DISCRETE SOLVABLE MATRIX GROUPS

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Let $GL(n, R)$ denote the real general linear group with the usual topology. A subgroup of $GL(n, R)$ will be called an n-matrix group. We will say that a subgroup G of $GL(n, R)$ is a discrete group if it is discrete in the induced topology. If for any group G, $[G, G]$ denotes the commutator subgroup of G, we will say that a group S is solvable provided the sequence of groups $S=S_1$, $[S_1, S_1]=S_2, \cdots$, $[S_{k-1}, S_{k-1}]=S_k$, terminates in the identity element for some k. If $S_k=e$ and $S_{k-1}\neq e$, where e is the identity in S, then we will say that S is k step solvable and k is called also the index of solvability. Then in [1] and [4] we have independent proofs of the following theorem:

THEOREM. *Let $k(S)$ denote the index of solvability of S, where S is a solvable n-matrix group. Then there exists an integer valued function of n, $f(n)$, such that $k(S)<f(n)$.*

This note has as its purpose the proof of the following theorem:

THEOREM. *Let S be a discrete solvable n-matrix group. Then S is finitely generated and, if $\#(S)$ denotes the minimal cardinality of a set of generators for S, there exists an integer valued function $g(n)$ such that $\#(S)<g(n)$.*

PROOF. Theorem 1 in [1] tells us that there exists $S'\subset S$ such that

(a) S' is of finite index, $I(S/S')$, in S.

(b) There exists a function $g_1(n)$ such that $I(S/S')<g_1(n)$.

(c) S' can be simultaneously triangulated over the complex field.

Hence if we can now show that S' is finitely generated and $\#(S')<g_2(n)$ we would be done. But by Theorem 1 [3] $S'\supset S''$ where $I(S'/S'')<\infty$ and S'' is a fundamental group of a compact solvmanifold whose dimension is no more than $n(n+1)/2$. By the structure theorem for the fundamental groups of solvmanifolds, we see that S'' is finitely generated and $\#(S'')\leqq n(n+1)/2$. Hence all that we have to show is that $\#(S'/S'')$ is a bounded function of n. But using the Theorem 1 of Wang and Theorem 3.1 of [2] we see that S'/S'' is a discrete subgroup of torus of dimension $2n$. Hence $\#(S'/S'')\leqq 2n$. This completes the proof of this theorem.

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CONDITIONS IMPLYING CONTINUITY OF FUNCTIONS

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In the study of functions on certain types of spaces, the question naturally arises as to what additional conditions may imply that the functions are continuous. Several papers, mainly [**2**; **3**; **4**], have considered this problem. In this note, some further results of this type are developed.

To avoid repetition, a function f will be at least on a Hausdorff space X onto a Hausdorff space Y with additional restrictions stated as needed. Also f is compact preserving (connected) if when K is a compact (connected) subset of X, $f(K)$ is a compact (connected) subset of Y; f has closed point inverses if for each $y \in Y$, $f^{-1}(y)$ is closed and f is monotone if $f^{-1}(y)$ is connected. The rest of the terminology is standard.

In [**1**], it was shown that if X is regular, Y compact and if f is closed with closed point inverses, f is continuous. Combining this with Theorem 3.1 of [**4**], one has the result:

THEOREM 1. *If f is a closed monotone connected function on a regular space X onto a compact space Y, then f is continuous.*

It is easy to see that without the assumption that Y is compact, the conclusion need no longer hold.

THEOREM 2. *If X is locally compact, then if f is compact preserving and point inverses are closed, f is continuous.*

Consider any point $x \in X$. Since X is locally compact, x has a neighborhood U_0 with a compact closure Cl U_0. Because continuity is a local property, one need only consider f restricted to Cl U_0. On Cl U_0, f is closed and $f(\text{Cl } U_0)$ is compact. Hence the conditions of Theorem 3 of [**1**] are satisfied and f is continuous at x.

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