EMBEDDING HOMOTOPY CELLS

JAMES J. ANDREWS

Let $W_k$ be a combinatorial $k$-manifold with boundary the $(k-1)$-sphere and also let $\pi_n(W_k)$ be trivial. The Poincaré Conjecture implies $W_k = I^k$ and hence by [2] and [3] the only cases of interest are $k = 3$ and 4.

**Theorem.** $W_k \times S^{5-k} \ (k = 3, 4)$ is embeddable in $S^6$.

**Proof.** Let $B = Bd (W_k \times I^{5-k})$ and $A_1$ and $A_2$ be two copies of $W_k \times I^{5-k}$. Also let $B^1 = Bd (W_k \times I^{5-k})$. Then $B = A_1 \cup A_2$, where $A_1 \cap A_2 = B^1$. We shall now wish to calculate $\pi_1(B)$ and $\pi_2(B)$. Now $\pi_1(W_k \times I^{5-k}) = \pi_1(W_k) = 1$ and $A_1 \cap A_2$ is connected. Hence by Van Kampen’s theorem $\pi_1(B) = 0$. By Hurewicz’s theorem $H_2(B) = \pi_2(B)$. Again $H_2(W_k \times I^{5-k}) = H_2(W_k)$ and from the Mayer-Vietoris sequence of the proper triad $(B, A_1, A_2)$ we see that $\pi_2(B) = H_2(B) = 0$.

It follows from [3] that $B = S^8$. Now

$$B = (W_k \times S^{5-k}) \cup (S^{k-1} \times I^{6-k}) = S^8.$$  

Hence $W_k \times S^{5-k}$ may be embedded in $S^6$.

Note that the same argument shows $B^* = Bd (W_k \times I^{5-k}) = S^8$ and since $B$ has a product neighborhood in $B^*$ it follows from [1] that $W_k \times I^{5-k} = I^k$.

**References**


University of Wisconsin

Received by the editors October 24, 1960.

917