By a tree we mean a continuum (= compact connected Hausdorff space) in which every pair of distinct points is separated by a third point. It is well known [4] that a dendrite is simply a metric tree. In this note we give a new characterization of dendrites and trees in terms of partially ordered spaces.

Recall from [3] that a POTS (= partially ordered topological space) is a topological space \( X \) endowed with a partial order, \( \leq \), which is semicontinuous in the sense that \( L(x) = \{ a : a \leq x \} \) and \( M(x) = \{ a : x \leq a \} \) are closed sets for each \( x \in X \). A partial order is order dense if \( x < y \) implies there is a \( z \) such that \( x < z < y \). A chain of a partially ordered set is a subset which is linear with respect to the partial order. An element in a partially ordered set is minimal (maximal) if it has no proper predecessor (successor).

Suppose that \( K \) is a locally connected continuum and \( e \in K \). Define a relation, \( \leq \), in \( K \) by \( x \leq y \) if, and only if, \( x = e \), or \( x = y \), or \( x \) separates \( e \) and \( y \) in \( K \). From [3] we have

**Lemma 1.** The relation \( \leq \) is a semicontinuous partial order.

We shall require two more results from [3]. Lemma 2 is originally due to Wallace [2].

**Lemma 2.** A compact POTS contains a maximal element.

**Lemma 3.** A compact order dense POTS is connected if its set of minimal elements is connected.

**Lemma 4.** A tree is locally connected.

**Proof.** Let \( X \) be a tree; by [1, p. 140] \( X \) is regular in the sense that if \( x \in U \subseteq X \), \( U \) open, then there is a neighborhood \( V \) of \( x \), with \( \overline{V} \subseteq U \) and such that the frontier of \( V \) is finite. By [4, p. 19] \( X \) is locally connected. Since the above arguments are valid in our situation, the proof is complete.

We may now prove our chief result.

**Theorem 1.** Let \( X \) be a compact Hausdorff space. A necessary and sufficient condition that \( X \) be a tree is that \( X \) admit a partial order, \( \leq \), satisfying

(i) \( \leq \) is semicontinuous,

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(ii) \( \leq \) is order dense,
(iii) for \( x \in X, y \in X \), it follows that \( L(x) \cap L(y) \) is a non-null chain,
(iv) \( M(x) - x \) is an open set, for each \( x \in X \).

**Proof.** Let \( X \) be a tree; choose \( e \in X \) and let \( X \) have the semi-
continuous partial order of Lemma 1. To establish (ii), let \( x < y \) and
suppose \( z \) separates \( x \) and \( y \), i.e.,

\[
X - z = C \cup D, \quad C \cap D, \quad x \in C, \quad y \in D.
\]

If \( x = e \), then, by definition, \( x < z < y \). Otherwise, we have

\[
X - x = A \cup B, \quad A \cap B, \quad e \in A, \quad y \in B.
\]

It is easy to see that \( x \in C \) and \( y \in B \) imply \( A \subset C \). Therefore \( e \in C \),
so that \( z < y \). Likewise, \( z \in B \), so that \( x < z \). To prove (iii), we first
note that by definition of the partial order, \( e \in L(x) \cap L(y) \) for any
\( x \) and \( y \) in \( X \). It remains to show that \( L(x) \) is a chain. If \( e < p_1, p_2 < x \),
we have

\[
X - p_i = A_i \cup B_i, \quad A_i \cap B_i, \quad x \in B_1 \cap B_2, \quad e \in A_1 \cap A_2.
\]

It is clear that \( p_1 \in A_2 \) implies \( p_1 \leq p_2 \) and \( p_1 \in B_2 \) implies \( p_2 \leq p_1 \). To
establish (iv), we note simply that \( M(x) - x = X - x \) if \( x = e \), and if \( x \neq e \), then

\[
M(x) - x = \bigcup \{ B_\alpha : X - x = A_\alpha \cup B_\alpha, \ A_\alpha \cap B_\alpha, \ e \in A_\alpha \}.
\]

In either case, \( M(x) - x \) is open. Thus the condition is necessary.

Suppose now that \( X \) admits a partial order satisfying (i)–(iv).
By Lemma 3, \( X \) is connected and hence is a continuum. If \( x < y \),
then by (ii) there is \( z \in X \) such that \( x < z < y \). By (i) and (iv), \( M(z) \) is
closed and \( M(z) - z \) is open, so that \( z \) separates \( x \) and \( y \). If \( x \) and \( y \) are
not comparable, then by (iii) and Lemma 2, there exists \( z = \max (L(x) \cap L(y)) \).
Choose \( t \) such that \( z < t < x \). Then \( x \in M(t) - t \) and \( y \in X - M(t) \), whence \( t \) separates \( x \) and \( y \). This completes the proof.

By Theorem 1 and Lemma 4, we have at once:

**Corollary.** Let \( X \) be a compactum. A necessary and sufficient condi-
tion that \( X \) be a dendrite is that \( X \) admit a partial order satisfying
(i)–(iv) of Theorem 1.

**Bibliography**

CORRECTION TO "UNIQUENESS OF THE PROJECTIVE PLANE WITH 57 POINTS"

MARSHALL HALL, JR.

Professor Gunter Pickert has pointed out to me that there is an error in my paper [Proc. Amer. Math. Soc. vol. 4 (1953) pp. 912–916]. The equation on page 915 which reads \(12 + 3s + 2t = 24\) should read \(12 + 3s + 2t + u = 24\). This invalidates my conclusion \(u = 0\) from which I deduce \(U = 0\), which is necessary for the rest of the paper. I give here a new proof that \(U = 0\).

We must show that in a 57 point plane a line containing points \(ACC\) is not possible, or that \(U = 0\). The proof is by showing that if there is a line \(ACC\) we reach a contradiction. We may take \(ACC\) as \(A_1C_4C_6\) by numbering points appropriately. We now reletter by the following rule:

\[
\begin{align*}
A_1 & A_2 A_3 A_4 B_1 B_2 B_3 C_4 C_6 \\
A_1 & B_1 B_6 A_0 B_4 B_2 A_2 B_3 B_6
\end{align*}
\]

and obtain the following configuration:

\[
\begin{align*}
A_1 & B_1 B_2 B_3 \\
A_0 & B_1 B_6 B_6 \\
A_0 & A_1 A_2 \\
A_1 & B_1 B_4 \\
A_1 & B_2 B_6 \\
A_1 & B_3 B_6 \\
A_2 & B_1 B_6 \\
A_2 & B_2 B_6 \\
A_2 & B_3 B_4
\end{align*}
\]