ON ENTIRE FUNCTIONS DEFINED BY A DIRICHLET SERIES: CORRECTION

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1. As pointed out by Sunyer i Balaguer in the preceding paper the proofs of Theorem 1 and of the second part of Theorem 2 of our paper [1] are faulty. We observe that if we impose the additional hypothesis that \( M_S(\sigma) = \max_{|t-t_0| \leq \pi a} |f(\sigma + it)| (a > D) \), is a nonincreasing function for sufficiently small \( \sigma \) then the proofs can be made to work. After correction Theorem 1 and the second part of Theorem 2 may be stated as follows.

**Theorem A.** If \( M_S(\sigma) = \max_{|t-t_0| \leq \pi a} |f(\sigma + it)| (a > D) \), is a nonincreasing function for sufficiently small \( \sigma \) then the lower order \( \lambda_S \) of \( f(s) \) in each horizontal strip \( S(\pi a) \), with \( a > D \), is equal to the lower order \( \lambda \) of \( f(s) \).

**Theorem B.** If \( h = \infty \), \( M_S(\sigma) = \max_{|t-t_0| \leq \pi a} |f(\sigma + it)| (a > 0) \) is a nonincreasing function for sufficiently small \( \sigma \) then the lower type \( \tau_S \) of \( f(s) \) in each horizontal strip \( S(\pi a) \), with \( a > 0 \), satisfies \( \tau_S \leq e^{-\pi a \tau} \).

2. Proof of Theorem A. In the notations of [1], \( \sigma_j^* = \sigma_j + k_j \), where \( |k_j| \leq \pi a \). By hypothesis \( M_S(\sigma) = \max_{|t-t_0| \leq \pi a} |f(\sigma + it)| (a > D) \) is nonincreasing for sufficiently small \( \sigma \) and therefore for \( \sigma_j < \sigma' \) [1, p. 215, line 3 (correcting the obvious misprint)]

\[
\log M_S(\sigma_j - \pi a) \geq \log M_S(\sigma_j^*) \geq \log \mu(\sigma_j + P) - K > \log M(\sigma_j + P + \epsilon) - \log K_1 - K.
\]

We can now conclude that \( \lambda_S \geq \lambda \). The fact that \( \lambda_S \leq \lambda \) completes the proof.

Proof of Theorem B. \( M_S(\sigma) = \max_{|t-t_0| \leq \pi a} |f(\sigma + it)| (a > 0) \) is by assumption a nonincreasing function for sufficiently small \( \sigma \) and therefore for \( \sigma_j < \sigma'' \) [1, p. 215, line 18]

\[
\frac{\log M_S(\sigma_j - \pi a)}{e^{-\rho_S(\sigma_j - \pi a)}} \geq \frac{\log M_S(\sigma_j^*)}{e^{-\rho_S(\sigma_j - \pi a)}} \geq e^{-\rho(\pi a + \epsilon + \epsilon')} \log M(\sigma_j + \epsilon + \epsilon) - \log K_1 + K.
\]

It follows that \( \tau_S \leq e^{-\pi a \tau} \).

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By the method of Sunyer i Balaguer [see Theorem A of the preceding paper] Theorem B can be improved and we can prove:

**Theorem C.** Under the conditions of Theorem B, \( \tau_S = \tau \).

The following printing mistake in [1] may be noted:
In line 21 on page 215 \( e^{(z_1 - z') - t} \) may be corrected to read \( e^{(k_1 - z' - t)} \).

**Reference**


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**THE UNIVERSAL REPRESENTATION KERNEL OF A LIE GROUP**

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Let \( G \) be a connected real Lie group. The universal representation kernel, \( K_\sigma \), of \( G \) is defined as the intersection of all kernels of continuous finite dimensional representations of \( G \). Evidently, \( K_\sigma \) is a closed normal subgroup of \( G \), and it is known from a theorem due to Goto (cf. [1, Theorem 7.1]) that \( G/K_\sigma \) has a faithful continuous finite dimensional representation. Thus \( K_\sigma \) is the smallest normal closed subgroup \( P \) of \( G \) such that \( G/P \) is isomorphic with a real analytic subgroup of a full linear group. The known criteria for the existence of a faithful representation lead to a determination of \( K_\sigma \) which we wish to record here.

Suppose first that \( G \) is semisimple. Let \( \mathfrak{G} \) denote the Lie algebra of \( G \). Let \( C \) stand for the field of the complex numbers, and denote by \( \mathfrak{G}^C \) the complexification of \( \mathfrak{G} \), i.e., the semisimple Lie algebra over \( C \) that is obtained by forming the tensor product, over the real field, of \( \mathfrak{G} \) with \( C \). Denote by \( S(\mathfrak{G}) \) and \( S(\mathfrak{G}^C) \) the simply connected Lie groups whose Lie algebras are \( \mathfrak{G} \) and \( \mathfrak{G}^C \), respectively. The injection \( \mathfrak{G} \to \mathfrak{G}^C \) is the differential of a uniquely determined continuous homomorphism \( \gamma \) of \( S(\mathfrak{G}) \) into \( S(\mathfrak{G}^C) \). The kernel \( P \) of \( \gamma \) is a discrete central subgroup of \( S(\mathfrak{G}) \). Let \( \phi \) denote the covering epimorphism of \( S(\mathfrak{G}) \) onto \( G \). We claim that \( K_\sigma = \phi(P) \), i.e., the universal representation kernel of the semisimple connected Lie group \( G \) is the image, under the universal covering epimorphism, of the kernel of the canonical homomorphism \( S(\mathfrak{G}) \to S(\mathfrak{G}^C) \).

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