax deposit via valid major credit card. All reservations are subject to a 72 hour cancellation policy. If an individual reservation is cancelled within 72 hours of arrival it will be subject to a loss of deposit. Individual guaranteed reservations that "no show" will forfeit their first night's room and tax deposit and the reserved room will be released back for general sale. The deadline for reservations at this rate is **September 20, 2018**.

Hyatt House Emeryville/San Francisco Bay Area, 5800 Shellmound St., Emeryville, CA 94608; (510) 601-5880; <https://emeryville.house.hyatt.com/en/hotel/home.html>. Rate is **US\$229** per night for a standard guest room. Additional charges of **US\$10** per person

will apply for triple or quadruple occupancy. Amenities include suites with a fully fitted kitchen, living room area and separate bedroom with either one king bed or two queen beds. On-site parking is US\$15 per day per vehicle. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Amenities include free breakfast, free Wi-Fi, fitness center, pool, and hot tub. This property is located about 18 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. Individual cancellations must be made 24 hours prior to arrival. Any cancellations within 24 hours of arrival or no-shows will be subject to a charge of one night room and tax to the individual guest. The deadline for reservations at this rate is **September 26, 2018**.

Hyatt Regency San Francisco Airport, 1333 Old Bayshore Hwy., Burlingame, CA 94010; (650) 347-1234; <https://sanfranciscoairport.regency.hyatt.com/en/hotel/home.html>. Rates are US\$159 per night for a guest room with single or double occupancy. Amenities include 24-hour complimentary shuttle service to the airport, Hertz rental car service desk on-site, free trolley to downtown Burlingame, fitness center, outdoor pool and whirlpool. This property is located about 13 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 28, 2018**.

Marriott San Mateo San Francisco Airport, 1770 S Amphlett Blvd., San Mateo, CA 94402; (650) 653-6000; <https://www.marriott.com/hotels/travel/sfosa-san-mateo-marriott-san-francisco-airport>. Rates are US\$199 for a standard guest room. Amenities include on-site breakfast and room service, fitness center, complimentary airport transportation and complimentary guest room Wi-Fi for those in the AMS room block. This property is located about 23 miles from campus. Self-parking is available at discounted rate for those in the room block at US\$12 per day. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. A cancellation less than 48 hours prior to arrival will result in a charge equal to one night room and tax. Early departures and no-shows will result in the same consequence. The deadline for reservations at this rate is **September 26, 2018**.

Marriott Oakland City Center, 1001 Broadway, Oakland, CA 94607; (510) 451-4000; <https://www.marriott.com/hotels/travel/oakdt-oakland-marriott-city-center>. Rates are US\$146 for a deluxe guest room. Amenities include on-site Iron & Oak bar and restaurant, fitness center and outdoor pool. This property is located about 20 miles from campus. Self-parking is available for overnight guests at US\$35 per day with no in and out privileges. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary

and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 19, 2018**.

Marriott San Francisco Airport Waterfront, 11800 Bayshore Hwy., Burlingame, CA 94010; (650) 692-9100; <https://www.marriott.com/hotels/travel/sfobg-san-francisco-airport-marriott-waterfront>. Rates are US\$199 for a standard guest room. Amenities include complimentary shuttle service to and from a BART subway station and San Francisco International Airport. This property is located about 14 miles from campus. Self-parking is available for overnight guests at US\$26 per day. The parking rate is subject to change and the parking fee in effect during the dates of your stay will be charged. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 28, 2018**.

The AMS is not holding room blocks at the following properties. These properties are located near the university and are listed for your convenience. The AMS is not responsible for rate charges or for the quality of the accommodations.

Alpine Inn & Suites, 560 Carter St., Daly City, CA 94014; (415) 334-6969; <https://www.alpineinnsuites.com>. This property is located about 5 miles from campus. Please contact the property or visit their website for room rates and availability.

Mirage Inn and Suites, 2600 Sloat Blvd., San Francisco, CA 94116; (415) 665-9000; mirageinnandsuites.com/. This property is located about 2 miles from campus. Please contact the property or visit their website for room rates and availability.

Ocean Park Hotel, 2690 46th Ave., San Francisco, CA 94116; (415) 566-7020; www.oceanparkmotel.com. This property is located about 2 miles from campus. Please contact the property or visit their website for room rates and availability.

Seal Rock Inn, 545 Point Lobos Ave., San Francisco, CA 94121; (415) 752-8000; sealrockinn.com. This property is located about 5 miles from campus. Please contact the property or visit their website for room rates and availability.

Great Highway Inn, 1234 Great Hwy., San Francisco, CA 94112; (415) 731-6644; www.greathwy.com/. The hotel is located approximately 4 miles from campus. Please contact the property or visit their website for room rates and availability.

Mission Inn, 5630 Mission St., San Francisco, CA 94112; (415) 584-5020; <https://www.sfmissioninn.com>. This hotel is located about 2 miles from campus. Please contact the property or visit their website for room rates and availability.

Serrano Hotel, 405 Taylor St., San Francisco, CA 94102; (415) 885-2500; www.serranohotel.com. This property is steps from Union Square and is located about 10 miles from campus. Please contact the property or visit their website for room rates and availability.

Housing Warning:

Please beware of aggressive housing bureaus that target potential attendees of a meeting. They are sometimes called “room poachers” or “room-block pirates” and these companies generally position themselves as a meeting’s housing bureau, convincing attendees to unknowingly book outside the official room block. They call people who they think will more likely than not attend a meeting and lure them with room rates that are significantly less than the published group rate—for a limited time only. And people who find this offer tempting may hand over their credit card data, believing they have scored a great rate and their housing is a done deal. Unfortunately, this often turns out to be the start of a long, costly nightmare. These housing bureaus are not affiliated with the American Mathematical Society or any of its meetings, in any way. The AMS would never call anyone to solicit reservations for a meeting. The only way to book a room at a rate negotiated for an AMS Sectional Meeting is through AMS Sectional Meetings pages. The AMS cannot be responsible for any damages incurred as a result of hotel bookings made with unofficial housing bureaus.

Food Services

On Campus: The San Francisco State campus has many dining options to suit your mood and budget. You can order a variety of sandwiches, enjoy a wide selection of ethnic cuisines—Mexican, Italian, Chinese, and more—or sip a cappuccino from one of the food vendors at the Cesar Chavez Student Center. For a full list of food vendors please visit <https://ucorp.sfsu.edu/food-vendors>.

Cafe Rosso, 1600 Holloway Ave. (plaza behind Burk Hall); (415) 405-0923; open Saturday 8:00 am to 3:00 pm; serves fun and creative dishes with a Mediterranean flair.

Clean Bites, Mashouf Wellness Center; (415) 404-9756; open 11:00 am to 10:00 pm on Saturday and Sunday; serves nutritious and energy-boosting juices, smoothies, beverages, bowls and wraps.

iNoodles, Cesar Chavez Student Center, Recreation & Dining Level; (415) 338-6338; open 10:00 am to 5:00 pm on Saturday and Sunday; specializes in traditional Asian-style noodles and rice plates.

Peet’s Coffee & Tea, first floor of the J. Paul Leonard Library; open 9:00 am to 9:00 pm on Saturday and Sunday; serving hand-crafted beverages, premium salads, bagels and sandwiches.

Quickly, Cesar Chavez Student Center, West Plaza; (415) 338-6484; open Saturday and Sunday from 11:00 am to 4:00 pm; serving Boba teas and snacks.

Station Cafe, 19th Ave. (in front of HSS Bldg); (415) 405-0463; open 8:00 am to 3:00 pm on Saturday; serving on-the-go sandwiches, salads, snacks, coffee and espresso.

Subway, Village at Centennial Square; (415) 405-3499; open 8:00 am to 3:00 am on Saturday and 9:00 am to midnight on Sunday; providing fast, healthy food, including made-to-order sandwiches.

Village Market & Pizza, Village at Centennial Square; (415) 405-2292; open 10:00 am to 3:00 am on Saturday;

specialty pizzas, signature sandwiches and pasta; offering groceries, sundries, and a full espresso bar.

Off Campus: The San Francisco dining scene is one of the best in the country. From hearty comfort food to Michelin-starred fine dining, there are restaurants for every palate. Please go to the San Francisco Travel site for a complete restaurant guide at www.sfsttravel.com/explore/dining.

Some options for coffee include:

Spessa Coffee Bar, 51 Cambon Dr.; (415) 239-1520; open 7:00 am to 7:00 pm Saturday and Sunday; serving coffee and pastries.

Starbucks, 3251 20th Ave. (Stonestown Galleria); (415) 665-4956; Seattle-based coffeehouse chain known for its signature roasts, light bites and Wi-Fi availability.

Some options for dining include:

Hall of Flame Burger, 73 Cambon Dr.; (415) 584-4444; open Saturday and Sunday for lunch and dinner; offers premium burgers, sandwiches and salads.

Taqueria Dos Charros, 55 Cambon Dr.; (415) 334-6075; open Saturday and Sunday until 10:00 pm; specializing in a combination of international foods.

Papa John’s, 69 Cambon Dr.; (415) 586-7272; open until 11:30 pm on Saturday and Sunday; take-out and delivery chain offering specialty pizzas, wings, bread sticks and desserts.

Olive Garden, 3251 20th Ave. (Stonestown Galleria); (415) 661-6770; www.olivegarden.com; open until 10:00 pm on Saturday and Sunday; lively, family-friendly chain featuring Italian standards such as pastas and salads, with a full bar.

Sorabol Korean BBQ, 3251 20th Ave. (Stonestown Galleria); (415) 753-5959; www.sorabolrestaurants.com; counter-serve chain dishing up Korean BBQ, plus Asian noodle dishes and soups.

Original Joe’s Westlake, 11 Glenwood Ave., Daly City; (650) 755-7400; www.originaljoessf.com; open until 10:00 pm on Saturday and Sunday; neighborhood fixture serving traditional, old-school Italian entrees in spacious environs.

Registration and Meeting Information

Advance Registration: Advance registration for this meeting opens on **August 1, 2018**. Advance registration fees will be **US\$63** for AMS members, **US\$95** for nonmembers, and **US\$10** for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing mmsb@ams.org. The deadline to cancel is the first day of the meeting.

On-site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located in Thornton Hall, Room 404. The Invited Address lectures, Special Sessions and Contributed Paper Sessions will take place in the nearby classrooms. Please look for additional information about specific session room locations on the web and in the printed program. For further information on building locations, a campus map is available at https://www.sfsu.edu/~sfsu-map/graphics/sfsu_map_color.pdf.

The registration desk will be open on Saturday, October 27, 7:30 am to 4:00 pm and Sunday, October 28, 8:00 am to 12:00 pm. The same fees listed above apply for on-site registration and are payable with cash, check or credit card.

Other Activities

Book Sales: Stop by the on-site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS and MAA members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Membership Activities: Stop by the AMS Membership Exhibit to learn about the benefits of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

Complimentary Refreshments will be served courtesy in part by the AMS Membership Department.

Special Needs

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- Registering early for the meeting
- Checking the appropriate box on the registration form, and
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AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Harassment is a form of misconduct that undermines the integrity of AMS activities and mission.

The AMS will make every effort to maintain an environment that is free of harassment, even though it does not control the behavior of third parties. A commitment to a welcoming environment is expected of all attendees at AMS activities, including mathematicians, students, guests, staff, contractors and exhibitors, and participants in scientific sessions and social events. To this end, the

AMS will include a statement concerning its expectations towards maintaining a welcoming environment in registration materials for all its meetings, and has put in place a mechanism for reporting violations. Violations may be reported confidentially and anonymously to 855-282-5703 or at www.mathsociety.ethicspoint.com. The reporting mechanism ensures the respect of privacy while alerting the AMS to the situation. Violations may also be brought to the attention of the coordinator for the meeting (who is usually at the meeting registration desk), and that person can provide advice about how to proceed.

For AMS policy statements concerning discrimination and harassment, see the AMS Anti-Harassment Policy at see the www.ams.org/about-us/governance/policy-statements/anti-harassment-policy.

Questions about this welcoming environment policy should be directed to the AMS Secretary at www.ams.org/about-us/governance/sec-contact.

Local Information and Maps

This meeting will take place on the *San Francisco State University* campus. A campus map can be found at https://www.sfsu.edu/~sfsumap/graphics/sfsu_map_color.pdf. Information about the *San Francisco State University Mathematics Department* can be found at math.sfsu.edu. Please visit the university website at <https://www.sfsu.edu> for additional information on the campus.

Please watch the AMS website at www.ams.org/meetings/sectional/sectional.html for additional information on this meeting.

Parking

SF State provides parking for students, faculty, staff and visitors. The cost to park at different facilities and parking locations for each type of user is located at <https://parking.sfsu.edu/sfsu-parking/campus-parking>. See the parking map for the locations of each of these lots available at https://parking.sfsu.edu/sites/default/files/maps/614_color_map_PARKING_0.pdf. A valid daily or semester permit is required at ALL times in every lot. After 5:00 pm and on weekends, all lots are open to the public.

The gymnasium parking structure, *Lot 6*, is the closest to the meeting registration area. On weekends the structure is open to the public. Parking fees are **US\$5** for two hours or **US\$8** for a day of parking. Daily parking permits expire at midnight. Please note that exact change is required; machines do not give change. Pay stations will accept \$1, \$5 and \$10 bills as well as credit/debit cards and the SF State OneCard. If credit/debit card is used there is a 2.75% credit card fee.

For parking information please contact the parking office at (415) 338-1441 or email parking@sfsu.edu.

Travel

This meeting will take place on the main campus of *San Francisco State University* located in San Francisco, California.

MEETINGS & CONFERENCES

By Air:

San Francisco International Airport (SFO) is an international airport 13 miles south of downtown San Francisco. SFO airport is only 12 miles from the campus. Road closures currently affect traffic to SFO airport. Attendees are advised to allow extra travel time. The San Bruno Canal Bridge at South Airport Boulevard is currently closed and the new bridge is expected to open by the end of 2018. Please visit the airport web site for a list of airlines and lists of cities with daily direct flights; <https://www.flysfo.com>.

There are several options available for transportation to and from the airport.

App-Based Ride Services provide prearranged transportation between SFO and passenger-specified locations. Passengers arrange service online, typically using a smartphone app. Drivers use their personal, non-commercial vehicles to transport passengers. App-Based Ride Services pick up and drop off passengers at the terminal curbs on the Departures/Ticketing Level of all terminals. Fare and travel times ranges vary. Customers should contact the company for details.

Bay Area Rapid Transit (BART) operates rapid rail service to northern San Mateo County, San Francisco and the East Bay directly from SFO. The SFO *BART Station* is located on the Departures/Ticketing Level of the International Terminal (Boarding Area G side). *BART* is easily accessed from any terminal by riding SFO's *AirTrain* to the Garage G/BART Station stop. There is no charge to board *AirTrain*. Tickets can be purchased at the SFO BART Station upon arrival. *BART* provides a connection to Caltrain at the Millbrae station, but before 9:05pm on weekdays, passengers must change *BART* trains at *BART's* San Bruno station.

Travelers are advised to purchase *BART* train vouchers in advance. Vouchers can only be redeemed at the SFO Information Booth located in the International Terminal G-side, near the *AirTrain* escalator, during the hours of 8:00 am to midnight daily. The *BART* fare range is US\$8.40 to US\$8.95. For more information visit www.bart.gov.

SamTrans Public Bus Service provides 24-hour bus service connecting SFO with San Mateo County and parts of San Francisco. Routes KX, 292, 397, 398 and 399 stop directly at the Airport terminals. Routes 140, 292, 397 and 399 stop at the Rental Car Center and the United Airlines Maintenance Center. Routes 292, 397 and 399 stop on North McDonnell Road at the Airport Business Center and West Field Road. *SamTrans* buses stop at Terminal 2 Arrivals/Baggage Claim Level, Terminal 3 Arrivals/Baggage Claim Level and International Terminal Level 1. The *SamTrans* fares range is US\$2 to US\$5. For more information on schedules and maps please visit www.samtrans.com.

San Francisco International Airport's Rental Car Center allows travelers to pick up and drop off rental cars at one convenient, central location. To access the Rental Car Center from the airport terminals, take the *AirTrain* Blue Line. *AirTrain*, SFO's fully automated people mover system, operates 24 hours every day and provides frequent service throughout SFO. *AirTrain* stations are located in

all terminals, terminal parking garages, the *Rental Car Center* and SFO's *BART* station.

By Bus:

A *Greyhound* station is located at 200 Folsom St. in San Francisco, about 11 mile from campus. For tickets and travel information visit their website at www.greyhound.com/default.aspx.

By Car:

From the North:

Take Highway 101 South, cross the Golden Gate Bridge. Take 19th Avenue/Highway 1 exit. Follow 19th Avenue to the main campus at Holloway Avenue.

From the South:

Take I-280 North, exit at 19th Avenue. Take Junipero Serra Boulevard to Holloway Avenue, turn left on Holloway Avenue to campus at 19th Avenue.

From the East:

Take I-80 West across the Bay Bridge to Highway 101 South. Take 101 South to I-280 toward Daly City. Take the San Jose Avenue/Mission St. exit (immediately after the Ocean Avenue exit), bearing right onto Sagamore Street to Brotherhood Way to Junipero Serra Boulevard North. Take Junipero Serra Boulevard to Holloway Avenue, turn left on Holloway Avenue to campus at 19th Avenue.

Car Rental: *Hertz* is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount", and type in our convention number (CV): CV#04N30008. You can also call *Hertz* directly at 800-654-2240 (US and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other *Hertz* rates and you will be quoted the best comparable rate available.

For directions to campus, inquire at your rental car counter.

Local Transportation

Walking, biking and personal cars are recommended to get around campus and San Francisco.

By Bus: The campus is served by the *Muni* 28 bus line. The cost per trip is US\$2.25 if paid in cash. For more information and a map of bus routes go to <https://parking.sfsu.edu/transit/muni>.

Biking: SF State encourages biking as a commute option. SF State was named a Bronze-level Bicycle Friendly University by the *League of American Bicyclists* in 2016. For the safety of all members of the community, the core of campus is designated as a pedestrian only zone. Please see the Bike Parking Map at parking.sfsu.edu/sites/default/files/614%20color%20map%20BIKE%20PARKING%20revised.pdf for designated on-campus bike routes. Riding your bike in prohibited areas may result in a citation.

Other options: Other local transportation options include *Uber*; www.uber.com and *Lyft*; www.lyft.com. *Zipcars* are available on campus, located in the Lot 20 garage on the street level accessed by State Drive and at University

Park North, on Winston Drive. For more information go to zipcar.com/sfsu.

Weather

The weather in San Francisco, California during the month of October is great for touring and other activities. In the afternoons throughout the month, the temperature typically stays mild around 60 degrees Fahrenheit. Although the mornings and evenings will normally be foggy and chilly, the afternoons have plenty of sunshine. As always in San Francisco, it can be windy near the coastline. Attendees are advised to wear layered clothing. The weather can be unpredictable so umbrellas are also recommended.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at <https://travel.state.gov/content/travel/en.html>. If you need a preliminary conference invitation in order to secure a visa, please send your request to cro@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of “binding” or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor’s home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Fayetteville, Arkansas

University of Arkansas

November 3–4, 2018

Saturday – Sunday

Meeting #1142

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: July 2018

Program first available on AMS website: August 16, 2018

Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: Expired

For abstracts: September 4, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mihalis Dafermos, Princeton University, *On Falling Into Black Holes*

Jonathan Hauenstein, University of Notre Dame, *Numerical Algebraic Geometry and Optimization*.

Kathryn Mann, Brown University, *Group actions, geometry and rigidity*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Birational Geometry (Code: SS 11A), **Roi Docampo**, University of Oklahoma, and **Lance Edward Miller** and **Wenbo Niu**, University of Arkansas.

Commutative Algebra (Code: SS 3A), **Alessandro De Stefani**, University of Nebraska–Lincoln, **Paolo Mantero**, University of Arkansas, and **Thomas Polstra**, University of Utah.

Groups in Low-dimensional Topology and Dynamics (Code: SS 7A), **Matt Clay**, University of Arkansas, and **Kathryn Mann**, Brown University.

Harmonic Analysis and Partial Differential Equations (Code: SS 8A), **Ariel Barton**, University of Arkansas, and **Simon Bortz**, University of Minnesota.

Interactions Between Combinatorics and Commutative Algebra (Code: SS 10A), **Ashwini Bhat**, **Chris Francisco**, and **Jeffrey Mermin**, Oklahoma State University.

MEETINGS & CONFERENCES

Interactions Between Contact and Symplectic Geometry and Low-dimensional Topology (Code: SS 5A), **Jeremy Van Horn-Morris**, University of Arkansas, and **David Shea Vela-Vick**, Louisiana State University.

Non-associative Algebraic Structures and their (Co)homology Theories (Code: SS 12A), **Michael Kinyon**, University of Denver, **Jozef H Przytycki**, The George Washington University, and **Petr Vojtechovsky** and **Seung Yeop Yang**, University of Denver.

Numerical Methods for Nonlinear Systems (Code: SS 9A), **Jonathan Hauenstein** and **Tingting Tang**, University of Notre Dame.

Operator Theory and Function Spaces of Analytic Functions (Code: SS 13A), **Daniel Luecking** and **Maria Tjani**, University of Arkansas.

Partial Differential Equations in Several Complex Variables (Code: SS 2A), **Phillip Harrington** and **Andrew Raich**, University of Arkansas.

Recent Advances in Mathematical Fluid Mechanics (Code: SS 1A), **Zachary Bradshaw**, University of Arkansas.

Recent Developments on Fluid Turbulence (Code: SS 6A), **Eleftherios Gkioulekas**, University of Texas Rio Grande Valley.

The Geometry of Curves and Applications (Code: SS 14A), **Jason Cantarella** and **Philipp Reiter**, University of Georgia.

Validation and Verification Strategies in Multiphysics Problems (Code: SS 4A), **Tulin Kaman**, University of Arkansas.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include a 13.75% occupancy tax. Participants must state that they are with the **American Mathematical Society (AMS) Meeting at the University of Arkansas/ AMS Fall Southeastern Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Inn at Carnall Hall (.5 miles from Champions Hall on campus), 465 Arkansas Avenue, Fayetteville, AR 72701; (479)582-0400; <https://www.innatcarnallhall.com>. Rates are US\$119 per night for a room with a king bed, and US\$149 per night for a room with two queen beds. The Inn at Carnall Hall offers complimentary high speed Wi-Fi and parking for guests of the hotel. Guests also receive access to the University of Arkansas fitness center. Ella's Table, the on-site restaurant, is open for breakfast, lunch (Monday-Friday, only) and dinner. The hotel's Lambeth Lounge is open every evening.

The deadline for reservations at this rate is **October 5, 2018**.

The Chancellor Hotel (.9 miles from Champions Hall on campus), 70 N. East Avenue, Fayetteville, AR 72703; (479) 442-5555, www.hotelchancellor.com/?gclid=CjwKCAjw06LZBRBNEiwA2vgMVTBNBmAnk-prh0vk2RXFa6Q

-2DtW0J5rwtARdx7xahN4s-MTQAqv8RoCckAQAvD_BwE. Rates are US\$139.00 for all room types. To make a reservation online, please use this link: bookings.ihotelier.com/bookings.jsp?groupID=2106178&hotelID=96837a. The Chancellor Hotel offers complimentary Wi-Fi in all guest rooms and public spaces as well as a complimentary shuttle within the Fayetteville area. Valet parking is US\$10 per day and garage parking is US\$4 per day. The Chancellor Hotel also has an indoor/outdoor pool and fitness facilities for their guests. 6494 Bistro and Lounge, the hotel's on-site casual dining, is open for breakfast, lunch and dinner.

The deadline for reservations at this rate is **October 3, 2018**.

Candlewood Suites (1.7 miles from Champions Hall on campus), 2270 W Martin Luther King, Jr. Boulevard, Fayetteville, AR 72701; (479) 856-6262, https://www.ihg.com/candlewood/hotels/us/en/fayetteville/fyvfs/hoteldetail?cm_mmc=GoogleMaps--CW--US--FYVFS. Rates are US\$81 for a room with one queen bed; US\$86 for a room with two double beds; US\$86 for a room with one king bed; US\$100 for a suite with one queen bed; and US\$105 for a suite with one king bed. Please note: there is a 7-day cancellation policy at Candlewood Suites. The hotel's amenities include complimentary wireless internet service, a fitness center and a 24-hour business center.

The deadline for reservations at this rate is **October 2, 2018**.

Staybridge Suites (1.8 miles from Champions Hall on campus), 1577 West 15th Street, Fayetteville, AR 72701; (479) 695-2400, <https://www.ihg.com/staybridge/hotels/us/en/fayetteville/fyvar/hoteldetail>. Rates are US\$112 for a room with a queen bed and US\$125 for a room with two double beds. To make a reservation online, please use this link: <https://www.staybridge.com/redirect?path=hd&brandCode=SB&localeCode=en®ionCode=1&hotelCode=FYVAR&PMID=99801505&GPC=AMS&viewfullsite=true>. Amenities offered at Staybridge Suites include complimentary Wi-Fi, complimentary hot breakfast buffet, fitness room and complimentary laundry facilities.

The deadline for reservations at this rate is **October 18, 2018**.

Econo Lodge (1.9 miles from Champions Hall on campus), 1000 S. Futrall Drive, Fayetteville, AR 72701; (479) 442-3041; <https://www.choicehotels.com/arkansas/fayetteville/econo-lodge-hotels/ar229?source=gyxt>. Rates are US\$69 per night for a room with two queen beds and US\$65 for a room with one king or one queen bed. Please note: there is a 7-day cancellation policy at the Econo Lodge. The hotel's amenities include complimentary Wi-Fi and complimentary continental breakfast, as well as free parking for guests.

The deadline for reservations at this rate is **October 2, 2018**.

Housing Warning:

Please beware of aggressive housing bureaus that target potential attendees of a meeting. They are sometimes called “room poachers” or “room-block pirates” and these companies generally position themselves as a meeting’s housing bureau, convincing attendees to unknowingly book outside the official room block. They call people who they think will more likely than not attend a meeting and lure them with room rates that are significantly less than the published group rate—for a limited time only. And people who find this offer tempting may hand over their credit card data, believing they have scored a great rate and their housing is a done deal. Unfortunately, this often turns out to be the start of a long, costly nightmare.

These housing bureaus are not affiliated with the American Mathematical Society or any of its meetings, in any way. The AMS would never call anyone to solicit reservations for a meeting. The only way to book a room at a rate negotiated for an AMS Sectional Meeting is through AMS Sectional Meetings pages. The AMS cannot be responsible for any damages incurred as a result of hotel bookings made with unofficial housing bureaus.

Food Services

On Campus: There are many dining options on campus within walking distance of the meeting. The hours of the on-campus restaurants vary on the weekends. Please see a complete listing here: <https://www.dineoncampus.com/razorbacks/campus-dining-map>.

Off Campus: Fayetteville offers many dining options of all types of cuisine. Dickson Street which runs through campus, is famous for its restaurants, bars and grills. For more information on dining throughout Fayetteville, please visit, <https://www.visitfayettevillenc.com/things-to-do/dining>. And for more information on visiting Fayetteville in general please visit, <https://www.visitfayettevillenc.com>.

Some of the nearby off-campus dining options include:
A Taste of Thai, 31 East Center Street, #100; originalatasteofthai.com

Petra Cafe, 31 East Center Street, #101; thepetracafe.com

Vetro, 17 East Center Street; vetro1925.com

Hugo’s, 2 1/2 North Block Avenue; hugosfayetteville.com

Hammontree’s, 326 Northwest Avenue, #8; hammontree-scheese.com

Emelia’s Kitchen, 309 West Dickson Street; lese1nashville.com

Wasabi, 313 West Dickson Street, #105; wasabifayetteville.com

US Pizza, 202 West Dickson Street; uspizzaco.net

Registration and Meeting Information

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On-site Information and Registration: The registration desk, AMS book exhibit, and coffee service will be located in the 3rd floor lobby of Champions Hall. The Invited Addresses will be held in Gearhart Hall, Room 0026. Special Sessions and Contributed Paper Sessions will take place in classrooms in Science Engineering Hall.

For additional information on building locations and a printable campus map, visit https://campusmaps.uark.edu/?pn1_disp=Y

The registration desk will be open on Saturday, November 3, 7:30 am–4:00 pm and Sunday, November 5, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

Other Activities

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free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Harassment is a form of misconduct that undermines the integrity of AMS activities and mission.

The AMS will make every effort to maintain an environment that is free of harassment, even though it does not control the behavior of third parties. A commitment to a welcoming environment is expected of all attendees at AMS activities, including mathematicians, students, guests, staff, contractors and exhibitors, and participants in scientific sessions and social events. To this end, the AMS will include a statement concerning its expectations towards maintaining a welcoming environment in registration materials for all its meetings, and has put in place a mechanism for reporting violations. Violations may be reported confidentially and anonymously to 855-282-5703 or at www.mathsociety.ethicspoint.com. The reporting mechanism ensures the respect of privacy while alerting the AMS to the situation. Violations may also be brought to the attention of the coordinator for the meeting (who is usually at the meeting registration desk), and that person can provide advice about how to proceed.

For AMS policy statements concerning discrimination and harassment, see the AMS Anti-Harassment Policy at see the www.ams.org/about-us/governance/policy-statements/anti-harassment-policy.

Questions about this welcoming environment policy should be directed to the AMS Secretary at www.ams.org/about-us/governance/sec-contact.

Local Information and Maps

This sectional will take place on the Main Campus of the University of Arkansas in Fayetteville, Arkansas. A campus map can be found at https://campusmaps.uark.edu/?pn1_disp=Y. Information about the University of Arkansas Department of Mathematical Sciences can be found at <https://fulbright.uark.edu/departments/math>. For additional information about the University please visit the University of Arkansas website at <https://www.uark.edu>.

The meeting will occupy space in Science Engineering Hall, Gearhart Hall, and Champions Hall. A campus map for the University of Arkansas can be found here, campusmaps.uark.edu/?pn1_disp=Y.

Parking

Parking is available at the Harmon Garage (847 Williams Street) just a few blocks away from Champion Hall, Gearhart Hall, and Science Engineering Hall. A map showing the location of Harmon Garage can be found here, <https://www.google.com/maps/dir/36.0652781,-94.1725431/@36.0689086,-94.1774083,1500m/data=!3m1!1e3>. The cost for parking is \$16 per day.

Travel

The University of Arkansas in Fayetteville, Arkansas.

By Air:

Northwest Arkansas Regional Airport (XNA) is the closest airport to the University of Arkansas. Northwest Arkansas Regional Airport is located approximately 30 miles from the University.

Rental cars are available at XNA can be found here, www.flyxna.com/getting-around/rental-cars. Taxi, limousine and shuttle services are also available at the airport. For a complete listing, please visit www.flyxna.com/getting-around/rental-cars. For more information, please visit the Northwest Arkansas Regional Airport website: www.flyxna.com.

To drive from the Northwest Arkansas Regional Airport to the meeting at the University of Arkansas:

Upon leaving the airport, turn left onto Tower Drive and then turn right onto Regional Avenue. Then, turn left onto Airport Boulevard and turn left onto AR-265/Healing Spring Road. In 3.8 miles, turn right onto AR-112/AR-264/North Main Street. Turn left onto AR-264/East Lowell Avenue. Take ramp right for I-49 South toward Fayetteville/Springdale and follow for 11.6 miles. At Exit 66, take ramp right for AR-112 South toward Fayetteville and then, turn right onto AR-112/North Garland Avenue/North Highway 112. Bear right onto North Garland Avenue and then turn left onto West Dickson Street. Then, turn right onto North Duncan Avenue and your destination is on the right. 201 North Duncan Avenue (Harmon Garage, parking lot on campus near the meeting).

By Train: Amtrak does not have a train station in Fayetteville. One can take a Greyhound bus from the Little Rock Amtrak Station to the Fayetteville Greyhound Station. Please contact Amtrak at Tel: 800-USA-RAIL, website: www.amtrak.com

By Bus: The Greyhound Bus station located at 3075 West Wedington Drive, Fayetteville, AR 72704 is located about 2.5 miles from the meeting at the University of Arkansas. Please contact Greyhound Tel: 1-800-231-2222; Website: www.greyhound.com.

By Car: All driving directions will end at the Harmon Parking Garage on the campus of the University of Arkansas.

From the north: Take I-49 South toward Fayetteville. At exit 66, take ramp right for AR-112 South toward Fayetteville and then turn right onto AR-112/North Garland Ave/N Highway 112. Bear right onto North Garland Avenue and then turn left onto West Dickson Street. Then, turn right onto North Duncan Avenue and your destination is on the right. 201 North Duncan Avenue (Harmon Garage, parking lot on campus near the meeting).

From the south: Take I-49 North toward Fayetteville for 40 miles. At exit 60, take ramp right for AR-265 toward Fayetteville Executive Airport. Turn right onto AR-265/South Cato Springs Road and then keep straight onto South Razorback Road. Keep straight onto AR-16 and then turn right onto West Indian Trail. Turn left onto South Paris Avenue/South Stadium Drive. The road name

changes to Sout Virginia Avenue. Turn right onto South California Boulevard/West Clinton Drive. The road name changes to West Center Street. Then, turn right onto North Duncan Avenue and your destination is on the right. 201 North Duncan Avenue (Harmon Garage, parking lot on campus near the meeting).

From the east: Take US-412 West to AR-45. Stay on AR-45 West for 18.6 miles. Turn right onto US-71 BR/North College Avenue and then immediately turn left onto East Lafayette Street. Road name changes to West Lafayette Street. Turn left onto North Gregg Avenue and then turn right onto West Dickson Street. Then, turn left onto North Duncan Avenue and your destination is on the right. 201 North Duncan Avenue (Harmon Garage, parking lot on campus near the meeting).

From the west: Take US-412 East. Bear right onto AR-16 SPUR / East Kenwood Street and then turn right onto AR-16/Progress Avenue. Keep straight onto AR-112 SPUR/AR-16 and then turn right onto AR-112/North Garland Avenue. Bear right onto North Garland Avenue and then turn left onto West Dickson Street. Then, turn right onto North Duncan Avenue and your destination is on the right. 201 North Duncan Avenue (Harmon Garage, parking lot on campus near the meeting).

Local Transportation

Car Rental: *Hertz* is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount," and type in our convention number (CV): 04N30008. You can also call *Hertz* directly at 800-654-2240 (US and Canada) or 405-749-4434 (other countries). At the time of your reservation, the meeting rates will be automatically compared to other *Hertz* rates and you will be quoted the best comparable rate available. Meeting rates include unlimited mileage and are subject to availability. Advance reservations are recommended, blackout dates may apply.

Taxi Service: Licensed, metered taxis are available in Fayetteville through *Dynasty Taxi Service*. Tel: 479-521-8294; Website: www.dynastytaxi.com.

Both *Lyft* and *Uber* also operate in the Fayetteville area.

Bus Service: The University of Arkansas Razorback Transit System has several bus stops in the area of the meeting. Service is free. For a map and schedule, please visit: <https://parking.ark.edu/transit-services/transit-operations/maps-and-schedules.php>.

Weather

The average high temperature in Fayetteville for November is in the low 60s, Fahrenheit and the average low is in the upper 30s/lower 40s, Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at <https://travel.state.gov/content/travel/en.html>. If you need a preliminary conference invitation in order to secure a visa, please send your request to mac@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

- * Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

- * Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

- * Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

- * Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

- * If travel plans will depend on early approval of the visa application, specify this at the time of the application;

- * Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Meeting #1145

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and

MEETINGS & CONFERENCES

the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2018

Program first available on AMS website: November 1, 2018

Issue of *Abstracts*: Volume 40, Issue 1

Deadlines

For organizers: Expired

For abstracts: September 25, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/national.html.

Joint Invited Addresses

Sarah Koch, University of Michigan, *Title to be announced* (AMS-MAA Invited Address).

Bryna Kra, Northwestern University, *Title to be announced* (AWM-AMS Noether Lecture).

Cathy O'Neil, ORCAA, *Title to be announced* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

Daniel Spielman, Yale University, *Title to be announced* (AMS-MAA Invited Address).

AMS Invited Addresses

Jesus De Loera, University of California Davis, *Title to be announced*.

Benedict Gross, Harvard University, *Title to be announced* (AMS Colloquium Lectures: Lecture I).

Benedict Gross, Harvard University, *Title to be announced* (AMS Colloquium Lectures: Lecture II).

Benedict Gross, Harvard University, *Title to be announced* (AMS Colloquium Lectures: Lecture III).

Peter Oszvath, Princeton University, *Title to be announced*.

Lior Pachter, University of California Berkeley, *Title to be announced*.

Karen Parshall, University of Virginia, *Title to be announced*.

Alan Perelson, Los Alamos National Laboratory, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

Lillian Pierce, Duke University, *Title to be announced*.

AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

25 years of Conferences for African-American Researchers in the Mathematical Sciences (CAARMS times 25) (Code: SS 82A), **William A. Massey**, Princeton University.

A Showcase of Number Theory at Undergraduate Institutions (Code: SS 76A), **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

Advances and Applications in Integral and Differential Equations (Code: SS 40A), **Jeffrey T. Neugebauer**, Eastern Kentucky University, and **Min Wang**, Rowan University.

Advances by Early Career Women in Discrete Mathematics (Code: SS 75A), **Jessalyn Bolkema**, State University of New York at Oswego, and **Jessica De Silva**, California State University, Stanislaus.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups (Code: SS 30A), **Joseph Ball**, Virginia Tech, **Marat Markin**, California State University, Fresno, **Igor Nikolaev**, St. John University, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

Advances in Quantum Walks, Quantum Simulations, and related Quantum Theory (Code: SS 18A), **Radhakrishnan Balu**, US Army Research Lab, **Chaobin Liu**, Bowie State University, and **Takuya Machida**, Nihon University, Japan.

Agent-based Modeling in Biological and Social Systems (a Mathematics Research Communities Session) (Code: SS 15A), .

Algebraic Structures Motivated by Knot Theory (Code: SS 51A), **Mikhail Khovanov**, Columbia University, and **Jozef H. Przytycki** and **Alexander Shumakovitch**, George Washington University.

Algebraic and Geometric Methods in Discrete Optimization (Code: SS 2A), **Amitabh Basu**, Johns Hopkins University, and **Jesus De Loera**, University of California, Davis.

Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology (Code: SS 43A), **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

Algorithmic Dimensions and Fractal Geometry (Code: SS 37A), **Jack H. Lutz**, Iowa State University, and **Elvira Mayordomo**, University of Zaragoza, Spain (AMS-ASL).

Analysis and Geometry of Nonlinear Evolution Equations (Code: SS 65A), **Marius Beceanu**, University at Albany, State University of New York, and **Dan-Andrei Geba**, University of Rochester.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems with Applications (Code: SS 19A), **John R. Graef**, University of Tennessee at Chattanooga, **G. S. Ladde**, University of South Florida, and **A. S. Vatsala**, University of Louisiana at Lafayette.

Analytic Number Theory (Code: SS 33A), **Thomas A. Hulse**, Morgan State University, **Angel V. Kumchev** and **Nathan McNew**, Towson University, and **John Miller**, The Johns Hopkins University.

Arithmetic Statistics (Code: SS 53A), **Michael Chou** and **Robert Lemke Oliver**, Tufts University, and **Ari Shnidman**, Center for Communications Research-Princeton.

Bifurcations of Difference Equations and Discrete Dynamical Systems with Applications (Code: SS 72A), **Arzu Bilgin**, Recep Tayyip Erdogan University, Turkey, and **Toufik Khyat**, Trinity College.

Commutative Ring Theory: Research for Undergraduate and Early Graduate Students (Code: SS 60A), **Nicholas**

Baeth, Franklin and Marshall College, and **Branden Stone**, Hamilton College.

Continued Fractions (Code: SS 35A), **Geremias Polanco Encarnacion**, Hampshire College, **James McLaughlin**, West Chester University, **Barry Smith**, Lebanon Valley College, and **Nancy J. Wyshinski**, Trinity College.

Counting Methods in Number Theory (Code: SS 6A), **Lillian Pierce**, Duke University, **Arindam Roy**, Rice University, and **Jiyua Wang**, University of Wisconsin.

Definability and Decidability Problems in Number Theory (Code: SS 24A), **Kirsten Eisentrager**, Pennsylvania State University, **Deidre Haskell**, McMaster University, Ontario, Canada, **Jennifer Park**, University of Michigan, and **Alexandra Shlapentokh**, East Carolina University (AMS-ASL).

Differential Equations on Fractals (Code: SS 87A), **Patricia Alonso-Ruiz**, University of Connecticut, **Joe Chen**, Colgate University, **Luke Rogers**, University of Connecticut, **Robert Strichartz**, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Enumerative Combinatorics (Code: SS 26A), **Miklos Bona**, University of Florida, and **Cheyne Homberger**, University of Maryland, Baltimore County.

Financial Mathematics (Code: SS 66A), **Maxim Bichuch**, Johns Hopkins University, **Anja Richter**, Baruch College, City University of New York, and **Stephan Sturm**, Worcester Polytechnic Institute.

Geometric and Topological Combinatorics (Code: SS 39A), **Anastasia Chavez** and **Jamie Haddock**, University of California, Davis, and **Annie Raymond**, University of Massachusetts, Amherst.

Geometric and Topological Generalization of Groups (Code: SS 74A), **Amrita Acharyya**, University of Toledo, and **Bikash C. Das**, University of North Georgia.

Geometry Labs United: Research, Visualization, and Outreach (Code: SS 57A), **Marianne Kortén**, Kansas State University, and **Sean Lawton** and **Anton Lukyanenko**, George Mason University.

Geometry and Dynamics of Continued Fractions (Code: SS 47A), **Anton Lukyanenko**, George Mason University, and **Joseph Vandehey**, Ohio State University.

Geometry of Representation Spaces (Code: SS 58A), **Sean Lawton**, George Mason University, **Chris Manon**, University of Kentucky, and **Daniel Ramras**, Indiana University-Purdue University Indianapolis.

Group Representation Theory and Character Theory (Code: SS 38A), **Mohammad Reza Darafsheh**, University of Tehran, Iran, and **Manouchehr Misaghian**, Prairie View A&M University.

Harmonic Analysis, Partial Differential Equations, and Applications (Code: SS 44A), **Russell Brown**, University of Kentucky, and **Irina Mitrea**, Temple University.

Harmonic Analysis: New Developments in Oscillatory Integrals (a Mathematics Research Communities Session) (Code: SS 12A), .

History of Mathematics (Code: SS 42A), **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Daniel E. Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College (AMS-MAA-ICHM).

Hopf Algebras and Tensor Categories (Code: SS 62A), **Siu-Hung Ng**, Louisiana State University, **Julia Plavnik**, Texas A&M University, and **Henry Tucker**, University of California, San Diego.

How to Guard an Art Gallery and Other Discrete Mathematical Adventures (In Memory of T. S. Michael, 1960 to 2016) (Code: SS 16A), **Joseph Bonin**, The George Washington University, **Carolyn Chun**, US Naval Academy, and **Nancy Neudauer**, Pacific University.

If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences (Code: SS 54A), **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

Latinx in Math (Code: SS 34A), **Alexander Diaz-Lopez**, Villanova University, **Laura Escobar**, University of Illinois, and **Juanita Pinzon-Caicedo**, North Carolina State University.

Lattice Path Combinatorics and Applications (Code: SS 68A), **Christian Krattenthaler**, University of Vienna, Austria, and **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Localization and Delocalization for Disordered Quantum Systems (Code: SS 83A), **Peter D. Hislop**, University of Kentucky, **Christoph A. Marx**, Oberlin College, and **Jeffery Schenker**, Michigan State University.

Low Complexity Models in Data Analysis and Machine Learning (Code: SS 55A), **Emily J. King**, University of Bremen, Germany, **Nate Strawn**, Georgetown University, and **Soledad Villar**, New York University.

Mappings on Metric and Banach Spaces with Applications to Fixed Point Theory (Code: SS 63A), **Torrey M. Gallagher**, Bucknell University, and **Christopher J. Lennard**, University of Pittsburgh.

Mathematical Analysis in Fluid Dynamics (Code: SS 31A), **Yanqiu Guo**, Florida International University, **Jinkai Li**, South China Normal University, **Jing Tian**, Towson University, and **Yuncheng You**, University of South Florida.

Mathematical Investigations of Spatial Ecology and Epidemiology (Code: SS 79A), **Leah Shaw** and **Junping Shi**, College of William and Mary, and **Zhisheng Shuai**, University of Central Florida.

Mathematical Models in Ecology, Epidemiology, and Medicine (Code: SS 85A), **Paula Grajdeanu**, Shenandoah University, and **Najat Ziyadi**, Morgan State University.

Mathematicians at Sea (in the Sky, or on Land): Defense Applications of Mathematics (Code: SS 21A), **Tegan Emerson**, **Timothy Doster**, and **George Stantchev**, Naval Research Laboratory.

Mathematics in the Realm of Cyber Research (Code: SS 22A), **Daniel Bennett**, Army Cyber Institute, **Paul Goethals**, United States Military Academy, and **Natalie Scala**, Towson University.

Mathematics of Coding Theory and Applications (Code: SS 78A), **Hiram Lopez-Valdez**, **Felice Manganiello**, and **Gretchen L. Matthews**, Clemson University.

Mathematics of Gravity and Light (a Mathematics Research Communities Session) (Code: SS 11A), .

Multiscale Problems in the Calculus of Variations (Code: SS 46A), **Elisa Davoli**, University of Vienna, Austria, and

MEETINGS & CONFERENCES

Rita Ferreira, King Abdullah University of Science and Technology, Saudi Arabia.

Natural Resources Modeling (Code: SS 56A), **Julie Blackwood**, Williams College, and **Shandelle M. Henson**, Andrews University.

Network Science (Code: SS 52A), **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng Bill Shi**, University of North Carolina.

New Directions in the Theory of Complex Multiplication (Code: SS 1A), **Henri Darmon**, McGill University, **Samit Dasgupta**, University of California, Santa Cruz, and **Benedict Gross**, Harvard University.

Nonlinear Evolution Equations and Their Applications (Code: SS 20A), **Mingchao Cai**, Morgan State University, **Gisele Mophou Loudjom**, University of French West Indies, Guadeloupe, France, and **Gaston NGuerekata**, **Alexander Pankov**, **Xuming Xie**, and **Guoping Zhang**, Morgan State University.

Not KNERds: A Community for Knot Theory (Code: SS 77A), **Moshe Cohen**, Vassar College, **Elizabeth Denne**, Washington and Lee University, and **Adam Lowrance**, Vassar College.

Number Theoretic Methods in Hyperbolic Geometry (a Mathematics Research Communities Session) (Code: SS 14A), **Samantha Fairchild**, University of Washington, **Junxian Li**, University of Illinois Urbana Champaign, and **Richard Vradenburgh**, University of Virginia.

Number Theory, Arithmetic Geometry, and Computation (Code: SS 61A), **Brendan Hassett**, Brown University, **Drew Sutherland**, Massachusetts Institute of Technology, and **John Voight**, Dartmouth College.

Numerical Methods for PDEs and Applications (Code: SS 41A), **Wenrui Hao**, **Qingguo Hong**, and **Jinchao Xu**, Pennsylvania State University.

Optimal Methods in Applicable Analysis: Variational Inequalities, Low Rank Matrix Approximations, Systems Engineering, Cyber Security (Code: SS 81A), **Aritra Dutta**, King Abdullah University of Science and Technology, Saudi Arabia, **Ram Mohapatra**, University of Central Florida, **Gayatri Pany**, Singapore University of Technology and Design, Singapore, and **Nabin Kumar Sahu**, Dhirubhai Ambani Institute of Information and Communication Technology, India.

Orthogonal Polynomials, Quantum Probability, Harmonic and Stochastic Analysis (Code: SS 27A), **Nobuhiro Asai**, Aichi University of Education, Kariya, Japan, **Rodica Costin**, The Ohio State University, **Aurel I. Stan**, The Ohio State University at Marion, and **Hiroaki Yoshida**, Ochanomizu University, Tokyo, Japan.

Partition Theory and Related Topics (Code: SS 80A), **Dennis Eichhorn**, University of California, Irvine, **Tim Huber**, University of Texas, Rio Grande Valley, and **Amita Malik**, Rutgers University.

Problems in Partial Differential Equations (Code: SS 36A), **Alex Himonas**, University of Notre Dame, and **Curtis Holliman**, The Catholic University of America.

Quantum Symmetries: Subfactors and Fusion Categories (a Mathematics Research Communities Session) (Code: SS 13A), **Cain Edie-Michell**, Australian National University,

Lauren Ruth, UC Riverside, and **Yilong Wang**, Ohio State University.

Quaternions (Code: SS 28A), **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

Recent Advancements in Mathematical Modeling of Cancer (Code: SS 49A), **Kamila Larripa**, Humboldt State University, and **Hwayeon Ryu**, University of Hartford.

Recent Advances and Trends in Computable Structure Theory (in honor of J. Remmel) (Code: SS 64A), **Jennifer Chubb**, University of San Francisco, and **Tim McNicholl**, Iowa State University.

Recent Advances in Biological Modeling and Related Dynamical Analysis (Code: SS 69A), **Joshi Raj Hem**, Xavier University, and **Yanyu Xiao**, University of Cincinnati.

Recent Advances in Homological and Commutative Algebra (Code: SS 70A), **Neil Epstein**, George Mason University, **Claudiu Raicu**, Notre Dame University, and **Alexandra Seceleanu**, University of Nebraska.

Recent Advances in Inverse Problems and Imaging (Code: SS 25A), **Kui Ren**, University of Texas at Austin, and **Yang Yang**, Michigan State University.

Recent Advances in Regularity Lemmas (Code: SS 71A), **Gabriel Conant**, University of Notre Dame, **Rehana Patel**, and **Julia Wolf**, University of Bristol, UK.

Recent Progress in Multivariable Operator Theory (Code: SS 86A), **Dmitry Kaliuzhnyi-Verbovetsky** and **Hugo Woerdeman**, Drexel University.

Research in Mathematics by Early Career Graduate Students (Code: SS 84A), **Marat Markin**, **Morgan Rodgers**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 32A), **Darren A. Narayan**, Rochester Institute of Technology, **Khang Tran**, California State University, Fresno, **Mark David Ward**, Purdue University, and **John Wierman**, The Johns Hopkins University (AMS-MAA-SIAM).

Riordan Arrays (Code: SS 50A), **Alexander Burstein** and **Dennis Davenport**, Howard University, **Asamoah Nkwanta**, Morgan State University, **Lou Shapiro**, Howard University, and **Leon Woodson**, Morgan State University.

Statistical, Variational, and Learning Techniques in Image Analysis and their Applications to Biomedical, Hyperspectral, and Other Imaging (Code: SS 45A), **Justin Marks**, Gonzaga University, **Laramie Paxton**, Washington State University, and **Viktorija Taroudaki**, Eastern Washington University.

Stochastic Analysis and Applications in Finance, Actuarial Science and Related Fields (Code: SS 17A), **Julius N. Esunge**, University of Mary Washington, **See Keong Lee**, University of the Sciences, Malaysia, and **Aurel I. Stan**, The Ohio State University at Marion.

Stochastic Differential Equations and Applications (Code: SS 59A), **Carey Caginalp**, University of Pittsburgh.

Symbolic Dynamics (Code: SS 9A), **Van Cyr**, Bucknell University, and **Bryna Kra**, Northwestern University.

The Mathematics of Historically Black Colleges and Universities (HBCUs) in the Mid-Atlantic (Code: SS 88A), **Edray Goins**, Purdue University, **Janis Oldham**, North Carolina A&T, **Talitha Washington**, Howard University, and **Scott Williams**, University at Buffalo, State University of New York.

Topological Data Analysis: Theory and Applications (Code: SS 73A), **Justin Curry**, University at Albany, State University of New York, **Mikael Vejdemo-Johansson**, College of Staten Island, City University of New York, and **Sara Kalisnik Verovsek**, Wesleyan University.

Topology, Structure and Symmetry in Graph Theory (Code: SS 48A), **Lowell Abrams**, George Washington University, and **Mark Ellingham**, Vanderbilt University.

Using Modeling to Motivate the Study of Differential Equations (Code: SS 29A), **Robert Kennedy**, Centennial High School, Ellicott City MD, **Audrey Malagon**, Virginia Wesleyan University, **Brian Winkel**, SIMIODE, Cornwall NY, and **Dina Yagodich**, Frederick Community College.

Women in Topology (Code: SS 67A), **Jocelyn Bell**, Hobart and William Smith Colleges, **Rosemary Guzman**, University of Chicago, **Candice Price**, University of San Diego, and **Arunima Ray**, Max Planck Institute for Mathematics, Germany.

Auburn, Alabama

Auburn University

March 15–17, 2019

Friday – Sunday

Meeting #1146

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: January 31, 2019

Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: August 15, 2018

For abstracts: January 22, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Grigoriy Blekherman, Georgia Institute of Technology, *To be announced.*

Carina Curto, Pennsylvania State University, *To be announced.*

Ming Liao, Auburn University, *To be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the ab-

stract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Combinatorial Matrix Theory (Code: SS 2A), **Zhongshan Li**, Georgia State University, and **Xavier Martínez-Rivera**, Auburn University.

Commutative and Combinatorial Algebra (Code: SS 3A), **Selvi Kara Beyarslan**, University of South Alabama, and **Alessandra Costantini**, Purdue University.

Developments in Commutative Algebra (Code: SS 1A), **Eloísa Grifo**, University of Michigan, and **Patricia Klein**, University of Kentucky.

Graph Theory in Honor of Robert E. Jamison's 70th Birthday (Code: SS 4A), **Robert A Beller**, East Tennessee State University, and **Gretchen Matthews** and **Beth Novick**, Clemson University.

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019

Friday – Sunday

Meeting #1147

Central Section

Associate secretaries: Georgia Benkart and Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: February 7, 2019

Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired

For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Barry Mazur, Harvard University, *On the Arithmetic of Curves* (Einstein Public Lecture in Mathematics).

Aaron Naber, Northwestern University, *Analysis of Geometric Nonlinear Partial Differential Equations.*

Deanna Needell, University of California, Los Angeles, *Simple approaches to complicated data analysis.*

Katherine Stange, University of Colorado, Boulder, *Title to be announced.*

Andrew Suk, University of California, San Diego, *Title to be announced.*

Hartford, Connecticut

University of Connecticut Hartford
(Hartford Regional Campus)

April 13–14, 2019

Saturday – Sunday

Meeting #1148

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: February 21, 2019

Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: September 13, 2018

For abstracts: February 5, 2019

*The scientific information listed below may be dated.
For the latest information, see www.ams.org/amsmtgs/sectional.html.*

Invited Addresses

Olivier Bernardi, Brandeis University, *Title to be announced.*

Brian Hall, Notre Dame University, *Title to be announced.*

Christina Sormani, City University of New York, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis, Geometry, and PDEs in Non-smooth Metric Spaces (Code: SS 1A), **Vyron Vellis**, University of Connecticut, **Xiaodan Zhou**, Worcester Polytechnic Institute, and **Scott Zimmerman**, University of Connecticut.

Computability Theory (Code: SS 2A), **Damir Dzhafarov** and **Reed Solomon**, University of Connecticut, and **Linda Brown Westrick**, Pennsylvania State University.

Modeling and Qualitative Study of PDEs from Materials Science and Geometry. (Code: SS 6A), **Yung-Sze Choi**, **Changfeng Gui**, and **Xiaodong Yan**, University of Connecticut.

Recent Development of Geometric Analysis and Nonlinear PDEs (Code: SS 3A), **Ovidiu Munteanu**, **Lihan Wang**, and **Ling Xiao**, University of Connecticut.

Special Session on Regularity Theory of PDEs and Calculus of Variations on Domains with Rough Boundaries (Code: SS 5A), **Murat Akman**, University of Connecticut, and **Zihui Zhao**, University of Washington.

Stochastic Processes, Random Walks, and Heat Kernels (Code: SS 4A), **Patricia Alonso Ruiz**, University of Connecticut, and **Phanuel Mariano**, Purdue University.

Quy Nhon City, Vietnam

Quy Nhon University

June 10–13, 2019

Monday – Thursday

Meeting #1149

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 30, 2018

For abstracts: To be announced

*The scientific information listed below may be dated.
For the latest information, see www.ams.org/amsmtgs/internmtgs.html.*

Invited Addresses

Henry Cohn, Microsoft Research, *To be announced.*

Robert Guralnick, University of Southern California, *To be announced.*

Le Tuan Hoa, Hanoi Institute of Mathematics, *To be announced.*

Nguyen Dong Yen, Hanoi Institute of Mathematics, *To be announced.*

Zhiwei Yun, Massachusetts Institute of Technology, *To be announced.*

Nguyen Tien Zung, Toulouse Mathematics Institute, *To be announced.*

Madison, Wisconsin

University of Wisconsin-Madison

September 14–15, 2019

Saturday – Sunday

Meeting #1150

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced

Program first available on AMS website: July 23, 2019

Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: February 14, 2019

For abstracts: July 16, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Nathan Dunfield, University of Illinois, Urbana-Champaign, *Title to be announced.*

Teena Gerhardt, Michigan State University, *Title to be announced.*

Lauren Williams, University of California, Berkeley, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Homological and Characteristic $p > 0$ Methods in Commutative Algebra (Code: SS 1A), **Michael Brown**, University of Wisconsin-Madison, and **Eric Canton**, University of Michigan.

Binghamton, New York

Binghamton University

October 12–13, 2019

Saturday – Sunday

Meeting #1151

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: August 29, 2019

Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: March 12, 2019

For abstracts: August 20, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Richard Kenyon, Brown University, *Title to be announced.*

Tony Pantev, University of Pennsylvania, *Title to be announced.*

Lai-Sang Young, New York University, *Title to be announced.*

Gainesville, Florida

University of Florida

November 2–3, 2019

Saturday – Sunday

Meeting #1152

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: September 19, 2019

Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 2, 2019

For abstracts: September 10, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jonathan Mattingly, Duke University, *To be announced.*

Isabella Novik, University of Washington, *To be announced.*

Eduardo Teixeira, University of Central Florida, *To be announced.*

Riverside, California

University of California, Riverside

November 9–10, 2019

Saturday – Sunday

Meeting #1153

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: September 12, 2019

Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 9, 2019

For abstracts: September 3, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Robert Boltje, University of California, Santa Cruz, *Title to be announced.*

Jonathan Novak, University of California, San Diego,
Title to be announced.

Anna Skripka, University of New Mexico, Albuquerque,
Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Topics in Operator Theory (Code: SS 1A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Denver, Colorado

Colorado Convention Center

January 15–18, 2020

Wednesday – Saturday

Meeting #1154

Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2019

Program first available on AMS website: November 1, 2019

Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2019

For abstracts: To be announced

Charlottesville, Virginia

University of Virginia

March 13–15, 2020

Friday – Sunday

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Fresno, California

California State University, Fresno

May 2–3, 2020

Saturday – Sunday

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021

Wednesday – Saturday

Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: October 2020

Program first available on AMS website: November 1, 2020

Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2020

For abstracts: To be announced

Grenoble, France

Université Grenoble Alpes

July 5–9, 2021

Monday – Friday

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Buenos Aires, Argentina

The University of Buenos Aires

July 19–23, 2021

Monday – Friday

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Seattle, Washington

*Washington State Convention Center and
the Sheraton Seattle Hotel*

January 5–8, 2022

Wednesday – Saturday

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: October 2021

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

Boston, Massachusetts

*John B. Hynes Veterans Memorial Con-
vention Center, Boston Marriott Hotel, and
Boston Sheraton Hotel*

January 4–7, 2023

Wednesday – Saturday

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2022

Program first available on AMS website: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

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THE BACK PAGE

In an article titled “Adventures in Fine Hall” (the Princeton math department) in the *Princeton Alumni Weekly* (January 10, 2018, paw.princeton.edu), Elyse Graham recounts how a woman “strolled up to Einstein, ... ruffled his hair, ... and said, ‘I have always wanted to do that.’” Another time, when he tried to learn ping pong, the ball ended up in his hair.

James Alexander left his second story office window ajar so that he could climb in if the building was locked.

John von Neumann finished papers at 3 am and then immediately phoned colleagues.

Solomon Lefschetz, famous for his comments in lectures, inspired the verse: “When he’s at last beneath the sod, he’ll then begin to heckle God.”

Kurt Gödel delivered an entire lecture facing the blackboard, which he didn’t even write on.

ALGEBRA License Plate



Submitted by FH family.

Send in your favorite image of a math-related license plate. If *Notices* uses it, we'll send you a prize.

Measuring Tree Height

At the University of Papua New Guinea in 1974, I asked my first-year class to determine the height of the tall gum tree outside. Most groups measured shadows and angles using homemade protractors. One group climbed up in the tree as high as they could and extended a stick with a large rope attached (see figure). They were awarded full marks.

Submitted by Deane Arganbright



What crazy things happen to you? Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: noti-backpage@ams.org.

IN THE NEXT ISSUE OF NOTICES



SEPTEMBER 2018

Hispanic Heritage Month (September 15–October 15)

is a national observance that recognizes the contributions made by an estimated 57 million Latinos and Hispanics in the US. The *Notices* September issue will highlight selected new mathematical contributions by Latinos and Hispanics.

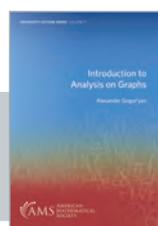
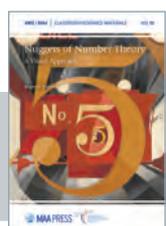
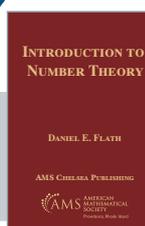


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RECENT RELEASES

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-  = Textbook
-  = Applied Mathematics
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Introduction to Number Theory



Daniel E. Flath

Suitable for use as a textbook in a course or self-study by advanced undergraduates or graduate students who possess a basic familiarity with abstract algebra, this text focuses on Gauss's theory of binary quadratic forms.

AMS Chelsea Publishing, Volume 384; 2018; 212 pages; Hardcover; ISBN: 978-1-4704-4694-9; List US\$53; AMS members US\$42.40; MAA members US\$47.70; Order code CHEL/384.H

Nuggets of Number Theory



A Visual Approach

Roger B. Nelsen, *Lewis & Clark College, Portland, OR*

Containing hundreds of visual explanations of results from elementary number theory, this volume will attract fans of visual thinking, number theory, and surprising connections.

Classroom Resource Materials, Volume 55; 2018; 153 pages; Softcover; ISBN: 978-1-4704-4398-6; List US\$45; Individual member US\$33.75; MAA members US\$33.75; Order code CLRM/55

Divisors and Sandpiles



An Introduction to Chip-Firing

Scott Corry, *Lawrence University, Appleton, WI*, and David Perkinson, *Reed College, Portland, OR*

Divisors and Sandpiles provides an introduction to the combinatorial theory of chip-firing on finite graphs.

2018; 329 pages; Softcover; ISBN: 978-1-4704-4218-7; List US\$79; AMS members US\$63.20; MAA members US\$71.10; Order code MBK/114

Extremal Problems for Finite Sets



Peter Frankl, *Rényi Institute, Budapest, Hungary*, and Norihide Tokushige, *Ryukyu University, Okinawa, Japan*

Written by two of the leading researchers in external set theory, this book is aimed at mathematically mature undergraduates and highlights the elegance and power of this field of study.

Student Mathematical Library, Volume 86; 2018; 224 pages; Softcover; ISBN: 978-1-4704-4039-8; List US\$52; All individuals US\$41.60; Order code STML/86

Introduction to

Analysis on Graphs



Alexander Grigor'yan, *University of Bielefeld, Germany*

Suitable for beginners in analysis on graphs and accessible to undergraduate and graduate students with a background in linear algebra I and analysis I, this book will help the reader to reach a level of understanding sufficient to start pursuing research in this exciting area.

University Lecture Series, Volume 71; 2018; 168 pages; Softcover; ISBN: 978-1-4704-4397-9; List US\$49; AMS members US\$39.20; MAA members US\$44.10; Order code ULECT/71

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