R. L. Moore's Axiom 1' and Metrization

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Let $S$ be a Hausdorff space for which there exists a simple sequence $G_1, G_2, \cdots$ of open coverings such that (1) for each $n$, $G_n \supseteq G_{n+1}$, and (2) if $H$ and $K$ are nonintersecting closed subsets of $S$ one of which is compact, then for some $n$ no element of $G_n$ intersects both $H$ and $K$. At the 1957 Summer Meeting of the Society the question arose in connection with Mr. Armentrout's paper, A study of certain plane-like spaces without the use of arcs, as to whether or not $S$ when satisfying certain rather complicated axioms was metric. I remarked that there did exist such nonmetric spaces. This observation was incorrect.

**Theorem.** The space $S$ is metric.

**Proof.** Let $p$ be a point of an open set $R$. There exists a natural number $n$ such that if $g, h \in G_n$, $p \in g$, and $g \cdot h \neq 0$, then $g + h \subseteq R$. For suppose, on the contrary, that for each natural number $n$, there exist $g_n, h_n \in G_n$, $p \in g_n$, $g_n \cdot h_n \neq 0$ and $(g_n + h_n) \cdot (S - R) \neq 0$; let $p_n$ be a point of $g_n \cdot h_n$. Obviously $p_1, p_2, \cdots$ converges to $p$. Let $H = R \cdot (p + p_1 + p_2 + \cdots)$ and let $K = S - R$. Both $H$ and $K$ are closed and $H$ is compact. Furthermore, for each $n$ some element of $G_n$ intersects both $H$ and $K$. This is a contradiction.

It now follows from Moore's metrization theorem [1] that $S$ is metric.

**References**


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Presented to the Society, November 30, 1957; received by the editors, November 9, 1957.

1 A National Science Foundation Senior Postdoctoral Fellow.
2 Cf., Moore's Axiom 1' in [2, p. 324].
3 Abstract number 797, Bull. Amer. Math. Soc. vol. 63 (1957) p. 403