R. H. Bing on p. 341 of [1] raises the following question:

**Question.** Does there exist a positive integer \( n \) such that the following result holds for each continuous curve \( M \), each positive number \( \epsilon \), and each pair of mutually exclusive closed subsets \( H \) and \( K \) of \( M \)? If \( R \) is a finite subset of \( M \) such that each point of \( R \) belongs to an arc in \( M \) of diameter less than \( \epsilon \) that intersects \( H + K \), there are two collections \( A_H \) and \( A_K \) of arcs satisfying the following conditions:

(a) Each element of \( A_H \) intersects \( H \) but not \( K \) and each element of \( A_K \) intersects \( K \) but not \( H \) nor any element of \( A_H \). (b) Each element of \( R \) belongs to an element of \( A_H + A_K \). (c) Each element of \( A_H + A_K \) is of diameter less than \( n \epsilon \).

If for some integer \( n \) the answer is yes, then E. E. Moise's method of partitioning would be validated (see [2] and [3]). Also, a simple technique yielding an affirmative answer would allow a more direct proof of partitioning by Bing's method and could probably be used to advantage on other problems.

Bing [1] gives an example to show that the answer is no for \( n = 1 \). The present paper gives an example to show that the answer is no for \( n = 2 \).

In the example given below, a metric will be defined such that with this metric the given point set has the desired metric property. The example was originally considered with a homeomorphism to Euclidean three space, where the image had the desired property. The metric given here follows a suggestion of R. H. Bing.

**Example.** The example is described at the top of the following page.

**The metric.** Consider the example as a finite graph \( G \), the sum of a finite number of segments, \( s_1, s_2, \ldots, s_n \), such that if \( s_i \) and \( s_j \) have a point in common, \( s_i \cap s_j \) is an end point of both \( s_i \) and \( s_j \), and such that each element \( a_i \) is the sum of elements of \( s_1, s_2, s_3, \ldots, s_n \). Let each element \( s_i \) be of length slightly less than \( \epsilon \), say \( \epsilon - \delta \). Now if \( v_i, v_j \) are vertices of segments of \( G \), set \( d(v_i, v_j) = 0 \) if \( i = j \), \( d(v_i, v_j) = N(\epsilon - \delta) \) if \( i \neq j \), where \( N \) is the minimum number of \( a_i \)'s whose sum is a continuum containing \( v_i \) and \( v_j \). If \( x, y \) belongs to the same \( a_i \), let \( d(x, y) = \min \) (distance \( x \) to \( y \) along the line segment, \( \epsilon - \delta \)).

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Concerning partitioning

Example

\[ R = \sum_{i=1}^{21} p_i \quad H = h_i (i = 1, 2, 3, 7, 8, 9, 16, 17, 18) \]
\[ K = k_i (i = 4, 5, 6, 10, 11, 12, 13, 14, 15, 19, 20, 21) \]
\[ a_i = \text{the arc } p_i \text{ to } h_i \text{ (or } k_i) \], \( i = 1 \) to 21.

Then for arbitrary \( x, y \) in \( G \) set

\[ d(x, y) = \min (d(x, p) + d(p, q) + d(q, y)) \]

where minimum is taken over all \( p, q \) where \( d(p, q) \) has already been defined.

**Indication of a proof.** Assume that \( n \) is two. Consider arcs \( a_1, a_2, a_3 \) (from \( p_1 \) to \( h_1 \), \( p_2 \) to \( h_2 \), and \( p_3 \) to \( h_3 \)). Note that since \( n = 2 \), \( p_3 \) must belong to an arc in \( A_H \) that is a subset of \( a_3 \) or \( a_4 + a_1 \). Then \( p_1 \) belongs to an arc of \( A_H \) as does \( p_2 \). This means that either:

- **Case I.** \( a_1 \) belongs to \( A_{H^*} \); or,
- **Case II.** \( a_2 \) belongs to \( A_{H^*} \).

Suppose Case I. Then \( a_4 \) must belong to \( A_{K^*} \), \( a_7 \) to \( A_{H^*} \), and \( a_{10} \) to \( A_{K^*} \), but \( a_{10} \) intersects \( a_1 \), which gives a contradiction. A similar line of reasoning leads to a contradiction for Case II.

**References**