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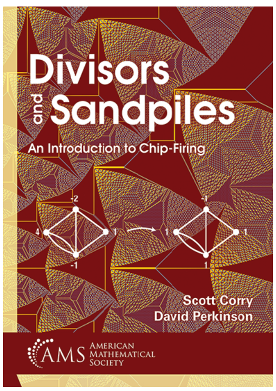

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Divisors and Sandpiles: An Introduction to Chip-Firing
by Scott Corry and David Perkinson

This book introduces the theory of *chip-firing* on finite graphs and its analogs in the theory of divisors on Riemann surfaces and fixed-energy sandpile Markov chains. The authors develop the theory around two concrete and intuitively appealing toy models (the dollar game and sandpiles) which are real-life relatable interpretations of the combinatorics and dynamics of the chip-firing process. After explaining the setups of the two models, the authors reinterpret them first in terms of a Laplacian operator, and then in the context of the geometry of Riemann surfaces in algebraic geometry and Markov processes in probability theory, in this way highlighting unexpected relationships between these two seemingly disparate fields The discussion leads naturally to other topics and generalizations, such as harmonic morphisms between graphs, M-matrices, matroids and the Tutte polynomial, and higher-dimensional chip-firing on simplicial complexes.

The two toy models used in the book can be understood at a variety of levels, both for education and for entertainment: the essential rules are accessible to someone with a middle school math background, and the myriad possibilities make for intriguing exploration, computer simulations and patterns. The dollar game can be thought of as a solitaire game, where you imagine people arranged (figuratively) on vertices of a graph with some integer amount of money (a negative value meaning that amount is owed). At each move (or chip-firing) you choose a vertex and the person on the vertex either gives a dollar to each adjacent player or takes one dollar from each adjacent player. You win the game if you can remove everyone's debt in a finite set of moves. Under what initial conditions is the game winnable? As the book explains, these questions can be studied via discrete algebraic geometry, by translating winnability to properties of the complete linear system of the divisor defined by the initial state and by applying the Riemann-Roch theorem.

The second model puts piles of sand at each vertex of a finite graph and assigns one designated vertex the property that sand can never accumulate there…making it a sort of sinkhole. If at some vertex, the sandpile becomes too high, for example it has more grains of sand than the degree of the vertex, then the sandpile topples sending a grain of sand onto each of the piles of neighboring vertices. If the graph is connected, then because of the sinkhole, the sandpiles eventually reach a stable state, and one can show that this state is uniquely determined by the initial one. Amongst stable states, there are two types: recurrent, which are the most likely to occur, and non-recurrent or transient states. A natural question to ask is which stable states are recurrent for a given graph. As with the dollar game scenario, the answer is elegant, involving an underlying abelian group structure on the space of stable recurrent states.

The book is written in an engaging way and only requires a minimal background in linear algebra, group theory and probability theory. At the same time the techniques described in the book have the potential to be useful in a broad range of research areas. Because of the far-reaching applications of its subject, it would be of interest to both undergraduate and graduate students and could be used as a textbook for a senior capstone course, or for self-study.

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