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SUBSETS OF AN ABSOLUTE RETRACT

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Introduction. In 1939 [**6**] Wojdyslawski asked whether the space $S(X)$ of subsets of an absolute retract X is an absolute retract. This was answered in 1939 [**6**] for the case where X is a bounded closed interval and in 1955 [**2**] for a Peano space. In this note the question is answered for all compact Hausdorff spaces. The author wishes to express her appreciation for the suggestions of Professor W. L. Strother.

Preliminaries. If Y is a space then $S(Y)$ denotes the set of non-null closed subsets of Y with the usual topology [**3**]. (The results of this paper are valid for $S(Y)$ topologized in any other way in which the united extension defined in the following paragraph preserves continuity.) A compact space Y is called a CAR^* if every continuous function from a closed subset M of a normal space N to Y can be extended to a continuous function from N to Y. A definition for M-CAR^* is obtained by replacing functions by multi-valued functions. The following are equivalent [**5**]: (1) X is a CAR^*, (2) X is homeomorphic to a retract of a Tychonoff cube, (3) every cube in which X can be homeomorphically embedded as a closed subset X_0 can be retracted onto X_0.

A multi-valued function $F: X \to Y$ is said to be continuous at x_0 if (1) V open and $F(x_0) \cap V \neq \emptyset$ implies that there is an open set U containing x_0 such that, for all $x \in U$, $F(x) \cap V \neq \emptyset$ and (2) V open and $F(x_0) \subset V$ implies that there is an open set U containing x_0 such that $F(U) \subset V$. If $f: X \to Y$ is continuous then $f_s: S(X) \to S(X)$ defined by $f_s(A) = \bigcup \{f(a) \mid a \in A\}$ is called the **united extension** of f. $S_1(X)$ denotes $S(X)$ and $S_n(X)$ is defined inductively by $S_n(X) = S[S_{n-1}(X)]$.

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Retracts.

LEMMA. *If T is a Tychonoff cube then $S(T)$ is a CAR^*.*

PROOF. The space $S(T)$ is compact Hausdorff and hence homeomorphic to a closed subset of some Tychonoff cube R [1]. The cubes T and R are, respectively, homeomorphic to subsets of $S(T)$ and $S(R)$. Identifying homeomorphic sets, one can write $T \subset S(T) \subset R \subset S(R)$. Since T is a CAR^* contained in the cube R, there exists a retraction p of R onto T. If $f: X \to Y$ is continuous then the united extension $f_s: S(X) \to S(X)$ is continuous [2]. In case f is a retraction of X onto Y it follows that f_s is a retraction of $S(X)$ onto $S(Y)$. Hence corresponding to the retraction p of R onto T there is a retraction p_s of $S(R)$ onto $S(T)$. Then p_s restricted to R is a retraction of R onto $S(T)$. This shows that $S(T)$ is homeomorphic to a retract of a cube R and hence $S(T)$ is a CAR^*.

THEOREM. *If X is a CAR^* then $S(X)$ is a CAR^*.*

PROOF. Embed X in a cube T. Then X is a retract of T by definition of CAR^*. By united extension, $S(X)$ is a retract of $S(T)$. But $S(T)$ is a CAR^* by the lemma, and CAR^* is a retraction invariant [5]. Hence $S(X)$ is a CAR^*.

COROLLARY 1. *Every CAR^* is an M-CAR^*.*

PROOF. It is known [2] that X is an M-CAR^* if and only if $S(X)$ is a CAR^*.

COROLLARY 2. *If X is a CAR^* then $S_n(X)$ is a CAR^*.*

This theorem also provides a new proof for the theorem that the space of subsets of a CAR^* has the fixed point property [4]. Whether X having the fixed point property implies that $S(X)$ has the fixed point property remains open.

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