\[ H^*(\Omega G_2, \mathbb{Z}) \cong \mathbb{Z}[x, h(x), t(x)] \otimes \mathbb{Z}\langle y \rangle \]

where \( h(x) = \infty \) and \( t(x) \) is defined by the greatest divisors \( g(x^n) = n! / g(u^n) \). In particular, \( g(x^2) = 1 \) so that (1.15) fails for \( x^2 \) and \( H^*(\Omega G_2, \mathbb{Z}) \) has no system of divided powers.

**Bibliography**


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**ANOTHER CUTPOINT THEOREM FOR PLANE CONTINUA**

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If the subcontinuum \( M \) of a topological 2-sphere \( S \) does not separate \( S \) and is *locally connected*, then each pair of points of \( M \), which are not separated in \( M \) by a point of \( M \), belongs to the closure of a connected domain (of \( S \)) lying in \( M \). This is true because each such pair of points belongs to a simple closed curve \( J \) lying in \( M \) and one of the complementary domains of \( J \) is a subset of \( M \). However, without local connectedness such a simple closed curve may fail to exist. In fact, the proposition would then be false because (to take an extreme case) of the existence of indecomposable subcontinua of \( S \) which fail to separate \( S \). While no point of an indecomposable continuum *separates* it, every point of it *cuts* it. Recently I showed \[1\] that this stronger form of separation (or rather the lack of it) is sufficient to restore the validity of the above proposition in the absence of local connectedness if a certain restriction were placed upon

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the pair of points under consideration. It is the purpose of this paper to remove even this restriction.

Notation and terminology. If $p$ and $q$ are points of a continuum $M$ and $x$ is a point of $M - (p + q)$, $x$ is said to cut $p$ from $q$ in $M$ if every subcontinuum of $M$ which contains $p + q$ contains $x$. By an interior point of $M$ is meant a nonboundary point of $M$. By "the plane" is meant the Euclidean number plane with $d$ denoting the usual Pythagorean distance function.

Theorem. Suppose that $M$ is a compact subcontinuum of the plane $S$ which does not separate $S$. If no point of $M$ cuts the point $p$ from the point $q$ in $M$ then some component of the set of interior points of $M$ contains both $p$ and $q$ in its closure.

Indication of proof. If either $p$ or $q$ is an interior point of $M$, the theorem follows from a previous result [1]. So we have left to prove the theorem for the case when both $p$ and $q$ are boundary points of $M$.

Suppose that $\varepsilon$ is a positive number such that $2\varepsilon < d(p, q)$. Let $C_p(\varepsilon)$ and $C_q(\varepsilon)$ denote circles of radius $\varepsilon$ centered on $p$ and $q$ respectively. There exists a simple domain $I(\varepsilon)$ which contains $M$ such that if $J(\varepsilon)$ denotes the boundary of $I(\varepsilon)$, $y$ is a boundary point of $M$ and $z$ is a point of $I(\varepsilon) + J(\varepsilon)$ then $d[y, J(\varepsilon)] < \varepsilon$ and $d(z, M) < \varepsilon$. There exist arcs $T_p(\varepsilon)$ and $T_q(\varepsilon)$ in $C_p(\varepsilon)$ and $C_q(\varepsilon)$ respectively such that each is minimal with respect to separating $p$ from $q$ in $I(\varepsilon) + J(\varepsilon)$ and $T_p(\varepsilon)$ separates $p$ from $q + T_q(\varepsilon)$ in $I(\varepsilon) + J(\varepsilon)$. It follows that $T_q(\varepsilon)$ separates $p + T_p(\varepsilon)$ from $q$ in $I(\varepsilon) + J(\varepsilon)$.

Since $T_p(\varepsilon)$ and $T_q(\varepsilon)$ have only their endpoints in $J(\varepsilon)$, there exist in $J(\varepsilon)$ two nonintersecting arcs $A(\varepsilon)$ and $B(\varepsilon)$ such that $T_p(\varepsilon) + A(\varepsilon) + T_q(\varepsilon) + B(\varepsilon)$ is a simple closed curve $H(\varepsilon)$. Let $D(\varepsilon)$ denote the bounded complementary domain of $H(\varepsilon)$. If $z$ is a point of $D(\varepsilon) + H(\varepsilon)$, then $d(z, M) < \varepsilon$. Any subcontinuum of $M$ which contains $p + q$ contains a subcontinuum irreducible from $T_p(\varepsilon)$ to $T_q(\varepsilon)$ which lies in $T_p(\varepsilon) + D(\varepsilon) + T_q(\varepsilon)$.

Now let $L(\varepsilon)$ denote a continuum lying in $T_p(\varepsilon) + D(\varepsilon) + T_q(\varepsilon)$ which intersects both $T_p(\varepsilon)$ and $T_q(\varepsilon)$ such that if $z$ belongs to $L(\varepsilon)$, then $d[z, A(\varepsilon)] = d[z, B(\varepsilon)]$. There exists a simple infinite sequence $\alpha$ of values of $\varepsilon$ such that $D(\varepsilon) + H(\varepsilon)$ converges to a subset of $M$ and $L(\varepsilon) \to L$ as $\varepsilon \to 0$ in $\alpha$. The set $L$ has the following properties:

(a) $L$ is a continuum containing both $p$ and $q$,
(b) $L$ is a subset of $M$, and
(c) every point of $L - (p + q)$ is an interior point of $M$. Properties (a) and (b) are evident. So it remains only to prove property (c).

Let $x$ be a point of $L - (p + q)$. Since $x$ does not cut $p$ from $q$ in
there exists a subcontinuum $K$ of $M$ which contains $p \oplus q$ but not $x$. Let $\delta$ be a positive number such that $4\delta = d(x, K)$ and let $U_\delta(x)$ and $U_{3\delta}(x)$ be the circular regions centered on $x$ of radius $\delta$ and $3\delta$ respectively. When $\epsilon$ (in $\alpha$) is sufficiently small $[T_p(\epsilon) + T_q(\epsilon)] \cdot [U_{3\delta}(x)] = 0$ but $L(\epsilon) \cdot U_\delta(x) \neq 0$. Let $y$ be some point of $L(\epsilon) \cdot U_\delta(x)$, let $r = \delta + d(x, y)$ and let $U_r(y)$ be a circular region of radius $r$ and center $y$. Obviously $U_{3\delta}(x) \supset U_r(y) \supset U_\delta(x)$. So $[T_p(\epsilon) + T_q(\epsilon)] \cdot U_r(y) = 0$. If $A(\epsilon) \cdot U_r(y) \neq 0$, let $f$ be a point of $A(\epsilon) \cdot U_r(y)$ such that $d(f, y) = d[y, A(\epsilon)]$. But $y$ belongs to $L(\epsilon)$. Hence there exists in $U_r(y)$ a point $g$ of $B(\epsilon)$ such that $d(g, y) = d[g, B(\epsilon)] = d(f, y)$. The sum of the straight line intervals from $y$ to $f$ and from $y$ to $g$ is an arc $T_y$ lying in $U_r(y)$, having only its endpoints $f$ and $g$ in $H(\epsilon)$, and containing the point $y$ of $D(\epsilon)$. Hence $T_y - (f + g) \subset D(\epsilon)$. But $T_y \cdot K = 0$ and $K$ contains a continuum lying in $T_p(\epsilon) + D(\epsilon) + T_q(\epsilon)$ irreducible from $T_p(\epsilon)$ to $T_q(\epsilon)$. Since the points $f$ and $g$ separate $T_p(\epsilon)$ from $T_q(\epsilon)$ in $H(\epsilon)$ this involves a contradiction [2, Theorem 17, p. 167]. Hence $U_r(y) \cdot H(\epsilon) = 0$ and since $y$ belongs to $D(\epsilon)$, $U_r(y) \subset D(\epsilon)$; so for sufficiently small values of $\epsilon$ (in $\alpha$), $U_\delta(x) \subset D(\epsilon)$. Consequently $U_\delta(x)$ is a subset of $M$ and $x$ is an interior point of $M$.

The continuum $L$ contains a subcontinuum $N$ which is irreducible from $p$ to $q$; $N - (p \oplus q)$ is connected and each of its points is an interior point of $M$. The component of the set of interior points of $M$ which contains $N - (p \oplus q)$ has both $p$ and $q$ in its closure.

**Counterexample.** The converse of the theorem is false. Let $L$ be the closure of the graph of $y = \sin 1/x$ ($-\pi \leq x \leq \pi$) together with one arc (the lower one) of the square whose vertices are $(\pm \pi, \pm \pi)$ so that this arc joins the endpoints of the graph, and let $M$ denote $L$ together with its bounded complementary domain $D$. Obviously $D = M$ but $(0, -1)$ cuts $(0, -\pi)$ from $(0, 1)$ in $M$.

**Bibliography**


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