

## REFERENCES

1. M. K. Fort, Jr., *Images of plane continua*, Amer. J. Math. vol. 81 (1959) pp. 541-546.
2. O. Hanner, *Retraction and extension of mappings of metric and non-metric spaces*, Ark. Mat. vol. 2 (1954) pp. 315-360.
3. P. J. Hilton, *An introduction to homotopy theory*, Cambridge Tracts in Mathematics and Mathematical Physics, no. 43, Cambridge University Press, 1953.
4. Ernest Michael, *Continuous selections*. II, Ann. of Math. vol. 64 (1956) pp. 562-580.
5. Norman Steenrod, *The topology of fibre bundles*, Princeton University Press, 1951.

THE UNIVERSITY OF GEORGIA

## A REMARK ON THE CHARACTERIZATION OF HOMOTHETIC TRANSFORMATION AND INVERSION<sup>1</sup>

CHIN-SHUI HSÜ

Given two closed orientable  $C^2$  surfaces  $S, \bar{S}$  in  $E^3$ , let  $h: S \rightarrow \bar{S}$  be a differentiable homeomorphism, such that referring to a suitable origin  $O$

$$(1) \quad \bar{X} = kX,$$

with  $k$  nonzero and  $C^2$  at points at which  $X \neq 0, \bar{X} \neq 0$ . Write  $p = -X \cdot N, \bar{p} = -\bar{X} \cdot \bar{N}$ . Suppose further that  $S, \bar{S}$  contains no pieces of cones with vertex  $O$ . We use the form  $b = (\bar{N} \times X) \cdot dX$ , introduced in [1]. We generalize results of [1] to

**THEOREM 1'.** *If  $\iint_S (H \mp k\bar{H}) p dA = 0$ , then  $k = \text{const.}$ , i.e. the map  $h$  is a homothetic transformation with center  $O$ .*

**PROOF.** The formulae  $\bar{p} d\bar{A} = k^3 p dA$  and  $db = (2/k^2) \bar{p} \bar{H} d\bar{A} + 2\bar{N} \cdot NdA$  yield  $\iint_S N \cdot \bar{N} dA = -\iint_S k \bar{H} p dA$ . Combining with  $\iint_S dA = \iint_S H p dA$  gives

$$\iint_S (1 \mp N \cdot \bar{N}) dA = \iint_S (H \pm k\bar{H}) p dA,$$

and the theorem follows as in [1].

Received by the editors May 15, 1959.

<sup>1</sup> The author thanks Professor Hans Samelson for his suggestions for revision.

THEOREM 3'. If  $\iint_S (H \pm k(\bar{H} + 2\bar{X} \cdot \bar{N} / \bar{X} \cdot \bar{X})) dA = 0$  and  $O \in S$ ,  $\bar{S}$  then  $h$  is an inversion with center  $O$ .

PROOF. This follows from Theorem 1' as Theorem 3 in [1] followed from Theorem 1.

COROLLARY 1. Given a closed orientable  $C^2$  surface  $S$  and an interior point  $O$  such that the reciprocal of the Gaussian curvature at each point equals its square distance from  $O$ . Then  $S$  is a sphere with center  $O$ .

PROOF. Taking  $O$  as origin we have

$$1/K = 1/X \cdot X$$

and then

$$\iint_S (H + X \cdot N / X \cdot X) dA = \iint_S (H - pK) dA = 0,$$

that is

$$\iint_S \{H + (H + 2X \cdot N / X \cdot X)\} dA = 0.$$

By Theorem 3' the identity map on  $S$  is an inversion. Therefore  $S$  is a sphere.

COROLLARY 2. Given a closed orientable  $C^2$  surface  $S$ , at each point  $P$  on  $S$  define  $\theta$  as the angle between  $OP$  and the outward normal at  $P$ . If  $OP = (1/H) \cos \theta$ , then  $S$  is a sphere with center  $O$ .

PROOF. Taking  $O$  as origin our condition becomes

$$H + X \cdot N / X \cdot X = 0.$$

Then the proof is the same as in Corollary 1.

#### REFERENCE

1. Chin-shui Hsü, *Characterization of some elementary transformations*, Proc. Amer. Math. Soc. vol. 10 (1959) pp. 324-328.

NATIONAL TAIWAN UNIVERSITY, TAIPEI, TAIWAN, CHINA