

## Dark Matter May Be a Bose-Einstein Condensate of Axions

*Kay L. Kirkpatrick*

“Dark matter” is a misnomer. This inferred but unobserved substance is clear and transparent, rather than dark, and makes up about 25% of the universe. (This exposition owes a lot to *The Disordered Cosmos: A Journey into Dark Matter, Spacetime, and Dreams Deferred*, by Chanda Prescod-Weinstein. Dark energy probably makes up another 70% of the universe; and atomic matter, which is what we can observe easily, about 5%.) Dark matter does not reflect or absorb light but can influence light and atomic matter primarily through gravity.

Dark matter has been hypothesized since the 1930s when Fritz Zwicky observed that galaxies did not seem to have enough visible mass to hold them together and inferred its existence. A couple of decades later, Vera Rubin did clever spectroscopy showing that the outer stars of the Milky Way are moving faster than they should be moving, based on the gravity of visible matter in our galaxy. There has been another hypothesis too, that our model of gravity requires another correction in addition to general relativity. While gravity might indeed require another correction, physicists remain almost sure that dark matter exists and cannot be explained away by modifying gravity.

More evidence that dark matter exists began with observations about the Bullet Cluster of galaxies, which has a lens in its midst that bends light coming from the other side. Called gravitational lensing, myriad effects like the Bullet Cluster have been observed all over the sky, including subtle examples called microlensing as well as dramatic examples such as the sixfold magnification of a distant galaxy. Something massive and transparent almost surely exists in the midst of these galaxy clusters, something that distorts light passing through it.

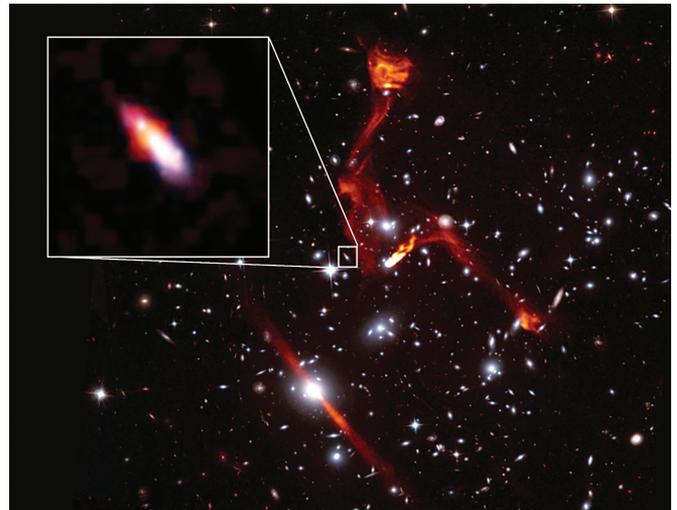
Recently we have observed galaxies that seem to have no dark matter, but there are very few of those in light of the many billions of galaxies that we can observe, which suggests again that dark matter is common and that it is matter

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**Figure 1.** Small box: VLA radio image of the distant galaxy VLAHFF-J071736.66 + 374506.4, which is more than 8 billion light-years away and is 0.3% the mass of the Milky Way. Main box: Hubble image of the foreground galaxy group MACSJ0717.5 + 3745, including its orange-red radio shock wave remnants, which is more than 5 billion light-years away and likely has a dark matter lens in its midst that magnifies the distant galaxy sixfold.

rather than just a modification to gravity. The cosmic microwave background also provides evidence of dark matter.

What kind of substance might dark matter be? There are many models of dark matter that theorists have created and used to make predictions for astronomical observations or experimental particle sensors and accelerators. Dark matter particles might be like neutrinos, but cold and slow, compared to the near-light-speed neutrinos that have been observed. Dark matter might be made of black holes that formed in the primordial universe and have not yet evaporated. Dark matter might convert to or from dark energy as in the Lambda Cold Dark Matter model. But perhaps the most intriguing model for dark matter is the axion, with its connections to patching the Standard Model and to ultra-low-temperature physics where quantum effects can be seen with the naked eye.

Axions are hypothetical pseudoscalar particles proposed in the 1970s by Roberto Peccei and Helen Quinn to solve the strong Charge+Parity problem: Why does quantum chromodynamics appear to preserve Charge-conjugation Parity symmetry? Peccei and Quinn took a previously constant parameter in QCD and turned it into a function, known occasionally as the Higglet and now usually called the axion. Coincidentally, this patch for QCD also provides candidates for dark matter particles.

Axions are quantum particles, so they can be thought of in terms of waves or smeared particles, and macroscopically they are described mathematically by a non-linear Schrödinger (NLS) equation and a second equation that

includes gravity, together called variously NLS-Poisson, NLS-Newton, or Gross-Pitaevskii-Poisson equations. Modulo many details that need to be sorted out by more research, axions in the early universe likely formed clusters with gravitationally bound cores that can be described by the solitons of the NLS-Poisson equations, and these soliton cores involve axions condensed into a phase called a Bose-Einstein condensate (BEC).

A BEC is an ultra-cold phase of matter in which quantum particles called bosons condense into the same state and behave as if they are one big quantum particle. BECs were predicted in the 1920s, experimentally produced in the 1990s, and heuristically described by Gross and Pitaevskii in the 1960s as having a one-particle macroscopic state that evolves according to the NLS:

$$i\partial_t\psi = -\Delta\psi + \lambda|\psi|^2\psi. \tag{1}$$

Here  $\lambda$  is the self-interaction strength and can have any sign, because laboratory BECs can be tuned for attractive or repulsive self-interactions. There are now rigorous connections between the physics of the interacting quantum particles and the mathematics of the NLS. For instance, we can show that quantum many-body systems justify mean-field descriptions such as the NLS, and we can analyze fluctuations and large deviations around that mean (e.g., “A large deviation principle in many-body quantum dynamics,” arXiv:2010.13754, and related references).

For models of laboratory BECs, we may not need to consider gravity, but when we think about BECs the size of a star (called a Bose star) or a galaxy, we need to include gravity in some way. One way is the NLS-Poisson model:

$$i\partial_t\psi = -\Delta\psi/2m + U\psi + \lambda|\psi|^2\psi, \tag{2}$$

$$\Delta U = 4\pi Gm^2(|\psi|^2 - n). \tag{3}$$

Here  $m$  is the mass of the axions,  $U$  is the Newtonian gravitational potential,  $G$  is the gravitational constant, and  $n$  is the mean density of the field. For models of axion BECs, the coefficient  $\lambda$  is believed to be positive for attractive self-interactions.

One thing a theory needs is plausibility for observations within our lifetimes. In work with Anthony Mirasola and Chanda Prescod-Weinstein, we have figured out that the condensation of axions into a Bose star is driven by gravity and should be able to happen within the lifetime of the universe (“Relaxation times for Bose-Einstein condensation in axion miniclusters,” MR4186382). Research like this will aid the direct observation of star- or galaxy-sized clusters of dark matter.

Coincidentally, certain laboratory BECs undergo a collapse similar to a supernova in an event called a Bosenova. This may foreshadow research on how dark matter could influence the explosion of an actual supernova. Dark matter may also interact with semiconductors, including

organic semiconductors. Many of the mysteries about this substance should become clear over the next few decades.



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**Credits**

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