From this it follows readily that the consistency of $Z_c'$ is provable in $Z_c$. (Cf. Theorem IV, p. 260, of [2].)

**BIBLIOGRAPHY**


**AN EXTENSION THEOREM FOR SOLUTIONS OF $d\omega = \Omega$**

**HARLEY FLANDERS**

Let $U$ and $V$ be open sets in $E_n$ such that $V \subset U$ and $U$ is connected and homologically trivial, i.e., all homology groups of $U$ beyond the zero-dimensional case vanish. Let $\Omega$ be an exterior differential form of degree $p$ on $E_n$ with infinitely differentiable coefficients whose exterior derivative vanishes: $d\Omega = 0$. The well known converse to the Lemma of Poincaré asserts that $\omega = \theta \omega$ where $\omega$ is an infinitely differentiable $p-1$ form on $E_n$. Let us suppose however that we are merely given a $p-1$ form $\alpha$ on $U$ such that $d\alpha = \omega$ on $U$. The question immediately arises as to whether it is possible to prolong $\alpha$ to all of $E_n$. The example $U = \{(x, y) | x > 0\}$, $\alpha = (x dy - y dx)/(x^2 + y^2)$, $\Omega = 0$ shows us that the answer is negative. Nevertheless, there exists a $p-1$ form $\beta$ on $E_n$ such that $\beta = \alpha$ on $V$ and $d\beta = \Omega$ on $E_n$.

To prove this, we shall take for granted the existence of an infinitely differentiable function $f$ on $E_n$ such that $f = 1$ on $V$ and $f = 0$ outside of a closed subset of $U$. We have the form $\omega$ on $E_n$ such that $d\omega = \Omega$ on $E_n$ and the form $\alpha$ on $U$ such that $d\alpha = \omega$ on $U$. Thus $d(\alpha - \omega) = 0$ on $U$ and so it follows from the hypotheses on $U$ and what is essentially de Rham's second theorem that $\alpha - \omega = d\lambda$ on $U$.

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where \( \lambda \) is an infinitely differentiable \( p-2 \) form on \( U \). One sets \( \mu = f \lambda \) on \( U \), \( \mu = 0 \) outside \( U \) so that \( \mu \) is a \( p-2 \) form on \( E_n \) such that \( d \mu = d(f \lambda) = d \lambda = \alpha - \omega \) on \( V \). The form \( \beta = \omega + d \mu \) solves the problem, for \( d \beta = d \omega = \Omega \) on \( E_n \) and \( \beta = \omega + (\alpha - \omega) = \alpha \) on \( V \).

If any doubt remains about the case \( p = 1 \), it is quickly settled when one notes that \( \alpha - \omega \) is a function and so \( d(\alpha - \omega) = 0 \) on \( U \) implies that \( \alpha - \omega = c \), a constant, on \( U \). Now \( \beta = \omega + c \) is the solution.

This result and method of proof can be extended to multiply connected regions; we give a single instance:

Let \( \Omega \) be a \( p \)-form on \( E_n \) such that \( d \Omega = 0 \) and suppose that \( \alpha \) is a \( (p-1) \)-form on \( r > 0 \) \( (r^2 = x_1^2 + \cdots + x_n^2) \) such that \( d \alpha = \Omega \) on \( r > 0 \). Finally, suppose that

\[
\lim_{r \to 0} \int_{r \leq r} \alpha = 0
\]

in the case \( p = n \). Then given any \( \epsilon > 0 \), there exists a \( (p-1) \)-form \( \beta \) on \( E_n \) such that \( d \beta = \Omega \) on \( E_n \) and \( \beta = \alpha \) on \( r > \epsilon \).

1 This form of de Rham's theorem is given in the paper of A. Weil, Sur les théorèmes de de Rham, Comment. Math. Helv. vol. 26 (1952). It is pointed out on p. 138 that in case \( U \) is convex, then an elementary proof is possible. Such a proof is given for the case in which \( U \) is a cell on p. 94 of the second edition of W. V. D. Hodge, Harmonic integrals.