EMBEDDING HOMOTOPY CELLS

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Let W_k be a combinatorial k-manifold with boundary the $(k-1)$-sphere and also let $\pi_n(W_k)$ be trivial. The Poincaré Conjecture implies $W_k = I^k$ and hence by [2] and [3] the only cases of interest are $k=3$ and 4.

THEOREM. *$W_k \times S^{5-k}$ $(k=3, 4)$ is embeddable in S^5.*

PROOF. Let $B = \mathrm{Bd}\,(W_k \times I^{6-k})$ and A_1 and A_2 be two copies of $W_k \times I^{5-k}$. Also let $B^1 = \mathrm{Bd}\,(W_k \times I^{5-k})$. Then $B = A_1 \cup A_2$, where $A_1 \cap A_2 = B^1$. We shall now wish to calculate $\pi_1(B)$ and $\pi_2(B)$. Now $\pi_1(W_k \times I^{5-k}) = \pi_1(W_k) = 1$ and $A_1 \cap A_2$ is connected. Hence by Van Kampen's theorem $\pi_1(B) = 0$. By Hurewicz's theorem $H_2(B) = \pi_2(B)$. Again $H_2(W_k \times I^{5-k}) = H_2(W_k)$ and from the Mayer-Vietoris sequence of the proper triad (B, A_1, A_2) we see that $\pi_2(B) = H_2(B) = 0$.

It follows from [3] that $B = S^5$. Now

$$B = (W_k \times S^{5-k}) \cup (S^{k-1} \times I^{6-k}) = S^5.$$

Hence $W_k \times S^{5-k}$ may be embedded in S^5.

Note that the same argument shows $B^* = \mathrm{Bd}\,(W_k \times I^{7-k}) = S^6$ and since B has a product neighborhood in B^* it follows from [1] that $W_k \times I^{6-k} = I^6$.

REFERENCES

1. Morton Brown, *A proof of the generalized Schoenflies theorem*, Bull. Amer. Math. Soc. vol. 66 (1960) pp. 74–76.

2. John Stallings, *Polyhedral homotopy spheres*, to appear.

3. E. C. Zeeman, *The Poincaré conjecture for $n \geq 5$*, to appear.

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