ON AN AXIOM OF BOURBAKI

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It is the purpose of this paper to show that Axiom A2 in Bourbaki’s axiomatic system for set theory can be replaced by the weaker statement that every term $x$ defines a set $\{x\}$ of which $x$ is the only element. All references in the paper are to [1], and the terminology and notation of [1] are used.

We place ourselves into a theory with specific signs $=$ and $\in$, in which S1 through S8 are schemas, and A1, A4 and the statement

$$(\forall x) \text{Coll}_y (y = x)$$

are axioms. The theory may contain other specific signs, schemas and axioms.

Axiom (1) allows us to define a set $\{x\} = E_y (y = x)$. Criterion C51 (p. 65) then can be proved. It follows that there is a set $\phi$ such that $(\forall x)(x \in \phi)$ is true. Sets $\{\phi\}$ and, using A4, $\mathcal{P}(\{\phi\})$ can then be constructed. It follows that $\phi \in \mathcal{P}(\{\phi\})$, $\{\phi\} \in \mathcal{P}(\{\phi\})$, and $\phi \neq \{\phi\}$.

Let now $R$ in S8 (p. 64) be the relation

$$(x = u \text{ and } y = \phi) \text{ or } (x = v \text{ and } y = \{\phi\}).$$

The relation

$$(\forall y)(\exists X)(\forall x)(R \Rightarrow (x \in X))$$

is true, as we may put $X = \{u\}$ for $y = \phi$ and $X = \{v\}$ for $y \neq \phi$. Then

$$\text{Coll}_x ((\exists y)(y \in \mathcal{P}(\{\phi\}) \text{ and } R))$$

is true, by S8 and C30 (p. 37). Since

$$(\exists y)(y \in \mathcal{P}(\{\phi\}) \text{ and } R) \iff (x = u \text{ or } x = v),$$

we have $\text{Coll}_x (x = u \text{ or } x = v)$, and hence, using C27 (p. 36),

$$(\forall u)(\forall v) \text{Coll}_x (x = u \text{ or } x = v).$$

This is a restatement of A2.

REFERENCE


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