2018 Fields Medals

The Fields Medal is awarded to recognize outstanding mathematical achievement for existing work and for the promise of future achievement. On August 1, 2018, the 2018 Fields Medals were awarded at the opening ceremony of the International Congress of Mathematicians (ICM) in Rio de Janeiro, Brazil. Following are the official prize citations issued by the International Mathematical Union. A future issue of the *Notices* will highlight the recipients of other awards given at ICM.

The Work of Caucher Birkar

Caucher Birkar

Caucher Birkar has made fundamental contributions to birational geometry in two particular areas: the *minimal model program* (MMP) and the boundedness of Fano varieties. The original MMP involves two kinds of projective varieties Y with so-called terminal singularities whose canonical divisors K have opposite properties: for a *minimal model* K is non-negative on curves on Y; while for a Fano fibering Y has a surjective morphism onto a lower dimensional projective variety with $-K$ relatively ample. The MMP attempts to construct for each smooth projective variety a birational map to either a minimal model or a Fano fibering.

Although the MMP is not always known to work, Birkar, jointly with Cascini, Hacon, and McKernan, made a stunning contribution: a special version of the MMP works for complex varieties of arbitrary dimension whose canonical divisor is either big or not pseudo effective, a situation which covers many important cases. They actually established the MMP for a wider class of singularities, which was essential for the induction on dimension in the proof, and it implies many important consequences, such as the finite generation of canonical rings of arbitrary smooth projective varieties. The MMP is now a fundamental tool which is used extensively.

It was Birkar who further proved that complex Fano varieties (i.e., Fano fiberings over a point) of arbitrary fixed dimension with terminal singularities are parametrized by a (possibly reducible) algebraic variety. Since these Fano varieties constitute one of the main outputs of MMP as applied to smooth projective varieties, their boundedness, previously considered unreachable, is fundamentally important. Birkar has settled the more general Borisov-Alexeev-Borisov conjecture building upon results by Hacon, McKernan, Xu, and others. Birkar's boundedness will be crucial as a paradigm for the full MMP.

Biographical Sketch

Caucher Birkar's dedication to the winding and multidimensional world of algebraic geometry, with its ellipses, lemniscates, Cassini ovals, among so many other forms defined by equations, granted him the Philip Leverhulme Prize in 2010 for exceptional scholars whose greatest achievement is yet to come. Eight years later, the Cambridge University researcher joins the select group of Fields Medal winners at the age of forty. Birkar, who was recognized earlier this year with the Whitehead Prize of the London Mathematical Society, was born in 1978 in Marivan, a Kurdish province in Iran bordering Iraq with about 200,000 inhabitants. His curiosity was awakened by algebraic geometry, an area that had attracted the attention of Omar Khayyam (1048–1131) and Sharaf al-Din al-Tusi (1135–1213) in previous centuries.

After graduating in mathematics from Tehran University, Birkar sought refugee status in the United Kingdom, where he became a British citizen. In 2004, he completed his PhD at the University of Nottingham with the thesis "Topics in Modern Algebraic Geometry." Throughout his trajectory, birational geometry has stood out as his main area of interest. He has devoted himself to the fundamental aspects of key problems in modern mathematics—such as minimal models, Fano varieties, and singularities. His theories have solved long-standing conjectures. In 2010 Birkar wrote (alongside Paolo Cascini, Imperial College London, Christopher Hacon, University of Utah, and James McKernan, University of California, San Diego) an article called "Existence of Minimal Models for Varieties of General Log Type" that revolutionized the field. The article earned the quartet the AMS Moore Prize in 2016.

The Work of Alessio Figalli

Alessio Figalli

ALESSIO FIGALLI has made multiple fundamental advances in the theory of optimal transport, while also applying this theory in novel ways to other areas of mathematics. Only a few of his numerous results in these areas are described here.

Figalli's joint work with De Philippis on regularity for the Monge-Ampère equation is a groundbreaking result filling the gap between gradient estimates discovered by Caffarelli and full Sobolev regularity of the second derivatives of the convex solution of the Monge-Ampère equation with merely bounded right-hand side. The result is almost optimal in view of existing counterexamples. It has direct implications on regularity of the optimal transport maps and on regularity to semigeostrophic equations.

Figalli initiated the study of the singular set of optimal transport maps and obtained the first definite results in this direction: he showed that it has null Lebesgue measure in full generality. He has also given significant contributions to the theory of obstacles problems, introducing new methods to analyze the structure of the free boundary.

Figalli and his coauthors have also applied optimal transport methods in a striking fashion to obtain sharp quantitative stability results for several fundamental geometric inequalities, such as the isoperimetric and Brunn-Minkowski inequalities, without any additional assumptions of regularity on the objects to which these inequalities are applied; the methods are also not reliant on Euclidean symmetries, extending in particular to the Wulff inequality to yield a quantitative description of the low-energy states of crystals.

Biographical Sketch

Born in Naples, Italy, on April 2, 1984, Alessio Figalli discovered an interest in science later than some. Until high school, he was more interested in playing football. Preparation for the International Mathematical Olympiad (IMO) awakened his interest in the subject, and he chose to study math when he joined the Scuola Normale Superiore di Pisa. Figalli completed his PhD in 2007 at the École Normale Supérieure de Lyon in France, with the guidance of Fields Medal laureate Cédric Villani. He has worked at the French National Center for Scientific Research, École Polytechnique, the University of Texas, and ETH Zürich. A specialist in calculating variations and partial differential equations, he was invited to speak at the 2014 ICM in Seoul. He has won several awards, including the Peccot-Vimont Prize (2011) and Cours Peccot (2012) of the College of France, the European Mathematical Society Prize (2012), the Stampacchia Gold Medal of the Italian Mathematical Union (2015), and the Feltrinelli Prize (2017).

The Work of Peter Scholze

Peter Scholze

PETER SCHOLZE has transformed arithmetic algebraic geometry over p-adic fields.

Scholze's theory of perfectoid spaces has profoundly altered the subject of p-adic geometry by relating it to geometry in characteristic p. Making use of this theory, Peter Scholze proved Deligne's weight-monodromy conjecture for complete intersections. As a further application, he constructed Galois representations that are attached to torsion cohomology classes of locally symmetric spaces, resolving a long-standing conjecture.

Scholze's version of p-adic Hodge theory extends to general p-adic rigid spaces. Together with Bhatt and Morrow, Scholze developed an integral version of p-adic Hodge theory that establishes a relation between the torsion in Betti and crystalline cohomologies.

On the way to the revolution that he launched in arithmetic geometry, Scholze took up a variety of topics that he reshaped, such as algebraic topology and topological Hochschild homology.

Scholze developed new cohomological methods. Beyond p-adic fields, Scholze's vision of a cohomology theory over the integers has become a guideline that fascinates the entire mathematical community.

Biographical Sketch

Peter Scholze was born in Dresden, Germany, on December 11, 1987. At thirty years old, he is already considered by the scientific community as one of the most influential mathematicians in the world. In 2012, at age twenty-four, he became a full professor at the University of Bonn. Scholze impresses his colleagues with the intellectual ability he has shown since was a teenager, when he won four medals—three gold and one silver—at the International Mathematical Olympiad (IMO). The German mathematician completed his university graduate and master's in record time—five semesters—and gained notoriety at the age of twenty-two, when he simplified a complex mathematical proof of number theory from 288 to 37 pages. A specialist in arithmetic algebraic geometry, he stands out for his ability to understand the nature of mathematical phenomena and to simplify them during presentations.

At age sixteen, still a student at the Heinrich-Hertz-Gymnasium—a school with a strong scientific focus—Scholze decided to study Andrew Wiles's solution to Fermat's Last Theorem. Faced with the complexity of the result, he realized that he was on the right track in choosing mathematics as a profession. He was a guest speaker at ICM 2014 in Seoul, South Korea, and was a plenary member this year at ICM 2018. Scholze is well decorated for his contributions to arithmetic algebraic geometry, and he has collected several major mathematics awards, such as the Prix and Cours Peccot of the College de France (2012), the SASTRA Ramanujan Prize (2013), a Clay Research Award

(2014), the Frank Nelson Cole Prize of the AMS (2015), the Fermat Prize (2015), the Ostrowski Prize (2015), the European Mathematical Prize (2016), and the Leibniz Prize (2016). In 2018 he was appointed a director of the Max Planck Institute for Mathematics in Bonn.

The Work of Akshay Venkatesh

Akshay Venkatesh

Akshay Venkatesh has made profound contributions to an exceptionally broad range of subjects in mathematics, including number theory, homogeneous dynamics, representation theory, and arithmetic geometry. He solved many long-standing problems by combining methods from seemingly unrelated areas, presented novel viewpoints on classical problems, and produced strikingly far-reaching conjectures.

What follows is a small sample of his major achievements.

Venkatesh introduced a general and unifying technique based on representation theory and homogeneous dynamics in the subconvexity problem for L-functions and (partly in collaboration with Michel) used these ideas to give a complete treatment of all cases of subconvexity for $GL(2)$ over number fields.

He made major progress on the local-global principle for the representations of one quadratic lattice by another, in joint work with Ellenberg.

In joint work with Einsiedler, Lindenstrauss, and Michel, Venkatesh proved equidistribution of the periodic torus orbits in $SL(3, Z)\backslash SL(3, R)$ that are attached to the ideal classes of totally real cubic number fields as the discriminant tends to infinity.

Venkatesh established effective equidistribution of periodic orbits of many semisimple groups both in the local and adelic settings, in joint work with Einsiedler, Margulis, and in part with Mohammadi.

With Ellenberg and Westerland, Venkatesh established significant special cases of the Cohen–Lenstra conjectures concerning class groups in the function field setting.

Biographical Sketch

Conquering the greatest honor among the world's mathematicians before the age of forty is a notable accomplishment, but the life of Akshay Venkatesh was already marked with precocious feats. Born in New Delhi in 1981 and raised in Australia, Venkatesh became a medalist at the International Mathematical Olympiad when he was twelve years old. This propelled him into the world of mathematics and kickstarted his illustrious career. He began his bachelor's degree in mathematics and physics at the University of Western Australia at the age of thirteen, and seven years later, age twenty, had completed his PhD at Princeton University. Venkatesh was appointed [as a C.L.E. Moore instructor] at the Massachusetts Institute of Technology (MIT), a prestigious position offered to recent graduates in the area of pure mathematics, previously occupied by prominent figures such as John Nash. Upon leaving MIT in 2004, he became a Clay Research Fellow and was appointed associate professor at the Courant Institute of Mathematical Sciences at New York University. He became a professor at Stanford University at the age of twenty-seven and, as of this year, is a faculty member at the Institute for Advanced Study (IAS).

Venkatesh researches number theory—an area that deals with abstract issues and had no known application until the arrival of cryptography in the late 1970s—and he roves with ease through related topics, such as theory of representation, ergodic theory, and automorphic forms. Armed with a meticulous, investigative, and creative approach to research, detecting impressive connections between diverse areas, Venkatesh's contributions are fundamental to several fields of research in mathematics. He has won several accolades for his work, including the Salem Prize (2007), the SASTRA Ramanujan Prize (2008), the Infosys Prize (2016), and the Ostrowski Prize (2017). Previously a guest speaker at the 2010 ICM, Venkatesh was invited to speak at ICM 2018.

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