

PROCEEDINGS  
OF THE  
AMERICAN MATHEMATICAL SOCIETY

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# Journals Published by the American Mathematical Society

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This journal is the official organ of the Society. It reports official acts of the Society and details of its meetings. It contains some of the officially invited addresses presented before the Society, research announcements and problems, and reviews of advanced mathematical books.

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## Journals Published by the American Mathematical Society

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### Mathematical Reviews

This journal contains abstracts and reviews of the current mathematical literature of the world. It is sponsored by thirteen mathematical organizations, located both in the United States and abroad.

The publication of MATHEMATICAL REVIEWS was begun in 1940. Starting in 1961, it appears monthly, in two parts. Prior to 1961 it appeared in eleven single issues. Orders for complete volumes only are accepted. Volumes 10 is not available. Volumes 1-9, 11-21 are available at the following prices: Vols. 1-14 (1940-1953), \$24.00 each; all other volumes \$50.00. In 1962, it will be published in two volumes; the price of each volume will be \$50.00.

### Soviet Mathematics—Doklady

This journal contains the entire pure mathematics section of the DOKLADY AKADEMII NAUK SSSR in translation. It appears six times a year, each bimonthly issue corresponding to one volume of the Soviet DOKLADY. (The DOKLADY AKADEMII NAUK SSSR is issued three times a month, six issues constituting a volume.)

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### Notices of the American Mathematical Society

This journal announces the programs of the meetings of the Society. It carries the abstracts of all contributed papers presented at the meetings of the Society and publishes news items of interest to mathematical scientists.

The subscription price is \$7.00 per annual volume of 7 numbers. A single copy is \$2.00.

All communications should be addressed to the Editor, 190 Hope Street, Providence 6, R. I. News items and insertions for each issue must be in the hands of the editor on or before the deadline for the abstracts of the papers to be presented in the meetings announced in that issue. These deadlines are published regularly on the back of the table of contents.

### Memoirs of the American Mathematical Society

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