ON ENTIRE FUNCTIONS DEFINED BY A DIRICHLET SERIES: CORRECTION

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1. As pointed out by Sunyer i Balaguer in the preceding paper the proofs of Theorem 1 and of the second part of Theorem 2 of our paper [1] are faulty. We observe that if we impose the additional hypothesis that $M_S(\sigma)=\max_{|t-t_0|\leq\pi a}|f(\sigma+it)|\,(a>D)$, is a nonincreasing function for sufficiently small σ then the proofs can be made to work. After correction Theorem 1 and the second part of Theorem 2 may be stated as follows.

THEOREM A. *If $M_S(\sigma)=\max_{|t-t_0|\leq\pi a}|f(\sigma+it)|\,(a>D)$, is a nonincreasing function for sufficiently small σ then the lower order λ_S of $f(s)$ in each horizontal strip $S(\pi a)$, with $a>D$, is equal to the lower order λ of $f(s)$.*

THEOREM B. *If $h=\infty$, $M_S(\sigma)=\max_{|t-t_0|\leq\pi a}|f(\sigma+it)|\,(a>0)$ is a nonincreasing function for sufficiently small σ then the lower type τ_S of $f(s)$ in each horizontal strip $S(\pi a)$, with $a>0$, satisfies $\tau_S\geqq e^{-\pi\rho a}\tau$.*

2. PROOF OF THEOREM A. In the notations of [1], $\sigma_j^*=\sigma_j+k_j$, where $|k_j|\leqq\pi a$. By hypothesis $M_S(\sigma)=\max_{|t-t_0|\leq\pi a}|f(\sigma+it)|\,(a>D)$ is nonincreasing for sufficiently small σ and therefore for $\sigma_j<\sigma'$ [1, p. 215, line 3 (correcting the obvious misprint)]

$$\log M_S(\sigma_j-\pi a)\geqq\log M_S(\sigma_j^*)\geqq\log\mu(\sigma_j+P)$$
$$-K>\log M(\sigma_j+P+\epsilon)-\log K_1-K.$$

We can now conclude that $\lambda_S\geqq\lambda$. The fact that $\lambda_S\leqq\lambda$ completes the proof.

PROOF OF THEOREM B. $M_S(\sigma)=\max_{|t-t_0|\leq\pi a}|f(\sigma+it)|\,(a>0)$ is by assumption a nonincreasing function for sufficiently small σ and therefore for $\sigma_j<\sigma''$ [1, p. 215, line 18]

$$\frac{\log M_S(\sigma_j-\pi a)}{e^{-\rho_S(\sigma_j-\pi a)}}\geqq\frac{\log M_S(\sigma_j^*)}{e^{-\rho_S(\sigma_j-\pi a)}}$$
$$>e^{-\rho(\pi a+\epsilon'+\epsilon)}\frac{\log M(\sigma_j+\epsilon'+\epsilon)}{e^{-\rho(\sigma_j+\epsilon'+\epsilon)}}-\frac{\log K_1+K}{e^{-\rho_S(\sigma_j-\pi a)}}.$$

It follows that $\tau_S\geqq e^{-\pi\rho a}\tau$.

Received by the editors October 9, 1959.

By the method of Sunyer i Balaguer [see Theorem A of the preceding paper] Theorem B can be improved and we can prove:

THEOREM C. *Under the conditions of Theorem* B, $\tau_S = \tau$.

The following printing mistake in [1] may be noted:
In line 21 on page 215 $e^{p(\sigma_j - \epsilon' - \epsilon)}$ may be corrected to read $e^{p(k_j - \epsilon' - \epsilon)}$.

REFERENCE

1. Q. I. Rahman, *On entire functions defined by a Dirichlet series*, Proc. Amer. Math. Soc. vol. 10 (1959) pp. 213–215.

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THE UNIVERSAL REPRESENTATION KERNEL OF A LIE GROUP

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Let G be a connected real Lie group. The universal representation kernel, K_G, of G is defined as the intersection of all kernels of continuous finite dimensional representations of G. Evidently, K_G is a closed normal subgroup of G, and it is known from a theorem due to Goto (cf. [1, Theorem 7.1]) that G/K_G has a faithful continuous finite dimensional representation. Thus K_G is the smallest normal closed subgroup P of G such that G/P is isomorphic with a real analytic subgroup of a full linear group. The known criteria for the existence of a faithful representation lead to a determination of K_G which we wish to record here.

Suppose first that G is semisimple. Let $\mathfrak{G}$ denote the Lie algebra of G. Let C stand for the field of the complex numbers, and denote by $\mathfrak{G}^C$ the complexification of $\mathfrak{G}$, i.e., the semisimple Lie algebra over C that is obtained by forming the tensor product, over the real field, of $\mathfrak{G}$ with C. Denote by $S(\mathfrak{G})$ and $S(\mathfrak{G}^C)$ the simply connected Lie groups whose Lie algebras are $\mathfrak{G}$ and $\mathfrak{G}^C$, respectively. The injection $\mathfrak{G} \rightarrow \mathfrak{G}^C$ is the differential of a uniquely determined continuous homomorphism γ of $S(\mathfrak{G})$ into $S(\mathfrak{G}^C)$. The kernel P of γ is a discrete central subgroup of $S(\mathfrak{G})$. Let ϕ denote the covering epimorphism of $S(\mathfrak{G})$ onto G. We claim that $K_G = \phi(P)$, i.e., *the universal representation kernel of the semisimple connected Lie group G is the image, under the universal covering epimorphism, of the kernel of the canonical homomorphism* $S(\mathfrak{G}) \rightarrow S(\mathfrak{G}^C)$.

Received by the editors September 18, 1959.