ON A CONVERGENCE PROBLEM

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Research Problem 25 [1] reads as follows:
If \( a_1 < a_2 < \cdots \) are positive integers, if \( C \) is compact, and if \( \sin a_n x \to 0 \) for all \( x \) in \( C \), prove that the convergence must actually be uniform.

The following example shows that the "theorem" is false.
Let
\[
a_n = 2^n \quad (n = 1, 2, \cdots);
\]
let \( C \) be the compact set of points
\[
(0, \pi, \pi/2, \pi/4, \pi/8, \cdots).
\]

Obviously \( \sin a_n x \to 0 \) for each \( x \) in \( C \), but for any \( \epsilon < 1 \) and any positive integer, \( N \), there is an \( x \) in \( C \) (namely, \( x = \pi/2^{N+2} \)) such that
\[
| \sin a_{N+1} x | = | \sin \pi/2 | = 1 > \epsilon,
\]
so that the convergence is not uniform.

REFERENCE


AIRCRAFT ARMAMENTS, INC.

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