be equal by the Cauchy integral theorem (by [1] and a result in the author’s dissertation not yet published).

Theorem 1 follows directly from Theorem 3.

**BIBLIOGRAPHY**


**REMARK ON A FORMULA FOR THE BERNOULLI NUMBERS**

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Some years ago Garabedian [1] proved the following formula:

\[ B_{k+1} = \frac{(-1)^{k+1}(k + 1)}{2^{k+1} - 1} \sum_{r=0}^{k} (-1)^r \frac{\Delta^r 1^k}{2r+1} \quad (k \geq 0), \]

where the even suffix notation is employed for the Bernoulli numbers. The proof of (1) made use of the sum of a certain divergent series.

We wish to point out that (1) is not new. It can be found (in somewhat different notation) in [3, p. 224, formula (68)].

It may be of interest to give a short proof of (1). We use the formula [2, p. 28]

\[ C_k = 2^{k+1}(1 - 2^{k+1}) \frac{B_{k+1}}{k+1}, \]

where the \( C_k \) are the coefficients in the Euler polynomial:

\[ E_k(x) = \left( x + \frac{C_k}{2} \right)^k = \sum_{s=0}^{k} \binom{k}{s} 2^{-s} C_s x^{k-s}. \]

Then in view of

\[ E_k(x + 1) + E_k(x) = 2x^k, \]

we have

\[ E_k(x) = \left( 1 + \frac{1}{2} \Delta \right)^{-1} x^k = \sum_{s=0}^{k} (-1)^s 2^{-s} \Delta^s x^k. \]

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If we take $x = 1$ in (5) and use (3) and (4), we get
\[ C_k = -2^k E_k(1) = -\sum_{r=0}^{k} (-1)^r 2^{k-r} \Delta^r 1. \]
Substitution in (2) leads at once to (1).

In a similar way we can prove
\[ B_{k+1} = \frac{(-1)^{k+1}(k + 1)}{2^{k+1} - 1} \sum_{r=0}^{k} (-1)^r 2^{-r} \Delta^r 0. \]

References