ON ENTIRE FUNCTIONS DEFINED BY A DIRICHLET SERIES: CORRECTION

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1. As pointed out by Sunyer i Balaguer in the preceding paper the proofs of Theorem 1 and of the second part of Theorem 2 of our paper [1] are faulty. We observe that if we impose the additional hypothesis that $M_S(\sigma) = \max_{t \in [-t_0, t_0]} |f(\sigma + it)| (a > D)$, is a nonincreasing function for sufficiently small $\sigma$ then the proofs can be made to work. After correction Theorem 1 and the second part of Theorem 2 may be stated as follows.

**Theorem A.** If $M_S(\sigma) = \max_{t \in [-t_0, t_0]} |f(\sigma + it)| (a > D)$, is a nonincreasing function for sufficiently small $\sigma$ then the lower order $\lambda_S$ of $f(s)$ in each horizontal strip $S(\pi a)$, with $a > D$, is equal to the lower order $\lambda$ of $f(s)$.

**Theorem B.** If $h = \infty$, $M_S(\sigma) = \max_{t \in [-t_0, t_0]} |f(\sigma + it)| (a > 0)$ is a nonincreasing function for sufficiently small $\sigma$ then the lower type $\tau_S$ of $f(s)$ in each horizontal strip $S(\pi a)$, with $a > 0$, satisfies $\tau_S \geq e^{-\rho a^2}$.

2. Proof of Theorem A. In the notations of [1], $\sigma_j^* = \sigma_j + k_j$, where $|k_j| \leq \pi a$. By hypothesis $M_S(\sigma) = \max_{t \in [-t_0, t_0]} |f(\sigma + it)| (a > D)$ is nonincreasing for sufficiently small $\sigma$ and therefore for $\sigma_j < \sigma_j^*$ [1, p. 215, line 3 (correcting the obvious misprint)]

$$
\log M_S(\sigma_j - \pi a) \geq \log M_S(\sigma_j^*) \geq \log \mu(\sigma_j + P) - K > \log M(\sigma_j + P + \epsilon) - \log K_1 - K.
$$

We can now conclude that $\lambda_S \geq \lambda$. The fact that $\lambda_S \leq \lambda$ completes the proof.

Proof of Theorem B. $M_S(\sigma) = \max_{t \in [-t_0, t_0]} |f(\sigma + it)| (a > 0)$ is by assumption a nonincreasing function for sufficiently small $\sigma$ and therefore for $\sigma_j < \sigma_j^*$ [1, p. 215, line 18]

$$
\frac{\log M_S(\sigma_j - \pi a)}{e^{-\rho a(\sigma_j - \pi a)}} \geq \frac{\log M_S(\sigma_j^*)}{e^{-\rho a(\sigma_j - \pi a)}} > e^{-\rho(\pi a + \epsilon')} \frac{\log M(\sigma_j + \epsilon' + \epsilon)}{e^{-\rho(\sigma_j + \epsilon' + \epsilon)}} - \frac{\log K_1 + K}{e^{-\rho a(\sigma_j - \pi a)}}.
$$

It follows that $\tau_S \geq e^{-\rho a^2}$.

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By the method of Sunyer i Balaguer [see Theorem A of the preceeding paper] Theorem B can be improved and we can prove:

**Theorem C.** Under the conditions of Theorem B, $\tau_S = \tau$.

The following printing mistake in [1] may be noted:
In line 21 on page 215 $e^{(t_1-t')e}$ may be corrected to read $e^{(k_1-k)'}$.

**Reference**


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**The Universal Representation Kernel of a Lie Group**

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Let $G$ be a connected real Lie group. The universal representation kernel, $K_G$, of $G$ is defined as the intersection of all kernels of continuous finite dimensional representations of $G$. Evidently, $K_G$ is a closed normal subgroup of $G$, and it is known from a theorem due to Goto (cf. [1, Theorem 7.1]) that $G/K_G$ has a faithful continuous finite dimensional representation. Thus $K_G$ is the smallest normal closed subgroup $P$ of $G$ such that $G/P$ is isomorphic with a real analytic subgroup of a full linear group. The known criteria for the existence of a faithful representation lead to a determination of $K_G$ which we wish to record here.

Suppose first that $G$ is semisimple. Let $\mathfrak{g}$ denote the Lie algebra of $G$. Let $C$ stand for the field of the complex numbers, and denote by $\mathfrak{g}^C$ the complexification of $\mathfrak{g}$, i.e., the semisimple Lie algebra over $C$ that is obtained by forming the tensor product, over the real field, of $\mathfrak{g}$ with $C$. Denote by $S(\mathfrak{g})$ and $S(\mathfrak{g}^C)$ the simply connected Lie groups whose Lie algebras are $\mathfrak{g}$ and $\mathfrak{g}^C$, respectively. The injection $\mathfrak{g} \to \mathfrak{g}^C$ is the differential of a uniquely determined continuous homomorphism $\gamma$ of $S(\mathfrak{g})$ into $S(\mathfrak{g}^C)$. The kernel $P$ of $\gamma$ is a discrete central subgroup of $S(\mathfrak{g})$. Let $\phi$ denote the covering epimorphism of $S(\mathfrak{g})$ onto $G$. We claim that $K_G = \phi(P)$, i.e., the universal representation kernel of the semisimple connected Lie group $G$ is the image, under the universal covering epimorphism, of the kernel of the canonical homomorphism $S(\mathfrak{g}) \to S(\mathfrak{g}^C)$.

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