Making Dance, Making Math: Parallels

Sophia D. Merow

Mathematicians have no monopoly on problem solving. Anesthesiologists do it, as do sound designers and air traffic controllers. Dancers, too. In a piece called *necessary and sufficient: a dance of mathematics* performed in Seattle last spring, math educator and dance improviser Katherine Cook explored the parallels between how humans puzzle their way out of tight spots, be it in math or in dance improvisation.

Cook has intimate, expert knowledge of both disciplines. She holds a masters degree in mathematics from the University of Washington and is creative director of Seattle-based Math For Love, an organization committed to transforming how math is taught and learned worldwide. She develops curricula and games, writes about math and math education. When not helping learners of all ages develop a meaningful relationship to math, Cook teaches and performs across the country as a dance improviser and conducts dance improvisation research with collaborators at the Institute for the Study of Somatic Communication and the Seattle CI Lab. (CI stands for Contact Improvisation, a dance form nurtured in the post-modern experiments of the 1970s.)

Sophia D. Merow is special projects editor and Notices *assistant. Her email address is* `merow.notices@gmail.com`.

DOI: https://dx.doi.org/10.1090/noti1802

Katherine Cook conceived of and performed in *necessary and sufficient: a dance of mathematics.*

Dancers think through their bodies, says Cook—and mathematicians do, too.

Part of Cook's motivation for staging *necessary and sufficient* was to give the audience a glimpse of the sparsely populated niche she occupies.

"I feel like I live all of the time in the overlap between dance and mathematics," she explains, "and I really sort of selfishly just wanted to invite other people in."

Mathematically inclined viewers stand to gain, Cook believes, from recognizing and mulling over the commonalities between dance improvisation and the core work of mathematics.

"I think understanding how one field solves its creative problems is of great use to another field," she says. Someone who spends lots of time stuck on math problems might be curious, then, how someone facing a dance problem intuits her way to a path forward.

If your only experience of dance is stumbling through waltz basics or bopping to the beat (more or less) in a club, the idea of a "dance problem" may mystify you. It's doesn't refer—in Cook's dance improvisation context, anyway—to having two left feet or no sense of rhythm.

A dance improvisation performance is created in real time, with the choreography or *score* of the piece not dictating the dancers' movements but instead directing their attention, guiding their thought. In one segment of *necessary and sufficient*, for example, the score stipulates that the four dancers "not not mimic"—no, that double negative isn't a typo—one another. It's up to them to figure out what that means and how to do it, and their actions naturally differ from one performance to the next.

"The score is like a set of axioms," wrote Cook in the *necessary and sufficient* program, "giving a shape, a skeleton, which the dancers fill out with creations and discoveries."

A dance improvisation problem is like a mathematical one, then, in that it involves determining what is possible under a given set of constraints. A dancer is constrained by the score's instructions and the performance space, by her body and the physics of our world.

"Exploring the intuitive and creative act of finding our way through systems that begin with constraints is what we do in both mathematics and in dance [improvisation]," Cook says. And dancers and mathematicians alike can be surprised by what a particular set of constraints can produce: a beguilingly dynamic interaction between a quartet of performers, for instance, or hyperbolic geometry.

Some math-inspired works of dance respond to surface features. They may meet math at a purely notational level by, for example, translating the symbols delineating a sheet of calculus problems into a sequence of body configurations. In conceiving *necessary and sufficient*, Cook wanted to delve deeper, to bypass the mathematical shallows, avoid getting

bogged down in the discipline's manifold technicalities (which she personally appreciates, but knows can be rough going for a lay audience), and instead access "some really meaty ideas in the schema of mathematics." She settled on *equivalence and morphism*.

Cook's notion of *morphism* derives from category theory's generalization of the structure-preserving maps found throughout mathematics: functions between sets; homomorphisms between groups, rings, or fields; linear transformations between vector spaces; and so on. In the *necessary and sufficient program*, Cook called morphisms "the mathematician's metaphors." "They are arrows connecting disparate objects and structures," she wrote, "highlighting similarities that might be missed elsewhere." In category theory, objects are completely defined by the morphisms between them, making the morphisms, in a way, the stars of the show. In *necessary and sufficient*, dancers play the role of morphisms.

"Everything the dancer does [in the piece] is a morphism," Cook explains, "a transformation from perceived phenomena to generated phenomena. The dancers are trying to preserve structure in these transformations and the result is an exploration of equivalence and sameness."

Through a category theoretic lens, equivalences can be considered morphisms with special properties, but Cook found it impracticable to convey this in her chosen medium. Though she toyed with trying to incorporate into the piece the conditions for an equivalence relation, she couldn't reconcile the time independence of mathematics with the ephemerality of dance. While a mathematical structure persists stably as long as one cares to study it, a dance structure dissipates as soon as it arises. Try establishing even something as straightforward as reflexivity under such conditions! Cook ended up "playing a little fast and loose with the language of *equivalence*" and interpreted the word largely in its colloquial sense.

A site-specific work, *necessary and sufficient* played out in an art gallery—first Seattle's Center on Contemporary Art, then Schack Art Center in Everett, WA—during the run of the exhibit Art ∩ Math, curated by Cook and Math For Love founder Dan Finkel. The piece opens with the four performers filing into the gallery and standing abreast before one of the show's installations. Here the exploration of morphism begins, as each dancer responds bodily to the work in view, becomes a locus of transformation from one

necessary and sufficient asked dancers (left to right: Brandin Steffensen, Cook, Corrie Befort, Aaron Swartzman) and onlookers alike to ponder flexible notions of sameness.

In parts of the piece, Cook and her fellow performers respond to the visual art on display, become morphisms taking art as input and yielding movement as output.

kind of stimulus to another, from a piece of visual art to a spontaneously generated movement phrase.

"The dancers are working like functions, essentially, from the set of visual stimuli to kind of movement and sound phenomena," says Cook.

Soon the dancers-as-morphisms are chaining themselves together into compositions (as do category theory's morphisms, according to composition laws). One dancer looks at a sculpture, painting, or print, and transforms it into movement; another dancer observes that movement and transforms it into another movement.

Responding to one another rather than to the art is what the performers do in the aforementioned not-not-mimic segment of the piece, which probes notions of equivalence. Cook shied away from writing a straight-up mimic instruction into the score because she is interested in how two things can be deemed the same when in some way they clearly are not. While there are obvious visual similarities between what the performers are doing at the outset of the not-not-mimic portion of the program, it invariably morphs into a deeper exploration of a more sophisticated, nuanced sense of sameness. Akin, perhaps, to the one that equates coffee cups and doughnuts.

Cook sees elegance in the view that subfields of mathematics are defined by their definitions of equivalence and suggests that there's a sense in which much of math comes down to determining when things are the same. "It's almost like different domains are hard at work sorting the objects in their universe according to those ideas of equivalence," she says. The not-not-mimic directive burdens the dancers with trying to find natures of equivalence; it asks the performers to do in their discipline what mathematicians do so tirelessly in theirs.

Press coverage of *necessary and sufficient* (see `https://bit.ly/2y35wsy`) played on the work's challenge to the perception of math and dance as enterprises without overlap. For Cook, though, the commonality between her two passions is not incidental, but fundamental: they derive from the same source.

necessary and sufficient is a celebration of both math and dance, "and of their shared provenance which, after all, is centered in humanity," Cook wrote in the program. "Our bodies are entangled with our thoughts, and both dance and mathematics spring from this entanglement."

Dancers think through their bodies, Cook observes, and any mathematician who feels she doesn't should remember the location of her brain. "All of our thinking happens in the body that we have," Cook says. "That's just the nature of being a person." Cook believes that embracing and interrogating thought as an embodied phenomenon could enrich and inform humankind's mathematical endeavors.[1]

In one regard, of course, mathematics benefits from a lack of physicality. Mathematics can only maintain its "obscene amount of precision and rigor," Cook concedes, because "the world of [pure] mathematics is imagined, so we don't have to deal with all the messy reality that bodies or anything else suggests."

But if the mathematically inclined could relax their strict standards temporarily, could for a period dispense with the restrictive exactitude of the field, perhaps they could come to appreciate a deeper union between mathematics and other human strivings.

Cook suspects that mathematicians may be loath to let go, even briefly, of the technical machinery of mathematics, even if doing so could afford them insight into what lies beneath. "But I think it's a grave mistake not to do that from time to time," she says, "because there's so much there."

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[1]*Pressed to elaborate, Cook notes that children formalize mathematical understanding more successfully when they use their bodies, when they manipulate blocks, count on their fingers, move in space. She also references a 2014 study (see* `https://bit.ly/2D4k8Nm`*) that found that moving in accordance with a math problem—going up (e.g., in an elevator) or walking rightward when adding, for instance—can facilitate its solution. Watching dancers problem-solve their way through a score can help observers recognize and appreciate the role of the body in cognition.*

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