1. Introduction. In recent years there have appeared two algebraizations of the first-order predicate calculus; i.e., the polyadic algebras of Halmos [1; 2], and the cylindric algebras of Tarski [3; 4]. While polyadic algebras are the algebraic version of the pure first-order calculus, cylindric algebras yield an algebraization of the first-order calculus with equality. Since the pure calculus does not contain any identifiable predicate, one cannot expect to find the algebraic analogue of an equality predicate in a general polyadic algebra. It is reasonable, however, to consider "adjoining" an equality predicate, in some sense, to a polyadic algebra, and ask if one then obtains a cylindric algebra. This is the procedure followed here. An e-algebra is defined as a polyadic algebra with an equality predicate. We show that every e-algebra is in a natural way a cylindric algebra. Conversely, it is shown that in the presence of an infinite supply of variables and a local finiteness condition, cylindric algebras are in a natural way e-algebras, and the correspondence obtained in this way between e-algebras and cylindric algebras is one-to-one.

2. Polyadic algebras. A quantifier (or, more explicitly, an existential quantifier) on a Boolean algebra $A$ is a mapping $\exists: A \rightarrow A$ such that

1. $\exists 0 = 0$,
2. $\exists p \leq \exists q$, and
3. $\exists(p \land \exists q) = \exists p \land \exists q$ for all $p, q \in A$.

A polyadic algebra is a quadruple $(A, I, S, \exists)$, where $A$ is a Boolean algebra, $I$ an arbitrary set whose elements are called variables, $S$ is a mapping from transformations of $I$ into itself to Boolean endomorphisms on $A$ (the transformations need not be one-to-one nor onto), and $\exists$ is a mapping from subsets of $I$ to quantifiers on $A$, satisfying the following conditions:

$\mathbf{P}_{1}$ $\exists(\emptyset)p = p$ for all $p \in A$ (\emptyset shall denote the empty set throughout).

$\mathbf{P}_{2}$ $\exists(J \cup K) = \exists(J) \exists(K)$ for all subsets $J$ and $K$ of $I$.

$\mathbf{P}_{3}$ $S(\delta) = f$ (where $\delta$ is the identity transformation on $I$ and $f$ is the identity endomorphism on $A$).

$\mathbf{P}_{4}$ $S(\sigma)S(\tau) = S(\sigma\tau)$ for all transformations $\sigma$ and $\tau$ on $I$.

$\mathbf{P}_{5}$ If $J \subseteq I$ and $\sigma$ and $\tau$ are transformations on $I$ which agree outside $J$, then $S(\sigma) \exists(J) = S(\tau) \exists(J)$.

$\mathbf{P}_{6}$ If $J \subseteq I$ and $\tau$ is a transformation which is one-to-one on $\tau^{-1}J$, then $\exists(J)S(\tau) = S(\tau) \exists(\tau^{-1}J)$.

Presented to the Society, April 22, 1955; received by the editors July 22, 1955 and, in revised form, March 28, 1956.

176
If \( p \in A \), then \( p \) will be said to be **supported by** the set \( J \) if \( \exists (I - J)p = p \). We will say that \( p \) is **independent** of the set \( K \) if \( \exists (K)p = p \), so that \( J \) supports \( p \) if and only if \( p \) is independent of \( I - J \). A polyadic algebra will be called **locally finite** if each element \( p \) of the algebra is supported by some finite set \( J_p \). A transformation \( \tau \) will be called **finite** if \( \tau \) agrees with \( \delta \) outside some finite set. If \( i \) and \( j \) are elements of \( I \), the transformation which maps \( i \) onto \( j \) and every other element of \( I \) (including \( j \)) onto itself will be called a **replacement** and denoted by \((i/j)\). If \( I \) is infinite, the algebra will be said to have **infinite degree**. A quasi-polyadic algebra is a quadruple \((A, I, S, \exists)\), where \( A \) is a Boolean algebra, \( I \) a set, \( S \) a mapping from finite transformations on \( I \) to Boolean endomorphisms on \( A \), and \( \exists \) a mapping from finite subsets of \( I \) to quantifiers on \( A \), satisfying the conditions:

1. \( S(\emptyset)p = p \) whenever \( p \in A \).
2. \( \exists (J \cup K) = \exists (J) \exists (K) \) whenever \( J \) and \( K \) are finite subsets of \( I \).
3. \( S(\delta) = f \).
4. \( S(\sigma)S(\tau) = S(\sigma\tau) \) whenever \( \sigma \) and \( \tau \) are finite transformations on \( I \).
5. If \( \sigma \) and \( \tau \) are finite transformations on \( I \), if \( J \) is a finite subset of \( I \), and \( \sigma = \tau \) outside \( J \), then \( S(\sigma) \exists (J) = S(\tau) \exists (J) \).
6. If \( \tau \) is a finite transformation on \( I \), if \( J \) is a finite subset of \( I \), and if \( \tau \) is one-to-one on \( \tau^{-1}J \), then \( \exists (J)S(\tau) = S(\tau) \exists (\tau^{-1}J) \).
7. If \( p \in A \), then there exists a cofinite set \( J \) (i.e., \( I - J \) is a finite set) such that \( \exists (K)p = p \) whenever \( K \) is a finite subset of \( J \).

We shall need the following result concerning quasi-polyadic algebras from [2].

**Theorem.** If \((A, I, S, \exists)\) is a quasi-polyadic algebra, then (i) there exists a mapping \( S^* \) from transformations on \( I \) to Boolean endomorphisms of \( A \) such that \( S^*(\tau) = S(\tau) \) whenever \( \tau \) is a finite transformation, (ii) there exists a mapping \( \exists^* \) from subsets of \( I \) to quantifiers on \( A \) such that \( \exists^*(J) = \exists (J) \) whenever \( J \) is a finite set, (iii) the quadruple \((A, I, S^*, \exists^*)\) is a locally finite polyadic algebra, and (iv) the mappings \( S^* \) and \( \exists^* \) are uniquely determined by (i), (ii), and (iii).

We shall also need the fact, established in [2], that if \( \tau \) is a finite transformation on \( I \) and \( J \) is a finite subset of \( I \), then there is a finite ordered collection \( \{\tau_1, \ldots, \tau_n\} \) of replacements on \( I \) such that \( \tau = \tau_1 \cdots \tau_n \) on \( J \).

**3. \( e \)-Algebras.** If \( e(\ , \) \) is the equality predicate for the first-order functional calculus with equality, then it is well known that \( e(\ , \) \) is characterized by the reflexive and substitution properties. Moreover,
if we have \(e(x, y)\), then the transformation which maps \(x\) onto \(z\) yields the equality of \(z\) and \(y\); i.e., \(e(z, y)\). More generally, the effect on \(e(x, y)\) of a transformation on the variables is obtained by allowing the transformation to act on the variables \(x\) and \(y\) directly. These considerations furnish the motivation for Definitions 1 and 2. (Condition (2) of Definition 2 asserts essentially that if \(p\) is true and \(i = j\), then \(p\) is true with \(i\) replaced by \(j\); i.e., the substitution property.)

**Definition 1.** Let \((A, I, S, \exists)\) be a polyadic algebra. A **binary predicate** for \(A\) is a function \(p : I \times I \to A\) such that \(S(\tau)p(i, j) = p(\tau i, \tau j)\) for every transformation \(\tau\) on \(I\).

**Definition 2.** A polyadic algebra with equality (or, an \(e\)-algebra) is a polyadic algebra \((A, I, S, \exists)\) for which there exists a binary predicate \(e\) for \(A\) such that (1) \(e(i, i) = 1\) for all \(i \in I\), and (2) \(p \land e(i, j) \leq S(i/j)p\) for all \(i, j \in I\) and \(p \in A\). We shall denote the \(e\)-algebra by \((A, I, S, \exists, e)\).

**Definition 3.** A cylindric algebra is a Boolean algebra \(A\), together with a function \(C\) from a set \(I\) to quantifiers on \(A\), and a function \(d : I \times I \to A\) such that (1) \(C(h)C(j) = C(j)C(h)\), (2) \(d(i, i) = 1\), (3) \(d(i, j) = C(k)[d(i, k) \land d(j, k)]\), and (4) \(C(i)[p \land d(i, k)] \lor C(i)[p' \land d(i, k)] = 0\) whenever \(i, j, h, k\) are elements of \(I\) such that \(i \neq k\) and \(j \neq k\). The cylindric algebra will be denoted by \((A, I, C, d)\).

**Definition 4.** A cylindric algebra \((A, I, C, d)\) will be called **locally finite** if for each \(p \in A\), the set \(\{i \in I \mid C(i)p = p\}\) is cofinite.

We note that Definitions 3 and 4 are in an obvious way equivalent to the definitions given by Tarski in [3].

**Definition 5.** An \(e\)-algebra \((A, I, S, \exists, e)\) will be called **cylindricizable** if there exists a cylindric algebra \((A_1, I_1, C, d)\) such that \(A_1 = A\), \(I_1 = I\), \(d = e\), and \(C(i) = \exists(i)\) for all \(i \in I\).

**Definition 6.** A cylindric algebra \((A, I, C, d)\) will be called **equalizable** if there exists an \(e\)-algebra \((A_1, I_1, S, \exists, e)\) such that \(A_1 = A\), \(I_1 = I\), \(e = d\), \(S(i/j)p = C(i)[p \land d(i, j)]\) whenever \(i \neq j\), and \(\exists(i) = C(i)\) for all \(i \in I\).

Let \((A, I, S, \exists, e)\) be an \(e\)-algebra. We shall need the following lemmas.

**Lemma 1.** Whenever \(i \neq j\), \(S(i/j)p = \exists(i)[p \land e(i, j)]\) for all \(p \in A\).

**Proof.** \(\exists(i)[p \land e(i, j)] \leq \exists(i)S(i/j)p = S(i/j)p\), since \(i \neq j\). Also, \(S(i/j)p = S(i/j)p \land e(j, j) = S(i/j)[p \land e(i, j)] \leq S(i/j) \exists(i)[p \land e(i, j)] = \exists(i)[p \land e(i, j)].\)

**Lemma 2.** For all \(i, j \in I\), \(e(i, j) = e(j, i)\).
Proof. By symmetry, it is sufficient to show that \( e(i, j) \leq e(j, i) \) for all \( i, j \in I \). But \( e'(i, i) \wedge e(i, j) \leq S(i/j)e'(j, i) = e'(j, j) = 0 \).

Theorem 1. Every \( e \)-algebra is cylindrizable.

Proof. Let \( (A, I, S, \exists, e) \) be an \( e \)-algebra. We define \( d = e \), and let \( C(k) = \exists(k) \) for all \( k \in I \). It is clear that \( C \) maps \( I \) into quantifiers on \( A \) which commute, and that \( d(i, i) = e(i, i) = 1 \) for all \( i \in I \). If \( i \neq k \), \( j \neq k \), then, by Lemmas 1 and 2, \( C(k) [d(i, k) \wedge d(j, k)] = \exists(k) [e(i, k) \wedge e(j, k)] = S(k/j)e(i, k) = e(i, j) = d(i, j) \). Finally, if \( i \neq k \) and \( p \in A \), we have \( \exists(i) [p \wedge d(i, k)] \wedge \exists(i) [p' \wedge d(i, k)] = S(i/k)p \wedge S(i/k)p' = S(i/k)(p \wedge p') = 0 \), so that \( (A, I, C, d) \) is a cylindric algebra.

4. Cylindric algebras. Let \( (A, I, C, d) \) be a locally finite cylindric algebra with \( I \) infinite. We shall show that \( (A, I, C, d) \) is equalizable. We let \( e = d \), \( \exists(\emptyset)p = p \), \( \exists(j) = C(j) \), \( S(j/i)p = p \) and \( S(i/k)p = C(i)[p \wedge d(i, k)] \) whenever \( p \in A \), and \( i, j, k \in I \) such that \( i \neq k \), and we define \( S(\tau)p \) for \( p \in A \) and \( \tau \) a finite transformation on \( I \), by finding a finite set of replacements of \( I \) by \( \{\tau_1, \ldots, \tau_n\} \) such that \( \tau = \tau_1 \cdots \tau_n \) on some finite support of \( p \) and letting \( S(\tau)p = S(\tau_1) \cdots S(\tau_n)p \). Such a finite set of replacements exists, as we have remarked above, but it will be necessary to show that the definition is unambiguous. If \( J = \{j_1, \ldots, j_n\} \) is a finite subset of \( I \), we define \( \exists(J) \) by the equation \( \exists(J) = C(j_1) \cdots C(j_n) \). Since the values of \( C \) commute, and since (as is easily verified) the product of two commuting quantifiers is again a quantifier, \( \exists(J) \) is unambiguously defined and is a quantifier.

The proofs of the next four lemmas consist of straightforward computations, and are omitted.

Lemma 3. If \( i \neq j \), then \( S(i/j) \) is a Boolean endomorphism on \( A \).

Lemma 4. (1) Whenever \( i \neq j, k \neq j \), \( S(i/k)\exists(j) = \exists(j)S(i/k) \), (2) \( S(j/i)\exists(j) = \exists(j) \) for all \( i, j \in I \), and (3) \( \exists(j)S(j/i) = S(j/i) \) whenever \( i \neq j \).

Lemma 5. If \( i, j, k, h \) are distinct elements of \( I \), then (1) \( S(i/j)S(k/h) = S(k/h)S(i/j) \), (2) \( S(k/h)S(k/j) = S(k/j) \), (3) \( S(k/j)S(k/j) = S(k/j) \), (4) \( S(i/j)S(k/i) = S(k/j)S(i/j) \), (5) \( S(i/j)S(k/j) = S(k/j)S(i/j) \).

Lemma 6. If \( \exists(j)p = p \), then \( S(j/i)S(i/j)p = p \) whenever \( p \in A \) and \( i, j \in I \).

Definition 7. Let \( \alpha \) be an ordered collection consisting of an even number of replacements on \( I \), say \( \alpha = \{\alpha_1, \ldots, \alpha_{2n}\} \), \( n \geq 0 \), and \( J \) a finite subset of \( I \). We shall say that \( \alpha \) is \( J \)-normal if there are distinct
elements \( k_1, \ldots, k_n \in J \), distinct elements \( i_1, \ldots, i_n \in I - J \), and (not necessarily distinct) elements \( j_1, \ldots, j_n \in J \) such that (1) \( \alpha_r = (i_r/j_r) \), and (2) \( \alpha_{n+r} = (k_r/i_r) \), \( r = 1, 2, \ldots, n \).

**Definition 8.** If \( \alpha = \{\alpha_1, \ldots, \alpha_n\} \) and \( \beta = \{\beta_1, \ldots, \beta_m\} \) are finite ordered collections of replacements on \( I, J \) any subset of \( I \), and \( \rho \in A \), we will say that \( \alpha \) and \( \beta \) are \((\rho, J)\)-equivalent if (1) \( \alpha_1 \cdots \alpha_n j = \beta_1 \cdots \beta_m j \) whenever \( j \in J \), and (2) \( S(\alpha) \rho = S(\beta) \rho \), where \( S(\alpha) = S(\alpha_1) \cdots S(\alpha_n) \) and \( S(\beta) = S(\beta_1) \cdots S(\beta_m) \). If \( \alpha \) and \( \beta \) are \((\rho, I)\)-equivalent for every \( \rho \in A \), we shall say that \( \alpha \) and \( \beta \) are equivalent.

**Definition 9.** If \((i/j)\) is a replacement on \( I \), we shall refer to \( i \) as the essential domain of \((i/j)\), and to \( j \) as the essential range of \((i/j)\).

Lemma 7 enables one to study the effect of a finite transformation \( \tau \) on a finite set by examining the image of each element separately. The method is one commonly used in mathematical logic; i.e., mapping the element \( i \) first into another element \( j \) far from the scene of the action, and then mapping \( j \) into \( \tau(i) \).

**Lemma 7.** Let \( \alpha \) be a finite ordered collection of replacements, \( \rho \in A \), and \( J \) a finite support of \( \rho \) which contains all essential domains and essential ranges of elements of \( \alpha \). Then there exists a finite ordered collection \( \phi \) of replacements which is \( J \)-normal and \((\rho, J)\)-equivalent to \( \alpha \). Moreover, if \( \phi = \{\phi_1, \cdots, \phi_{2m}\} \), the essential domains of \( \phi_1, \cdots, \phi_m \) may be chosen arbitrarily from \( I - J \), provided they are distinct, and the essential domains of \( \phi_{m+1}, \cdots, \phi_{2m} \) are all elements of \( J \).

**Proof.** The proof consists of successively transforming \( \alpha \) into various ordered collections, the last of which is \( \phi \), with the property that each is \((\rho, J)\)-equivalent to the preceding one. The details are omitted.

Lemma 8 states essentially that if two transformations agree on a (finite) set \( P \) which supports an element \( q \) of \( A \), except possibly on a subset \( K \) of \( P \) of which \( q \) is independent, then they produce the same effect on \( q \).

**Lemma 8.** Let \( \alpha = \{\alpha_1, \cdots, \alpha_n\} \) and \( \alpha^* = \{\alpha_1^*, \cdots, \alpha_m^*\} \) be finite ordered collections of replacements on \( I, \rho \in A, P \) a finite support of \( \rho \), and \( K \) any finite subset of \( I \), such that \( \alpha_1 \cdots \alpha_n j = \alpha_1^* \cdots \alpha_m^* j \) whenever \( j \in P - K \). Then \( S(\alpha) \exists(K) \rho = S(\alpha^*) \exists(K) \rho \) (cf. Definition 8).

**Proof.** Applying Lemma 7 to \( \alpha \) and \( \alpha^* \) and a finite set \( J \) which contains \( P \) and satisfies the hypotheses of Lemma 7 with respect to \( \alpha \) and \( \alpha^* \), we obtain finite ordered collections \( \beta \) and \( \beta^* \). These in turn can be transformed into collections \( \gamma \) and \( \gamma^* \) such that \( \gamma = \gamma^* \). Since \( \alpha, \beta, \gamma \) and \( \alpha^*, \beta^*, \gamma^* \) are seen to be \((\exists(K) \rho, J)\)-equivalent, it
Corollary 1. The definition of \( S(\tau) \) is unambiguous for every finite transformation \( \tau \).

Proof. Let \( \alpha = \{\alpha_1, \ldots, \alpha_n\} \) and \( \alpha^* = \{\alpha_1^*, \ldots, \alpha_m^*\} \) be finite ordered collections of replacements such that \( \alpha_1 \cdots \alpha_n = \tau \) on the finite support \( Q \) of \( p \), and \( \alpha_1^* \cdots \alpha_m^* = \tau \) on the finite support \( Q^* \) of \( p \). Then \( P = Q \cap Q^* \) is a finite support of \( p \), and \( \alpha_1 \cdots \alpha_n = \tau = \alpha_1^* \cdots \alpha_m^* \) on \( P \). We apply Lemma 8 with \( K = \emptyset \), to obtain \( S(\alpha)p = S(\alpha^*)p \).

Corollary 2. If \( \sigma, \tau \) are finite transformations on \( I \) which agree outside a finite set \( K \), then \( S(\sigma)\exists(K) = S(\tau)\exists(K) \).

Proof. Let \( p \in A \). We find a finite ordered collection \( \alpha = \{\alpha_1, \ldots, \alpha_n\} \) of replacements such that \( \alpha_1 \cdots \alpha_n = \sigma \) on a finite support \( P_1 \) of \( p \), and a finite ordered collection \( \beta = \{\beta_1, \ldots, \beta_m\} \) such that \( \beta_1 \cdots \beta_m = \tau \) on a finite support \( P_2 \) of \( p \). Then \( \alpha_1 \cdots \alpha_n = \beta_1 \cdots \beta_m \) on \( P = P_1 \cap P_2 \). It follows from Lemma 8 that \( S(\sigma)\exists(K)p = S(\alpha)\exists(K)p = S(\beta)\exists(K)p = S(\tau)\exists(K)p \).

Lemma 9. Let \( \tau \) be a finite transformation on \( I \), \( J \) a finite subset of \( I \). If \( \tau \) is one-to-one on \( \tau^{-1}J \), then \( S(\tau)\exists(\tau^{-1}J) = \exists(J)S(\tau) \).

Proof. If \( J = \emptyset \), the lemma is trivial. Assume first that \( J = \{j\} \). Let \( p \in A \), and let \( \alpha \) be a finite ordered collection of replacements, \( \alpha = \{\alpha_1, \ldots, \alpha_n\} \), such that (1) \( \alpha_1 \cdots \alpha_n = \tau \) on a support \( K_1 \) of \( p \), and (2) \( (\alpha_1 \cdots \alpha_n)^{-1}j = \tau^{-1}j \). (It is possible to find such a collection, for example, by considering \( \tau \mid (I - J) \).) Let \( K \) be a finite support of \( p \) which includes all essential domains and essential ranges of elements of \( \alpha \), as well as \( j \) and \( k = \tau^{-1}j = (\alpha_1 \cdots \alpha_n)^{-1} j \). By Lemma 7, we can find a finite ordered collection of replacements \( \beta = \{\beta_1, \ldots, \beta_{2m}\} \) which is \( K \)-normal and \( (\exists(j)p, K) \)-equivalent to \( \alpha \). The proof for \( J = \{j\} \) then follows from Lemmas 4, 5, and 6 by consideration of the cases \( j = k \) and \( j \neq k \).

If \( J = \{j_1, \ldots, j_n\}, n \geq 1 \), and the lemma holds for all sets \( J_1 \) with fewer than \( n \) elements, let \( \tau \) be one-to-one on \( \tau^{-1}J \). Then \( \tau \) is one-to-one on \( \tau^{-1}j_1 \) and \( \tau^{-1}J_1 \), where \( J_1 = J - \{j_1\} \), and \( \exists(J)S(\tau) = \exists(J_1)S(\tau) = \exists(J_1)S(\tau)S(\tau^{-1}j_1) = S(\tau)S(\tau^{-1}J_1) = S(\tau)S(\tau)S(\tau^{-1}j_1) = S(\tau)S(\tau)S(\tau)S(\tau) \).

We are now in a position to prove the principal theorem.

Theorem 2. Every locally finite cylindrical algebra of infinite degree is equalizable.
Proof. Let \((A, I, C, d)\) be a locally finite cylindric algebra with \(I\) infinite. We define \(e, S, \text{ and } \exists(j)\) for \(j \in I\) as in Definition 6, and for \(J\) finite, \(J = \{j_1, \ldots, j_n\}\), we define \(\exists(J) = C(j_1) \cdots C(j_n)\). It follows from our earlier remarks and Corollary 1 to Lemma 8 that the definitions of \(\exists(J)\) and \(S(\tau)\) are unambiguous and that \(\exists(J)\) is a quantifier for any finite subset \(J\) of \(I\) and any finite transformation \(\tau\) on \(I\). An easy induction based on Lemma 3 shows that \(S(\tau)\) is a Boolean endomorphism for any finite \(\tau\). We shall see that the postulates for a quasi-polyadic algebra are satisfied by \((A, I, S, \exists)\), and it will follow from the theorem on quasi-polyadic algebras quoted above that \((A, I, S, \exists)\) determines a unique polyadic algebra.

Since \(S(j/j)p = p\) for all \(j \in I\) and \(p \in A\), it follows that \(Q_1\) holds. Postulates \(Q_2, Q_3, Q_4,\) and \(Q_7\) follow immediately from the definitions, and \(Q_6\) is Corollary 2 to Lemma 8, while \(Q_8\) is Lemma 9. We must show that \(e\) is a binary predicate satisfying conditions (1) and (2) of Definition 2. Since \(e = d\), we know that \(e\) maps \(IXI\) into \(A\), and \(e(i, i) = 1\) for all \(i \in I\). To show that \(S(\tau)e(i, j) = e(\tau i, \tau j)\), it follows from the definition of \(S\) that it is sufficient to verify the equation \(S(k/h)e(i, j) = e((k/h)i, (k/h)j)\) for all \(i, j, h, k \in I\). If \(k \notin \{i, j\}\) or if \(k = h\), then the equation holds trivially, since \(e(i, j)\) is supported by \(\{i, j\}\). Suppose, then, that \(k = i, k \neq h\). Then \(S(k/h)e(i, j) = \exists(i)[d(i, j) \land d(i, h)] = \exists(i)[d(j, i) \land d(h, i)] = d(j, h) = d(h, j) = e((k/h)i, (k/h)j)\). The case \(k = j, k \neq h\) is similar. Now suppose \(p \in A\). Then \(p \land e(i, j) \leq \exists(i)[p \land d(i, j)] = S(i/j)p\) whenever \(i \neq j\), and the inequality holds trivially when \(i = j\).

Theorem 3. Let \(\mathfrak{A} = (A, I, S, \exists, e)\) be a locally finite e-algebra of infinite degree. Let \(\mathfrak{B} = (A, I, C, d)\) be the locally finite cylindric algebra of infinite degree arising from \(\mathfrak{A}\) by means of Definition 5 (cf. Theorem 1). Let \(\mathfrak{B}_1\) be the e-algebra arising from \(\mathfrak{B}\) by means of Definition 6 (cf. Theorem 2). Then \(\mathfrak{B}_1 \cong \mathfrak{A}\).

Proof. Let \(\mathfrak{A}_1 = (A, I, \exists_1, S_1, e_1)\). It follows from definitions that \(\exists_1(k) = C(k) = \exists(k)\) for all \(k \in I\), and therefore that \(\exists_1(J) = \exists(J)\) for any finite \(J\). Also, we have \(e_1 = d = e\). If \(p \in A\) and \(i, j \in I, i \neq j\), then \(S_1(i/j)p = C(i)[p \land d(i, j)] = \exists(i)[p \land e(i, j)] = S(i/j)p\) by the definition of \(S_1\) and Lemma 1. An easy induction shows that \(S_1(\tau) = S(\tau)\) for any finite \(\tau\). The theorem follows from the uniqueness assertion of the theorem on quasi-polyadic algebras.

Theorem 4. Let \(\mathfrak{B} = (A, I, C, d)\) be a locally finite cylindric algebra of infinite degree. Let \(\mathfrak{A} = (A, I, S, \exists, e)\) be the e-algebra arising from \(\mathfrak{B}\) by means of Definition 6 (cf. Theorem 2). Let \(\mathfrak{B}_1 = (A, I, C_1, d_1)\) be the
cylindric algebra arising from \( \mathfrak{A} \) by means of Definition 5 (cf. Theorem 1). Then \( \mathfrak{B} = \mathfrak{B}_1 \).

Proof. From the definitions, we have \( d_1 = e = d \), and \( C_1(k) = \exists(k) = C(k) \) for all \( k \in I \).

Bibliography


University of Chicago