CHOICE FUNCTIONS AND TYCHONOFF'S THEOREM

W. H. GOTTSCHALK

The purpose of this note is to point out that the following set-theoretic theorem [R. Rado, Axiomatic treatment of rank in infinite sets, Canadian Journal of Mathematics vol. 1 (1949) pp. 337–343] is an easy consequence of Tychonoff’s theorem that the cartesian product of a family of compact spaces is compact.

**Theorem.** Let \((X_\alpha|\alpha \in I)\) be a family of finite sets, let \(A\) be the class of all finite subsets of \(I\), and for each \(A \in A\) let \(\phi_A\) be a choice function of \((X_\alpha|\alpha \in A)\). Then there exists a choice function \(\phi\) of \((X_\alpha|\alpha \in I)\) such that \(A \in A\) implies the existence of \(B \in A\) such that \(B \supset A\) and \(\alpha \phi = \alpha \phi_B\ (\alpha \in A)\).

**Proof.** For \(A \in A\) let \(E_A\) be the set of all \(\phi \in X = \prod_{\alpha \in I} X_\alpha\) such that \(\alpha \phi = \alpha \phi_B\ (\alpha \in A)\) for some \(B \in A\) with \(B \supset A\). Provide each \(X_\alpha (\alpha \in I)\) with its discrete topology. Since \(X\) is compact and \([E_A|A \in A]\) is a class of nonvacuous closed subsets of \(X\) with the finite intersection property, there exists \(\phi \in \bigcap_{A \in A} E_A\). The proof is completed.

**Corollary.** A family of finite sets has a one-to-one choice function if and only if each of its finite subfamilies has a one-to-one choice function. [Cf. C. J. Everett and G. Whaples, Representations of sequences of sets, Amer. J. Math. vol. 71 (1949) pp. 287–293.]

University of Pennsylvania

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