2021 *Election*

Nominations for President

Photo provided by David Jerison

Nomination of David Jerison

by Charles Fefferman and Carlos E. Kenig

It is a pleasure to nominate David Jerison for the presidency of the AMS. We have both known David for over four decades. Over the course of his distinguished career David has made fundamental contributions to Fourier analysis and partial differential equations. David has served the profession through his work on many committees of the AMS, the vice presidency of the AMS, his many administrative positions at MIT, and his unwavering commitment to teaching and research. We are convinced that David would be an outstanding choice for president of the American Mathematical Society.

David Jerison's Background

David grew up in West Lafayette, Indiana, where his father was a professor of mathematics at Purdue University. David became interested in mathematics early on, and he was an enthusiastic participant in A. Ross's famous program for gifted high school students at Ohio State University. He obtained his AB degree from Harvard in 1975. After a year visiting the University of Paris, he started his graduate studies at Princeton, where he obtained his PhD in 1980 under the direction of E. M. Stein. David was an NSF Postdoctoral Fellow at the University of Chicago in 1980–1981, and in 1981 he moved to MIT, where he has remained ever since. At MIT, David has been involved in a number of administrative positions. He was the chair of the Undergraduate Mathematics Committee (1988–1991), the chair of the Pure Mathematics Committee (2002–2004 and 2009–2011), and co-chair of the Graduate Students Committee (2007–2009). David has a love of teaching and a lifelong commitment to it. He was instrumental in the redesign of the Open Courseware for Single Variable Calculus 18.01 and for Multi Variable Calculus 18.02 at MIT. He also directs SPUR, the Summer Program for Undergraduate Research, dedicated to MIT undergraduates, and he organizes the mathematics section of RSI, the Research Summer Institute for high school students, at MIT. David has served on many committees of the AMS, and as vice president of the AMS, 2017–2020.

David has been widely recognized for his scientific achievements and for his teaching. He received an NSF Postdoctoral Fellowship (1980–1983), a Sloan Research Fellowship (1985–1989), a Presidential Young Investigator Award (1985–1990), a Simons Fellowship (2018), and a Guggenheim Fellowship (2019). He was elected a Fellow of the American Academy of Arts and Sciences in 1999 and a Fellow of the AMS in 2012. In 2012, David received (with J. Lee) the AMS Bergman Prize for their work in CR geometry. In 2004, he was elected a MacVicar Fellow at MIT, in recognition of his work in revamping the calculus classes 18.01 and 18.02. David was co-awarded (with Gigliola Staffilani) the MIT Class of 1960 Fellowship (2014–2016) for efforts to bring 18.01 to the MITx platform, and he was a co-recipient (with the edx group) of the first MITx Prize for teaching in MOOC (massive open online courses) given by MIT digital. Over the years, David has delivered many invited and distinguished lectures all over the world. For instance, he gave an invited address at the AMS summer meeting in Salt Lake City (1987) and an invited address at the International Congress of Mathematicians (ICM) in Zurich (1984).

David Jerison's Mathematics

David's research is in partial differential equations and Fourier analysis, areas to which he has made landmark, transformative contributions. David has worked on a broad range of topics. They include the Dirichlet problem on the Heisenberg group, the Yamabe problem in CR geometry, the study of potential theory and boundary value problems under optimal regularity conditions, and analogs of the Minkowski problem in differential geometry, involving capacity and harmonic measure. Some other research topics to which David has made seminal contributions are the study of internal Diffusion-Limited Aggregation (internal DLA), and the understanding of the shape of Neumann and Dirichlet eigenfunctions and related questions concerning the regularity, geometry, and topology of free boundaries and minimal surfaces. To this amazingly broad array of difficult and fundamental issues, David has brought new insights and perspectives, and has made deep, lasting contributions. A brief description of some of David's breakthroughs follows.

Optimal regularity potential theory and boundary value problems

The 1950s saw the beginning of a spectacular development of harmonic analysis and partial differential equations in higher dimensions, through the use of real variable methods that replaced the classical complex analysis methods. This was in large part accomplished through the works of Calderón, Zygmund, and Stein, and their collaborators. Two of the results that had important influence in this endeavor were the extension to higher dimensions of the classical theorem of Fatou on the almost everywhere existence of nontangential limits for bounded harmonic functions (due to Calderón) and the related higher dimensional result on "square functions" which had two parts, one due to Calderón and the other one to Stein. These results highlighted the use of "saw-tooth" regions and the study of harmonic functions in them. A further influential result in this direction was due to Carleson in the early 60s, which extended Fatou's theorem to nonnegative harmonic functions (one-sided boundedness) through a careful study of "harmonic measure" and nonnegative harmonic functions in "saw-tooth" regions. "Saw-tooth" regions are examples of Lipschitz domains, and in the early 70s Hunt and Wheeden initiated the study of nonnegative harmonic functions on Lipschitz domains in higher dimensions. They proved the analogs of the results just mentioned, but with the exceptional sets having zero "harmonic measure." In the late 70s, B. Dahlberg proved the mutual absolute continuity of "harmonic measure" and surface measure on Lipschitz domains. It is against this backdrop that one can place the deep work of David Jerison in this area. This work started jointly with one of us (CEK) at the time when David was a graduate student at Princeton and CEK a beginning instructor there. The first result obtained was a new proof of Dahlberg's theorem, through the use of an integral identity, now known as the "Rellich identity," because as it turned out, it had been previously discovered by Rellich in the 1940s in his work on eigenfunctions. The idea of using such an identity to prove mutual absolute continuity results of this kind has been very influential and has led to many results by many researchers. David Jerison (and CEK) later used it to initiate the study of the Neumann problem on Lipschitz domains. It was also later used by the same authors to obtain a subtle optimal regularity result for the density of "harmonic measure" on C^1 domains, which had been conjectured by Dahlberg. A very important contribution of David's in this context is a kind of converse of this result, which he obtained in the early 90s, and which has generated a large amount of research in free boundary problems "below the continuous threshold." Another influential work of David's (with CEK) was the development of NTA domains ("nontangentially accessible domains") and their potential theory, especially the study of nonnegative harmonic functions on them. These domains generalize the Lipschitz ones, but they don't necessarily have a surface measure, and in this study, "harmonic measure" replaces it. It turns out that this class of domains appears naturally in the study of free boundary problems and in geometric measure theory, and their study has influenced a flourishing research enterprise in these fields.

David Jerison's work described above has been pioneering, foundational and enduring, and of great impact.

The joint work of Jerison and Jack Lee on the CR Yamabe problem

A major achievement of geometric analysis was the solution of the Yamabe problem: Given a compact Riemannian manifold (M^n,g) there exists a unique smooth conformal deformation of the metric, $g = (u^4/(n-2))g$ with constant scalar curvature. The relevant PDE is Laplacian$(u) - (n-2)/4(n-1)Ru + \mu u^{(n+2)/(n-2)} = 0$, where Laplacian denotes the Laplace-Beltrami operator and R denotes the scalar curvature, both associated to g, for a constant μ called the Yamabe invariant of (M,g), denoted $Y(M,g)$. The sphere S with its standard round metric g_0 has the highest Yamabe invariant among all compact manifolds of a given dimension. The Yamabe problem was first solved by Yamabe and Aubin in the case of manifolds with $Y(M,g)$ strictly less than $Y(S,g_0)$. R. Schoen completed the solution by showing that $Y(M,g)$ is strictly less than $Y(S,g_0)$ unless (M,g) is isometric to (S,g_0). At about the same time, the fundamental notion of CR manifolds arose from several complex variables. The basic example of a CR manifold M is the boundary of a smooth domain D in C^n. When $n > 1$, the n Cauchy-Riemann equations on D give rise to $n-1$ Cauchy-Riemann equations on M, which define the CR structure. It is natural to impose convexity conditions on M, and the simplest case is that of strictly pseudoconvex CR manifolds. A CR manifold gives rise to an equivalence class of contact 1-forms, specified up to multiplication by a nonzero smooth function. A particular choice of the contact form within that equivalence class gives rise to a scalar called the Webster curvature. Together with J. Lee, Jerison posed the CR Yamabe problem: Given a compact orientable strictly pseudoconvex CR manifold, can one find a choice of contact form for which the Webster curvature is constant? If so, that constant Webster curvature is equal to $\mu(M)$, the CR Yamabe invariant of M, defined by a variational problem. Jerison and Lee proved that $\mu(M)$ is maximized among all CR manifolds of a given dimension by taking M to be the round sphere S, and that the CR Yamabe problem admits a unique smooth solution whenever $\mu(M)$ is strictly less than $\mu(S)$. That's the CR analogue of the classical work of Yamabe and Aubin. The work of Jerison and Lee is very deep. Whereas the classical Yamabe problem centers on an elliptic PDE with a critical nonlinearity, the CR Yamabe problem involves a nonelliptic PDE with a critical nonlinearity. Much, but by no means all, of the analysis by Jerison and Lee marries the classical work on the Yamabe problem with developments in several complex variables going back to the work of E. M. Stein. At a key point the classical approach breaks down. Jerison and Lee get around the problem by adapting work of K. Uhlenbeck. In retrospect, we see a far-reaching analogy between CR manifolds and classical conformal geometry. The work of Jerison and Lee is arguably the high point of that analogy.

Remarks on Jerison's work on eigenfunctions and on free boundaries

Over the last decade, Jerison has done deep work on free boundary problems. Here, we single out his paper with T. Beck and S. Raynor on boundary regularity for solutions of a two-phase problem arising in fluid mechanics and in optimal shape design. The unknown function lives on a convex domain D in R^n. The goal is to prove that the minimizer for the relevant variational principle is Lipschitz up to the boundary. Beck, Jerison, and Raynor succeed in proving Lipschitz continuity provided D satisfies an estimate (a "Dini condition") on the rate at which the boundary bD approaches its tangent cone at small scales.

The Dini condition holds for all convex domains in R^2 and for all $C^{1,\alpha}$ domains in R^n. It is remarkably hard to find a convex domain in R^n for which the Dini condition fails, but Beck, Jerison, and Raynor produced an example. The Dini condition is a fascinating new idea in the geometry of convex sets.

We close our selection of Jerison's works with a remarkable joint paper by Arnold, David, Filoche, Jerison, and Mayboroda on localization of eigenfunctions of Schrödinger operators and their close relatives. Localization of eigenfunctions of Schrödinger operators with highly disordered potentials has been an important idea in mathematics and physics for half a century. The phenomenon is well established from physical and numerical experiments, but rigorous understanding has been slow and hard-won. The discovery by Filoche and Mayboroda of the role of the "Landscape Function" was a fundamental new idea, but it too was confirmed only by simulations, without rigorous justification.

For a Schrödinger operator H with a nonnegative potential, the Landscape Function is simply the solution of the equation $Hu = 1$. Eigenfunctions of u like to localize around the local minima of the "effective potential" $1/u$.

The paper of Jerison and his co-authors proves an estimate that guarantees exponential decrease of an eigenfunction outside an "island" where $1/u$ is small, ringed by "walls" where $1/u$ is somewhat larger. One easily verifies the formation of such islands and walls in particular cases, and the islands are seen to be precisely the regions to which eigenfunctions are localized. Rigorous understanding of the formation of the islands and walls is still missing, but the work of Jerison et al. is a big step forward.

Conclusion

As the remarks above show, David Jerison has made fundamental contributions to teaching and research, and to the administrative aspects of our profession. This experience, and his sterling character, would serve the AMS and the whole mathematical community very well should he be elected to the presidency of the AMS.

Photo provided by Bryna Kra

Nomination of Bryna Kra

by Terence Tao and Amie Wilkinson

We are excited to nominate Bryna Kra for the presidency of the American Mathematical Society. We have known Kra as a friend and colleague for many years and have witnessed firsthand her formidable strengths as a mathematician, organizer, and leader, skills that will make her a first-rate president.

Kra received her PhD in mathematics from Stanford in 1995. She is currently the Sarah Rebecca Roland Professor at the Department of Mathematics at Northwestern University and also served as chair of that department from 2009 to 2012. She is a member of the National Academy of Sciences and a Fellow of both the American Academy of Arts and Sciences and the American Mathematical Society. Among her recognitions, she has received the Levi L. Conant Prize from the AMS for outstanding exposition and was a speaker at the 2006 International Congress of Mathematicians.

Research Contributions

Kra has made multiple contributions to several areas of dynamics, particularly ergodic theory and topological dynamics, and her work has seen important applications to combinatorics and number theory. Perhaps her best known result is the *Host-Kra structure theorem*, established with Bernard Host and published in the *Annals of Mathematics* in 2005, which gives a deep and powerful structural description of an arbitrary ergodic measure-preserving system. Roughly speaking, it shows that any such system (X, μ, T) contains a factor, now known as the *Host-Kra characteristic factor*, that controls the recurrence and multiple recurrence properties of the original system, in the sense that any multiple ergodic average such as

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \int_X \prod_{i=1}^{k} (f_i \circ T^{in})\, d\mu \tag{1}$$

involving one or more bounded measurable functions $f_1, \ldots, f_k$ of the original system can be evaluated by first projecting each of the f_i to the characteristic factor and then computing the resulting average. Importantly, this characteristic factor also has a very useful algebraic structure; it is the inverse limit of *nilsystems*, which are the measure-preserving systems formed from translation actions on nilmanifolds.

By combining these two facts, many questions concerning convergence or recurrence properties in ergodic theory can be reduced to verifying some version of these questions on nilsystems, which can often be resolved by using the well-understood equidistribution theory of such systems. For instance, Host and Kra used their theorem to give one of the first solutions to the long-standing question of establishing convergence in norm of the limit (1) for arbitrary measure-preserving systems (X, μ, T) and arbitrary bounded measurable functions $f_1, \ldots, f_k$. Through a fundamental connection between ergodic theory and combinatorics, known as the *Furstenberg correspondence principle*, this structure theorem had several applications (both direct and indirect) to questions in combinatorics, particularly in the area of density Ramsey theory in which one seeks to locate patterns such as arithmetic progressions inside reasonably dense sets of integers (or other groups).

For instance, the Host-Kra theorem directly inspired an analogous combinatorial conjecture known as the *inverse conjecture for the Gowers uniformity norm*, the subsequent proof of which had numerous applications in combinatorics and number theory (including for instance a sweeping generalization of the so-called "Green-Tao theorem" that asserts that the primes contain arbitrarily long arithmetic progressions). In addition, analogues of the Host-Kra theorem over finite fields have had applications to theoretical computer science, for example in showing that the property of a boolean function being correlated with a low-degree polynomial is efficiently testable. Kra and her co-authors have been involved in developing several applications and extensions of this theory, most notably in extending the theory to cover polynomial averages or averages involving multiple commuting shifts, or when the shift parameter n is restricted to a set of number-theoretic interest, such as the set of primes.

In recent years, Kra (in collaboration with Van Cyr) has been working extensively on the study of shift systems of slow growth, which from a combinatorial perspective can be viewed as infinite strings of letters in some finite alphabet, with the property that the number of distinct subwords of length k grows slowly with k. The theory of shifts of linear growth is fairly well understood; for instance, there is a classical theorem of Morse and Hedlund that a string is periodic if and only if there is a k such that the number of subwords of length k is at most k. An influential conjecture of Nivat asserts a two-dimensional version: if a two-dimensional infinite array of letters has the property that the number of subarrays in a $k \times n$ rectangle is at most kn for some k, n, then the array is periodic. This remains open, but through a delicate combinatorial and geometric analysis, Kra and Cyr obtained one of the strongest known partial results towards this conjecture, verifying it when the number of subarrays in a $k \times n$ rectangle is at most $kn/2$. Work by Kra and Cyr has revealed further intriguing structural properties of shift systems of slow growth, for instance that their group of automorphisms is surprisingly small.

Service

Kra has an extensive service record with the American Mathematical Society; she has served on over two dozen AMS committees, and chaired the Colloquium Lecture Committee, the Prize Venue Committee, the Central Section Program Committee, the Development Committee, and the Executive Committee and Board of Trustees Nominating Committee. Kra also chaired the Board of Trustees of the AMS, and was a regional co-chair for the Fund for the Next Generation.

In addition to service within the AMS, Kra has also served on high-level committees at many other mathematical institutions, including the Board of Trustees for the Institute for Pure and Applied Mathematics, the Board on the Mathematical Sciences and their Applications at the National Academies, the Executive Committee for the Association for Women in Mathematics, and the Steering Committee for the Park City Mathematics Institute. She has also been on organizational committees for over a dozen conferences and programs and held over a dozen editorial positions, including being executive editor at *Ergodic Theory and Dynamical Systems* since 2012.

Kra founded and organized the Women in Mathematics groups at both Pennsylvania State University and Northwestern University. In 2015, Kra co-founded the Graduate Research Opportunities for Women (GROW) conference for women-identified students interested in graduate school in the mathematical sciences. This successful conference is now held annually at different universities, and was recently recognized by the American Mathematical Society with the Programs that Make a Difference Award. Kra also co-founded the Causeway Postbaccalaureate program at Northwestern University to provide mentorship, foundational training, and evaluation for mathematics graduate students in historically underrepresented groups.

Conclusion

Bryna Kra is an outstanding mathematician who has been extensively involved at all levels with the American Mathematical Society, and with the mathematical community at large. She is committed to mathematical excellence, and to providing opportunities to all prospective mathematicians, with special attention to those from underserved communities. We believe she would be a superbly qualified and effective president of the Society.