Three Countries, Two Continents: Common Challenges and Opportunities in Teaching First-Year Mathematics

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Every year, for tens of thousands of first-year university students across Canada, taking a mathematics course is one of their initial university academic experiences. Serving as both a gatekeeper and a requirement for programs ranging from economics to science to engineering, first-year mathematics courses come with numerous challenges for students, instructors, and mathematics departments.

Present-day first-year students share many characteristics with incoming students from a generation ago: a good number of them lack the necessary mathematical knowledge and skills that they need to succeed and thus find themselves placed into various remedial environments; they sit in large classes and are exposed to mathematical pedagogies that no longer work; they do not see the value of what they are supposed to learn in their mathematics classes, do not seem to be engaged, and often do not attend lectures.

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However, the two generations do differ in a significant way. Psychologist and college instructor Linda Bips [Bip10] wrote: "Many of today's students lack resilience and at the first sign of difficulty are unable to summon strategies to cope. The hardship can be a failing grade on a test, a cut from the team, or a romantic breakup. At the first sign of trouble many become unable to function and persevere. Often they even anticipate difficulties and their anxiety alone paralyzes them."

For a large number of incoming university students, taking a fast-paced, topic-packed first-year mathematics course presents a significant obstacle, and many of these students cannot successfully meet the expectations and standards of such courses and their instructors. This puts instructors teaching these courses in the position of having to reconcile two seemingly contradictory tasks. On the one hand, they have a responsibility to teach their course at the appropriate level of rigour and to be academically demanding. On the other hand, they play an important role in welcoming, encouraging, and supporting a heterogeneous group of students during one of the major transitions in their lives.

In addition, course instructors are expected to keep up with teaching and learning technologies: from handling ever-evolving learning management systems to creating and/or managing electronic learning resources, to using in-class technology for delivering or enhancing their lectures, to facing competition from a variety of freely available online resources.

To stay relevant, mathematics departments are forced to reassess the ways of delivery and the content of their courses and academic programs. This includes introducing new pedagogies to meet the needs of the increasingly diverse (in every sense of the word) classes of incoming students and modifying existing courses and/or creating new ones that address the needs of their programs and the programs that they are expected to "serve."

Ample evidence suggests that university mathematical teaching communities in Australia and the United States are facing similar challenges and opportunities as their Canadian colleagues.

In the last few years, we, the authors of this article, have organized and participated in a number of events that critically examined teaching and learning of mathematics and statistics at a national level in Canada. To share and inform others about our conversations, discussions, and decisions, we reached beyond our Canadian community of first-year instructors. The result is this article, whose purpose is to briefly summarize several recent initiatives in Australia, the United States, and Canada that have the common goal of constructively addressing aspects of the current situation and the future status of teaching first-year university mathematics courses. (We wish to emphasize that this is in no way a comprehensive overview but rather a limited selection of such initiatives.)

In Australia, between 2012 and 2014, Deborah King from the University of Melbourne led the First Year in Maths (FYiMaths) project. This project, funded by the Australian Government Office for Learning and Teaching, aimed to promote and support strategic changes and improvements in teaching first-year mathematics courses and enhancing the overall student-learning experience. The issues addressed by the project team ranged from discussing the challenges that students face during their transition from high school to university to arguing for a well-defined and supportive departmental teaching stream faculty position that would coordinate first-year mathematics courses.

One of the outcomes of the FYiMaths project was the establishment of the FYiMaths Network, "a 'community of practice' providing an informal and supportive group for academics and educators to access information, exchange ideas, address shared challenges and collaborate on research and policy questions" (`https://fyimaths.org.au`). Currently, the network includes academics and educators teaching undergraduate mathematics, individuals that provide additional mathematics support, and secondary mathematics teachers in Australia and New Zealand.

The FYiMaths project final report [Kin15] is quite revealing of the current state of first-year mathematics teaching in Australia. The report provided a list of challenges that Australian university instructors face when teaching first-year mathematics courses, which mirrors the challenges of their North American colleagues. These teaching challenges are further validated in a recent article in the *Notices of the AMS* [Buc19] in which the author summarizes the "most significant results, events, and developments in undergraduate mathematics education of the last decade" (p. 46).

In the United States, the Mathematical Association of America (MAA) coordinated two large studies of introductory mathematics courses, centered around calculus [Bre16, MAA10]; see also [Bre15]. In 2016, the conference Precalculus to Calculus: Insights and Innovations was held in St. Paul, Minnesota. The conference "shared insights gained from the MAA studies of calculus instruction, highlighted some of the many innovations now underway around the country, and facilitated networking of peer institutions that are seeking to improve student successes in precalculus and calculus" [MAA16].

In 2015, AMATYC, AMS, ASA, MAA, and SIAM jointly sponsored "The Common Vision" project with the ultimate goal to "galvanize the mathematical sciences community around a modern vision for undergraduate programs and to spur grassroots efforts within the community as a foundation for addressing the collective challenges we face" ([Sax16, p. vi]). One of the outcomes of this project was a far-reaching document titled *A Common Vision for Undergraduate Mathematical Science Programs in 2025* [Sax16]. The consensus among all five organizations is that the status quo is unacceptable, and the document calls for significant action in the areas of "curricula, course structure, workforce preparation, and faculty development" (p. vi).

In 2018, a resource *Guide to Evidence-Based Instructional Practices in Undergraduate Mathematics* [Abe18] was published, with an aim to support faculty seeking to improve students' learning outcomes. The authors of the guide summarized their work and findings in the initial section "Manifesto: A Declaration of Values," a passionate call for a "community wide transformation toward improved learning experiences and equitable access to mathematics for all students."

In Canada, the efforts to address at least some of the issues around teaching first-year mathematics courses have intensified, and recently the conversations and action have emerged as a genuine grassroots movement. In 2017, two of the coauthors of this article published their "Call for national dialogue: The present and future of teaching first year mathematics at Canadian universities" [Jun17]. The article called for instructors from across Canada to "communicate, share our experiences, coordinate our efforts and work together."

The response of the Canadian mathematics teaching community was overwhelming. Since 2017, several meetings on the topic of teaching the first-year mathematics courses have been organized at a national level. In addition, in 2017 the *First Year Mathematics and Statistics Courses Repository*, a resource supporting this ongoing national dialogue, was created and introduced to our community of

first-year instructors. This shareable dynamic online database (https://firstyearmath.ca) contains extensive data about first-year mathematics and statistics courses collected from instructors across Canada. The database includes course content, information about resource and technology use, learning outcomes, modes of delivery, connections with other courses, as well as informal descriptions of various practices in teaching these courses.

The attendees at the national meetings represent a cross section of the Canadian mathematics and statistics teaching community. The audiences comprised postsecondary teaching practitioners from all walks of academic life: graduate students, contract/sessional faculty, teaching faculty, research faculty, and undergraduate associate chairs.

About a half of the attendees were female, which reflects the fact that at the university level a large group of female mathematicians and statisticians are involved in teaching lower-level courses. This implies that when we discuss the gender gap in our mathematics and statistics postsecondary community, then we—faculty, departments, as well as relevant professional organizations—need to recognize and value teaching (and teachers!) at a higher degree than presently. Of course, when we talk about "women in mathematics and statistics" we must celebrate all great achievements by "women researchers in mathematics and statistics," and we have to continue pointing out that they are still significantly underrepresented in academia. What, in our view, is missing is the full recognition of the tremendous contributions that a large group of female mathematics and statistics instructors make on a daily basis in their classrooms across Canada (and, we assume, in Australia and the USA). Besides being on the front line of postsecondary education, these talented, knowledgeable, and hard-working women are the true role models and inspiration for all their students.

The Canadian mathematics and statistics teaching community must advocate for a major shift in the way mathematics and statistics departments view and treat their teaching faculty, in particular the large number of instructors who are in precarious contract, part-time, or limited-term positions. And yet it is these instructors who are routinely tasked with teaching important first-year courses. Mathematics departments should create a respectful and supportive environment for the teaching faculty and offer full-time, permanent employment while allowing for a small number of emergency short-term contractual positions.

A further barrier to a significant change is the well-known gap between mathematics education research and teaching practice (see, for instance, [Art99], where the author articulates the reasons why this gap naturally emerges). Our meetings have demonstrated that the number of Canadian postsecondary mathematics and statistics instructors who are familiar with the relevant education research is rather small. In this light, thinking about evidence-based teaching, we are curious to know how the research findings could be identified (when and if they exist) and how to ensure that they reach those who are actually teaching. We believe that our conferences, the Repository, and the newsletter (https://firstyearmath.ca/newsletter) we started publishing are small steps toward achieving this goal.

Our meetings have witnessed numerous calls for examining the unsustainably dominant role of calculus in the university mathematics curriculum. We have heard strong views suggesting that perhaps the time has come to abandon calculus and restructure first-year mathematics around a different paradigm, such as mathematical modeling, problem solving, or mathematical thinking.

Somewhat unexpectedly, it became apparent that in Canada responses to the set of common challenges in teaching first-year mathematics courses may depend on a number of external parameters, such as the size of the university (large versus small), institution's mandate (research intense versus teaching intense), financial model (public versus private), and geographical location (have versus have-not provinces). Our meetings made us more aware that the outcomes of what we do in our classes may be directly affected by the dynamics of the economic, social, and political environments in which our students and we, their instructors, live and work.

In summary, recent documents and conversations related to teaching first-year courses across the three countries convey a sense of urgency and the need for significant changes. The challenges, problems, objectives, and strategies that have been discussed are definitely not new: they have been around for decades (see, for instance, [Com97]). As it seems that our previous efforts have not managed to alleviate any of the dominant problems, we have to ask ourselves: should we, postsecondary teaching practitioners and our students, hope that this time will be successful? Our answer is yes, and our optimism is based on the fact that there is a widespread consensus that "the status quo is unacceptable." The recent initiatives indicate that there is growing awareness that we, the global mathematics and statistics teaching community, have to put additional effort into supporting our current and future students to learn mathematics and to achieve their academic goals. By using proven, evidence-based ways to transfer our knowledge of and passion for mathematics to our students, we can meet our share of the responsibility to ensure that the next generation of scholars and instructors holds high the torch of mathematics and statistics.

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