DISCRETE SOLVABLE MATRIX GROUPS

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Let $GL(n, R)$ denote the real general linear group with the usual topology. A subgroup of $GL(n, R)$ will be called an $n$-matrix group. We will say that a subgroup $G$ of $GL(n, R)$ is a discrete group if it is discrete in the induced topology. If for any group $G$, $[G, G]$ denotes the commutator subgroup of $G$, we will say that a group $S$ is solvable provided the sequence of groups $S = S_1, [S_1, S_1] = S_2, \cdots, [S_{k-1}, S_{k-1}] = S_k$, terminates in the identity element for some $k$. If $S_k = e$ and $S_{k-1} \neq e$, where $e$ is the identity in $S$, then we will say that $S$ is $k$ step solvable and $k$ is called also the index of solvability. Then in [1] and [4] we have independent proofs of the following theorem:

**Theorem.** Let $k(S)$ denote the index of solvability of $S$, where $S$ is a solvable $n$-matrix group. Then there exists an integer valued function of $n, f(n)$, such that $k(S) < f(n)$.

This note has as its purpose the proof of the following theorem:

**Theorem.** Let $S$ be a discrete solvable $n$-matrix group. Then $S$ is finitely generated and, if $(S)$ denotes the minimal cardinality of a set of generators for $S$, there exists an integer valued function $g(n)$ such that $(S) < g(n)$.

**Proof.** Theorem 1 in [1] tells us that there exists $S' \subseteq S$ such that

(a) $S'$ is of finite index, $I(S'/S')$, in $S$.

(b) There exists a function $g_1(n)$ such that $I(S'/S') < g_1(n)$.

(c) $S'$ can be simultaneously triangulated over the complex field.

Hence if we can now show that $S'$ is finitely generated and $(S') < g_2(n)$ we would be done. But by Theorem 1 [3] $S' \supset S''$ where $I(S'/S'') < \infty$ and $S''$ is a fundamental group of a compact solvmanifold whose dimension is no more than $n(n+1)/2$. By the structure theorem for the fundamental groups of solvmanifolds, we see that $S''$ is finitely generated and $(S'') \leq n(n+1)/2$. Hence all that we have to show is that $(S'/S'')$ is a bounded function of $n$. But using the Theorem 1 of Wang and Theorem 3.1 of [2] we see that $S'/S''$ is a discrete subgroup of torus of dimension $2n$. Hence $(S'/S'') \leq 2n$. This completes the proof of this theorem.

**References**


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In the study of functions on certain types of spaces, the question naturally arises as to what additional conditions may imply that the functions are continuous. Several papers, mainly [2; 3; 4], have considered this problem. In this note, some further results of this type are developed.

To avoid repetition, a function \( f \) will be at least on a Hausdorff space \( X \) onto a Hausdorff space \( Y \) with additional restrictions stated as needed. Also \( f \) is compact preserving (connected) if when \( K \) is a compact (connected) subset of \( X \), \( f(K) \) is a compact (connected) subset of \( Y \); \( f \) has closed point inverses if for each \( y \in Y \), \( f^{-1}(y) \) is closed and \( f \) is monotone if \( f^{-1}(y) \) is connected. The rest of the terminology is standard.

In [1], it was shown that if \( X \) is regular, \( Y \) compact and if \( f \) is closed with closed point inverses, \( f \) is continuous. Combining this with Theorem 3.1 of [4], one has the result:

**Theorem 1.** If \( f \) is a closed monotone connected function on a regular space \( X \) onto a compact space \( Y \), then \( f \) is continuous.

It is easy to see that without the assumption that \( Y \) is compact, the conclusion need no longer hold.

**Theorem 2.** If \( X \) is locally compact, then if \( f \) is compact preserving and point inverses are closed, \( f \) is continuous.

Consider any point \( x \in X \). Since \( X \) is locally compact, \( x \) has a neighborhood \( U_0 \) with a compact closure \( \text{Cl} \ U_0 \). Because continuity is a local property, one need only consider \( f \) restricted to \( \text{Cl} \ U_0 \). On \( \text{Cl} \ U_0 \), \( f \) is closed and \( f(\text{Cl} \ U_0) \) is compact. Hence the conditions of Theorem 3 of [1] are satisfied and \( f \) is continuous at \( x \).

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